Library
Weather Bureau

A 512 st

59202

Digitized by the Internet Archive
in 2026

https://archive.org/details/american-society-of-civil-engineers-transactions_1935_100

189091
agr
7-11

PROCEEDINGS

TWO PARTS PART 2

American Society
of
Civil Engineers

OCTOBER
1935

TRANSACTIONS NUMBER

1935

VOLUME 61 No. 8, PART 2

eather Bureau
/Room
81
case 4

PROCEEDINGS

OF THE

AMERICAN SOCIETY OF CIVIL ENGINEERS

VOL. 61 OCTOBER, 1935 NO. 8, PART 2

TRANSACTIONS
NUMBER 100
1935

59202

OCT 1 1935

PUBLISHED BY THE SOCIETY
1935

Entered as Second-Class Matter, December 28, 1931,
at the Post Office at Albany, N. Y., under the
Act of March 3, 1879. Acceptance for
mailing at special rate of postage
provided for in Section 1103,
Act of October 3, 1917,
authorized on
July 5, 1918

Published at 99–129 North Broadway, Albany, N. Y.,
by the American Society of Civil Engineers

NOTE.—The Society is not responsible for any statement made or opinion expressed in its publications.

CONTENTS

PAPERS

No. Page

1891 SOME SOIL PRESSURE TESTS.

By H. de B. Parsons 1

Discussion:

By J. C. Meem 14
Eugene E. Halmos 17
L. C. Wilcoxen 19
O. K. Froehlich 21
H. L. Thackwell 24
Jacob Feld 26
T. Farrance Davey 27
D. P. Krynine 30
Charles Terzaghi 35
R. L. Vaughn 39
M. Hirschthal 43
H. de B. Parsons 45

1892 WATER-BEARING MEMBERS OF ARTICULATED BUTTRESS DAMS.

By Hakan D. Birke 55

Discussion:

By R. A. Sutherland 70
P. Wilhelm Werner 71
Charles P. Williams 76
Paul Baumann 81
A. Floris 84
Hakan D. Birke 85

1893 LINCOLN HIGHWAY FROM JERSEY CITY TO ELIZABETH, NEW JERSEY.

By Sigvald Johannesson 91

Discussion:

By Fred Lavis 115
Theodore Belzner 116
J. B. French 116

1894 PRACTICAL RIVER LABORATORY HYDRAULICS.

By Herbert D. Vogel 118

Discussion:

By Lorenz G. Straub 145
Paul W. Thompson 151
Ralph W. Powell 155
K. D. Nichols 159
Frank W. Edwards 163
L. H. Patty 166
Charles S. Bennett 168
Kenneth C. Reynolds 170
V. V. Tchikoff 170
Samuel Shulits 175
Charles D. Curran 177
J. B. Egiazaroff 178
Herbert D. Vogel 180

No. Page

1895 MODEL OF CALDERWOOD ARCH DAM.
By A. V. Karpov and R. L. Templin 185

Discussion:
By A. W. Simonds 220
Lars R. Jorgensen 224
A. C. Janni 227
P. Wilhelm Werner 231
Eldred D. Smith 235
Elmer O. Bergman 240
E. Probst 244
A. V. Karpov and R. L. Templin 247

1896 FORMATION OF FLOC BY FERRIC COAGULENTS.
By Edward Bartow, A. P. Black, and Walter E. Sansbury 263

Discussion:
By Edward S. Hopkins 277
W. D. Hatfield 278
L. B. Miller 279
Linn H. Enslow 279
C. J. Brockman 282

1897 MODIFYING THE PHYSIOGRAPHICAL BALANCE BY CONSERVATION METHODS.
By A. L. Sonderegger 284

Discussion:
By H. H. Chapman 305
E. B. Debler 306
Frank E. Bonner 307
C. S. Jarvis 315
W. P. Rowe 317
J. C. Stevens 324
Gerard H. Matthes 326
J. B. Lippincott 328
Rhodes E. Rule 335
A. L. Sonderegger 337

1898 AN APPROACH TO DETERMINATE STREAM FLOW.
By Merrill M. Bernard 347

Discussion:
By C. S. Jarvis 363
LeRoy K. Sherman 365
W. W. Horner 368
C. H. Eiffert 370
Charles S. Bennett 371
R. L. Gregory and C. E. Arnold 372
Gordon R. Williams 383
W. G. Hoyt, L. L. Harrold, and F. F. Snyder 384
Merrill M. Bernard 389

1899 DISCHARGE FORMULA AND TABLES FOR SHARP-CRESTED SUPPRESSED WEIRS.
By C. G. Cline, Esq 396

Discussion:
By Jasper O. Draffin 414
W. Bruce McMillan 415
C. G. Cline 422

No. PAGE

1900 LABORATORY TESTS OF MULTIPLE-SPAN REINFORCED CONCRETE ARCH BRIDGES.
By **Wilbur M. Wilson** 424

Discussion:
By C. B. McCullough 455
Carroll L. Mann, Jr 461
M. Hirschthal 465
Wilbur M. Wilson 467

1901 RENEWAL OF MITER-GATE BEARINGS, PANAMA CANAL.
By **Clinton Morse** 471

Discussion:
By E. S. Randolph 496
Clinton Morse 496

1902 WATER-POWER DEVELOPMENT OF THE ST. LAWRENCE RIVER.
By **Daniel W. Mead** 498

Discussion:
By T. H. Hogg 517
Theron M. Ripley 529
W. S. Lee 531
Frank E. Bonner 532
Rufus W. Putnam 539
James W. Rickey 540
Walter M. Smith 541
J. W. Beardsley 545
L. F. Harza 547
Adolph J. Ackerman 551
Daniel W. Mead 552

1903 LOSS OF HEAD IN ACTIVATED SLUDGE AERATION CHANNELS.
By **Darwin Wadsworth Townsend** 558

Discussion:
By H. L. Thackwell 567
Henry R. King 573
M. H. Klegerman 575
Darwin Wadsworth Townsend 578

1904 WILLIOT EQUATIONS FOR STATICALLY INDETERMINATE STRUCTURES IN COMBINATION WITH MOMENT EQUATIONS IN TERMS OF ANGULAR DISPLACEMENTS.
By **Charles A. Ellis** 580

Discussion:
By Ralph A. Tudor 599
Leon S. Moisseiff 601
A. A. Eremin 604
Charles A. Ellis 605

1905 RAINFALL STUDIES FOR NEW YORK, N. Y.
By **S. D. Bleich** 609

Discussion:
By H. Alden Foster 628
J. J. Slade, Jr 628
Charles W. Sherman 630
Merrill M. Bernard 632
Clifford Seaver 634
Jose Garcia Montes, Jr 638
Rhodes E. Rule 640
S. D. Bleich 642

No. PAGE

1906 FLEXIBLE "FIRST-STORY" CONSTRUCTION FOR EARTHQUAKE RESISTANCE.
By **Norman B. Green** 645

Discussion:
By LEE H. JOHNSON 653
EDWARD J. BEDNARSKI 657
MERIT P. WHITE AND PAUL L. KARTZKE 660
HOWARD G. SMITS 662
H. A. WILLIAMS 665
NORMAN B. GREEN 669

1907 INVESTIGATION OF WEB BUCKLING IN STEEL BEAMS.
By **Inge Lyse and H. J. Godfrey** 675

Discussion:
By R. L. MOORE AND E. C. HARTMANN 696
INGE LYSE AND H. J. GODFREY 701

1908 ANALYSIS OF SHEET-PILE BULKHEADS.
By **Paul Baumann** 707

Discussion:
By JACOB FELD 741
R. L. VAUGHN 744
M. A. DRUCKER 753
RAYMOND P. PENNOYER 761
D. P. KRYNINE 766
E. h. O. FRANZIUS 771
THEODOR VON KÁRMÁN 774
E. LOHMEYER 776
PAUL BAUMANN 785

1909 SAND MIXTURES AND SAND MOVEMENT IN FLUVIAL MODELS.
By **Hans Kramer** 798

Discussion:
By JOHN LEIGHLY 839
PAUL W. THOMPSON 842
GERARD H. MATTHES 844
R. H. KEAYS 848
F. T. MAVIS 849
V. V. TCHIKOFF 852
MORROUGH P. O'BRIEN AND BRUCE D. RINDLAUB 858
HERBERT D. VOGEL 860
JOSEPH B. TIFFANY, JR., AND CARL E. BENTZEL 861
LORENZ G. STRAUB 867
HANS KRAMER 873

1910 THE RESERVOIR AS A FLOOD CONTROL STRUCTURE.
By **George R. Clemens** 879

Discussion:
By C. MAXWELL STANLEY 914
F. KNAPP 917
CHARLES S. BENNETT 918
GERARD H. MATTHES 919
GEORGE R. CLEMENS 923

1911 STRESSES IN SPACE STRUCTURES.
By **F. H. Constant** 928

Discussion:
By WILLIAM R. OSGOOD 935
L. E. GRINTER 936
CHARLES M. SPOFFORD 941
A. H. FINLAY 942
F. H. CONSTANT 944

No. PAGE

1912 EXPERIMENTS WITH CONCRETE IN TORSION.
By Paul Andersen 949

Discussion:
By E. MIRABELLI 961
FRANK M. RUSSELL 964
LESLIE TURNER 967
A. W. FISCHER 969
H. J. GILKEY 970
A. A. EREMIN 980
PAUL ANDERSEN 981

1913 WAVE PRESSURES ON SEA-WALLS AND BREAKWATERS.
By David A. Molitor 984

Discussion:
By CHARLES E. FOWLER 1003
CHARLES T. LEEDS 1004
A. A. EREMIN 1009
T. L. CONDRON AND CHESTER L. POST 1010
DAVID A. MOLITOR 1014

1914 FLOW OF WATER AROUND BENDS IN PIPES.
By David L. Yarnell and Floyd A. Nagler 1018

Discussion:
By D. BENJAMEN GUMENSKY 1033
WALLACE M. LANSFORD 1036
F. T. MAVIS 1040
DAVID L. YARNELL 1042

1915 STREET THOROUGHFARES: A SYMPOSIUM 1045

FOREWORD
By George H. Herrold 1046

DEFINITIONS, GENERAL PRINCIPLES, AND RECOMMENDATIONS: REPORT OF THE COMMITTEE OF THE CITY PLANNING DIVISION ON STREET THOROUGHFARES MANUAL.
W. W. Crosby, Chairman 1048

RELATION OF STREET THOROUGHFARES TO OTHER ARTERIES OF TRANSPORTATION.
By Jacob L. Crane, Jr 1056

DESIGN OF BOULEVARDS AND PARKWAYS.
By Jay Downer 1060

SUPER-HIGHWAYS.
By J. P. Hallihan 1062

THOROUGHFARE LAYOUTS IN RELATION TO DRAINAGE SYSTEMS.
By W. W. Horner 1064

EXPRESS HIGHWAYS AND BY-PASS RESULTS.
By Daniel L. Turner 1066

PAYING FOR STREET THOROUGHFARES.
By George H. Herrold 1068

No. PAGE

BIBLIOGRAPHY ON STREET THOROUGHFARES.
By the Committee of the City Planning Division on Street Thoroughfares Manual 1076

Discussion:
By H. SHIFRIN 1079
DONALD M. BAKER 1080
FREDERIC BASS 1083
HUGH E. YOUNG 1083
COMMITTEE OF CITY PLANNING DIVISION ON STREET THOROUGHFARES MANUAL 1084

1916 ECCENTRIC RIVETED CONNECTIONS.
By Eugene A. Dubin 1086

Discussion:
By CARLTON T. BISHOP 1094
JAMES R. BOLE 1094
ALBERT WERTHEIMER 1095
KENNETH L. DEBLOIS 1096
JONATHAN JONES 1097
ARMIN ROZMAN 1097
A. E. R. DE JONGE 1099
FANG-YIN TSAI 1103
ODD ALBERT 1104
EUGENE A. DUBIN 1108

1917 DETERMINATION OF TRAPEZOIDAL PROFILES FOR RETAINING WALLS.
By A. J. Sutton Pippard 1116

Discussion:
By E. S. LINDLEY 1128
KENNETH L. DEBLOIS 1128
M. A. DRUCKER 1129
A. J. SUTTON PIPPARD 1131

1918 A GENERALIZED DEFLECTION THEORY FOR SUSPENSION BRIDGES.
By D. B. Steinman 1133

Discussion:
By E. L. PAVLO 1171
JONATHAN JONES 1174
A. MÜLLENHOFF 1175
H. CECIL BOOTH 1177
JACOB FELD 1178
GLENN B. WOODRUFF, HOWARD C. WOOD, AND RALPH A. TUDOR 1179
L. J. MENSCH 1185
A. A. EREMIN 1185
HANS H. BLEICH 1187
F. H. FRANKLAND 1190
GUSTAV LINDENTHAL 1190
JULIAN W. SHIELDS 1195
A. W. FISCHER 1197
J. M. FRANKLAND 1201
FREDRIK VOGT 1204
LEON S. MOISSEIFF 1205
A. MITCHELL AND G. T. PARKIN 1209
STERLING JOHNSTON 1210
HAROLD E. WESSMAN 1213
C. H. GRONQUIST 1214
W. R. FREDERICK, JR. 1215
WILLIAM H. YATES 1222
D. B. STEINMAN 1222

No. PAGE

1919 SECURITY FROM UNDER-SEEPAGE—MASONRY DAMS ON EARTH FOUNDATIONS.

By E. W. Lane 1235

Discussion:

By WILLIAM P. CREAGER 1273
L. F. HARZA 1275
JOEL D. JUSTIN 1276
LOUIS E. AYRES 1278
W. M. GRIFFITH 1280
E. MCKENZIE TAYLOR 1283
DONALD J. HEBERT 1285
ARTHUR CASAGRANDE 1289
CALVIN V. DAVIS 1294
C. H. EIFFERT 1299
E. H. BURROUGHS, JR. 1300
ALEXANDER POTTER 1318
ROBERT E. KENNEDY 1320
A. N. KHOSLA 1320
A. STREIFF 1325
E. W. LANE 1334

1920 UPLIFT AND SEEPAGE UNDER DAMS ON SAND

By L. F. Harza 1352

Discussion:

By EDWARD GODFREY 1386
HIBBERT M. HILL 1386
JOEL D. JUSTIN 1388
CHARLES TERZAGHI 1391
HARRY H. HATCH 1395
DONALD J. HEBERT 1396
C. A. MOCKMORE AND JOHN W. DOUGHERTY 1396
L. F. HARZA 1401

1921 THE DEVELOPMENT OF THE AMERICAN SOCIETY OF CIVIL ENGINEERS: ADDRESS AT THE ANNUAL CONVENTION, LOS ANGELES, CALIFORNIA, JULY 3, 1935.

By Arthur S. Tuttle 1407

1922 FINAL REPORT OF THE SPECIAL COMMITTEE ON CONCRETE AND REINFORCED CONCRETE ARCHES 1427

MEMOIRS OF DECEASED MEMBERS

PAGE

ARTHUR POWELL DAVIS, PAST-PRESIDENT, AM. SOC. C. E. 1582
CARL EWALD GRUNSKY, PAST-PRESIDENT, AM. SOC. C. E. 1591
KOI FURUICHI, HON. M. AM. SOC. C. E. 1595
OCTAVIO AUGUSTO ACEVEDO, M. AM. SOC. C. E. 1597
FREDERICK WHITNEY ADGATE, M. AM. SOC. C. E. 1599
JULIUS BAIER, M. AM. SOC. C. E. 1600
KORT BERLE, M. AM. SOC. C. E. 1602
LYMAN EDGAR BISHOP, M. AM. SOC. C. E. 1603
GEORGE MEADE BOND, M. AM. SOC. C. E. 1605
ALFRED VICTOR BOUILLON, M. AM. SOC. C. E. 1608
WILBUR GAYLE BROWN, M. AM. SOC. C. E. 1610
JAMES PARKER BROWNELL, M. AM. SOC. C. E. 1611
GEORGE BRYAN, JR., M. AM. SOC. C. E. 1613

Page

Edward Morris Burd, M. Am. Soc. C. E. 1614
William Hubert Burr, M. Am. Soc. C. E. 1617
Morton Lewis Byers, M. Am. Soc. C. E. 1621
Walter John Cahill, M. Am. Soc. C. E. 1623
Henry Hall Carter, M. Am. Soc. C. E. 1624
William Arnold Christian, M. Am. Soc. C. E. 1627
Charles Samuel Churchill, M. Am. Soc. C. E. 1629
George Hallett Clark, M. Am. Soc. C. E. 1631
John Adams Cole, M. Am. Soc. C. E. 1634
Eugene Hunter Coleman, M. Am. Soc. C. E. 1635
Charles Francis Conn, M. Am. Soc. C. E. 1637
Lorenzo Dana Cornish, M. Am. Soc. C. E. 1638
Clyde Maxwell Cram, M. Am. Soc. C. E. 1640
Benjamin John Curtis, M. Am. Soc. C. E. 1642
William Clarence Davidson, M. Am. Soc. C. E. 1644
Frederick Eaton, M. Am. Soc. C. E. 1645
Eric Gustaf Ericson, M. Am. Soc. C. E. 1648
Harry Jacob Finebaum, M. Am. Soc. C. E. 1649
Walter Linder Foster, M. Am. Soc. C. E. 1651
George Warren Fuller, M. Am. Soc. C. E. 1653
William Gore, M. Am. Soc. C. E. 1660
William Albert Grover, M. Am. Soc. C. E. 1662
John Wendell Hall, M. Am. Soc. C. E. 1663
Thomas Chalkley Hatton, M. Am. Soc. C. E. 1665
John Edward Hill, M. Am. Soc. C. E. 1667
Burt Bradley Hodgman, M. Am. Soc. C. E. 1672
Elmer Guy Hooper, M. Am. Soc. C. E. 1673
George Leonard Hosmer, M. Am. Soc. C. E. 1676
George Edward Howe, M. Am. Soc. C. E. 1677
George Arthur Johnson, M. Am. Soc. C. E. 1678
Guy Robert Johnston, M. Am. Soc. C. E. 1681
Maurice Edwin Kernot, M. Am. Soc. C. E. 1683
Victor Hugo Kriegshaber, M. Am. Soc. C. E. 1685
Oliver Howard Lang, M. Am. Soc. C. E. 1687
William Campbell Langfitt, M. Am. Soc. C. E. 1688
James Robinson McClintock, M. Am. Soc. C. E. 1693
Archibald McLean, M. Am. Soc. C. E. 1694
Edward Charles Murphy, M. Am. Soc. C. E. 1697
Floyd August Nagler, M. Am. Soc. C. E. 1698
Lars Netland, M. Am. Soc. C. E. 1701
Maury Nicholson, M. Am. Soc. C. E. 1702
Emil Louis Nuebling, M. Am. Soc. C. E. 1704
Francis Joseph O'Hara, M. Am. Soc. C. E. 1706
John Francis O'Rourke, M. Am. Soc. C. E. 1707
Michael Maurice O'Shaughnessy, M. Am. Soc. C. E. 1710
William Bancroft Potter, M. Am. Soc. C. E. 1713
Henry Sewall Prichard, M. Am. Soc. C. E. 1716
James Ritchie, M. Am. Soc. C. E. 1718
Clarence Mord Rogers, M. Am. Soc. C. E. 1719
Hiram Abiff Schofield, M. Am. Soc. C. E. 1721
Paul Albert Nicolas Seurot, M. Am. Soc. C. E. 1723
George Harry Thornton Shaw, M. Am. Soc. C. E. 1728
Joseph Mansfield Slater, M. Am. Soc. C. E. 1733
Kenneth Irving Small, M. Am. Soc. C. E. 1734
Theodore Speiden, M. Am. Soc. C. E. 1735
Frank Julian Sprague, M. Am. Soc. C. E. 1736
Gustave Arnold Stierlin, M. Am. Soc. C. E. 1741
Aaron Howell Van Cleve, M. Am. Soc. C. E. 1742
Alexander Wilson Vars, M. Am. Soc. C. E. 1746
Isaac Spurr Voorhees, M. Am. Soc. C. E. 1748
John Douglas Waldrop, M. Am. Soc. C. E. 1749
Edwin John Weigand, M. Am. Soc. C. E. 1754
Paul Willis, M. Am. Soc. C. E. 1756

	Page
Herbert Alva Wilson, M. Am. Soc. C. E.	1757
George Marshall Ames, Assoc. M. Am. Soc. C. E.	1759
Frederic Morris Faude, Assoc. M. Am. Soc. C. E.	1762
Harry Foster Ferguson, Assoc. M. Am. Soc. C. E.	1763
Elbert Allan Gibbs, Assoc. M. Am. Soc. C. E.	1765
Elmer Clark Goodwin, Assoc. M. Am. Soc. C. E.	1767
John Francis Grady, Assoc. M. Am. Soc. C. E.	1768
Françis Vincent Harney, Assoc. M. Am. Soc. C. E.	1770
George Waldo Hillman, Assoc. M. Am. Soc. C. E.	1771
John Jay Lafayette Houston, Assoc. M. Am. Soc. C. E.	1772
Warren Raymond King, Assoc. M. Am. Soc. C. E.	1775
Issac Henry Kirby, Assoc. M. Am. Soc. C. E.	1776
James Brownson McClain, Assoc. M. Am. Soc. C. E.	1778
Arthur Sawyer Mahony, Assoc. M. Am. Soc. C. E.	1778
Thomas Vincent Moore, Assoc. M. Am. Soc. C. E.	1780
George Abel Pierce, Assoc. M. Am. Soc. C. E.	1781
Herbert Allan Rice, Assoc. M. Am. Soc. C. E.	1783
Edson Oliver Sessions, Assoc. M. Am. Soc. C. E.	1785
Burtis Paul Thomas, Assoc. M. Am. Soc. C. E.	1787
Ralph Penny Thompson, Assoc. M. Am. Soc. C. E.	1788
John Alexander Jameson, Jr., Jun. Am. Soc. C. E.	1789
Oliver Bertrand Johnson, Jun. Am. Soc. C. E.	1790
Herman Greig Veeder, Jr., Jun. Am. Soc. C. E.	1790
Patrick McGovern, Affiliate, Am. Soc. C. E.	1791
Alfred Watts Kiddle, Fellow, Am. Soc. C. E.	1796

ERRATA

Transactions, Vol. 98 (1933)

Page 116, Line 28, the sentence, "The normal ring stress due to radial forces, from Equation (30), equals $f_3 = \frac{43.4 \times 60}{60}\left(1 + 6.04 \times \frac{12}{12.25 + 1.51}\right) = 1\,020$. Since all these stresses occur combined at the horizontal diameter the maximum fiber stress, (tension), in the supporting ring becomes, $f = f_1 + f_2 + f_3 = 16\,380$", should read, as follows: "The normal ring stress due to radial forces, from Equation (30), equals $f_3 = \frac{43.4 \times 60}{12}\left(1 + 6.04 \times \frac{12}{12.25 + 1.51}\right) = 1\,360$. Since all these stresses occur combined at the horizontal diameter, the maximum fiber stress, (tension), in the supporting ring becomes, $f = f_1 + f_2 + f_3 = 16\,720$".

Transactions, Vol. 99 (1934)

Page V (Contents), Lines 27 and 32, for "C. Stanley Maxwell," read "C. Maxwell Stanley."

Page 1330, Line 4, for "J. T. Moore, M. Am. Soc. C. E.," read "J. T. More, Assoc. M. Am. Soc. C. E."

PAPERS AND REPORTS OF SPECIAL COMMITTEES PUBLISHED IN PROCEEDINGS, AM. SOC. C. E. AUGUST, 1934, TO MAY, 1935, INCLUSIVE, NOT PUBLISHED IN TRANSACTIONS VOL. 100

	Number of *Proceedings.*
AN ASYMMETRIC PROBABILITY FUNCTION. By **J. J. Slade, Jr**	October, 1934
ANALYSIS OF CONTINUOUS STRUCTURES BY TRAVERSING THE ELASTIC CURVES. By **Ralph W. Stewart**	October, 1934
RELATION BETWEEN RAINFALL AND RUN-OFF FROM SMALL URBAN AREAS. By **W. W. Horner** and **F. L. Flynt**	October, 1934
THE SILT PROBLEM. By **J. C. Stevens**	October, 1934
EFFECT OF SECONDARY STRESSES UPON ULTIMATE STRENGTH. By **John I. Parcel** and **Eldred B. Murer**	November, 1934
THE SPRINGWELLS FILTRATION PLANT, DETROIT, MICHIGAN. By **Eugene A. Hardin**	November, 1934
ANALYSIS OF MULTIPLE ARCHES. By **Alexander Hrennikoff**	December, 1934
RATIONAL DESIGN OF STEEL COLUMNS. By **D. H. Young**	December, 1934
A DIRECT METHOD OF MOMENT DISTRIBUTION. By **T. Y. Lin**	December, 1934
WATER SUPPLY ENGINEERING: PROGRESS REPORT OF THE COMMITTEE OF THE SANITARY ENGINEERING DIVISION	December, 1934*
ELASTIC PROPERTIES OF RIVETED CONNECTIONS. By **J. Charles Rathbun**	January, 1935
ANALYSIS OF THICK ARCH DAMS, INCLUDING ABUTMENT YIELD. By **Philip Cravitz**	January, 1935
HYDRAULIC LABORATORY RESULTS AND THEIR VERIFICATION IN NATURE By **Herbert D. Vogel**	January, 1935
THE HYDRAULIC JUMP IN TERMS OF DYNAMIC SIMILARITY. By **Boris A. Bakhmeteff** and **Arthur E. Matzke**	February, 1935
FRICTIONAL RESISTANCE IN ARTIFICIALLY ROUGHENED PIPES. By **Victor L. Streeter**	February, 1935
STABILIZING CONSTRUCTED MASONRY DAMS BY MEANS OF CEMENT INJECTIONS. By **D. W. Cole**	February, 1935
WEIGHTS OF METAL IN STEEL TRUSSES. By **J. A. L. Waddell**	February, 1935
LINE LOAD ACTION ON THIN CYLINDRICAL SHELLS. By **Herman Schorer**	March, 1935
UNDERGROUND CORROSION. By **K. H. Logan**	March, 1935

---

* Published in *Proceedings* only.

	Number of *Proceedings.*
FLOOD PROTECTION DATA: PROGRESS REPORT OF THE COMMITTEE	March, 1935*
MASONRY AND REINFORCED CONCRETE: PROGRESS REPORT OF THE COMMITTEE OF THE STRUCTURAL DIVISION	March, 1935*
WATER SUPPLY ENGINEERING: THIRD REPORT OF THE COMMITTEE OF THE SANITARY ENGINEERING DIVISION	April, 1935*
THE ADJUSTMENT OF A LEVEL NET. By **George H. Dell**	April, 1935
STRUCTURAL BEAMS IN TORSION. By **Inge Lyse and Bruce G. Johnston**	April, 1935
PHOTO-ELASTIC DETERMINATION OF SHRINKAGE STRESSES. By **Howard G. Smits**	May, 1935
THE SHEAR-AREA METHOD. By **Horace B. Compton and Clayton O. Dohrenwend**	May, 1935

* Published in *Proceedings* only.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 1891

SOME SOIL PRESSURE TESTS

By H. de B. Parsons[1], M. Am. Soc. C. E.

With Discussion by Messrs. J. C. Meem, Eugene E. Halmos, L. C. Wilcoxen, O. K. Froehlich, H. L. Thackwell, Jacob Feld, T. Farrance Davey, D. P. Krynine, Charles Terzaghi, R. L. Vaughn, M. Hirschthal, and H. de B. Parsons.

Synopsis

This research is a sequel to a previous paper entitled "Hydrostatic Uplift in Pervious Soils".[2] Measurements were made of the horizontal pressures on a stationary bulkhead, resulting from bank sand and bank gravel. The fills were 7 ft deep, and alternately dry, fully saturated with water, and drained. The results indicated that the horizontal pressures for a pervious soil are more dependent on the angle of internal friction than on the angle of repose. At the beginning of irrigation, the pressures became less until the hydraulic pressure, due to the rising water, began to increase more than the relatively diminishing pressures from the soil.

General

This research was made to verify the horizontal pressure due to a fill completely saturated with water. Such conditions may occur behind retaining walls and dams. Writers of textbooks have assumed that the horizontal pressure equalled the hydraulic pressure plus the soil pressure when using the weight of soil in water and not in air. These tests on sand and fine gravel indicated that the value of ϕ should be that of the angle of internal friction and not the angle of repose. The tests also developed a novel and interesting phenomenon, namely, at the beginning of irrigation the horizontal pressures (due to the fill and to the water near the bottom) first

---

Note.—Published in November, 1933, *Proceedings*.

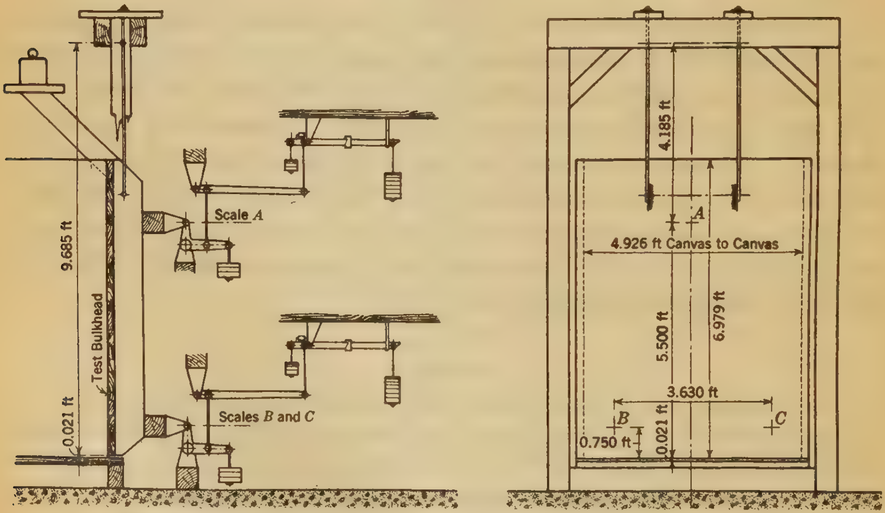
[1] Prof. Emeritus, Rensselaer Polytechnic Inst., Troy, N. Y.; Cons. Engr., New York, N. Y. Mr. Parsons died on January 26, 1935.

[2] *Transactions*, Am. Soc. C. E., Vol. 93 (1929), p. 1317.

decreased and then began to increase as the hydraulic pressure from the rising water increased. This change of pressure will be mentioned more fully later.

The testing apparatus used in this investigation is described in detail at the end of this paper. The front end of the testing apparatus was closed by a suspended bulkhead, sealed at the edges with painted canvas (see Fig. 1). This bulkhead was supported by steel rods and links, and was carefully balanced to hang vertically after the scales were attached. When in position, the lower edge was $\frac{1}{4}$ in. above the floor of the apparatus. The area exposed to the fill was 4.926 ft wide by 6.979 ft high. Three scales, *A*, *B*, and *C*, of the lever and movable-weight type, with knife-edges and ball-bearings, were attached to its outside face. Each scale could weigh 3 000 lb, or a total of 9 000 lb for the set. The scales were located so that each might take practically the same loading when the apparatus was full. The end levers had a freedom of movement at their tips of $\frac{1}{2}$ in, and the measuring leverage was 125 to 1. In consequence, movement of the bulkhead was restricted to 0.004 in. Therefore, the bulkhead can be considered as stationary at all loadings.

Fig. 1 illustrates the face of the bulkhead exposed to the pressure of the fill, and also gives a side view with the scales attached. The crosses on the face (Points *A*, *B*, and *C*, Fig. 1 (*b*)) mark the centers of the scale attachments. The dimensions are in feet, as determined from the finished bulkhead. The canvas, sealing the edges, is not shown, but will be described later. The sum of the three scale readings gave the horizontal pressure, and the center of the resultant pressure was determined from the individual scale readings. The calculations, by moments and levers, to find the centers of pressure were



(*a*) SECTION AT TEST BULKHEAD SHOWING SCALE ARRANGEMENT

(*b*) VIEW OF TEST BULKHEAD SHOWING LOCATION OF SCALES

FIG. 1.—TESTING BULKHEAD, SHOWING SECTION AND FACE

made always from the links, while the distances from the centers to the bottom of the bulkhead were found by subtraction.

The apparatus was standardized by filling it with water. A series of tests were made with water at various depths on the bulkhead, and the calculated results at each depth (Column (1)) are given in Column (3) and the averaged results by the scales in Column (5), of Table 1. The theoretical hydraulic pressures, using weight of water at 62.22 lb, on the area of bulkhead found by multiplying the actual depth by a width of 4.926 ft, were approximately 5.1% greater than the sums of the scale pressures, Column (5); that is, 5.1% of the water pressure was required to overcome the stiffness of the bulkhead and the influence of the canvas sealing its edges. It was also found that the actual location of the center of pressure fell somewhat below the theoretical, indicating a reduction in the head acting on the bulkhead. To compensate for the reduced pressure and to harmonize the discrepancy in the location of the center of pressure, it was assumed that the water acted on an "effective" area 5.1% less than its total area exposed to the water. As both the width and height had to be diminished, this assumption results in a slight error which, however, was thought permissible.

By inspection, the effective areas had an approximate width of 4.75 ft, and effective depths of 0.073 ft less than the actual ones. These reductions in dimensions would not have remained constant for shallower depths of water, but that is immaterial as only the actual depth (6.979 ft), was used in the soil tests.

TABLE 1.—STANDARDIZING TESTS; WATER PRESSURES—CALCULATED AND BY SCALES

HEAD OF WATER ON BULKHEAD, IN FEET		Theoretical water pressure, in pounds	PRESSURE ON BULKHEAD, IN POUNDS		CENTER OF RESULTANT PRESSURE ABOVE BOTTOM OF BULKHEAD, IN FEET	
Actual (1)	Effective (2)	(3)	Reduced from theoretical (4)	Average by scales (5)	Calculated (6)	Average by scales (7)
3.979	3.906	2 390	2 268	2 272	1.302	1.321
4.979	4.906	3 784	3 590	3 575	1.635	1.656
5.979	5.906	5 469	5 190	5 155	1.968	1.978
6.979	6.906	7 460	7 079	7 080	2.302	2.300

In Table 1, the effective heads of water, given in Column (2), were determined by deducting 0.073 ft from the actual head in Column (1). The figures in Column (4) represent the water pressures on the effective areas, while those in Column (5) were obtained from the averaged scale readings. The figures in Column (6) are one-third of those in Column (2), while the figures in Column (7) are the averaged results of calculating the centers of resultant pressures from the scale readings. A comparison of the figures in Column (4) with those in Column (5) and, also, of the figures in Column (6) with those in Column (7), show the agreement between the working of the scales and the calculations based on the effective area of the bulkhead.

To get the true pressure of the fill in any test, the sum of the pressures as recorded by the scales should be increased in the ratio of 100 to 94.9. This "corrected pressure" should be divided by 4.926 ft, the width of the bulkhead from canvas to canvas, to get the "corrected pressure" per foot width, as mentioned later.

The horizontal pressures recorded are for dry, saturated, and drained fills, and all had a depth of 7 ft, which, with the $\frac{1}{4}$-in. clearance under the bulkhead, made the 6.979-ft depth on the bulkhead. While the scales recorded the pressures due to the dry and drained (wet) fills, those for the saturated fills were the sum of the full hydrostatic pressure plus the pressure due to the irrigated soil.[2] The recorded pressures do not include frost action, as freezing creates another set of conditions and likewise of pressure.

Many of the calculations were made with the slide-rule. When made numerically, long decimals were shortened. Soil tests, such as those described in this paper, will not repeat with strict accuracy, and only a close approximation was essential.

MATERIALS TESTED

The soils tested were bank sand and bank gravel. Three samples of each were passed through a set of Tyler sieves, and Fig. 2 shows the mechanical analyses in graphic form. It will be noted that the bank gravel contained a

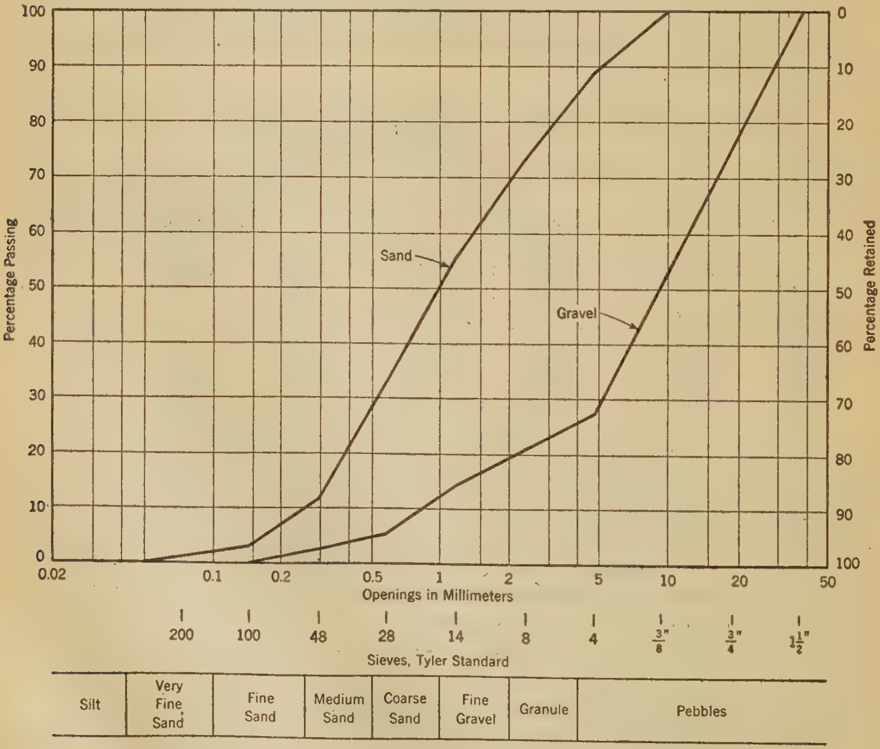


FIG. 2.—MECHANICAL ANALYSES

considerable proportion of small particles. Samples were taken of the bank sand as received, and its loss of weight (moisture) was determined by drying at 5.45 per cent. With the gravel, the loss was about 5.5 per cent. All the tests herein recorded as "dry" really contained these percentages of moisture.

Careful tests were made to find the weights and other characteristics of the sand and gravel, which are given in Table 2.

The samples were "rodded" according to the rules of the American Society for Testing Materials, Specifications C–29–27. The samples were rammed by a 20-lb bar, dropped 6 in. The bottom of the bar had an area of 6 sq in., in order to conform with the "rammed" material in the pressure tests, in which the material was placed in 1-ft layers and compacted with a 50-lb tamp with a fall of 6 in., each layer being rammed twice. The tamp was always used with the flat of the base parallel to the slope; that is, the handle was at a right angle to the surface of the slope.

TABLE 2.—GENERAL DATA FOR MATERIALS TESTED

Description	Sand	Gravel
Weight of loose material, in pounds per cubic foot	101.5	115
Weight of rodded material, in pounds per cubic foot	111	122
Weight of rammed material, in pounds per cubic foot	112.5	122
Weight in water, in pounds per cubic foot	67.75	74.5
Specific gravity	2.59	2.60
Percentages of voids on rodded material	30.48	23.55
Weight of water, in pounds per cubic foot, at average of temperatures used	62.22	

The low percentage of voids in the gravel was due undoubtedly to the fact that the gravel was well graded, the spaces between the larger particles being filled with the finer material.

PRESSURE TESTS

Seven series of tests were made, with seven tests in each series, or forty-nine in all. In each series, the soil was put into the testing apparatus, without surcharge, its surface being always level and 7 ft above the bottom of the apparatus. It was then alternately irrigated and drained. The water was allowed to enter the apparatus in the open space behind the cross-partition; it rose at an approximate rate of 1 ft in 5 min, and this space was kept full until the water became flush with the surface of the soil. Records of the scale pressures were made before water was admitted, and afterward at intervals of from 3 to 5 min, until the soils were fully saturated and the scale readings became constant. The apparatus was then drained at the approximate rate of 6 in. in 5 min until 2 ft was drawn off, after which the rate was increased to 2 ft in 5 min, as measured in the open space. As before, the scale readings were taken at intervals of from 3 to 5 min, and continued until the readings became constant. This procedure was repeated for three irrigations and two drainings. At the end of the last irrigation, the apparatus remained full of soil and water for 18 hr, or longer, and then was allowed to drain for 12 hr. The water did not rise and fall at the bulkhead at the aforementioned rates, because it was admitted to, and drained from, the space behind the plank partition (see Fig. 4), and had to leak through the partition and seep through the soil (which required time) before it could rise and fall at the bulkhead. The time for a full irrigation was about 2 hr, and for a drainage about 1 hr and 15 min. There was also an interval of several hours between some of the drainages and the next irrigations, but these intervals had no appreciable effect on the results as the scale pressures were similar to those of tests between which there were no intervals.

In the first series, the sand was shoveled loose into the apparatus, without any effort to compact it. When the first irrigation was completed the sand had settled 4 in. of its original 7-ft depth, or nearly 5 per cent. As this settlement altered comparisons with the results of the other six series, in which the soils were compacted, only a few results of loose, dry sand are reported, thus:

Depth of sand on bulkhead, in feet	6.979
Load on Scale *A*, in pounds	760
Loads on Scales *B* and *C*, in pounds	1 751
Total load on all scales, in pounds	2 511
Corrected total load, in pounds per foot of width	536
Center of pressure above bottom of bulkhead, in feet	2.187
Theoretical center at one-third of effective depth, in feet	2.302
$\tan^2 (45° - \frac{1}{2}\phi)$	0.218
Angle ϕ	40°

Shoveling the sand into the testing apparatus probably increased its weight per cubic foot in the lower portions and thus brought the center of pressure below the theoretical one-third of effective depth.

TABLE 3.—PRESSURES, IN POUNDS PER FOOT OF WIDTH (CORRECTED)*

Series	Description of test (2)	Initial dry (3)	First irrigation (4)	First drainage (5)	Second irrigation (6)	Second drainage (7)	Third irrigation (8)	Final drainage (9)
2	Sand compacted in horizontal layers.	514	1 645 (135)	1 092	1 621 (111)	1 184	1 625 (115)	1 138
4	Sand compacted in horizontal layers.	524	1 678 (168)	1 057	1 657 (147)	1 075	1 640 (130)	1 166
3	Sand compacted in layers sloping from bulkhead	472	1 697 (187)	1 014	1 624 (114)	1 114	1 622 (112)	1 129
5	Sand compacted in layers sloping from bulkhead	536	1 625 (115)	1 054	1 624 (114)	1 133	1 606 (96)	1 162
6	Sand compacted in layers sloping toward bulkhead	572	1 656 (146)	964	1 626 (116)	1 044	1 630 (120)	1 077
7	Gravel compacted in horizontal layers	456	1 729 (219)	978	1 683 (173)	1 076	1 683 (173)	.....

* Values in parentheses are pressures due to the soil: that is, total pressures less the hydraulic pressure.

In each of the other six series of tests, the procedure was identical with the foregoing except that the apparatus was filled with the soil in 1-ft layers and each layer was compacted until the 7-ft depth was completed. After the last drainage, a final reading of the scales was taken at the end of 12 hr, and the series was ended.

These series were designated by numbers from 2 to 7, inclusive, and were compacted in the various ways described in Column (2), Table 3. In each series the compacting was done by the 50-lb tamp, as already described. The depth of the soil at the bulkhead was always 6.979 ft, and the effective head was 6.906 ft, as determined from the standardizing tests with water.

The measured pressures (Table 3) are those of the scales after the readings had become constant and showed that the repeated irrigations and drainages destroyed the effects from compacting. The 12-in. layers were too thick for any enduring effect under irrigations and drainages, and the results when the layers sloped either from or toward the bulkhead did not show any appreciably distinct characteristics from those when the layers were horizontal. The compacting, however, did consolidate the soils, as shrinkage, on application of water, was practically nil.

From an inspection of the results in Table 4, it will be seen that the horizontal pressure from the sand or gravel when dry (that is, as received from the bank and containing somewhat more than 5% of moisture), was 523 lb and 456 lb, respectively, per foot of width, but only 131 lb (1 641 − 1 510) and 196 lb (1 706 − 1 510) when fully irrigated; that is, saturated with water. These latter pressures are the differences between the saturated soil pressures and the water pressure only.

TABLE 4.—AVERAGE RESULTS OF TESTS

Item No.	Description (2)	Water only, no soil* (3)	SAND			GRAVEL		
			Dry (4)	Saturated (5)	Drained (6)	Dry (7)	Saturated (8)	Drained (9)
1	Total scale pressure on bulkhead, in pounds..........	7 080	2 453	7 691	5 052	2 136	7 998	4 816
2	Resultant pressure acting above bottom of bulkhead, in feet....................	2.300	2.236	2.296	2.273	2.343	2.356	2.532
3	Difference between point of resultant and one-third of effective height, in feet†	−0.002	−0.066	−0.006	−0.029	+0.041	+0.054	+0.230
4	Corrected‡ total pressure per foot of width, in pounds .	1 510	523	1 641	1 077	456	1 706	1 027
5	Values of $\tan^2 (45° - \frac{1}{2}\phi)$.....	......	0.195	0.093	0.356	0.155	0.125	0.310
6	Angle of Internal Friction:							
	Computed................	......	42° 10′	58° 0′	27° 10′	46° 40′	53° 10′	30° 0′
	By experiment...........	......	48° 40′	61° 40′	......	48° 0′	58° 0′	......

* For comparison. † Positive values greater than, and negative values less than, the theoretical.
‡ An allowance of 5.1% was made for stiffness of bulkhead and influence of canvas at edges.

As the apparatus stood long enough for the water to saturate fully the mass (by rising from the bottom to the top), the water acted on the bulkhead both through the voids and by transmitting pressure through the soil particles. Therefore, the excess pressure recorded by the scales over that due to water only, was the pressure from the weight of the soil. Applying the

Rankine or Coulomb theory, for a fill without surcharge, the angles between the horizontal and the planes of internal friction for the dry soils and for the fully saturated soils varied. The angles always were smallest for the drained soils, intermediate for the dry soils, and largest for the saturated soils. It will be remembered that the intensity of the horizontal pressure varies inversely with the angle of internal friction.

The figures in Item No. 5, Table 4, were determined from the equation, horizontal pressure $= \frac{1}{2}\, w \cdot h^2 \tan^2 (45° - \frac{1}{2}\,\phi)$, in which the pressures are given in Item No. 4, the values of w in Table 2, and h taken as the effective heads. In the drained conditions, water remained in the interstices between the particles, although drained for 12 hr, and made the soils heavier, which would increase the horizontal pressures. For the drained conditions, an allowance for increase in weight was made by assuming that one-half the voids were full of water; that is, for the drained sand the weight was taken at 120.4 lb per cu ft, and for the gravel at 129.3 lb. An error in this assumption will not cause a serious change in the results quoted. It was not necessary to be too exact, as soils are not homogeneous, but differ throughout their mass. The computed angles of internal friction, Item No. 6, Table 4, correspond to the tangents in Item No. 5. It is difficult to duplicate the exact conditions of a large test mass of a soil in a much smaller sample, but an effort to do so was made which gave the angles "by experiment," as mentioned in Item No. 6. This was done by putting the compacted soils in two boxes and measuring the force to pull one over the other, allowances being made for the weight of the upper box and for the friction between the boxes. This method gave the tangents corresponding to the angles quoted. The soils were tested both dry and saturated.

During drainage the velocity of the water between the particles must have been appreciable. It would seem that the retreating water removed some support from the particles, which it maintained while the mass was saturated in a quiescent state. On the next irrigation, the apparent support of the particles by the water was restored.

The recorded scale pressures developed an interesting phenomenon as irrigation of the soil progressed, which the writer believes to be novel, and reported for the first time. After an initial reading of soil pressure was made, and the admission of water commenced, the combined pressure decreased until the water rose about 1 ft, or somewhat less, in the soil. Then, as water continued to rise in the soil, the pressure increased and continued to increase until complete saturation had taken place, with water standing at the level of the top of the fill. This phenomenon was repeated in most of the irrigation tests, and Table 5 gives the scale records for two of the tests.

The height of soil on the bulkhead was constant at 6.979 ft, while the depth of water was variable, rising from 0 ft at the initial pressure record to the top of the soil at the last record. It is not clear why this fall of pressure at the beginning of irrigation should occur. As a possible explanation the following suggestion is offered: Below the water level, the soil weight would be lighter (due to its weight in water), while above the water level, the soil would retain its dry weight. This would reduce the horizontal pressure due to the soil. As the irrigation-water level rose, the increase in hydrostatic pressure would be greater than this reduction of soil pressure. Furthermore, introduction of the water increased the internal angle of friction of the soil, resulting in a decrease of the volume of the soil wedge causing the horizontal pressure. This reduction in pressure from the soil would be relatively greater when the water levels were low and the hydrostatic pressure was small.

TABLE 5.—SCALE PRESSURES IN TWO TESTS AS WATER RISES IN THE SOIL, IN POUNDS

Sand			Gravel		
Scale *A*	Scales *B* and *C*	Total	Scale *A*	Scales *B* and *C*	Total
821	1 860	2 681	716	1 420	2 136
.....	.....	.....	702	1 387	2 089
812	1 847	2 659	708	1 650	2 358
808	1 804	2 612	720	1 772	2 492
800	1 796	2 596	850	2 317	3 167
1 015	2 960	3 975	2 300	5 097	7 397
2 250	5 013	7 263	2 698	5 310	8 008
.....	.....	.....	2 730	5 375	8 105
2 590	5 172	7 762	.....	.....	.....

When drainage commenced, the pressure fell until about 2 ft of water remained in the soil, and, thereafter, it decreased very slowly until the final drained reading was recorded. A hypothetical calculation of the pressures, as drainage progressed, indicated that the regimen of decrease in pressure would be similar to that for rise in pressure due to irrigation; that is, the pressure would decrease below and then increase to the final drained pressure. The hypothetical calculation was based on the assumption that, at any intermediate stage of the water level, the horizontal pressure was the sum of the

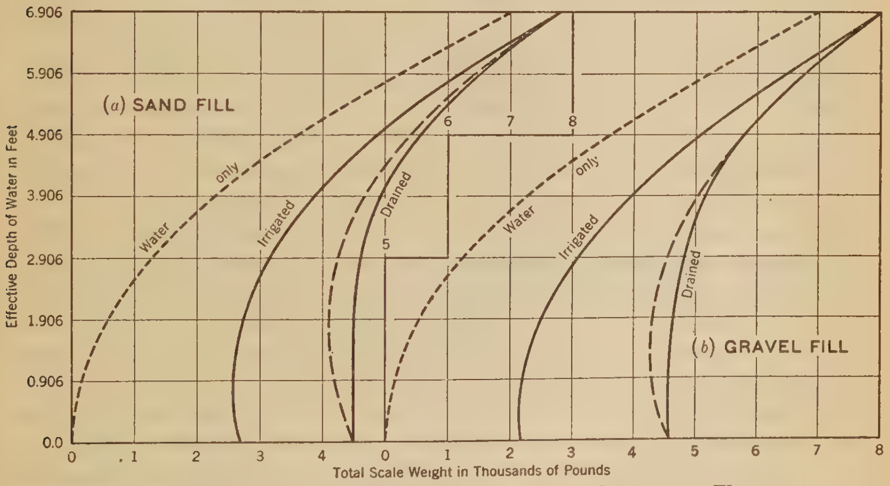


FIG. 3.—TYPICAL PRESSURE CURVES AT VARIABLE DEPTHS OF WATER

pressure due to the dry fill above the water level, of the pressure due to the fill below the water level, and of the hydrostatic pressure. It was realized that the value of $\tan^2 (45° - \frac{1}{2} \phi)$ did not remain constant as the water rose, and a compromise was made by plotting the values for dry and saturated conditions, with water depths as ordinates, and joining the points by a straight line. Then, values for each foot of depth of water could be determined. Probably the change in values of the tangents did not vary as this linear assumption, but the interpolations were approximate.

The pressures for Series 6 with sand and for Series 7 with gravel are illustrated in Fig. 3, on which curves of pressure due to water only have been drawn. The broken lines for the drainage curves are computed values. The horizontal distance between the water curve and either of the remaining curves, measures the pressure from the soil. It will be noted that the difference between the water curve and the irrigation curve steadily decreases as the water rises. In other words, the part of the total pressure due to the soil becomes less and less as irrigation progresses until, at full saturation, the pressure due to the soil is only a small percentage of that due to water alone. Furthermore, it is only a small portion of that due to the soil without any water (initial pressure).

DESCRIPTION OF APPARATUS

The testing apparatus, Fig. 4, was constructed of 2-in. dressed yellow pine planks, grooved and fitted with splines for water-tightness. It was stiffened by 6 by 4-in. uprights at the sides and sleepers across the bottom and by steel tie-rods across the top. The box stood on a level concrete floor, and its interior width was tapered to reduce any arching of the soils. It was lined with building paper, and the front end was the free bulkhead shown in Fig. 1, suspended by two steel rods, linked into shorter rods having washers and nuts, that were supported by cross-pieces. The cross-pieces, in turn, were supported by braced posts. There was opportunity in the supporting arrangement to adjust the bulkhead to the front end of the testing box.

The supporting rods were practically in the plane of the center of gravity. The three scales were attached and supported, as shown in Fig. 1. To balance the bulkhead in a truly vertical position, counterweights attached to the top of the bulkhead were provided. The openings at the edges of the bulkhead were closed by canvas tacked to the sides and bottom, then miter-folded at the bottom corners, and tacked to the bulkhead. The canvas was in two layers and painted to prevent leakage. When the apparatus was empty and ready for testing, the scales were balanced at zero readings, and were very sensitive to pressure on the bulkhead. The rear end of the box was arranged to open for the easier removal of the soils between each series of tests. The box was designed long enough for a more extended research, but such additional work was not undertaken. To reduce the quantity of soil handled, a loose plank partition was placed 7 ft back from the bulkhead as shown in Fig. 4. This was far enough so as not to interfere with the planes of rupture of the soils. Water for irrigation was admitted to the empty space in the rear of this partition, and it flowed through the open joints and the soil to the bulkhead. There was also a valve for drainage.

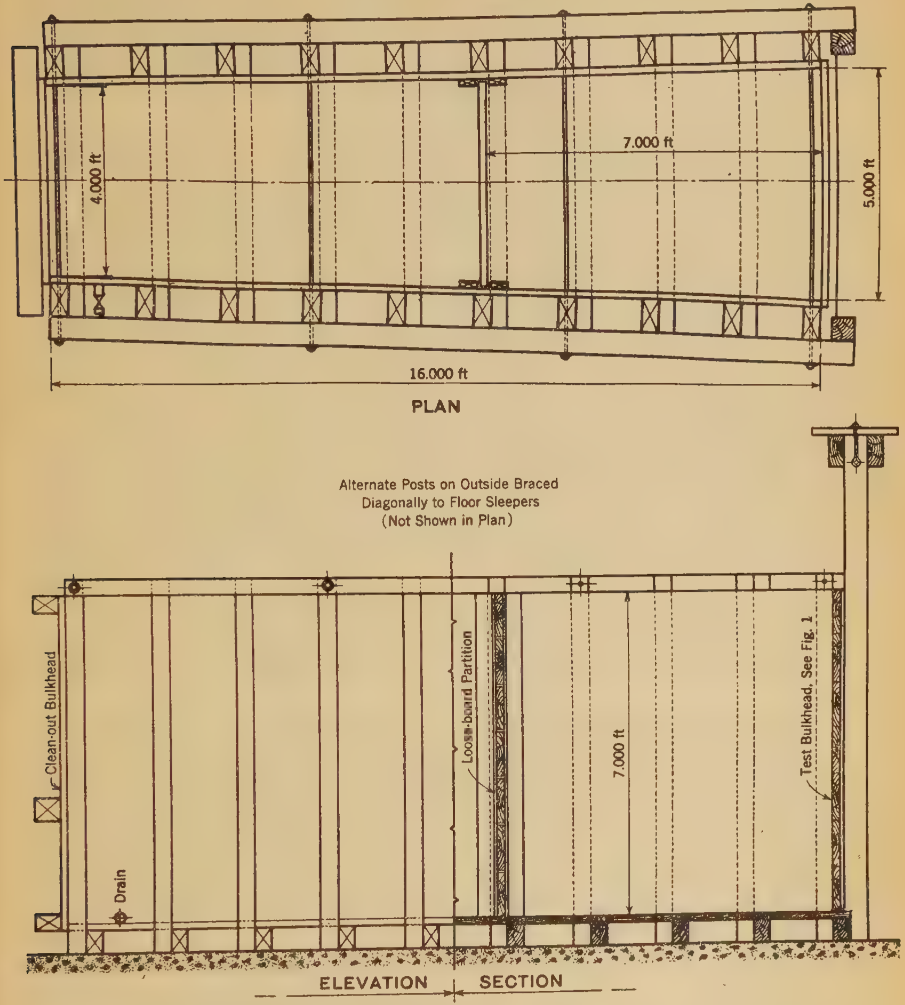

7.000 ft
4.000 ft
5.000 ft
16.000 ft
PLAN
Alternate Posts on Outside Braced
Diagonally to Floor Sleepers
(Not Shown in Plan)
Clean-out Bulkhead
Drain
Loose-board Partition
7.000 ft
Test Bulkhead, See Fig. 1
ELEVATION
SECTION

FIG. 4.—TESTING APPARATUS

SUMMARY

1.—There are many variables in soil problems, and only by the agreement of a number of researches can actual facts become established, which is the excuse for publishing this work.

2.—It may be said that "arching" affected the results of these tests. Arching might have occurred in the "dry" tests, but the repeated irrigations and drainages which followed gave such uniform results that arching did not appear to be a factor.

3.—The results of these tests approximate the Rankine and the Coulomb theory for soils without surcharge, provided ϕ is the angle of internal friction and not the angle of repose. Field observations of unsupported banks, in which the soil had some moisture content, show a surface of rupture that is not a true plane, but is nearly vertical at the surface and becomes more horizontal near the bottom. Such observations confirm the test results, namely, that the wedge causing the pressure is smaller than in theory, which uses the angle of repose.

4.—These tests indicated that the center of pressure, on repeated irrigations and drainages, approximated the theoretical $\frac{1}{3}h$, except for the drained gravel where the center was considerably higher than the theoretical (see Table 4). It is not apparent why the point of resultant pressure was high for the gravel, especially in the drained state. It is conceivable that the plane of rupture in this coarser material was a warped or curved surface, which reduced the section of the wedge from the theoretical triangular shape and placed its center of gravity above that of the triangle. The wet condition after drainage may have increased the cohesion of the gravel particles and thus produced a smaller wedge by a curved rupture plane.

5.—When the soil was fully irrigated, the pressure consisted of the hydrostatic pressure plus a pressure due to the soil. The portion of the pressure which was due to the soil was less than that which would have been expected when using the angle of internal friction as deduced from the dry material (that is, containing about 5% moisture), and making an allowance for the reduced weight of the material; that is, the ratio of its weight in water to its weight in air. It may be that under saturated conditions, the plane of rupture is curved, rather than flat, as explained herein under Summary Item 4.

6.—Under fully irrigated conditions it would seem from Table 4 that the pressure due to the sand was 131 lb (total pressure, 1 641 lb, less hydraulic pressure, 1 510 lb). This soil pressure was about 25% of the dry soil pressure, namely, 523 lb. In the same way, for the gravel, the total pressure was 1 706 lb less the hydraulic pressure, 1 510 lb, leaving 196 lb for the gravel portion of the total. This latter value is about 43% of the dry gravel pressure of 456 lb. It will be noted that these soil-pressure percentages are much less than the percentages of the weight of the sand or gravel in water to their weight in air, which were about 61 in both cases.

7.—Under drained conditions, the horizontal pressures of the soils were high and the computed angles of internal friction were low, when compared to the results for dry and saturated conditions. This was probably due to the increased weight of the water retained in the interstices, and also to a possible hydrostatic pressure resulting from the "head" corresponding to the water thus held by the particles. Ordinarily, wet soils are stiffer than dry soils due to cohesion between the particles, but these drained soils possibly retained more water than necessary for maximum cohesive effect.

8.—The scale pressures as given in the horizontal lines of Table 5, were taken at similar intervals, and, therefore, the sequence of decrease and subsequent increase of pressures is comparable for both the sand and gravel tests. Probably the gravel, being the coarser, allowed the water to seep through its mass faster than through the sand and rise more quickly on the bulkhead. Thus, the lowest pressure for the gravel occurred at the first interval, after the initial, while for the sand the decrease continued until the fourth interval.

ACKNOWLEDGMENTS

The writer acknowledges his thanks to the Trustees of Rensselaer Polytechnic Institute, Troy, N. Y., for partial financial assistance, and to Palmer C. Ricketts, Hon. M. Am. Soc. C. E., for space for the apparatus. He is indebted, furthermore, to T. R. Lawson, M. Am. Soc. C. E., and Professor Edward J. Kilcawley, for determining the weights of the soils; to Lieut. Hunt V. Martin, C. E., U. S. N., a Post-Graduate Student at the Institute, for the manipulation of the tests; and to Eugene E. Halmos, M. Am. Soc. C. E., for assistance in the analytical work.

DISCUSSION

J. C. MEEM,[3] M. AM. SOC. C. E. (by letter).—It is not an agreeable task which has been self-assigned to the writer—that of criticizing adversely this carefully and painstakingly prepared paper. There is one criticism, however, which is fundamental in its application to a soil pressure test of this scale. Due to the normal lateral pressure of soil, the results of any tests must be misleading if not valueless unless they are made on a scale sufficiently large to preclude the possibility of lateral arching, either vertical or horizontal. This applies even with greater force to kiln-dried sand which, in the writer's judgment, is the only material in which such tests should be made and final results of value computed.

The fallacy of expecting results of value from apparatus having a face of even 5 ft in width can be shown by making the following simple test or experiment. A box of 1-ft cube is constructed without top or bottom. A bottom plate, slightly larger in area than that of the box (say, 14 by 14 in.), carrying four 15-in. bolts, one near each corner, forms the loose unattached bottom. The box is filled level full with kiln-dried sand and a top plate about 10 by 10 in. in area, with bolt holes, forms the top. The bottom plate is brought to bear firmly against the bottom edges of the box by tightening the washer bearing nuts at the top. As these nuts are tightened, the sand in the box is firmly compacted; but instead of transmitting this pressure to the bottom, it is transmitted laterally, and friction is developed between the sand and the sides.

It will be seen that the development of this friction can readily be made sufficient to carry the sub-arch sand bearing on the bottom, at which time the box can be lifted as a whole and carried about without danger of collapse. If the inside faces of the box are not too smooth, the friction, of course, will be developed more readily.

It is necessarily true that if this can be done with a 1-ft cube, it can be done with a 5-ft cube; except that it is relatively more difficult to transmit by hand the pressure necessary to develop the required friction to support the 1 000-lb load on the bottom of a 5-ft cube than to support the 8-lb load on the 1-ft cube. Under natural conditions this is developed by increased friction and cohesion due to moisture and to increase of depth sufficient to provide relatively larger frictional area.

If it can be shown, therefore, that lateral pressure on a 5-ft cube may be fully developed even in kiln-dried sand, the results of tests made in a box with only a 5-ft opening must necessarily be misleading and without definite value. This is true not only with kiln-dried sand, but with moist sand, since it can be readily shown that one side of a 5-ft cube of ordinary moist or moulding sand may be cut vertically without developing any side pressure whatever.

---

[3] Cons. Engr., Brooklyn, N. Y.

For a long time the writer has advocated and proposed the making of soil tests on a scale sufficiently large to obviate the possibility of erroneous results from lateral arching or other causes. The tests or examinations, as suggested, are hereinafter noted.

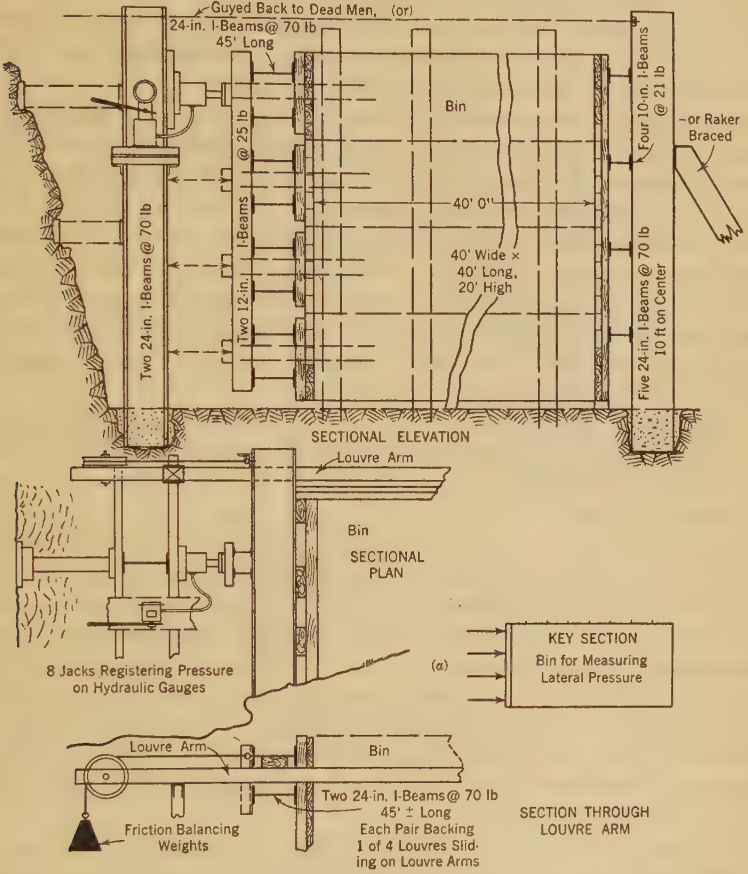

Guyed Back to Dead Men, (or)
24-in. I-Beams @ 70 lb
45' Long
Bin
Four 10-in. I-Beams @ 21 lb
-or Raker Braced
Two 12-in. I-Beams @ 25 lb
Two 24-in. I-Beams @ 70 lb
40' 0"
40' Wide × 40' Long, 20' High
Five 24-in. I-Beams @ 70 lb 10 ft on Center
SECTIONAL ELEVATION
Louvre Arm
Bin
SECTIONAL PLAN
8 Jacks Registering Pressure on Hydraulic Gauges
(a)
KEY SECTION
Bin for Measuring Lateral Pressure
Louvre Arm
Bin
Two 24-in. I-Beams @ 70 lb 45' ± Long Each Pair Backing 1 of 4 Louvres Sliding on Louvre Arms
Friction Balancing Weights
SECTION THROUGH LOUVRE ARM

FIG. 5.—DETAILS OF SAND BIN FOR MEASURING LATERAL EARTH PRESSURES.

Test 1.—Determination of Force and Value of Lateral Earth Pressure.—In order that this determination may be definite and convincing, it must be made on a scale at least approximately as large as that shown, otherwise the cohesiveness of soil particles, as well as the lateral arching properties of the soil will make the tests of little or no value.

Fig. 5 shows an apparatus consisting of a bin 20 ft high, 40 ft wide, and 40 ft long, net inside dimensions, in a pit or in the open, and braced and tied across so as to leave it internally clear and free of all tie-bolts. The face of the bin is made of four louvers, each 5 by 40 ft, and backed by two 24-in., 70-lb beams to insure stability and stiffness. These beams are faced and connected so that the top one of each pair slides on a horizontal arm in order that no pressure is exerted on the one next below it or on the bottom itself. Eight hydraulic pressure gauges or dynamometers are used to measure the pressure that is transmitted directly to these gauges. In order to overcome friction and to insure exact data, the pressure on all gauges is raised equally to, say, 10 lb (or as noted), by means of eight weights. Each of these weights, acting through and over a pulley, is attached outwardly to each end of a louver. It is thus seen that any pressure from the soil will be transmitted directly through each louver to the gauge.

A light roof or cover should be provided to insure the maintenance of dryness, particularly for the kiln-dried sand with which the bin is first filled. As soon as the bin is filled, and while it is being filled, the gauges are read to note the varying and continuing pressures. As soon as this test has been repeated to insure against errors, the bin is left level full with dry sand and the readings are noted. Uniformly spaced and properly located superimposed loads are then erected on the top of the sand and the readings of the various gauges noted as the loads are increased. The bin is then emptied and filled with normally dry sand and finally emptied and filled with normally dry sand and gravel, and the various readings are noted.

It is essential that this test be made with superimposed louvers sliding on arms in contact with, but independent of, each other, backed by heavy beams to insure against distortion and having friction balanced by independently acting weights.

Test 2.—The next test is to determine the arching value of soil in its relation to pressure on a tunnel roof at varying depths. A pit about 15 by 20 ft at the bottom, is excavated to a depth of 20 ft, with sides as steep as the ground will stand. A heavy box platform about 15 by 15 ft by 4 ft high, opening into a braced shaft at one side of the pit (see Fig. 6 (*b*)), has a 10 by 10-ft opening in the top and is arranged with a false bottom below and inside the top. It is supported by four hydraulic jacks with gauges. The bin is then filled with kiln-dried sand and the increased pressure on the gauges is noted as the depth of sand is increased, or until no further increase of pressure is observed.

If or when this point or height is reached, superimposed loads are added and increases of pressure (if any) are noted. This test may be repeated with moist or normally dry sand or gravel, but it is believed that the best results will be noted with the kiln-dried sand.

Test 3.—A large pit, say, 30 by 50 ft at the top (see Fig. 6(*b*)) and perhaps 20 by 40 ft, at the bottom, and about 20 ft deep, is dug in firm non-porous soil. A coffer-dam of the master sheet-pile type is then driven into the bottom soil with the master-piling penetrating about 5 ft and the sheet-piling just into or below the bottom. The master piles are then braced at the top with adjustable bracing.

Fill the pit outside the coffer-dam with sand, then flood it to above the level of the sand. Replace the bracing with bracing carrying pressure gauges and note the pressure. Then gradually pump the water from inside the coffer-dam and note the increase of pressure (if any) due to the lowering of the water. This test should be of incalculable value to those who contemplate building coffer-dams.

It is believed that the tremendous importance of the results from such tests would lead the author to consider the possibility of making them with co-ordination between such facilities as he may have at his command. It is in effect simply bringing his own tests out of the danger zone of a scale too small to be of value.

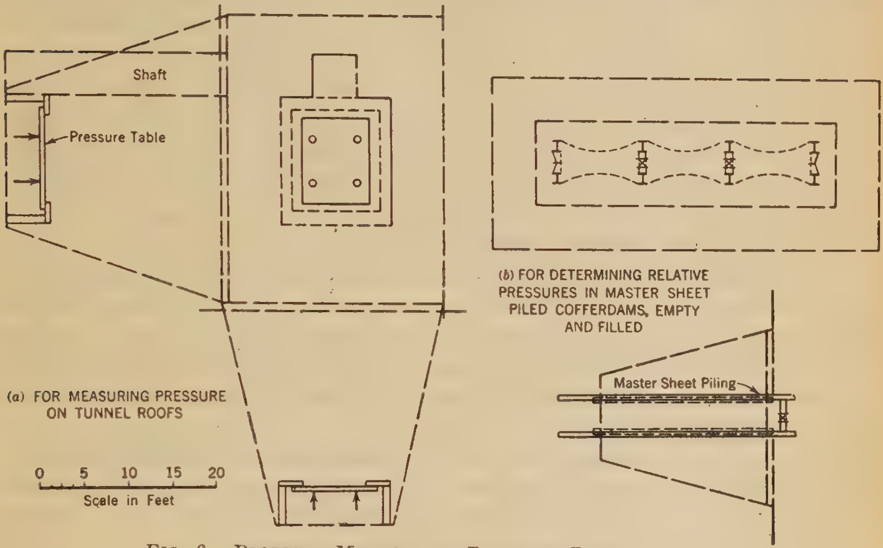

Shaft
Pressure Table
(a) FOR MEASURING PRESSURE ON TUNNEL ROOFS
0 5 10 15 20
Scale in Feet
(b) FOR DETERMINING RELATIVE PRESSURES IN MASTER SHEET PILED COFFERDAMS, EMPTY AND FILLED
Master Sheet Piling

FIG. 6.—PROPOSED METHODS IN PRESSURE DETERMINATIONS.

Concerning the question of the use of the term, angle of repose, the author finds some objection because the plane of repose or rupture for normally dry or moist soils is found usually to have a curved instead of a rectilinear cross-section. The angle of repose is that angle between the normal slope of the soil after the soil above the plane of repose has slid away, and the vertical. If the line measuring this angle is curved instead of rectilinear, the angle of repose may be taken as the mean of the sum of all the angles between the tangents and the vertical.

It is certain that the only pressure possible in a retaining wall from sandy soil must come from that volume of soil sliding or tending to slide, and the measure of its cross-sectional area is that between the plane of repose and the vertical.

EUGENE E. HALMOS,[4] M. AM. SOC. C. E. (by letter).—A large amount of work was done in gathering data for this paper, both in the laboratory and in the office, and the writer wishes to pay his compliments to Mr. Parsons' ingenuity for having been able to condense the voluminous information collected into such a short paper. All the essential findings of the tests and the author's analytical studies have been included in a manner which can be easily digested by the reader.

---

[4] Res. Engr., Parsons, Klapp, Brinckerhoff & Douglas, Sutherland Project, North Platte, Nebr.

Tests to determine the pressure of earth on retaining walls have been in progress for nearly a century, the purpose of such tests being, first, to develop formulas for the design of retaining walls, and, later, when the theory of earth pressure was more fully developed, to determine the value of the constants entering into such formulas. Most of the previous tests were conducted with dry materials and, in many cases, the materials were selected to be truly granular so as to come more closely within the scope of cohesionless semi-fluids. The writer has no knowledge of any tests conducted along the lines described by Mr. Parsons and believes that these are the first in which the influence of irrigation and drainage was determined throughout the entire height of a fill consisting of "run-of-bank" material.

The writer agrees with Mr. Parsons that considerably more data will have to be collected and studied before certain peculiar characteristics disclosed by the tests can be fully explained. Such a peculiarity is the fall of the pressure at the first stages of the irrigation; another is the material decrease of that part of the pressure attributed to the earth alone as the irrigation approaches the full height of the fill. This latter may be due to "arching" or "bin action" transferring part of the weight of the fill to the side walls of the apparatus by means of friction. It is not clear why such action should be accentuated under irrigated condition. Neglect of either or both of these phenomena, however, results in a conservative design.

The most interesting and the most important practical disclosure is that described in Conclusion (7) which states the fact that after full drainage the residual pressure was found to be much greater than that of the originally dry earth. This should have an appreciable bearing on the assumptions underlying the safe design of retaining walls in which the supported bank has no positive surface drainage and, therefore, it is conceivable that after heavy rains the earth fill may be in the same condition of wet consistency as that of the tested fill after drainage was completed.

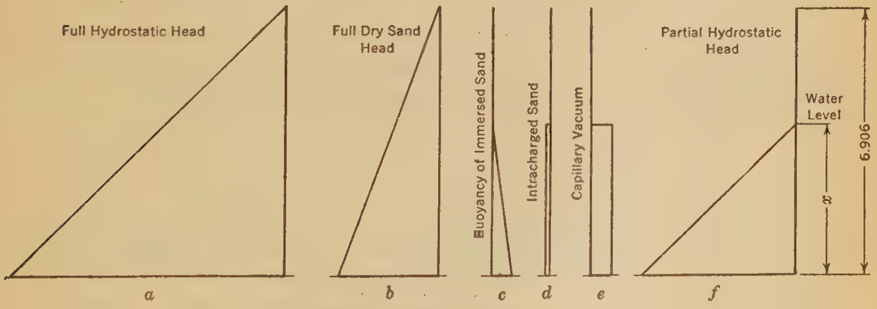
Retaining walls are designed usually on the assumption that as long as weep-holes, about 3 in. in diameter and about 25 ft apart, are placed in the lower part of the wall, the earth pressure during and after rains will remain the same as that estimated for the dry condition of the back-fill. A substantial proportion of retaining wall failures have occurred after heavy storms, indicating that more knowledge is needed on the pressure exerted by a back-fill alternately wetted and drained. Mr. Parson's experiments show that this pressure is about twice the assumed thrust, when the draining is from a fully saturated state.

Further tests should be made, simulating the effect of heavy and extended rain, with weep-holes inserted in the bottom of the test wall, or with sub-drains at the back of the wall, in order to determine how much additional pressure should be assumed in the design to compensate for the wet consistency of the back-fill resulting from any expected rate of rainfall.

It may be argued that the test results referred to in Conclusion (7) are brought forth by the "passive" resistance of the fill which prevents the bulkhead from moving toward the fill when the outward pressure is diminished. The writer believes that the effect of this resistance should be negligible, considering the extremely small movements of the bulkhead. However, if the tests are repeated, it is suggested that measurements be made to determine the vertical component of the earth pressure and the magnitude of the elastic deformation of both the bulkhead and the side walls of the apparatus.

L. C. Wilcoxen,[5] Assoc. M. Am. Soc. C. E. (by letter).—The total pressure of sand on a bulkhead is reduced when the lower 1-ft stratum is immersed. This is a paradox that the author demonstrates positively. In view of the fact that he does not give a satisfactory explanation of the cause, the writer will relate a simple test that gives the clue to the nature of the phenomenon. An analysis based thereon follows: A glass jar was partitioned vertically and on one side a dry mortar sand was rammed level with the top. Water was then poured slowly into the open side. It was found that the water in the sand rose and remained the full 6 in. above the level on the open side. The difference in levels was due to the capillary action of the sand. This is the cue to a physical action that takes place when sand behind a bulkhead is immersed.

Consider the case of a hypothetical bulkhead, 1 ft wide and 6.903 ft high, backed by a sand weighing 112.5 lb per cu ft, having a void ratio of 30.48% (solid ratio, 69.52%), and a horizontal reaction when dry of 21.88 lb per sq ft per ft of depth; that is, the ratio of the horizontal to vertical reaction is 0.194. The reaction diagram of this sand is shown (Fig. 7 (*b*)) in comparison with a similar one for water of the same depth (Fig. 7 (*a*)).



Horizontal Reactions on the Bulkhead

Fig. 7.

If the water level is raised to any height on the bulkhead, capillary action will cause it to rise, say, 6 in. higher in the sand behind. Several results accrue from the introduction of the water: First, there is the buoyancy of the immersed sand with its proportionate reduction of the horizontal reaction; second, the sand is intracharged at the surface of the water, by the weight of water suspended by capillary action (this proportionally increases the horizontal reaction of the sand on the bulkhead); third, downward from the level of the water in the sand, the capillary action induces a vacuum pressure equal to the height of water supported by it; and, fourth, from the same level downward the hydrostatic head is effective.

---

[5] Asst. Civ. Engr., City Engr.'s Office, Detroit, Mich.

Assuming a capillary head of 6 in. and the weight of water as 62.22, the quantitative results will be as follows: First, the buoyancy of the immersed sand will be (0.6952 × 62.22 =) 43.25 lb per cu ft, and the resulting reduction in the horizontal reaction will be (43.25 × 0.194 =) 8.39 lb per sq ft per ft of depth (Fig. 7 (*c*)); second, the intracharge due to the 6 in. of water suspended at its surface in the sand, will be (0.5 × 62.22 =) 31.11 lb per sq ft, resulting in an increased horizontal sand reaction of (31.11 × 0.194 =) 6.03 lb per sq ft (Fig. 7 (*d*)); third, the vacuum pressure due to capillary action will be (0.5 × 62.22 =) 31.11 lb per sq ft (Fig. 7 (*e*)); and, fourth, the hydrostatic head will be 62.22 lb per sq ft of depth (Fig. 7 (*f*)).

From this it may be shown that the total horizontal reactions, in pounds, will be: $536 - 25.08x + 26.92x^2$, when x is the height of the water surface in the sand. This indicates that as the lower stratum of sand is immersed, the total pressure is decreased, the minimum pressure occurring when $x = 0.44$. At this point the decreased pressure is 5.8 lb. The computed pressure curves of water only and the irrigated sand for the author's bulkhead are shown (Fig. 8). They check closely the experimental results of Fig. 3. At the full head, the author observes a difference between the total reactions of water and irrigated sand of (7 691 − 7 080 =) 611 lb, whereas the writer's computed difference from the equation is (8 041 − 7 309 =) 732 lb.

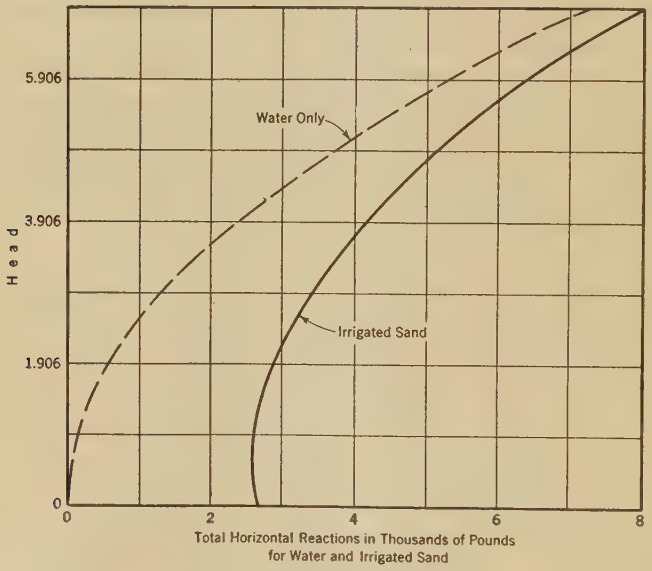


FIG. 8.

The author is to be complimented for the accuracy of his tests, which have brought to light so interesting a phenomenon.

Dr. Ing. O. K. Froehlich[6] (by letter).—When attempting to interpret qualitatively and, as far as possible, quantitatively, the results of soil pressure tests, dealt with in Mr. Parson's valuable paper, one must keep in mind that:

(1) The well-known, existing, so-called "classical" theories on earth pressures relate to a two-dimensional problem, such as an infinitely long retaining wall.

(2) The earth pressure problem of a sand or gravel fill, contained in the apparatus illustrated in Fig. 1, is three-dimensional.

(3) The bulkhead and side walls of the apparatus in Fig. 1 were not absolutely rigid, but were flexible to some extent, and the deflections were not measured.

(4) The deflections, caused by water only (no soil), did not influence the distribution of water pressures.

(5) The "dry" fills were not placed regularly in thin layers in the apparatus, but were shoveled in, so that the equilibrium of the lower portion probably was affected by dynamic influences.

(6) The measured values of the weight of material (loose and rodded) per cubic foot are only approximate average figures.

(7) The vertical wall friction was not measured.

(8) The difference between the quantity of water used for irrigation and the quantity that drained, was not measured.

Consequently, it seems to be advisable to avoid exaggerated mathematical rigidity in interpreting the most important results of these earth-pressure tests. First, consider the simple case of an infinitely long vertical retaining wall with a height, h, and supporting a horizontal, cohesionless back-fill, weighing w units per unit of volume. Let ϕ be the angle of internal friction, and ϕ', the angle of friction between the wall and the fill. According to the general wedge theory, the horizontal component, H_a, of the active lateral earth pressure (on the assumption that $\phi = 40°$ and $\phi' = 37° \, 30'$) amounts to:

$$H_a = 0.207 \left(\tfrac{1}{2} w h^2\right) \cos \phi' = 0.164 \left(\tfrac{1}{2} w h^2\right) \ldots\ldots\ldots\ldots (1)$$

Leaving the inclination, ϕ', of the total earth pressure unchanged, the horizontal component, H_p, of the passive earth pressure is:

$$H_p = 1.11 \left(\tfrac{1}{2} w h^2\right) \cos \phi' = 0.880 \left(\tfrac{1}{2} w h^2\right) \ldots\ldots\ldots\ldots (2)$$

Active and passive pressures are created by slight wall movements away from, and toward, the fill, respectively. Between these values, H_a and H_p, there is a "neutral" earth pressure,[7] H_i, which corresponds to the motionless, or rigid, wall and amounts to:

$$H_i = 0.5 \left(\tfrac{1}{2} w h^2\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

---

[6] The Hague, The Netherlands.

[7] "Drukverdeeling in Bouwgrond," *De Ingenieur*, 1932, No. 16; 1933, No. 25.

The ratios between active and "neutral" and between passive and "neutral" pressure, according to Equations (1), (2), and (3), are, respectively:

$$\frac{H_a}{H_i} = 0.328 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

and,

$$\frac{H_p}{H_i} = 1.76 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

Assume that these ratios also hold true for the three-dimensional problem of the bin, having flexible side walls. At present, this assumption can only be checked empirically, as the three-dimensional general wedge theory has not yet been developed.

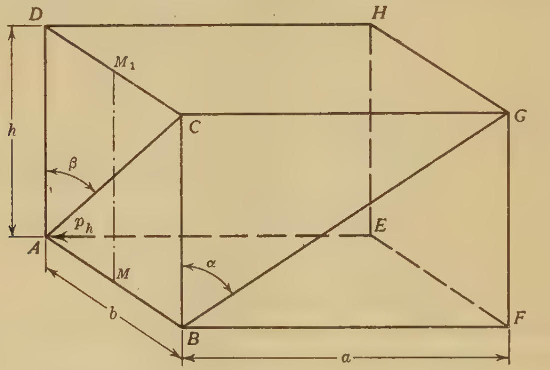


FIG. 9.—COMPUTATION OF "NEUTRAL" EARTH PRESSURE

The theory of the three-dimensional "neutral" earth pressure problem[8] enables one to determine the lateral pressure, exerted by the granular fill of a rectangular bin having a section of 7.0 by 5.0 ft, and a height of 7.0 ft, if the side walls and bottom of this bin are practically rigid.

The horizontal pressure, p_h, per unit of area at Point A of the side wall, $b \times h$, is approximately given by the following expression (see Fig. 9):

$$p_h = \frac{w\,b}{2\,\pi}\left\{ \log\left(\cot\frac{\beta}{2}\tan\frac{\alpha}{2}\right) + \cos\alpha \right\}\cot\phi' \ldots\ldots\ldots\ldots (6)$$

By applying Equation (6) to the points of the lines, $A\ D$ and $M\ M_1$, Fig. 9, at depths of 1, 2, 3, . . ., 7 ft, under the surface, $C\ D\ H\ G$, of the fill in the bin, the unit pressures, p_h, in Table 6, are obtained by multiplying each value by $\frac{w\,b}{2\,\pi}\cot\phi'$. The averages in Table 6 are obtained, as follows (at a depth of 1 ft. for example: $0.199 + \frac{2}{3}(0.390 - 0.199) = 0.327$.

---

[8] "Drukverdeeling in silo's * * *", *Polytechnisch Weekblad*, 1933, Nos. 38–39.

Using the values, $w = 101.5$ lb per cu ft for sand; $b = 5$ ft; and $\phi' = 37° 30'$; the total thrust is:

$$\frac{101.5 \times 5}{2\pi} \cot 37° 30' \frac{(2 \times 6.641 - 1.372) \times 1 \times 5}{2} = 3\,130 \text{ lb}$$

If an allowance of 5.1% is made for the influence of the canvas at the edges, etc., $H_t = 3\,130 - 160 = 2\,970$ lb. From Table 4, the total scale pressure on the bulkhead for dry sand was 2 453 lb; that is, 17.4% less than the "neutral" earth pressure. This reduction is due to the inevitable movement of the bulkhead of (maximum) 0.004 in. = 0.1 mm.

TABLE 6.—"NEUTRAL" UNIT PRESSURES, p_h, ON BULKHEAD

Depth, in feet	Side line, $A\ D$	Center line, $M\ M_1$	Average
1	0.199	0.390	0.327
2	0.383	0.726	0.611
3	0.547	0.995	0.844
4	0.685	1.202	1.029
5	0.799	1.363	1.175
6	0.889	1.481	1.283
7	0.960	1.578	1.372
Total	....	....	6.641

As regards the magnitude of the pressures on irrigations and drainages, the writer offers the following explanation: The water pressure on the side walls of the apparatus, which is approximately three times as great as the earth thrust, produces wall deflections—however small these may be — and allows the fill to follow an "active" tendency. When the water is drained away, the deflections are restored and the four side walls of the bin exert a passive pressure on the fill.

To develop this idea, one must keep in mind: First, that the unit weight of the sand immersed in water was 61% of the weight in air; and, second, that the principle of superposition of pressures must remain valid in the present case.

According to Equation (4), the sand pressure against the bulkhead, if the sand is saturated, must be:

$H_a = 0.328 \times 0.61 \times 2\,970$.......... = 594 lb
Water pressure according to Table 4.......... = 7 080 lb

Total.................. = 7 674 lb

Table 4 gives 7 691 lb as the average result. After drainage the sand pressure against the bulkhead is passive, and Equation (5) yields: $H_p = 1.76 \times 2\,970 = 5\,230$ lb. Table 4 gives 5 052 lb as the average result.

These figures would confirm the previously mentioned opinion, that slight deflections of the side walls took place during irrigation and were restored during drainage. In an absolutely rigid apparatus the following total pressures might be expected: Dry, 2 970 lb; saturated, $0.61 \times 2\,970 + 7\,080 = 8\,890$ lb; and, drained, 2 970 lb.

H. L. THACKWELL,[9] ASSOC. M. AM. SOC. C. E. (by letter).—Due to the paucity of reliable information on a subject that deals with the forces exerted on non-homogeneous materials, such as earth, sands, clays, and gravels, and a combination of all such materials under various conditions of saturation, the average practicing engineer has resorted either to personal judgment, or to "rule-of-thumb" methods, for assigning the forces assumed to act against walls, the sides of tanks, and bins. This excellent paper describing soils tested under saturated and drained conditions, with measured lateral pressures, is a valuable addition to a subject in which many engineers have been groping for a method of rational analysis.

If the conditions of loading, and the physical properties of such non-homogeneous, granular, and plastic materials, could be ascertained accurately, it would be possible to develop a correct theory for the particular case under consideration. Correct theory is only possible when based on inductive reasoning (from the particular to the general); the particulars must be studied by experiment, and such experiments must be based on conditions that are likely to be encountered in practice. The classical theories of Rankine and Coulomb have served very well in the design of walls back-filled with dry materials of uniform nature. With these formulas, ϕ is the angle of repose, although Boussinesq has pointed out that the angle of internal friction would be more applicable.

In the frequent case of a wall to be designed with a vertical back and with the requirement that it retain a level bank of earth, the customary formula is:

$$P = \frac{w}{2} h^2 \tan^2 \left(45^\circ - \frac{\phi}{2}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots (7)$$

If this formula is correct in theory, it is necessary, in order to use it properly, to assign the correct values for w and ϕ, which are the only unknowns. If the conditions of back-filling are known in advance, it would be possible, by experiment, to obtain the weight of the material, or combination of materials, to be used in the fill, and also the angle, ϕ, which should be the angle of internal friction.

In practice, it is seldom that the engineer who is entrusted with the problem of designing simple concrete retaining walls, will have the time or the money to make the necessary experiments for determining the correct values of the aforementioned variables. For example, consider that an engineer has been retained to design, and supervise the construction of, a sewage disposal plant for a city of 5 000 population. The usual procedure is to select a suitable site for the plant, and then to make a topographic map of it, including a few borings for soil and sub-surface drainage conditions. This map, with the borings for a guide, then becomes the field for office locations of all tanks, walls, etc., that will be incorporated in the design. The designing engineer, after a few trial layouts, will finally select the exact positions for the various buried tanks and the retaining walls for filter enclosures. It is impractical to expect that the engineer entrusted with the design of a small project, and receiving only a 5% fee for engineering work, will make a sufficient number of tests of the filling material and of the angles of internal friction, which will occur at various positions behind the structures, in order to obtain accurate values that will eliminate the unknowns of the equation.

---

[9] Cons. and Const. Engr., Longview, Tex.

Moreover, the conditions after back-filling are subject to change, due to a rise in the ground-water table, rises and falls in temperature, freezing of the surface layers, or to the peculiar properties of certain (black land) clays which, on drying out, shrink away from the surface of structures.

The usual methods of back-filling around tank walls and filter retaining walls, are to dump the previously excavated material, reserved for that purpose, into the excavation and puddle it with water. Under these conditions the materials have become more or less mixed in the process of excavating and removal for storage, which storage pile is later used for back-fill and probably removed some distance from the locality where it was excavated.

With all these conditions in mind, and with "post" reflections on completed walls, the writer, against engineering approval, has been guilty of using the unscientific method of "equivalent fluid pressure" for approximating earth thrusts on minor walls and structures. With the "equivalent fluid method," of course, considerable is left to the judgment of the engineer. Table 7 is offered in an attempt to justify the "fluid method" of approximate thrusts, using Rankine's formula for the bouyant earth pressures under complete saturation, and considering the water pressure added to the lateral earth pressures, ϕ being the angle of internal friction of the material. The head used is 6.979 ft as in the soil test apparatus of the paper, the angle, ϕ, being 58° for the lightest material and increasing 1° for each 10 lb of weight, until a maximum of 63° is reached for a weight of 120 lb of dry material. The specific gravity was assumed as 2.60, and the weight of water taken as 62.22 lb per cu ft.

It can be seen from Table 7 that, in a fully saturated soil, the resultant combined lateral pressures of earth and water are practically the same for all possible weights of soil. It is quite logical then to assume that an equivalent fluid pressure may be adopted, which will give the same results as the more

TABLE 7.—EFFECT OF VARIOUS WEIGHTS OF SATURATED SOILS ON LATERAL PRESSURES, USING RANKINE'S FORMULA WITH A VARIABLE VALUE OF ϕ.

Description	Dry weight of material, in pounds	Void in soil, percentage	Weight of contained water, in pounds	Combined weight under saturation, in pounds	Weight of earth in water, in pounds	$\frac{w}{2}$	$\tan^2$	h^2	p, due to earth pressure	P, due to water pressure	Total lateral pressures
Loose fine sand..	80	50.6	31.5	111.5	49.28	24.64	0.082	48.75	98.47	1 510	1 608.5
Coarse sand....	90	44.4	27.6	117.6	55.38	27.69	0.077	48.75	103.84	1 510	1 613.8
Graded sand....	100	38.3	23.8	123.8	61.58	30.79	0.072	48.75	108.71	1 510	1 618.7
Packed sand....	110	32.1	20.0	130.0	67.78	33.89	0.067	48.75	110.66	1 510	1 620.7
Packed sand and gravel........	120	25.9	16.1	136.1	73.88	36.94	0.062	48.75	111.35	1 510	1 621.3

refined and exact method, provided the angle of internal friction has only slight variations with different weights of soil, which are likely to become incorporated in the fill.

In this case of full saturation the equivalent fluid would have a weight of 66 lb per cu ft, and with a drained condition would have a weight of 44.2 lb per cu ft, this latter pressure being obtained by using the corrected total pressure of 1 077 lb as shown in Column (6), Table 4. The writer has been using a value of 40 lb for drained conditions, and 70 lb for a full fluid condition of back-fill, for East Texas sands and clays.

The phenomenon illustrated in Fig. 3 of decreasing pressures as the water rises up to the 1-ft mark, may be due to air binding. Water rising under a sand filter would have the effect of compressing air in the voids of the sand, and until this compressed air reached a pressure sufficient to overcome the frictional resistance of the sand grains it would not be released Under the condition of partial air compression, some uplift would occur which would be reflected in the lateral pressures as measured.

The writer agrees with the author in his "Summary" of conclusions, and is grateful for this valuable addition to the meager information on this subject, now at the disposal of practicing engineers.

JACOB FELD,[10] M. AM. SOC. C. E. (by letter).—The description of tests to determine the lateral pressure of sand and gravel with various degrees of saturation, as offered by Mr. Parsons, is clear and complete. To permit comparison with the paper and with other tests, the present remarks are divided into discussion of the apparatus, results, and conclusions.

Apparatus.—It is unfortunate that the apparatus was not equipped with provision for measuring the vertical components. Since the pressure of the water on the vertical test wall is horizontal, the effect of the soil in water at varying heights would have been more striking. The calibration of the apparatus with water cannot be questioned. It was found that the effective width of the wall for water was 4.75 ft, and it is assumed that the same applies for the soils. Such assumption cannot be accepted without further proof, especially because of the converging sides of the bin. Such convergence has no effect on water pressure, but must have a decreasing effect on soil pressure. The volume of the wedge of rupture (actual or fictitious) is reduced. The pressures on the end vertical strips of the test wall are very much less than those on the inner strips. A further test can give some valuable data; namely, to add soil to a bin full of water and determine the increase in pressure under these conditions. In this type of test, the variation in the vertical component would give important information.

Results.—The material used is described as a bank sand and gravel. The effect of water on this material is striking. When water was added, complete saturation could not occur immediately. A time interval must pass before the entrapped air is expelled. During this period the pressure decreased—the amount of decrease being less for gravel than for sand. In spite of a smaller percentage of voids, the individual interstices in the gravel were larger than in the sand, and the air could leave faster. Without complete saturation and a continuous film of water, hydrostatic pressure could not be recorded. Apparently, additional moisture content under partial saturation decreased the pressure because of an increase in the coefficient of internal resistance of the material.

---

[10] Cons. Engr., New York, N. Y.

Experiments reported by the writer[11] on sand containing from 5 to 9% moisture indicated a uniform relationship between the coefficient of internal resistance and lateral pressure. From the amount of shrinkage of loose fill (see "Pressure Tests"), the sand used in these tests could hold a considerable percentage of water. In the compacted state, as a result of flooding or ramming, the coefficient of resistance will be greater than in the loose state.

For the loose sand, the author reports a value of 40° for the angle of internal resistance, or a coefficient of 0.839. An increase of this coefficient by less than 10% to 1.00, or 45°, would mean a decrease in lateral pressure of 22½% (theoretically).

The low values for the position of the resultant pressure are caused partly by the shape of the bin, eliminating larger parts of the wedge of rupture in the upper levels than in the lower levels.

Conclusions.—The most important conclusion is that, the belief of many "practical" engineers notwithstanding, a lateral pressure greater than the hydrostatic pressure of water can occur.

The "arching" effect was entirely disproved by the tests at the University of Cincinnati[11] where identical results were obtained with rigid and free side walls. The discussion of theoretical and practical reasons for the elimination of the angle of repose from any consideration is also included in the paper reporting these tests.[11]

The point of application of the resultant pressure for consolidated fills should be above the one-third point because consolidation is equivalent to surcharge. The effects of saturation and drainage must cause internal re-adjustments in the fill—more so in sand than in gravel, with greater variation in results from theoretical values for sand than for gravel.

T. FARRANCE DAVEY,[12] ASSOC. M. AM. SOC. C. E. (by letter).—As the author states it is only by the agreement of a number of researches that actual facts become established. In like manner, it is only by the observed results of failures (which, by their implications, are likely to be obscured), that full-scale results can be established. This is the writer's justification for introducing a record of several failures that have a bearing on Mr. Parsons' paper.

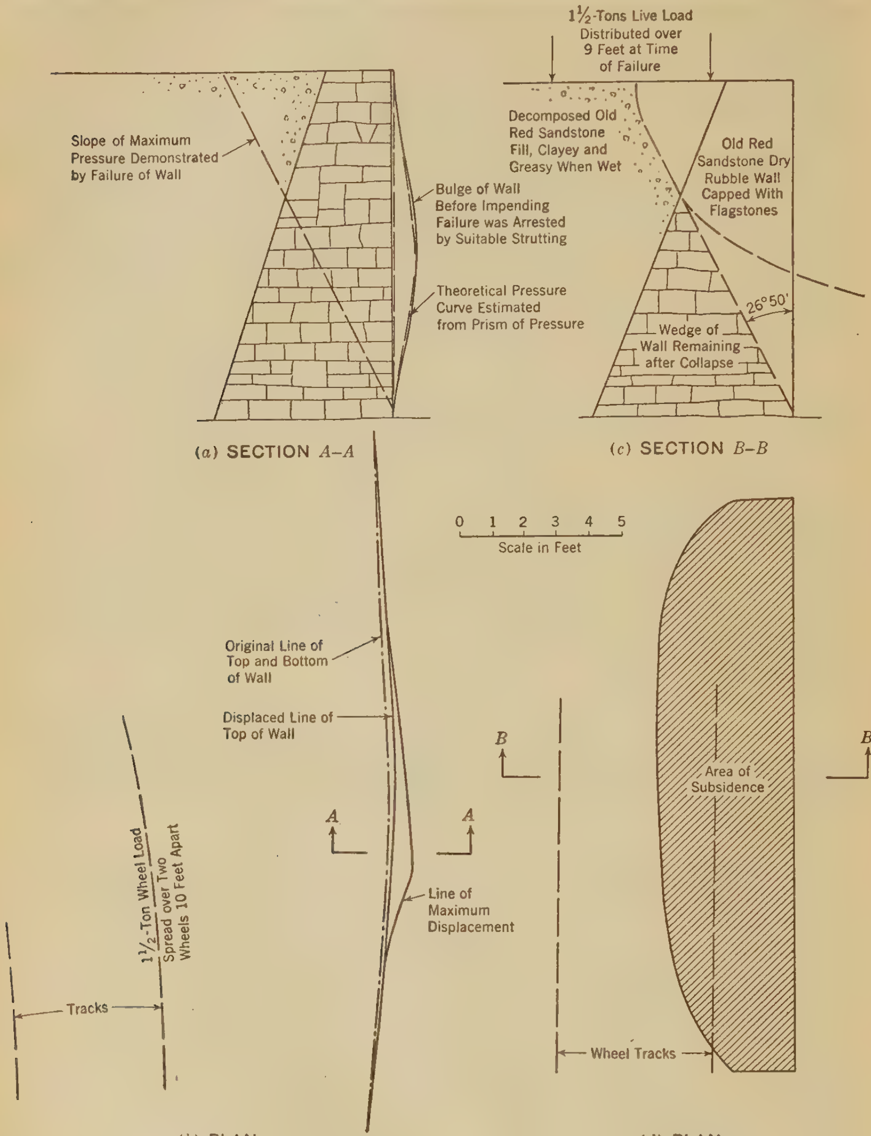
Fig. 10 refers to two sections of a dry, rubble, masonry retaining wall, about 10 ft high, built of old red sandstone and supporting a fill of old, decomposed red sandstone. The failures occurred when the material had been wetted by rain. The decomposed material had a somewhat clayey coating as had also some of the sandstone in the wall itself. The immediate cause of the failure was the superimposed moving loads, the compacting of the fill due to live load, and the vibration of this load.

---

[11] *Transactions*, Am. Soc. C. E., Vol. LXXXVI (1923), p. 1448.

[12] With City of Birmingham Elec. Supply Dept., Birmingham, England.

Fig. 10 shows the wall after it had bulged, and at the time further bulging was arrested by suitable shoring. In this instance it is interesting to compare the pressure curve obtained from the "prism of maximum pressure"[13] and


1½-Tons Live Load Distributed over 9 Feet at Time of Failure
Slope of Maximum Pressure Demonstrated by Failure of Wall
Bulge of Wall Before Impending Failure was Arrested by Suitable Strutting
Theoretical Pressure Curve Estimated from Prism of Pressure
Decomposed Old Red Sandstone Fill, Clayey and Greasy When Wet
Old Red Sandstone Dry Rubble Wall Capped With Flagstones
26°50'
Wedge of Wall Remaining after Collapse
0 1 2 3 4 5
Scale in Feet
Original Line of Top and Bottom of Wall
Displaced Line of Top of Wall
A
A
B
B
Line of Maximum Displacement
1½-Ton Wheel Load Spread over Two Wheels 10 Feet Apart
Tracks
Area of Subsidence
Wheel Tracks

(*a*) SECTION *A–A* (*c*) SECTION *B–B*

(*b*) PLAN (*d*) PLAN

FIG. 10.

the actual vertical contour of the bulged wall (Fig. 10(*a*)), although there would seem to be little mathematical relationship between them unless the wall were assumed to be a plastic body in which deformation is proportional to the load applied, and to the time in which it is applied.

---

[13] "Reinforced Concrete Construction," by George A. Hool, Vol. II, Second Edition, p. 433.

Fig. 10 (*c*), Fig. 10(*d*), and Fig. 11 show the wall in another section after it had bulged to complete failure. The conditions seem to correspond somewhat to what Mr. Parsons describes as a "drained" condition. In this instance, the surface of rupture was along a straight slope of maximum pressure (see Fig. 10(*c*)), although (due to the concentrated loads that caused the failure) the actual mass displaced was somewhat conical. This is contrary to Mr. Parsons' observation as to unsupported banks, probably because of a lack of cohesion between the large pieces of rock. The finer material behind the wall conformed with the author's observations and failed along a plane nearly perpendicular at the surface.

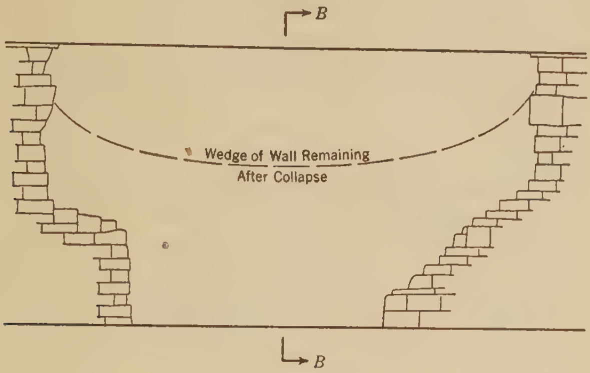
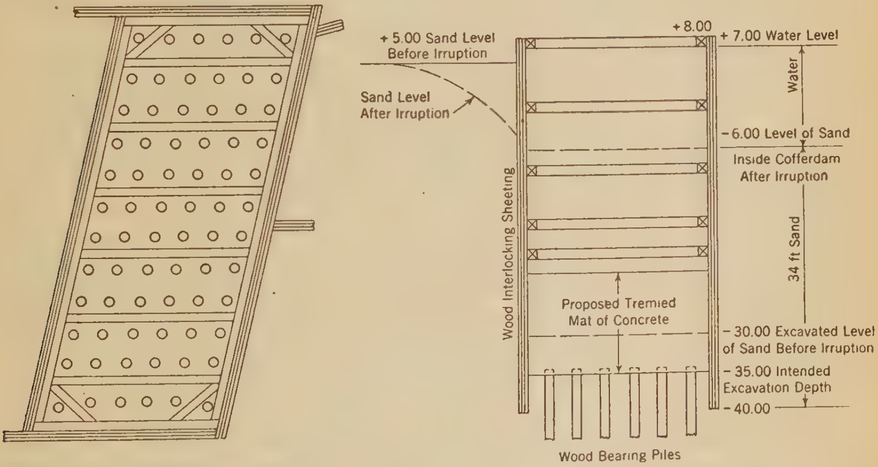

B
Wedge of Wall Remaining
After Collapse
B

FIG. 11.


+5.00 Sand Level
Before Irruption
Sand Level
After Irruption
+8.00
+7.00 Water Level
Water
-6.00 Level of Sand
Inside Cofferdam
After Irruption
34 ft Sand
Wood Interlocking Sheeting
Proposed Tremied
Mat of Concrete
-30.00 Excavated Level
of Sand Before Irruption
-35.00 Intended
Excavation Depth
-40.00
Wood Bearing Piles

(*a*) PLAN (*b*) ELEVATION

FIG. 12.

With reference to Conclusion 6 of Mr. Parsons' "Summary" the failure of the seal of an interlocked wood sheet-pile coffer-dam near Long Beach, Calif., in 1928, is of interest. Because sand of "quick" behavior had been found at near-by points, the coffer-dam (Fig. 12) was excavated by jetting and pumping with the entire site flooded. Bearing piles were driven first and were followed down to their required depths by a "dolly." When the coffer-dam had been excavated, but while it was still flooded, a number of piles had to be driven where they had previously been omitted. The result was that the seal at the bottom was broken and the sand, mixed with water, became quite liquid, and invaded the excavated site. From the final levels (which can be assumed to have been obtained while the material was effectively liquid, due to pile-driving), a simple calculation can be made of the pressure exerted vertically by a liquid sand.

Treating the inside and outside of the coffer-dam as the legs of a manometer and using X as the unit weight (pounds per cubic foot) of the fluid sand (see Fig. 12): $40\ X = 34\ X + 14 \times 62$, or, $X = 145$ lb. Possibly, the decrease in pressure on first irrigating, as reported by Mr. Parsons, was due to the sand falling toward the center of the mass during irrigation as it compacts 5% of its depth (see under the heading, "Pressure Tests"), and thus falling away from the wall.

D. P. KRYNINE,[14] M. AM. SOC. C. E. (by letter).—Notwithstanding the existence of a long series of experiments on earth pressure against retaining walls, this paper represents an able addition to the technical literature in this field. The value of Mr. Parsons' experiments is indisputable.

Behavior of Soil in Containers.—To the eight conclusions offered, the writer would like to add one more, namely: These experiments clearly show that present knowledge as to the behavior of soil in containers is extremely limited; and until a sound theory of such behavior is developed, quantitative results of smaller soil tests can be extended to larger structures only with great care.

In order to clarify this idea, it is necessary to realize that a soil mass in an experimental box possesses six faces, each of which may be considered as the surface. Hence, surface phenomena may develop on six faces, while a soil mass in practice possesses one face, or two faces, but seldom more. For instance, the soil mass under a building is to be considered as possessing one face only—the soil surface under the foundation.

The problem of how a plate acts on a soil mass in which there is only one horizontal face, is still not definitely solved, although it has become evident that the soil condition in this case depends largely on both the size and the rigidity of the acting plate. Hence, it may be realized how complicated the stress distribution is, in a box with five or six plates acting on the same soil mass. It is quite logical to believe that if, in the case of a single plate, the stress condition depends on the size and the rigidity of the acting plate, the same is true in the case of five or six plates acting simultaneously or, in other words, in the case of an experimental box.

---

[14] Research Associate in Soil Mechanics, Dept. of Civ. Eng., Yale Univ., New Haven, Conn.

Experiments show that when a plate acts on the horizontal surface of a soil mass, a disturbed zone may develop under that plate.[15] An immediate conclusion follows that a disturbed zone exists at the bottom of each experimental box filled with earth, since the bottom reaction is quite similar to a force transmitted by a plate. Perhaps disturbed zones close to the walls and the bottom of an experimental box are responsible—at least partly—for the obscure phenomenon of sand arching.

The interesting fact that there is a decrease of horizontal pressures at the beginning of irrigation, as discovered by the author, can be explained by surface phenomena as stated subsequently; but before passing to that phase of the discussion, the writer wishes to emphasize that even in a full-sized earth-pressure test in a box the influence of the walls may be so considerable as to disturb the entire picture of stress distribution. Smaller samples sometimes furnish more reliable results than larger ones as, for instance, in the case of the well-known Terzaghi compression device, in which samples should be as thin as possible.

Until a theory as to the behavior of the soil in containers is advanced, experimental apparatus should be adjusted by trial. For instance, the loose-board partition of the box in Fig. 4 should have been placed at variable distances from the bulkhead, and the scale readings compared. The correct distance would then be determined by the stabilization of the reading at subsequent movements of the partition. The width of the wall is to be determined in a similar way. It would be interesting to know how the length and the width of the experimental box (Fig. 4) were actually determined by the author.

Decrease of Horizontal Pressures at the Beginning of Irrigation.—This noteworthy fact discovered by the author is explained in the paper as the result of: (*a*) A decrease in the weight of the soil below the water level, while above the soil, the dry weight is retained; and (*b*) a reduction of the angle of internal friction in the submerged section. The angle of internal friction of sand changes only slightly under water; and the apparent loss of weight in water cannot influence the walls of the container since, in the points of contact with those walls, the mass "sand and water" develops its full weight.

The phenomenon may be explained by the action of the surface tension of the capillary water. In the beginning of irrigation, capillary water rises rapidly, preceding the gravitational water. This contributes to the partial contraction of the sand mass, connected with its partial removal from the walls and consequent decrease of pressure. In order to demonstrate this, the writer made the following experiments using small portions of dry soil powder placed in crucibles. The crucible shown in Fig. 13 is simply a porcelain container with some small perforations in the bottom. The price of a crucible is negligible and the experiments are so simple that they can easily be checked by one who does not even possess laboratory training. A little round piece of filter or blotting paper should be placed on the bottom of the crucible in order to prevent fine soil particles from dropping through.

---

[15] "Some Shear Phenomena in a Loaded Soil Mass," by D. P. Krynine, M. Am. Soc. C. E., *Civil Engineering*, Vol. 3, No. 10, October, 1933, p. 574 *et seq.*

Experiment 1.—The crucible was filled with dry fine New Haven sand (6.5% remaining on Sieve No. 100) and placed in water about $\frac{1}{4}$ in. deep. Even before the visible capillary moisture reached the top, a clearly defined fissure, $\frac{1}{50}$ in. wide, appeared between the soil mass and the walls of the crucible. The sand column was standing by itself and did not exert any lateral pressure on the walls of the crucible. This lasted $2\frac{1}{2}$ min, after which an excess of capillary moisture softened the soil column, and the fissure disappeared.

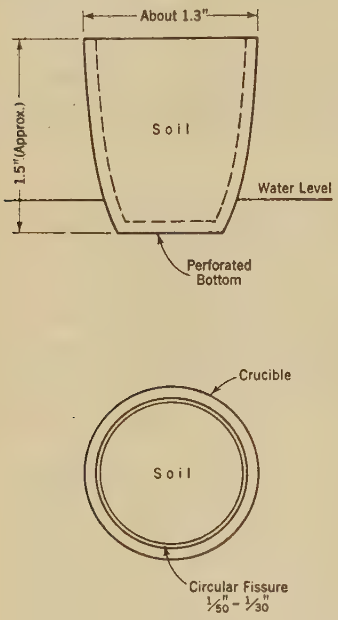

About 1.3"
1.5" (Approx.)
Soil
Water Level
Perforated Bottom
Crucible
Soil
Circular Fissure
1/50" - 1/30"

FIG. 13.

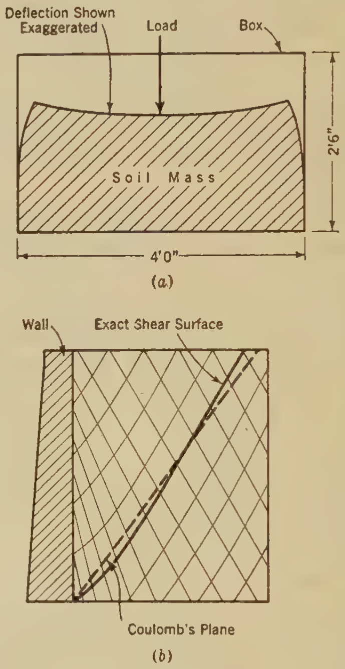

Deflection Shown Exaggerated
Load
Box
2'6"
Soil Mass
4'0"
(a)
Wall
Exact Shear Surface
Coulomb's Plane
(b)

FIG. 14.

Experiment 2.—Experiment 1 was repeated with the difference that when the fissure appeared, the crucible was removed from the water hastily so that no excess of capillary moisture was permitted to enter the soil mass. A fissure, $\frac{1}{50}$ in. wide, was clearly visible for 1 hr thereafter.

Experiment 3.—The crucible was filled with dry clay powder containing about 24% of particles smaller than 0.005 mm, with a plasticity index of about 15. A fissure, $\frac{1}{30}$ in. wide, was observed prior to the appearance of the visible capillary moisture at the top. The crucible was left in water throughout the entire experiment, and the fissure did not close for 1 hr thereafter.

Experiment 4.—The crucible was filled with Ottawa sand. No fissure could be distinguished on its moist surface owing to the coarse texture of the tested material. The crucible was removed from the water and slightly tapped with a finger. The sand mass then separated from the walls and the excess of water dropped through the bottom of the crucible. Vertical settlements of the soil mass were observed in each of the four experiments.

Basing his conclusions on these experiments, the writer believes that when the box in Fig. 4 was being filled with water, the capillary moisture was rising rapidly above the water level, thus binding the particles as described. This caused a certain decrease of pressure (A). As the water level rose, the pressure increased to a variable amount (B), due to the weight of the water. When the water level was low, Pressure A was greater than Pressure B, and the total pressure on the retaining wall decreased; but quite soon the contrary occurred ($B > A$), and the factor, A, lost its influence on the general shape of the pressure curve, although it ceased to exist only when the water level came close to the top of the experimental box. The experimental sand contained about 10%, and the gravel about 3%, of fine particles, and these were primarily responsible for the partial rigid binding of the entire soil mass.

As another proof of his belief, the writer wishes to describe a field observation made by him in the summer of 1933, in Cornwall, Conn. A wooden box, 4 ft by 4 ft by 2 ft 6 in., was partly filled with loam soil for certain measuring purposes. A heavy rain obliged him to postpone the work and to leave the filled box outside. The rain lasted an hour or more. The water drained through the soil mass, and some moisture adhered to the particles. An examination of the box 15 hr after the rain, disclosed that the particular soil had become rubber-like and obviously exerted no lateral pressure on the walls of the box. Fig. 14(a) represents the deformation of the soil mass in question under the weight of a man standing on it.

Pressure of the Drained Material.—In the drained condition water remained in the interstices for 12 hr, and the author assumed that, after this period of time, one-half the voids were still full of water. According to Mr. Parsons an error in this assumption will not cause a serious change in the results. This may be quite true when moisture distribution is more or less uniform throughout the soil mass. The writer believes, however, that the actual moisture content was insignificant close to the top and, on the contrary, that it was considerable close to the bottom of the experimental box. Several investigations along this line have been made by soil physicists. Mr. F. H. King,[16] for instance, filled several galvanized-iron cylinders, 8 ft high and 5 in. in diameter, with sand, made water percolate through the sand for 190 hr, and then kept the cylinders for 2½ yr under conditions of no surface evaporation and no addition of water. Moisture content was obtained by cutting the tubes down in 3-in. sections and removing and drying the sand at 110° C until constant weight was reached as this is usually done in laboratories.

Table 8 represents some of Mr. King's data.[17] A similar moisture distribution would perhaps mean that another method of computing the angle of internal friction for the drained material is required.

---

[16] "Principles and Conditions of the Movement of Ground-Waters," by F. H. King, U. S. Geological Survey, 19th Annual Rept., Pt. 2 (1897-98).

[17] *Loc. cit.*, p. 89.

TABLE 8

Distance from bottom of tube, in inches	MOISTURE CONTENT OF SAND (PERCENTAGES BY WEIGHT) FOR EFFECTIVE SIZES, IN MILLIMETER, OF:		Distance from bottom of tube, in inches	MOISTURE CONTENT OF SAND (PERCENTAGES BY WEIGHT) FOR EFFECTIVE SIZES, IN MILLIMETER, OF:	
	0.5	0.1		0.5	0.1
96	0.3	3.4	12	16.1	22.7
60	1.8	6.8	3	21.6	24.1
30	2.6	17.2	..	....	....

Coulomb and Rankine Formulas.—The author should be highly commended for definitely identifying both the Coulomb and the Rankine formulas. Actually, the Rankine ratio, u, between the principal stresses for the case of a horizontal ground surface, $\left(u = \frac{1 - \sin \phi}{1 + \sin \phi}\right)$, may be reduced easily to the expression:

$$u = \tan^2\left(45^\circ - \frac{\phi}{2}\right) \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

in which, ϕ is the angle of friction. Furthermore, for a ground surface that slopes at an angle, i, with the horizon:

$$u = \frac{\cos i - \sqrt{\cos^2 i - \cos^2 \phi}}{\cos i + \sqrt{\cos^2 i - \cos^2 \phi}} \ldots\ldots\ldots\ldots\ldots\ldots (9)$$

Introduce an auxiliary angle, j, which would satisfy the condition:

$$\cos j = \frac{\cos \phi}{\cos i} \ldots\ldots\ldots\ldots\ldots\ldots (10)$$

and divide both the nominator and the denominator of Equation (9) by $\cos i$. Then:

$$u = \frac{1 - \sin j}{1 + \sin j} = \tan^2\left(45^\circ - \frac{j}{2}\right) \ldots\ldots\ldots\ldots (11)$$

It should be noted, however, that the Coulomb and the Rankine formulas can be applied to the so-called "one-phase soil systems" only, such as dry cohesionless sand. They do not yield accurate results when cohesion acts in addition to friction. The angles given by the author in Table 4 are rather "angles of internal resistance."[18]

Shape of the Wedge Behind a Retaining Wall.—Apparently, Mr. Parsons admits that the theoretical wedge behind a retaining wall is bounded by a true plane. The writer understands that the existence of such a true plane was merely an assumption on the part of Coulomb, and that exact computations reveal that the wedge in question is slightly curved, even in the case of quite cohesionless sand. Such computations were made, for instance, by H.

---

[18] "Lateral Earth Pressure," by Jacob Feld, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. LXXXVI (1923), p. 1448 *et seq.*

Reissner[19] and Th. v. Kármán.[20] Complicated mathematics (omitted herein) led the latter investigator to the conclusion illustrated in Fig. 14(*b*) (for $\phi = 30°$). Obviously, such a small difference may be neglected. Besides these strictly exact solutions, there are many approximate theories of curved shearing surfaces in dry sand, without mentioning those dealing with cohesive soils. Careful experiments were made by F. Takabeya,[21] who used dry sand for this purpose and obtained shearing surfaces behind a retaining wall that were true planes, perhaps slightly curved at the bottom. Professor Takabeya also made very interesting experiments using moist sand.

CHARLES TERZAGHI,[22] M. AM. SOC. C. E. (by letter).—The essential results of Mr. Parsons' tests are assembled in Table 4, the most important being the values of $\tan^2 (45 - \frac{1}{2} \phi)$ and of ϕ (Items Nos. 5 and 6, Table 4). The values of $\tan^2 (45 - \frac{1}{2} \phi)$ are also known as "lateral pressure coefficients," because they are equal to the ratio between the measured horizontal component, H, of the lateral pressure of the back-fill and the lateral pressure exerted by a liquid, the unit weight of which, s, is equal to that of the back-fill. For a saturated back-fill the value, H, is equal to the difference between the measured lateral pressure and the full water pressure, and the unit weight, s, is equal to the unit weight of the dry back-fill reduced by the full hydrostatic uplift. In the following discussion, $\tan^2 (45 - \frac{1}{2} \phi)$ will be denoted by k.

At first, Items Nos. 5 and 6, Table 4, are startling because they indicate that the lateral pressure of the drained back-fill was more than twice as great as that exerted by the dry fill; yet, in the light of previous experience, they are exactly what one should expect to obtain from experiments performed with Mr. Parsons' apparatus.

In 1920, in his first paper on earth-pressure tests,[23] the writer called attention to the fact that the value of the lateral pressure coefficient, k, depends not only on the value of the coefficient of internal friction but, also, to a very important extent, on the distance, l, through which the wall is allowed to yield during the test. In 1929, the writer had an opportunity to verify his conclusions on a large scale, by means of the earth-pressure apparatus at the Massachusetts Institute of Technology, in Cambridge, Mass.[24] This apparatus was constructed so that the model retaining wall can be considered perfectly rigid, and its movements are measured by means of Ames dials with direct readings of 0.0001 in. Results of tests on this apparatus have been published elsewhere.[25] Fig. 15 represents a brief abstract of all those test results obtained with dry sand, which have a direct bearing on the topic of the discussion. The abscissas indicate the average distance through which the wall yielded (actual distance, in millimeters, and distance expressed as a fraction of the depth, *h*, of the back-fill), and the ordinates, the hydrostatic-pressure ratio, *k*, of the earth pressure. The solid line corresponds to an outward movement of the wall and the dotted line to a subsequent movement of the wall toward the fill. During this second part of the process, a movement of the wall through an average distance of 0.5 mm was sufficient to increase the value of *k* from 0.12 to 0.30!

---

[19] "Zum Erddruckproblem," von H. Reissner, *Sitzungsberichte* der Berliner Math. Ges., 1924, pp. 14 to 45.

[20] "Ueber elastische Grenzzustände," von Th. v. Kármán, *Proceedings*, Second International Congress of Applied Mechanics, 1926, pp. 23 to 32.

[21] "Experimental Investigations on Surface of Rupture in Sand," by F. Takabeya, *Memoirs*, Faculty of Eng., Hokkaido, Imperial Univ., Vol. 3, No. 2, October, 1932, pp. 65 to 76.

[22] Dr. Ing.; Prof., Technische Hochschule, Vienna, Austria.

[23] "Old Earth Pressure Theories and New Test Results," by Charles Terzaghi M. Am. Soc. C. E., *Engineering News-Record*, September 30, 1920.

[24] "Record Earth Pressure Testing Machine," by Charles Terzaghi, M. Am. Soc. C. E., *Engineering News-Record*, September 29, 1932.

[25] *Engineering News-Record*, February 1, 1934, p. 136, particularly Figs. 2 and 6.

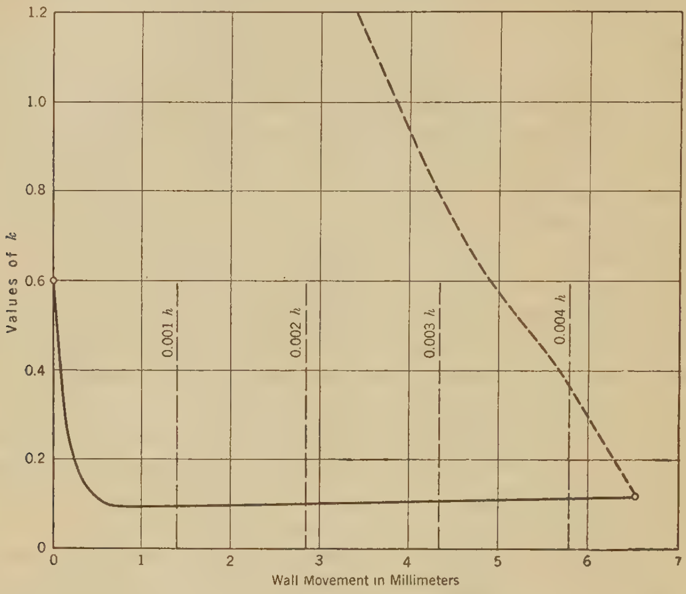


FIG. 15.

Fig. 16 represents the result of a test during which the back-fill of the wall was alternately saturated and drained. The depth of the back-fill was equal to 7.0 ft. The light solid lines show the relation between *k* and *l* for the drained condition and the light broken lines for the saturated condition. The heavy vertical lines represent the change in the value of *k* while the wall remained stationary. Thus, for instance, at Condition *d*, Fig. 16 (*a*), the value of *k* was 0.133 and the angle of wall friction, δ, was 30° 10′. Eighteen hours later (see Point *e*, Fig. 16 (*a*)), the value, *k*, had increased to 0.184, and the angle, δ, had decreased to 22°, while the wall had not changed its position. When the water was admitted into the fill, *k* assumed a still greater value of 0.225 (Point *f*, Fig. 16 (*a*)), which, during the following 21 hr increased to 0.254 (Point *g*). Subsequent drainage caused an insignificant increase of the value of *k* from 0.254 to 0.258.

Fig. 16 shows that the process of saturation invariably caused a slight increase in the value of *k*, while according to Mr. Parsons' Table 4 saturation of the back-fill caused an important decrease of *k*. On the other hand, drainage at Point *g*, Fig. 16, had no influence on the value of *k*, while the back-fill drainage in Mr. Parsons' tests caused *k* to rise to almost four times its original value.

Knowing the facts represented by Fig. 16,[26] there can be no doubt as to the cause of the facts revealed in Table 4 (Items Nos. 5 and 6). The tests by Mr. Parsons and by the writer were made with practically identical materials, the writer's sand being intermediate between the sand and gravel used by Mr. Parsons (effective size, 0.54 mm). For the test plotted in Fig. 16, the depth of the back-fill was the same as in Mr. Parsons' test; and yet the results of the writer's tests were normal in every respect, while those of Mr. Parsons exhibited the striking anomalies previously mentioned. Therefore, the cause of these anomalies must be in the arrangement of Mr. Parsons' testing apparatus. When comparing the two devices the following differences are noted: The writer's model retaining wall was made of a 5-in. reinforced concrete lining, stiffened by extra heavy 24-in. I-beams. It weighed 13 tons, while the bulkhead described by Mr. Parsons consisted of 2-in. planks, bearing against a framework of wooden stiffeners and could certainly be lifted by two men. Furthermore, the width of Mr. Parsons' bin was only 5 ft as against a width of 14 ft for the writer's bin.

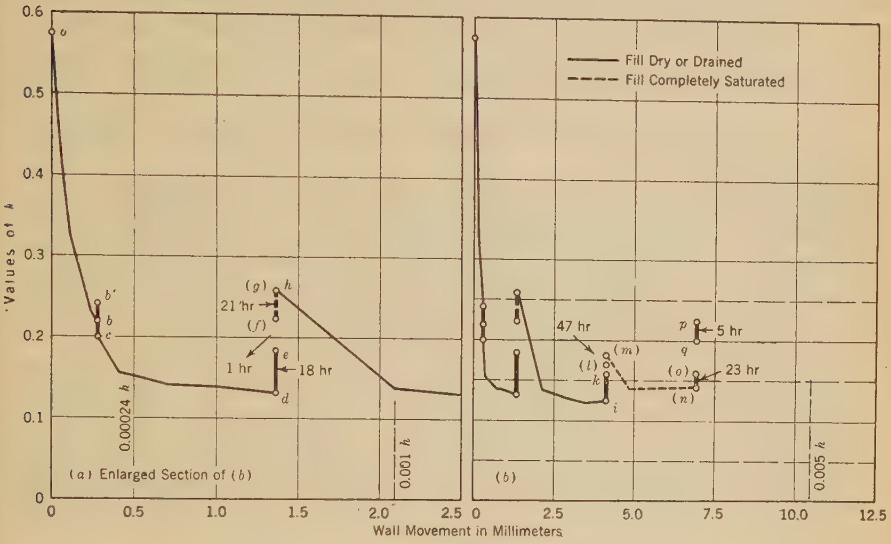


FIG. 16.

Hence, Mr. Parsons' bulkhead was very flexible if compared with the writer's model retaining wall. The relatively small width of Mr. Parsons' bin still further helped to increase the effect of the deflections of the wall on the value of the lateral pressure coefficient, *k*, because in a narrow bin a slight lateral expansion of the fill not only mobilizes the internal friction of the fill, but also the friction between the fill and the wall. Considering, in addition, the facts represented in Fig. 16, it is easy to visualize the causes of Mr. Parsons' startling results.

---

[26] See, also, *Engineering News-Record*, February 1, 1934, p. 136.

Let H (= 2 453 lb) be the horizontal pressure exerted by the dry sand back-fill; H_1 (= 7 691 lb), the horizontal pressure of the saturated sand back-fill, with water pressure included; H_2 (= 5 052 lb), the horizontal pressure of the drained sand back-fill; x, the average deflection of the bulkhead under the influence of the horizontal force, H; and, $x_1 = x\frac{H_1}{H}$ and $x_2 = x\frac{H_2}{H}$, the average deflections of the bulkhead under the influence of a lateral pressure, H_1 and H_2, respectively.

The values, x, x_1, and x_2, are supposed to represent the deflections of the bulkhead with reference to the plane through the points of support, *A*, *B*, and *C*, in Mr. Parsons' Fig. 1 (*b*). Therefore, it is proper to assume that the values, x, increase in simple proportion with the intensity of the lateral pressure. For a quantitative study of the phenomena it would also be necessary to consider, in addition, the movements of the points of support.

Since H_1 is greater than H, saturation caused an increase of the deflection of the wall and a corresponding decrease of k. On the other hand, due to H_2 being smaller than H_1, drainage caused the bulkhead to press back again toward the fill, which produced a substantial increase of the value of k. The curves in Fig. 17 are the values of $k = \tan^2 (45 - \frac{1}{2}\phi)$ from Table 4 plotted against the deflections, x, of the wall.

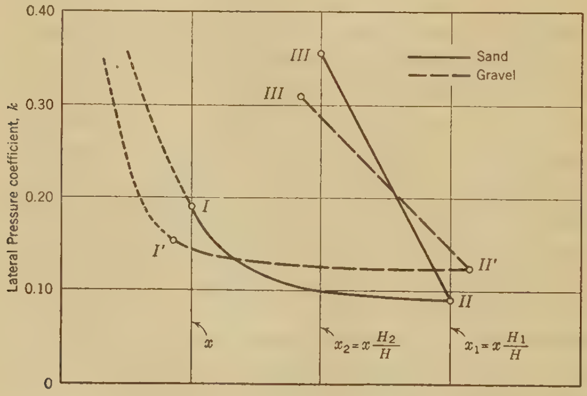


FIG. 17.

Knowing the importance of the effect of wall movements, the writer insisted on providing the aforementioned apparatus with a practically rigid wall, in spite of the expense involved. The braces and stiffeners of this wall were computed in such a way as to make certain that their deflections would have practically no effect on the intensity of the lateral pressure. The computation was based on the results of previous experiments. Unfortunately, Mr. Parson's paper does not contain the exact dimensions and the spacing of the bulkhead stiffeners required for determining the values of x in Fig. 17. For obtaining reliable data concerning this subject the writer suggests that the movements of Mr. Parsons' bulkhead be measured by means of Ames dials and that the deflections of its various structural members under the influence of water pressure, and, if possible, of the lateral pressure of a dry sand-fill, be determined likewise. A rough estimate based on the results of measuring the dimensions of the members of Mr. Parsons' retaining wall as contained in his diagrams, indicates a good agreement between the results of his tests and those of the Massachusetts Institute of Technology represented by Figs. 15 and 16.

Mr. Parsons' test arrangement for measuring the coefficient of internal friction must have been somewhat inadequate, since he obtained coefficients of internal friction of 64° 40′ and 58° 0′ against 48° 40′ and 48° 0′ for the dry materials. The latest American and European friction tests with first-class apparatus have demonstrated conclusively that the angle of internal friction for saturated sands it either equal to, or a trifle smaller than, the angle for the dry material.

The tests by Mr. Parsons have taught engineers that a repeated in-and-out movement of the wall leads to a kind of reversible state, in which the lateral earth pressure corresponding to the extreme positions of the wall becomes practically independent of the number of repetitions. This finding represents a valuable contribution to present knowledge of earth pressure.

R. L. VAUGHN,[27] M. AM. SOC. C. E. (by letter).—One important distinction made in this paper is that ϕ is the angle of internal friction, not necessarily the angle of repose. Other writers have recognized this fact in recent years, but it is one which should be emphasized. The well-known formula for horizontal earth pressure, $P = \frac{1}{2}\, w\, h^2 \tan^2 (45° - \frac{1}{2}\, \phi)$, was evolved long ago. The derivation was for an ideal granular material presenting no other internal resistance than that due to friction. For such a material it could be demonstrated that the angle of internal friction was equal to the angle of repose.

At a somewhat later date the proposition was demonstrated that when such an ideal material was submerged the same formula for horizontal earth pressure would apply, providing that, in such case, w be taken as the weight of material when submerged, in addition to which there would be full hydrostatic pressure corresponding to the given head of water. The present series of experiments marks the first attempt, of which the writer has any knowledge, to verify the last conclusion. The question is of importance in connection with the design of bulkheads and quay walls to retain submerged fills.

The author refers to ϕ as the angle of internal friction. Exception might be taken to this definition, and it is suggested that a better term would be "angle of internal resistance." It would be difficult to conceive of any manner in which the internal friction in compacted sand could be markedly increased when the sand was saturated, and it is evident that some property other than friction is brought into play by the inundation. This additional property is generally referred to as "cohesion," and it is supposed to be due to capillary action of the water in the small passages between the grains of sand. If this is the true explanation, then one might expect to find a much greater increase in internal resistance when sand was saturated than when gravel was saturated, because the water passages in the latter are much larger. The results of the author's tests tend to confirm this explanation.

---

[27] Cons. Engr., San Francisco, Calif.

If it is granted that the angle of resistance is due to a combination of friction and cohesion, it follows at once that the value of ϕ will vary with the depth of material, and the mathematics of the investigation become quite complicated. In addition, there is no experimental proof, as far as the writer knows, that either the friction or the cohesion remains constant with a varying water pressure. For instance, if the bank of sand 7 ft high, the pressure of which was measured by the author, had been resting against the base of a dam 200 ft high, the results might have been far different. It scarcely seems likely that such would have been the case, but the contrary has never been proved.

While in no wise wishing to disparage the author's results, which are valuable and interesting, the writer desires to point out that the tests are on a bank of sand only 7 ft high. They show a most marked change in the coefficient of internal resistance, for sand, upon inundation. This may be caused by the development of additional internal resistance due to capillary action. The numerical value of such capillary resistance, presumably, is limited. If this is true, its effect upon the coefficient of internal resistance would decrease as the total of such internal resistance, due to all causes, increased; that is, the additional resistance due to cohesion in the test case caused a change of 100% in the coefficient of internal resistance; but if this same amount of cohesion had been introduced into a bank of sand 100 ft high in which the true internal friction was many times as great, then upon inundation the change in the coefficient of resistance might have been relatively insignificant.

Actual experimental proof is urgently needed as to whether this relative change in the value of tan ϕ upon inundation holds good for high banks of soil, or whether as the depth becomes greater the importance of the observed effect decreases. Of even greater importance is the need for exhaustive experiments regarding the behavior of the cohesion in finer grained soils, such as silts and clays, under heavy pressures.

These questions are anything but academic. There is at present in prospect a resumption of hydraulic mining in California. Existing laws make mandatory the construction of dams to retain the débris from such operations, in order to avoid the silting up of the main rivers. There is a wide divergence of opinion among engineers as to the basis upon which such dams should be designed. One school claims that the pressure against such dams will actually be less than that due to a full head of water. Another group of engineers, in a position of some authority, insists that such dams shall be designed to withstand a pressure due to a liquid having a weight of 160 lb. per cu ft. In many cases, if not all, the decision as to whether or not mining shall be resumed will depend upon the cost of the necessary débris dam. Some of these dams may be considerably higher than 100 ft. No actual information concerning earth pressures, whether or not of saturated material, involving heights even approaching these, is known.

Direct measurements of ϕ by the method described in the paper, or by any other method, are difficult to make with any degree of accuracy. The fact that in this instance such direct measurements gave higher results than those derived from the observed pressures suggests the possibility that cohesion was present in the material when the direct measurements were taken. The effect of such cohesion would be to increase the value of tan ϕ for light loadings. Had the upper box in the measuring apparatus been weighted to give the average unit pressure that existed in the bin, the agreement between observed and derived values might have been closer.

It does not seem logical to suppose that the loss in total pressure upon partial inundation could be due to loss of weight in the submerged material, as the author suggests. It is true that when submerged the material loses weight, and that, if the value of ϕ were constant at all times, the pressure from the submerged earth would be less than that from the same earth when not submerged. However, the water of submergence exerts an additional pressure and, upon the principle that action and reaction must be equal, it would be impossible for the water, through its buoyant action, to take away more pressure than it itself added through its presence.

It is suggested that a more plausible explanation of this unexpected phenomenon may be found in the varying value of ϕ. The total pressure against the bulkhead may be regarded as made up of four parts: (*a*) The pressure of the dry earth, above the line of submergence, against that part of the bulkhead with which it comes in contact; (*b*) the pressure of the submerged earth, due to its own weight only, against that part of the bulkhead with which it comes in contact; (*c*) the pressure of the water against the submerged part of the bulkhead; and (*d*) the pressure against the submerged part of the bulkhead, exerted through the submerged earth, due to the load or surcharge upon that earth, caused by the dry earth above. The weight of this surcharge would not be the weight of the earth above, but would be that weight decreased by the amount of the friction of such earth against the face of the bulkhead, and the distribution of this imaginary surcharge would vary with the distance from the bulkhead. In computing Part (*a*) w and ϕ for dry sand would be used; for Part (*b*) the corresponding values for the inundated condition would be used; while for Part (*d*) the amount of the surcharge and its distribution would require further investigation and the value of ϕ for the submerged condition would be used. To put the idea in other words: When some of the material is submerged that portion loses weight, and there is a consequent loss in pressure. Furthermore, since ϕ is increased, the pressure due to the weight is further decreased; and, again, the weight of the unsubmerged earth above causes an additional pressure to be transmitted to the bulkhead through the submerged earth; but if the value of ϕ is increased, then the pressure so transmitted will be decreased. On the other hand, the water of submergence itself exerts a pressure against the bulkhead. Until some considerable depth of submergence is reached, the sum of the three losses may exceed the gain due to the presence of the water.

The suggestion that ϕ may show a marked and hitherto unsuspected increase on submersion, at least in the case of sands, is most interesting. If this result should be confirmed by further experiments and for greater depths, it might help to explain several observed facts which have not been accounted for in a completely satisfactory manner. Hydraulic dredge operators have long known that an otherwise deficient bulkhead could be relieved by pumping an embankment against it before completing the remainder of the fill. Just why this should be so, particularly where the material was a permeable one, such as a sand, has not been entirely clear, since before the fill could be completed the water level must be raised to the top, apparently giving a full hydrostatic head against the bulkhead in any event. The most common idea has been that the embankment first deposited had time to settle, consolidate, and develop cohesion. Undoubtedly, these factors had some effect, but the explanation has never been completely satisfactory to the writer and he has been inclined to suspect that leakage through the bulkhead, together with the well-known loss of head experienced by water flowing through earth, materially reduced the hydraulic head effective against the bulkhead. In other words, after the bulkhead had been protected by some width of earth the cracks and leaks in it served the same purpose as the weep-holes that every one knows should be placed in all retaining walls. The author's results indicate that, particularly in the case of fills which were partly submerged, an unexpectedly high value for ϕ might furnish a partial explanation. However, such a consideration could not explain how some bulkheads which evidently could not withstand a full water head alone, not to mention an additional load due to earth, could retain a dredger fill, and the writer still thinks that loss of head due to leakage is an important factor. It would be interesting to run another set of tests with the author's apparatus, leaving a number of small leaks in the bulkhead.

Most bulkhead design is an empirical affair at best. Some modern types of steel sheet-pile bulkhead are practically water-tight. If a steel bulkhead were designed so that each part of it were mechanically equal in strength to the corresponding part of some wooden bulkhead which experience had shown to be adequate, the designer might get into trouble because there was no leakage to relieve the water pressure.

Bulkheads depend on the passive resistance of the earth outside their toe for stability. Through a process of reasoning similar to that by which the pressure formula was derived, it can be shown that this resistance (sometimes called "passive pressure") is equal to: $R = \frac{1}{2}\, w\, h^2 \tan^2 (45° + \frac{1}{2}\, \phi)$. Both these formulas are not even approximately true for cases in which cohesion plays an important part, unless the pressure is measured first and then ϕ is computed to suit; in other words, if the answer is known, the formula can be "patched up." Any values of ϕ obtained by taking published "angles of repose," or by measurement of surfaces of repose, or by direct measurement of the same material under different conditions of moisture content and unit pressure, will give seriously erroneous results when substituted in either of these formulas, if cohesion is present at any time.

A number of reliable European experimental investigators have reported that the passive resistance was at least twice that given by the formula, for any sand or gravel having an angle of repose not flatter than two horizontal to one vertical. No one has ever proved that there was any inherent relation between the angle of repose and the angle of internal resistance (which governs pressures), except in some ideal case that probably never would be encountered in Nature. Furthermore, there was nothing in the accepted theory to account for this added resistance. It has seemed dangerous to rely upon such a factor in preparing a design, at least until some explanation was advanced which would indicate that the effect would be present invariably. The writer has been inclined to believe that the effect was reliable, and was due to friction on the face of the bulkhead, which would tend to increase the effective weight of the material that furnishes the passive resistance. However, in some cases at least, the effect could be explained through unsuspected variations in the value of ϕ which the author's tests indicate may be more common than might be supposed.

Comment is made in the paper concerning the slope assumed in Nature by banks of the material tested. This is by no means a rare phenomenon. It is a matter of every-day occurrence in the dredging business, in which case the sides of channels generally assume some kind of a curve, concave upward, and the upper part of the curve may approach the vertical. Such slopes bear scant resemblance to the natural side slopes not flatter than (say) two to one frequently mentioned in the specifications, and shown by straight inclined lines on the cross-sections. There have been many arguments about side slopes between dredging contractors and engineers. These curved slopes are doubtless due to the effect of cohesion or some similar property; theoretically a true granular, cohesionless material should assume a straight slope, but few materials ever do so under water.

M. Hirschthal,[28] M. Am. Soc. C. E. (by letter).—The difficulties encountered when making earth-pressure tests to obtain results that simulate actual conditions in the field are suggested in this paper describing Mr. Parsons' painstaking tests. For example, there is an utter lack of test data as a basis for studying the action or variation of earth pressure with increasing heights of fill and for different types of surcharge on such fills.

From the standpoint of the railroad engineer this particular problem is of great importance since a wide variation of conditions is encountered in railroad embankments. Observations on structures built in connection with such embankments have shown ample evidence of varying angles of repose in the same material for varying heights of embankment. Furthermore, examination of earlier structures built on the Delaware, Lackawanna and Western Railroad clearly indicates a flattening of the slopes with increase in heights of embankment such that the slopes in Table 9 have been adopted for use in the design of retaining walls of various heights, and in the determination of the dimensions of structures supporting such embankment.

---

[28] Concrete Engr., D. L. & W. R. R., Hoboken, N. J.

TABLE 9.—DESIGN DATA FOR EARTH PRESSURES

Height, in feet	$\tan^2 \frac{1}{2} (90 - \phi)$	Angle of repose	Slope
Up to 20........	0.286	33° 41′	1.0 on 1.5
20 to 30........	0.300	32° 33′	1.0 on 1.57
30 to 40........	0.315	31° 24′	1.0 on 1.64
More than 40....	0.333	30° 00′	1.0 on 1.732

It was found that even with this flattening of slope the fill would "slop" over the wing-walls, or over the parapets of supporting arch barrels, so that present practice is to make provision for a slope of 1.0 on 1.7 for all embankments between 20 ft and 40 ft, increasing the allowance for heights greater than 40 ft.

These results are bound up with the effect of sloping surcharge. The effects of both uniform and sloping surcharge have not been sufficiently analyzed and determined. It is quite certain that a surcharge of both types produces a change in the angle of "internal friction" as well as in the angle of repose of the underlying material in the course of the compaction of this material.

Mr. Parsons calls attention to the fact that the Rankine or Coulomb theory, for a fill without surcharge, yielded varying results for the angles between the horizontal and the planes of internal friction for dry soils and fully saturated soils. As far back as 1912 the writer emphasized[29] the inconsistency of results obtained by the application of either the Rankine or the Coulomb theory of earth pressures to uniform, as compared with sloping, surcharge for certain conditions.

The only reliable results obtainable as a guide for future design are to be sought (and found) through the installation of pressure-cells in actual construction, so as to register the pressure of the filling material against the retaining walls and abutments for the various conditions, including that of a surcharge that may obtain at a definite site and the resulting soil pressure on the foundations.

Engineers entrusted with such design and construction are earnestly urged to induce their clients to install these cells, in order to obtain the very valuable resulting information as a guide for subsequent design.

The question of weight of material, of course, is an interesting and important one in connection with the pressures against the structure, and no doubt the various degrees of saturation modify the results. There is an anomalous condition, however, produced by saturation, inasmuch as complete saturation may produce pressures apparently greater than hydrostatic pressures due to the greater apparent weight of the filling material than that of water; whereas the water produces an upward pressure causing buoyancy and a reduction in the weight of the filling material. This compensating effect of buoyancy would naturally reduce the resulting horizontal pressure and under certain conditions, possibly, bring it within the range of the pressure exerted by the dry material. These results will be greatly modified, of course, by the grading of the filling material.

---

[29] *Engineering News*, April 25, 1912, p. 799.

The weight of the filling material is of secondary importance in the design of retaining structures since it also plays the part of aiding the stability of the structure by means of the vertical component of the resultant pressure where the design involves a base or footing projecting into the fill, a batter in the direction of the fill, or a sloping surcharge.

Retaining walls never fail in shear through any section, and infrequently by sliding on the base. Most failures are due to excess of pressures at the toe (or abutment) resulting in possible overturning. A slight difference in the location of the resultant at the base will produce a marked difference in the maximum soil pressure at the toe, and this difference in the location of the resultant is traceable to the assumption of the angle of "internal friction" (or angle of repose) of the filling material in its action causing the overturning moment due to the horizontal thrust.

The question of internal friction is another puzzling one when it is considered that the filling material may be a uniform sand, each granule being an independent little mass more or less globular; or, it may be a rock fill laid in layers, with dirt filling the voids.

All these points indicate the need of further research on this important subject of earth or soil pressures—including foundation pressures, which are really passive resistance of the soil to superimposed pressures.

H. DE B. PARSONS,[30] M. AM. SOC. C. E. (by letter).—This research was made to study the horizontal pressure of a back-fill against a bulkhead or retaining wall. The numerical values apparently reflected a movement of the bulkhead. The results demonstrated (1) that there was a reduction in pressure at early stages of irrigation; (2) that the repetitions of irrigation and drainage cause no permanent change in the pressures; and (3) that there is a difference between internal friction and internal resistance of soil grains under wetted conditions. The second result is akin to a quay wall against which a tide rises and falls.

While compiling the test results, the writer conceived the theory that atmospheric pressure may have caused the reduction of the soil pressure when the soil was saturated. This theory will be elaborated more fully hereafter.

In Mr. Meem's box of 1-ft cube, the forces set up by tightening the bolts do not correspond with the pressures discussed in the paper. Mr. Meem evidently refers to "arching" as affecting the horizontal pressures of the tests. The repeated tests gave uniform results, and had arching existed to any marked extent, the results would have shown greater variation. Mr. Meem's idea of an apparatus having a wide width would help to avoid doubt as to arching, although increasing the expense of a test. The writer did consider a testing apparatus having water or oil rams for measuring the pressures, but he abandoned the rams because they were not as delicate as the scales for recording small differences in pressures.

---

[30] Prof. Emeritus, Rensselaer Polytechnic Inst., Troy, N. Y.; Cons. Engr., New York, N. Y. Mr. Parsons died on January 26, 1935.

Mr. Meem is correct in stating that the plane of rupture is curved, but he is in error in describing the angle of repose as between the vertical and the plane of repose. It is the angle between the horizontal and the plane of repose; and the planes of repose and of rupture are not identical.

Mr. Halmos was familiar with the manuscript of the paper, and concurred with the writer that minute possible movement of the bulkhead would not materially affect the results. Neither he nor the writer knew, at that time, the results of the experiments conducted at the Massachusetts Institute of Technology,[31] and no doubt Mr. Halmos would modify his opinion, in the light of these experiments, which showed that a slight movement of a bulkhead, or wall, decreased the active pressure.

Mr. Wilcoxen's analysis of the reduction in the pressure of the bulkhead, when the lower strata of the back-fill were immersed, is interesting; and his idea of capillarity may account for some soil phenomena. The writer believes that the irrigating water entered too fast for active capillary action to occur. Capillary water rises above gravity water by creeping between the particles due to some attractive force. Voids, left by the upward-moving capillary water, would be re-filled by water, and this water movement would be resisted by its surface tension and by friction of the water moving over the surfaces of the soil particles. This is a slow process, and capillarity requires time.

The unit weight of the sand in the test bin averaged more nearly the rodded weight of 111 lb than the rammed weight of 112.5 lb, as used by Mr. Wilcoxen. As the effective height of 6.906 ft was used, he should also have used the effective width of 4.75 ft instead of the full width of 4.926 ft from canvas to canvas.

It is of interest that Mr. Wilcoxen has referred to vacua as affecting the pressures in a mass of soil, and that Professor Terzaghi, in analyzing the Massachusetts experiments,[32] has also referred to vacua in his "suction-head", as both these ideas are analogous to the writer's vacuum theory,[33] which conceives a change due to atmospheric pressure in a soil under certain conditions, as will be mentioned more fully elsewhere.

Dr. Froehlich's eight premises are not open to argument, except that the weight of the soils as recorded were close average values rather than approximate. The "'rodded" weights, in Table 2, were considered to be closer to the average actual unit weights of the compacted materials in the testing bin, than the "rammed" unit weights, and the former were used by the writer. Dr. Froehlich considered that the bulkhead and bin walls were not rigid, and that movements, both outward and inward, took place during the tests. The side walls were strongly braced by waling-pieces and outside diagonal braces; they were always wet after the standardizing tests with water; and they had an outward thrust against them from the compacted fills, and this thrust was not removed by drainages although it was reduced. The bulkhead had a theoretical movement of 0.004 in. (0.1 mm) due to the scale attachments; and, if it did deflect, the actual movement is not known. During the tests, the writer assumed that there was no side-wall movement, and that the possible movement of the bulkhead was too minute to affect the horizontal thrusts of the back-fill materially. However, he must admit that some movement did take place and regrets that it was not actually measured. This change of opinion is due to the experiments made at the Massachusetts Institute of Technology.[31] In his discussion Professor Terzaghi has made a direct comparison with his Massachusetts experiments, and has shown the similarity between both sets of tests. The ratios found by the Massachusetts experiments are not numerically applicable to soils that differ from the cohesionless, coarse sand used in those experiments, but they do indicate the tendency.

---

[31] "Large Retaining Wall Tests", by Charles Terzaghi, M. Am. Soc. C. E., *Engineering News-Record*, February 1, and 22, 1934; March 8 and 29, 1934; and April 19, 1934.

[32] *Engineering News-Record*, March 19, 1934, p. 404.

[33] "Vacuum in Soil Mechanics", by H. de B. Parsons, M. Am. Soc. C. E., *Civil Engineering*, December, 1933, p. 690.

Dr. Froehlich believes it probable that the "passive" resistance on drainage accounts for the high values of the horizontal pressures as reported in the paper. When the water was drained, the full hydraulic pressure was released, and the bulkhead might have moved inward, causing the soil to resist being pushed back along its plane of rupture. This reverse action is known as the passive resistance, in contra to the active pressure of the soil wedge forcing the bulkhead outward.

Mr. Thackwell advocates using an "equivalent fluid pressure" method to find the approximate horizontal thrust of a soil wedge on a wall. Usually, this fluid method is based upon the assumption that the material acts like a liquid having a unit weight equal to that of the material, and that the result should be multiplied by a factor determined by experience. For fills without surcharge, this factor is the value of $\tan^2 (45° - \frac{1}{2} \phi)$, and it varies between dry, saturated, and drained conditions. In the Massachusetts experiments, Professor Terzaghi used an hydrostatic pressure ratio, namely, the horizontal component of the soil pressure divided by the horizontal pressure that would be exerted by a liquid the unit weight of which is equal to the unit weight of the back-fill. In other words, the expected soil pressure would equal "$k \times$ liquid pressure". These experiments showed how k varied for slight movements of the wall. It is possible that a series of values may be established in the future for the tangent-square, or k, corresponding to varying weights of soils, arranged according to their characteristics, uniformity, dryness, saturation, and wetness (drained), to help obtain results sufficiently accurate for ordinary purposes.

Mr. Thackwell used a depth of 6.979 ft when computing his Table 7. It would have been more consistent to have used the effective head of 6.906 ft, as he used the water pressure at the effective head in the next to the last column of his table.

The writer does not believe that air binding caused the decreasing pressures during the beginning of the irrigations. He thinks it was due to the loss of weight (buoyancy) of the submerged soil and to the increase in the angle of internal resistance of that portion of the soil which was submerged. As the irrigating water rose in the back-fill the hydrostatic pressure increased sufficiently to offset this loss.

Mr. Feld's objection to the converging sides of the bin as causing a decrease of the pressure of the soils on the bulkhead, especially at the sides, is more theoretical than actual. The writer feels that the slope given to the sides materially reduced arching effects, as shown by the consistent results of the repeating tests, and that the benefits from the slope outweighed the disadvantages.

The slope of one side was 0.5 ft in 16 ft, or 0.031 ft per ft of length. The canvas nailed to one side was 0.037 ft in thickness; therefore, the slope of one side did not narrow the back-fill as much as the canvas thickness in the first 1.2 ft as measured on the surface of the fill back from the bulkhead. As a plane of rupture slopes from the surface toward the bulkhead, the narrowing of the back-fill by the contraction of the sides was progressively decreasing from the surface downward. Furthermore, an actual plane of rupture, in a soil having cohesion, is a curved surface concave to the bulkhead; it is not a true plane, and the distance from the bulkhead to the place where a curved rupture plane cuts the surface of a back-fill is less than the distance where the theoretical true plane of rupture would cut the surface. Furthermore, the layer of sand along the side walls, equal to the canvas thickness, helped to give more accurate results than if the bulkhead had been the full width from side wall to side wall. In other words, the layer of sand along the bottom (the $\frac{1}{4}$ in. below the bottom edge of the bulkhead) and the layers of sand along the sides bounded the wedge causing the pressures by soil and not by the bottom and the sides of the bin.

The writer believes the low values (for sand especially) for the position of resultant pressure were not caused by the shape of the bin, but by a movement of the bulkhead, the effect of which was not anticipated when the apparatus was designed.

Regarding loss of pressure at early stages of irrigation, the writer agrees with Mr. Feld in his statement that "additional moisture content under partial saturation decreased the pressure because of an increase in the coefficient of internal resistance of the material".

Mr. Davey's descriptions of actual failures are most interesting. Masonry is not homogeneous, and a stone wall is not of equal strength throughout its length. A break in a retaining wall would only approximate the theoretical slip of a fill or bank. The illustrations, however, are quite representative of the active forces.

Mr. Davey suggests that possibly the decrease in pressure on first irrigating was due to the compression of the soils, thus permitting them to fall away from the bulkhead. This suggestion is not the writer's idea. The soils were in compacted states, and settlement was not noticed. If any settlement did occur, it was minute. The 5% settlement, to which Mr. Davey refers, was with loose, not compacted, sand.

The writer accepts Professor Krynine's additional conclusion that quantitative results of the "behavior of soil in containers" can be extended to large actual structures only with great care. When accurate test results are correctly interpreted, the conclusions do present knowledge for the design of engineering structures. If the mathematical analysis is correctly applied, using approximately correct data, theoretical and actual results will be close enough to be covered by the usual factor of safety. The rear partition in the bin was too far back from the rupture plane to have had any influence on the pressures on the bulkhead, and the $\frac{1}{4}$-in. layer of soil below the lower edge of the bulkhead helped to remove the "bottom face" and tended to make the tests resemble natural conditions.

The experimental box was made to the dimensions as marked (Fig. 4), which were checked by a tape. The sides were held securely to the dimensions by the tie-rods across the top and by the bottom flooring. Each foot of depth above the bottom was marked on the sides. As the bulkhead was suspended $\frac{1}{4}$ in. above the bottom, the full depth of fill (as measured on the bulkhead) was 6.979 ft. In their discussions, both Dr. Froehlich and Professor Terzaghi state that there was deflection or movement. Measurements were not made to determine deflection of the sides. The bulkhead had a possible movement of 0.004 in. (0.1 mm) due to the scale attachments, and if it did deflect, such additional movement was not measured.

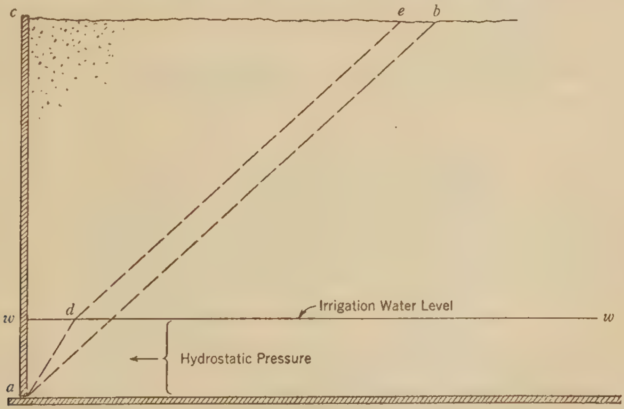


FIG. 18.

Professor Krynine's idea that surface tension of the capillary water causes the decrease of horizontal pressure at the beginning of irrigation is similar to that of Mr. Wilcoxen, mentioned previously. The tests showed that the angle of internal resistance increased when the soils were irrigated, and Fig. 18 illustrates the writer's idea of the loss of pressure at early stages of irrigation. Assuming for the sake of simplicity that the critical planes are true planes, let *ab* be the plane of rupture of the dry material; *abc*, the wedge causing pressure on the bulkhead; and let *ww* represent the level of the irrigation water at an early stage. The angle of internal resistance of the wetted material, *aw*, increased so that the critical plane of this material would be *ad*, while that of the dryer material above would be *de*. The wedge causing pressure would be *adec* smaller than the original wedge. In addition, the weight of the material in the wedge, *adw*, would be its weight in water. The bulkhead pressure would be increased by the hydrostatic pressure having a head, *aw*. As the irrigation water level rises the total pressure will increase faster than the reduction of pressure due to the soil. This reduction in pressure due to the "soil only" is shown in the horizontal distances, Fig. 3, between the curves marked "Water only" and "Irrigated". The writer has advanced the theory that the angle of internal resistance of an irrigated compacted soil is greater than its angle of internal friction, due to the action of atmospheric pressure.[33]

In the drained material, Professor Krynine believes that the water remaining was not uniformly distributed, but that the water content was more concentrated near the bottom. This is true if the material stands long enough in a drained state. The writer's assumption was made to avoid complication.

Professor Krynine is correct in stating that the calculated angles in Table 4 are angles of internal resistance, rather than of internal friction.

The writer believes that the wedge behind a retaining wall is bounded by a curved rupture plane. He has referred to a true plane only for the sake of more easily studying the pressure phenomena. In Nature, curvature of a rupture plane probably varies according to the amount of cohesion in a soil. A true plane can be conceived for simplicity as one which is an average of the curved surface.

Professor Terzaghi's discussion is of interest because he has explained many phenomena which were puzzling. The experimental apparatus at the Massachusetts Institute of Technology[31] was capable of great refinement, since it had means of measuring, closely, the movement of the wall. Professor Terzaghi's experimental wall weighed 13 tons, while the writer's bulkhead weighed about 800 lb, exclusive of the supporting rods.

The $\frac{1}{4}$-in. layer of soil below the bulkhead was arranged in order that the lower strata of soil would rest on soil and not on the floor of the bin. The effect of this layer on the results certainly tended toward accuracy.

A comparison with the results of the experiments at the Massachusetts Institute of Technology has been made by Professor Terzaghi and shows a general agreement. The Massachusetts experiments were made with artificially dried, cohesionless sand, having an effective size of 0.54 mm, and an uniformity coefficient of 1.70; while the soils in the writer's tests contained about $5\frac{1}{2}\%$ of moisture, were not cohesionless, had effective sizes of 0.25 mm for sand, and of 0.57 mm for gravel, and had uniformity coefficients of 5.8 for sand and of 21.0 for gravel.

In the Massachusetts experiments, the value of k (horizontal pressure as measured divided by the horizontal pressure of a liquid the unit weight of which is equal to the unit weight of the back-fill) became smaller as the movable wall yielded outward. The bulkhead of the writer's apparatus vibrated while the back-fill was being placed and compacted. To protect the scale bearings and knife-edges, the scale lever arms were wedged. When the bin was filled, these wedges were withdrawn, so that the scales could operate. The movement of the bulkhead was limited to 0.004 in. (or 0.1 mm) which is equivalent to 0.00005 h (in which, h = depth of fill). At the time of testing, it seemed to the writer that this movement was too small to affect the results, but, in the light of Professor Terzaghi's experiments which were made to record the effect of a movable wall, it is probable that a movement of the bulkhead did cause a reduction in the horizontal pressures and in the values of k. The movements were so slight that slips and settlements were not noticed on the soil surface in the bin.

The writer's values for k are 0.22 for loose sand, 0.20 for compacted sand, and 0.16 for compacted gravel. These values are computed from the "corrected" results given for loose sand, and for compacted sand and gravel in Column (4), Table 4, using the rodded weights. The Massachusetts experiments recorded $k = 0.36$ for its loose sand, with an outward tilting movement of the wall of 0.00005 h; and for its compacted sand, with the wall yielding outward and parallel to its original position these experiments recorded values of $k = 0.31$ at 0.00005 h; $k = 0.21$ at 0.0001 h; and $k = 0.18$ at 0.00015 h. As the bulkhead described in the paper was hung from links above (Fig. 1), any movement resembled the Massachusetts Test No. 2, in which the wall yielded parallel to its original position.

In the Massachusetts experiments, the centers of pressure were higher than in the writer's tests, but this may have resulted from the fact that the writer's bulkhead moved outward more at the bottom than at the top, whereas the wall of the Massachusetts experiments moved in the reverse direction (see Tests Nos. 1 and 3)[34], or equally at top and bottom (see Test No. 2).

According to Professor Terzaghi,[34] the horizontal pressure ratios, k, depend on the intensity of the frictional stresses that act within the wedges, and the greater these stresses the smaller is the corresponding value for k. This is the same conclusion that the writer's tests illustrated, and the effect of atmospheric pressure may offer a possible explanation, especially for wet soils, as will be explained hereafter.

Six Goldbeck cells were used by the writer, but were not mentioned in the paper, because they did not "make and break" contact sharply so as to give results sufficiently accurate for his research. As Professor Terzaghi used Goldbeck cells in the Massachusetts experiments[35], it would increase general knowledge to record the writer's experience.

The cells were in pairs, three on approximately the center line of the bulkhead and three on one side wall 15 in. back from the bulkhead. The upper pair were 2 ft below the surface of the back-fill, the middle pair, 3 ft 6 in., and the bottom pair, 6 ft. Each cell was set in a mortise and the space around it was tightly packed, making the face of the cell flush with the inner surface of the bin. The air and electric connections entered from the outside, and the pressures were recorded in inches of mercury. The cells were calibrated during the standardizing tests with water, and corrections found for each. It is possible that the corrections for water did not hold true for sand, and, also, as the cells stood vertically, the sand pressures recorded might not have been the true horizontal pressures, but diagonal pressures of an unknown degree.

---

[34] *Engineering News-Record*, February 1, 1934, p. 140.

[35] *Loc. cit.*, February 22, 1934, p. 262.

The pressures, in inches of mercury, as averaged from the records of four dry tests, ten irrigated tests, and eight drained tests of compacted sand, are given in Table 10. The pressures for "sand only" under saturated conditions were found by subtracting from the cell readings for saturated sand the readings for water under the same head as the depth of back-fill. Table 10 gives the readings for the cells on the bulkhead, and on the side wall, as mentioned. The average pressures are plotted in Fig. 19, and the pressure curves drawn. These curves reflect any movement of the writer's bulkhead which may have made the pressures on the bulkhead cells generally lower than those on the side-wall cells (see Table 10).

TABLE 10.—PRESSURE, IN INCHES OF MERCURY, DETERMINED BY GOLDBECK CELLS.

DESCRIPTION OF SAND	DRY			SATURATED SAND ONLY			DRAINED		
Cells	Bulk-head	Side wall	Average	Bulk-head	Side wall	Average	Bulk-head	Side wall	Average
Upper........	1.24	1.33	1.285	0.94	0.88	0.910	1.42	1.31	1.365
Middle.......	1.45	1.86	1.655	0.72	0.88	0.800	1.51	2.66	2.085
Bottom......	2.25	2.45	2.350	0.63	0.70	0.665	2.76	3.17	2.915

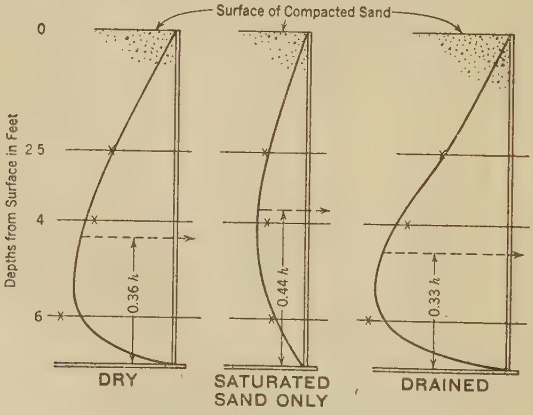


FIG. 19.—PRESSURES AS RECORDED BY GOLDBECK CELLS.

The writer recognized, before reading Mr. Vaughn's discussion, that there seems to be a difference between "internal friction" and "internal resistance", and has used the latter expression in his closure. There seems to be more than "friction" between the grains of a compacted and saturated fill before slipping can occur on a critical plane. Under conditions of a compacted fill that is saturated (not super-saturated) the voids are filled with water, and the writer's theory[33] is that the partial vacua created when the grains tend to, or do, move apart on a plane of rupture cause a portion of the atmospheric pressure to resist movements and to hold the grains from separating. The water in the voids cannot expand (like a gas) when the grains try to move away from their neighbors. However, time and the surface tension of the water may affect the results. Atmospheric pressure is similar to hydrostatic pressure, and acts through the grains of the soil where they are in intimate contact, and also on the water in the interstices between the grains. In consequence, a critical plane, to cause slipping, would have to be steeper for a saturated soil than for the same soil in dry condition; that is, the angle, ϕ, commonly called the "angle of internal resistance", would be greater for saturated than for dry conditions.

To illustrate, let MF (Fig. 20(*a*)), represent a part of a plane of rupture for a dry soil. Draw the vertical, MG, to denote the weight of the soil above M, and GF normal to the plane. Then, MF would be the shear or internal friction, just causing rupture. If the same soil is saturated and partial vacua tend to form at M when the grains above the plane of rupture try to move, a portion of the atmospheric pressure would act along MA, and MX would be its vertical component. Then, lay off XW to denote the weight above M, of the soil in water; and draw the normal, WF_1. The new shear, MF_1, would be less than that required for rupture. If the plane of rupture is made steeper (see MF_1, Fig. 20(*b*)), and MA represents the same unit value for air pressure as before, MX, its new vertical component, and XW, the weight of soil in water as before, the value of MF_1, or shear, will be equal to or greater than MF in Fig. 20(*a*), and rupture will occur. The line, HF_1, represents the horizontal pressure, and will be smaller under

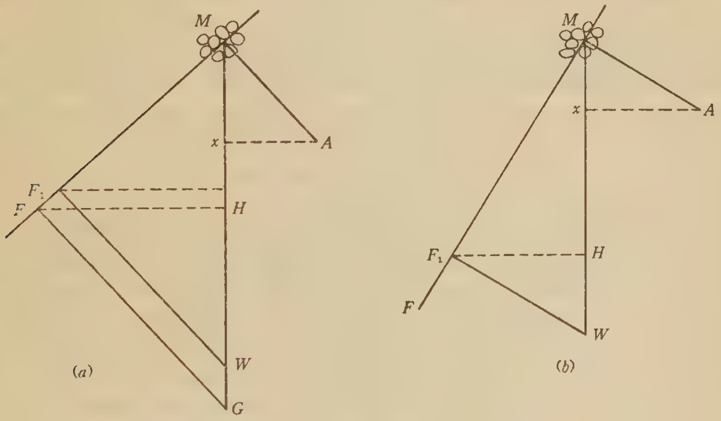


FIG. 20.

saturated conditions than *HF* under dry conditions in Fig. 20(*a*), which is the result shown by the tests.

The writer has given values to these lines from his test records, and it would appear that the air pressure under saturated conditions which would just cause rupture in Fig. 20 (*b*), would be less than one-quarter of an atmosphere.

From an inspection of Fig. 20, it is seen that, near the surface of a back-fill, the weight, *MG,* is small and the plane of rupture must be steep to cause slipping. Farther down in the fill the weight, *MG,* is greater, and the plane can be less steep to create an equal shearing force. This may explain why banks cave on curved surfaces, and why surfaces of rupture for soils are curved and not true planes.

With a saturated back-fill or deposit, against the lower part of the back of a dam which is submerged by the water in the reservoir (the voids being full of water with the grains in a compacted state and not forced apart by super-saturation), it would seem that the same conditions apply as described for a saturated back-fill, the surface of which is exposed to atmospheric pressure. The wedge of the submerged back-fill exerting pressure on the dam would be bounded by its plane of rupture. The total pressure would act with full effect through the mass and would tend to prevent the grains from separating on a normal plane of rupture causing an increase in the size of the voids. Therefore, the plane of rupture would have to be steeper before gravity could separate the grains and slip could occur. If the mass of the fill is super-saturated—that is, if the grains are separated by super-saturation—it is conceivable that the water would act as a lubricant and diminish the angle of internal resistance.

It would seem that atmospheric pressure has a more important part in many soil phenomena than has been recognized heretofore, and its action is more than a relation to capillarity. The tests show that drainages removed the "vacua" support, while subsequent irrigations restored it. The repetitions of loss and recovery of support were very consistent in the tests.

Referring to Mr. Vaughn's discussion of the reduction in horizontal pressure during the first stages of irrigation, the writer mentioned in his paper that the phenomenon was probably caused by the loss of weight of that part of the back-fill which was submerged and to a change in the angle, ϕ, due to the irrigation water. An increase in the angle of resistance would reduce the pressure from the irrigated portion of the fill, both by reduced weight of soil and size of wedge, and would also reduce the size of the wedge of the dry part above. At early stages of irrigation, these reductions might be less than the additional hydrostatic pressure, as explained in connection with Fig. 18.

Mr. Hirschthal mentioned soil-pressure cells, and the writer's results of pressures recorded by the cells used in his research are mentioned herein. In further explanation of Mr. Hirschthal's reference to internal friction, the writer calls attention to the effect of the movement of the bulkhead holding the back-fill, and to the atmsopheric pressure acting internally in the soil mass, thus increasing "internal friction" to "internal resistance". Both these subjects have been discussed herein.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 1892

WATER-BEARING MEMBERS OF ARTICULATED BUTTRESS DAMS

BY HAKAN D. BIRKE,[1] JUN. AM. SOC. C. E.

WITH DISCUSSION BY MESSRS. R. A. SUTHERLAND, P. WILHELM WERNER, CHARLES P. WILLIAMS, PAUL BAUMANN, A. FLORIS, AND HAKAN D. BIRKE.

SYNOPSIS

This paper presents a scientific method of determining mathematically, the most economical proportions of the water-bearing members of the articulated type of buttress dam.

INTRODUCTION

Wide buttress spacings combined with massive masonry sections are improvements in buttress dam design that have accompanied the recent trend toward the use of dams of great height. Greater safety and economy have been attained through these improvements, and through the more scientific methods of design that have been developed at the same time. At first, the use of a wide buttress spacing was confined to the multiple-arch and multiple-dome types of dam. However, the unsuitability of these types to faulted, or otherwise poor, foundations has led to the improvements of the well-known flexible, "articulated," type of dam, using thick, widely spaced buttresses instead of the relatively thin, closely spaced buttresses that have been common to this type. Fortunately, the characteristics of massiveness and wide spacing have been found to be as economical for articulated dams as they were for the rigid types and, in addition, the flexibility of this type makes it suitable for sites with poor foundations, such as are frequently encountered, and for those subjected to earthquake shocks or other foundation movements.

The use of thick buttresses and wide spacing has introduced new problems in the design of the water-bearing members or decks and the haunches. Two types of flexible water-bearing members are suitable for use in connection with long spans, the first of which (Fig. 1(*b*)), consists of a gradually

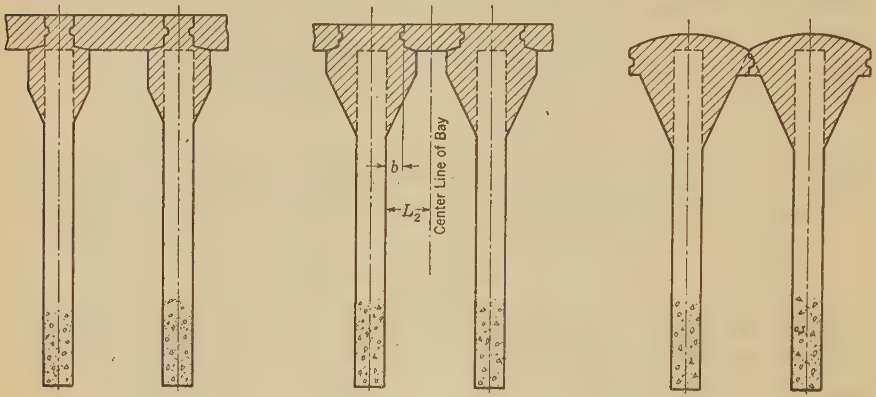
---

NOTE.—Published in September, 1933, *Proceedings*.

[1] Designing Engr., Ambursen Dam Co., New York, N. Y.

flaring buttress section connected with a short reinforced concrete slab[2]; and the other (Fig. 1(c)), is the round-head type.[3] For purposes of comparison a standard Ambursen type deck is shown in Fig. 1(a). It will be noted (Fig. 1(b)) that for the dimension, $b = 0$, this type will be identical with the original type shown in Fig. 1(a), and for $b = L_2$ (see Fig. 1(b)), the deck will be replaced entirely by the enlarged haunches shown in (Fig. 1(c)).

Heretofore, the economic limits of each of the three types shown in Fig. 1 have been determined by the tedious method of trial designs. It is a matter of considerable economic importance to determine the correct relations between length of span, bearing surface of slab, depth of haunch, slope of haunch, the height at which the deck slab may be omitted, and the height at which the flared buttress sections may be brought in contact to form a round-head type of water-bearing member. In order to facilitate the economic design of these flexible water-bearing members, equations and curves have been developed, through which the relations outlined in the foregoing are easily and quickly obtained.



(a) (b) (c)

FIG. 1.

NOTATION

The following symbols adopted for use in this paper, are listed for the guidance of discussers:

a = width of haunch bearing (Fig. 2).

b = one-half the difference between tongue and buttress thickness (Fig. 1(b)).

c = one-half the buttress thickness (Fig. 4(b)).

d = effective depth: d_1, at the outer face; and d_2, the depth of deck (Fig. 4(b)).

e = (a ratio, multiplied by 100) the increase in percentage of steel due to bent-up bars and hooks: e_1, haunch steel; and e_2, deck steel (Equation (34)).

---

[2] "Recent Advances in Buttress Type Dams," by Calvin V. Davis, M. Am. Soc. C. E., *Civil Engineering*, February, 1931.

[3] "The Design and Construction of Dams," by Edward Wegmann, M. Am. Soc. C. E.

f = allowable unit stress: f_c, in concrete; and f_s, in steel (Equation (35)).
i = depth of cover for steel: i_1, in haunch; and i_2, in deck (Equation (18)).
j = factor of distribution of shear: j_1, in haunch; and j_2, in deck.
p = unit loading: p_0, maximum on haunch, from the deck (Equation (1)).
s = tangent of the slope of the under side of the haunch on sections normal to the deck (Fig. 2).
x = variable distance along an X-axis.
A = substitution factor, with appropriate subscript (Equation (8)).
B = substitution factor, with appropriate subscript (Equation (25)).
C = total cost, in dollars: C_c, of concrete in deck and haunch; C_h, of steel in haunch; C_d, of steel in deck; C_t, of concrete and steel in deck and haunch.
D = unit cost: D_c, of concrete, in dollars per unit volume; D'_c, of concrete, in dollars per cubic yard; D_s, of steel, in dollars per unit volume; D'_s, of steel, in dollars per pound.
E = modulus of elasticity.
F = substitution factor, with appropriate subscript.
G = substitution factor, with appropriate subscript.
H = depth below maximum high-water level to the under side of the deck.
J = substitution factor, with appropriate subscript.
K = substitution factor, with appropriate subscript.
L = one-half span: L_1 = one-half the clear deck span; and L_2, one-half the clear buttress spacing.
M = bending moment.
N = substitution factor, with appropriate subscript.
Q = substitution factor, with appropriate subscript.
R = area of steel: R_1 = area required for a given bending moment.
V = volume: V_c, of concrete in deck and haunch; V_h, of steel in haunch; and V_d, of steel in deck.
W = loading on deck per unit area.
τ = allowable shear in haunch.
μ = coefficient of friction.
ν = allowable shear in deck; $\nu_{max.}$ = maximum shear.
σ = principal stress: σ_1 equals first principal stress in buttress.

The symbols for costs and volumes of materials in haunch and deck are given for one-half a bay, and per unit length of deck and haunch (see Fig. 4).

RELATION OF HAUNCH DIMENSIONS TO LOADING DISTRIBUTION AND HAUNCH SLOPE

The haunch may be considered as a cantilever beam with length, a (see Fig. 2). The loading from the deck is assumed to be triangularly distributed with a maximum value of p_0. The effective depth of the haunch at the outside face is equal to d_1 and the cover for the steel is equal to i_1. As the length of this beam will be small compared with the depth, the maximum allowable shear will govern the dimensions. The location of the section in the haunch where the shear is maximum must be determined, and the haunch designed considering the shear at this section. For example, consider the part of the haunch between the plane, *A-A* (Fig. 2), and the outer face of the haunch.

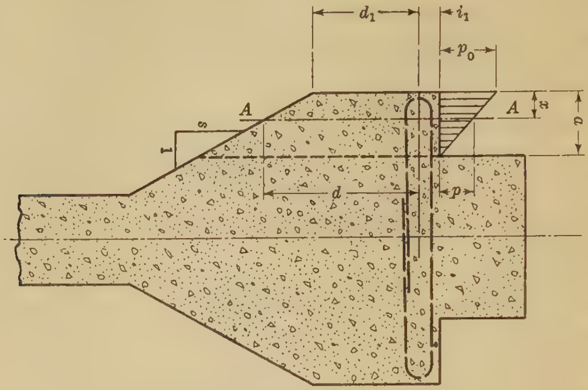


FIG. 2.—NORMAL SECTION THROUGH HAUNCH.

Let τ = shear along the plane, *A-A*; p = unit loading at the plane, *A-A*; and d = effective depth of haunch at the plane, *A-A*. Then,

$$p = p_o - \frac{x}{a}\, p_o \ldots\ldots (1)$$

and,

$$d = d_1 + sx \ldots\ldots (2)$$

Summing up the vertical forces for the element of haunch limited by the plane, *A-A,* will give Equation (3):

$$\tfrac{1}{2}\,(p + p_o)\, x - j_1\, \tau\, d = 0 \ldots\ldots (3)$$

Combining Equations (1), (2), and (3) and re-arranging,

$$j_1\, \tau\, d_1 = x\, (p_o - \tau\, j_1\, s) - \tfrac{1}{2}\, p_o\, \frac{x^2}{a} \ldots\ldots (4)$$

The distance, x, should be determined so that the depth, d_1, becomes a maximum. Therefore,

$$\frac{\partial d_1}{\partial x} = 0 \ldots\ldots (5)$$

From Equations (4) and (5), by differentiating and solving for x,

$$x = \left(1 - \frac{j_1\, \tau\, s}{p_o}\right) a \ldots\ldots (6)$$

Substituting this value of x in Equation (4) and solving for d_1 will give,

$$d_1 = \tfrac{1}{2}\, \frac{j_1\, \tau}{p_o} \left(\frac{p_o}{j_1\, \tau} - s\right)^2 a = Aa \ldots\ldots (7)$$

Cancelling a in Equation (7):

$$2\, \frac{p_o}{j_1\, \tau}\, A = \left(\frac{p_o}{j_1\, \tau} - s\right)^2 \ldots\ldots (8)$$

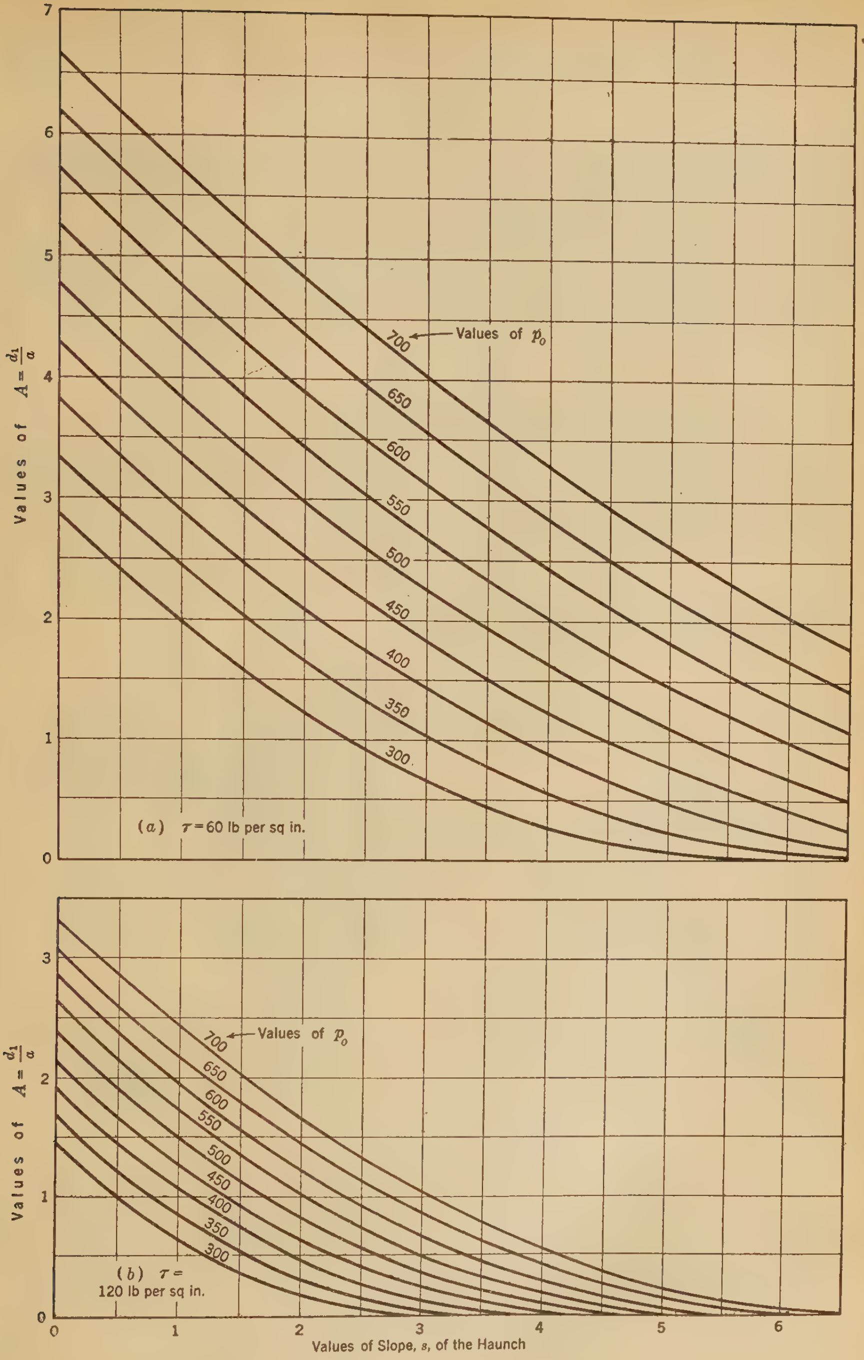


FIG. 3.—VALUES OF A IN EQUATION (8).

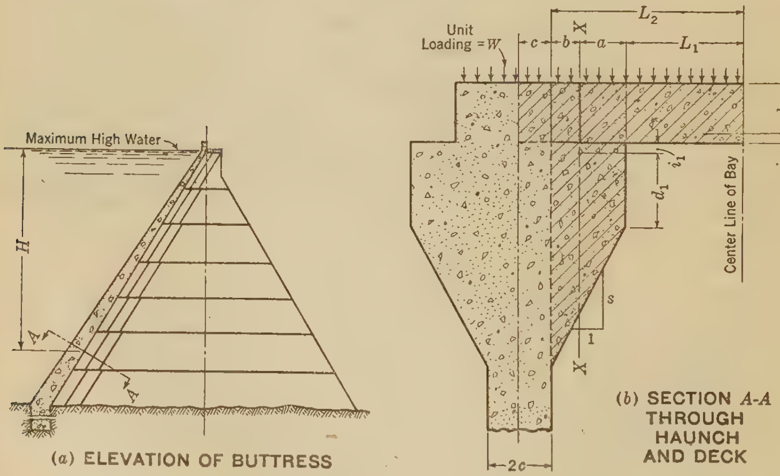
For p_0, j_1, and τ being constants, and A and s being variables, Equation (8) will represent analytically a parabola with its top in the point, $s = \frac{p_0}{j_1 \tau}$ and $A = 0$.

In Fig. 3, Equation (8) is represented in curves for two cases of allowable shear in the haunch, namely, $\tau = 60$ lb per sq in., and $\tau = 120$ lb per sq in. When the maximum bearing pressure, p_0, is known, the factor, A, may be found conveniently from these charts for different values of shear, τ, and haunch slope, s. The factor, j_1, for shear distribution has been assumed constant at $j_1 = 0.875$. When the value of A is known, the dimensions of the haunch are determined by the aid of $d_1 = Aa$ (Equation (7)).

Economical Relations of Deck and Haunch Dimensions to Height of Dam

In the following analysis of the economic proportions of water-bearing members, only the costs of the concrete and steel will be considered because, with the same buttress spacing, the cost of forms will remain practically constant for the three types under discussion.

It will be convenient to introduce the distance, b (Fig. 4), as a variable and then express other variable dimensions as functions of b. The total volume of concrete and steel may be expressed as functions of b, and so may also the total costs of these elements. When the total cost, C_t, is expressed as a function of b, the problem will be confined to the determination of the value of b that will make C_t a minimum; that is,



(a) ELEVATION OF BUTTRESS

(b) SECTION A-A THROUGH HAUNCH AND DECK

FIG. 4.

$$\frac{\partial C_t}{\partial b} = 0 \quad \text{.............................} \quad (9)$$

Let V_c = the volume of concrete in deck and haunch; D_c, the cost of concrete per unit volume; and C_c the total cost of concrete in deck and haunch. Then,

$$C_c = D_cV_c \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

In the same manner, the cost of the steel in the haunch will be expressed by the equation,

$$C_h = D_sV_h \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (11)$$

in which, V_h = the volume of steel in the haunch; D_s, the cost per unit volume of steel; and C_h, the cost of steel in the haunch. For the deck steel, the equation will be:

$$C_d = D_sV_d \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (12)$$

in which, V_d = the volume of steel in the deck; and C_d, the cost of the steel in the deck. The total cost, C_t, of concrete and steel in haunch and deck will be determined by the equation,

$$C_t = C_c + C_h + C_d = D_cV_c + D_s\,(V_h + V_d) \ldots\ldots (13)$$

Applying Equation (9) to Equation (13),

$$D_c\,\frac{\partial V_c}{\partial b} + D_s\left(\frac{\partial V_h}{\partial b} + \frac{\partial V_d}{\partial b}\right) = 0 \ldots\ldots\ldots\ldots (14)$$

SOLUTION OF EQUATION (14)

There will be two general cases of analysis: Case I, in which shear governs the thickness of the deck; and Case II, in which bending governs the thickness of the deck.

Case I: Determination of $\dfrac{\partial V_c}{\partial b}$.—The volume of concrete in haunch and deck per unit of length (see hatched area in Fig. 4(*b*)) = V_c:

$$V_c = (L_2 + c)\,i_2 + (L_2 + c)\,d_2 + (a + b)\,i_1 + (a + b)\,d_1 + \tfrac{1}{2}\,(a + b)^2\,s \,..\, (15)$$

Let,

$$i_1 + d_1 = K_1\,d_1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (16)$$

then, Equation (15) becomes,

$$V_c = (L_2 + c)\,i_2 + (L_2 + c)\,d_2 + (a + b)\,K_1\,d_1 + \tfrac{1}{2}\,(a + b)^2 s \,..\, (17)$$

The width of the haunch bearing, a, is determined by the equation,

$$a = \frac{2\,W\,(L_2 - b)}{p_0} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (18)$$

One-half the clear deck span is determined by the equation,

$$L_1 = L_2 - (a + b) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (19)$$

Substituting the value of a (Equation (18)), in Equation (19),

$$L_1 = \frac{(L_2 - b)\,(p_0 - 2\,W)}{p_0} \ldots\ldots\ldots\ldots\ldots\ldots (20)$$

The effective depth, d_2, of the deck as required for maximum shear, ν, is determined by the usual equation,

$$d_2 = \frac{W L_1}{\nu\, j_2} = \frac{W}{\nu\, j_2}\; \frac{(L_2 - b)\;(p_0 - 2\,W)}{p_0} \qquad \ldots\ldots\ldots\ldots\ldots (21)$$

The effective depth, d_1, of the haunch is determined by Equation (7), thus $d_1 = A a$. Substituting the value of a (Equation (18)), in Equation (7) will give, as a result, the equation,

$$d_1 = A\, \frac{2\,W\,(L_2 - b)}{p_0} \qquad \ldots\ldots\ldots\ldots\ldots (22)$$

Combining Equations (18), (20), (21), and (22) with Equation (17), and re-arranging,

$$V_c = \left(1 + \frac{c}{L_2}\right) i_2\, L_2 + (p_0 - 2\,W)\, \frac{W}{p_0^2} \left[B + G\, \frac{b}{L_2} - \tfrac{1}{2}\, J\, \frac{b^2}{L_2^2} \right] L_2^2 \ldots (23)$$

in which,

$$B = \frac{4\,K_1\,A + 2\,s}{p_0 - 2\,W}\; W + \frac{p_0}{\nu_{\text{max.}}\; j_2} \left(1 + \frac{c}{L_2}\right) \qquad \ldots\ldots\ldots\ldots (24)$$

$$G = 2\,K_1\,A\; \frac{p_0 - 4\,W}{p_0 - 2\,W} + 2\,s - \frac{1 + \dfrac{c}{L_2}}{j_2}\; \frac{p_0}{\nu_{\text{max.}}} \ldots\ldots\ldots\ldots (25)$$

and,

$$J = 4\,K_1\,A - s\left(\frac{p_0}{W} - 2\right) \qquad \ldots\ldots\ldots\ldots\ldots (26)$$

Differentiating V_c with respect to b will result in the equation,

$$\frac{\partial V_c}{\partial b} = (p_0 - 2\,W)\, \frac{W}{p_0^2} \left[G - J\, \frac{b}{L_2} \right] L_2 \ldots\ldots\ldots\ldots (27)$$

Case I: Determination of $\dfrac{\partial V_h}{\partial b}$.—Let M equal the bending moment in the haunch at Section X-X (Fig. 4(b)), and d, the effective depth of the haunch at Section X-X; then,

$$M = (L_2 - b)\, W\, \frac{2\,a}{3} \qquad \ldots\ldots\ldots\ldots\ldots (28)$$

and,

$$d = d_1 + a\, s \qquad \ldots\ldots\ldots\ldots\ldots (29)$$

or, by substituting the value for d_1 from Equation (7), $d = a\,(A + s)$.

Let R_1 equal the steel area required for the bending moment,

$$R_1 = \frac{2}{3}\; \frac{(L_2 - b)\; W}{(A + s)\; j_1\, f_s} \qquad \ldots\ldots\ldots\ldots\ldots (30)$$

in which, j_1 is the distribution factor for shear, and f_s, the allowable tensile stress in steel. (For other symbols, see Fig. 4.)

For movements in the deck, caused by temperature changes, etc., a pull will be exerted on the haunch due to friction between the deck and the haunch surfaces. If μ is the coefficient of friction between deck and haunch and if R_2 is the steel area required for this pull, the following equation results for the steel area required,

$$R_2 = \frac{\mu\,(L_2 - b)\,W}{f_s} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (31)$$

Let R_h equal the total steel area required in the haunch for bending moment and pull from the deck. Then,

$$R_h = R_1 + R_2 = \frac{L_2 - b}{f_s}\,W\left[\frac{2}{3\,j_1\,(A + s)} + \mu\right] \ldots\ldots\ldots\ldots (32)$$

Now the volume of the haunch steel is determined by the equation,

$$V_h = (1 + e_1)\,(a + b + c)\,\frac{L_2 - b}{f_s}\,W\left[\frac{2}{3\,j_1\,(A + s)} + \mu\right] \ldots\ldots (33)$$

in which, e_1 is a factor for the increase in steel due to hooks, shear bars, etc., in the haunch. Let,

$$N = (1 + e_1)\,\frac{p_0}{f_s}\left[\frac{2}{3\,j_1\,(A + s)} + \mu\right] \ldots\ldots\ldots\ldots\ldots\ldots (34)$$

Introducing the value of a in Equation (33), it may be written,

$$V_h = (p_0 - 2\,W)\,\frac{W}{p_0^2}\left[\frac{p_0\,\dfrac{c}{L_2} + 2\,W}{p_0 - 2\,W} + \frac{p_0\left(1 - \dfrac{c}{L_2}\right) - 4\,W}{p_0 - 2\,W}\,\frac{b}{L_2} - \frac{b^2}{L_2^2}\right] N\,L_2^2 \,..(35)$$

Differentiating V_h with respect to b:

$$\frac{\partial V_h}{\partial b} = (p_0 - 2\,W)\,\frac{W}{p_0^2}\left[\frac{p_0\left(1 - \dfrac{c}{L_2}\right) - 4\,W}{p_0 - 2\,W} - 2\,\frac{b}{L_2}\right] N\,L_2 \ldots\ldots (36)$$

Case I: Determination of $\dfrac{\partial V_d}{\partial b}$.—The bending moment in the center of the deck span is expressed by,

$$M = \frac{1}{8}\,W\,[2\,(L_2 - b) - a]^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (37)$$

and the steel area required for this bending moment is,

$$R_d = \frac{W\,[2\,(L_2 - b) - a]^2}{8\,j_2\,d_2\,f_s} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (38)$$

Introducing values for a and d_2 from Equations (18) and (21), and simplifying:

$$R_d = \frac{1}{2\,f_s}\,\frac{\nu}{p_0}\,\frac{(p_0 - W)^2}{p_0 - 2\,W}\,(L_2 - b) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (39)$$

Let V_d equal the volume of steel in the deck slab; then,

$$V_d = \frac{1}{2}\,(p_0 - 2\,W)\,\frac{W}{p_0^2}\,F\,(L_2 - b)^2 \ldots\ldots\ldots\ldots\ldots\ldots (40)$$

in which,

$$F = (1 + e_2)\,\frac{\nu}{f_s}\,\frac{p_0}{W}\left(\frac{p_0 - W}{p_0 - 2\,W}\right)^2 \ldots\ldots\ldots\ldots\ldots\ldots (41)$$

and e_2 is the factor for increase in steel due to hooks and bent-up bars in the deck slab.

Differentiating Equation (40) with respect to b:

$$\frac{\partial V_d}{\partial b} = -\,(p_0 - 2\,W)\,\frac{W}{p_0^2}\,F\left(1 - \frac{b}{L_2}\right)L_2 \ldots\ldots\ldots\ldots (42)$$

Introducing the values of $\frac{\partial V_c}{\partial b}$, $\frac{\partial V_h}{\partial b}$, and $\frac{\partial V_d}{\partial b}$ from Equations (27), (36) and (42) into Equation (14), will result in the following equation:

$$(p_0 - 2\,W)\,\frac{W}{p_0^2}\left[D_c\left(G - J\,\frac{b}{L_2}\right) + D_s\left(N\,\frac{p_0\left(1 - \frac{c}{L_2}\right) - 4\,W}{p_0 - 2\,W} - 2\,N\,\frac{b}{L_2} + F\,\frac{b}{L_2} - F\right)\right]L_2 = 0 \ldots\ldots\ldots\ldots\ldots\ldots (43)$$

From the expression within the square brackets, b, may be solved, thus:

$$b = \frac{G + \left(\frac{p_0\left(1 - \frac{c}{L_2}\right) - 4\,W}{p_0 - 2\,W}\,N - F\right)\frac{D_s}{D_c}}{J + (2\,N - F)\,\frac{D_s}{D_c}}\,L_2 \ldots\ldots\ldots\ldots (44)$$

Case II.—Determination of $\frac{\partial V_c}{\partial b}$.—Let L_b equal the bending span for the deck slab. Then,

$$L_b = 2\left(L_1 + \frac{a}{2}\right) = \frac{2}{p_0}\,(p_0 - W)\,(L_2 - b) \ldots\ldots\ldots (45)$$

The maximum bending moment for the deck is expressed by,

$$M = \frac{1}{2}\,\frac{W}{p_0^2}\,(p_0 - W)^2\,(L_2 - b)^2 \ldots\ldots\ldots\ldots\ldots\ldots (46)$$

The deck thickness, d_2, may now be determined from the usual formula,

$$M = K\,b\,d_2^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (47)$$

Combining Equations (46) and (47) and solving for d_2 will result in the equation,

$$d_2 = \frac{p_0 - W}{p_0}\,\sqrt{\frac{W}{2\,K}}\,(L_2 - b) \ldots\ldots\ldots\ldots\ldots\ldots (48)$$

Introducing this value of d_2 in Equation (17) will result in an equation identical with Equation (23), but in which:

$$B' = \frac{W}{p_0 - 2\,W}\left[4\,K_1\,A + 2\,s + (p_0 - W)\,\frac{p_0}{W^2}\sqrt{\frac{W}{2\,K}}\left(1 + \frac{c}{L_2}\right)\right] \ldots (49)$$

and,

$$G' = -\frac{p_0 - W}{p_0 - 2W}\,\frac{p_0}{W}\sqrt{\frac{W}{2K}}\left(1 + \frac{c}{L_2}\right) + 2K_1\;A\;\frac{p_0 - 4W}{p_0 - 2W} + 2s\ldots(50)$$

Differentiating V_c with respect to b as before, will result in an equation identical with Equation (27), but in which the values of B', G', and J, in Equations (49), (50), and (26), respectively, are substituted. The volume of steel in the haunch will have the same value as before under Case I.

Case II.—Determination of $\frac{\partial V_d}{\partial b}$.—The steel area in the deck required for bending moment will be,

$$R_d = \frac{1}{j_2 f_s}\,\frac{p_0 - W}{p_0}\sqrt{\frac{WK}{2}}\,(L_2 - b)\ldots\ldots\ldots\ldots\ldots(51)$$

The volume of the steel in the deck will then have the value,

$$V_d = \tfrac{1}{2}\,(p_0 - 2W)\,\frac{W}{p^2_0}\,F'\,(L_2 - b)^2\ldots\ldots\ldots\ldots\ldots(52)$$

in which,

$$F' = 2\,(1 + e_2)\,\frac{p_0 - W}{p_0 - 2W}\,\frac{p_0}{W}\sqrt{\frac{W}{2K}}\,\frac{K}{j_2 f_s}\ldots\ldots\ldots\ldots\ldots(53)$$

Differentiating:

$$\frac{\partial V_d}{\partial b} = -\,(p_0 - 2W)\,\frac{W}{p^2_0}\,F'\,(L_2 - b)\ldots\ldots\ldots\ldots\ldots(54)$$

Substituting these values for $\frac{\partial V_c}{\partial b}$, $\frac{\partial V_h}{\partial b}$, and $\frac{\partial V_d}{\partial b}$, in Equation (14) and solving for b, an equation identical to Equation (44) is derived, but in which F' from Equation (53) is substituted.

Equation (44) determines the value of b that will result in the most economical design. It will be noted, that for a constant head the ratio, $\frac{b}{L_2}$, will be independent of the clear buttress spacing. Therefore, for a certain set of stresses and values of the coefficients, this ratio may be solved as a function of the height, H, and then used for any buttress spacing desired.

In the following is presented a typical example, illustrating the solution of Equation (44) for varying heights of the dam and when using values of stresses and coefficients that are customary for this type of structure.

The following stresses and values of the coefficients have been used: $p_0 = 600$ lb per sq in.; $f_c = 650$ lb per sq in.; $f_s = 16\,000$ lb per sq in.; $\tau = 120$ lb per sq in.; $\nu = 120$ lb per sq in.; $\sigma_1 = 500$ lb per sq in.; $K = 107.7$; $j_1 = 0.875$; $j_2 = 0.875$; $\mu = 0.33$; $e_1 = 0.5$; $e_2 = 0.2$; $K_1 = 1.1$; and $\frac{D'_s}{D'_c} = 0.005$, in which, D'_s is the cost of steel, in dollars per pound, and D'_c is the cost of concrete, in dollars per cubic yard. Then $\frac{D_s}{D_c} = 13\,200\,\frac{D'_s}{D'_c} = 66.$

The loading on the deck, W, in pounds per square inch, may be written,

$$W = 0.434\ H \quad \text{(55)}$$

in which, H is the height, in feet, from the under side of the deck to maximum high-water level. For a general case, the buttress thickness, $2c$, may be determined approximately from the equation,

$$(L_2 + c)\ W = c\sigma_1 \quad \text{(56)}$$

in which, σ_1 is the first principal stress in the buttress. Solving for $\frac{c}{L_2}$ there results,

$$\frac{c}{L_2} = \frac{W}{\sigma_1 - W} \quad \text{(57)}$$

The maximum allowable bending stresses will determine the deck thickness at the top of the dam. The height of dam below which shear governs the deck thickness may be determined by combining Equations (21) and (48), which will result in the equation,

$$\frac{W}{\nu j_2}\ \frac{(L_2 - b)\ (p_0 - 2\ W)}{p_0} = \frac{p_0 - W}{p_0}\sqrt{\frac{W}{2\ K}}\ (L_2 - b) \quad \text{(58)}$$

This height does not depend upon the span as the factor, $L_2 - b$, will disappear. Solving for W and remembering that $W = 0.434H$, the value of the height below which the shear governs the deck thickness will be for the conditions assumed, namely, $H = 155$ ft. Therefore, b may be determined from Equation (44), with substitution factors, G, J, N, and F, for $H < 155$ ft, and from Equation (44) for $H > 155$ ft, with substitution factors, G', F', J, and N. The results obtained by solving Equation (44) are shown graphically by Fig. 5, for five values of s. For a certain slope of the under side of the haunch, these curves will give directly the most economical ratio of $\frac{b}{L_2}$.

Fig. 5 shows how the ratio, $\frac{b}{L_2}$, for a constant haunch slope is increasing with the head. This indicates that the use of enlarged haunches and shortened deck will be more economical as the height of the dam increases until, finally, below a certain height the best economy will be obtained by omitting the deck entirely and using the round-head type. These limiting heights, below which the round-head type is the most economical, are indicated by horizontal dashed lines in Fig. 5.

The equation for total cost, C_t, of haunch and deck will be of the second degree with respect to b. Therefore, it will have only one minimum and C_t will increase continuously if the value of b is departing from that determined by Equation (44), or as shown by Fig. 5. Thus, if for practical reasons this value of b cannot be used, the best economy will be attained by having the actual value of b approach as nearly as possible the value of b determined from the equations.

A comparison of the cost of the water-bearing members for different values of the ratio, $\frac{b}{L_2}$, would indicate the saving effected by using values as determined by the equations. It may be done by introducing the values, V_c, V_h, and V_d, in Equation (13) from which the total cost, C_t, may be solved under the different assumptions. This equation will be of the form,

$$C_t = \left(1 + \frac{c}{L_2}\right) i_2 D_c L_2 + f\left(\frac{b}{L_2}\right) L_2^2 \quad \ldots\ldots\ldots\ldots\ldots (59)$$

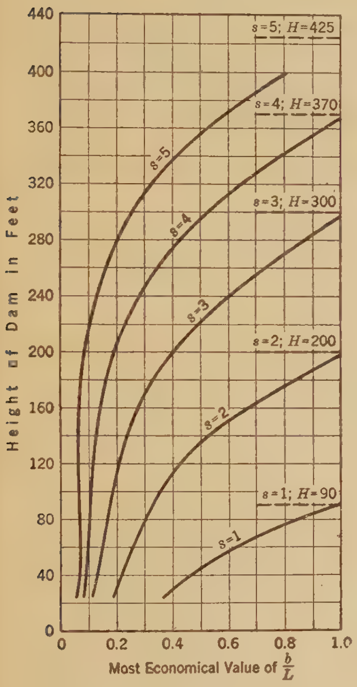


FIG. 5.—MOST ECONOMICAL VALUE OF $\frac{b}{L_2}$.

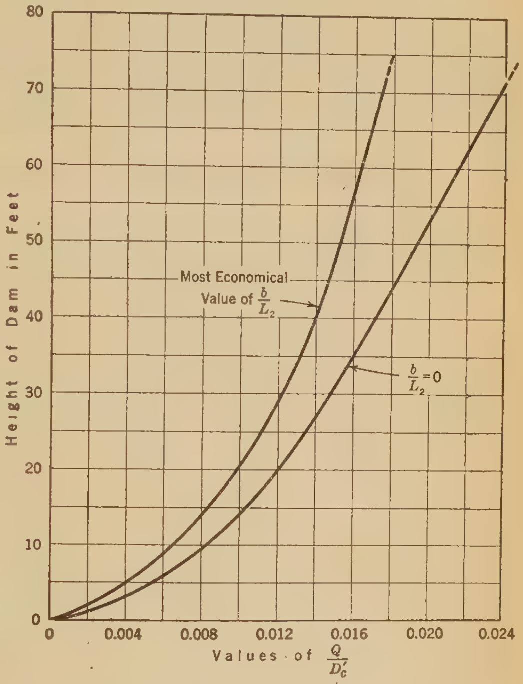


FIG. 6.—VALUES OF Q FOR $s = 1$.

The first term for the value of C_t will be small in comparison with the second one; also, it will be independent of the ratio, $\frac{b}{L_2}$, and, therefore, it may be omitted for this comparison; hence,

$$C_t = f\left(\frac{b}{L_2}\right) L_2^2 = Q L_2^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots (60)$$

Fig. 6 shows the values of Q for $s = 1$, the values of Q being for the following values of the ratio, $\frac{b}{L_2}$: First, $\frac{b}{L_2} = 0$; that is, the tongue has the same thickness as the buttress (Fig. (1(a)); and, second, $\frac{b}{L_2}$ as determined from Equation (44). This coefficient will give the minimum cost when introduced in Equation (60). Curves similar to Fig. 6 for other values of s can be prepared as the need occurs.

When the cost of concrete, D'_c (in dollars per cubic yard), is known, the coefficient, Q, is determined by the curves in Fig. 6. Introducing this value of Q in Equation (60), this equation will give the cost, C_t, directly in dollars per foot length of deck and haunch for any span, L_2, desired (L_2 = one-half the clear buttress spacing, in feet). Therefore, the curves in Fig. 6 will indicate directly the savings effected by economical proportioning of the deck and haunch dimensions, as outlined in this paper. Table 1 shows, for different heights and haunch slopes, the magnitudes of these savings in percentage of the cost of the water-bearing members for the case, $\frac{b}{L_2} = 0$. These savings are given as a percentage of the cost of the water-bearing members. The cost, C_t, of these members is a function of the square of the buttress spacing (see Equation (60)), and when the buttress spacing is small, say, 15 to 20 ft, this cost will be relatively small; but as the buttress spacing increases, the cost C_t, of the deck and haunches will increase rapidly (in proportion to the square of the buttress spacing), and will be an important factor in the total cost of the dam. Therefore, in the case of wide buttress spacing, the savings in cost of the water-bearing members made possible by proportioning them according to the procedure outlined in this paper will materially affect the total cost of the dam.

TABLE 1.—SAVINGS EFFECTED BY ECONOMICAL PROPORTIONS OF DECK AND HAUNCH DIMENSIONS (IN PERCENTAGE OF COST OF WATER-BEARING MEMBERS, FOR THE CASE, $\frac{b}{L_2} = 0$).

Haunch slope, s	PERCENTAGE SAVINGS CORRESPONDING TO HEIGHTS, H, IN FEET							
	24	48	72	96	120	144	192	240
1............	17	20	27	...	...	.....	...	....
3............	5.3	5.2	5.0	5.2	5.2	5.4	8.1	12.8

CONCLUSIONS

The equations developed in this paper are sufficient for the precise determination of the most economical proportions of the decks and haunches of articulated buttress dams. The most economical relation between the span of the deck and the clear buttress spacing is shown to be independent of the buttress spacing for a given height of dam. Furthermore, the quantities of concrete and steel in the decks and haunches, as well as the total cost of these members, are shown to be in direct proportion to the square of the buttress spacing. The examples clearly indicate that for dams of great height for which the buttress spacing generally is large, the savings effected by the proportioning as outlined are of major economical importance.

Although the mathematical developments may seem formidable the results as presented in the final simplified equations, and curves may be applied easily and quickly to actual design problems. Fortunately, the flexible, articulated buttress dam is subject to an exact mathematical analysis of both economy and stresses, as there are no statically indeterminate elements introducing mathematical and structural uncertainties.

DISCUSSION

R. A. SUTHERLAND,[4] ASSOC. M. AM. SOC. C. E. (by letter).—Articulated buttress dams are applicable to a wide variety of foundation conditions, showing in many cases greater economy and in almost all cases greater security than gravity dams. In certain cases, the choice of this type may be indicated by considerations not entirely economic (such, for example, as the likelihood of earth movements), and it is in such cases that the value of the author's results is greatest, in that they then enable the parts of the structure to be proportioned in the best manner.

In other cases, a comparison between the buttress type of dam and other types (in particular the gravity dam), may be the deciding consideration, and the writer recently had occasion to make such a comparison. He was impressed by the fact that the shape of the dam site largely influences the relative merits of the two types on the score of economy. In Fig. 5, certain heights are shown concerning which the author states "the best economy will be obtained by omitting the deck entirely and using the round-head type." The writer will confine his remarks to a comparison between a round-head type of dam and a gravity dam. The dimensions of the round-head type for comparison are shown in Fig. 7.[5]

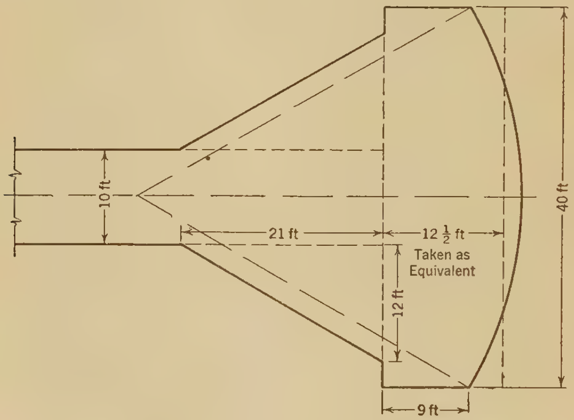


FIG. 7.—SECTION OF ROUND HEAD AT CREST LEVEL.

In Table 2 one buttress and round head is compared with a 40-ft slice of a gravity dam with total batter equal to 0.8. The batter of the buttress is 0.6 both up stream and down stream; the crest thickness is 10 ft, increasing at the rate of 1 ft for every 30 ft of height. In both cases, no consideration is given to additional material in the crest of dam, it being assumed that the extra material in the crest of the gravity dam will cost about the same as the roadway, etc., on the crest of the buttress dam. The volume of the haunches and the buttress proper has been computed by the prismoidal rule.

---

[4] Berkeley, Calif.

[5] *Engineering News-Record*, February 11, 1932, p. 212, Fig. 3.

It will be seen that if the height of the dam were uniform throughout, economy of material is shown only for structures higher than 95 ft, and economy of cost only for even greater heights. No dam, however, is of uniform height throughout, and economy in the entire dam will depend greatly on whether a large part of it is of a considerable height, or whether it spans a relatively narrow gulch with long flanks of lower height; in other words, economy will depend on the value of n as used by the writer elsewhere[6] as an index of the shape of the site. Using a procedure described elsewhere[6] the writer was able to develop a simple expression to determine the height above which the buttress dam showed a given degree of economy, as compared with a gravity dam. However, this was only for the assumption of parallel-sided buttresses, as the consideration of tapered buttresses led to expressions which are not readily integrable.

TABLE 2.—COMPARISON OF BUTTRESS AND ROUND-HEAD DAM, WITH A 40-FOOT SLICE OF A GRAVITY DAM

Height of buttress, in feet	Volume of buttress, etc., in cubic feet	Volume of gravity slice, in cubic feet
30	31 260	14 400
60	74 080	57 600
90	130 700	129 600
120	202 200	230 400

On grounds similar to the foregoing comparison with a gravity dam, the writer would suggest that the limits shown in Fig. 5 (limits above which the round-head type should be used), while perfectly legitimate in considering an individual buttress, should be modified in considering the entire dam, for the reason that if the round head is decided on, the entire dam must generally be of that type. The round-head buttress does not lend itself to combination with the slab type, nor with a gravity dam. The writer would be glad to know whether the author has considered the desirability of reducing buttress spacing toward the flanks of the dam, particularly with the round-head type. With this type there seems to be no inherent reason for a uniform spacing, if economical considerations dictate otherwise.

P. WILHELM WERNER,[7] ASSOC. M. AM. SOC. C. E. (by letter).—The economic principles involved in this study are of importance for structures other than buttress dams. The author has assumed the factor of distribution of shear, j, to be constant. It is questionable, however, whether this is true for the haunches, the depth, d, of which is great in relation to their span, a. Investigations made by the writer seem to indicate that for such beams the shear distribution is dependent to a considerable degree, on the ratio, $\frac{d}{a}$, although sufficient investigations are not yet available to give a definite quantitative answer.

---

[6] "Some Aspects of Water Conservation," *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 157.

[7] Civ. Engr., Stockholm, Sweden.

In his determination of $\frac{\partial V_h}{\partial b}$, Mr. Birke has included in the cost of the haunch a steel area, R_2, for the pull due to the friction between the deck and the haunch surfaces to allow for a possible movement in the deck (Equation (31)). As far as the writer is aware, no corresponding steel area is included in the cost of the deck (Equation (39)), where it is evident that there must be a reactive pull of the same magnitude due to the pull in the haunches. Probably, there is some reason for omitting the corresponding steel in the deck, and, if so, it would be interesting to know it.

Mr. Birke has investigated the relative economy of the deck and the haunches, without going into the economy of each member separately, except to use stresses that are customary for the type of structures under consideration. It is well known that, in a slab in flexure, there is a certain economic relation between the steel and the concrete. It would seem logical in this connection to consider this relation, especially in view of the fact that for the type of structures in question the space available for the depth of the deck is not restricted by influences other than those of economy. The point may also be of interest in view of the present trend toward revising standard specifications for maximum allowable stresses in reinforced concrete.

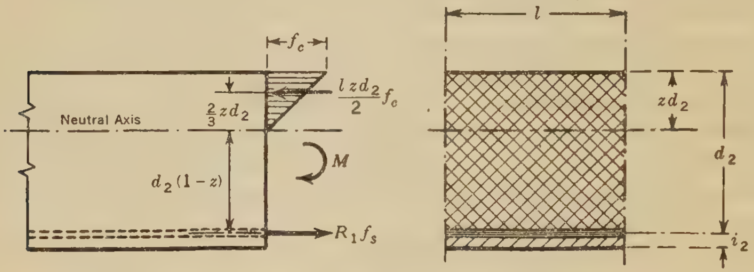


FIG. 8.

Considering the slab shown in Fig. 8, and using the nomenclature of the paper as far as possible, the following well-known relation is obtained,

$$\frac{f_s}{f_c} = \frac{E_s}{E_c} \frac{1 - z}{z} \qquad \text{.........................(61)}$$

or, with $\frac{f_s}{f_c} = \beta$ and $\frac{E_s}{E_c} = n$,

$$\beta = n \frac{1 - z}{z} \qquad \text{.........................(62)}$$

The equilibrium condition for the moments gives, $M = z\,(3 - z)\,\frac{l\,d_2^2}{6}\,f_c$; from which,

$$d_2 = \sqrt{\frac{6\,M}{l\,f_c}}\;\frac{1}{\sqrt{z\,(3 - z)}} \qquad \text{....................(63)}$$

The equilibrium condition for horizontal forces gives, $\frac{l\,z\,d_2}{2}\,f_c = R_1\,f_s$; from which,

$$R_1 = \frac{l\,z\,d_2}{2\,\beta} \quad \dots\dots\dots\dots\dots\dots\dots\dots (64)$$

The cost of the concrete, C_{cd}, is determined by Equation (65) (the volume occupied by the steel and the steel cover, i_2, which is practically constant, are not taken into account):

$$C_{cd} = d_2\,l\,L\,D_c \dots\dots\dots\dots\dots\dots\dots\dots (65)$$

and the cost of the steel, C_{sd}, by the equation:

$$C_{sd} = (1 + e_2)\,\frac{z\,d_2}{2\,\beta}\,l\,L\,D_s \dots\dots\dots\dots\dots\dots (66)$$

Thus, the total cost, C_{td}, of the slab, except the cost of the forms and other practically constant items, is:

$$C_{td} = C_{cd} + C_{sd} = d_2\left[1 + (1 + e_2)\,\frac{D_s}{D_c}\,\frac{z}{2\,\beta}\right] l\,L\,D_c \dots\dots\dots (67)$$

Letting,

$$\kappa = \frac{1 + e_2}{2\,n}\,\frac{D_s}{D_c} \quad \dots\dots\dots\dots\dots\dots\dots\dots (68)$$

and substituting the values of β (Equation (62)), and of d_2 (Equation (63)), in Equation (67):

$$C_{td} = \frac{1}{\sqrt{z\,(3 - z)}}\left(1 + \kappa\,\frac{z^2}{1 - z}\right)\sqrt{\frac{6\,M}{l\,f_c}}\,l\,L\,D_c \dots\dots\dots (69)$$

or, if $Z = \frac{1}{\sqrt{z\,(3 - z)}}\left(1 + \kappa\,\frac{z^2}{1 - z}\right)$.

$$C_{td} = Z\,\sqrt{\frac{6\,M}{l\,f_c}}\,l\,L\,D_c \quad \dots\dots\dots\dots\dots\dots\dots\dots (70)$$

Equation (70) is evaluated in Fig. 9, which shows the cost of the slab for various ratios, $\frac{D_s}{D_c}$, at constant value of f_c.

The value of z that will make C_{td} a minimum is controlled by the condition,

$$\frac{\partial C_{td}}{\partial z} = 0 \quad \dots\dots\dots\dots\dots\dots\dots\dots (71)$$

By differentiating the expression for C_{td} in Equation (69) (in which, f_c is assumed constant), substituting in Equation (71), and reducing, the following equation is obtained:

$$(5\,\kappa - 2)\,z^3 + (7 - 9\,\kappa)\,z^2 - 8\,z + 3 = 0 \quad \dots\dots\dots\dots (72)$$

For $e = 0.2$ and $n = 15$, Equation (68) becomes,

$$\kappa = 0.04\,\frac{D_s}{D_c} \quad \dots\dots\dots\dots\dots\dots\dots\dots (73)$$

For different values of the ratio, $\frac{D_s}{D_c}$, the most economical ratio, $\beta = \frac{f_s}{f_c}$, at constant f_c, can now be computed by solving Equation (72) for z and substituting these values of z in Equation (62). The result is shown in Fig. 10, for values of the ratio, $\frac{D_s}{D_c}$, between 20 and 100, which probably represent practical limits of the ratio between the costs of steel and concrete.

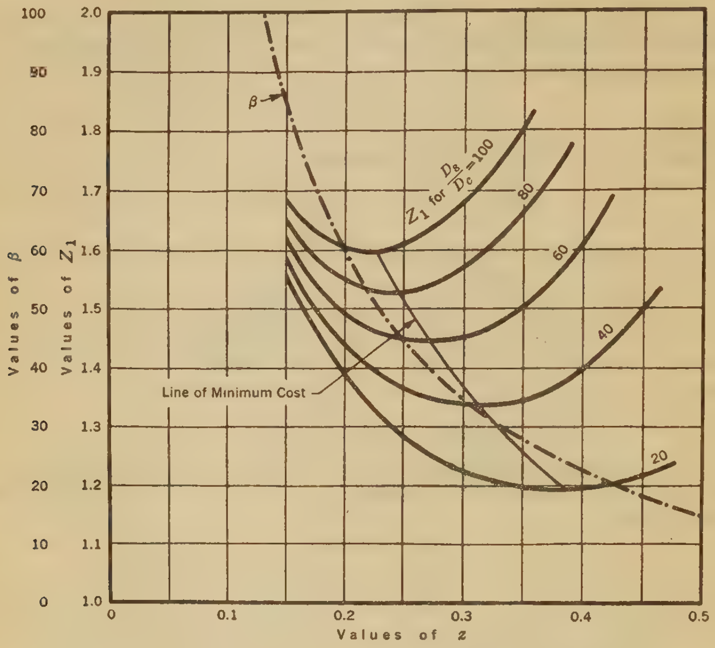


FIG. 9.—COST OF DECK SLAB IN FLEXURE FOR VARIOUS RATIOS OF $\frac{D_s}{D_c}$ AT CONSTANT f_c

Taking $\frac{D_s}{D_c} = 60$, for instance, it is seen from Fig. 9 that the most economical ratio between the stresses, f_s and f_c, is $\beta = 40$. With a concrete stress of $f_c = 650$ lb per sq in., as used by the author, the steel stress would then be $f_s = 24\,000$ lb per sq in. For $\frac{D_s}{D_c} = 40$, the corresponding values would be $\beta = 34$ and $f_s = 22\,000$ lb per sq in. These values exceed, greatly, the allowable steel stress of 16 000 lb per sq in. used, and are in any case out of the question in structures of the type under consideration, especially in view of the necessity of avoiding cracks in the concrete. It is, therefore, of greater interest in this connection to investigate the problem under the assumption of a constant steel stress.

By substituting the value of f_c (Equation (62)), in Equation (69), C_{td} becomes,

$$C_{td} = \frac{1}{z}\sqrt{\frac{1-z}{3-z}}\left(1+\kappa\,\frac{z^2}{1-z}\right)\sqrt{\frac{6\,M}{l\,f_s}}\;n\,l\,L\,D_c \quad \text{........(74)}$$

in which, $\frac{1}{z}\sqrt{\frac{1-z}{3-z}}\left(1+\kappa\,\frac{z^2}{1-z}\right) = Z$

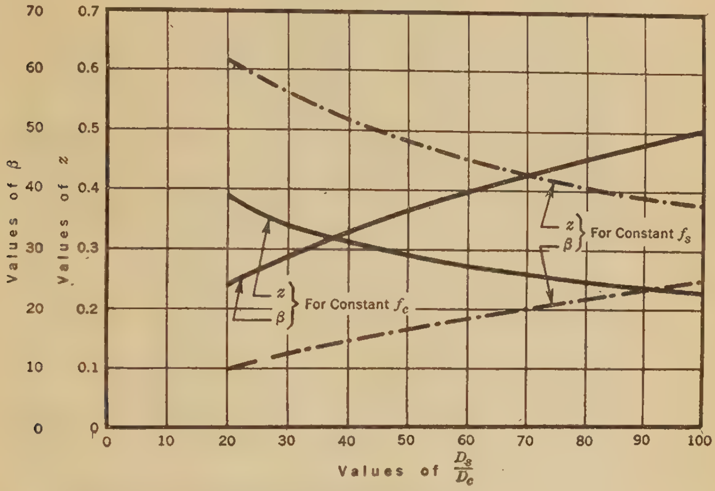


FIG. 10.—ECONOMIC DIMENSION AND STRESS RELATION OF DECK SLAB IN FLEXURE AT CONSTANT f_c OR CONSTANT f_s

This equation is evaluated in Fig. 11, showing the cost of the slab for various ratios, $\frac{D_s}{D_c}$, at constant f_s.

The value of z that will make C_{td} in Equation (74) a minimum is obtained under the condition expressed by Equation (71). By differentiating the expression for C_{td} in Equation (74) (in which, f_s is assumed constant), and substituting in Equation (71), and reducing, the following equation is obtained:

$$(1 - 2\,\kappa)\,z^3 + (3\,\kappa - 4)\,z^2 + 6\,z - 3 = 0 \quad \text{............(75)}$$

Using the value of κ given in Equation (73) the most economical ratio, $\beta = \frac{f_s}{f_c}$, at constant f_s, can now be obtained by solving Equation (75) for z and substituting the resultant values of z in Equation (62). The result is shown by dash-dotted lines in Fig. 10.

In the typical example introduced by Mr. Birke, with $f_s = 16\,000$ and $f_c = 650$ lb per sq in., the value of β is $\beta = 24.6$. Thus, for $\frac{D_s}{D_c} = 66$, the point, A, in Fig. 11, is obtained. The minimum cost is obtained for $\beta = 20$, corresponding to the point, B, in the diagram. It is seen that the values of the stresses used by the author very nearly give the best economy with the ratio, $\frac{D_s}{D_c} = 66$. It should be noted that using a lower value of β than about 20 (Point *B*, Fig. 11) will only increase the cost.

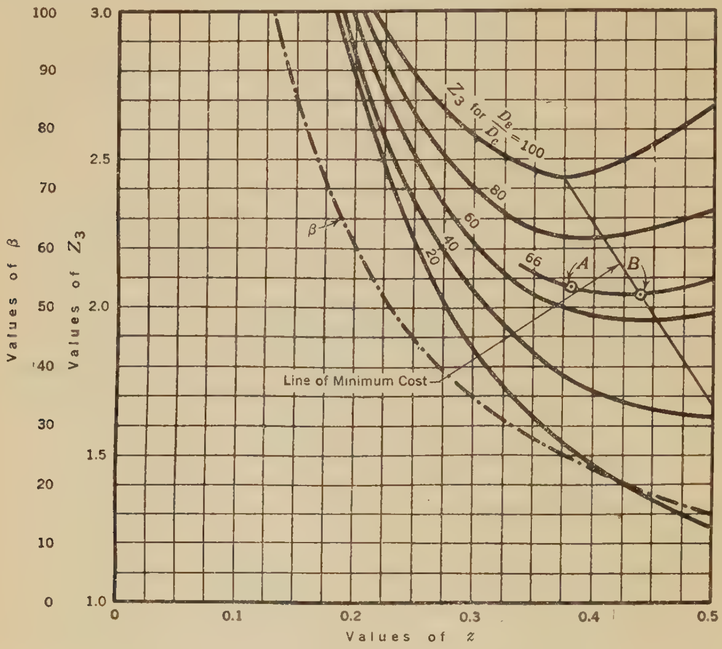


FIG. 11.—COST OF DECK SLAB IN FLEXURE FOR VARIOUS RATIOS, $\frac{D_s}{D_c}$, AT CONSTANT f_s

CHARLES P. WILLIAMS,[8] M. AM. SOC. C. E. (by letter).—Only within the last few years, have attempts been made to determine directly the most economic proportions of the class of structures treated in this paper, and economy of design has been secured only by the comparison of several designs, in which many of the controlling dimensions were selected arbitrarily. It has been the custom to select an "out-to-out" length of deck span equal to the clear spacing of buttresses. The author demonstrates that, in many cases, this is not economical, and, moreover, that at a determinable distance below the top of the dam, it is more economical to omit the deck altogether, substituting for the "articulated buttress" type of dam, the "round-head buttress" type, devised by the late Fred A. Noetzli, M. Am. Soc. C. E.

While Mr. Birke's analysis is presented in a logical and ingenious manner, a few of his assumptions are in error. His analysis of haunch stresses is based on the commonly used formulas for a beam of uniform depth, which, when a unit width of beam is considered, are as follows:

$$j = 1 - \frac{k}{3} = 1 - \frac{n f_c}{3\,(f_s + n f_c)} \quad \ldots\ldots\ldots\ldots\ldots\ldots (76)$$

---

[8] Cons. Engr., Los Angeles. Calif.

$$\tau \text{ (max.)} = \frac{S}{j\,d} \quad \ldots\ldots\ldots\ldots (77)$$

$$f_c = \frac{2\,M}{j\,k\,d^2} \quad \ldots\ldots\ldots\ldots (78)$$

and,

$$R_1\,f_s = \frac{M}{j\,d} \quad \ldots\ldots\ldots\ldots (79)$$

in which, in addition to the notation of the paper: n = the ratio of the coefficient of elasticity of steel to that of concrete; and k = a coefficient such that $k\,d$ equals the distance from the neutral axis to the extreme compression element, measured in the direction of the applied external load.

Equations (76), (77), (78), and (79) are applicable only when the tensile steel and the extreme compression filament are each perpendicular to the direction of the external load. The late William Cain, M. Am. Soc. C. E., has given[9] an approximate solution of the case, in which either or both the tensile steel and the extreme compression filament are inclined to the direction of the external load.

In the following discussion, the terms, shear, unit shear, and total shear, signify shear in the plane, *A-A*, Fig. 12, unless otherwise expressly stated.

In a beam having parallel tensile and compressive elements, each perpendicular to the direction of the external load, the unit shear in any plane parallel to the direction of the loading (assuming the concrete to carry no tension) is uniform on the tensile side of the neutral axis, and is the maximum unit shear in the section. On the opposite side of the neutral axis, the unit shear decreases at a rate proportional to the square of the distance from that axis, becoming zero at the extreme compression filament.

In the case of a beam having the tensile element perpendicular to the direction of the external load, but having its compression face inclined thereto, making the angle, ϕ, with the tensile element, as shown in Fig. 12, it can be shown that the factor, j, the unit shear, τ_1, at all points on the tension side of the neutral axis, the unit shear, τ_2, on the opposite side of the neutral axis, at a point distant y therefrom, and the unit shear, τ_3, at the extreme compression filament, are given by the following equations:

$$j = 1 - \frac{k}{3} = 1 - \frac{n\,f_c \cos^2\phi}{3\,(f_s + n\,f_c \cos^2\phi)} \quad \ldots\ldots\ldots\ldots (80)$$

$$\tau_1 = \frac{1}{j\,d}\left(S - \frac{M}{d}\tan\phi\right) \quad \ldots\ldots\ldots\ldots (81)$$

$$\tau_2 = \tau_1 - \frac{\tau_1}{k^2\,d^2}\,y^2 + \frac{2\,M\tan\phi\,y}{k^2\,j\,d^3} \quad \ldots\ldots\ldots\ldots (82)$$

and,

$$\tau_3 = f_c \sin\phi\cos\phi = \frac{2\,M\tan\phi}{k\,j\,d^2} \quad \ldots\ldots\ldots\ldots (83)$$

---

[9] "Earth Pressures, Retaining Walls, and Bins," by William Cain, p. 239 *et seq.*, N. Y., John Wiley & Sons.

The values of τ_1 and τ_2, in Equations (81) and (82), are approximations, based on the assumption that j is constant throughout a strip of infinitesimal width at the section, A-A, considered. The total shear at the section is S. The error in the approximation may be indicated by computing the total shear from the equations for unit shear. Designating by S_c, the total shear, computed from Equations (80) and (81), the following equation is obtained:

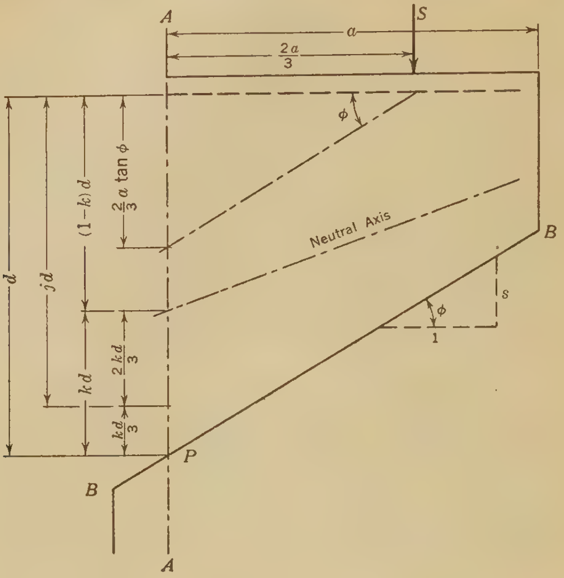


FIG. 12.

$$S_c = (1-k)\,d\,\tau_1 + \int_0^{kd} \tau_2\,\partial y \text{ .... } (84)$$

Substituting for τ_1 and τ_2, their approximate values, performing the operations indicated, and reducing, the computed total shear is found to be:

$$S_c = S + \frac{1-j}{j\,d} M \tan \phi \text{ .... } (85)$$

Substituting for M its value, $\frac{2}{3}\,a\,S$, and reducing:

$$S_c = S \left\{ 1 + \left(\frac{1-j}{j} \right) \frac{2\,a \tan \phi}{3\,d} \right\} \text{ .... } (86)$$

Assuming $f_c = 650$ lb per sq in.; $f_s = 16\,000$ lb per sq in.; and $n = 15$, the value of j will range from 0.922 for $s = \tan \phi = 1$, to 0.992 for $s = 5$. For s ranging from 1 to 5, then, the value of $\frac{1-j}{j}$ will range from 0.085 to 0.008. From Fig. 12, it will be observed that, $a \tan \phi < d$; hence, $\frac{2\,a \tan \phi}{3\,d} < \frac{2}{3}$. The error in the computed value of S_c, then, is less than 0.057 S when $s = 1$, and less than 0.005 S, when $s = 5$. For haunches in which $\frac{d}{a}$ is large, the error will be much less.

The distance from the neutral axis, corresponding to the point of maximum shear, is found from Equation (82) by placing the first derivative (as to y) of τ_2, equal to zero and solving for y. Performing these operations, it is found that the value of y at which the shear is a maximum, is:

$$y = \frac{M \tan \phi}{S - \frac{M}{d} \tan \phi} = \frac{M \tan \phi}{j\, d\, \tau_1} \quad \ldots\ldots\ldots\ldots\ldots\ldots (87)$$

It may be found that the point, determined by this value of y, is outside the structure, in which case the maximum unit shear is τ_3, as given in Equation (83). If, however, the point falls within the structure, the value of y is substituted for y in Equation (82), and reducing, the maximum unit shear is found to be:

$$\tau_3\,(\text{max.}) = \tau_1 + \frac{(M \tan \phi)^2}{k^2\, j^2\, d^4\, \tau_1} \quad \ldots\ldots\ldots\ldots\ldots\ldots (88)$$

The maximum unit shear may exceed greatly the unit shear on the tension side of the neutral axis.

At the point, P, in the section, A-A, Fig. 12, the compressive stress is the first principal stress. The direction of the maximum unit shear is at an angle of 45° with the direction of the principal stress. If, then, $\phi = 45°$, the maximum unit shear at P is in the plane, A-A. Between P and the neutral axis the maximum unit shear is approximately in that plane. If, however, ϕ is greater or less than 45°, the unit shear in the plane, A-A, at the point, P, will not be the maximum unit shear at that point, and the same will be true approximately for other points in the plane, A-A, between P and the neutral axis.

From the foregoing it might appear that there is greater danger of failure from excessive shear on the compression side of the neutral axis than on the tension side. This is not the case, however, since usually failure occurs from diagonal tension, incident to the shear, and not from direct shear.

The compressive forces below the neutral axis may be resolved into two components, one normal to the plane, A-A, and causing compression, and one parallel thereto and causing shear in the direction of the plane, and also in the direction perpendicular to it. The variation of shear below the neutral axis is due to the compressive forces, which are assumed to increase uniformly from the neutral axis to the extreme filament. The uniform increase in the normal component results in a decrease in the unit shear, in proportion to the square of the distance from the neutral axis, and the uniform increase in the parallel component results in a uniform increase in the unit shear. Any net increase in the unit shear is due to the excess of the effect of the parallel component over that of the normal component. Now, it can be shown that these compressive forces do not cause diagonal tension, hence any increase in shear below the neutral axis will not result in diagonal tension. The unit shear above the neutral axis, as given by Equation (81), may be taken as a measure of the diagonal tension.

The author determines the depth of the haunch, d_1, by Equation (7), assuming values for j_1 and τ. He then determines the area of steel, R_1, required to resist the bending moment, by Equation (30), assuming a value for f_s. Not more than two of the three variables, j, τ, and f_s can be assumed, which is demonstrated as follows: In order to simplify the equations, assume a beam with parallel tension and compression faces; that is, $\phi = 0$; then,

$$\tau = \frac{S}{j\,d} \qquad \text{(89)}$$

and,

$$M = \frac{f_c}{2}\,kd\;jd = \frac{3}{2}\,f_c\,(1 - j)\,jd^2 \qquad \text{(90)}$$

From Equation (89),

$$d = \frac{S}{j\,\tau} \qquad \text{(91)}$$

and from Equation (90):

$$d_2 = \frac{2\,M}{3\,f_c\,(1 - j)\,j} = \frac{4\,a\,S}{9\,f_c\,(1 - j)\,j} \qquad \text{(92)}$$

Hence,

$$\frac{S}{\tau^2\,j} = \frac{4\,a}{9\,f_c\,(1 - j)} \qquad \text{(93)}$$

and,

$$f_c = \frac{4\,a\,\tau^2\,j}{9\,(1 - j)\,S} \qquad \text{(94)}$$

The value of j is determined by the equation:

$$j = 1 - \frac{n\,f_c}{3\,(f_s + n\,f_c)} \qquad \text{(95)}$$

from which may be derived the equation:

$$f_c = \frac{3\,(1 - j)\,f_s}{n\,(3j - 2)} \qquad \text{(96)}$$

Substituting this value of f_c for f_c in Equation (94):

$$\frac{3\,(1 - j)\,f_s}{n\,(3j - 2)} = \frac{4\,a\,\tau^2\,j}{9\,(1 - j)\,S} \qquad \text{(97)}$$

The variables in Equation (97) are j, τ, and f_s. Therefore, when any two are assumed, the other is fixed.

In Equation (55), Mr. Birke assumes the load per unit area of deck to be the water-load only. The haunch must carry, not only the water-load, but also the normal component of the weight of the deck and of the haunch itself. For an Ambursen dam, having a buttress spacing of 22 ft, center to center, the normal component of the weight of the deck varies from 0.16 W, at 30 ft below the top, to 0.05 W, at a depth of 180 ft. The weight of the deck is a function not only of the depth below the water surface, but also of the span. An increase in the dimension, b, will result in a decrease of the thickness of the deck, and its consequent weight; however, the weight of the haunch will be increased.

Referring to Equation (44), the author states that "for a constant head, the ratio, $\frac{b}{L_2}$, will be independent of the clear buttress spacing." For this to be true, $\frac{c}{L_2}$ must be constant. It is constant as to the water-load, but not as to the weight of the deck.

PAUL BAUMANN,[10] M. AM. SOC. C. E. (by letter).—The deductions in this paper are based on the premise of a triangular distribution of bearing pressure on the haunches. All mathematical operations, based on this premise, lead to correct results, provided the premise is correct. The exact solution of the bearing-pressure problem on yielding supports is no simple one and leads to terms that would be entirely too complicated for the purpose of showing economy features of water-bearing members of buttressed dams.

The use of a simple, geometrical shape of the bearing pressure area, therefore, is justified, provided this shape—in this case a triangle—is a close approximation of the actual shape. If it is a close approximation, then the yield of the haunch, due to p at any distance, x, from a Y-axis through the end of the face slab (Fig. 4(b)) must be substantially equal to the deflection of the face slab due to p and W.

For the elastic line of the face slab between $X = 0$ and $X = a$, due to bending:

$$E_1\,I\,\frac{d^2y}{dx^2} = -\,M \quad \text{.......................(98)}$$

and for $M = \frac{p\,x^3}{6} - \frac{W\,x^2}{2}$:

$$E_1\,I\,\frac{d^2y}{dx^2} = -\,\frac{1}{2}\left\{\frac{p_0\,x^4}{3\,a} - W\,x^2\right\} \quad \text{..................(99)}$$

in which, $\frac{p_0\,x}{a} = p$. Integrating Equation (99):

$$E_1\,I\,\frac{dy}{dk} = -\,\frac{1}{2}\left\{\frac{p_0\,x^5}{15\,a} - \frac{W\,x^3}{3}\right\}_0^a + C_1 \quad \text{.............(100)}$$

For $x = a$, the slope, $\frac{dy}{dk}$, may be determined by cutting the face slab in two at mid-span and by applying the stresses there as external forces the resultant of which is the maximum moment; thus:

$$M_0 = \frac{W}{6}\left[L_1\,(s\,a + 3\,L_1) - a^2\right] \quad \text{................(101)}$$

---

[10] Asst. Engr., in Chg., Mountain Div., Los Angeles County Flood Control Dist., Los Angeles, Calif..

which differs from the author's Equations (37) and (46), because the latter are based on a uniform distribution of bearing pressure which is in contradiction to the basic assumption of a triangular distribution.

The moment at any section between $x = a$ and $x = L_1 + a$, is:

$$M = M_0 - \frac{W\,(L_1 + a - x)^2}{2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (102)$$

Therefore,

$$E_1\,I\,\frac{d^2y}{dx^2} = \left\{M_0 - \frac{W}{2}\left[(L_1 + a)^2 - 2\,(L_1 + a)\,x + x^2\right]\right\}$$

and,

$$E_1\,I\,\frac{dy}{dx} = \left\{M_0\,x - \frac{W}{2}\left[(L_1 + a)^2\,x - (L_1 + a)\,x^2 + \frac{x^3}{3}\right]\right\}_a^{L_1+a} + C \,..(103)$$

The constant, C, follows from the condition that $\frac{dy}{dx}$ at mid-span must be zero; thus:

$$C = (L_1 - a)\left[\frac{W}{6}\,(L_1 + a)^2 - M_0\right] \ldots\ldots\ldots\ldots\ldots (104)$$

and for $x = a$:

$$E_1\,I\,\frac{dy}{dx} = -\Bigg\{a\Bigg[M_0 - \frac{W}{2}\Bigg((L_1 + a)^2 - (L_1 + a)\,a + \frac{a^2}{3}\Bigg)\Bigg]$$
$$+ (L_1 + a)\left[\frac{W}{6}\,(L_1 + a)^2 - M_0\right]\Bigg\} \ldots\ldots\ldots\ldots\ldots (105)$$

Introducing Equation (105) in Equation (100):

$$C_1 = \frac{1}{E_1 I}\Bigg\{a\Bigg[M_0 - \frac{W}{2}\Bigg((L_1 + a)^2 - (L_1 + a)\,a + \frac{a^2}{3}\Bigg)$$
$$+ (L_1 + a)\left[\frac{W}{6}\,(L_1 + a)^2 - M_0\right] + \frac{1}{2}\left[\frac{p_0\,a^4}{15} - \frac{W a^3}{3}\right]\Bigg\} \ldots..(106)$$

Integrating again, the resulting simplified formula is the equation of the elastic line of the face slab due to bending; thus:

$$E_1 I\,y = -\frac{x^4}{12}\left\{\frac{p_0\,x^2}{15\,a} - \frac{W}{2}\right\} + x\left\{\frac{W}{6}\left[L_1^3 - a^3 + 12\,L_1\,a^2\right] - M_0\,L_1 + \frac{p_0\,a^4}{30}\right\} ..(107)$$

The deflection, $\Delta\,y_s$, due to shear of the face slab is:

$$\Delta\,y_s = \frac{1}{j_1\,G_1\,d_2}\left\{\frac{p_1\,x^4}{8\,a} - \frac{W\,x^2}{2}\right\} \ldots\ldots\ldots\ldots\ldots\ldots (108)$$

Adding Equation (108) to the deflection due to bending:

$$E_1\,I\,(y + \Delta\,y_s) = E_1\,I\,y_F = -\frac{x^4}{12}\left\{\frac{p_0\,x^2}{15\,a} - \frac{W}{2}\right\} + x\Bigg\{\frac{W}{6}\,(L_1^3 - a^3 + 12\,L_1\,a^2)$$
$$- M_0\,L_1 + \frac{p_0\,a^4}{30}\Bigg\} + \frac{1}{0.30\,j_1\,d_2}\left\{\frac{p_0\,x^4}{8\,a} - \frac{W\,x^2}{2}\right\} \ldots\ldots\ldots (109)$$

Neglecting the influence of bending as suggested by the author, the elastic yield of the haunch due to shear, Q, is:

$$y_H = -\frac{1}{j_2\,G_2}\int_0^a \frac{Q\,Q_A}{d}\,dx \dots\dots\dots\dots\dots\dots (110)$$

in which, $Q = \frac{p_0\ (a^2 - x^2)}{2\,a}$; $Q_A = 1$; and, $d = d_1 + S\ (a - x)$. Introducing these values in Equation (110):

$$y = \frac{-\,p_0}{2\,a j_2\,G_2}\int_0^a \frac{a^2 - x^2}{d_1 + s\ (a - x)}\,dx = -\,K\int_0^a \frac{a^2 - x^2}{d_1 + s\ (a - x)}\,dx$$

$$= -\,K\Bigg\{\int \frac{a^2}{d_1 = s\ (a - x)}\,dx - \int \frac{x^2}{d + s\ (a - x)}\,dx\Bigg\} \dots\dots (111)$$

Considering each integral separately:

$$\int \frac{a^2}{d_1 + s\ (a - x)}\,dx = -\frac{1}{s}\,l\,n\Big\{d_1 + s\ (a - x)\Big\} \dots\dots\dots (112)$$

and, if v is substituted for $a - x$:

$$\int \frac{x^2}{d_1 + s\ (a - x)}\,dx = a^2\int \frac{-\,dv}{d_1 + sv} - 2a\int \frac{-v\,dv}{d_1 + sv} + \int \frac{v^2\,dv}{d_1 + sv}\ .(113)$$

Then Equation (111) may be written:

$$y = -\,K\Bigg[-\frac{2\,a}{s^2}\Bigg\{d_1 + s\ (a - x) - d_1\,l\,n\Bigg[d_1 + s\ (a - x)\Bigg]\Bigg\}$$

$$+\frac{1}{s^3}\Bigg\{\frac{1}{2}\Bigg[d_1 + s\ (a - x)\Bigg]^2 - 2\,d_1\Bigg[d_1 + s\ (a - x)\Bigg]$$

$$+\,d^2_1\,l\,n\Bigg[d_1 + s\ (a - x)\Bigg]\Bigg\} + C \dots\dots\dots\dots\dots\dots (114)$$

For $x = 0$, $y = 0$, and:

$$C = \frac{2\,a}{s^2}\Bigg\{d_1 + s\,a - d_1\,l\,n\Bigg[d_1 + s\,a\Bigg]\Bigg\} - \frac{1}{s^3}\Bigg\{\frac{1}{2}\Bigg[d_1 + s\,a\Bigg]^2$$

$$-\,2\,d_1\Bigg[d_1 + s\,a\Bigg] + d^2_1\,l\,n\Bigg[d_1 + s\,a\Bigg]\Bigg\} \dots\dots\dots\dots (115)$$

After reducing and re-arranging:

$$y_H = \frac{-\,p_0}{2\,a\,j_2\,G_2\,s}\Bigg\{\frac{d_1}{s}\Bigg[l\,n\,\frac{d_1 + s\,(s - x)}{d_1 + s\,a}\Bigg]\Bigg(\frac{d_1}{s} + 2\,a\Bigg)$$

$$+\Bigg[\frac{x^2}{2} - a\,x + \frac{d_1\,x}{s}\Bigg]\Bigg\} \dots\dots\dots\dots\dots\dots\dots\dots (116)$$

which obviously does not conform to Equation (109).

A numerical example will serve to check this. In Fig. 3 (a), let $H = 100$ ft; $W = 43.40$ lb per sq in.; $s = 3$; $p_0 = 700$ lb per sq in.; $\tau = 60$ lb per sq in.; and, $L_2 = 240$ in. Furthermore, let $A = \frac{d_1}{a} = 4$ and $b = 0$; then, from Equation (18), $a = \frac{2\,W\,L_2}{p_0} = 29.80$ in., and, therefore, $d_1 = 4 \times 29.80 = 119.20$ in. With these values, Equation (109) gives, for $x = a$:

$$E_1\,y_F = \left\{\frac{3\,400}{I} + \frac{8.70}{d_2}\right\} 10^6$$

With sufficient approximation:

$$I = \frac{(d_2 + e_2)^3}{12} = \frac{(0.0625\,\sqrt{M_0} + 2.5)^3}{12} = 24\,200 \text{ in.}^4$$

in which, $0.0625\,\sqrt{M_0} = d_2 = 63.70$ in. Finally, $E_1\,y_F = 0.278 \times 10^6$ lb per in.

Introducing the same values in Equation (116) with $x = a$ and $G_2 = G_1 = 0.30\,E_1$:

$$E_1\,y_H = \frac{-\,700}{0.60 \times 29.80 \times 0.875 \times 3}\left\{-\,9\,509\right\} = 0.1415 \times 10^6 \text{ lb per in.}$$

which is roughly one-half as great as $E_1\,y_F$. The deflection of the haunch due to bending is of the order of magnitude of 0.002×10^6 and, therefore, would not appreciably adjust the discrepancy.

The distribution of the bearing pressure, p, on the haunch could readily be ascertained by means of photo-elastic tests on models for various, relative rigidities of haunch and face slab, and new equations for the configuration of the haunch could be derived based on the results thereof, which would establish compatibility between the two members of the structure.

A. FLORIS,[11] ESQ. (by letter).—Some of the statements in this interesting paper, do not appear to be well founded. The claim, for instance, that buttressed dams with deck slabs are less expensive that multiple-arch dams, is not quite obvious. The very purpose of arches is to reduce the bending moment caused by the water pressure; whereas deck slabs must develop considerable resisting moment to support the same pressure. If the mass concrete of buttressed dams is more economical, multiple-arch dams can also be built with thick buttresses using the same kind of concrete.

That the buttressed dams with freely supported deck slabs will be safer in an earthquake than multiple-arch dams is not at all a proved fact. Loosely connected members of a structure cannot be stronger against lateral forces than a monolithic structure. If the earthquake movement is propagated along the longitudinal axis of a multiple-arch dam, the flexible arches will act like springs or shock absorbers, thus relieving the structure of excessive stresses. The same action cannot be claimed in the case of buttressed dams.

In his analysis Mr. Birke determines the minimum cost of the deck slabs supported by buttresses with large heads. He considers these heads as cantilevers and applies the beam theory of prismatic bars. It cannot be taken for granted that the stress distribution in these large buttress heads will be the same as in a simple cantilever beam. Therefore, the conclusion that buttressed types of dams (including round-head buttresses), can be subjected to an exact mathematical analysis, may be seriously questioned. This being the case, the painstaking cost analysis of buttressed dams given in this paper appears to be of restricted usefulness to engineers engaged in the design of dams.

---

[11] Los Angeles, Calif.

HAKAN D. BIRKE,[12] ASSOC. M. AM. SOC. C. E. (by letter).—An excellent group of discussions has been presented in connection with this paper. It is gratifying that the importance of establishing the principles of economic design of articulated buttress dams has been recognized by such competent authorities.

Aside from its merits in the field of economy and its high factor of safety, the principal advantage of the articulated buttress dam, as pointed out by Mr. Sutherland, is its adaptability to a wide variety of foundation conditions. The increasing scarcity of sites with sound rock foundations will undoubtedly lead to a more widespread use of this type of dam and, therefore, a detailed knowledge of its economic limits becomes of major importance.

In attempting to establish such economic limits Mr. Sutherland has compared a round-head buttress type dam, 120 ft. high, having a buttress spacing of 40 ft, center to center, with a gravity type dam of the same height. The buttress dam is found to be somewhat more economical than the gravity dam although Mr. Sutherland found the reverse to be true for all heights less than 90 ft. His analysis gave rise to the inquiry as to whether a reduction of buttress spacing at the flanks of the dam would be more economical than the use of the same buttress spacing throughout.

In answer to this inquiry it may be stated that there are several definite objections to changing the buttress spacing. In the first place, a change of spacing throws a heavy eccentric loading on the buttress at the point of transition. Such a transition is necessarily a heavy structure and the additional concrete required goes a long way toward offsetting the small economy that would be gained by changing the buttress spacing. A second and perhaps more serious objection is that a change in spacing would increase the cost of form work considerably. One of the advantages in the buttress type of construction is that standard forms may be used throughout the entire dam. This leads to an appreciable economy in construction costs.

In determining the most economic type and buttress spacing for a dam at a given site, the proper procedure is first to establish the mean height of the dam. Comparisons may be made of the mean heights for the various types; but it will be found, in general, that these heights will agree closely and will not be far enough apart to affect the selection of type. The mean height may then be used to determine the most economical height and the correct buttress spacing.

If Mr. Sutherland had investigated other types and buttress spacings, he would have found that a considerably closer spacing than 40 ft and an Ambursen type of deck and haunch would have shown a much greater saving than the round-head dam or the gravity dam used in his example for heights even as low as 25 ft.

---

[12] Designing Engr., Ambursen Dam Co., New York, N. Y. Elected an Associate Member, Am. Soc. C. E., on July 25, 1932.

Mr. Werner makes an analysis of the economical stress relations in concrete and steel (f_c and f_s) for a deck slab in flexure. By his analysis he demonstrates that in the example introduced by the writer (in which, $f_s = 16\,000$; $f_c = 650$ lb per sq in.; and $\beta = 24.6$), better economy would have been obtained by using $\beta = 20$; but this value of β will yield a concrete stress, that is in contradiction to the assumption. Mr. Werner's analysis is based on a constant value of f_s, and, therefore, using his suggested value of β would make $f_c = \dfrac{16\,000}{20} = 800$ lb per sq in., which is in excess of the assumed maximum allowable concrete stress, $f_c = 650$ lb per sq in.

Generally, in design problems, maximum allowable stresses are used regardless of their economic relations. Mr. Werner has shown how the greatest economy for a deck slab in flexure may be attained when one of the stresses is kept constant and the other, varied. It is of equal interest to determine the economic effect of varying the stresses independently of each other. Consider, for example, Equation (69), as developed by Mr. Werner, and let,

$$Y = \frac{1}{\sqrt{z\,(3 - z)}} \left(1 + H\,\frac{z^2}{1 - z}\right) \frac{1}{\sqrt{f_c}} \quad \ldots\ldots\ldots\ldots(117)$$

or, by introducing:

$$z = \frac{n}{\beta + n} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(118)$$

the result will be:

$$Y = \frac{\beta\,(\beta + n) + H\,n^2}{\beta\,\sqrt{n\,(3\,\beta + z\,n)\,f_c}} \quad \ldots\ldots\ldots\ldots\ldots\ldots(119)$$

and,

$$C_{td} = Y\,\sqrt{\frac{6\,M}{l}}\;l\,L\,D_c \quad \ldots\ldots\ldots\ldots\ldots\ldots(120)$$

For a certain ratio, $\dfrac{D_s}{D_c}$, the cost of the slab may be determined from Equation (119) for various stress values.

Assuming that $C = 0.2$, $n = 15$, and $\dfrac{D_s}{D_c} = 66$, Equation (119) has been evaluated for $f_s = 10\,000$ to $20\,000$ lb per sq in., and $f_c = 400$ to 800 lb per sq in. The results have been plotted and then curves of equal costs have been drawn, as shown by Fig. 13.

The stress condition introduced by the writer in his typical example (that is, $f_s = 16\,000$ and $f_c = 650$ lb per sq in.), is represented by Point A in Fig. 13. It may be seen that reducing any or both of the stresses would only increase the cost of the slab. Any increase in the stresses is out of the question as these values were assumed to be the highest allowable. Thus, it is proved that the stress condition used by the writer was the most economical one.

Fig. 13 demonstrates some interesting facts about the economy of different stress relations for a slab in flexure. Thus, for a low value of the stress in concrete, such as, $f_c = 450$ lb per sq in., little economy would be gained by stressing the steel above 14 000 lb per sq in. In the same way, for a value of $f_c = 13\,000$ lb per sq in., no economy will be gained by stressing the concrete above 600 lb per sq in.

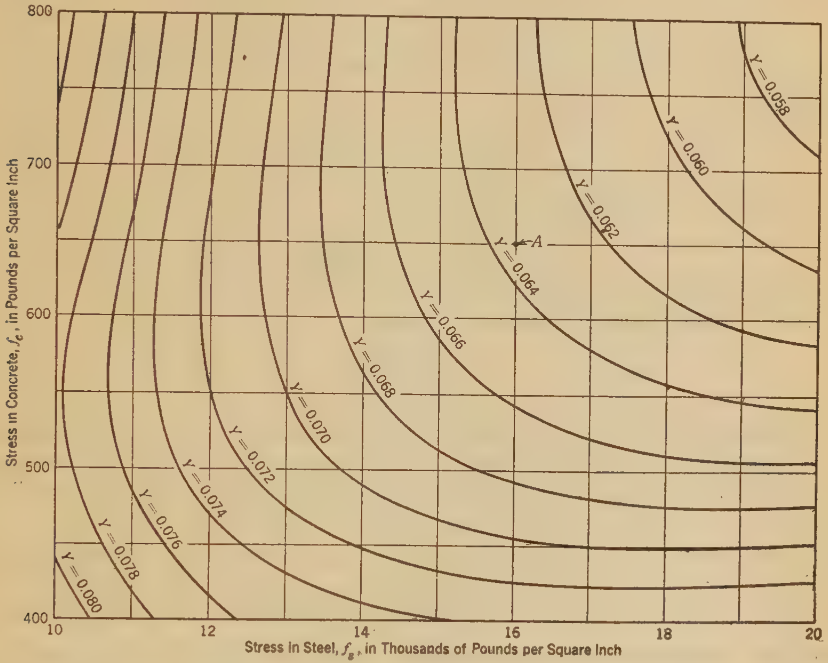


Fig. 13.—Curves of Equal Cost for Deck Slab in Flexure at Various Values of Stresses in Steel and Concrete Ratio, $\frac{D_s}{D_c} = 66$.

The object of the paper was to demonstrate the conditions under which maximum economy would be attained for the water-bearing members of articulated buttress dams. The economic equations were based on ordinary designing assumptions which, in some cases, are somewhat approximate. This has been pointed out by some of the discussers, and, therefore, it is necessary to consider the designing methods presented by the writer in the light of these criticisms.

Mr. Williams has pointed out that the equations developed in the paper are based on the conventional formulas for designing beams of uniform depth, and then demonstrates the differences between these and the formula for wedge-shaped beams as developed by the late William Cain, M. Am. Soc. C. E.

If the haunch is designed as a wedged beam by the methods introduced by Professor Cain, the shear stress on the tensile side of the neutral axis is determined by Equation (81), as presented by Mr. Williams. This shear stress,

τ_1, generally will control the depth of the section because the shear stresses on the compressive side of the neutral axis are combined with compressive normal stresses and hence will not cause any diagonal tension.

The total shear, S, at Section A-A, Fig. 12, will be equal to $\frac{1}{2}\, p\, o\, a$ and the moment, M, will be equal to $\frac{1}{3}\, p\, o\, a^2$. If these values are introduced in Equation (81), and d is eliminated by combining Equations (81) and (83):

$$\tau_1 = \frac{\cos\,\phi}{2}\sqrt{\frac{3\,k\,f_c\,p_0}{2\,j}} - \frac{k\,f_c \sin\phi\cos\phi}{2} \quad \text{..........(121)}$$

If the haunch is designed as a beam of uniform depth the shear, τ, will have the value:

$$\tau = \frac{S}{j_1\,d} = \frac{p_0\,a}{2\,j_1\,d} \quad \text{..........(122)}$$

or, by combining with Equation (83),

$$\tau = \frac{\cos\,\phi}{2\,j_1}\sqrt{\frac{3\,k\,j\,p_0\,f_c}{2}} \quad \text{..........(123)}$$

From Equations (80), (121), and (123):

$$\frac{\tau_1}{\tau} = \frac{j_1}{j} - j_1\sqrt{\frac{6\,f_s\,(1-j)^2}{p_0\,n\,j\,(3\,j-2)}}\,\tan\phi \quad \text{..........(124)}$$

In Equation (124), τ_1 designates the shear when the haunch is designed as a wedged beam and, τ, the shear in the haunch computed as for a beam with uniform depth. Thus, Equation (124) gives the relation between the controlling shear stresses under these different assumptions.

In the economy problem, the factors, f_s, p_o, n, and ϕ are constants and thus the only variables in the expression for $\frac{\tau_1}{\tau}$ (Equation 124) will be j_1 and j, which are the distribution factors for shear. As the variations in j_1 and j will be negligible, the relation will be nearly constant. If the haunch is to be designed for a certain value of τ_1 (controlling shear when computed as a wedged beam), from Equation (124) the corresponding value of τ (controlling shear when the haunch is designed as a beam of uniform depth) may be determined. Using this value of shear, the depth of the haunch may be determined as if it were a beam of uniform depth, and the formulas developed will still apply.

Mr. Williams demonstrates, that in determining the depth of the haunch only two of the three factors j, τ, and f_s can be assumed. However, that should not restrict the use of Equation (30) for determining the steel area in the haunch for bending moment. For instance, if the expression, $S = \frac{p_0\ a}{2}$, is introduced in Equation (97), as presented by Mr. Williams, the result will be:

$$\frac{(1-j)^2}{3\,j-2} = \frac{8}{27}\,\frac{n\ \tau^2}{f_s\ p_0} \quad \text{..........(125)}$$

in which, the maximum pressure p_o, on the haunch, is constant. If the values of τ and f_s are assumed, it is a simple matter to solve Equation (125) and determine the value of j. There is then no objection to using Equation (30).

The load the deck must carry, in pounds per square inch, may be written as follows (see Fig. 4(*a*), and Fig. 14):

Water Load:

$$62.5\ (H - t \cos \alpha) \tag{126}$$

Weight of Structure:

$$150\ t \cos \alpha \tag{127}$$

Total load:

$$W = 62.5\ H + 87.5\ t \cos \alpha \tag{128}$$

If, as an approximation, the total load is assumed as, $W = 62.5\ H$, it does not mean, as stated by Mr. Williams, that the water-load alone has been taken into account (compare Equation (126)). It only means that the last term in Equation (128) has been neglected. For the Ambursen dam introduced as an example by Mr. Williams, having a buttress spacing of 22 ft, on centers, the last term in Equation (128) varies from the value 0.06 W, at 30 ft below the top, to 0.03 W at a depth of 180 ft. At greater heights, where the cost of the water-bearing members is a more important item, the last term will be entirely negligible when compared to the first.

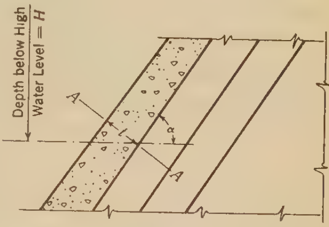


Fig. 14.—Vertical Section Through Deck of Ambursen Dam.

Mr. Baumann has demonstrated the difficulties of determining the exact bearing-pressure distribution on the haunch. The writer assumed a triangular bearing pressure area according to the common practice for this kind of structure. Actually, the character of this pressure distribution will vary with the relative rigidities of the haunch and deck. The writer is of the opinion, however, that an assumption of triangular distribution is on the safe side.

Mr. Baumann has made a check on the result of this assumption by comparing the deflection of the deck with the yield of the haunch at corresponding points. It may be noted that the formulas for these deflections are based on an assumed straight linear distribution of the stresses. Actually, in the deck above the bearing area of the haunch, the distribution of the stresses is not nearly as simple as straight-lined diagrams would indicate. In view of this, the results of Mr. Baumann's computations do not offer sufficient evidence to justify changing the assumption of triangular bearing pressure on the haunches.

It is quite true, as Mr. Floris states, "that the buttressed dams with freely supported deck slabs will be safer in an earthquake than multiple-arch dams. is not at all a proved fact." Neither is the converse a proved fact. Under some conditions, loosely connected members of a structure will not be as strong against lateral forces as a monolithic structure, but it is not a fact that loosely connected members cannot be stronger, against all forces acting, than a monolithic structure. To be stable a monolithic structure must have sufficient strength, in all its parts, to resist all the loads to which it will be subjected. In many cases, construction joints, properly located, render a structure safe, whereas, if the joints were not provided, the structure would be unsafe. This is a well recognized and generally accepted fact in structural design.

Mr. Floris states that: "If the earthquake movement is propagated along the longitudinal axis of a multiple-arch dam, the flexible arches will act like springs or shock absorbers, thus relieving the structure of excessive stresses". This description might be correct, possibly, if the earth tremors were slight; however, the arches could scarcely withstand a severe shock without being overstressed, and it is improbable that they could withstand the torsional stresses that frequently accompany earthquakes. Even assuming that the only earth movement is in the direction of the longitudinal axis, the members connecting the buttresses, in transmitting the shock, would act as columns. It is inconceivable that a curved column composed of arch barrels would transmit a shock more safely than a straight column composed of flat-slab decks.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 1893

LINCOLN HIGHWAY FROM JERSEY CITY TO ELIZABETH, NEW JERSEY

By Sigvald Johannesson,[1] M. Am. Soc. C. E.

With Discussion by Messrs. Fred Lavis, Theodore Belzner, and J. B. French.

Synopsis

In order to take care of the vehicular traffic concentrating at the west end of the Holland Tunnel, between Jersey City, N. J., and New York, N. Y., the New Jersey State Highway Commission began in 1924 the construction of a "super-highway," leading from the tunnel through Jersey City, Kearny, Newark, and Elizabeth, to connection with existing or new State highways. The construction of this highway was completed in 1932.

As this "super-highway" necessarily had to go through built-up sections of cities, and through other sections highly developed for railroad and industrial purposes, and as it was expected to carry a large volume of traffic, it was decided at the inception of the work to build it so as to avoid grade crossings throughout the greater part of its length.

In this paper are discussed the general aspects of the design of this highway, which included embankments, viaducts, high-level bridges, and a subway, with their ramp connections to the surface streets.

General Description

The last link of the New Jersey State Highway, designated the Lincoln Highway, leading from the west end of the Holland Tunnel through Jersey City, Kearny, Newark, and Elizabeth, to connection with the main highways to Trenton, Philadelphia, Pa., and beyond, was completed eight years after the inception of the work, and was opened for traffic on November 24, 1932.

Note.—Published in November, 1933, *Proceedings*.

[1] Designing Engr., State Highway Comm., Trenton, N. J.

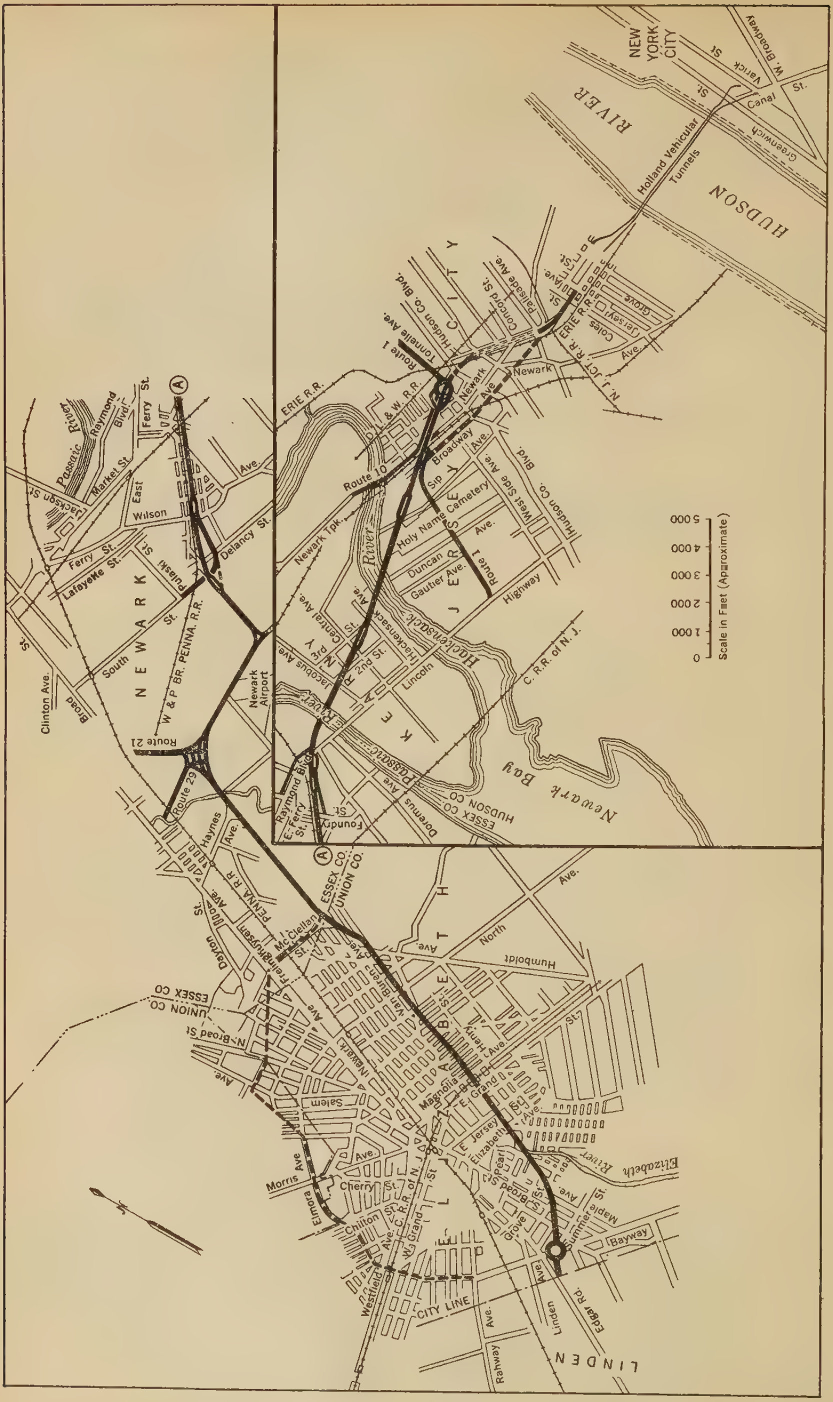

NEW YORK CITY
HUDSON RIVER
W. Broadway
Varick St.
Canal St.
Greenwich St.
Holland Vehicular Tunnels
JERSEY CITY
Route 1
Tonnelle Ave.
Hudson Co. Blvd.
Concord St.
Palisade Ave.
ERIE R.R.
N. J. JCT. R.R.
Coles
Jersey
Grove
Newark Ave.
D. L. & W. R.R.
Route 10
Broadway
Sip Ave.
Holy Name Cemetery
West Side Ave.
Duncan Ave.
Gautier Ave.
Highway
Hackensack River
Newark Tpk.
Central Ave.
Hackensack Ave.
2nd St.
Jacobus Ave.
Lincoln
KEARNY
C. R.R. of N. J.
Newark Bay
Passaic River
Doremus Ave.
HUDSON CO.
ESSEX CO.
Raymond Blvd.
E. Ferry St.
Foundry St.
Scale in Feet (Approximate)
0
1 000
2 000
3 000
4 000
5 000
A
Passaic River
Raymond Blvd.
Ferry St.
Market St.
Jackson St.
East
Wilson
Delancy St.
Ave.
Ferry St.
Lafayette St.
Pulaski St.
NEWARK
South St.
W & P BR. PENNA. R.R.
Newark Airport
Clinton Ave.
Broad St.
Route 21
Route 29
Haynes Ave.
PENNA. R.R.
Dayton St.
Frelinghuysen Ave.
McClellan St.
ESSEX CO.
UNION CO.
Van Buren Ave.
North Ave.
Humboldt
ELIZABETH
N. Broad St.
Newark Ave.
Salem
Magnolia
Henry
E. Grand St.
E. Jersey St.
Elizabeth Ave.
Pearl St.
Broad St.
Elizabeth River
Morris Ave.
Cherry St.
Elmora Ave.
Chilton St.
C. R.R. of N. J.
W. Grand St.
Westfield Ave.
CITY LINE
Grove St.
Summer St.
Maple Ave.
Bayway
Linden Ave.
Edgar Rd.
Rahway Ave.
LINDEN

Fig. 1.—Alignment of Route 25 Through Jersey City, Kearny, Newark, and Elizabeth, New Jersey

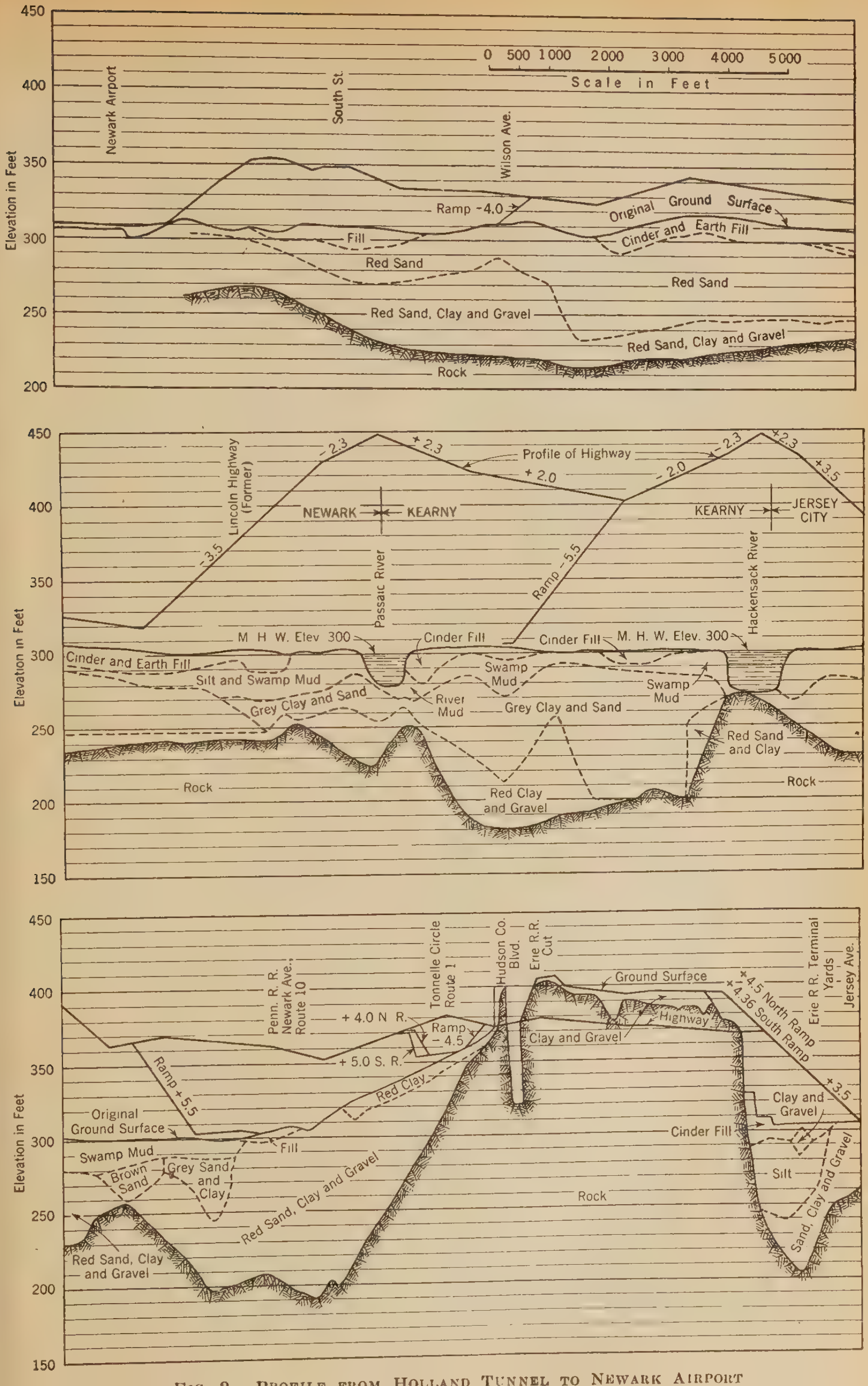

450
400
350
300
250
200
Elevation in Feet
0 500 1000 2000 3000 4000 5000
Scale in Feet
Newark Airport
South St.
Wilson Ave.
Ramp -4.0
Original Ground Surface
Cinder and Earth Fill
Fill
Red Sand
Red Sand
Red Sand, Clay and Gravel
Red Sand, Clay and Gravel
Rock
450
400
350
300
250
200
150
Elevation in Feet
Lincoln Highway (Former)
-2.3
+2.3
Profile of Highway
+2.0
-2.0
-2.3
+2.3
+3.5
NEWARK
KEARNY
KEARNY
JERSEY CITY
-3.5
Passaic River
Ramp -5.5
Hackensack River
M H W. Elev 300
Cinder Fill
Cinder Fill
M. H. W. Elev. 300
Cinder and Earth Fill
Silt and Swamp Mud
Swamp Mud
Swamp Mud
Grey Clay and Sand
River Mud
Grey Clay and Sand
Red Sand and Clay
Rock
Rock
Red Clay and Gravel
450
400
350
300
250
200
150
Elevation in Feet
Penn. R. R. Newark Ave. Route 10
Tonnelle Circle Route 1
Hudson Co. Blvd.
Erie R.R. Cut
Ground Surface
Erie R R. Terminal Yards
Jersey Ave.
+4.0 N R.
Ramp -4.5
+5.0 S. R.
Highway
Clay and Gravel
+4.5 North Ramp
+4.36 South Ramp
Ramp +5.5
Red Clay
+3.5
Original Ground Surface
Fill
Clay and Gravel
Cinder Fill
Swamp Mud
Brown Sand
Grey Sand and Clay
Silt
Rock
Red Sand, Clay and Gravel
Sand, Clay and Gravel
Red Sand, Clay and Gravel

FIG. 2.—PROFILE FROM HOLLAND TUNNEL TO NEWARK AIRPORT

This highway, the alignment and partial profile of which are shown in Figs. 1 and 2, respectively, is about 12 miles long, and its cost has been approximately $40 000 000.

At present (1933) this highway carries traffic at the rate of 11 000 000 vehicles per year and is expected to become one of the most heavily traveled roads in the world. For the greater part of its length it goes through built-up sections of cities with heavy street traffic, or through land developed for industrial and railroad purposes, and it crosses three navigable rivers.

When the project was originally conceived it was appreciated that these conditions would affect, materially, its economics as applied to construction and use, and, for that reason, a careful study of this phase was made prior to actual design, so as to co-ordinate, as far as possible, the cost of vehicle operation with the cost of construction. This study indicated clearly, that it would be of economic advantage to design the highway throughout the greater part of its length so as to avoid grade intersections with railroads and with other highways and streets. At first, consideration was given to movable bridges for river crossings but, subsequently, it was decided for economic reasons to provide fixed bridges, high enough above the water level to clear all river traffic.

This decision established the principle, that the section from the Holland Tunnel in Jersey City to Newark Airport (a distance of 7¼ miles) should be designed so as to avoid all interference from cross-traffic. Beyond the Airport, the route passes through partly undeveloped territory with little immediate need of grade separation, and it was possible, therefore, to design the greater part of this section as a surface road.

The various political, physical, and economic considerations that led to the final location of the highway are described in a previous paper[2] and will not be touched upon herein except to add, that no attempt was made to subordinate the alignment to possible variations in the sub-surface conditions for the purpose of making a saving in cost of construction. This was partly due to the probability that such savings in the cost of foundation construction, as might be attained by a change of alignment, would be small; but for the most part it was due to the fact that any such change would have increased the cost of vehicle operation out of proportion to the possible saving in construction cost.

No attempt was made to distort the alignment for the purpose of improving the angle of a crossing over highways, railroads, and rivers, because any saving in construction cost would have been overbalanced greatly by the consequent additional cost of vehicle operation. In fact, the highway crosses the Hackensack River at an angle of about 50°, and the Elizabeth River, with an S-turn, is crossed three times by the viaduct carrying the highway.

To avoid grade crossings with other traffic arteries, the highway in general has been built above them at such elevations as were necessary to provide the proper under-clearance for the type of traffic crossed. For highways and streets, the standard minimum clearance was 14 ft and for railways, 23 ft.

---

[2] "Highways as Elements of Transportation," by Fred Lavis, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 95 (1931), p. 1020.

At the crossings of the Hackensack and Passaic Rivers, which rivers carry sea-going vessels, the under-clearance height was made 135 ft at mean high water across the full width of the dredged channel; while at the crossings of the Elizabeth River, the under-clearance height was made 22 ft at mean high water. At the crossing points, the Elizabeth River is only a small stream with practically no channel depth at low water, but, nevertheless, it is classified as a navigable river.

An exception to the general method of locating the highway above the surface traffic was made in Jersey City, where it crosses a hill about 100 ft high and about ¾ mile wide. In order to avoid excessive rise and fall on the profile with the consequent additional cost of vehicle operation, the line at this place was driven through a subway below the surface streets.

As previously stated, there are no grade intersections with any kind of traffic routes between the Holland Tunnel and Newark Airport; but, with the exception of certain isolated places, beyond the Airport there are no railway crossings, and the intersecting vehicular traffic is not of sufficient importance to warrant grade separation. The greater part of the line between Newark Airport and the southern boundary of the City of Elizabeth, therefore, is designed and built as a surface road.

In due time, however, it is probable that the vehicular traffic will develop sufficiently to justify a grade separation. For this reason a strip of land, 120 ft wide, has been acquired throughout this section. At present, the roadway occupies the central 50-ft strip of this right of way. The remainder, which is intended for sidewalks and local roadways, is separated from the highway by narrow longitudinal "islands." When the crossing traffic has developed sufficiently to warrant the change, it is contemplated to raise the grade above the cross-traffic level, leaving the local roadways at the present grade to serve the surface traffic along the line of the highway.

Physical, topographical, and economic conditions have necessitated the use of many types of structures. Generally none of these types is novel or unusual in itself, but many of the designs, which now seem entirely obvious and simple, were the final results of many studies with various plans. The work on the highway was started in 1924 and completed in 1932, and during the intervening years, design was continuously in progress on some section of the highway. In details, the design of similar structures was not always identical if they were developed at different periods, because of new ideas and additional knowledge and experience gained.

Alignment and Profile of Roadway and Ramps

At the beginning of the work the width of the roadway was established at approximately 50 ft. In the earlier designs the width actually was made 48.5 ft, but, later, the roadway was made 50 ft wide. The maximum gradient was established at 3.5%, which value was maintained throughout the work for the main roadway. At grade breaks vertical curves were introduced, which were made quite long in the earlier period; but it was found difficult at times to provide proper drainage with these long curves, and gradually it became customary to make the length 67 ft for each 1% of grade change at ridges, and from 33 to 50 ft at valleys. In special cases, where these lengths have been decreased, because of physical conditions, an easy riding surface and a sight distance of not less than 500 ft have been maintained.

It was desired from the beginning, that the radii of horizontal curves should not be less than 1 000 ft., and, generally, this minimum requirement was maintained, although in some cases it was made less. For the designs made during the more recent years, the tendency was to increase the minimum radius to 1 500 ft, and, whenever possible, the curve radii have been made from 5 000 to 10 000 ft.

Ramp connections have been provided at intervals, leading from the adjacent roadway surfaces to the new level; and usually they enter at the middle of the highway, which is widened and divided where such ramps occur. At first, the reverse curves at these wide sections were generally made with radii of 1 000 ft. In one case, where local conditions were unfavorable, the reverse curves were designed with radii of less than 500 ft, but the result was not satisfactory. Later, the radii were increased to 2 000 ft for the convex part, and 1 000 ft for the concave part, of the reverse curve, which permitted the traffic to move with greater smoothness and safety, and the additional cost of construction was generally relatively small.

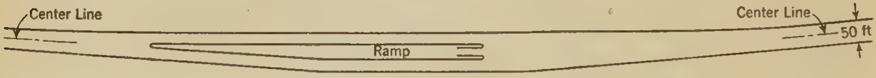


FIG. 3.—OUTLINE OF RAMP CONNECTION, KEARNY, N. J.

Wherever it was possible, it was found advantageous to place the head of a center ramp at a curve in the main line in order to avoid reverse curves. Such an arrangement is indicated in Fig. 3, which shows the outline of the ramp connection in Kearny. The curve angle of the alignment at this point is small, but it was possible, nevertheless, to construct the roadways without reverse curves.

It was necessary, of course, to introduce a vertical curve in the profile at the head of a ramp. Where the profile of the main line is descending in the same direction as the ramp, there is no difficulty, but if it is pitched in the opposite direction, it is necessary to continue the rise on the ramp for some distance. This may add materially to the length, and, consequently, to the cost of the ramp. For the sake of economy, it was found advisable in

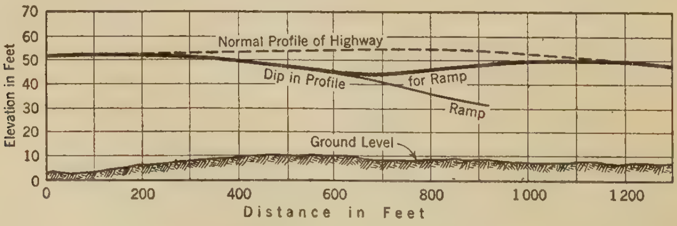


FIG. 4.—DIP IN NORMAL PROFILE TO PROVIDE FOR RAMP CONNECTION

certain cases, therefore, to dip the profile of the main highway at the head of the ramp. Such a dip was made near Newark Airport, as shown in Fig. 4, and a material saving in construction cost was gained by this expedient; but the appearance of the viaduct, as seen from the side, is not as satisfactory as might be desired, because the reason for the dip is not apparent.

Center ramps usually have been made wide enough for two lanes, one for entering, and one for leaving, traffic. The actual designed width of the ramps has been changed from time to time, but in the later designs it was 24 ft. In some cases the gradients have been made a maximum of 5.5%, in order to shorten the length of the ramp.

As it was intended that no U-turns should be made at the head of the ramps, each center ramp would serve incoming traffic in one direction, and outgoing traffic in the opposite direction only. The original plan of connections, therefore, contemplated that alternate ramps should serve the traffic in alternate directions, but in certain cases others were subsequently introduced at important street crossings.

This was done, for example, in Jersey City, where State Highway Route No. 1 leads from the George Washington Bridge south to Bayonne. When the highway was designed originally, the intention was to provide a ramp connection at this point for eastbound entrance and westbound exit. It was intended also to construct a surface highway leading west from the bottom of the ramp, so that, in fact, there would be a surface grade crossing at this point. As it was expected that the traffic would be heavy, a traffic circle was designed for the surface intersection to minimize as much as possible the inevitable delays as well as the chances of accidents. The diameter of the center line of the circular roadway was made 240 ft, which is rather small, but local conditions did not permit a larger diameter without heavy additional cost, and with a simple crossing this diameter should have been sufficient for satisfactory vehicle operation. A diagram of the traffic circle, with the originally contemplated street connections indicated in full lines, is shown in Fig. 5.

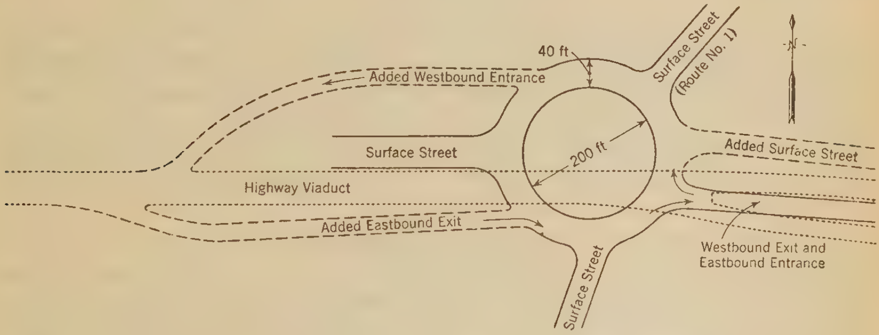


FIG. 5.—TRAFFIC CIRCLE, JERSEY CITY, NEW JERSEY

In the meantime, however, an additional street connection was deemed desirable on the east side (see dotted lines, Fig. 5), the introduction of which decreased the weaving distance necessary for proper vehicle operation, and, consequently, the efficient use of the circle. Later, it was decided further to add two side ramps from the west side of the traffic circle to the highway viaduct so as to provide a direct westbound connection with Route No. 1, in addition to the eastbound connection (see dotted lines, Fig. 5).

It is quite apparent that with these additional roadways the traffic circle cannot function as originally intended; in fact, traffic control by signal lights and traffic officers has become necessary. With this control the flow of traffic is reasonably satisfactory, but it is probable that the circle would have been made larger or that some other arrangement would have been made, if all the roadway connections had been considered at the time of the original design.

There has been some doubt as to the advisability of using center ramps rather than side ramps, mostly on account of the delay that may occur when a slow-moving truck arrives at the head of the ramp and attempts to weave through the fast-moving inner-lane traffic to the lane alongside the curb. No doubt, some delay may occur at times; but, on the other hand, the traffic probably moves more smoothly if the curb line is not interrupted, as would be necessary if side ramps were used. In addition, the cost of the work is often materially increased with side ramps, particularly where the highway profile is high above the surface level.

Pile Foundations

As indicated in Fig. 2, the sub-surface conditions varied from place to place throughout the line of the highway. When rock or other firm materials was found near the surface level, the foundation problem usually was simple. Otherwise, it was frequently necessary to use either timber or concrete piles for the support of the foundations. Concrete piles (either pre-cast or cast in place) were used where necessary, either to carry greater loads, or to extend beyond the ground-water level.

Pre-cast piles were usually 16 in. square, with beveled corners. Cast-in-place piles were used in the foundations of the piers for the viaduct portion of the highway across the Jersey City meadows east of the Hackensack River. Where the lengths required were in excess of the maximum available for this type, composite piles were used, having a lower pre-cast section and an upper cast-in-place section.

Batter piles were required whenever there was any question as to the lateral stability of the foundations, but merely as an extra precaution because foundations of this type were used only where the ground could be expected to resist such lateral pressures as might be developed.

In one case, special arrangements were provided in the design to prevent displacement of foundations on concrete piles due to lateral pressures that might be caused by mud waves in the surrounding ground. This was in the meadow section in Jersey City, east of the Hackensack River, where the ground surface is just about the level of mean high water. The upper 10 ft or more of the meadow is soft silt, covered with vegetable growths, the roots of which form a tough fibrous mat. Below the silt are various mixtures of clay and sand, extending to rock, which is about 60 or 70 feet below the surface level.

The concrete foundations for this section were designed to extend 10 ft or more below the surface level and to be supported on concrete piles. With this arrangement, the top of the piles would generally be below the silt and the underlying material was expected to provide proper lateral support for the full length of the piles. It was thought, however, that future disturbance of the overlying silt, caused by filling in the meadows, or by other construction work in the vicinity of the highway, might possibly cause lateral mud waves, which would tend to displace the foundations. For this reason it was decided to include in the contract for the foundations a design for the two earth dikes, one at each side of the right of way of the highway, but outside the foundation sites. The design provided that strips of the vegetable mat, 18 in. wide, should be cut and removed at each side of each dike prior to the placing of the fill. This was done to prevent the fill from being suspended on the tough vegetable mat and to insure its settling through the silt, displacing and consolidating it so as to form dams, which would prevent mud waves from reaching the foundations.

As the dikes were used as roadways to provide access to the working site while the work was in progress, their construction is not believed to have added to the cost. If they had not been included in the contract, the Contractors would have had to make similar fills in order to provide access to their work, and the cost would necessarily have been included in some of the pay items under the contract. On the other hand, by including the construction of the dikes in the contract as a pay item, this necessary work was under the control of the Engineer, and the fill was placed in such a manner that it served a useful purpose, not only during the construction period, but also in the permanent structure.

Caisson Foundations

Where soft ground without material lateral stability was encountered, caisson foundations were used instead of piles, as, for example, at the foot of the steep east slope of the hill in Jersey City. Except for some surface fill, the borings at this place indicated that the sub-surface material was soft silt to within 8 or 10 ft of the top of the underlying rock, which was about 70 ft below the surface level. The viaduct structure at this point was to be about 80 ft above the surface level, and rather long spans were necessary on account of the track arrangement of the railroad yard, over which the viaduct was to be built. Taking all these matters into consideration, caisson foundations were deemed advisable.

The foundations were designed as cylindrical concrete piers, some of which were 8 ft and some 10.5 ft in diameter, and individual piers were used for each viaduct column to be supported. The caissons were designed as hollow concrete cylinders with an outside diameter equal to that of the finished pier and having a minimum wall thickness of 2 ft and a minimum specified reinforcement. After the caissons had come to a proper bearing, the construction was completed by filling them with concrete so that the finished structure was in effect a solid concrete cylinder.

A more extensive use of caisson foundations was made in the construction of the highway viaduct between and adjacent to the Hackensack and Passaic Rivers. The sub-surface strata, as indicated by borings, are shown in Fig. 2. It will be seen that the ground surface throughout this section is at or near the level of the water of the rivers and that the materials overlying the rock are soft and unsuitable for foundations. The surface of the rock, which at the east side of the Hackensack River is about 80 ft below the ground surface, rises rapidly toward the river. The original rock line at certain points under the river was perhaps less than 20 ft below the water level before the channel was deepened. West of the Hackensack River there is a rapid drop of the rock level which, near the west shore, is more than 90 ft below the water level. This somewhat peculiar formation appears to be due to a subterranean rock ridge, running in a north and west direction and intersecting the river at the location of the highway. The rock surface continues to drop until, at one point about midway between the Hackensack and Passaic Rivers, the level is about 140 ft below the ground (see Fig. 2). From this point the rock begins to rise again, so that at the crossing of the Passaic River the depth is about 60 ft, and beyond this river the elevation is from 50 to 60 ft below the surface.

Owing to the sub-surface conditions it was deemed necessary to use caisson foundations throughout the section from the east side of the Hackensack River to the west side of the Passaic River, and the design for the foundations was prepared accordingly. The drawings, however, did not show the details of the design of the caissons, but only the outlines of the finished work required, namely, concrete piers of certain horizontal cross-sections and extending from a given elevation near the ground surface to such a depth as would be necessary to attain proper bearing on sound rock. The reason for not showing the details was that, after all, it is the finished product that is desired, and as long as the work is finished in a safe and suitable manner, the method is generally of minor importance. In this way the individual contractor was also given an opportunity to design and construct the caissons according to such methods as his experience dictated might be most suitable and economical. The specifications provided, however, that,

> "Detail drawings and descriptions of the caissons and of the methods and details of construction, which the contractor proposes to use, shall be submitted to the engineer for his approval before the proposed work is commenced."

The caisson work for this section was divided into two contracts and the two contractors submitted different designs for the caisson construction according to their individual preferences. One of them proposed a complete wooden caisson, with the idea of placing the caisson concrete during the sinking operation, and leaving the wooden sheathing permanently in place. The other preferred to construct section after section of a concrete caisson, removing the wooden forms before sinking. Both designs were acceptable to the Engineer, and in both cases the work was done satisfactorily.

For this work, as well as for all other caisson work, the specifications stipulated that compressed air should be used when the cutting-edge of the caissons had reached a level a few feet above the rock surface, and that the caissons should be sealed to rock under compressed air. When the design was being prepared, consideration was given to the fact that, for some of the deeper caissons, the air pressure needed to seal the caissons might be greater than that permitted by law, if the ground should be found to be of an open character. An examination of the boring samples had indicated, however, that the material to be penetrated would be of a stiff, clayey, and impermeable character when exposed to the air pressure. It was believed, therefore, that if the work were done properly, the necessary air pressure would be materially less than the theoretical hydrostatic head indicated by the depth and that it would be within the legal limit. This belief was found to be correct, because the maximum air pressure used at any time during the construction of the caissons was 48 lb per sq in., and in the greater number of cases it was considerably less. The method of construction was to sink the caissons by open dredging until the cutting-edge was near the rock surface. The wells were then roofed over, and the remainder of the excavation was done under compressed air.

Owing to the depth to which the caissons were to be sunk, as well as to the height of the superstructure and to the character of the material to be penetrated, all the caisson foundations between the east side of the Hackensack River and the west side of the Passaic River were designed so as to provide a single pier for each viaduct bent. For the greater number of the piers the over-all horizontal dimensions were 25 ft wide by 60 ft long, while the piers for the main spans across the two rivers were 30 ft wide by 87 ft long.

On the west side of the Passaic River the conditions were somewhat different, as the average depth to rock was only about 60 ft, and the material overlying the rock was of such a character that it might be expected to afford some lateral stability. For these reasons each bent foundation was designed as two separate caissons, connected by means of a heavy concrete tie just below the surface level. The average size of these caissons was 14 ft wide by 16 ft long.

As the sub-surface conditions, as disclosed by the borings, indicated that either piling or caisson foundations might be used for this section, alternative designs were prepared and advertised for bids, one design involving caisson construction and the other concrete piling. The low bids received for the two plans were within 2% of the same amount, the bid for the caisson construction being the lower.

Subway Structure

As previously stated, the part of the highway that crosses the hill in Jersey City is built as a subway under the level of the existing surface streets (see Fig. 6). It was located north of, and quite close to, both the old Erie Railroad Tunnel, and the more recent cut made by the Erie Railroad Company for its passenger trains (see Figs. 1 and 2). In fact, this location was selected largely in deference to the public opinion of the City of Jersey City, which considered the Erie Cut as a scar across the face of the city. It was desired, if another cut were to be made across the hill, that this be adjacent and parallel to that already made, and the location was determined accordingly.

Borings made previous to the design of this subway indicated that the ground immediately underlying the surface consisted of clay and sandy clay; below this there was rock. As an average, a little more than one-half the depth of the subway excavation was indicated to be in soft ground, and the remainder in rock. The design provided for a concrete wall at the north side of the structure, the upper part of which was designed as a retaining wall for the soft ground, and the lower part as a facing wall for rock.

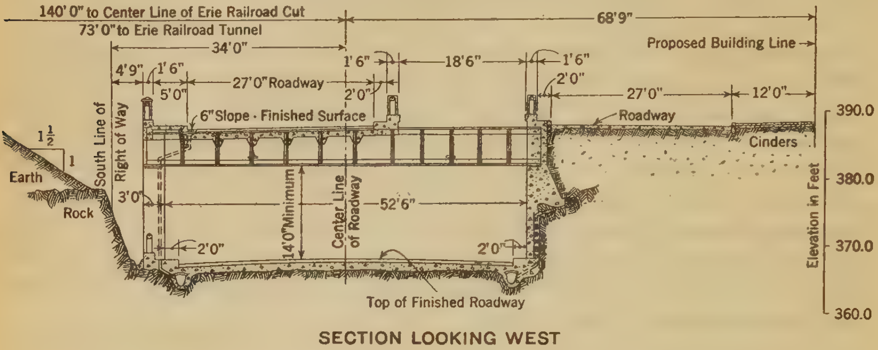

140' 0" to Center Line of Erie Railroad Cut
73' 0" to Erie Railroad Tunnel
68'9"
Proposed Building Line
34'0"
4'9"
1'6"
5'0"
27'0" Roadway
1'6"
2'0"
18'6"
1'6"
2'0"
27'0"
12'0"
6" Slope · Finished Surface
Roadway
Cinders
1½
1
Earth
Rock
South Line of Right of Way
3'0"
14'0" Minimum
Center Line of Roadway
52'6"
2'0"
2'0"
Top of Finished Roadway
Elevation in Feet
390.0
380.0
370.0
360.0
SECTION LOOKING WEST

Fig. 6.—Cross-Section of Subway, Jersey City, New Jersey

In order to provide proper ventilation, the excavation was sloped away from the highway at the south, but as it was desired to carry a surface road over the subway, concrete columns were provided at intervals on this side to support the roof of the subway. To provide additional ventilation facilities the roof was designed so as to cover only a part of the full width of the highway. All the steel work of the subway roof was designed to be covered with concrete, and reinforced concrete balustrades were provided around the ventilation opening at the north side of the structure, as well as along the south edge of the upper roadway.

In addition to the surface roadway constructed above the highway, another surface roadway was built north of, and immediately adjacent to, the subway. This roadway is used for westbound traffic while that built above the subway is for eastbound traffic, and together the two roadways form a wide, well paved, and much needed boulevard connecting the east and west parts of the city.

EMBANKMENTS

Earth embankments were used for the highway structure either where the roadway elevation is slightly above the surface level (as, for example, west of Newark Airport, where the elevation is only a few feet above the level of the meadow crossed), or where the physical conditions warranted embankments rather than viaduct structures. A condition like this occurred in Newark, where for a distance of about 1½ miles the highway is parallel, and immediately adjacent, to an existing railroad embankment.

In order to avoid settlement the sequence of the construction work was usually arranged in such a manner that any embankment fill would be completed a year in advance of the pavement placed on it. Nevertheless, some settlement almost invariably occurs after the pavement is built, but usually this is not of any serious importance, except immediately adjacent to an abutment where an abrupt change of surface elevation will occur. To compensate for this change of elevation, a reinforced concrete slab was introduced at the abutment during the earlier part of the work, which was capable of bridging a distance of about 20 ft. The design of this slab provided that one end of it would be supported on the abutment and the other on the earth embankment, the theory being that the end on the earth embankment might settle, but the concrete slab would form a runway to the higher elevation of the abutment, which would cause no inconvenience to traffic.

This arrangement in some cases did not prove satisfactory, although it would function as contemplated. The difficulty was that the settlement would cause changes in the roadway profile, which were too great to maintain comfortable riding conditions. The fact is that fairly small settlements are sufficient to produce excessive distortion of the profile in the short length available.

In later designs no attempt was made to introduce this concrete slab, but instead, the permanent pavement was omitted for a distance of about 50 ft immediately behind the abutment, and a temporary bituminous surface was used in its place.

VIADUCTS

Although concrete viaducts were used in a few cases, where the span lengths were moderate, the greater part of the viaduct structures was built of steel, preferably steel plate girders. The largest girder was 144 ft long, 11 ft 4½ in. deep, and weighed 60 tons.

Expansion was provided for mostly by means of double columns at the expansion joints; in fact, no sliding joints were used, throughout the length of the highway, where it was possible to avoid them. There were two reasons for attempting to avoid sliding joints, the first being that they are subject to wear and consequent expensive maintenance. Where it was necessary to use them, bronze sliding plates were provided so as to reduce wear and corrosion as much as possible. The other reason was that the steel structures as a rule were covered with a concrete protection, and it was somewhat difficult under these circumstances to arrange a sliding joint which would have no water-pockets and which could be readily maintained.

The policy has been to cover steel structures with concrete or "gunite" in order to decrease the cost of maintenance. There were exceptions to this rule, however; for example, where through truss construction was used, the steel below the level of the deck was covered, while the part above was left uncovered and was painted. The principal exception, however, was the high-level and long-span viaduct between Jersey City and Newark. For this structure the additional steel required for carrying the weight of the protection would have increased the cost so materially that it did not appear to be warranted economically.

Generally speaking, concrete covering was used on columns, brackets, fascia girders, and other places exposed to view, while "gunite" was used in less exposed places. The reason therefor was that, in the earlier stages of the work, the latter was difficult to obtain with sufficiently pleasing outlines to permit its use where exposed to view. In later years, however, the art of placing has improved considerably, and at the present time this type of covering can be obtained with sharp, straight lines and even surfaces.

Members that were to be encased in "gunite," were designed with cross-sections to which the covering could be readily and economically applied, and the results indicated that material savings in cost could be made by suitable design of the members to be covered.

The thickness was generally specified as 1½ in., but this thickness can probably be materially decreased without impairing the value of the covering, and with considerable decrease in the cost, not only on account of the smaller amount of material used, but also on account of the smaller tonnage of steel required to carry the extra weight of this type of covering. Sufficient evidence has not as yet been gathered, however, to warrant a decrease materially below the standard thickness of 1½ in.

The high-level viaduct structure between Jersey City and Newark, for which the span lengths varied from about 200 to 550 ft, was designed as a deck-truss structure, except at the river crossings where through-truss construction was used. The structure throughout is a series of cantilever spans with intervening suspended spans. The deck structure consists (*a*) of cross-girders set on the top chords of the main trusses at their panel points, which are normally 25 ft apart; (*b*) of stringers framed into the cross-girders about 8 ft apart; and (*c*) of secondary cross-beams supported on the stringers. This arrangement was made to permit the main reinforcement to be placed lengthwise of the structure rather than transversely. The secondary cross-beams were spaced at varying distances apart ranging, in a 25-ft panel, from 3 ft 6½ in. at the ends to 4 ft 8½ in. at the center of the panel, so as to equalize the moments in the reinforced concrete roadway slab, which rests directly on the secondary cross-beams.

In order to get the necessary under-clearance across the dredged channels of the two rivers crossed, without undue rise and fall of the highway profile, the channel spans were designed as through trusses and, for the purpose of obtaining harmonious lines, the flanking spans were carried gradually from the higher level of the through trusses to the lower level of the deck trusses, as shown in Fig. 7. As the through trusses are 57 ft, center to center, and the deck trusses are 36 ft, center to center, it was necessary to make a horizontal break in the truss line of the flanking spans at the point where they reach the level of the deck trusses. This break was quite simple in design, and is not noticeable when the structure is seen at a distance; but from a point on the ground near the side of the viaduct, looking lengthwise to the point where the break occurs, it appears as if the curved lower chord is out of alignment. This condition was appreciated in making the design, but the only way that it might have been avoided was by tapering the trusses horizontally throughout the entire length of the flanking spans, which would have added materially to the cost of construction.

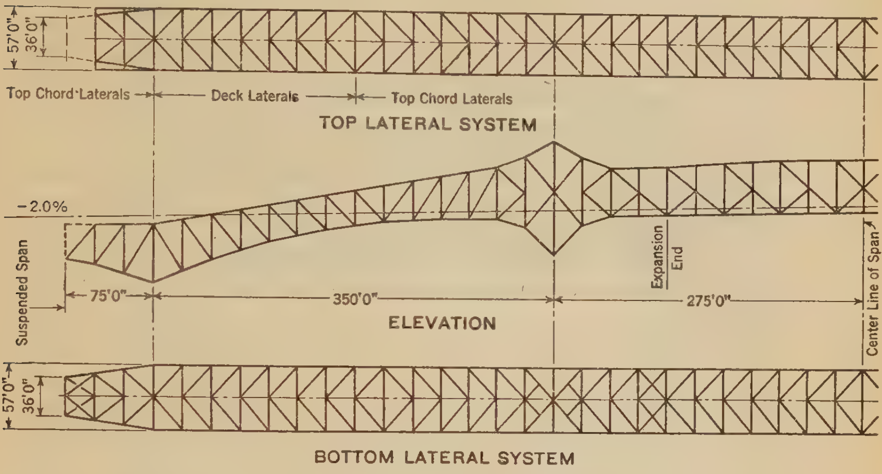


FIG. 7.—THROUGH TRUSS SYSTEM, FOR RIVER CROSSING

The trusses are supported on concrete piers by shoes of heavy steel castings with forged steel pins. On alternate piers the shoes are set on rollers to allow for changes in length due to changes in temperature.

The suspended spans are fastened at one end to the supporting cantilever arm by a hinged connection in the top chord. Actually, a simple riveted connection was used, because it was found that the deflection angle of the cantilever arm was practically the same as that of the suspended span. In other words, there is a point of contraflexure at the point of connection. The expansion end of the suspended span was supported by a vertical strut with pin connections at both ends. The reason for using a strut rather than a hanger was principally to maintain the same arrangement of diagonals throughout. If a hanger had been used, it would have been necessary to reverse the direction of the diagonal at this point.

The steel superstructure was designed to be supported on concrete piers. Each truss was supported on a separate shaft and, as there were normally two trusses to be supported, each pier bent had two pier shafts. At the location of the center ramps, however, where the structure was widened and was supported on three or four trusses, the number of pier shafts was increased accordingly. The shafts were designed with vertical outlines throughout their height, except near the bottom where they were flared out so as to form a base. Where the height was such as to require it, the shafts were connected with a horizontal strut, and, at greater heights, a truss was used instead of the strut. Both the truss and the strut were designed as steel members encased in concrete, but in order to conceal the steel outlines for the sake of appearance, the concrete covering was made so heavy that when finished, they had the appearance of masonry rather than of steel covered with concrete. Fig. 8 shows one of these pier bents.

All viaduct structures were designed for an H-20 loading. For the high-level viaduct between Jersey City and Newark the specifications used were those of the American Association of State Highway Officials for Steel Bridges, which were published about the time this work was begun. The specifications used prior to that time were those of the American Railway Engineering Association.

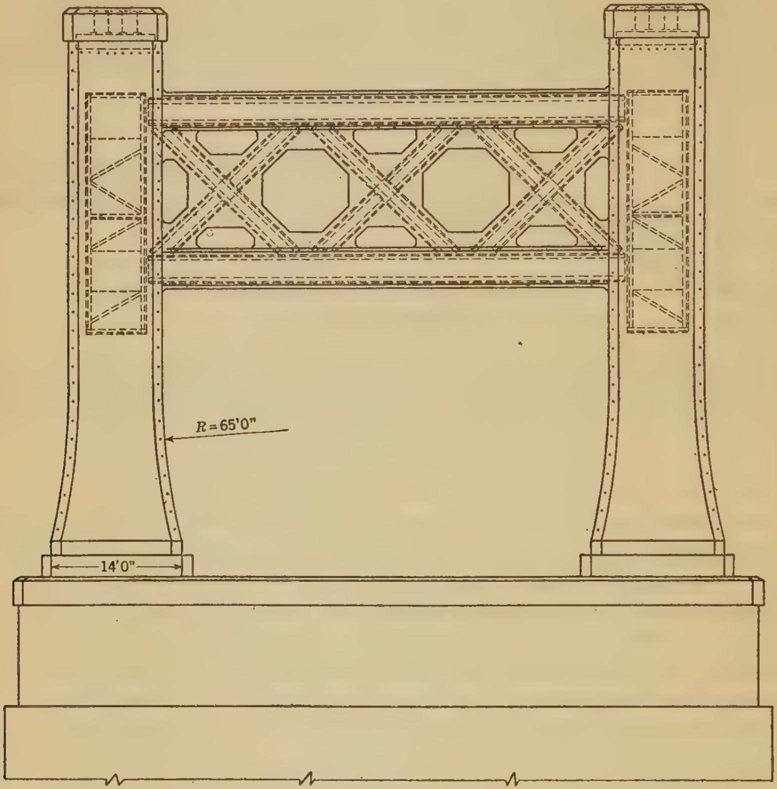


FIG. 8.—CROSS-SECTION OF PIER BENT, FOR STEEL SUPERSTRUCTURE

EXPANSION JOINTS

For the high-level steel viaduct structure just described, the expansion joints in the main structure are normally about 600 ft apart, requiring provisions to be made at the joint for a movement of 6 in., or more. The type of expansion joints designed for the roadway slab is shown in Fig. 9, Type 1. It consists of two sets of steel castings provided with a series of parallel fingers, each about 1¾ in. wide, and deep enough to carry the load that may come upon them. One set of these "finger castings" is placed at the one side of the expansion joint and the other set at the other side so that the fingers intermesh and provide a continuous surface at any position the structure may take due to expansion or contraction. The finger castings, which extend all the way across the roadway, were designed to be made in sections from 4 to 6 ft in length, and the openings between the teeth were about $\frac{1}{4}$ in. wider than the teeth so as to insure free movement of the joint. Similar castings were designed and used for the joints in the curb and sidewalk.

For viaduct structures of shorter span lengths, and where it was desired to provide tight joints, several types of expansion joints were used, but were changed in subsequent designs because the earlier designs did not prove to be as satisfactory as desired. The earliest type (Fig. 9, Type 2) was designed for the surface roadway over the subway in Jersey City. This type was found to be unsatisfactory for several reasons: First, the rivets in the checker-plate would wear loose, causing the plate to rattle in an undesirable manner, whenever a vehicle passed over it; and, furthermore, in certain cases, the entire expansion construction at one side of the joint would wear loose. This was due to the asphalt roadway surface wearing down adjacent to the concrete behind the steel expansion joint. The result was a constant pounding of the concrete by wheels striking against the exposed edge and forcing it against the steel of the expansion joint, which would gradually pulverize and destroy the concrete. Finally, the gutter between the twin girders would fill up with dust and dirt, and difficulty was experienced in removing this accumulation. It was possible for a man to enter the space between the two girders for the purpose of cleaning, but he could not make a visual inspection of it; all he could do was to feel his way along.

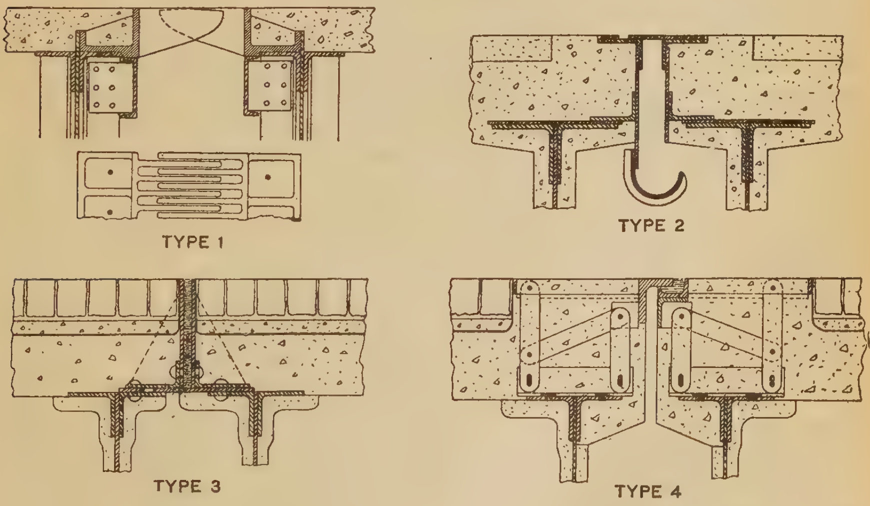

TYPE 1
TYPE 2
TYPE 3
TYPE 4

FIG. 9.—TYPES OF EXPANSION JOINTS

For subsequent work, therefore, the expansion joint, shown as Type 3 in Fig. 9, was designed. This type was found not to be entirely satisfactory either, partly because of the difficulties of making an adequate enclosure at the ends of the joint, and partly because of the large volume changes that would occur in the space filled with mastic whenever the joint opened or closed due to temperature changes. If the joint opened, the mastic would drop below the roadway level, and if it closed, large quantities of the mastic would be squeezed out. Furthermore, it was difficult to prevent the comparatively large mass of mastic from seeking a level surface, with the result that it flowed out above the roadway surface near the curb lines and dropped below at the crown of the roadway.

The expansion joint shown as Type 4 in Fig. 9, was developed finally. It consists essentially of a small steel channel set at one side of the joint with the flanges turned up. One flange extends to the surface of the roadway and the other is cut down so as to permit the horizontal flange of a heavy steel angle, attached to the other side of the joint, to slide over it with its top surface at the roadway level. The channel is filled with mastic. The advantage is that only a small volume of mastic is needed to fill the joint, and the greater part of this is unaffected by the changes of volumes caused by temperature changes.

In order to anchor this joint to the structure, and to reinforce the roadway surface at the joint, flat steel bars, the top edges of which conformed to the roadway surface, were welded to the channel and angle. Light frameworks of flat bars were provided as shown, which not only were used for anchorage, but were also used to secure the expansion joint in place before the concrete was poured. This expansion joint has been in service for several years and has been found satisfactory.

Roadway Surfaces

At the beginning of the work the policy was established that the roadway surface should be paved with granite blocks, and this policy was followed for several years; but it was modified subsequently, and several other types of pavement were used on the highway. Surface roadways paved with granite blocks were designed to be laid on a concrete foundation. In the early designs, the foundation, due to the expected heavy truck loads, was made 10 in. thick, with top and bottom reinforcement. In later designs, this was changed to a concrete slab, 8 in. thick, without reinforcement, as this appeared to be sufficient where the sub-surface was firm.

The granite blocks, which have an average depth of 5 in., were set on a cement mortar cushion with a mastic filler in the joints. The surface was parabolic in cross-section, with a 6-in. crown for a 50-ft roadway. The mastic filler was not entirely satisfactory in all cases, because in hot weather it had a tendency to flow and cover the stone blocks, and eventually found its way to the low level of the roadway at the curb. For this reason later designs provided for a cement-mortar filler, except at intervals of about 30 ft, where three or four cross-joints, as well as the adjacent longitudinal joints, were filled with mastic. This type of construction has been used for only a comparatively short time, so that nothing definite can be said about it; thus far, however, it appears to be satisfactory.

In certain places where the highway occupies previously existing streets, the roadway was paved with a bituminous concrete surface on a concrete foundation. This was in deference to the wishes of authorities in the local municipalities; but it has a tendency to become wavy under the heavy traffic. It has the advantage, of course, that whenever openings must be made in it for construction or repairs of sub-surface utilities, it can be replaced without leaving an appreciable mark on the surface. On the other hand, as the work has been designed so as to keep sub-surface utilities outside the area of the paved highway, as far as possible, there should be little if any reason for making openings in the pavement.

A bituminous concrete pavement on a macadam foundation was found suitable as a temporary expedient when placed on fill that has not been entirely compacted. Of course, it does not remain smooth under heavy traffic, but it appears to stand up for a few years without undue maintenance. In some parts of the earlier work attempts were made to use surface-treated macadam for such temporary pavements, but it would not stand up under the heavy traffic.

A concrete pavement was adopted for the section adjacent to Newark Airport. The slab was made 10 in. thick, with top and bottom reinforcement. This heavy construction was used because there was some doubt whether the fill on which it was to be constructed, had finished settling. At other places, where the sub-surface conditions were more favorable, concrete pavements were designed with a thickness of 8 in. and a single top reinforcement. They were built with a V-crown and with a side slope of 1½ in. in 10 ft. There has been a tendency to decrease the slope to about 1 in. in 10 ft.

Where granite block pavement was used on viaduct structures, the concrete floor-slab was water-proofed and the water-proofing was protected with a layer of mortar or concrete. The pavement was placed on this protective course in a manner similar to that used on surface roads.

An asphalt block pavement was adopted on a part of the highway viaduct in Jersey City, largely for the purpose of comparing this type with others. In this case, the concrete floor-slab was water-proofed in a manner similar to that specified for granite block pavements.

For the high-level viaduct between Jersey City and Newark, considerable study was given to the selection of the most suitable type of pavement. Granite block pavement was not favored on account of its heavy weight, and, for a time, it was not considered advisable to use a type of pavement for which the wearing surface was an integral part of the carrying slab. For that reason serious consideration was given to paving the roadway with some asphaltic surface, on top of the concrete floor-slab. In the meantime, several of the bridges built by the Port of New York Authority and other important bridges had been provided with a concrete surface integral with the floor-slab, and the results appeared to be quite satisfactory. Based on this evidence, as well as on that furnished by the many concrete roadway surfaces in the State of New Jersey and elsewhere, it was decided finally to use a concrete roadway surface integral with the concrete floor-slab.

While this study of pavement type was under way, the design and fabrication of the steel structure had progressed so far that it became necessary to establish a definite slab thickness, before the exact type of floor-slab had been decided upon, and, for that reason, a thickness of 8½ in, was adopted as being that which might be used for a standard reinforced concrete slab.

Following this an intensive study was made of the various types of slabs that might be used, and several types were found suitable. The choice, therefore, depended on which would prove to be the most economical, and, for that reason, designs for four alternative types were prepared and submitted for bids.

Type 1 (Fig. 10), which was chosen eventually, is a concrete slab, 8½ in. thick, reinforced with fabricated trusses, composed of round bars welded together. Essentially, it consists of a top and a bottom chord of round bars with a web member made of a bar bent so as to form the diagonals of the truss, the various pieces being connected by pressure welding. In some cases the trusses are fabribated with single bars for top and bottom chords and with the web member placed between the two chords, while in other cases each of the chord members consists of two bars welded to the sides of the web member at its panel points.

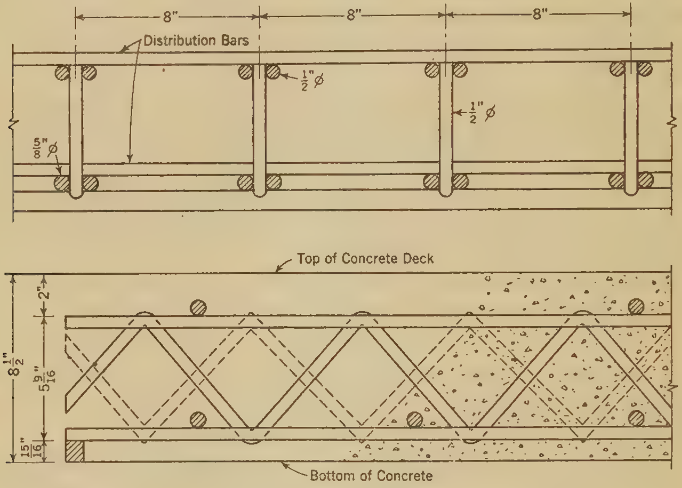


Fig. 10.—Cross-Sections of Types of Floor-Slabs for Roadway Surfaces

Type 2 was a series of inverted 5-in. T-beams, or half 10-in. H-beams, set closely together and tack welded so as to form a solid steel surface at the bottom of the slab, and filled with concrete to a height of 2 in. above the top of the T-beams. This type of slab reinforcement is usually fabricated in depths ranging from 2½ to 3½ in. The 5-in. depth was required because it was thought that while the slab of a lesser depth might be designed to carry the required load, it was not entirely certain that the concrete in it would not be crushed, in time, by constant use.

Type 3 was a solid-bottom plate with ribs at intervals and a steel grating attached to them with its top surface at a height of 5 in. from the bottom of the plate; concrete was placed on top of the plate to a depth of 2 in. above the steel. Type 4 was a solid-bottom plate with welded trusses, 5 in. deep, attached as ribs, and concrete was placed to a depth of 2 in. above the top of the ribs.

In all four cases the reinforcement also involved transverse top bars secured in place in various ways and, in addition, transverse bottom bars were required for Type 1. Each type was carefully investigated and found to be suitable for the purpose. For Types 2, 3, and 4, however (which had an over-all depth of 7 in.), it was necessary to introduce fillers, 1½ in. deep, between the steel support and the floor-slab, in order to make them conform to the depth of slab for which the structure had been designed. This, naturally, added to the cost of these three types although, on the other hand, the volume of concrete required was less.

The trusses in Type 1 were designed to be spaced on 8-in. centers and to be 5 in. deep over all. The two top bars and the web member were ½ in., and the two bottom bars, ⅝ in., in diameter. Rectangular spacing bars, as shown, were welded to the cross-beams on which the trusses were supported, and round top and bottom distribution bars, extending the full width of the roadway, were provided, as shown. In order to facilitate the work of erection and to insure exact spacing of the reinforcement, it was required that the trusses should be assembled in mats prior to erection by welding them together by the spacing and distribution bars. The length of the mats was required to conform to the distance between the primary floor-beams, which distance normally was 25 ft. The width was not specified, but left to the option of the Contractor, subject to the approval of the Engineer. The width of the mats actually furnished by the Contractor was normally 10 ft, but it is probable that the erection would have been easier if the width had been about one-half that actually used. The reinforcement trusses were arranged to be placed longitudinally on the structure, because better wheel-load distribution could be obtained in this manner than with a transverse arrangement of the trusses.

The concrete for the slab was required to have a crushing strength of not less than 4 000 lb per sq. in., when 28 days old. Actually, the concrete furnished had an average strength of about 4 800 lb. In computing the strength of the slab, the upper 2 in. of the concrete was not taken into account. When the time comes that it may be necessary to renew the wearing surface of the roadway it will thus be possible to remove this top layer without affecting the carrying capacity of the slab, and in this way a new wearing surface can be placed. It was thought that this new surface might be either concrete, or some type of asphaltic surface—or possibly some entirely new type of surface pavement which has not as yet been developed.

Curbs, Sidewalks, and Railings

No provisions have been made on any of the viaduct structures for sidewalks for pedestrians. Provisions have been made, however, for footwalks about 2 ft wide on both sides of the roadway, to be used in connection with the policing and maintenance of the highway and for emergency purposes. Except on the high-level viaduct the curb height of these footwalks is normally 8 in. above the roadway.

The general details of the footwalk as used on all steel structures, (except on the high-level viaduct between Jersey City and Newark) are shown in Fig. 11. In the earlier designs no provisions were made for any support of the curb angle during construction, but it was found that the contractors had some difficulties in aligning the curb angles properly and in securing them during the pouring of the concrete. In later designs, therefore, steel supports were provided, as shown in Fig. 11, which furnished a direct connection between the curb angle and the steel structure. These supports materially facilitated the proper erection of the curb angles.

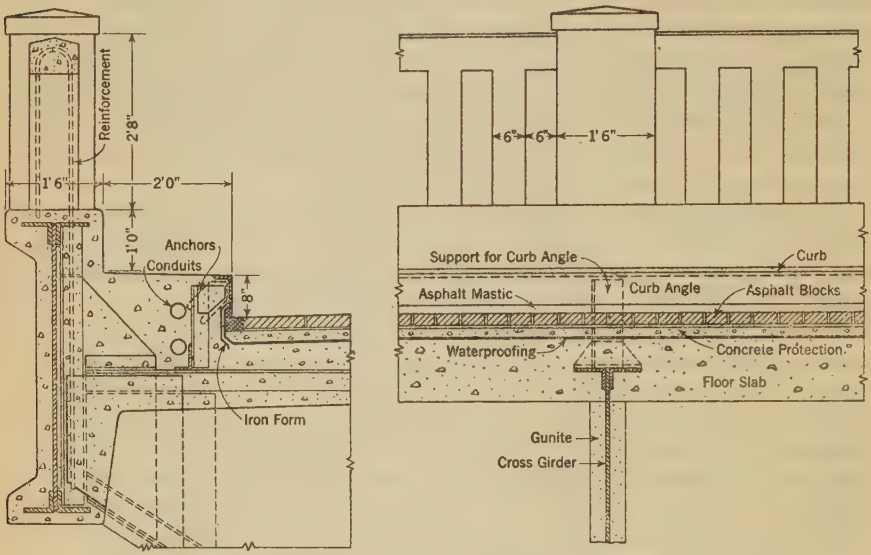

Reinforcement
2'8"
1'6"
2'0"
1'0"
Anchors
Conduits
8"
Iron Form
6"
6"
1'6"
Support for Curb Angle
Curb
Asphalt Mastic
Curb Angle
Asphalt Blocks
Waterproofing
Concrete Protection
Floor Slab
Gunite
Cross Girder

FIG. 11.—DETAILS OF FOOTWALK USED ON STEEL STRUCTURES.

For further facility of erection the design provided for the placing of a sheet-metal form reaching from below the curb angle to the top of the concrete roadway slab. In conjunction with the curb angle, this metal form usually made it possible to pour the concrete of the roadway and footwalk slab without additional form work, and further provided a means of flashing the roadway water-proofing against the face of the curb.

The railings on all viaducts were made of concrete as shown in Fig. 11. On account of the narrow footwalk, it was considered essential to provide a railing strong enough to withstand the impact of any vehicle that might accidentally mount the curb and strike against it. Therefore, it was made quite substantial, and every one of the vertical posts was securely anchored against overturning.

On the high-level viaduct it was appreciated that special precautions should be taken, not only to prevent vehicles from falling from the viaduct, but also to allay the fear of such nervous drivers as are not able to look down from great heights without a qualm. These considerations were particularly important on this structure because, due to the narrow footwalk, the roadway is quite close to the side of the structure.

For these reasons, the curb height was made 16 in. above the roadway surface, so as to make it practically impossible for a vehicle to mount the footwalk. Behind the curb and forming the footwalk a horizontal plate girder was attached to the main steel structure at each panel point of the supporting trusses and covered on top with concrete. The vertical face of the curb consisted of a steel plate attached to the flange of the horizontal girder, and this plate extended to within 5 in. of the roadway surface, leaving an opening to permit drainage over its edge.

The top of the railing was made 4 ft 5 in. above the footwalk level, or 5 ft 5 in. above the roadway level at the curb. With this height it is not possible for a driver of a passenger car to look over the railing to the meadows below and, if he is properly attending to his driving, he cannot look through the railing between the posts; in fact, the railing appears to him as a solid wall. On the other hand, if the car is moving at a reasonably fast rate of speed, the passengers in the car, who are able to look squarely at the railing, will be able to look through the railing as if there were no posts at all.

DRAINAGE

The profile of the highway has generally been laid on a minimum gradient of 0.5%, so as to provide for proper drainage of the roadway. In undeveloped territory the cross-section was designed so as to permit the water to flow over the sides and, when necessary, side ditches were provided to carry the water to existing drainage channels. In due time, however, it may be necessary to provide curbs, and storm sewers must be built at that time. Preliminary plans of the future sewer were prepared, therefore, and the cross-drains necessary according to these plans were constructed prior to the permanent paving of the roadways.

Where the highway was constructed on embankment with curbed roadways, a complete drainage system was provided and connected either to the municipal sewer systems or to specially constructed drainage systems. Frequently, in the earlier designs, a 12-in. pipe was used, but experience indicated that this size was rather too small, and during the later years the minimum diameter was increased to 18 in.

The catch-basin heads, used on curbed surface roads, have a curb inlet and a grating that is slightly dished for the purpose of collecting the water more effectively. This is the standard design of the Highway Department, but it might be better to make the gratings flat, because drivers of cars have the tendency to avoid driving over the dished gratings, with the result that 2 or 3 ft of the width of the roadway is wasted. On a viaduct, catch-basins of a similar type, but without curb inlet, were generally used.

The basins are connected with 8-in. cast-iron pipes attached to the columns of the structure, which carry the drainage to the sub-surface sewer system.

In order to carry these pipes from the catch-basin down the structure and past the column base it was frequently necessary to introduce several bends or other fittings in the run. As each fitting appears to be a potential location for stoppage of the flow of drainage water, and as each additional fitting rapidly increases the difficulty of cleaning the drain, the writer has considered in future work, where conditions will permit such an arrangement, to place the catch-basin some distance away from the column bent and to carry the pipe vertically down to the ground level. Of course, this would necessitate a special support for the pipe, but it would facilitate the maintenance of the drains materially.

In the subway, the drainage was extended to two longitudinal drains built under the sidewalks as shown in Fig. 6. These drains divert the water to the ends of the subway, where it emerges in the open on the slopes of the hill through which the subway section was built, and from these points the drainage water flows through pipes to the existing sewer systems.

ORGANIZATION

The work of which the design has been described, was carried out under the direction of the New Jersey State Highway Commission and the State Highway Engineer. W. G. Sloan, M. Am. Soc. C. E., was the State Highway Engineer from the inception of the work until 1929, when he was succeeded by J. L. Bauer, M. Am. Soc. C. E. Fred Lavis, M. Am. Soc. C. E., was in charge of the work from the beginning until 1928, when he was succeeded by H. W. Hudson, M. Am. Soc. C. E. The writer was Designing Engineer from the inception of the work to its completion.

DISCUSSION

Fred Lavis,[3] M. Am. Soc. C. E. (by letter).—So thoroughly has the author covered his subject that there is very little opportunity for comment or discussion. Nevertheless, emphasis may be placed on certain phases of the work, particularly in regard to those which affect engineers and engineering.

The author states that eight years elapsed from the "inception" of the work until its completion. These eight years of active work, however, were preceded by almost two years of preliminary studies[4] by an Advisory Board, under the Chairmanship of William G. Sloan, M. Am. Soc. C. E., the Highway Commission, and the State Legislature, and it seems worth while to emphasize the fact of this lapse of time and also of the almost incessant negotiations with all kinds of people and entities whose interests were affected throughout almost the entire period. The burden of these negotiations fell very largely on the Engineering Department of the State Highway Commission and on the counties and municipalities affected. The point to be emphasized is the importance of recognizing this as a part of engineering work and that this latter is by no means confined to computing the size of an I-beam or the proper mixture of concrete.

In the valuations of public utilities and other enterprises, due credit and allowance are seldom given for the time involved and the necessary costs of these factors; that is, first, the item of "Interest during Construction" and, second, the "Costs of Engineering."

It should be made quite clear, also, that there are two elements, or two major types, of design in a project of this nature: (1) The design of the route; and, (2) the design of the structure which carries the route. In both these designs the element of engineering expense is greatly increased by the need of meeting the numerous and varying demands of the many interested parties. It should not be thought that this engineering work and these demands are unnecessary or unavoidable, but their cost and the time necessary to meet them should be recognized as an important element in a project of this nature.

The engineers in charge of this work visualized its general nature, importance, and effect from the very beginning, as is witnessed by the report of the Advisory Board; but naturally these factors were not so clear to the public and other interested entities. An infinitude of details also developed as soon as actual surveys and designs were started, and their effect on the studies continued throughout the course of the work.

The writer has previously mentioned[5] the various routes through Jersey City of which more or less detailed studies were made. This was also true of the route through Elizabeth. In the matter of structures, a number of schemes, including a viaduct of reinforced concrete arches of varying length of span was worked out for the meadows crossing where now the high viaduct has been built. Fairly detailed studies were made for lift spans for the river crossings, one of which would have been, if built, the heaviest lift span in existence, and these are only instances of the many phases of the studies.

---

[3] Cons. Engr., New York, N. Y.

[4] Rept. of Advisory Board to the New Jersey State Highway Comm., August 8, 1923.

[5] *Transactions*, Am. Soc. C. E., Vol. 95 (1931), p. 1020.

It is generally true, as the author points out, that there is nothing particularly novel or unusual in any of the types of structures finally adopted; but the fact should not be lost sight of that the project as a whole was new and unusual and that it required not only great patience and not a little ingenuity in the negotiations. All these things required the expenditure of much time and considerable money.

It has been the writer's experience, and that of many engineers, that all this work of preparing sketches, plans, detailed studies, innumerable estimates of costs, etc., for various schemes eventually to be discarded, is apt to be entirely forgotten after the structure is completed; or, it is assumed that the engineers should have brought forth, full fledged, the final design at the beginning.

Even in private enterprises, where there may be less diversity of opinion, this is not possible or practical. It is much less so in public enterprises where State, county, and municipal officials, Chambers of Commerce, other public bodies, and the public itself, all have ideas, often many different ideas, to all of which some attention must be paid. It is quite difficult to obtain recognition by public officials of this phase of engineering, and the work involved in the development of these projects, and it seems important, therefore, to record this and to give due recognition to the fact that the State Highway Commission of New Jersey and the State Highway Engineer never failed to give those in charge of the project complete support and full recognition of the work required, and which was accomplished, in working out the problems of this important enterprise.

THEODORE BELZNER,[6] AFFILIATE, AM. SOC. C. E. (by letter).—The author has covered his subject so thoroughly that discussion is limited to a few remarks on the subject of expansion joints.

The writer has had occasion to observe the behavior of the expansion joints shown in Fig. 9, and of many other types, all of them giving similar results, under heavy vehicular traffic.

Of them all, the writer believes firmly that Type 4, Fig. 9, is not only an ideal, efficient, and economical one from a maintenance point of view, but that it is the most serviceable type for heavy vehicular traffic on viaduct structures.

J. B. FRENCH,[7] M. AM. SOC. C. E. (by letter).—The clearness and value of a paper on such an extensive and varied piece of construction are largely dependent on the good judgment with which the descriptive and illustrative material is selected to emphasize the salient and most important features without the inclusion of too much subordinate detail; and this paper deserves high praise on this score.

---

[6] Insp. of Steel and Bridge Insp. in Chg., Brooklyn Bridge, Dept. of Plant and Structures, City of New York, Brooklyn, N. Y.

[7] Cons. Engr., New York, N. Y.

Due to the almost unique importance of this "super-highway" as a trunk line of intense motor-vehicle traffic between New York City and the South and West, structural details, otherwise worth description, are quite overshadowed by questions of alignment, grades, roadway widths, and wearing surfaces, and all those details which have the most direct effect in facilitating the safe, fast, and uninterrupted flow of traffic on the elevated structure itself and which, at the same time, cause the least possible interference with the street, railroad, and water-borne traffic below.

These considerations, of course, enhance, rather than reduce, the importance of adequate foundations, solid and enduring types of construction, and the reduction of maintenance operations (particularly those that are likely to interrupt traffic) to the lowest practicable limits, and the paper gives ample evidence that all these matters have had competent and thorough-going attention.

In the viaduct structures, the use of heavy reinforced concrete floor, curb, and railing construction, the encasing in concrete of all structural steelwork below the roadway level (except in the case of the long-span trusses), the careful attention given to the collection of the drainage on the roadways, and the carefully designed expansion joints to insure smooth riding at all temperatures—all these appear admirably designed to meet the requirements imposed and to satisfy the standards of up-to-date practice.

In noting the omission of concrete covering and the painting of the steelwork above the roadway level and in the long-span trusses, the author justifies the procedure by stating that "the additional steel required for carrying the weight of the protection would have increased the cost so materially that it did not appear to be warranted economically;" and the question naturally arises whether, after making the painting of steelwork on a fairly large scale a necessary maintenance operation, it would not have been consistent to apply the same reasoning further, and thereby to have reduced proportionately the cost of both the superstructure and the foundations.

It also seems pertinent to point out, without criticism of the construction described in this paper, that recent advancement in the design and construction of open-grid metal flooring; in the use of light-weight non-corrosive metal alloys in floor construction; and the more extensive and successful use of welding processes, will, it is to be hoped, make practicable much lighter and less expensive floor, curb, sidewalk, and railing construction without involving increased maintenance cost.

Another economy that would obviously follow the adoption of open flooring would result from allowing rain and snow to go through the floor, like a sieve, in practically no more concentrated form than if the floor were not there, thereby eliminating the costly necessity of keeping acres of roadway surface water-tight and of providing a complicated and expensive system of catch-basins and water pipes, over at least large parts of viaducts where the use of the land or water below needs no special protection from the weather.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 1894

PRACTICAL RIVER LABORATORY HYDRAULICS

By Herbert D. Vogel[1], Assoc. M. Am. Soc. C. E.

With Discussion by Messrs. Lorenz G. Straub, Paul W. Thompson, Ralph W. Powell, K. D. Nichols, Frank W. Edwards, I. H. Patty, Charles S. Bennett, Kenneth C. Reynolds, V. V. Tchikoff, Samuel Shulits, Charles D. Curran, J. B. Egiazaroff, and Herbert D. Vogel.

Synopsis

The science of laboratory hydraulics as applied to determinations of problems relating to the control of rivers is of only recent origin, although hydraulic experiments of other kinds have been common for many years. Since the beginning of the Twentieth Century, however, considerable progress has been made, and while much still remains to be learned of limiting laws and conditions, it appears, nevertheless, that the time has come to take stock of the practical information available.

It is the purpose of this paper to summarize, as briefly as possible, existing data and to describe proven methods applicable to the solution of river hydraulics problems in the laboratory. By so doing, it should serve as a basis of procedure for the laboratory technician and as a guide to the field engineer who wishes to know what specific types of problems may be reasonably subjected to laboratory treatment.

Introduction

The utilization of models as a means for the determination of full-scale performances was conceived nearly 250 years ago when Sir Isaac Newton in his monumental treatise, the "Principia," stated in a single paragraph the Principle of Similitude. For more than 175 years this idea lay dormant; then it was seized upon by Froude, an English Naval Engineer, who applied it to studies aimed to ascertain the best design for a ship's hull. Numerous

---

Note.—Published in November, 1933, *Proceedings*.

[1] Lieut., Corps of Engrs., U. S. A.; The Command and Gen. Staff School, Fort Leavenworth, Kans. (Formerly Director, U. S. Waterways Experiment Station, Vicksburg, Miss.)

models were constructed by Froude, and in a narrow, water-filled tank they were towed at varying velocities, while accurate measurements were made with a view to determining the tractive resistances encountered. These experiments with moving objects in still water quickly suggested the possibility of investigating, through the use of small-scale replicas, the effects of flowing water on stationary objects, and, similarly, the effects of engineering structures on the regimen of rivers.

In 1875, the first true river models were constructed in France by Fargue, who attempted to determine effects resulting in prototypes a hundred times as large as the miniatures with which he dealt. His models were of the Garonne River and his problem was to determine methods of improving the channel of that stream for navigation. Ten years after the experiments by Fargue, Professor Osborne Reynolds, of the University of Manchester, Manchester, England, invoked the Principle of Similitude by utilizing scale models in the investigations of flow in a tidal estuary of the River Mersey, at Liverpool. About a year later (1886), Sir William Vernon-Harcourt applied these principles in experiments for the improvement of navigation near the mouth of the River Seine in France, but his investigations, like those of his predecessors, were conducted haphazardly, without suitable equipment and without a thorough understanding of the basic principles of model research.

A renaissance era in the science of river research was initiated by Professor Hubert Engels, of the Technical University at Dresden, Germany, at the beginning of the Twentieth Century as the result of a visit to America a few years earlier. Professor Engels began his investigational work in 1891, but at that time possessed no laboratory and no organized staff for research. In the University of Michigan, at Ann Arbor, Mich., he found an embryo hydraulic laboratory with demonstration equipment then used for classroom instruction, which so inspired him that he returned to his own University with definite plans in mind for a permanent and well-ordered hydraulic research institution. His first laboratory was a modest affair installed in the basement of the University in 1898. Here, he gradually increased his range of experiments and a few years later rebuilt his laboratory in larger form. The utility of the methods which he introduced quickly attracted the attention of other German engineering colleges, where new and larger laboratories were built—guided often by the friendly personal counsel of Professor Engels.

In 1903 there appeared on the horizon a new personality destined to influence largely the development of hydraulic laboratory methods and the promulgation of the laboratory idea. This was Dr. G. de Thierry, who, in that year, received his appointment as Professor of Canal and Harbor Engineering and of Hydraulics at the Technical University in Berlin, Germany. Professor de Thierry founded an efficient student laboratory at his institution of learning and, assisted by Professor Ludin, of the same University, did much toward promoting the development of the Experiment Institute for Hydraulics and Naval Architecture in Berlin. This Institute built in 1902 and 1903 by the Prussian Government, carries definitely associated with it the name of Hans Detlef Krey, who, until his death in 1928, stood pre-eminent in the field of practical hydrodynamics. Under the supervision of Professor de Thierry and the sponsorship of the late John R. Freeman, Past-President and Hon. M. Am. Soc. C. E., the monumental treatise, "Wasserbaulaboratorien Europas," came into being in 1926 as a summary by leading directors of the accomplishments of their respective laboratories. The same book translated into English and published in 1929 under the title, "Hydraulic Laboratory Practice," stands to-day as a leading reference in the subject of hydraulic research. Because of the comprehensive manner in which this work has treated the subject it is deemed unnecessary to elaborate here on the details of the many laboratories now existent in the world as offspring of the parent institution founded by Professor Engels. Suffice it to say that the idea has been received with considerable favor by engineers and that the mathematical doctrine stated somewhat obscurely by Newton, in a single paragraph, has been expanded and elaborated to cover a wide range of problems, from turbine design to the improvement of harbors. Researches carried on by means of the principle relate to: (*a*) The improvement of both the efficiency and capacity of hydraulic turbines; (*b*) the finding of remedies for the evils of cavitation in turbines; (*c*) improvements in efficiency and capacity of centrifugal pumps and propeller pumps; (*d*) the improvement of the designs of spillways for water-power dams; and (*e*) the designing of great sluice-ways so as to present a minimum loss of head. Another important problem is that of finding the best means of dissipating energy of flood waters flowing over a high dam where it is tending to cause damaging scour in the river bed below the dam.

The scope of hydraulic laboratory practice has been extended since about 1918 to include studies of bed movement in rivers, and methods of changing or influencing such movement; to investigations aimed at the determination of effects of cut-offs in large alluvial rivers; to experiments dealing with means of preventing damage to railway embankments during overflow by flood waters; and to hundreds of similar studies and investigations. In spite of this rapid expansion of the utility of the hydraulic laboratory, it appears probable that its greatest service to hydraulic science remains yet unperformed. The practical field engineer has had a new and most important tool placed at his disposal, but until he has made further advances toward mastering the technique of this science and adapting it to his purposes, it will not fulfill its full potential value. For this reason it is of the utmost importance that field forces be encouraged to maintain constant contact with tests in progress at the laboratory and that they be trained to observe, for themselves, the model phenomena and their interpretation in terms of Nature. Such observations may be of much greater benefit than a formal written report of the tests.

HYDRAULIC SIMILITUDE

General.—If two geometric figures are drawn so that all corresponding lines are of the same ratio and their corresponding angles are of equal degree, it will be found that their areas bear a fixed relationship, one to the other, that may be expressed by the square of the basic linear ratio. The two figures are then said to be similar, and from the one may be determined all the properties of the other. As defined by the dictionary, for a geometric sense, similitude is "the relation of identity between two figures irrespective of magnitude."

From an hydraulic standpoint, strict geometric similarity may often be subordinated to the necessity of preserving other, more important, relationships, and hydraulic similitude has been defined as referring to those "conditions which must be observed in experiments with a model in order to reproduce faithfully the phenomena occurring in full-scale performance." Where then, in such cases, a sacrifice of true geometric similarity becomes necessary it is on the basis of preserving dynamic or kinematic similarity as a means toward obtaining hydraulic similitude.

True geometric similarity implies undistorted linear relationships; that is, it requires that all corresponding linear dimensions in the full-scale and model must be in the same ratio. More generally speaking, however, it may be said that a kind of form similarity exists even in cases where the model is distorted, provided the distortion is a result of some definite and consistent plan.

Dynamic similarity requires that at corresponding times in full-scale and model the accelerations of corresponding mass elements must be in constant ratio to each other, and must be directed in the same relative directions. Since it is physically impossible to modify the effect of gravity in a model, the same reduction of accelerations, according to a fixed ratio, will fail to satisfy completely the foregoing requirements of dynamic similarity, but from the relationship existing between force and the product of mass and acceleration, it is evident that with corresponding forces bearing a constant ratio to each other, such similarity may be attained by making corresponding accelerations equal in model and prototype. Briefly stated, therefore, dynamic similarity is governed by the requirement that at corresponding times any two corresponding forces in full-scale and model must be in the same ratio, and to obtain this condition the conditions for mechanical and kinematic similarity must also be satisfied. The first of these requires corresponding weights in the two systems to possess a definite fixed ratio one to the other; the latter demands that the full-scale and model must be geometrically similar in all respects, at corresponding times; or, in the case of hydraulic models, the elements of flow must simulate corresponding elements in Nature with respect to both direction and time.[2] Difficulties naturally arise in the interpretation of results obtained from model studies because it is almost impossible to simulate all conditions and properties of Nature. Theoretically, this might be done in certain special cases if it were possible to control independently, and at will, all the properties of the materials in the model as well as the forces involved. These properties include the density, viscosity, and surface tension of the fluid used in the model; and the size, shape, cohesion, and density of the solid materials. The external forces include gravity, friction, and barometric pressure.

---

[2] See "Die Grundlagen der Aehnlichkeitsmechanik und ihre Verwertung bei Modellversuchen, unter besonderer Berücksichtigung Schiffbau technischer Anwendungen," von M. Weber, Charlottenburg, Germany, 1918. For a slightly different conception refer to "Strömungslehre der Rohre und Offene Gerinne," von Schiller und Eisner, Leipzig, 1932.

Scale Relationships and Nomenclature.—Although a number of papers have been presented on the subject of similitude, it is extremely difficult in many cases to follow the reasoning involved, due to complex systems of nomenclature. These systems are obviously the result of haphazard growth rather than predetermined plan, and English, German, and Greek letters will be found indiscriminately mixed.

At the United States Waterways Experiment Station, at Vicksburg, Miss., capital letters with the subscripts, m and n, are taken to indicate corresponding quantities in the model and in Nature. The ratio of these quantities expressed in the order, model to Nature, is denoted by the same letter, uncapitalized. Thus, the length scale is, $l = \frac{L_m}{L_n}$, and is always a fraction except in the case of the slope scale, which is an integral. This is in accordance with conventional designations and the usual understanding of scale values pertinent to other sciences. The scale of a map, for example, is conventionally expressed as 1:25 000 rather than 25 000 which, by itself, constitutes only a meaningless number. The following letters are taken to represent commonly used hydraulic elements:

A = area of cross-section.
B = a distribution factor pertaining to relative fineness of sand.
C = a roughness coefficient (see, also, N).
D = depth, or head; also, diameter.
E = diameter of sand grains.
F = tractive force per unit area.
G = acceleration of gravity.
H = head (see, also, D).
J = weight of fluid per unit of volume.
K = a constant for practical purposes $\left(\text{in Equation (4), } K = \frac{Q}{L D^{\frac{3}{2}}}\right)$.
L = length, or breadth.
M = modulus of viscosity.
N = coefficient of roughness.
P = wetted perimeter.
Q = discharge.
R = hydraulic radius.
$\mathbf{R}$ = Reynolds' number.
S = slope.
T = time.
V = velocity.
Z = turbulence criterion.
α = a basic scale ratio.
$\boldsymbol{a}$ = a scale ratio of areas, $\frac{A_m}{A}$.

d = a scale ratio of depth, $\frac{D_m}{D_n}$.

e = a scale ratio of sand-grain diameters, $\frac{E_m}{E_n}$.

j = the scale ratio, $\frac{J_m}{J_n}$.

j_1 = unit weight of bed material; j_2 = unit weight of water.

l = a scale ratio of length, $\frac{L_m}{L_n}$.

m = a subscript denoting a model characteristic.

n = a scale ratio of roughness coefficient; also, a subscript denoting a prototype characteristic (n, for Nature).

p = a scale ratio of the wetted perimeters, $\frac{P_m}{P_n}$; also, density.

q = a scale ratio of discharge, $\frac{Q_m}{Q_n}$.

r = a scale ratio of hydraulic radius, $\frac{a}{p}$.

s = a scale ratio of slope, $\frac{S_m}{S_n}$.

t = a scale ratio of times, $\frac{T_m}{T_n}$.

u = viscosity coefficient.

v = a scale ratio of velocity, $\frac{V_m}{V_n}$.

This system is designed to apply particularly to practical hydraulic research and is not essential to the case of ideal similarity, such as will now be discussed.

Similarity of perfect fluids requires only a consideration of the force of gravity and, therefore, all others, such as friction and viscosity, may be neglected. When this is true, the involved quantities may be expressed as simple functions of the basic scale ratio, which will be denoted by α. The following list is based on such a conception:

Quantities	Scale values
Angular accelerations	$\alpha^{-1.0}$
Revolutions per unit of time	$\alpha^{-0.5}$
Accelerations, constants	$\alpha^{0.0}$
Velocity and time	$\alpha^{0.5}$
Linear dimensions	$\alpha^{1.0}$
Discharge per unit time per unit width	$\alpha^{1.5}$
Areas	$\alpha^{2.0}$
Discharge per unit time	$\alpha^{2.5}$
Volumes, masses, forces, weights	$\alpha^{3.0}$
Power, momentum, impulse	$\alpha^{3.5}$
Work, energy, moments	$\alpha^{4.0}$

The values in this tabulation are directly obtainable from the formula for velocity produced in a freely falling body by the influence of gravity, ($V = \sqrt{2GH}$). Beginning with this relationship:

$$\frac{V_m}{V_n} = \frac{\sqrt{2GH_m}}{\sqrt{2GH_n}} = \frac{\sqrt{2GD_m}}{\sqrt{2GD_n}} = \frac{\sqrt{D_m}}{\sqrt{D_n}} \quad \ldots\ldots\ldots\ldots (1)$$

Whence,

$$v = \sqrt{d} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

or, expressed in words, the velocity scale is equal to the square root of the vertical scale. This relationship, together with that for the time scale, $t = \sqrt{d}$, is known as Froude's law and is the basic equation for the determination of velocity scales in hydraulics and other sciences, in which gravity is the controlling influence. It should be noted, however, that Equation (2) is for the effects of gravity only and is not applicable to conditions involving frictional or other coefficients. From it is derived the discharge scale:

$$q = l\,d^{\frac{3}{2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

obtained by multiplying scale values for area and velocity.

In case the model is designed to show the effects of discharge over dikes, dams, or other structures functioning primarily by gravity, it is obvious that only the relationship expressed by Equation (3), is directly applicable to an expression of the natural phenomenon involved. In the case of a weir this relationship may be determined in another way, by applying the principles of similitude to standard weir formulas of the form,

$$Q = K L D^{\frac{3}{2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

Factor K in this case disappearing as a constant in the resulting formula for similitude. For practical purposes, K may be assumed. Actually, experiments indicate that K is a function of D to a certain extent. Here is a first indication of the complexities that may be encountered in attempting to simulate, completely, the effects of both main forces (gravity and friction) in connection with any hydraulic model study. Fortunately, however (and generally), in practice, one of the forces may be eliminated from consideration, as in the present instance. In so far as K is not a constant, but presents frictional characteristics varying with D, a slight error will be introduced by Equation (4), which was derived solely from a gravitational relationship.

The validity of the Froudian relationship, even in cases where the frictional variances may be fairly large, is indicated by the results of experiments with full-scale and small-scale railroad embankments at the U. S. Waterways Experiment Station. These tests were conducted to determine the erosive action and general destructive effects of flood waters and to devise methods of protecting the embankment, economically, against such effects; but since several models were used they served nearly equally well to show existing scale relationships. Table 1 of readings at corresponding times, illustrates the observed similarity.

TABLE 1.—SCALE RELATIONSHIPS BETWEEN MODEL AND FULL-SCALE EMBANKMENTS

Item No. (1)	Description (2)	Full-scale embankment (3)	1:5-scale embankment (4)	1:5-model results transferred to full scale (5)
	Gauge Reading 30 Ft (Distance in Full Scale) from Center Line of Track:			
1....	(a) Up stream....................	3.0	0.60	3.0
2....		3.2	0.63	3.2
3....	(b) Down stream..................	2.4	0.48	2.4
4....		2.8	0.59	3.0
5....	Velocity, in feet per second...........	5.4	2.6	5.8
6....		5.2	2.4	5.4

Open-Channel Models, Undistorted.—As previously stated, fulfillment of the conditions establishing perfect similarity is seldom had in connection with problems pertinent to flowing water, and except where gravity might be assumed to act alone, as in the case of flow over dams, weirs, etc., modifications must be made according to the nature of the predominant forces.

Where a movement of water occurs in relatively shallow channels, as is frequently the case in Nature, the factor of bed roughness assumes such proportions as to make friction practically a controlling consideration, and no longer can the relations, previously cited, be adhered to unless secondary factors are introduced for their modification. Rather than attempt this, however, it is often better to determine scale values from well-known hydraulic formulas, such as those developed by Kutter and Ganguillet, Bazin, and Manning. Making use of the formula of the latter:

$$Q = A\ V = \left(\frac{1.486\ A}{N}\right) R^{\frac{2}{3}}\ S^{\frac{1}{2}}$$

whence,

$$q = \frac{Q_m}{Q_n} = \frac{A_m}{A_n}\left(\frac{N_n}{N_m}\right)\left(\frac{(R^{\frac{2}{3}}{}_m\ S^{\frac{1}{2}}{}_m)}{R^{\frac{2}{3}}{}_n\ S^{\frac{1}{2}}{}_n}\right) = \frac{a\ r^{\frac{2}{3}}\ s^{\frac{1}{2}}}{n} = \frac{a^{\frac{5}{3}}\ s^{\frac{1}{2}}}{n\ p^{\frac{2}{3}}}$$

By correcting this relationship for frictional variations in channels of varying shapes, the same is then directly applicable to problems involving distorted scales.

If no distortion is involved, horizontal and vertical scales will be equal, and the formula for discharge will be reduced to $q = \frac{l^{\frac{10}{3}}}{n\ p^{\frac{2}{3}}}$, or, assuming the hydraulic radius equal to the mean depth,

$$q = \frac{l^{\frac{8}{3}}}{n} \quad \text{..............................(5)}$$

Since the velocity scale, $(v) = \frac{q}{a} = \frac{q}{l^2}$,

$$v = \frac{l^{\frac{2}{3}}}{n} \quad \text{..............................(6)}$$

Other functions of the model may be determined in the same manner on a basis of the Manning formula or, if desired, from the Chezy or other relationships.

Although many European experimenters have expressed themselves unmistakably as opposed to the use of distorted models, nevertheless it is impossible in many cases to adhere to true similarity of reproduction on a small scale. As a matter of fact, there is scarcely a conceivable problem with respect to the Mississippi, or any other alluvial river, for which a solution can be obtained without resort to distortion because, where depths bear small ratios to widths, a reduction of linear elements by the same scale results in shallows in the model that are similar in their functioning (through the phenomena of surface tension and adhesion) to nothing that exists in Nature. Moreover, the velocity as deduced by Froude's law may be so small as to cause stream-line flow in the model, whereas flow in Nature is almost invariably turbulent. In turbulent flow the surface slope of the fluid (and, therefore, the friction head) increases in proportion to the square of the mean velocity, while in stream-line flow it increases only as the first power of the velocity. It is plain, therefore, that the characteristics of flow will be entirely different under the two conditions, and it becomes necessary to provide, in each case, that natural phenomena are accurately simulated.

Much has been said about geometric distortion in models and little about the distortion of other properties which occurs when the scale is reduced even if the model is not distorted geometrically. In reality, every model is distorted in some respects. The investigator must not be misled into overlooking, complacently, the less obvious distortions of properties and forces while he concentrates on avoiding geometrical distortion. The question is not whether distorted models will be used; it is rather, how can the distortion that is certain to occur in every model be made least harmful?

Referring once more to the Manning formula a relationship for similarity of discharge may be derived for distorted models in the same way as previously indicated. By dropping constants and substituting scale values for the hydraulic radius, *R*, slope, *S*, area, *A*, and the coefficient, *N*, there is evolved, as an expression of the discharge scale:

$$q = \frac{r^{\frac{2}{3}}\ s^{\frac{1}{2}}\ a}{n} \quad \ldots\ldots\ldots\ldots\ldots\ldots (7)$$

If the model has been built according to principles of simple distortion—that is, with no artificial adjustment of slope—and if the cross-sections are flat enough to permit substitution of mean depth, *d*, for *r*, it is found that,

$$q = \frac{l^{\frac{1}{2}}\ d^{\frac{13}{6}}}{n} \text{ (approximately)} \ldots\ldots\ldots\ldots (8)$$

Where an additional slope is given to the model to facilitate bed-load movement, the equation becomes:

$$q = \frac{l d^{\frac{5}{3}}\ s^{\frac{1}{2}}}{n} \quad \ldots\ldots\ldots\ldots\ldots\ldots (9)$$

These relationships are of value mainly as means of determining before the construction of the model, the quantity of water it will be necessary to supply. In an active laboratory with many models in operation it is frequently necessary to conserve the water or to regulate its distribution so that each model will be adequately served. It is essential, therefore, to know in advance approximately what discharge will be required. After the model has been put into operation, exact scales for discharge, velocity, and time may be determined experimentally in a manner to be described subsequently. To use Equation (8), however, even as a basis for estimates and design, an approximate evaluation must be made of the ratio, n, which is never accurately determinable. Fortunately, it has been found by experiment that for small-scale models (those in which $l < \frac{1}{1\,000}$), built with cement mortar, and for models of intermediate scale (those in which $\frac{1}{1\,000} < l < \frac{1}{500}$), built with a sand bed, the value of n is nearly unity. Since this n is the ratio between N of the natural river and the N of the model, there would appear at first to be an anomaly, because standard tables place the value of N in a reasonably good stretch of river at about 0.030 and at about one-half that in a concrete flume. The controlling factors of slope and hydraulic radius must be taken into consideration, however, and even casual thought will reveal that in a model river of small cross-section much resistance will be encountered even when the surface is relatively smooth.

Under the assumption that n may be taken as equal to unity, the following formula is obtained for discharge in the model of a river:

$$q = l^{\frac{1}{2}}\, d^{\frac{13}{6}} \qquad (10)$$

The applicability of Equation (10) as a means of determining the required discharge in a model is shown in Table 2, which gives data in round numbers on experiments conducted at the U. S. Waterways Experiment Station. A further interesting proof of the formula is found in its application to the results of European experiments. In the case of models built to a slope scale that does not result from normal distortion, the formula must be applied in the form, $q = l\, d^{\frac{5}{3}}\, s^{\frac{1}{2}}$. Testing it for the Elbe River model of the Preussische Versuchsanstalt, where $l = 1:200$, $d = 1:40$, and $s = 8:1$, q is found to equal 1: 33 000. The value of q as determined by Professor Krey's experiments was 1: 30 000.[8]

The discharge scales given in the last line of Table 2 were determined for each model by varying the flow over the weir and adjusting the tail-gate until the flow line measured in Nature had been definitely reproduced. The ratio between the measured discharge of the model and the corresponding measured discharge in Nature was taken as the true scale for discharge. As shown by Table 2, these values are in close agreement with the theoretical determinations for channel flow, but are considerably at variance with the determinations by the gravity formula.

---

[8] Unpublished report of the Preussische Versuchsanstalt für Wasserbau und Schiffbau, Berlin, 1928.

The velocity scale is obtained as previously indicated, of course, by dividing the scale for discharge by the scale for area, whence:

$$v = l^{-\frac{1}{2}} d^{\frac{7}{6}} \quad \ldots\ldots (11)$$

TABLE 2.—Comparison of Theoretical and Actual Discharge
(Data Obtained from Tests Conducted at the U. S. Waterways Experiment Station)

Scale ratios	Small-Scale, Concrete Surface				Sand Bed	
	A	*B*	*C*	*D*	*E*	*F*
$l =$	1:4 800	1:4 800	1:2 400	1:1 200	1:720	1:720
$d =$	1:360	1:200	1:120	1:48	1:72	1:72
$q = l^{\frac{1}{2}} d^{\frac{13}{6}}$	1:24 000 000	1:6 700 000	1:1 567 000	1:152 000	1:284 000	1:284 000
$q = l d^{\frac{3}{2}}$	1:32 800 000	1:13 600 000	1:3 155 000	1:399 000	1:440 000	1:440 000
Measured $q =$	1:21 000 000	1:7 200 000	1:1 570 000	1:150 000	1:300 000	1:280 000

The time scale as referred to hydraulic discharge is determined from the relationship: $\text{Discharge} = \frac{\text{Volume}}{\text{Unit time}}$. Whence, the time scale equals the volume scale divided by the discharge scale, or,

$$t = \frac{l^2 d}{q} \quad \ldots\ldots (12)$$

This relationship is particularly applicable to reservoir problems, or to flood-control problems where the time element is important, such as in the filling of back-water basins and other areas of storage. It is not applicable to problems of channel contraction, sedimentation, or similar investigations involving rates of bed development. Such rates must be determined by actual tests conducted in Nature and in the laboratory. There is no known "short-cut" to such determinations, and formulas taking into account the transporting power of flowing water and the resistance to motion of grain particles in suspension or otherwise, are almost invariably misleading.

Similitude as Applied to Bed Movement in Open-Channel Models

General.—In the preceding pages it has been shown that problems encountered in a river hydraulics laboratory fall naturally into two general groups. In the case of spillways, submerged dikes, or weirs, for which it is desired to determine coefficients of discharge, head over the structure, or other similar data, gravity may be considered as the controlling force; but in the case of open-channel models, bed and wall friction becomes of increasing importance. A compromise is frequently necessary in instances where dikes must be considered as a means of channel improvement, and seldom is it possible to apply any one set of laws indiscriminately to such a problem. Where the investigation is designed for the determination of back-water limits, or the drawdown resulting from the removal of some obstacle resisting the flow of the stream, it may be that a fixed-bed model will suffice, but all too frequently it is necessary to build the replica with a bed of movable material that will be acted upon by various existent currents. Experiments with movable-bed models being of the most difficult type to perform, it follows that investigations designed to determine effects of dikes on river beds must be approached with caution and understanding.

In choosing a sand for the bed of the model it must be conceded that close adherence to principles of geometric similitude is entirely impossible because, assuming the model to be built to a scale of 1:500, the same particles of Nature would be so reduced in size as to present a microscopic appearance. The impalpable powder could not be used in the model under any operating condition because, due to its extreme fineness, it might either float, be transported in suspension as sedimentary matter, or become compacted so as to resist, completely, the tractive power of the model stream. Fine, uniform sands have been used throughout many experiments, but a cardinal objection is that riffles of the bed invariably form. These riffles, of course, bear a certain similarity to the sand waves of Nature, but their height may be so grossly exaggerated as to detract from the accuracy of the model results. In practice, the difficulty may be partly rectified by smoothing out, in the model, the ranges on which soundings are taken.

Certain limiting conditions must always be considered whenever it is planned to experiment with a movable-bed model. It is obviously impossible to simulate entirely all physical properties of a natural stream. Distortion of linear dimensions is sometimes necessary to produce measurable depths and proper turbulence. Excessive distortion may result, in certain types of models, in an extreme and undesirable dissimilarity of flow phenomena. In addition to the factors that influence bed development (such as discharge, slope, etc.), there is also to be considered the guiding of the transported bed load. Preservation of geometric similarity, on the other hand, is practically impossible for the reasons given, namely, that a scale reproduction of the transported bed material would result in a fine material, subject to excessive riffling should any movement whatever occur. Just as it is often necessary to depart from conditions of true geometric similarity in the design of the model in order to keep it within workable limits and, at the same time, to obtain measurable depths and the proper character of flow, it is also necessary to exaggerate the size of sand grains, and, concurrently, it is frequently advisable to exaggerate the model slope in order to obtain similarity of the bed-load movement.

In departing from the laws of geometric similarity by distorting the model and utilizing a coarser grained bed material, thought must be given to those natural laws that are regarded as *a priori* principles of stream-bed development; that is, the plan of the stream must be unchanged from that of Nature and all curvatures must be reproduced in their exact relationships. If the radii of curvature of the channel are altered from one point of inflection to the next so that they are smallest at the vertex of the curve and then increase gradually (from the vertex to each point of inflection) to infinity again at the points of inflection, then the pools will be deepest in the vertex of the curve and will gradually decrease in depth as the inflection points are approached. In the same way, it is essential that the length of the curve be made to correspond to the inherent nature of the stream, by matching the deflection points with those of a well-developed reach of the natural river investigated.

The result of a well-guided curvature is generally to produce a satisfactory channel in which the thalweg passes through the middle of the cross-sections at points of inflection and crosses in an easy curve to the outside of the next bend. The close relationship between radii of curvature and pool depths must be kept in mind whenever experiments with models are undertaken because, since the purpose of such experiments is to determine methods for procuring a good, navigable channel, the flow must be made to follow smoothly from one bend to the next and thus secure navigable depths at all points.

As it is scarcely possible in theory to predetermine how deep the pools in a natural stream will become as a result of contractions, or whether dredging may be required, so also is it difficult to predict with assurance what depths may be anticipated unless the entire investigation is subjected to model study. Where a depth scale is chosen, as, for example, 1:50, there is no reason to expect that the same scale will be exactly applicable to the bed formations. By trial and accident this may be true, but in many cases compensatory factors must be determined and applied if the results are to be susceptible of quantitative as well as qualitative interpretation. The method most generally followed is to shape the model bed flat and, running through typical hydrographs for sufficient time to produce equilibrium in the bed movement, make observations as to the development, and comparisons as to its similarity with the corresponding developments of Nature. In many cases the natural development of the model is so nearly like that of Nature, that it is difficult to distinguish between maps of the two.

When a model is constructed to very small scale (as is frequently necessary) investigators are confronted with the aforementioned difficulty that sand, sufficiently fine to simulate (in scale) materials of the natural stream, is neither obtainable nor satisfactory, due to resulting riffles. To avoid the formation of riffles, many experiments have been made in America and abroad to determine the practicability of using other materials, such as ground lignite, saw-dust from pressure-creosoted timber, crushed coal, and other likely substances. A more satisfactory procedure has been found, however, in the use of coarser sand and increased tractive forces through greater slopes. In doing this the velocity of flow, of course, must not be made to exceed the upper limit for turbulence, above which shooting flow will be produced. This type of flow will result in the formation of standing waves, which are not characteristic of the type of flow ordinarily encountered in rivers.

If the water depth in the model is represented by D_m, the sand-grain diameter by E_m, and the roughness factor by C_m, then according to Professor

Krey, the limiting values for slope are: $\frac{E_m}{8\,D_m} < S_m < \frac{G}{C^2_m}$. Assuming the average value of C to be 80, then $S < \frac{1}{200}$. This value of 0.005 for the slope is much too great, wherefore, with the small values for D_m and E_m, riffles must undoubtedly appear when S_m is taken approximately equal to $\frac{G}{C^2_m}$; but, if a coarse sand (with a small value of C_m) is used, the obstacle will be largely eliminated. It appears, and is, possible, therefore, to choose E_m and S_m so that, instead of riffles, true sand banks will develop. In this same connection Armin Schoklitsch[4] states that the model sand should be compounded of various grain sizes. Natural sand banks are characterized by a diversity of grain sizes and, therefore, are firmer, for the reason that the spaces between the coarse and fine sand grains are in part filled and cemented with fine sedimentary material. To insure streaming (non-shooting) flow, S_m must be less than $\frac{G}{C^2_m}$, as is discussed in a subsequent paragraph.

Experiments Involving Movable Beds.—Material particles on the bed surface of a stream are acted upon by a number of conflicting forces, including those developed by the flowing water and those of a re-active nature arising from mass resistance and friction. Turbulence of the water may result in the exertion of component forces in several directions and may even produce an actual lifting of any reasonably light particle. This is the phenomenon known as transportation by suspension and is worthy of considerable study in connection with problems of sedimentation and kindred effects. From the standpoint of the practical river engineer, however, the phenomenon meriting most serious consideration is that of the movement of material particles along the stream bottom by hydraulic traction because, if the tractive force is sufficient in any case to overcome the inertia of a particle and its frictional resistance, movement in a down-stream direction will occur. When all bed particles, including the largest and heaviest, have been put into motion, general movement is said to have taken place, and, corollary to this, the critical tractive force for any particular bed load is defined as that force which will produce general movement of the material. ("Geschiebe" is a concise and descriptive German word used to designate bed material. Literally, it means, "that which is shoved.")

The tractive force of flowing water is nothing more than the component in the direction of movement of the weight of the water. Application of the laws of mechanics shows that this tractive force, per unit of area on the river bottom, is expressed by:

$$F = J \times D \times S \qquad (13)$$

in which, F is the tractive force per unit area; J is the weight of water per unit volume; S is the slope (non-dimensional); and D is the depth. Tractive force, then, is a function only of slope and depth. This is a very important consideration and should be kept in mind.

---

[4] "Geschiebebewegung in Flüssen und an Stauwerken," von Dr. Techn. Armin Schoklitsch Wien, 1926.

Suppose that it is desired to design a model involving a movable bed. The usual considerations of space, water supply, etc., have caused the adoption of definite horizontal and vertical scale ratios, assumed for the sake of argument, as 1:500 and 1:125. It is requisite that the bed load in the model move in a manner similar to that in its prototype; hence, if general bed movement in Nature begins at a 30-ft stage, it should begin at a corresponding stage in the model; but at what stage does bed movement begin in Nature? It is seldom practicable to determine this by actual observation, due to the depth and turbidity of the average river. Fortunately, a solution by laboratory methods is readily found by applying the law of M. P. du Boys[5] as given previously herein, since it may be demonstrated that for any particular bed material the critical tractive force is constant; that is, if the slope is doubled, the depth may be reduced by one-half and movement will still obtain.

To return to the problem of determining where general movement occurs in Nature: Suppose a representative sample of bed material is obtained from the actual river and moulded to a constant depth in a tilting flume. Obviously, if the flume is tilted to the average slope of the natural bed, to get movement it would be necessary for water to flow over it at a depth equal to that at which movement occurs in Nature. Since this depth could not be obtained in the average laboratory flume the critical tractive force must be determined by increasing the slope in the flume until general movement begins at less depth. Then, knowing the slope in that particular reach of the river, and by substituting this in Equation (13), it is possible to determine the depth in the river at which general movement begins. Computations and procedure for the hypothetical case are, as follows:

Obtaining a large sample of bed-load material from Nature at the locality in question, where it so happens that the average slope is 0.0001, it is placed level in the flume to a depth of about 4 or 5 in., after which the flume is tilted to a slope of 20 × 0.0001, or 0.002. Then water is passed over the bed material, keeping the water-surface slope equal to the bed slope and increasing the depth until movement occurs. Assume that general movement occurs at a depth of 0.5 ft. This indicates that movement in Nature will occur at a depth of about 20 × 0.5, or 10 ft. Therefore, movement in the model must occur at a model depth of $\frac{10}{125}$, or 0.08 ft.

Next, select the sand it is desired to use for the model "geschiebe." Suppose, for example, that it is desired to use Mississippi River sand from a reach of the river near Vicksburg. In the tilting flume, it is found that for general movement a tractive force of 0.008 lb per sq ft is required with this material. It has already been stated that movement is desired in the model at a depth of 0.08 ft; hence, in Equation (13): F = critical tractive force = 0.008 lb per sq ft; J = weight of water per cubic foot = 62.5 lb; D_m = depth = 0.08 ft; and, therefore, S_m = 0.0016 ft per ft. From this it follows that proper movement in the model will result if it is given a bottom slope of 0.0016, or sixteen times that of the natural river reach. Since there is a vertical distortion of 4 to 1, the slope of the model would normally be 4 × 0.0001, or 0.0004. Thus, it appears necessary to supply the additional slope by some approved method. The final slope distortion will be, $\frac{0.0016}{0.0001} = 16$.

---

[5] *Annales des Ponts et Chaussees*, Tome XVIII, 1879.

It is to be noted here that a tacit assumption was made, to the effect that the river bed is parallel to the water surface. That assumption is no doubt true for all practical purposes considering a long reach of a river. All it implies is that the mean depth of the river is fairly constant. However, there are pools and crossings in the river, and the bottom is not parallel to the water surface, except for average conditions over a long reach. Since one cannot hope for sufficient data to design for each crossing and each shoal, the best that can be done is to take averages, and, in the general case, such procedure should result in reasonable accuracy. Consider in this connection, that the model, too, has its pools and crossings and, hence, the average condition designed for will no doubt cause movement in the model at about the same time and place as in the prototype.

To insure proper movement of the model bed material the foregoing procedure may be epitomized, as follows:

1.—Select vertical and horizontal scales so as to produce as little distortion as may be consistent with the production of measurable depths and turbulent flow.

2.—Make a flume test of the sample of bed-load material obtained from the actual locality to be represented by the model, and determine the critical tractive force necessary to insure its transportation. Should it prove impracticable to obtain the bed-load sample in quantities sufficient for testing in the flume, or in the event equipment is not available, the critical tractive force may be obtained by the use of any one of several formulas. Probably the best of these formulas is that of Kramer,[6] namely, $F = \frac{100}{6}\left(\frac{(j_1 - j_2)}{B}\right)E$, in which, j_1 is the unit weight of the bed material; j_2, the unit weight of water; E, the mean grain diameter; and B, a sand uniformity factor obtained by plotting the curve of the mechanical analysis of the bed material, separating the area under the curve into two areas—above and below the 50% line—and determining their ratio.

3.—From a consideration of the results of Step 2 and of the average slope of the natural river, determine the depth at which bed movement occurs in Nature.

---

[6] "Modellgeschiebe und Schleppkraft," von Hans Kramer, *Mitteilungen* der Preussischen Versuchsanstalt für Wasserbau und Schiffbau, Berlin, Heft 9, 1932.

4.—Determine the critical tractive force required for the bed material which will be used in the model. The method to be adopted may be either of those described in Step 2.

5.—From Step 3, plus a consideration of the vertical scale, the depth in the model at which movement is to occur, is known. From a consideration of this model depth and of the results of Step 4 determine the slope necessary for the model.

6.—Tilt the model sufficiently to obtain the slope found in Step 5.

TURBULENCE IN OPEN-CHANNEL MODELS

Requirements for Hydraulic Similitude.—In hydrodynamics many problems arise that defy purely mathematical solutions; however, hydraulic models now provide ready means for investigating such problems. The principles of hydraulic similitude insure that the results obtained from investigation by means of the model will be transferable, quantitatively or qualitatively, to the prototype.

Geometric similitude is concerned with the relations between linear dimensions of model and prototype. Chief among the factors influencing the selection of the horizontal (length and width) scale for the model are the area and shape of the prototype, laboratory space available for the model, and cost of construction. Prominent among the factors influencing the selection of the vertical (depth) scale is the range in elevations of the prototype as against the depths necessary in the model. The latter limitation leads to a consideration of dynamic similitude.

Kinematic similitude requires that every particle in model and prototype shall describe geometrically similar paths in corresponding time intervals; and, closely akin to this, dynamic similitude requires that these motions shall be produced by forces acting in a definite model-to-prototype ratio. For all practical purposes, dynamic and kinematic similitude exists when the flow of water in the model is of the same character as the flow in Nature. If laminar or stream-line flow exists in Nature, it must be reproduced in the model, whereas, if turbulent flow occurs in Nature that kind of flow must be reproduced in the model. As a matter of fact, flow in natural channels is almost invariably turbulent. There are index numbers, derived experimentally, by means of which model and prototype may be examined to find if there is dynamic similitude with respect to any particular force.

It is important to note that both gravity and friction act on flowing water, whereas, in general, dynamic similarity can exist only in respect to a single force. Evidently, this gives rise to a conflict of requirements, and perfect similarity is not to be attained. The best that can be done is to select that force which is of predominant importance, and to seek dynamic similitude with respect to that force. The modifying effect of the other force will prevent the accomplishment of absolute similitude, which in practice is not necessary. Fortunately, the effects of one or the other of the forces is usually so predominant that little error results in considering it as acting alone. Thus, in experiments in which, a high degree of turbulence results, internal friction may be neglected and the laws of gravitation considered as governing.

Reynolds' Criterion for Turbulent Flow.—The index by which turbulence may be measured, was discovered in 1883 by the great English experimenter whose name it bears. The experiments of Osborne Reynolds were confined to small, smooth pipes, but sufficient evidence was obtained to show that a rather definite transition occurs between laminar and turbulent flow. It was further proved that this transition is indicated by a definite value of the non-dimensional expression, $\frac{VD}{K}$ (since called Reynolds' number), in which, V equals velocity; D equals pipe diameter; and K, the coefficient of kinematic viscosity.

From the expression, $\mathsf{R} = \frac{VD}{K}$:

$$\mathsf{R} = \frac{VDp}{u} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)$$

in which, the coefficient, K, is replaced by its equivalent value, $\frac{u}{p}$ (viscosity divided by density). Since the dimension of V (velocity) is length per unit time, or $L\,T^{-1}$; of D (diameter) is length, or L; of p (density) is weight per unit volume, or $M\ L^{-3}$; and of u (viscosity) is $M\ L^{-1}\ T^{-1}$, Reynolds' number has no dimensions; that is, $\frac{L\,T^{-1} \times L \times M\,L^{-3}}{M\,L^{-1}\,T^{-1}} = M^{\circ}L^{\circ} = 1$, and the modulus may be shown to be merely a number, or ratio.

Table 3 gives values for absolute viscosity and kinematic viscosity of water for temperatures within possible working ranges.

TABLE 3.—VISCOSITY OF WATER

Temperature, in degrees Centigrade	Absolute viscosity, u, in English units	Kinematic viscosity, $K = \frac{u}{p}$
5	0.001020	0.00001639
10	0.000880	0.00001411
15	0.000766	0.00001230*
20	0.000600	0.00001085
30	0.000538	0.00000865
40	0.000442	0.00000713
50	0.000370	0.00000599

* Value generally used in computations.

As the experiments of Reynolds were confined to small glass pipes, his determinations for the value of the viscosity modulus are not strictly applicable to open channels, but it is worth noting that in the case of pipes the modulus is equal to about 2 500. To write the expression so that it may be used in open-channel determinations, it is necessary, of course, to change pipe diameters into hydraulic radii. It should be noted that as these are both linear dimensions no change is made in the dimensions of the modulus.

From $\mathsf{R} = \frac{V D}{K} = 2\,500$, it follows that $\mathsf{R} = \frac{4\, V R}{K} = 2\ 500$; or, $M = \frac{V R}{K} = \frac{2\,500}{4} = 625$, in which, M is the viscosity modulus for open channels, corresponding to Reynolds' number for pipes.

The value for the modulus (625) as determined from the Reynolds' number, is the subject of some disagreement among modern investigators, Schoklitsch setting its value at 900, and Hopf at 320. Probably, the safest value for its higher working limit is that established by the Prussian Research Institute for Hydraulics, Marine Design, and Earth Testing, which gives 6 000, with a notation that an ample factor of safety is included. Since the range of turbulence is wide between laminar and shooting flow, it is obviously better to choose a safe value if it can be done without too greatly increasing the cost of the model. Accepting tentatively the value of the Prussian Institute: $M = \frac{V R}{0.0000125} = 6\ 000$; or, $V\ R = 0.075$, in which, R is expressed in feet, and V, in feet per second. From this it may be assumed that turbulent flow will always exist when the product of velocity and hydraulic radius (or mean depth) is greater than 0.075.

Recent experiments conducted at the U. S. Waterways Experiment Station have thrown additional light upon the conditions necessary for production of turbulent flow in a small stream or model. The tests were made with a tilting flume, lined with concrete of the same general roughness that is used in models built at the Station, and curves were determined to show the variations of velocity with depth when slope was maintained constant. A straight-line relationship was observed to the point where turbulent flow occurred, the curve beyond this point becoming representative of a second degree function. Thus, the break from straight to curved line indicated the transition from laminar to turbulent flow and served to supply a more accurate criterion for turbulence than had been available previously. By averaging the results of many tests at different slopes, the value of the product of depth and velocity was found to indicate definitely the character of flow produced and, where the value was greater than 0.02 (in English units), turbulence prevailed. Expressed mathematically the criterion for turbulent flow is:

$$Z = V\ D > 0.02 \quad \ldots\ldots\ldots\ldots\ldots\ldots (15)$$

D being expressed in feet and V, in feet per second.

It is possible, of course, for flow in the model to pass upward through the turbulent stage and into shooting flow. This possibility must be investigated, and, since the velocity of shooting flow is the same as wave velocity, $\sqrt{G\,D}$, in which, G is the acceleration due to gravity, the necessary condition to insure non-shooting flow is that the velocity be less than $\sqrt{G\,D}$. To summarize, turbulent (streaming) flow exists in the model when $\frac{0.020}{D} < V < \sqrt{G\,D}$.

The discussion of geometric similitude has developed the fact that the depths necessary in the model govern the selection of the vertical scale. The discussion of Reynolds' criterion reveals the connection between model depths (hydraulic radii) and turbulent flow. To insure a high degree of turbulence in models of large rivers, such as the Mississippi, it is almost invariably necessary to use a scale value of depths larger than that for horizontal dimensions. This results in distortion of the model.

To summarize briefly: The horizontal dimension scale of a model is arbitrarily selected, usually with a view to making the model fit in the space available for it. The vertical scale is more intimately connected with the requirements for dynamic similitude, and must usually be made larger than the horizontal scale. On the typical river model, turbulence is to be tested by use of Reynolds' number, the viscosity modulus, M, or the turbulence criterion, Z.

To make application of the foregoing principles clearer the reader is referred to an actual example published elsewhere[7] by the U. S. Waterways Experiment Station, which demonstrates how turbulence was considered in connection with a model of the Ohio River.

Practical Laboratory Expedients

Necessity being the mother of invention it would seem far better to complete the work of equipping a laboratory over an extended period as demand indicated than to complete the equipment at the very beginning. Heavy, fixed equipment is expensive, difficult to manipulate, and, unless chosen with discretion by a well-informed and experienced individual, may be inapplicable to the special needs of the proposed institution. Of course, in the case of laboratories designed for instruction of University students, it is more necessary to install a certain amount of fixed equipment in the beginning, and, in general, past experience may be used as a guide to such purchases. The set-up of a practical river laboratory, on the other hand, must be so flexible as to permit it to deal with any problem falling within its general scope, and the first essential to this is adequate and unencumbered space.

Special flumes, designed to carry river models are relatively worthless for the reason that they are adaptable only to special problems. Where one model may be 12 ft wide and 100 ft long, the next required may be 40 ft wide by 60 ft long; and, obviously, the same flume would be inapplicable to both. Moreover, the reach of the river under investigation may bend back upon itself in such a way as to preclude the use of a uniformly tilting flume. For a reasonably straight reach of river the tilting flume would be desirable in dealing with models requiring a slope greater than that resulting from vertical distortion; but if the river winds materially such slope must be acquired by the placement of successive templates. Even so, it is sometimes difficult to apply slope corrections, the situation being practically impossible when the river meanders and when over-bank flow must be considered. If flood stages are to be reproduced no slope may be given the model except that normally resulting from distortions, but if less than bank-full flow is the important thing, or if the reach to be investigated is reasonably straight, practically any increase in slope may be given. Fortunately, those problems requiring experimentation with extensive models are generally concerned with hydraulics of flow instead of bed movements, and increased slopes are non-essential to their satisfactory solution. In any case a temporary wooden box of sufficient size will make a container for the model, which may be built of sand and concrete. Alternate types of containers are built of concrete or brick and mortar.

---

[7] "Model Study of Effects of Dikes on the River Bed at Walker's Bar, Ohio River," *Paper L,* U. S. Waterways Experiment Station, 1932.

The simplest method of constructing a model is to cut templates from sheet iron to conform to river and over-bank sections; level the containing box; and place the templates to grade by means of a level. The templates may then be braced and filled to within an inch of their tops with moist sand, packed in place. A coating of concrete, placed level to the tops of the templates and moulded to conform in shape with the sections, then completes the model proper, which must be equipped subsequently with an adjustable tail-gate, a measuring weir, an entrance chamber, and baffles. All these devices are extremely simple and except for the weir may be made by any good handy-man or carpenter. Practically any type of tail-gate is satisfactory to produce a back-water similar to that induced by resistance to flow of the entire river below the lowest point represented in the model. Fig. 1 is a model of a tail-gate, showing a valve outlet for fine adjustments.

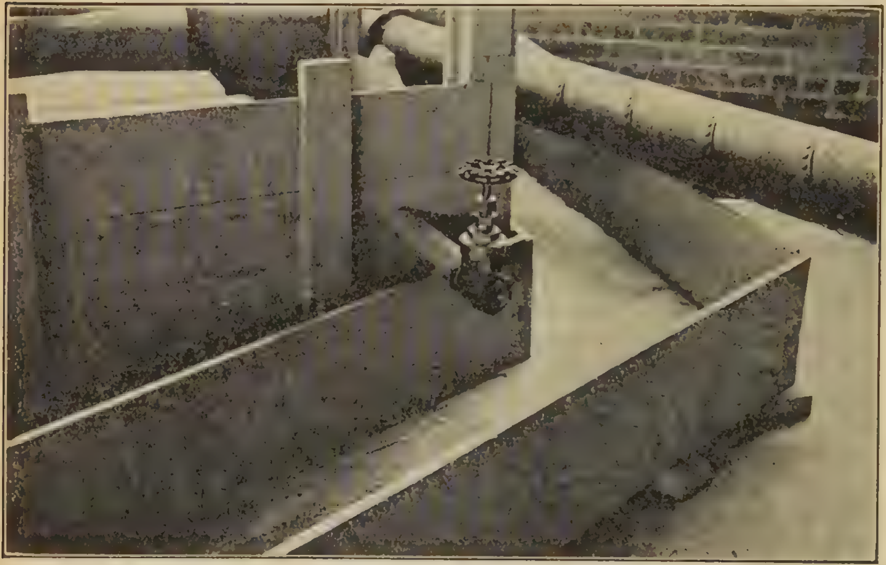


FIG. 1.—TAIL-GATE OF RACE-TRACK TOWHEAD MODEL, SHOWING VALVE OUTLET FOR FINE ADJUSTMENTS

Special flumes may be needed, of course, in a river laboratory for control tests and for tests of dams, weirs, spillways, and various measuring devices.

In this connection it is well to make the flume as long as the building will permit and of as great cross-section as is required to accommodate the maximum flow of water that can be allowed. Flumes of steel are generally more suitable than those of concrete because cracks and leakage may be developed in the latter as possible settlements in the building's foundations or as temperature changes occur. The former, moreover, may later be cut at any point should it be desired to insert plate-glass sections, or manometer connections. A smaller tilting flume may be built of wood or other material if suitably mounted on steel I-beams, or similarly rigid supports.

Having provided a laboratory building with expensive and unencumbered floor space upon which models may be constructed, and having designed an inlet and outlet system for the water, together with a diversion to some measurement chamber, little more need be taken into account until experiments actually have been begun. After work has started various needs will present themselves and ingenuity may be given free play in meeting requirements. If a reservoir system is used it may be unnecessary to provide pumping equipment, but nearly always it will be desirable to arrange for some pumping and circulation of water. This is particularly essential when the reservoir is supplied by run-off of rainfall from adjacent territory and droughts may occur during summer months. Whenever a pumping and circulating system is used it is necessary to provide also for: A deep (or storage) reservoir with overflow; pumps of required capacity; a constant-head reservoir; and incidental piping.

The capacity of the pumps, size of the pipe, and dimensions of the high and low reservoir are naturally determined by the character of work it is proposed to perform in the laboratory, it being unnecessary to arrange for great volumes of flow under high head if only normal river problems are anticipated. The constant-head reservoir—having as its purpose (as its name implies) the supplying of water from pumps to models under an unvarying pressure and, therefore, without fluctuations in flow—must have a length of overflow crest great enough to eliminate, entirely, such variations as will result from pump pulsations. A good value to use as a basis of design is 20 lin ft of overflow crest for each second-foot of water pumped.

In subsequent paragraphs other accessories of a modern river laboratory are described. These descriptions are general and it should be kept in mind that any other workable design may be equally good in any case. The personnel should be encouraged to think and develop new ideas as new problems present themselves. Listed briefly, the accessories are:

(*a*).—*Level Pedestal.*—Since a level must be used continually for setting templates, checking elevation of model beds, and setting gauges for zero readings, a fixed pedestal mount is extremely convenient and helpful.

(*b*).—*Profilograph.*—This is a labor-saving device for obtaining cross-sections of the model river bed after running (see Fig. 2). It consists of a carriage that travels along a rack and holds a vertically sliding shaft equipped with a pencil at its upper end. The shaft is counterweighted and placed between nearly frictionless rollers so that as a roller at its lower end is moved across the model bed the shaft, with its pencil, will rise and fall. Geared to the main carriage is a secondary carriage fastened to a vertical board on which a sheet of cross-section paper may be mounted. Movement of the main carriage results in reduced movement of the secondary carriage in relation to it, so that with the pencil moving vertically and the paper horizontally a section of bed is traced to full vertical and reduced horizontal scale. A more direct and efficient method of obtaining the same result has been recently developed, by utilizing a special sounding rod equipped with a pivoted foot and an adjustable T-head that rides on a leveled cross-beam.

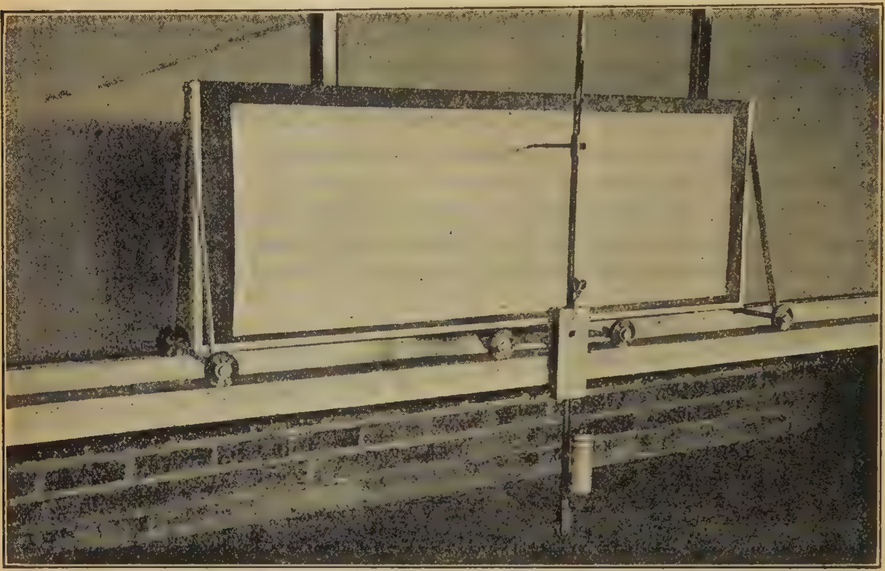


Fig. 2.—Profilograph for Taking Cross-Sections of Model River Beds

(*c*).—*"Geschiebe" Meter.*—This is a simple device designed to feed bed material to the model at a fixed rate and in accordance with the movement of such material in the model. Operating on the hour-glass principle with funnels of various sizes to be used interchangeably, it is free from moving parts and is very effective. The bed material is washed into the model by a small stream of water brought in by a rubber tube.

(*d*).—*Current-Measuring Devices.*—These devices are of many types and designs, the simplest expedient for velocity determinations being the use of floats of varying weights and densities. Other devices are pendulums actuated by the velocity of flow, small propellers mounted between jeweled bearings, and Pitot tubes.

(*e*).—*Depth Meter.*—Difficulties are frequently encountered in determining depths of flow in very shallow reaches of a model, where, as in a floodway problem, many such determinations must be made. By placing round-headed gutter nails in the model so that the heads project but slightly through the concrete, a needle-gauge mounted on a U-frame may be used to determine differences between elevations of the nail-heads and the water surface. Data are obtained for each nail-head and with the depth meter measuring the distance between the head of the nail and the water surface in each case, elevations referred to a common datum are readily determinable.

(*f*).—*Traveling Crane.*—Traveling cranes of many types and purposes are made commercially and may be purchased in the open market. Purchase should not be made, however, until a definite need has been revealed, after which the choice of type will be obvious.

(*g*).—*Multiple-Gauge Rack and Wells.*—This is a simple device to make one hook-gauge serve the purpose of several (see Fig. 3). It consists of a single gauge mounted rigidly to a carrier which slides along a carefully machined steel bar. The bar being leveled in place with leveling screws, one zero is obtained for the gauge and applied to all readings obtained.

(*h*).—*Dye Injector.*—A long thin nozzle through which dye may be introduced under pressure to the model is often advantageous in making observations as to the character of flow obtaining in various reaches.

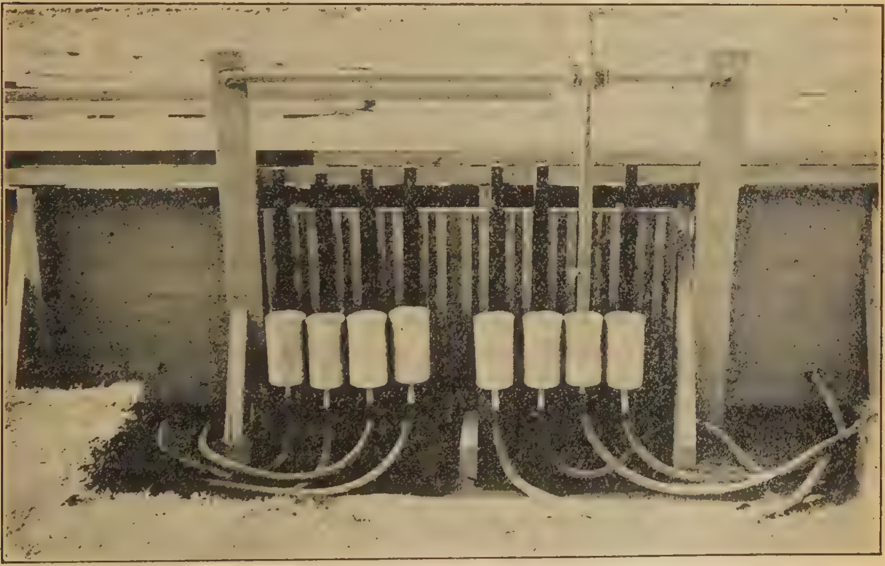


FIG. 3.—GAUGE RACK WITH MULTIPLE-WELL ARRANGEMENT.

(*i*).—*Racks and Baffles.*—These may be of many types and their design in any case should be the result of trial, experience, and experiment. Wooden gratings, crushed rock, coarse gravel, baffle-plates, wire screening, and swinging boards may all be used as the case demands.

(*j*).—*Valve Outlet at Tail-Gate of Model.*—This arrangement, shown in Fig. 1, is often advantageous in controlling the flow from the model. After a rough setting of the gate, fine adjustments of flow are possible by regulating the value.

Typical Experiments Encountered in a Rivers and Harbors Laboratory

Classification of Model Types

Fluvial models may be classified according to the character of their beds and according to the nature of their predominant forces, thus:

1.—Fixed Bed Models.—

(*A*) Those for which a consideration of gravitational force is paramount, and to which Froude's law must be applied, either: (*a*) Strictly; or (*b*), where channel friction is of decisive influence, by application of a factor to adjust the velocity and discharge scales.

(*B*) Those for which a consideration of internal friction is paramount and to which Reynolds' law must be applied.

2.—Movable Bed Models.—

Same as Item 1(*A*) and Item 1(*B*). For this type of model, consideration must also be given to a development of tractive force comparable to that existing in Nature according to the kind of sand used.

The preceding outline classifies in the briefest possible form the several types of models that may be used for different investigations. However, since every problem will require a different treatment it does not follow that a model must be built in one of several ways and that no other methods are permissible under any circumstances. It is intended to show, on the other hand, that many possible combinations of conditions must be considered and that while there are typical model forms, these must be varied sufficiently in each case to meet the peculiarities of the problem. Several general types of models suggested by the outline will now be discussed.

Type 1(A(a)).—Fixed Bed Models to Which Froude's Law Must Be Strictly Applied.—This type of model is exemplified by those used for determinations of: (1) Discharge capacities of weirs, spillways, siphons, etc.; (2) effects of sills on aprons of overflow structures; (3) factors influencing the production of an hydraulic jump; and (4) other hydraulic phenomena resulting from the gravitational fall of water when friction may be largely neglected.

Type 1(A(b)).—Fixed Bed Models in Which Channel Friction Is of Decisive Influence.—Typical of experiments requiring this class of model are those designed for determinations of effects of cut-offs and limits of backwater. These problems, being practically independent of channel changes, may be solved satisfactorily with a fixed bed in which experimental runs may be repeated time after time for the assurance of accuracy. When it is desired to obtain effects of silting or erosion, critical parts of the model may be built in of sedimentary materials. The problem of back-water or its corollary is frequently offered to a laboratory for solution, and it will be worth while for the reader to study two brief reports of the U. S. Waterways Experiment Station on this subject.[8]

---

[8] "Experiment to Determine the Limit of Backwater Influence in the Illinois River," *Paper Y*, 1931, and "Experiment to Determine the Effects of Mississippi River Backwater on the Red River," *Paper 10*, 1933.

The problems described in these reports resolved themselves into simple determinations of water-surface profiles after adjustment of the models had been accomplished, and no question of bed movement was involved in the investigations. Cut-off problems and similar determinations involving the addition or subtraction of resistance to flow are pursued in an identical manner, the questions of erosion and sedimentary deposition being subordinated to questions of effects on stages within the reaches considered.

Type 2(A(a)).—Movable Bed Models in Which Gravity May be Considered the Controlling Force.—Experiments with models of this nature will generally follow the lines of those mentioned in connection with models of Type 1(*A*(*a*)), the main difference being that erosive effects must be taken into consideration. Investigations conducted by Antonin Smrček, in Brno, Czecho-Slovakia,[9] and by Theodor Rehbock, in Karlsruhe, Germany,[10] to determine relative scour below spillway aprons, involved typical models of this classification.

Type 2(A(b)).—Movable Bed Models with Channel Friction the Controlling Force.—Models built for the determination of design of contraction works of permeable type are illustrative of this classification. In the general case, where qualitative effects are sought in preference to less easily obtainable quantitative data this type of model will suffice adequately. Thus, if it is desired only to choose between several proposed plans of contraction works, a movable bed model, built with primary regard to consideration of the force of traction, is indicated. In this case rates of bed movement in model and Nature are observed, and corresponding bed changes under the various imposed conditions are compared.

In practical experimentation there occur instances in which the foregoing classifications cannot be applied rigidly. Most common is the case of flow over a regulating work, where the question arises as to whether the channel friction is of appreciable influence. For a method by which compatibility between the laws may be attained in a model, the reader is referred to a discussion by Lieut. John Paul Dean, U. S. Corps of Engineers, and the writer.[11]

The types of study indicated under 1(*A*(*b*)) and 2(*A*(*b*)) of the preceding classification are those in which the forces of internal friction of the fluid are predominant. Typical of such studies is the problem of the resistance to motion of a deeply submerged body, such as a submarine. The laws governing this type of experimentation are not compatible with those governing the cases heretofore discussed unless fluid different from that of the prototype is used in the model. Fortunately, in the field of practical river hydraulics, the forces of internal friction of the liquid are relatively small and may be neglected without appreciable error resulting. It may be noted that the more turbulent the regimen of a stream becomes the more nearly negligible are the forces of internal friction.

---

[9] Wilsonova Přehrada Na Rece Tennessee, Alabama, U. S. A., by Prof. Ing. Antonin Smrcek, v Praze, 1929.

[10] "Hydraulic Laboratory Practice," by the late John R. Freeman, Past-President and Hon. M. Am. Soc. C. E., 1929.

[11] "Geometric Versus Hydraulic Similitude," *Civil Engineering*, August, 1932, p. 467.

Problems Susceptible to Laboratory Treatment

Experience indicates that the following general types of problems may be prosecuted successfully in river hydraulics laboratories: (1) Effects of dikes or other contractive works on the river bed; (2) effects of cut-offs of river bends; (3) back-water influences; (4) effects of channel dredging; (5) relationship existing between mean discharge and river sections; (6) discharge over navigation dams and spillways; (7) design of locks and dams; (8) determinations of velocities and current directions in rivers after placement of corrective works; (9) flow through siphons, siphon spillways, etc.; (10) effect of baffle-piers on spillway aprons; (11) loss through trash-racks and screens; (12) effect of storage basins on flood flow; (13) progression of flood waves; (14) flow through diversion floodways; (15) effect of bends in a river; (16) harbor entrance problems; (17) resistance of, and flow around, cylinders and bridge piers in streams; (18) effects of flow over highways and railroad embankments; (19) experiments to determine effects of obstructions placed in a stream (submerged sills, etc.); (20) flow of water through sluices of various kinds; (21) towing experiments and experiments on bodies falling in water; (22) experiments relating to transportation of sedimentary and bed materials; (23) design of stilling-pools; (24) movement of sedimentary material and its distribution at river divisions; and (25) seepage in earth embankments and levees.

Types of investigations of the future are difficult to predict, but the definite tendency is toward more and more work with models of the movable bed type. In this field the surface has been scratched by experiments similar to that performed to obtain a solution of the difficulties encountered at Island No. 35, Mississippi River, but infinitely more remains to be accomplished. However, that may be, it still seems assured that much work will remain at all times for every hydraulic laboratory in the land.

Acknowledgments

It is desired to express appreciation to H. W. King, M. Am. Soc. C. E., and C. O. Wisler, Assoc. M. Am. Soc. C. E., for carefully reviewing this paper in an earlier form, and to acknowledge gratefully the loyal assistance and co-operation rendered by the Staff of the U. S. Waterways Experiment Station in developing new methods of laboratory technique described in the preceding pages.

DISCUSSION

LORENZ G. STRAUB,[12] ASSOC. M. AM. SOC. C. E. (by letter).—The treatment given by Lieut. Vogel to practical river laboratory hydraulics is timely and especially interesting because of the author's connection with the development of the U. S. Waterways Laboratory. He points out in a clear manner the value of the laboratory in solving a variety of river-improvement problems.

The writer has used a nomenclature similar to that proposed by Lieut. Vogel and finds it especially convenient. Most nomenclatures used heretofore in presenting the principles of mechanical similitude are complicated with numerous Greek and German letters which have no apparent relation to each other, entirely different symbols being used for homologous items in the prototype and model. Such a set of symbols makes it difficult to follow the literature which has been presented on this subject particularly because each writer uses his own set of letters. In keeping with the general scheme proposed by the author it might be well to use the symbol, l, for the basic scale ratio instead of α. The writer prefers the inverse of the ratio used by the author; that is, the ratio of the length, time period, velocity, etc., in the prototype to that in the model; this provides integers instead of fractions for the ratios and seems somewhat more convenient in application when dealing with a series of models of different size.

Frequently, it is not understood that although the usual mechanics are practically unlimited in applicability in so far as pure dynamic occurrences are concerned, the applicability of the mechanics of similitude has certain limitations. Not all motion occurrences can be imitated by means of a model, and therefore, not all dynamic problems met with in practice can be solved by means of the mechanics of similitude. In fact, relatively few motion occurrences can be imitated accurately by means of a model. Obviously, therefore, the first step in the design of a model for the solution of a practical hydraulics problem necessitates a study of whether or not the problem can be treated at all by the method of similitude.

An analysis must be made to determine what physical forces cause the occurrence. The nature of these forces fixes the "model law" under which the model must operate, if it is to be similar to the occurrence in the prototype. The physical forces in the sense here used are those that cause acceleration of the particles forming the system concerned. Some of the more important are as follows: The earth's gravitational forces, frictional forces between liquid particles, capillary forces, elastic forces, normal and tangential forces (in solid bodies), and many others. Each one of these forces defines a model law; that is, the relation between the length ratio and time ratio for the prototype and model.

The most important physical forces that cause fluid motion are the earth's gravitational forces, friction between fluid particles, and capillary forces. It is seldom that any one of these is the sole cause of the occurrence. Consequently, perfect similitude is almost never obtainable; the motion of particles in the hydraulic model is almost never perfectly similar to that in the prototype. The solution of design problems by means of models, therefore, resolves itself largely into determining the limitations of the applicability of various models.

---

[12] Associate Prof. of Hydraulics, and Head, Hydraulics Div., Experimental Eng. Laboratories, Univ. of Minnesota, Minneapolis, Minn.

The degree of similarity obtainable between the model and the prototype depends upon the nature of individual circumstances. If the movements in the prototype and model are influenced by the action of only one type of physical force, the model occurrences may be made to take place in a mechanically similar fashion to the prototype occurrences. The model law that must be followed in order to obtain mechanical similitude may be determined from various equations. The most natural procedure seems to be to determine the relation of the basic ratios,[13] l, t, and f, in a manner somewhat as follows. The ratio of the inertia or mass accelerating forces may be represented by:

$$f = \frac{M_m \; B_m}{M_n \; B_n} \dots\dots\dots\dots\dots\dots\dots\dots (16)$$

in which, M_m and M_n represent the masses of homologous volumes in the model and prototype, respectively, and B_m and B_n represent the corresponding accelerations of these masses. (The writer prefers the inverse of the ratios herein presented; the ratios are set up to correspond to the author's paper for the sake of uniformity.) Inasmuch as, for geometric similarity, the masses vary as the cube of the linear dimension, the ratio, $\frac{M_m}{M_n}$, is equal to $\frac{\Gamma_m}{\Gamma_n} l^3$; also, since $B = \frac{d^2 X}{d T^2}$, the ratio, $\frac{B_m}{B_n} = \frac{l}{t^2}$. Therefore, in terms of the scale ratios, the ratio of the mass accelerating forces may be represented by,

$$f = \frac{\Gamma_m}{\Gamma_n} l^3 \frac{l}{t^2} \frac{\Gamma_m}{\Gamma_n} \frac{l^4}{t^2} = \phi_1 (l, t) \dots\dots\dots\dots\dots\dots (17)$$

For similarity, the ratio of the inertia forces must be equal to the ratio of the physical forces producing the occurrence; hence,

$$f = \frac{F_m}{F_n} \dots\dots\dots\dots\dots\dots\dots\dots\dots (18)$$

in which, F_m and F_n are the physical forces (such as the force of gravity) acting upon homologous masses in the model and prototype, respectively. This equation may be written in the form:

$$f = \frac{F_m}{F_n} = \phi_2 \; (l, \; t) \dots\dots\dots\dots\dots\dots\dots (19)$$

The value of the force ratio, f, may be eliminated from Equations (17) and (19), thus obtaining the "model law" in question in the basic form,

$$t = \Omega \;\, (l) \dots\dots\dots\dots\dots\dots\dots\dots\dots (20)$$

---

[13] See "Nomenclature," pp. 122–123.

Here, an arbitrarily chosen length ratio, l (ratio of length between model and prototype), fixes the time ratio, t (ratio of time required for an occurrence to take place in the model to the time required for a corresponding occurrence to take place in the prototype). It will be observed that an infinite number of models (of different scales) are possible for this case in which only one physical force comes into consideration. It will also be noted, from Equation (17), that the model law is influenced by the ratio of the densities of the mediums used, respectively, for the model and prototype. Equation (20) is of general form for all model laws, provided only one physical force comes into consideration.

Consideration will not be given at this time to various model laws which might be derived from the foregoing equations; suffice it to use the force of gravity as the physical force causing the occurrence in order to exemplify the method of analysis. Equation (17) will be written in the form,

$$f = \frac{M_m B_m}{M_n B_n} = \frac{\Gamma_m}{\Gamma_n} \frac{l^4}{t^2} \quad \ldots\ldots\ldots\ldots \text{(21)}$$

and Equation (19) in the form,

$$f = \frac{F_{gm}}{F_{gn}} = \frac{W_m v_m}{W_n v_n} = \frac{W_m}{W_n} l^3 \quad \ldots\ldots\ldots\ldots \text{(22)}$$

in which, W_m and W_n are the unit weights of the mediums of the model and prototype. These unit weights may be represented by $W_m = \Gamma_m G_m$ and $W_n = \Gamma_n G_n$, in which, G_m and G_n are the homologous accelerations of gravity in the similar occurrences. The expressions, v_m and v_n, are homologous volumes. Of course, normally, G_m and G_n are equal, the value being 32.09 at the equator, 32.17 at the 45° parallel, and 32.25 at the poles.

By equating the two expressions for f given in Equations (21) and (22), and solving for the ratio of time periods:

$$t = \sqrt{l \frac{\Gamma_m \div W_m}{\Gamma_n \div W_n}} \quad \ldots\ldots\ldots\ldots \text{(23)}$$

so that the model law (relation of time ratio to length ratio), if the force of gravity is the only physical force influencing the occurrence, is given by,

$$t = \sqrt{l \frac{G_n}{G_m}} \quad \ldots\ldots\ldots\ldots \text{(24)}$$

This relation is referred to as the Froude model law. If G is the same for both prototype and model, Equation (24) takes the form,

$$t = \sqrt{l} \quad \ldots\ldots\ldots\ldots \text{(25)}$$

A similar method of analysis might be used in determining the model law for other physical forces. Once having the model law one can readily determine the relation of homologous discharges, homologous velocities, homologous accelerations, etc., of the prototype and model.

However, as pointed out by Lieut. Vogel, ordinarily perfect similarity is not obtainable; this is due to the fact that more than one physical force influences the motion occurrences. It will be assumed, for example, that two physical forces effect the flow conditions, possibly the force of gravity and frictional forces between liquid particles. Three equations are then obtained between the force ratio, time ratio, and length ratio, as follows:

Ratio of inertia forces:

$$f = \phi_1 (l, t) \qquad (26)$$

Ratio of gravity forces:

$$f = \phi_2 (l, t) \qquad (27)$$

Ratio of frictional forces:

$$f = \phi_3 (l, t) \qquad (28)$$

Since there are three equations with three unknowns (the ratios, l, t, and f), none of the unknowns may be chosen arbitrarily as in the simple case of only one type of force influencing the occurrence; each unknown has a definite value. Therefore, when once the medium of the model is fixed, there is only one size model that will provide perfect mechanical similarity. If the medium in both prototype and model is the same (for example, if it is water in both cases) then the model must be of the same size as the prototype for perfect similarity.

Since perfect similarity in models is seldom if ever obtainable in the development of practical hydraulic designs, recourse must be had to certain approximations. This is particularly true in the case of working models of rivers and harbors. For these, in the apt words of the author, "the question is not whether distorted models will be used; it is rather how can the distortion that is certain to occur in every model be made least harmful?" What are the advantages and shortcomings of different types of distortion?

In river models, similarity based upon the empirical Manning formula apparently gives better agreement with experimental results than either Reynolds' model law or Froude's model law. In this case the predominant forces influencing the occurrence are fluid friction and the force of gravity. However, similarity cannot be completely satisfied, ordinarily, for either of these forces if the most workable model is to be obtained. Usually, in well-designed river models, Reynolds' number is smaller than in the prototype. In some laboratories, the practice has been to design such models to satisfy Froude's number as far as velocities are concerned and to limit the scale ratio in such a manner that turbulent flow will occur in the model; also, in the case of movable bed models, the scale ratio has been limited so that sediment transportation will occur. Table 2 indicates that Manning's formula gives a better value for the velocity and discharge than Froude's number.

The necessity of geometrical distortion becomes particularly evident when designing models of large alluvial rivers having relatively fine bed materials. In many cases it will be found necessary to make a model of a size covering several acres if recourse is not taken to some kind of geometrical distortion. The large size often imposed in order to obtain a geometrical non-distorted workable model would result in excessive cost of construction of the model and very cumbersome operation.

For broad river channels the hydraulic radius, for all practical purposes, is proportional to the mean depth of the channel; in the case of a model in which the vertical scale is different from the horizontal scale, therefore, the hydraulic radius will here be assumed equal to the mean depth. Using the author's nomenclature and Manning's formula as a basis, for a vertically distorted model the velocity ratio becomes:

$$v = \frac{d^{\frac{2}{3}} s^{\frac{1}{2}}}{n} \dots\dots\dots\dots\dots\dots\dots\dots (29)$$

If the amount of distortion is represented by $k = \frac{d}{l}$, it will be found that the velocity in the distorted model may be represented by:

$$v = \frac{1}{n} d^{\frac{2}{3}} \left(\frac{d}{l}\right)^{\frac{1}{2}} = \frac{1}{n} k^{\frac{7}{6}} l^{\frac{2}{3}} \dots\dots\dots\dots\dots\dots (30)$$

Thus, the velocity increases as the seven-sixth power of the distortion. This condition is particularly desirable inasmuch as the velocity in small scale models frequently becomes too low.

Vertical distortion also results in a definite increase in the tractive force of the stream in moving sediment along the bottom of the model river. Assuming the ratio of the tractive forces of the model and prototype to be represented by f, the ratio may be expressed by the linear ratios, by:

$$f = d\,s = d\,\frac{d}{l} = k^2\,l \dots\dots\dots\dots\dots\dots\dots (31)$$

showing that the tractive force increases as the square of the vertical distortion. In a similar fashion, it may be shown that the ratio of the Reynolds' numbers for the model and prototype is represented by:

$$\frac{\mathsf{R}_m}{\mathsf{R}_n} = k^{\frac{13}{6}} l^{\frac{5}{3}} \dots\dots\dots\dots\dots\dots\dots\dots (32)$$

Reynolds' number thus increasing as the thirteen-sixth power of the distortion for a given horizontal scale ratio. This is a particularly desirable condition because the Reynolds' number of the model is ordinarily lower than that of the prototype in the case of river models. Distortion thus results in a degree of turbulence more nearly that of the prototype. These advantages of vertical distortion may be summarized as follows:

1.—The velocity in the model is increased by distortion, thus resulting in more suitable Reynolds' numbers and making possible more precise measurements in consequence of the increased discharge.

2.—Tractive forces (and consequent suitability of models for use in experiments involving movable beds) are increased markedly.

3.—Great economies arise in the cost of construction and operation of the model because the necessary horizontal scale may thereby be greatly reduced.

Counterbalancing these advantages are a number of disadvantages, particularly where the amount of distortion attempted is very large. For various reasons accurate similarity of the bed elevations in the distorted model as compared to the prototype are not possible. This is particularly true where the erosion in the prototype causes the bottom to stand at approximately the natural under-water angle of repose of the sedimentary material. For example, in Fig. 4 let the shaded area represent the idealized scour that occurs in the non-distorted model. In the distorted model the corresponding scour should be represented by the triangle, *ABC*; but, since it is not possible for the slope to be greater than is allowed by the natural angle of repose, the scoured pool would take the form represented by the triangle, *DEC*, if the scoured depth is to correspond to the depth ratio. It is conceivable, therefore, that where the vertical distortion is great, considerable deviation between the results of an undistorted model and the conditions in the prototype might obtain. A similar analysis could be made in the case of sedimentation; in fact, it might be shown that the model could indicate erosion where sedimentation would occur in the prototype.

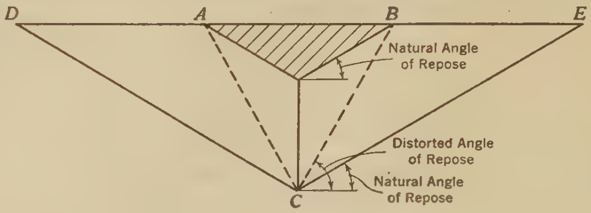


FIG. 4.—ANALYSIS OF EROSION CONDITIONS IN VERTICALLY DISTORTED MODELS

Other conditions brought about as a result of distortion which might be harmful are the formation of fixed eddies (frequently referred to as "rollers"). Distortion may result in the elimination of these fixed eddies in some localities and the introduction of some in other localities, inasmuch as there are limits to the amount that a "roller" may be distorted without breaking it into counteracting "rollers."

The foregoing are some of the shortcomings that cannot be eliminated, nor can their effect be determined accurately without comparative experiments in the individual cases concerned. The observer must be constantly alert to avoid making erroneous deductions as a result of his experiments on models constructed to vertically distorted scales.

It has sometimes been suggested that the proper type of distortion consists in merely increasing the slope of the model—that is, by tilting the model to a slope steeper than that of the prototype while the horizontal length ratio and vertical length ratio remain equal. Such an arrangement has certain advantages in that it brings about the desired greater velocities and greater transporting force (although not to as marked a degree as vertical distortion) and greatly eliminates the disadvantage of producing "rollers" where they should not occur. On the other hand, in the case of a broad river model, inasmuch as the flow takes place in a serpentine rather than in a rectilinear direction, the model would have to be warped in order to obtain uniform distortion in slope.

In connection with experiments concerning the regimen of the Upper Rhine River, Dr. Theodore Rehbock, of Karlsruhe, Germany, found that certain advantages were obtained by using a sedimentary material composed of volcanic ash. The specific gravity of this material was considerably less than ordinary sand, being about 1.6. In this material no difficulty was encountered with riffle formation; bars formed in the model river channel in a manner quite similar to that actually occurring in the Rhine River.

Paul W. Thompson,[14] Jun. Am. Soc. C. E. (by letter).—The study of problems in open-channel regulation by use of small-scale models has received great impetus in the United States during recent years. This impetus is due in no small part to the wisdom and generosity of the late John R. Freeman, Past-President and Hon. M. Am. Soc. C. E., who provided for the Fellowships which have resulted in making available in this country the best of foreign hydraulic laboratory practice. Another powerful factor has been the increased use by the Engineer Department of the Federal Government of the small-scale model as an aid in the solution of certain problems arising in the administration of the rivers and harbors of the country. The U. S. Waterways Experiment Station at Vicksburg, Miss., where most of the model studies for the Engineer Department are made, has been, for the past few years, among the most active laboratories in the world. It is encouraging to note that many of the principles followed and developed at that Station have been made available through the medium of this paper, which is a valuable addition to the literature of experimental hydraulics.

The author's remarks on the effects and importance of turbulence in open-channel models will serve to clarify this little understood subject. Turbulence is that fortunate condition of flow which justifies the assumptions that: (*a*) The forces of internal friction in the fluid are negligible; and (*b*) the force of gravity is predominant. The consistency of these two assumptions with the requirements of the law of Reeche-Froude (generally called the law of Froude) will be observed at once. The observation is substantiated by consideration of the facts that: (1) In turbulent flow, velocity is a function of the square root of the depth; while, similarly, (2) under the law of Froude, the velocity scale is a function of the square root of the depth scale.

The evaluation of a turbulence index as given by Equation (15) is a noteworthy addition to the list of practical aids to the experimentalist. The value of $Z = 0.02$, should be considered as an absolute permissible minimum. In most of the open-channel models at the U. S. Waterways Experiment Station the product of the actual mean velocity by the actual mean depth has proved to be several times this value. Turbulence is a comparative phenomenon—that is, there are degrees of turbulence. Thus, in a regimen in which the value of Z is, say, 0.06, the effects of the forces of internal friction will be more nearly negligible than in a regimen in which Z has a value less than 0.6. Short of going into "shooting flow," the experimentalist will seek as high a degree of turbulence as possible, since he will try to render the effects of internal friction as nearly negligible as possible.

---

[14] Second Lieut., Corps of Engrs., U. S. Army; Military Asst., Dist. Engr., Omaha, Nebr. (Asst. to Director, U. S. Waterways Experiment Station, June, 1932, until November, 1933.)

Lieut. Vogel has touched on most of the practical difficulties that confront the small-scale model experimentalist. Perhaps the most important of these difficulties concerns the adjustment, and the need for the adjustment, of the roughness of the model. The writer, who has been privileged to co-labor with Lieut. Vogel on much of the work at the U. S. Waterways Experiment Station, believes that a few more words may be justified in regard to this matter of model roughness.

The basic decision that the experimentalist must make in planning his model study concerns the forces with respect to which he is to seek similitude. The author has discussed the reasoning which, in the case of open-channel models, usually results in the decision to seek similitude with respect to the force of gravity. This decision having been made, the question of model roughness immediately assumes important proportions.

The decision to seek similitude with respect to the force of gravity presumes, in effect, that water is a perfect fluid. The result is the law of Reeche-Froude; namely,

$$t = \sqrt{d} \quad \text{..........} \quad (33)$$

From this law follows immediately the relations expressed by Equations (2) and (3) of the paper. Consider, now, the difficulties that arise in an attempt to realize the fundamental relationships expressed by Equations (2), (3), and (33).

Assume that flow in an open channel in Nature is in accordance with the Chezy law:

$$V_n = C_n \sqrt{D_n\, S_n} \quad \text{..........} \quad (34)$$

and that flow in a model of that channel also accords with that law, as follows:

$$V_m = C_m \sqrt{D_m\, S_m} \quad \text{..........} \quad (35)$$

It immediately follows, using the nomenclature of the author, that,

$$v = c \sqrt{ds} \quad \text{..........} \quad (36)$$

Thus, if the fundamental relationship of Equation (2) (Froude's law) is to obtain, it is necessary that,

$$c \sqrt{s} = 1 \quad \text{..........} \quad (37)$$

The experimentalist will endeavor to maintain an "undistorted' slope; that is, he will endeavor to keep s equal to unity. This is true because of the impossibility of analyzing completely all the effects due to any form of distortion. However, if s is to equal unity, then according to Equation (37), c must equal unity. Since c is simply the quotient of the C for the model (C_m) divided by the C for Nature (C_n), this is tantamount to stating that,

$$C_m = C_n \quad \text{..........} \quad (38)$$

Consider now the position of the experimentalist who is designing an open-channel model, and who is aiming for similitude with respect to the force of gravity. He knows that the accuracy of his results will be a function of the degree to which he can satisfy the law of Equation (38). The degree of his success in satisfying this law must rest on: (*a*) The accuracy to which the value of C_n may be determined; and (*b*) the degree of control, or manipulation, that may be exercised on C_m. The experimentalist will proceed to study all the data available concerning the laws of flow in the prototype, and he will thus arrive at his "best value" for C_n. He will then consult his own records, and the records of others, in an endeavor to determine just how he may insure a value of C_m which approaches the "best value" for C_n. Thus, as an idealized illustration, suppose that the experimentalist has determined that $C_n = 40$. From his own files and from the files of others, he finds records of models, similar to the one he contemplates, as follows: (*a*) For Model *A*, made of rough mortar, $C_m = 30$; (*b*) for Model *B*, made of glazed plaster of Paris, $C_m = 46$; and (*c*) for Model *C*, made of smooth mortar, $C_m = 41$. Obviously, the experimentalist decides to construct his model of smooth cement mortar, since, in the light of past experiences, it offers the best chance of satisfying the law of Equation (38).

Actually, the problems will never be so simple of solution as indicated in the illustration. First, the "best value" for C_n often is not a very good value. Indeed, in many instances—especially those in which proposed construction will exert great influence on the value of C_n—the "best value," in reality, may be the "best estimate." Second, such records of model roughness coefficients as are now available are scarcely ever extensive enough to justify absolute dependence on them; in fact, in many instances, such records as are available form little more than general indications or guides. In such instances, the experimentalist must keep before him the factors that influence the value of *C*. Thus, taking the Manning relationship (because it is used by the author), the experimentalist considers that $C \propto \frac{D^{\frac{1}{6}}}{N}$. Herein are indicated the effects of the reduced linear dimensions of the model. Thus, *N* is a value that increases with increasing roughness; but *D* is a value that decreases as the linear scale ratio, model to Nature, decreases. It can thus be seen that, if Equation (38) is to be satisfied, the model, as it is made progressively smaller, must be made correspondingly smoother.

After his model is built, the experimentalist will proceed to "adjust" it; that is, he will run many tests, under various conditions, and obtain, thereby, actual values for C_m. He will be able, by smoothing or roughening the model, to manipulate his values within reasonable limits. He will continue his "adjustments" until he is satisfied that the requirements of Equation (38) are as well fulfilled as is practicable.

This discussion has concerned itself with the problem of satisfying Froude's law in the case of an "undistorted" model; but, as pointed out and explained by the author, most of the problems arising from the great alluvial rivers of this country require models in which the horizontal linear scale ratio is considerably smaller than the corresponding ratio for vertical dimensions. In building such models, it is customary (although not essential) to make the slope of the model greater than the slope of the prototype by the value, $\frac{d}{l}$. In other words, usually, $s = \frac{d}{l}$. The significance of such a condition, as effecting the subject under discussion, is apparent from a consideration of Equation (37). Thus, consider the values given for Model *F*, in Table 2: $l = \frac{1}{720}$; and, $d = \frac{1}{72}$. For Model *F*, $s = 10$; hence, in this particular case, if the law of Froude were to be satisfied, $c = \frac{1}{\sqrt{10}}$ or,

$$C_m = \frac{1}{\sqrt{10}} C_n \text{ .... } (39)$$

Considering that the value of C decreases as roughness increases, Equation (39) serves to explain an apparent anomaly. Thus, the fact that the dimensions of individual roughnesses cannot be reduced to the same extent as other dimensions, would justify the expectation that small-scale models, in general, would be too rough. However, in the case of distortion of scales, the slope factor enters, as illustrated in Equation (39), and often operates to make the model too smooth. A case in point is Model *F*, already noted. Table 2 shows that the actual discharge scale of the model was $\frac{1}{280\,000}$, while, according to the Froude law, it should have been $\frac{1}{440\,000}$. Obviously, the model was too smooth; that is, C_m was too large.

The author presents, as Equation (10), an expression for discharge scale obtained by the simple process of applying the Manning formula to the discharges of model and Nature, respectively, and dividing the former result by the latter. Implicit in his operations are the assumptions of $n = 1$, and $s = \frac{d}{l}$. Obviously, if the Froudian law is to hold, peculiar and definite relationship must exist between the d and the l of Equation (10). This relationship may be easily determined by writing the Froudian expression for discharge scale, Equation (3), and solving it simultaneously with Equation (10). This operation was reported in another connection,[15] wherein it was found that, if $d = l^{\frac{3}{4}}$, there is no conflict between Equation (10) and the Froude law. This requirement has been called the "law of compatibility." Making the same assumptions that the author makes in deriving Equation (10), ($n = 1, s = \frac{d}{l}$), and inserting the Manning expression for C in Equation (37), it follows as before that $C = l^{\frac{3}{4}}$. Thus, the law of compatibility is an adaptation, under certain given conditions, of the general requirement, $c\sqrt{s} = 1$.

---

[15] *Civil Engineering*, August, 1932, p. 467.

In this connection it can be noted that a judicious manipulation of the slope ratio, s, might serve as a means of obtaining satisfaction of Equation (38). This notation is entirely correct, but it should be observed that such manipulation introduces the indeterminate effects of a distortion in the model results. Furthermore, it should be observed that the slope ratio must be determined before the model is built, and may not be altered after construction. Since it is impossible to foresee exactly the effects of reduced dimensions, altered slopes, etc., it follows that manipulation of the slope ratio alone will not insure satisfaction of the law. Resort must still be had to the methods already described.

It must not be concluded that failure to obtain exact satisfaction of the law of Equation (38) invalidates the model results. Such failure, of course, does affect the results, and must be taken into careful consideration when data from the model study are analyzed and interpreted. A small-scale model represents at best a series of compromises—a structure in which theoretical considerations have conflicted, at many points, with practical possibilities. The word, model, is itself almost too exact an appellation to apply to the small-scale replica of a natural stream. More strictly appropriate would be some title, such as "approximate reproduction."

The writer is a firm believer in the advisability of the use of small-scale models in the solution of open-channel problems where large sums of money are involved. His belief, indeed, is so firm that he does not deem it necessary to overstate the case in favor of the use of models by a single iota. The open-channel laboratory can stand on its own feet, and is more and more able, as time goes on, to point to the record books for its justification. With all its limitations, the small-scale model is still a valuable tool for the river hydraulician who understands its use.

Ralph W. Powell,[16] M. Am. Soc. C. E. (by letter).—The great increase during the last few years in the use of models for the solution of various problems connected with rivers, makes this paper very timely, and the author deserves the thanks of the profession for making the results of his experience available.

There are only a few minor criticisms. Where so many quantities are involved there seems a distinct advantage in making use of the Greek alphabet, especially where such use is already familiar. It would seem better not to break with such established usage as ρ for density, μ for coefficient of viscosity, and ν for coefficient of kinematic viscosity. It must be discouraging to those who are working for uniformity in the use of symbols to see one of the best established ones —g for the acceleration of gravity— replaced by G. Furthermore, M is called "the viscosity modulus for open channels." It would seem that "critical value of Reynolds' number for open channels" would better express the idea. However, the writer has endeavored to abide by the author's nomenclature in what follows.

The statement preceding Equation (10), that small-scale models of cement mortar and intermediate-scale models of sand, have about the same coefficient of roughness as the natural river, is interesting. Certainly Kutter and Ganguillet, Bazin, and Manning meant the coefficient of roughness in their formulas to depend only on the material, and to be independent of the size of the channel cross-section; but it is also probably true that none of the formulas actually holds over a wide range of sizes. The increasing use of small-scale models gives the question an importance it had not had before, and creates an urgent demand for a new and better formula for channel flow. As a preliminary step in that direction, the writer offers the following theoretical considerations.

---

[16] Associate Prof. of Mechanics, Ohio State Univ., Columbus, Ohio.

Suppose that the average velocity over the cross-section of the channel is V, and that it depends on the following seven factors only: S, the surface slope; D, the mean depth = cross-sectional area divided by surface width; W, the surface width; G, the acceleration of gravity; p, the density of the liquid; u, the coefficient of viscosity of the liquid; and, E, a measure of the roughness, such as the average diameter of the sand grains, the average height of the projections which produce the roughness of a cement model, or some such quantity. Then, the formula desired will be of the form:

$$f\ (V, S, D, W, G, p, u, E) = 0 \ldots\ldots\ldots\ldots\ldots\ldots (40)$$

By a method developed independently by D. Riabouchinsky[17] and by Edgar Buckingham,[18] this can be replaced by the equation:

$$\phi\ (\pi_1, \pi_2, \pi_3, \pi_4, \pi_5) = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (41)$$

in which all of the π's are dimensionless, and, $\pi_1 = V^{x1}\ D^{y1}\ u^{z1}\ G^{-1}$; $\pi_2 = V^{x2}\ D^{y2}\ u^{z2}\ p$; $\pi_3 = \dfrac{E}{D}$; $\pi_4 = \dfrac{W}{D}$; and, $\pi_5 = S$.

The dimensions of the quantities in Equation (40) are:

$[V] = [L\ T^{-1}]$ $\qquad$ $[G] = [L\ T^{-2}]$

$[S] = [L^0]$ (dimensionless) $\qquad$ $[p] = [M\ L^{-3}]$

$[D] = [L]$ $\qquad$ $[u] = [M\ L^{-1}\ T^{-1}]$

$[W] = [L]$ $\qquad$ $[E] = [L]$

The values of the x's, y's, and z's are found by applying the principle of dimensional homogeneity to the expressions for the π's; π_1 gives,

$$[\pi_1] = 0 = [L\ T^{-1}]^{x1}\ [L]^{y1}\ [M\ L^{-1}\ T^{-1}]^{z1}\ [L\ T^{-2}]^{-1} \ldots\ldots\ldots\ldots (42)$$

Equating the exponents of M:

$$z_1 = 0$$

---

[17] "Methode des Variables de Dimensions Zero," par D. Riabouchinsky, *L'Aerophile*, September 1, 1911.

[18] "On Physically Similar Systems," by Edgar Buckingham, *Physical Review*, Vol. 4, pp. 345–376 (1914); also, "Model Experiments and the Forms of Empirical Equations," *Transactions*, Am. Soc. Mech. Engrs., Vol. 37, pp. 263–296 (June, 1915); and "Notes on the Method of Dimensions," *Philosophical Magazine*, 6th Ser., Vol. 42 (November, 1921), pp. 696–719.

Equating the exponents of T:

$$-x_1 - z_1 + 2 = 0$$

therefore, $x_1 = 2$.

Equating the exponents of L:

$$x_1 + y_1 - z_1 - 1 = 0$$

therefore, $y_1 = -1$, and $\pi_1 = \dfrac{V^2}{D\,G}$. This is "Froude's number" and is usually represented by the symbol, F.

The expression for π_2 gives:

$$[\pi_2] = 0\,[L\,T^{-1}]^{x_2}\,[L]^{y_2}\,[M\,L^{-1}\,T^{-1}]^{z_2}\,[M\,L^{-3}] \ldots\ldots\ldots\ldots\ldots (43)$$

Equating the exponents of M,

$$z_2 + 1 = 0$$

therefore, $z_2 = -1$.

Equating the exponents of T,

$$-x_2 - z_2 = 0$$

therefore, $x_2 = 1$.

Equating the exponents of L,

$$x_2 + y_2 - z_2 - 3 = 0$$

therefore, $y_2 = 1$ and $\pi_2 = \dfrac{V\,D\,p}{u}$. This is Reynolds' number, R. If, similarly, $\pi_3 = \dfrac{E}{D}$ is represented by E, and $\pi_4 = \dfrac{W}{D}$ by W, Equation (41) becomes $\phi\,(\mathsf{F}, \mathsf{R}, \mathsf{E}, \mathsf{W}, S) = 0$, or,

$$S = \Phi\,(\mathsf{F}, \mathsf{R}, \mathsf{E}, \mathsf{W}) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (44)$$

By experiment it is known that F is by far the most important of the four factors, and that as far as it is concerned the function is of the simplest sort; therefore,

$$S = k\,\mathsf{F} = \frac{k\,V^2}{G\,D} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (45)$$

in which $k = \theta\,(\mathsf{R}, \mathsf{E}, \mathsf{W})$. The classical formulas are in terms of R, the hydraulic radius, instead of D, the mean depth, but in this discussion the writer follows the author in using them more or less synonymously. Then, the Chezy formula is Equation (45), with $\dfrac{G}{k} = C^2$. Bazin's formula, $C = \dfrac{87}{0.552 + \dfrac{m}{\sqrt{R}}}$, would reduce to:

$$k = \frac{G\,(0.305 + \dfrac{1.104\,m}{\sqrt{R}} + \dfrac{m^2}{R}}{7\,569} \ldots\ldots\ldots\ldots\ldots\ldots (46)$$

which is obviously not dimensionally homogeneous and, therefore, cannot be the ideal formula that is being sought. The same applies to Kutter's formula. Manning's formula may be put in the form, $S = \frac{N^2 V^2}{2.208 R^{\frac{4}{3}}}$, which is Equation (45), if $R = D$ and $k = \frac{N^2 G}{2.208 D^{\frac{1}{3}}}$. This shows that the dimensions of Manning's N are $[T L^{-\frac{1}{3}}]$ and that his formula cannot be expected to give the same value of N for the same surface in different sizes. The Williams and Hazen formula, $V = 1.318 c R^{0.63} S^{0.54}$, reduces to Equation (45) if $R = D$ and $k = \frac{G}{1.737 c^2 D^{0.26} S^{0.05}}$. This makes the dimensions of the Williams and Hazen $c = [T^{-1} L^{0.37}]$, so that it would be expected to vary with the size slightly more than Manning's N.

Until the true form of the Φ-function in Equation (44) is known the simplest assumption to make is that it is of the form, $S = k \mathsf{F} \mathsf{R}^{-x} \mathsf{E}^y \mathsf{W}^z$, which reduces to:

$$S = \frac{k K^z E^y W^z V^{2-x}}{G D^{1+x+y+z}} \qquad \text{.......................(46)}$$

in which, K is the coefficient of kinematic viscosity $= \frac{u}{p}$ and k is an absolute dimensionless constant. From the analogy with pipes it is quite certain that x is between 0.00 and 0.10; but a great amount of careful research would be required to get a more accurate value and to find y and z. Furthermore, it might be found that no equation of the form of Equation (46) would accurately represent the facts, because x is probably not a constant for all roughnesses, and probably the velocity distribution, as determined by bends in the channel, etc., is quite important; and it has not been brought into the formula at all.

If, however, it is assumed that at least Equation (46) will approximately represent the facts, the requirement for similarity between model and prototype would be:

$$\frac{S_m}{S_n} = \frac{d}{l} = \frac{K^x{}_m E^y{}_m W^z{}_m V_m^{2-x} D_n{}^{1+x+y+z}}{K^x{}_n E^y{}_n W^z{}_n V_n{}^{2-x} D_m{}^{1+x+y+z}} \qquad \text{............(47)}$$

If $K_m = K_n$, $\frac{d}{l} = \frac{e^y l^z v^{2-x}}{d^{1+x+y+z}}$, or,

$$v^{2-x} = \frac{d^{2+x+y+z}}{l^{1+z} e^y} \qquad \text{........................(48)}$$

For a non-distorted model, in which $l = d$,

$$v^{z-x} = \frac{d^{1+x+y}}{e^y} \qquad \text{........................(49)}$$

Multiplying by the area scale, $a = l\ d$, to get discharge, q for the distorted model is expressed by,

$$q^{2-x} = \frac{d^{4+y+z}\ l^{1-x-z}}{e^{y}} \qquad \text{(50)}$$

and for no distortion,

$$q^{2-x} = \frac{d^{5-x+y}}{e^{y}} \qquad \text{(51)}$$

Assuming that $x = 0.04$, Equation (50) becomes,

$$q^{1.96} = \frac{d^{4+y+z}\ l^{0.96-z}}{e^{y}}$$

or,

$$q = \frac{d^{0.51(4+y+z)}\ l^{0.51(0.96-z)}}{e^{0.51\,y}} \qquad \text{(52)}$$

Then n in the author's Equation (8) will be:

$$n = l^{0.010+0.51z}\ d^{0.127-0.51\,(y+z)}\ e^{0.51y} \qquad \text{(53)}$$

The values of n as computed from Table 2 are:

Test	Value of n	Test	Value of n
A	0.875	*D*	0.987
B	1.075	*E*	1.056
C	1.002	*F*	0.986

Using the values of n, l, and d for Tests *A*, *B*, *C*, and *D*, and solving by the method of least squares for the most probable values of y, z, and e, gives $y = 0.40$, $z = -0.85$, and $e = 1:2\,800$; and $n = l^{-0.424}\ d^{0.357}\ e^{0.204}$. Then, Tests *E* and *F* give $e = 1:374$ and $1:523$, respectively. Taking the average as 1: 450, it can be said that these tests indicate that the sand was about 6.25 times as coarse as the equivalent size of the cement surface, and that the natural river was 450 times as coarse as the sand. Undoubtedly, these values are incorrect, but they are near enough within the range of probability to indicate that this method of attack, supported by extensive tests on channels of different scales and distortions, and definite roughnesses, might lead to a satisfactory formula for the flow in open channels, and, therefore, to a true law of model similarity.

K. D. NICHOLS,[19] JUN. AM. SOC. C. E. (by letter).—Although this is a valuable contribution to the science of experimental hydraulics and covers the entire field relating to the solution of river hydraulic problems in the laboratory in an excellent manner, there are many points that should be explained more completely.

A pertinent case relates to the selection of a suitable sand for use in a movable bed model. Under the heading "Experiment Involving Movable Beds," Lieut. Vogel has epitomized in six steps the procedure to be followed in order to insure proper movement of the model bed material. This procedure insures movement of sand in the model. However, it gives no index of the amount of movement to be expected. An improved method is to select a sand that, in addition to fulfilling all the requirements as listed by Lieut. Vogel, will also give adequate movement. This selection must be based on flume tests to determine the rate of movement for any given tractive force.

---

[19] Lieut., Corps of Engrs., U. S. Army, Berlin Germany (Formerly Asst. Director, U. S. Waterways Experiment Station, Vicksburg, Miss.)

Fig. 5 and Table 4 illustrate the mechanical sieve analyses of four sands that have been tested in a tilting flume at the U. S. Waterways Experiment Station. The theoretical critical tractive force has also been determined by formula for each sand and is listed. On the basis of the theoretical critical tractive force necessary to insure movement Sand *A*, with the least tractive force required for general movement, is the best sand, followed in order by Sands *B*, *C*, and *D*.

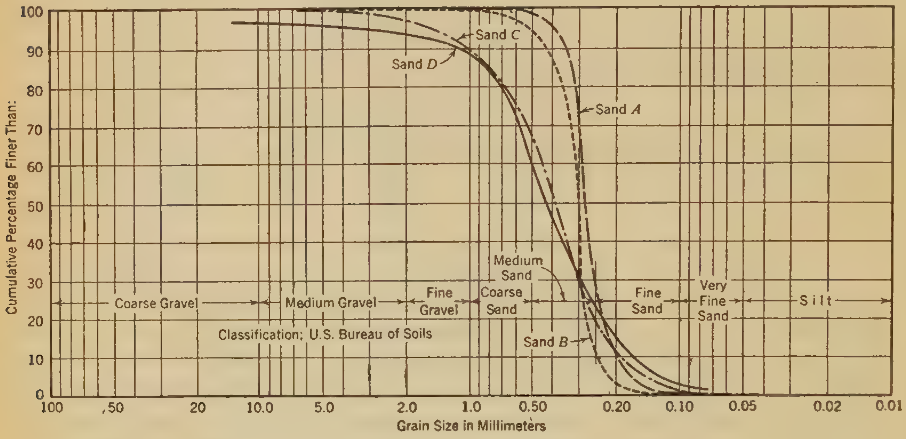


FIG. 5.—MECHANICAL ANALYSIS DIAGRAM

TABLE 4.—CHARACTERISTICS OF SAND SPECIMENS

Item No.	Symbol	Definition	SAND SPECIMEN			
			A	*B*	*C*	*D*
1	E	Mean grain diameter	0.2842	0.3470	0.5584	1.0257
2	D_m	Median grain diameter	0.2714	0.3367	0.3984	0.4358
3	B	Sand uniformity factor	0.6482	0.6428	0.3222	0.1460
4	F	Critical tractive force, in grams per square meter	11.91	14.84	46.80	193.19
5	j_1	Specific gravity of bed material	2.63	2.65	2.62	2.65

Fig. 6 illustrates the results of tests made in a tilting flume to determine the rates of movement for the four sands. The tests indicate that Sand *C* has a much higher rate of movement for any given tractive force than either Sand *A* or Sand *B*. Sand *D*, while having in general a more rapid rate of movement than Sand *C*, requires a higher tractive force for the beginning of general movement.

One of the chief advantages of using a more rapidly moving sand is the saving in time required for the operation of a river model. Consider, for example, a model of a reach in the Mississippi River with scales, as follows: $l = \frac{1}{1\,000}$, $d = \frac{1}{125}$, and $s = 17.4$. The use of Sand *C* in lieu of Sand *A* reduces considerably the time of operation of the model to represent a given time in the prototype. Table 5 compares the rates of movement of Sands *A* and *C* to be expected at different stages. The values in Column (2) are the products of the measured slope and the mean depth in the model and the values in Columns (3) and (4) are read from Fig. 6.

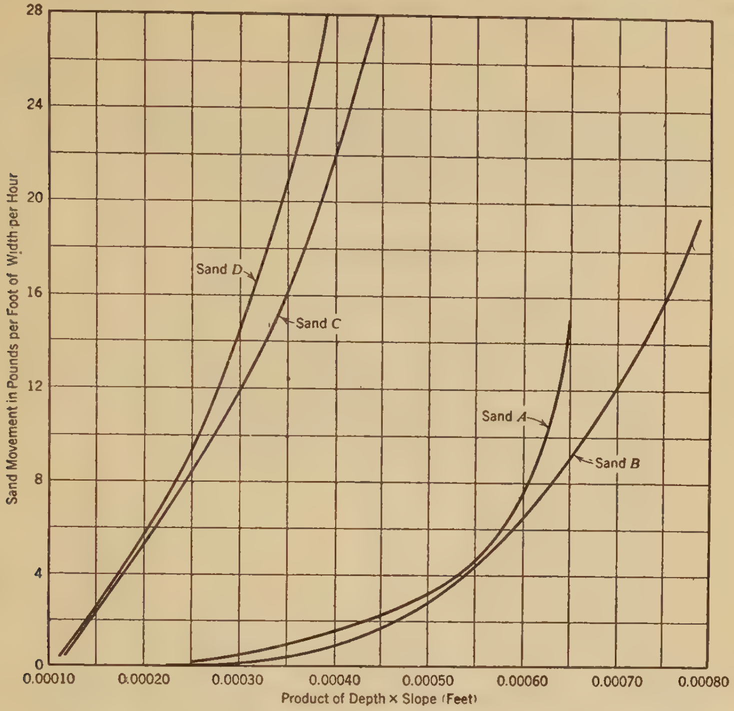


FIG. 6.—RELATION OF RATE OF SAND MOVEMENT TO PRODUCT OF SLOPE AND DEPTH

TABLE 5.—RATE OF MOVEMENT OF BED LOAD MATERIAL FOR DEPTHS AND SLOPES ENCOUNTERED IN A TYPICAL MISSISSIPPI RIVER MODEL

Stage (1)	$D \times S$ (2)	Rate of Movement, in Pounds per Hour per Foot of Width: Sand *A* (3)	Rate of Movement, in Pounds per Hour per Foot of Width: Sand *C* (4)	Ratio of movement, $C \div A$ (5)
0	0.000252	0.2	8.5	42.5 : 1
5	0.000308	0.6	12.0	20.0 : 1
10	0.000360	1.1	16.9	15.4 : 1
15	0.000412	1.8	23.5	13.1 : 1
20	0.000458	2.5	28.5	11.4 : 1
25	0.000504	3.3	....	......
35	0.000596	7.2	....	......

It should be noted that the ratio of the rate of movement of Sand *C* to Sand *A* is not a constant for different stages. For a 5-ft stage twenty times as much sand moves if Sand *C* rather than Sand *A* is used, whereas for a 15-ft stage the ratio of the rates of movement is only 13.1:1. Similarly, for other sands, the ratio of rates of movement varies according to some as yet unknown law. This variance for any two sands emphasizes Lieut. Vogel's statement that formulas representing a time scale for movable bed models based on the theoretical transporting power of flowing water and the resistance to motion of grain particles in suspension or otherwise, are almost invariably misleading. A time scale that applies to a low stage will not apply to a higher stage.

This suggests that perhaps the best method of approach for deriving a time scale is to determine experimentally, by means of flume tests, a time scale for each stage at which the model is to be operated.

TABLE 6

$$\left(l = \frac{1}{1\,000};\ d = \frac{1}{125};\ s = 17.4.\right)$$

Stage, in feet (1)	$D \times S$, in Nature (2)	$D \times S$, in model (3)	Rate of Movement, in Pounds per Hour per Foot, of Width		q (sand) (6)	t (sand) (7)
			Nature sand (4)	Model sand (C) (5)		
0.....	0.00181	0.000252	900	8.5	$\frac{1}{106\,500}$	$\frac{1}{1\,170}$
5.....	0.00222	0.000308	2 200	12.0	$\frac{1}{182\,000}$	$\frac{1}{690}$
10.....	0.00259	0.000360	4 150	16.9	$\frac{1}{244\,000}$	$\frac{1}{510}$
15.....	0.00296	0.000412	7 500	23.5	$\frac{1}{323\,000}$	$\frac{1}{390}$

The computations compiled in Table 6 are obtained, as follows (data are for river model with scales of $l = \frac{1}{1\,000}$, $d = \frac{1}{125}$, and $s = 17.4$):

1.—Compute the product of depth times slope in the prototype for various stages, for example, at 5-ft increments (Column (2), Table 6).

2.—Compute the product of depth times slope for corresponding stages in the model (Column (3), Table 6).

3.—Determine by flume test the rate of movement of the prototype sand for values of $D \times S$ computed in Step 1 (Column (4), Table 6).

4.—Determine by flume test the rate of movement of the model sand for values of $D \times S$ computed in Step 2 (Column (5), Table 6).

5.—Determine the discharge scale value for bed-load movement (Column (6), Table 6), by the formula,

$$q \text{ (sand)} = \frac{\text{Pounds of model sand per foot of width per hour}}{\text{Pounds of Nature sand per foot of width per hour}} \times l$$

6.—Compute the time scale for each stage, by the formula:

$$\text{Time scale, } t \text{ (sand)} = \frac{\text{Volume scale}}{\text{Discharge scale}}$$

For example (see Column (7), Table 6), for the 5-ft stage,

$$t\ (\text{sand}) = \frac{l^2\, d}{q\ (\text{sand})} = \frac{\dfrac{1}{125\,000\,000}}{\dfrac{1}{182\,000}} = \frac{1}{690}$$

Table 6 illustrates the variation in time scale for various stages, for a typical movable bed model. It should be noted that the values in Column (4), Table 6, have been obtained by extrapolation of existing data for a Mississippi sand and probably are only approximately correct. However, it is possible to determine the proper data experimentally and more work should be done along this line as soon as possible. These values should be obtained by flume tests for the particular sand in the prototype.

Having obtained, experimentally, the time scale for the various stages at which the model is to be operated, a model operating chart representing a cycle of one year is computed on the basis of allowing a time interval for each stage equal to the product obtained by multiplying the time scale by the time duration of that stage in the prototype during a typical or average year.

The procedure described herein is only one of various methods of attacking this difficult problem.

FRANK W. EDWARDS,[20] JUN. AM. SOC. C. E. (by letter).—As the possibilities offered by model studies have gradually become apparent to engineers in the United States, the application of the principle of hydraulic similitude has attracted more attention. A number of papers have been written on the subject, some of which have been highly theoretical. These have their place in engineering literature, of course, but it has seemed that the purely theoretical papers should be supplemented with practical suggestions for actual model design, operation, and interpretation. The author has furnished the needed supplement in his practical paper, the purpose of which, as set forth in its "Synopsis," has been fulfilled.

It is the purpose of this discussion to bring out a few points that may be of interest to engineers engaged in model study. The following points are discussed: First, the author's Equation (10); second, the possibility of selecting the horizontal and vertical scales, for a proposed model, in such a manner that the discharge ratio, q, will more nearly satisfy both the gravity and friction formulas; and third, the feasibility of controlling, within limits, the roughness factor in the model.

In Table 2 of the author's paper, Equation (10) has been compared with the actual discharge ratio, as determined experimentally in several models. The relatively close agreement between the ratio, as determined by the formula and by experimentation, is rather surprising. This formula is restricted in three ways: (a) The roughness factors in the model and its prototype must be equal; (b) the slope of the model must not be distorted artificially (that is, the slope in the model should equal the slope in Nature multiplied by the scale distortion); and (*c*) the cross-sections of both the model and its prototype must be such that the hydraulic radius can be replaced by the mean depth without causing appreciable error. It is possible to meet the requirements imposed by the first two restrictions, but the third offers difficulties. In the majority of cases the substitution of d, the scale ratio for the depth, for r, the scale ratio for the hydraulic radius, would introduce a considerable error. In a cross-section of a river which is wide in comparison with its depth, the hydraulic radius may be replaced by the mean depth. The resulting error will be negligible. When this same section is reproduced in a model having a distorted scale, the depth, which is greatly exaggerated, is no longer insignificant when compared with the width. In some cases, when a large distortion is used in a model, the mean depth in the model cross-section might equal twice the value of the hydraulic radius for the same section. This reasoning suggests that, in the majority of cases, the actual ratio, r, as determined from cross-sections in the model and in Nature, should be used in the formula for interpreting certain results of model operation. The formula would then be written,

---

[20] Asst. Engr., U. S. Corps of Engrs., Zanesville, Ohio.

$$q = l^{\frac{1}{2}}\, d^{\frac{3}{2}}\, r^{\frac{2}{3}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (54)$$

It should be noted, also, that using the ratio, r, introduces more difficulties. This ratio will not be the same for all stages of the river. For the lower river stages, R_m, the hydraulic radius in the model, would approach D_m, the mean depth in the model; but, more often than not, the high stages are the subject of experimental investigation. For certain types of problems, the scale ratios should be chosen to keep the variation of the ratio, r, within definite limits for all the river stages to be simulated in the model.

As to the second point: When a model study involves, at the same time, flow through an open channel and flow over a structure, such as a spillway or submerged wing-dam, both gravity and friction should be considered. The author's Equation (3) follows from Froude's law, and Equation (10) from the Manning formula. In a specific problem, the correct ratio for r could be used in the formula instead of replacing r with d. To satisfy both laws, $l\, d^{\frac{3}{2}} = l^{\frac{1}{2}}\, d^{\frac{13}{6}}$, and,

$$d = l^{\frac{3}{4}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (55)$$

This means, according to the Manning formula, that the depth scale must equal the length scale raised to the three-fourths power, if both the gravity and friction formulas are to be satisfied.

Using a similar procedure with the Kutter formula as a basis, it is learned that the value of the exponent for l varies for each specific problem. For ordinary models, having scale ratios within the limits indicated by the author's paper for a roughness factor ratio of unity, the Kutter formula gives the following results when equated to the gravity formula: $l^{0.65} > d > l^{0.75}$. The correct exponent is determined by the specific problem.

It is interesting to compare the results obtained from the Manning and Kutter formulas with those obtained from a formula derived by Richard

Winkel, of the Technical University at Danzig. He suggests that the coefficient in the Chezy formula varies with the characteristic value, $VR\rho$ (modified Reynold's number).[10] With this as a basis, he obtained,

$$q = l\, d^{\frac{12}{7}}\, s^{\frac{4}{7}} \qquad (56)$$

in which, s = the slope ratio, model to Nature. For no artificial distortion of the slope,

$$q = l^{\frac{3}{7}}\, d^{\frac{16}{7}} \qquad (57)$$

Then, using the foregoing procedure, to satisfy the gravity and friction formulas,

$$d = l^{0.728} \qquad (58)$$

This equation gives an exponent for l that is smaller than that obtained with the Manning formula, but is within the limits indicated by the Kutter formula.

To illustrate the formulas just derived, assume a proposed model study that involves both gravity and friction. For this problem let $D_n = 40$ ft, $S_n = 0.0001$, and $N_n = 0.030$. Suppose a horizontal scale of $\frac{1}{500}$ appears suitable. According to the Manning formula, $d = l^{0.75} = \frac{1}{106}$; according to the Kutter formula, $d = l^{0.69} = \frac{1}{73}$; and according to the Winkel formula, $d = l^{0.728} = \frac{1}{92}$. Experimental data alone can provide evidence for determining which formula is the most satisfactory.

The third point to be discussed herein is the control of the roughness factor of the model. When it is assumed that N_m, the model roughness factor, and N_n, the prototype roughness factor, are equal, a determined effort should be made to produce this result. It is practically impossible to predict the roughness that will apply to a proposed model. This factor, which is actually affected by a number of items besides channel roughness, has a relatively high value in small-scale models. Probably the short radius of curvature and the small sections encountered in the model contribute most to the large values. When constructing a fixed bed model, the surface should be left as rough as practical until tests to determine the model roughness factor can be made. These tests can then be used as a basis for finishing the surface of the model to produce the desired roughness factor within the limits of its determination in the prototype.

Very little can be done in the case of a movable bed model. However, if more than one bed material is equally suitable for satisfactory bed-load movement, the material that provides the proper roughness should be selected. Usually, the sides of a movable bed model are made of concrete; that is, they are fixed, while the movable bed material is confined to the bottom of the channel. This fixed surface offers another opportunity for varying the roughness factor to some extent.

I. H. Patty,[21] Esq. (by letter).—In this excellent paper Lieut. Vogel presents Equation (12) as a time scale for models, stating that this formula is applicable to reservoir problems, or to flood-control problems where the time element is important, such as in the filling of back-water basins and other storage areas. The author further states that the formula is not applicable in problems involving bed-movement development; that no formulas have been devised which reliably indicate the relationship for such development; and that such ratios must be determined by actual tests.

During the short space of time that has elapsed since the establishment of the U. S. Waterways Experiment Station, distinct advances have been made in the technique of designing and operating hydraulic models. This is particularly true in connection with experiments on models of the movable bed type. Such models have beds moulded of sand or other transportable material and are used when it is desired to determine the effects of a stream on its bed and banks. They are useful in problems involving river regulation where it is desired to study effects of dikes, channel alignment, methods of increasing depths for navigation, effects of cut-offs, caving banks, and general channel stabilization. The technique of experimentation with these models has advanced to the stage that quantitative results are regularly attained with a fair degree of accuracy. Naturally, the value of the findings are increased enormously if the time of their accomplishment in Nature can be predicted. To make such predictions requires a scale value indicating the relationship between the time required to produce given effects in the model and corresponding effects in the prototype.

There are two methods whereby a time scale for a movable bed model may be developed. A time scale may be established when at least two surveys, taken at different times, are available of the territory represented by the model. The procedure is to determine by actual observation the length of time required for the model to reproduce known changes that have occurred in its prototype. The scale is then established as the ratio between the observed time required by the model and the prototype to effect these changes. This is the method that has been most commonly followed by experimenters in the past, and it is fairly accurate where the necessary survey data are available.

For many years attempts have been made to devise time scales by straight mathematical procedure. Such formulas usually have had factors representing the relationships for length, depth, slope, and mean grain diameter of bed materials. When applied to models for which scales have been derived experimentally, they have invariably indicated relationships so widely at variance, and so conflicting, that they have come to be considered, not only as having little value, but as being misleading.

A formula for a time scale has been derived, mathematically, and has been tested on the movable bed models built at the U. S. Waterways Experiment Station. On several of these models, experimentally derived time scales were also developed, and were found to agree with those determined by the theoretical method, indicating the latter to be both accurate and dependable.

---

[21] Asst. Engr., U. S. Waterways Experiment Station, Vicksburg, Miss.

A stream of water (either a model stream, or its prototype) performs work and loses energy as it flows down-hill, transporting its burden of sedimentary matter, and shifting the materials of its bed to change its hydrographic features. The ability of a movable bed model to simulate the hydrographic configurations of its prototype was demonstrated more than fifty years ago by Fargue, and such results are now regarded as routine technique in hydraulic laboratory practice. Scales for such characteristics in the model as length, depth, slope, velocity, discharge, etc., are derived by direct linear relationships. The time scale developed at the Vicksburg Laboratory after Lieut. Vogel's paper was written, shows the ratio between the time required for a model and its prototype to perform similar work.

Work is done by a force moving a body through any distance. The product of the numerical value of the force and the distance is a measure of the work, or it may be expressed by the equation:

$$W = F\,L \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (59)$$

in which, W = work; F = force, and L = length traveled. Equation (59) may be adapted to express the work done by a flowing stream. It was shown by du Boys, in 1879, that the tractive force of flowing water may be expressed by the formula:

$$J\,D\,S \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (60)$$

in which, D = depth; S = slope; and J = the weight of a unit volume of the liquid. Placing $J\,D\,S$ equal to F in Equation (59):

$$W = J\,D\,S\,L \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (61)$$

Similarly, a work scale, W, denoting the ratio between the work done by a model and its prototype per similar unit length, can be derived if the respective values for the model are divided by those for the full scale:

$$\frac{W_m}{W_n} = \frac{J_m\,L_m\,D_m\,S_m}{J_n\,L_n\,D_n\,S_n} = l\,d\,s = w \ldots\ldots\ldots\ldots (62)$$

The scale, $l\,d\,s$, would also denote the ratio between the time required for the model and the prototype to perform similar work if the velocity of flow in each were the same. This is never true in river models, but if the work scale, $l\,d\,s$, is divided by the velocity scale, v, the result will be a unit-time-work scale, t_w:

$$t_w = \frac{l\,d\,s}{v} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (63)$$

From Manning's formula the scale value for velocity in models has been found about equal to $d^{\frac{2}{3}}\,s^{\frac{1}{2}}$, the expression, $\frac{1.486}{n}$, being cancelled since it is quite nearly equal to unity. In Equation (63), let $v = d^{\frac{2}{3}} s^{\frac{1}{2}}$; then:

$$t_w = \frac{l\,d\,s}{v} = \frac{l\,d\,s}{d^{\frac{2}{3}} s^{\frac{1}{2}}} = l\,d^{\frac{1}{3}} s^{\frac{1}{2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (64)$$

Equation (64) is a complete expression for a unit-time-work scale, showing the ratio between the time required for a model and its prototype to perform similar work. It presumes the bed material in each to be identical, or nearly so, and when they are dissimilar a factor, *f*, denoting the relative transportability of the two materials, as determined by one of several existing formulas or by flume test, must be added and the scale computed for each of the stage heights at which the model is operated. The complete formula, then, will be:

$$t_w = f\,l\,d^{\frac{1}{3}} s^{\frac{1}{2}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (65)$$

The mathematical soundness of Equation (64) may be further demonstrated by developing it from a different angle. A tractive force scale, $d\,s$, may be derived as follows:

$$d\,s = \frac{J_m\,D_m\,S_m}{J_n\,D_n\,S_n} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (66)$$

in which, the factor, *J*, being equal to unity, cancels out. A unit-time-distance scale, t_d, showing the ratio between the distance traveled in the model and in Nature, in unit time, may be derived by dividing the length scale, *l*, by the velocity scale, *v*, or,

$$t_d = \frac{l}{v} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (67)$$

Then the product of the unit-time-distance scale, $\frac{l}{v}$, and the tractive force scale, $d\,s$, will give the unit-time-work scale, or,

$$t_w = \frac{l}{v}\;d\,s = \frac{l\,d\,s}{v} = l\,d^{\frac{1}{3}} s^{\frac{1}{2}} \ldots\ldots\ldots\ldots\ldots\ldots (68)$$

Most of the movable bed models constructed at the U. S. Waterways Experiment Station have horizontal scales (*l*) of between 1 : 1 000 and 1 : 500, and vertical scales (*d*) of from 1 : 150 to 1 : 100. On such models the average time of operation, indicated to simulate 1 year in Nature, is from 6 to 16 hours.

Experience gained on nearly one hundred model studies has demonstrated the reliability of the various formulas given by Lieut. Vogel in his paper. With the exception of a formula for determining the time relationship for movable bed models they comprise all the scale values commonly needed in a river hydraulics laboratory. Other methods of deriving a time scale for such models are being investigated.

CHARLES S. BENNETT,[22] M. AM. SOC. C. E. (by letter).—Engineers engaged in the field of applied hydraulics should welcome this excellent compilation of data on laboratory methods of solving numerous hydraulic problems. The use of models in the study of certain hydraulics phenomena furnishes the only practical and reliable solution to many problems confronted by designers of hydraulic structures.

---

[22] Engr., The Miami Conservancy Dist., Dayton, Ohio.

The writer wishes to call attention to the fact that expensive equipment and elaborate laboratories are not always necessary for the application of some of the studies outlined by Lieut. Vogel. It is often possible to secure valuable data on specific problems by the use of a small amount of equipment and some ingenuity.

The improved channel of the Miami River at Dayton, Ohio, is designed to carry a regulated maximum flood flow with a free-board of 3 ft on the levees flanking the channel. Several years ago it was proposed to erect several low overflow dams in this channel to provide slack water during periods of low flow, primarily to enhance the appearance of the stream and to provide facilities for boating and swimming. Before such structures could be approved it was necessary to determine what effect they might have on river stages at times of high flow. As various shapes and types of dams were under consideration it did not seem feasible or safe to rely entirely on calculations of the effects of such structures.

It was decided to construct a small model of the section of the stream involved and to measure the effects of various types of dams, using flows corresponding to great floods. The reach of river channel to be investigated was about 6 000 ft long, with an average width of about 600 ft. Space available for the model limited the scale to 1 : 600 horizontal and 1 : 120 vertical.

A specially constructed table top of laminated wood, about 4 by 10 ft, was mounted on trestles set up on a concrete garage floor. The table was carefully and accurately leveled and adjusted with clamps so as to maintain its level position. The model was constructed on this table, using the top surface as a datum plane. Templates made of cardboard coated with shellac and spaced 200 ft apart were used. The model was made of a concrete consisting of cement and commercial asbestos in about equal proportions. This material can be cut with a knife, nails can be driven in it, and it is relatively light in weight. Two paintings with a liquid water-proofing material made it impervious.

Water was furnished by a 2-in. direct-connected centrifugal pump, discharging into a baffled weir box in which was a 90° V-notch of steel plate. Water flowing through the channel passed out at the lower end under an adjustable tail-gate and emptied into a storage trough under the table, from which it flowed back to the pump. The maximum quantity of flow required was about 0.15 sec-ft.

Measurements of surface elevations in the channel were made with point gauges, each mounted on a beam spanning the channel. These cross-beams slid on long steel-bar beams mounted parallel to the channel on each side. The point gauges were equipped with verniers and could be read to 0.0005 ft. During tests an engineer's level was used to check all adjustable parts of the model set-up to insure accuracy.

Models of dams were cast of babbitt metal to fit the bed of the stream at pre-determined points. The various models could be interchanged as required.

The cost of this model was relatively small. The pump and motor cost \$60 (second-hand) and were available for other uses. The total cost of materials was about \$85. All labor was performed by regular employees who were handy with tools. Even the point gauges were "home-made," the verniers being salvaged from discarded level rods. Many valuable data were secured by the use of the model and most of the material was salvaged for subsequent experiments.

Kenneth C. Reynolds,[23] Assoc. M. Am. Soc. C. E. (by letter).—The U. S. Waterways Experiment Station is carrying on excellent researches and the presentation of methods and equipment which have proved to be satisfactory at this Station, is of inestimable value to other experimenters in the field of river hydraulics.

The writer agrees as to the desirability of having uniform nomenclature although he can not agree with some notations used by the author. If some Greek letters have been frequently used for certain factors, they might well be considered as standard. Thus, ν is used by many for the kinematic coefficient of viscosity.

Immediately preceding Equation (5), consideration is given to an undistorted model, and it is shown that the ratio of discharges is, $q = \frac{a\, r^{\frac{2}{3}}\, s^{\frac{1}{2}}}{n}$. In deducing Equation (5) it is assumed that the hydraulic radius is equal to the mean depth. This assumption is unnecessary since the hydraulic radius is an area divided by a wetted perimeter or is a linear dimension for an undistorted model. The ratio of these linear dimensions is l, from which Equation (5) follows.

In the paragraph preceding Equation (10) Lieut. Vogel states that it has been found from experiments with distorted models of rivers that the ratio of the roughness coefficients for model and prototype is unity, both for models built of cement mortar, if the scale ratio of length is less than 1 to 1 000, and also, for models with a sand bed when the scale length is between 1 to 1 000 and 1 to 500. This information is of real value. In 1925, Erik G. W. Lindquist, M. Am. Soc. C. E., determined[3] this ratio of coefficients analytically for an undistorted model with non-uniform flow to be the sixth root of the scale of length.

V. V. Tchikoff,[24] M. Am. Soc. C. E. (by letter).—In this extremely valuable paper the author covers, quite completely, the theory and practice of river laboratory hydraulics. For a model and the prototype to be hydraulically similar it is theoretically necessary that they have geometrical, kinetic, and dynamic similitude. However, when a river model is designed, the cross-section is usually distorted and, referring to the last two mentioned conditions, the author states: "For all practical purposes, dynamic and kinematic similitude exists when the flow of water in the model is of the same character as the flow in Nature."

---

[23] Asst. Prof. of Hydraulics, Dept. of Civ. and San. Eng., Mass. Inst. Tech., Cambridge. Mass.

[24] Cons. Engr., New York, N. Y.

The question now arises as to how to judge when the flow in the model is of the same character as that in Nature. The best and, perhaps, the only practical criterion is whether the character of transportation is the same for the same silt in both channels. If the prototype and model meet these requirements, their flow will have the same character and there will be hydraulic similitude.

The study of the phenomena of silting leads to two inevitable conclusions: First, the prototype and the model must have the same kinetic flow factor; and, second, their discharges should be in proportion to the hydraulic radius, cubed. As will be shown, subsequently, the first condition provides the same turbulence of water, and the second, determines the precise distortion of the cross-section.

Kinetic Flow Factor.—This important idea was introduced by B. A. Bakhmeteff, M. Am. Soc. C. E.,[25] in connection with his "specific energy of flow" expressed by the equation:

$$\epsilon_d = D + \frac{V^2}{2\,g} = D\,(1 + \tfrac{1}{2}\,K_d) \dots\dots\dots\dots\dots (69)$$

in which, ϵ_d is the average velocity head (energy per unit weight of liquid) referred to the bottom of the canal cross-section; and K_d is the kinetic flow factor, a dimensionless number which is equal to:

$$K_d = \frac{V^2}{g\,D} = 2\,\frac{\frac{V^2}{2\,g}}{D} \dots\dots\dots\dots\dots\dots\dots\dots (70)$$

or twice the ratio of the kinetic energy head to the potential energy head.

From Equation (70):

$$V = \sqrt{K_d\,g\,D} \dots\dots\dots\dots\dots\dots\dots\dots (71)$$

which is no more than the author's Equation (2), or Froude's law derived from the formula for velocity of a body falling freely under the influence of gravity. It applies also in the case of a channel of indefinite width.

Fluid friction, however, becomes of considerable importance in the usual channel and most of the formulas for uniform flow in channels and pipes are connected with the hydraulic radius. Therefore, if one refers the energy head to the hydraulic bottom instead of to the bottom of the cross-section as in Equation (70), the result is a new expression for the specific energy:

$$\epsilon_r = R + \frac{V^2}{2\,g} = R\,(1 + \tfrac{1}{2}\,K_r) \dots\dots\dots\dots\dots\dots (72)$$

in which,

$$K_r = \frac{V^2}{gR} = 2\,\frac{\frac{V^2}{2\,g}}{R} \dots\dots\dots\dots\dots\dots\dots\dots (73)$$

---

[25] "Hydraulics of Open Channels," by Boris A. Bakhmeteff, p. 64.

is a factor of turbulence, or is the kinetic factor of flow; but in the more general hydraulic understanding of the liquid flow, this is twice the ratio of the velocity head to the hydraulic radius.

If a curve of ϵ_r is constructed as a function of the hydraulic radius, R, it will be similar to that of Equation (70), with D substituted for the hydraulic radius, R. The first derivative of Equation (72) will give the critical flow conditions depending on the form of the cross-section. In the case of a channel of indefinite width when $R = D$, it is not difficult to obtain the well-known equation for critical depths:

$$D_{cr} = \sqrt[3]{\frac{q^2_b}{g}} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (74)$$

in which, q_b equals the discharge per unit of channel width.

Similar to Equation (71) the value of V, from Equation (73), is equal to:

$$V = \sqrt{g K_r R} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (75)$$

The correctness of Equation (75) and the general incorrectness of Equation (71) is proved by practical data on non-silting channels. This problem has been studied thoroughly by engineers in British India and the equation for the critical non-silting velocity,

$$V_s = b X^c \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (76)$$

passed through the evolution illustrated by Table 7.

TABLE 7.—VALUES OF NON-SILTING AND NON-SCOURING VELOCITY, V_s, BRITISH INDIA (VALUES IN EQUATION (76))

Item No.	Authority	UPPER BARI DOAB CANAL			LOWER CHENAB CANAL			MADRAS GODAVERI WESTERN DELTA CANAL		
		b	X	c	b	X	c	b	X	c
1	Kennedy	0.84	D	0.64	....	....	....	0.67	D	0.55
2	Lindley	....	..	....	0.95	D	0.57	....	....	....
3	Griffith	0.97	D_m	0.57	....	....	....	0.69	D	0.57
4	Hawley	....	....	....	....	....	....	0.72	D	0.55
5	Lacey	1.17	R	0.50	1.13	R	0.50	0.79	R	0.50
6	The writer	1.15	R	0.50	1.13	R	0.50	0.81	R	0.50

One of the most recent students of this problem is Gerald Lacey[26] and his formulas (see Item No. 5, Table 7) are now generally adopted. Mr. Lacey's equation completely coincides with the writer's Equation (75) developed from theoretical considerations. Item No. 6, Table 7, contains the comparable elements of the writer's formulas for the three canals mentioned, calculated on the basis of mean value of the kinetic factor. For the three canals, V_s equals

---

[26] "Stable Channels in Alluvium," by Gerald Lacey, *Minutes of Proceedings*, Inst. C. E., Vol. 229, Session 1920–30, Pt. I.

4.15%, 3.98%, and 2.04%, respectively. The writer's data yield a quantitative expression of kineticity of flow (that is, turbulence) and, of course, are directly related to the size of the silt particles. Thus, to secure hydraulic similitude, the same kinetic factor in the model and prototype is the first prerequisite.

Hydraulic Similitude.—There are many channels, however, with the same kinetic factor. It is easy to fulfill this condition in designing the model, but this is not sufficient to obtain the definite dimension of a channel. A second condition is that the discharges in the prototype and model must be in proportion to their hydraulic radius, cubed. The reason for this is again the necessity of preserving the same character of silt transportation.

The discharge of water is represented geometrically by a solid with a plan area, $A = PR$, and a height, $V = \sqrt{g K_r R}$. The relation of discharge for the same kinetic factor will be:

$$\frac{Q}{Q_1} = \frac{A V}{A_1 V_1} = \frac{A R^{0.5}}{A_1 R_1^{0.5}}$$

The area, A, may be represented as,

$$A = PR = y R^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (77)$$

in which, $y = \frac{P}{R}$ depends on the shape of the cross-section; and the relation of the discharge will be,

$$\frac{Q}{Q_1} = \frac{y R^{2.5}}{y_1 R_1^{2.5}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (78)$$

If the areas are geometrically similar, y will equal y_1 and the discharges will be in proportion to $R^{2.5}$. This condition occurs in designing non-distorted hydraulic models of channels. In this case, it is necessary to change the other hydraulic elements accordingly.

Thus, keeping the similitude of cross-sections, the relationship of the discharge will be as $R^{2.5}$, which means that the discharges represented as volumes are not geometrically similar. In order that they may be similar, even if only hydraulically, it is necessary that they be comparable in relation to the hydraulic radius cubed; thus:

$$\frac{Q}{Q_1} = \frac{R^3}{R^3_1} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (79)$$

or, in view of Equation (75):

$$\frac{Q}{Q_1} = \frac{V^6}{V^6_1} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (80)$$

In other words, the discharges vary as the sixth power of velocity. This may be considered as a well-established law for turbulent flow, as was pointed out by Mr. Lacey[27] in reducing the same formula, although by a different method. Comparing Equations (78) and (79), it is found that the relation of the coefficient, y, to the square root of the hydraulic radius must be a constant and,

$$\frac{y}{y_1} = \frac{R^{0.5}}{R_1^{0.5}} = \frac{V}{V_1} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (81)$$

---

[27] *Minutes of Proceedings*, Inst. C. E., Vol. 229, Session 1929–30, Pt. I, p. 271.

This shows that, in order to have hydraulic similitude of discharges, it is necessary to distort the cross-section of the channel, so that the coefficient, $y = \frac{P}{R}$, varies according to the velocity. The necessity of this relationship has been shown by Dr. H. E. Hurst.[28]

The discharges for the channels of the same hydraulic similitude can be designated by,

$$Q = Y\,R^3 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (82)$$

in which, Y is the constant coefficient for the channels with the same kinetic factor and the same transportation characteristics.

If one constructs the relation of Q as a function of R, using the data of the three non-silting canals given by Lacey, the result is: $Q = 6.93\ R^3$ (Upper Bari Doab); $Q = 9.45\ R^3$ (Lower Chenab); and, $Q = 4.77\ R^3$ (Madras Godaveri). This is the practical confirmation of Equation (82). Coefficient Y is known in the prototype.

Thus, in order to preserve the same character of flow in the model, or the same transportation characteristics, it is necessary to fulfill both conditions, namely, the same constant kinetic factor and the relationship of discharges as R^3. This will give definite data for designing the model which will be precisely distorted.

Viscosity.—The smaller the model the more important is viscosity, the second flow factor. Entirely similar to the kinetic factor it is advisable to introduce the viscosity factor of flow which, according to Poiseulle's law, is:

$$U = \frac{Y\,V}{g\,R^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (83)$$

in which, U is the viscosity factor and Y, the coefficient of kinematic viscosity. Comparing Equations (73) and (83), it is clear that in the model it is impossible to keep both factors. The preservation of the kinetic flow factor is possible up to a certain size of model when viscosity must be considered. Reynolds' number serves as the criterion. Having two factors, kineticity and viscosity, this number is obtained by dividing one by the other:

$$\mathsf{R} = \frac{K_r}{U} = \frac{V R}{Y} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (84)$$

This is a simple and clear definition of Reynolds' number. Its numerical values are discussed by Lieut. Vogel.

Manning's Exponent.—The writer would also like to make some remarks about the variation of Manning's exponent of $\frac{1}{6}$. That this exponent is not constant, varying with the nature of the soil of the channel, is a well-established fact. This has no particular importance in the use of Manning's formula for normal practical conditions when the hydraulic radius is, say, between 0.8 ft and 10.0 ft. However, in designing the model of a river the variation of the hydraulic radius is much greater. To find the relationship of Manning's exponent to N, the coefficient of roughness, the writer constructed the value of Chezy's C as function of R for various Kutter N-values and found that the following formula represents this relation:

---

[28] *Minutes of Proceedings*, Inst. C. E., Vol. 229, Session 1929–30, Pt. I, p. 330.

$$z = 6\,N + 0.05 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (85)$$

in which, z is Manning's exponent.

For Kutter's $N = 0.010$, one has $z = 0.11$ (about $\frac{1}{9}$) and for $N = 0.040$, $z = 0.29$ (about $\frac{1}{3.4}$). This shows considerable variation in comparison with the usual Manning's exponent, $z = \frac{1}{6} = 0.167$. The last value of z corresponds with $N = 0.0194$, when only Manning's exponent is correct. Therefore, in developing relations for the model while using Manning's formula, Equation (85) must be taken into consideration.

Some comparisons can be made of the author's and writer's formulas in reference to exponential dimensions. Equation (74) is similar to the author's Equation (3), both having the general exponent, 2.5. Equation (10) is based on Manning's exponent, $\frac{1}{6}$. In another paper[29] Lieut. Vogel gives the value for N for the Mississippi River as 0.037. The corresponding value of Manning's exponent from Equation (85) will be equal to 0.272 instead of 0.167. If the general exponent, 2.67, in Equation (10), is corrected, it becomes 2.77 as compared with 3.00 in the writer's Equation (82).

Samuel Shulits,[30] Jun. Am. Soc. C. E. (by letter).—Equation (10) of this instructive paper is a special case of a more general law for the discharge scale of distorted models. Consider the well-known formula for the loss of head in pipes:

$$H = \lambda\,\frac{L}{D}\,\frac{V^2}{2\,G} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (86)$$

in which, λ is a coefficient of friction, and the other symbols are as defined in the paper. This form of equation is highly desirable because of the non-dimensional coefficient, λ. It can, and should, be adapted for open-channel flow. This can be accomplished easily by inserting the hydraulic radius, R, for D. The fact that $D = 4\,R$ is knowingly disregarded, because it would only necessitate the inclusion of the numeral, 4. Equation (86), rewritten, then becomes:

$$V = \sqrt{\frac{2\,G}{\lambda}}\,\sqrt{RS} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (87)$$

It is evident that the term, $\sqrt{\frac{2\,G}{\lambda}}$, is nothing more than the Chezy roughness coefficient, C. The relation between the non-dimensional coefficient and the Chezy number is $\lambda = \frac{2\,G}{C^2}$, in which, C has the dimension, $\frac{L^{0.5}}{T}$.

---

[29] "Geometric *versus* Hydraulic Similitude," by Herbert D. Vogel, Assoc. M. Am. Soc. C. E., *Civil Engineering*, August, 1932, p. 467.

[30] Asst. Engr., U. S. Bureau of Reclamation, Denver, Colo.

It is important to note that the use of the coefficient, λ, has become general practice in Europe, especially in hydraulic laboratories. If Equation (87) is used instead of the Manning formula, the discharge scale becomes:

$$q = \frac{\lambda_n{}^{0.50}}{\lambda_m{}^{0.50}} \, a \, r^{0.50} \, s^{0.50} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (88)$$

With the same assumptions as in the original paper, it is found, finally, that,

$$q = \frac{\lambda_n{}^{0.50}}{\lambda_m{}^{0.50}} \, l^{0.50} \, d^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (89)$$

Since $C = \frac{1.486\, R^{\frac{1}{6}}}{N}$, the relation between λ and N is:

$$\lambda = \frac{2\, G N^2}{(1.486)^2 R^{\frac{1}{3}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (90)$$

Substitution of Equation (90) in Equation (89) yields the author's Equation (8) which proves that the latter equation is only a special case. Had the Ganguillet and Kutter formula been used instead of the simple one of Manning, a very complex equation, indeed, would have been obtained for the discharge scale.

Lieut. Vogel's formula for the discharge scale is a valuable tool for the laboratory technician because it will enable him to estimate the scale when experience data are lacking; but it must be emphasized that the rule was derived by using the Manning flow formula, and for models and prototypes for which n is approximately equal to 1.

A more rational method now in use in many laboratories consists in collecting values of λ_m, the non-dimensional coefficient of roughness, for the various surfaces and types of models encountered in the laboratory during its operation, and plotting these values against Reynolds' number. Sometimes over-bank roughness caused by trees during floods is simulated in the model by means of pebbles. Values of λ_m computed from such models with different values of R would facilitate greatly the determination of the discharge scale for future models, since roughness coefficients for such conditions are not to be found in any engineering tables. Equation (88) or Equation (89) is used for this purpose. It is desirable, of course, that the ratio of λ_m to λ_n be as close to 1 as possible. A very instructive and useful table on scales is given by Eisner.[81]

Turbulence Criterion.—The criterion for turbulent flow, Equation (15), should be used with some care. To obviate the inconvenience of the Reynolds' number with its string of zeros, Krey[82] introduced what is known to-day as the Krey number, $V D \rho$, in which, ρ is a modified form of the kinematic viscosity, K; that is, $\rho = \frac{1.2}{10^6\, K}$ in the metric system. The value of ρ is chosen so that at a mean temperature of 13.2° C, it has a value of 1; but at 5° C, it is

---

[81] "Theory of Flow in Open Channels," by F. Eisner, Leipzig, 1932.

[82] "Grenzen der Übertragbarkeit der Versuchsergebnisse und Modellähnlichkeit bei praktischen Flussbauversuchen," *Zeitschrift* für angewandte Mathematik und Mechanik, 1925, Vol. 5, No. 6, p. 484.

0.79, and at 25° C it is 1.32 sec. per sq m. Consequently, if the value of VD was determined at the higher temperature, the corresponding metric Krey number would be $1.32 \times 0.00186 = 0.00245$. With water at a temperature of 41° F (5° C), as sometimes occurs in the Denver Laboratory, VD would have to be as high as 0.00311 to yield the same Krey number. This is an increase of 67% over the value in Equation (15) (in metric units, $Z = VD > 0.00186$). The use of VD alone as a criterion probably would result in laminar or transitional flow. In models with very shallow depths of flow in warm climates, the effect of temperature may be considerable.

Bed-Load Scale.—Krey gives the following approximate relation for the bed-load scale, which will be designated by β:

$$\beta = l\, d^6\, s^6 \qquad (91)$$

This relation is based on the fact that according to experimental data the bed load per unit width of channel varies directly, and approximately, as the sixth power of the tractive force, F, in Equation (13).

Charles D. Curran,[33] Assoc. M. Am. Soc. C. E. (by letter).—Doubtless, this is the most complete treatise on the subject of river laboratory hydraulics presented in recent years. It gives concisely those tenets of earlier research still followed and presents clearly the latest developments in the science as practiced to-day in the United States, and especially at the U. S. Waterways Laboratory, at Vicksburg, Miss.

Since this art is relatively young in this country those interested in it will have widely ranging opinions of what problems have been solved adequately and what problems are in urgent need of solution. It must first be remembered that the practice of river laboratory hydraulics has a dual goal; (1) The prompt, accurate, and practical solution of the problem presented; and (2) the development of more precise laws of the elements affecting both rivers and models.

The first is especially important in Federal work. A waterways engineer is confronted with a problem that calls for immediate solution. Therefore, the laboratory solving the problem must work as expeditiously as possible to give him the best answer it can in time for him to use it. The principles of reproduction are followed and a laboratory representation of the river or harbor in question is constructed. Because of the impossibility of exact undistorted reduction of every dimension and every force, a true model cannot be built and truly quantitative results cannot be obtained. The writer was pleasantly surprised to learn[34] that experiments of the U. S. Waterways Experiment Station have given accurate quantitative results in the solution of some river problems. However, he believes that, as yet, answers found must be presented by laboratory workers as qualitative and, likewise, must be accepted by field workers as qualitative. Neither must expect that laboratory experiments will show that such and such an island will be scoured away 50 ft in 2 years and 3 months if certain dikes are constructed. An experiment will show rather that with such construction the island will recede, stay fixed, or advance in a short time or a long time. The attempt to learn of conditions that will exist in the year 2000 in the Rangoon River Delta[35] is probably expecting too much from a study of a laboratory model, and surely problems being studied in a limited time should not be expected to be so precisely solved. When more laws are more exactly known and when unlimited time may be spent in model study, then perhaps one can expect answers that are quantitatively precise.

---

[33] Lieut., Corps of Engrs., U. S. Army, Brooklyn, N. Y.

[34] *Civil Engineering,* March, 1934, p. 118.

Hand in hand with the practical solution of river problems must go work on the problems of determining various hydraulic laws. Among such latter problems is that, met in movable bed models, of tractive force on the bed load. Work on this problem is being undertaken at many laboratories. Equation (13) is a simple solution of this problem. Its veracity has been proved in German experiments. However, as the author states, it is based on the assumption that the bottom slope is parallel to the surface slope. Furthermore, Lieut. Vogel states that this assumption is not absolutely accurate. Presumably this will make only a negligible difference in average reaches of river at normal stages. However, in a rising river, as a flood approaches, surface slope increases; and as the flood recedes, surface slope decreases. It is true that depth varies likewise. However, bottom slope does not necessarily remain parallel to surface slope. Does the du Boys formula (Equation (13)) then hold true? It is possible to find a stretch of river, practically unvarying in bottom slope, which, at different seasons, will have different surface slopes but equal depth.

When surface slope is parallel to bottom slope it can be seen from consideration of the Chezy formula and Equation (13) that $F \propto V^2$. Does this relationship hold true in a rising and in a falling stream?

Kennedy's[36] experiments give evidence of a kind that indicates such a variation. He stated that power of suspension varied as V^3, theoretically; yet because of bottom movement he found this suspension power varying as $V^{2.56}$. If he considered all material moved by the stream as in suspension, it may well have been that because $F \propto V^2$ the combining of bed-load material with suspended material gave him the exponent of 2.56. At present, the writer is inclined to give greatest credence to du Boys' equation. However, he is not yet willing to disregard entirely Gilbert's[37] work and other works that give tractive force varying as high as the sixth power of velocity. From Lieut. Vogel's paper it may be judged that the du Boys and Kramer formulas give sufficiently accurate results.

J. B. EGIAZAROFF,[38] ASSOC. M. AM. SOC. C. E. (by letter).—A number of very interesting and important questions are developed in this excellent paper. Its special merit lies in the fact that its author has brought to a conclusion the answer to practical questions that formerly were explained only in a general way.

---

[35] *Civil Engineering*, February, 1933, p. 89.

[36] *Minutes of Proceedings*, Inst. C. E., Vol. 119, p. 281.

[37] *Professional Paper No. 86*, U. S. Geological Survey, 1914.

[38] Prof., Electrotechnical Inst. in Leningrad, Leningrad, Union of Socialistic Soviet Republics.

Two aspects of the problem should be emphasized: (1) The value of the criterion for turbulent flow in models; and (2) conditions to be met in devising fluvial models to accommodate a movable bed load.

(1) The Value of the Criterion for Turbulent Flow.—The author has shown that, for the Reynolds' criterion as proposed by Professor H. Krey, $V D > 0.075$, can be substituted by $V D > 0.02$, even for a smooth concrete-lined bed.

Consequently, for a model with a movable bed load (sand), this condition will give results even more favorable, on account of the greater roughness of the bed. Therefore, it becomes possible to experiment with non-distorted models which, according to Professor Krey, it was necessary, formerly, to distort.

Of great influence in the choice of the scale of a model is that part of its cross-section, that is in active motion. The entire cross-section of a river does not take part in this motion; in a part of the channel (especially in cases when water flow is retarded by a dam) the water has a mild eddying motion. The application of the Reynolds' criterion to the entire cross-section, including those parts not actively involved in the motion of the stream, seems to be incorrect.

(2) Conditions to Be Met in Designing River Models.—Of exceptional interest is the question of similitude in models designed to accommodate a movable bed. This paper and others emanating from the U. S. Waterways Laboratory, indicate clearly how to choose sand as to size and quantities, and the corresponding rate of flow for any experiment. Hans Kramer, Assoc. M. Am. Soc. C. E., particularly, has contributed valuable information in this field.[39]

One serious difficulty remains, however, the task of obtaining sufficient data as to the movement of the bed load in Nature (the prototype), so as to create the same conditions in a laboratory flume. This problem is especially serious in the forebays of hydro-electric plants where, after sedimentation, particles are put in motion, which could not be considered as present in the normal bed load of the stream.

From 1931 to 1933 experiments were performed in the Hydro-Electric Laboratory in Leningrad,[40] on three models of head-works with intakes. Observations were made of the behavior of a movable bed load in a river with a steep slope, and in a laboratory flume. All these experiments showed, that when the size of sand particles chosen for the model was limited to 0.5 mm to 2 mm and when the average effective diameter did not exceed 1.5 mm, the new bed of the river, in the forebay, assumed a slope that was practically independent of the size of the sand particles.

This new bed slope, however, was greatly influenced by the ratio of the quantity of sand to the quantity of water that passed through the model at the same time. The results thus obtained make easier the selection of model

---

[39] *Proceedings*, Am. Soc. C. E., April, 1934, p. 443.

[40] See, "Description of the Laboratories of the State Scientific Research Institution on Hydrotechnics in Leningrad," 1933, and *Proceedings* of the same Institute, Vol. IX and X (Engineering Societies Library).

"geschiebe" for torrential streams retarded by dams, than would be necessary for freely flowing streams, with relatively flat slopes and transporting a finer silt.

Naturally, comparisons of observations on a model and prototype are of the utmost importance. Such a comparison was made in 1933 at the Hydro-Electric Laboratory for the hydro-electric plant at Zemo-Avtchally, U.S.S.R. The scale ratio of the model (non-distorted) was 1:100. The forebay of the prototype was fully silted after two years, with little change in the succeeding three years; the material was composed of fine sand in suspension, without gravel or boulders. In the model the sedimentation was entirely the result of "geschiebe" movement. Comparisons of the silted sections of prototype and model indicated a discrepancy of 6 to 8% near the head-works and intake structure, and as much as 25 to 35% farther up stream where a sharp curvature of the channel may have influenced the result.

HERBERT D. VOGEL,[41] ASSOC. M. AM. SOC. C. E. (by letter).—Inasmuch as the main purpose in presenting a paper on practical river laboratory hydraulics was to draw out constructive thought on the subject, the many discussions which ensued have proved most gratifying. The fact that several have emanated from young men in the profession is also encouraging because, as the science itself is young, its full future development will rest upon engineers who have years of growth ahead of them. The principal point of disagreement among those offering discussions, fortunately, is one which should be easily settled by the newly appointed Special Committee of the Society on Hydraulic Research. No one individual can lay down symbols and nomenclature which will be universally acceptable, but an impartial committee, acting on the matter, should be able to arrive at a generally satisfactory terminology.

Professor Straub favors the designations used in the paper with the reservation that symbols for scale should represent whole numbers instead of proper fractions. This makes for a certain degree of simplification, but may cause confusion in the minds of those who have habitually expressed scales—as of maps—as proper fractions. The effects of distortion have been well brought out by Professor Straub and recent tests conducted at the U. S. Waterways Experiment Station have served to verify a number of his points. In the case of a model of the St. Clair River from which it was desired to obtain the hydraulic effects of submerged sills, it was found that a distortion of about 3 to 1 caused a sufficient departure from the Froudian law to give results materially different from those obtained in the case of a non-distorted model constructed subsequently. Changing the roughness of the distorted model served to bring the results into much closer agreement with those of the non-distorted model, but practicable limits of roughness could not be exceeded. Tests on the non-distorted model with discharges varying from Froudian relationships relatively the same degree as on the distorted model, were in close agreement with the results obtained on the latter, thus indicating that if Froudian relationships had been adhered to the distorted model would have given accurate results. In this case, the practicable limits of roughness having been reached, a decrease in slope throughout the model (to be obtained by progressively raising each template from head to foot of the model sufficiently to eliminate part of the increase in slope due to geometric distortion) was indicated as a necessity to comply with Froudian relationships.

---

[41] Lieut., Corps of Engrs., U. S. A.; Student, The Command and General Staff School, Fort Leavenworth, Kans.

Had the model been built for the determination of resulting bed configurations—that is, had it required a movable sand bed—an increased slope (additional to that imposed by distortion) or depth would have been necessary to develop adequate tractive force, and a greater departure from Froudian laws would have been necessary. Relatively few movable bed models comply with Froudian relationships, however, and if proper verification tests are conducted this departure is unimportant.[42] In a movable bed model the principal objection to distortion is that angles of repose of sand banks are substantially the same as in Nature, while distortion requires that they be maintained much steeper. Professor Straub has explained this point to advantage.

Lieut. Thompson has introduced a new note in striking upon the Chezy coefficient as a criterion for controlling roughness in the model. This entire subject is one upon which much future research should be centered. At present, it can only be said that the sum of knowledge relative to roughness coefficients of natural rivers or models is pitifully small and that there is no index for the actual measurement of roughness. The symbols, n, m, or C, as commonly used, are factors introduced to make the answer "come out right", and include resistance of bends, loss in eddies, effects of changes of cross-section, unknown obstructions, such as wrecks of vessels, etc.

Professor Powell voices the attitude just expressed in stating the need for a "new and better formula for channel flow". His theoretical treatment of the problem from a basis of dimensional analysis is extremely interesting, although not necessarily conducive to aid in the practical design and operation of river models. As stated in his closing paragraph, however, the method is the principal contribution that can be advanced at this time, and any really satisfactory formula must follow from extensive tests on channels of varying characteristics. The American Committee of the World Power Conference, in 1932, undertook the task of co-ordinating research activities in this field.

The efforts of Lieut. Nichols and Mr. Patty to develop a suitable and dependable time scale are wholly laudable, but in spite of all present evidence to the contrary the writer still believes that, as was previously stated (see paragraph following Equation (12)),

> "There is no known 'short-cut' to such determinations, and formulas taking into account the transporting power of flowing water and the resistance to motion of grain particles, in suspension or otherwise, are almost invariably misleading."

---

[42] See "Sand Mixtures and Sand Movement in Fluvial Models", by Hans Kramer. Assoc. M. Am. Soc. C. E., *Proceedings*, Am. Soc. C. E., April, 1934, p. 443.

Recognizing that for every condition of bank alignment or curvature there will result for each stage an ultimate bed development of unchanging nature, it seems more reasonable to assume a cycle of such duration as will fit into a normal working day, and then, for each condition to be tested, to repeat the cycle a sufficient number of times to insure the attainment of stable conditions. Both Mr. Patty and Lieut. Nichols recognize this fact, and the latter has made it particularly plain by his discussion that practical difficulties exist, which make an empirical formula nearly impossible of achievement.

The discussion by Mr. Edwards points out once more the possibility of reconciling the Froudian law with established laws of friction flow in open channels. This was tried in the case of the St. Clair River experiment, previously mentioned, and again in the canal-crossing experiment at Freeport, Tex. In the latter experiment the silt-transporting capacity of the model river had to be reconciled with the natural laws of hydraulic flow as established by Froudian principles. This was accomplished by designing the model so as to obtain a discharge scale based on the Manning formula, and, subsequently, roughening the concrete bed so as to obtain a value for Manning's n which would bring the discharge scale in perfect agreement with both the Froudian law and the Manning formula. A heavily stuccoed concrete bed produced the required frictional resistance ($n = 0.022$), and no other expedients were necessary to obtain compatibility. Another method of adjustment is to change the form of the relationship, $d = l^{\frac{3}{4}}$, or to add negative supplementary tilt to the model. Much experimental work must be done before laws can be established to meet the manifold ramifications of compatibility requirements. Eventually, research will point the way to true reconciliation of the requirements of Froudian laws, turbulence, and bed-load transportation, but for the present the first thing to be recognized is that, as a rule, each model is designed for a particular purpose—that is, to procure similarity of behavior with respect to a single type of phenomenon.

Mr. Bennett's discussion serves a valuable purpose in showing how model tests may be accomplished at very low cost when limited funds are available to the experimenter. The accuracy of experimental data is generally greater for models of large scale than for those of small dimensions, but frequently a model of modest proportions will serve to supply necessary information within the range of required accuracy. It would appear foolish to forego all model testing simply because the goal of achievement has been set too high. When funds are not available for extended research and experimentation, it is still possible that a less ambitious program will yield results sufficiently accurate for practical purposes.

Professor Reynolds joins with Professor Powell in protesting departure from the use of standard Greek symbols to designate viscosity, kinematic viscosity, and density. Since these symbols are not strictly a part of scale designations, it would seem wise to follow their suggestion and adhere a while longer to the more easily recognized notations.

Mr. Tchikoff offers a number of valuable ideas, all of which are interesting and few of which are subject to serious controversy. He expands somewhat on the statement in the paper (see heading, "Turbulence in Open-Channel Models") that: "For all practical purposes dynamic and kinematic similitude exists when the flow of water in the model is of the same character as the flow in Nature." He then raises the question, "How is one to judge when the two flows are of the same character?" This is answered by the presentation of a practical criterion, based upon the Bakhmeteff theory of hydraulic flow,[25] according to which the prototype and model must have the same kinetic flow factor and the discharges must be in proportion to the cubes of the hydraulic radii. As a basis for the foregoing criterion, it can be assumed that the character of silt transportation must be the same for the same silt in both channels. Mr. Tchikoff, however, does not make clear his definition of silt, or his reason for believing it necessary to use the same material in the model as in Nature. If by "silt" bed material is meant, the factors of slope and depth will control; hence, by judicious control of the model slope and by careful selection of the model sand, similarity of action is readily procured. It is desired to emphasize here, as elsewhere, that in general a model should be designed to fulfill a single specific mission; that the controlling factors should be carefully taken into account in each case; and that if more than a singe purpose is to be fulfilled a conscious check must be made to determine whether the original model—modified perhaps—will suffice, or whether an entirely new model should be constructed.

Mr. Shulits' discussion of the friction factor and its effects is rational and interesting. The method of simulating over-bank roughness by means of pebbles, has long been in use at the U. S. Waterways Experiment Station, but in recent months (1934) wire screens have been found more satisfactory, especially when it becomes necessary to represent floods of considerable height which would overtop small stones on the over-bank. Data are gradually being accumulated in all laboratories to determine values of the non-dimensional roughness coefficient, but the combinations of surfaces and types of models are so many as to make the problem a lengthy one. In the meantime, the situation is far from hopeless if adequate data are known of the river investigated. One method is to reproduce a water-surface profile in the model corresponding to less than bank-full flow in Nature; then, by comparing discharges in model and in Nature, to determine a definite discharge scale; next, using the discharge scale thus determined, to reproduce a flood of considerable known magnitude, for which gauge readings were taken in Nature; and, finally, to adjust the over-bank roughness on the model until a water-surface profile is procured which coincides with that of the natural river. Referring to Mr. Shulits' comments relative to the effects of temperature on the turbulence criterion, it is agreed that considerable importance should be attached to this item. In the general case, however, turbulence is not an all-important factor because if bed movement is a requirement there will always be high turbulence when slopes and depths are such as to result in the production of adequate tractive force.

Lieut. Curran states a timely warning in advising against a too optimistic interpretation of model results; but, nevertheless, recent progress has been such as to make quantitative answers frequently obtainable. In cases where exact reproductions of natural developments appear impossible, corrective factors may be applied to the model results with nearly complete satisfaction.

Professor Egiazaroff points out that, assuming turbulence to result whenever the product of velocity and depth exceeds 0.02, there remains little need for distortion of models, especially in the case of a movable bed. This would be true if turbulence were the only requirement, but, if the bed is of sand, then sand movement is another matter for consideration, and distortion may be necessary for attainment of the required tractive force. The rapid development of Russian laboratories and the excellent work being performed at Leningrad, make Professor Egiazaroff's remarks of particular interest, and his citation of recent experiments gives a valuable sidelight to his discussion.

During the past summer (1934) it has been the writer's privilege to revisit leading hydraulic ("flussbau") laboratories of Germany and, mentally, to compare their present installations and methods with those of 1929. While considerable progress has been made, particularly in the field of lock models, the advancement has been not nearly as rapid, or upon as broad a front, as in the United States. Probably the most important single contribution to the science is the work of Hans Kramer, Assoc. M. Am. Soc. C. E., on the movement of bed material in fluvial models.[42] Investigations supplementary to the studies of Captain Kramer are being carried on at present by the U. S. Waterways Experiment Station and at the Berlin Laboratory of the Prussian Government (Preussische Versuchsanstalt für Wasserbau, Schiffbau, und Erdbau) by Captain Hugh J. Casey, Corps of Engineers, U. S. Army, M. Am. Soc. C. E., but the original work by itself has opened an entirely new field for model investigations. That this is true is evidenced by the fact that nearly every laboratory in the United States has recognized the importance of tractive force studies and is carrying the work forward along individual lines.

What the future will bring forth is difficult, if not impossible, to predict, but it seems certain that American engineers are coming more and more to appreciate the advantages following from tests of river and harbor models. In nearly every section of the country where important public works are being constructed, there can now be found the proposed works in miniature and under test by competent, well-trained, hydraulic-model engineers. It is to be hoped that the growth will be steady and sure, with a background of common sense and practical applicability constantly maintained, and that, as a result, model studies will take their proper place with other forms of engineering activities, not exaggerated in importance, not regarded as a cure-all for all troubles, but respected as an auxiliary method available to the practical engineer in connection with problems not readily susceptible to a purely theoretical analysis.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 1895

MODEL OF CALDERWOOD ARCH DAM

By A. V. Karpov[1] and R. L. Templin,[2] Members, Am. Soc. C. E.

With Discussion by Messrs. A. W. Simonds, Lars R. Jorgensen, A. C. Janni, P. Wilhelm Werner, Eldred D. Smith, Elmer O. Bergman, E. Probst, and A. V. Karpov and R. L. Templin.

Synopsis

A model built in the Aluminum Research Laboratories in New Kensington, Pa., of the Calderwood Arch Dam on the Little Tennessee River, south of Knoxville, Tenn., is the subject of the paper. The writers present a discussion of the theoretical principles on which the model study is based, how these principles are incorporated in the model, and the results of a series of tests made on the model in conjunction with some of the tests made on the prototype.

Theoretical Considerations

The behavior of an arch dam is so complex that in order to make possible a model study it is essential to determine a simple relationship between the two. This relationship is included in a set of conditions that are usually designated as similarity conditions. The writers have referred elsewhere to: (*a*) The mathematical derivation of the similarity conditions and the specifications that the material for an arch dam model must meet in order to satisfy these conditions[3]; (*b*) the discussions of these similarity conditions[4]; and (*c*) the work done in developing a proper material.[5]

Note.—Published in December, 1933, *Proceedings*.

[1] Designing Engr., Hydr. Dept., Aluminum Co. of America, Pittsburgh, Pa.

[2] Chf. Engr. of Tests, Aluminum Co. of America, New Kensington, Pa.

[3] "Theory of Similarity and Models," by B. F. Groat, M. Am. Soc. C. E., *Transactions* Am. Soc. C. E., Vol. 96 (1932), Discussion by A. V. Karpov, M. Am. Soc. C. E., pp. 308–325.

[4] "Building and Testing an Arch Dam Model," by A. V. Karpov and R. L Templin, Members, Am. Soc. C. E., *Civil Engineering*, January, 1932, p. 11.

[5] "The Flexing of Rubber Products" by E. G. Kimmich, Am. Soc. for Testing Materials, 1932, Symposium on Rubber; and "Elastic Behavior of Vulcanized Rubber," by Prof. Hencky, *Transactions*, Am. Soc. M. E., 1933, APM-55-8, Discussions by A. V. Karpov and R. L. Templin, Members, Am. Soc. C. E.

The relationship between the behavior of the prototype and its model may be given by the ratios of the deflections and the stresses. Introducing the ratio of the corresponding linear dimensions of the model and prototype as the scale of the model, R, the similarity conditions stated in Reference (a) show that the simplest possible relationship of deflections and stresses are: First, to have the ratio of deflections of the prototype and its model equal to the square of the inverse scale of the model, $\frac{1}{R^2}$, multiplied by the ratio of the specific gravities of the materials of the prototype and its model, $\frac{\rho_p}{\rho_m}$, multiplied by the inverse ratio of the moduli of elasticity of the prototype and its model, $\frac{E_m}{E_p}$; and, second, to have the ratio of the stresses of the prototype and its model equal to the inverse scale of the model, $\frac{1}{R}$, multiplied by the ratio of the specific gravities of the materials of the prototype and its model, $\frac{\rho_p}{\rho_m}$.

In order to have such simple relationship, the following similarity conditions stated in Reference (a) must be satisfied:

1.—The materials of the prototype and model must follow Hooke's law, which means that the modulus of elasticity, E, is constant in the prototype and the model. (A better expression for that condition is: The materials of the prototype and its model for their respective ranges of stress must exhibit a straight-line relationship between stress and strain.)

2.—The ratio of the specific gravities of the material of the prototype and its loading fluid, $\frac{\rho_p}{\gamma_p}$, must be equal to the same ratio of the model, $\frac{\rho_m}{\gamma_m}$.

3.—Poisson's ratios of the prototype and model materials must be equal at corresponding stresses, $\mu_p = \mu_m$.

4.—The ratio of the moduli of elasticity of the model and prototype must be equal to the scale of the model, R, multiplied by the ratio of specific gravities of the loading liquids of the model and prototype, $\frac{\gamma_m}{\gamma_p}$.

5.—The ratio between the ultimate unit tensile stresses that can be carried by the horizontal joints of the model and prototype must be equal to the linear scale of the model, R, multiplied by the ratio of the specific gravities of the loading liquids of the model and prototype, $\frac{\gamma_m}{\gamma_p}$.

6.—The ratio of the width of the vertical construction joints in the model and prototype, $\frac{b_m}{b_p}$, must be equal to the square of the scale of the model, R^2, multiplied by the ratio of the specific gravities of the loading liquids, $\frac{\gamma_m}{\gamma_p}$, and by the inverse ratio of the moduli of elasticity, $\frac{E_p}{E_m}$, of the model and prototype.

7.—The action of the indefinitely large foundation rock may be represented by a block of elastic material, of some definite dimensions, which has a fairly low modulus of elasticity, so that by supporting it on a base made of a material having a high modulus of elasticity, the conditions of an elastic foundation block supported on an unyielding non-elastic base can be represented.

8.—The ratio of the temperature difference of the model material, between the time of testing and building of the model, and of the temperature difference of the material of the prototype, between the time for which the investigations are made and the time of the building of the prototype, $\frac{T_m}{T_p}$ must be equal to the inverse ratio of the coefficients of thermal expansion of the materials of the model and the prototype, $\frac{c_p}{c_m}$, provided these coefficients are sufficiently uniform to be assumed as constant.

In applying these conditions to the correct design of a model, it is more convenient to divide them into the following classes: (I) Conditions Governing the Methods of Loading the Model; (II) Conditions Governing the Properties of the Model Material; and, (III) Conditions Governing the Method of Building the Model.

(I) Conditions Governing the Methods of Loading the Model

There are two possible ways to load the model. The first is to simulate exactly the loading conditions of the prototype and to load the model by action of gravity forces (weights) of the model material and the loading liquid. The second is to attempt to simulate the loading conditions of the prototype by loading the model with properly distributed forces. A number of methods may be devised for this purpose, such as loading the model by means of a number of weights or springs, or by placing it in a centrifuge and, by rotating it, produce the load by centrifugal forces.[6]

By the first method a comparatively simple relationship may be established between the behavior of the prototype and its model, depending on the fulfillment of Similarity Conditions 1 to 8, and only water or mercury can be considered. Since mercury is excluded on account of the specific gravity limitation, water is the only suitable liquid. Any attempt to establish exact similarity conditions for any model loaded in accordance with the second method will result in such a complicated relationship between the behavior of the prototype and its model that it probably could not be applied to the general study of the behavior of an arch dam.

(II) Conditions Governing the Properties of the Model Material

The established similarity conditions, stated in Reference (*a*), show that a simple relationship between the behavior of the prototype and its model is possible only when the ratios of the specific gravities of the loading liquid to the material in the prototype are the same as in the model. To realize the importance of this similarity condition, the principle of superposition should be applied properly to a model dam. This well-known principle may be stated as follows:

---

[6] "Use of Models for the Study of Mining Problems," by Philip B. Bucky, *Technical Publication 425*, Am. Inst. of Min. and Metallurgical Engrs.

> Any increase or decrease in the loading will cause a proportional increase or decrease of the stresses and deflections, provided the deflections are sufficiently small.

If applied properly to the model this principle would mean that if the loading is increased by a simultaneous increase of the specific gravities, both of the loading liquid and the material of the model, the deflection and stresses will change proportionally. Under such conditions the corresponding deflections and stresses of the prototype can be readily evaluated.

An improper application would be to assume that the specific gravities of the loading fluid and material of the model may be changed independently and that the stresses and deflections would still remain proportional. If the principle of superposition could be applied in such a manner, the ratio of specific gravity requirement would be of no importance, and it would be a simple matter to recompute the stresses and deflections of a model that is built with an improper ratio of specific gravities of loading liquid and material and to predict the stresses and deflections of the prototype. For several reasons, however, the inter-relation between the weights of the loading liquid and the dam material must be taken into consideration.

The first reason is that, by taking the strictly theoretical point of view, and considering the vertical stresses in a vertical element of an arch dam (that is, the so-called cantilever stresses), if the straight-line distribution of stress is assumed, the principle of superposition could be applied independently and an increase or decrease in the specific gravity of the loading liquid or the material would simply increase or decrease the stresses in the proper ratio. However, the assumption of straight-line distribution of stress does not represent the actual stress conditions. An illustration of this point may be of interest. For such illustration, it is preferable to consider a dam with a cross-section, as shown on Fig. 1, which differs somewhat from the conventional. The difference consists in introducing fillets at the sections where the faces of the dam meet the foundation by making the foundation surface a continuation of these faces. Such a shape removes the lines of discontinuity that occur in the conventional dam cross-sections at points where its faces meet the foundation, and makes it easier to visualize the stress conditions.

If an attempt is made in such a case to follow the vertical stresses along the line, *A-B*, at the foundation, they are definitely known at only two points: At *A*, the vertical stress is equal to the total water pressure, $w\ h$, and at *B* it equals zero. No methods are known by which the stresses can be evaluated between these two points, but it is clear that in order to retain static equilibrium, a distribution of vertical stress of the order illustrated on Fig. 1 is necessary, and this stress distribution may be assumed as giving a fairly correct picture without attempting to show its relative magnitude. If, now, the specific gravity of the material of the dam is reduced and that of the loading fluid remains the same, the stress shown along the line, *A-B*, will change and the maximum tension and compression will increase as indicated in Fig. 1 by dotted lines. However, the stresses at A and B will remain unchanged. In other words, there will be no proportionality of any kind in change of stress and, consequently, of deflection.

In order to retain the proportionality between stresses, and, consequently, between deflections, of the prototype and its model, it is necessary, in case of a change in the specific gravity of the material, to change it in the same ratio as the specific gravity of the loading fluid. Then the pressure of the fluid, $w_f\ h$, will change in the same proportion as the stresses, and the necessary proportionality will be retained between the stresses, and also between the deflections of the prototype and its model. The same reasoning applies to the horizontal stresses at the points at which the horizontal arches meet the abutments or the bed-rock of the canyon.

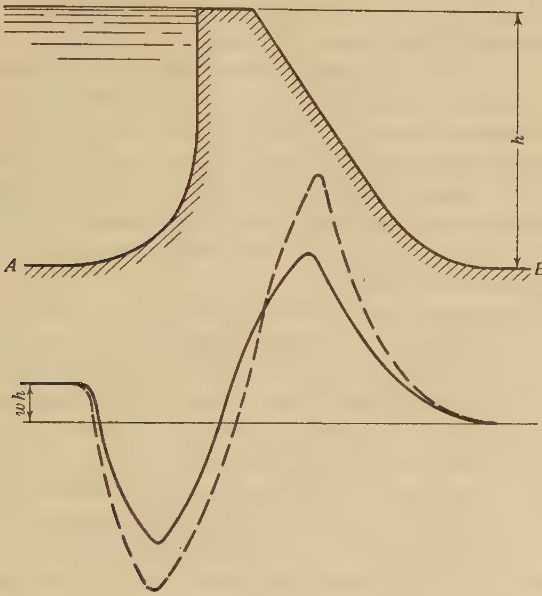


Fig. 1.—Distribution of Vertical Stresses in a Vertical Element of a Dam

The second reason for considering the relative weights of the loading liquid and the dam material is that the effect of the ratio between the specific gravities of the material of the model and the loading fluid is important when the resistance to overturning is considered for the dam as a whole. When subjected to water load an arch dam deforms elastically and tends to tip over as a unit. This tendency is opposed mostly by the resistance of the arch abutments. The ratio of the specific gravities will influence the proportion of load carried by the arches and transmitted to the abutments.

The third reason is a more practical consideration. Even assuming a straight-line distribution of stress, it is clear that if the ratios of specific gravities of the loading liquid and the material of the dam differ considerably in the prototype and its model, a reversal of stress will occur at some points; that is, tension will appear in the model instead of compression at the corresponding point of the prototype, or *vice versa;* and, under actual conditions of stress distribution, similar to those in Fig. 1, more pronounced reversal of stress would occur. Only when the ratio of the specific gravities is the same in the prototype and in its model, will there be a complete similarity of stress distribution in sign and magnitude. If this ratio is different, a dissimilarity of sign of the stress will occur and will increase in proportion to the difference between the specific gravity ratios. The joints of the prototype practically exclude tension resistance at a number of points, but if tension appears at these points the joints will act as planes of discontinuity, changing the stress distribution radically. For a model loaded with a heavy liquid, tension will appear at a number of points and tension resistance must be introduced to save it from destruction. This necessitates the building of a model without joints, so that it can withstand tension, which again violates the similarity conditions considerably.

From the theoretical point of view, there is no fundamental difference between the model of a gravity dam and the model of an arch dam. The same similarity conditions cover both cases since the straight gravity dam is an arch dam with an infinite radius of curvature. However, in designing a model, consideration should be given to the relative importance of the various similarity conditions, which may differ in the two types of dams.

As long as the prototype is such that gravity action is subdued (as, for instance, in a very thin arch dam, with a short radius of curvature, and in which most of the load is carried by the horizontal arches), the similarity condition pertaining to the ratios of specific gravities may be of little importance. This ratio becomes more important with an increase in the thickness of the dam or in the radius of curvature, until it is of paramount importance in a straight gravity type of dam. The only conclusion that can be derived from a model of a gravity dam, in which the ratio of the specific gravity of loading liquid to the material is many times higher than in the prototype, is that every gravity dam will fail. The impossibility of arriving at correct conclusions concerning the behavior of a gravity dam based on such a model study applies also to thick arch dams.

Agreement between the calculated and measured stresses and deflections on models loaded with mercury has generally been considered sufficient proof that this particular similarity condition is of little or no importance. This statement is rather convincing, but analyzing it more carefully, it can be shown that it is probably incorrect since no attempt was made to compare directly the calculated deflections and stresses in an actual dam with the results of the model tests. Such calculations were made only for the model, which did not have vertical joints as in the prototype, and was loaded with mercury; consequently, the gravity action was considerably subdued and the arch action exaggerated.

Rubber-Litharge Compound Developed as Suitable Model Material.—The specifications covering a material suitable for the model may be derived from Similarity Conditions 1, 2, 3, and 4. Using water as the loading liquid, the specific gravity of the model material is definitely fixed (in accordance with Similarity Condition 2) as 2.4; that is, the same specific gravity as the concrete in the prototype.

Similarity Condition 4 is of quite minor importance. It is derived from the requirement of exact geometrical similarity between the prototype and its model before and after the loads are applied. If the deflections of the model are not excessive, so that the loaded model does not change its geometrical shape appreciably, these conditions can be violated with the assurance that such violation will have little influence on the behavior of the model; furthermore, increased deflections facilitate model study.

Consequently, the following specifications for the material of the model were laid down in Reference (*a*)[7]: (1) Specific gravity of 2.4; (2) uniform elastic properties without pronounced directional qualities; (3) absence of skin or internal stresses; (4) low modulus of elasticity; (5) reasonably uniform values of the modulus of elasticity under varying unit stresses; (6) low permanent set; (7) low plastic properties; (8) properties that will permit the material to be provided readily in the required irregular shapes; (9) Poisson's ratio close to 0.20; and (10) uniform coefficient of thermal expansion.

The rubber-litharge compound used in the model of Calderwood Dam meets all these specifications reasonably close except Specification (9), in which Poisson's ratio is 0.50.

The question then arises, "How will a model be influenced by this difference in Poisson's ratios?" If a gravity dam is located or constructed in such a way that the vertical joints permit enough lateral expansion to provide for lateral bulge of the material, the test results on a model built of such material will be close to actual conditions, the principal variation being the difference in deflections due to different shear moduli. If the conditions are such that they do not permit expansion of the material at the vertical joints, it is only necessary to determine the stresses in the model at a sufficient number of places, after which they can be expressed readily in terms of the prototype. In other words, for a gravity dam, the non-fulfillment of Similarity Condition 9 is of little importance. For an arch dam it may be of more importance, but the deviations in the behavior of the model will be comparatively small and can be accounted for to a large extent.

It was thought that this new material would permit the building of an arch dam model that would be not only a scalar representation of the prototype, but also an engineering model, the structural behavior of which is similar in every respect to the behavior of the prototype.

(III) Conditions Governing the Method of Building the Model

In the building of an engineering model of a dam, consideration should be given to the methods used in the building of the prototype and to the necessary adaptations of the model that should be made to insure the proper similarity of all conditions.

---

[7] *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 322.

Similarity Conditions 5, 6, and 7 govern the building of the model. The exact fulfillment of these conditions is somewhat difficult, due to a number of uncertainties inherent in any arch dam, such as: Uncertainties of the dam proper (Similarity Conditions 5 and 6); and uncertain boundary conditions of the dam where it meets the foundation and abutments (Similarity Condition 7).

Uncertainties of the Dam Proper.—Similarity Condition 5 establishes the ultimate unit tensile strength across the horizontal joints of the model and depends on the strength of the horizontal joints of the prototype. All existing arch dams are designed so that stresses will be considerably below the ultimate strength of the concrete. The horizontal construction joints, however, are usually subjected at a number of points to tensile stress that is higher than can be carried across such joints and, as a consequence, the joints open or crack, thus relieving such stress.

The tension that can be carried across a horizontal joint is rather uncertain and depends largely on the time interval between the placing of two successive layers of concrete. To ascertain the conditions at Calderwood Dam a few preliminary tests were made under conditions that simulated as closely as possible the actual conditions in the dam. These tests showed that the ultimate tensile strength of the horizontal construction joints that were tested, ranged from 60 to 88 lb per sq in. This justifies the conclusion that 25% of the ultimate tensile strength of concrete is about the maximum stress that can be carried in this manner and, in many cases, a much lower stress will open the joint. The horizontal joints in the prototype were spaced about 10 ft apart vertically and under such conditions no vertical tensile stress of any magnitude can be built up in the prototype. In order to simulate these conditions in the model, it was built so that no tensile stresses could be carried across the horizontal joints; but the number of joints was reduced, so that they correspond to a 20-ft vertical spacing in the model instead of the actual 10 ft in the prototype.

Similarity Condition 6 defines the width of the vertical construction joints of the model as compared with the prototype. It is certain that these joints in the prototype cannot carry any stress, but their width is somewhat uncertain.

In Calderwood Dam, as in every commercial dam, the concrete was not placed continuously as in monolithic construction, but in independent blocks, by placing in alternate piers. Considerable time elapsed from the beginning to the end of the concrete work. This procedure resulted in vertical construction joints, formed by the contraction of the concrete in the adjoining blocks. Customary methods were used to fill them by subsequent pressure grouting,[8] but due to the large size of the individual blocks, the time interval between the placing of the concrete and the application of the water load caused internal stress to be developed in the blocks of the prototype. Considering the large size of the blocks and the considerable tension that the concrete can withstand, it is probable that the internal tensile stress developed in these blocks is rather high. Pressure grouting reduces the size of these joints, but probably has little, if any, effect on the internal stress developed in each block.

---

[8] "Grouting Dam Foundations and Construction Joints," by James B. Hays, M. Am. Soc. C. E., *Civil Engineering*, November, 1933, p. 606.

When the water load is applied, the concrete blocks are subjected to compression and then they contract. This contraction will be greater in a block that has a preponderance of internal tensile stress than in one without such stress. If the zero readings are taken before the first application of load, this additional contraction will be included in the measurements. The same conditions of increased contraction could be produced by building a dam of blocks without internal stress, but with wider vertical construction joints.

The rubber compound used in the blocks of the model dam had practically no internal stress when they were assembled. Any subsequent stress that has been introduced is due to differences in temperature of the model at the time of its construction and at the time of loading. Such stress, if any, is so small as to be negligible.

In order to retain the similarity conditions between the prototype, in which the concrete has internal tensile stress, and the model, in which the material has no such stress, the size of the vertical construction joints of the model must be made relatively larger than the theoretical size stipulated by the similarity conditions. The size of vertical construction joints is also influenced by the differences in Poisson's ratio for the material of the prototype and its model.

Uncertain Boundary Conditions of the Dam Where It Meets the Foundation and Abutments.—Similarity Condition 7 covers the condition at the boundaries; the deflections of the foundation and abutments of the model must be similar and in a proper ratio to the deflections of the prototype. The fulfillment of this condition would be fairly simple if sufficient information were available concerning the deflections of foundations and abutments in general and of the Calderwood site in particular. Little is known, however, about the deformation and stress in the bed-rock due to the dead load of a massive structure like a high dam and the additional stress and deflection superimposed by the subsequent water loading of the dam. The scarcity of information made it necessary to include a study of foundation deformation in the testing program.

In order to make such a study possible, the model was arranged so that the deflection of the foundation could be adjusted over a fairly wide range, and with as little disturbing influence on the model *per se* as possible. This increased flexibility of the foundation was obtained by applying screw-jacks, not under the model itself, but on the rubber compound foundation strip on which the model rested, the idea being that any unnatural change in stress or deflection resulting from such adjustments would be kept away from the model or at least smoothed out by the action of the flexible foundation strip.

The lower part of Fig. 2 shows the arrangement of the vertical and horizontal jacks which permitted increased flexibility in vertical and horizontal directions.

Comparative Prototype-Model Table.—Assuming the scale of the model as 1 : 50 and basing it on the theoretical considerations discussed previously, as well as on the properties of the material of the prototype (see Appendix I) and the properties of the material of the model (see Appendix II), the major factors are arranged for comparison in Table 1.

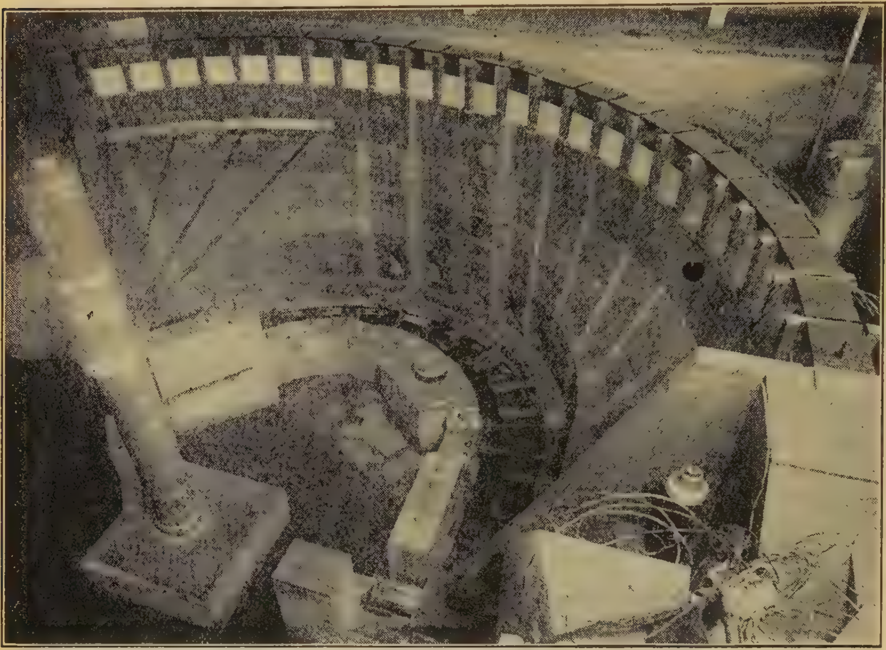


FIG. 2.—HORIZONTAL JACKS TO CHANGE THE SIZE OF VERTICAL CONSTRUCTION JOINTS AND HORIZONTAL AND VERTICAL JACKS TO REGULATE THE FLEXIBILITY OF MODEL FOUNDATIONS.

TABLE 1.—COMPARISON OF PROTOTYPE AND MODEL

Item No.	Description	Prototype	MODEL		Ratio, Prototype: Model
			Actual	Required	
1	Deflections	....	....	....	1:2.68*
2	Stresses	....	....	....	50:1*
3	Strains	....	....	....	1:134*
4	Specific Gravities:				
	Loading liquids	1	1	1	1:1†
	Materials	2.375	2.380	2.375	1:1†
5	Modulus of elasticity, in pounds per square inch	3 770 000	563	563	....
6	Shear modulus, in pounds per square inch ..	1 640 000	188	245	....
7	Poisson's ratio	0.15	0.50	0.15	....
8	Coefficient of thermal expansion per degree Fahrenheit	0.000006	0.00008	....	....

* Theoretical. † Actual.

MEASUREMENTS MADE ON CALDERWOOD DAM

In connection with the study of the model of Calderwood Dam, four sets of measurements on the prototype are available, as follows: Measurements made in April, 1930, before, during, and immediately after the filling period; and those made in September, 1930; May, 1931; and September, 1931. As the lake elevation has not been lowered materially since the initial filling in April, 1930, the dam has been continuously and quite uniformly loaded.

During each set of measurements the lake was maintained at approximately its maximum level, so that as regards the loading conditions, all four sets are similar. There is, however, a difference in temperature conditions.

The noticeable difference in the behavior of the dam as determined by these four sets of measurements can scarcely be attributed entirely to the temperature difference and indicates clearly that a number of additional factors influence the behavior of a dam. These factors may be referred to as "time effect," and the difference between the results of the subsequent tests probably represents the influence of such "time effect." The recognized factors that may produce "time effect" are: The plastic flow of the foundation, abutments, and the concrete of the dam; the difference in concrete temperatures caused by change in climatic conditions; the aging of the concrete; the volumetric changes in concrete caused by shrinkage due to aging; swelling due to the absorption of moisture from the lake or air; and shrinkage due to evaporation of water from the concrete. After a sufficiently long time a continuously and uniformly loaded dam should attain the final or permanent condition. After that, the behavior of the dam will depend only on the changes in climatic conditions.

The storage capacity of the lake behind Calderwood Dam is relatively small and it took less than six days to fill the lake and to load the dam fully.

Since the zero readings were taken immediately prior to the closure of the dam, when the water first began to rise, and the full load readings were taken immediately after the lake was full, the elapsed time between the dates of the two readings was short. The influence of the "time effect" can be noticed only after the dam has been loaded for some time. This combination justifies the assumption that, because of the short filling period, the "time effect" on the dam was so small that it did not influence its behavior. Such an assumption makes it possible to separate the "time effect" and study it in the light of subsequent measurements.

The present model study was made under the assumption of no "time effect" in conjunction with the measurements made on the prototype in April, 1930. The intention is to continue the model study in conjunction with subsequent measurements on the prototype, so as to obtain a better understanding of the influence of the "time effect."

During the first filling, three kinds of measurements were made on the prototype: Deflection, strain, and temperature measurements.

Deflection Measurements.—When the dam was built, provision was made for obtaining deflection measurements. A concrete pier was placed at the center of the crest circle and from this pier the distances were measured to a number of points on the down-stream face of the dam by means of a special apparatus and a measuring tape.

The apparatus consists essentially of the three-legged frame shown in Figs. 3 and 4, in which a swivel-frame is swung. The frame, in turn, supports a pulley over which the tape is reeved. The reference points for measurements are drilled on a block. Suspended by the tape, from the pulley, is a 120-lb weight hung from another pulley similar to that fastened to the swivel arm. When raised from the floor of the pit in which it is suspended this weight subjects the tape to a constant tension of 60 lb. Ball-bearings are provided at either end of the frame as well as for both the tape and weight pulleys. The pulleys and the tape reel were made of aluminum alloys. The reel was constructed with internal reduction gears so as to reduce the force required to raise the weight, and was provided with a spring-brake to prevent the tape from leaving the reel or becoming loose and tangling. Means were provided to lock the reel so that it was not necessary to touch the apparatus while readings were being taken.

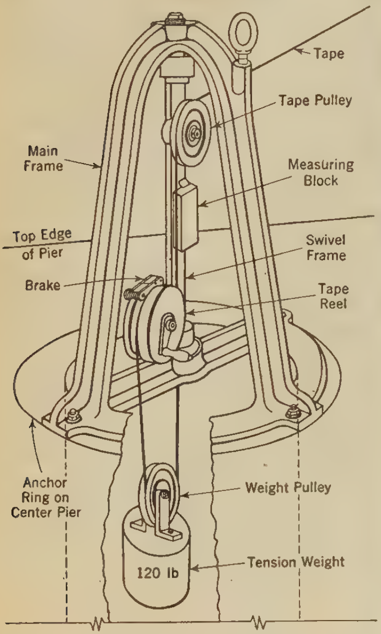


FIG. 3.—DETAILS OF DEFLECTION MEASURING APPARATUS.

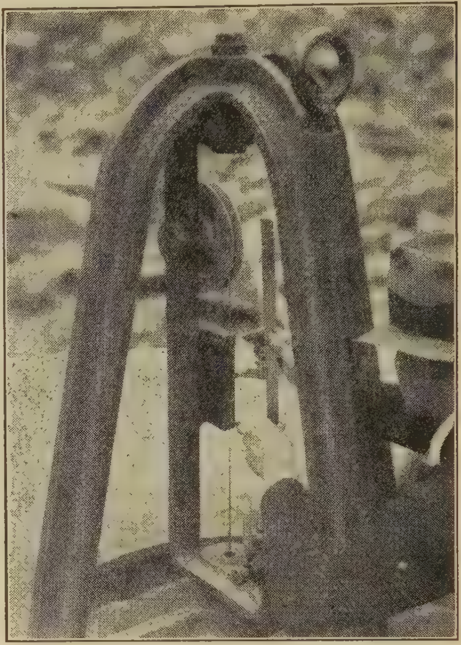


FIG. 4.—OBSERVER MEASURING MOVEMENT OF TAPE

The measuring or reference block bolted to the swivel-frame had a shallow slot along one edge in which the tape moved. Four reference holes were drilled in this block, with a No. 57 drill (0.043 in. in diameter), one on either side of, but close to, the slot at the top and bottom of the block. The swivel arm was offset in such a manner that the edge of the tape reeved over the pulley, passed through the hole in the bottom of the swivel-frame and the center of the cross-member of the outer frame, lay in the slot of the measuring block, and coincided with the center of rotation. The other end of the tape was provided with a hook that was inserted in stainless steel eye-bolts set into the down-stream face of the dam at each deflection measurement station.

Fig. 4 shows the offset of this swivel-frame and also serves to illustrate the method of measuring the movement of the tape, which corresponded to a movement or change in the deflection of the station on the down-stream face of the dam to which the tape was hooked. Each brass sleeve on which the foot graduations of the steel tape appear, was marked with suitable punch holes. The distance between the nearest of these marks and a corresponding reference hole on the measuring block of the apparatus was determined, using a standard vernier caliper, to each jaw of which was fastened a detachable hardened steel point, conical in shape. This change in distance could be determined to the nearest 0.001 in. Since the change in distance from the deflection station to the center pier was the desired measurement, the difference in reading of the vernier caliper, when adjusted to the same footmark and reference point each time, and properly corrected for temperature, angularity, etc., was a measure of the deflection of the dam at that point.

Making the deflection measurements at night and in fair weather it was possible to obtain the unusual over-all accuracy of ± 0.02 in. Since a single reference pier was used, the deflections could be measured in one direction only; that is, from the point on the down-stream face of the dam toward the reference pier. No provision was made to measure vertical displacements; hence it was necessary to assume that all deflection was horizontal, and to correct for the deviation. Due to the topographical conditions it was impractical to measure deflections at the lower elevations of the dam.

In this paper only the deflection measurements on the prototype made in April, 1930, are considered, the zero readings being taken after the dam was completed and before the water load was applied, and the load readings after the dam was loaded by the water in the full reservoir. That means that the measurements included in this paper show deflections due to water load only; or, in other words, the deflections caused by the dead weight of the dam are not considered.

Strain Measurements.—Strain measurements on the prototype were made on the down-stream and the up-stream faces of the dam. Those on the down-stream face were made at a number of rosettes where the strain was measured in four directions, vertically, horizontally, and in two diagonal 45° directions. These rosettes were set when the concrete was placed; each rosette consisted of eight steel inserts fastened to the inside of the forms on a gauge circle 10 in. in diameter. When the forms were removed, these inserts were anchored in the concrete. A small hole was drilled in each insert, and any change in distance between the gauge holes in two opposite inserts was measured with a 10-in. Whittemore strain-gauge.

The state of stress on the surface of a uniform solid may be obtained from strain measurements on three intersecting gauge lines, but at every rosette a fourth measurement was made which served as a check on the other three, and thus a number of inconsistent readings was eliminated.

Strain measurements on the up-stream, submerged face of the dam have not been made heretofore, although they are just as important—and may be even more important—than those on the down-stream face. Since there was no previous experience as a guide, an attempt was made to make such measurements at a few points.

The method consisted essentially of using a comparatively long gauge length (10 ft), at one end of which a piece of 0.091-in. invar wire was anchored to a heavy pin which was set about 2 ft into the concrete. (The invar wire had a modulus of elasticity of 20 300 000 lb per sq in.; a proportional limit of 14 000 lb per sq in.; a yield strength of 38 000 lb per sq in.; and an ultimate strength of 66 000 lb per sq in.) At the other end of the gauge length, this wire was attached to the short arm of a bell-crank lever anchored to the face of the dam by another pin which was machined to provide proper bearing. This bell-crank lever had a ratio of 5 to 1. To the long end of the lever was attached a piece of 0.048-in. galvanized hard steel wire which was carried almost vertically along the face of the dam and through a protecting pipe to the crest where the upper end of the wire was again attached to a lever and dead-weight arrangement so as to maintain a constant tension in the entire lever and wire system. (The hard steel wire had a modulus of elasticity of 29 000 000 lb per sq in.; a proportional limit of 140 000 lb per sq in.; and an ultimate strength of 180 000 lb per sq in.)

Corresponding to each vertical wire a second wire was fastened at its lower end to the lever fulcrum and at its upper end to a second lever-weight system. This second wire furnished a zero reference point in every case. The two wires leading up from any single gauge line were close enough to each other so that a microscope could be focused on first one and then the other, without moving the barrel of the microscope, except in the direction of the travel of the focusing screws. The use of the second wire automatically eliminated temperature corrections as well as corrections for movement of the top measuring point relative to the fulcrum of the lever. It was also possible with this set-up to measure the strains at all points with the same microscope by simply transferring the mounting. This mounting consisted of a vertical standard set into a block which could be clamped to the side of the guard-pipe by a hinged strap and a thumb-screw. A swinging arm fitted on the shouldered top of the vertical standard. The microscope was clamped directly to this swinging arm.

Fig 5 shows the arrangement of the pins and levers in the rosettes; Fig. 6 shows the wires entering the guard-pipe; and Fig. 7 shows the lever arrangement and microscope mounting at the top of the guard-pipe.

Strains were measured on three gauge lines at each station, one vertical, one horizontal, and one at 45° to the other two. A measurement consisted of determining the change in the difference in elevation between two scratches, one made on the stationary or dead vertical wire fastened to the fulcrum of the bell-crank, and the other made on the moving wire attached to the long arm of the bell-crank lever. Measurements were made with a precision filar microscope, which was mounted on the top of the guard-pipe in

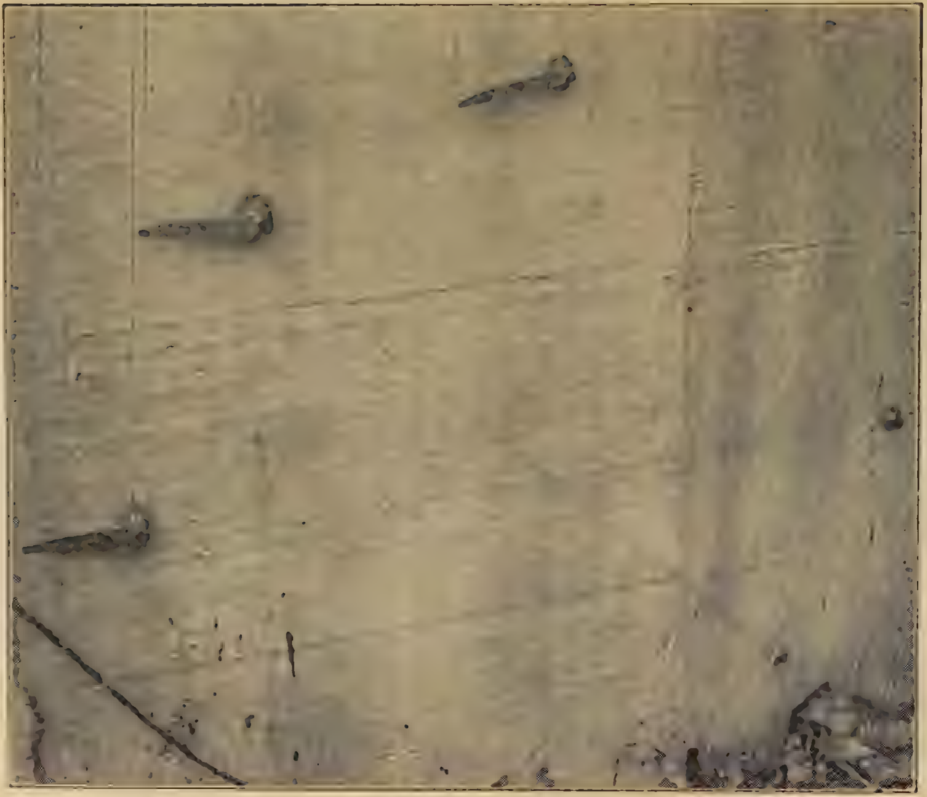


FIG. 5.—ARRANGEMENT OF PINS AND LEVERS IN THE ROSETTES

FIG. 6.—WIRES ENTERING GUARD-PIPE

such a way that it could be focused on any pair of wires without changing the elevation of the microscope mounting. The interval of time between reading the live or moving wire and reading the dead or fixed wire was made as short as possible to eliminate any changes that might be caused from rising or falling temperature. One reading was made on each pair of wires of a rosette and repeated at least three times before moving to the next rosette. The results obtained showed conclusively that such measurements are possible and feasible if properly designed instruments are used.

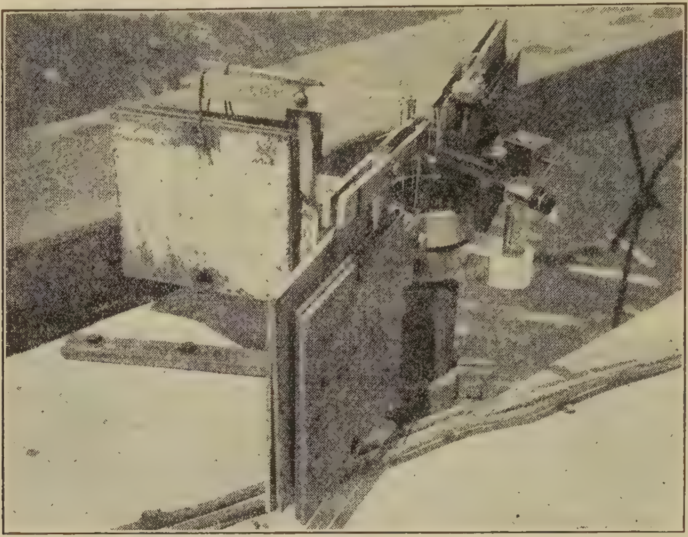


Fig. 7.—Lever Arrangement and Microscope Mounting at the Top of the Guard-Pipe.

In one case the strains were measured 160 ft below the surface. However, the limited number of these measurements made it impossible to form a correct picture of the stress distribution on the up-stream face of the dam.

As in the case of the deflection measurements, the strain measurements considered in this paper on both the down-stream and the up-stream faces of the dam were made so that the zero readings were taken after the dam was completed, but before the reservoir was filled. The load readings were taken immediately after the reservoir was filled. In so far as strains are concerned, the readings designated as "zero readings" do not represent the true no-strain conditions, but, on the contrary, the strain conditions, due to the weight of the concrete in the dam above the rosettes. In other words, the results of the strain measurements do not give the actual strains, but only the difference in strains which resulted from the application of the water load. Similarly, after the strains are reduced to stresses, they do not represent the actual stresses in the dam but only the difference due to the water load.

Temperature Measurements.—By means of a number of thermo-couples distributed through the prototype at different elevations and at different distances from the faces of the dam, a rather complete picture was obtained of the temperature conditions. The temperatures were recorded automatically at definite time intervals during each set of measurements. The temperature differences of the concrete during the filling period, except at the surfaces, were small and, consequently, were neglected as were the other "time effects."

Measurements Made on the Model

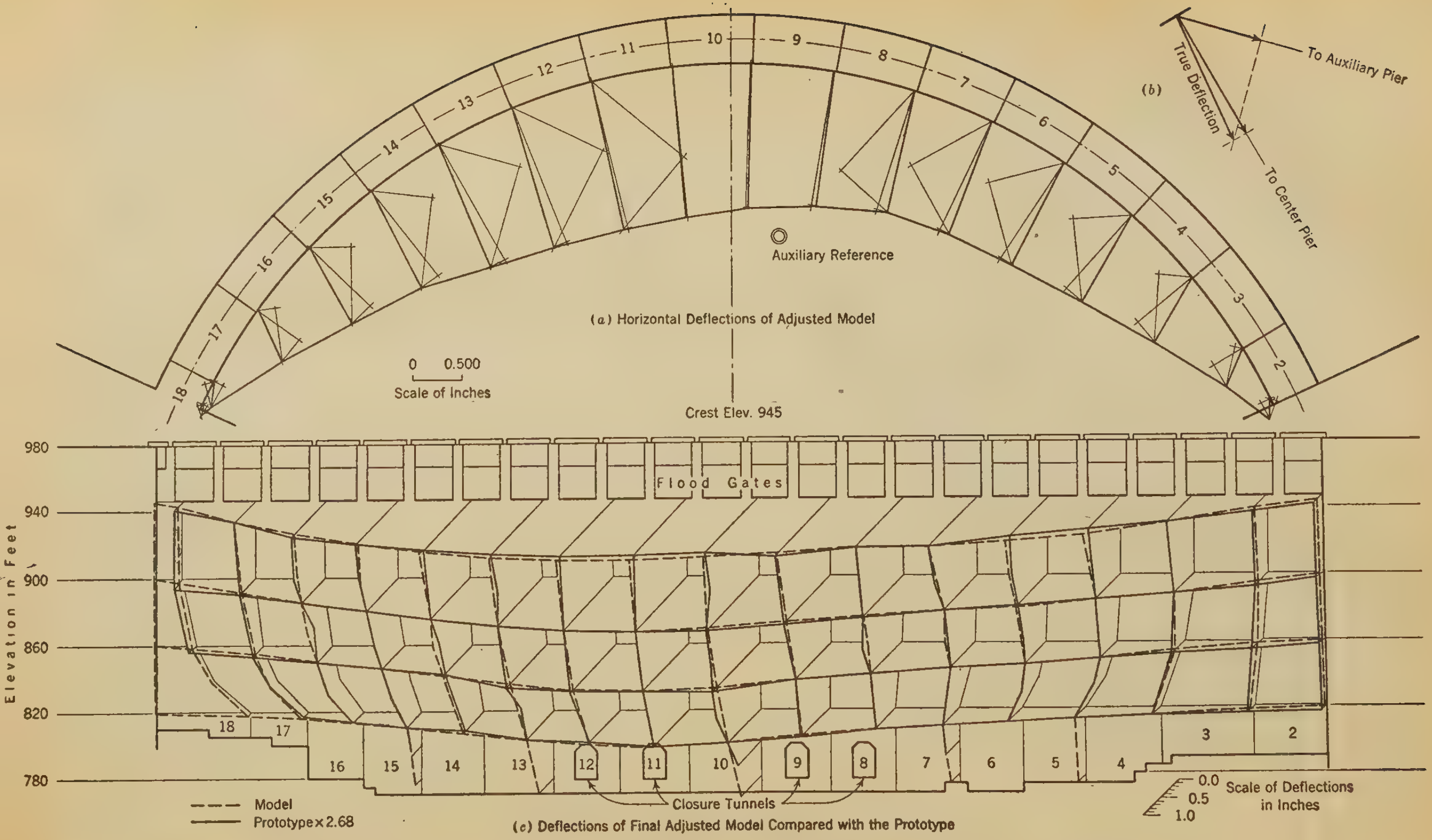
In devising the method of making measurements on the model, it was kept in mind: (1) That such measurements could be simulated, reasonably well, on the prototype; and (2) that a number of additional measurements which could not be made on the prototype could be made on the model. Three kinds of measurements were made on the model, namely, deflection, strain, and temperature measurements, and these will be discussd separately.

Deflection Measurements.—Unlike the prototype, deflection measurements on the model could be made in any desired direction, so that true deflections could be evaluated. Fig. 8 and Fig. 9 afford a clear conception of the deflection measurements. Fig. 8(*a*) shows the section of the model and prototype at the elevation corresponding to Elevation 945.0 on the prototype, and gives the correct horizontal deflections of the model. The individual blocks are numbered and the vertical construction joints are indicated. The true deflections were determined by making measurements from two reference piers, one a permanent (center) pier and the other an auxiliary (removable) pier.

The deflection of a number of points on the down-stream face of the model was determined and deflection vectors similar to Fig. 8(*b*) were drawn to an exaggerated scale in the proper direction. The end points of these vectors were connected by straight lines, thus giving a diagrammatic representation of the deflection of the down-stream face of the model. Fig. 8(*a*) shows that in the middle horizontal section of the model, where the deflections are nearly radial, those measured toward the center pier approximate, closely, the true deflections. However, toward the abutments there is an appreciable difference between the radial and the true horizontal deflections.

Fig. 9 shows the condition at Block 10. The down-stream face of this block, indicated by the heavy full line, is drawn and the corresponding elevations of the prototype are indicated. As shown in Fig 9(*a*), the true deflection of a point at the down-stream face is determined by making two deflection measurements, one horizontal and the other at an angle to the horizontal. The deflection vectors of a number of points on the down-stream face of the model are drawn to an exaggerated scale in the proper direction and the end points of these vectors are connected by straight lines.

The dotted line in Fig. 9 gives the horizontal deflections on the model; the dashed line the true deflections which can be determined, provided the vertical deflections are considered. The solid line gives the deflections as measured on the prototype, multiplied by 2.68, the theoretical ratio of the model to the prototype deflections, to make them directly comparable with the model deflections. The prototype deflections are horizontal only. Here, again, it appears that near the top of the model the difference between the hori-


To Auxiliary Pier
To Center Pier
True Deflection
(b)
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
Auxiliary Reference
0 0.500
Scale of Inches
Crest Elev. 945
980
940
900
860
820
780
Elevation in Feet
Flood Gates
Closure Tunnels
0.0
0.5
1.0
Scale of Deflections in Inches
Model
Prototype × 2.68

(a) Horizontal Deflections of Adjusted Model

(c) Deflections of Final Adjusted Model Compared with the Prototype

FIG. 8.—DEFLECTION MEASUREMENTS, CALDERWOOD DAM MODEL AND PROTOTYPE; (a) HORIZONTAL DEFLECTION OF ADJUSTED MODEL; (b) VECTOR DIAGRAM; AND (c) DEFLECTIONS OF FINAL ADJUSTED MODEL COMPARED WITH PROTOTYPE.

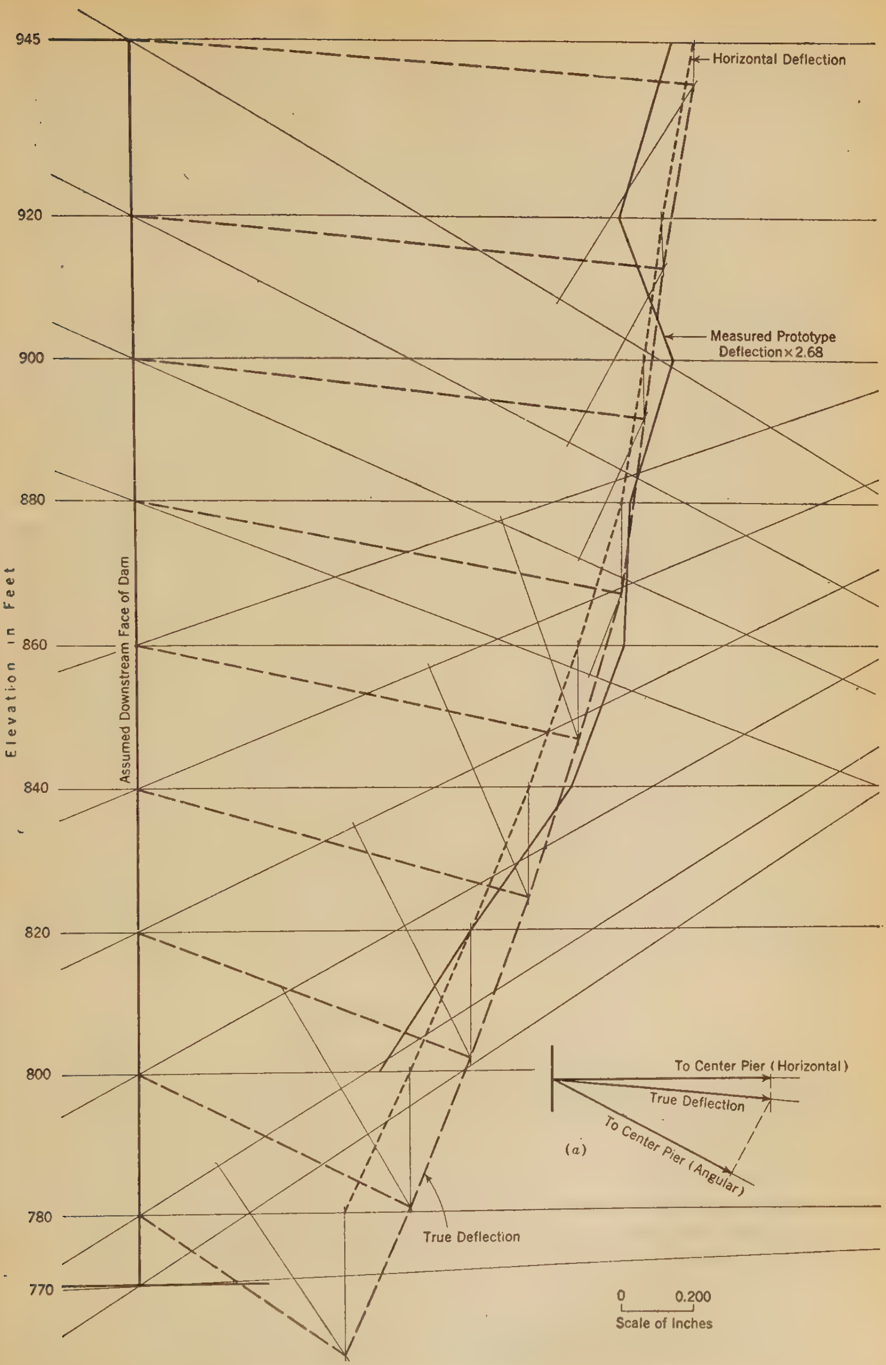


FIG. 9.—DEFLECTION AT BLOCK 10 OF ADJUSTED MODEL.

zontal and the true deflections is rather small, but it increases toward the lower parts of the model. In order to maintain as true a comparison as possible, the horizontal-radial, rather than the true, deflections of the model were used for comparison with the measured deflections of the prototype.

The location of the center reference pier of the model corresponds exactly to the location of the center pier of the prototype. In determining the horizontal-radial deflections of the model, the horizontal distances were measured from this pier to different points of the down-stream face of the model.

Strain Measurements.—In making strain measurements on the model at its down-stream face, the general method used on the prototype was followed. Instead of steel inserts, eight bent metal pins were set in the model at each measuring station, in a 2-in. gauge circle. However, a compound filar microscope, supported by a special traveler, was used instead of the 10-in. Whittemore strain-gauge (see Reference (*b*)[4]). The down-stream strain-measuring stations were distributed along the entire face of the model.

Strain measurements at the up-stream face were made with specially designed strain-gauges that gave measurements simultaneously in three directions. These gauges, which were beneath the surface of the water, were read by means of a flash-light (see Reference (*b*)[4]). The up-stream stations were distributed only over one-half the up-stream face of the dam between Blocks 10 and 18.

After the model was built, the zero readings, which included the dead load stresses, were taken. The full load readings were taken after the water load was applied. In this way the measured strains represent the strain or stress decrements due to water load only and correspond to the strain measurements on the prototype.

Temperature Measurements.—As in the prototype, a number of thermocouples were distributed in the model at different distances from the faces. Due to the low thermal conductivity of the material of the model, it was comparatively simple to keep the temperature variations within such narrow limits that it was not considered necessary to introduce temperature corrections.

ADJUSTMENT OF THE MODEL

One of the factors that increases, considerably, the value of the model is its high sensitivity. Extremely slight changes in the conditions of the model produce noticeable differences in deflections and corresponding variations in strains and stresses. At the same time, the model is very stable, and loadings repeated under similar conditions result in very exact repetition of the deflection and strain measurements. The high sensitivity of the model requires that all the conditions be properly adjusted, and makes possible the study, with unusual exactness, of the different factors that influence the behavior of a dam.

The uncertainty of the deflections of the rock foundation on which the prototype rests, made it impossible to determine in advance the dimensions of the foundation of the model so that the foundation deflections in the model would correspond to those in the prototype. Consequently, it was necessary to devise some method of adjustment that would make it possible to vary the flexibility of the foundation of the model, so as to make its deflections agree with those of the prototype which contacts the foundation in two ways. In its central part the dam rests directly on the rock foundation and at the ends it contacts the abutments which, in turn, rest on the rock and re-act against it.

The model was built so that the arch proper, and its abutments, rested on a rubber-compound foundation strip of limited size, and the abutments of the prototype were reproduced to scale in the model.

The deflections and stresses developed in the prototype may be considered as due to two factors: (1) Those that were developed after the dam was completed, but before the water load was applied (these are the dead load deflections and stresses); and (2), those that were developed by application of the water load (these are the water-load deflections and stresses and are the only ones considered in this paper).

The stress measurements on the prototype showed that the abutments of the dam received only a small part of the water load so that the resulting deflection of the foundation due to such load was limited. On the contrary, in the arch section the water load influenced considerably the deflections and stresses of the foundation. Consequently, in adjusting the model it was necessary only to introduce additional flexibility in the foundation which supported the arch section, without introducing additional flexibility in the foundation of the abutments.

The next adjustments pertained to the vertical construction joints, the size of which was increased by application of horizontal jacks shown on the upper part of the model in Fig. 2. These jacks produced the necessary upstream movement of the model and a corresponding increase in width of the vertical joints. Additional adjustments were made by introducing thin plate fillers in some of the vertical joints. The extent of such adjustment was determined by a number of preliminary tests. The final zero and full load readings were taken after these jacks had been removed and all the fillers had been introduced.

Comparison Between the Behavior of the Prototype and Its Model

If all similarity conditions are fulfilled, the behavior of the prototype and its model should be in exact agreement; but if some of these conditions are violated, a disagreement in relative behavior is introduced, which will be unimportant for minor violations, but will increase as the more important conditions are violated.

Table 1 shows that in so far as the rubber compound in the model is concerned, there is a difference between the actual and theoretical values of the shear modulus and of Poisson's ratio. The difference in shear moduli is not very great; hence this violation is unimportant; but probably the difference in the Poisson ratios is of more importance. There is also some uncertainty in so far as the adjustments of the foundation and the size of construction joints are concerned. This combination of different factors makes it difficult

TABLE 2.—STRAINS AND STRESSES IN THE PROTOTYPE AND ADJUSTED MODEL

ELEVATION		BLOCK NUMBER									
		17	16	14	12	10	9	8	6	4	2
(*a*) STRAINS, IN MILLIONTHS OF AN INCH PER INCH; DOWN-STREAM FACE OF PROTOTYPE (FILLING PERIOD, APRIL, 1930)											
940.0	Gauge Lines:										
	1	......	......	+10	+30	+10	......	......	+60	+24	+50
	2	......	......	−30	−30	−65	......	......	−20	−10	+20
	3	......	......	−50	−100	−135	......	......	−100	−23	−25
	4	......	......	−25	−40	−65	......	......	−40	+11	+15
920.0	Gauge Lines:										
	1	......	......	......	......	+15	......	......	+20	......	......
	2	......	......	......	......	−40	......	......	−45	......	......
	3	......	......	......	......	−75	......	......	−90	......	......
	4	......	......	......	......	−10	......	......	−25	......	......
900.0	Gauge Lines:										
	1	......	+13	+75	+83	+55	......	......	+45	+13	+20
	2	......	−57	+15	+28	+5	......	......	−35	+3	+60
	3	......	−18	−13	−28	−45	......	......	−110	−48	−5
	4	......	+63	+63	+32	+20	......	......	−25	−38	−25
880.0	Gauge Lines:										
	1	......	......	......	......	+85	......	......	......	......	+15
	2	......	......	......	......	+22	......	......	......	......	+5
	3	......	......	......	......	−2	......	......	......	......	0
	4	......	......	......	......	+70	......	......	......	......	+10
860.0	Gauge Lines:										
	1	......	......	......	+47	+87	......	+65	+45	−35	−10
	2	......	......	......	+2	+67	......	+40	−5	−20	+25
	3	......	......	......	−30	+17	......	−10	−155	−65	−5
	4	......	......	......	+40	+48	......	+15	−100	−80	−20
840.0	Gauge Lines:										
	1	......	......	......	......	+70	......	......	......	......	−30
	2	......	......	......	......	+55	......	......	......	......	−10
	3	......	......	......	......	+40	......	......	......	......	−15
	4	......	......	......	......	+40	......	......	......	......	−35
820.0	Gauge Lines:										
	1	......	......	......	−3	−12	......	......	−15	−8	......
	2	......	......	......	−58	−52	......	......	−13	−13	......
	3	......	......	......	−23	−80	......	......	−70	−28	......
	4	......	......	......	+23	−40	......	......	−73	−23	......
800.0	Gauge Lines:										
	1	......	......	......	......	......	−18	......	......	......	......
	2	......	......	......	......	......	−13	......	......	......	......
	3	......	......	......	......	......	−12	......	......	......	......
	4	......	......	......	......	......	−17	......	......	......	......
792.0	Gauge Lines:										
	1	......	......	......	......	......	−32	......	......	......	......
	2	......	......	......	......	......	−19	......	......	......	......
	3	......	......	......	......	......	−7	......	......	......	......
	4	......	......	......	......	......	−20	......	......	......	......
(*b*) STRESSES,* IN POUNDS PER SQUARE INCH; DOWN-STREAM FACE OF PROTOTYPE (FILLING PERIOD, APRIL, 1930)											
940.0	Stresses:										
	V	......	......	+9	+58	−41	......	......	+173	+79	+178
	H	......	......	−187	−369	−516	......	......	−352	−76	−69
	S	......	......	98	213	238	......	......	266	85	125
920.0	Stresses:										
	V	......	......	......	......	+14	......	......	+26	......	......
	H	......	......	......	......	−283	......	......	−337	......	......
	S	......	......	......	......	157	......	......	184	......	......
900.0	Stresses:										
	V	......	+39	+283	+304	+188	......	......	+109	+22	+75
	H	......	−62	−5	−62	−141	......	......	−400	−177	−9
	S	......	205	164	184	167	......	......	246	118	144
880.0	Stresses:										
	V	......	......	......	......	+327	......	......	......	......	+58
	H	......	......	......	......	+41	......	......	......	......	+9
	S	......	......	......	......	164	......	......	......	......	26
860.0	Stresses:										
	V	......	......	......	+164	+348	......	+245	+82	−173	−41
	H	......	......	......	−88	+117	......	0	−574	−271	−26
	S	......	......	......	139	118	......	131	358	112	75
840.0	Stresses:										
	V	......	......	......	......	+295	......	......	......	......	−124
	H	......	......	......	......	+196	......	......	......	......	−75
	S	......	......	......	......	56	......	......	......	......	49
820.0	Stresses:										
	V	......	......	......	−25	−93	......	......	−101	−48	......
	H	......	......	......	−91	−316	......	......	−280	−114	......
	S	......	......	......	138	115	......	......	131	36	......
800.0	Stresses:										
	V	......	......	......	......	......	−75	......	......	......	......
	H	......	......	......	......	......	−48	......	......	......	......
	S	......	......	......	......	......	10	......	......	......	......

TABLE 2.—(*Continued*)

ELEVATION		BLOCK NUMBER									
		17	16	14	12	10	9	8	6	4	2
	(*c*) STRAINS, IN THOUSANDTHS OF AN INCH PER INCH; DOWN-STREAM FACE OF MODEL										
940.0	Gauge Lines:										
	1	+4.8	......	+6.3	+7.5	+9.5	......	......	+6.7	+2.5	+4.9
	2	—4.6	......	—2.8	—2.3	—2.0	......	......	—3.8	+0.6	......
	3	—8.5	......	—11.1	—11.5	—15.2	......	......	—11.4	—4.4	—10.7
	4	+0.3	......	—1.9	—2.4	—3.5	......	......	—1.9	—2.2	......
920.0	Gauge Lines:										
	1	......	......	......	......	+5.9	......	......	+7.9	......	......
	2	......	......	......	......	—1.8	......	......	—0.9	......	......
	3	......	......	......	......	—9.1	......	......	—10.2	......	......
	4	......	......	......	......	—1.0	......	......	—2.8	......	......
900.0	Gauge Lines:										
	1	......	+3.9	+9.0	+8.0	+13.1	......	......	+5.8	+5.9	+2.8
	2	......	—3.7	—8.3	—1.4	+2.4	......	......	+2.5	......	......
	3	......	—9.3	—11.8	—6.6	—10.9	......	......	—9.8	—7.2	—7.1
	4	......	0.0	+4.3	+2.7	—2.4	......	......	—6.9	......	......
880.0	Gauge Lines:										
	1	+0.3	......	......	......	+6.3	......	......	......	......	+1.2
	2	—8.4	......	......	......	—0.5	......	......	......	......	+1.6
	3	—4.4	......	......	......	—8.9	......	......	......	......	—3.9
	4	+3.8	......	......	......	—2.0	......	......	......	......	—3.5
860.0	Gauge Lines:										
	1	......	+0.3	+1.9	+6.8	+8.3	......	+4.4	+6.3	+2.8	+4.9
	2	......	—3.7	—5.0	—1.3	0.0	......	+0.8	—1.5	+0.3	+1.6
	3	......	—3.9	—4.7	—9.1	—8.1	......	—7.3	—8.9	—4.1	—4.5
	4	......	+0.6	+2.6	—0.1	—2.0	......	—3.7	—3.0	—1.1	—1.7
840.0	Gauge Lines:										
	1	+5.2	......	......	......	+11.2	......	......	......	......	+3.5
	2	+10.2	......	......	......	0.0	......	......	......	......	+1.9
	3	—8.1	......	......	......	—10.2	......	......	......	......	—4.2
	4	—11.0	......	......	......	+1.2	......	......	......	......	—1.2
820.0	Gauge Lines:										
	1	......	......	+3.0	+4.7	+7.5	......	......	+0.9	—0.5	......
	2	......	......	—3.8	+1.6	+0.9	......	......	+0.8	—0.9	......
	3	......	......	—5.6	—4.3	—6.3	......	......	—2.7	—0.9	......
	4	......	......	+2.0	—2.2	+0.2	......	......	—3.2	—1.0	......
800.0	Gauge Lines:										
	1	......	......	......	......	+4.9	......	......	—1.8	......	......
	2	......	......	......	......	+2.8	......	......	+2.9	......	......
	3	......	......	......	......	—1.8	......	......	—4.2	......	......
	4	......	......	......	......	+0.5	......	......	—8.9	......	......
	(*d*) STRESSES,* IN POUNDS PER SQUARE INCH; DOWN-STREAM FACE OF MODEL										
940.0	Stresses:										
	V	+0.41	......	+0.56	+1.31	+1.43	......	......	+0.75	+0.23	—0.34
	H	—4.58	......	—5.96	—5.81	—7.86	......	......	—6.04	—2.36	—6.19
	S	2.60	......	3.26	3.54	4.58	......	......	3.35	1.40	......
920.0	Stresses:										
	V	......	......	......	......	+1.01	......	......	+2.10	......	......
	H	......	......	......	......	—4.62	......	......	—4.69	......	......
	S	......	......	......	......	2.79	......	......	3.35	......	......
900.0	Stresses:										
	V	......	—0.56	+2.33	+3.53	+5.74	......	......	+0.68	+1.73	—0.56
	H	......	—5.51	—5.48	—1.95	—3.26	......	......	—5.18	—3.19	—4.28
	S	......	2.53	4.50	2.79	4.54	......	......	3.35	......	......
880.0	Stresses:										
	V	—1.43	......	......	......	+1.39	......	......	......	......	—0.56
	H	—3.19	......	......	......	—4.31	......	......	......	......	—2.48
	S	2.42	......	......	......	2.83	......	......	......	......	1.34
860.0	Stresses:										
	V	......	—1.24	—0.34	+1.69	+3.19	......	+0.56	+1.39	+0.56	+0.99
	H	......	—2.81	—2.81	—4.28	—2.96	......	—3.83	—4.31	—2.02	—1.54
	S	......	1.12	1.86	2.94	3.07	......	2.32	2.94	1.30	1.86
840.0	Stresses:										
	V	+0.86	......	......	......	+4.58	......	......	......	......	+1.05
	H	—4.13	......	......	......	—3.45	......	......	......	......	—1.84
	S	4.65	......	......	......	3.98	......	......	......	......	1.53
820.0	Stresses:										
	V	......	......	+0.15	+1.91	+3.26	......	......	—0.34	—0.71	......
	H	......	......	—3.08	—1.46	—1.91	......	......	—1.69	—0.86	......
	S	......	......	1.94	1.83	2.57	......	......	0.97	0.08	......
800.0	Stresses:										
	V	......	......	......	......	+3.00	......	......	—2.93	......	......
	H	......	......	......	......	+0.49	......	......	—3.83	......	......
	S	......	......	......	......	1.34	......	......	2.27	......	......

to set up any strict criteria as to the way in which the adjustment of the model should be made. It is a step-by-step procedure in which the influence of the different factors on the behavior of the model must be studied and the combination found which gives the most satisfactory results.

Deflections.—Fig. 8(*c*) shows the comparison of the deflections of the prototype and the adjusted model and represents the developed down-stream face of the model. The elevations correspond to those of the prototype. The block numbers are shown at the lower part of the dam. The deflections measured at the down-stream face of the model are shown by dotted lines. In order to make the prototype deflections directly comparable with those of the model, they were multiplied by 2.68, which is the theoretical ratio of model to prototype deflection. The deflections of the prototype are shown by solid lines.

Fig. 8(*c*) shows that, following the procedure just indicated, the deflections of the adjusted model are in a very satisfactory agreement with those of the prototype.

Strains and Stresses.—The data pertaining to the strains and stresses in the prototype and adjusted model are given in Tables 2 and 3. (In Table 2(*a*) and Table 2(*c*) Gauge Line 1 is vertical; Gauge Line 2 is at an angle of 45°; Gauge Line 3 is horizontal; and Gauge Line 4 is at an angle of 135 degrees. In Table 2(*b*) and Table 2(*d*), *V*, *H*, and *S* denote vertical and horizontal stresses, and maximum shears, respectively.) Using these data, the strains and stresses in the adjusted model and the comparison between the prototype and its model may be shown by a number of diagrams, which represent the developed down-stream and up-stream faces of the model, on which are indicated the corresponding elevations of the prototype and the block numbers.

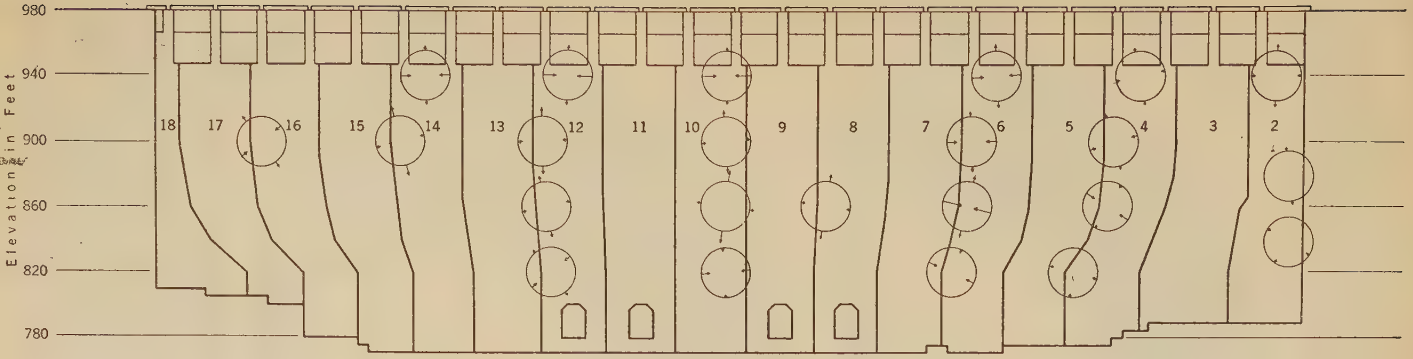
TABLE 3.—STRAINS AND STRESSES IN UP-STREAM FACE OF MODEL

Elevation	Block	STRAINS, IN THOUSANDTHS OF AN INCH PER INCH			STRESSES†, IN POUNDS PER SQUARE INCH	
		Vertical	Diagonal*	Horizontal	Vertical	Horizontal
920.0	10	+7.8	—0.3	—14.5	+0.07	—8.34
847.0	10	+6.5	+0.3	— 8.0	+0.87	—4.61
787.0	10	+3.5	—1.3	— 1.5	+0.57	—1.31
916.0	12	+7.3	—5.4	— 2.8	+4.01	+0.26
876.0	13	+4.3	—9.5	— 9.5	—1.14	—6.31
919.0	14	+5.3	—1.3	— 5.3	+1.63	—2.41
807.0	14	+6.0	+0.3	0.0	+3.14	+0.89
845.0	16	+3.5	0.0	— 2.8	+0.55	—1.86
920.0	17	+1.5	—0.4	— 0.3	+0.67	0.00

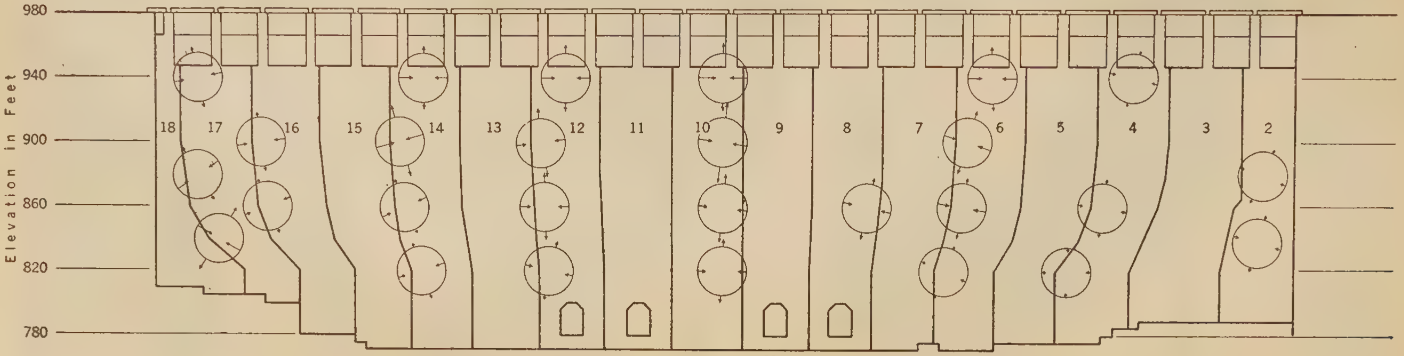
* 135° from vertical. † Compression is negative and tension is positive.

Fig. 10 shows both the direction and value of the principal strains on the down-stream face of the prototype and its model. With a few exceptions, the satisfactory agreement in the direction of the principal strains should be noted.

In order to make possible a direct comparison, all the measured data in Table 2 may be presented in a diagram in the terms of the model. Such diagrams will show the actual strains as measured on the model, but those


980
940
900
860
820
780
Elevation in Feet
18
17
16
15
14
13
12
11
10
9
8
7
6
5
4
3
2

(*a*) Prototype; Scale of Unit Strains (in Millionths) 0 200 Measured from Either End of Diameter


980
940
900
860
820
780
Elevation in Feet
18
17
16
15
14
13
12
11
10
9
8
7
6
5
4
3
2

(*b*) Model; Scale of Unit Strains (in Thousandths) 0 20 Measured from Either End of Diameter

FIG. 10.—PRINCIPAL STRAINS AT THE FACES OF THE PROTOTYPE AND MODEL.

of the prototype were multiplied by 134 which is the theoretical model: Prototype strain ratio.

In computing stress as measured by strain on the non-submerged surfaces of the dam and its model, the bi-axial stress relation was taken into account.

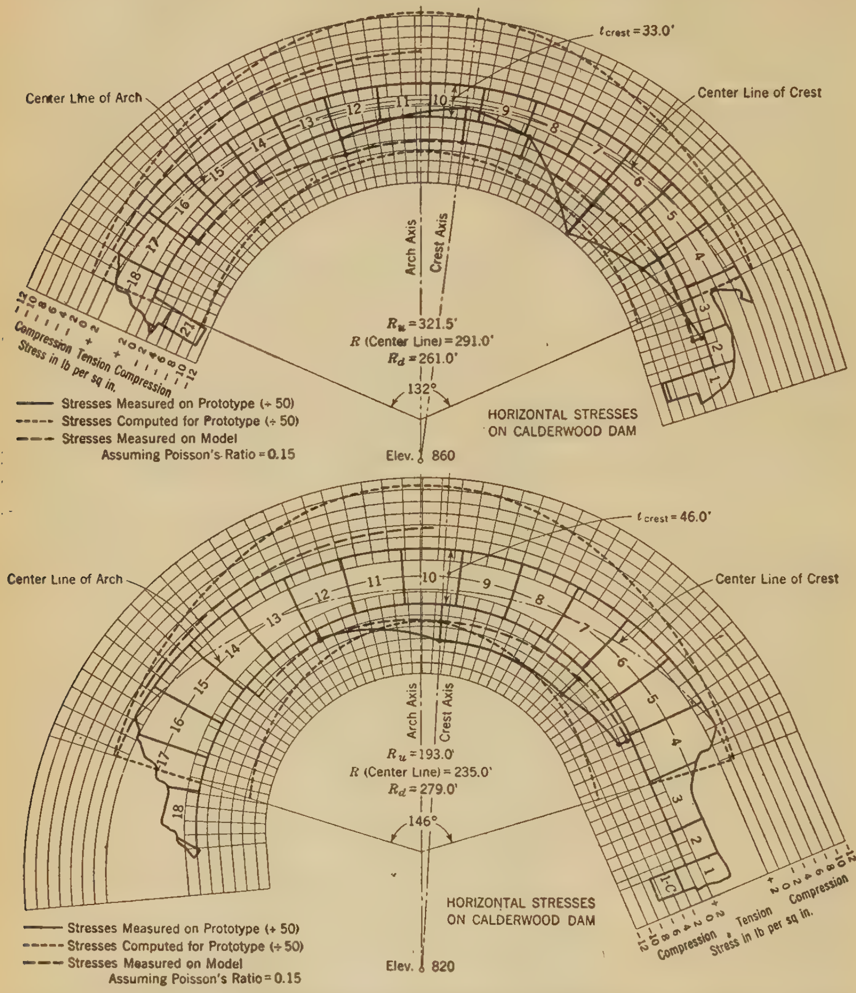

t_{crest} = 33.0'
Center Line of Arch
Center Line of Crest
Arch Axis
Crest Axis
R_u = 321.5'
R (Center Line) = 291.0'
R_d = 261.0'
132°
Compression Tension Compression
Stress in lb per sq in.
12 10 8 6 4 2 0 2 4 6 8 10 12
Stresses Measured on Prototype (÷ 50)
Stresses Computed for Prototype (÷ 50)
Stresses Measured on Model
Assuming Poisson's Ratio = 0.15
HORIZONTAL STRESSES ON CALDERWOOD DAM
Elev. 860
t_{crest} = 46.0'
Center Line of Arch
Center Line of Crest
Arch Axis
Crest Axis
R_u = 193.0'
R (Center Line) = 235.0'
R_d = 279.0'
146°
HORIZONTAL STRESSES ON CALDERWOOD DAM
Compression Tension Compression
Stress in lb per sq in.
Stresses Measured on Prototype (÷ 50)
Stresses Computed for Prototype (÷ 50)
Stresses Measured on Model
Assuming Poisson's Ratio = 0.15
Elev. 820

FIG. 11.—COMPARATIVE DIAGRAMS OF HORIZONTAL STRESSES ON CALDERWOOD DAM AT ELEVATION 820 AND ELEVATION 860.

On the submerged surfaces the normal component due to direct water pressure was also taken into consideration.

The vertical stresses at the up-stream and the down-stream faces of the model are of particular interest. In many cases, tension, due to the action of water load, was superimposed on the original dead load stresses on both the up-stream and down-stream faces of the model. If the straight-line distribution of stress were correct, and if tension were superimposed on one face, compression should be superimposed on the other face. The comparison of the vertical stresses actually superimposed on both faces of the model shows that the assumption of straight-line distribution of stress is incorrect, not only at the lower elevations, but probably also at the higher elevations. This is important because often the assertion is made that the straight-line assumption does not apply at the lower elevations, but is close to the actual conditions at the higher elevations, which are influenced less by the foundations.

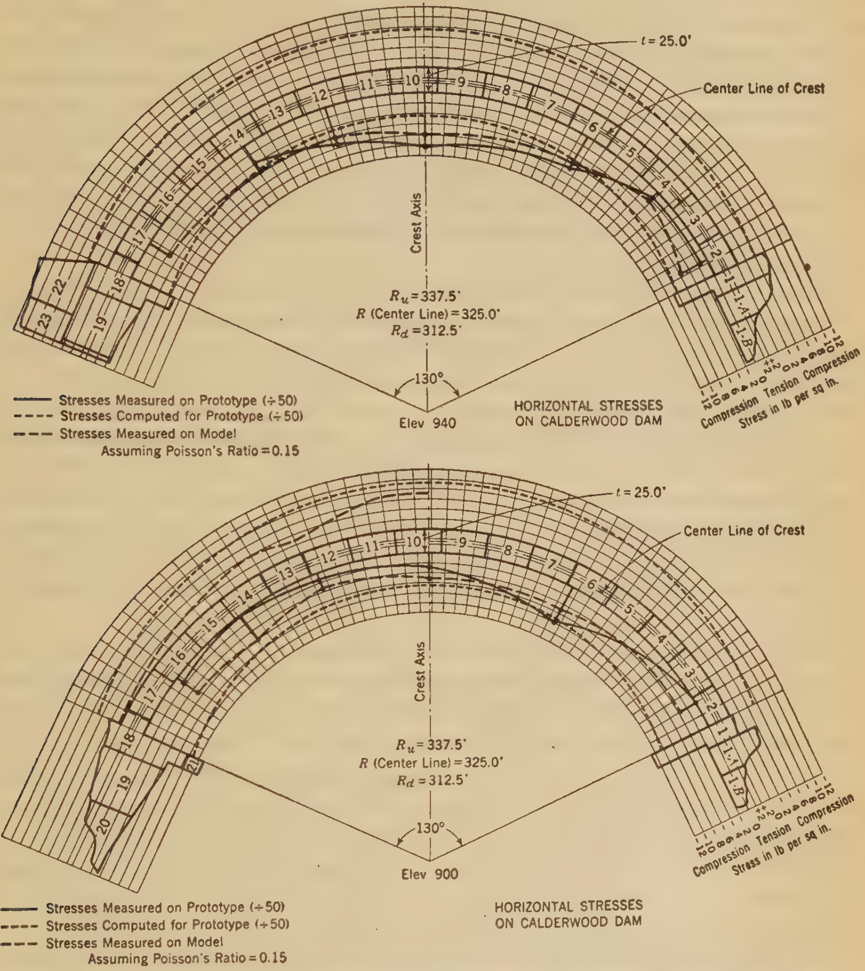

t = 25.0'
Center Line of Crest
Crest Axis
R_u = 337.5'
R (Center Line) = 325.0'
R_d = 312.5'
130°
Elev 940
HORIZONTAL STRESSES ON CALDERWOOD DAM
Compression Tension Compression
Stress in lb per sq in.
Stresses Measured on Prototype (÷50)
Stresses Computed for Prototype (÷50)
Stresses Measured on Model Assuming Poisson's Ratio = 0.15
t = 25.0'
Center Line of Crest
Crest Axis
R_u = 337.5'
R (Center Line) = 325.0'
R_d = 312.5'
130°
Elev 900
HORIZONTAL STRESSES ON CALDERWOOD DAM
Compression Tension Compression
Stress in lb per sq in.
Stresses Measured on Prototype (÷50)
Stresses Computed for Prototype (÷50)
Stresses Measured on Model Assuming Poisson's Ratio = 0.15

FIG. 12.—COMPARATIVE DIAGRAMS OF HORIZONTAL STRESSES ON CALDERWOOD DAM AT ELEVATION 900 AND ELEVATION 940.

In order to eliminate, at least partly, the discrepancies due to the different Poisson ratios, the strains in the model may be reduced to stresses by taking Poisson's ratio for the model to be 0.15, the same as in the prototype; the stresses of the prototype should be divided by 50 to have them correspond to the stresses in the model.

The agreement between the stresses is less satisfactory than in the case of the deflections. This discrepancy raises a question as to the reliability of the strain measurements. In so far as the model is concerned, it can be safely assumed that they are reasonably reliable, but the strain measurements on the prototype are somewhat doubtful. As discussed in Appendix III there is a possibility that the strains measured on the prototype are only partly due to actual stresses and partly to other phenomena. Under such conditions, a certain deviation may be expected between the actual stresses of the prototype and those evaluated from strain readings. On the other hand, this discrepancy may be due to the partial non-fulfillment of a number of similarity conditions, such as the flexibility of foundations, size of the joints, and difference in Poisson's ratios.

Figs. 11 and 12 show a comparison of the computed horizontal stresses as measured on the prototype and on its model. To construct these diagrams, the cross-sections of the prototype at Elevations 820, 860, 900, and 940 were drawn. Instead of the customary rectangular system of co-ordinates, a system of concentric circles and radial lines was used. A separate system was adopted for the stresses on the up-stream and the down-stream faces of the dam. These systems were chosen in such way that the up-stream and down-stream faces were taken as the zero lines of the corresponding stress diagrams. All prototype stresses were divided by 50 and the model stresses were computed, on the basis of Poisson's ratio of the rubber compound, to be 0.15. The resulting curves based on this assumption should be indicative at least.

The computed stresses are those used in the design of the prototype. In computing these stresses, gravity action was neglected and the design was made under the assumption that the dam consists of a number of horizontal independent arches with fixed abutments. Figs. 11 and 12 show that:

1.—The computed stresses are radically different from those measured both on the prototype and on its model. This is particularly noticeable at the down-stream face, where at the abutments the computed stresses show a maximum and the measured stresses a minimum, and *vice versa* toward the crown. These differences are most pronounced at the lower elevations where the thickness of the horizontal arches is increased toward the abutments.

2.—The curves of horizontal stress at the prototype are of particular interest since they indicate a considerable increase of stress at some of the vertical joints, which, in turn, indicates that large bending moments in the arches twist the individual blocks and produce a large concentration of stress at certain points. This increase is most pronounced at the sections which, due to the increased thickness, have horizontal arches that are very stiff at the abutments.

3.—In many cases the stress as measured on the prototype and on the model shows a fairly close numerical agreement and there is a similarity in the general shape of the stress curves.

4.—The non-uniform distribution of stress and in particular the reduction of stress toward the abutments on the prototype indicate that conditions might be improved by changing the shape of the horizontal arches.[9]

CONCLUSIONS

In order that a model study may be made to determine the behavior of a commercial arch dam, it is necessary to satisfy the major similarity conditions. The model described in this paper fulfills the most important ones and can be designated, with a considerable degree of accuracy, an engineering model. The pronounced similarity in behavior between the prototype and its model makes it possible to apply to the prototype the conclusions based on a study of the model.

The purpose of a proper design is to create a structure that is safe and economical. An arch dam, like any other structure, is safe and economical if the stresses are not only within the allowable maximum, but are uniform, and if, throughout the structure, they approach, closely, the allowable maximum stress. Too low stress at any point may indicate that the structure is not economical, and too high stress may indicate that it is unsafe. Finally, a non-uniform distribution of stress may make it necessary to reduce the average stress in order to keep the maximum stress low enough, and this may result in an uneconomical structure.

The tests on the prototype and its model show that there are a number of points where the stresses are too high and also too low, thus indicating that considerable improvement in the design of future structures can be made.

From the viewpoint of design, the following conclusions may be drawn:

(1) The foundation conditions have such influence on the behavior of an arch dam that a design which assumes a non-yielding foundation cannot satisfy the actual conditions and, therefore, is inadequate.

(2) The assumption of straight-line distribution of stress is not in agreement with actual conditions, and a design which is based on such assumption appears to be inadequate.

(3) The bending moments in the horizontal arches produce such heavy concentration of stress at some of the vertical construction joints that a design which neglects the influence of such joints and assumes a monolithic structure is inadequate.

(4) The circular shape of the conventional arch dam seems to be responsible for the non-uniformity of the horizontal stresses and introduces additional stresses which should be avoided. Presumably, this can be accomplished by substituting for the circular shape one that will better fit the conditions of the particular canyon.

(5) A design in which gravity action is neglected and all load is assumed to be taken by arch action results in a considerable and unwarranted increase in thickness of the horizontal arches at the abutments; the computed stress in such designs is radically different from the actual stress.

---

[9] "The Compensated Arch Dam," by A. V. Karpov, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 1309; also, *Transactions*, Am. Soc. C. E., Vol 99 (1934), p. 951.

(6) Vertical tension on both faces of the dam seems to be much larger than is usually assumed. The reduction of such stress merits attention.

From the viewpoint of building and testing a model, the following conclusions may be drawn:

(7) The adopted rubber compound permits the building of a model of any dam, reproducing its geometrical shape, the foundation contours, and the construction joints.

(8) Such a model will repeat its behavior under repeated loadings.

(9) The low modulus of elasticity of the rubber compound gives the model a high degree of sensitivity, and the relatively large distortions occurring under load permit the ready measurement of deflections and strains with a high degree of accuracy.

(10) The method of measuring deflections from three references (not in a plane) gives the absolute movements of any point at which deflections are measured.

(11) The method devised for measuring strains on the up-stream face, under water, and the arrangement for measuring strains on the down-stream face proved quite satisfactory as evidenced by the exactness with which readings could be repeated.

ACKNOWLEDGMENTS

The work described in the paper is a part of the arch dam investigation that was initiated by E. S. Fickes, M. Am. Soc. C. E., in an endeavor to put the design of such dams on a more scientific basis. The investigation was conducted by the writers, under the general direction of James W. Rickey and J. P. Growdon, Members, Am. Soc. C. E. H. N. Hill, Jun. Am. Soc. C. E., was directly responsible for the measurements made on both the prototype and the model. Recognition is also due to R. G. Sturm, Assoc. M. Am. Soc C. E., and to Messrs. C. Dumont and R. L. Moore, of the Research Staff of the Aluminum Company of America, for valuable assistance given in connection with the work.

APPENDIX I

PROPERTIES OF THE PROTOTYPE MATERIAL

In order to determine the proper relationship between the prototype and its model, it was necessary to secure reliable information concerning the behavior of the concrete of the prototype. The tests to determine the physical properties of this concrete were made on cylinders 2 ft. in diameter and 4 ft long, prepared as follows.

The concrete from which the specimens were made was taken from the batches that were being placed in the dam at the time of moulding the cylinders. It was dumped from the mixer into a 4-yd bucket, taken to the dam, and dumped on a flat-car, which was shifted to a side-track where the concrete was shoveled into the moulds, first removing all stone larger than 6 in. The concrete was placed in three layers, each of which was vibrated for 1½ min. Immediately after moulding, the cylinders were covered with planks to prevent excess evaporation and checking. After 18 to 24 hours, water was allowed to run over the tops of all cylinders for 14 days, the forms being stripped within 1 to 3 days. The cylinders were then allowed to weather until ready for shipment to the National Bureau of Standards, United States Department of Commerce, Washington, D. C., where they were tested.

Modulus of Elasticity.—Stress-strain determinations were made on eight cylinders to determine the modulus of elasticity of the concrete. The average results were found to be, as follows:

Stress, in pounds per square inch	Average strain, in inch per inch
0	0.0
100	0.0000279
200	0.0000542
400	0.0001063
600	0.0001615
800	0.0002125
1 000	0.0002665

The stress-strain curve represents a practically straight line and the modulus of elasticity of the concrete of the prototype was found to be, $E_p = 3\,770\,000$ lb per sq in.

The exceptional regularity of this stress-strain curve seems to indicate that if the average values of the moduli of elasticity are considered, the material of the prototype satisfies perfectly the similarity requirement that the material must follow Hooke's law. At the same time it leaves the impression that for stress less than 1 000 lb per sq in., a properly mixed and vibrated concrete is a much more uniform product than is usually assumed.

Poisson's Ratio and Other Factors.—The average value of Poisson's ratio of the concrete was determined and was found to be 0.15. The shear modulus was not determined, but was computed in accordance with the formula,

$$S_p = \frac{E_p}{2\,(1 + \mu_p)} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

in which, E_p is the modulus of elasticity, and μ_p, Poisson's ratio of the concrete; and, from which, $S_p = 1\,640\,000$ lb per sq in.

The average specific gravity of the concrete was determined by weighing twenty-four test cylinders, and was found to be, $\rho_p = 2.375$. The coefficient of thermal expansion per degree Fahrenheit of the concrete was not determined, but was assumed as, $c_p = 0.000006$.

APPENDIX II

PROPERTIES OF THE MODEL MATERIAL

In securing the material for the model, it was clear from the tests that the laboratory specimens of the rubber-litharge compound satisfied the necessary requirements. The laboratory testing methods have been described elsewhere.[10] The method of testing this compound was to cut three cylinders from the material in three different directions and apply the proper tests. This gave the necessary information regarding the uniformity of the material, but such methods are not well adapted to shop inspection and are practically unknown to the rubber industry. Therefore, it was necessary to develop a method, suitable for shop inspection, which would be familiar to the manufacturers; this method should be such as would insure a much higher uniformity of material than is customary in the commercial rubber compounds.

A number of preliminary tests indicated that for the inspection of a rubber compound manufactured under definite procedure and delivered in sheets of equal thickness, "durometer" readings give a satisfactory indication as to uniformity of the material.

Consequently, the uniformity criterion was established within narrow limits of variation in thickness, specific gravity, and durometer readings. These limits were, as follows:

Specific gravity.............................. 2.38 ± 0.03

Thickness of rubber slabs, in inches............ 1 ± $\frac{1}{32}$

In the total number of durometer readings:

Not less than 90% must be within ± 2% from the average reading
Not more than 10% may be within ± 3% from the average reading

Durometer readings were taken at each 4 by 4-in. square on both surfaces, and one reading at intervals of 4 in. along the edges of each slab.

After the material was received from the factory, a number of test cylinders were cut from slabs in three directions and the elastic properties were determined as described fully elsewhere.[10]

Modulus of Elasticity.—Stress-strain determinations were made on six sets of cylinders to determine the modulus of elasticity of the rubber compound. Each set consisted of three cylinders cut from the same slab. The average results of these determinations are, as follows:

Stress, in pounds per square inch	Average strain, in inch per inch	Stress, in pounds per square inch	Average strain, in inch per inch
0.0	0.00	10.25	0.01835
2.56	0.00461	12.81	0.02290
5.12	0.00921	15.37	0.02740
7.68	0.01380	17.92	0.03190

The stress-strain curve drawn from the foregoing data is practically a straight line, and the modulus of elasticity was found to be, $E_m = 563$ lb per sq in. Strain-stress curves of these data compared with those for the prototype show that in both cases the curves are straight lines within the limits of stress of the prototype and its model.

---

[10] "Methods for Determining the Physical Properties of Certain Rubber Compounds at Low Stresses," by R. L. Templin, M. Am. Soc. C. E., and R. G. Sturm, Assoc. M. Am. Soc. C. E., *Proceedings*, Am. Soc. for Testing Materials, Vol. 31, 1931.

Poisson's Ratio and Other Factors.—The average value of Poisson's ratio of the model material was found to be 0.5. The shear modulus was not determined, but was computed and found to be $S_m = 188$ lb per sq in. The average specific gravity of the model material was determined by a number of tests, and was found to be $\rho_m = 2.38$.

For the material of the model the coefficient of thermal expansion per degree Fahrenheit was determined by a number of tests and found to be, $c_m = 0.00008$. Specimens of the model material were soaked in water from 10 to 14 days. The increase in weight and elongation due to this increase was negligibly small.

APPENDIX III

Suggestions Based on Experience Gained During the Work on the Prototype and Model

Considerable experience, gained during the progress of the work on the prototype and its model, was applied to the investigation, but some of it was gained too late to apply directly to this work *per se*. Nevertheless, this experience is of general interest, and the following suggestions may be helpful to those who expect to carry on similar investigation.

Measurements on the Prototype

First.—Deflection Measurements.—The method used to make the measurements from a central concrete pier by means of an invar or steel tape, gives satisfactory results, but this work must be done under favorable weather conditions, preferably at night. Measurement of deflection in the direction of only one pier does not seem to be sufficient; the use of two or perhaps three piers is recommended.

There is some indication that after the water load is applied, a certain movement of the entire foundation takes place in the vicinity of the dam, so that it is essential to tie in the reference points used for deflection measurements with other points at such a distance from the dam that they cannot be influenced by movement of the foundation.

Because of the foundation movement, deflection measurements made at the lowest elevations of the dam and, perhaps of the ledge adjoining the dam, should be of considerable value.

Second.—Strain Measurements.—The generally accepted methods of strain measurement seem to give results that are much less reliable than the deflection measurements. Even if it is possible to arrange the measurements in such a way that only a comparatively short time elapses between the zero and the full load readings, the changes that take place in the concrete may influence the strain readings considerably, so that the measured strains are due partly to stress and partly to the swelling and contraction of the concrete. This is probably a particularly disturbing element during the first loading. If it were possible to load and unload the structure a number of times and take a number of zero and full load readings during the subsequent no-load and full-load conditions, this uncertainty could be removed, at least partly. Even then it is practically impossible to determine the actual stress in the concrete; probably only the stress change due to water load could be determined.

It seems necessary to devise a method by which it would be possible to determine, under any conditions, the load and zero strain readings at short time intervals. The best method would be to take a complete set of strain measurements at any desired point of the dam without the necessity of making any previous measurements under no load conditions. The method suggested for that purpose is illustrated in Fig. 13. The procedure would be as follows:

Drill at the proper places the necessary holes in the concrete of the dam and grout short steel plugs in these holes. The plugs are to be used as measuring points for the strain-gauge. Instead of drilling holes in the concrete, it might be preferable to fasten to the surface of the concrete, using proper adhesive, the required number of thin, small-diameter, pieces of metal, with suitable indentations for the gauge points. The full load readings would be made under full load conditions. Next, using suitable equipment, cut a circular groove on a diameter larger than that of the gauge-line circle. The depth of the groove would be determined by check-strain readings. When a further increase of the depth of the groove does not influence such readings the desired condition of no surfaces stresses has been obtained.

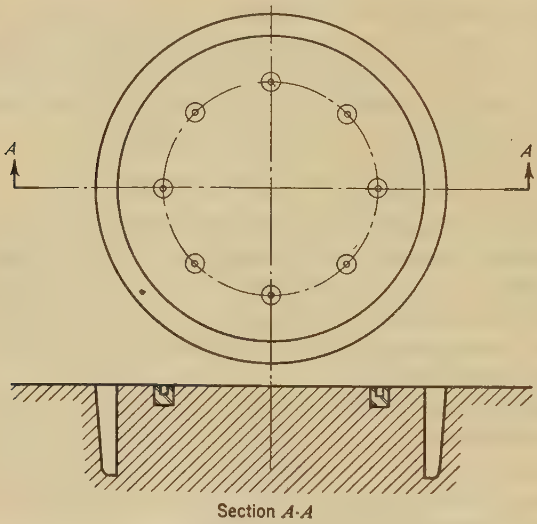


FIG. 13.—PROPOSED METHOD OF MEASURING STRAINS AT THE FACE OF A DAM

The groove thus constructed will remove all stress from the surface of the concrete within the boundaries of the groove and, in this way, the zero readings may be obtained, not only under full water-load conditions, but without being influenced by the aging of the concrete, or by temperature changes, since the time interval between the full load and no load measurements may be short. Strain measured in this way will represent more closely strain due to stress alone. If the modulus of elasticity of the concrete is unknown a test cylinder may be cut from the dam at the same time and the modulus determined in the laboratory.

It is believed that the foregoing method would make it possible to determine the actual stress at any surface point not covered with water, on any existing dam, with great accuracy.

If it is deemed advisable to make strain measurements in the usual way, much longer gauge lines are suggested, and instead of the usual 10-in. gauge line, 5-ft, or even 10-ft, gauge lines are possible and preferable. Since such long distances cannot be measured by ordinary gauges it would be necessary to devise special instruments, which could be arranged so that the readings may be made at the top of the dam. It is of utmost importance to make strain measurements on both the up-stream, water-covered face of the dam and on the down-stream face.

Model Work

The rubber-litharge compound developed for the model of the Calderwood Dam seems to be superior to any other known material, but it still has the drawback of a high value of Poisson's ratio. Future progress in model work, which is so essential for the proper design of arch and gravity dams, depends to a large extent on developing a material similar in its properties to this compound, but with a much lower value of Poisson's ratio.

In designing a model the importance of a sufficiently extended foundation strip, particularly in the horizontal direction, should not be under-estimated. Improved methods of adjusting foundation deflections seem to be important. It does not seem necessary to reproduce a large part of the canyon, but rather to provide arrangements for adjustment which will make possible the simulation of a wide range of conditions.

DISCUSSION

A. W. SIMONDS,[11] ASSOC. M. AM. SOC. C. E. (by letter).—With the construction of arch dams of heights of 200 ft, or more, in recent years the problems involving design have become of increasing importance. Any research pertaining to the action of such structures should be welcomed by members of the Engineering Profession.

Models have long been of value in studying the action of indeterminate structures, such as arch dams, arch bridges, rigid frame structures, etc., and considerable can be learned from them; but it is not sufficient to build a model of a structure and test it and then, as a result of such tests, to magnify the model fifty or one hundred times, or more, as a basis of design. There are too many uncertainties involved, and a mathematical analysis of reasonable exactness should be used as a basis for the design of such structures.

The greatest uncertainty in building a model of an arch dam is in the foundation conditions. How thick should the base be and how should the elastic properties of the base and the abutments compare with those of the model? Actual field conditions of the foundations and abutments can not be reproduced exactly. In their tests the authors state that screw-jacks were used under the foundation strip and that the model was tested under a fairly wide range of foundation conditions. What effect did a change of foundation conditions have on the action of the model? Were the deflection and the strain distribution affected materially by a change in the adjustment of the jacks? Was there sufficient material in the abutments to allow any shear deflection to occur; and what was the effect of jacking on such deflection? Having the results of observations on the Calderwood Dam available for comparison, the authors were able to select a foundation condition for the model which would furnish a close agreement between the deflections of the model and those of the dam. What were the foundation conditions that were used in obtaining the deflection curves shown in Fig. 8? If observations on the action of Calderwood Dam had not been available at the time of making the model tests, what foundation conditions would have been used in the final results of the tests?

The choice of a suitable material for a model is an uncertainty. The rubber and litharge compound is nice to work with, but the material itself is not like concrete. The high value of Poisson's ratio for this material makes it altogether different from concrete in its action, in that it deforms at constant volume. This may lead to peculiar results in tests of models built of this material, which would not be encountered in tests of models built of other materials having Poisson's ratios nearer that of concrete. For example, the writer was recently engaged in measuring the radial deflection of a model of a thick arch dam built of this rubber and litharge compound. As was customary, the deflection measurements were made at the down-stream face of the model. However, in this case, small holes had been drilled in the model and invar steel rods were anchored at the center lines of the arch elements and at the up-stream face. It was found that the deflection at the center line was somewhat greater than at the down-stream face, while the deflection at the up-stream face was about twice that at the down-stream face. This squeezing has been noted in models built of other materials, but not to such a marked extent.

---

[11] Engr., U. S. Bureau of Reclamation, Boulder, Colo.

A further investigation of the squeezing effect showed that both the foundation and the abutments deformed when the load was on the arch. Vertical deflection measurements showed that the model deflected upward at both the up-stream and the down-stream faces, the vertical deflection at the top of the model being about 20% of the horizontal radial deflection. This excessive vertical deflection actually stretched both the up-stream and down-stream faces in a vertical direction and, as a result, strain measurements indicated tensile cantilever strains on both faces simultaneously. This fact confirms the authors' statement that "in many cases, tension, due to the action of the water load, was superimposed on the original dead load stresses on both the up-stream and down-stream faces of the model." Further research by the writer involved the testing of a model of a thick arch dam composed of a compound of commercial building plaster and diatomaceous earth, a material having a Poisson's ratio of 0.20. Some squeezing was noted of the up-stream face of this model toward the down-stream face in the deflection measurements, but it was not so pronounced as in the rubber and litharge model. This was accompanied by a similar vertical deflection of the model, but it was rather small.

For partial loads, the measurements indicated tensile cantilever strains on the down-stream face. For this condition, the maximum deflection of the cantilever elements occurred below the top of the model; as the elevation of the surface of the load was increased, the maximum deflection of the cantilever elements occurred nearer the top of the model and the area of tensile cantilever strains became smaller. Finally, a point was reached at which there was no contraflexure in the cantilever deflection curves, the maximum deflection occurring at the top of the model. For this condition no tensile strains were found on the down-stream face of the model.

Apparently, the phenomenon of tension appearing simultaneously on both the up-stream and down-stream faces of the cantilever elements is found only in a material having an abnormally high Poisson's ratio. Further investigation is being made by the writer to ascertain whether models built of other materials will show tension on both faces of the cantilever elements simultaneously.

Eldred D. Smith, Jun. Am. Soc. C. E., has built and tested a model of a gravity dam composed of a compound of finishing plaster and a commercially treated diatomaceous earth having a weight of 42 lb per cu ft and a Poisson's ratio of 0.20.[12] This model was loaded with a film of mercury applied in a rubber bag. In order to secure a correct strain distribution, weights were applied to increase the unit weight to 13.6 times that of concrete. The results of the strain measurements showed that the vertical stress along the foundation line did not vary as a straight line. Above the foundation, however, this variation became less, and in the upper elevations where the model was thinner, it followed closely to a straight line. This was the result of tests of a gravity section which had no arch thrust. If a thrust had been applied the distribution of vertical stress might have been changed somewhat, but not to any great extent. In view of the results of these tests, the writer is inclined to disagree with the authors' statement that "the comparison of the vertical stresses actually superimposed on both faces of the model shows that the assumption of straight-line distribution of stress is incorrect, not only at the lower elevations, but probably also at the higher elevations."

---

[12] A Thesis by E. D. Smith, Jun. Am. Soc. C. E., at the Univ. of Colorado, in partial fulfillment of the requirements for the degree of Master of Science.

An arch dam is acted on by a complex system of forces. The water load acts normal to the up-stream face. The thrust of the arch tends to produce rib-shortening. The weight of the structure is downward. The resistance of the structure to deflection introduces couples. What effect do these forces have on an arch dam built of a material with a high Poisson's ratio, such as the rubber and litharge compound? Since rubber deforms at a constant volume, the material in the model deflects in the direction of least resistance which, in this case, is vertical. The resulting measured cantilever strains therefore, are tensile. Would this condition occur in a concrete arch dam?

The conversion of such strains to stresses is likely to lead to errors of considerable magnitude. To change the value of Poisson's ratio from 0.5 to 0.15 is not justifiable even for the purpose of comparison. This ratio is too important in the calculation of stresses as can be seen in the formulas for vertical and horizontal stress:

$$\sigma_v = \frac{E}{1 - \mu^2} (\epsilon_v + \mu \, \epsilon_h) \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

and,

$$\sigma_h = \frac{E}{1 - \mu^2} (\epsilon_h + \mu \, \epsilon_v) \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

in which, σ_v is the vertical stress; σ_h is the horizontal stress; ϵ_v, the measured vertical strain; ϵ_h, the measured horizontal strain; μ is Poisson's ratio; and E is the modulus of elasticity.

It has never been the fortune of the writer to build and test a model of a variable radius arch dam. In making some mathematical analyses of such dams, however, it was noticed that the maximum compressive cantilever stresses usually occurred in the lower elevations of the cantilever elements. Were the cantilever strains measured in the model of Calderwood Dam near Elevation 780 and what was their magnitude? Did the strain measurements show a concentration of stress around the closure tunnels? No indications of the strains in these locations are shown in Fig. 10. If the tunnels were effectively plugged no great concentration of stress should occur around these openings.

The chief advantage in using the rubber and litharge compound for models of dams is that strain-gauges that operate under water can be mounted on the up-stream face. It is presumed that the up-stream strains were measured with the apparatus illustrated in a previous paper[13] by the authors. What degree of precision can be expected of these strain-gauges when operating under water? It has been the writer's experience that some variation in results can be expected in regions of considerable deformation. The gauges were mounted on pins driven in the rubber. If the depth to which the pins were driven varied, a variation of measurement was encountered. Even the use of a jig in driving the pins was not altogether satisfactory as the pins had a tendency to spring up after driving.

The discussion of the properties of the model material given in Appendix II is of particular interest. On account of its excessive deformation, soft rubber is somewhat difficult to test for elastic properties. The usual apparatus found in most laboratories is not at all suitable for testing this material. The test specimens frequently present unusual problems.

In testing the rubber and litharge compound for physical characteristics, the writer has observed that it has pronounced directional properties. For compression tests, the specimens used were 6 by 6 by 12-in. rectangular prisms which were built up from slabs of 1 in. in thickness. These specimens were built up in several ways. One type was made from a single 24 by 24 by 1-in. slab. In building these specimens care was taken to keep the direction of the grain of the rubber the same in each lift. In another type, the direction of the grain of the rubber was alternated in each succeeding lift. In the third type the specimens were built of 6 by 6 by 1-in. slabs, with no attention paid to the direction of the grain of the rubber. Tension and torsion specimens consisted of 3 by 12-in. cylinders of two types, one with longitudinal and the other with transverse joints.

The results of the compression tests showed that the material had three different moduli of elasticity and six different Poisson's ratios depending on the direction of measurement. The moduli of elasticity varied from 320 to 420 lb per sq in. and Poisson's ratio from 0.38 to 0.64. The results of the tension tests disclosed that the moduli of elasticity of the different types of specimens varied from 350 to 395 lb per sq in. and Poisson's ratios from 0.39 to 0.51. In torsion tests, the shearing modulus varied from 120 to 140 lb per sq in. These values were obtained after correcting for the change in dimensions of the specimen for both strain and stress. The coefficient of thermal expansion per degree Fahrenheit was not constant but varied with the temperature at which the tests were made. It varied from 0.000065 to 0.000078 which is slightly less than the value obtained by the authors.

From these tests it was found that the average value of Poisson's ratio for any one specimen was about 0.5. In some cases, however, it was slightly greater, in one case being as much as 0.544. The writer is fully aware of the fact that for a truly isotropic material a value of Poisson's ratio greater than 0.5 indicates an increase in volume under a uniform pressure which is not possible. According to Karl Pearson[14] and St. Venant[15] it is possible for a non-isotropic material to increase in volume when subjected to pressure. An increase in volume of soft rubber compounds in compression tests has been noted by R. Ariano.[16]

---

[13] "Building and Testing an Arch Dam Model," *Civil Engineering*, January, 1932, p. 11.

Whether the value of Poisson's ratio is 0.5, or slightly greater than 0.5, is not of material consequence and should not reflect on the manner of making the tests for physical properties. Either value should serve as a warning that soft rubber is materially different from concrete and that the conversion of strains measured on soft rubber to predicted stresses in concrete is questionable.

LARS R. JORGENSEN,[17] M. AM. SOC. C. E. (by letter).—A most interesting account of the most unusual performance of a model of an arch dam and the prototype is contained in this paper. The stresses given in Table 2 for the Calderwood Dam and its model are amazing, since they differ greatly from what would ordinarily be expected especially for the prototype.

On the Calderwood Dam there is 20 ft of water on top of the overflow crest, held back by Stoney gates and, therefore, the arch is heavily loaded from the very crest. The stress sheet shows vertical tension on the downstream face of the dam for the upper 100 ft, with a maximum of 348 lb per sq in. at Elevation 860, and 295 lb at Elevation 840, Block 10, where considerable compression would always be expected. There could be zero vertical surface stress in the down-stream face of the Calderwood Dam and the cantilever would still be capable of supporting some load on the up-stream face. In such a case the horizontal shear would have a parabolic distribution in the triangular cross-section in which the apex was approximately at the maximum water level. The vertical surface stress being high in tension, however, the cantilever action has evidently been reversed from that which is usually expected and the arch, therefore, will be required to carry all the water load, in addition to giving an up-stream push to the cantilever, or holding it from deflecting down stream as much as it would otherwise.

The arches presumably must carry more than their design load, and, nevertheless, develop horizontal arch stresses less than those calculated (except at Elevations 860 to 840 at the down-stream face at the crown, where tensions of 117 and 196 lb per sq in. are shown to exist), and a somewhat higher compression in Block 6, Elevation 860, than was anticipated.

At the abutments the stresses are much less than those calculated, and it appears as if the abutment stresses had been transferred to the points of contraflexure by the action of punching shear, or in some other manner. That the horizontal arch stresses in general are less than those calculated might indicate a better stress distribution than a straight-line variation but it is difficult to explain why the horizontal arch stresses should be so much less at the abutments than farther out on the arch, near the points of contraflexure.

---

[14] "A History of Elasticity and Strength of Materials," by Todhunter and Parsons, Vol. II, Pt. I, p. 110.

[15] *Loc. cit.*, p. 208.

[16] "The Resistance of Rubber to Compression," by R. Ariano, *The India Rubber Journal,* August 11, 1928, p. 8.

[17] Cons. Hydro-Elec. Engr., Berkeley, Calif.

The maximum arch stresses are not horizontal, as shown in Fig. 10, but meet the abutments at an angle with the horizontal, as would be expected and, therefore, are not given directly in Table 2. The difference between the measured horizontal arch stresses and the calculated or anticipated arch stresses at the abutments is so large, however, that it can not be explained thereby.

The greatest departure from what would ordinarily be expected is that the cantilevers instead of helping to carry the load, apparently act in the opposite direction practically from the crest of the dam to about 30 ft above the river bottom. Presumably, they must be held back by the arch. If the deflection of an arch dam under load can be kept small by efficient grouting while low temperatures prevail in the structure, the dam can be considered almost monolithic and this should always be the aim.

The contraction joints of the Calderwood Dam were grouted but, being a diversion dam, from which it is impossible to remove the load after the structure has taken its permanent set and has cooled off, the grouting could only be, say, 50% effective. Many dams are in the same class, and one has to be contented with the best possible grouting under the circumstances. Nevertheless, the fact that there was less than 100% efficiency in grouting does not explain the peculiar cantilever action of this dam. The writer will be much interested in future check measurements on account of the unexpected behavior of this dam. When the plans were made, the design was regarded as good and the action disclosed by Table 2 was not anticipated.

Cutting the arch into narrow voussoirs, which will adjust themselves under load relative to each other, will most likely produce an ideal arch; that is, an arch that will carry a greater proportion of the total load as pure arch action without bending, than any other kind,[18] but the question arises, whether it is not more practical to keep the individual voussoirs not less than, say, 30 ft long and take advantage of about 50% of the shearing strength of the concrete, as well as the smaller number of joint details. In the Calderwood Dam the shear stresses seem to be the dominating load carriers at the abutments.

Castigliano[19] has shown that it was possible in a certain arch bridge to reduce the maximum stress from 61 kg per sq cm to 41 kg per sq cm at the abutments by using mortar of a much lower modulus of elasticity (100 times) than that of the granite voussoirs between 11 voussoirs at the abutments and 10 at the crown (of a total of 48). He had the thickness of the mortar wedge-shaped, with the larger thickness in the heavy compression zone as did Mr. Karpov in a previous paper.[20]

Castigliano's calculations have led the writer to experiment with a grout mixture consisting of plaster of Paris and Portland cement, which, when setting, is practically without volume change and is quite plastic especially at first. Therefore, it has a tendency to flow from places of higher stress to places of lower stress, as the load increases on the structure. The plaster of

---

[18] "The Compensated Arch Dam," by A. V. Karpov, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 98 (1933), p. 1309.

[19] "Elastic Stresses in Structures," tr. by Ewart S. Andrews, Lond., 1919, p. 357.

[20] *Transactions*, Am. Soc. C. E., Vol. 98 (1933), Fig. 8(*a*), p. 1328.

Paris crystallizes faster than the cement hydrates and takes most of the mixing water. Later, the cement finishes hydration by means of capillary action some time after the water covers the up-stream face of the dam, and the grout gets some elastic properties, or it may deteriorate, in which case it expands. The grout is between two water-stops and several recesses on the face of the contraction joint wall and can not escape no matter how plastic it is.

The authors' Conclusion (5) is that "a design in which gravity action is neglected * * * results in a considerable and unwarranted increase in thickness of the horizontal arches at the abutments." The purpose for which the thickness is increased at the abutments at lower elevations does not require this increase in thickness to be extended entirely to the middle of the dam; it can generally be tapered toward the points of contraflexure and does not require so much material. It may look better architecturally to taper this added thickness gradually from a maximum at the abutments to zero in the middle of the dam at the down-stream face, as was done on the Calderwood Dam. According to Table 2, the arch stresses in the Calderwood Dam do not increase as expected from the points of contraflexure toward the abutments, the abutments having, so to speak, been moved to the points of contraflexure. This is a rather strange performance, but still more unexplainable is the phenomenon that the cantilever action (also according to Table 2) is opposite to that which has always been assumed. This is difficult to understand and, therefore, still more difficult to take care of in a design. Such a vertical tension as 348 lb per sq in. at Elevation 860 in the down-stream face of the Calderwood Dam, the writer can scarcely comprehend.

It is true, that when an arch dam is designed by dimensioning each horizontal arch slice separately, an approximation is made by assuming that individual slices can slide relative to each other. It seems that this assumption should be on the side of safety, and this is important for an uncertain problem. The shearing strength between the imaginary slices would make possible a transfer of stress from high to low points, compensating for some irrigularities of the canyon sides. The radial shear stresses are greatest at the abutments and this may warrant increasing the thickness for that reason alone. These shear stresses can not be measured or, at least, have not been measured. It seems reasonable to conclude that to widen the contact area between the structure and the rock foundation by thickening the dam in the joint zone, would be material well spent, serving as a "filler" between the structure and the infinite base.

It is surprising to find the measured arch stresses generally less than the calculated arch stresses inasmuch as the cantilever stresses evidently are opposite to what they were assumed. The designers of the Calderwood Dam did not plan the cantilever to help the arch carry the water load; it was not suspected, however, that the arch would have to carry extra load because the cantilever leaned back on it. This is a rather startling discovery if it is actually the case.

Additional measurements on the Calderwood Dam are most desirable in order to clear up several doubtful points. It seems entirely unreasonable, for instance, that a cantilever subject to bending one way or the other should have tension in both faces at the same time, or even that it should have extensive tension in the down-stream face at any time when loaded.

The writer has always believed that an arch dam should be designed so as to carry all the load with a reasonable margin of safety. Unless theory and experiments agree better than they have in the past, it is not on the safe side to "skin" the arch by assuming or figuring that the cantilever will support part of the load. The paper does not inspire confidence in the prediction of accurate actual stresses by means of calculation.

A. C. JANNI,[21] M. AM. SOC. C. E. (by letter).—The care with which these tests were conducted, has produced results that deserve the attention of the profession. The writer has called attention,[22] in a general way, to the principal uncertainties attending the analysis of an arch dam, and these uncertainties seem to be confirmed, once more, by the tests and conclusions described in this paper.

The trouble is neither with the designer, nor his method of analysis; but with the assumptions he is compelled to make, and which he knows are not correct. Indeed, some of the assumptions are entirely contrary to the laws of equilibrium. Consequently, the results obtained by analysis, including the uncertainties that creep in with questionable assumptions, render them illusory, to a great extent.

The paper is concerned with measurements of the deformation of a rubber litharge model of Calderwood Dam, caused by hydrostatic pressure. From these measurements the authors reach the conclusion that the stresses that actually occur in arch dams are not the same as those determined by theory. The writer is in agreement with this conclusion. One of the reasons for this difference is that theory takes temperature effects into consideration, which cannot be measured by tests, because they do not deform the structure.

Every engineer knows that the stresses caused by temperature change in an arch dam are often more important than those due to the hydrostatic pressure. Dams are exposed to the most sudden and violent changes of temperature, and this fact is due to their location. For example, in a mountainous region, where dams are usually built, a sudden and important temperature change can occur almost over-night. Then the dam will have its down-stream face exposed to the surrounding temperature of the air, whereas its up-stream face is in contact with a body of water the temperature of which is quite different from that of the air. In other words, while the up-stream face of the dam has a temperature of about 32° F, the down-stream face may have a temperature of, say, $-$ 20° F. Every engineer can realize the unfavorable condition to which a concrete structure is subjected, when such temperature drops occur (as they often do).

Furthermore, the law of temperature transmission, in large masses of concrete, is not known, and this adds still more uncertainty to the analysis of temperature stresses. Assume, for example, that the temperature is the same for the entire structure, and that it is known. Although this assumption is almost certain never to occur, it will be made here, to simplify the argument.

---

[21] Cons. Engr., New York, N. Y.

[22] "A New Type of Dam," by A. C. Janni, M. Am. Soc. C. E., *Proceedings*, Am. Concrete Inst., December, 1933.

Let t_e be the change of temperature at the extrados and t_i that at the intrados of the elemental arch of the dam. As a further simplification, assume that the law of transmission of temperature from the intrados to the extrados, or *vice versa,* is a linear one. (The latter assumption is known to be incorrect as applied to large masses of concrete.)

The static effect caused by a change in temperature may be considered as the algebraic summation of that caused by a uniform change, $\frac{1}{2}(t_e + t_2) = t_0$, plus that caused by a change, $\frac{1}{2}(t_e - t_i)$, at the extrados, and $-\frac{1}{2}(t_e - t_i)$, at the intrados. Assume that both t_e and t_i represent a temperature increase and that $t_e > t_i$. Let *AB,* in Fig. 14, be the elemental arch of the dam, and *CDEF* (Fig. 15), a voussoir of it. If End *A* were free, Section *CE* would take the position, $C_1 E_1$, after the arch expands, because $CC_1 = c\ t_e\ \Delta s$ and $EE_1 = c\ t_i\ \Delta s$. This total deformation consists of a displacement of the section, *CE,* to $C_2 E_2$, *CE* and $C_2 E_2$ being parallel (caused by the uniform change of temperature, $\frac{1}{2}(t_e + t_i) = t_0$, and by a rotation, $d(\Delta\phi)$), bringing the section, $C_2 E_2$, into the position, $C_1 E_1$, which is given by:[23]

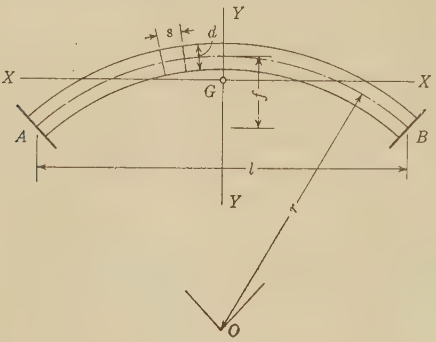


FIG. 14.

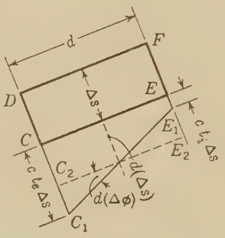


FIG. 15.

$$d(\Delta\phi)\ \frac{c\,(t_e - t_i)\,\Delta s}{d} = \frac{c\,\Delta t\,\Delta s}{d} \quad \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

The first deformation (to $C_2 E_2$) would cause a displacement of *A* (Fig. 14) to the left, an amount equal to $c\ \frac{t_e + t_i}{2}\ l = c\ t_0\ l$.

However, End *A* of the arch is fixed and, therefore, there will be a reaction at that abutment of the same nature as that caused by the hydrostatic pressure. In other words, since Point *A* will not rotate, the reaction at that abutment will act along the *X*-axis and to the right (Point *G* being at the elastic center of the arch). The value of that reaction will be, $H_t = \frac{c\ t_0\ l}{I_x}$, which, after some substitutions and transformations, becomes: $H_t = E\ c\ t_0\ \frac{d^3}{6\ \delta\ r^2}$, in which, all the letters have the usual meanings, and,

---

[23] "Statica delle dighe per Laghi Artificiali," by C. Guidi.

$$\delta = \frac{s}{l} + \frac{r - f}{r} - \frac{2\ l}{s} + \frac{d^2}{6\ r^2}\left(2\ \frac{s}{l} - \frac{r - f}{r}\right)$$

Concerning the second part of the deformation, it is observed that if End A were free, then it, together with the elastic center, G (assumed to be rigidly connected with A), would rotate through an angle given by Equation (4), when that equation is extended to the entire arch, that is: $\Delta\phi = c\ \Delta t\ \Sigma\ \frac{\Delta s}{d}$.

At the same time, Center G would be displaced in a direction, the components of which, along the X and Y-axes, respectively, are:

$$\Delta x = c\ \Delta t\ \Sigma\ \frac{\Delta s}{d}\ y \quad \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

and,

$$\Delta y = c\ \Delta t\ \Sigma\ \frac{\Delta s}{d}\ x \quad \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

Since d is constant for the entire arch, the center of gravity of the arch will coincide with the elastic center, G (center of gravity of the elastic weights of the entire arch). In fact, the elastic weight of an element of the arch is given by $dG = \frac{ds}{E\,I}$, in which, $I = \frac{1}{12} \times 1 \times d^3$ = a constant; therefore, dG is proportional to ds; and the two centers of gravity of the arch will coincide. Because of this fact, Equations (5) and (6) will give:

$$\Sigma y\ \Delta s = \Sigma x\ (\Delta s) = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots (7)$$

and,

$$\Delta y = \Delta s = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

Equation (8) reveals that, during the second part of the deformation of the arch to $C_1\ E_1$ (Fig. 15), if End A were not fixed, it would rotate about the elastic center, G. However, End A is fixed and, therefore, it will develop a reaction which, according to the theory of the ellipse of elasticity, will be a force infinitely small and infinitely far from Abutment A; that is, a couple, M. Since, according to the theory of the ellipse of elasticity, $\Delta\phi$ is always equal to $G\ M$, the rotation will be given by:

$$M = E\ \frac{\Sigma d\ (\Delta\phi)}{\sum \frac{\Delta s}{I}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (9)$$

In the presence of Equation (4), with d constant:

$$M = \frac{E\,I\,c\,\Delta t\,\Sigma\,(\Delta s)}{d\,\Sigma\,(\Delta s)} = \frac{E\,I\,c\,\Delta t}{d}$$

but, since $I = \frac{d^3}{12}$:

$$M = \frac{E\,c\,\Delta t}{12}\,d^2 \qquad \text{.........................(10)}$$

Thus far, this discussion has been concerned with an analysis of stresses due to a temperature change as it applies to the conclusions offered by Messrs. Karpov and Templin. Next, consider the nature of the deformation caused by the foregoing temperature stresses.

As stated previously, the effect upon the dam of that part of change in temperature, which has been considered as uniform (to $C_2\,E_2$, in Fig. 15), is perfectly similar to the effect upon the dam caused by hydrostatic pressure. As in the latter case, the unit shortening of the geometrical axis of the arch is given by $\frac{p\,r}{E\,d}$, in which, p = hydrostatic pressure, and r = radius, so, in the case of a uniform change in temperature, t_0, the same axis will be translated a unit distance, caused by $\pm\,c\,t_0$.

Therefore, if the equation for the deflection of the crown of the elemental arch of the dam, caused by hydrostatic pressure, is:[24]

$$\Delta f_p = \frac{p}{E}\,\frac{r}{d}\,f\left\{1 + \frac{1}{\delta}\left[2\,\frac{l}{s} - \frac{l^2}{4\,f\,r}\left(1 - \frac{d^2}{6\,r^2}\right)\right]\right\} \qquad \text{......(11)}$$

and if $c\,t_0$ is substituted for $\frac{p\,r}{E\,d}$, the final result will be:

$$\Delta f_t = \Delta f_p\,\frac{E\,c\,t_0\,d}{p\,r} \qquad \text{......................(12)}$$

which is the formula for the vertical deflection of the crown of the arch, caused by the uniform change in temperature.

For the second part of the deformation (to $C_1\,E_1$, in Fig. 15), the deflection of the crown of the arch is zero. In fact, imagining, again, that End A of the arch is free, the two sections, CE and DF (Fig. 15), would rotate, with respect to each other, at an angle, $d\,(\Delta\phi)$, as given in Equation (4); but End A is fixed, which creates a couple that acts upon every element of the arch; and the value of its moment is given by Equation (10). Furthermore, the rotation of the two sections, CE and DF, is given by:

$$d(\Delta\phi) = \frac{M}{E\,I}\,\Delta s = \frac{\frac{E\,c\,\Delta t\,d^2}{12}}{E\,\frac{d^3}{12}}\,\Delta s = \frac{c\,\Delta t\,\Delta s}{d}$$

which expression coincides with Equation (4), and demonstrates that the elastic deformation caused by the reaction at End A is equal and opposite to the thermal deformation. This fact is very important because it demonstrates that, although stresses are developed, the geometrical axis of the arch does not deform.

Obviously, models are useless for the determination of these last stresses; on the other hand, while theory shows that those stresses may be very important, the assumptions upon which the theory is based are known to be uncertain. At first, this phenomenon may appear inconsistent; but if a cube of concrete is imagined as held firmly between the plates of a compression machine, and if by some means the temperature of the cube is increased, the concrete is then known to be subjected to considerable compression, whereas its height is not changed.

By their conclusions, the authors have given an additional warning to the profession; and they have made a worthy contribution to the literature on dams.

P. Wilhelm Werner,[24] Assoc. M. Am. Soc. C. E. (by letter).—Many important points pertaining to the design and construction of high dams, and to the application of tests on models of such dams are contained in this paper. Under "Measurements Made on the Model (Temperature Measurements)", the authors state that for the model "it was comparatively simple to keep the temperature variations within such narrow limits that it was not considered necessary to introduce temperature corrections." Referring to Similarity Condition 8,

$$T_m = T_p \frac{c_p}{c_m} \quad \text{.........................} (13)$$

it would be of value to know in what manner the temperature difference, T_m, for the model was, and, for future measurements, will be, controlled in the special case under consideration. Even if it may be fairly safe to conclude that the coefficients, c_p and c_m, are sufficiently uniform to be assumed constant, it seems rather difficult to make the temperature difference, T_m, comply with Equation (13) at a sufficient number of points, when considering the irregular distribution of temperatures that occurs in a structure the size of the prototype. Because of the different thermal conductivity, and probably also, the different emissivity, of the materials in the prototype and the model, it would scarcely prove adequate to control only the temperatures of the media surrounding the model.

For a prospective dam there would appear a further complication in that the distribution of the temperatures in the prototype is not known beforehand. This latter obstacle, however, will probably be overcome in the future through the knowledge obtained by the increasing number of temperature measurements on existing dams and on dams in course of construction.

Similarity Condition 8 will be of importance especially when it comes to determining the effect of the progressive dissipation of the chemical heat in the concrete, and of the changes in the climatic conditions. It is well known (although probably not always fully realized even in the most modern dam designs) that an irregular distribution of the temperature may have a marked influence on, and may even be the dominating factor in, the elastic behavior and stress conditions of a structure of the type in question.

---

[24] Civ. Engr., Stockholm, Sweden.

The state of stress in an arch dam due to an irregular distribution of the temperature is very complex in its nature. The behavior of an elementary arch subjected to a differential temperature with a linear distribution between the faces has been referred to elsewhere.[25] Another aspect of the problem is demonstrated in Fig. 16, representing a vertical element of an arch dam with triangular cross-section. Assume that, with the mean temperature kept constant, a certain difference in temperature (ΔT) is introduced between the up-stream and down-stream fibers. (Note that this is not the same as the difference in temperature between the media surrounding the dam, as there will be a drop in temperature also along the contact surfaces between the media and the dam as shown in Fig. 16(*b*)). Whatever the law of transmission of temperature between the up-stream and down-stream fibers may be, it is evident that the difference in temperature will cause a tilting of the cantilever. If $\Delta T < 0$, that is, if the up-stream fiber is cooler than the down-stream fiber, the tilting will be in the up-stream direction, which means that the horizontal arches will be stressed in tension and, probably, the vertical joints open up (if they are not already open). This, in turn, means that the cantilevers will be required to carry a still greater part of the load than otherwise, before the arches take any load. Theoretical considerations, based on some simplifying assumptions as regards the distribution of temperature, will show that this condition may be of great importance in practical cases.

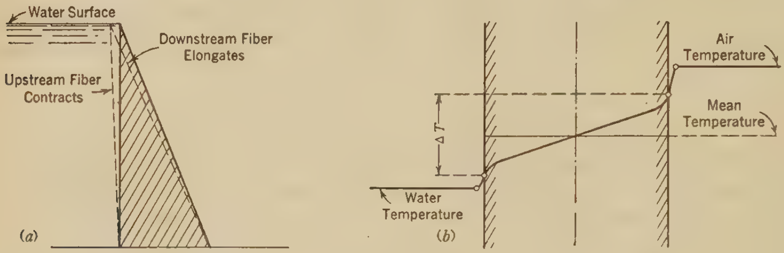


FIG. 16.

When analyzing the results of the measurements on the prototype, it is well to remember the uniform and ordinary elastic properties of the concrete used (see Appendix I of the paper). That the cantilevers in an arch dam built of this material should have tension on both sides simultaneously, due to water load, is very improbable. On the other hand, it is quite possible that an irregular distribution of the temperature, or irregular volumetric changes due to shrinkage, etc., may cause tensile stresses on both sides of a cantilever simultaneously, and these stresses may be of such magnitude that they will outweigh those due to water load and dead load.

---

[25] "Die Staumauern", by N. Kelen, Berlin, 1926.

As far as the writer is aware, the phenomenon of tensile stresses occurring simultaneously on both sides of the cantilever was definitely observed by the authors for the model only. In this case, the question enters as to whether an irregular distribution of the temperature was responsible to some degree for the unexpected results of the measurements. If, on the other hand, the explanation should lie wholly with some peculiar elastic or plastic properties of the model material, then the results obtained when using this material for model study of arch dams may be questioned. This point is of considerable importance, and it would seem desirable that the authors make an attempt to present some theoretical analysis for the explanation of the phenomenon.

That the cantilevers in a concrete arch dam should have excessive tension on the down-stream face due to water load, may also seem unusual and is considered unreasonable by some engineers. In the writer's opinion, however, this condition does not seem impossible although the magnitude of the tensile stresses found experimentally by the authors is rather startling. The so-called cantilever stresses may be looked upon as bending stresses in vertical beams which are supported by an elastic medium constituted by the horizontal arches. It is easily conceivable that under certain conditions contraflexure may occur in the upper parts of the beams. This may be elucidated by another loading case, which is similar in principle, but which is much simpler and, therefore, much more easily and clearly realized than an arch dam. Such a loading case is shown in Fig. 17(*a*), representing a cylindrical vessel subjected to hydrostatic pressure.

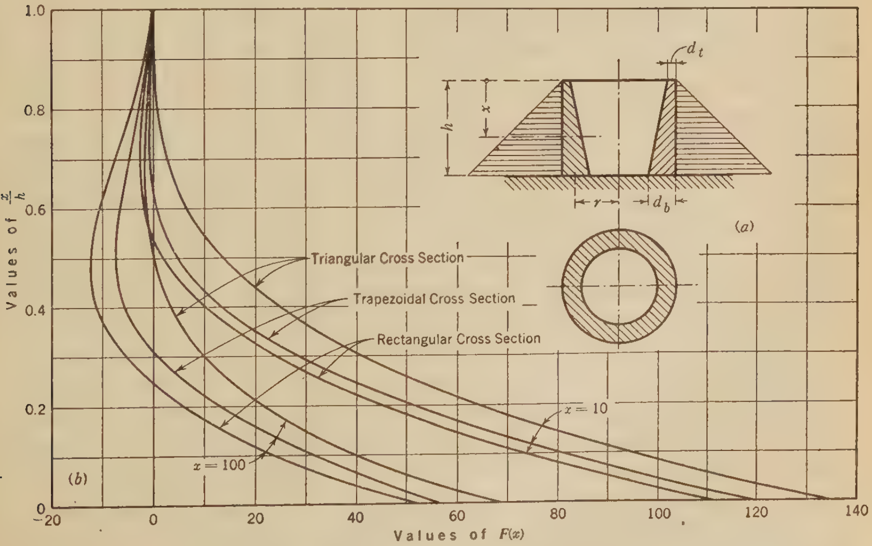


FIG. 17.—CANTILEVER BENDING MOMENTS IN A CYLINDRICAL VESSEL SUBJECTED TO HYDROSTATIC PRESSURE, $M_x = a\ F(x)\ h^3$.

The upper edge of the vessel is free like the crest of the arch dam, whereas the lower edge is entirely fixed so as to make the cantilever action more pronounced. Evidently, cantilever tensile stresses on the inner side of this vessel could not be expected any more than on the down-stream face of an arch dam. Nevertheless, it is easily shown that such stresses may occur in the vessel under certain conditions.

The problem of finding the stresses in the cylindrical vessel (bending in the vertical cantilevers and direct stress in the horizontal arches) may be solved, with sufficient accuracy, by a method developed by Dr.-Ing. Th. Pöschl[26], and based on the principle of least work. Pöschl shows that the elastic behavior of the cylindrical vessel is dependent to a considerable degree upon a dimensionless quantity:

$$\kappa = \frac{12\, h^4\, (1 - \mu^2)}{r^2\, d_b^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)$$

in which (see Fig. 17(*a*)), h = the height of the vessel; r, the mean radius of the vessel; d_b, the wall thickness at the bottom of the vessel; and μ, Poisson's ratio.

The bending moment in the cantilevers may be written,

$$M_x = a\, F(x)\, h^3 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (15)$$

in which, a is a constant and $F(x)$ is a function of x. This function has been determined by Pöschl for triangular, trapezoidal ($d_t = 0.5\, d_b$,) and rectangular cross-sections, and is graphically represented in Fig. 17(*b*) for $\kappa = 10$ and $\kappa = 100$.

Fig. 17 is interesting inasmuch as it shows that, under certain conditions, considerable positive moments may occur in the cantilever, causing extensive tensile stresses on the inner face of the vessel. The writer is fully aware of the fact that the arch dam is different from the vessel in that the former has bending moments in the horizontal arches, whereas the latter has no such moments; but, nevertheless, the character of an arch dam (especially the part near the crown) is quite comparable with that of the cylindrical vessel. Consequently it would seem rather natural, under similar conditions, to find cantilever tensile stresses at the down-stream face of an arch dam due to water load. The general dimensions of the Calderwood (arch) Dam also imply that a minute analysis of the dam would probably show this tendency, under the assumption that the dam would act as a monolithic structure.

However, as the grouting of the vertical joints in the dam probably is less than 100% effective, it is scarcely to be expected that the dam will act as a monolithic structure. The vertical elements will then, to a greater extent than otherwise, act as true cantilevers, with compression or decreased tension on the down-stream side, and tension or decreased compression on the up-stream side. A yielding foundation under the base of the dam would probably have the reverse effect, and thereby would cause an increase in the tensile stresses on the down-stream side. The relative effect of these two factors depends, of course, upon local conditions.

---

[26] "Armierter Beton", by Th. Pöschl, Berlin, 1912, p. 169.

As indicated in the foregoing discussion, the stress distribution may also have been influenced by other factors, namely, temperature changes, swelling, shrinkage, etc. The authors state, that all time effects were excluded from the investigations. In this connection it would be of interest to know more exactly the relative dates of the closing of the dam, the filling of the reservoir, the taking of measurements, and the temperature variations during these periods.

In Appendix I, the authors state that the material in the prototype follows Hooke's law very closely and that it gives the impression of being a very uniform product. Under these conditions the distribution of the stresses could not depart very much from a straight line in the upper part of a triangular element loaded with hydrostatic pressure up to the crest. However, in the cantilevers of an arch dam there are several factors that will cause a redistribution of the stresses: The cross-section may not be exactly triangular; the distribution of the part of the load taken by the cantilevers will be far from the shape of the hydrostatic pressure; etc. The effect of these factors is difficult to determine; but they are of minor importance when the apex angle of the cross-section is comparatively small, so that the "depth" of the cantilevers (base width of dam) can be considered as small in relation to its "span" (height of dam) at any given elevation. In view of this the writer believes that for the upper parts of the Calderwood Dam the assumption of a straight-line distribution of stresses, due to water load, would not involve great uncertainties and, in any case, would cause far less discrepancies between actual and calculated stresses than, for instance, disregarding the stresses due to an irregular distribution of the temperatures.

ELDRED D. SMITH,[27] JUN. AM. SOC. C. E. (by letter).—A clear and concise statement of the similarity conditions involved in the problem of arch dam model analysis is presented in this paper. It is apparent that the authors are attempting to make a model test by which the action of the prototype can be predicted from the action of the model by applying a single factor of conversion. In attaining this result, it was necessary to violate some of the similarity conditions in order to satisfy others, which may have considerable effect on the final similarity of their results.

In their similarity conditions, the authors state (and go to considerable pains to prove) that the specific gravity of the model and its loading liquid must be in the same proportion as in the prototype. In other words, for a water loading, the specific gravity of the model must be 2.4. However, the manner in which this much emphasized property is used is rather obscure. Their results contain no reference to dead load stresses or deflections which are the only stresses or deflections affected by the specific gravity of the structure. Theoretically, this specific gravity requirement is quite desirable, but in order to utilize it, it presupposes the ability to measure simultaneously the dead load strains along with the live load strains as a single quantity. If this cannot be done the advantage of the specific gravity requirement is lost.

---

[27] Care, U. S. Bureau of Reclamation, Denver, Colo.

In proving the specific gravity requirement the authors state a condition of the principle of superposition:

> "Any increase or decrease in the loading will cause a proportional increase or decrease of the stresses and deflections, provided the deflections are sufficiently small."

The principle in general states that: "The resultant effect of all forces acting on a structure is equal to the sum of the component effects of each force." If the first-named condition holds, the stresses and deflections produced in a structure by a given system of forces, is independent of the stresses and deflections produced by another set of forces. To illustrate, consider the section of a dam as shown in Fig. 18 (not drawn to scale), in which, W = the water load and D, the weight of the dam.

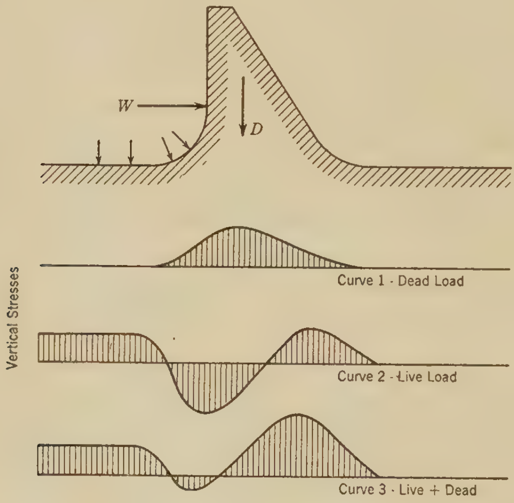


FIG. 18.

Curve 1 shows the stress at the base caused by the weight of the structure; Curve 2 shows the stress at the base produced by hydrostatic pressure; and Curve 3 shows the resultant stress, which is the sum of Curves 1 and 2. Curve 1 and Curve 2 are absolutely independent if the condition of superposition holds. Therefore, the stress distribution due to the water load on the structure is independent of the specific gravity of the material in the structure.

This fact has been proved by three separate experimental investigations of gravity dams on which the writer has been engaged. The models were made of a very light-weight material, but by properly distributed external loads, the equivalent specific gravity was increased to 2.4 × 13.6, or 32.64. The live load liquid was mercury, with a specific gravity of 13.6, so that the specified ratio of specific gravities was maintained. In each of these studies, the effect of the mercury pressure on the structure was the same, whether or not the dead loads were applied. In these tests it was possible to measure the effect due to dead load only, to live load only, and to the combined live and dead loads. Naturally, if the total stress only is to be measured, the theoretical proportion between live load and specific gravity of the structure should be maintained; but, if the dead load stress can be measured as a separate quantity, and the live load as a separate quantity, the experimenter will have considerable latitude in the selection of his model materials, providing the similarity of the elastic properties is maintained.

Consider a model arch dam of a scale, $R = \frac{1}{60}$, with reference to the prototype. The model to be used to predict the stresses in the prototype has a specific gravity of 2.4 and will be loaded with water. All other similarity conditions are assumed to be fulfilled. The ratio of stresses between model and prototype for dead load stresses will be:

$$\frac{1}{R} \times \frac{\rho_p}{\rho_m} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (16)$$

in which, ρ_p and ρ_m are the specific gravities of the prototype and model, respectively. Equation (16) with the specific gravities of the loading liquids substituted, will give the ratio for the live load stresses. Assume that strains can be measured from a zero or weightless condition (this involves many practical difficulties) so that both dead load and live load strains can be obtained. At a certain point assume that the strain measurements indicate a compressive stress due to dead load only of 10 lb per sq in. and a tensile stress due to live load only of 5 lb per sq in. The stresses in the prototype predicted from these measurements, applying Equation (16) to the model stresses, will be:

$$\text{Dead load} = 10 \times \frac{1}{R} \times \frac{\rho_p}{\rho_m} = 10 \times 60 \times \frac{2.4}{2.4} = 600 \text{ lb per sq in.}$$

and,

$$\text{Live load} = -5 \times 60 \times \frac{1}{1} = -300 \text{ lb per sq in.}$$

This gives a resultant total stress of 300 lb per sq in., compression.

Next, consider the same scale model as before, but loaded with mercury of a specific gravity of 13.6 and, in order to change the proportionality of weights of dam and loading, the material may have a specific gravity of 1.2, or one-half that of concrete. For the same point under consideration, by applying the principle of superposition, the dead load stress in the model will be one-half the previous value, or 5 lb per sq in., compression. By the same token, the live load stress must be 13.6 times that obtained before, or $5 \times 13.6 = 68$ lb per sq in., tension. These stresses will now be referred to the prototype, using the new constants of specific gravity:

$$\text{Dead load} = 5 \times 60 \times \frac{2.4}{1.2} = 600 \text{ lb per sq in., compression}$$

and,

$$\text{Live load} = -\,68 \times 60 \times \frac{1}{13.6} = -\,300 \text{ lb per sq in., tension}$$

This gives a resultant total stress of 300 lb per sq in., compression, the same result as obtained before. In this latter case, the specific gravity requirement, as specified by the authors, has been grossly violated, but the stresses predicted for the prototype are the same.

Thus, the question of the fulfillment of a definite specific gravity requirement rests on the ability of the experimenter to measure dead load stresses. It is obvious that to measure the stresses in a structure due to its own weight is a difficult matter. Under "Measurements Made on the Model: Strain Measurements", the authors state:

> "After the model was built, the zero readings, which included the dead load stresses, were taken. The full load readings were taken after the water load was applied. In this way the measured strains represent the strain or stress decrements due to water load only and correspond to the strain measurements on the prototype."

Although the reference to dead load stress given in this statement is not clear, it is plain that the results include only the effect of the water load. Therefore, from the principle of superposition, which states the independence of forces, the writer contends that the authors would have measured the same strains and deflections on their model, regardless of the specific gravity of the material, providing the elastic constants of their material remained the same.

If an attempt to measure dead load stresses in an arch dam model is made, there are two methods which may be followed, without applying external dead loads. The first method is to mount strain-gauges on the layers of material as the model is built so that they will measure the accumulated strain as the height of the model increases. This process would require considerable time, and the accuracy of the results would be affected by: (1) Changes in temperature during the time of construction; (2) the shrinkage or volume changes of the cementing material; and (3), especially in rubber compounds, by the time yield or plastic deformation of the material. Furthermore, if this method were used on the Calderwood model, the strain would be considerably affected by the jacking of the foundation. The second process of determining dead load stresses is a reverse of the first. When all the tests on the model are completed, gauges are mounted on the faces of the model and their change in deflection noted as the model is removed, layer by layer. These readings will be subject to some of the inaccuracies mentioned previously. The time effect will be less and the shrinkage of the cement will have become constant. The latter method was used on a rubber-litharge model on which the writer was engaged.

The apparent goal of the authors was to obtain complete similitude between the action of the model and its prototype; but if the model is to be used as a step in the final design of the structure, this may not be desirable, especially in regard to the construction joints. High tensile stresses in any direction are undesirable in a dam, and the design should eliminate them as far as possible. Therefore, it is more useful to construct the model as a monolith so that no tension will be relieved by the construction joints. In this way, the strain measurements will indicate the areas of tension, and, if necessary, a re-design of the structure may be made to eliminate them. If the model is not constructed as a monolith, the joints will be free to open to relieve tension; therefore, the strain-gauges, unless they should be placed across a joint, would not locate these tension areas. If the gauge were placed across a joint, there would be a question as to how much of the deflection was an actual strain and how much represented a crack opening.

An interesting comparison between two model tests which involved a check with the mathematical theory of arch dams was developed in the laboratory where the writer was engaged. An arch dam model was constructed of a mixture of plaster of Paris and diatomaceous earth and was loaded with mercury to determine the live load stresses and deflections. The trial-load method of analysis was used to determine the distribution of live load between arch and cantilever elements which would produce the measured deflections. Then, as a separate experiment,[28] a model of the crown cantilever alone was constructed to the same scale and of the same material as the model previously tested. This model was loaded by weights with the component of the mercury load which was computed to act on the cantilever element. It was found that this load produced practically the same deflection of the cantilever section as the full mercury load produced in the complete model. The slight difference was accounted for from the larger foundation used in the cantilever model and its freedom from lateral restraint. The most interesting point in this second test was that the cantilever model was loaded artificially by weights acting through pins in the model to produce an equivalent specific gravity of 32.64, so that the relationship specified by the authors was fulfilled; yet the live load deflection of this model checked the deflection of the complete model which was loaded with mercury only.

To summarize this discussion a few points of importance in relation to model studies are offered:

1.—In two-dimensional stress, the stress distribution is independent of the elastic properties of the material, providing Hooke's law holds. Therefore, any elastic material is theoretically suitable for models under two-dimensional stress.

---

[28] "An Investigation of the Crown Cantilever Section of the Boulder Dam by Means of a Plaster-Celite Model", by Charles W. Fletcher, Jun. Am. Soc. C. E.; thesis presented to the Univ. of Colorado in partial fulfillment of the requirements for the degree of Master of Science.

2.—In three-dimensional stress, the stress distribution is not independent of the elastic properties of the materials, but is dependent on the volumetric changes of the material under stress. This requires that Poisson's ratio be the same for both model and prototype if the same distribution of stress is to be obtained.

3.—If the effect of external loads only is to be investigated, the specific gravity of the material of the model is not a factor.

4.—If the static effect of the weight of the structure itself is to be measured, only the ratio of the specific gravities of the model and prototype need be known.

5.—If the combined effect of dead weight and external forces is to be measured as a single quantity, the specific gravity of the model and its external loads must be in the same proportion as in the prototype.

ELMER O. BERGMAN,[29] ASSOC. M. AM. SOC. C. E. (by letter).—The literature of model testing has been enriched by this comprehensive paper on the Calderwood Arch Dam, and the Engineering Profession is indebted to the authors for their efforts in making available such a complete record of their methods and results. The testing of models of dams is a relatively new field of engineering research and, therefore, it is important that the assumptions, theory, and methods which are proposed, be subjected to careful scrutiny in order that their validity may be established before they are accepted as the basis for further work in the field.

The first of the questions which the writer wishes to raise has to do with the authors' insistence on Similarity Condition 2 which they state, as follows: "The ratio of the specific gravities of the material of the prototype and its loading fluid, $\frac{\rho_p}{\gamma_p}$, must be equal to the same ratio of the model, $\frac{\rho_m}{\gamma_m}$."

If the deflections and stresses to be determined are those due to combined dead and live load, the writer agrees that this is a necessary condition. It would require that the zero readings be taken on a weightless dam and the final readings after the weight of the dam and the water load have been applied. The authors point out the difficulty of such a procedure. If the strain and deflection measurements are to represent the effect of water load only, then the dimensional equation[30] reduces to:

$$\delta_p = \psi' \, (\gamma_p, E_p, g, l) = l_p \frac{(\gamma_p \, g \, l_p)^u}{E_p} \quad \ldots\ldots\ldots\ldots\ldots (17)$$

in which, as before, the exponent, u, $= 1$ and $\delta_p =$ the deflection of the prototype; $\psi' =$ an unknown dimensionless function; $\gamma_p =$ mass per unit volume of the loading liquid of the dam (water); $E_p =$ modulus of elasticity in flexure, of the material of the dam; g = acceleration due to gravity; and l = length scale, so that,

---

[29] Assoc. Prof. of Civ. Eng., Dept. of Civ. Eng., Univ. of Colorado, Boulder, Colo.

[30] "Theory of Similarity and Models", by Benjamin F. Groat, M. Am. Soc. C. E.; Discussion by A. V. Karpov, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 311.

$$\delta_p = \frac{\gamma_p \, g \, l^2_p}{E_p} \qquad (18)$$

and,

$$\frac{\delta_m}{\delta_p} = \frac{(l_m)^2}{(l_p)} \frac{\gamma_m}{\gamma_p} \frac{E_p}{E_m} \qquad (19)$$

without any requirement that $\frac{\rho_m}{\rho_p} = \frac{\gamma_m}{\gamma_p}$.

The law of superposition may be used to prove that Similarity Condition 2 must be satisfied in order that correct values of combined dead and live load may be obtained. It can not be used to prove that the maintenance of the specific gravity is necessary when only live load effects are to be determined. On the contrary, the law of superposition may be used to prove just the opposite; namely, that any ratio of $\frac{\rho_m}{\rho_p}$ may be used in determining the effect of live load, as long as the condition that the deflections are small is not violated.

A dam is subjected to two different systems of loading. One is the weight of the dam itself, which is proportional to the specific gravity of the material of the dam, and the other is the pressure of the loading liquid, which is proportional to its specific gravity. These systems may be compared to a load, P, at a point, A, in a beam and a load, Q, at a point, B. The deflections and strains in the beam can be measured by taking zero readings before either load is applied and final readings after both loads are applied. Obviously, in accordance with Similarity Condition 2, the deflections and strains due to P and Q could be obtained by applying the loads, $2P$ and $2Q$, and then dividing the measured results by two.

If, however, loads of $2P$ and $3Q$ are applied and, as before, zero readings with both loads off, and final readings with both loads on, the beam are taken, there is no way in which the readings can be adjusted to yield the deflections and strains that are produced by the loads, P and Q. On the other hand, if zero readings are taken with both loads off, intermediate readings with the load, $2P$, on, and final readings with both the loads, $2P$ and $3Q$, on the beam, the effect produced by P and Q is equal to one-half the increment of reading, due to $2P$, plus one-third the increment due to $3Q$. The argument is the same as that used in the proof of Maxwell's theory of reciprocal deflections. There is nothing unique about the action of a dam that would serve to exempt it from obeying such a universal principle.

It is difficult to see how Fig. 1 proves anything more than that Similarity Condition 2 holds for combined loads. The diagram itself is likely to be misleading. The area between the solid line, AB, and the dotted line, AB, represents the vertical stress at the foundation line due to an unstated percentage of the dead load. If the vertical distances between the two curves are plotted on a horizontal base line, the curve shown in Fig. 19 is obtained.

This would indicate that the dead load weight of the dam produces tension along the down-stream part of the foundation line, which is unusual to say the least.

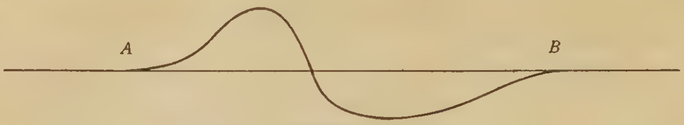


FIG. 19.

The writer also questions the validity of the second reason for considering the relative weights of the loading liquid and the dam material. Provided the deflections remain small, it would seem that doubling the density of the material in the dam would cause a uniform 2:1 increase in the foundation reactions along the entire periphery of the dam.

The third reason seems to be well taken. However, the same change from tension to compression may occur when there is such a marked increase in Poisson's ratio as that from 0.15 to 0.50.

The authors next make the statement (see "Conditions Governing the Properties of the Model Material") that,

> "The only conclusion that can be derived from a model of a gravity dam, in which the ratio of the specific gravity of loading liquid to the material is many times higher than in the prototype, is that every gravity dam will fail."

By what process do they arrive at such a conclusion?

To the ten specifications for a material suitable for constructing a model should be added the requirement that the relation of the tensile strength to the compressive strength should be the same as for concrete. This requirement is of major importance.

The authors do not give sufficient weight to the condition that the value of Poisson's ratio should be the same for the material of the model and the prototype. Such easy acquiescence in regard to a factor of major importance is not in keeping with their insistence on the maintenance of the specific-gravity ratio. It is true that a small divergence might be passed over as insignificant in its effect on deflections and stresses, but can such a difference as that between 0.15 and 0.50 be passed over lightly? Moreover, it should be noted that a material with a Poisson ratio of 0.50 presents a special case. For such a value of the Poisson ratio the material deforms with no change in volume, so that if it is compressed at one point, it must expand at another. The deflections, stresses, and strains in a model built of such a material will differ markedly from those in a model built of a material that will undergo considerable change in volume under load, except perhaps for a dam of thin section. It seems doubtful whether reducing the strains in a rubber model to stresses by using a value of Poisson's ratio of 0.15 will give stresses that are even indicative of those in the prototype, as the authors suggest in connection with Figs. 11 and 12.

In connection with the "Uncertainties of the Dam Proper," it is stated that the time interval between the placing of the concrete and the application of the water load causes rather high internal tensile stress to be developed in the blocks of the prototype. Furthermore, the authors state that when the water load is applied, the blocks undergo a contraction under compression which is greater in a block that has a preponderance of internal tensile stress than in one without such stress. Will the authors explain these statements?

In testing the model an unknown factor was introduced by the jacks that were used to change the position and shape of the model and its foundation. The jacks were adjusted until deflection readings at a considerable number of stations were obtained, which corresponded closely with the deflections measured on the prototype, as shown in Fig. 8(*c*). It is then assumed, at least implicitly, that a reasonably close correspondence will exist between the strains and stresses in the model and in the prototype.[31] It is interesting to check the validity of such an assumption when applied to a simply supported beam. Consider a beam, *A*, loaded with a total uniformly distributed load of 5 600 lb and a similar beam, *B*, loaded with a central load of 3 500 lb. The ratio of the deflection of Beam *B* to that of Beam *A* is 1 at the center line and $\frac{55}{57}$ at the quarter-points, while the ratio of the extreme fiber stress is $\frac{5}{4}$ at the center line and $\frac{5}{6}$ at the quarter-points. If loads of 3 200 lb are placed at the quarter-points and a load of 1 300 lb at the center line, the deflections are the same as those of Beam *A* at the quarter-points as well as at the center line; but now the ratios of the stresses in Beam *B* to those in Beam *A* are $\frac{45}{21}$ at the center line and $\frac{11}{6}$ at the quarter-points. Evidently corresponding deflections at five points along the beam do not produce corresponding strains and stresses. There must be correspondence at a sufficient number of points in order that the loads on Beam *B* (model) shall become approximately equivalent to the load on Beam *A* (prototype). Whether the number of points shown in Fig. 8(*c*) is enough to insure reasonable correspondence of strain and stress values between the model and its prototype is at least debatable, particularly in view of the lack of agreement shown in Figs. 11 and 12.

The sum of the horizontal and vertical strains at a point is equal to the sum of the strains in the 45° and 135° directions. This relation may be applied to the data in Table 2 to check their consistency. Deviations may arise from the fact that the measured strains are the average over the gauge line instead of the strain at the point where the gauge lines intersect, and to instrumental and personal errors. Table 2 shows reasonably good agreement. The agreement is better in the model than in the prototype.

With regard to Conclusion (7), the writer believes that a material composed of a mixture of plaster and diatomaceous earth and having a low modulus of elasticity is a better model material than rubber, because its strength properties resemble more closely those of concrete. The chief advantage of rubber as a model material is that it enables strain measurements to be made on the up-stream face.

---

[31] See the authors' discussion of a similar case in "Actual Deflections and Temperatures in a Trial-Load Arch Dam", by A. T. Larned and W. S. Merrill, Members Am. Soc. C. E.; Discussion by A. V. Karpov and R. L. Templin, Members Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 99 (1934), p. 951.

DR. ING. E. PROBST[32] (by letter).—The intention of the authors to check the theoretical bases of the design of an arch dam by the use of a model is remarkable in many respects. During recent years a number of measurements have been made on small and larger models of different proposed structures. There are fields, however, in which no direct conclusions can be made based on such tests. Dams belong in that field because they represent structures, the design of which is influenced by the ground conditions to a much greater extent than any other kind of structure. The model can not represent, properly, the condition at the base and abutments of the dam.

In their Conclusion (1), the authors state that a design based on the assumption of non-yielding foundations does not come close to the actual conditions. Differences between the elastic properties of the bed-rock and the material of the dam will change the stress distribution at the base, and that change will increase with the increase of the height of the dam; but the same conclusion can be reached by theoretical considerations.

If, in order to make possible improvements in future designs, it is intended to determine the over-stressed and under-stressed parts of the dam, then the representation in the model of the connections between the dam and the abutments is of particular importance.

The most formidable difficulty in designing the model is the choice of the material, which should have properties different from those of the prototype. The relatively smaller scale of the model makes it impossible to use the same material as in the prototype. The elastic properties and the strength of the concrete are influenced to the greatest degree by the ratio of the different ingredients; these conditions cannot be properly represented even in a large sized model.

Even if it is perfectly mixed and placed, concrete is non-uniform due to the non-uniform distribution of particles of different size. It can be stated that in not a single vertical cross-section of a dam is concrete uniform from the top to the bottom, even if its composition is the same. If the attempt is made to simulate such a complex substance by substituting a uniform material in the model, the conditions of the model will be quite different from those that would be necessary to represent the actual dam. This difference will be in addition to that due to the impossibility of representing, properly, the conditions at the abutments. These considerations and the circular shape of the horizontal cross-section seem to be responsible for the non-uniformity of the horizontal stresses measured on the model. The distribution of horizontal stresses could be improved or could be made uniform by changing this shape.

---

[32] Karlsruhe, Germany.

Quite properly, the authors state in Conclusion (5) that to design an arch dam neglecting gravity action will result in an unwarranted increase in the thickness of the arch at the abutments; furthermore, the evaluated stresses will be quite different from the actual ones.

The authors compare the measurments made on the model with those made on the prototype. In many cases, the deflection measurements may be of some value as an addition to the strain measurements as, for example, in bridges or in structures in which the measuring apparatus is influenced by climatic conditions to a less degree than in dams. The writer knows of cases of deflection measurements on dams which, due to such influences, appeared so unreasonable that they could not be used. Furthermore, all the deflections were so small that, in many cases, even with the greatest possible precision, the small deflection changes could not be determined.

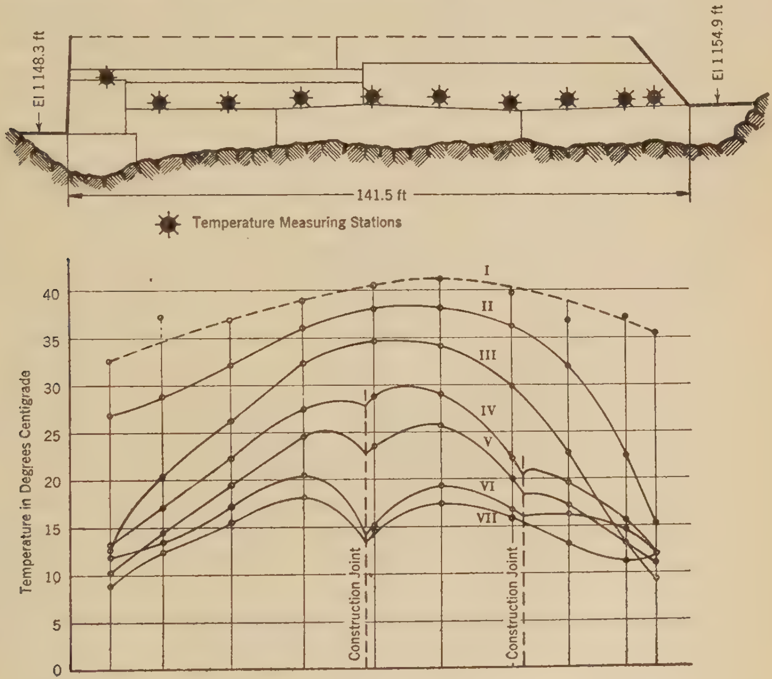


FIG. 20.

The influence of climatic conditions is most pronounced if the strain measurements are made at the faces of a dam as was the case in the measurements made by the authors. Such measurements cannot disclose the strains and stresses within the dam. Even if the measurements made at the faces gave the proper values of the strains, these measurements could not be used to support conclusions regarding the actual stresses in a cross-section of the dam.

All efforts to increase current knowledge concerning the bases of design, necessarily will lead to the necessity of making measurements on the structure itself. Of course, such measurements must be carefully planned based on previous experience. They must be painstakingly accurate, from the beginning of the construction work and must be continued as long as possible after the dam has been put in service. Such measurements can be made only from a distant reference point so arranged as to eliminate the influence of the weather on the measurements.

If it is intended to form a picture of the deformations and stresses within the dam, measurement methods must be used that will disclose the influence of all forces within the thickness of the structure. Besides the influence that is due to the outside and inside temperatures, the stresses at each point will be influenced to a considerable degree by the change in moisture content, due to the continuous setting of the concrete.

A detailed report of such measurements made on the Bleiloch Dam, in Thuringen, Germany (207 ft high), has been presented by the writer elsewhere.[33] At a depth of 197 ft below the crest of this dam, thirty telemeters were inserted at ten stations. Some new data on this subject are shown in Fig. 20, with supplementary information concerning temperature measurements, as follows:

Curve No. (see Fig. 20)	Date of measurements	Time elapsed, in years, since date of first measurement
I	May 15, 1931	0.0
II	October 10, 1931	0.5
III	April 29, 1932	1.0
IV	November 7, 1932	1.5
V	May 5, 1933	2.0
VI	November 6, 1933	2.5
VII	May 24, 1934	3.0

The strains, measured 10 ft above the ground-line joint, show that the changes in strain due to temperature influences are such that no conclusions can be drawn about the strains inside the dam, based on measurements made on the surface. It was shown also that the final conditions in a cross-section were not arrived at three years after the beginning of the construction and nearly two years after the dam was put into service. The final conditions will be reached after some time that depends on the thickness of the dam; but no matter what the conditions, no proper conclusions can be drawn concerning the distribution of stress inside the dam based on measurements made at the face.

The best guaranty of a safe and economical design is to create conditions under which the design assumptions can be controlled clearly and simply. This objective can scarcely be accomplished by model studies.

Comparative measurements on the actual structure should not be limited to surface measurements since the distribution of deformations throughout the cross-section are of the utmost importance as far as the value and location of the unfavorable or objectionable stresses are concerned.

Based on these considerations, the writer would like to suggest that, in the future, measurements be made on each large dam instead of on models.

---

[33] First International Conference on Dams, Stockholm, Sweden.

A. V. KARPOV[34] AND R. L. TEMPLIN,[35] MEMBERS, AM. SOC. C. E. (by letter).—Seven discussions were published applying to this paper, two of which came from abroad. The writers are much impressed with the exceptionally high character of each of these contributions. The different points developed in the discussion can be broadly summarized as follows: (I) The use of models in investigation of arch dam problems; (II) similarity conditions; (III) type of models; (IV) results of tests on Calderwood and other models; and, (V) design methods and theories. It will better answer the purpose of the closure to discuss each of these topics separately, rather than to consider individually the contribution of each discusser.

(I) The Use of Models in Investigation of Arch Dam Problems.—Two discussers, Mr. Janni and Dr. Probst, question the advisability of using models in the design of arch dams. Mr. Janni considers the temperature stress of utmost importance and does not believe that this stress can be measured on a model.

In a number of points the writers disagree with Mr. Janni. In so far as the theoretical computations are concerned, they are correct so long as the assumption can be made that there is a monolithic arch between fixed abutments on a non-yielding foundation. It seems that the usual simplified assumption of fixed abutments and a non-yielding foundation is quite firmly fixed in the engineering mind. For a number of structures such an assumption does not involve any considerable mistakes; but for an arch dam, consisting of a number of blocks between abutments, such simplified assumption leads to radically wrong conclusions.

Thinking in terms of the total deflections of an arch dam, the abutment, and particularly the foundation, deflections are large and the abutments and foundation should be considered as very flexible. Fig. 9 makes clear the comparative magnitude of the deflections of the foundations and the dam proper. Under such conditions the conclusions based on the formulas by Mr. Janni are incorrect. On the contrary, temperature variations may involve changes in deflections that are accompanied by insignificant changes in stress.

TABLE 4.—DATA PERTAINING TO TEMPERATURE CURVES IN FIGS. 21 AND 22

Description	Block 6	Block 10
Thickness of block, at temperature station, in feet	58	26
Elevation of temperature station above datum,* in feet	795	895
Concrete placed at temperature station	July 9, 1929	July 30, 1929
Concrete placed at adjacent blocks	July 26, 1929	October 17, 1929
Block exposed to water	April 15, 1930	April 17, 1930
Distances from Down-Stream Face of Block, in Feet:		
Test 1	31	14
Test 2	14	22
Test 3	2	..

* Datum is 122.62 ft above mean sea level.

Figs. 21 and 22, with Table 4, show the results of temperature measurements, extending over a period of nearly two years, on the Calderwood Dam.

---

[34] Designing Engr., Hydr. Dept., Aluminum Co. of America, Pittsburgh, Pa.

[35] Chf. Engr. of Tests, Aluminum Co. of America, New Kensington, Pa.

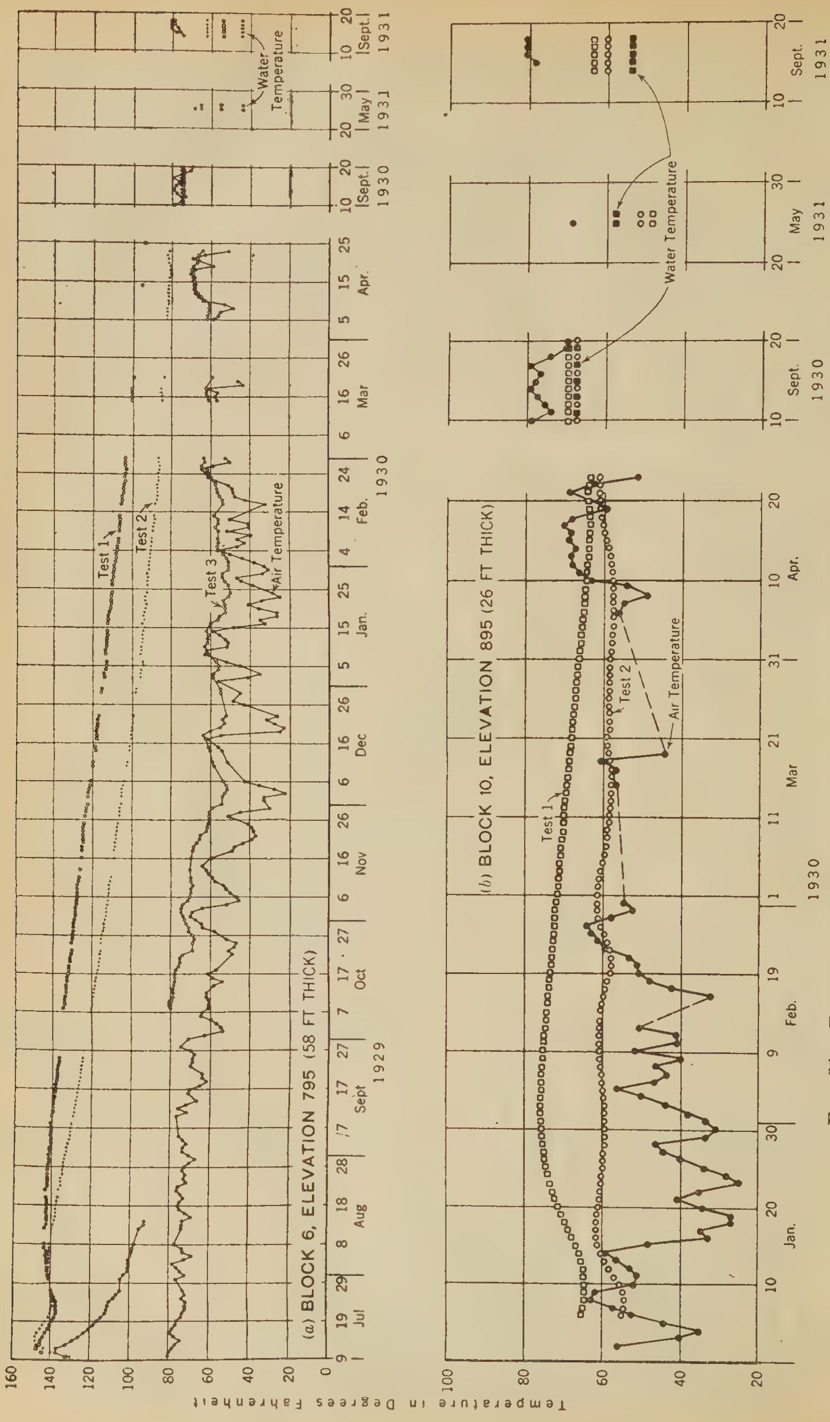


FIG. 21.—TEMPERATURE MEASUREMENTS, CALDERWOOD ARCH DAM; DAILY AVERAGES.

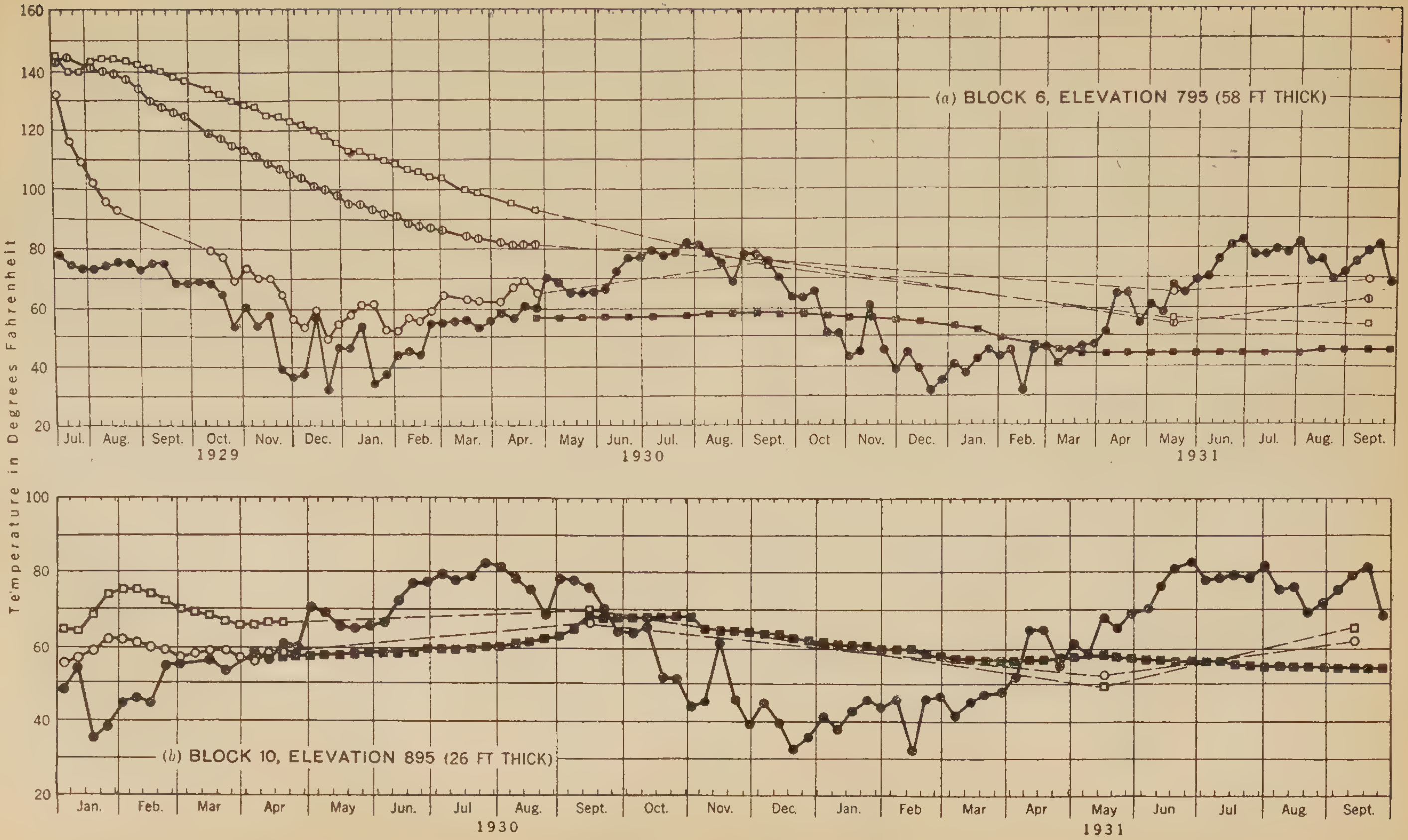


FIG. 22.—TEMPERATURE CURVES, CALDERWOOD ARCH DAM; WEEKLY AVERAGES. (FOR IDENTIFICATION OF CURVES SEE SIMILAR CURVES IN FIG. 21.)

During that time four sets of measurements were made; the first set was begun at the time of placing the concrete until, and including, April, 1930; after that, tests were made in September, 1930, May, 1931, and September, 1931.

Figs. 21(*a*) and 22(*a*) show the daily and weekly averages at Elevation 795 of Block 6 (see Fig. 8). The temperature measuring stations were located 31, 14, and 2 ft from the down-stream face of a block, 58 ft thick. Figs. 21(*b*) and 22(*b*) show similar temperature measurements at Elevation 895 of Block 10, the stations being located 14 and 22 ft from the down-stream face of a block, 26 ft thick. The time when the concrete blocks were completely surrounded by adjoining blocks, as well as the time since, when they were exposed to water at the up-stream faces, is indicated in Table 4.

A study of these temperature curves, as well as of temperature curves of any dam, will show that the concrete only a few inches from the surface does not undergo sudden temperature changes. On the contrary, the temperature within the dam changes very slowly; only a few feet from its face, the temperature changes are due principally to the latent heat of the concrete and its rate of dissipation, and to a much less extent to the changing air or water temperatures. Nevertheless, if any stresses are developed in the prototype, they can be simulated and measured on a model properly built and operated. Some method analogous to the one described in connection with Fig. 13 will disclose strain and stress irrespective of the actual deformation.

In his discussion Dr. Probst presented, quite ably, the general arguments against the use of models in arch dam design. The writers are in full agreement with a number of points that are brought out in his discussion, although they cannot agree with his conclusions. In conformity with the ideas expressed in the paper he states that the ground and abutment conditions are of paramount importance in so far as the behavior of the dam is concerned. Furthermore, he declares that these conditions cannot be properly represented in a model. That was one of the ideas on which models of dams were built in the past; to-day, however, it is clear that such a statement cannot be justified because the ground and abutment conditions, as well as the conditions of the dam proper, can be represented in a model. Fig. 8 of the paper, giving the comparative model-prototype deflections, is a striking proof of that statement. Investigators know better now than in the past how to represent the ground conditions in a model, and with further study the methods of ground representation will be improved.

As the matter now stands, there are no particular difficulties in representing the ground conditions. The main difficulty lies in determining the properties of the ground; and, in that respect, the work done during the last few years, particularly in Germany, in applying the vibration method for determining the properties of the ground, seems to be of cardinal importance. If such work on actual ground is continued and is followed by model studies, engineering science will dispose of one more mystery, and the ground material will be treated as every other material used in engineering.

As an argument against the use of models, Dr. Probst makes the statement that the stresses within the structure are of more importance than the surface stresses and only measurements at the dam itself can disclose the stresses within the structure. The writers scarcely can agree with that statement. First, the stresses within the structure are undoubtedly of importance and should be studied and determined; but why they should be more important than surface stresses is not clear. After all, considering the practical point of view, the deterioration of concrete starts at the surface and the surface stresses are at least as important as the stresses within the dam. Second, the stresses within the structure can be determined with considerable saving of time and much more reliably, if the measurements made on the dam are supplemented by a model study.

Dr. Probst considers also the long-discussed question of the non-uniformity of concrete. Of course, concrete is non-uniform to a greater degree than, for instance, many commercial metals; but, again, the concrete of to-day is much more uniform than the concrete of yesterday. It is rather surprising to what extent the uniformity of concrete can be improved by proportioning and mixing in a central plant with rigidly controlled ingredients, proper water-cement ratio, and the liberal use of vibrators. Concrete aggregates in large masses act as a uniform material even if the aggregates are non-uniform. The best possible illustration is probably the one taken from the ordinary commercially used metals. Every metal is composed of crystals that may have different elastic and pronounced directional properties. Nevertheless, so long as these crystals are distributed at random, but not preferably oriented, the material has a high degree of uniformity in pieces that are large when compared to the size of a single crystal. The elastic properties of such materials represent some average of the integrated values of the properties of the individual crystals. Proper proportioning, mixing, placing, and vibrating of the concrete will prevent any preferential orientation of the particles and in that respect a large mass of concrete probably is quite uniform, relatively speaking.

Dr. Probst arrives at the conclusion that since it is impossible to build a proper model of a gravity dam or an arch dam, no model tests should be attempted, but rather that measurements should be made on actual dams. The writers do not believe that measurements made on a prototype exclude the necessity of measurements on a model, or *vice versa*. From a theoretical viewpoint, measurements made on a prototype are the best. From a practical viewpoint, considering the magnitude of a dam, the inevitable limitations of such testing program and the impossibility of varying arbitrarily the loading conditions, such measurements are of relatively little importance and scarcely justify the time, work, and expense involved. These measurements, however, become of the utmost importance if accompanied by an adequate model study.

Taking as an illustration the measurements on the Bleiloch Dam mentioned by Dr. Probst: These measurements undoubtedly will give interesting results, but the practical application of these results is uncertain unless they are used in connection with a model study. Every investigation of this kind should disclose how improvements can be made and how safer dams can be built with the least possible material. Measurements made on a dam scarcely can give an answer to these questions unless supplemented by a model study.

Contrary to the opinions of the past, one cannot expect to use the results of measurements to develop an improved theoretical method of design. The question of to-day is not how to supplant one arbitrary set of assumptions by another just as arbitrary, but to find a method that will give results approaching the actual conditions and in this manner make possible an improved design.

Fig. 20 of Dr. Probst's discussion is of considerable interest, but does not support the contention that the stresses within the dam cannot be predicted from measurements on its faces. First, should be noted the unusual distribution (from the American viewpoint) of the construction joints. Second, the changes in the character of the temperature curves should be studied. Curves I, II, and III show the temperature before the dam was put in service and Curves IV, V, VI, and VII, after water was stored above the dam. The breaks in these latter curves are an indication that water circulated through the construction joints. Such circulation increased when the concrete blocks cooled and the size of the joints increased; this is manifested by the shape of the breaks of the temperature curves at the left construction joint shown on the diagram. If these joints were differently arranged and better grouted, no water would circulate through them and no breaks would have occurred in the temperature curves. Even under present conditions, however, after the concrete has cooled and the dam has attained its final service conditions, the breaks in the temperature curves should disappear. The regular shape of the temperature curve (Curve VII) at the right construction joint, is a rather convincing argument to that effect.

(*II*) *Similarity Conditions.*—There seems to be an agreement of opinion that the similarity conditions established in connection with the Calderwood model are complete and that a perfect model must meet every one of them. Mr. Smith is perfectly correct when he states that the writers took pains to prove the importance of the specific gravity requirements. It is rather discouraging, therefore, to the writers that this question is raised again, and attempts are made to minimize the importance of these conditions.

Considerable discussion comes from those who advocate the advisability of building an imperfect or only a partly correct model. In connection with such model, the relative importance of the different similarity conditions are discussed. Messrs. Simonds, Smith, and Bergman allege that the specific gravity requirements are of importance only if both the dead load and the live load stresses are measured, but that this requirement is of no importance if only live load stresses are measured and hence an imperfect model that neglects the specific gravity requirements is satisfactory for practical purposes.

The writers emphasize the dangers of the misapplication of the dimensional theory by the omission of a quantity that is difficult to handle, as was done by Professor Bergman in the derivation of his Equations (17) to (19).

By using such a method, almost any desired statement can be proved and any improper similarity condition can be established. In order to apply the dimensional theory properly, all quantities entering into the problem must be considered. Only quantities that are negligible in their effect can be omitted without appreciable error. The omission of such an important quantity as the weight of the material of the dam will result in similarity conditions applicable to a weightless dam. Such similarity conditions could be used if a dam is investigated, the material of which has a very small specific gravity as compared with the loading liquid. These conditions, however, cannot be used with reference to an actual dam, the material of which has an appreciable specific gravity.

Fig. 18 of Mr. Smith's discussion and his elaboration on the subject as well as Equations (17) to (19) of Professor Bergman's discussion and his Fig. 19 are all based on the fallacious assumption that it is admissible to evaluate, separately, the dead load and live load stresses and to arrive at the total stress by simply adding these stresses.

A proper understanding of the more complicated three-dimensional stress problems is possible only if the principle of superposition is properly applied. The simple addition of the gravity and live load stresses would be permissible if a straight-line distribution of stress could be assumed. In addition to Fig. 1, Fig. 23,[36] shows clearly the incorrectness of the straight-line assumption.

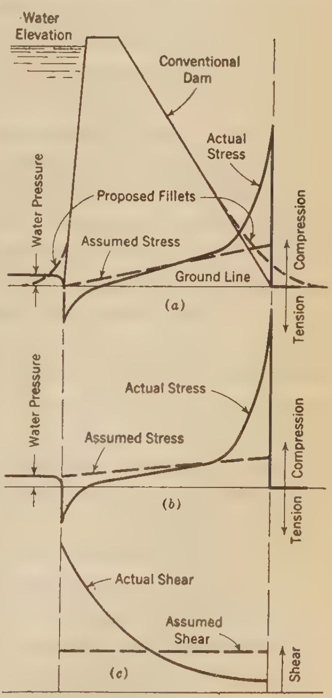


FIG. 23.—VERTICAL STRESS AND SHEAR AT THE GROUND LINE.

Professor Bergman states: "That a material with a Poisson ratio of 0.50 presents a special case." So far as a structural model is concerned, only a practically non-existent material with a Poisson ratio of zero could be considered as a special case. In such a material the application of stress in one direction would not produce strains or stresses in any other direction.

Due to the application of stress in one direction, any material having a Poisson ratio of 0.15 or 0.50 will induce strains or stresses in other directions, the only difference being in the magnitude of the strain or stress. In that connection it would seem that Professor Bergman's statement, "for such a value of the Poisson ratio [0.50] the material deforms with no change in volume, so that if it is compressed at one point, it must expand at another," is unfortunately worded, because there is no difference in the behavior of the material having a Poisson ratio of 0.15 or even 0.50. There is no question that "it must expand," but in both cases it will have the tendency to expand; if restrained from expansion both materials will develop stresses similarly, the only difference being in the amount of expansion and in the value of the induced stresses.

---

[36] "Stability of Straight Concrete Gravity Dams," by D. C. Henny, M. Am. Soc. C. E. Discussion by A. V. Karpov, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 99 (1934), Fig. 12, p. 1083.

All the discussers who stress the importance of Poisson's ratio and the non-importance of the specific gravity requirements apparently overlook the importance of the proper representation of the foundations, abutments, and joints in the model. Mr. Smith even goes so far as to make the statement that the representation of joints in a model is undesirable and advocates the building of models that intentionally violate this similarity condition. All of which brings the entire problem back to the fundamental question : What is a modern structural model?

(*III*) *Type of Models.*—There seems to be a considerable difference in opinion as to the purpose of structural models and what constitutes an up-to-date engineering model. It should be realized that present engineering knowledge is limited[37] and that in many structures a wide discrepancy exists between the actual stress in the structure and the stress evaluated by the use of conventional methods. This discrepancy is very pronounced in statically complicated structures, the design of which is based upon assumptions that were satisfactory in the design of simple structures. Consequently, a rapid change in the conception of the theory of stress is currently taking place. Many assumptions that were considered axiomatic a few years ago are now not only challenged but often discarded.

The major problem of modern engineering is to determine the correct design assumptions which will suit the particular structure under consideration. The next in importance is the selection of a proper mathematical treatment which will make it possible to evaluate the different phases of the behavior of the structure. The limitations of present mathematical knowledge makes it impossible to obtain a complete and exact solution of the differential equations set up by introducing the theory of stress. Consequently, the strains, deflections, and stresses can be evaluated only approximately. The discrepancy between the actual and the theoretically determined behavior of a structure depends upon the correctness of the design assumptions and upon the approximations involved in the particular mathematical method used.

A correctly designed model has the important advantage that it eliminates the approximations of the mathematical methods and gives results that depend only on the correctness of the design assumptions, thus making possible an independent solution of the major engineering problem.

The uncertainties regarding some of the conditions of an arch dam, and particularly with respect to the foundation conditions, are used by Mr. Simonds as an argument against the use of models and in favor of the substitution of a mathematical analysis as the basis of design. It should be realized that the same uncertainties that enter into the design, the building, and the testing of a model, also enter into the mathematical analysis of the prototype.

---

[37] "The Use of Engineering Models," by A. V. Karpov, M. Am. Soc. C. E., *The Military Engineer,* November–December, 1934.

If, for instance, the actual ground conditions are not known, some assumptions regarding these conditions must be made and applied equally to the mathematical analysis of the prototype and to the building of the model. Any reasonable design assumption can be represented in an adequate model; but there is this radical difference between the mathematical analysis and the model study: The uncertain assumptions upon which the mathematical analysis is based will be imposed upon the ground assumptions which are used in the mathematical analysis. Due to the uncertainty of the ground assumptions and of the mathematical method the final results will differ from actual conditions. The part of the discrepancy that is due to the uncertain ground assumptions may be small when compared with the part that is due to the currently accepted mathematical methods. On the other hand, the same ground assumptions, if properly incorporated in an otherwise correct model, will be the only uncertainty, and the discrepancy between the actual behavior of the prototype and the predicted behavior, based on the model study, will depend only upon the correctness of the ground assumptions.

It was not the intention of the writers even to suggest the elimination of the mathematical analysis. The results of proper model studies can, and should, be used to develop and check a reliable design theory which could be finally incorporated in a mathematical design method. The writers contend that a mathematical theory which is checked by an incorrectly designed model cannot be considered as satisfactory; only a theory that can be checked on a correctly designed model may be considered satisfactory and hence should be used.

By the use of models there are two methods of approach to the major problem. The first method, often used in the past, is to make a set of assumptions which may appear to be either more simple or more reliable, and thereby attempt to obtain proof of the more complicated or less reliable assumptions. In an arch dam investigation numerous assumptions of this type are possible. The mercury loaded plaster-celite model of an arch dam is based on a series of such assumptions, some of which are: (*a*) That the total stress can be determined from the summation of the dead load stress, determined without considering the hydrostatic pressure, and the live load stress, determined under zero weight conditions; (*b*) that the behavior of an arch dam model with fixed foundations and abutments is similar to that of a prototype arch dam with flexible foundations and abutments, so that tests made on a model with fixed foundations and abutments will determine the condition of a prototype built on flexible foundations and abutments; and (*c*) that a dam having joints will behave similarly to a dam that is built as a monolithic structure. This method is a simple one. The assumptions can be chosen so that no particular attention need be paid to any similarity conditions. The mercury loaded models are usually scalar representations of the prototype; but even that is not necessary and an arbitrarily shaped model could be used. However, it should be realized that this method does not give any proof of the accuracy of the assumptions made; it merely substitutes one more or less arbitrary set of assumptions for another.

The second method is based on the omission of all arbitrary assumptions. This is accomplished by the determination of the similarity conditions that will establish a natural relation between the behavior of the model and its prototype. By building a model that satisfies these similarity conditions and its testing, the actual behavior of the prototype can be determined and the correctness of the design assumptions can be checked. A concise definition of such an up-to-date engineering model is given by Mr. Arthur C. Ruge,[38] who states:

> "The true criterion is that it must be possible to deduce the behavior of the prototype, under an arbitrary set of conditions, from the behavior of the model when subjected to a similar and properly proportioned set of conditions."

Some of the discussers prefer mercury-loaded models because they consider the high value of Poisson's ratio of the rubber-litharge compound a serious drawback of the water-loaded rubber-litharge model.

Table 5 permits a comparison with respect to the specific gravities of the model materials used in the three kinds of models referred to in the discussions.

TABLE 5.—SIMILARITY CONDITIONS WITH RESPECT TO SPECIFIC GRAVITY OF THE MODEL MATERIALS

Kind of model	Specific Gravity		Ratio of actual to required weight (percentages)
	Required	Actual	
Mercury-loaded plaster-celite	32.84	0.7	2.1
Mercury-loaded concrete	32.84	2.4	7.3
Water-loaded rubber-litharge	2.4	2.4	100.0

Comparing the mercury-loaded, plaster-celite model with the water-loaded, rubber-litharge model, it should be realized that the comparison is to be made between a model having only 2% of the required weight and without a proper representation of the foundations, abutments, and joints, and a model having 100% of the required weight and built of a reasonably uniform material, with properly represented foundations, abutments, and joints, but having a deficiency in Poisson's ratio. The difference between these two kinds of models can be visualized clearly by comparing three dams of the same size—an actual dam and two imaginary dams. The first imaginary dam is assumed to be built as a monolithic structure, on foundations and abutments that are very many times as stiff as the actual foundations and abutments, and using a material that has only 2% of the weight of the concrete of the actual dam. The second imaginary dam is assumed to be built in exactly the same way as the actual dam, on the same foundations and between the same abutments, using a material that has the same properties as the concrete of the actual dam, except the high value of Poisson's ratio.

---

[38] "The Determination of Earthquake Stresses in Elastic Structures by Means of Models," by Arthur C. Ruge, *Bulletin*, Seismological Soc. of America, Vol. 24, No. 3, July, 1934.

The behavior of the first imaginary dam would be radically different from that of the actual dam, and it would be impossible to establish a rational inter-relation between the respective stresses and deflections of this imaginary dam and the actual dam. The behavior of the second imaginary dam would be substantially the same as that of the actual dam, the only difference being due to the different Poisson ratios. Applying secondary corrections, that will be necessary to account for the difference in Poisson ratios, it would be possible to compare directly the respective stresses and strains of the second imaginary dam and the actual dam.

In comparing the plaster-celite and rubber-litharge models a number of statements were made in the discussion concerning the uniformity of the plaster celite and the non-uniformity of the rubber litharge. There are no supporting data to confirm the statements to the effect that the plaster-celite material is uniform. It is known[39] that this material is unstable in so far as its elastic properties are concerned. The presence of moisture in the model material, or high humidity of the surrounding air, may result in pronounced plastic instead of the presumed elastic properties. Such qualities make the conditions uncertain. The statements regarding uniformity should be supported by results of tests properly made under conditions and on specimens that would assure that the test results are representative in so far as the material in the model itself is concerned.

The rubber litharge used in the Calderwood model was reasonably uniform. In so far as statements concerning the non-uniformity of the rubber-litharge compound of the Boulder Dam model are concerned, it should be kept in mind that this compound requires special technique in testing because if tested by conventional methods it is quite probable that incorrect results will be obtained.[10] The maximum value of Poisson's ratio of 0.64, stated by Mr. Simonds, makes the writers somewhat doubtful as to the reliability of the procedure and testing methods used.

The statement by Professor Bergman that "the chief advantage of rubber as a model material is that it enables strain measurements to be made on the up-stream face" is incorrect. The possibility of using strain-gauges on the water-covered face of the model is merely an incidental advantage, but it cannot be termed the "chief advantage." The chief advantage in using the rubber-litharge material is that measurements made on a water-loaded model will enable the prediction of the behavior of the prototype with satisfactory accuracy. No such predictions are possible where the measurements are made on a mercury-loaded plaster-celite model.

For future models it is very desirable to use a material that has a low modulus of elasticity, a specific gravity of 2.4, and a Poisson ratio between 0.15 and 0.20.[39] It would also be desirable to have a more uniform material, that is, with less than ±15% variation in the modulus of elasticity. Such refinements would improve future model work.

One of the most significant features of the work done on Calderwood Dam and its model is the proof of the important influence that the behavior of the ground and abutments exerts on the behavior of an arch dam. An arch dam cannot be considered as an isolated independent structure, but must be investigated as an integral part of the surrounding canyon. Considering the present status of the art, any model that neglects the proper representation of the foundation and abutments cannot be considered as an up-to-date model.

---

[39] "Materials for Use in Structural Engineering Models, with Special Reference to Rubber Compounds," by A. V. Karpov, M. Am. Soc. C. E., *Proceedings*, Am. Soc. for Testing Materials, Vol. 34, Pt. II, 1934.

Mr. Simonds asks a number of pertinent questions regarding the representation of the foundations in the Calderwood model. It was stated in the paper that the investigation of the foundation conditions was necessarily one of the objectives. In studying the Calderwood model, the writers had at their disposal the measurements made on the prototype and could adjust the foundations and abutments accordingly. If such measurements were not available, or in case of the building of a new model on a foundation for which sufficient information is not available, the writers probably would attempt to determine the behavior of the model in a number of steps, each of which would disclose the behavior of the dam under different flexibility conditions of the foundations. Having at their disposal the results of the tests on the Calderwood model, it is assumed that for any other prototype, model conditions could be obtained that would closely approximate the actual ones.

Professor Bergman's discussion as to the possible difference between the stress in a beam having the same deflections, but different loadings, is perfectly correct. However, under the conditions of a model having the proper specific gravity ratio, an additional condition must be introduced; that is, the impossibility of arbitrarily varying the loads. If that is taken into consideration the conclusions based on the example of the action of a straight beam will be quite different.

Referring to Mr. Werner's inquiry concerning the temperature control, the writers had in mind to utilize the low thermal conductivity of the rubber-litharge compound. If and when the testing of the model is resumed, it is planned to conduct a number of tests keeping the room temperature high for a period of time sufficiently long to have the temperature of the model material uniformly high. Next a series of tests should be run with lower room temperature and with a considerably lower water temperature. It seems that such a method of testing will result in a series of deflection and strain readings that will correspond to different temperature conditions of the dam, air, and water. The low thermal conductivity of the rubber-litharge compound should make it possible to obtain distribution of temperatures throughout the model quite similar to that of the prototype.

So far as Mr. Smith's work is concerned, the writers again point to the danger of the indiscriminate use of models. To build a model of a material that weighs only 42 lb per cu ft and, by applying external weights, increase its total weight about forty-nine times, or to about 2 050 lb per cu ft, would seem to indicate a serious violation of fundamental similarity conditions. At present, such models cannot be justified, because the procedure is contrary to modern ideas regarding correct model design and particularly since materials of proper specific gravity for use in water-loaded models are now readily obtainable.

(*IV*) *Results of Tests on Calderwood and Other Models.*—The results of tests on the Calderwood model differ from expectations based on conventional design ideas and also from results of tests made on mercury-loaded models.

The results of measurements of the vertical stress on the up-stream and down-stream faces of the model are probably the ones that raised the most questions. These questions cannot be answered under conventional assumptions; but if these assumptions are disregarded and if it is conceded that there is no straight-line distribution of stress and that the total stress is not equal to the sum of the dead load and live load stress determined separately, the results of these measurements are very logical. The application of the live load results in a redistribution of the dead load stress; the character of this redistribution may be rather complicated on account of the influence of the foundations and abutments.

The present design methods do not disclose such redistribution of stress; neither can a model in which the weight of the material is subdued, and the weight of the loading fluid considerably exaggerated, show the redistribution of stress that will correspond to the actual conditions in the prototype.

Mr. Werner gives a convincing answer to the doubts raised with reference to the possibility of having vertical tension stress on the down-stream face of the dam. The instance of a cylindrical vessel cited in his discussion could not be improved upon by the writers.

Mr. Simonds refers to the measurements on the up-stream face of the model. It is only natural that there should be a difference in measured strains if pins of different lengths are used. Since the measurements should give the strains at the face of the dam, the shortest possible pins should be used. Indeed it may be advisable to substitute for the pins some device that would make it possible to attach the gauge points directly to the surface of the model.

Mr. Jorgensen's conclusion that the vertical elements of the dam impose additional loads on the upper arches and reduce the loads on the lower arches is correct. In so far as the deviation of the principal stresses from the vertical-horizontal directions is concerned, Fig. 10 shows that such a deviation is much smaller than is usually assumed and is pronounced only near the abutments. For dams that do not have a considerable increase of thickness toward the abutments, the deviation of the principal stresses from vertical-horizontal directions will be still less. Mr. Jorgensen believes that the assumption customarily made in designs neglecting the effect of gravity—that the horizontal arches slide frictionless between the adjoining arches—is on the side of safety. The writers would like to state that in the design of a complicated structure the only safe procedure is to base the design on correct assumptions and to eliminate all incorrect assumptions. If in such designs incorrect assumptions are made, it is impossile to foresee the complications that may arise, and it is very unlikely that an incorrect assumption can be made which will bring the design on the side of safety. The effect of gravity was neglected in the design of Calderwood Dam. This assumption, which was supposed to be on the safe side, resulted in the increased stiffness of the dam toward the abutments in the lower elevations. Fig. 11 shows clearly that this stiffness is responsible for the increase of stress between Blocks 6 and 7 at Elevation 860. Such increase of stress, of course, cannot be considered as making the dam safer.

Mr. Simonds' Equations (2) and (3) are correct only if the vertical and horizontal stresses are, at the same time, the principal stresses. If that is not the case, more complicated expressions should be used.[40]

Professor Bergman questions the method used to compare the stresses that were measured on the prototype and the model. Figs. 11 and 12 speak for themselves and show a reasonably close agreement between these stresses. The writers do not contend that this is the correct method but, as stated in the paper, believe that these diagrams are "indicative at least."

The writers are very much impressed by the two model tests described in Mr. Smith's discussion. First, a complete plaster-celite model of an arch dam was loaded with mercury. The deficiency in weight of that model was 98 per cent. The deflections of the crown cantilever were measured. Second, a slice model at the crown cantilever was built using the same plaster-celite material, but loading it by additional weights to about forty-nine times the weight of the original material. This model was loaded, not with mercury, but by using a wire pulley and weight arrangement, so as to have the load correspond to the mercury load that was carried by the cantilever. These loads were evaluated by the trial-load method. The cantilever deflections in both cases were found to be practically the same. To the writers the procedure seems wholly inadequate and was made necessary on account of the type of model used.

Nevertheless, Mr. Smith arrives at the conclusion that the trial-load method is correct and that the weight of the model material is of no importance. Actually, the deflections of the cantilever depend on the ground conditions and on the weight of the material of the dam. There does not seem to be any rational inter-relation between the two models and between these models and the prototype.

(*V*) *Design Methods and Theories.*—The different methods used in the design of arch dams are summarized in a paper by one of the writers.[18] Typical cross-sections are given[41] of a number of dams designed by the two design methods that are most used at present. One design neglects the gravity action and the other uses the trial-load method in which an attempt is made to consider the gravity action.

The measurements made on Calderwood Dam and the model work described in the paper seem to indicate that the neglect of the gravity action leads to a radical discrepancy between the actual and the design conditions. From the current engineering point of view such a method cannot be considered satisfactory. The trial-load method represents some improvement.[41] However, this method is far from being satisfactory or even reasonably approaching a method that should find general recognition.

A tendency seems to be developing in arch dam design to consider everything highly satisfactory and to disregard any evidence to the contrary. The supporters of the trial-load method seem to be going very far in that direction. It may be of interest to survey the present conditions. More or less satisfactory measurements were made on two commercial dams: Calderwood Dam, on which the most extended series of deflection and stress measurements were made, and Ariel Dam, on which deflection measurements were made, but stress measurements were omitted. On the Gibson Dam preparation for making measurements was included, but unfortunately no measurements of value could be taken. The measurements made on Calderwood Dam could not be verified by the use of the "trial-load" method. The Ariel Dam was designed by the use of the trial-load method and the deflection measurements disclosed a rather startling difference in the evaluated and measured deflections.[31]

---

[40] "The Determination of Stresses from Strains on Three Intersecting Gage Lines," by William R. Osgood and Rolland G. Sturm, Assoc. Members, Am. Soc. C. E., *Research Paper No. 559*, National Bureau of Standards, *Journal of Research*, Vol. 10, May, 1933.

[41] "The Compensated Arch Dam," by A. V. Karpov, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 98 (1933), Figs. 1 and 2, pp. 1310–1311.

The only tests by which the trial-load method could be verified are the model tests made on mercury-loaded models on which the specific gravity requirement was grossly violated and on which neither the foundation and abutment conditions nor the joint conditions were properly represented. Under such conditions it seems that the statements so often made in recent years, that the designs of arch dams are based on proven test data, are over-optimistic. It is no secret to the Engineering Profession, that the safety of high arch dams is not obtained on account of sufficient engineering knowledge, but rather to the contrary: The present limited engineering knowledge makes it necessary to use in such dams a much larger amount of material—unsatisfactorily distributed—than would be necessary if the design could be based on proved assumptions. In that respect the horizontal cracks or the openings of horizontal construction joints in arch dams are very significant. In the latest designs, and in particular in the trial-load designs, vertical tension stress that could produce cracks or joint openings of such character presumably are absent; consequently, the tendency is to attribute such cracks to faulty construction methods, contraction of the concrete, etc. In fact, such an explanation was accepted a few years ago on one of the arch dams with which the writers were connected. The tests on Calderwood Dam and its model show very clearly that such cracks, not caused by faulty construction methods or by shrinking concrete, may be caused by high vertical tension stress at both the up-stream and the down-stream face of the dam. Such stresses cannot be evaluated by using one of the conventional design methods; nor can they be disclosed on mercury-loaded models.

The proper design of an arch dam should give consideration to the following items: (1) Flexibility of foundations and abutments; (2) non-linear distribution of stress; (3) substitution of properly shaped horizontal arches instead of the arbitrarily chosen circular arches; (4) influence and proper utilization of the construction joints; and (5) necessity of liberal use of fillets.

These items can be investigated on a model in which the specific gravity requirements are satisfied. A low value of Poisson's ratio would make a more satisfactory model, but even when using a material with a high Poisson ratio satisfactory conclusions can be drawn.

The mercury-loaded plaster-celite model does not make possible the proper investigation of any of these most important requirements and, consequently, no satisfactory design theory can be developed from studies on such models.

Conclusion.—In conclusion, the writers would like to emphasize a few points which often seem to be neglected, but which are of considerable importance in the model study of structural problems.

1.—It should be realized that the new conceptions in physical and chemical sciences necessarily involve engineering. Designers of important structures should take cognizance of the fact that the conventional stress theories do not represent the actual conditions, and that, in the building up of the new theories, many engineering assumptions must be revised and many rejected, notwithstanding the fact that until recently they were considered perfectly sound.

2.—The present design methods do not disclose the importance of obtaining reasonably flexible structures. No matter how massive or stiff an engineering structure may be, it acts substantially as an elastic structure. The applied load introduces stresses that result in strains, deformations, and deflections. The value of the actual stress can be determined accurately if very refined design methods, based on proper assumptions, are utilized. Very often, due mostly to lack of knowledge, less refined design methods combined with unreliable assumptions are used. Then the actual stresses, at least in some parts of the structure, will be higher than the evaluated stresses; and stress concentrations that were not foreseen in the design, will occur.

Comparing two similarly designed structures of a different degree of flexibility, the stress concentrations in a more flexible structure will be not as high as in a stiffer structure, the more flexible structure having a better opportunity to distribute the stress more uniformly.

In some cases an increase in material in some parts of the structure may increase considerably the stiffness and thus result in much higher stress concentrations. Under such conditions, contrary to the design data, the actual stress may be higher in some parts of a structure in which more material is used as compared with a structure in which less material is used. Designs obviously should not be used in which the stresses are increased on account of the use of an increased amount of material.

3.—A properly designed model, if used discriminately, is one of the most valuable of engineering tools and permits the correct solution, among others, of the most complicated structural problems; but models are not miraculous tools, and valuable results are obtainable only if theoretically correct models are used.

4.—In any model study it is necessary to approach the problem without preconceived ideas to be proved or confirmed. Only a model which is given a fair chance to act naturally and in a manner similar to the prototype will give reliable results. Models that are forced to act in a certain way dissimilar to the action of the prototype, in nearly all cases will be misleading, and should not be used.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 1896

FORMATION OF FLOC BY FERRIC COAGULANTS

By Edward Bartow,[1] M. Am. Soc. C. E., A. P. Black,[2] Esq.,
and Walter E. Sansbury,[3] Esq.

With Discussion by Messrs. Edward S. Hopkins, W. D. Hatfield,
L. B. Miller, Linn H. Enslow, and C. J. Brockman.

Synopsis

Ferrous sulfate (oxidized by atmospheric air in the presence of alkali), ferric chloride, chlorinated copperas, and ferric sulfate are used as coagulants in water purification. Natural waters contain varying quantities of sulfate ion, chloride ion, sodium ion, calcium ion, etc. An experimental determination of the action of these ions on the formation of floc with ferric salts at different pH-values shows the following results: (1) On the acid side the sulfate ion has a much greater effect on coagulation than the chloride ion; (2) using from 25 to 250 ppm of sulfate ion, there is little change in the effect produced; (3) between a pH-value of 6.5 and a pH-value of 8.5, there is a zone in which ferric floc forms slowly or not at all; and (4) in and beyond this zone, sodium and calcium ions are most effective in coagulation.

The assumption of a change in the sign of the colloidal ferric floc from positive, where it is more affected by sulfate or chloride ions, to negative will explain the zone of no floc formation and the more effective action of the sodium and calcium ions beyond this zone.

The quantity of residual iron in solution is roughly proportional to the time required for the floc to form.

Note.—Presented at the meeting of the Sanitary Engineering Division, New York, N. Y., January 19, 1933, and published in December, 1933, *Proceedings*.

[1] Prof. of Chemistry, and Head, Dept. of Chemistry and Chemical Eng., State Univ. of Iowa, Iowa City, Iowa.

[2] Prof. of Agricultural Chemistry, Univ. of Florida, Gainesville, Fla.

[3] Fellow in Chemistry, Univ. of Florida, Gainesville, Fla.

Ferric Coagulants

It has long been recognized that ferric compounds may be utilized in the formation of floc for the removal of color or turbidity from natural waters, but it is only in recent years that they have become available in adequate quantity and at a price sufficiently low to compete with alum.

With the possibility of increased use of these ferric coagulants, the writers have undertaken a study similar to that made by Messrs. Bartow and Ben. H. Peterson[4] and by Messrs. Black, Rice, and Bartow[5] on aluminium sulfate. It is known that aluminium salts and iron salts differ in their action when used as coagulants in water treatment, and the purpose of this paper is to try to determine and explain these differences.

Ferrous Sulfate as a Coagulant.—Ferrous sulfate with lime was first used by Mr. W. B. Bull, at Quincy, Ill. According to Mr. Dow R. Gwinn[6] the experiments were begun in July, 1898, the ferrous sulfate being made by burning sulfur and passing the gas into water and over scrap iron. Chlorinated copperas made by adding chlorine to a solution of ferrous sulfate was used by Messrs. F. W. Mohlman and J. R. Palmer,[7] at Chicago, Ill., in the coagulation of sewage sludge. It has since been adapted to water purification.

Since the coagulation takes place after oxidation to ferric salt by the oxygen of the air, ferrous sulfate is really a ferric coagulant. It is unsuitable for the treatment of certain types of waters. Soft, colored waters, or soft, turbid waters, are best coagulated on the acid side of the neutral point—at pH-values of less than 7. (pH is equal to the log of the reciprocal of the hydrogen-ion concentration.) The color appears to become fixed or set by the addition of alkali. The use of ferrous sulfate is limited, therefore, to those waters in which alkalinity will not interfere with color removal. It has been shown by Mr. L. B. Miller[8] and, later, by Messrs. Jacob Cornog and Albert Hershberger[9] that when alkali is added to a dilute solution of ferrous sulfate, a small fraction of one equivalent increases the pH to about 7.5. It remains practically constant at this figure until nine-tenths of an equivalent of alkali has been added; then it rises again rapidly to a pH of about 11.0. Further addition of alkali up to the theoretical two equivalents produces little change in the hydrogen-ion concentration. According to Mr. Miller (see Fig. 1), the first zone of fairly constant hydrogen-ion concentration is due to the formation of a basic salt. It is obvious, therefore, that ferrous salts are precipitated by alkali only at comparatively high pH-values.

A more important factor is the atmospheric oxidation of the ferrous coagulant to the ferric state. Messrs. J. A. N. Friend and E. G. K. Pritchett[10] and Messrs. Cornog and Hershberger[9] have shown that the rate of oxidation of ferrous salts by atmospheric oxygen depends upon both the concentration of the ferrous salt and the hydrogen-ion concentration. For the exceedingly low concentrations met with in water-works practice, the rate of oxidation increases as the hydrogen-ion concentration decreases; that is, the rate increases with increased pH-values.

---

[4] *Industrial and Engineering Chemistry,* Vol. 20 (1928), p. 51.

[5] *Loc cit.,* Vol. 25 (1933), p. 811.

[6] *Engineering News,* Vol. 43 (1900), p. 351; *Engineering Record,* Vol. 39 (1899), p. 595.

[7] *Engineering News-Record,* Vol. 100 (1928), p. 147.

[8] Public Health Repts., U. S. Public Health Service, Vol. 40 (1925), p. 1413.

[9] Thesis for Master's Degree, State Univ. of Iowa, Iowa City, Iowa, 1928.

[10] *Journal,* Chemical Soc., 1928, p. 3227.

Mr. C. V. Smythe[11] has shown that certain anions greatly influence the rate of atmospheric oxidation of ferrous compounds. The theory is advanced that the ferrous ion forms un-ionized complexes with the anions in question and that it is easier to remove an electron from such a neutral complex than from a doubly charged ferrous ion. In terms of this theory the oxidation of ferrous salts in alkaline solution would involve the formation of insoluble neutral molecules of ferrous hydroxide which are far more readily oxidized than the ferrous ion. These considerations also apply in the removal of iron from natural waters by aeration.

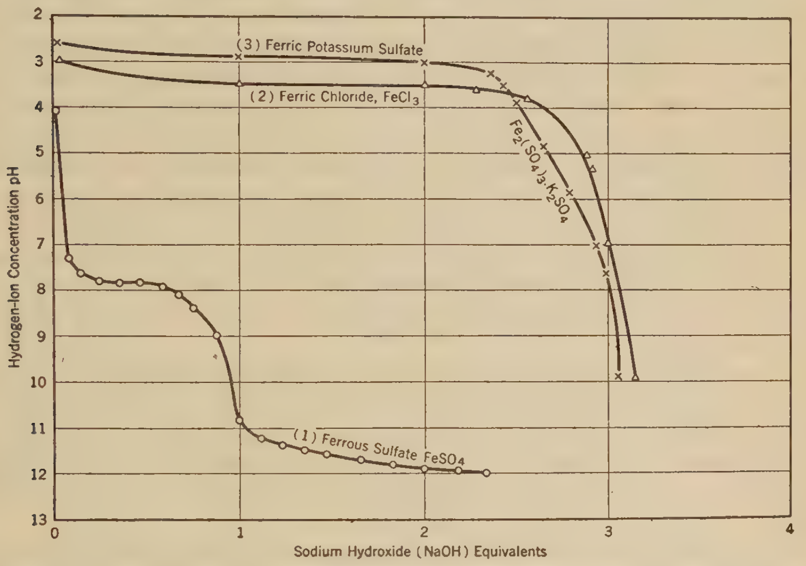


FIG. 1.—COMPARISON OF CHANGE IN HYDROGEN-ION CONCENTRATION (pH), DURING PRECIPITATION OF FERROUS HYDROXIDE AND FERRIC HYDROXIDE BY SODIUM HYDROXIDE.

Ferric Salts as Coagulants.—Ferric chloride made by treating scrap iron with gaseous chlorine obtained directly from a chlorine cell was used experimentally in 1910 in Chicago and in Toledo, Ohio, by Mr. Bull.[12] Ferric chloride in concentrated solution, shipped in rubber-lined tank cars, has been available since about 1930. Ferric sulfate in dry form was placed on the market in 1931.

In coagulation or formation of floc, the action of ferric salts is very different from that of ferrous sulfate. According to Mr. Miller[8] a very dilute solution (0.01 M) of ferric sulfate, when treated with an alkali, has a pH-value of 3 when two equivalents of alkali have been added; of 4 when two and one-half equivalents have been added; and of 7 when the three equivalents theoretically required have been added. (The symbol, M, denotes mols, or molecules per liter.) With ferric chloride the pH-values are slightly higher. (See Fig. 1.) This means that a ferric floc may be formed in solutions that are much more acid than would normally be encountered in waterworks practice.

---

[11] *Journal of Biological Chemistry,* Vol. 95 (1931), p. 251.

[12] *Proceedings,* Illinois Water Supply Assoc., Vol. 3 (1911), p. 66; and *Municipal Engineering,* Vol. 42 (1911), p. 159.

Since soft, colored waters and soft, turbid waters may usually be coagulated more satisfactorily on the acid side, alum or a combination of alum and sulfuric acid has been used satisfactorily as a coagulant with such waters. However, since aluminium hydroxide is amphoteric (that is, exhibits both acid and alkaline characteristics), its solubility increases as the hydrogen-ion concentration increases. Messrs. E. J. Theriault and W. M. Clark,[13] Mr. Miller,[14] J. R. Baylis, Assoc. M. Am. Soc. C. E.,[15] and W. D. Hatfield, Assoc. M. Am. Soc. C. E.,[16] state that from the acid side precipitation becomes complete at pH 5.4. At less than pH 5.4 alum floc will re-dissolve and pass the filters unless secondary coagulation is utilized following color removal. However, with many highly colored waters, successful clarification may be obtained only at much lower pH-values. Mr. L. L. Hedgepeth and W. C. Olsen, Assoc. M. Am. Soc. C. E., report[17] an optimum pH-value of 4.9 for Elizabeth City, N. C., and L. H. Enslow, Assoc. M. Am. Soc. C. E., gives[18] the optimum zone for highly colored swamp waters as pH 3.8 to 4.7. Such acidities make coagulation with alum impossible, but satisfactory results may be obtained with ferric salts.

Ferric floc is not only more insoluble at lower pH-values than alum floc, but is also more insoluble at high pH-values. The possible pH-range of floc formation is, therefore, decidedly broader on both sides of neutrality for ferric salts than for alum. The ferric floc as ordinarily formed has a higher specific gravity than alum floc and, therefore, requires shorter sedimentation periods.

Explanations differ as to the exact mechanism of the process by which turbidity and certain types of color are removed. While it has long been thought that the highly adsorptive gelatinous floc plays the leading rôle, Mr. Miller[19] in his fundamental work indicates that color removal is most often accomplished by neutralization of the negatively charged color colloid by the positively charged trivalent aluminium or ferric ion and that as a rule "alum floc" and "iron floc" play relatively unimportant parts.

Effect of Various Anions on Floc Formation

Waters apparently similar in character may respond in very different ways to the same method of treatment. This is particularly true of the hydrogen-ion concentration, at which optimum floc formation is secured with a given dose of coagulant. The point of optimum coagulation may be as low as pH 3.8, in the case of highly colored swamp waters, or it may be greater than

---

[13] Public Health Repts., U. S. Public Health Service, Vol. 38 (1923), p. 181.

[14] *Loc. cit.*, p. 1995; and Vol. 40 (1925), p. 351.

[15] *Journal*, Am. Water Works Assoc., Vol. 10 (1923), p. 365.

[16] *Loc. cit.*, Vol. 11 (1924), p. 554.

[17] *Loc. cit.*, Vol. 20 (1928), p. 467.

[18] *Proceedings*, First Virginia Conference on Water Purification and Sewage Treatment, 1929, and *Municipal News and Water Works*, Vol. 76 (1929), p. 227.

[19] Public Health Repts., U. S. Public Health Service, Vol. 40 (1925), p. 1472.

8.0 with other types of waters. It is often true that the pH-range within which good flocs may be formed rapidly is relatively narrow and in some cases the pH-values must be adjusted closely.

The investigations of Messrs. Bartow and Peterson[4] and of Messrs. Black, Rice, and Bartow[5] have explained in part at least the "individuality" possessed by most natural waters. These investigations have shown that the anions present in the water treated, particularly the sulfate and chloride ions, influence to a marked degree the pH-zone of optimum floc formation when alum is used as a coagulant. The bivalent sulfate ion extends the pH-zone of optimum floc formation far to the acid side and very slightly to the alkaline side, the effect increasing, as would be expected, with increasing quantities of sulfate ion. The monovalent chloride ion has much less influence than the bivalent sulfate ion. It does broaden the pH-zone somewhat, and it is to be noted that its effect is somewhat greater on the alkaline side than that of the sulfate ion.

Messrs. Black, Rice, and Bartow,[5] at the experimental water treatment plant of the University of Florida, have shown that results obtained in jar tests are likewise obtained when large quantities of water are treated.

In view of the marked effects exerted by the sulfate and chloride ions on the pH-zone of optimum alum floc formation, it seemed desirable to determine the effect of these ions on the formation of ferric floc with various ferric coagulants.

Three ferric coagulants are available to-day—chlorinated copperas, ferric chloride, and soluble ferric sulfate—although chlorinated copperas is not sold as a ferric salt. It is a mixture of ferric chloride and ferric sulfate obtained by oxidizing ferrous sulfate (copperas) with chlorine; hence its name. The reaction is as follows: $6FeSO_4 + 3Cl_2 \rightarrow 2Fe_2(SO_4)_3 + 2FeCl_3$.

In practice, 1 lb of chlorine is required for 8 lb of copperas. As stated previously, chlorinated copperas instead of ferric chloride was first used by Messrs. Mohlman and Palmer[7] for dewatering sewage sludge. It proved greatly superior to alum. It was first used in water purification by Messrs. Hedgepeth and Olsen[17] in 1928 for the treatment of the highly colored water supply of Elizabeth City, N. C. The so-called "color floc" produced at low pH-values by the action of the trivalent ferric ion on the negatively charged color colloid was stated by them to have adsorptive properties far superior to those of a true ferric hydroxide floc. Following its highly successful use at Elizabeth City, trials were made elsewhere, and several papers describing the results obtained are available in the literature. A. C. Decker, M. Am. Soc. C. E., and Mr. H. G. Menke[20] used the process at Birmingham, Ala. Mr. E. S. Hopkins,[21] at Baltimore, Md., studied the floc produced by this material.

Messrs. E. C. Craig and E. H. Bean,[22] at Providence, R. I., have used the three ferric coagulants interchangeably and conclude that there is less than 10% variation in the results obtained with any of them when they are compared on the basis of equal iron content.

---

[20] *American Journal of Public Health*, Vol. 20 (1930), p. 357.; and *Municipal News and Water Works*, Vol. 76 (1929), p. 246.

[21] *Industrial and Engineering Chemistry*, Vol. 21 (1929), p. 581, and Vol. 22 (1930), p. 79.

[22] *Water-Works and Sewerage*, Vol. 79 (1932), p. 301.

EXPERIMENTAL

Coagulation experiments have been made in the stirring apparatus used by Messrs. Black, Rice, and Bartow[5] in the study of the effect of various anions on the formation of aluminium floc.

For these experiments six battery jars (3.5 liter capacity) were used; 2 liters of water were used for each test. Glass stirrers were used to mix the water in the battery jars. They were composed of glass blades, each 3 by 15 cm, attached to vertical glass rods, 1 by 25 cm, which, in turn, were attached to vertical steel drive shafts by heavy walled rubber tubing, so that each might be disconnected and the jars removed during a run. The vertical shafts were supported on an iron framework and on the upper end had pulleys, all driven by one continuous belt from a small electric motor. Identical conditions of illumination and the formation of Tyndall cones were produced by rays from 100-watt electric light bulbs, controlled by a single switch, the light from which passed through $\frac{3}{8}$-in. holes in a long wooden box placed behind the jars. The Tyndall cones produced were so sharp that it was usually possible to duplicate the shorter floc times to 15 sec in successive experiments. To exclude heating effect from the bulbs, the side of the box nearest the jars was lined with asbestos board which, together with a 1-in. air gap between the box and the jars, caused a maximum rise of only 1°C during the longest runs. The apparatus with the exception of the motor and gear reducer was built in the machine shops of the University of Florida at little cost. G. E. Willcomb, Assoc. M. Am. Soc. C. E., describes[23] the construction of laboratory stirring machines of this general type.

Method of Procedure.—The determined quantities of alkaline solution and salt solution to be used, were placed in a 2-liter volumetric flask, made up to a volume of 1 975 milliliters, with distilled water, thoroughly mixed and placed in the jars. Stirring was begun and 25 milliliters of coagulant solution was added to each jar. The time of addition was noted with a stopwatch. The time at which well defined floc formed in each jar was carefully noted, and the total elapsed time for each jar was corrected for the time required to add the coagulent.

The equivalent of 1.6 grains of coagulant, calculated as $Fe_2(SO_4)_3 \cdot 9H_2O$, was used for each test. This is the approximate molar equivalent of the 2 grains of alum used in previous work. The quantities of alkali in the various tests required to adjust the solutions to the desired pH-values, were determined by experience.

Standard Solutions.—The purest analytical grade of chemicals was used for the preparation of all solutions, which were made of such strength that convenient small volumes, when diluted to a final volume of 2 liters, gave known concentrations of the desired constituent. In the case of the coagulants, 25 milliliters of working solution represented a dose of 1.6 grains per gal., calculated as ferric sulfate. The solutions of sodium chloride, sodium sulfate, sodium bicarbonate, and calcium chloride were of such strength that each milliliter contained 10 mg of the respective anion. The chlorinated copperas was prepared by passing chlorine gas through a ferrous sulfate solution until oxidation was complete, as shown by testing with potassium ferricyanide. Air was then bubbled through the solution until starch iodide paper showed the absence of excess of free chlorine. Concentrated stock solutions of ferric sulfate and chlorinated copperas were prepared and the exact strengths determined by titration with standard potassium dichromate solution. Since the dilute working solutions were unstable, they were prepared from the stock solutions in small quantities as needed.

---

[23] *Journal*, Am. Water Works Assoc., Vol. 24 (1932), p. 1416.

As soon as the floc had formed in a jar, this jar was removed from the machine without interrupting the stirring of the other jars. A portion of the water from the jar was placed in a clean Erlenmeyer flask, tightly stoppered, and allowed to settle until it was clear and bright. The pH-values of all solutions were determined by the isohydric colorimetric method fully described elsewhere.[5] Samples of solution (10 milliliters) were carefully pipetted into color comparison tubes, filled with air from which carbon dioxide had been removed, each containing 0.5 milliliter of an indicator solution of chosen pH-value. Thorough mixing was secured, without shaking, by allowing the water to flow from the pipette tip into the indicator solution in the tubes. The tubes were stoppered to exclude carbon dioxide and were then compared with buffer-color standards in a comparator. Ordinarily, the pH-value of each sample was determined with indicator solutions of three different pH-values. From the data a value for the hydrogen-ion concentration could be obtained which was reliable to 0.05 pH-unit.

Residual iron determinations were made by the thiocyanate method on samples of the water filtered through two thicknesses of ashless filter paper.

Floc Formation at Varying pH-Values

The time of floc formation was determined at varying pH-values in solutions of coagulant and distilled water, in which the alkalinity was furnished by sodium hydroxide in one case and sodium bicarbonate in the other. The pH-zone in which a ferric floc would form under these conditions was relatively narrow, pH 5.0 to pH 7.0. This is entirely on the acid side of the neutral point. (See Fig. 2.) The minimum time for floc formation occurred at about the same place, pH 6.1 to pH 6.4, but the zone was somewhat wider in the case of sodium bicarbonate.

Effect of Sulfate Ion on Floc Formation Using Ferric Sulfate Coagulant.—Sulfate ion as pure sodium sulfate, $Na_2SO_4 \cdot 10H_2O$, was added to the water in dosages of 25, 50, 125, and 250 ppm, and its effect on the pH-zone of floc formation determined. It was interesting to note: (1) That (see Fig. 3, Curve 1). the zone of rapid floc formation was greatly widened on the acid side (floc formed in 4½ min at a pH-value of 3.9); (2) that the effect of a dose of 25 ppm of sulfate ion (see Curve 2, Fig. 3) was almost as great as that of a dose ten times as large (Curve 5), which was not found to be the case with alum[4]; (3) that the time of floc formation increased markedly in the pH-range, 7.0 to 8.5, after which it again decreased as the pH increased to the maximum used in this work, 9.6; and (4) that the solution containing 25 ppm of sulfate ion, which flocculated rapidly at all pH-values between 4.0

and 7.0, failed to flocculate in 45 min between pH-values of 7.0 and 8.0. This interesting and unexpected behavior of these solutions in this pH-range will be discussed later. (Points marked with an arrow signify that no floc formed within 45 min at the designated pH-value.)

Effect of Chloride Ion on Floc Formation Using Ferric Sulfate Coagulant.—Chloride ion furnished by sodium chloride was added to the water in dosages of 25, 50, 125, and 250 ppm, and its effect on the pH-zone of floc formation determined in the same manner as for the sulfate ion. (See Fig. 4.)

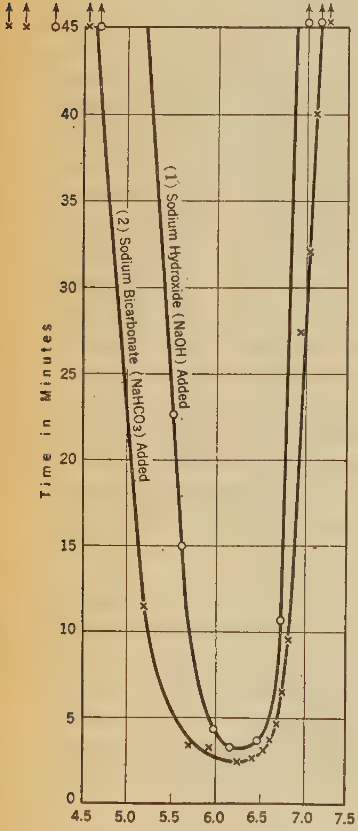


Fig. 2.—Rate of Floc Formation by Ferric Sulfate with Hydrogen-Ion Concentration Adjusted by: (1) Sodium Hydroxide (NaOH); and (2) Sodium Bicarbonate ($NaHCO_3$).

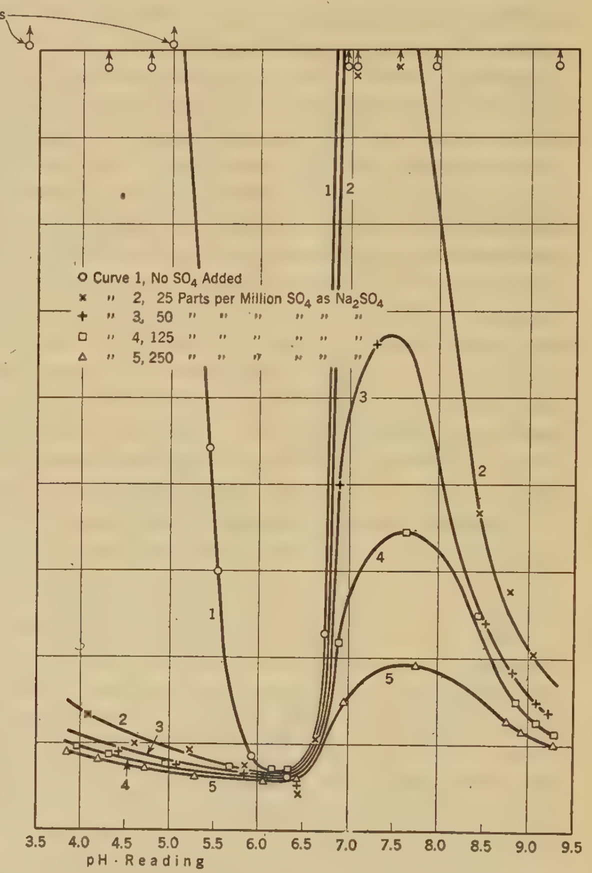


Fig. 3.—Effect of SO_4 as Sodium Sulfate (Na_2SO_4) on the Zone of Floc Formation by Ferric Sulfate ($Fe_2(SO_4)_3$) at Varying pH-Values.

On the acid side the chloride ion was not nearly as effective as the sulfate ion in influencing the formation of floc. While the zone of floc formation was greatly broadened on the acid side, the time required for the floc to form was considerably longer in all cases. This behavior is quite simply explained when it is remembered that the floc forms first as a positive

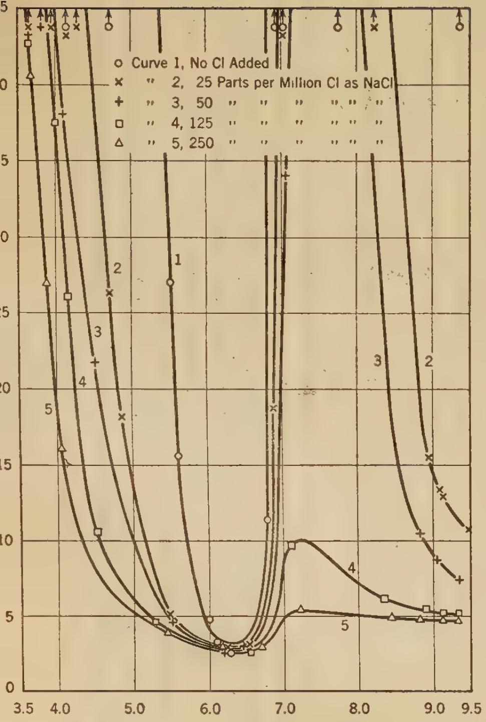


FIG. 4.—EFFECT OF CHLORIDE-ION AS NaCl, ON THE ZONE OF FLOC FORMATION WITH FERRIC SULFATE.

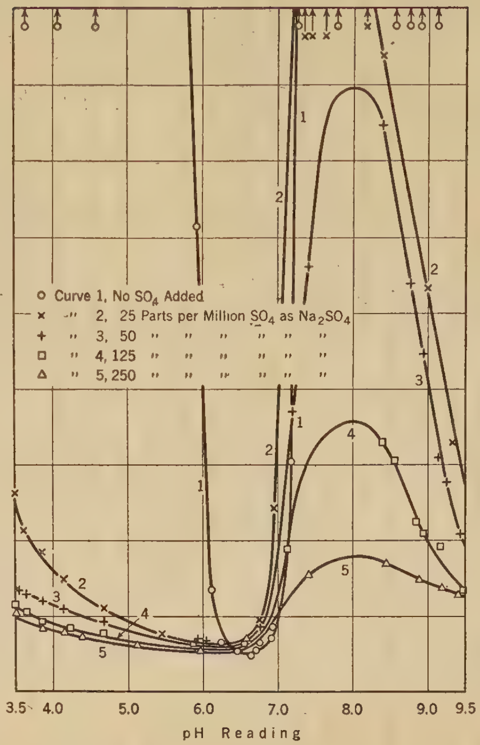


FIG. 5.—EFFECT OF SULFATE-ION AS Na_2SO_4 ON ZONE OF FLOC FORMATION WITH CHLORINATED COPPERAS.

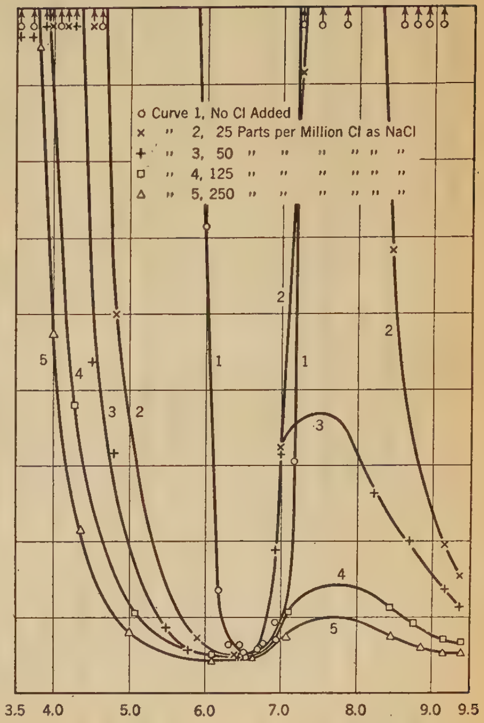


FIG. 6.—EFFECT OF CHLORIDE-ION AS NaCl ON ZONE OF FLOC FORMATION WITH CHLORINATED COPPERAS.

colloid. The precipitating effect of negative ions, therefore, should depend largely upon their valence and should follow the Schultz rule.[14] While this rule explains qualitatively the relative effects observed in the case of sulfate and chloride ions, water-works chemists are in need of exact data giving the quantitative effect of these and other anions over a considerable range of hydrogen-ion concentrations in the various ion concentrations likely to be encountered in water treatment. This investigation furnishes some of these data.

As in the case of sulfate ion mentioned previously, the floc time increased in the pH-zone, 7.0 to 8.5, and decreased again above pH 8.5 (see Fig. 3). It was noted, however, that in concentrations of 125 to 250 ppm, the chloride ion appeared to exert a greater effect on the alkaline side than the sulfate ion. (Compare Fig. 3 and Fig. 4.)

Effect of Sulfate and Chloride Ions on Floc Formation, Using Chlorinated Copperas Coagulant.—In order to check these results, the entire series of experiments was repeated using chlorinated copperas as coagulant in place of ferric sulfate. The quantity of ferric ion added to the solutions was the same in both cases. In the low dosages used, it is evident that the slight differences in the chloride and sulfate content of the two coagulant solutions were not significant. The procedure for obtaining the data was identical in every respect with that described for ferric sulfate. The action was practically identical with that when ferric sulfate was used as a coagulant. (See Fig. 5 and Fig. 6.) The zone of rapid floc formation was greatly broadened on the acid side by the sulfate ion and to a much less degree by the chloride ion. The floc formation was markedly retarded in the pH-zone, 7.0 to 8.5. At the higher concentrations, the effect of the chloride ion appeared to be greater than that of the sulfate ion.

Effect of Sodium and Calcium Ions on Floc Formation.—A possible explanation of this seemingly anomalous behavior of the chloride and sulfate ions at pH-values greater than 7.0 is that the sign of the colloidal floc particles reverses at pH-values in the neighborhood of 6.5. Mr. Miller[19] showed by cataphoresis experiments that the colloidal flocs with which he worked almost invariably bore a positive charge relative to the solution in which they were formed. In certain of his experiments with colored waters, he did obtain negatively charged aggregates which he attributed to the combination of the positive floc with an excess of the negatively charged color colloid.

In studying the adsorption of phosphate ion by ferric oxide hydrosols from solutions of mono-sodium, di-sodium, and tri-sodium phosphate, Messrs. W. Stollenwerk and M. von Wrabgel[24] found that adsorption decreased markedly as the concentration of the hydroxyl-ion increased. They explained it by assuming a reversal of the colloidal charge in the more alkaline solutions probably due to the influence of the hydroxyl-ion.

If the sign of the charge on the colloidal ferric hydroxide floc did reverse as has been assumed, there should be a point or zone in which the floc bears no charge. This should be the point of minimum stability at which precipitation should take place most rapidly. On the alkaline side of this point or zone, the cations present in solution, and not the anions, should act as precipitating agents. Sodium chloride contains approximately 40% of sodium, whereas anhydrous sodium sulfate contains only 32.4%, and the apparently anomalous behavior of the chloride and sulfate ions might be due, not to these anions, but to the greater concentration of sodium ion furnished by the sodium chloride. If this is true, a bivalent cation should exert a greater effect than the monovalent sodium cation at pH-values above 6.5.

---

[24] *Zeitschrift für Electrochemie,* Vol. 33 (1927), p. 501.

In order to test the theory, two series of experiments were made with ferric sulfate as coagulant. In one set chloride ion was supplied in the form of sodium chloride and in the other, in an equal amount, in the form of calcium chloride. The results obtained furnish strong evidence for the correctness of the theory. Flocs formed more quickly in solutions containing only 5 ppm of chloride from calcium chloride than in solutions containing 50 ppm of chloride from sodium chloride. The solutions containing 25 to 50 ppm of chloride from calcium chloride formed more quickly than any other flocs worked with in the entire investigation. (See Fig. 7.) To test the theory further two series of experiments were made using only distilled water and coagulant, supplying the alkali in one series with sodium hydroxide and in

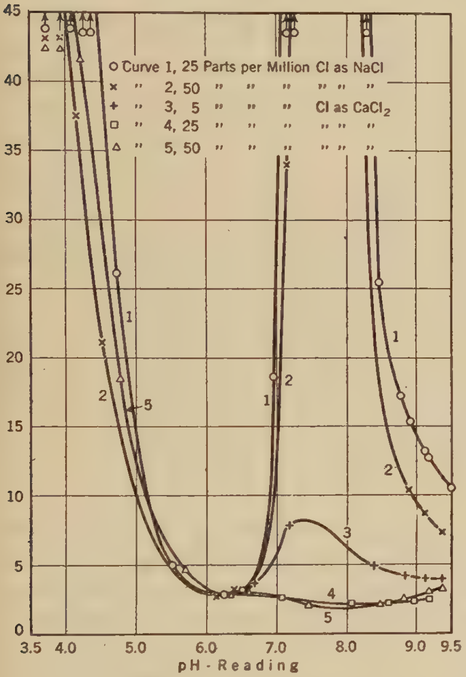


FIG. 7.—COMPARISON OF EFFECT OF SODIUM CHLORIDE AND CALCIUM CHLORIDE ON FLOC FORMATION WITH FERRIC SULFATE.

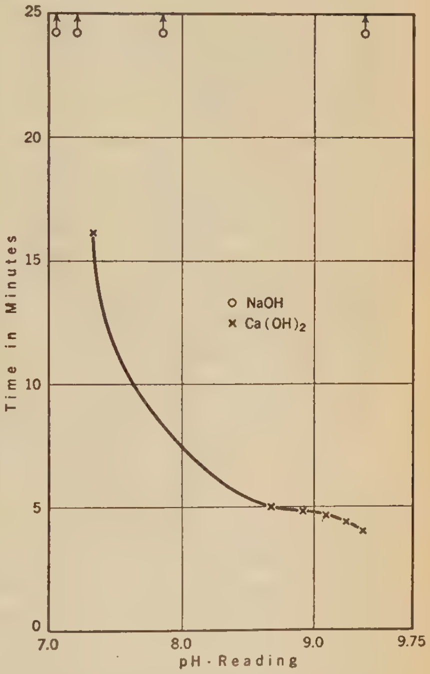


FIG. 8.—COMPARISON OF SODIUM AND CALCIUM IONS ON FLOC FORMATION WITH FERRIC SULFATE.

the other series with calcium hydroxide. Where sodium hydroxide was used, no floc formed in 45 min at pH-values between 7 and 9.0, whereas when calcium hydroxide was used, a floc formed in 16 min at a pH-value of 7.3, and in 5 min between pH-values of 8.6 and 9.4. (See Fig. 8.)

Messrs. Bartow and Peterson,[4] in one set of experiments, made a comparison of the relative effects of equal dosages of sodium and calcium sulfates on the time of formation of alum floc. Their results (see Fig. 9) reveal the interesting fact that, while the differences are not marked, calcium sulfate gave wider zones of floc formation on the alkaline side than were given by sodium sulfate, used in equivalent amounts.

Colloid chemists make a distinction between the rapid coagulation and the slow coagulation of hydrophobic sols. In both cases the ion of charge opposite to that carried by the colloidal particles is the effective agent in causing precipitation. In rapid coagulation there are no differences due to the nature or concentrations of the coagulating ions. In slow coagulation on the other hand, the nature and concentrations of the ions must be considered. M. V. Smoluchowski[25] developed a theory of coagulation from the kinetic point of view, which is confirmed by observations. The colloidal particle is assumed to consist of a central charged particle surrounded by an envelope of ions of opposite charge. This envelope of ions may extend rather deeply into the liquid phase, and the double layer so produced gives rise to a potential difference termed the ζ-potential. The repulsive forces between the sol particles are due to this ζ-potential and in the range of rapid coagulation it is abolished. He developed the following formula:

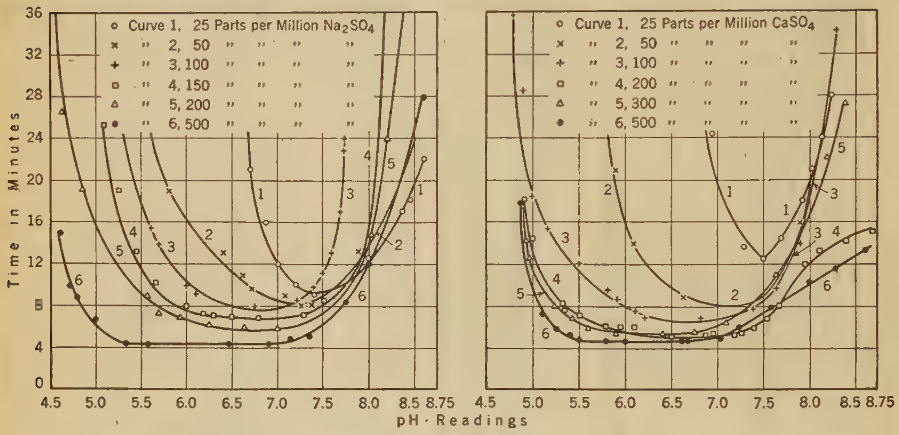


FIG. 9.—COMPARISON OF EFFECT OF SODIUM SULFATE AND CALCIUM SULFATE ON COAGULATION TIME OF ALUM FLOC.

$$K_r = 4\pi D l \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

in which, K_r is the velocity constant, assuming that the coagulation takes place as a dimolecular reaction: D, the diffusion constant of the particles;

---

[25] *Zeitschrift für Physikalische Chemie,* Vol. 92 (1917), p. 155.

and l, the distance from each other to which two particles must approach, in order that the collision between them becomes effective. There is no factor introducing the nature and concentration of the electrolyte producing coagulation.

For slow coagulation, Mr. Smoluchowski[25] introduces an additional factor into the equation. In this case, the formula is stated:

$$K_s = 4\pi D l \xi \qquad (2)$$

in which, ξ signifies that only a certain fraction, ξ, of all collisions results in coagulation. This fraction, ξ, depends very markedly upon the nature and concentration of the electrolyte. The ion of charge similar to that of the ionic envelope tends to diminish the depth of the envelope, the resulting distortion giving rise to attractive forces. The ion may also be adsorbed on the surface of the particle or even re-act chemically with it. In the zone of slow coagulation, only a certain fraction of the collisions is effective, due possibly to the fact that the particles do not possess sufficient kinetic energy to overcome the ζ-potential. If it is assumed that the distribution of the kinetic velocities of the sol particles follows Maxwell's curve, then the results are as would be expected.

In terms of this theory, the behavior of the ferric hydroxide floc might possibly be explained as follows: On the acid side the colloid consists of positively charged particles of ferric hydroxide plus basic ferric sulfate or ferric chloride, surrounded by an envelope of sulfate or chloride ions. In the zone of slow coagulation on the acid side, the sulfate ion is far more effective than the chloride ion and both are effective in proportion to their concentrations. At a pH of about 6.5, at which point rapid coagulation takes place, both ions are equally effective. On the alkaline side of this point there is a second zone of slow coagulation in which the colloidal particles bear a negative charge due possibly to adsorption of hydroxide ions from the solution. In this zone (pH = 6.5 to 8.5), the bivalent calcium ion is far more effective than the monovalent sodium ion, and both are effective in proportion to their concentrations. At still higher pH-values, a second zone of rapid coagulation is approached in which zone both are equally effective. In order to establish definitely the sign of the charge carried by the colloidal floc at various pH-values on both sides of the neutral point, cataphoresis experiments have been begun.

If further work should confirm this apparent change in the sign of the colloidal floc on the alkaline side. it might serve to explain in part the difficulty that is often experienced in color removal in this pH-range, 6.5 to 8.5.

Color is usually present in natural waters in the form of a negative colloid, and it has been assumed that it is sufficiently acid to be appreciably soluble at the higher alkaline pH-values often used in coagulation. While this seems well established, yet the difficulties inherent in the removal of a color colloid by a colloidal floc of the same sign are obvious.

Residual Iron in Solution at Varying pH-Values.—Experiments were made to determine the quantity of iron remaining in solution at various pH-values. A series of runs was made in the usual manner with chlorinated copperas as coagulant. Sulfate ion (25 ppm) was added in the form of sodium sulfate and the hydrogen-ion concentration adjusted with sodium hydroxide at various pH-values from 3.5 to 9.4. The iron remaining in the solution (see Fig. 10), at various pH-values, correlates with the times of floc formation. There is a wide range on the acid side in which little or no residual iron is present. Beyond pH 9.0, when the floc again begins to form, the quantities of residual iron decrease rapidly with decreasing time of floc formation. Determinations were not made beyond pH 9.4 since Mr. Hopkins[21] has shown that the floc is highly insoluble in the pH-range, 9.4 to 13.0. The very high values which he obtained at lower pH-values (11.0 ppm at pH 6.0) are undoubtedly due not to solubility of the ferric hydroxide floc, but to the incomplete oxidation of the ferrous sulfate that he used as coagulant.

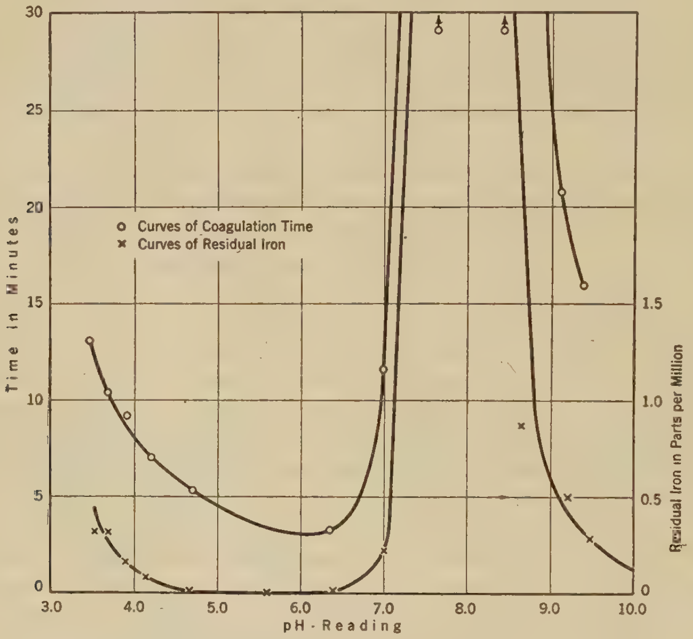


FIG. 10.—RESIDUAL IRON FROM FORMATION OF FERRIC FLOC AT VARYING pH-VALUES, USING 1.6 GRAINS OF CHLORINATED COPPERAS AND 25 PARTS PER MILLION ADDITIONAL SULFATE.

No attempt has been made to investigate the chemical composition of the floc formed at various pH-values, since Messrs. Miller[14] and Hopkins[21] showed rather definitely that the floc is a basic sulfate or chloride until three equivalents of alkali have been added. The molecular ratios, $\frac{SO_4}{Fe}$ and $\frac{Cl}{Fe}$, increase as the pH decreases. When three equivalents of alkali are present, the floc may be washed free of sulfate or chloride and appears to be true hydrated ferric oxide.

DISCUSSION

Edward S. Hopkins,[26] Esq. (by letter).—Practically applied, this paper shows the necessity of completely dispersing and oxidizing the iron salt before the addition of the alkali to produce floc. A fairly high velocity (1 ft to 2 ft per sec) of mixing must be maintained for complete dispersion, followed by a short period (5 min) of agitation to oxidize the iron by aeration. Upon the completion of this initial mixing the alkali should be added to form floc. Upon addition of the alkali a colloidal iron solution is immediately formed and by continued mixing at high velocity, coagulation occurs. A decreasing velocity through the basin beyond this point will produce a compact floc of greater tenacity than if uniform speed is maintained. Such a floc settles readily. This is an important factor in mixing-basin design and should be considered in future plans. It is believed that trouble with "pin-head" floc can be frequently traced to insufficient velocity or time of mixing. Microscopic examinations indicate that the physical properties of floc depend in a large measure on mixing conditions.

Coagulation of color in waters is a different problem chemically from that of turbidity. The sulfate ion, obtained from either alum or ferrous sulfate, is an important component of the floc and is of equal value to the iron or aluminum ion for coagulation of turbid waters. Color combines only with the aluminum or iron ion forming the flocculated organic particles of color floc. This coagulation is best at very low pH-values (4.0 to 5.0). When alum is used, it is becoming good operation practice to settle the color floc, then add coagulant and alkali, thereby raising the pH, with additional mixing to coagulate the turbidity followed by secondary sedimentation, the final effluent passing to the filters. The data presented in this paper, together with information from the Montebello Laboratory, indicate that if the color is coagulated by iron salts in the initial part of the mixing basin, alkali may be added in the latter part, necessitating only one sedimentation period. Analysis of floc thus obtained gives a formula of $Fe_2O_3 \cdot 3H_2O \cdot 2SiO_2 \cdot 4X$, when X represents the organic color material in combination. These results indicate that iron salts may be more advantageous for color coagulation than alum.

This study indicates that coagulation with iron salts reaches a maximum between pH-values of 6.0 to 6.8. As stated by the authors, it is at variance with previous work done in the Montebello Laboratory. Their assumption that insufficient oxidation of these salts explains this difference appears to be correct and should be accepted unless future work reverses it. Floc formation with chlorinated copperas showed clearly that when the iron was completely oxidized by chlorine, precipitation occurred at very low pH-values (4.0). Iron and lime coagulation is utilized in the Baltimore plant to remove manganese from the supply. This element is not adsorbed by this treatment below a pH-value of 9.0; therefore, no contemporary data are available relating to the effect of oxidation at lower values.

---

[26] Prin. San. Chemist, Montebello Filters, Baltimore, Md.

By presenting fundamental data of this type to a group of engineers, the authors have shown clearly the necessity of trained chemical personnel in charge of plant operation. This is particularly true for the small water plant; the large ones have such personnel. An excellent junior engineer in charge of plant construction is frequently given its operation after completion, with inefficient results. Under such circumstances the individual is not at fault; it is the wrong type of training that he possesses. Coagulation control is a highly complex problem in colloidal chemistry; therefore, operators should be fundamentally trained in this science thereby giving them a better understanding of the problems involved. Considerable improvement in operation would also result if a closer co-operation could be secured between the designing engineer and the chemical engineer. As is well known, the characteristics of every water are different and, therefore, each source of supply, apart from its engineering problem, is a chemical problem. The advice of the chemical engineer relating to details of plant design would be of mutual benefit and would assure a more smoothly operated plant than is now found in many localities. The architect is needed to provide a beautiful plant, the chemical engineer to minimize operating troubles.

W. D. HATFIELD,[27] ASSOC. M. AM. SOC. C. E. (by letter).—Until recently the coagulation of natural waters by means of iron, lime, and aluminum salts, before sedimentation and filtration, has been more of an art than a science. Although a well-trained chemist was best fitted to study the idiosyncracies of water treatment, even he had to use "cut-and-try" methods which, combined with hard-earned experience, would produce a satisfactory drinking water with a minimum of color, odor, and taste. Each water has been a distinct problem (some more than others) and will remain a distinct problem for some time to come. However, in recent years, material advances have been made in the knowledge and theory of inter-ionic activities. R. S. Weston, M. Am. Soc. C. E., first used pre-chlorination to aid in color removal. Soon afterward numerous investigators began studying the important action of the concentration of hydrogen ions on iron and aluminum flocs and color colloids. For a while the pH, or hydrogen-ion concentration, was believed to be the true governing factor in coagulation, but, later, this guide was found to vary with different types of water; in some cases the pH-zone of coagulation was narrow; in others, it was wide; in some cases, the optimum pH for coagulation was 6.3, in others it was 5.3, and in some water it was as low as 4. In the latter case iron had to be used as the coagulant because alum would not coagulate satisfactorily at a pH of 4.0. When one coagulant would not work another, and then another, was tried.

The individuality of a natural water is due to the compounds it contains in solution, and to the equilibrium between these compounds and the coagulant added. This phase of water treatment has been sadly neglected. The authors have made a real contribution to this field, by showing the effect of the chloride, sulfate, sodium, and calcium ions in conjunction with hydrogen and hydroxyl-ions on the coagulation of the ferric ion. Such contributions as these will soon completely remove water purification (coagulation) from an art to a science which is based on a definite knowledge of the part played by each individual component of the solution.

---

[27] Supt., Sewage Disposal Plant, Decatur, Ill.

The use of ferric salts in water treatment followed its use by Messrs. F. W. Mohlman and J. R. Palmer[7] in the coagulation of activated sludge in sewage treatment. The practical problems in water, sewage, and sludge treatment differ greatly, but fundamentally the theory on which coagulation of iron salts rests, must be basic and applicable in both fields. It is hoped that the authors will carry to a conclusion these ionic studies, using all the ions found in waters, sewages, and sludge liquors. Perhaps even the sewage field may some day rest on a firmer theoretical basis, due to a clearer knowledge of these ionic activities.

L. B. MILLER,[28] ESQ. (by letter).—In addition to summarizing clearly the pertinent work previously done on ferric coagulants for water clarification, its significance, and limitations, the authors have made a most significant contribution to the theory and practice of the use of ferric coagulants. The zone of non-floc formation between the pH-values of approximately 7.0 to 8.0 could not be anticipated; and, indeed, in some of the earlier work in which more concentrated solutions were used than those by the authors, it remained unnoted. The importance of this zone of non-floc formation in the practical application of ferric coagulants to water clarification cannot be over-estimated. It indicates clearly the conditions under which the water-works engineer must operate, in order to achieve optimum results.

The theory of the use of additional salts, such as those of alkali and lime, to control, partly, the formation of floc in the zones in which ferric coagulants exhibit a reluctance to floc formation, is interesting, and the practical significance of the application of this theory may be of considerable importance in certain cases.

The comparison of alum with ferric coagulants in the clarification of waters highly colored by organic matter, where clarification must be carried out at low pH-values, the advantages, and limitations of each of these types of coagulants, are likewise very opportunely emphasized.

LINN H. ENSLOW,[29] ASSOC. M. AM. SOC. C. E. (by letter).—In Nature a certain amount of natural coagulation is constantly going on, in which iron salts play an important rôle. Soluble iron is contributed through reducing reactions on iron compounds in the rocks and soil. Later, upon becoming oxidized, the iron thus brought into solution flocculates in running streams and at or near the surface of water bodies at rest. It is highly probable that such natural coagulation is responsible, to a considerable extent, for the gradual lessening of organic color in new reservoirs, more particularly the deep ones. In the latter the same iron is returned to the solution stage after having settled to the bottom and is released to the upper strata to flocculate anew.

---

[28] With Johns-Manville Research Laboratories, Manville, N. J.

[29] San. Engr., The Chlorine Inst., Inc.; Editor, *Water Works and Sewerage*, New York, N. Y.

Diffusion, wind action, and turnover of reservoir contents due to wind action and temperature changes, serve to complete the precipitation-solution-precipitation cycles.

Whenever the writer notes references in the literature that laud the work of Mr. L. B. Miller, who in 1923 to 1925 studied coagulation effects and the composition of the floc produced under varying conditions, he is always hopeful that some one will look a little farther back and give some of the deserved recognition to an earlier worker, Dr. Frank E. Hale, who did not have the new "constellation," pH, to serve as the remarkable tool that it is in coagulation studies and control. Hale, working with various waters, reached the conclusion that color removal was due primarily to the formation of a complex compound from the acidic color compound and the amphoteric hydrate of alumina. He explained why effective flocculation and precipitation did not necessitate the theoretical quantities of alkali in certain waters, and his paper[30] abounds in theoretical discussion and conclusions which have been substantiated in practice or confirmed by subsequent investigators—but rarely, if ever, with acknowledgment.

Early Work with Chlorinated Copperas.—The origin of chlorinated ferrous sulfate (chlorinated copperas) can be traced to the laboratories of the Chicago, Ill., Sanitary District. Ferric chloride had proved considerably superior to alum and other coagulants for conditioning sewage sludge for dewatering. Its "drug store" price was the chief obstacle to its use. The simple expedient of oxidizing ferrous sulfate solutions to produce a mixture of ferric sulfate and ferric chloride was the outgrowth of a search for the most inexpensive ferric coagulant obtainable. Its efficiency was lower than that of ferric chloride, but the over-all cost to obtain desired effects was sufficiently less to cause its adoption at Houston, Tex., for activated sludge dewatering. Shortly afterward, the City of Milwaukee, Wis., followed in quick order and although chlorinated copperas can still be used at a lower cost at Houston, Milwaukee found it less costly to purchase, on long-time contract, a supply of ferric chloride in solution which is shipped in rubber-lined tank cars. Since then several sewage plants—notably Gastonia, High Point, and Charlotte, N. C., and Pasadena, Calif.—have supplanted other conditioning agents with ferric chloride.

Coagulation of Water.—Following the successful adaptation of chlorinated copperas to sludge dewatering, the writer made several tests of its use as a coagulant for water and industrial wastes. It was soon apparent that it was a coagulant considerably superior to others in the low pH range (5.5 and less) and the pH range greater than 8.5. The outcome of this was its introduction at Elizabeth City, N. C., where studies were conducted on a plant scale by L. L. Hedgepeth, in co-operation with the writer.[18] Its use at Chickasaw, Ala., and at other points followed, notably its adoption at Providence, R. I.,[22] and Dallas, Tex. It can be stated with reasonable assurance that ferric iron salts are superior coagulants in dealing with color removal from water and with organic matter removal from other liquids, such as trade wastes and sewages. This statement applies in the matter of costs as well as efficiency.

---

[30] "The Relation Between Aluminum Sulphate and Color in Mechanical Filtration," by Dr. Frank E. Hale, *Journal of Industrial and Engineering Chemistry*, Vol. 6, August, 1914, p. 632.

In some instances, notably when oxygen is present to a sufficient extent and the pH is 9.0, or more, ferrous salts will serve. The chief drawback to the use of unoxidized iron is the fact that effective coagulation is slow, and frequently indifferent. The fact that ferrous hydroxide must first be produced and that thereafter it changes to the insoluble ferric hydroxide, or to some more complex compound, requires more time and longer mixing periods, than is the case where the iron is pre-oxidized, in part if not in whole, before introduction to the liquid being treated. In coagulating creamery wastes ferrous sulfate coagulates primarily as a light and feathery floc of ferrous hydroxide, and but little reaches the ferric state because of lack of oxygen.

Relationship Between pH-Values, Effectiveness of Coagulation, and Unprecipitated (Residual) Iron.—Of particular interest to the writer are the observations of the authors as illustrated in Fig. 10, revealing the pH-zone of quickest floc formation and the most perfect precipitation of the iron added in the form of chlorinated copperas. The curves, indicating that in the zone between pH 4.5 and pH 6.5 iron removal was complete and practically complete between pH 3.8 and pH 7.0, and, similarly, that floc formation was most rapid over the same range, might have been drawn by plotting actual operating data at the filter plant in Elizabeth City, N. C., during the periods of operation with which the writer is familiar.

Using colorless water fortified with calcium chloride the studies made in the laboratory of the University of Iowa show that the floc, if formed at all, was very poor between pH 7 and pH 9.5, but the residual iron was satisfactorily eliminated at pH 9.5 and beyond. Such high pH-values can not be considered as of practical use, in dealing with soft organic waters. Color removal would show a minus efficiency because alkalinization not only disperses color, but intensifies it, due to the fact that some of the organic compounds met with in water act as pH indicators to the extent of showing deeper color with increasing pH-values.

In practice, when coagulating with ferric salts at pH less than 7, the appearance of residual iron indicates the need for increased dosage of coagulant; and that is actually done in practice to meet the circumstance rather than reducing the iron applied or increasing the alkali, if the latter is used. It has been demonstrated in laundry waste treatment that acidification with sulfuric acid serves almost the same purpose, provided the necessary minimum of ferric iron is present to produce the floc effectively and give it the high gravity desired.

The authors made some reference to the fact that the quantity of ferric iron was the all-important consideration, regardless of the form in which it is used. In water treatment and sewage coagulation that appears to be approximately correct, but in sewage-sludge conditioning, to the acid side on the pH-scale, it does not hold. Ferric chloride remains superior to chlorinated copperas which probably has 66% of the iron present in the sulfate form, whereas only 33% is converted to the chloride during oxidation of the copperas with chlorine.

New Sources of Ferric Iron.—During the past year there have appeared on the market two ferric sulfate coagulants—one of which is being successfully used at Providence, R. I.[31] Both these products are dry and subject to handling and feeding in the same manner as pulverized sulfate of alumina—filter alum.

C. J. Brockman,[32] Esq. (by letter).—For too many years the purification of water through the use of chemicals has been thought to be merely the addition of alum, aluminum sulfate, or iron sulfate. Water-works operators in many cities and towns to-day believe that, when some of these chemicals have been added in the treatment process, nothing more can be done to improve the efficiency of the filtering process or the quality of the water.

It has not been many years since the term, pH, was introduced. The pioneer work of Messrs. W. M. Clark and E. J. Theriault, L. B. Miller, and others now too numerous to mention, have afforded an insight into the chemistry of the coagulating process in water-works control.

In the light of the work of these three pioneers, it appeared to be necessary to control the pH of the water within certain limits in order to coagulate the floc efficiently. Unfortunately, this simple procedure is not adequate as has been so ably shown by the authors. "Alkalinity determination" is no longer sufficient. It is now known that "alkalinity," pH, also chloride and sulfate content must be determined before efficient coagulation can be accomplished. It was indeed fortunate that the cheapest aluminum salt in the early days of water treatment was the sulfate and not the chloride, because the latter is more difficult to floc than the former. Consequently, water-treatment processes would have developed much more slowly.

Messrs. Bartow, Black, and Sansbury have shown that the iron floc formation in even such small concentrations as 25 ppm of sulfate ion takes only one-fifth the time as in an equivalent concentration of chloride ion when the pH is about 3.5 to 4.0, as will be the case with many colored waters. With larger values (8 to 9.5) for pH, the presence of sulfate ions also permits more rapid coagulation. They also show that, compared with previous knowledge, chlorinated copperas is more easily made than ferrous sulfate.

Over and above this, the most interesting phenomenon discovered is that the metal ion content of the water has a pronounced influence on the time of floc formation. This is a most outstanding fact of this contribution. In the range of pH of 7 to 9, it is shown that a solution of 5 ppm of chloride, as calcium chloride, is many times more satisfactory than 25 ppm of sodium chloride, and the coagulation range is wider. In the case of sodium sulfate as compared with calcium sulfate, the differences are not so marked; in the lower concentration ranges, the former permits more rapid coagulation and a wider band of floc formation on the acid side, but calcium sulfate allows a wider band on the alkaline side.

---

[31] "Advances in Iron Coagulation and Coagulants," by E. L. Bean, *Journal*, New England Water Works Assoc., Vol. 47, September, 1933, p. 273.

[32] Assoc. Prof. of Chemistry, Univ. of Georgia, Athens, Ga.

The implication which these facts have on water treatment is that the control of the process has developed into a complicated chemical problem. An efficiently operated water treatment plant must be in the care of a trained chemist, or, at least, under the not-too-distant supervision of some one capable of the necessary directive measures. Accurate control is now more than ever, of prime importance in the water-works.

Water-works chemists will await with interest a completion of this work which will establish definitely the sign of a change carried by the colloidal floc at various pH-values on both sides of the neutral point, because the treatment of colored water is still a serious problem.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 1897

MODIFYING THE PHYSIOGRAPHICAL BALANCE BY CONSERVATION MEASURES

By A. L. Sonderegger,[1] M. Am. Soc. C. E.

With Discussion by Messrs. H. H. Chapman, E. B. Debler, Frank E. Bonner, C. S. Jarvis, W. P. Rowe, J. C. Stevens, Gerard H. Matthes, J. B. Lippincott, Rhodes E. Rule, and A. L. Sonderegger.

Synopsis

In Nature, precipitation, erosion, and débris transportation and deposition by stream flow combine to produce, in some measure, a physiographical balance which, while not permanent, undergoes only gradual changes. This balance is expressed in the topography and cover of the mountain water-shed as well as in that of the flood-plain of a drainage area.

Interference with the natural processes of erosion, débris transportation, and disposal of flood waters of the stream of a water-shed by partial or complete regulation, may result in a modification of this physiographical balance to such an extent as to menace existing or future development. This modification of balance may be a slow process, apparent only after a considerable period of time and, for this reason, may be overlooked or considered as negligible; yet it may prove to be the undoing of otherwise well-laid plans.

The effect of interference with natural conditions is accentuated in the semi-arid regions because of the greater extremes of precipitation and other climatic factors which tend toward the accumulation of débris during dry periods. In the Southwestern United States, the regulation of water supply and the control of flood waters has brought about the erection of works of unprecedented size, which regulate large floods and permit the artificial disposal of the entire stream flow.

This paper is intended to call attention to the effect, on the physiographical balance, of the following conservation measures: (*a*) Changes in the water-shed cover; (*b*) effect of regulation on the natural balance of a

---

Note.—Published in December, 1933, *Proceedings*.

[1] Cons. Engr., Los Angeles, Calif.

stream system; and (*c*) effect of débris barriers on the stability of the stream bed on the débris cone.

In the examples presented, it was not so much the intention to discuss in great detail the phenomena which are affected by the modifying of the physiographical balance, but rather to illustrate the theory. For this reason, the problems arising in the mountain water-shed and those which arise in the transportation of débris are not discussed exhaustively.

1.—The Physiographical Balance

The physiographical appearance of a country is generally the combined result of orogenic (mountain-forming) movement and climatic conditions. Crustal movement is responsible for the basic physical features of a country. This basic relief is, however, gradually and continuously modified by climatic factors.

Of the climatic factors, precipitation and the processes accompanying its disposal, are principally responsible for the existing aspects of a country. Although the distribution of precipitation may vary in intensity with seasons and periods, past performance indicates, nevertheless, that over long periods its general characteristics and effects may be adjusted to a mean of conditions governing changes in terrestrial appearance which, to the human eye, is seemingly more or less permanent and unchanging. In other words, despite their erratic character, climatic factors in general, and precipitation in particular, produce in Nature what might be termed a state of equilibrium, or balance, expressed in a certain type and density of native vegetation, in more or less stable gradients of stream beds, in the disposal of débris, in the slopes of the plains, etc. It is a balance of natural forces which is seemingly maintained while, at the same time, the phenomenon of stream erosion gradually produces changes on land surfaces.

Any long-sustained change in the mean precipitation would naturally be followed by an adjustment to new conditions. A decrease in rainfall of 20% may transform habitable semi-arid areas into uninhabitable deserts. An increase in the mean precipitation by the same amount would result in less severe droughts, increased mean water supply, and in flood conditions of much greater severity. The vegetal cover of the water-shed would adapt itself gradually to the more abundant supply, becoming more profuse, but possibly less drought-resistant. Changed run-off conditions would gradually alter the regimen of the watercourses, including their gradients, stream beds, and character and volume of débris. Slopes of the plains, too, would be affected as well as the vegetation thereon; and ground-waters would rise above their former levels. In other words, the face of the country would alter gradually, but inevitably, until the new balance of geophysical forces becomes seemingly established, in conformity with the changed conditions.

The effect on human life of a long-sustained, substantial increase in wetness might be as far-reaching as that of greater prevalence of drought. In fact, it is conceivable that the disturbances during the period of transition may make human habitation in specific localities difficult, if not impossible, to be maintained.

If, then, there is a natural balance of physical effects, as the result of ages of adjustment to existing conditions, it is logical to infer that any interference therewith by human agencies is likely to result in disturbances. Those disturbances may, or may not, become apparent immediately, but are sure to follow and to remain as long as the cause exists. The very fact that such results are not immediate, or become felt only gradually, may be the reason why they are sometimes overlooked or under-estimated until they have assumed proportions that require heroic counter-measures, or may even have passed beyond control—comparable to the experience of the sorcerer's apprentice who conjured the spirits of witchcraft, but was unable to dismiss them.

A simple, every-day occurrence may serve to explain the continued and relatively far-reaching effect of interference with Nature's equilibrium. Under natural conditions, the gently sloping sides of the foothills are covered with certain types of vegetation which prevent the washing of soil and which are inducive to absorption of rainfall, producing a normal condition preventive of flood run-off, thus affording a physiographical balance on these foothills. If these slopes are cleared and cultivated, erosive action will immediately be noticeable, even if protection by cover crops is provided. It is not uncommon, therefore, that after some years the plow may strike an irrigation pipe line which originally had been buried the customary 2 ft under the surface.

Precipitation causes innumerable rivulets and barrancas on such slopes; flood waste is increased, while deep penetration is diminished, contrary to the opinion sometimes advanced that plowing favors larger absorption of rainfall. Where such slopes are devoted to residential settlement, rainfall penetration will be diminished further, augmenting erosion and flood waste to such proportions as to necessitate costly terracing and the construction of lined storm channels. The one effect which may not be immediately apparent, but may be the most far-reaching, in the long run, is accelerated erosion.

It is concluded that the physiographical condition of an area is the result of the activity and balance of natural processes of precipitation, erosion, and débris transportation and deposition by stream flow over a long period. A pronounced or protracted modification of these phenomena will initiate changes tending to establish a different physiographic balance. Although these changes may appear to be gradual and negligible, their effect, nevertheless, is irresistible and in due time will upset existing conditions.

2.—Changes in the Water-Shed Cover and Their Effect on Run-Off Conditions

Plans for the "improvement" of the water-shed for the purpose of increasing the water yield have been advanced of late years from many quarters. Radical changes in the water-shed cover have a more or less protracted effect on débris production, run-off, and the capacity of the stream to move its load. Aside from the timely agitation for the reforestation of cut-over or burnt-over timber lands, the proposals may generally be segregated into two classes:

First.—"Super-forestation"[2] to the effect of producing a denser growth of native or other vegetation on the water-shed; to "keep the water supply in the water-shed," as the saying goes; and to induce more copious precipitation.

Second.—"Light deforestation" to restrict the vegetation to a minimum compatible with effective prevention of erosion, with a view to reducing the evapo-transpiration requirements of the cover.

Super-forestation (and forestation for that matter) is analyzed as an effort to increase the storage of meteoric water in the water-shed by promoting profuse vegetation which naturally tends to absorb rainfall and diminish flood run-off. By this means surface soils may be better protected from washing, and débris production may be reduced. However, a dense vegetation may also be the cause of more intense forest fires and greater difficulties pertaining to fire protection and fire-fighting. Indirectly, the more complete destruction of plant life by fire, as a result of super-forestation, may be the cause of retarding re-growth and exposing the water-shed to a longer period of abnormal erosion.

Under the caption of reforestation comes the proposal of replacing chaparral by trees. However, since chaparral recovers naturally and more rapidly from fire than trees which would have to be replanted, the effect on the water-shed cover would be decidedly disadvantageous and would be most likely to protract the period of abnormal erosion after a fire.

Efforts at "super-forestation" on a large scale are not likely to meet with success. The general type of the forest cover of a region is indigenous to it and reflects its climatical conditions—mean temperatures, mean precipitation, and other climatic factors—while the range in precipitation and temperatures is reflected in the tolerance of the species, all to the effect that Nature has produced the vegetal cover which, both as to species and density, is best adapted to prevailing conditions. Any attempt, therefore, to induce denser growth is likely to fail, due to the great "weeding-out" process expressed in the law of the survival of the fittest.

Light deforestation is to be considered an effort to reduce underground storage in the water-shed and the evapo-transpiration losses of the cover, at the expense, however, of a more rapid, flashy, surface run-off and correspondingly increased erosive action. Under this category may be grouped: (*a*) The systematic burning of the water-shed; (*b*) the substitution of light water-using species for heavy water users; and (*c*) the cutting of trees which grow along the watercourses, such as alders, willows, sycamores, and the like. Similar results as to light deforestation would result from a diminished vigilance relative to forest fires.

The practicability of these proposals has never been put to a test. The most far-reaching plan in its effect on conditions of stream flow and débris production, and the one entailing the greatest risk, would be the systematic and periodic burning of the chaparral. This has been under discussion and advocated in some parts of California—first, with a view of increasing the water supply; and, second, to prevent the accumulation, over long periods, of masses of highly inflammable débris which results under the existing official policy of protection of the water-shed from fire. Burning is proposed to be done in checker-board fashion at the end of the wet season when humidity is relatively high and when soil moisture is sufficient to produce new sprouts of brush, grasses, and herbs.

---

[2] "Forests and Stream Flow," by W. G. Hoyt, M. Am. Soc. C. E., and H. C. Troxell, Assoc. M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 99 (1934), p. 1.

This proposal and others is of a highly controversial character. Although advocated in certain localities with considerable popular and political pressure, those in charge of water-sheds and responsible for damages, should the burning get beyond control, have been slow in yielding to the demand. In a report of the Ventura County (California) Farm Bureau[3] relative to the burning, it is stated:

> "The general consensus of opinion among those with great experience in fire-fighting is that when it is dry enough to burn, it is too dangerous to burn, and when it is safe to burn, the vegetation will not burn."

The effect of denudation by fire manifested in flashy, heavy flood flow and abnormal débris load, the possible destruction of surface storage, and the flooding of the plains, is too well understood to require discussion. A point sometimes overlooked is the sealing effect of ashes on the stream bed, which greatly reduces natural seepage, to the extent of more than offsetting the increase in the water yield of the burnt-over water-shed.

However, forest fires will be a factor as long as the public is allowed free access to the mountain areas. They will have to be reckoned with to an increasing degree in the planning of flood-control and conservation measures.

The forest cover unquestionably has an important constructive function in Nature, which may not be fully appreciated because it is not well understood. Its preservation should be uppermost in the minds of water conservationists because of the far-reaching and lasting effects that destruction or denudation of the water-shed will have on flood flow and on the production and movement of débris. It is not generally recognized that domestic water supply, irrigation, and hydro-electric enterprise have rights in the control of the water-shed. In settled regions, the value of the water-shed for natural storage and regulation of run-off may be superior to any other uses, such as grazing, recreation, or even the commercial exploitation of its timber and, therefore, should receive due consideration.

In the past there has been little or no co-operation between the forester, the engineer, and the water user, except to the extent of protection of the water-shed from fire hazard. A step toward a better understanding of the common problem was undertaken by the Committee on Water Conservation of the Irrigation Division of the Society, by calling a conference[4] of research workers, engineers, foresters, and agriculturists. This meeting resulted in the outlining of a program to correlate the work of the various groups relative to conservation of water supply.

---

[3] Mimeographed Rept., Conservation Committee, Ventura County Farm Bureau, April 8, 1931.

[4] Mimeographed Rept. on "Conference of Research Problems in Consumptive Use of Water and Conservation of Rainfall, Los Angeles, Calif., March 27, 28, and 29, 1930."

As yet, the function of the water-shed in its ramifications is not well understood. The United States Forest Service in late years has initiated a comprehensive program of research with a view of increasing the water yield and decreasing the fire hazard. When these investigations have progressed to the point where definite conclusions can be drawn as to the practicability of the proposed plans and their ultimate effect on the water yield, a logical solution may present itself. From the standpoint of general economics and efficiency of flood control and conservation, proper co-ordination of the improvements of the water-shed to those of the flood channel becomes a necessity.

3.—Effect of Regulation Upon the Natural Balance of a Stream System

The work performed by a stream or stream system may be segregated into three distinct functions: First, erosive action on the mountain water-shed; second, the transportation of débris; and, third, the building of débris slopes, or flood-plains.

With undisturbed conditions, each of these activities depends directly upon the combined effect of stream flow and available débris. Since both stream flow and volume of débris are subject to great and, sometimes, erratic fluctuations, it becomes apparent that the magnitude of these uncontrolled forces is forever variable, without attaining uniformity for any great length of time. However, available records indicate that, considering cycles of several years, these stream activities occur between limits that are defined by droughts and capital floods, so that in the intervals between such extremes there is a constant tendency to 'iron out" the effects of individual floods or periods of drought. There results a state of balance which is characteristic for a stream, or stream system, termed its "regimen." It finds expression in the gradient of the flood-plain and the character of its débris and alluvium.

Stream activity follows lines of least resistance and, therefore, will tend to establish the steepest gradient consistent with the basic factors—stream flow and débris. Only long-sustained climatic changes or the exposure of different geological formations, or a change in botanical cover over large areas of the water-shed, would materially affect the normal gradients and change the character of the débris of the flood-plain. The characteristics of a stream system, therefore, can be established by a study of its deposits and of its gradients.

Generally speaking, and in comparison with the time allotted individual human experience, water-shed erosion and the construction of the flood-plain are relatively slow, but ceaseless, processes.

Nevertheless, if, as a result of the construction of an impounding reservoir, the transportation of the débris is interrupted for a period of years, or if it is stopped, a permanent effect on the stream bed and flood-plain below is certain to follow.

If the stream bed is used for the diversion of the impounded water, the clarified effluent—having the capacity to load itself anew with sediment—will begin to scour and meander, reducing its gradient and changing its wetted perimeter. These new conditions, acting on the alluvium of the former natural stream, therefore, will establish a new state of equilibrium, in due time, expressed in flatter gradients. A further and vital change will occur at the point below the reservoir where the stream intercepts the first major tributary.

With a stream system functioning normally, there is a balance of forces between the flow of the main stream, its gradients, and the débris carried by it and those similar functions of its tributaries. The addition of a tributary brings about a flatter gradient in the main stream for some distance above the junction, while at and below the junction such steeper slope is established as may be required for the movement of the added débris. The tributary, as a rule, has a lesser run-off and steeper gradient than the main stream, its characteristics being reflected in its own flood-plain where it has established the most favorable conditions for the transportation of its own débris under its own conditions of flow.

If the main stream is regulated while the tributary remains unregulated, sooner or later conditions at the junction of the two will become critical because of the absence of the flood flow in the main stream, which formerly was available for the removal of the sediments and débris from the tributary. The tributary, now having to assume the rôle of the main stream, cannot cope with the wider stream bed and flatter grade of the latter, hence the deposition of sediments, a raising of both stream beds, and inundations at the junction.

A natural correction of these conditions cannot take place unless storage is released at intervals and in sufficient quantities to cause the movement of the excess sediment. However, since this remedy may not prove effective without serious infringement on storage, the only other solutions will be: (*a*) To improve conditions of flow in the main stream below the reservoir by straightening the alignment, to line the channel to meet the flow requirements of the tributary, or both; (*b*) to regulate the tributary itself by means of surface storage, as in the case of the main stream; and (*c*) to construct débris barriers on the tributary for the purpose of reducing its load.

In any event, if tributaries are of sufficient magnitude, the problem may prove to be a baffling one and not easily solved. If it can be solved at all, it may require the expenditure of relatively large sums, or the periodical removal of débris.

The depth of deposits when spread over a wide flood-plain even during a period of years, is relatively insignificant and its absence would not be noticeable or of material effect in a life time. From the geological standpoint, the volume of débris stored in the largest reservoirs is small.

Some observations pertinent to the foregoing discussion have been made in Southern California. During a series of dry years, the San Gabriel River with 218 sq miles of drainage area may have little or no flood run-off, because all the normal flow has been diverted, so that the stream bed remains dry over long distances below the mouth of the canyon. During the same period, the tributaries flowing in more or less trained channels and receiving the run-off from urban areas, have high, flashy silt-laden flows with every rain. At the point of junction with the river, this silt is deposited in the river bed, filling it enough so that inundation of adjacent lands during floods becomes a possibility.

In the Western States, the regulation, and, in some cases, the ultimate complete diversion of the flow, of large rivers form the bases of many reclamation and flood-control projects.

The regulation of a stream presents one problem and the effect of this regulation upon the tributaries becomes the cause of another problem. Although this effect may be gradual and over a period of normal stream flow create no menace, a new condition has been initiated which must be met sooner or later. In planning regulation works, this phase should receive the engineer's attention, as affecting either the quantity of storage that may become available for beneficial use, or requiring measures for the improvement of tributaries. The question arises in particular, as to whether it is feasible to regulate the capital flood run-off for irrigation or domestic use. In some cases this has been advocated regardless of costs when such flood flow might be wasted more profitably for the removal of débris.

Experience on the Rio Grande, in Texas, as a result of the construction of the Elephant Butte Dam, will serve to illustrate the foregoing analysis. In that case the problem is complicated by the use of the river channel below the reservoir for the conveyance of irrigation supplies.

4.—The Effect of Regulation of Rio Grande on Flood Conditions Below Elephant Butte Dam

The Rio Grande drainage basin above El Paso, Tex., occupies the south central part of Colorado and the central part of New Mexico. Drainage areas are divided as follows: Above San Marcial, N. Mex., 26 700 sq miles; above Elephant Butte Dam, 28 360 sq miles; and above El Paso, 30 460 sq miles. Fig. 1 shows the principal geographic and topographic features of Rio Grande Valley from San Marcial (Elevation 4 458), at the north end of Elephant Butte Reservoir, to Fabens, Tex. (Elevation 3 619) on the Rio Grande, 26 miles below El Paso.

Elephant Butte Dam is located across the bed of the stream about 125 miles above El Paso, and creates a reservoir of about 2 600 000 acre-ft capacity. The mean annual run-off at San Marcial, at the head of this reservoir, is roughly 1 200 000 acre-ft. The entire discharge of the Rio Grande has been retained in the reservoir (except as released for irrigation use) since storage operations started in 1915. The highest recorded discharge at San Marcial prior to 1930, was 33 000 sec-ft and the highest passing El Paso prior to the construction of Elephant Butte Dam was 24 000 sec-ft. The highest discharge at El Paso since the construction of Elephant Butte Dam is 13 500 sec-ft. The large floods which occurred at intervals of several

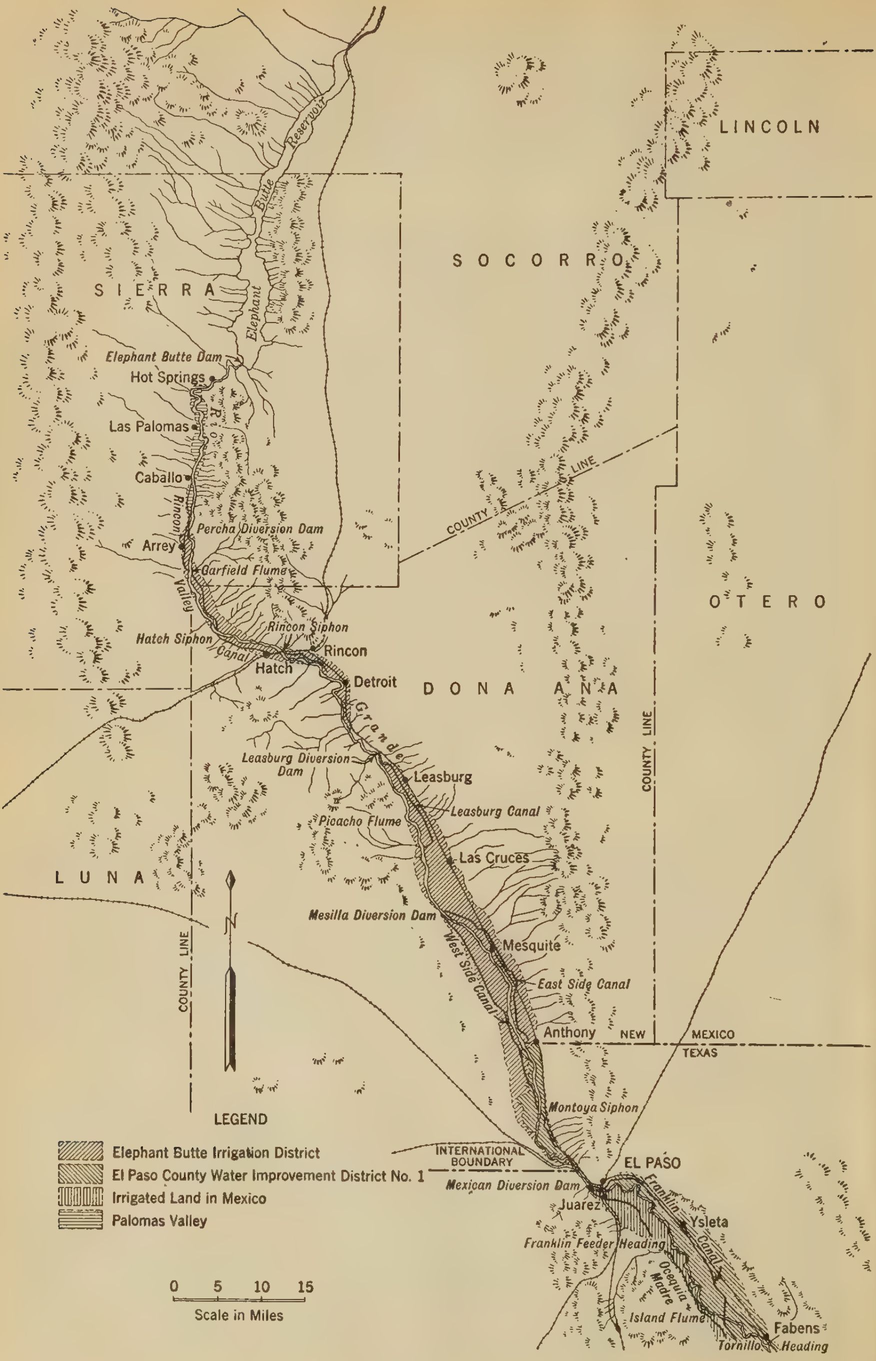

LINCOLN
SOCORRO
SIERRA
Elephant Butte Reservoir
Elephant Butte Dam
Hot Springs
Rio
Las Palomas
Caballo
Rincon
Percha Diversion Dam
Arrey
Garfield Flume
Valley
COUNTY LINE
OTERO
Rincon Siphon
Hatch Siphon
Canal
Rincon
Hatch
Detroit
DONA ANA
Grande
Leasburg Diversion Dam
Leasburg
COUNTY LINE
Leasburg Canal
Picacho Flume
Las Cruces
LUNA
N
Mesilla Diversion Dam
West Side Canal
Mesquite
COUNTY LINE
East Side Canal
Anthony
NEW MEXICO
TEXAS
Montoya Siphon
LEGEND
▨ Elephant Butte Irrigation District
▧ El Paso County Water Improvement District No. 1
▥ Irrigated Land in Mexico
▤ Palomas Valley
INTERNATIONAL BOUNDARY
EL PASO
Mexican Diversion Dam
Juarez
Franklin Canal
Ysleta
Franklin Feeder Heading
Acequia Madre
Island Flume
Fabens
Tornillo Heading
0 5 10 15
Scale in Miles

Fig. 1—Rio Grande Drainage Basin.

years, prior to the construction of the dam, covered the valley lands and often established new locations for the main river channel. In 1912, a peak of 16 000 sec-ft at El Paso flooded an area of 15 900 acres of land in the El Paso Valley. Except for overflow confinement by road and canal embankments the flooded area would have been much greater.

Prior to the construction of the dam the channel was shifting, due to its lengthening during years of low flood and to erosion and change in alignment during higher floods. Silt entering above the dam is 1.6% of the flow, by volume, the average annual silt burden being 20 000 acre-ft. The slope of the river below Elephant Butte Dam measured along the thread of the stream (as determined by a survey made in 1917) was about 4.00 ft per mile for the first 100 miles, or to the Mesilla Dam; 3.50 ft per mile from Mesilla Dam to Canutillo, Tex., 30 miles; and 2.8 ft per mile from Canutillo to El Paso, 15 miles. Below El Paso, the river had gradients as determined from the survey made in 1917 of from 2.45 ft per mile for the 6 miles immediately below the International Dam, to 1.45 ft per mile to the vicinity of Fabens, 50 river-miles below. River gradients through El Paso Valley, as determined from a survey made in 1907, varied from a slope of 1.83 ft per mile for the first 20 miles below the International Dam to 1.54 ft per mile near the 60-mile point. Other stretches had gradients of as much as 2.70 ft per mile.

River gradients as determined from the survey of 1932 show marked changes from the foregoing. In the reaches immediately below the dam heavy detritus from arroyos which the present river flow is unable to move, has formed rapids, resulting in a decrease in gradient. There is usually a decrease in gradient below the dam, although this is doubtless due to scour. Several artificial cut-offs have been made and these have caused scour above and filling below. Immediately below the International Dam, at El Paso, filling has taken place so that now the river bed is 12 ft higher than in 1907 and 6 ft higher than in 1917. This is in marked contrast to the action at the other diversion dams where scour below has resulted, this being approximately 6 ft at Percha Dam, 2 ft at Leasburg Dam; and 1 ft at Mesilla Dam; all as compared to the 1917 record.

The regulated discharge for irrigation demand as released from Elephant Butte Reservoir varies from 500 to 2 500 sec-ft. The Rio Grande is used as the main carriage channel of the Rio Grande Project with four diversion dams at the heads of as many irrigation divisions in the 200-mile length of the river through the project. In addition to the flow released at the dam, the river receives drainage water and run-off from arroyos, which drain the 2 100 sq miles between the dam and El Paso. This latter run-off is sporadic. In 1916, the flow from a combination of arroyos was 8 000 sec-ft. On September 1, 1925, rains on drainage areas of several arroyos caused a discharge of 13 500 sec-ft at El Paso. Large fans of eroded material from these arroyos now (1933) extend into the river or valley lands, according to topography. These materials were formerly carried away by the river. At points where they reach the river, the irrigation supply is now continuously loaded with fine sand by the action of the comparatively clear water from

Elephant Butte Reservoir on these accumulations. The concentrated discharge from one arroyo, resulting from an isolated cloudburst, has often equalled the discharge of all arroyos during a general rain. The drainage areas of these arroyos varying from a few square miles to 400 sq miles, are partly denuded of growth.

With the completion of the storage works, and the assurance of water supply throughout the irrigation season, the irrigated area was enlarged. Drainage canal embankments and lateral canals constructed near the river channel confined the flow to a more definite location and the deposition of sediment to a limited area. The former meandering river channel became the main canal for the project. Maximum flows were reduced more than 50%, the mean annual flows throughout the project were reduced 30%, and the run-off extended throughout the length of the irrigation season of nine months, rather than being concentrated in the spring or fall.

Project diversion works, as well as principal canal structures, were provided with skimming weirs and canal wasteways to the river, which are constructed and operated to return the sand sediment to the river and to prevent large maintenance costs. As these desilting devices were perfected and general sluicing operations became more effective, sand accumulations were concentrated in the river channel. River profiles show the effect of such deposits at diversion points and at the outlets of the wasteways.

Cross-sections of the river channel before and after construction of the storage unit are of interest in expressing the actual effect of the control and regulation of discharge on the former river channel. Table 1 gives the change in areas of cross-sections below a given elevation.

TABLE 1.—CROSS-SECTIONS OF RIO GRANDE BELOW ELEPHANT BUTTE DAM (Areas, in Square Feet, Below a Given Elevation)

Section	Miles below Elephant Butte Dam	1915	1918	1932	Difference	Bottom		Side	
						Scour	Fill	Erosion	Fill
Hot Springs, N. Mex....	7	4 970	.....	4 671	—299	626	.....	.....	925
Garfield, N. Mex.......	36	.....	1 528	2 228	+700	700	.....	.....	.....
Hatch, N. Mex.........	54	.....	1 669	2 066	+397	579	.....	.....	182
Picacho, N. Mex.......	87	738	.....	1 178	+440	440	.....	.....	.....
Line 34...............	93	.....	5 359	4 301	—1 058	449	.....	.....	1 507
Line 30...............	97	.....	9 568	8 393	—1 175	.....	1 175	.....	.....
Line 25...............	105	.....	1 642	1 928	+286	286	.....	.....	.....
Line 13...............	120	.....	1 410	1 812	+402	356	.....	46	.....
Line 5................	129	.....	1 663	2 066	+403	414	.....	.....	11
International Dam.....	143	.....	.....	.....	......	......	.....	.....	.....
Colorado..............	144	.....	(1929) 2 213	1 769	—444	.....	448	4	.....
Socorro, N. Mex.......	168	.....	(1927) 320	162	—158	.....	158	.....	.....
Flume.................	184	.....	(1920) 1 548	1 270	—278	.....	278	.....	.....
Line 61...............	236	.....	(1926) 696	443	—253	.....	218	.....	35
Line 65...............	267	.....	679	519	—160	.....	140	.....	20

As may be expected, the decrease in flow at the lower end of the project, due to the consumption for irrigation purposes, has resulted in a closing in of the channel. This is shown in Table 1 for all sections below the International Dam. Furthermore, side fill is shown in many of the sections and immediately above the Mesilla Dam, "Line 30," bottom fill has resulted due probably to the checking effect at that dam. The absence of large destructive floods has afforded no opportunity for the former scouring action, and the river channel below El Paso has filled with sand sediment to a point where it is higher in many places than the adjoining farm area. As the channel thus conforms itself to the normal irrigation flow, floods from excess rainfall on the drainage area below the dam cause the overflow of the low-lying areas adjoining the river channel. The elevation of the river channel, of course, is no new condition. Practically all the valley lands were formed by river deposits of silt and sand from upper drainage areas.

Thus, the outstanding characteristics of the present regimen of the Rio Grande are: (1) The entire absence of large floods to scour the bottom of the river bed; (2) continuous flow in the river of a quantity sufficient to cover the necessities of irrigation, a flow that produces a continuous accumulation of sediment which is deposited in that part of the valley below El Paso, where the decrease of the flow and the reduction of the slope favor this deposit; (3) diminution of the flood area of the river in the lower parts of the valley, in consequence of this continuous deposit of sediments; (4) a river bed at present inadequate to permit the free passage of the waste waters of floods that may rise from the combination of the simultaneous discharge into the river from the various tributary arroyos below Elephant Butte Dam; and (5) greater actual danger of inundation of the lower river lands caused by the waste waters of the present smaller floods than those that occurred before the construction of the Elephant Butte Dam.

The proposed solution of the problem consists of shortening the river from 155 miles to 88 miles between El Paso and Quitman Canyon, by making cuts across bends. In this manner the slope of the channel is changed from 1.8 ft to 3.2 ft per mile.

As the silt content of the river at El Paso is only about 500 acre-ft per year, it is anticipated that following this rectification of the main channel, the flood and normal irrigation flow of tributaries will be of sufficient magnitude to produce scouring and to establish a new stable channel.

5.—Effect of Débris Barriers on the Stability of the Stream Bed on the Débris Cone

The stabilizing of stream courses on the cone has become a factor of importance in the semi-arid areas of the Southwest because the hillside and the cone have been proved to enjoy better protection from frosts than valley areas and, hence, to be better adapted to high-class residential development and to the culture of semi-tropical fruits. Furthermore, the availability of deep gravel deposits and the general commanding elevation above irrigable areas make the cone the most desirable location for the conservation of flood waste by water-spreading in contour ditches or ponds. Hence, the necessity of diverting flood waters on the cone by means of diversion dams into canals and ditches and of confining flood areas by levees for the protection of spreading grounds and residential areas.

Stream Action on the Cone.—The débris cone of a stream is defined herein as the alluvial deposit accumulated below the mouth of its canyon and over which the stream periodically meanders. When the flood stream emerges from the confining walls of the canyon, the path is clear for a lateral spreading. Velocities, therefore, will decrease and so will the transporting power of the stream; all to the effect that sediments and detritus will be deposited on the cone, more or less graded from the coarsest at the apex to fine materials at its base. This alluvial action constantly obstructs and tends to force the stream into other paths. The location of the stream bed on the cone is, therefore, inherently unstable, because there is no longer a balance between stream flow and the débris in traction.

As a rule, streams arrive at the base of the cone in broad sheets, which, having delivered themselves of their burden, then again collect into one, or possibly more, channels. As this concentration of flow takes place, velocities increase, scouring activity is again initiated, and, eventually, a single stream bed in cut is produced. Thus, a cycle of events is established: Erosive action in the mountain water-shed, the deposition of the débris on the cone, and the issuance from the cone of a stream delivered of its normal load and again capable of scouring its bed.

The apex of the cone being at the mouth of the canyon, shifting of the stream bed is likely to follow more or less along lines issuing radially from the apex, so that any structure or improvement on the cone is exposed to the danger of being flooded, washed out, or covered with débris. Little discussion is needed to show that any attempt to establish a balance on the cone between stream flow and débris by confining the stream between levees, can only be temporarily successful or, at best, will merely shift the location of deposit to lower areas.

It is generally accepted that a riparian owner may lawfully protect his property from the ravages of floods by means of levees, while diversion or other interference with natural conditions of stream flow is at the manipulator's risk. Considering then the instability of a stream on the cone, its manipulation will involve responsibilities which may assume unexpected proportions.

However, there are localities (and they are numerous in California) in which cones are among the choicest frost-free foothill lands, suitable for high-class agriculture or residences. Large areas have thus been improved, representing investments of thousands of dollars per acre and owners, therefore, demand protection from potential hazards, whether caused from natural or from artificial features.

Débris Barriers.—Débris deposition being the basic cause of the instability of stream courses on the cone, the apparent remedy is storage of the debris in the canyon above. A stream thus delivered of its burden has increased capacity for erosive action below, picking up its "normal load" on its course down the cone. In this manner, it may reverse its action from alluvial to erosive and automatically stabilize its alignment by deepening the bed.

A point to be given due consideration is the location of the barrier relative to the mouth of the canyon. If débris storage is accomplished at points only a short distance above the mouth, the stream will reach the cone practically free of débris. In heavy floods, such a stream would have the capacity to pick up and move even the largest type of débris. The general effect would be a deepening of the stream bed in the upper part of the cone, a re-grading of the materials removed, and their transport to points beyond the former margin of the cone. It is conceivable that this last-named activity might be the cause of property damage.

On the other hand, if the barrier is placed at some distance above the mouth of the canyon, the scouring action of the de-loaded stream would be primarily on the intervening section of the river bed, with the effect on the cone less accentuated. This result may be expected in cases where the stream intercepts below the barrier a number of tributaries which furnish a substantial supply of débris.

Over a period of moderate floods, the changes in the stream bed below the débris storage may be minor and gradual, while during capital floods they may be expected to become very marked and may be permanent. The essential fact remains that débris storage in the canyon involves a radical change of natural processes and hence will have its lasting effects on the action of the stream on the cone.

Practical Application of Débris Storage.—The practicability of débris storage behind dams was recognized and adopted years ago by the California Débris Commission to prevent devastation of valley lands by the waste from hydraulic mining operations. Numerous débris dams have been constructed and are being operated successfully in the western foothills of the Sierra Nevada. Of late years, débris barriers have also been used as flood-control measures in Los Angeles County, California, and in Utah.[5]

In the report of the Special Flood Commission[6] appointed by Governor George H. Dern, of Utah, it is stated that the present situation in Davis County is such as to call for steps: (*a*) To erect control works at the mouths of the canyons that have flooded, commensurate with the property values involved; (*b*) to control the débris that will be washed on to the farm lands and other property at the mouths of the canyons by high water in the spring, and by summer freshets; and (*c*) in so far as possible, to control summer floods that may occur until such time as adequate vegetation conditions have been restored on the water-sheds.

The essential requisite for débris storage is an economic dam site, since a reservoir site such as that for economic water storage, is not the governing requirement. The slope which débris will assume behind a barrier can be estimated from a study of canyons with alluvial fills. In such canyons, the gradient of the stream is the combined result of stream flow and débris. These two factors are not affected by the construction of a dam or barrier, which accounts for the tendency of the stream to re-establish the gradient it had before. As a matter of fact, the cone below the canyon mouth assumes the function of the barrier, above which the canyon fill has accumulated, and it is reasonable, therefore, to conclude that the slope ultimately produced above a barrier will not be materially different from that maintained by the old stream bed, except as modified by the greater width of the canyon. The effect of the latter would tend to produce a steeper slope because of less favorable channel properties.

---

[5] "Check Dams Control Débris Movements on Mountain Streams," by L. M. Winsor, M. Am. Soc. C. E., *Engineering News-Record*, August 20, 1931.

[6] "Torrential Floods in Northern Utah," *Bulletin*, Agricultural Experiment Station, Logan, Utah, 1930.

In selecting a site for a barrier, this fact has a bearing on the ultimate débris capacity in that a location below a wide section of the canyon would increase the available space, not only laterally but also vertically, because the new deposit would assume a steeper gradient. Fig. 2 shows the effect of a dam 6 ft high (built in 1921 across one of the active flood channels) on the cone of the Santa Ana River near San Bernardino, in Southern California, for the purpose of diverting water on to adjacent spreading grounds. The original gradient was approximately 3.2 per cent. The débris is exceptionally coarse, consisting mainly of large boulders. The survey of 1932 shows that the channel above the dam has been filled to a gradient of 3% for the first 1 000 ft.

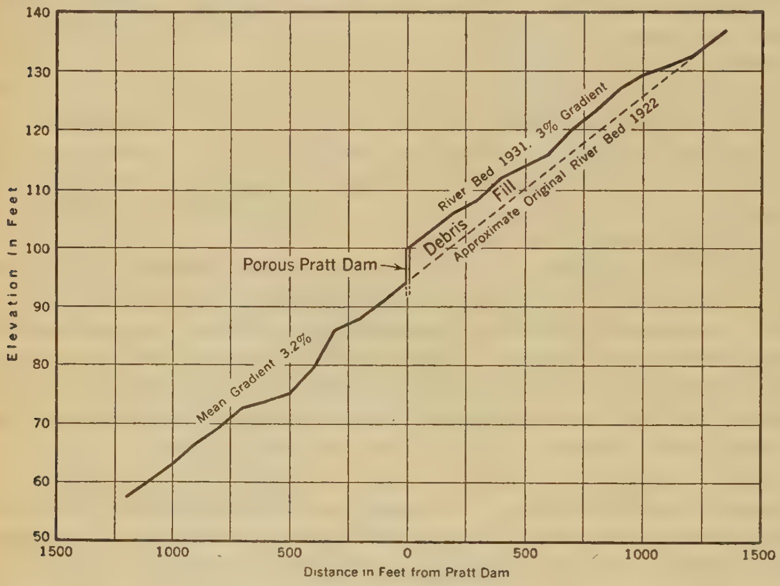


FIG. 2.—EFFECT OF INSTALLATION OF PRATT POROUS DAM ON CONE OF SANTA ANA RIVER, SOUTHERN CALIFORNIA.

The following examples will indicate what gradient may be expected. The Santa Ana River, above Mentone, in San Bernardino County, Southern California, has a well forested water-shed of 189 sq miles. Elevations range from 2 000 to 11 000 ft; and mean run-off is 75 000 acre-ft per annum, the estimated peak flood flow being 29 100 sec-ft, with a mean velocity of 13 ft. The width of the canyon varies for the first 4 miles from 500 to 700 ft. Bed-rock is at 70 ft, more or less. Débris is granitic and exceedingly coarse, with large boulders predominating. The stream gradients are as follows:

Location	Santa Ana River, Southern California:	
	Elevation, in feet	Gradient of stream bed (percentages)
Cone:		
Mile Two below canyon	1 550	3.8
Mile One below canyon	1 800	3.0
Canyon:		
Mouth of canyon	1 875	1.2
Mile One above mouth of canyon	1 960	2.2
Mile Two above mouth of canyon	2 100	3.8
Mile Three above mouth of canyon	2 320	5.6
Mile Four above mouth of canyon	2 570	4.7
Mile Five above mouth of canyon	2 750	3.4

The San Gabriel River above Azusa, in Southern California, has a water-shed of 220 sq miles, an average run-off of 135 000 acre-ft, and an estimated flood peak of 40 000 sec-ft. Its canyon above the mouth varies in width from 300 to 800 ft. Bed-rock is at 60 ft, or more. The water-shed is poorly forested and steep, rising in elevation to 10 000 ft. Débris is granitic, and less coarse than in Santa Ana Canyon, although boulders of several cubic yards may be found.

The gradients are, as follows:

Location	San Gabriel River, Southern California:	
	Elevation, in feet	Gradient of stream bed (percentages)
Cone:		
Four miles below canyon	500	1.4
Two miles below canyon	625	1.2
Canyon:		
Mouth of canyon	750	1.2
Mile Two, up stream	875	1.6
Mile Four, up stream	1 050	0.9
Mile Six, up stream	1 150	1.4
Mile Eight, up stream	1 300	1.6

Depending on the character of the débris, gradients as steep as 4% may ultimately be realized. The steep slopes of streams running on bed-rock cannot be maintained with alluvial fills. Where alluvial fills alternate with bed-rock exposures, the latter are characterized, as a rule, by steeper slopes.

The manner in which débris will be deposited above a dam is governed by the general laws of stream flow. In describing the process, the California Débris Commission states[7] that when the first of the débris arrives at the head of the (water) reservoir, where the current is relatively low, it will tend to pile up and thus flatten the slope. This will cause a progressive deposit up stream until the entire bed and slope of the river up stream from the reservior are raised. Thereafter, according to the Commission, feeding into the reservoir will come from material carried along or scoured from this bed.

---

[7] "Hydraulic Mining Investigations in California," Senate Doc. No. 90, 70th Cong., 1st Session, p. 20.

When the débris enters the reservoir, all the tractional load will settle immediately, assuming a steep natural slope down stream, with its top surface in prolongation of the surface of the débris bed in the river above. The suspended material will enter the reservoir with the initial velocity approaching that of the stream, and all but the lightest material will then begin to settle immediately, a portion on the slope of the tractional material and the remainder prolonging and flattening the toe of this slope on the bed of the reservoir to a distance depending on the fineness and specific gravity of the material. The reservoir will thus be filled from the upper end by an extension of the bed of the river into the reservoir, similar to the formation of a delta, with a relatively steep slope at its down-stream end. "The cross-section of the reservoir down stream from this slope," to quote the Commission directly, "will then be reduced only by the deposit of silt until the toe of the delta slope reaches the dam."

It is clear, without further explanation, that the final stable surface of the débris will be that assumed under maximum flood velocities. The foregoing discussion indicates that under suitable conditions, débris storage over a period of years may assume large proportions and that with a series of dams the stream may be stabilized on the cone proportionately as deposit areas are made available.

It is considered imperative that barriers be of a permanent type of dam construction. The sudden release of large quantities of alluvia, as a result of dam failure, might create conditions far worse than the natural meandering of streams on the cone.

As with all flood-control structures financed by public funds, the people assume, and possibly justly so, that the works are of a permanent type, that they do control floods, that they will afford effective protection (any intention of the planner or any publicity to the contrary notwithstanding), and that, therefore, the use of areas formerly menaced by floods may proceed with safety. This psychological effect is accentuated where structures are beyond the easy access by the general public, and it gains with time as minor floods are handled successfully. For this reason, temporary structures that are likely to fail during major floods may augment the dangers incident to the ravages of floods which occur only at long intervals. Débris barriers should be built to maximum heights consistent with funds, and should be provided with spillways and drain outlets. Economic dam sites are found in almost any canyon.

Rate of Débris Production.—Data on débris production in mountain water-sheds are meager. Table 2 was compiled from various sources.[8] Referring to Item No. 2, Column (7), the capacity of Castlewood Reservoir, which was originally 5 267 acre-ft, is said to have been greatly reduced after thirty-eight years. Had the reservoir been filled in this period, the rate of filling would have been 0.83 acre-ft per sq mile, annually.[9] Of the many streams mentioned in Table 2, all except Buckhorn Creek and Cherry Creek drain alluvial plains. Information on these two streams obtained by courtesy of Ralph I. Meeker, M. Am. Soc. C. E., is as follows:

---

[8] *Transactions*, Am. Soc. C. E., Vol. 93 (1929), p. 1732, and Vol. 95 (1931), p. 1061.

TABLE 2.—SILT DEPOSITS IN RESERVOIRS

Item No. (1)	Reservoir (2)	Stream (3)	State (4)	Drainage area, in square miles (5)	Record, in years (6)	Silt deposit, in acre-feet per acre per square mile (7)	Silt by volume, in percentage of annual run-off (8)
1	Buckhorn	Buckhorn Creek	Colorado	130	18	0.24	....
2	Castlewood	Cherry Creek	Colorado	165	38	0.83	....
3	Elephant Butte	Rio Grande	New Mexico	30 000	11	0.665	1.66
4	Roosevelt	Salt	Arizona	5 760	20	0.875	....
5	White Rock	....	Texas	114	5	1.19	....
6	Zuni	Pecos	New Mexico	500	22	0.88	1.88
7	Lake Wichita	Holliday Creek	Texas	135	25	Very little	....
8	Lake Penick	Clark Fork, Brazos River	Texas	2 250	7	0.06	....
9	McMillan	Pecos River	New Mexico	22 000	10	0.06	0.8
10	Worth	Trinity River	Texas	1 870	13	0.57	....

Buckhorn (Item No. 1) is a channel reservoir on Buckhorn Creek, a tributary of the Big Thompson River. Its contributing drainage area extends from Elevation 10 500 to Elevation 5 300 at the water surface, but most of it is below Elevation 7 500. The annual precipitation is from 20.00 to 16.00 in. The foothills are grass-covered with some bushes, while the mountain area is covered with pines, spruces, and quaking aspen. Geologically, the foothills are characterized by sedimentaries, ranging from cretaceous to carboniferous material and the mountain area is composed of metamorphic rocks, granites, and predominant schists. The 10 miles of stream channel immediately above the Buckhorn Reservoir traverses sedimentaries that are the chief source of erosion. There are no measured records of flow, the average of 5-year estimates by the Water Commissioner ranging from 2 000 to 51 000 acre-ft per year. The yearly average run-off may be accepted at 19 000 acre-ft.

Castlewood (Item No. 2) is a channel reservoir on Cherry Creek, a tributary of South Platte River. Its contributing drainage area extends from Elevation 7 600 to Elevation 6 500 at the water surface. The annual precipitation is ± 18 in. The cover is mostly grass, with small areas of oak brush and some pines. The surface geology is characterized by sedimentaries in the tertiary group, with loam soil and some rock outcrops. There are no measured records of flow, but it has been estimated by Mr. Meeker at 5 000 to 10 000 acre-ft, with an average of 7 500 acre-ft. Some lands in the basin are dry-farmed.

As to McMillan Reservoir, the period available for the determination of the silt load may not be representative, because the reservoir has been in operation more than 40 years; it has been enlarged twice and loss in capacity may not have been in direct relation to run-off at all times.

---

[9] *Transactions*, Am. Soc. C. E., Vol. 93 (1929), p. 1732.

Water Storage in Débris.—The material deposited in reservoirs draining alluvial plains may consist largely of silt and clay with initial porosity that may run as high as 60 or 70 per cent. It is quite different from that produced in mountain water-sheds of crystalline formation and composed of sand, gravel, and boulders, the pore space of which may vary from 8% for materials in which large boulders are predominant to 35% for sand deposits. The following values were determined experimentally for the San Gabriel Wash, in San Gabriel Valley, Southern California,[10] mentioned previously:

Miles below mouth of canyon	Material	Percentage of voids
2	Boulders, gravel, and sand	12
7	Coarse gravel and sand	17
10	Medium gravel and sand	20
12	Fine gravel and sand	28

A reservoir intercepting a stream such as the San Gabriel River would also receive small quantities of silt that otherwise would be deposited in the plains below the cone. The percentage of fine materials might increase considerably if the reservoir should intercept the débris of burnt-over areas, which as a rule deliver large quantities of silt and ashes, depending on the severity of the rain storm.

In semi-arid regions, the water storage in the débris above the barrier may assume commercial value. The tendency of the water-table is to assume a slope approximately parallel to the surface of the débris fill and, for this reason, an appreciable increment of ground-water over and above free-water storage is available. Even with relatively high rates of percolation, a retarding effect is produced and eventually seasonal water storage takes place. Provision for controlled drainage of the débris mass, therefore, should be made at the time the works are planned.

It is interesting to note that, offsetting a diminished water storage in reservoirs resulting from débris deposition, water storage in accumulated débris behind barriers increases in proportion to the quantity of débris.

It is concluded that a feasible method of stabilizing the course of the stream on the débris cone is available in the construction of suitable débris storage above the cone. In planning the location of such storage, it should be borne in mind that the débris plain is the combined result of stream flow and available débris. The complete elimination of débris will reverse stream activity from alluvial to erosive. The effect may turn out to be more radical and far-reaching than expected because of the steep gradients generally encountered and the large volume of the periodic floods. A location at some distance up stream from the head of the cone, which will intercept a part of the débris produced by the water-shed without completely eliminating it, may be worth consideration.

---

[10] "San Gabriel Investigation," by Harold Conkling, M. Am. Soc. C. E., *Bulletin No. 7*, Dept. of Public Works, State of California.

6.—Conclusions

(*1*).—*Physiographical Balance.*—The great constructive processes of Nature expressed in the functions of the water-shed cover, the stream, the débris cone, and the flood-plain, combine in maintaining a physiographical balance. Any attempt to modify, or reverse, these processes by flood-control or conservation measures will re-act, in due time, in disturbing this balance. Although the effect may be slow, and although it may appear to a fast-moving world as imperceptible or negligible, the change of natural forces thus initiated by permanent improvements may be lasting and irresistible; hence, they present new problems to solve.

(*2*).—*Physiographical Inter-Relation of Mountain Water-Shed and Flood-Plain.*—In the development of flood control or conservation programs, the agency undertaking the control of streams should be in a position to consider improvements in the mountain water-shed as well as on the flood-plain and to treat the stream system of a drainage area as a unit, because interference with natural conditions at one point on the stream may have its reaction on the physiographical balance at points below and eventually create a new menace which, sooner or later, must be met.

(*3*).—*Co-Operation of Governmental Agencies Necessary to Attainment of Maximum Yield of Water-Sheds.*—The demand for additional water supplies is responsible for numerous plans advanced from many quarters for the improvement of the water-shed, with a view to increasing its water yield. Some of these plans are considered impracticable, while others entail a risk tending to unbalance débris production and removal. The United States Forest Service and affiliated State agencies, while not approving some of these projects, have initiated systematic studies of the water-shed, however, with a view to increasing run-off and decreasing fire hazard. It is also noted that engineers, investigators, and agriculturists have come to realize that the water-shed is their common problem and that co-operation will lead to a rational solution of water conservation in the water-shed.

(*4*).—*Storage Regulation May Produce Flood Menace.*—The regulation by storage of a major stream, in time, may become the cause of new inundation problems at the point of junction of tributaries below the reservoir, because the regulated flow may lack the power to move the load which an uncontrolled tributary may carry to the point of junction. This problem may present itself particularly in cases where storage of the capital flood flow is proposed. The effect of the Elephant Butte Dam on flood conditions of the Lower Rio Grande is an example.

(*5*).—*Effectiveness of Débris Barriers.*—The checking of débris transportation by a débris barrier will change stream activity below until a new load and a modified stream bed and gradients balance the carrying capacity of the stream. This principle may be applied to the stabilization of stream courses on débris cones and on the flood-plain, in that alluvial stream action may be lessened, stopped, or changed to erosive action.

Single barriers built to considerable height (50 ft, or more) are far more effective in intercepting abnormal débris loads which may result from denudation or other causes, than large numbers of check dams built to nominal heights of about 6 ft.

(6).—*Temporary Structures to Check Débris Not Advisable.*—The checking of débris by temporary structures in the mountain water-shed is adverse to best public policy:

(*a*) For the physical reason that an abnormal débris load caused by failure of structures may unbalance stream activity and cause a shifting of the course of the stream, for which the agency erecting the barrier would be responsible.

(*b*) For the psychological reason that the people assume flood-control structures to be permanent and effective for any magnitude of flood. They take for granted that the natural flood channel below such works may be encroached upon and improved without risk.

ACKNOWLEDGMENTS

For the statement, "(4) The Effect of Regulation of Rio Grande on Flood Conditions Below Elephant Butte Dam," the writer is indebted to L. M. Lawson, M. Am. Soc. C. E., for many years Project Superintendent of the Rio Grande Project of the United States Bureau of Reclamation, and, at present (1933), Engineer of the United States Department of the Interior, and United States Boundary Commissioner.

The writer is indebted to S. T. Harding and Charles H. Lee, Members, Am. Soc. C. E., and Harry F. Blaney, Assoc. M. Am. Soc. C. E., members of the Water Conservation Committee of the Irrigation Division, and to L. M. Winsor, Assoc. M. Am. Soc. C. E., for critical review of this paper and constructive suggestions; also to R. L. Egenhoff, Assoc. M. Am. Soc. C. E., for valuable suggestions and for information relative to the operation of mining débris barriers.

DISCUSSION

H. H. CHAPMAN,[11] ESQ. (by letter).—That changes in the water-shed cover upset the state of equilibrium or balance expressed in a certain type and density of vegetation, and that the one effect which may not be immediately apparent, but may be the most far-reaching in the long run is accelerated erosion, as stated by Mr. Sonderegger, are facts worthy of closest scrutiny in their far-reaching effect on irrigation, storage reservoirs, and water control. This balance is maintained over huge areas in the West by grasses and herbage. Profound disturbance of this equilibrium has taken place through excessive over-grazing. Extensive channel cutting has accompanied this phenomenon in the Southwest and intermountain region. That over-grazing is the principal cause of this arroyo formation is proved by the fact, attested by Dr. Earl Morris, Division of Historical Research, Carnegie Institute, Washington, D. C., that the cycle of degradation now operating is of recent origin. Throughout the occupation of the San Juan Area, in Colorado, by agricultural aborigines, the valley floors were rising instead of being cut away. The recent cuttings have laid bare stratified exposures recapitulating in reverse order the entire culture series as it is known to have existed in the area in question. Other outstanding areas showing the same series are Chaco Canyon and Canyon de Chelly. The destruction of all vegetation by grazing animals, the consequent prevention of reseeding, and the beating out of the roots of grass and other plants, are given as the dominant causes of arroyo cutting in this region by this authority on aboriginal American history.

An exceptionally valuable piece of historical evidence as to the condition of the Southwestern rivers in the period preceding the coming of white civilization is found in the diary[12] of the noted explorer, James M. Pattie. Of the Gila River in Arizona, now transformed into a typical sandy wash, he says, on December 14, 1824, "we started early and crossed the river, here a beautiful clear stream about 30 yards in width, running over a rocky bottom and filled with fish"; and "on the morning of February 1, 1826, we began to ascend the Black River. It is a most beautiful stream to the point where it forks in the mountains, that is to say, about 80 miles from its mouth." During January, Pattie trapped the Gila to its junction with the Red, or Colorado River, and on February 2, 1826, says,

> "At twelve, we started up the Red River, which is between 200 and 300 yards wide, a deep bold stream, and the water at this point is entirely clear. The bottoms are a mile in general width. The timber of the bottoms is very heavy and the grass rank and high."

On April 15, 1826, after passing the Grand Canyon, he says, "we returned to the banks of the Red River, which is here a clear beautiful stream."

---

[11] Prof. of Forest Management, School of Forestry, Yale Univ., New Haven, Conn.

[12] "Early Western Travels," by Reuben Gold Thwaites, Vol. XVIII, pp. 87, 131-132.

The cumulative effects of erosion following disturbance of vegetative equilibrium have been discussed[13] in great detail. B. P. Fleming, M. Am. Soc. C. E., has discussed the problem of water erosion in New Mexico, on the Rio Grande area covered by Mr. Sonderegger's paper, and attributes disastrous changes in stream conditions and erosion to overgrazing.[14]

There are incalculable quantities of loose or unconsolidated soils on all water-sheds, capable of being eroded at an accelerated pace once the vegetative balance is destroyed, and of upsetting the calculations of engineers on storage capacity and life of reservoirs.

E. B. Debler,[15] M. Am. Soc. C. E. (by letter).—The author has devoted not a little space to forestation and rightly so. It is noted that the Forest Service is stated to be engaged upon a program of research with a view, among other objects, of increasing the water yield. Such research, if conducted without prejudice and with due regard to all interests involved, should receive every possible encouragement. Dogmatic propaganda for tree planting without regard to results other than the growing of trees will injure the cause of reforestation. The effects on grazing, recreation, and water production must be given full consideration.

Silt problems arising from accentuated erosion and from stream depletion merit careful consideration by every one. The mining of soils and their wholesale erosion with or without the aid of Man should not be viewed complacently. As with forestation there must be careful consideration of the attendant risks. Silt barriers are being erected or promoted, wholesale, throughout the United States. Many of these erosion checks or barriers are being built of short-lived materials. Who will maintain these barriers? Are they not likely to lead to even greater damage than now occurring when some future flood sweeps through a series of such structures weakened by age and overwhelms an area built up on a false sense of security? Permanent construction and watchful maintenance are essential to the success of such structures with the latter particularly difficult to secure. The efforts of the author and others to arouse interest in the erosion problem are no longer falling on deaf ears; the public is awakening to this menace, but more remains to be done before the selfish interests of the cattle owner, the sheep herder, the agriculturist, and the miner give way to an enlightened public policy.

The writer cannot fully agree with the conclusions presented in the case of the Rio Grande Project. From the Elephant Butte Dam almost to El Paso, Tex., the author's data show increases in river sections, except only where the sections were formerly much greater than the average. The increase in area is by means of bottom scour, occurring even in some of the sections, which show a net loss. The only area that did not show such a scour was one with a total area fully three times the average, and so unnatural in size that the adjustment of river sections and gradients naturally resulted in a reduction under conditions of regulated flow. Section changes have resulted in increased capacities.

---

[13] Sen. Doc. No. 12, 73d Cong., pp. 298-462 (Copeland Rept.).

[14] "The Southwestern Conservation League," *Bulletin No. 1*, Conservation Series, Univ. of New Mexico, December 1, 1933.

[15] Hydr. Engr., U. S. Bureau of Reclamation, Denver, Colo.

The statement that arroyo fans have formed rapids, is also open to question. Bottom scour with the usual increase in section and the lowering of the stream bed at the dams, supports a conclusion that the rapids are the remains of older fans of coarse materials which have resisted recent removal, whereas the softer bottom material between these rapids has been removed. The writer is unaware of data showing the relative silt deposition in the valley above El Paso before and after the construction of the Elephant Butte Reservoir. It is likely that less silt is being carried away from the project than formerly, but he doubts whether the difference is equal to the amount being deposited in the reservoir.

At the International Dam, near El Paso, a silt plug is in constant process of formation through the desilting of diverted waters, little water normally passing this point. Each flood carries away a part of the silt plug. For lack of such floods for a number of years, this plug reached menacing levels prior to 1925 when two floods of local origin restored favorable conditions. The favorable river-bed level of 1907, mentioned by the author, was probably due to severe scouring resulting from the heavy run-off then current, and particularly the high floods of the latter part of 1905.

The stabilized river channel has gradually approached El Paso and should reach there in the near future, with anticipated improvement in local conditions. Thereafter, local floods and surplus waters entering Elephant Butte Reservoir, which have been missing in recent years of sub-normal run-off, no doubt, will clear the river channel at El Paso with sufficient frequency to avoid the extreme high levels of 1925.

Below El Paso, there has been a loss of capacity such as is to be expected in a channel carrying little water for a number of years. This situation will shortly be improved by channel rectification. Unavoidable waste and surplus waters should have little difficulty in maintaining the new channel free of silt accretions provided meandering is prevented.

A feature of stream development not mentioned by the author, and which may warrant much consideration, is the concentration of salts through re-diversion. Rarely is the quality of waters improved in passing through the soils. There is usually a marked increase in deleterious salts, with drain waters in some cases carrying several times the agricultural tolerance limit for ordinary crops. This feature must receive increasing attention as the utilization of Western streams progresses toward complete consumption of all ordinary flows.

Frank E. Bonner,[16] M. Am. Soc. C. E. (by letter).—An excellent review of problems involved in water-shed development and control is presented in this paper. Although relating primarily to the special conditions of the arid Southwest, some features have more or less general application. The meagerness of data bearing on the rates of débris production has been pointed out by the author. It is unfortunate that published information on the subject is so fragmentary in character. In an endeavor to provide a basis for general estimates, the writer has been compiling, for some time, such records as have been made available for representative drainage basins of the United States. The results thus far are presented in Table 3, which has been compiled from information supplied by the U. S. Department of Agriculture,[17] the U. S. Army Engineer Corps,[18] U. S. Geological Survey,[19] American Society of Civil Engineers,[20] *Engineering News-Record,*[21] U. S. Reclamation Service,[22] the late A. J. Wiley M. Am. Soc. C. E.,[23] and R. V. Meikle, M. Am. Soc. C. E.[24] Wide variation is evident, but the unit values exhibit reasonable consistency with the different climatical and surface geological factors which constitute the basic influences on erosin.

---

[16] Cons. Engr., Piedmont, Calif.

Some uncertainty is involved in the correlation of the results of the various investigations because of different methods adopted for silt measurements. Many have used the dry-weight basis, while others have recorded determinations in the less certain terms of volume. The weight-volume ratio varies considerably because of the wide range of density in alluvial deposits. The variation is due not only to the size and grading of the particles, depending on the manner of deposition, but also on the degree of consolidation resulting from exposure to the atmosphere. Coarse débris, consisting of a mixture of boulders, gravel, sand, and silt, characteristic of the load laid down at the head of débris cones of swift mountain streams, will have high density. On the other hand, fine particles of sand or silt carried long distances in suspension and finally dropped in slowly moving water form relatively porous deposits. Between these two extremes endless combinations are represented. On exposure to the atmosphere and loss of the moisture contant, deposits of finely divided silt undergo substantial shrinkage of volume. Visible evidence of this process of consolidation is found everywhere in the pattern of wide cracks developed in drying mud-banks. Renewal of saturation fails to produce corresponding expansion. Tests show that the specific gravity of the solid material in alluvial deposits generally ranges from 2.6 to 2.7, depending largely on variation in the small proportion of organic material ordinarily present. Obviously, the deviation of specific gravity is too small to affect the weight-volume ratio to any extent.

The dry weight of material ordinarily found in silt deposits ranges from 30 to 106 lb per cu ft. This corresponds to a range in place (saturated) of 81 to 128 lb per cu ft. Sometimes even these wide limits are exceeded. Harold Conkling, M. Am. Soc. C. E., reports some coarse detrital material in San Gabriel Wash, California, with only 12% voids[25] which corresponds to a

---

[17] *Technical Bulletins Nos. 1430, 382,* and *67.*

[18] H. R. Doc. No. 328, 1930.

[19] *Water Supply Papers 274* and *617,* and *Professional Paper No. 105.*

[20] *Transactions,* Am. Soc. C. E., Vol. 93 (1929), and Vol. 95 (1931).

[21] September 7, 1933, p. 280.

[22] Weymouth Rept., p. 70.

[23] Rept., 1922, p. 20.

[24] Unpublished data.

[25] *Bulletin No. 7,* California Div. of Water Resources, p. 44.

TABLE 3.—SILT LOAD OF REPRESENTATIVE STREAMS

Stream	Measuring point	Tributary area, in square miles	Annual run-off, in acre-feet	Silt record, in years	ANNUAL SILT LOAD		Loads,* in percentage of run-off	WATER-SHED COEFFICIENT	
					Acre-feet	Tons		Acre-feet per square mile	Tons per square mile
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
(a) NORTH ATLANTIC BASINS									
Hudson River	Peekskill, N. Y.	13 366	15 000 000	......	....	240 000	0.0009	0.0097‖	18
Susquehanna River	Danville, Pa	9 530	10 000 000	......	....	240 150	0.0013	0.0135‖	25
(b) SOUTH ATLANTIC BASINS									
Savannah River	Augusta, Ga	7 294	7 700 000	......	1 093	....	0.014	0.150	278‖
Yadkin River	Salisbury, N. C	3 400	3 600 000	......	....	925 000	0.014	0.147‖	272
(c) MISSISSIPPI BASIN									
Upper Mississippi River	Minneapolis, Minn.	19 585	3 600 000	......	....	117 000	0.0017	0.0032‖	6
Missouri River	Ruegg, Mo	528 700	72 400 000	......	....	176 000 000	0.131	0.180‖	333
Tennessee River	At mouth	40 600	48 500 000	......	....	8 120 000	0.009	0.108‖	200
Kansas River	Lecompton, Kans.	58 600	4 300 000	......	8 740	....	0.203	0.149	276‖
Arkansas River	Little Rock, Ark.	148 000	40 000 000	......	....	40 000 000	0.054	0.146‖	270
North Fork, Red River	Hendrick, Okla.	5 470	400 000	1.7	....	1 600 000	0.216	0.158‖	293
South Platte River	Buckhorn Reservoir	130	19 000	18	31.4	....	0.165	0.242	448‖
Cherry Creek	Castlewood Reservoir	165	7 500	43	42.6	....	0.568	0.258	477‖
Big Horn River	Fort Custer, Mont.	20 700	3 600 000	1	....	7 000 000	0.105	0.183‖	338
(d) WESTERN GULF									
Brazos River	Waco, Tex	28 500	1 629 000	3	....	19 054 000	0.632	0.361‖	669
Colorado River (Texas)	Austin, Tex	38 000	1 827 000	3	11 500	9 000 000	0.629	0.303	237
Trinity River	Lake Worth, Tex.	1 870	....	11	1 000	1 013 000	....	0.535	542
Medina River	Medina Reservoir	587	....	13	207	135 250	....	0.353	229
Pecos River	McMillan Reservoir, New Mexico	22 000	316 000	31.4	1 430	....	0.452	0.065	120‖
Rio Grande	Elephant Butte, Reservoir	28 360	1 200 000	9	20 470	....	1.706	0.722	1 336‖
(e) COLORADO BASIN									
Upper Colorado River	Cisco, Utah	24 100	6 600 000	......	8 175	....	0.124	0.339	628‖
Green River	Green River, Utah	40 600	5 650 000	1	6 770	....	0.120	0.167	309‖
San Juan River	At mouth	26 000	2 700 000	......	29 000	....	1.074	1.115	1 206‖
Gila River	Dome, Ariz	52 340†	1 070 000	......	16 000	....	1.495	0.306	420‖
Zuni River	Zuni Reservoir, New Mexico	650	26 400	22	543	....	2.057	0.836	1 547‖
Salt River	Roosevelt Reservoir	5 760	800 000	20	5 050	....	0.631	0.862	1 623‖
Colorado River	Yuma, Ariz	244 000	16 200 000	18	154 000	224 000 000	0.951	0.631	918
(f) GREAT BASIN									
Truckee River	Derby, Nev	1 250‡	900 000	1	....	87 000	0.005	0.038‖	70
Carson River	Hazen, Nev	1 700	430 000	1	....	97 000	0.012	0.031‖	57
Owens River	Round Valley, Calif.	400	210 000	1	....	13 400	0.003	0.018‖	34
(g) CALIFORNIA PACIFIC									
Sacramento River	Red Bluff, Calif	9 260	9 200 000	1.7	....	1 360 000	0.008	0.079‖	147
Feather River	Oroville, Calif	3 625	4 800 000	1.6	....	680 000	0.008	0.114‖	188
Yuba River	Smartsville, Calif	1 042‡	2 240 000	......	310§	....	0.014	0.297	550‖
Stanislaus River	Melones, Calif	744‡	1 375 000	0.3	125	....	0.009	0.168	311‖
Tuolumne River	La Grange, Calif	1 350	2 096 000	13	314	....	0.015	0.232	430
Putah Creek	Winters, Calif	655	420 000	1.2	....	279 000	0.036	0.230‖	426‖
(h) COLUMBIA BASIN									
Columbia River	The Dalles, Ore	237 000	146 000 000	......	6 040	....	0.004	0.0266	49‖

* Measured by deposit volume. † Exclusive of Salt River Basin tributary to Roovevelt Reservoir.
‡ Exclusive of area tributary to upper lakes and reservoirs. § Exclusive of hydraulic mining débris.
‖ Computed on basis of 85 lb per cu ft.

weight of 144 lb per cu ft. Orville A. Faris, M. Am. Soc. C. E., found several samples from Lake Worth, Texas, to weigh only 18.7 to 22.6 lb. (dry weight) per cu ft.[26] On the basis of his comprehensive study of conditions in Texas, Mr. Faris concluded that the density of silt in that locality would be controlled primarily by the degree of exposure, and he estimated[27] the average dry weight per cubic foot of reservoir deposits, as follows: (*a*) 30 lb when continuously submerged; (*b*) 70 lb in reservoirs emptied occasionally; and (*c*) 90 lb in reservoirs standing empty most of the time.

As a result of their detailed investigation of the Colorado River silt problem the late Samuel Fortier, M. Am. Soc. C. E., and Harry F. Blaney, Assoc. M. Am. Soc. C. E., concluded[28] that 85 lb per cu ft would represent a fair average for the combination of suspended and bed-load sediment deposited in a reservoir near the lower end of the Canyon Section of the river.

From the foregoing it will be apparent that selection of a value to represent average conditions is subject to no small degree of conjecture. Considering, however, that most deposits occur along river channels and in storage reservoirs with fluctuating levels it is believed that 85 lb per cu ft offers a fair estimate, and this ratio has been adopted for conversion in those cases listed in Table 3 where the investigations failed to establish a relation between the weight and volume of the ultimate deposit. Accumulations of the coarse débris carried by mountain streams will be heavier on the average, while the fine silt deposited in lakes, bays, and reservoirs with constant level will be considerably lighter.

Some of the determinations recorded in Table 3 are derived from measurements of accumulations in reservoirs over long periods, while others are based on systematic sampling and calculation of the amount of solids carried in suspension at different river stages. In connection with the latter there is no feasible method of measuring the bed load which in certain situations is known to be a substantial fraction of the total load. Messrs. Fortier and Blaney estimated[29] the bed load of the Colorado River at Yuma, Ariz., to be about 20% of the total load. Mr. Faris cites[30] a similar estimate relating to the Rio Grande and an estimate that the bed load of the Mississippi River amounts to 11% of the suspended load. Determinations relating to the movement of coarse débris from the mountain water-sheds of the Sierra Nevada, in California, give somewhat higher results.

It might be assumed, therefore, that reservoir measurement would provide more accurate appraisal than river sampling. This will depend, however, on the effectiveness of the reservoir as a desilting agency which, in turn, depends largely on the relation between the dimensions of the reservoir and the extent of the tributary water-shed. Mr. Faris shows[31] that the average annual silt accumulation (20-year period) in the small Lake Austin Reservoir, on the Colorado River of Texas, amounted to less than one-fourth the average suspended load of the stream. In that case, the reservoir capacity was comparatively small in proportion to the area of the drainage basin above. That estimates by river sampling provide reliable indices to the probable siltation of large reservoirs is well demonstrated by the work of the late W. W. Follett, M. Am. Soc. C. E., on the Rio Grande. On the basis of stream sampling at San Marcial, from 1905 to 1912, he estimated the average annual suspended load of the river at 19 739 acre-ft,[32] whereas measurement by the U. S. Bureau of Reclamation of the siltation of Elephant Butte Reservoir from 1916 to 1925 indicates an average annual accumulation of 20 470 acre-ft.[33]

---

[26] *Technical Bulletin No. 382,* U. S. Dept. of Agriculture, p. 36.

[27] *Loc. cit.,* p. 55.

[28] *Loc. cit., No. 67,* U. S. Dept. of Agriculture, p. 72.

[29] *Loc. cit.,* U. S. Dept. of Agriculture, p. 53.

[30] *Loc. cit., No. 382,* U. S. Dept. of Agriculture, p. 9.

[31] *Loc. cit.,* p. 45.

Comparison of the values in Column (10), Table 3, show plainly the regional variations of soil erosion. It is least active in the permeable soils of the northern part of the United States and reaches a maximum on the soft sandstone and friable shale formations of the arid Southwest. In the Rio Grande and Lower Colorado Basins the removal of material progresses at an annual rate in excess of 1 000 tons per sq mile and over certain areas it is evident that the rates are much higher. As may be observed from the tabulation, investigations show an average of about 1 600 tons per sq mile for the Salt River and Zuni River water-sheds. Herman Stabler, M. Am. Soc. C. E.,[34] estimates an average annual removal of 3 000 tons per sq mile from the barren desert region of 60 000 sq miles drained by the San Juan and Little Colorado Rivers.

These high rates of débris removal over large areas are exceeded only by those for several basins of the Sierra Nevada in California where hydraulic mining of auriferous gravel was practiced extensively. Although this form of gold mining was brought to a sudden halt by Federal Court decree in 1884 the rivers of that section are still struggling to rid themselves of the enormous load of alluvial débris dumped into the channels during the thirty years previous. The hydraulic mining operations were carried out extensively on all the streams from Feather River south to Tuolumne River, but the greatest activity centered in the Yuba Basin, where a total of about 700 000 000 cu yd of material, or nearly one-half the total for the entire region, was dislodged. This great disturbance of natural conditions and the progressive movement of the huge load of detritus down the river channels to eventual deposit in the tidal bays, supplied a most unusual opportunity for observation and study of river hydraulics in relation to débris transportation, but the results are beyond the bounds of the present discussion, except for certain features. Through the notable investigation by Dr. G. K. Gilbert[35] it was demonstrated that "the apex of the débris flood, leaving the mines in 1883, passed the mouth of the canyon in about the year 1900 and the mouth of the Yuba River in about 1905." Dr. Gilbert's comprehensive measurements showed that the detrital load carried past the mouth of the canyon in 1905 amounted to 7 200 000 cu yd. Nearly 70% of the total consisted of bed load. The tributary area of the Yuba, exclusive of that part which drains to head-water lakes and reservoirs, amounts to 1 042 sq miles, thus, the 1905 load (at 85 lb per cu ft) was equivalent to 7 930 tons per sq mile. Recent investigations by the writer in connection with plans for a dam in the Lower Yuba Canyon indicate that the normal load of the river has now (1934) decreased to approximately 1 700 000 cu yd, or about 1 870 tons per sq mile. Natural erosion accounts for only 30% of the total, the remainder coming from the diminishing remnants of the old mining deposits scattered throughout the water-shed.

---

[32] *Technical Bulletin No. 67,* U. S. Dept. of Agriculture, p. 4.

[33] *Transactions,* Am. Soc. C. E., Vol. 93 (1929), pp. 1690 and 1727.

[34] *Civil Engineering,* September, 1932, p. 542.

[35] *Professional Paper No. 105,* U. S. Geological Survey, 1917.

Comparison of the heavy detrital load carried from the Sierra Nevada mining water-sheds with the débris produced by natural erosion elsewhere serves to emphasize the possible acceleration of surface degradation through violent disturbance of the natural processes by human agencies. Of course, the hydraulic mining areas offer a highly exaggerated illustration, since no other possible activity by Man could result in comparable removal of surface material. Under certain conditions agricultural activities contribute measurably to soil waste. Farming on comparatively level land or on coarse permeable soils probably has but little influence; but instances have been cited of excessive erosion from cultivation of light soils on moderate slopes. On the other hand, some farming operations may serve to build up the soil. This is true of irrigation enterprises in which the sediment of the diverted waters is carried through the canals and deposited on the fields. In California, for example, it is estimated that total soil accretion through irrigation exceeds the erosion losses from all the unirrigated land in cultivation.

Native vegetation constitutes an important control on erosion and its destruction naturally increases the normal rate of soil removal. Various destructive agencies arise from human activity. Smelter fumes have destroyed the vegetation of certain areas in Tennessee, California, and Montana, with consequent increase of erosion. Fires and over-grazing are also destructive in some localities. The latter is probably more serious from an erosion standpoint since the effect is likely to be more widespread and continuous. The scars left by fire are quickly healed by volunteer growth of brush, grass, or weeds, and excessive erosion results only when the burned area is subjected to torrential downpours before new growth is well started. Such occurrences are infrequent and mainly of local significance. The construction of improvements, such as railroads, canals, highways, etc., serve to increase erosion along their course, but the regional result is relatively slight. In the broad view, human activity can have but little effect either to accelerate or to retard the slow but inevitable weathering of the earth's surface.

Depletion of reservoir storage capacity through siltation comprises one of the effects of surface erosion that in some cases may be of economic importance. Instances have occurred in which small reservoirs were rapidly filled with silt and lost their usefulness. Even in the case of some larger reservoirs, the depletion of storage capacity through siltation has reached such proportions as to interfere with the purpose for which the reservoir was intended. Studies demonstrate that silt removal is more costly than provision of additional storage capacity, and thus it is obvious that original designs that fail to take into consideration and provide for the progressive silt accumulation are defective. It is not difficult for the engineer to estimate the effects of silting within reasonable limits, and to overlook this feature may not only jeopardize the economic success of the enterprise through early loss of capacity, but also the stability of the dam itself may be threatened by improper appraisal of the up-stream pressures. The horizontal forces acting against a dam through combination of the silt and water load may amount to 50% more than those from the water load alone.

Instances of excessive silting are chiefly in connection with reservoirs disproportionately small as compared with the area of the tributary basins. Lake Austin, Texas,[36] is an outstanding example. Naturally, also, if the formations of the water-shed are of such character as to yield extraordinarily large amounts of débris like at Zuni Reservoir, New Mexico,[37] rapid depletion is unavoidable. Effective preventive measures cost more as a rule than the construction of new storage capacity. The problem of rapid depletion seems to be confined largely to the Southwest. In California, several small reservoirs (used primarily for diversion purposes) have filled with silt at a rapid rate but, in general, the water storage reservoirs, have been proportioned to their respective drainage basins so that the percentage of depletion through siltation will be negligible for at least a century to come. The situation is illustrated by Table 4 which shows the siltation rates of representative South-

TABLE 4.—STORAGE RESERVOIR DEPLETION THROUGH SILTATION

Reservoir (1)	Stream (2)	Tributary area, in square miles (3)	Original capacity, in acre-feet (4)	Annual siltation, in acre-feet (5)	Years for total depletion (6)
(*a*) ARIZONA, NEW MEXICO, AND TEXAS					
Boulder	Colorado River	169 000	30 500 000	137 000	223
Elephant Butte	Rio Grande	28 360	2 638 860	20 470	129
Roosevelt	Salt River	5 760	1 637 000	5 050	324
Zuni, New Mexico	Zuni River	650	10 230	440	23
Medina Lake	Medina River, Texas	587	254 000	207	1 230
Lake Worth	Trinity River, Texas	1 870	47 177	1 000	47
Lake Austin (old)	Colorado River, Texas	38 000	49 300	3 490	14
Lake Austin (new)	Colorado River, Texas	38 000	32 029	2 350	14
Lake Pennick	Clear Fork, Brazos River	2 250	3 094	123	24
(*b*) CALIFORNIA					
Almanor	North Fork, Feather River	503	1 308 000	51	25 647
Bucks	Bucks Creek	28	101 926	3	33 975
Bowman	Canyon Creek	19*	63 000	5	12 600
Spaulding	South Yuba River	88*	74 448	24	3 102
Fordyce	Fordyce Creek	28*	46 662	8	5 883
Twin Lakes	South Fork, American River	13	21 250	3	7 083
Salt Springs	Mokelumne River	150*	130 000	35	3 714
Pardee	Mokelumne River	387*	222 000	90	2 467
Strawberry	South Fork, Stanislaus River	27	18 500	6	3 083
Melones	Stanislaus River	744*	112 500	125	900
Eleanor	Eleanor Creek	79	27 800	18	1 544
Hetch Hetchy	Tuolumne River	459	206 000	107	1 925
Don Pedro	Tuolumne River	962*	292 000	223	1 309
Exchequer	Merced River	1 027	281 000	238	1 180
Crane Valley	West Fork, San Joaquin River	55	45 000	13	3 461
Florence	South Fork, San Joaquin River	171	64 406	40	1 610
Huntington	Big Creek	79	88 834	18	4 935
Shaver	Stevenson Creek	30	135 283	7	19 326
Pillsbury	South Eel River	268	73 163	62	1 180
Stony Gorge	Stony Creek	210*	50 200	48	1 046

* Exclusive of area tributary to upper lakes and reservoirs.

---

[36] *Transactions*, Am. Soc. C. E., Vol. 93 (1929), p. 1681.

[37] *Loc. cit.*, p. 1695.

western reservoirs as compared with twenty of the main storage reservoirs in California. The latter group is shown to have more enduring usefulness. Only one of the twenty has an indicated life of less than 1 000 yr. This means that during the first 100 yr of use the decrease of effective storage capacity will not exceed 10 per cent. Under the circumstances, it is clear that in the operation of these reservoirs the siltation problem will be of no consequence.

It probably should be pointed out that none of these reservoirs is in Southern California. A number of reservoirs have been constructed in that section for flood control, irrigation, and domestic supply purposes, but the water-shed conditions are essentially different from those of the remainder of the State. The area of the South Pacific slope (south of San Luis Obispo) amounts to 13 600 sq miles, about evenly divided between mountain water-sheds and coastal plains. Although comprising less than 9% of the State area and yielding less than 5% of the run-off of California's streams, the water-shed problems are important because of the dense population and intensive water utilization. Rainfall is erratic and occasional severe storms produce flood flows carrying heavy loads of débris from the steep mountain water-sheds. Considerable damage results to limited areas where human habitation has crowded upon active débris cones, but over the locality as a whole the erosion rate is not exceptional.

TABLE 5.—ANNUAL SILT PRODUCTION OF THE UNITED STATES

Item No.	Drainage basin (1)	Area, in square miles (2)	Tons of silt per square mile (3)	Total silt, in millions of tons (4)	*Years for erosion depth of 1 in. (5)	Remarks (6)
1	St. Lawrence	128 000	20	2.56	6 150	Area, exclusive of a part in Canada
2	North Atlantic	150 075	80	12.00	1 537	....
3	South Atlantic	291 151	250	72.79	492	Area extends north to, and including, James River Basin, in Virginia
4	Mississippi	1 231 348	300	369.40	410	Area is exclusive of 11 000 sq miles in Canada
5	Western Gulf Basins	321 800	600	193.08	205	Area is exclusive of a part of the Rio Grande in Mexico
6	Colorado Basins	244 000	918	223.99	134	....
7	Great Basin	201 692	50	10.08	2 440	....
8	California Pacific	104 297	300	31.28	410	....
9	Columbia Basin	220 800	50	11.04	2 460	Area is exclusive of 38 700 sq miles in British Columbia
10	Pacific Northwest miscellaneous	51 626	50	2.58	2 460	....
11	Hudson Bay	67 850	40	2.71	3 075	....
12	Arizona and New Mexico to Mexico	14 300	100	1.43	1 230	....
13	Total, United States	3 026 939	....	932.94	....	....
14	Weighted average	....	308	......	400	....

*Computed on the basis of 106 lb per cu ft in place.

Consideration of rates of débris production in various sections naturally leads to speculation as to regional comparisons and quantitative estimates for the country as a whole. Available data can scarcely be considered adequate for accurate determinations, but the values in Table 3 are sufficiently well distributed to serve as a basis for general approximation. Study of the data permits an estimate of the average annual waste per unit of area for each of the main drainage basins and in combination with the tributary areas, the total weight of material removed and the depth of degradation have been computed. The results are presented in Table 5.

The estimated annual total of 933 000 000 tons for the United States as a whole amounts to an average of 308 tons per sq mile. Assuming a weight of 106 lb per cu ft for the material in original place, the average wearing off would be 0.0025 in. per yr. It thus appears that the elements are planing down the surface of the country at an average rate of 1 in. in about 400 yr.

It should be understood, however, that this material is not lost. Very little escapes to the oceans. Dr. Gilbert found[38] that of the 2 375 000 000 cu yd of alluvial débris originating from lands tributary to San Francisco Bay, during the 65 yr, from 1849 to 1914, only 50 000 000 cu yd were carried through the Golden Gate and deposited in the ocean. For the most part eroded material is re-deposited along river channels, on flooded bottom-lands, and in lakes, reservoirs, and bays. New land is thereby formed in the same way that the fertile low lands have been built up through countless geologic ages. The foregoing estimate fails to consider the constant building up of soil through the decay of vegetation. Over large areas this agency probably more than offsets the degrading effects of erosion. On the other hand, no consideration has been given to material carried away in solution. Investigations disclose that, on some streams,[39] the amount of dissolved solids exceeds the weight of the suspended matter. As a rough approximation, it seems reasonable to assume that the losses of dissolved solids are balanced by the gains from vegetation decay, and the estimate based on silt movement provides a fair measure of the general rate of surface leveling.

C. S. Jarvis,[40] M. Am. Soc. C. E. (by letter).—The author has undertaken a calm, methodical analysis of the natural forces as related to run-off and erosion, together with the influence manifest by works of Man in altering the definite balance that presumably existed under long-established natural conditions. Engineers need more of such serious consideration, before committing themselves to policies that have a decided trend against both public welfare and individual security. A great service has been rendered to the profession, and thereby to communities at large, by bringing the subject and its ramifications into such relief at this time, when it is most needed. Although the writer is in accord generally with the views and conclusions in this paper, he must take exception to some of the minor details.

When long records are studied—especially graphic charts of rainfall and run-off—the most impressive feature seems to be the wide variety of precipitation and stream flow that may occur in a locality. With long-term records, such as 50-yr periods, the mean annual rainfall at stations of most dependable supply in the United States may represent about 1.4 times the minimum yearly; and a like ratio is approached by the maximum in comparison with the mean. A wider disparity exists between minimum and mean or maximum seasonal rainfall; and a still wider disparity, if compared on a monthly basis. Furthermore, these ranges are greatly increased in the field of run-off, the minimum annual yield from a water-shed being perhaps one-tenth of the long-term maximum, while the variation among monthly or daily discharges may exceed 100 as a ratio, even in humid areas. In arid regions, both precipitation and stream flow are still more erratic and flashy.

---

[38] *Professional Paper No. 105*, U. S. Geological Survey, 1917, p. 50.

[39] *Water Supply Paper No. 274*, U. S. Geological Survey.

[40] Senior Hydr. Engr., U. S. Geological Survey, Washington, D. C.

Man's main efforts with respect to altering conditions of natural stream flow and erosion may be symbolized by his treatment of rough land: He reduces the knolls and fills in the gullies, striving to obtain a more nearly even and dependable profile. Taking the example of the Rio Grande immediately below the Elephant Butte Dam as representative of thorough regulation, it is difficult to find any serious up-setting of the physiographic balance. The data of Table 1, relating to comparative scour and fill of the Rio Grande channel below the dam are enlightening, particularly when expressed as totals or subtotals. Thus, the changes therein noted between the years 1915, or 1918, and 1932, in the cross-sections of the river channel selected at the first ten of the tabulated stations, amount to totals of 2 628 sq ft of scouring as compared with 2 332 sq ft of fill. Therefore, in the 143 miles between the Elephant Butte Dam and International Dam, near El Paso, Tex., the regulated stream, devoid of the customary silt burden that had prevailed during the natural and primitive conditions, was actually deepening its channel. It is not surprising, however, that the inclusion of the final five stations for this comparison would have shown an increase of fill amounting to 1 493 sq ft, or nearly 300 sq ft per station, far below the main canal diversions. This silting of the channel is small in comparison with what has been observed near San Marcial, N. Mex., as a result of back-water influence from the reservoir. Moreover, it was demonstrated early in August, 1925, that destructive floods could form as run-off from the usually arid region between Elephant Butte and El Paso, representing more than one-half the peak flow of record from the unregulated stream.[41]

The writer's observations do not uphold the idea expressed in Article 1, to the effect that a decrease of 20% in rainfall may "transform habitable semi-arid areas into uninhabitable deserts"; and that, an increase by the same amount would necessarily result in "flood conditions of much greater severity." The proof lies largely in the record of variations previously mentioned herein. The sub-normal precipitation occasionally ranges to a minor part of the mean annual, and the excessive rainfall ranges to more than double the mean. Even a succession of several years with rainfall far removed from the ordinary either above or below normal, has not apparently wrought the corresponding changes. Neglecting those years during which the excess or the deficiency occurred in the winter or non-growing season, and centering attention on those growing seasons with either sub-normal or excessive rainfall to the amount of only 20% of the mean, intensive cultivation and mulching to conserve the soil moisture may compensate fully for such a deficiency; and the increased capillary action of the soil during periods of abundant rainfall seems to increase the permeability thereof, thus allowing larger percentages to enter underground storage than if the surface were bone dry. Under such conditions, floods of greater severity do not always follow. They are more likely to occur from a torrential downpour on baked, drought-hardened soil than on moderately moist earth, supporting a luxuriant seasonal growth of vegetation to retard surface flow and to encourage or facilitate absorption, and transpiration thereafter.

---

[41] "Flood Flow Characteristics," by C. S. Jarvis, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 89 (1926), p. 1098.

Granting that débris barriers of moderate height, 50 ft, or more, are more effective for intercepting abnormal débris loads than a large number of 6-ft check dams would be, the writer's experience and observation would justify the adoption of numerous small structures rather than single high dams, for the majority of conservation projects. The potential hazards are so much less with the smaller barriers; their construction is much simpler; they are drowned out and cease to cause intensive local erosion when the flood volume increases greatly; and the failure of one or more need not greatly disturb the others of a series, nor damage the property which they are intended to protect. In many cases, they are better investments; they may be undermined or overturned, yet they remain obstacles, better adapted for maintaining the physiographic balance than the higher structures.

W. P. Rowe,[42] Assoc. M. Am. Soc. C. E. (by letter).—The subject of this paper is timely for Southern California, because of the extensive flood-control and water-conservation works being built under the Federal Unemployment Relief Administration in this area. The author sub-divides the effect on the physiographical balance of conservation measures, under three headings: (*a*) Changes in the water-shed cover; (*b*) effect of regulation on the natural balance of a stream system; and (*c*) effect of débris barriers on the stability of the stream bed on the débris cone. He assumes that these three effects are the work of Man. The water-shed cover is the work of Nature, and it is axiomatic that Nature is never static. Climatic conditions are a governing factor in water-shed cover and one over which Man has no control.

The disaster in the La Crescenta-Montrose area of Southern California (New Year's Day, 1934), resulting from an intense rain (averaging more than 12 in. for the 2-day storm, and with a fairly uniform maximum intensity of about 1 in. per hr for 3 hr over the entire area), on a small water-shed which had been recently denuded by fire, has concentrated attention on water-shed cover. All the measures for fire prevention, advocated by the Forest Service in dealing with brush-covered areas, such as paved highways, motor-ways, fire trails, fire breaks, fire apparatus, and hundreds of men from near-by unemployment relief camps were available, and still the fire burned unchecked until the area was burned over. The fire was held at the crest of the drainage area by back-firing, which is nothing more nor less than controlled burning. When a torrential rain fell on this area, the catastrophe resulted. The natural phenomena accompanying this disaster offer the clearest examples as to how the many enormous débris fans have been formed at the mouths of Southern California streams. The occurrence of these fans is evidence of similar but heavier floods in the past.

---

[42] Cons. Engr., San Bernardino, Calif.

Segregating proposals for improving the water-shed cover Mr. Sonderegger, in Article 2, suggests two classes. Regarding the first of these (as it relates to the capacity of the water-shed cover to "induce more copious precipitation"), the rain in the La Crescenta-Montrose area, where there was apparently as much rainfall on deforested areas as on those which were heavily forested with brush, would seem to disprove this theory; at least, as regards local conditions in Southern California.

In treating the second class it should be emphasized that reduction of evapo-transpiration loss is not the primary reason for the advocation of light deforestation or controlled burning. The fire in the La Crescenta-Montrose area demonstrates the advisability of some method of light deforestation in the brush-covered areas, which would permit no more than 50% of the water-shed cover to be denuded at any one time. Where the entire water-shed cover was destroyed, the resulting damage was heavy, but in the adjacent areas, where less than 50% of the water-shed area was burned over, there was little material damage. The fact that this area was denuded, in the face of all the preventive measures recommended and used by the Forest Service, would indicate that the present policy is wrong. If light deforestation were advocated simply as a means of decreasing evapo-transpiration losses and thereby increasing the water supply, the present Forest Service policy of fostering a fire hazard, which eventually results in the temporary destruction of the entire water-shed cover when the inevitable fire occurs, offers the best medium. The inevitable needs no advocate.

That the effect of ashes is to seal the stream bed, is a moot question. The presence of ashes in an absorptive stream bed, after a forest fire, is an indication that the water has percolated through and left the ashes behind. Any flood will stir up the stream bed and carry away the transportable material, the size of which will depend on the severity of the flood. When the peak of a flood has passed, lighter and still lighter materials will be dropped as the flow decreases, until, eventually, only the finer materials are left to be deposited when the water sinks. The argument relating to ashes has been used in the past as regards the presence of silt in a stream bed after a flood. The present methods of maintaining silt-suspending velocities in the spreading ditches have solved this problem. It is significant that no large deposits of ashes or silt have ever been found, as far as the writer knows, in excavations made in active stream channels on Southern California débris cones.

In connection with the acts of Man on the effects of over-balancing the regimen of the stream, Mr. Sonderegger has pointed out the effects of the deposition of débris resulting from the construction of dams or barriers across the channels. The Government's activities for unemployment relief in Southern California have resulted in large sums of money being spent on water conservation works on the débris cones at the mouths of Southern California streams. Dams have been built across them without any regard for the future effects of deposition of additional débris.

In 1926, a spreading dam, 2 000 ft long, was constructed across the mouth of Lytle Creek. This dam raised the bed of the active channel at least 10 ft. In succeeding years, floods re-graded the stream to such an extent that it has been raised at least 2 ft at the U. S. Geological Survey gauging station 1.5 miles up stream. The future effect of this dam would have been a shifting of the stream to a highly improved area on its débris cone west of the old active channel, if an expensive training levee confining the stream to its former active channel, had not been constructed. This reconstruction resulted in the abandonment of about 1 750 ft of the original dam.

In 1920, a spreading dam, 3 000 ft long, was constructed on the débris cone of the Whitewater River. This dam was built on an approximate contour to a height of 5 ft, but with the maximum height limited to 3.5 ft above any of the active channels traversing the cone. The other 1.5 ft of height was buried. Floods which followed the construction of the dam resulted in the re-grading of the stream beds up stream, until the water was forced out of its former channels and spread over an area which the stream had not occupied for at least twenty years. Precise profile lines run across the cone from this dam in 1920, 1924, and 1927, show that each flood gradually raised the bed up stream until, after the capital flood of 1927, the re-grading effect was noticed at least 1 mile up stream. This was one of the primary reasons for the construction of this dam, as no danger could result from such additional overflow of adjacent lands.

A careful survey of the débris cone was made in 1927 by running an axial line down the center of the cone and parallel radial lines on the arcs of circles having their apexes at a common point where the stream first begins to fan out. This survey showed that there was a true cone between the confining banks of the wash. Future dams are planned to take advantage of this condition by installing movable crests so that the stream bed may be raised or scoured at will. In addition to the re-grading effect up stream, there was a scour down stream due to the erosion of the 1.5 ft of material between the original stream bed and the spillway mat 5 ft below the crest of the dam. This scouring effect extended down stream for a distance of 700 ft.

The author has shown correctly that the effect of the construction of bridges, culverts, dikes, and roadway dips across débris-laden streams is not a simple matter, but one that should receive careful study, so that the various structures along the same stream shall be co-ordinated in their effect on the deposition and scouring of the débris.

Another factor in the deposition from débris-laden streams is the elimination of artesian areas through which such streams may flow. In the San Bernardino Basin, the artesian rim in 1916 extended to a point 5 miles, or more, above the Bunker Hill dike, and no deposition took place in this stretch of the stream while the artesian rim remained at this limit. However, with increasing use of water and years of sub-normal stream flow, the artesian area shrank until, in 1934, there are more than 3 miles of absorptive channel where there was formerly a rising stream. This has resulted in deposition of débris by the diminution of the stream flow due to absorption. At the Gage Canal intake, 2 miles above the Bunker Hill dike, there is a sand-trap in the canal which formerly drained water and sand back to the stream level. One can now stand in this sand-trap, and his head will be below the level of the present stream channel.

In designing barriers across such streams, the effect of scouring during flood periods has not been considered in many cases. The writer has found tin cans and other signs of recent origin at a depth of 5 ft below the active stream bed, indicating scouring to at least this depth during flood. A sawed plank was found during the excavation for Boulder Dam at a depth of 50 ft below the present bed of the Colorado River.[43] Stream gaugings at Yuma, Ariz., indicate that the Colorado River scours to a depth of more than 30 ft during capital floods.[44] When a permanent barrier is placed across such a stream, the flow that was formerly transported through a scoured section is forced to pass over the barrier, thus increasing the wetted area above that barrier. In many cases the only way in which this increased flow can be accommodated is by overflowing adjacent lands which were formerly in no danger from floods.

Another active phenomenon that influences the physiographical balance of streams in Southern California is the effect of changing climatic conditions. The Whitewater River flows into the Salton Sea, which in recent years (but not within the memory of Man), extended to an elevation 40 ft above sea level. The present level of the Salton Sea is about 250 ft below sea level. While it was at its former level of + 40 ft, the stream bed of the Whitewater River was graded to fit this condition; since the lowering of the sea level, the stream is accommodating itself to the changed condition by scouring its bed. This scouring is an almost imperceptible process because of the long intervals between capital floods. Whenever such a capital flood occurs the erosion proceeds up stream as a series of cataracts.

In former years the Mojave River adapted its bed to fit the conditions created by the ancient Manix Lake into which it flowed. This lake had a surface area large enough to evaporate more than five times the present mean annual flow of the river. After the barrier that formed one side of the lake was washed away, the water in the lake escaped and scoured a deep outlet. The stream is still adapting its grade to this new condition, and in this case, also, it is slow because of the infrequency of capital floods.

The physiographical balance on débris cones is a delicate one. There is ample proof that slow climatic changes (imperceptible from year to year, but apparent over long periods of time) are modifying this balance. Annual precipitation data are misleading. A year of well-distributed rainfall will move a certain quantity of débris and will deposit it as the water sinks into the cone. Another year of equal rainfall, but torrential in character for a few days or hours, will have a far more devastating effect.

Professor Ellsworth Huntington has given a vivid description of alluvial terraces as proof of climatic changes.[45] Similar evidences of change are to be noted on the various coastal streams of Southern California. Changes of stream-channel conditions, that can not be attributed to over-grazing, forest fires, or any act of Man, have occurred within the memory of man.

---

[43] *Engineering News-Record*, December 21, 1933, p. 762.

[44] "The Colorado River Problem," by William Kelly, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 88 (1925), p. 322.

[45] "The Climatic Factor," by Ellsworth Huntington, *Publication No. 192*, Carnegie Institution, pp. 23–36.

The New Year's Day, 1934, flood in Pickens Canyon in the La Crescenta-Montrose area exposed evidence of similar floods and canyon-side slumping in previous years. At one place, a log about 12 in. in diameter, that had been buried long enough to rot through, is exposed. At another place, a flood had carried down an oak tree and buried it in a horizontal position. Its roots were buried and several sprouts came up from the trunk. These sprouts had developed into trees as large as 4 in. in diameter before the last flood exposed their origin. The fact that some agency was instrumental in building more than a hundred check dams in this narrow bed-rock canyon would also indicate that some one with previous experience had recognized the danger from débris or mud flows.

The carrying capacity of a stream loaded with sand, gravel, and boulders may be calculated, but when a large quantity of mud increases the specific gravity of the conveying fluid, the carrying capacity is tremendously increased. A flood of clear water of sufficient force to move some of the boulders found on the local débris cones is inconceivable, but when there is a flow of mud accompanying the flood one can easily account for the presence of these boulders.

Mr. Sonderegger has mentioned débris barriers above the canyon mouths as a means for preventing the débris from reaching the cone. The Haines Canyon débris basin, which was credited with such miraculous powers during the La Crescenta-Montrose disaster, was simply a gravel-pit in the stream channel. Its lower end was closed with the waste sand from the screening plant. The confining wall had been lined with wire and rock protection and equipped with a shallow flat spillway lined with concrete. All the storage capacity was below the original stream bed at the outlet end. The excavation of rock and gravel at the upper end of the pit had created a drop of more than 30 ft at the entrance of the stream channel. When the flood occurred, the water that flowed over this loose alluvial bank soon cut a new grade and deposited the eroded material in the basin. This recession of grade worked up stream through several wire and rock check dams and was stopped just below the gauging station about ½ mile up stream. At this point the grade was lowered about 5 ft. The material eroded in this section of the stream accounted for most of the débris deposited in the basin. Had the barrier been erected above the stream bed, most of this scouring would not have occurred, and there would have been more storage space for débris from the upper water-shed. In such a case, the dam and spillway would have to be of the safest and most permanent construction. It would probably be cheaper to re-excavate the débris after each flood than to construct such expensive works. This would depend on the frequency of débris-carrying floods.

The high peak flow of 4 000 or 5 000 sec-ft per sq mile credited to Haines Canyon[46] was undoubtedly caused by log jams in the canyon backing up the water and suddenly giving way. That this flow was of short duration is proved by the outflow through the spillway of the débris basin. The area of the Haines Canyon water-shed above the débris basin is approximately 1.2 sq miles, about one-half of which was burned over. At the critical section of the spillway, the average depth of water was 2.2 ft and the average width was 45 ft. The concrete approach channel is level, but just below the critical section at the crest there is a drop of about 1 ft in 10 ft. Computations based on these data indicate that the outflow did not exceed 400 sec-ft.

---

[46] "Notes on the Recent Flood," by E. C. Eaton, M. Am. Soc. C. E. (Not published.)

The area of the Haines Canyon débris basin below the spillway level was not greater than 2.5 acres before the flood. During the flood a débris cone was built at the upper end of the basin by inflowing débris. The present area (1934) at the high-water mark is 1.9 acres. According to an eye witness, the water level in the basin was 3.5 ft below the spillway level at the time the peak flow arrived, so that there was a capacity of about 9 acre-ft for storage before overflow began. The float-well at the U. S. Geological Survey gauging station silted up during the peak flow. A total of 7 acre-ft passed the gauging station during the three days preceding the peak. This value, less seepage losses, is an indication of the original storage below the aforementioned 3.5-ft mark. Between the spillway level and the high-water mark there would have been about 5 acre-ft of storage while the peak outflow was occurring. If the inflow was assumed to be 4 000 sec-ft for only 3 min. and the outflow was 400 sec-ft for the same period, the available storage would balance the flows.

The same witness stated that the outflow continued for 2.5 hr. The inflow during this period could not have exceeded the outflow as there was no available storage for regulation. If all the rain, falling on a burned-over area of 300 acres, at a rate of 1 in. per hr, became available as run-off and one-fourth of the same amount of rain falling on 400 acres of brush-covered area joined it, one would have a run-off figure consistent with the peak outflow as computed previously herein. It is evident that the high peaks were caused by the sudden release of dammed water and that steps should be taken to prevent the recurrence of such a condition in the future.

The approach to the spillway of the Haines Canyon débris basin is crossed by a railroad trestle for the operation of the gravel pit. The trees carried down by the flood partly blocked the approach and it is conceivable that the outlet might have been blocked completely by trees and other drift, in which case the débris barrier would have failed, resulting in an enormous damage. The elimination of similar hazards to the proposed small débris basins will be expensive unless the hazard is eliminated at the source. The saving of evapo-transpiration losses from trees removed from stream bottoms would pay the cost of removal. It would not be necessary to remove all the trees. A selection could be made and the trees with deep root systems and least water requirements, preserved for recreational purposes. Under the present policy of the Forest Service in closing most of these similar areas to recreational uses, in direct contradiction of the policy on which this Service was established. it would not be necessary to save even these few trees.

The maximum depth of débris deposition in the basin amounted to about 18 ft and was of such a mixture of mud, sand, gravel, and boulders that there would have been little drainable void space for underground storage, especially near the outlet. If the débris deposited behind a barrier is to be utilized for underground water storage it would be advisable to provide some means of passing mud during the flood.

In Article 5, the author gives two examples of the major débris-carrying streams in Southern California. A study of these data and those of other local streams would indicate that the grade of the débris cone just below the mouth of the canyon is an indication of the flow, the steeper cones being below the mouths of the streams of lesser flow. These examples could be used as a substantiation of the argument of Professor Huntington relating to profiles of débris cones in relation to stream flow.[47]

Mr. Sonderegger is clairvoyant in describing the results of dam failures on a débris cone (see paragraphs in Article 5 preceding "Rate of Débris Production"). These paragraphs, written as they were several months before the La Crescenta-Montrose flood, are prophetic. Pickens Canyon, having the major water-shed (1.6 sq miles) of the streams causing the damage, traverses a region of faulted and easily eroded rock. It has built a large débris cone below its mouth from previously eroded material.

The main branch of Pickens Canyon flows in a narrow trough, ranging from 50 to 100 ft deep. Before the flood it was crossed with a multitude of wire and rock check dams averaging 6 ft high. Most of these dams were founded on bed-rock bottom, or very near to it. The sides of the canyon, for a distance of at least ½ mile above its mouth, are very precipitous, and the soil is held in place by the large masses of roots of the oaks and other vegetation. The trees in the canyon bottom were not burned by the fire preceding the flood. About ½ mile above the mouth of the canyon there are typical slides which accompany faulting along such streams in Southern California. The bank material is loose rock with large masses of clayey earth. The rainfall of more than 12 in. for the storm, saturated this material, causing it to slump into the stream, carrying trees, rocks, and earth into the canyon bottom, where the trees lodged and formed temporary dams, behind which the heavier débris lodged. There would have been a débris flow from Pickens Canyon even if the water-shed cover had not been burned.

The check dams were washed out as soon as the confining wires were broken by the erosive action of the material in the stream. The débris accumulated behind these check dams was added to the slumped material and carried down the canyon. There was very little scouring of the channel below the original canyon bottom because of the shallow fill above bed-rock. At the mouth of the canyon, the stream debouches on a steep débris cone built up by previous floods. The stream channel across this cone above the highway was also crossed by numerous check dams. These dams accumulated large masses of débris before their failure, but when they failed, the channel was scoured to depths ranging from 5 to 10 ft below the original bed. The hugh boulders deposited in the residential area below the State highway were carried down from this débris cone and their presence on the cone is evidence of similar but heavier floods in the past, before there was any over-grazing or forest reserve.

---

[47] "The Climatic Theory," by Ellsworth Huntington, *Publication No. 192*, Carnegie Institution, p. 33.

The check dams that withstood the flood in Haines Canyon, offer an example of what must have happened to the check dams on the various débris cones above La Crescenta and Montrose. Unlike Pickens Canyon, Haines Canyon has a broad flat débris-filled bed. The flood passing over the check dams in Haines Canyon scoured the channel below the toe of the dams and deposited the eroded material behind the next dam down stream.

The check dams on the Pickens Canyon débris cone remained intact long enough to raise the stream bed and cause overflow of the banks. Vast amounts of débris were then diverted out of the original channel. The culvert at the State highway was so well constructed that it did not wash out. Its capacity, however, was so reduced by a 6-ft boulder lodging at its entrance, that most of the débris coming down the channel passed over the highway, which acted as a spreading dam and spread the débris over a broad area.

The latest spreading works on the Upper Santa Ana River are designed to by-pass the silt back into the active channel, where the stream will have a flow diminished by the amount of absorption above. This action will tend to increase the silt load beyond normal conditions. When the stream reaches the absorptive area in the lower river any further absorption will raise the stream bed by the deposition of this silt. It is necessary, if spreading works are to be maintained at their maximum efficiency in such stream channels, that large flows be permitted to pass through them at intervals, in order that the silt may be scoured out, even if large flows may escape into the ocean. Professional conservationists, in their desire to impress the taxpayer with the importance of their endeavors, frequently cite this "large waste of water into the ocean" as the argument for more extensive works to preserve this water, even if the cost of the works outweighs the value of the water thus escaping. They fail to take account of Nature's laws and unwittingly accelerate the overthrowing of the physiographical balance.

J. C. Stevens,[48] M. Am. Soc. C. E. (by letter).—The author postulates his thesis on the theory that the physiographical factors combine to produce a balance. Such a general statement may well be challenged.

Nature herself seldom remains quiescent long enough to permit the establishment of anything like a permanent or even a gradually changing balance of these factors. She builds consistently for a few years and then with wild abandon utterly wrecks her handiwork. Man's efforts are puny in comparison, yet so conceited is he that he likes to believe that his is the mighty hand.

One factor at man's command is the construction of dams. Another is the straightening of stream channels. He may remove timber, cultivate lands, divert water for power and irrigation, and graze stock. Beyond these, his efforts are futile. Even his control is haphazard and uncertain, because his highest dams fail and his diversions get out of control, as witness the St. Francis Dam failure and the flooding of the Imperial Valley by the Colorado River. He cannot control, in the slightest degree, rains, frost, winds, tides, landslides, earthquakes, fire, and vulcanism. Nature buffets him mercilessly with these forces.

---

[48] Cons. Hydr. Engr. (Stevens & Koon), Portland, Ore.

Man's works are temporary. Left without persistent and painstaking maintenance and repair, they soon become mere ruins, failing utterly in their intended purpose. Nature's forces never rest. These are such platitudes, it is almost a waste of space to recount them; yet, in the face of these facts, how can any one conceive of a physiographical balance "that undergoes only gradual changes"?

Was the Gros Ventre landslide (Wyoming), that put a dam in Gros Ventre River 1 000 ft high and, later, wrecked the entire landscape below it, a gradual change? Are the rather frequent Southern California floods caused by a foot or more of rainfall in a day factors for gradual changes? Is it fair or consistent, or rational, to assign the burning of a scattered growth of chaparral as the cause? If so, what about the recent unprecedented floods in the Pacific Northwest from the most heavily forested areas in this country? The changes effected were not gradual, but sudden and mighty. What about the Yellow River of China that, during the flood of 1851, suddenly left its channel, inundated an empire in spite of elaborate works of man, snuffed out a million lives, and found a new outlet to the sea 500 miles to the north? The entire physiographical complexion of China is thus suddenly changed every few generations. There is no balance.

A protracted wind storm a few years ago laid low thousands of acres of standing timber in Oregon. A more ruthless destruction, man, with all his machinery and ingenuity, has not equalled. In the summer of 1933 a forest fire on the west slope of the Coast Range in Oregon destroyed an enormous area of heavy virgin timber. That fire was carelessly set by man; yet, throughout the Northwest are untold acres similarly destroyed by fire from lightning before man's occupancy.

These few examples illustrate the impotency of man's control and raise a serious doubt as to whether any permanent relief can be predicated on a theory that there is a physiographical balance of Nature's factors. It seems that the author's physiographical balance is only Nature's quiescence between upheavals.

Erosion has come to claim the attention of scientists, pseudo-scientists, "yes men," and politicians. A struggle for the control of the grazing areas of the public domain has invaded Congress. Who gets that control has funds, personnel, jobs, influence. Propaganda in waves, broadcast by speeches, radio, articles in scientific and lay journals, and by newspapers, points to the same thing, "unprecedented and accelerated erosion due to over-grazing."

The writer believes in a National control of grazing and forested areas, but not for the reasons often assigned. Control of forest areas to perpetuate a timber supply and for recreational purposes is reason enough. The water-conserving properties are insignificant, unimportant, and enormously exaggerated. Control of grazing areas for maintenance of stock feed affords abundant reasons why it should be exercised. If grazing areas are so controlled for that purpose the erosion feature will be cared for automatically.

Erosion does not result from the mere removal of timber or the mere grazing of animals. It results from the cultural works of man. Roads, trails, and logways, are incipient causes of erosion. Field cultivation is another. On grazing areas erosion occurs at places of animal concentrations, such as waterholes, trails, and feeding and bedding grounds. On both forested and grazing areas such incipient erosion can be eliminated only in so far as these cultural activities can be eliminated or effectively controlled.

The writer agrees heartily with the author that temporary débris barriers, silt dams, flood channels, and works for erosion control of non-permanent materials are almost worse than none. When they fail, as they surely will, without well planned and well financed maintenance expenditures, the results are more disastrous than if they had never been constructed.

GERARD H. MATTHES,[49] M. AM. SOC. C. E. (by letter).—The author has focused attention on a much neglected phase of water conservation in the United States. In Europe, the evils resulting from disturbing the physiographic balance which exists in any water-shed between erosion, run-off, and the débris-carrying capacity of streams, have attracted considerable attention. The situation on the Rio Grande, described by the author, shows clearly that the time has come for American engineers to look ahead and profit from European experience. This is the more important in view of the large number of flood-control and water conservation projects now under consideration. As far as the writer is informed, little thought appears to have been given, in any of these projects, regarding the ultimate effect of their operation on the stream channels below them.

Channel deterioration is of two distinct kinds:

(*a*) Excessive bed scour in the reach immediately below a reservoir, caused by lack of sufficient débris load to absorb the energy of the released water normally available for débris transportation; this assumes all the bed load and, at least, part of the suspended load to lodge in the reservoir.

(*b*) Loss of cross-sectional area in the lower reaches due to the inability of the reduced stream flow to remove débris accumulations contributed by uncontrolled tributaries emptying below the reservoir. As illustrated by the Rio Grande case, both forms of deterioration, although of opposite types, may occur in the same stream. In fact, the conditions cited under Class (*a*) operate to aggravate those under Class (*b*). In either case, the hydraulic gradient eventually becomes affected, after which restoration to normal carrying capacities for both water and débris cannot be accomplished except at considerable cost. It is to be noted that the conditions under Class (*a*) are readily checked by providing suitable sill dams, whereas conditions under Class (*b*) are not easily rectified; and if they are not checked at the start they lead to serious losses of channel capacity.

---

[49] Prin. Engr., Office of Pres., Mississippi River Comm., Vicksburg, Miss.

In most projects interference with the physiographic balance, as described, can be minimized if not completely eliminated by careful *a priori* evaluation of soil erosion and run-off conditions on the one hand, and by the débris-carrying capacities of streams on the other hand. The principle involved is that any stream, viewed as a débris carrier, requires for its channel maintenance the flushing action of bank-full, or nearly bank-full, discharges at appropriate times. Furthermore, intelligent operation of reservoirs can provide such discharge if included as an essential feature in project design and operation. The reason why comparatively little channel deterioration has resulted in the Eastern United States from failure to observe these requirements, is that water utilization there is still in its infancy. Most reservoirs are not large enough to prevent major floods from spilling large volumes into the river below, and, therefore, do not materially curtail the flushing action of flood flows. In the arid Western States where for economic reasons water conservation has been extended farther, reservoirs large enough to absorb most of the flood flow are found on many streams. It is in such cases that the effects of channel deterioration are most noticeable. The author has given a clear portrayal of the problem as it affects these Western conditions.

One factor enters into the problem, which heretofore has received little attention. This is the resistance to scour and transportation by water on the part of débris accumulations which have aged and become compacted. Compaction of this kind is much more pronounced in Eastern than in Western streams, a circumstance which tends to mitigate channel deterioration in the latter. While bank-full flow is practically a necessity for removing débris accumulations in Eastern streams, it is probable that much less than bank-full discharge does effect such removal from streams in the arid West. Accurate data on this latter aspect are greatly needed. Certain it is, that the physiographic balance in Eastern streams is dependent on more frequent high rates of discharge than in the arid West, and that the streams of the latter during high stages carry a vastly greater débris load than Eastern rivers in flood.

An important difference appears to lie in the relative degrees of compaction of the débris. In the East, stream flow is perennial, and any materials left to lie undisturbed under water compact at a rapid rate, especially during the summer and fall when organic matter of all kinds is very active, and bed-load movement practically ceases. Agglutination of the sediment particles results, and this, added to the mechanical bond brought about by the puddling action of water which tends to fill the voids between large particles with smaller ones, enhances cohesion to a marked degree. The finer silts and clays compact more firmly than the coarser sands and gravels. In the arid regions, stream beds are dry or partly dry for many months each year. Oxidation of organic matter, sunlight, and wind then operate to reduce compaction. As flood flow in such streams usually subsides rapidly, débris deposits do not lie submerged long enough for compaction to reach a high stage of development; and this will account for the notoriously mobile character of the materials composing the dry beds of Western arroyos and rivers. This mobility facilitates their transportation when flow resumes.

Deforestation, over-grazing, and soil erosion have received increasing attention of late years, but the necessity of maintaining unimpaired débris-carrying capacity in streams has not come in for the attention that is due it. Evidently, it has not forced itself sufficiently on the attention of those who have the power to correct it. Impairment of natural channels in its incipient stages, is not easily recognized and, like an insidious disease, its symptoms are not manifest until an advanced stage is reached. This makes it all the more important for the Engineering Profession to be alert in this matter.

J. B. Lippincott,[50] M. Am. Soc. C. E. (by letter).—The effect of structures on the natural channels in the streams in the southwestern part of the United States, is well described in this paper. The deductions are of interest and value.

Temporary check dams composed of stones wrapped in wire netting and placed on the débris in the canyons or cones, to catch detritus and reduce stream gradients, are a menace rather than a benefit. Following the severe storm on New Year's Day, 1934, in the drainage basin of Verdugo Creek, 12 miles north of Los Angeles, Calif., all such structures inspected, which had not been built on bed-rock, failed, releasing impounded material. They are likely to mislead the public into believing in false safety.

In Article 2 the author states that "the forest cover unquestionably has an important constructive function in Nature which may not be fully appreciated because it is not well understood." The beneficial effect of forests in regulating floods has been generally accepted, but its profound extent has not been appreciated until recent investigations by foresters in California have made available its quantitative effect. A review of some of these determinations appears to be justified. The general conclusion of field tests has been sustained and emphasized by the startling results of the New Year's flood disaster in the drainage basin of Verdugo Creek.

In Southern California stream gradients of the mountain drainage basins are steep and the prevailing crystalline rocks are of a fragmentary and friable nature subject to rapid erosion. The sedimentary formations of the foothill areas also are rapidly cut by floods. When exposed to violent storms, floods are projected through the canyons on to the valleys, carrying with them surprisingly large quantities of débris which are deposited as delta cones below the mouths of the canyons. The depth of the valley fills is often in excess of 1 000 ft. Floods travel over these deltas in changing and uncertain channels. Communities located in the plains are menaced both by the floods and also, to a greater extent, by the débris. Occasionally, these violent floods contain surprisingly large percentages of mud, sand, and gravel as a thick emulsion, their high flow marks indicating a discharge too great to appear rational, if they were considered as composed of water only. For instance, in the summer of 1931, a flood occurred in Lone Tree Canyon, in the Tehachepi Mountains, discharging into the Mojave Desert, 14 miles northeast of the Town of Mojave. This was computed from high-water marks, to have been 51 500 cu ft per sec from a drainage basin of 35 sq miles as estimated by engineers of the Department of Water and Power of the City of Los Angeles. This drainage basin rises from 3 000 to 6 708 ft in elevation. The lower half is practically bare. Above 4 500 ft it is sparsely timbered and has little brush cover. Floods exceeding 2 000 sec-ft per sq mile were indicated for certain neighboring basins of from 2 to 4 sq miles. Fig. 3 shows the character of the lower part of the drainage basin and the flood channel of Lone Tree Canyon.

---

[50] Cons. Hydr. Engr., Los Angeles, Calif.

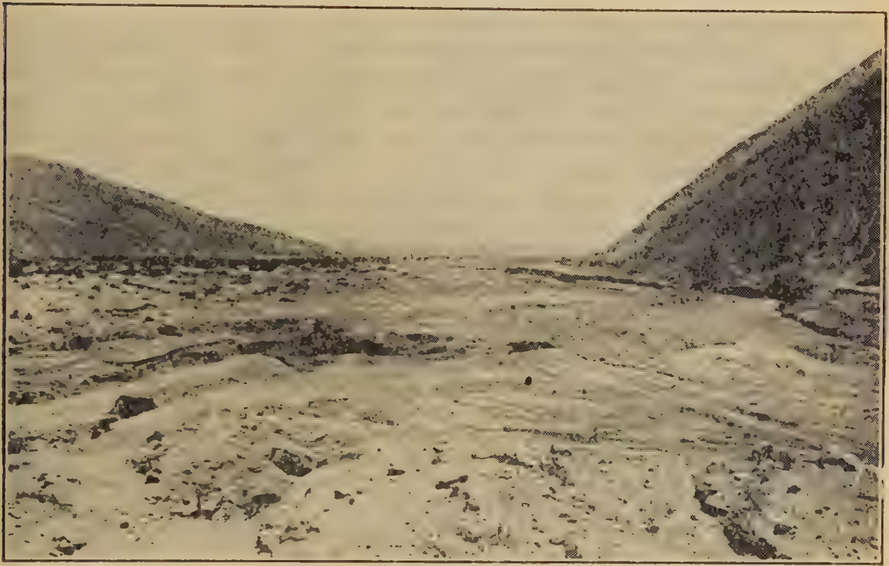


FIG. 3.—LONE TREE CANYON, SHOWING CHARACTER OF CHANNEL FOLLOWING FLOOD OF AUGUST, 1931, WHERE SECTIONS AND GRADES WERE TAKEN. NOTE BARE CHARACTER OF DRAINAGE BASIN.

Another flood on September 30, 1932, in the Tehachepi Pass left marks of indicated flows of 19 200 cu ft per sec from 33.8 sq miles on the headwaters of Tehachepi Creek. This basin is from 3 000 to 5 000 ft in elevation and is about one-half bare of cover. The remaining part has a few scattered oak and pine trees and little brush cover. A flow of 43 500 cu ft per sec was estimated from 20.2 sq miles on Oak Creek.[61] This basin is 10 miles west of Mojave. It rises from 3 600 to 7 950 ft in elevation, and is about three-fourths bare of cover. These three drainage basins are relatively smooth mountain areas. The floods were flashy, resulting from so-called cloudbursts.

During the past few years, the State and National Forest Services have been making intensive studies both in the laboratory and in the field, to find the relationship between water and débris discharge from brush-covered and burned areas.

---

[61] Estimates for Tehachepi and Oak Creeks by the California Div. of Water Resources Rept. (January, 1933), on Flood of September 30, 1932, on Tehachepi Creek.

The storm of December 30 and 31, 1933, and January 1, 1934, which occurred in the Los Angeles area, produced large floods of water and débris from the upper parts of Verdugo Creek. These floods were projected from the steep slopes of the Sierra Madre Range on to the Canada Valley, 12 miles north of Central Los Angeles, causing the death of 34 people, the demolition of 200 homes, and damage to much other property. The results from this storm, accentuated the importance of the study of the foresters of the relation between flows from denuded and forested areas. The data that have been compiled are of such impressive nature to the writer, that it appears of interest to digest some of the records in this discussion.

W. C. Lowdermilk, Senior Silviculturist of the California Forest Experiment Station, at Berkeley, Calif., has conducted a series of laboratory tests for the purpose of throwing light on this relationship. Observations have also been made, under his direction, of the run-off and eroded material from two duplicate small adjacent plots of land at North Fork, on the western slope of the Sierra Nevada, east of Madera, Calif. Two of these plots ($\frac{1}{40}$ acre, each) had normal brush cover, and on two of them this cover was burned. The ground slopes were from 30 to 60 per cent. Recording gauges and sand-traps were used to determine quantities. Dr. Lowdermilk states[52] the following conclusions from this test:

> "Two significant results appear in the data of rainfall and runoff from the plots covered with a natural mantle of vegetation, and from plots burned bare of this vegetation. They are (1) that under intensities of rain up to as much as 2.4 inches per hour no surficial runoff was measured from the covered plots, except where snow drifted into baffle troughs, and (2) that the soil surface burned bare yielded rates of runoff up to 0.30 second feet per acre, representing 200 second feet per square mile. * * * Eroded material from the burned plots reached maximum totals of 3 cubic yards per acre for the rainy season of 1929–30. This material was predominantly mineral soil, whereas material from the covered plots was essentially the organic material of litter."

TABLE 6.—RUN-OFF AND EROSION, IN CUBIC FEET PER ACRE, FROM NORTH FORK EXPERIMENTAL PLOTS; STORM OF DECEMBER 30 AND 31, 1933, AND JANUARY 1, 1934

Area	Unburned	Burned once, 1930	Burned annually 1929–31–32–33
Run-off	4	56	1 854
Ratio	1	14	463
Erosion	0	0.1	100
Ratio	0	1	1 000

The eroded material from the burned plots was 241 times as much as from the covered area. One pair of plots is in undisturbed natural forest-brush cover characteristics of the Sierra foothills. Two of the plots are on an area on which all cover was cut and burned completely in the fall of 1929.

---

[52] *Agricultural Engineering*, April, 1931.

Table 6 gives the results of the test for the storm of December 30 and 31, 1933, and January 1, 1934, on these North Fork plots.[53] The total rainfall for the storm was 2.92 in. Its maximum intensities for 10-min and 60-min periods were 0.18 and 0.58 in., respectively. It will be noted that the ratio of run-off for the period of the storm for the area that was burned once, was 14 times that from the unburned area and 463 times that from the area burned annually four times. The eroded material was zero from the unburned area and 100 cu ft per acre for the area that was burned four times.

Similar observations were made from small plots in San Dimas Canyon, which is on the southern slopes of the Sierra Madre Mountains, in Los Angeles County. These plots are from 1 000 to 1 200 sq ft in area, equipped with inset borders to prevent the incursion of surficial run-off from each side and to prevent the escape of run-off except through the outlets. Automatic measuring devices of the flow are installed. The forest cover consists of brush in the undisturbed plots, the other adjacent plots being burned off. The plots (0.022 acre, each) are established in pairs. The results obtained from observations on the storm of December 30 and 31, 1933, to January 1, 1934, are shown in Table 7. The total rainfall for this storm was 11.68 in.

TABLE 7.—SAN DIMAS RUN-OFF AND EROSION PLOTS; SUMMARY OF STORM, DECEMBER 30, 1933, TO JANUARY 1, 1934

Description	Covered plots		Burned plots	
Plot No.	301	302	303	304
Precipitation per plot, in cubic feet	932.5	932.5	932.5	932.5
Run-off per plot, in cubic feet	1.055	0.702	34.65	37.26
Run-off as percentage of precipitation	0.11	0.08	3.72	3.99
Ratio of run-off	1		40.9	
Maximum rate of run-off expressed as second-feet per square mile	10.2	11.4	223.4	200.7
Eroded material (dry weight, in grams)*	935	446	8 367	16 595
Eroded material (average)	690		12 481	
Eroded material expressed in moisture-free weight, in pounds per acre	68		1 250	
Ratio of erosion	1		18.4	

* Material was dried at 110° C for 40 hr.

Its maximum intensities for 5-min, 60-min, and 1 440-min periods were 0.12, 0.97, and 9.55 in., respectively.

The cover in the unburned area consists of a dense growth of brush approximately 10 ft in height. It will be noted that the ratio of run-off from the burned plots was 40.9 times that of the unburned plots, and that the ratio or erosion from the burned plots was 18.4 times the erosion from the unburned plots. These measurements were made by the California Forest Experiment Station.

Table 8 shows the rain that occurred during the storm of December 30, 1933, to January 1, 1934, along the southerly face of the Sierra Madre Range.

---

[53] From paper by Charles J. Kraebel, Senior Silviculturist, California Experiment Station, before a Conservation Conference in Los Angeles, Calif., March 23, 1934.

TABLE 8.—MAXIMUM RAINFALL INTENSITIES

Station	Inches of Rainfall		Location	Total for storm, in inches
	10-min. period	1-hr. period		
Mt. Lukens	0.30	0.88	Summit of burn	11.04
Haines Canyon	0.27	0.94	West edge of burn	12.08
Flintridge	0.34	1.33	1 mile southeast of burn	14.03
San Dimas Water-Shed	0.25	0.97	25 miles east of burn	10.8
Pomona City	0.28	1.29	30 miles southeast of burn	10.56

While quite uniform it was somewhat unusual in that there was no marked increase in precipitation with rise in elevation. The first station, Mt. Lukens, is at the crest of the Verdugo Water-Shed, Haines Canyon adjoins it on the west and Flintridge to the east. San Dimas is near the base of the range, 26 miles to the east.

TABLE 9.—RESULTS OF NEW YEAR'S STORM; RUN-OFF AND EROSION FROM SOUTHERN SLOPES OF THE SIERRA MADRE RANGE, LOS ANGELES COUNTY, CALIFORNIA

Water-Shed	Rainfall, in inches	Area, in square miles	Percentage of Water-Shed		Peak run-off, in cubic feet per second per square mile	Erosion, in cubic yards per square mile
			Burned	Unburned		
BURNED WATER-SHEDS						
Verdugo *	12.5	19.3	33	67	350	30 700
Pickens †	12.5	0.48	100	0	1 000	50 000
UNBURNED WATER-SHEDS						
Arroyo Seco	12.3	16.24	0§	99.4	58	(No record)
San Dimas	10.8	16.85	0	100	53	56
Fern and Bell ‡	12.4	0.30	0	100	25	52

* Area and run-off computed at Wabasso Way. † Area, etc., computed above junction with east fork. This is a mountainous part of the Verdugo Water-Shed. ‡ Average of four experimental water-sheds, except area, which is total. § The area burned, 58 acres, in backfiring was too small and too lightly burned to affect the run-off from more than 16 sq. miles.

Table 9, taken from Mr. Kraebel's paper,[53] gives run-off and erosion figures from five small neighboring drainage basins. It also shows the extent of burning that occurred in the Verdugo Basin in the fall of 1934.

The increase in stream run-off and eroded material from the burned range as compared to the unburned areas is startling. The comparison from the small Pickens (burned) Mountain area of 0.48 sq miles and the Fern and Bell (unburned) area is illuminating. The Bell Canyon figures are actual measurements, but the Pickens Canyon quantities are estimated. The estimates by the engineers of the Los Angeles County Flood Control of peak flows from some adjacent small canyons in the burned area were much larger. The rainfall on the different basins was quite similar. The Arroyo Seco Basin adjoins the Verdugo Basin to the east. The San Dimas Basin is 26 miles east of the Verdugo Basin. Their topography and exposure are quite similar except that about 6 sq miles of the Verdugo Basin is the steep absorbent canyon floor. Probably the greater damage to life and property in the Verdugo area was caused by the débris that was projected from the burned areas more than by the water.

The volume of the flows in the Verdugo Basin is not possible of close estimation because of the high percentage of débris carried by the water and the erosion of the channels. It is also difficult to make accurate estimates of the quantity of débris discharged on to the valley floor. It has been computed from surveys as more than 500 000 cu yd, mostly from the burned part of the Verdugo Basin.

On the Arroyo Seco, which adjoins the Verdugo Canyon on the east, the peak run-off of which was caught in a reservoir, there occurred only one-sixth the quantity per square mile of the 19.3 sq miles of the Verdugo, the higher mountainous portion of which had been burned over in the fall of 1933. There is no record of the eroded material from the Arroyo Seco, but it was relatively insignificant. The violence of the flood in the headwaters of the Verdugo Canyon is demonstrated by the size of some of the boulders that were left standing by the storm on concrete street pavements, the largest of these boulders being estimated at 59.5 tons (see Fig. 4). It probably was pushed or rolled along by a mud flow.

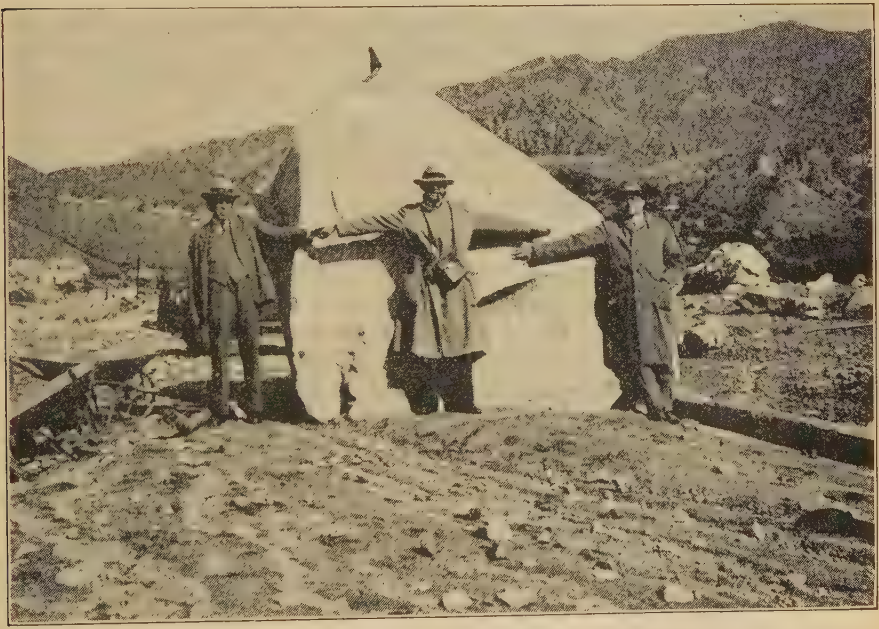


Fig. 4.—Flood Damage, Montrose District, Large Boulder Displaced near Mouth of Dunsmuir Canyon. This Boulder Is Resting on Street Pavement.

It will be noted that only one-third of the Verdugo water-shed was burned over, but this was in a high mountain part of the basin. It will be observed also that La Canada Valley, which has been filled by previous storms with steep sloping débris cones, constitutes another third of the Verdugo Basin. The remainder and unburned part consists of the Verdugo brush-covered mountains and hills. It would be more reasonable to expect less peak run-off per square mile from the Verdugo water-shed had it been brush-covered than from the Arroyo Seco, or other basins to the east, both because of its topography and exposure.

Many "check dams" which consist of rock covered with wire netting, built in the beds of the canyons, to heights of from 6 to 10 ft in the Verdugo water-shed, were destroyed by this flood, demonstrating that such structures, which have been advocated by many, are failures as retarding elements for such floods. If these dams were built on bed-rock and of masonry to heights of 25 to 50 ft, they would have beneficial effect.

Efforts at afforestation in the Sierra Madre Mountains have been failures except on high crests and northern slopes. The brush cover has proved efficient for flood and débris control. It has the advantage of reproducing itself by sprouting from its roots after fires, which is not true of large trees. The so-called "forest reserves" are not used for, and have little value as, a source of lumber.

The lesson to be learned from these floods demonstrates the value of brush cover in controlling floods, preventing erosions, and indirectly aiding in maintaining leveed channels and reservoirs from filling with débris in Southern California. While this a self-evident fact, its full importance has not been quantitatively shown until recently. Greater effort should be made than in the past to preserve the brush cover from fire. Gordon E. Ellis, Assistant Supervisor of the Angeles National Forest, reports that since 1905, 448 fires have occurred in and adjacent to that reserve, which have burned off a total of 227 710 acres, including 102 fires outside the boundaries of the National Forest, the latter totaling 18 704 acres burned. The expenditure by the Federal Government in fighting these fires has been $448 277. In addition to the work of the National Forestry Service, both the State of California and the County of Los Angeles have active organizations which co-operate in forestry work with the Federal Government. Fires occur during the heat of the dry summer and spread through the dense brush with rapidity. It is extremely difficult, if not impossible to extinguish them unless the effort is made at their incipiency. It is essential to reach the fire quickly. Permanent outlooks are maintained in the Forest Reserves and they are also patrolled by airplane to detect fires.

The most effective way of fighting fire is by the construction of a system of roads in the Forest Reserves that will permit of men and equipment being promptly delivered to them when the fire is reported. This can best be accomplished by building trunk roads through the Reserves, which may be used as public highways, from which branch forest roads may be constructed to strategic points, which laterals may be kept closed except in times of emergency. Such road systems are now (1934) being constructed in most of the Southern California Forest Reserves.

The adopted policy of building substantial masonry dams on the principal stream lines for the purpose of flood control and conservation is desirable but very expensive. They have demonstrated their effectiveness. Usually, the capacities of the reservoirs are small and unless their drainage basins are protected by forest or brush cover so as to prevent large erosion in their tributary basins, the duration of their usefulness is rapidly reduced.

The author has properly called attention to the necessity of careful consideration of stream gradients in relation to constructive conservation works. The protection of the forest cover is a still more basic requirement for the maintenance of permanent flood control.

In Conclusion (3) the author states that "engineers, investigators, and agriculturists have come to realize that the water-shed is their common problem and that co-operation will lead to a rational solution of water conservation in the water-shed." The writer heartily concurs in these sentiments which were endorsed at a large meeting of conservationists, including engineers, foresters, and civic organizations, held in Los Angeles on March 23, 1934.

RHODES E. RULE.[54] ASSOC. M. AM. SOC. C. E. (by letter).—The result of mature deliberation on an important but usually neglected phase of conservation engineering is presented in this paper. In so doing the author displays that rarest of qualities, the long-range viewpoint. Although many may find exceptions to detailed opinions expressed, none can question the fundamental wisdom of Mr. Sonderegger's proposition.

The author's physiographical balance is not a condition of true equilibrium; it is rather a relative uniformity in the time rate of change of the physiographic condition. As he states, this uniformity is of long-term trend rather than short. However, this uniformity may be over-stressed. Within human experience rivers have made major changes in their courses. Mountains have actually "fallen", forming gigantic natural dams. Other exceptions to the rule that geologic processes are in general non-cataclysmic are abundant. There is a common tendency to exaggerate the importance of normal geologic processes when they occur infrequently.

When going to extremes Nature has a habit of doing a thorough job of it. The late Allen Hazen, M. Am. Soc. C. E., has stated;[55]

> "In considering the whole flood problem there is the remarkable condition that a few very large floods are so high in comparison with all the rest that the question is raised as to whether they belong in the same class or whether they are outside and follow some law of their own which comes into operation only at very long intervals and with extraordinary floods."

Recent statistical studies made by the writer tend to bear out this hypothesis that extreme storms—and, hence, floods—follow a law of their own. The tentative conclusion is that great floods are more probable of occurrence than is indicated by ordinary probability studies.

---

[54] Civ. Engr., (Rule & Rule), Los Angeles, Calif.

[55] "Flood Flows", by Allen Hazen, p. 53, John Wiley & Sons, New York, N. Y.

Of recent years it has become the vogue to credit many extreme floods to human interference in the form of fires, timber removal, and over-grazing. The conclusion that removal of this interference will reduce materially the possible maximum flood must be treated cautiously. On the other hand, there is no doubt that these human activities have a marked effect on flood conditions and, in some cases, are the controlling factors.

In the case of the destructive floods of 1923 and 1930, at the westerly base of the Wasatch Range, between Salt Lake City and Ogden, Utah, there is strong geologic evidence to support the theory that the rate of erosion resulting from these floods was much greater than at any time since the passing of Lake Bonneville, a matter of tens of thousands of years. This excessive erosion rate is probably properly attributed to denudation by fire and over-grazing.

Similarly, the La Crescenta-Montrose mud flow in Southern California, on January 1, 1934, was the result of a severe precipitation concentrated on a relatively small but almost completely fire-denuded area of steep watershed. Undoubtedly, some human hand was responsible for setting this fire and was, therefore, responsible for upsetting the physiographic balance which resulted in destructive floods and mud flows, and loss of life and property. It is significant, however, that this took place despite a most elaborate and highly organized system of fire-patrol. This raises the question as to how far human effort can go toward stemming the tide of natural forces.

Before civilized man came on the scene detrital flows of the same nature as those occurring in the La Crescenta-Montrose area were of frequent happenings. In no other way is it possible to account for the alluvial fans at the bases of the Southern California mountain ranges. Detritus hundreds of feet in depth has been deposited in the space of relatively short geologic time. To have accomplished this vast task of transportation required many great floods because it is only in major floods that coarse material, such as that comprising these fans, can be transported in any appreciable quantity. It is doubtful whether works of Man can do much in an economic manner toward controlling such floods to the point that occupation of these alluvial fans can be made safe. Granting that débris dams may be built to accomplish the purpose for a term of years, is the cost balanced by the benefits? Should an entire county be taxed for protecting the property of a small special class of property owners? The major portion of the damage in the La Crescenta-Montrose area was done to improvements lying in the direct paths of countless previous floods.

The author's treatment of the effect of regulation upon the natural balance of a stream system deserves serious consideration. In Southern California the equilibrium between aggradation and degradation is accomplished by many seasons of the former and few of the latter. When storage is provided on a stream for all but the capital floods the burden of establishing a new gradient is imposed upon the stream below the storage. The periods of degradation become much fewer and of less ability to do work of transportation. Add to this the continually growing flood discharges from paved streets in the metropolitan area, and the situation becomes acute. The effect of this discharge from improved areas is to increase the discharge and carrying power of the tributary streams without adding much to that of the main stream. It seems that unless proper precautions are taken the arrival of the rare capital floods will find conditions right for overflow of much highly improved real estate.

A. L. SONDEREGGER,[56] M. AM. SOC. C. E. (by letter).—More than gratifying have been the discussions in that they have brought out many points of interest and importance in stream control and conservation. Like the subject of the paper, the discussions have covered a large range of thought and observation. Most of the discussers appreciate the necessity of a correct evaluation of the effect of modern works on natural processes and the physiographical balance. Engineering science has made great strides in the theoretical analysis of strains, stresses, and forces, and has succeeded in clothing important results of research in convenient formulas for the ready use of a learned and busy profession. On the other hand, the study of natural phenomena and a correct appraisal thereof proves to be a matter of experience rather than of learning and, as a rule, is not acquired without reprimand by failure. Hence, it would not be astonishing if subjects like the disturbance of the physiographical balance would find more appreciation with the older members of the profession than with the recent graduate who is justly proud of his newly acquired knowledge.

The importance of the water-shed cover and its effect on run-off and débris production is touched upon by several discussers. Mr. Lippincott's discussion is particularly illuminating in this respect, in that it is supported by comparative data relative to the effect of the vegetal cover in its natural condition and after one or more burnings. He concludes that "greater effort should be made than in the past to preserve the brush cover from fire." While this is a desirable objective, it must be borne in mind that the preservation of the brush cover over a long period of years promotes the accumulation of highly inflammable materials which, when set on fire, are difficult of control. Fires will continue to occur; in fact, with an ever growing population and greater facilities of access to the water-shed, the odds are in favor of an increasing fire hazard.

These considerations may have induced Mr. Rowe's point of view, namely, that of periodical controlled burning. Possibly the solution to this perplexing problem is already found in the present policy of the responsible authorities, of maintaining efficient fire-fighting organizations and completing the network of mountain roads which will permit the rapid conveyance of crews and apparatus to strategical points while fires are in their incipiency. While the number of fires may continue to increase, the probabilities are that, with improved methods and perfected organizations, damages will be confined to smaller areas. Thus, a system of rotation of burning may evolve which will eliminate major conflagrations, at the same time relieving authorities of the risks and responsibilities that attend controlled burning.

---

[56] Cons. Engr., Los Angeles, Calif.

The La Crescenta-Montrose flood discussed by Messrs. Lippincott and Rowe, and referred to by others, has brought out several distinct facts bearing on the flood-control problem of Southern California: (1) That brush fires are a permanent menace at any point; (2) that run-off from tributaries may assume such proportions as to overtax existing flood channels of main streams; (3) that débris production from burnt-over areas may be of such magnitude as to be a menace to foothill areas and to the storage space of flood-control detention reservoirs; and (4) that a material increase in the peak discharge of both water and débris may result from the failure of temporary check dams.

Mr. Jarvis takes exception to the writer's statement that a permanent decrease or increase of 20% of the mean rainfall of semi-arid areas might result in far-reaching climatological or physiographic changes, while Mr. Stevens challenges the general statement that physiographical factors combine to produce a balance, even under existing conditions. The viewpoints are distinctly at variance.

It is admitted that, in the Southwest, cyclic fluctuations of seasonal rainfall in the past, which resulted in wet and dry periods of 10 to 15 yr each, seemingly had no effect on the development of the country, or on the earth's surface. As far as farm husbandry is concerned this is explained, in part, by the utilization of underground storage which, over large areas, yielded abundant supplementary supplies for the relatively short period of droughts. With a permanent reduction of 20% in the mean seasonal rainfall, the periods of relatively low water supply would necessarily be extended and periods of recharge correspondingly shortened, leaving no doubt as to the ultimate effect on the safe water yield of large areas.

The natural vegetal cover of the water-shed, as a rule, is a fair index of the mean seasonal rainfall, expressed in the prevailing species, as well as in the vigor and size of individual plants. In fact, with some experience in these matters, it is feasible to estimate with a fair degree of accuracy, the seasonal rainfall by the character of the vegetal cover. The difference in appearance of water-sheds receiving 12 in., 15 in., and 18 in. of mean seasonal rainfall is striking enough to leave no doubt as to Nature's adjustment to mean climatic conditions. Similarly, general topographic conditions must be the result of some combination of the effect of the controlling causes. If the latter undergo permanent changes, a reflection in effect is inevitable. Nature, like the "mills of the Gods", may grind slowly, yet it grinds exceedingly fine.

Mr. Stevens' standpoint is scarcely justified. Nature's adjustments must be to some mean condition which undergoes only gradual changes; otherwise, human settlement would be forced to make way to a nomadic existence. Exceptions to this rule, due to local disturbances by reason of floods, wind storms, fires, etc., no doubt take place, and may result in radical changes of physiographical features — often temporary, sometimes permanent — but they effect a relatively insignificant portion of the inhabitable areas.

Professor Chapman makes some rather startling assertions in that he claims (1) that the physiographical balance is maintained over large areas in the West by grasses and herbage; (2) that profound disturbances of the equilibrium have taken place through extensive over-grazing; (3) that over-grazing is the principal cause of modern arroyo formation and that the cycle of degradation now operating is of recent origin; and (4) that historical evidence shows that both the Gila and the Colorado were clear streams near Yuma, Ariz., in 1824.

It is not disputed that over-grazing within the water-shed of the Colorado River has reduced the grass carpet over large areas, thus promoting erosion and denudation. This phase has been discussed by Ralph I. Meeker,[57] M. Am. Soc. C. E., and by F. H. Olmstead,[58] M. Am. Soc. C. E. However, it is questioned whether over-grazing is a major phenomenon affecting the physiographic balance of the West. On the subject of the origin of silt the following quotation from the comprehensive study of silt production in the Colorado River water-shed is submitted:[59]

> "The aridity of the climate and the consequent lack of vegetation is one of the main causes. Because the territory is sparsely settled, relatively few climatic records have been kept, and in many parts the precipitation is not known accurately. Roughly estimated, 40% of the total area of the basin has a precipitation of less than 10 inches a year, in 50 per cent the precipitation ranges from 10 to 17 inches, and in the remaining 10 per cent it is from 17 to 25 inches and higher in the high mountains. Fully one-half of the basin is either bare or but scantily covered with desert shrubs and grasses."

Relative to the silt production of the Green River and Upper Colorado, where the major grazing areas are located, as compared with that of the Little Colorado and San Juan Rivers, data given in the report[60] reveal a much greater proportion of silt in the water of the lower tributaries which drain desert areas.

Professor Chapman's reference to the diary of the explorer, James M. Pattie, in which the Gila and Colorado Rivers are described as clear streams, can scarcely be accepted as conclusive historical evidence that the muddy appearance of these streams is a modern phenomenon. Other historians have left records which emphasize conditions to the contrary. Quoting from F. S. Dellembaugh[61] who was a member of the Second Powell Expedition through the Colorado River Canyon, in 1872:

> "In 1539, the Spanish explorer, Francisco de Ulloa, leaving from Acapulo, sailed to the head of the Gulf of California where he made this observation—'and thus sailing we always found more shallow water and the sea thick, black and very muddy'.
>
> * * * * * * * * * *
>
> "In 1609, Don Juan de Onate, traveling west from San Juan on the Rio Grande *via* Zuni, '10 leagues beyond Moki, they crossed a stream flowing

---

[57] "Silt in the Colorado River and Its Relation to Irrigation", by the late Samuel Fortier, M. Am. Soc. C. E., and H. F. Blaney, Assoc. M. Am. Soc. C. E., *Technical Bulletin No. 67*, U. S. Dept. of Agriculture, pp. 7 and 19.

[58] "Gila River Flood Control", 1919, Senate Doc. No. 436, 65th Cong., 3d Session.

[59] *Technical Bulletin No. 67*, U. S. Dept. of Agriculture, p. 5.

[60] *Loc. cit.*, Tables 2 and 3.

[61] "The Romance of the Colorado River", pp. 7, 78, 120, 215, and 218.

northwesterly which was called "Colorado" from the color of its water.' This has been identified as the Little Colorado.

* * * * * * * * * *

"In 1826, Lieut. R. W. Hardy of the British Navy sailed up the Gulf of California and encountered 'a vein of red water' which later was proved to have been the Colorado.

* * * * * * * * * *

"On Powell's first trip down the Colorado, the tributary Fremont River, in Southeastern Utah, was given the name 'Dirty Devil' from the character of its water.

* * * * * * * * * *

"In 1869, Powell's party found the Little Colorado at its mouth 'small, muddy and saline'."

Lieut. R. S. Williamson[63] writes:

"The water [in the Colorado River, at Yuma, in 1853] was highly charged with fine red mud which gave it a decided red color and opacity. * * * The amount thus annually transported to the Gulf of California by this river must be very large, and very considerable additions to and alterations of its delta must result."

Last, but not least, the statement is here presented that the Grand Canyon of the Colorado, cut to a depth of 1 mile into the plateau during past ages, has been the basic cause for the degradation of the tributaries and their water-sheds during the same period, with the great Colorado Delta a silent but potent witness to a process in which the works of Man had no part.

The discussion by Mr. Bonner of the rate of débris production of representative streams, storage depletion through siltation, and annual silt production in the United States, illustrated in Tables 3, 4, and 5, is of general interest and no doubt is widely appreciated by engineers occupied with problems of water supply, conservation, and flood control. Table 3, in particular, has filled a long-felt want relative to the silt load of streams in that it covers a wide range of drainage areas, both as to size and geographical location. The assumption of a mean weight of 85 lb per cu ft for alluvial deposits furnishes a convenient justifiable ratio for the establishment of the relation between weight and volume and for comparison of data when different methods of measurement have been adopted. The compilation of records of storage reserve depletion is gratifying in that it demonstrates for a large majority of cases a remarkably low rate of depletion.

Reference has been made to the conditions on the South Pacific slope which form rather the exception to this rule. Because of steep gradients of streams and, in many cases, unsuitable foundations, economic reservoir sites are few, and the available storage capacity, almost without exception, is deficient for flood control. Flood regulation, therefore, remains imperfect, being circumscribed by available detention storage. On many streams, the one practical reservoir site has been utilized and the channel capacity below it has been adjusted to the resulting peak flow. Although, under normal conditions of the water-shed cover, débris production may not be a disturbing factor, there remains the ever-present danger of abnormal erosion from burned areas. Under such conditions, a preconceived plan of débris storage for the protection of water storage becomes imperative.

---

[63] Report of Explorations in California, 1853, Senate Doc. No. 78, 33d Cong., 2d Session, p. 112.

Check Dams.—Mr. Jarvis comments on the relative merits of high débris barriers and check dams for conservation projects. Check dams were an important feature in the La Crescenta-Montrose flood discussed by Messrs. Rowe, Lippincott, and Rule.

These check dams were of a temporary type of construction. Many had been placed in the small canyons of the La Crescenta-Montrose area by the Los Angeles County Flood Control District several years prior to the fire. Immediately after the fire of November, 1933, other agencies in the vicinity were engaged to build additional checks, both in the canyons and on the débris cones. In Pickens Canyon, checks had been placed on the cone from the State Highway to the mouth of the canyon and from there in the rock formation to upper reaches of the creek. This work was of such magnitude as to be generally known and to create in the local population the conviction that the problem of flood control had been satisfactorily solved. As an added precaution, the Forestry Service had sowed a large part of the burned-over area with a variety of mustard used in Southern California for cover crops. The criticism has been advanced that a widespread use of temporary structures in the mountain areas tends to create a false sense of security on the part of the population of the area below. This unfortunate psychological effect, and the dangers inherent in temporary types of structures have also been brought out by Mr. Debler.

In this particular case, the checks undoubtedly released a considerable quantity of débris which, with permanent structures, might have remained in the mountains. However, with a subsequent flood, their efficiency would have been negligible in any case because they would have been either completely back-filled or buried. The latter effect was observed in 1924 in Sawpit Canyon where the stream bed, raised by a series of checks, assumed its original gradient.

Check dams have been an element in the flood protection and conservation program of Southern California. Thousands have been built and although those subjected to severe tests have failed, the popular faith in and demand for this type of structure has not abated.

The conception has been that check dams "hold the water in the mountains" for future release, affecting not only conservation, but also regulation of flood flow. So firmly has this idea been entrenched in the public mind that check dams have lost their technical significance and in Southern California have become a political issue. For this reason, it is considered justifiable to devote some space to an analysis of the function of this type of structure.

As a flood-control feature check dams were proposed in Southern California after the capital floods of 1914 and 1916, the idea being to regulate the flow of the mountain streams at the source. The prototype of the check-dam system was the débris barrier of the Swiss Alps. Unfortunately, in applying the Swiss system to conditions in Southern California, the true functions of the structures were misunderstood, and the guiding principles of construction disregarded. In the Alps, the barrier systems were utilized to control abnormal débris production in regions where the formation consists of soft slates, marls, and chalks, which are easily erodible. The Alpine barriers were built of massive dry masonry, as a rule, with the two top courses of pointed rock laid in cement mortar, and were from 20 to 100 ft in height. Particular care was exercised to secure the foundation by building on bed-rock, or by providing one or more stilling-basins as a protection.

The purpose of these barriers is first to stop erosion of the stream bed and then to cause the back-fill of the barrier and thereby protect the adjacent slopes from undercutting and slides. After the slopes are stabilized, they are further consolidated by planting. The massive type of dry masonry is well demonstrated in reports of Swiss engineers on the improvement of mountain streams.[63]

In the application of checks to Southern California conditions, the intention was primarily to regulate the flow of water during flood peaks and to effect conservation by holding water temporarily in the back-fill of the dams. This was to be accomplished by the erection of systems of check dams of a nominal height of 6 ft, distributed throughout the length of the stream bed. In some canyons several hundred checks were erected. Débris control, or storage, has not been considered the major objective.

The original plan contemplated the construction of dry masonry arches, built carefully to bring the arch action into play. However, lack of skilled labor, unsuitable materials, and the absence of proper foundations soon caused a modification in design. Arches were abandoned, and the dry masonry degenerated into dry rock walls, built by unskilled labor. Cases are known in which, in the absence of suitable quarry rock, boulders from the bed of the creek were used. After numerous failures, the practice of wrapping the structures with triangular wire netting was adopted, so that, to-day, the strength of the rock wall rests with the strength of the wire mesh. Large numbers of such dams have been built in Southern California. The contention has always been made that these dams can be built for from $25 to $50, although experience has shown that they cost more and that the cost of wire-bound rock check dams may be as high as $500, depending on the length. It is a feature of this check system that the dams are built approximately the same height regardless of the size of the water-shed and the depth of the stream flow, and that the spacing is more or less arbitrary. The systems, as constructed deviate further from the theoretical plan in that, in many canyons, dams cannot be built in the uppermost and steep parts where the slopes may be from 40 to 60 per cent.

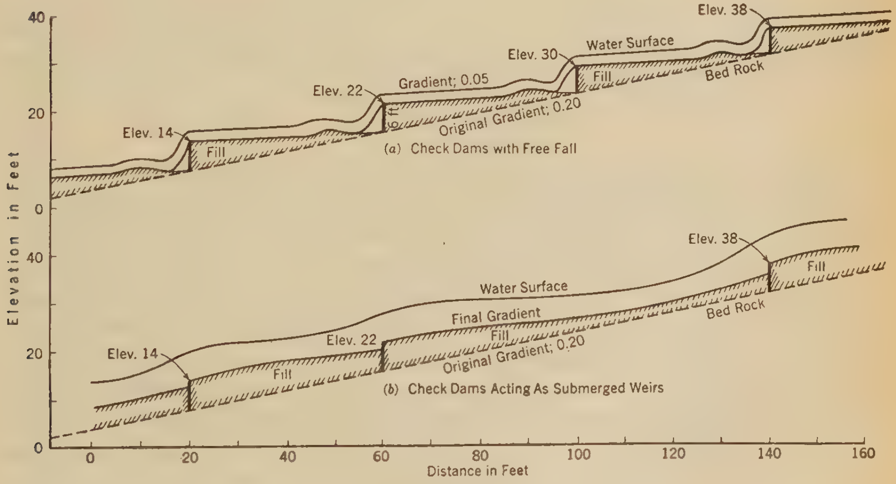
It is apparent that the function of the barrier systems in Southern California is fundamentally different from that in the Swiss Alps and, furthermore, that the rugged construction of the latter has been abandoned for a comparatively fragile and unstable type.

---

[63] "Wildbachverbauungen und Flusskorrektionen in der Schweiz", 1914, and *Proceedings*, Swiss Engrs. and Archts. Assoc., 1903.

Flood Regulation by Check Dams.—The popular view that "check dams hold the water in the rocky formation of mountains" may be explained by the erroneous concept that the time consumed in transit of the water from the water-shed to the valley is materially increased. It should be clear that checks do not affect the rate at which water collects from the canyon slopes so that the period of retardation is determined mainly by the amount of possible channel storage within the checked area.

Conditions are conceivable in which the construction of a proper number of correctly proportioned small checks may reduce the velocities of a mountain stream to such an extent that a temporary storage in the channel will result and, hence, a material reduction of the peak flow. A first requirement is that the height of check dams is in correct proportion to the depth of overflow (see Fig. 5(*a*)). Next, comes the spacing of check dams which is determined theoretically by the point of intersection of the original slope (20%) with the slope of the back-fill of the check (5%). In other words, the spacing must be a function (*a*) of the height of the check; (*b*) of the original gradient of the stream; and (*c*) of the gradient normally assumed by the back-fill material. This proportioning holds for a depth of overflow which will produce a free fall and a destruction of the energy by fall and turbulence. However, if the depth of overflow is such as to cause the check to act as a submerged weir, the effect of the drop is largely lost. This, for example, was the case with some of the check dams in the La Crescenta-Montrose area (see Fig. 5(*b*)).


40
20
0
40
20
0
Elevation in Feet
Elev. 38
Water Surface
Elev. 30
Fill
Bed Rock
Elev. 22
Gradient; 0.05
Original Gradient; 0.20
Elev. 14
Fill
Elev. 38
Fill
Water Surface
Bed Rock
Final Gradient
Fill
Elev. 22
Original Gradient; 0.20
Elev. 14
Fill
0
20
40
60
80
100
120
140
160
Distance in Feet

(*a*) Check Dams with Free Fall

(*b*) Check Dams Acting As Submerged Weirs

FIG. 5.

Aside from any question of design, regulation is a function of the total reduction in velocity during peak periods. This reduction results from decrease in gradient, greater width of channel, and loss of energy at the various drops. Theoretically, the reduction of the gradient from 20% to 5% would reduce the mean velocity by about one-third; the flat section above checks tends further to retard the flow, while the increased depth of water in the narrower parts of the channel below the check increases the theoretical gradient. The resulting channel storage will be in proportion to the length of the checked section and the reduction of velocity.

Apparently, the most efficient reduction in gradient compatible with economy would result from high barriers spaced correspondingly, for the reason that the narrow section of the channel would be only a small part of the total length between checks, leaving a large stretch for the adjustment of velocities. With barriers of a height of 20 ft, or more, the width of channel at the check would probably be in excess of 50 ft. Under such conditions, channel storage would attain a maximum.

Assume the theoretical case of a canyon being visited by a rain of maximum intensity. In such an event, the hydrograph of an unchecked canyon would assume the line, *A-B*, shown in Fig. 6, with a peak discharge represented by Line *B-C* and the run out by Line *C-D*, the period of the storm being indicated by the distance, *T*. If, as a result of proper checking, peak velocities have been reduced, the time represented by Curve *A-B* becomes longer and if the peak is delayed beyond Point *C*, it will also be reduced as indicated by Curve A-B_1-C_1. In other words, factors remaining equal, the shorter the time represented by Line *T* (that is, the duration of the storm), the greater the probability that checking would be effective. On the other hand, if the duration of the storm represented by the distance, *T*, and the peak flow, *B-C*, are extended beyond B_1, checking would have no effect on the size of the peak; the effect would be a delayed peak of shorter duration. In any case, the delay could not be longer than the difference in time of transit for the water from the upper end of the canyon to the mouth.

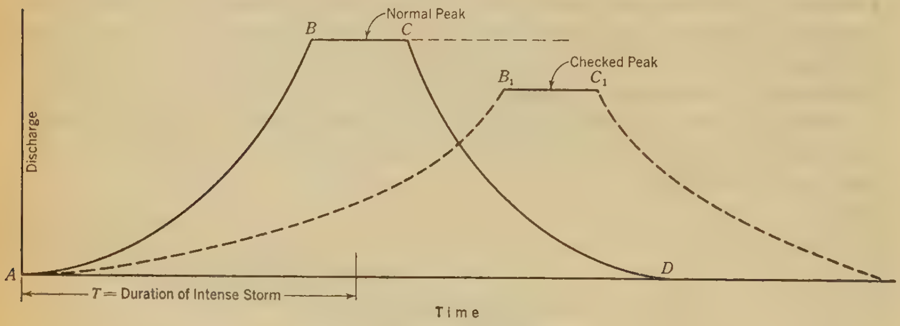


FIG. 6.

The effect of checking can best be explained by an example. Assume a mountain stream with hydraulic properties, as given in Table 10. While the total difference in channel storage is 100 000 cu ft, the proportion of this extra storage which affects the peak flow depends on the depth of stream flow prior to the period of intense rainfall and, consequently, can be only a portion of the 100 000 cu ft. Furthermore, if the time of the peak discharge exceeds 833 sec, the maximum flow at the canyon mouth remains the same in both instances.

In Southern California, the small tributary canyons, in which checks are placed, are short, ranging in length from ½ mile to 1½ miles, with gradients of from 10 to 40%; hence, velocities are high and time of transit relatively short—probably less than 30 min for severe storms. Cases are recorded where two periods of intense rainfall occurred at short intervals. Under such conditions the second peak might catch up with the delayed first peak of a checked canyon, resulting in a greater flood than would happen under natural stream flow.

TABLE 10.—HYDRAULIC PROPERTIES OF AN ASSUMED MOUNTAIN STREAM

Description	Unchecked stream	Checked stream
Length of bed, in feet	5 000	5 000
Peak flow, in Cubic Feet per Second:		
Upper end	100	100
Lower end	500	500
Mean velocity, in feet per second	10	6
Mean Wetted Area, in Square Feet:		
Upper end	10	16⅔
Lower end	50	83⅓
Channel storage, in cubic feet	150 000	250 000
Time of transit, in seconds	500	833

The difficulty in estimating the effect of check dams lies in the lack of reliable data as to the intensity and duration of storms. It is unwise, therefore, to place too much, if any, reliance on the effective flood protection by systems of checks, even if properly designed and constructed. In fact, it is the writer's opinion that, during capital floods, conditions may be such as to render regulation by check dams ineffective.

Water Conservation by Check Dams.—In an appraisal of the efficiency of check dams for purposes of water conservation, differentiation must be made between streams flowing on bed-rock and those flowing on an alluvial fill.

Mountain streams issuing from bed-rock canyons act essentially as drains, intercepting the water which percolates down the slopes in fissures, cracks, and in the soil cover. If the creek bed is crossed by deep-seated fissures, the stream will be carrying the unabsorbed water. This function of the stream as a drain is not changed by the construction of a series of check dams, except that the débris behind the checks will hold some of the water temporarily, which then will be exposed to evaporation from the saturated mass, or will promote plant growth. Absorption into deep-seated fissures is assisted by this process only to a small extent.

A stream flowing on porous alluvium, with a water-table at effective depths, is naturally a losing stream. If checked by a system of dams, absorption will increase in proportion to the additional width of the wetted perimeter. In both cases checks should be credited with the storage in the débris which, although small, nevertheless remains an element to be considered. However, the débris storage and its water capacity increase more than the square of the height of the dam; hence, conservation favors the higher dams.

Summarizing on the relative merits of low and high check dams, it is concluded that for the consolidation of the stream bed in an erodible formation, low checks or fixed sills offer a satisfactory solution; that for débris storage, the consolidation of slopes, and, for conservation, high barriers may be found to be more efficient; and that for purposes of regulation, barriers with negligible water storage cannot be relied upon at times of critical discharge.

Unfortunately, the popular conception may attribute flood regulation to a system of checks designed for conservation, or débris control, and induce encroachment on the over-flow areas of a stream. Unless such areas are definitely zoned as dangerous for habitation, destruction of human life and property is inevitable.

Design of Check Dams.—To be effective check dams must have a height in proper relation: (*a*) To the depth of overflow; (*b*) to the gradient assumed by the stored débris; and (*c*) to the distance between checks. These matters would be taken into consideration were the problem one of designing a small number of high barriers; yet they are equally important with small checks.

Another feature of the design would be the correct thickness of the dam relative to the overflow wherein must be considered such forces or combined forces as extreme high water, débris, and mud flow, resulting in an extremely high specific gravity for the material passing over the dam and the impact of large rocks. Depending on the magnitude of slides, the stream may become temporarily blocked or dammed until a small lake is formed. When such slides are topped they may be released suddenly, advancing down the canyon as a mud-flow or débris wave. Such conditions were observed in the La Crescenta-Montrose flood. To withstand such forces in steep mountain canyons will require structures quite different from a dam consisting of loosely piled rock wrapped with triangular wire netting.

The conclusion is reached that in order to be permanent a 6-ft check dam must be constructed to withstand strains and forces out of proportion to its size, and, therefore, if properly built, will prove likely to be the most expensive type of control works instead of the cheapest, as has often been claimed.

The discussions of the paper necessarily centered on conditions in the arid Southwest. The writer is indebted to Mr. Matthes for his comparison of the problem in Eastern and Western streams. Attention is called to a paper by Mr. Matthes, which bears on certain phases of the physiographical balance manifested in stream dynamics.[64]

It was the purpose of this paper to awaken the consciousness of the Engineering Profession to the dangers accompanying the disturbance of the physiographic balance. The interest thus aroused is gratifying.

---

[64] "Floods and Their Economic Importance", National Research Council, *Transactions*, Am. Geophysical Union, Pt. II, 1934, p. 427.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 1898

AN APPROACH TO DETERMINATE STREAM FLOW

BY MERRILL M. BERNARD,[1] M. AM. SOC. C. E.

WITH DISCUSSION BY MESSRS. C. S. JARVIS, LEROY K. SHERMAN, W. W. HORNER, C. H. EIFFERT, CHARLES S. BENNETT, R. L. GREGORY AND C. E. ARNOLD, GORDON R. WILLIAMS, W. G. HOYT, L. L. HARROLD, AND F. F. SNYDER, AND MERRILL M. BERNARD.

SYNOPSIS

In a paper on the determination of stream flow from rainfall records, LeRoy K. Sherman, M. Am. Soc. C. E.[2] undertook, through the summation of daily increments of run-off, not only to establish the maximum flow resulting from a recorded rainfall, but to produce the stream hydrograph, a process that to a remarkable degree parallels the phenomenon of stream flow produced from storm run-off.

The results of a study, the outgrowth of an interest in Mr. Sherman's paper, is presented herewith. It treats of other aspects of the problem and offers a procedure that is believed to be an approach to determinate stream flow. The transition from rainfall to stream flow is accomplished through the medium of a "distribution graph," which is found to be a function of water-shed characteristics. The possibility of developing the distribution graph without resorting to the stream-flow records is shown. The "pluviagraph," or maximum hydrograph, is discussed, and its value to design in the field of hydraulic engineering is demonstrated.

The study utilizes six water-sheds of from 500 to 6 000 sq miles and embraces practically the entire State of Ohio. These water-sheds were selected from the unregulated streams served by the United States Geological Survey, and represent an acceptable range in area and water-shed characteristics.

NOTE.—Published in January, 1934, *Proceedings*.

[1] Cons. Civ. Engr., Crowley, La.

[2] "Streamflow from Rainfall by Unit-Graph Method," *Engineering News-Record*, April 7, 1932.

Only such illustrations are included as are necessary to demonstrate the points raised in the paper. A complete copy of the original paper, with all data and drawings, is filed for reference in Engineering Societies Library, New York, N. Y.

Introduction

A general interest in the flow of a stream divides itself into: (1) The conservation of water for such useful purposes as irrigation and power development; and (2) the disposal of flood-waters without loss or damage. In the latter case the objective will be the determination of maximum values of flood volume or stream stage, depending on the nature of the problem and the degree of protection sought.

The ideal situation is that of finding a long-term flow record applicable to the location of the structure to be designed. If the project is one of flood protection, however, or of channel improvement, that disturbs the regimen of the stream; or if the water-shed is subject to further development, even this record must be modified to meet the new conditions of delivery and concentration of flood-water.

As is more generally the case, the designer is forced to deal with a short-term record on the stream itself, must extrapolate a record of a neighboring stream having similar water-shed characteristics, or must attempt to establish the magnitude and frequency of various flood flows by analyzing the factors that produce them. In dealing with such problems, complexities have discouraged all except a comparatively superficial treatment of the problem.

The evolution of this science is interesting. The best of the older empirical formulas expressed flood flow as a function of area only; others introduced rainfall rate and slope factors. More recent studies are to be divided into two groups. One group, utilizing recorded stream flow, relates flood flow to area by means of coefficients and exponents which tend to become constant for localities of considerable extent. The other group attempts to express flood flow as a function of storm intensity, more recently giving values to such factors as storm frequency and duration, and water-shed characteristics. Further development of the former method is restricted by a data source rendered inadequate through poor distribution and short-record period. The latter group, advocating the rational method, or some modification of it, must develop, through research, values for several important factors now indeterminate.

The objective in this field may be stated thus: A method of determining flood flow in terms of magnitude, duration, and frequency, at any point on any stream, within a region having the same range in climatological, topographical, hydrological, and geological conditions. The method must give determinate values to all principal factors affecting run-off and must group the minor influences into coefficients that approach constant values of regional application. It must utilize available data only, which, in this country, are limited to the published records of the United States Weather Bureau and the United States Geological Survey. The method must meet the test of reproducing the stream hydrograph accurately.

BASIC DATA

The foregoing "specification" has stipulated that the proposed method must be capable of utilizing published Weather Bureau and Geological Survey records. This assumes their application to the larger water-sheds on which the time unit for the unit graph or for the distribution graph is one day or more. The advantages of general availability and distribution far outweigh the disadvantages and objections which are set forth, with suggested remedies, in a recent Committee report.[3] Other improvements in this service, admittedly not easy of accomplishment, would be: (1) All stations to take readings on the same hour; (2) notation of hour and depth at end of period of continuous downpour, within the 24-hr interval; and (3) improved typography and arrangement in the monthly publication of climatological data.

The Unit Graph.—Defining the theory of the unit graph, LeRoy K. Sherman, M. Am. Soc. C. E., states[4] that, for the same drainage area, there is a definite total flood period corresponding to a given rainfall, and that the length of the base of the hydrograph will be the same for all 1-day rainfalls, regardless of intensity.

According to Mr. Sherman, "if a given one-day rainfall produces a 1-in. depth of run-off over the given drainage area, the hydrograph showing the rates at which the run-off occurred can be considered a unit-graph for that water-shed."

The unit graph is constructed by segregating the daily stream flow resulting from a single downpour of 1 day (or the other time unit), from the flow occasioned by previous and subsequent rainfall. This, by proportion, is converted into run-off, in feet per second, equivalent to a depth of 1 in. over the area, and becomes the unit graph of the water-shed.

The hydrograph is developed through the following procedure: (*a*) Daily rainfall, properly weighted, is listed by date; (*b*) a coefficient is applied, reducing rainfall to run-off, in inches of depth; (*c*) the unit-graph value for each day, multiplied by the run-off depth, gives the increment of stream flow attributable on that day to the daily rainfall so affected; and (*d*) the horizontal summation of the run-off increments, day by day, produces the total flow and becomes the ordinates of the hydrograph.

The Distribution Graph.—The writer's conception of the theory on which the unit graph is based, is as follows: For a particular water-shed, two storm periods confined to the same time interval and uninfluenced by preceding and subsequent rainfall will become effective to stream flow at the same proportional rate, expressed as a percentage of the area contributing its run-off to the flow of the stream, at the point of observation, day by day. This is to be conceived as a time contour, which, having developed around the point of concentration, recedes to the outer limits of the water-shed, progressively enveloping the various portions as they contribute to the flow, reaching the extremities when all parts have contributed and the area under the distribution graph is equal to 100 per cent.

---

[3] Progress Rept. of Special Committee on Meteorological Data, *Proceedings*, Am. Soc. C. E., January, 1933, p. 153.

[4] "Streamflow from Rainfall by Unit-Graph Method," *Engineering News-Record*, April 7, 1932, p. 50.

Here, as in the unit-graph method, the concentration period may be defined as a time interval, at the end of which all parts of the water-shed are contributing to the flow at the point of observation. This definition should apply, as well, to the time interval used in the rational method to establish the critical average rainfall intensity; but it will be found that the concentration period, as usually estimated or calculated, is much shorter. The calculated time in transit of flood-waters from head-water to outlet on the Tuscarawas River, at Newcomerstown, Ohio, is between 3 000 and 4 000 min, while its distribution graph shows that only about one-half its area has become effective to the stream flow in that period. The error lies in a misconception of the meaning of "remote area." It is usual to consider that part of the water-shed in the vicinity of the head-waters as establishing concentration time because of its distance from the outlet. As a matter of fact, the efficiency of the main channel, through higher velocities, places head-water areas much closer to the outlet in terms of time than extensive areas nearer but less advantageously located.

It is difficult to select from the rainfall record, a storm of the type which will produce a hydrograph suited to the construction of the distribution graph of the water-shed. The task requires boundless patience and the application of unusual care and judgment. Upon first attempt one is convinced that any result can be obtained. A storm period, to produce the distribution graph of the water-shed, must have the following characteristics: (1) The rain must have fallen within the recording time interval, or the time unit, as the day (or hour); (2) the storm must have been well distributed over the water-shed, all stations showing an appreciable depth of rainfall; (3) the run-off following the storm must have been uninterrupted by the effects of low temperatures and unaccompanied by melting snow and ice; and (4) the storm period must occupy a place of comparative isolation in the record. It should follow a period of low stream flow and there should be no further rainfall until the flood peak is well passed.

Certain unavoidable sources of error and inaccuracy must be recognized and accepted in dealing with Weather Bureau data. At some rainfall stations readings are made at 8:00 A. M.; at others, at 5:00 P. M. It is highly probable that at many of the co-operative stations readings are taken at irregular intervals, the record going in as a forenoon or an afternoon observation. A continuous storm of 24 hr, or less, may be divided by the recording hour, appearing in the record as a 2-day storm. Particularly on small water-sheds it is possible that the entire rainfall may have fallen in the first hour, the flood peak on the twelfth hour, and the flow, officially recorded on the twenty-fourth hour, may be only a small fraction of the maximum. Stream flow is recorded as a 24-hr average, obscuring true maximum values, particularly on small water-sheds.

Table 1 includes all data used in developing the distribution graphs for the six water-sheds embraced by this paper. In every case but one at least three storm periods were found which gave acceptable graphs, the average being taken as the distribution graph for the water-shed.

TABLE 1.—ARRANGEMENT OF DATA FOR DEVELOPMENT OF DISTRIBUTION GRAPHS

(Units in Cubic Feet per Second)

STORM NO. 1					STORM NO. 2					STORM NO. 3					DISTRIBUTION GRAPH, IN PERCENTAGE OF TOTAL FLOW ON THE WATER-SHED	
Date	Observed stream flow	Base flow	Segregated flow	Distribution (percentage of total segregated flow)	Date	Observed stream flow	Base flow	Segregated flow	Distribution (percentage of total segregated flow)	Date	Observed stream flow	Base flow	Segregated flow	Distribution (percentage of total segregated flow)	Average of Storms Nos. 1, 2, and 3	Taken from Fig. 2
(a) RACCOON CREEK, AT ADAMSVILLE, OHIO																
April, 1925:					June, 1927:											
21	282	282	0	......	18	530	530	0	......	....	......	.....	......	......	....	....
22	2 150	250	1 900	17	19	1 300	400	900	15	....	......	.....	......	......	16	17
23	3 290	225	3 065	28	20	2 070	330	1 740	29	....	......	.....	......	......	29	32
24	2 840	198	2 642	24	21	1 720	290	1 430	24	....	......	.....	......	......	24	16
25	1 300	170	1 130	10	22	950	252	698	12	....	......	.....	......	......	11	9
26	750	125	625	6	23*	540	229	311	5	....	......	.....	......	......	5	6
27*	550	75	475	4	24	405	200	205	3	....	......	.....	......	......	4	4
28	410	40	370	3	25	335	172	163	3	....	......	.....	......	......	3	3
29	340	36	304	3	26	290	140	150	3	....	......	.....	......	......	3	3
30	290	29	261	2	27	255	123	132	2	....	......	.....	......	......	2	2
May, 1925:																
1	255	24	231	2	28	230	107	123	2	....	......	.....	......	......	2	2
2	219	19	200	1	29	200	86	114	2	....	......	.....	......	......	1	1
....	.....	.....	11 203	100	....	.....	.....	4 966	100	....	......	.....	......	......	100	95
(b) LICKING RIVER, AT TOBOSO, OHIO																
July, 1924:					July, 1923:					June, 1924:						
12	218	218	0	......	2	130	130	0	......	22	374	374	0	......	....	....
13	2 600	200	2 400	65	3	1 680	122	1 558	66	23	3 400	277	3 123	68	66	66
14	630	190	440	12	4	457	117	340	14	24	790	243	557	13	13	16
15	600	180	320	9	5	261	114	147	7	25	541	226	315	7	8	8
16	374	170	204	6	6	198	107	92	4	26	401	203	198	4	5	5
17	277	164	113	3	7	176	104	72	3	27*	374	190	184	3	3	3
18	243	160	83	2	8*	162	104	58	2	28	277	180	97	2	2	2
19	226	153	73	2	9	156	104	52	2	29	243	170	73	2	2	2
20	203	146	57	1	10	130	100	30	2	30	226	160	66	1	1	1
....	.....	.....	3 690	100	....	.....	.....	2 349	100	....	......	.....	4 613	100	100	103

* Day record abandoned.

TABLE 1.—(*Continued*)

STORM NO. 1					STORM NO. 2					STORM NO. 3					DISTRIBUTION GRAPH, IN PERCENTAGE OF TOTAL FLOW ON THE WATER-SHED	
Date	Observed stream flow	Base flow	Segregated flow	Distribution (percentage of total segregated flow)	Date	Observed stream flow	Base flow	Segregated flow	Distribution (percentage of total segregated flow)	Date	Observed stream flow	Base flow	Segregated flow	Distribution (percentage of total segregated flow)	Average of Storms Nos. 1, 2, and 3	Taken from Fig. 2
(*c*) HOCKING RIVER, AT ATHENS, OHIO																
June, 1927:					June, 1927:					Oct., 1926:						
3	435	435	0	......	17	1 380	1 380	0	......	1	740	740	0	......	....	......
4	1 100	400	700	11	18	2 360	900	1 460	10	2	1 240	650	590	8	10	14
5	2 780	375	2 405	37	19	7 100	700	6 400	45	3	4 120	520	3 600	44	42	34
6	1 940	350	1 590	26	20	3 560	530	3 030	22	4	2 290	470	1 820	23	24	17
7	770	320	450	8	21	1 660	470	1 190	8	5	1 100	420	680	8	8	10
8*	535	310	225	4	22	1 170	420	750	5	6*	895	380	515	6	5	6
9	480	290	190	3	23*	900	390	510	4	7	600	340	260	3	3	4
10	425	242	183	3	24	700	360	340	2	8	510	310	200	2	2	3
11	397	220	177	3	25	530	340	190	2	9	460	290	170	2	2	3
12	370	200	170	3	26	470	320	150	1	10	410	260	150	2	2	2
13	345	181	164	2	27	420	310	110	1	11	380	240	140	2	2	2
....	.....	.....	6 254	100	....	.....	.....	14 130	100	....	......	.....	8 125	100	100	95
(*d*) WALHONDING RIVER, AT POMERENE, OHIO																
July, 1922:					July, 1927:					June, 1927:						
18	582	582	0	......	21	502	502	0	......	3	1 660	1 660	0	......	....	....
19	633	540	93	9	22	1 010	465	545	11	4	3 470	1 320	2 150	11	11	9
20	905	475	430	42	23	2 280	438	1 842	38	5	11 100	1 210	9 890	48	41	41
21	599	440	159	16	24	1 160	408	752	16	6	3 830	1 060	2 770	14	16	21
22	499	410	89	9	25	775	360	415	9	7	2 410	890	1 520	7	8	12
23*	460	385	75	7	26	608	330	278	6	8	1 900	810	1 090	5	6	8
24	397	340	57	5	27*	536	310	226	5	9	1 540	765	775	4	5	5
25	360	320	40	4	28	470	300	170	4	10	1 320	720	600	3	4	4
26	330	300	30	3	29	438	280	158	3	11	1 210	635	575	3	3	3
27	310	290	20	2	30	408	250	148	3	12	1 060	605	455	2	3	2
28	300	280	20	2	31	360	220	140	3	13*	890	580	310	2	2	2
29	290	275	15	1	Aug., 1927: 1	330	195	135	2	14	810	530	280	1	1	1
....	.....	.....	......	......					......	....	......	.....	......	......	....	....
....	.....	.....	1 028	100	....	.....	.....	4 809	100	....	......	.....	20 415	100	100	108

* Day record abandoned.

Storm No. 1					Storm No. 2					Storm No. 3					Distribution Graph, in Percentage of Total Flow on the Water-Shed	
Date	Observed stream flow	Base flow	Segregated flow	Distribution (percentage of total segregated flow)	Date	Observed stream flow	Base flow	Segregated flow	Distribution (percentage of total segregated flow)	Date	Observed stream flow	Base flow	Segregated flow	Distribution (percentage of total segregated flow)	Average of Storms Nos. 1, 2, and 3	Taken from Fig. 2
(e) Tuscarawas River, at Newcomerstown, Ohio																
July, 1922:					May, 1925:					June, 1927:						
17	542	542	0	......	23	980	980	0	......	4	1 560	1 560	0	......	....	....
18	644	485	159	4	24	1 080	745	335	4	5	3 110	1 420	1 690	7	5	6
19	1 540	420	1 120	25	25	2 280	615	1 665	19	6	8 250	1 315	6 935	26	24	24
20	1 300	360	940	21	26	2 280	505	1 775	20	7	7 490	1 245	6 245	24	23	25
21	886	330	556	13	27	1 530	419	1 111	13	8	4 530	1 200	3 330	13	13	14
22	682	315	367	8	28	1 240	367	873	10	9	3 280	1 100	2 180	8	8	9
23	609	298	311	7	29	1 030	355	675	8	10	2 440	1 035	1 405	5	6	6
24	542	280	262	6	30	932	319	613	7	11	2 120	985	1 135	4	5	5
25*	485	270	215	5	31	790	308	482	5	12	1 830	935	895	3	4	4
26	420	265	155	4	June, 1925:				4	13*	1 590	900	690	3	4	3
27	360	260	100	2	1	700	300	400	4	14	1 440	870	570	2	3	2
28	330	255	75	2	2*	600	290	310	3	15	1 330	855	475	2	2	2
29	315	250	65	2	3	550	280	270	......	16	1 250	830	420	2	2	2
30	298	250	48	1	4	450	270	180	2	17	1 210	800	410	1	1	1
....	.....	.....	......	......	5	400	260	140	1	....	......	.....	......	......	....	....
....	.....	.....	4 373	100	....	.....	.....	8 829	100	....	......	.....	26 380	100	100	103
(f) Muskingum River, at Dresden, Ohio																
July, 1927:					July, 1925:					Feb., 1929						
21	1 680	1 680	0	......	9	2 390	2 390	0	......	25	2 520	2 520	0	......	....	....
22	2 140	1 500	640	2	10	3 560	2 200	1 360	3	26	18 700	2 300	16 400	5	3	3
23	4 950	1 370	3 580	13	11	7 990	2 100	5 890	15	27	45 600	2 150	43 450	14	14	11
24	7 450	1 270	6 180	22	12	10 600	1 960	8 640	22	28	52 200	2 030	50 170	16	20	24
25	6 070	1 160	4 910	18	13	8 920	1 850	7 070	18	Mar., 1929						
26	3 990	1 050	2 940	11	14	6 400	1 720	4 680	12	1	50 300	1 910	48 390	15	17	19
27	2 660	980	1 680	6	15	3 980	1 500	2 480	7	2	45 200	1 800	43 400	14	12	13
28	2 140	930	1 210	4	16	2 780	1 400	1 380	4	3	32 200	1 600	30 600	9	7	9
29*	2 000	870	1 130	4	17*	2 430	1 280	1 150	3	4	19 800	1 450	18 350	6	5	7
30	1 880	820	1 060	4	18	2 230	1 190	1 040	3	5*	17 000	1 350	15 650	5	4	5
31	1 800	780	1 020	4	19	2 110	1 080	1 030	3	6	14 500	1 230	13 270	4	4	4
Aug., 1927:					20	1 980	990	990	2	7	11 000	1 130	9 870	3	3	3
1	1 600	740	860	3	21	1 870	940	930	2	8	8 350	1 030	7 320	2	3	3
2	1 450	700	750	3	22	1 770	890	860	2	9	7 450	960	6 490	2	2	2
3	1 350	680	670	2	23	1 570	850	720	2	10	5 500	910	4 590	2	2	2
4	1 230	680	550	2	24	1 430	810	620	1	11	4 300	860	3 440	1	2	2
5	1 130	680	450	1	25	1 330	780	550	1	12	3 970	810	3 160	1	1	2
6	1 020	680	340	1	....	.....	.....	......	......	13	3 700	775	2 925	1	1	1
....	.....	.....	27 970	100	....	.....	.....	39 390	100	....	......	.....	317 475	100	100	110

* Day record abandoned.

The daily value of the distribution graph is given a proportional value instead of a quantitative value as in the unit graph. In developing it, the rainfall record is only necessary to check the isolation, duration, and distribution of the storm period and to determine the date on which subsequent rainfall affected the flow of the stream, at which point the record is to be abandoned and the further recession of the flood flow estimated.

By taking a number of storm periods followed by intervals of no rainfall, a composite "recession curve" can be prepared which gives a dependable idea of how the base flow would have fallen and the flood flow would have receded had no subsequent rainfall occurred.

THE WATER-SHED FACTOR

As the area under all distribution graphs is equal to 100%, the differences between water-sheds are reflected in the respective shapes of their distribution graphs. It is apparent, too, that the distribution graph demonstrates the inadequacy of area as the basis of comparison between water-sheds, as illustrated by Fig. 1. Here, arises the need for some means of evaluating the characteristics of a water-shed which so markedly change the shape of the distribution graph on water-sheds of approximately the same area.

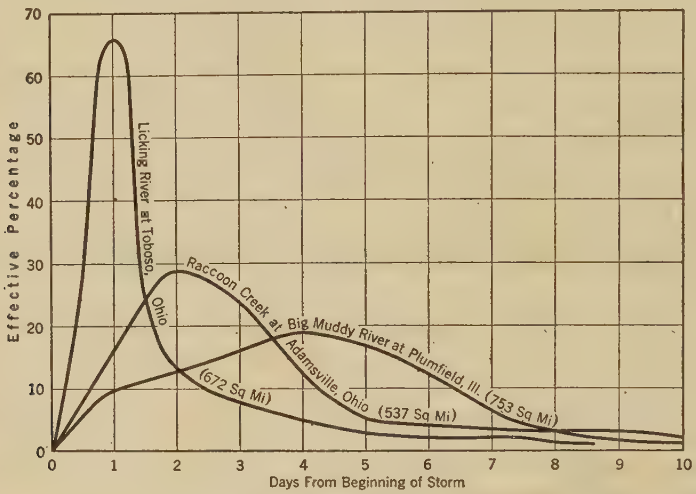


FIG. 1.—DISTRIBUTION GRAPHS OF THREE WATER-SHEDS.

Elsewhere[5] the writer has segregated from the general expression of run-off[6] the factors representing water-shed characteristics into the expression:

$$U = \left(\frac{60\,P}{L}\right)^{4eg} F^{8eg}\,\frac{S^{1.5eg}}{1\,000^{2eg}} \qquad \text{(1)}$$

in which, P is a constant, depending on the shape of the area of the water-shed and its manner of concentration; L, the length, in feet, which water must traverse in running from the most remote portion of the water-shed to its outlet; *F*, a constant, depending on the shape and condition of the main channel of flow; *S*, the fall, in feet per 1 000 ft of main channel of flow; and *e*, a positive fractional exponent of *t*, the duration of rainfall intensity for the locality.

---

[5] *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 1154.

[6] *Loc. cit.*, p. 1059.

Table 2 gives the values of the various factors entering into the determination of *U* in Equation (1). In its further use on large water-sheds the factor, *P*, should be re-tabulated and its range of values checked. The values given for the factor, *F*, in Table 2 are known to be inaccurate, the writer having no direct data at hand regarding channel shape and condition. The values of *L* and *S* were taken from the topographic maps of the U. S. Geological Survey.

TABLE 2.—COMPUTATION DATA

Description	Raccoon Creek	Licking River	Hocking River	Walhonding River	Tuscarawas River	Muskingum River
M, in square miles	537	672	944	1 490	2 430	5 980
L, in feet	380 000	234 300	401 200	475 000	486 000	706 000
P (a constant)	0.543	0.469	0.513	0.510	0.480	0.470
F (a constant)	4.85	4.79	4.69	4.56	4.41	4.30
S, in feet per 1 000 ft	0.89	2.22	1.05	0.64	0.65	0.62
U (a constant)	0.0000610	0.0001570	0.0000540	0.000042	0.0000320	0.0000195

If the influences of the water-shed upon run-off can be separated from the influences of the storm, and if these water-shed influences are given a determinate value such as *U*, then it is possible to prepare a chart, as shown in Fig. 2, applicable to any region over which geological and climatological conditions are, in general, the same. This enables the designer, after having determined the value of *U*, to select, at any point on any stream within the region, a distribution graph for its water-shed. The value of this procedure to stream-flow problems, at points where no flow records are available, can be appreciated.

The relationships between the value of *U*, the daily value of the distribution graph, and the progressive duration of the storm period, for the region embraced by the study, can be stated as follows: For any particular day within the storm period the daily value of the distribution graph increases directly as the five-thirds power of *U* to a maximum daily value; then it recedes inversely as the two-thirds power of *U*. Furthermore, for a particular value of *U*, the daily value of the distribution graph increases directly as the square of the storm duration, in days, to a maximum value, then it recedes inversely as the square of the storm duration, in days.

Consistent results from the use of the chart are shown in Table 1, the last column in the table giving a distribution graph for each water-shed, the values being taken from the curves of the chart.

REPRODUCING THE HYDROGRAPH

The reproduced hydrograph serves the principal purpose of demonstrating the utility of the method proposed and is the means of developing coefficients.

Table 3 demonstrates the arrangement of the computation sheet for the Tuscarawas River, at Newcomerstown. The procedure is, as follows:

1.—Daily rainfall is taken from the monthly climatological record, weighted[7] according to station-area distribution and listed opposite the proper date in Column (2).

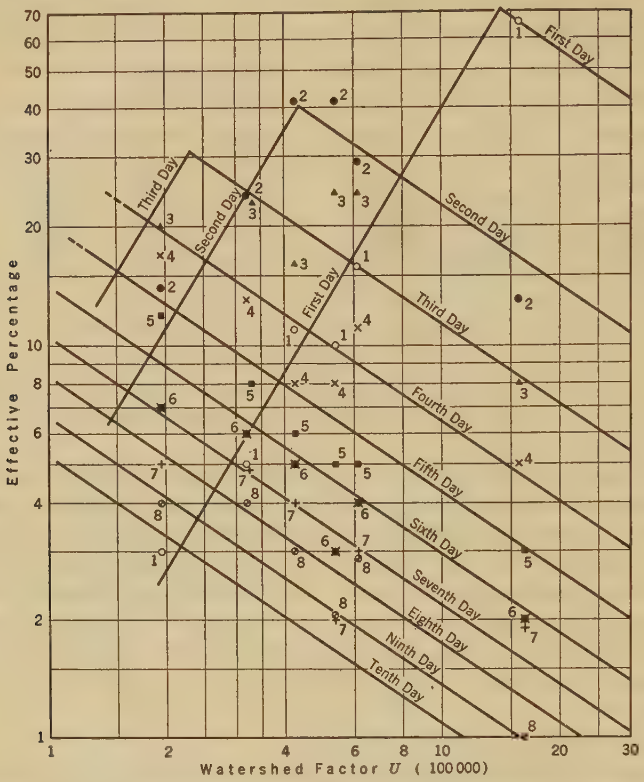


FIG. 2.—RELATION OF WATER-SHED FACTOR *U*, TO DAILY INCREMENTS OF THE VARIOUS DISTRIBUTION GRAPHS.

2.—The distribution graph is applied to the weighted daily rainfall and listed in Columns (3) to (13), inclusive, beginning the distribution opposite the date on which the rainfall occurred.

3.—The rainfall increments are added horizontally and the summation, the effective rainfall depth, *D*, is listed in Column (14).

4.—The 24-hr average observed stream flow is taken from the Geological Survey record and listed according to date in Column (15).

5.—Stream flow, in second-feet-days, is converted into depth of run-off, in inches, by the conversion equation,

$$D = \frac{0.03718\,Q}{M} \quad \text{.........................(2)}$$

---

[7] *Engineering News-Record*, August 2, 1917, p. 211.

TABLE 3.—COMPUTATION SHEET, TUSCARAWAS RIVER, AT NEWCOMERSTOWN, OHIO

Date	Weighted daily rainfall, in inches	DISTRIBUTION GRAPH OF THE WATER-SHED APPLIED TO WEIGHTED DAILY RAINFALL, AS FOLLOWS: Day........ 1 2 3 4 5 6 7 8 9 10 11 12 Effective percentage.... 6 24 23 12 8 6 5 4 4 3 3 2											Effective rainfall depth, I, in inches	24-hour average observed stream flow, Q, in feet per second	Run-off depth, $D = \frac{0.03718\,Q}{\text{square miles}} = \frac{0.03718\,Q}{2\ 430}$, in inches	Retention coefficient, $C = \frac{\text{Ave. } D}{\text{Ave. } I}$	Computed $D = CI$, in inches	Computed Q, in feet per second	Pluviagraph values, in feet per second
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
July 27......	0.18	0.011																	
July 28......	0.54	0.043	0.032																
July 29......	....	0.041	0.130																
July 30......	....	0.022	0.124																
July 31......	....	0.014	0.065																
August 1....	0.09	0.011	0.043	0.005									0.059	386	0.006	$\frac{0.0057}{0.067} = 0.085$	0.005	330	3 860
August 2....	0.07	0.009	0.032	0.022	0.004								0.067	355	0.005		0.006	390	4 380
August 3....	0.09	0.007	0.027	0.021	0.017	0.005							0.072	330	0.005		0.007	460	5 035
August 4....	0.05	0.007	0.022	0.011	0.016	0.022	0.003						0.077	336	0.005		0.007	460	5 300
August 5....	0.22	0.005	0.022	0.007	0.008	0.021	0.012	0.013					0.084	373	0.006		0.007	460	5 760
August 6....	0.09	0.005	0.016	0.005	0.006	0.011	0.012	0.053	0.005				0.113	367	0.006		0.010	650	7 390
August 7....	0.03	0.003	0.016	0.005	0.004	0.007	0.006	0.051	0.022	0.002			0.116	380	0.006		0.010	650	7 590
August 8....	....	.....	0.011	0.004	0.004	0.005	0.004	0.026	0.021	0.007			0.082	615	0.009		0.007	460	5 360
August 9....	....	.....	.....	0.004	0.003	0.005	0.003	0.018	0.011	0.007			Storm period 0.051	520	0.008		0.005	330	3 340
August 10...	....	.....	.....	0.003	0.003	0.004	0.003	0.013	0.007	0.004			0.037	380	0.006		0.003	200	2 420
August 11...	0.01	.....	.....	0.003	0.003	0.004	0.002	0.011	0.005	0.002	0.001		0.030	297	0.004		0.003	200	1 963
August 12...	0.06	.....	.....	0.002	0.002	0.003	0.002	0.009	0.005	0.002	0.002	0.004	0.031	275	0.004		0.003	200	2 030
August 13...	0.05	0.003	.....	.....	0.001	0.003	0.002	0.009	0.004	0.002	0.002	0.014	0.040	265	0.004		0.004	260	2 615
August 14...	0.82	0.012	0.049	.....	.....	0.002	0.002	0.007	0.004	0.001	0.001	0.014	0.092	292	0.005	$\frac{0.014}{0.236} = 0.059$	0.005	330	6 020
August 15...	0.34	0.012	0.197	0.020	.....	.....	0.001	0.007	0.003	0.001	0.001	0.007	0.249	575	0.009		0.015	980	16 300
August 16...	0.40	0.006	0.189	0.082	0.024	.....	.....	0.004	0.003	0.001	0.001	0.005	0.315	1 030	0.016		0.019	1 240	20 620
August 17...	0.22	0.004	0.098	0.078	0.096	0.013	.....	.....	0.002	0.001	0.001	0.004	0.297	885	0.014		0.017	1 110	19 430
August 18...	0.32	0.003	0.066	0.041	0.092	0.053	0.019	.....	.....	.....	0.001	0.003	0.278	1 140	0.017		0.016	1 050	18 200
August 19...	0.08	0.003	0.049	0.027	0.048	0.051	0.077	0.005	.....	.....	.....	0.002	0.262	1 240	0.019		0.015	980	17 150
August 20...	....	0.002	0.041	0.020	0.032	0.026	0.074	0.019	.....	.....	.....	0.002	0.216	1 140	0.017		0.013	850	14 130
August 21...	0.43	0.002	0.033	0.017	0.024	0.018	0.038	0.018	0.026	.....	.....	0.002	0.178	838	0.013		0.011	720	11 650
August 22...	0.09	0.002	0.033	0.014	0.020	0.013	0.026	0.010	0.103	0.005	.....	0.002	0.228	1 080	0.017	$\frac{0.0126}{0.116} = 0.109$	0.025	1 630	14 910
August 23...	....	0.002	0.025	0.014	0.016	0.011	0.019	0.006	0.099	0.022	.....	0.001	0.215	1 650	0.025		0.023	1 500	14 070
August 24...	0.18	0.001	0.025	0.010	0.016	0.009	0.016	0.005	0.052	0.021	0.011	.....	0.166	1 360	0.021		0.018	1 180	10 860
August 25...	0.01	.....	0.016	0.010	0.012	0.009	0.013	0.004	0.034	0.011	0.043	0.001	0.153	932	0.014		0.017	1 110	10 010
August 26...	....	.....	.....	0.007	0.012	0.007	0.013	0.003	0.026	0.007	0.041	0.002	0.118	700	0.011		0.013	850	7 720
August 27...	0.01	0.001	.....	.....	0.008	0.007	0.010	0.003	0.022	0.005	0.022	0.002	0.080	575	0.009		0.009	590	5 230
August 28...	....	0.002	.....	.....	.....	0.004	0.010	0.002	0.017	0.005	0.014	0.001	0.055	535	0.008		0.006	390	3 600
August 29...	0.08	0.002	0.005	.....	.....	.....	0.006	0.002	0.017	0.004	0.011	0.001	0.048	490	0.008		0.005	330	3 140
August 30...	....	0.001	0.019	.....	.....	.....	.....	0.002	0.013	0.004	0.009	0.001	0.049	433	0.007		0.005	330	3 205
August 31...	....	0.001	0.018	.....	.....	.....	.....	.....	0.013	0.003	0.007	0.001	0.043	386	0.006		0.005	330	2 813
September 1.	1.23	0.001	0.010	0.074	.....	.....	.....	.....	.....	0.003	0.007	0.001	0.105	373	0.006		0.021	1 370	6 870
September 2.	0.26	0.001	0.006	0.295	0.016	.....	.....	.....	.....	.....	0.002	0.005	0.325	475	0.007		0.064	4 180	21 270

in which, *Q* is the average 24-hr discharge and *M,* the area of the water-shed, in square miles. These values are listed in Column (16).

6.—The tabulation is divided into storm or peak periods, the divisions being easily identified in the column (Column (14)) of effective rainfall depths.

7.—Both rainfall and run-off are expressed in the same terms (that is, inches of depth). Within the storm period they bear a straight-line relationship to each other, and the coefficient is the ratio of their storm-period averages. These are to be found in Column (17).

8.—The coefficients are applied to the effective rainfall depths, *I,* computed values of *D* being obtained and recorded in Column (18).

9.—The computed values of *D* are converted into stream flow, in second-feet-days, and listed in Column (19).

10.—A graphic comparison of daily rainfall, effective rainfall depth, the observed hydrograph, and the computed hydrograph for the six water-sheds is finally plotted, as illustrated by Fig. 3 for the Licking River Water-Shed, and Fig. 4 for the Tuscarawas River Water-Shed.

COEFFICIENTS

The distribution graph is, in reality, a distribution coefficient, fulfilling that function in a manner much more in accord with the actual phenomenon than is the application of a single coefficient to the rainfall depth on the day in which it falls. The distribution coefficient breaks down rainfall depth and re-assembles it as it becomes effective to the flow of the stream. In this form there is a straight-line relationship between rainfall and run-off within the storm period, the slope of the curve becoming a coefficient that might be termed the "retention coefficient," because it is now called on to express the value of the losses to run-off only.

It will be seen that the retention coefficient marks the seasonal change in soil absorption. In addition, it is influenced by the degree of saturation immediately preceding the storm period and by the intensity of the storm itself. Beginning with a high value, throughout the storage period, January to March, it drops to a value of less than 0.10 through the growing season, May to September, increasing into the fall and winter months and reaching, in November, a value of approximately 0.50. When melting snow and ice are added to run-off, the value may exceed unity.

The seasonal fluctuation in the coefficient value shows marked stability over the entire region embraced by the study. It would seem that the way is opened for further refinement of the method by possible correlation of the coefficient with soil saturation in terms of previous rainfall, evaporation in terms of temperature, and with the average intensity of the storm which obviously affects the value of the coefficient.

THE PLUVIAGRAPH

If the factor that is used to convert depth over the water-shed to stream flow, in second-feet-days, is applied to the effective rainfall depths, *I,* a

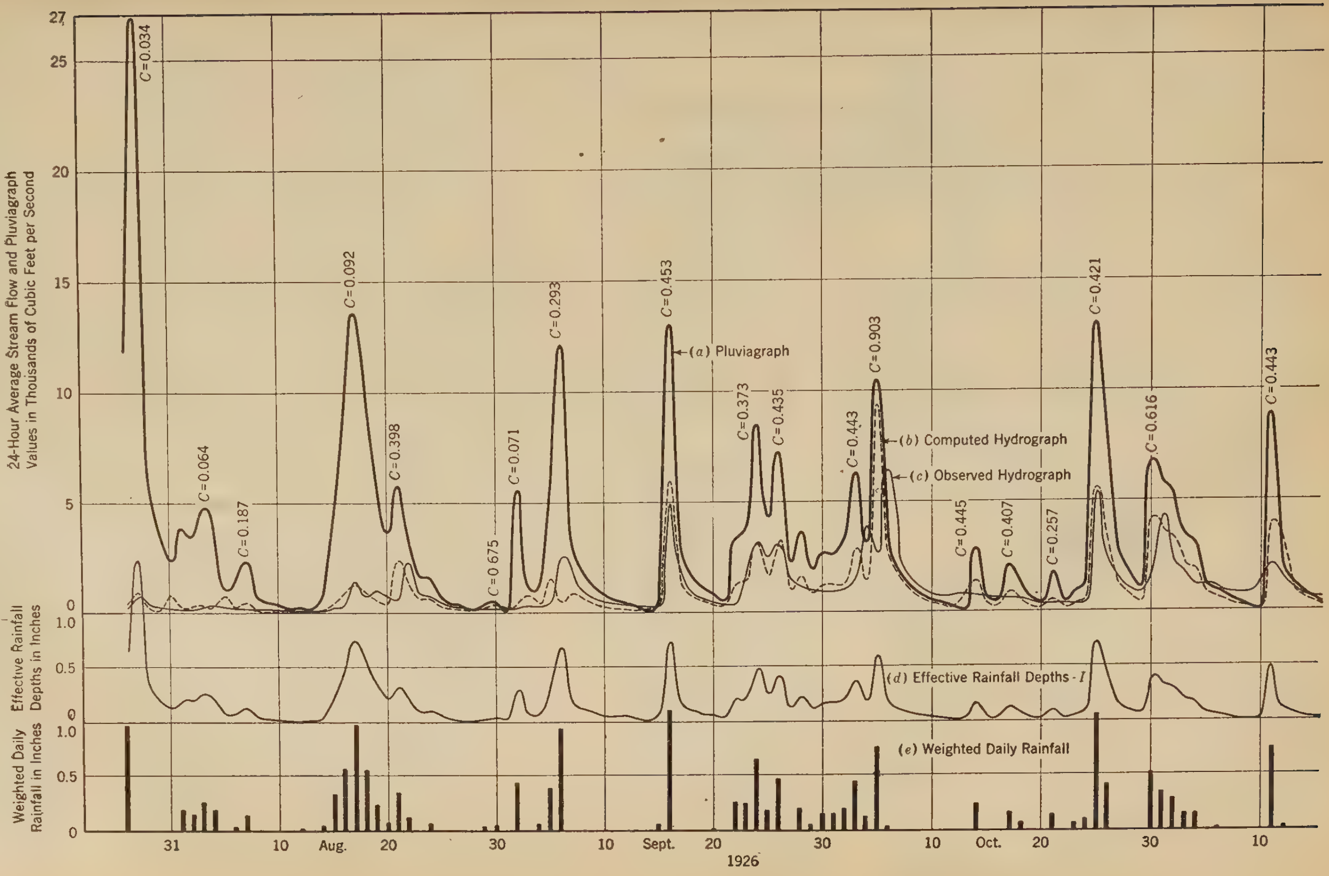


FIG. 5.—RAINFALL-STREAM-FLOW CHARACTERISTICS, LICKING RIVER, AT TOBOSO, OHIO.

maximum hydrograph (here termed the "pluviagraph") results (see Fig. 3(*a*)). The pluviagraph values for the Tuscarawas River appear in Column (20) of Table 3. The pluviagraph, when affected by the retention coefficient, becomes the stream hydrograph. In developing the coefficient,

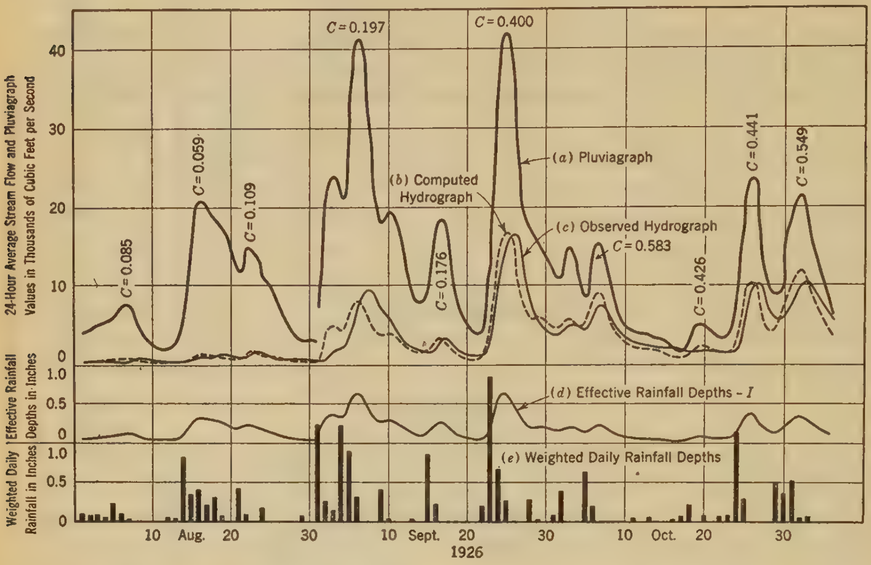


FIG. 4.—RAINFALL-STREAM-FLOW CHARACTERISTICS, TUSCARAWAS RIVER, NEWCOMERSTOWN, OHIO.

therefore, it is not necessary to reduce stream flow to run-off depth as the ratio may be stated as between the pluviagraph and the hydrograph values, both being expressed in the same unit—the second-foot-day.

The pluviagraph should be an important adjunct to hydraulic design because it becomes the means of expressing flood flow in terms of frequency.

The retention coefficient practically reaches unity during the winter and spring months of each year, and north of the snow line it will exceed unity by what appears to be 10 to 20 per cent. If, therefore, the unity, C, occurs with a frequency of at least once each year, and if the possibility of the concurrence of a maximum storm intensity and the unity, C, is the product of their respective frequencies, the pluviagraph of a storm of known frequency may be taken as the hydrograph that will be reached or exceeded with that frequency. From the writer's charts,[8] the rainfall intensity equation for the Tuscarawas River above Newcomerstown is,

$$i = \frac{30\, T^{0.21}}{t^{0.80}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

in which, i is the average rainfall intensity, in inches per hour; T, the frequency with which i will be reached or exceeded, in years; t, the duration of rainfall, exceeding 1 hr, in minutes; K, a coefficient depending for value on locality; and x and n are exponents depending for value on locality.

---

[8] *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 618, Figs. 18, 19, and 20.

When reduced by an area factor,[9] $(1 - 0.015 \sqrt[3]{A_m})$, Equation (3) becomes,

$$i = \frac{24\ T^{0.21}}{t^{0.80}} \qquad \text{(4)}$$

Reducing the average intensities of known frequency to daily rainfall depth and applying the distribution graph of the water-shed, results in the frequency-pluviagraph group shown in Fig. 5(*a*). These graphs are based on the assumption that the storm will parallel the rainfall intensity curve group, coming to a peak on the first day. That the storm curve may take any shape without materially affecting the pluviagraph is shown in Fig. 5(*b*), with Table 4, in which the peak is shifted to various positions in the storm period, without disturbing the average intensity or frequency of the storm. The effect is to increase the maximum pluviagraph values about 10 per cent.

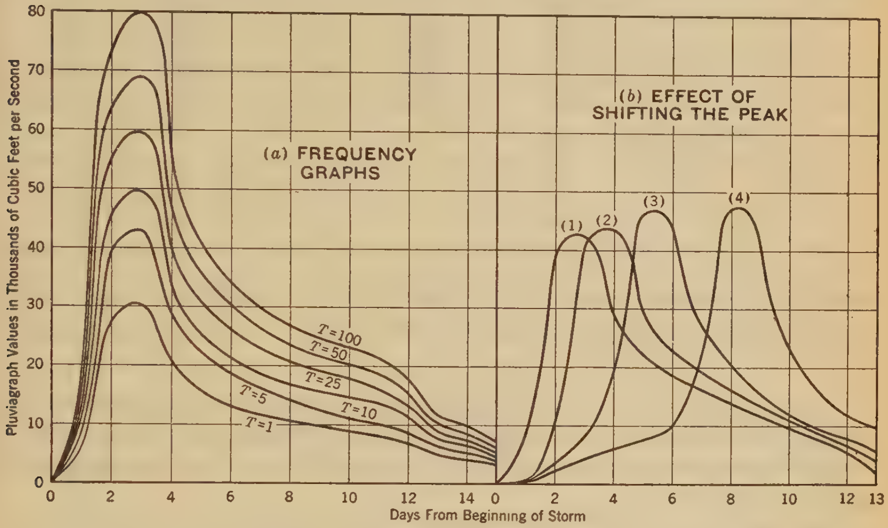


FIG. 5—PLUVIAGRAPHS, TUSCARAWAS RIVER, AT NEWCOMERSTOWN, OHIO

LIMITATIONS OF THE METHOD

While the theory of the unit and distribution graphs appears to be generally applicable, the units used, and, therefore, the Weather Bureau and Geological Survey data, have their limits of application.

That the writer's study approaches the upper limit of value of U, and the lower limit of area, is shown in Fig. 3, the rainfall-stream-flow characteristics of Licking River at Toboso, Ohio. The relatively poor agreement of the computed and observed hydrographs and the erratic relationship of the peak coefficients, show that the 24-hr interval is too great to measure unit changes in stream discharge and storm intensity. For the smaller areas, particularly where the shape and slope of the water-shed are conducive to the prompt delivery of run-off waters, the hour should become the unit interval of the rainfall and stream-flow record. From the satisfactory results gained on the largest water-shed in the study, (5 980 sq miles), the upper limit of application, apparently, has not been approached.

---

[9] *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 1150.

TABLE 4.—DISTRIBUTION FACTORS AND DAILY RAINFALL DEPTH CORRESPONDING TO FIG. 5(*b*)

Days	Distribution factors (effective percentage of total flow)	DAILY RAINFALL IN INCHES			
		Curve 1	Curve 2	Curve 3	Curve 4
1..............	6	2.40	0.12	0.17	0.12
2..............	24	0.36	2.40	0.24	0.13
3..............	23	0.24	0.36	0.36	0.14
4..............	12	0.17	0.24	2.40	0.17
5..............	8	0.14	0.17	0.14	0.24
6..............	6	0.13	0.14	0.13	0.36
7..............	5	0.12	0.13	0.12	2.40
8..............	4	....	....	....	....
9..............	4	....	....	....	....
10..............	3	....	....	....	....
11..............	3	....	....	....	....
12..............	2	....	....	....	....
Total.......	100	....	....	....	....

CONCLUSION

This paper has served its purpose if it has pointed to the possibility of stream-flow studies through the use of published Weather Bureau and Geological Survey data, by which regional coefficients may be developed and made available for use.

The idea fundamental to the method proposed might well be taken as common ground upon which to unify engineering thought in the important field of stream-flow determination.

DISCUSSION

C. S. JARVIS,[10] M. AM. SOC. C. E. (by letter).—This paper embodies several forward steps in a scientific field of much promise. The author has made practical application of some of the most recent developments toward logical and fairly dependable estimates of stream flow that should follow rainfall of given intensities and duration.

The assignment of proper and consistent values for the various constants seems to demand special skill or experience for guidance. The values of U in Table 2, ranging from 0.0000195 to 0.0001570 (or to eight times the smaller), need some kind of fundamental co-relation. Obviously, the factors in Table 2 most directly related are the length, L, and some function of the slope of the main channel, designated as S. Furthermore, they vary inversely, as shown by Equation (1). By inspection and trial, it was found that a fair approximation to the tabulated values of U for the six streams, as there listed, may be computed from the following expression,

$$20\,\frac{\sqrt{S}}{L} = U \text{ (approx.)} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

Slide-rule computations brought the results shown in Table 5. While the agreement is not all that could be desired, it may served as a first approximation.

TABLE 5.—VALUES OF U IN RELATION TO S AND L IN TABLE 2, IN MILLIONTHS.

River	From Table 2	Computed by Equation (5)	River	From Table 2	Computed by Equation (5)
Raccoon Creek	61.0	49.6	Walhonding River	42.0	33.7
Licking River	157.0	128.2	Tuscarawas River	32.0	33.2
Hocking River	54.0	50.8	Muskingum River	19.5	22.4

Examination of the extensive tabulations leading up to the desired hydrograph, as illustrated in Fig. 3, suggests the idea that a free-hand sketching of a curve under the pluviagraph (with ordinates varying from 1% to 30% of the corresponding pluviagraph values, and, with due regard to the time element for the given water-shed, the mounting percentage of run-off with higher degrees of saturation, after several days of storm, and whatever local data are available), may result in nearly as close approximations of the observed hydrograph as will normally follow from the more elaborate and detailed procedure. It is conceivable that the selection of the numerous constant values, the mastery of the extensive nomenclature, and the great number of computations involved in the author's method of determining stream flow, may be too much to expect of those who attempt to follow it. The value of dependable forecasts for different types of storms that habitually visit a region cannot be over-estimated; but it will require specialists in the science of hydrology and in the application of both meteorological and hydraulic data, to prepare such charts, with coefficients of variation to conform with seasonal and other changes of physical data affecting run-off.

---

[10] Senior Hydr. Engr., U. S. Geological Survey, Washington, D. C.

Before 1864 it was known that the annual run-off from the Mississippi water-shed was approximately "one-fourth of the downfall," or precipitation. Subsequent years of record, with gauging and meteorological stations multiplied in number, have demonstrated that the yield may range between approximately 14 and 31% of the rainfall; but these later records confirm the early investigations as to average percentage of yield. Therefore, a pluviagraph of this area would constitute a fairly reliable guide for a resulting hydrograph, with its peak covering the months of April and May, and its area between the curve and the horizontal axis representing about 25% of the area under the pluviagraph. For such a water-shed as that of the Mississippi, the human or personal equation would be kept within bounds more effectively by this means than if independent judgments were required to assign proper values to various constants in the several equations utilized, especially as some of these values constitute powers of certain factors. Furthermore, due regard to the grouping of rainfall days, and to the seasons in which they occurred, would justify a departure from the average value of 25% as representing run-off within the observed range of 14 to 31%; and, in this manner, the sketched hydrograph may prove to be as reliable as those resulting from voluminous computations, interspersed with occasional estimates that are just as difficult to apply and as far-reaching in their effects as those required in sketching the desired hydrograph from the pluviagraph.

After investigating or dealing with the entire category of reputable run-off formulas now available, the writer continues to incline toward simplification rather than toward multiplication of factors and processes. For first approximations to maximum, or for any other assumed rate of run-off corresponding to the probable frequency of recurrence, applicable to any given section of the United States, or of other countries, significant basic data have been presented by the writer.[11] The formula for maximum run-off which he has recommended is,

$$Q = 10\,000\, p \sqrt{M} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

in which, Q is the peak discharge, in cubic feet per second; M, the drainage area, in square miles, and p, the percentage rating on the Myers scale, roughly equal, numerically, to the percentage of rainfall that appears as run-off during the most severe storms and floods of that area, except as modified by geometrical shape of basin and alignment of main drainage channels, or other water-shed characteristics. The streams of Northern Ohio are found to range from about 5 to 12% on the Myers scale, while those of Southern Ohio are usually 20% or more. This disparity is traceable partly to the difference in rainfall habits, but, in greater degree, to the difference in soil and surface characteristics, channel gradients, and valley storage. The maximum discharges recorded in Table 1 are found to rate, respectively, only 1.4%; 1.3%; 2.3%; 2.9%; 1.7%; and 6.8% on the Myers scale. The last-named item applies to the Muskingum River, which attained a rating of 28.4% at Marietta, Ohio, in 1913. The foregoing percentages, therefore, represent run-offs of relatively low volume and correspondingly high frequency of recurrence, perhaps attainable nearly every year.

---

[11] "Flood Flow Characteristics," *Transactions*, Am. Soc. C. E., Vol. 89 (1926), p. 985; and "Rainfall Characteristics and Their Relation to Soils and Run-Off," *Loc cit.*, Vol. 95 (1931), p. 379.

LEROY K. SHERMAN,[12] M. AM. SOC. C. E. (by letter).—In this paper Mr. Bernard has confirmed the principle of the unit graph by analysis based on independent data. He has also developed certain new procedures.

The Distribution Graph.—The author expresses the typical graph of run-off, for a given drainage basin, in terms of percentages of distribution of the total volume of the run-off during a number of equal time intervals. The writer's unit graph[13] expresses this same drainage-basin characteristic in terms of distribution of rates of run-off due to a unit depth of effective rainfall (rainfall appearing as surface run-off), applied in a unit of time. The unit graph and the distribution graph are readily convertible one to another by applying the following formula:

$$q = p \times 26.88\, M \qquad (7)$$

in which, q is the average rate of run-off for a given day, read from the unit graph; p, the average percentage of run-off for the same time interval, read from the distribution graph; and M, the area of the drainage basin, in square miles.

The writer recognizes that the use of the distribution graph is preferable to the unit graph. It is a simpler mode of expression. Both graphs are predicated on the hypothesis that, for a given basin; (*a*) The distribution of percentages of run-off is constant for a run-off period due to rainfall occurring in a single day, or time unit; and (*b*) the duration of the run-off period, under aforesaid conditions, is a constant.[14] This hypothesis is confirmed by the seventeen cases given by Mr. Bernard in Table 1. It is also substantiated by the fair agreement between observed and computed hydrographs of stream flow made by Mr. Bernard and by the writer.

As a further check the writer recently computed, for comparison, two hydrographs based on the same duration of rainfalls on the same hypothetical drainage basin (see Fig. 6). One had a high rainfall intensity. The other had a low intensity of rainfall. Both were subject to infiltration loss and pondage effect. With a given stage-velocity curve for surface run-off, and with given channel elements of slope, cross-section, and coefficient of roughness, this becomes a problem in hydraulics. It was solved by the application of well-known and accepted laws. The stage-velocity curves for both overland flow and stream flow had large differences in velocity between high and low stages. Referring to Fig. 6, it will be noted that the duration of run-off was greater for the greater rainfall, but that at the time, 13 000 sec, the relative difference in rates of flow became insignificant; in other words, the run-off periods were practically the same. The total volumes of run-off (area of graphs) are 97.5 and 17.6 acre-ft, a ratio of 5.55 : 1. Curve (*c*) was obtained by dividing the flow ordinates of Curve (*b*) by 5.55, following the hypothesis used by Mr. Bernard. It will be noted that the peak of Curve (*c*) appears earlier than that of the smaller hydrograph, Curve (*a*). If Curve (*c*) is moved forward so that the peaks coincide, the two graphs will be found to be almost identical. This example illustrates an extreme range of conditions. The lack of synchronization is due to the fact that time of transit is shorter for the high stage. This effect is apparent in the examples of computed and observed flows made by both Mr. Bernard and the writer. At ordinary, and high, stages, especially on the many streams with almost vertical stage-velocity curves, the synchronization is almost perfect. Mr. Bernard's procedure of using the average of three observed distribution graphs is desirable. From an engineering standpoint small differences in time are immaterial, provided the peaks and troughs give correct rates of flow. If requirements warrant closer synchronization, a special low-flow distribution graph may be developed and applied.

---

[12] Cons. Engr., Chicago, Ill.

[13] *Engineering News-Record*, April 7, 1932.

[14] Rept. of the Committee, Boston Soc. of Civ. Engrs., on Floods, *Journal*, Boston Soc. of Civ. Engrs., Vol. XVII, September, 1930, No. 7.

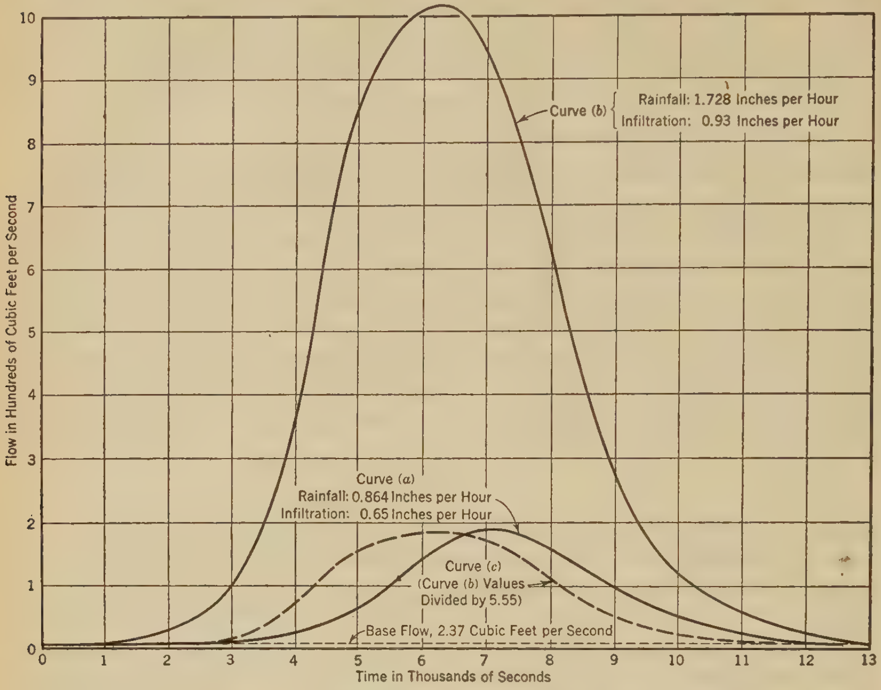


FIG. 6.—HYDROGRAPHS FROM HYPOTHETICAL BASIN OF 2.37 SQUARE MILES; BOTH RAIN DURATIONS OF 5 200 SECONDS.

The 24-hr rainfalls, furnished by the U. S. Weather Bureau, in Mr. Bernard's Table 1, may have occurred in 1 hr, 10 hr, or 20 hr. The close agreement of the three distribution graphs shows that this factor makes little difference on large areas.

The Water-Shed Factor.—The author has evaluated another procedure for determining the distribution graph of water-sheds approximately, where stream-flow data are not available. The writer views this accomplishment, in co-ordinating such complex factors, with sincere admiration. The frequent statement that certain phenomena of Nature are erratic is Man's comfortable and self-satisfying way of acknowledging his ignorance and bewilderment.

The Retention Coefficient.—In the procedure for reproducing a hydrograph of stream flow, Mr. Bernard uses, in Column (17), Table 3, a so-called "retention coefficient." This coefficient applied to rainfall depth gives the rate of stream flow. The latter is made up of surface run-off plus base flow, Although Mr. Bernard states, "the retention coefficient * * * expresses the value of the losses to run-off only," the numerator, D, in deriving the coefficient, includes base flow, as set forth in the U. S. Geological Survey records in Column (15), Table 3. In practice, this coefficient could not be derived from flow records. That record is the answer to be determined. The retention coefficient must be forecast.

The retention coefficient combines the two independent factors: Percentage or net amount of rainfall appearing as surface run-off, and base flow. The forecasting of this "retention coefficient" may be possible; it has not been done as yet. The writer deems the more usual procedure preferable, namely, application of the run-off percentage, or loss deduction, to rainfall, followed by the independent addition of the base flow. Refer, for example, to the computed run-off on October 7, 1926, as shown in Fig. 4. In Mr. Bernard's complete paper the summation sheet (a continuation of Table 3) gives the following values:

Distribution graph applied to weighted daily rainfall:

Column (3), Table 3	0.001
Column (4), Table 3	0.145
Column (5), Table 3	0.048
Column (10), Table 3	0.009
Column (11), Table 3	0.002
Column (12), Table 3	0.005
Column (13), Table 3	0.024

Effective rainfall depth, in inches (Column (14), Table 3)	0.234
Average 24-hr observed flow, in cubic feet per second (Column (15), Table 3)	7 680
Retention coefficient, C (Column (17), Table 3)	0.583
Computed stream flow, in cubic feet per second	8 900

The weighted daily rainfall in Column (2), Table 3, extended to include the period, September 28 to October 6, inclusive, is as follows: September 28, 0.29; September 29, 0.05; October 1, 0.09; October 2, 0.40; October 3, 0.01; October 5, 0.63; and October 6, 0.20. The computed flow for October 7 is seen to be 8 900 cu ft per sec as against an observed flow of 7 680 cu ft per sec. The writer finds as follows: The two hydrographs due to rainfalls of October 1 to 3 and October 5 and 6 can be segregated graphically. From this step, the percentages of run-off can be determined as 43 and 40. The base flow can be segregated, by the approximate method of Messrs. Meinzer and Houk,[15] which yields 1 400 cu ft per sec as of October 7. In the absence of the observed run-off these three figures could be approximated with fair accuracy by a competent hydrologist. Applying 43% to Mr. Bernard's data for October 7 in Columns (3), (10), (11), (12), and (13); and 40% to the values in Columns (4) and (5), results in a total surface run-off of 0.094 in., or 6 200 cu ft per sec, on October 7. Adding the base flow of 1 400 cu ft per sec gives the total stream flow as 7 600 cu ft per sec.

The problem of estimating infiltration, or the run-off coefficient, is a separate realm of hydrology. It is now in the process of evolution by a number of investigators. Ultimately, the best solution may come through an application of the principles noted[16] by Robert E. Horton, M. Am. Soc. C. E., in conjunction with a better forecasting of infiltration rates. The most accurate procedure probably will not be based on utilizing a coefficient of retention or run-off. It will be a deduction of infiltration rates from the data in each of Columns (3) to (13) Table 3, bearing in mind that infiltration is greatest at the beginning of a storm and practically ceases when surface run-off has reached the main stream.

The Pluviagraph.—The writer understands the pluviagraph to be the hydrograph of run-off if, or when, the surface of the drainage basin is impervious. This graph can be very useful in a study of the maximum rates of run-off due to a given storm under different and varying rates of infiltration. It is not necessary to assume the customary uniform rate of rainfall throughout the concentration period. Any possible sequence of rates of rainfall and infiltration may be compared. In Mr. Bernard's example (Fig. 5), the same maximum rainfall rate is used on different days. May not some other sequence in the 7-day storm produce a greater peak rate of flow? The way is provided for a better analysis of the rational method, the concentration period, and peak rate of flow.

The author is modest in his conclusions. Observed stream-flow records of long duration furnish the best guide for water supply or flood flow. Such stream data, however, are frequently lacking, whereas pertinent rainfall data for storms of maximum or other frequency are generally available. The method described by Mr. Bernard, in such cases, will furnish more reliable results than any of the existing empirical run-off formulas.

W. W. HORNER,[17] M. AM. SOC. C. E. (by letter).—Inspired (as he states) by Mr. Sherman's demonstration of the "unit graph," the author has taken another valuable step in systematizing the relation of rainfall and run-off. No doubt, the distribution graph will be accepted by hydrologists as a workable mechanism.

---

[15] *Water Supply Paper 638–C*, U. S. Geological Survey, p. 105.

[16] "The Rôle of Infiltration in the Hydrologic Cycle," by Robert E. Horton, *Transactions*, Am. Geophysical Union, 1933, p. 446.

[17] Cons. Engr., St. Louis, Mo.

Many students of this subject have been of the opinion that the present difficulty in establishing a rational relation between run-off and rainfall might be largely overcome if the effect of the fixed drainage-area characteristics, such as shape, slope, and channel condition, could be isolated. In another connection,[18] Mr. Bernard made a nice academic approach to the problem; in his paper, he indicates a practical means of achieving this result.

He refers to his work as an "approach to determinate stream flow"; the qualification should be kept well in mind. Apparently, he has examined a great mass of data and has subjected it to analysis by his method. To substantiate the hypothesis fully will require a study on a larger scale than could be undertaken by any one person; it is to be hoped that Mr. Bernard's effort will bring about the necessary organization.

The idea that, for rain within the chosen time unit, run-off diagrams will fall into definite patterns corresponding to salient water-shed characteristics, and that this hydrograph may be considered as independent of variations in rainfall intensity within the time unit, will undoubtedly require some qualifications, as the author suggests under "Limitations of the Method." It is possible that variations in ground condition will be found to account for nonconformities with the distribution graph; that is, as certain areas are relatively more retentive than others at certain seasons, the effect may be similar to a shift in the water-shed boundaries. In recent studies of small areas, the writer has encountered vagaries that might be accounted for in this manner.

The writer's experience with small areas leads him to feel that Mr. Bernard's viewpoint as to "Coefficients" is too optimistic. For these small areas a synthetic treatment of the rainfall resulted in the development of "100% run-off curves," corresponding to Mr. Bernard's pluviagraphs. The method was adjusted so that these curves were quite similar to the measured hydrographs. A simple relation between the ordinates of the two diagrams was anticipated, but the results were disappointing, the ratios covering a wide range of values without appreciable seasonal significance.

It is doubtful whether, even within a particular storm period, a straight-line variation between rainfall and run-off will be found; probably the relation will be more complex, and the seasonal fluctuation will be difficult to index. In passing, the writer suggests that the coefficient of retention be a "coefficient of non-retention"; it is very like the old "percentage of run-off."

The suggested preparation of frequency pluviagraphs needs further study; it does not seem to follow that, because a value of C equal to unity occurs over a year, "the pluviagraph of a storm of known frequency may be taken as a hydrograph that will be reached or exceeded with that frequency"; for this to be true, it would seem to be necessary that the unit, C, persist throughout the year. The writer's studies indicate that high run-off values are generally the result of a combination of high values of C with not excessive precipitation intensities, but sometimes from the alternate combination. He is doubtful whether any satisfactorily simple plan can be devised for studying probabilities of the concurrence of particular values.

---

[18] *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 1150.

Mr. Bernard's method appears to be most valuable, with the present lack of knowledge of coefficients, as a suggestion for analyzing existing information on a large scale. If such analysis should produce regular patterns and consistent factors, it may become the plan of construction for synthetic hydrographs.

C. H. EIFFERT,[19] M. AM. SOC. C. E. (by letter).—That the field of study along the line of correlating rainfall and run-off data is far from exhausted, is amply demonstrated in this valuable paper by Mr. Bernard. The author admits that his method has limitations and states in his "Conclusion" what he hopes it may do.

The writer would like to be able to believe that river discharges may be calculated by formula and that regional coefficients may be developed for such a formula, but to the present time his convictions point to quite the contrary.

Stream-flow or run-off formulas should be most valuable for use on drainage areas for which records are not available, or are few. If records are scarce, however, it will be difficult to arrive at the proper coefficient or formula for that region, and, therefore, the solution will be extremely uncertain. Furthermore, the time of year or season will cause a great variation in run-off due to the changes in the capacity of the ground to absorb rainfall. For instance, in the Miami Valley, the run-off from two very similar 1-day rains has varied from 1.25% to 114% of the rainfall.

The writer has not made any extensive use of Sherman's unit-graph method, but believes that the same statement would apply; that is, if records are scarce it will be difficult to arrive at the proper unit graph, especially if 1-day rainfalls from Weather Bureau records are used. Such rains sometimes are spread fairly uniformly over 24 hr, but often they occur in one-half as many hours, or less.

Of course, it must be acknowledged that no run-off figure can be arrived at without some kind of calculation; but in the last analysis the calculations that most nearly approach the correct result will contain a factor which depends largely on the engineering ability and judgment of the calculator. The best method of attack is through a more thorough and careful study of rainfall intensities and frequencies for the reason that rainfall records are much more voluminous than run-off records.

While engineers are using available records they should concentrate their efforts upon making these records better and more complete in the future. It is true, as stated by Mr. Bernard, that certain unavoidable sources of error must be recognized and accepted in using Weather Bureau data. Certainly, the engineer must recognize them and make the best possible use of the material, but he should not let it go at that. He should exert all possible influence to induce the Government and other departments collecting such information to make it more adaptable for the use of engineers. Considerable can be done along this line without necessarily increasing appropriations. Through better co-operation and co-ordination of data the various Federal

---

[19] Chf. Engr. and Gen. Mgr., The Miami Conservancy Dist., Dayton, Ohio.

Bureaus could make their records more useful to the Engineering Profession without limiting or reducing their usefulness for other purposes. Such matters as making time of readings uniform, obtaining better information on time limits of rainfall, and peak discharges of streams, instead of only the 24-hr maximum, should be easily adjusted. There are other condidtions that could be improved, such as the duplication of gauges by different departments, the breaking off of old records and the starting of others, and the more effective interrelated distribution of the Weather Bureau, Geological Survey, and Army stations.

Another fruitful line of study should be that of time of concentration on various sizes of drainage areas. Few data are available on this subject, except for metropolitan areas. This information will become available, of course, if rainfall periods and peak discharge are properly timed, measured, and recorded.

While studies such as those by Mr. Bernard ought to be encouraged, it must be remembered that nothing can take the place of adequate records. Engineers should be right "on their toes" to see that the errors or deficiencies of the past are not perpetuated.

Charles S. Bennett,[20] M. Am. Soc. C. E. (by letter).—Great ingenuity and much patience are evidenced in the work of Mr. Bernard in developing the approach to determinate stream flow in this paper. The writer is impressed with the logical development of the plan for constructing the "pluviagraph" and the "distribution graph", and is of the opinion that if basic data were available the method would be of great use to hydraulic engineers.

Unfortunately, there are serious handicaps. The rainfall data of the U. S. Weather Bureau, while complete in a quantitative sense, does not permit of selection of rainfall intensities of less than 24-hr duration. In most instances, there is no way of determining whether a given 24-hr. percipitation occurred in a few minutes, or was distributed over a period of hours. As pointed out by Mr. Bernard, this is a serious drawback in selecting data for the construction of the basic graphs.

The determination of the "water-shed factors" presents practical difficulties. So many coefficients need to be determined, each of which depends largely upon the trained guess of the computer, that a mathematical solution of the equation for determining this factor, *U*, seems to the writer to be wasted effort.

The application of the method seems limited to streams upon which some previous flow records, including flood flows, are available. This will eliminate the use of the method for many streams.

In the present state of the science of applied hydraulics and meteorological observations, much time would be saved (and likewise laborious calculation) by using simple direct relationships, based on data at hand and judgment, to determine such estimates of stream flow as are necessary for the solution of a given problem.

---

[20] Engr., The Miami Conservancy Dist., Dayton, Ohio.

These criticisms are not intended to deprecate the value of such painstaking research as have been done by Mr. Bernard and numerous other engineers who have been working on the problem of devising means of determining run-off. Great credit is due those who have led the way in attacking the problem. However, many factors involved are indeterminate, and this should be fully recognized by those who attempt to apply the various methods to specific problems.

R. L. GREGORY[21] AND C. E. ARNOLD,[22] ASSOC. MEMBERS, AM. SOC. C. E. (by letter).—Based on the assumption that certain statements contained, and procedure outlined, in a paper[23] by L. K. Sherman, M. Am. Soc. C. E., are correct, the author has presented an ingenuous method for the determination of stream flow, which utilizes some of the factors occurring in the writers' formulas.[24] The method constitutes an approach to the construction of hydrographs at points on streams for which adequate flow records are not available. Engineers in general, and the writers in particular, fully appreciate the value of any thoughts, data, or methods that tend to advance the solution of this problem.

Previously, in his discussion of the writers' paper,[25] the author had separated and combined the run-off factors pertaining strictly to the drainage area from those influenced more directly by rainfall. This combination, Equation (1), omits the three remaining run-off factors, C, A, and R_H, contained in the general run-off equation.[26]

There can be no doubt that Equation (1) may serve as a convenient instrument for aiding in the analyzation and presentation of drainage data. The author, through its use, has already presented valuable data for consideration by the profession. There has arisen in the minds of the writers, however, a doubt as to the wisdom of using Equation (1) in a procedure based on hypotheses that inherently prohibit consideration of changes in the hydrograph due to differences in the magnitudes of storm intensities.

In his paper,[23] Mr. Sherman states, "for the same drainage area, however, there is a definite total flood period corresponding to a given rainfall, and all one-day rainfalls, regardless of intensity, will give the same length of base of the hydrograph"; that is, the same total period of run-off. As far as the writers have been able to determine, data have not been presented to substantiate the truth of this unqualified assertion. The statement could not be true, since a difference in the magnitude of intensity for a given time period would alter the quantity of water flowing and, consequently, the time of concentration, and the relative shape and length of the graph for the total period of run-off.

---

[21] Chf. Insp., County of Los Angeles, Storm Drain Div., Los Angeles, Calif.

[22] Storm Drain and Constr. Engr., Los Angeles County Surveyor's Office, Los Angeles, Calif.

[23] "Streamflow from Rainfall by Unit Graph Method", *Engineering News-Record*, April 7, 1932, p. 501.

[24] *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 1038.

[25] *Loc. cit.*, p. 1150.

[26] *Loc. cit.*, p. 1059, Equation (24).

For like reasons, and others, inspection leads to the conclusion that Mr. Sherman's Fig. 1 and the equations immediately following convey erroneous conceptions of the manner in which water combines in a drainage area to cause run-off.[23] Therein exists a disregard of the fundamental principle that the velocity of flow of water in streams and channels is a function of the quantity flowing.

The author's method embodies the unit-graph procedure as exemplified in his Table 3.

It is comparatively easy to make or to deny assertions, but sometimes difficult and tedious to prove their truth or falsity. The purpose of this discussion is to test the fundamentality of the procedure that the author follows in his method and to arrive at some measure of its departure from the truth in the construction of hydrographs.

To ascertain the effect on a stream hydrograph due solely to a difference in the quantity of water running requires the complete elimination of all variables with the exception of intensity of precipitation. Due to the many and varying factors and conditions contributing to flow of a drainage area, it is almost impossible to find a single experiment wherein all variables are definitely known.

For their purpose the writers have chosen a hypothetical drainage area; and have stripped it of all pondage, channel constrictions, and other features, which tend only to confuse the issue.

Consider a triangular water-shed[27] with streams concentrating along lateral lines to the main channel of flow, to be divided, as in Fig. 7, into time-contours, all points on which are equi-distant in time from the outlet.

Assume any definite proportions for the water-shed (say, ratio of length, L, to bottom width is 5 to 3).. Then (all computations by slide-rule) since $L \propto A^{\frac{1}{2}}$, or $L = KA^{\frac{1}{2}}$, $K = 380.8$, in which, A = area, in acres, and L = length, in feet.

In a formula derived by the writers,[28]

$$v = 0.03162 F^2 \, S^{\frac{3}{8}} \, Q^{\frac{1}{4}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

Let $v = K_1 \, Q^{\frac{1}{4}} = 0.3944 \, Q^{\frac{1}{4}}$ (which is the equivalent of assuming that $F = 4.02$, with $S = 0.5$, or other combinations of these factors).

It has been shown[29] that for a triangular water-shed of which Fig. 7 is a special case, subjected to a uniform intensity, the value of $P = 0.5$, $Q \propto A$, and $v \propto t$, the run-off coefficient being assumed as constant. Since $v \propto t$ and $v \propto Q^{\frac{1}{4}}$, then $t \propto Q^{\frac{1}{4}}$, or $Q \propto t^4$. To simplify computations and eliminate confusion, the run-off coefficient is herein assumed as equal to 1; that is, all rain that falls runs off.

For a given intensity of precipitation, the water from Area ACB, Fig. 7, will be first to contribute its maximum rate of run-off. If it rains long enough, Areas *DFE, GIH, JLK,* and *MON,* Fig. 7, will reach their maximum contributions in sequence as time increases.

---

[27] *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 1068, Fig. 5(*a*).

[28] *Loc. cit.*, p. 1094, Equation (50).

[29] *Loc. cit.*, p. 1068.

Case 1.—Assume a uniform intensity of $\frac{1}{4}$ in. per hr throughout the period of concentration of the entire area, and assign definite values to the area and its subdivisions. The hydrograph for the period of concentration is formed by Table 6(*a*), the rate, in cubic feet per second, and the time, in days, being shown in the part of the graph to the left of the peak discharge point in Fig. 8 ("Case 1: Calculated True Graph").

Case 1(a).—If the rain ceases when the full area concentrates, much water will be left on the shed in transit to the outlet. Its measure is easily found for this case. Since (as shown previously), $Q \propto t^4$,

$$Q = K_2\, t^4 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

K_2 being a constant. The volume of water that has passed the outlet at the time the full area concentrates, is,

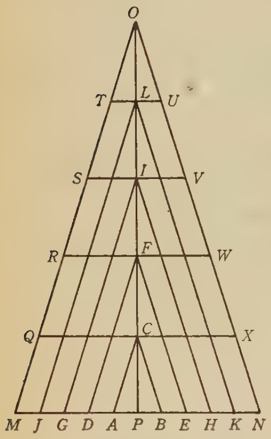


FIG. 7.—WATER-SHED.

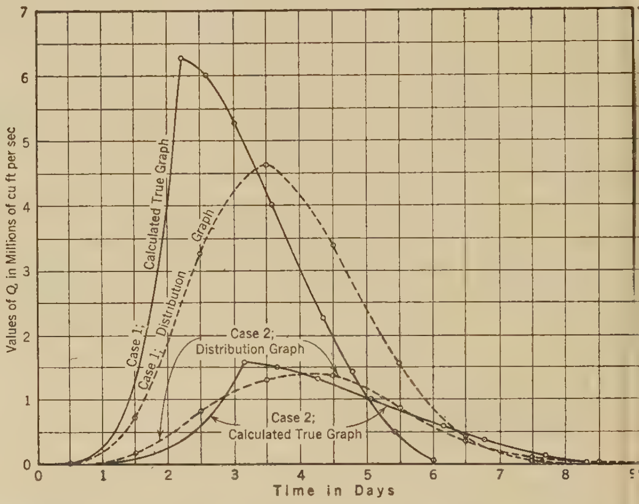


FIG. 8.—COMPARATIVE HYDROGRAPHS, CASE 1 AND CASE 2.

$$\int K_2\, t^4 = \frac{K_2\, t^5}{5} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

in which, t = time of concentration. Since $Q_{\max}\, t = (K_2\, t^4)\, t = K_2\, t^5$ = the volume of all rain that fell, one-fifth of the water has passed the outlet and four-fifths remains on the shed. Thus, using the data opposite the area, *MON,* of Table 6(*a*), the quantity remaining on the shed, in millions of cubic feet, is:

$$\frac{4}{5}\, Q_{\max}\, t = \frac{4}{5} \times 6\,250\,000 \times 193\,200 = 966\,000$$

The distribution of the water remaining on the shed at the time of concentration of the entire area may be ascertained by the following procedure: Find the quantities that have passed the various time-contour lines during the time of the rain. From the total rain, falling on the area above a given time-contour during the period, deduct the quantity passed, the remainder being the quantity left on the area.

TABLE 6.—VALUES FOR POINTS ON HYDROGRAPH FROM BEGINNING OF RAIN TO MAXIMUM RATE OF RUN-OFF

Area (see Fig. 7) (1)	Values of area, A, in acres (2)	Length of travel, $L = 380.8\ A^{\frac{1}{2}}$, in feet (3)	Flow, Q, in cubic feet per second (4)	Velocity, $v = P\ K_1 Q^{\frac{1}{4}}$, in feet per second (5)	TIME, $t = \frac{L}{v}$: In seconds (6)	TIME, $t = \frac{L}{v}$: In days (7)
			(*a*) CASE 1			
ACB	1 000 000	380 800	250 000	4.41	86 400	1
DFE	4 000 000	761 600	1 000 000	6.24	122 000	1.414
GIH	9 000 000	1 142 400	2 225 000	7.64	149 500	1.73
JLK	16 000 000	1 523 200	4 000 000	8.82	172 700	2.00
MON	25 000 000	1 904 000	6 250 000	9.86	193 200	2.236
			(*b*) CASE 2			
....	250 000	190 400	15 625	2.205	86 400	1
ACB	1 000 000	380 800	62 500	3.12	122 000	1.414
DFE	4 000 000	761 600	250 000	4.41	172 700	2
GIH	9 000 000	1 142 400	565 000	5.40	211 400	2.45
JLK	16 000 000	1 523 200	1 000 000	6.24	244 000	2.83
MON	25 000 000	1 904 000	1 562 500	6.975	272 800	3.1623

For example, consider the time-contour, *DFE* (Fig. 7). At the end of the first day of the rain, Area *GIHEFD* (A = 5 000 000 acres; Q = 1 000 000 cu ft per sec) was contributing its maximum quantity to the contour. At the end of 1.414 days of rain, Area *JLKEFD* (A = 12 000 000 acres; Q = 3 000 000 cu ft per sec) was contributing its maximum. The entire area (A = 21 000 000 acres; Q = 5 250 000 cu ft per sec) above the time-contour was contributing its maximum after 1.73 days of rain. The times of concentration, to the time-contour, of the three areas are determined from, and are the same as, those for the respective parts of the areas contributing to the main channel between *FI, FL,* and *FO* (see Fig. 7). In Table 7(*a*), showing times and quantities to the time of concentration to this time-contour, *DFE,* the values of Q' correspond to the time-rendering areas, the time having been calculated from Equation (9) with substitution of data opposite Area *ACB,* of Table 6(*a*) or, the time of concentration, in days, equals:

$$t = 0.04472\ (Q')^{\frac{1}{4}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (11)$$

TABLE 7.—EXPLANATORY DATA

(*a*) CASE 1: Flow, in Millions of Cubic Feet per Second: Q	(*a*) CASE 1: Flow, in Millions of Cubic Feet per Second: Q'	(*a*) CASE 1: Time, in days	(*b*) CASE 3: Flow, in Millions of Cubic Feet per Second: Q	(*b*) CASE 3: Flow, in Millions of Cubic Feet per Second: Q'	(*b*) CASE 3: Time, in days
0.44	0.04	0.633	0.21	0.01	0.4475
1.25	0.25	1.000	0.44	0.04	0.6325
3.00	1.00	1.414	0.69	0.09	0.775
5.25	2.25	1.73	0.96	0.16	0.8945
....	....	....	1.25	0.25	1.000

In Fig. 9 the area (under the curve formed by plotting the data of Table 7(*a*)) designated *ABC*, represents the volume of water that had passed the time-contour, *DFE* (Fig. 7), to the time (1.73 days) the area above it had contributed its maximum run-off. This maximum rate of flow continues to pass the time-contour until the end of the rain, or to the time of concentration of the entire area. Hence, the total quantity of water passing the time-contour during the period of rain (2.236 days) is equal to the summation of the area (*ACB*) under the curve and the rectangle, *BCDE*. In like manner data were calculated for other time-contours and plotted in Fig. 9. The various contours are represented by their respective, tributary, maximum *Q*-values. In Table 8(*a*) these maximum *Q*-values are shown opposite the quantities, passed beyond and left on the water-shed above, the various time-contours represented.

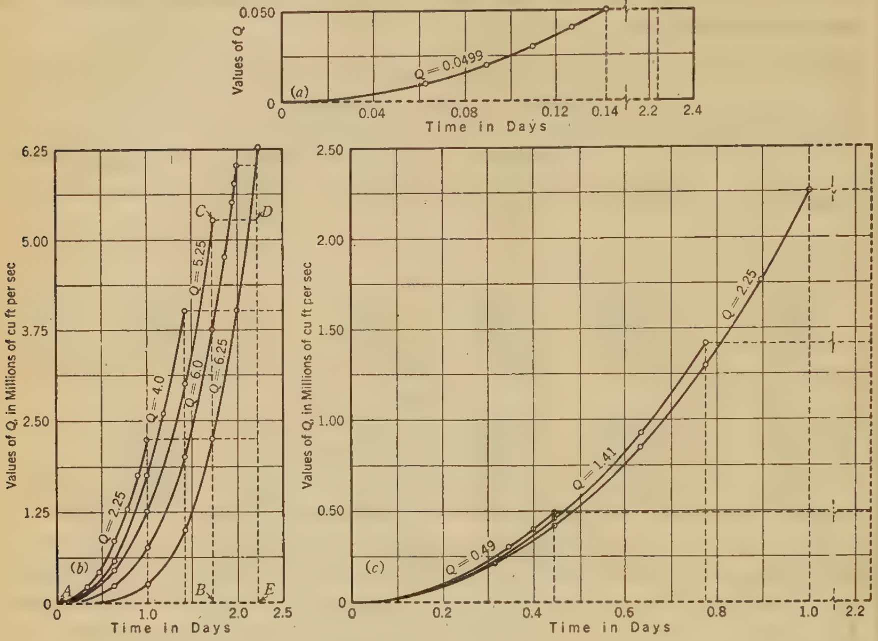


FIG. 9.—EXPLANATORY GRAPHS, CASE 1.

When a given value of *Q* (Column (1), Table 8(*a*)) has reached the outlet, its corresponding quantity (Column (4) Table 8(*a*)) will remain on the water-shed. If, therefore, a quantity (Column (4), Table 8(*a*)) is subtracted from the total quantity left on the entire water-shed at its time of concentration (shown hereinbefore to be 966 000 millions of cubic feet), the remainder will represent the quantity that has subsequently passed the outlet when the corresponding *Q*-value arrives.

These quantities, together with their corresponding *Q*-values, were used to construct the outflow curve as plotted to the right of the peak discharge in Fig. 8 ("Calculated True Graph"). This curve of Fig. 8 represents a true

TABLE 8.—DISTRIBUTION OF WATER LEFT ON A WATER-SHED AT THE END OF A RAIN

Flow, in millions of cubic feet per second	RAINFALL AND RUN-OFF, IN MILLIONS OF CUBIC FEET			Flow, in millions of cubic feet per second	RAINFALL AND RUN-OFF, IN MILLIONS OF CUBIC FEET		
	Total rainfall	Quantity passed	Quantity left on water-shed		Total rainfall	Quantity passed	Quantity left on water-shed
(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
(*a*) CASE 1				(*b*) CASE 2			
6.25	1 207 500	241 500	966 000	1.5625	426 900	85 400	341 500
6	1 159 000	377 000	782 000	1.5	410 000	134 400	275 600
5.25	1 016 000	446 000	570 000	1.3125	359 000	158 600	200 400
4	774 000	431 000	343 000	1.0	273 000	152 000	121 000
2.25	435 000	302 500	132 500	0.5625	153 600	106 900	46 700
1.41	273 000	209 300	63 700	0.3525	96 500	73 800	22 700
0.49	94 600	81 900	12 700	0.1225	33 500	29 000	4 500
0.0499	9 650	9 250	400	0.012475	3 410	3 260	150
(*c*) CASE 3				(*d*) CASE 4			
0.25	540 000	4 300	535 700	0.015625	135 000	270	134 730
0.75	518 400	18 400	500 000	0.046875	133 650	1 190	132 460
1.25	453 600	32 600	421 000	0.078125	129 600	2 090	127 510
1.75	345 600	47 100	298 500	0.140625	113 400	3 870	109 530
2.25	194 400	61 400	133 000	0.203125	86 400	5 690	80 710
1.41	121 800	58 200	63 600	0.265625	48 600	7 430	41 170
0.49	42 350	29 700	12 650	0.296875	25 650	8 400	17 250
0.0499	4 310	3 900	410	0.1225	10 580	6 100	4 480
				0.012475	1 078	937	140

hydrograph of the water-shed for the assumptions used, the area under the curve to the left of the peak discharge being one-fifth the total outflow of water and that to the right, four-fifths.

Case 2.—Assume a uniform intensity one-fourth that of Case 1, or $\frac{1}{16}$ in. per hr throughout the period of concentration of the entire area for this intensity. The hydrograph of this lighter intensity on the same area for its period of concentration is also shown in Fig. 8 ("Case 2: Calculated True Graph") as plotted from Table 6(*b*). Note that the time of concentration of the entire area (with the light intensity) is approximately 1.4 times as long as that for the higher one.

Case 2(a).—If the rain ceases when the full area concentrates, the same process is used in computing the data as for Case 1(*a*). The difference in results is due solely to the use of a lower intensity. Table 8(*b*) and Fig. 10 are to be compared with Table 8(*a*) and Fig. 9, respectively, of Case 1(*a*). The hydrograph is plotted in Fig. 8 and is labeled "Case 2: Calculated True Graph".

Equation (12) replaces Equation (11) of Case 1(*a*) since substitution of the data of Table 6(*b*) in Equation (9) renders:

$$t = 0.08944\ Q^{\frac{1}{4}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (12)$$

Case 3.—One-Day Duration of Rain; All Other Assumptions Identical with Case 1.—The procedure used in the solution for this case differs sufficiently from that for Case 1 to warrant a brief explanation. Note (see

Table 6(*a*) and Fig. 7) that the area, ACB = 1 000 000 acres, concentrates to a maximum rate of run-off of 250 000 cu ft per sec at the end of the first day of rain. During this period one-fifth the water that fell on this area had passed the outlet and four-fifths was left on the water-shed at the end of the day—from Equations (9) and (10). Thus, the total rainfall on Area $ACB = Q\ t$ = 250 000 × 86 400 = 21 600 millions of cubic feet, of which 4 300 and 17 300 millions had passed the outlet and remained on the water-shed, respectively; but, the total rainfall on the entire water-shed during the 1-day period (see Table 6(*a*)) $= \left(\frac{A}{4}\right) t$ = 6 250 000 × 86 400 = 540 000 millions of cubic feet. Therefore, the quantity remaining on the entire water-shed at the end of the first day of rain = 540 000 − 4 320 = 535 680 millions of cubic feet.

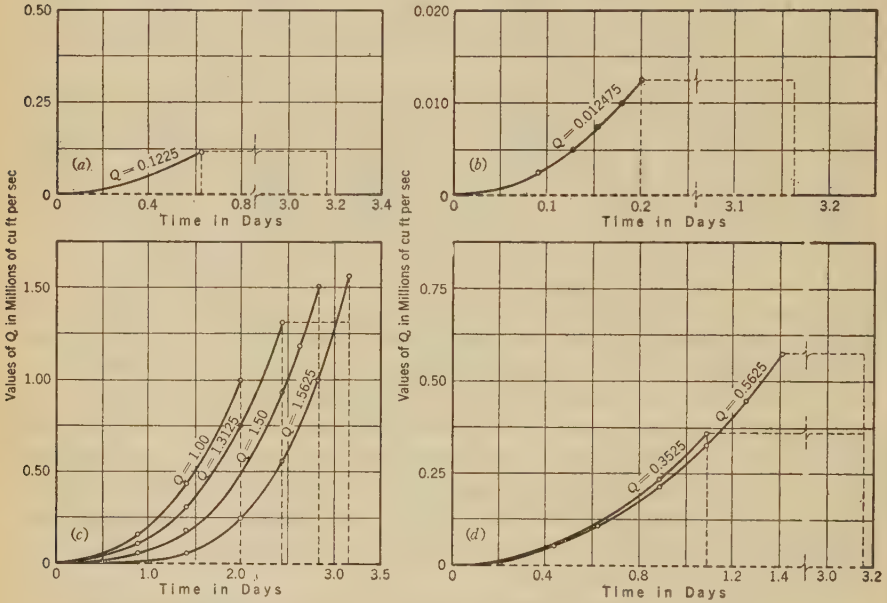


FIG. 10.—EXPLANATORY GRAPHS, CASE 2.

At the instant the rainfall ceases, the same rate of run-off, 250 000 cu ft per sec, will prevail at all points between L and P of Fig. 7. At this instant the maximum Q-values tributary to the time-contours, ACB, DFE, GIH, and JLK, are due to contributions from Areas $DACBEF$, $GDFEHI$, $JGIHKL$, and $MJLKNO$, respectively.

The quantity of water that has passed a given time-contour during the 1-day rain may be ascertained as illustrated by the example, which follows: Consider the time-contour, DFE (Fig. 7). Divide the tributary area (5 000 000 acres) into sub-contours, and determine the respective values of Q, Q', and t as illustrated under Case 1(*a*). As before, the time is found from Equation (11). These values are given in Table 7(*b*) and are plotted in Fig. 11(*a*) (see Curve *AB*).

In Fig. 11(*a*) the area, *ABC*, under Curve *AB*, represents the quantity of water having passed the time-contour, *DFE*, during the day's rain.

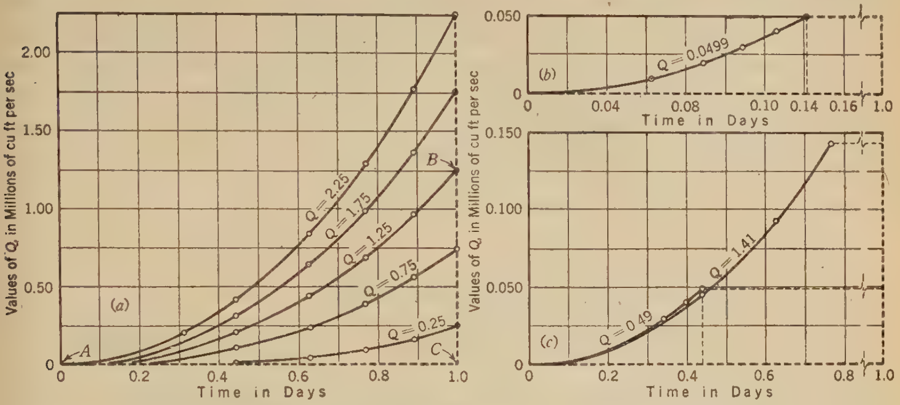


FIG. 11.—EXPLANATORY GRAPHS, CASE 3.

If this is deducted from the total rainfall on the water-shed above the contour, the result will be the quantity left on the water-shed at the end of the period.

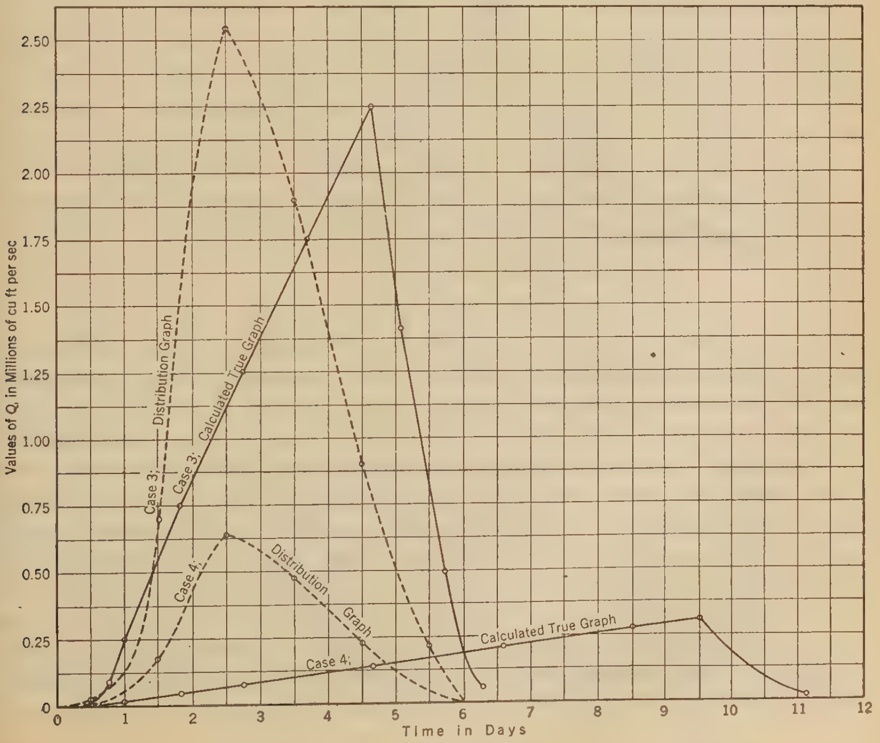


FIG. 12.—COMPARATIVE HYDROGRAPHS, CASE 3 AND CASE 4.

In Table 8(*c*) values are given for the total rainfalls on the area above, the quantities passed, and those left on the water-shed above, the various time-contours as represented by their respective maximum Q-values.

As before, a given quantity (Column (4), Table 8(*c*)) will still be on the shed when its corresponding Q-value (Column (1), Table 8(*c*)) arrives at the outlet. Hence, if a given quantity of Column (4) is deducted from the total quantity of water on the water-shed at the end of the rain (shown hereinbefore to be 535 700 millions of cubic feet), the result will be the quantity that has subsequently passed the outlet when the corresponding flow, Q (Column (1), Table 8(*c*)), arrives.

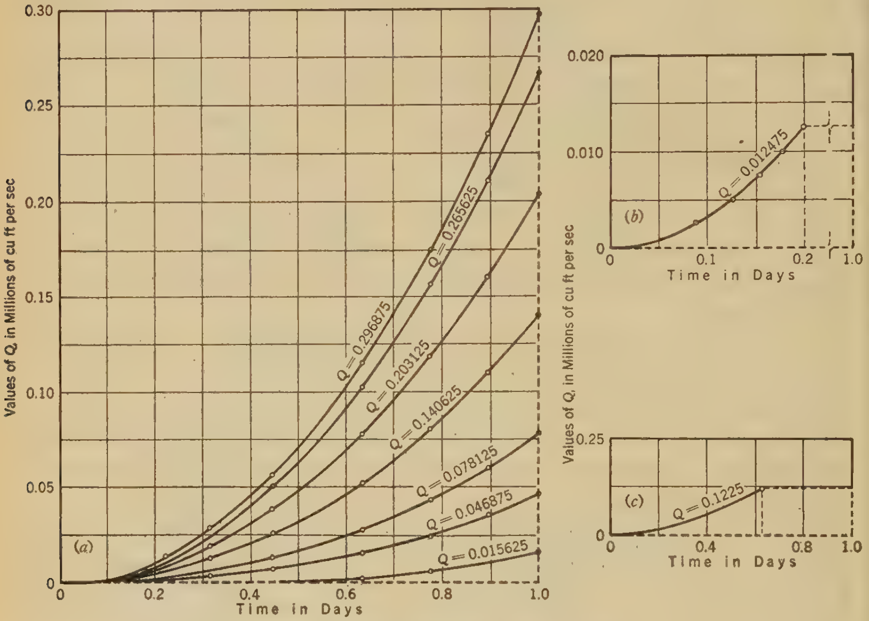


FIG. 13.—EXPLANATORY GRAPHS, CASE 4.

These quantities, with their corresponding Q-values, are plotted in Fig. 12 (see "Case 3: Calculated True Graph") which represents a true hydrograph for the assumptions of Case 3.

Case 4.—One-Day Duration of Rain; All Other Assumptions Identical with Case 2.—Following the procedure outlined for Case 3, Table 8(*d*) and Fig. 13 were constructed, due consideration being given to the effects of the lower intensity. These data are to be compared with Table 8(*c*) and Fig. 11 of Case 3, respectively. The hydrograph (see "Case 4: Calculated True Graph") platted in Fig. 12, is to be compared with that for Case 3, Fig. 12.

Comparisons.—The hydrograph (see "Calculated True Graph"), for Cases 3 and 4, may be compared directly through a study of Fig. 12. The two graphs differ radically both in length and in shape. They may be used to construct graphs following Mr. Sherman's procedure as outlined for

Fig. 1 of the "Unit-Graph Method"[23]. Marked differences in results will be noted, for the two cases, as to total length of graphs, times to maximum rate of run-off, and shapes of graphs. If these graphs, so constructed, are compared with the two hydrographs of Fig. 8 (see "Calculated True Graphs"), a more complete idea as to the magnitude of errors resulting from disregard of fundamentals may be gained. Conversions to the unit-graph method are omitted herein in order to conserve space.

The distribution graph method, as proposed by the author, is more pertinent to this discussion; and for the purpose of making comparisons, data for constructing a distribution graph were selected from the hydrograph of Fig 8 (see "Case 1: Calculated True Graph"). These data are shown in Table 9.

TABLE 9.—DISTRIBUTION GRAPH FOR CASE 1, FIG. 8

(Total Discharge = 1 207 500 Millions of Cubic Feet)

Day	Discharge, in millions of cubic feet	Approximate percentage of total discharge (distribution graph)	Day	Discharge, in millions of cubic feet	Approximate percentage of total discharge (distribution graph)
1	4 300	0.4	4	364 500	30.2
2	133 500	11.1	5	173 000	14.3
3	492 000	40.7	6	39 800	3.3

The distribution graph of Table 9 was then applied to Cases 1, 2, 3, and 4, using the data for rainfall assumed herein and following the author's method, as exemplified in his Table 3. The results are shown in Table 10 and the reproduced hydrographs are plotted in Figs. 8 and 12, as represented by the dashed curves. Little comment is necessary. Comparisons may be made directly with the calculated true hydrographs as platted on the same diagrams. In each case the variation is due solely to the complete disregard (which is inherent in the method) of the effects of a difference in intensity.

The variation incidental to the choice of the time-unit may be obviated by plotting the ordinates of the calculated true graphs in terms of the average flow, Q, per day. The resulting curves (plotted by the writers, but not published herein) still show marked variations from the distribution hydrographs.

Conclusion (a).—The water-shed selected, its size, shape, and manner of concentration, together with assumptions for intensity and characteristics, which rendered the foregoing variations, was chosen merely for the purpose of arriving at definite results for use in making comparisons. Another water-shed of any size and with other uniform intensities of the same ratio, but with the same shape, manner of concentration, and remaining assumptions, will render similar variations for comparison. Any given water-shed, similarly subjected to two different uniform intensities, will show results varying in the same general direction. Pondage, channel constrictions, and

TABLE 10.—APPLICATION OF DISTRIBUTION GRAPHS TO THE WATER-SHEDS FOR CASES 1, 2, 3, AND 4

Day No:	Daily rainfall, in inches	GRAPH DATA Day:* 1 2 3 4 5 6 Effective percentage:* 0.4 11.1 40.7 30.2 14.3 3.3				Effective rainfall depth, in inches	Computed flow, Q, in millions of cubic feet per second	Daily rainfall, in inches	Graph data	Effective rainfall depth, in inches	Computed flow, Q, in millions of cubic feet per second
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
			(a) CASE 1						*(c)* CASE 3		
1	6.000	0.024			.....	0.024	0.025	6.00	0.024	0.024	0.025
2	6.000	0.666	0.024		.....	0.690	0.720	.....	0.666	0.666	0.695
3	1.417	2.442	0.666	0.006	.....	3.114	3.250	.....	2.442	2.442	2.546
4	.....	1.812	2.442	0.157	.....	4.411	4.600	.....	1.812	1.812	1.890
5	.....	0.858	1.812	0.577	.....	3.247	3.390	.....	0.858	0.858	0.895
6	.....	0.198	0.858	0.428	.....	1.484	1.546	.....	0.198	0.198	0.204
7	.....	.....	0.198	0.202	.....	0.400	0.417	.....	.....	.....	....
8	.....	.....	.....	0.047	.....	0.047	0.049	.....	.....	.....	....
			(b) CASE 2						*(d)* CASE 4		
1	1.500	0.006				0.006	0.006	1.500	0.006	0.006	0.006
2	1.500	0.167	0.006			0.173	0.180	.....	0.167	0.167	0.174
3	1.500	0.611	0.167	0.006		0.784	0.816	.....	0.611	0.610	0.636
4	0.244	0.453	0.611	0.167	0.001	1.231	1.284	.....	0.453	0.453	0.472
5	.....	0.214	0.453	0.611	0.027	1.304	1.360	.....	0.214	0.214	0.225
6	.....	0.050	0.214	0.453	0.099	0.815	0.850	.....	0.050	0.050	0.052
7	.....	.....	0.050	0.214	0.074	0.338	0.352	.....	.....	.....	....
8	.....	.....	.....	0.050	0.035	0.085	0.089	.....	.....	.....	....
9	.....	.....	.....	.....	0.008	0.008	0.008	.....	.....	.....	....

* Derived from Fig. 8 (see, "Case 1: Calculated True Graph.")

the existence of water in the channels immediately before the rains would tend to "iron out" the peaks and generally to make the variations less marked. On the other hand, perviousness of the area, causing a greater difference in the ratio of resulting run-offs than that of applied intensities, would increase the variations above those obtained for the completely impervious area assumed for the foregoing cases.

Any method or system, proposed for devising hydrographs, should at least embody within its scope the opportunity for designers to use their judgment in allowing for the effects on run-off of all fundamental factors and relations.

Conclusion (b).—The procedure, outlined and followed in the foregoing computations for the calculated true graphs of Figs. 8 and 12, although utilized for special cases, for the purpose of obtaining results to be used in making comparisons with those of the author's method, may serve as a guide in the determination of hydrographs for points on streams where adequate flow records are not available. A few more comments may serve further to clarify the applicability of this procedure to any water-shed.

Usually, it will be desired in such cases to consider a rain that lasts as long as the time of concentration of the area. The time of concentration to the maximum rate of run-off and the resulting average intensity for the assumptions may be ascertained through use of the writers' formulas and methods.[24]

Any alterations of these times and intensities, due to pondage or other delays, will depend, of course, on the judgment of the designer, as guided by any information that he may possess. These formulas and methods will also be of service in dividing the water-shed into time-contours. The distribution of the water left on the shed at the end of the rain (being the total estimated run-off less the area under the hydrograph from the beginning to the end of the rain) may be determined through the procedure outlined under Case 1.

The ordinary water-shed, of course, is irregular in shape and manner of concentration, as are also the time-contours by which it may be divided. Instead of a simple equation (as Equation (11), for Case 1) for use in conjunction with time-rendering areas in determining the quantities that have passed a given time-contour, resort should be made to the special tables[30] of the factor P, previously introduced by the writers. Since $P = \frac{v}{V}$, and since the value of V can be computed for the known quantity of water and average slope, the average velocity, v, is readily obtained. The known length divided by v will render the required time.

The detail required for a given problem and any abridgment of the method may well be left to the judgment of the engineer.

Gordon R. Williams,[31] Jun. Am. Soc. C. E. (by letter).—The unit-graph method has interesting possibilities in the determination of flood flows. The author considers the feasibility of establishing magnitude-frequency relations through the use of an equation for rainfall intensities and the pluviagraph, thereby assuming a storm coefficient of unity. In regions where melting snow is a component of stream flow the assumption of a storm coefficient of unity or greater than unity is reasonable, but there are many places where such an assumption would not be in accord with hydrologic experience. In such places it might be possible, by the author's methods, to determine the frequency with which storm coefficients approach maximum values. The next question is to determine the frequency of flood-producing storms. This necessitates a consideration of the intensities of rainfall, the areas covered by rains of various intensities, the positions of the storm centers, and the directions of storm movement. If the task of combining these factors so that they can be properly expressed in relation to frequency could be accomplished, there still would remain the problem of combining this result with the frequency of maximum storm coefficients in order to attain the ultimate goal, which is the frequency of flood discharges of various magnitudes.

The unit-graph method appears to be applicable to the determination of rare floods that can be expected from moderate-sized drainage areas. The author states that the method has been applied for other purposes to drainage areas approximating 6 000 sq miles. The determination of the probable maximum flood is a preliminary step in deciding upon suitable designs for most hydraulic structures. It would be of value in almost any method for analyzing floods, even in purely theoretical procedures, such as that presented by Professor J. J. Slade, Jr.[32]

---

[30] *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 1048, Tables 3 and 4.

[31] Asst. Engr., Water Resources Branch, U. S. Geological Survey, Washington, D. C.

[32] *Proceedings*, Am. Soc. C. E., October, 1934, p. 1118.

Flood studies bring out the fact that the smaller the drainage area the greater the potentialities for producing high yields per unit area. Small drainage areas usually have the important requisites for producing floods; that is, they can be entirely surrounded by areas having higher rates of rainfall, they have steep slopes, and they lack the essentials for natural flood detention. Many of the most severe floods are those produced by so-called "cloudburst" storms. The periodicity of such floods may be very great when expressed in years, so great, in fact, that a study by statistical methods[33] of what might be considered a long record in the United States would not indicate that such a flood is likely to occur. Examples of these rare floods occurred on the Miami River, in Ohio, in 1913,[34] on the Arkansas River, at Pueblo, in 1921,[35] and on many rivers in Vermont in 1927.[36] The discharges in these floods were two or three times greater than would be expected from a study of a magnitude-frequency relation. For such floods, statistical methods appear to be inadequate. The unit-graph method, on the other hand, may make it possible to arrive at an approximation to the limiting flood through the application of high rainfall rates, together with high, but altogether possible, storm coefficients, and the placing of a hypothetical storm in a critical position.

On large drainage areas, such as that of the Ohio River or the Tennessee River, where the application of the unit-graph method might encounter difficulties, the statistical method seems to offer a partial solution, provided a long record is available. On such drainage areas the differences in percentage between the leading floods in the record are usually small in comparison to those from a small drainage area, thus making the periodicity a relatively less important factor in relation to magnitude. A frequency curve derived from perhaps a 40-yr to 50-yr record of a large drainage area might be extended to a periodicity of materially greater length, with the assurance that an approximation to the maximum flood had been reached.

W. G. Hoyt,[37] M. Am. Soc. C. E., and L. L. Harrold,[38] and F. F. Snyder,[39] Juniors, Am. Soc. C. E. (by letter).—In connection with general studies of relations between rainfall and run-off being made by the U. S. Geological Survey, in collaboration with the Water Planning Committee of the National Resources Board, the writers have had an opportunity to investigate the distribution graph devised by Mr. Bernard based on the unit graph set forth by Leroy K. Sherman,[40] M. Am. Soc. C. E., to study relations between precipitation and surface run-off. Although the studies have not progressed far enough to warrant definite conclusions, it is believed that a brief summary of some of the problems encountered will be of value to hydrologists and engineers who may be interested in water utilization. It should be recognized that the distribution graph described by Mr. Bernard effects the same purpose as Mr. Sherman's unit graph, and that, in theory at least, both relate wholly to direct surface run-off, because in the development of a unit graph all flow occasioned by precipitation previous to the storm under consideration is segregated from the flow resulting from the 1-day storm. The accuracy with which a unit graph and a distribution graph can be developed, therefore, depends upon the accuracy with which the stream flow resulting from antecedent rainfall can.be determined—a problem with which many hydrologists and hydraulic engineers have been concerned.

---

[33] See, "Flood Flows", by the late Allen Hazen, M. Am. Soc. C. E., John Wiley & Sons, Inc., 1930; "Theoretical Frequency Curves and Their Application to Engineering Problems", by H. Alden Foster, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 87 (1924) p. 142; and "Flow in California Streams", *Bulletin No. 5* (1923), California Dept. of Public Works, Div. of Eng. and Irrig.

[34] "The Ohio Valley Flood of March-April, 1913", by A. H. Horton, Assoc. M. Am. Soc. C. E., and H. J. Jackson, U. S. Geological Survey, *Water Supply Paper 334* (1913).

[35] "The Arkansas River Flood of June 3–5, 1921", by Robert Follansbee, M. Am. Soc. C. E., and Edward E. Jones, U. S. Geological Survey, *Water Supply Paper 487* (1922).

[36] "The New England Flood of November, 1927", by H. B. Kinnison, Assoc. M. Am. Soc. C. E., U. S. Geological Survey, *Water Supply Paper 636 C.*

[37] Hydr. Engr. (Prin.) Conservation Branch, U. S. Geological Survey, Washington, D. C.

[38] Asst. Hydr. Engr., U. S. Geological Survey, Washington, D. C.

[39] Junior Engr., U. S. Geological Survey, Washington, D. C.

[40] "Stream Flow from Rainfall by Unit-Graph Method," *Engineering News-Record*, April 7, 1932, p. 50.

By definition and example, Mr. Bernard develops a so-called "recession curve" by combining the recessions of the hydrograph of several storm periods followed by intervals of no rainfall, which he states "gives a dependable idea of how the base flow would have fallen and the flood flow would have receded had no subsequent rainfall occurred," and he uses this curve to determine the so-called "base flow". It would seem that this method properly excludes surface run-off resulting from antecedent rainfall, but does not take into account the facts that base flow—or, as otherwise defined, ground-water flow, or seepage flow—is a variable quantity and that recession curves based on hydrographs of stream flow, unless made up of segments of the hydrograph representing periods when no surface run-off is in the stream, may not be comparable with depletion curves of ground-water flow. For example, in drawing the base-flow curve for the Muskingum River at Dresden, Ohio, Mr. Bernard assumes that the base flow would have fallen from 1 500 to 700 cu ft per sec in 11 davs during March, 1929, and similarly during the two storms in July, 1925, and in July and August, 1927, that were used to determine the average-distribution graph. This use of a fixed "recession curve" does not take account of the fact that during March the water-table in the Muskingum Basin may be high enough to maintain a base flow between 2 000 and 5 000 cu ft per sec, whereas, in July, it may be only high enough to maintain a flow of 1 000 cu ft per sec, or less. Use of a fixed "recession curve" in the determination of the unit graph seems to rely on the accuracy of an average curve and to ignore departures therefrom due to variations in gound-water levels and ground-water flow and to recharge from the storm under consideration. If the storm producing the surface run-off occurs at a time when the ground-water flow corresponds to the lower end of the fixed recession curve, the error introduced may be negligible, but if the storm producing the surface run-off occurs when the ground-water is high, the error may be very appreciable. In each case, of course, the error depends on the magnitude of the storm run-off in relation to the ground-water flow. Different methods[41] have been used to segregate surface run-off from ground-water run-off, but apparently none of them is subject to exact analysis, and care must be taken that the method used eliminates, and does not include, any ground-water flow in the determination of the unit graph.

---

[41] R. E. Horton, M. Am. Soc. C. E., in his discussions of "The Rôle of Infiltration in the Hydrologic Cycle", pub. in the *Transactions* of the Am. Geophysical Union, National Research Council, of the National Academy of Science, June, 1933, p. 446, outlines methods that have been used to separate surface flow from ground-water flow and gives valuable references.

If the fundamental requirements are observed the writers have found, as Mr. Bernard did, that the theory of the unit graph or distribution graph seems to apply to a remarkable degree in actual practice. In their derivation, however, the following factors must receive due consideration:

1.—In deriving unit graphs it is necessary to select storms that conform as closely as possible to the ideal, which is rainfall of uniform intensity and 24-hr duration. Since such storms do not occur in Nature over any extended area, the peaks of unit graphs of 1-day storms vary somewhat, depending on whether the storm center is near or remote from the gauging station. Conversely, in the application of an average unit graph to actual precipitation, the results will differ from the recorded data to the extent that the storm pattern differs from the average condition used in determining the unit graph.

2.—The published records of the U. S. Weather Bureau do not indicate whether the recorded precipitation on any one day occurred during 1 hr, or was well distributed over the 24 hr. In a similar manner an ideal 24-hr storm may be recorded in equal amounts on two consecutive days. Unit graphs for storms lasting 1 hr may not be comparable with those for storms lasting 24 hr, and in the application of the determined unit graph the resultant pluviagraph will differ from the recorded flow to the extent that the unit graph of storms of less than 24-hr duration differs from that of storms of 24-hr duration. Published records of precipitation give the total for the day, and published discharge records give the average flow for the day. These limitations in the details of recording basic data make it impossible to determine comparable unit graphs from small basins, but in the opinion of the writers they do not disprove the essential theory of the unit graph. These limitations also make it difficult to combine or average the separate unit graphs for the same basin. For example, a unit graph of one storm may show no rise on the day the precipitation was recorded, while that of another storm may show an appreciable rise, a difference which may result from inadequacies in the refinement of recording, or from causes not now definitely determined, such as varying initial rates of infiltration and interception, or water required and held in initial surface wetting.

3.—Theoretically, it would seem that, as a result of differences in channel velocities, the unit graph of a 1-day storm occurring when the river is at a high stage would differ from that of a storm occurring when the river is at a low stage.

4.—Unit graphs of 1-day storms apparently cannot be obtained in basins where the infiltration capacity exceeds the rainfall rate, or where the direct surface run-off is substantially delayed either by artificial or by natural storage. To a moderate extent both these conditions occur in many basins, and the Rock River above Afton, Wis., is herewith discussed as an example of the difficulties which may be encountered in the preparation of distribution graphs of surface run-off resulting from 1-day storms. The drainage area is 3 190 sq miles. Lakes and marshes occupy between 10 and 15% of the area; channel slopes are small, and the soil is largely sandy loam. These conditions give rise to a large ground-water and sustained flow and a relatively small quantity of direct surface run-off. An examination of the daily hydrograph of the Rock River, at Afton, for the period, October, 1914, to September, 1933, failed to disclose an instance in which the rise in the hydrograph could be associated with a 1-day storm. All pronounced rises in the hydrograph were the result of rainy periods of from 4 to 10 days' duration. Although in this particular instance it might be possible to develop distribution graphs based on 5-day storms, they would not seem applicable to any 1-day storm.

Notwithstanding these apparent limitations, distribution graphs having the same general pattern, have been obtained by the writers from basins ranging in area from 1 000 to 40 000 sq miles. Although the distribution graphs follow the same general pattern, the variations between peak magnitudes are very appreciable. These variations may be a source of inconsistencies in the application of an average distribution graph to hydraulic problems.

After developing an average unit graph or distribution graph, Mr. Bernard transposes the total weighted daily precipitation on the basin into so-called "pluviagraph values" and, in his examples, compares these values with total run-off by means of "retention coefficients" obtained by dividing the total observed run-off for the several days making up the storm period by the total pluviagraph value or by distributed rainfall for the same period.

Two questions have puzzled the writers in regard to the pluviagraph: First, is it fundamentally logical to distribute total rainfall by means of a distribution graph based on direct surface run-off (rainfall minus infiltration and storage); and, second, is it logical to compare rainfall thus distributed with total recorded flow? In connection with the general studies previously mentioned, the writers found that the average annual direct surface run-off in all major sub-basins of the Mississippi River, above Keokuk, Iowa, ranged from about 2% of the precipitation for the Minnesota River Basin to about 26% of that for the Black River Basin. In other words, by the application of Mr. Bernard's method to the Minnesota River Basin, 98% of the pluviagraph would be built up of precipitation that did not appear as direct surface run-off. In the application to the Black River Basin, which apparently has the maximum surface run-off of all the larger basins in the Upper Mississippi River drainage area, use of Mr. Bernard's procedure would distribute about 74% of the precipitation that did not appear as surface run-off.

Of the precipitation that did not appear as direct surface run-off, about 1% in the Minnesota River Basin and about 6% in the Black River Basin eventually appeared as sustained or ground-water flow. It seems illogical to the writers to include in a pluviagraph precipitation which, even if it does eventually appear as stream flow, arrives there through the ground and manifestly follows laws different from those controlling direct surface run-off as reflected in the unit graph.

For isolated storms, Mr. Bernard's "retention coefficient" is the simple relation between the total storm precipitation and the total run-off. For the Tuscarawas River, at Newcomerstown, Ohio, which he has used as an example, it is the relation between the total pluviagraph value and the total recorded run-off for the period between consecutive troughs on the hydrograph.

More and more hydrologists are looking upon stream flow as made up of two distinct parts, one of which results from direct surface run-off and the other from that part of the precipitation which is either retarded on the surface, or which passes through the ground and eventually reaches the stream channel. For the direct surface run-off the time between the occurrence of the precipitation and its appearance at the gauging station is relatively short, as shown by the unit graph. For the sustained or ground-water flow the time is much longer and can probably be measured in terms of months rather than of days. For example, the late J. F. Hayford,[42] M. Am. Soc. C. E., found, by strict mathematical analysis, that in the Wagonwheel Gap area, in Colorado, the flow on any one day depended, in part at least, upon the meteorologic conditions existing during the preceding 257 days. To the extent that the total flow is made up of ground-water flow dependent on conditions antecedent to the time covered by the distribution graph, relations between total pluviagraph records and total run-off seem valueless. If ground-water is deducted, however, comparisons between total pluviagraph records and observed stream flow less ground-water flow may have value. If they can be carried back through the distribution graph, these relations might show, to the extent that the surface run-off followed the average distribution graph, the amount of infiltration day by day. Furthermore, it may be possible to correlate the relations with past and present conditions to an extent sufficient at least to make use of total pluviagraph records in determining surface run-off from storms of great magnitude.

The writers are studying both methods of approach, and the results, to date (1935), seem to indicate, as Mr. Sherman and others have found, that antecedent conditions, as well as amount and intensity of the particular rainfall considered, play an important part. For example, under similar storm conditions, low rates of infiltration with resulting high run-off relations, will apply on Ohio streams during normal spring periods, when soil moisture and ground-water levels are at a maximum, and high rates of infiltration with resulting low run-off relations will apply during normal Septembers, when conditions are reversed. However, if conditions are abnormal and moisture conditions in the spring are comparable to normal fall conditions, the infiltration and run-off relations for the two periods tend to be comparable. In so far as total surface run-off throughout the year is concerned, the applicability of the unit-graph method will depend upon the accuracy with which relations between several factors can be determined.

---

[42] "A New Method of Estimating Stream Flow" by the late J. F. Hayford, M. Am Soc. C. E., and J. A. Folse, *Publication 400*, Carnegie Inst., Washington, D. C.

With respect to the value of the "retention coefficient" in connection with flood flows, a somewhat different condition exists. Floods result when infiltration is at a minimum, with corresponding high run-off relations. If there is no snow on the ground the pluviagraph record should represent, within reasonable limits, the maximum surface run-off possible with zero infiltration. It is believed that studies and experiments will eventually disclose infiltration limits that may be reached and thus make possible the determination of limiting run-off rates which, when used with Mr. Bernard's pluviagraph, will make it possible to determine flood run-off in a rational manner.

The author is to be commended for clarifying the basic principles of the unit graph, for his development of the distribution graph, and for his conception of the pluviagraph. Although it is far from an exact process for computing stream flow, the writers have found that it yields reasonably comparable distribution graphs, and that when net rainfall, as used by Mr. Sherman, or total precipitation, as used by Mr. Bernard, was distributed by means of an average unit graph or distribution graph, the peaks of computed and observed surface run-off occurred on the same day or, if not, rarely more than one day apart. The concepts developed are of material assistance in the analysis of run-off in relation to its causes, thus promoting a better understanding of the occurrence of trends and the possible affects of storage, land drainage, agriculture, and other factors that may be of importance in considering the wise utilization of water resources. They furnish a tool for analysis of rainfall and run-off that may be of practical value where detailed knowledge of hydrology is important, as in (*a*) manipulation of storage on large systems of river development for power and water supply; (*b*) obtaining definite knowledge of run-off characteristics of urban areas, where special conditions make such knowledge of exceptional value; and (*c*), analysis of potentialities of drainage basins for producing floods.

MERRILL M. BERNARD,[43] M. AM. SOC. C. E. (by letter).—In 1934 and 1935, the writer was associated with a group in the U. S. Geological Survey which has been engaged on a study of floods and flood frequencies, and rainfall, run-off, and inter-related factors. A large part of his time was devoted to the application of the unit-hydrograph method to run-off resulting from storms of high magnitude. Underlying principles and methods have been studied by other members of the group, as outlined in the discussion of Messrs. Hoyt, Harrold, and Snyder. In the preparation of his closing discussion, the writer has drawn on his own additional experience as well as on the ideas and studies of his associates.

---

[43] Cons. Civ. Engr., Crowley, La.

The simplified form of Equation (1), suggested by Mr. Jarvis in Equation (5), should be accceptable, in general, particularly in cases where channel characteristics are not known. The factor, U, is purely an abstract one, having its principal use in the construction of regional charts, as shown in Fig. 2.

The suggestion that the pluviagraph be used as a guide to the shape of the hydrograph has possibilities. Recent studies indicate that, where only definite flood hydrographs are considered, the ratio, $\frac{\Sigma \text{ hydrograph}}{\Sigma \text{ pluviagraph}}$, for any basin follows a rather well-defined trend throughout the seasons. Such a trend curve can be developed from a comparatively short record of flow, and would be a more dependable guide to run-off coefficients for a particular locality within the Mississippi Basin than the range of from 14 to 31%, suggested by Mr. Jarvis.

The writer is in accord with Mr. Sherman on the points covered by his discussion. Particularly does he join him in concurring with Mr. Horton and others in the belief that infiltration loss should be considered as a deduction from daily rainfall. It has been the writer's experience, however, that average or gradually decreasing infiltration deductions throughout periods of continuous or intermittent rainfall do not produce net rainfalls that can be converted directly into the stream hydrograph with the use of the distribution graph. For storm periods consisting of two or three days of appreciable rainfall, preceded by several days of light rainfall, all such light rainfall, together with the greater part of the first day of heavy rainfall, may have been consumed by infiltration, whereas the following days of heavy rainfall largely account for the resulting stream flow. It is also believed that the rainfall-recording interval of 24 hr obscures comparatively long periods between actual rainfalls which have been recorded as continuous, making the practical correlation of a daily infiltration deduction with antecedent conditions difficult.

The phenomenon of infiltration, which is the principal loss to rainfall, is so unlike that of surface run-off, that the two should not be directly combined to give the total loss. Infiltration activity is more logically associated with that of rainfall. Like rainfall, it can be applied to a unit of area and is practically accomplished within the duration of rainfall, whereas run-off, very shortly after it is created, is to be associated with surface and channel storage, and stream flow. In the case of a unit-time rainfall, producing a unit hydrograph, surface run-off has reached the stream system within that period, to a great extent, thus becoming identified with storage. With the exception of such minor losses as channel seepage and surface evaporation, withdrawals of water from the basin, as measured stream flow, constitute the changes in storage. The equation of these factors, including storage, could be stated as follows:

$$\underset{\text{(Daily rainfall)}}{\text{Inflow}} - \underset{\text{(Infiltration)}}{\text{Outflow}} = \text{Change in Storage} + \underset{\text{(Stream flow)}}{\text{Outflow}}$$

It would seem, then, that to preserve the rationality of the unit-hydrograph theory, the distribution of run-off or stream flow throughout the run-off period should be made by applying a distribution graph to rainfall excess, or average daily rainfall minus a depth lost through infiltration.

The pluviagraph is the hydrograph of 100% run-off; but it is not the graph of run-off plus infiltration in that such a definition would imply the distribution of infiltration loss throughout the period of run-off in proportion to run-off. The writer has found the pluviagraph useful as a limiting or enveloping hydrograph from which the hydrograph of a particular flood resulting from a continuous storm period can be reproduced by the application of two related coefficients. First, it must be recognized that the pluviagraph of a unit-time rainfall is nothing more than the mechanical distribution of such rainfall throughout the period of surface run-off in proportions fixed by the distribution graph, establishing the proportionality between the ordinates of the pluviagraph and those of the resulting unit hydrograph. The relation between the unit pluviagraph and the unit hydrograph is linear, therefore, making the cumulative plot of distributed rainfall (pluviagraph) and stream flow from surface run-off (unit hydrograph) a straight line, the slope of which

TABLE 11.—Relation Between Factors Involved in Development of Unit Graph and Distribution Graph, Susquehanna River, at Towanda, Pa. (Area, 7 770 Square Miles)

Date: September and October, 1924	Average daily rainfall, in inches	Flow, in Cubic Feet per Second			Distribution graph (percentages)	Pluviagraph, P		Flood coefficient, $C = \frac{Q_s}{P}$
		Observed stream flow, Q_o	Estimated base flow, Q_b	From surface run-off, Q_s		In inches	In cubic feet per second	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
29.........	....	1 780	1 780	0	0	0	0	.....
30.........	3.77	45 300	2 500	42 800	14.0	0.527	110 200	0.386
1.........	....	110 000	3 500	106 500	34.9	1.315	275 000	0.386
2.........	....	81 200	4 500	76 700	25.1	0.946	197 800	0.386
3.........	....	51 200	6 000	45 200	14.8	0.557	116 500	0.386
4.........	....	26 400	7 500	18 900	6.3	0.241	50 200	0.386
5.........	....	17 100	8 500	8 600	2.9	0.108	22 600	0.386
6.........	....	13 500	9 000	4 500	1.5	0.057	11 900	0.386
7.........	....	10 600	9 200	1 400	0.5	0.019	3 970	0.386
8.........	....	9 060	9 060	0	0	0	0	.....

Date: September and October, 1924	Inflow (Rainfall), I, —		Outflow (Infiltration), F, =		Storage, S, +		Outflow (Stream Flow) Q_s
	In inches	In cubic feet per second	In inches	In cubic feet per second	Cumulative, in cubic feet per second	Change, cubic feet per second	In cubic feet per second
(1)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
29.........	....	....	....	....	....	....	....
30.........	3.77	787 900	2.32	483 300	261 800	+261 800	42 800
1.........	....	0	....	0	155 300	—106 500	106 500
2.........	....	0	....	0	78 600	— 76 700	76 700
3.........	....	0	....	0	33 400	— 45 200	45 200
4.........	....	0	....	0	14 500	— 18 900	18 900
5.........	....	0	....	0	5 900	— 8 600	8 600
6.........	....	0	....	0	1 400	— 4 500	4 500
7.........	....	0	....	0	0	— 1 400	1 400
8.........	....	....	....	....	....	....	....

can be called a storm or flood coefficient. The application of such a coefficient to the pluviagraph of a day's rainfall is the same, in effect, as the distribution of the rainfall excess which remains after the deduction of infiltration from the day's rainfall, and which will produce the unit hydrograph direct.

The relation of the factors involved in the development of the unit graph and the distribution graph is given in Table 11, which presents as an example the effect of a completely isolated storm of September 29 and 30, 1924, on the Susquehanna River Basin above Towanda, Pa. This storm, although recorded in 2 days, is believed to have been confined to a continuous period approximating 24 hr. The factors of this unit hydrograph are shown in relationship in Fig. 14, together with the cumulative curves of rainfall, infiltration, run-off, and storage.

With less accuracy, but within practical limits, the pluviagraph of a multiple-day storm may be reduced to the approximate flood hydrograph by the application of a compound linear coefficient, as demonstrated with the typical storm of May 21 to 25, 1919, on the same basin, as reported in Table 12. The cumulative plot of the pluviagraph against that of surface run-off takes a

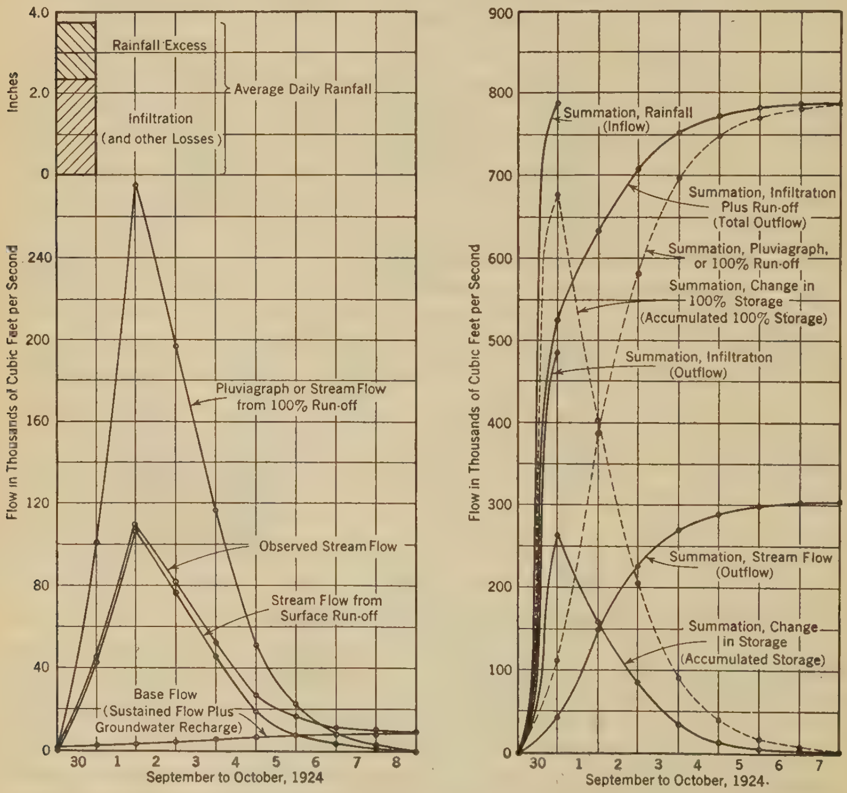


FIG. 14.—RELATIONS OF RAINFALL, INFILTRATION, STORAGE, AND STREAM FLOW, SUSQUEHANNA RIVER, AT TOWANDA, PA.

flatter slope for the first two days than that for the remainder of the run-off period, indicating the greater rate of loss through infiltration, which is to be expected. The accuracy with which these results can be obtained will depend largely on how near the prevailing rainfall, surface, and ground-water conditions are to those represented by the average distribution graph used.

TABLE 12.—DEVELOPMENT OF FLOOD COEFFICIENTS FOR A FIVE-DAY STORM PERIOD; SUSQUEHANNA RIVER AT TOWANDA, PA.

Date: May, 1919	Average daily rainfall, I, in inches	Average distribution graph (percentages)	PLUVIAGRAPH, P*		FLOW, IN CUBIC FEET PER SECOND			Flood coefficient, C	Computed flow, Q_c, in cubic feet per second
			In inches	In cubic feet per second	Observed stream flow, Q_o	Estimated base flow, Q_b	Flow from surface run-off, Q_s		
21......	0.41	16	0.143	29 900	14 400	6 800	7 600	0.28	8 400
22......	0.48	37	0.272	26 800	23 900	7 000	16 900	0.28	16 000
23......	0.56	23	0.386	80 600	54 300	7 000	47 300	0.65	52 400
24......	0.27	12	0.422	88 300	70 000	7 000	63 000	0.65	57 400
25......	0.14	6	0.338	70 600	49 000	6 900	42 100	0.65	45 900
....	....	3	0.225	47 000	37 500	6 800	30 700	0.65	30 600
....	....	2	0.125	26 100	26 400	6 700	19 700	0.65	17 000
....	....	1	.....	......	......	.....	......	....	......

* Includes the distribution of effective antecedent rainfall.

The writer made no attempt to correlate a run-off coefficient or deductive infiltration factor with antecedent conditions, except to comment on the apparent seasonal trend in the value of his "retention coefficient," which he now more properly refers to as a "flood coefficient." Subsequent experience discourages the hope that such correlations are simple. The method of estimating daily rainfall excess on a percentage basis, proposed by Mr. Sherman[2] in his original paper, has not, as far as the writer knows, been improved upon.

Mr. Horner is correct in his criticism of the frequency graphs in Fig. 5. These might be regarded as "possibility" or "limiting" graphs, accepting the assumption that it is physically possible for a flood coefficient equal to unity and the 100-yr storm to occur in the same season. More recent studies in which the unit-hydrograph method has been applied to limiting storms indicates its usefulness in this field.

As stated by Messrs. Eiffert and Bennett, time should not be lost in improving, in several ways, methods of gathering and recording climatological and hydrological data by the various governmental agencies. This is no criticism of the bureaus themselves, for they have "spread" their appropriations remarkably well, but rather is it a challenge to all interested engineers to co-operate more fully in the improvement of this service.

The discussion of Messrs. Gregory and Arnold, the writer believes, fails to make its point and does not disprove the theory of the unit hydrograph by proving that rainfall, run-off, and stream flow relate themselves in another manner. In the first place they have deviated from the principle of the unit-hydrograph method in developing their distribution graph from the 3-day storm of Case 1, thereby assuming a time unit of 3 days, after which they apply the resulting distribution graph to 1-day storms. Although the use of a distribution graph developed from their 1-day storm would not have brought the two methods into accord, the absence of any mention of this deviation from the principle of the unit hydrograph would confuse a reader hopelessly who, not being particularly conversant with the unit-hydrograph theory, would attempt to associate the discussion by Messrs. Gregory and Arnold with the writer's paper.

There is only one answer to the discussion by Messrs. Gregory and Arnold. The conclusions, as determined from their hypothetical example, have not been found in Nature. It is possible that their assumptions, among which are uniform rainfall intensity over a 3-day period, no losses to run-off, and no sustained or base flow, may contribute to the abnormal results obtained.

A recent study in which the writer has been engaged, has developed distribution graphs on about fifteen or twenty basins, ranging in area from 500 to 60 000 sq miles. The one nearest in size to the hypothetical problem given by Messrs. Gregory and Arnold (the Red River, at Denison, Tex., of 38 000 sq miles) is selected for the purpose of making the comparison between the natural phenomenon and that visualized by these discussers.

They have presented four conditions of rainfall: Case 1, a heavy rainfall for a duration of 3 days; Case 2, a light rainfall for a duration of 3 days; Case 3, a heavy rainfall for a duration of 1 day; and Case 4, a light rainfall for a duration of 1 day. From their analysis the conclusion is reached that the base of the unit hydrograph and the position of its peak will vary between wide limits, caused principally by the great variation through which average velocity passes in going from low to flood stage. The following list gives their results in terms of "lag" (this term being defined as the time interval between the center of mass of rainfall and the peak of the flood):

Case	Lag-days	Case	Lag-days
1..........	0.53	3..........	4.20
2..........	1.10	4..........	9.00

Under natural conditions it is found that the lag and hydrograph base for 1-day storm periods are approximately the same regardless of rainfall intensity. Furthermore, for continuous rainfall the lag does not change, regardless of intensity or deviations from the average. Fig. 15 shows, for the same basin, the lag of 3 days for rainfalls of various intensities for durations of 3, 4, and 5 days.

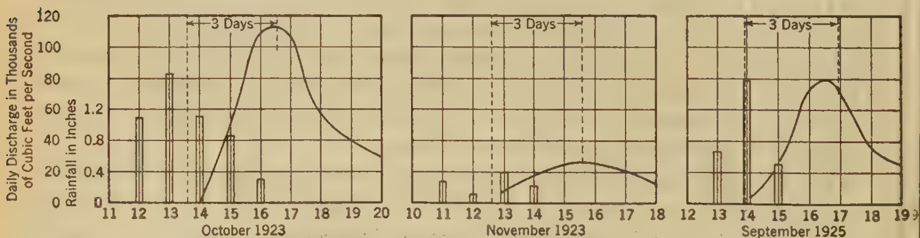


FIG. 15.—LAG OF PEAK RUN-OFF BEHIND CENTER OF MASS OF RAINFALL FOR THREE, FOUR, AND FIVE-DAY STORM PERIODS, RED RIVER, AT DENISON, TEX.

Table 13 shows the relations between rainfall intensity and lag, to the nearest day, for a group of unit hydrographs for the Red River at Denison, Tex.

TABLE 13.—RELATION BETWEEN RAINFALL INTENSITY AND LAG

Date of rainfall (1)	Rainfall, in inches (2)	Date of peak (3)	Lag, in days (4)
December 10 and 11, 1923	1.31	December 13	3
March 20 and 21, 1926	0.94	March 25	4
March 27 and 28, 1929	2.58	March 31	3
June 14 and 15, 1930	0.78	June 18	3
December 4 and 5, 1930	1.54	December 7	3
January 4, 1932	1.50	January 7	3
April 18, 1932	0.48	April 20 and 21	3
August 31 and September 1, 1932	1.41	September 5	4

Comparable results obtained by W. W. Horner, M. Am. Soc. C. E., and F. L. Flynt, Assoc. M. Am. Soc. C. E.,[44] also substantiate the unit-hydrograph theory and disprove the contentions of Messrs Gregory and Arnold.

Mr. Williams directs attention to the use of the unit-hydrograph method as the means of determining the limiting flood on any basin. Such procedure is possible in any case where a record of flow exists sufficiently long to yield the unit hydrographs (preferably not less than three) necessary to the development of the distribution graph for the basin and a sufficient number of well-defined flood periods to indicate the probable limiting value of the flood coefficient. With the distribution graph and the limiting coefficient at hand, outstanding storms that have visited the region may be superimposed upon the basin in a critical position and the resulting flood hydrograph obtained.

The studies of the unit-hydrograph method, referred to in the discussion of Messrs. Hoyt, Harrold, and Snyder, have contributed to the further development and refinement of the method. They have greatly improved the technique of constructing the distribution graph and have shown that seasonal changes in ground-water flow may seriously affect results, in many cases, if not taken into account.

In conclusion the writer expresses the hope that his paper, together with the discussions, is a step in the approach to a fuller and more rational conception of the relation of rainfall to run-off and stream flow. The unit hydrograph, as now understood, relates exclusively to surface run-off, a point to be kept in mind in any application of the method to hydraulic problems. The writer admits readily that such devices as the distribution graph, the pluviagraph, and the flood coefficient are not scientifically precise in all respects. They are tools of practical value, however, the complete explanation of their various functions being bound up in the further development of the unit-hydrograph method.

---

[44] "Relation between Rainfall and Run-Off from Small Urban Areas," *Proceedings*, Am. Soc. C. E., October, 1934, p. 1135.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 1899

DISCHARGE FORMULA AND TABLES FOR SHARP-CRESTED SUPPRESSED WEIRS

BY C. G. CLINE,[1] ESQ.

WITH DISCUSSION BY MESSRS. JASPER O. DRAFFIN, W. BRUCE MCMILLAN, AND C. G. CLINE.

SYNOPSIS

The new discharge formula presented in this paper applies to sharp-crested weirs that occupy the full width of the channel; that is, weirs with the end contractions suppressed. This formula is entirely empirical and was developed by a mathematical study of the results of a series of weir experiments[2] made at Cornell University by E. W. Schoder, M. Am. Soc. C. E., and the late Kenneth B. Turner, Assistant Professor of Hydraulics. It differs from those in use at present in that a variable exponent, increasing with the head, replaces the customary fixed exponent (1.5), thus making it possible to obtain accurate discharges, even at low heads, by a comparatively simple form of coefficient. Furthermore, correction for the velocity of approach, which has been developed, depends entirely on the physical dimensions of the weir and can be applied directly.

Curves in this paper offer a comparison between the results of the Schoder and Turner experiments in comparison with the new formula and with the three standard formulas of Francis, Bazin, and Rehbock. These three standard formulas differ from one another, but each is supported by the experimental data over certain ranges of head and height of weir; however, over the entire range of head and height of weir covered by the experiments, the new formula agrees with the data more closely than any of these standard formulas. A set of complete tables for use in applying the new formula has been placed on record in Engineering Societies Library, New York, N. Y.[2a]

NOTE.—Published in January, 1934, *Proceedings*.

[1] Senior Asst. Engr., Water Power and Hydrometric Bureau, Dept. of the Interior, Canada, Niagara Falls, Ont., Canada.

[2] "Precise Weir Measurements," *Transactions*, Am. Soc. C. E., Vol. 93 (1929), p. 999.

[2a] These tables were reproduced and placed on sale at Society Headquarters in January, 1934.

STATUS OF WEIR FORMULAS

The sharp-crested suppressed weir has been in use as a measuring device for many years, and a number of eminent hydraulic engineers have proposed formulas for computing its discharge. The paper by Professor Schoder and the late Professor Turner previously referred to, and the discussions thereon, together constitute an excellent review of the present status of this subject; the limitations of the various weir formulas in use are pointed out, with the net result that the question of a standard weir formula is left still unsettled. The weir experiments described in that paper comprise the most comprehensive set yet published, but the tentative formula proposed as an interpretation of these data does not seem convenient for practical use, since it requires such extensive measurements of the velocities in the approach channel as to constitute in themselves practically a measurement of the discharge. The formula presented in the present paper, based on part of the same data, can be used without any such auxiliary velocity measurements.

The paper by Professors Schoder and Turner contains a tabulation of the results of a large number of weir experiments. The set selected for this present analysis consists of Series D and O[3], in all of which the crest was a 12-in. brass plate. In making this selection, the intention was to have a set of accurate measurements all made under the same conditions except for the changes in the two variables, head and height of weir. Accordingly, all runs were omitted in which the natural flow of the water was disturbed by special fences; it was found necessary, also, to omit all those in which the head was less than 0.04 ft. In cases where two or more runs were made at practically the same head, mean values of head and discharge were used. Particulars as to height of weir, range of head, number of measurements, etc., taken mainly from the Schoder and Turner paper,[4] are listed in Table 1. All the weirs were 4.22 ft long, the full width of the channel, with no end contractions.

TABLE 1.—SUMMARY OF WEIR EXPERIMENTS SELECTED

Schoder and Turner series	Height of weir, in feet	Range of head, in feet	Original number of volumetric measurements	Number selected after combining duplicates and omitting specials
E	0.5	0.03 to 2.0	57	26
N	0.75	0.01 to 2.3	100	31
F	1.0	0.03 to 2.4	91	30
G	1.5	0.02 to 2.4	90	29
H	2.0	0.02 to 2.5	87	28
I	3.0	0.02 to 2.7	93	29
J	4.0	0.02 to 2.7	93	29
K and O	5.5	0.01 to 2.7	174	54
D, L, and M	7.5	0.01 to 2.7	220	79
Total	....	....	805	335

---

[3] *Transactions*, Am. Soc. C. E., Vol. 93 (1929), Appendices A and B, Tables 11, 12, 31, 32, 37, 38, 44, 45, and 55.

[4] *Loc. cit.*, Table 1, pp. 1000-1001.

NOTATION

In the metric formulas and tables of this paper, all lengths are in meters, and the discharges are in cubic meters per second. For purposes of conversion, note that 1 ft equals 0.3048 m. Unknown corrections (see Equation (10)) are expressed as *du, db,* etc. The following notation is introduced for the guidance of discussers:

a = a formula constant.
b = a formula constant (Equation (7)).
c = a constant coefficient (Equation (7)).
d = a substitution factor (Equation (7)).
f = a substitution factor (Equation (19)).
g = acceleration due to gravity.
h = head (see, also, H); h_v = head corresponding to mean velocity of approach for use in Equation (2).
i = a substitution factor (Equation (18)).
k = a function of $P = \log a + K$ (Equation (8)).
n = a formula constant (Equation (7)).
r = a residual.
u = a formula constant.
v = a formula constant (Equation (16)).
x = a variable exponent of H.
C = a constant $= c \sqrt{2\,g}$ (Equation (9)).
H = head on weir; ϕH = dimensionless function, with single argument, H; $\phi(P\ H)$ = dimensionless function with two arguments, P and H (this constitutes the velocity of approach correction (see, also, h)).
K = a function of $P = k - \log a$ (Equation (14)).
L = length of crest of weir.
P = height of weir; $\phi(P\ H)$ = dimensionless function, with two arguments, P and H (this constitutes the velocity of approach correction).
Q = discharge.
θ = a dimensionless multiplier.
ϕ = a symbol denoting function; ϕH = dimensionless function, with single argument, H; and $\phi(P\ H)$ = dimensionless function with two arguments, P and H (this constitutes the velocity of approach correction).

GENERAL ALGEBRAIC FORM OF EQUATION

With sufficient accurate experimental data available, the main difficulty in developing an accurate weir formula lies in selecting the proper algebraic form. Once this is done, it is comparatively simple, although sometimes somewhat laborious, to compute the best numerical values of the constants contained in the formula.

For a sharp-crested, suppressed weir, it is known from dimensional analysis that the equation for the discharge per foot of crest is of the form:

$$\frac{Q}{L} = \sqrt{2\,g}\;H^{\frac{3}{2}}\,\theta \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

in which, Q = total discharge of weir; L = length of crest; H = head; g = acceleration due to gravity; and θ = dimensionless multiplier.

Nearly all weir formulas are of this general type, the only difference being in the treatment of the term, θ, and in the "velocity of approach" correction. For purposes of comparison throughout this paper, three standard equations have been selected:

The Francis Formula:

$$\frac{Q}{L} = 3.33\left\{(H + h_v)^{\frac{3}{2}} - h_v^{\frac{3}{2}}\right\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

The Bazin Formula:

$$\frac{Q}{L} = \left(0.405 + \frac{0.00984}{H}\right)\left\{1 + 0.55\left(\frac{H}{H + P}\right)^2\right\}\sqrt{2\,g}\;H^{\frac{3}{2}} \ldots\ldots (3)$$

The Rehbock Formula:

$$\frac{Q}{L} = \frac{2}{3}\left(0.605 + \frac{1}{320\,H - 3} + 0.08\,\frac{H}{P}\right)\sqrt{2\,g}\;H^{\frac{3}{2}} \ldots\ldots\ldots (4)$$

in which, h_v is the head corresponding to the mean velocity of approach.

In the experiments summarized in Table 1, changes in the discharge, Q, are due either to changes in the head, H, or to changes in the height, P, of the weir crest above the bottom of the approach channel. All other conditions remain unchanged. In Series E and M, $P = 0.5$ and 7.5 ft, respectively. Simultaneous values of $\log \frac{Q}{L}$ and $\log H$ for these two series are shown graphically in Fig. 1. The points for each series fall on a separate curve, which is practically a straight line for small values of H, but which shows definite curvature for large values. Series E, with the smaller value of P, has the greater discharge, the difference increasing as the head increases, so that its graph has a greater curvature.

A consideration of these two graphs, and of the principles of dimensional analysis, led to the conclusion that for the experiments summarized in Table 1, θ may be written as the product of two dimensionless functions, as follows:

$$\theta = \phi H \times \phi(P\;H) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

in which, ϕH is a dimensionless function, with a single variable argument, H; and $\phi(P\;H)$, a dimensionless function, with two variable arguments, H and P. Using this expression for θ, Equation (1) may be written in the form:

$$\frac{Q}{L} = \phi H \times \sqrt{2\,g}\;H^{\frac{3}{2}} \times \phi\,(P\;H) \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

In the ordinary terminology of hydraulics, ϕH is the discharge coefficient, and $\phi(P\ H)$ is the "velocity of approach" correction, expressed as a separate multiplier. In the Francis formula, $\phi H = \frac{3.33}{\sqrt{2g}}$, which is constant for all values of H, and the velocity of approach correction is applied by an indirect method involving a series of approximations. In the Bazin formula, $\phi H = \left(0.405 + \frac{0.00984}{H}\right)$ and $\phi\ (P\ H) = \left\{1 + 0.55\left(\frac{H}{H + P}\right)^2\right\}$. In the Rehbock formula, $\phi H \times \phi(P\ H) = \frac{2}{3}\left(0.605 + \frac{1}{320\ H - 3} + 0.08\ \frac{H}{P}\right)$. Evidently, the Bazin formula is the only one that is of the same form as Equation (6).

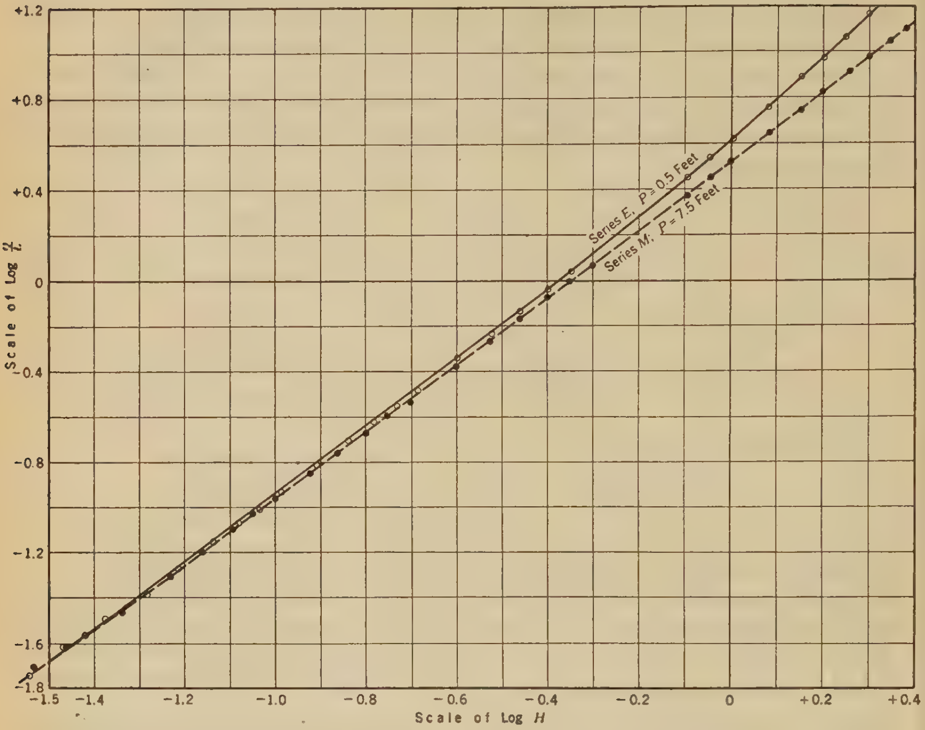


FIG. 1.—PLOTTING OF LOGARITHMIC VALUES OF $\frac{Q}{L}$ FROM SCHODER AND TURNER EXPERIMENTS, SERIES M AND E.

ALGEBRAIC FORMS OF ϕH AND $\phi(P\ H)$

The algebraic forms proposed in this paper for ϕH and $\phi(P\ H)$ are different from those used in the Bazin formula. They are entirely empirical, having been derived mainly by studying the results of the Schoder and Turner experiments, and without much dependence on purely theoretical considerations.

If a set of experiments were available with an infinite height of weir, P, so that the velocity of approach could be neglected ($\phi(P\ H) = 1$), the only unknown in Equation (6) would be the coefficient, ϕH, and for each observed value of H the corresponding value of ϕH could be computed. It should then be possible to determine an algebraic form with suitable numerical constants, such that the values of ϕH given by this formula would agree with those given by the experiments, within the limits of the experimental errors.

Similarly, by combining the results of such a set of experiments with others in which the value of P was not infinite, it would be possible to compute values of $\phi(P\ H)$ for various values of P and H; and thus determine an algebraic form for $\phi(P\ H)$ also.

A modification of this general method can be applied to the experiments of Table 1, by using a process of approximations that leads finally to reasonably accurate results. First, let it be assumed that part of the curvature of the graph of Series M in Fig. 1 is caused by $\phi H \times H^{\frac{3}{2}}$. The algebraic form selected for $\phi H \times H^{\frac{3}{2}}$ must then be such that log ϕH will give a curve when plotted against log H. If $\phi H \times H^{\frac{3}{2}}$ is to be of the form, $c\ H^x$, then, in order to produce the curvature, either c or x must change when H changes. As a first approximation, to be modified later if necessary, assume c constant for all values of H and write the exponent, x, in the form $n\ H^b$ ($= d$) so that it will change as H changes. Then, $\phi H \times H^{\frac{3}{2}}$ will be expressed as:

$$\phi H \times H^{\frac{3}{2}} = c\ H^d \quad \text{.......................(7)}$$

in which, d ($= n\ H^b$) is a substitution factor introduced to simplify typography and c, n, and b are arbitrary constants (n being very nearly equal to $\frac{3}{2}$ and b very small).

Similarly, one set of approximate values of $\phi(P\ H)$ can be found by subtracting log $\frac{Q}{L}$ for Series M from log $\frac{Q}{L}$ for Series E. This can be done directly, since there are a number of experiments in which the value of H is approximately the same in both series. When these values of log $\phi(P\ H)$ are plotted against H, all the points fall close to a straight line through the origin. Evidently, log $\phi(P\ H)$ can be written in the form:

$$\log \phi(P\ H) = kH \quad \text{.......................(8)}$$

in which, k is a constant when P is constant. The slope of the line gives an approximate value of k for Series E; and approximate values of k for the other series can be found in a similar manner.

APPLICATION OF METHOD OF LEAST SQUARES

The logarithmic equivalent of Equation (6) may now be written in the form:

$$\log \frac{Q}{L} = \log C + n\,H^b \log H + k\,H \quad \text{.................(9)}$$

in which, $\log C = \log c \sqrt{2g}$ and $\log C$, n, b, and k are unknown constants, k having a different value for each different value of P. By adopting Equation (9) as a general equation for all the experiments listed in Table 1, and applying the method of least squares to the entire list, it was possible to secure additional information, as follows: (1) To determine more precise values of k for each value of P covered by the experiments, so that the relation between k and P may be expressed algebraically; (2) to determine more precise values of $\log C$, n, and b for use in a second solution; and (3) to show that this form of equation is adequate to represent the data of Table 1, by completing the solution, computing the residuals, and examining them.

In each solution by the method of least squares, the same general procedure was followed. The logarithmic form of equation was used so that each experiment would be of equal weight. Continuing the process of approximations, a differential formula was established (by partial differentiations of the original logarithmic formula), which showed the proportionate change in $\log Q$ (or in the logarithmic residuals), that would result from small changes in each of the constants. The residuals were found by substituting in the original logarithmic equation, assumed approximate values of the unknown constants. Each experiment in Table 1 furnished one residual, and each residual an observation equation, so that there were 335 observation equations in all. From these, the normal equations were formed and solved, and the most probable set of corrections was found. The application of these corrections to the approximate values assumed for the constants in the beginning, furnished the most probable values of the constants. In the final solution, the probable error of each constant in the formula was computed; the probable error of a single observation was also computed, or, more correctly, the probable difference between an observed and a computed discharge. As logarithmic equations were used, some of the probable errors are expressed as percentages.

The practical application of this process involved considerable work, especially in computing the residuals and in forming and solving the normal equations. The computations were arranged in tabular form and numerical checks were carried along with them. Adding and calculating machines, and tables of products, were used whenever possible.

This general method of utilizing the data of the experiments involves the assumption that all observations of head are exactly correct, and that any discrepancies that occur are due to errors in the discharge. This assumption, of course, is not strictly correct. A solution based on the assumption that errors of head and errors of discharge are equally probable, might be expected to give slightly different values for the constants, but it is doubtful whether such a refinement is warranted in this case.

In the first solution, the differential observation equation derived from Equation (9), was as follows:

$$d\,(\log C) + H^{(b1)} \log H\, dn + 2.3026\, n_1 H^{(b1)} (\log H)^2\, db + H\, dk = r_1 \quad \text{..(10)}$$

in which, d (log C), dn, db, and dk are the unknown corrections which, when applied to the approximate values used in Equation (11), will give the most probable values of the unknown constants. Common logarithms are used in Equation (10), as in all other formulas in this paper. The residuals, r_1, are found from the equation:

$$r_1 = \log \frac{Q}{L} - (\log C_1 + n_1 H^{(b1)} \log H + kH) \dots\dots\dots (11)$$

in which, $\log \frac{Q}{L}$, and H are given by the experiment and $\log C_1$, n_1, $(b1)$, and k are the assumed approximate values of the unknown constants. For the first solution, $n_1 = 1.5$, $(b1) = 0.008$, and the values used for k were those found by the graphical method described in the preceding section. The corrected values obtained for log C, n, and b were used in computing residuals for the second solution, the small size of these residuals showing the suitability of the algebraic form used. The corrected values of k, for the various values of P, are listed in Table 2, from which it may be noted that there is a definite relation between k and P, although a curve plotted from these data indicates certain minor inconsistencies.

TABLE 2.—VALUES OF k FOR DIFFERENT VALUES OF P

Series	Height of weir, P	Value of k	Series	Height of weir, P	Value of k
E.....	0.5	0.0962	I...........	3.0	0.0154
N.....	0.75	0.0581	J...........	4.0	0.0123
F.....	1.0	0.0484	K and O.....	5.5	0.0116
G.....	1.5	0.0337	D, L, and M..	7.5	0.0105
H.....	2.0	0.0245	....	....	....

FINAL ALGEBRAIC FORMS

In order to complete the weir formula, it is necessary to express, algebraically, the relation between the values of P and k in Table 2. For the smaller values of P, k decreases quickly as P increases, but, for the larger values, there is little change in k for relatively large changes in P. Furthermore, when the corresponding values of log k and log P are plotted, the resulting graph is not straight, but curved. From these considerations, and continuing the process of approximations by substituting trial values for certain of the constants and solving for the others, the following algebraic formula was finally adopted:

$$k = \log a + \frac{u}{1 + v\,P^{\frac{3}{2}}} \dots\dots\dots\dots\dots\dots\dots (12)$$

By choosing suitable numerical values for the constants, Equation (12) can be made to agree well, although not perfectly, with the values of k in Table 2. That is the only reason for selecting this particular algebraic form, and it is quite possible that some other form might be found that would be more suitable.

Being a constant, log a is independent of P and, therefore, belongs in ϕH rather than in $\phi(P\ H)$, so that the final algebraic forms of ϕH and $\phi(P\ H)$ (see Equations (7) and (8)) are as follows:

$$\phi H = \frac{c\, a^{H}\, H^{a}}{H^{\frac{3}{2}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13)$$

and $\phi(P\ H) = 10^{KH}$, or $\log \phi(P\ H) = KH$, in which,

$$K = k - \log a = \frac{u}{1 + v\, P^{\frac{3}{2}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)$$

These algebraic forms give reasonably accurate results, but there is no absolute certainty that they make an exactly accurate division between ϕH and $\phi(P\ H)$.

FINAL LEAST SQUARES ADJUSTMENT

A final adjustment of the entire equation was made by a second least squares solution. For this, Equation (9) was re-written in the form:

$$\log \frac{Q}{L} = \log C + n H^{b} \log H + H \log a + \left(\frac{u}{1 + v\, P^{\frac{3}{2}}} \right) H \ldots\ldots\ldots (15)$$

and the corresponding differential and residual equations are, as follows:

$$d\,(\log C) + H^{(b2)} \log H\ dn + 2.3026\ n_2\ H^{(b2)}\ (\log H)^2\ db + H\ d\,(\log a)$$
$$+ \left(\frac{1}{1 + v\, P^{\frac{3}{2}}} \right) H\ du - \left\{ \frac{u\, P^{\frac{3}{2}}}{(1 + v\, P^{\frac{3}{2}})^2} \right\} H\ dv = r_2 \ \ldots\ldots\ldots (16)$$

and,

$$r_2 = \log \frac{Q}{L} - \left(\log C_2 + n_2 H^{(b2)} \log H + H \log a + \frac{u\, H}{1 + v\, P^{\frac{3}{2}}}\right) \ldots (17)$$

in which, the values of log C_2, n_2, and ($b2$) were given by the first solution and the values of log a, u, and v were those found by the trial method previously mentioned.

NUMERICAL VALUES

The numerical values of the constants obtained in this final adjustment, together with their probable errors, are given in Table 3.

TABLE 3.—NUMERICAL CONSTANTS IN WEIR FORMULA

Symbol	Numerical value	Logarithmic value	Probable error	Symbol	Numerical value	Logarithmic value	Probable error
$C = c\sqrt{2g}$.	3.2765	0.5154	±0.03%	b....	0.0081	....	±0.00015
a......	1.0195	0.0084	±0.03%	u....	0.2713	....	±0.00014
n......	1.5064	....	±0.00027	v....	6.1452	....	±0.00012

The complete formula, with the numerical values of the constants, may now be written in the logarithmic and numerical forms, as follows:

$$\log \frac{Q}{L} = 0.5154 + 0.0084\ H + 1.5064\ H^{0.0081} \log H + K\ H \ldots\ldots (18)$$

or,

$$\frac{Q}{L} = 3.276 \times (1.0195)^{H} \times H^{f} \times 10^{KH} \ldots\ldots\ldots\ldots\ldots (19)$$

in which, to simplify typography, $f = 1.5064 H^{0.0081}$, and,

$$K = \frac{0.2713}{1 + 6.1452\ P^{\frac{3}{2}}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (20$$

Furthermore,

$$\sqrt{2\,g}\ \phi H = \frac{3.276 \times (1.0195)^{H} \times H^{f}}{H^{\frac{3}{2}}} \ldots\ldots\ldots\ldots\ldots (21)$$

and,

$$\phi(P\ H) = 10^{KH} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (22)$$

Table 4 has been prepared to show, for various values of the argument, H, the corresponding numerical values of the variable exponent, $f = 1.5064\ H^{0.0081}$,

TABLE 4.—Numerical Values of Variable Exponent and Coefficients

Head, H, in feet (1)	Equation (19)		Equations (21) and (6)
	Exponent, $1.5064 H^{0.0081}$ (2)	Coefficient, $3.276\ (1.0195)^{H}$ (3)	Coefficient, $\sqrt{2\,g}\ \phi H$ (4)
0.1	1.478	3.282	3.449
0.2	1.487	3.289	3.360
0.3	1.492	3.295	3.327
0.4	1.495	3.302	3.316
0.5	1.498	3.308	3.313
0.6	1.500	3.314	3.314
0.7	1.502	3.321	3.318
0.8	1.504	3.327	3.324
0.9	1.505	3.334	3.332
1.0	1.506	3.340	3.340
1.2	1.509	3.353	3.359
1.4	1.510	3.366	3.379
1.6	1.512	3.378	3.399
1.8	1.514	3.392	3.420
2.0	1.515	3.406	3.441
2.5	1.517	3.439	3.495
3.0	1.520	3.472	3.549

and the variable coefficient, $3.276\ (1.0195)^{H}$, in Equation (19). This table also gives the coefficient $\sqrt{2\,g}\ \phi H$ from Equation (21), for use with the fixed exponent, 1.5, in Equation (6). The value of the variable exponent (Table 4) is 1.5 at 0.6 ft, 0.022 less than 1.5 at 0.1 ft, and 0.020 greater than 1.5 at 3.0 ft; so that the greatest differences from 1.5 are less than ±1.5 per cent. However, this comparatively small change in the exponent makes a considerable difference in the corresponding coefficient, as can be seen by plotting Column (1), Table 4, against Column (4). The coefficient for use with the variable exponent (Column (3), Table 4) plots as a straight line, whereas the coefficient for use with the constant exponent, 1.5 (Column (4), Table 4), is a somewhat complicated curve, intersecting the straight line at the two points, $H = 1.0$ ft and $H = 0.6$ ft, practically a straight line above $H = 1.0$ ft, but with rapidly increasing curvature below $H = 0.6$ ft.

This peculiarity of the coefficient for use with the fixed exponent, 1.5, is one of the most serious difficulties in devising an accurate weir formula. It is neglected entirely in the Francis formula, probably because Francis did not make any experiments with heads less than 0.6 ft; but it is reproduced more or less accurately in the Bazin and Rehbock formulas by the introduction of a term containing H in the denominator.

The coefficient, $3.276\ (1.0195)^H$, varies as a straight line. When expanded by the binomial theorem, neglecting the third term and all subsequent terms, it becomes $3.276\ (1 + 0.0195H)$ which is the equation of a straight line. The value of the neglected third term is only 0.1% for $H = 3$ ft, and less for smaller values of H; so that for all practical purposes the two expressions are equal and may be used interchangeably in Equation (19). However, the original form is more convenient for use with logarithms.

Accuracy of New Formula

The probable difference between an observed and a computed weir discharge, as given by the final least squares solution, is ±0.65 per cent. If the weir experiments could be regarded as exact, this would be the probable error of a single value of discharge computed by the formula, and its smallness shows that Equation (19) agrees very closely with the measurements. If the formula is regarded as exact, ±0.65% is the probable error of a single weir experiment, which is an indication of a high order of consistency in the experiments.

The agreement of the new formula (Equation (19)), with the weir experiments is shown graphically in Fig. 2, for three values of P. Similar agreement was found for intermediate values. In order to condense the figures as much as possible, the discharge per foot of crest, $\frac{Q}{L}$, has been divided by $H^{\frac{3}{2}}$, and the quotient for each experiment, plotted against the corresponding value of H, is shown by the circles.

The full line curves in Fig 2 represent the corresponding values given by Equation (19). All these individual curves have been assembled in Fig. 3. They constitute a "family" of curves that change shape and inclination to the H-axis when the value of P is changed. Each individual curve, in addition to fitting its own points as closely as possible, must also maintain a symmetrical position in Fig. 3. There are certain minor inconsistencies among the experiments themselves that make it impossible for any such symmetrical family of curves to fit all the points perfectly; this may be due partly to certain features of the weir equipment, which are discussed in another section of this paper. On that account, although the present set of curves differs from the experiments by more than 1% in some cases, it is felt that the search for an accurate weir formula has been carried as far as these experiments warrant. The method of least squares, of course, has resulted in the selection of the best numerical values of the coefficients in the particular type of formula proposed and any improvement could be made

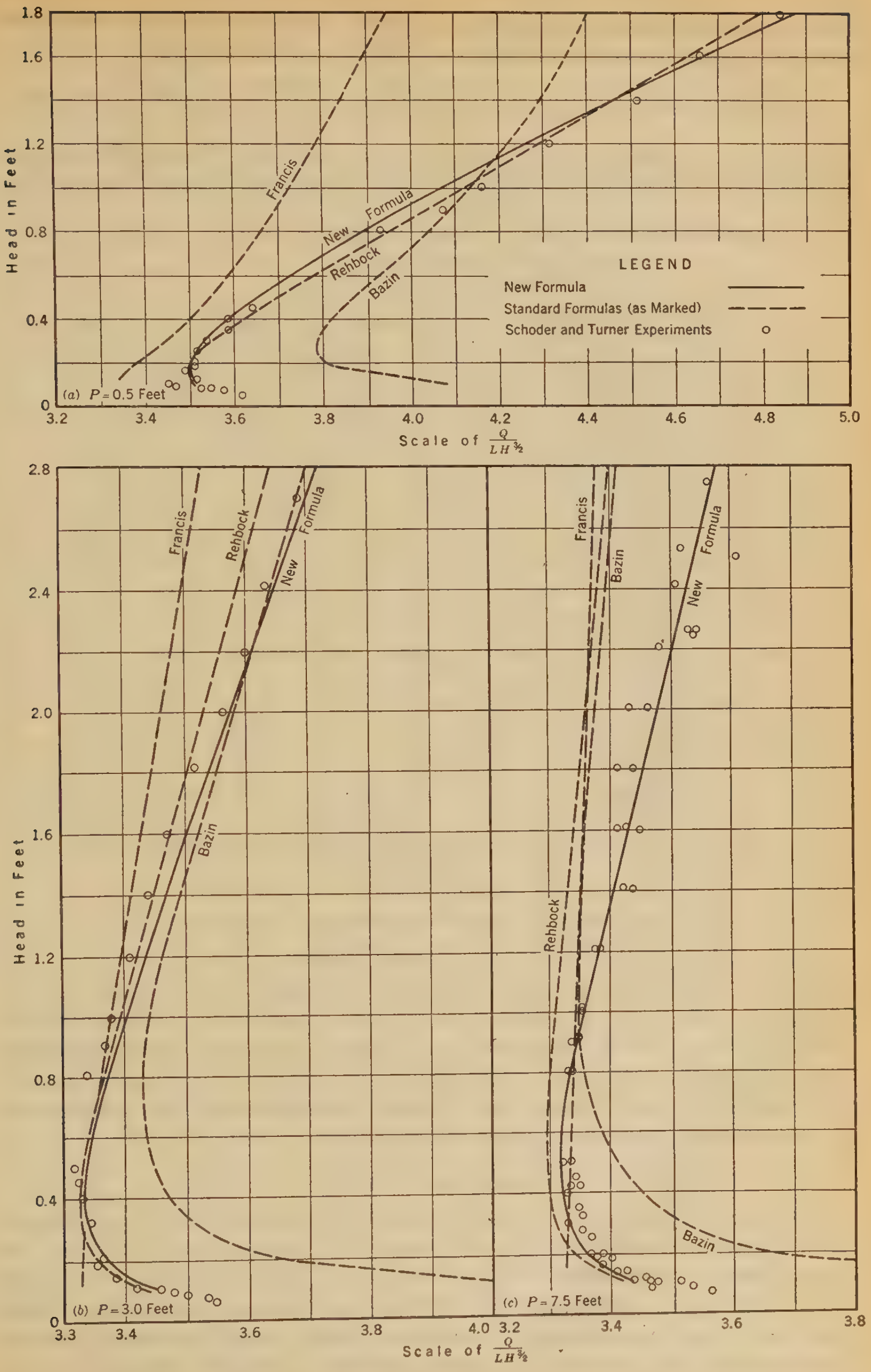


Fig. 2.—Plotted Values of $\frac{Q}{LH^{\frac{3}{2}}}$ from Schoder and Turner Experiments from Equation (19), and from Three Other Formulas.

only by some change in the algebraic form of Equation (19). Additional experiments will be required to show whether this is necessary.

The broken lines in Fig. 2 show the values of $\frac{Q}{L H^{\frac{3}{2}}}$ as given by the three standard weir formulas, Equations (2), (3), and (4). The differences in their algebraic form allow them to agree with one another for certain values of H and P and to disagree for other values. For the full range of H and P covered by the Schoder and Turner experiments, none of these formulas agrees with the experiments as well as the new formula (Equation (19)) proposed in this paper. However, for values of H not greater than 2 ft and values of P not greater than 4 ft, the Rehbock formula does agree with the experiments fully as well as the new formula.

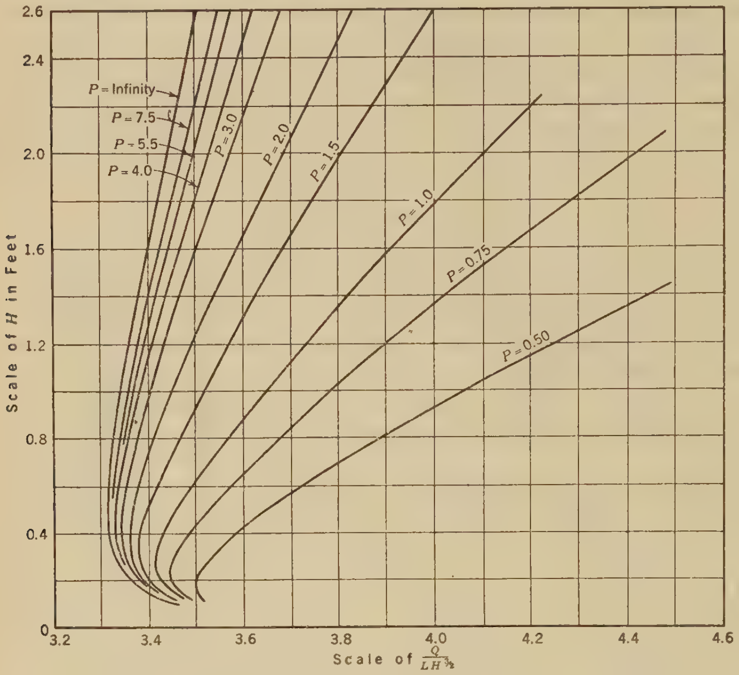


FIG. 3.—CURVES SHOWING RELATIONSHIP BETWEEN HEAD AND $\frac{Q}{L H^{\frac{3}{2}}}$ FOR VARIOUS VALUES OF P, FROM EQUATION (19).

DISCHARGE TABLES

For practical field and laboratory use, the writer has prepared a discharge table by solving Equation (18) for values of H at intervals of 0.01 ft. It shows directly the discharge, in cubic feet per second per linear foot of crest, in terms of the two arguments: H, the head on the weir, and P, the height of the weir above the channel of approach. Table 5 is a condensed form of this discharge table, suitable for plotting curves. The table was confined to the range of heads covered by the experiments, except that, for the higher weirs, it was extended to an even 3 ft, although the experiments go only to 2.7 ft. Column (11), Table 5, shows the discharge computed for a weir of infinite height; that is, with no velocity of approach.

TABLE 5.—DISCHARGE FOR SUPPRESSED WEIRS, IN CUBIC FEET PER SECOND PER FOOT OF CREST LENGTH

Head, in feet	HEIGHT OF WEIR, IN FEET									
(1)	0.5 (2)	1.0 (3)	2.0 (4)	3.0 (5)	4.0 (6)	5.0 (7)	6.0 (8)	7.0 (9)	8.0 (10)	Infinity (11)
0.05...	0.040	0.040	0.040	0.040	0.040	0.040	0.040	0.040	0.040	0.040
0.06...	0.053	0.052	0.052	0.052	0.052	0.052	0.052	0.052	0.052	0.052
0.07...	0.066	0.065	0.065	0.065	0.065	0.065	0.065	0.065	0.065	0.065
0.08...	0.080	0.079	0.079	0.079	0.079	0.079	0.079	0.079	0.079	0.079
0.09...	0.095	0.094	0.094	0.094	0.094	0.094	0.094	0.094	0.094	0.094
0.10...	0.111	0.110	0.109	0.109	0.109	0.109	0.109	0.109	0.109	0.109
0.11...	0.128	0.127	0.126	0.126	0.125	0.125	0.125	0.125	0.125	0.125
0.12...	0.146	0.144	0.143	0.143	0.142	0.142	0.142	0.142	0.142	0.142
0.13...	0.164	0.162	0.161	0.161	0.160	0.160	0.160	0.160	0.160	0.160
0.14...	0.183	0.181	0.179	0.179	0.179	0.179	0.179	0.179	0.178	0.178
0.15...	0.203	0.200	0.198	0.198	0.197	0.197	0.197	0.197	0.197	0.197
0.20...	0.313	0.306	0.303	0.302	0.301	0.301	0.301	0.300	0.300	0.300
0.30...	0.580	0.561	0.552	0.550	0.549	0.548	0.548	0.548	0.548	0.547
0.40...	0.908	0.869	0.851	0.845	0.843	0.842	0.841	0.841	0.841	0.839
0.50...	1.293	1.224	1.191	1.182	1.179	1.177	1.175	1.175	1.174	1.171
0.60...	1.73	1.62	1.57	1.56	1.55	1.55	1.55	1.54	1.54	1.54
0.70...	2.23	2.07	1.99	1.97	1.96	1.95	1.95	1.95	1.95	1.95
0.80...	2.78	2.55	2.44	2.42	2.40	2.40	2.39	2.39	2.39	2.38
0.90...	3.40	3.08	2.93	2.89	2.88	2.87	2.86	2.86	2.86	2.85
1.00...	4.07	3.65	3.46	3.40	3.38	3.37	3.36	3.36	3.35	3.34
1.10...	4.80	4.25	4.01	3.94	3.92	3.90	3.89	3.89	3.88	3.86
1.20...	5.59	4.91	4.60	4.52	4.48	4.46	4.45	4.45	4.44	4.42
1.30...	6.45	5.59	5.22	5.12	5.07	5.05	5.04	5.03	5.02	4.99
1.40...	7.37	6.33	5.87	5.75	5.70	5.67	5.65	5.64	5.63	5.60
1.50...	8.36	7.10	6.55	6.40	6.34	6.31	6.29	6.28	6.27	6.22
1.60...	9.42	7.91	7.26	7.09	7.01	6.98	6.95	6.94	6.93	6.88
1.70...	10.56	8.77	8.01	7.80	7.72	7.67	7.65	7.63	7.61	7.56
1.80...	11.77	9.67	8.78	8.54	8.45	8.39	8.36	8.34	8.33	8.26
1.90...	13.06	10.61	9.58	9.31	9.20	9.14	9.10	9.08	9.06	9.98
2.00...	14.43	11.59	10.42	10.11	9.98	9.91	9.87	9.84	9.82	9.73
2.10...	......	12.62	11.28	10.93	10.78	10.70	10.66	10.62	10.60	10.50
2.20...	......	13.70	12.18	11.78	11.61	11.52	11.47	11.44	11.41	11.30
2.30...	......	14.82	13.10	12.65	12.46	12.37	12.31	12.27	12.24	12.12
2.40...	......	......	14.06	13.56	13.35	13.24	13.17	13.13	13.09	12.95
2.50...	......	......	15.04	14.48	14.25	14.13	14.06	14.00	13.97	13.81
2.60...	......	......	......	15.44	15.18	15.04	14.96	14.90	14.87	14.70
2.70...	......	......	......	16.42	16.14	15.99	15.89	15.84	15.79	15.60
2.80...	......	......	......	17.43	17.11	16.95	16.85	16.78	16.74	16.53
2.90...	......	......	......	18.46	18.12	17.94	17.82	17.75	17.70	17.48
3.00...	......	......	......	19.51	19.14	18.94	18.82	18.74	18.68	18.44

Table 6 is a selected list of K-values computed by Equation (20), for values of P from 0.5 to 8.0 ft. A complete table, for intervals of $P = 0.1$ ft, is necessary, but for purposes of demonstration the trends indicated by Table 6 must suffice. Tables such as Table 4 (Column (4)) and Table 6, may be used for computing discharges independently of Table 5 by means of the following formula:

$$\frac{Q}{L} = \sqrt{2\,g}\,\phi\,H \times H^{\frac{3}{2}} \times 10^{KH} \qquad \text{(23)}$$

This method is useful for computing the discharge for low weirs with heights intermediate between those listed in the discharge table, in which simple interpolation between the columns will not give the correct discharge for high heads. Tables in metric units similar to Table 4 (Column (4)) and Table 6, in English units, are necessary in some computations and have been prepared by the writer.

TABLE 6.—SELECTED VALUES OF K BY EQUATION (20), IN ENGLISH UNITS, FOR USE IN EQUATION (23)

P, in feet	K	P, in feet	K	P, in feet	K	P, in feet	K	P, in feet	K	P, in feet	K
0.5....	0.0855	0.8	0.0502	1.2	0.0299	1.8	0.0171	3.0	0.0082	6.0	0.0030
0.6....	0.0703	0.9	0.0434	1.4	0.0243	2.0	0.0148	4.0	0.0054	7.0	0.0024
0.7....	0.0590	1.0	0.0380	1.6	0.0202	2.5	0.0107	5.0	0.0039	8.0	0.0019

WEIR DESIGN

Strictly speaking, the discharge tables offered for discussion in this paper should be used only with weirs constructed and arranged similarly to that on which the experiments were performed, a detailed description of which is given in the paper by Professors Schoder and Turner.[2] The weir plate was of rolled brass, 12 in. wide, with the top edge machine-beveled and with the up-stream face smooth. The head was measured, at a section 11.74 ft up stream from the crest, by direct measurement in the channel, with a steel tape and also by a gauge in a stilling-well. This distance of 11.74 ft is greater than the 6 ft used by Francis and less than the 16.4 ft used by Bazin.

Back-water must be avoided, of course; the jet must be fully aerated, and should not cling to the weir plate. At very low heads, a violation of this latter condition may introduce erratic results; on that account, the discharge table was not carried below 0.05 ft.

In the Schoder and Turner experiments, the water was supplied by a 30-in. pipe which emptied into the weir structure about 25 ft from the crest and only 5 ft below it. It is not the intention in this paper to suggest the duplication of this supply arrangement, which presumably was necessitated by local conditions, but rather to draw attention to the suggestions[5] of Professor Th. Rehbock that: (*a*) The approach channel should be of sufficient length and of the same width throughout; (*b*) the water should enter the approach channel axially; and (*c*) its free approach to the weir should not be disturbed by built-in structures. This provides a natural arrangement that can be duplicated readily.

The Schoder and Turner experiments, in which the approach conditions differed most from conditions suggested herein, namely, those in which special fences were used, were specifically excluded from consideration in this paper, but apparently some of the experiments that were used have been affected somewhat by other features of the special approach arrangements. Those with high heads and consequent high velocities of approach would be affected more than those with low heads and low velocities. If the experiments for high heads were omitted, the curves could be made to fit the remaining points better (see Fig. 2). Professor Rehbock's contention that the position of the supply pipe must have interfered with the free flow of water for weirs 5.5 and 7.5 ft high, may account for the fact that, for certain heads, the discharge is greater for the 7.5-ft weir than for the 4-ft weir.

---

[5] *Transactions*, Am. Soc. C. E., Vol. 93 (1929) p. 1143.

However, these inconsistencies are really of small magnitude (possibly of the order of ±1%), and their mention here should not detract seriously from the merit of the experiments; nor should they obscure the fact that the new formula agrees with the entire set of experiments with a probable error of only ±0.65 per cent.

STANDARD DISCHARGE TABLES

A standard set of discharge tables for suppressed weirs would be of great practical value. Having such tables, the practical man need not concern himself with the value of the discharge coefficient, the method of applying the velocity of approach correction, nor indeed with any particulars of the formula. The only question he need ask is whether the discharges given by the tables are accurate; the remainder he can leave to the computer who prepares the tables.

The discharge table, of which Table 5 is a sample, is submitted as a tentative discharge table for standard weirs with the approach conditions suggested by Professor Rehbock as stated previously, even if it is based on the Schoder and Turner experiments, which were made with somewhat different approach arrangements. It is probable that the table is not seriously in error from this cause, but it would be an advantage to have it extended to include higher heads; also, additional experiments would possibly allow a more precise determination of the formula. Accordingly, a program is suggested by means of which this tentative table might be extended and if necessary corrected, as follows: (1) Adopt a standard design of suppressed weir; (2) using weirs of this standard design, make a set of precise weir experiments, similar to those made by Professors Schoder and Turner; (3) select an algebraic form of equation that will be sufficiently flexible to agree with the results of the experiments over the entire range of all the variables introduced into the experiments; (4) use the method of least squares to determine the best values of the numerical constants in the formula; and (5) use the formula to compute a set of discharge tables covering the entire range of all the variables.

CHANGING TO METRIC UNITS

Discharges in metric units may be found from Table 5 by converting the head from meters to feet and the discharge from cubic feet per second to cubic meters per second, or liters per second. However, it would be more convenient to use a discharge table in metric units, and such a table can be computed by solving Equation (23), or its logarithmic equivalent, together with the values of $\sqrt{2\ g}\ \phi H$ and of K, which have been computed for the metric system and are on record in Engineering Societies Library as stated previously.

The discharge formula and the auxiliary terms, with the numerical values of the coefficients for the metric system, are given in the "Summary."

SUMMARY

The general type of discharge formula proposed in this paper for sharp-crested suppressed weirs (see Equation (6)) is as follows:

$$\frac{Q}{L} = \sqrt{2\,g}\,\phi\,H \times H^{\frac{3}{2}} \times \phi\,(P\,H) \dots\dots\dots\dots\dots (24)$$

in which (see Equation (13)), $\phi H = \dfrac{c \times a^{H} \times H^{d}}{H^{\frac{3}{2}}}$; $\phi\ (P\ H) = 10^{KH}$; and (see Equation (14)), $K = \dfrac{u}{1 + v\,P^{\frac{3}{2}}}$. Whence,

$$\frac{Q}{L} = C \times a^{H} \times H^{d} \times 10^{KH} \dots\dots\dots\dots\dots (25)$$

in which, $C = c\,\sqrt{2\,g}$.

Inserting the numerical values of the coefficients for the two systems of units, Equation (25) becomes in English units (feet and cubic feet per second):

$$\frac{Q}{L} = 3.276 \times (1.0195)^{H} \times H^{f} \times 10^{KH} \dots\dots\dots\dots\dots (26)$$

and (see Equations (18) and (19)),

$$\log \frac{Q}{L} = 0.5154 + 0.0084\,H + 1.5064\,H^{0.0081} \log H + K\,H \dots\dots (27)$$

in which, $K = \dfrac{0.2713}{1 + 6.1452\,P^{\frac{3}{2}}}$; and $f = 1.5064\ H^{0.0081}$. Furthermore (see Equation (21)):

$$\sqrt{2\,g}\,\phi\,H = \frac{3.276 \times (1.0195)^{H} \times H^{f}}{H^{\frac{3}{2}}} \dots\dots\dots\dots\dots (28)$$

In metric units, Equation (25) becomes,

$$\frac{Q}{L} = 0.3044 \times (1.0654)^{H} \times (3.2808\,H)^{i} \times 10^{KH} \dots\dots\dots\dots (29)$$

and,

$$\log \frac{Q}{L} = 1.4834 + 0.0275\,H + 1.5210\,H^{0.0081} (\log H + 0.5160) + K\,H \dots (30)$$

in which, $K = \dfrac{0.890}{1 + 36.52\,P^{\frac{3}{2}}}$; and $i = 1.5210\ H^{0.0081}$. Furthermore,

$$\sqrt{2\,g}\,\phi\,H = \frac{0.3044 \times (1.0654)^{H} \times (3.2808\,H)^{i}}{H^{\frac{3}{2}}} \dots\dots\dots\dots (31)$$

The complete discharge table at intervals of $H = 0.01$ ft is not published, a condensed form (Table 5) being considered adequate for purposes of discussion. It shows directly the discharge, in cubic feet per second per linear foot of crest, in terms of the two arguments, both expressed in feet, namely.

H, the head on the weir; and P, the height of the weir above the bottom of the channel of approach. A table compiled for intervals of $H = 0.01$ ft is probably sufficiently accurate for most practical purposes, but it has been shown how the results of future experiments may be used to make small corrections in the table, if necessary, and to extend it for higher heads.

Tabular values of $\sqrt{2\,g}\;\phi H$ and of K, are contained in Tables 4 and 6. Tables such as these, compiled for intervals of $H = 0.01$ ft and $P = 0.1$ ft, respectively, may be used in Equation (23) for computing discharges independently of the discharge table, or to supplement that table.

The adoption of a standard design for sharp-crested suppressed weirs and the preparation of an elaborate and accurate set of standard discharge tables, as advocated in this paper, should help to restore the lost prestige of the weir as an accurate device for measuring water.

DISCUSSION

Jasper O. Draffin,[6] M. Am. Soc. C. E. (by letter).—The view is held by some hydraulic engineers that a weir is not an acceptable device for measuring the flow of water, partly because no formula has been found which will give with sufficient accuracy the discharge for all heads and weir heights. In this paper the author derives a formula which he claims is more exact and easier to use than other existing formulas and which agrees with experiments. This still leaves untouched the question of whether or not a given sharp-crested weir will continue to discharge at the same rate after it has been in use for several years.

The formula proposed by Mr. Cline is more complex than the Rehbock formula, or that proposed by Schoder, but it may be used with ease when tables or curves have been constructed, as they have been by the author.

The Bazin and Francis formulas do not give results which agree with the Schoder and Turner experiments, except for limited ranges of head and weir height, but the results given by the Rehbock formula agree very closely with the measured quantities for heads not in excess of 2 ft and for weir heights of not more than 4 ft. Beyond these values the Rehbock formula is but little better than either the Francis or the Bazin formula and none of them agrees with the Schoder and Turner experiments. Rehbock accounts for this lack of agreement by claiming that "the data for the 5.5 and 7.5-ft weir [Series D, K, L, M, and O, Table 1] are obviously unreliable."[7] He attributes the lack of reliability to the approach channel conditions, measurement of head, and the possibility of insufficient aeration of the jet, and shows that the data are not consistent among themselves. Schoder, however, points out that the jet was fully aerated. Rehbock also states that "sharp-crested weirs more than 4 ft in height are almost never used in practice."[7]

The Cline formula gives results which are decidedly better than any of the other formulas when the whole range of heads and weir heights is considered. Even if the Schoder and Turner data for high heads and weir heights are somewhat erratic, the Cline formula gives a good average of the measured quantities. The fact that Equation (23) agrees so closely with the experimental quantities at the lower heads and weir heights, as well as at the higher ones, indicates that the form of the equation is correct. Of course, the weak point in it is the fact that the constants were derived to fit data which seem somewhat erratic and which might be different under different experimental conditions. However, the author does not propose the formula as a final one, but as one which may be modified as further experiments are made, and he has made clear the method which should be followed in making a revision. A series of experiments should be carefully planned and performed in which the approach channel, aeration of jet, and measurement of head will all be of an accepted standard condition. Then, if the Cline formula does not exactly fit the new data, the constants in it may be modified in the manner which he suggests.

---

[6] Associate Prof., Theoretical and Applied Mechanics, Univ. of Illinois, Urbana, Ill.

[7] "Precise Weir Measurements," *Transactions*, Am. Soc. C. E. Vol. 93 (1929), p. 1155.

Certainly, the author deserves much credit for his splendid approach to the problem and for the large amount of work which he has done in deriving the formula which he proposes, and in computing the tables.

W. Bruce McMillan,[8] M. Am. Soc. C. E. (by letter).—This paper again brings forth the important subject of measurement of water by weirs. The sharp-crested suppressed weir is the type that may be most readily standardized for more precise measurements. If the conditions under which the experiments are made, are duplicated, or made standard, the formulas that represent these experiments accurately may be said to be precise, as limited by the accuracy of observation.

The writer has derived a tentative formula by curve-fitting to substantially the same experiments (Series D to P) as those upon which the paper is based[9] and has arrived at a form similar in general to the Rehbock formula. This same general form may be reduced from the author's formula. Equation (19) may be expressed as follows,

$$\frac{Q}{L\,H^{\frac{3}{2}}} = 3.276\ \frac{H'}{H^{\frac{3}{2}}}\ [1.0195\ (10)^{K}]^{H} \quad \ldots\ldots\ldots\ldots\ldots (32)$$

The first bracket may be represented by:

$$\frac{H'}{H^{\frac{3}{2}}} = 0.9812 + \frac{0.0067}{H} + 0.01209\ H$$

and the second bracket by:

$$[1.0195\ (10)^{K}]^{H} = \left[1.017 + \frac{0.095}{P^{1.25}}\right]^{H}$$

Substituting in Equation (32) and expanding $(1.017)^{H}$ by the binomial theorem:

$$\frac{Q}{L\,H^{\frac{3}{2}}} = \left(3.212 + \frac{0.02405}{H} + 0.0949\ H\right)\left(1 + \frac{0.0934}{P^{1.25}}\right)^{H} \quad \ldots\ldots (33)$$

Expanding by the binomial theorem again and casting out the smaller terms:

$$\frac{Q}{L\,H^{\frac{3}{2}}} = 3.212 + \frac{0.02405}{H} + \left(0.0949 + \frac{0.30}{P^{1.25}}\right) H \quad \ldots\ldots\ldots (34)$$

Equation (34) is shown to complete the conversion, but it is not accurate. The studies leading to the writer's tentative formula,

$$\frac{Q}{L\,H^{\frac{3}{2}}} = 3.22 + \frac{0.021}{H} + \left(0.105 + \frac{0.31}{P^{1.38}}\right) H$$

were refined and were adjusted somewhat in deference to the author's determination by least squares, but with deliberate deviation in the case of the highest and lowest weirs for better accord with the experimental data (see Fig. 4). The writer's revised formula is:

---

[8] Cons. Engr., Palo Alto, Calif.

$$\frac{Q}{L H^{\frac{3}{2}}} = 3.222 + \frac{0.0222}{H} + \left(0.081 + \frac{0.340}{P^{1.25}}\right) H \ldots\ldots\ldots\ldots (35)$$

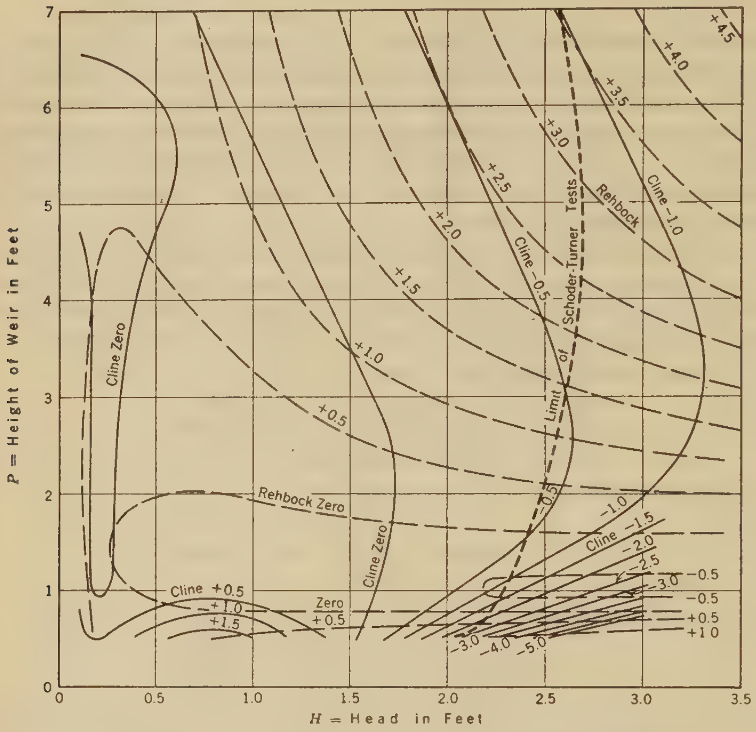


FIG. 4.—PERCENTAGE DISCHARGE BY EQUATION (35), GREATER OR LESS THAN CLINE AND REHBOCK FORMULAS.

In order to compare it with the Bazin and Rehbock equations in the form given by the author (Equations (3) and (4)), Equation (35) is expressed as:

$$\frac{Q}{L} = \frac{2}{3}\left[0.602 + \frac{0.00415}{H} + \left(0.0151 + \frac{0.0636}{P^{1.25}}\right) H\right] \sqrt{2\,g}\; H^{\frac{3}{2}} \ldots (36)$$

Equation (33) gives values of $\frac{Q}{L H^{\frac{3}{2}}}$ almost identical with those by the author's formula. Equation (35) is preferred for greater simplicity in computation as well as being in a more nearly rational form. Fig. 4 shows the percentage deviation of Equation (35) greater or less than by the author's and the Rehbock formulas. Fig. 5 shows a comparison of the same formulas for weirs 0.50 and 7.50 ft high, for extremely low heads. A curve is also shown for Equation (35) with an additional factor, $\left[1 - \frac{(1.122)^{\frac{1}{H}}}{200}\right]$, as a multiplier to the second term, $\frac{0.0222}{H}$. This factor forms an end point to the curves, where the nappe adheres, and follows the experimental data better for heads less than 0.06 ft; but aside from this it is a matter of interest rather than utility. The conformity of the curves of Fig. 5 is shown by comparison with the curves published by Schoder and Turner, showing measured discharge.[9]

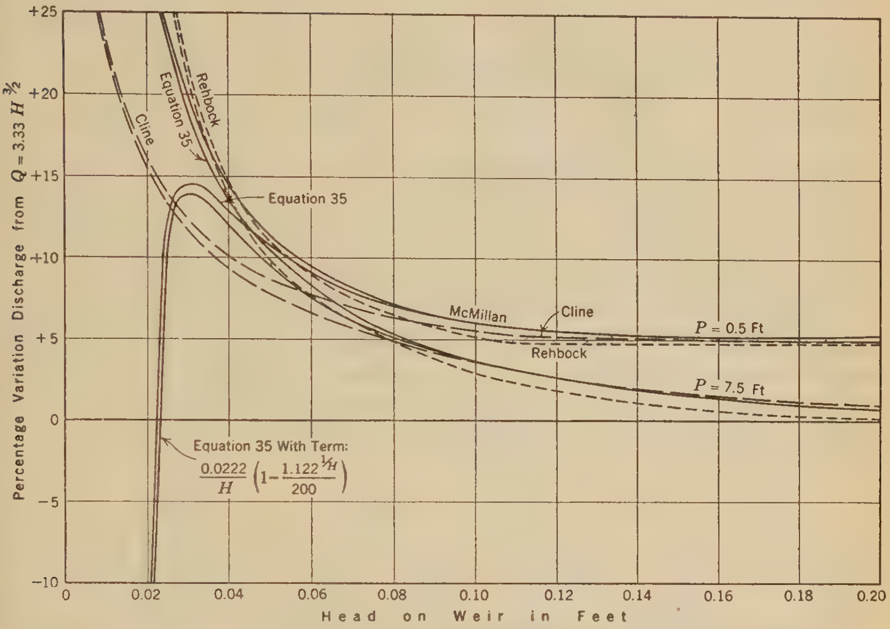


FIG. 5.—COMPARISON OF EQUATION (35) WITH CLINE AND REHBOCK FORMULAS FOR EXTREMELY LOW HEADS.

The generalized Francis formula,

$$\frac{Q}{L} = \frac{2}{3}\, C \left[\left(1 + \frac{\alpha\, h}{H}\right)^{\frac{3}{2}} - \frac{(\alpha\, h)^{\frac{3}{2}}}{H}\right] \sqrt{2\,g}\; H^{\frac{3}{2}} \ldots\ldots\ldots\ldots(37)$$

is the fundamental form from which most, if not all, weir formulas are derived and may be shown to be the genesis of Equation (35). Eliminating the second term within the bracket, Equation (37) may be expressed in very general terms,

$$\frac{Q}{L} = \frac{2}{3}\, C \left[1 + f\,(H,\, P)\right]^{\frac{3}{2}} \sqrt{2\,g}\; H^{\frac{3}{2}} + A + B \ldots\ldots\ldots(38)$$

By a process of curve-fitting and solving for the constants, together with consideration of the studies leading to the original formula and Equation (35), the following is obtained:

$$\frac{Q}{L} = \frac{2}{3}\,(0.602)\left(1 + \frac{0.0683\;H}{P^{1.20}}\right)^{\frac{3}{2}} \sqrt{2\,g}\; H^{\frac{3}{2}} + 0.081\;H^{\frac{5}{2}} + 0.0222\;H^{\frac{1}{2}}$$

---

[9] *Transactions*, Am. Soc. C. E., Vol. 93 (1929), Fig. 11, p. 1029.

or,

$$\frac{Q}{L\,H^{\frac{3}{2}}} = 3.222\left(1 + \frac{0.0683\ H}{P^{1.20}}\right)^{\frac{3}{2}} + 0.081\ H + \frac{0.0222}{H} \quad \ldots\ldots (39)$$

Through expansion of Equation (39) by the binomial theorem the form of Equation (35) is obtained. The second or third terms, or both, might be included in the bracket to the $\frac{3}{2}$ power, and the inclusion of these terms was tried with resulting expressions which followed the experimental data quite as well as Equation (39). Professor Rehbock suggests including the term corresponding to $\frac{0.0222}{H}$, ascribing this factor to be attributable to capillarity.[10] As shown subsequently, if this term is maintained outside the bracket, it appears in a form which suggests a crest condition controlled by sub-atmospheric pressure. It is probable that this term is more rational outside the bracket. The term, 0.081 H, is also probably more rational outside the bracket since it is independent of velocity of approach.

Equation (39) gives values of the coefficient within a fraction of a per cent. of those determined by Equation (35), except for low weirs where the values increase in somewhat the same manner as do those of the author's formula, but not to the same degree. Although Equation (39) is more rational, Equation (35) is preferred for simplicity of computation.

Equation (35) may be expressed as:

$$\frac{Q}{L} = 3.222\ H^{\frac{3}{2}} + 0.0222\ H^{\frac{1}{2}} + 0.081\ H^{\frac{5}{2}} + 0.340\left(\frac{H}{\sqrt{P}}\right)^{\frac{5}{2}} \quad \ldots . (40)$$

This form reveals a combination of simple elements of flow which may be dissected and analyzed. It may be noted, in passing, that Equation (40) lends itself to facility in computation through the use of commonly published tables of squares and the $\frac{3}{2}$ and $\frac{5}{2}$ powers of numbers. The first term, $q_1 = 3.222\ H^{\frac{3}{2}}$, is the theoretical or ideal discharge over a weir of unit width in the familiar form of the Francis formula without velocity of approach, and is the principal term.

The second term, $q_2 = 0.0222\ H^{\frac{1}{2}}$, is the theoretical discharge through an orifice under a head, H, and may be expressed as $q_2 = 0.00277\sqrt{2\,g\,H}$, which represents the discharge of an orifice, or a 0.00277-ft, or $\frac{1}{32}$-in., slit of unit width, without contraction or friction. This term accounts for an increasing value of the coefficient for heads decreasing from 0.20 ft (see Fig. 5). The term is apparently most affected by the top thickness and by the sharpness of the crest and, no doubt, to some extent by the condition of roughness of the up-stream face of the weir close to the crest. This is indicated by tests of Walker and Weidner[11] and may be explained by the condition of sub-atmospheric pressure between the top of the crest and the nappe, induced by the velocity of the water at this point, and a function of $H^{\frac{1}{2}}$. This reduced pressure and consequent orifice action becomes increasingly effective with an increase in thickness of the top of the crest.

---

[10] *Transactions*, Am. Soc. C. E., Vol. 93 (1929), Equations (15) to (18), pp. 1148–1149.

[11] *Loc. cit.*, p. 1046, Fig. 26.

The third term, $q_3 = 0.081\ H$, represents the theoretical discharge without contraction or friction over a triangular weir of constant $\frac{b}{H}$ per unit width, in which, $\frac{b}{H} = 0.0379$ in the familiar formula, $q_3 = \frac{4}{15}\sqrt{2\,g}\left(\frac{b}{H}\right)H^{\frac{5}{2}}$.

The fourth term is the theoretical discharge without contraction or friction over a triangular weir, in which, $\frac{b}{H} = \frac{0.159}{P^{1.25}}$, in the formula, $q_4 = \frac{4}{15}\sqrt{2\,g}\left(\frac{b}{H}\right)H^{\frac{5}{2}}$, and accounts for what is ordinarily referred to as velocity of approach.

These elements of the formula may be represented graphically (Fig. 6), to show a hypothetical distribution of velocity in the nappe. These are not necessarily the actual velocities, but the assumption that they do occur provides an insight to the mechanics of flow over sharp-crested weirs. The third and fourth terms representing discharge over triangular weirs are shown on Fig. 6 to be equivalent increased velocities as measured by the virtual triangular area, to give the same discharge. The parabola, OAX,

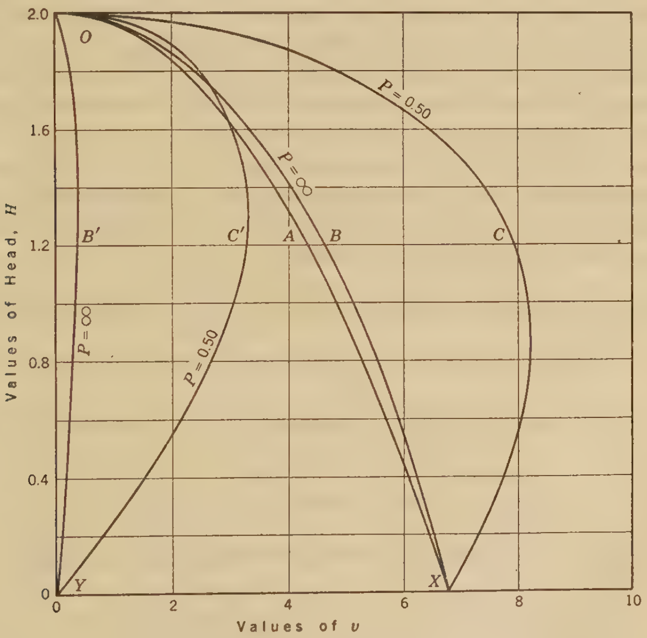


FIG. 6.—HYPOTHETICAL DISTRIBUTION OF VELOCITY IN NAPPE, EQUATION (40).

represents the velocity at any point at which, $v = 0.602\sqrt{2gy}$, and the area, $OAXY$, represents the discharge, $q = 3.222\ H^{\frac{3}{2}}$. The curve, $OB'Y$, is the hypothetical increased velocity due to the third term, $0.081\ H_{\frac{1}{2}}$, and OBX is the distribution of velocity due to this term and likewise the distribution of velocity when the height of the weir is great, at what is usually referred to as "zero velocity of approach". The curve, $OC'Y$, is the increased velocity due to the fourth term, $0.340\left(\frac{H}{\sqrt{P}}\right)^{\frac{5}{2}}$, and OCX is the distribution of velocity due to the third and fourth terms, in which, $P = 0.50$ ft, the curves being based on $H = 2.0$ ft. The area, $OCXY$, represents the discharge according to Equation (40) or Equation (35), neglecting the term, $\frac{0.0222}{H}$, which is nominal for a head of 2 ft. The velocity distribution is hypothetical, but the form is suggestive of the forces that act within the jet. Curve OBX for a weir of great or infinite height suggests the effect of the factor of curved flow, or the inertia of the stream filaments in curved flow, with special reference to the upward direction of the contraction filaments at the crest. Curve OBX is the form of distribution of velocity that would be expected rather than the parabolic distribution, OAX. Curve OCX for a weir of height, $P = 0.50$ ft, with greater velocity of approach and velocity of the filaments within the jet, indicates the effect of curved flow to be more pronounced. It would appear that this manifestation of velocity of approach does partly, if not entirely, obliterate the effect of the velocity head in the velocity of approach as it is accounted for in the Francis formula, the Fteley and Stearns formula, and in formulas of the Bazin type, including the Frese, Swiss Society, and King formulas.

The velocity of approach factor, $1 + \beta\left(\frac{H}{H + P}\right)^2$, in the formulas of the Bazin type, is derived from the generalized Francis formula (Equation 37), omitting the term, $-\left(\frac{\alpha h}{H}\right)^{\frac{3}{2}}$. Expanding by the binomial theorem, substituting $h = \frac{C^2 H^2}{2g(H + P)^2}$, and assigning a derived and definite value to α and C, or $\alpha\ C^2$, within the bracket, results in the formula,

$$Q = CLH^{\frac{3}{2}}\left[1 + \beta\left(\frac{H}{H + P}\right)^2\right]$$

in which, $C = \left(3.248 + \frac{0.079}{H}\right)$ in the Bazin formula and in various forms for others of the same type. As a matter of fact both α and C vary as a function of H and P and the writer's attempt to formulate these values, based on the Schoder and Turner measurements, has led to Equation (39).

Casting out a small term involving an error of less than 1%, the Bazin formula may be expressed as:

$$\frac{Q}{L\,H^{\frac{3}{2}}} = 3.248 + \frac{0.079}{H} + H\left[\frac{1.786\,H}{(H + P)^2}\right] \dots\dots\dots\dots (41)$$

If the envelope of the factor, $y = \dfrac{1.786\,H}{(H + P)^2}$, is substituted in its place, the modified formula will read:

$$\frac{Q}{L\,H^{\frac{3}{2}}} = 3.248 + \frac{0.079}{H} + \frac{0.447\,H}{P} \dots\dots\dots\dots (42)$$

The similarity to the Rehbock formula and Equation (35) is apparent. By introducing the factor, $\dfrac{4\,H\,P}{(H + P)^2}$, the Bazin formula is expressed in a different form:

$$\frac{Q}{L\,H^{\frac{3}{2}}} = 3.248 + 0.079 + 0.447\,\frac{H}{P}\left[\frac{4\,H\,P}{(H + P)^2}\right] \dots\dots (43)$$

When $\dfrac{H}{P}$ is unity in Equation (43), the factor within the bracket becomes unity and Equation (43) is identical with Equation (42). Furthermore, when $H = P$ the factor, $\dfrac{4\,H\,P}{(H + P)^2}$, is a maximum, which accounts for the point of inflection of the Bazin curve and others of the same type (see Fig. 2, for $P = 0.50$ ft). This is also the point of common tangency of the curves for Equations (41) and (42). Due to the fact that formulas of the Bazin type show this inflection while the experimental data show, if anything, a continued curvature or increase of coefficient with head (accounted for in the author's formula and to a lesser degree in Equation (39)), it would appear that formulas of the Bazin type, involving the velocity of approach factor, $\left[1 + \beta\left(\dfrac{H}{H + P}\right)^2\right]$, are incorrect.

It has been advocated many times, and again by the author, that a standard design of suppressed weir be adopted. This is important with reference to the control of the distribution of velocity in a short channel and particular study should be given to the design of a simple means of inducing water in a natural way to a short approach channel if it is not possible to obtain an approach of a length sufficient to insure axial undisturbed flow. A standard distance of point of observation of head becomes increasingly important in the case of high ratio of head over height of weir, due to the water surface draw-down of what approaches the hydraulic drop at the end of a horizontal flume. A standard crest of specific top thickness can readily be duplicated, but there should be no question as to the sufficiency of ventilation beneath the nappe.

The writer has presented several formulas and suggested variations, resulting from three separate studies in derivation. They all reduce most naturally to the general form of Equation (35). This formula, as well as the author's formula and tables, may be applied with confidence for general use. If a high degree of accuracy is required the crest must be truly sharp and $\frac{3}{32}$ in. thick at the top[12] and the approach channel and means of induction of water should conform to that used by Schoder and Turner. However, the importance of the duplication of approach conditions may be overstressed, due to the fact that the water in a normal unobstructed channel tends to adjust itself to a natural regimen on approaching the jet. Inspection of Fig. 4 shows the close conformation of Equation (35) to the Rehbock formula for low weirs. The Rehbock formula is based on careful and comprehensive experiments on weirs from 0.41 ft to 1.64 ft high under comparatively low heads, up to 1.362 ft. Reference to Fig. 2 for $P = 0.5$ ft, shows the Rehbock formula meeting the experimental data as well as, if not better than, the author's formula.

The Schoder and Turner experiments and the formulas based thereon show greater discharge for high weirs (low velocity of approach) than any of the recognized formulas. These older equations have been extrapolated for use not supported by tests. It would appear that it has been the prevailing supposition that high weirs under little or no velocity of approach, under substantial heads, discharge according to parabolic distribution of velocity in the jet—that is, simply at the $\frac{3}{2}$ power of the head. It is indicated herein that the distribution of velocity is reasonably not parabolic and that, due to the action of centrifugal forces in curved flow, the weir discharges an extra quantity above that accounted for by parabolic distribution, determined here to be measured by the term, $q = 0.081\, H^{\frac{5}{2}}$. The possible eccentric approach conditions which may have existed in the run of the Schoder and Turner experiments would be less effective for high weirs with concomitant lower velocity of approach than for low weirs under the same head, and it is reasonable to suppose that the experiments for high weirs are inherently the more accurate. These experiments, complete and comprehensive in range, must be presumed to be correct, at least until they are proved otherwise. The explanation of failure to conform to the recognized formulas is reasonable, and the formulas presented herein are properly substantiated for high as well as for low weirs.

C. G. Cline,[13] Esq. (by letter).—As pointed out by Mr. McMillan, the results of the Schoder and Turner experiments must be presumed to be correct until they are proved otherwise. The fact that they do not agree with many of the standard formulas must be taken as an indication that the formulas are at fault.

---

[12] *Transactions*, Am. Soc. C. E., Vol. 93 (1929), Fig. 3 (Series D to P), p. 1007.

[13] Senior Asst. Engr. Water Power and Hydrometric Bureau, Dept. of the Interior, Canada, Niagara Falls, Ont., Canada.

Professor Draffin discusses a number of formulas and shows how they compare with the Schoder and Turner experiments. Messrs. McMillan and Draffin both agree with the writer that a standard design of suppressed weir should be adopted, a set of carefully planned experiments should be made, and a standard discharge formula and tables computed to agree with the results of the experiments. The computations can be made by using the method of least squares to determine the best numerical values of the coefficients in the algebraic formula selected. This general procedure can be followed without necessarily retaining the particular type of formula used by the writer as an illustration. Mr. McMillan has shown that there need be no scarcity of algebraic formulas; his Equation (35) is relatively simple and yet seems to be sufficiently flexible to cover the entire range of head and height of weir.

In the meantime, as Mr. McMillan states, either his Equation (35), or the writer's Equation (18) and the tables that have been computed from it, may be applied with confidence for general use.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 1900

LABORATORY TESTS OF MULTIPLE-SPAN REINFORCED CONCRETE ARCH BRIDGES

By Wilbur M. Wilson,[1] M. Am. Soc. C. E.

With Discussion by Messrs. C. B. McCullough, Carroll L. Mann, Jr., M. Hirschthal, and Wilbur M. Wilson.

Synopsis

The multiple-span arch bridge differs from a single-span arch in that, for the latter, the ends are fixed, whereas, for the former, the fixed points of the structure are the outer ends of the two end spans and the bases of the intermediate piers. As a live load crosses a multiple-span arch bridge the uneven loading of the two spans adjacent to a pier produces a horizontal thrust and moment at the top of the pier which move the top even if the pier bases and abutments are fixed. This movement of the pier top produces reactions at the end of the span differing materially from those that would be produced by a fixed abutment supporting a similar single-span arch carrying the same load.

The structures tested in this investigation include: (1) A three-span structure consisting of a rib without deck; (2) two three-span structures each consisting of a rib, spandrel columns, and deck, the deck being a considerable distance above the rib of the crown; and (3) two three-span structures consisting of a rib, spandrel columns, and deck, the deck being so low as to be integral with the rib at the crown. Of the two structures constituting each of the latter two groups, one had expansion joints in the deck over the piers only, and the other had expansion joints over the piers and at intermediate joints near the one-third point of each span. These structures were tested at pier heights of 20 ft, 15 ft, and 10 ft, except the structures having a low deck, which were tested at a pier height of 20 ft only.

Note.—Presented at the Joint Meeting of the Structural Division, Am. Soc. C. E., and the Applied Mechanics Division, Am. Soc. M. E., Chicago, Ill., June 29, 1933. Published in April, 1934, *Proceedings*.

[1] Research Prof., Structural Eng., Univ. of Illinois, Urbana, Ill.

INTRODUCTION

Object and Scope of Investigation.—The objects of the tests of the three-span structure consisting of a rib without deck were to determine its load-carrying capacity and to compare the values of reactions and strains measured in the laboratory with values of the corresponding quantities obtained by the elastic theory. There are a number of reasons why this comparison is desirable.

The elastic theory is complicated when applied to a multiple-span arch series on high elastic piers, taking into account the deformation of the piers. Comparatively few engineers have acquired the mastery of this analysis that justifies them in using it with the complete confidence they should have in designing large important structures. It is believed that an experimental verification of this theory will be welcomed by engineers who are called upon to use it.

Tests of a multiple-span arch series having a rib without deck are also desirable to determine whether or not errors in the assumptions upon which the analysis is based, cause errors in the results. Engineers who have mastered the analysis have the same confidence in it that they have in that of simpler structures. In other words, they are confident that the results will be correct if the assumptions upon which the analysis is based are correct; but the assumptions upon which the analysis of an arch is based, are known to be in error. Tests of single-span arches with fixed ends[2] indicate that the errors in the assumptions upon which the analysis of single-span arches is based, do not seriously affect the results. It was hoped that, as a result of these tests, a similar statement could be made relative to multiple-span structures.

The structures consisting of a rib, spandrel columns, and a deck are not readily susceptible of algebraic analysis. An experimental study is especially desirable for these structures since many of this type have been built. In the investigation of these structures, the following information was determined: (1) The elastic constants for a single span; (2) the influence ordinates for fixed-end reactions for a single span; (3) the influence ordinates for reactions at the springing of all spans of the three-span structures; (4) the magnitude and position of the thrust due to the design load; (5) the effect of intermediate expansion joints in the deck upon the load-carrying capacity of the three-span structure; and (6) the maximum carrying capacity of the three-span structure.

Stresses were computed from influence ordinates, determined experimentally, and the maximum stress produced by a given live load was compared for the different structures, all having identical ribs but varying in pier height and type of deck. As a result of the investigation a combined experimental and algebraic method of analyzing multiple-span arch bridges with decks is proposed, which, it is believed, will be useful in designing structures of this type.

---

[2] "Co-Operative Investigation of Multiple-Span Reinforced Concrete Arch Bridges, Report No. 2," *Bulletins Nos. 202* and *226*, Eng. Experiment Station, Univ. of Illinois. Urbana, Ill.

DESCRIPTION OF SPECIMENS AND APPARATUS

Specimens.—The three-span structure consisting of a rib, spandrel columns, and a high deck, is shown in Fig. 1. The details of the ribs and piers (which were alike for all specimens) and the spandrel columns, and deck, are shown in Figs. 2 to 5, inclusive. Fig. 2(*a*) shows the specimens having a high deck and Fig. 2(*g*) those having a low deck. The concrete was designed to have a strength of 2 200 lb per sq in. at 28 days. A 1 : 3 : 3 mix having a 1.2 water-cement ratio (by volume) was used, the quantities for a batch being determined by weight. The sand passed a $\frac{1}{4}$-in. sieve and the gravel a 1-in. sieve. The proportions and the materials were the same for the concrete for all structures, except for the piers of the low-deck bridge. The concrete in these piers contained more water than was used in the other structures. The cylinders and arches were stripped when 10 days old, and the cylinders were stored in the laboratory near the arch so as to be subject to the same atmospheric conditions. The strength of the control cylinders at the time of the final test was 3 310 lb per sq in., for the structure without deck, and 3 412 lb per sq in., for the one with a high deck. The moduli of elasticity for the two structures were 3 293 500 lb per sq in. and 3 145 000 lb per sq in., respectively. For the structure with a low deck the concrete in the arches had an ultimate strength of 4 281 lb per sq in., and a modulus of elasticity of 4 243 000 lb per sq in. The corresponding values for the concrete that went into the piers were 2 871 lb per sq in. and 3 501 000 lb per sq in.

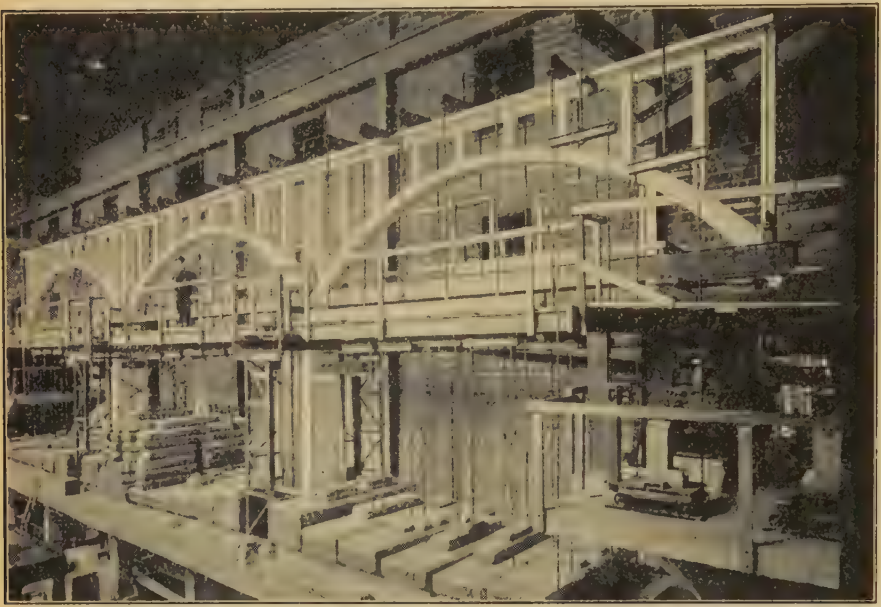


FIG. 1.—THE THREE-SPAN STRUCTURE, WITH A HIGH DECK.

The specimen is not a model, to a definite scale, of a larger structure; but the proportions were based on the design of an arch with a span 135 ft long.

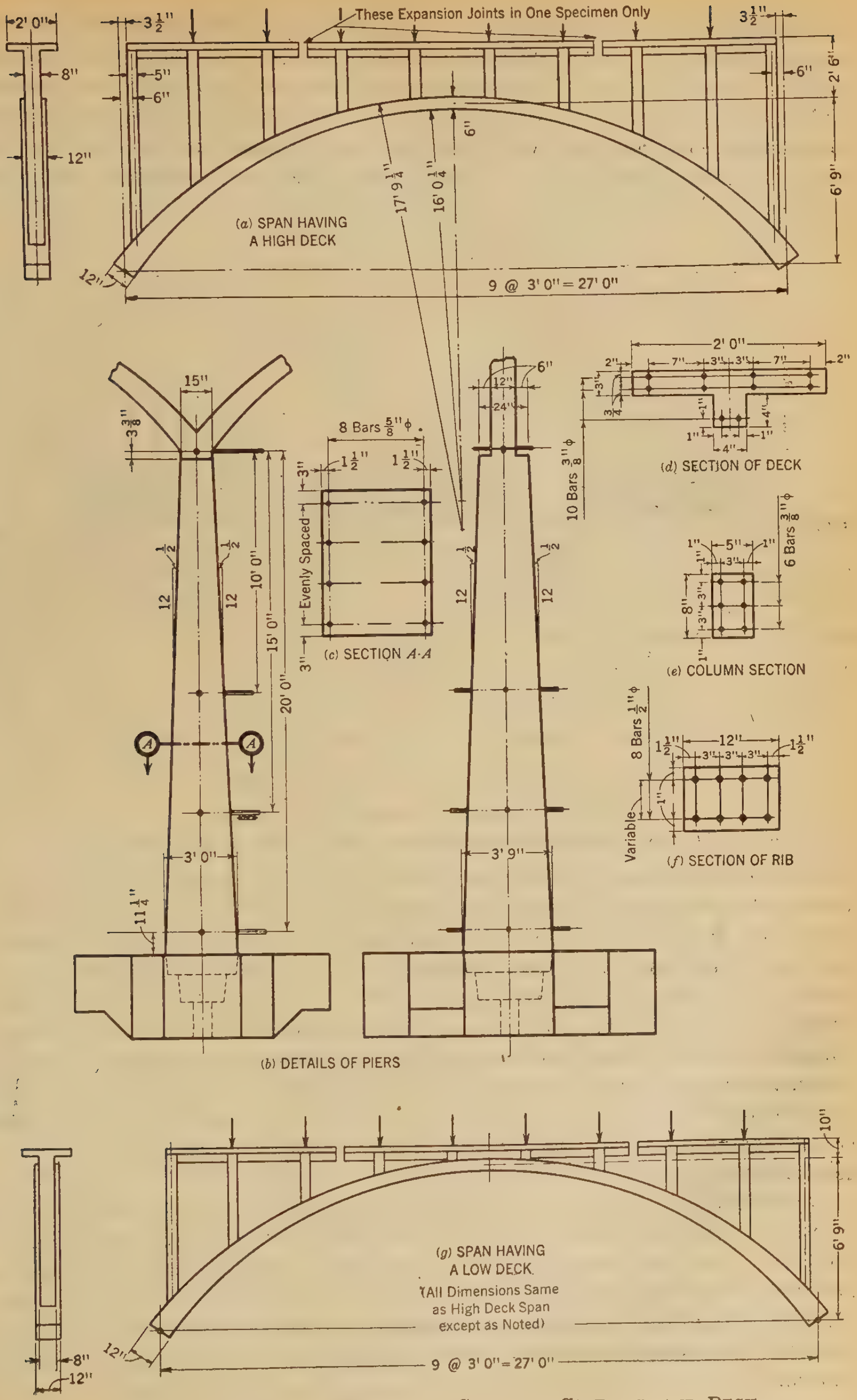

These Expansion Joints in One Specimen Only
2' 0"
8"
12"
3½"
5"
6"
6"
2' 6"
6' 9"
6"
17' 9¼"
16' 0¼"
(a) SPAN HAVING A HIGH DECK
12"
9 @ 3' 0" = 27' 0"
15"
3⅜"
½
12
10' 0"
15' 0"
20' 0"
A
3' 0"
11¼"
8 Bars ⅝"φ
1½"
3"
Evenly Spaced
(c) SECTION A-A
6"
12"
24"
3' 9"
(b) DETAILS OF PIERS
10 Bars ⅜"φ
2' 0"
2"
7"
3"
4"
1"
(d) SECTION OF DECK
6 Bars ⅜"φ
5"
8"
(e) COLUMN SECTION
8 Bars ½"φ
Variable
(f) SECTION OF RIB
10"
6' 9"
(g) SPAN HAVING A LOW DECK
(All Dimensions Same as High Deck Span except as Noted)
12"
8"
9 @ 3' 0" = 27' 0"

FIG. 2.—DETAILS OF PIERS, RIBS, SPANDREL COLUMNS, AND DECK.

five times the length of the spans tested. Likewise, the selection of the live load to be used in testing the arch was based on the live load that would be used in designing a 135-ft highway bridge with two ribs and two lanes of traffic. The weight of the specimen was not great enough to produce a dead-load stress commensurate with the dead-load stresses encountered in the design of a full-sized bridge. A load, designated as the design dead load, was selected, arbitrarily, therefore, distributed so as to have a thrust line within the kern of the rib, and of a magnitude such that the maximum dead-load stress would be of the order of the dead-load stresses existing in actual bridges. The design live load consisted of a distributed load of 960 lb per panel, with a single concentration of 1 800 lb superimposed upon the distributed load. The live load, as usual, was distributed for maximum stress at the section being studied in each instance. The design dead load and live load for maximum stress at Section C_4 for the rib without deck, are shown in Fig. 3. The live load shown is designated as "one L. L."; "two L. L.'s" and "three L. L.'s" are groups of loads two and three times, respectively, the group designated as "one L. L.", in Fig. 3.

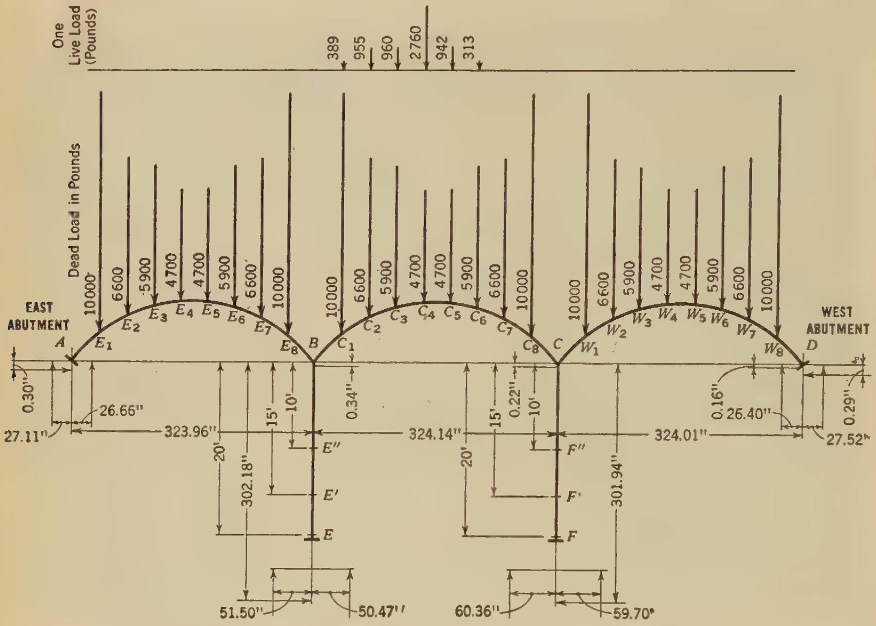

One Live Load (Pounds)
389
955
960
2760
942
313
Dead Load in Pounds
10000
6600
5900
4700
4700
5900
6600
10000
10000
6600
5900
4700
4700
5900
6600
10000
10000
6600
5900
4700
4700
5900
6600
10000
EAST ABUTMENT
WEST ABUTMENT
A
B
C
D
E_1 E_2 E_3 E_4 E_5 E_6 E_7 E_8
C_1 C_2 C_3 C_4 C_5 C_6 C_7 C_8
W_1 W_2 W_3 W_4 W_5 W_6 W_7 W_8
0.30"
27.11"
26.66"
323.96"
20'
302.18"
15'
10'
0.34"
E''
E'
E
324.14"
20'
15'
0.22"
10'
F''
F'
F
301.94"
324.01"
0.16"
26.40"
0.29"
27.52"
51.50"
50.47"
60.36"
59.70"

FIG. 3.—POSITION OF MEASURED REACTIONS, THREE-SPAN RIB WITHOUT DECK.

Apparatus.—The apparatus was built especially for this investigation and it was designed to perform certain specific functions. For example, the support for each terminal, abutment, or pier base, had to be capable of holding the terminal or of returning it to its normal position, and of giving the terminal each of the three component movements, x, y, and θ, in any predetermined amount. Instruments were provided for measuring each of these component movements.

The supports for the abutments and pier bases are shown in Fig. 4 and Fig. 5, respectively. Each abutment was supported on two vertical scales, as shown. The load was transmitted from the abutment to the scales by jacks, two for each scale—one on the north, and the other on the south, side of the abutment. The contact with the jacks is through knife-edges embedded in the abutment so that the line of action of the vertical forces weighed by each scale is accurately known. An abutment can be raised or lowered without rotation by extending or depressing all jacks the same amount; or it can be

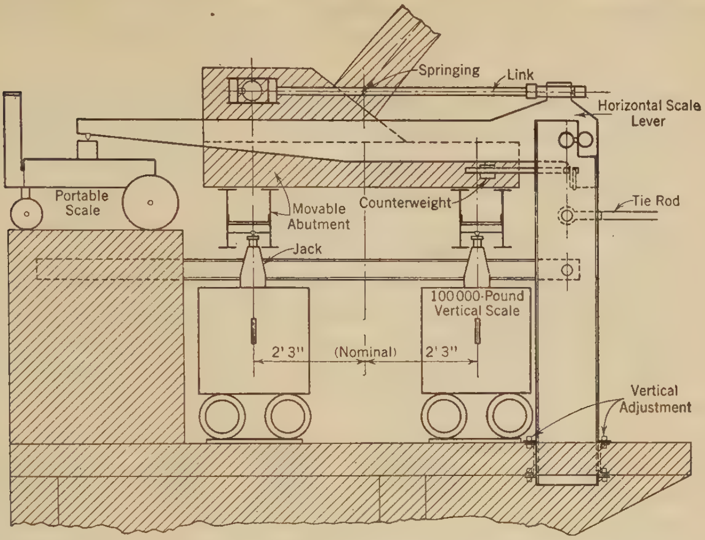

Springing
Link
Horizontal Scale Lever
Portable Scale
Movable Abutment
Counterweight
Tie Rod
Jack
100000-Pound Vertical Scale
2' 3"
(Nominal)
2' 3"
Vertical Adjustment

FIG. 4.—WEIGHING APPARATUS AT THE ABUTMENT.

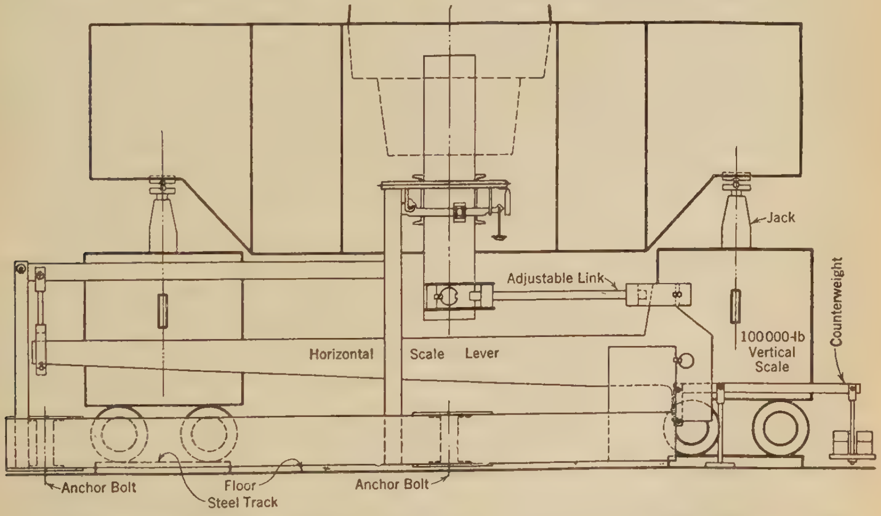

Jack
Adjustable Link
Horizontal
Scale
Lever
100000-lb Vertical Scale
Counterweight
Anchor Bolt
Floor
Steel Track
Anchor Bolt

FIG. 5.—APPARATUS FOR MEASURING REACTIONS OF PIER BASES, THREE-SPAN SERIES.

rotated about a horizontal north-and-south axis by extending both the east jacks and depressing both the west jacks, or the reverse. The vertical scales were mounted on carefully machined steel rollers, 10 in. in diameter, that ran upon a carefully machined track so that the vertical scales offered practically no resistance to horizontal motion.

The horizontal reaction of each abutment was measured by means of a scale, as shown in Fig. 4. This scale consists of two right-angle levers (bell cranks), one on the north, and the other on the south, side of the abutment, that received the horizontal thrust through links and converted it into a vertical force which was delivered to a platform scale. The link connecting an abutment and horizontal scale had knife-edge contacts at both ends, and was maintained in a horizontal position, and the line of action of the horizontal thrust was determined from the position of the knife-edge embedded in the abutment. An abutment could be moved horizontally by turning the link, since the latter had a right-hand thread at one end and a left-hand thread at the other.

The vertical reactions of the pier bases were measured with apparatus similar to that used for the abutments. The horizontal reaction of each pier base was measured with two horizontal scales, one on the north, and the other on the south, side of the structure, shown in Fig. 5.

The relative vertical movement of the piers and abutments was measured with a hydrostatic gauge. This consists of four hook-gauges, one attached to the structure at the top of each pier and one attached to each abutment, connected by a pipe line so that the water level was the same for all gauges at any instant. The vertical movement of the load points on the arch rib was measured relative to two *J*-and-*L* pony **I**-beams, one on each side of the rib, continuous from end to end of the structure. A $\frac{1}{2}$-in. vertical rod embedded in the concrete and projecting below the rib on its center line, was provided directly below each load point. The distance from the lower end of the rod to the **I**-beams was measured with an Ames dial mounted on a steel rod. The **I**-beams, from which the deflections were measured, were also used in measuring the horizontal movement of the west abutment and of the tops of both piers, relative to the east abutment. These beams were attached to a 1-in. pin that extended through the rib at the east springing of the east span in such a way as to cause the beams to move horizontally with the east abutment. The beams were supported at the pier tops and at the west abutment on rollers that enable the pier tops and west abutment to move horizontally relative to the beams. Ames dials attached to the beams, with their plungers bearing on pins through the ribs at the springings, indicated horizontal movement of the west abutment and of the tops of the piers, relative to the east abutment.

The horizontal movement of the pier bases relative to the east abutment, was measured by means of the frame shown in Fig. 6. The member, *A B C D*, consists of the **I**-beams from which vertical deflections were measured. The frame, *B C F E*, was suspended from these beams. The main members of the frame were 12-in. junior **I**-beams. Ames dials attached to the lower corners of this frame had plungers bearing upon the pins, *E* and *F*, which projected from the sides of the piers. The pins, *E* and *F*, were 20 ft below the springing of the arches; Pins *E′* and *F′* and Pins *E″* and *F″* were 15 ft and 10 ft, respectively, below the springing. The horizontal movement was measured at Pins *E* and *F*, *E′* and *F′*, and from Pins *E″* and *F″*, when testing the structure at pier heights of 20 ft, 15 ft, and 10 ft, respectively. In tests requiring one pier to be raised, shims were inserted at that pier between the beam, *A B C D,* and its support. When the pier was raised, the shims were removed so that Points *B* and *C* on the beam remained at the same level; the sides of the suspended frame remained in a vertical position; and the corners, *E* and *F,* did not move horizontally because of the vertical movement of the piers.

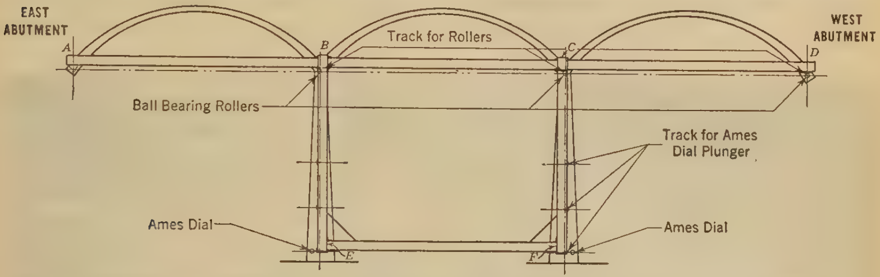


Fig. 6.—Frame for Measuring Horizontal Movement of Pier Bases Relative to Abutments, Three-Span Series.

The angular position of the abutments and pier bases was determined with level bubbles attached to the structure at the points where the rotation was to be measured. The bubbles at the abutments were carried on steel bars embedded in the concrete and projecting from the rib at its center line and normal to the axis of the rib at the springing. The bubbles for the piers were carried on horizontal bars embedded in the concrete, one at the top and one each at distances below the top of 20 ft, 15 ft, and 10 ft. The former was used to determine the rotation of the top of the pier and the latter, to determine the rotation of the base when testing the structure at various pier heights.

In tests for which the abutments and the bases of the piers were fixed, the bubbles were adjusted so as to be in their mid-position before the test began and, after the load was changed, the abutments and pier bases were rotated until the bubbles returned to their central position. In tests for which a predetermined angular movement was to be produced, the bubble was adjusted to its mid-position before the test began and then the piers and abutments were rotated by manipulating the jacks supporting them until the bubble was off center by an amount which, as shown by a previous calibration, corresponds to the desired rotation.

The strain in the concrete was measured with an 8-in. Berry strain-gauge. Readings were taken on two gauge lines on the intrados and two on the extrados at the section midway between each pair of adjacent load points. Strains were measured for the design load tests and for the tests to determine the load-carrying capacity of the structure.

The load on the structure was produced by suspending concrete blocks of known weight at the load points. For each load point there was one large concrete block suspended by four steel rods, as shown in Fig. 7, which served as a loading platform on which to place smaller blocks which, in combination with the loading platform, constitute the total load to be applied at a particular point. When the arch was unloaded, the loading platform rested on the concrete girders of a trestle provided for the purpose and the loading beam, which extended across the top of the arch above the loading shelf, was carried on the steel suspension rods acting as struts. To apply the load, the turnbuckles in the suspension rods were turned, shortening the rods, until the loading beam came into contact with the loading shelf. The turnbuckles were then turned, successively, by small amounts, in order, back and forth from one end of the structure to the other, thereby transferring the weight of the loading platforms gradually from the girders to the arch. Each loading shelf on the arch was capped with a steel plate containing a $\frac{3}{4}$-in. steel ball located in its top surface and at the point of application of the load. The loading beam had a small steel block attached at the center of the bottom flange. A small depression in the bottom of this block fitted over the top of the ball in the steel plate on the loading shelf, thus locating, accurately, the point of application of the load.

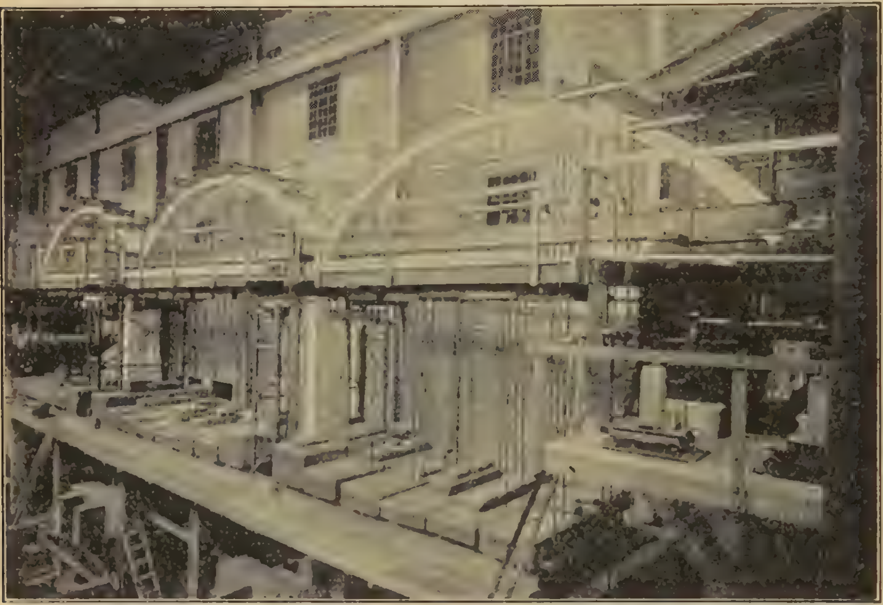


FIG. 7.—THREE-SPAN STRUCTURE CONSISTING OF A RIB WITHOUT DECK WITH APPARATUS ATTACHED READY FOR TESTING.

The load at a given load point, for a particular test, was obtained by placing concrete blocks of known weight on the loading platform until the desired load had been obtained.

DESCRIPTION OF TESTS

Influence Ordinates by Unit Loads.—The direct objective of many of the tests was to determine the influence ordinates for the reactions at the springings of the various spans since, with these influence ordinates known, each span is statically determinate under any system of loads.

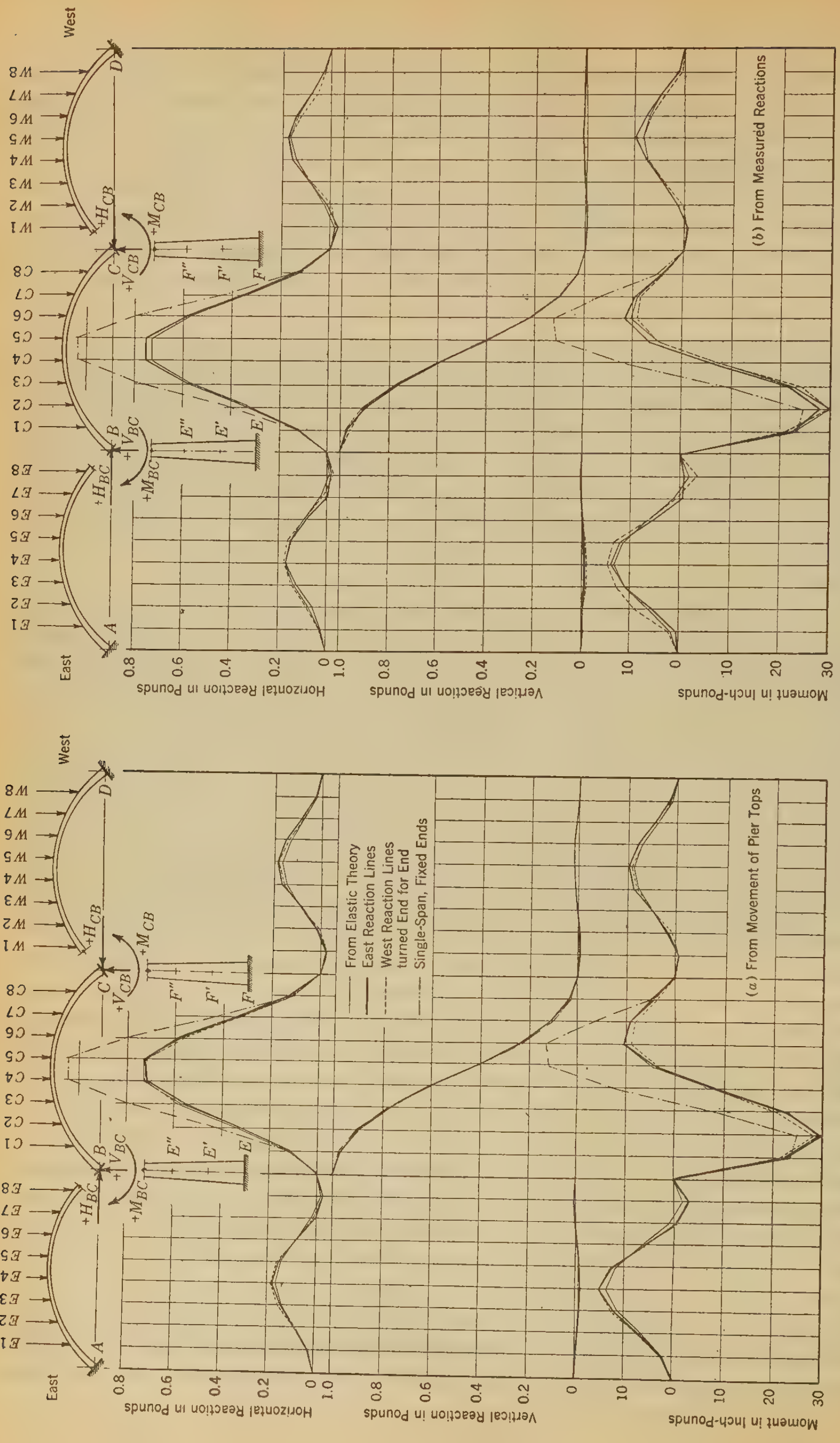
The influence ordinates were determined experimentally by measuring the changes that occurred when a unit load was applied and removed, successively, at the various load points. The unit load was 2 000 lb and the tests were made with the design dead load on the structure. The reactions at the springings were determined in two ways: (1) By weighing the reactions at the terminals, before and after the application and removal of the unit load, and computing the moment and the horizontal and vertical components of the reactions at the springings from the changes in the readings of the scales at the terminals; and (2) by measuring the components of the movement (x, y, and θ) of the pier tops, due to the application and removal of the unit load, and computing the moment and the horizontal and vertical components of the reactions corresponding to these movements. This presupposes that the fixed-end reactions due to a unit load and the elastic constants for the arches are known.

In making these tests all terminals were returned to their normal position before readings were taken so that the only changes would be those due to the application and removal of the load. Three sets of observations were made for each load point: Before the load was applied, after it was applied, and after it had been removed; that is, the changes in reactions, due to adding and removing the load, were determined separately. The average of the two results is reported in each instance. Tests were made for each load point over the entire length of the structure. Since the structure is symmetrical, values determined for load points on the east half were checked against values for corresponding points on the west half of the same structure.

The effective height of the pier is the vertical distance from the springings of the arches to the point on the pier that was returned to its original position before and after each load change. By attaching the instruments indicating the position of the pier base at different distances below the springings, the one structure could be tested at different pier heights. The influence ordinates for reactions at the springings were determined for structures with pier heights of 20 ft, 15 ft, and 10 ft, except that the low-deck structure was tested at a pier height of 20 ft only.

The fixed-end reactions for each of the two end spans were determined by reading all scales at the abutments before and after applying (and removing) a unit load of 2 000 lb at each load point in the end spans. For this test, the points to be returned to their normal position are the abutments and pier tops. The structure carried the design dead load throughout this series of tests.

The elastic constants for the end spans were determined by noting the changes in the readings of all scales at the abutments due to giving one end of a span, successively, each of the component movements (x, y, and θ). The



(a) From Movement of Pier Tops

(b) From Measured Reactions

FIG. 8.—INFLUENCE LINES BY UNIT LOADS; REACTIONS, TOPS OF 20-FOOT CENTER-SPAN PIERS.

design dead load was on the structure during this test, and one end of the span being tested was fixed and the other end restrained so as to have only the one component of movement being studied.

The elastic constants and the fixed-end reactions, determined in the manner just described, were used to compute the reactions at the springings from the movement of the pier tops accompanying the application and removal of a unit load.

The influence lines for the reactions at the springings of the center span for a three-span structure, consisting of a rib without deck and having 20-ft piers, determined from the movement of the pier tops, are shown in Fig. 8(*a*). The difference between the influence lines for the middle span of a three-span series and for a single span with fixed ends, is worthy of note. The influence lines of Fig 8(*b*) are for the same functions and for the same structure as those in Fig 8(*a*) but the lines derived by experiment were determined from the measured reactions at the terminals instead of from the movement of the pier tops. The light full lines representing the computed values are the same for the two diagrams.

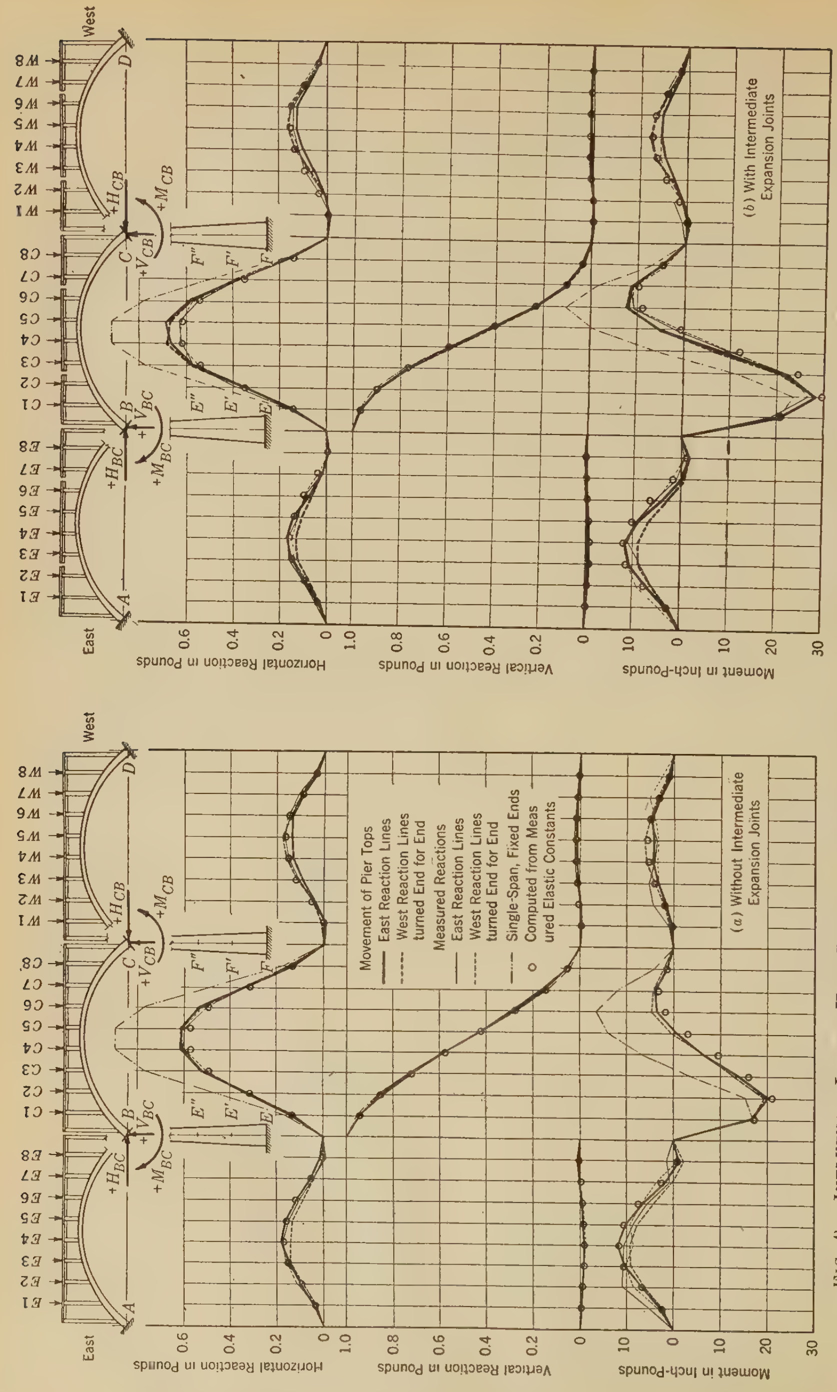
Diagrams similar to those shown in Fig. 8(*a*) and Fig. 8(*b*) were also constructed for the abutment, and for the pier reactions of the end spans. They were constructed for the structure having a rib without deck when tested, and analyzed with pier heights of 20 ft, 15 ft, and 10 ft. The same diagrams, except those based on the algebraic analysis, were constructed for the high-deck bridges both with and without intermediate expansion joints, and they were also constructed for the low-deck type but, for the latter, the diagrams were drawn for the structures having 20-ft piers only. Lack of space has excluded most of these diagrams from this paper, but the results of the tests they represent have influenced the conclusions in the "Summary of Results."

Algebraic Analysis Based on Elastic Constants Determined Experimentally.—The analysis of the three-span arch series involves the use of elastic constants for the pier and arch span. These constants can be determined algebraically for structures consisting of a rib without deck; but the corresponding constants for the arch span cannot readily be determined algebraically for a structure with spandrel columns and a deck. However, Donald E. Larsen, Jun. Am. Soc. C. E., under the supervision of the writer, has developed[8] a method for analyzing these structures by using experimentally determined elastic constants for the arch spans in the otherwise algebraic method. The small circles of Figs. 9 and 10, represent the influence ordinates obtained by this combination experimental and algebraic method.

It is believed that the algebraic analysis of a multiple-span structure in which experimentally determined elastic constants are used, gives accurate results. The fact should be borne in mind, however, that the results obtained by the algebraic method are based on the fixed-end reactions for a single span, and an error in these will introduce an error in the results of the analysis of the three-span structure.

---

[8] "A Study of Multiple-Span Arches"; A Thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in Civil Engineering in the Graduate School of the Univ. of Illinois, 1930.



(a) Without Intermediate Expansion Joints

(b) With Intermediate Expansion Joints

FIG. 9.—INFLUENCE LINES, HIGH-DECK SPANS, BY UNIT LOADS; REACTIONS, TOPS OF 20-FOOT CENTER-SPAN PIERS.

Capacity Tests.—Tests were made (at only one pier height, 20 ft) to determine the load-carrying capacity of all structures. For the structure consisting of a rib without deck, the capacity test was begun with no load on the structure, except its own weight. Readings were then taken from which to determine the reactions at the terminals, the vertical movement of points on the arch rib directly beneath the load points, and the strain in the concrete of the rib at sections midway between the load points. The first load increment was the design dead load. This was followed by increments of one live load until four live loads had been applied.

The structure consisting of ribs, spandrel columns, and a high deck, was first tested with no intermediate expansion joints in the deck, the method of testing being similar to that used in the test of the structure having a rib without deck. The structure was first loaded for maximum stress at Section *B* of Span *BC*. Five live loads were applied, and then the test was discontinued. The same structure was then tested in a similar manner, but the load was distributed to produce a maximum moment about the arch axis on a section through the load point, C_4. This test was also discontinued after five live loads had been applied. Intermediate expansion joints were then cut through the deck at the sections indicated in Fig. 2(*a*), and the resulting structure was subjected to a capacity test, the live load being distributed to produce a maximum stress in the rib at the section through the load point, C_3. The structure with a low deck was tested in the same manner as the one with a high deck.

Movement at Expansion Joints Due to a Change in Span.—The need of expansion joints in the deck of an arch bridge depends upon the movement that would take place if such joints were provided. Tests were made on the structures having a deck with intermediate expansion joints to determine the relation between a change in span and the corresponding opening and closing of the expansion joints.

For this test the structure carried the design dead load, and the initial readings were taken after the east and west abutments had each been moved inward 0.075 in., thereby making the total length of the structure 0.15 in. less than normal. Ames dials attached to the deck indicated the change in width of the expansion joints. After the initial readings had been recorded, the abutments were moved out 0.30 in., thereby increasing each span 0.10 in. The tops of the piers were brought to the same vertical and angular position as for the initial readings; and the Ames dials indicating the changes in span at the tops of the piers and the movement at the expansion joints, were read, thereby giving the change in span of each arch and the movement at each expansion joint due to the increase in span. The abutments were then returned to their initial position, and the readings were again recorded after the pier tops had been adjusted for vertical and angular position.

The ratio of the movement at an intermediate expansion joint to the change in span was 0.25 for the structure with a high deck and 0.11 for the structure with a low deck. For both structures the joint closed as the span increased.

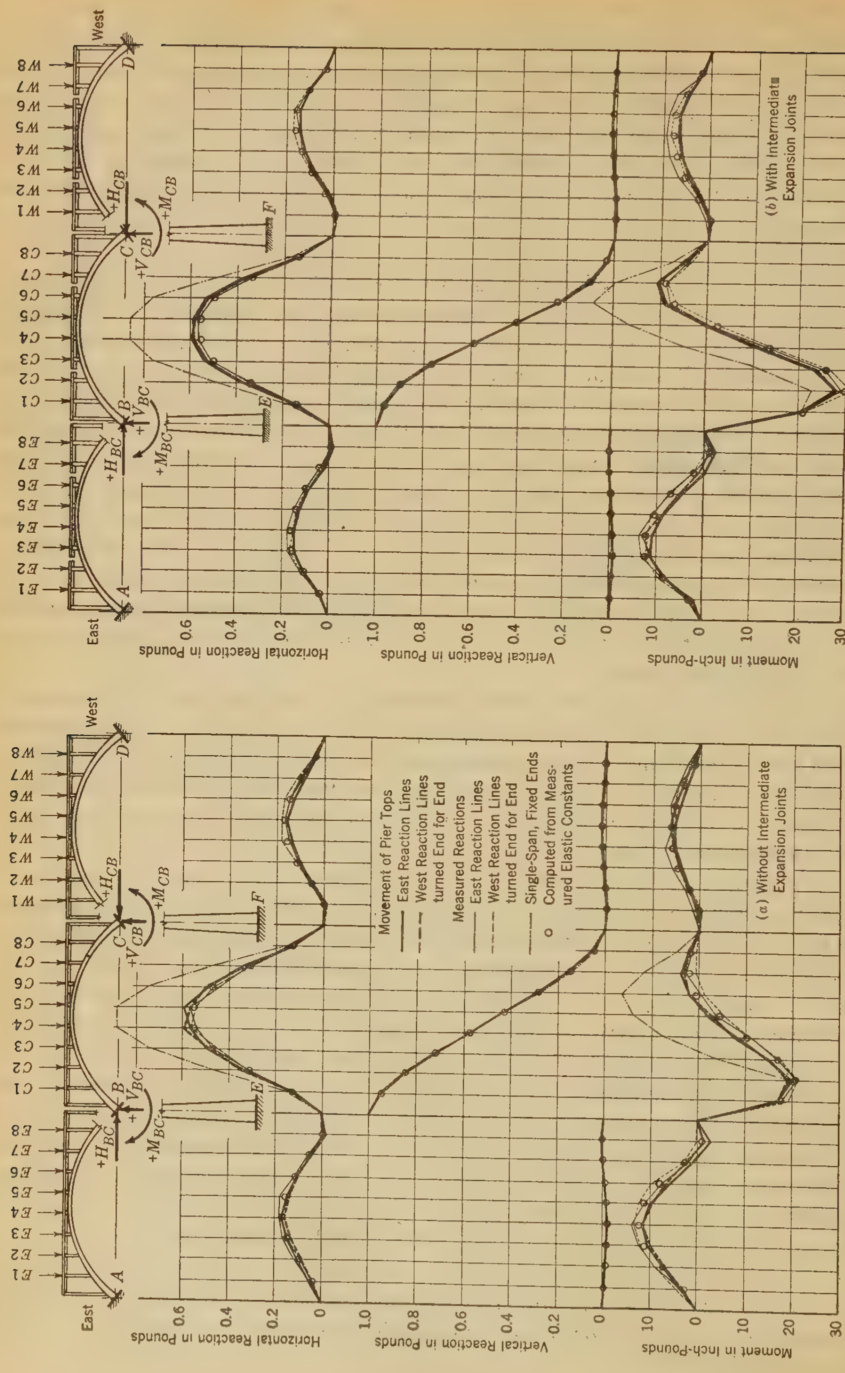


FIG. 10.—INFLUENCE LINES, LOW-DECK SPANS, BY UNIT LOADS; REACTIONS, TOPS OF 20-FOOT CENTER-SPAN PIERS.

Discussion of Results

Effect of Deck on Fixed-End Reactions.—The experimentally determined influence lines for fixed-end reactions of a rib without deck and an arch having a high deck, both with and without intermediate expansion joints, are compared in Fig. 11(*a*). A similar comparison for a rib without deck and an arch with a low deck is made in Fig. 11(*b*). The curves show that the values of the horizontal thrust for the four structures having a deck are nearly alike and that they are much less than the value for the rib without a deck. The vertical reactions for the structures having a deck, with expansion joints, are about the same as for a rib without deck, and both differ materially from the reactions for a structure having a deck, without expansion joints. The fixed-end moment is much less for the structure having a deck, without expansion joints, than it is for a rib without deck; and the fixed-end moment for the structure having a deck with expansion joints lies between the values of the same function for the other two structures. The fixed-end moment

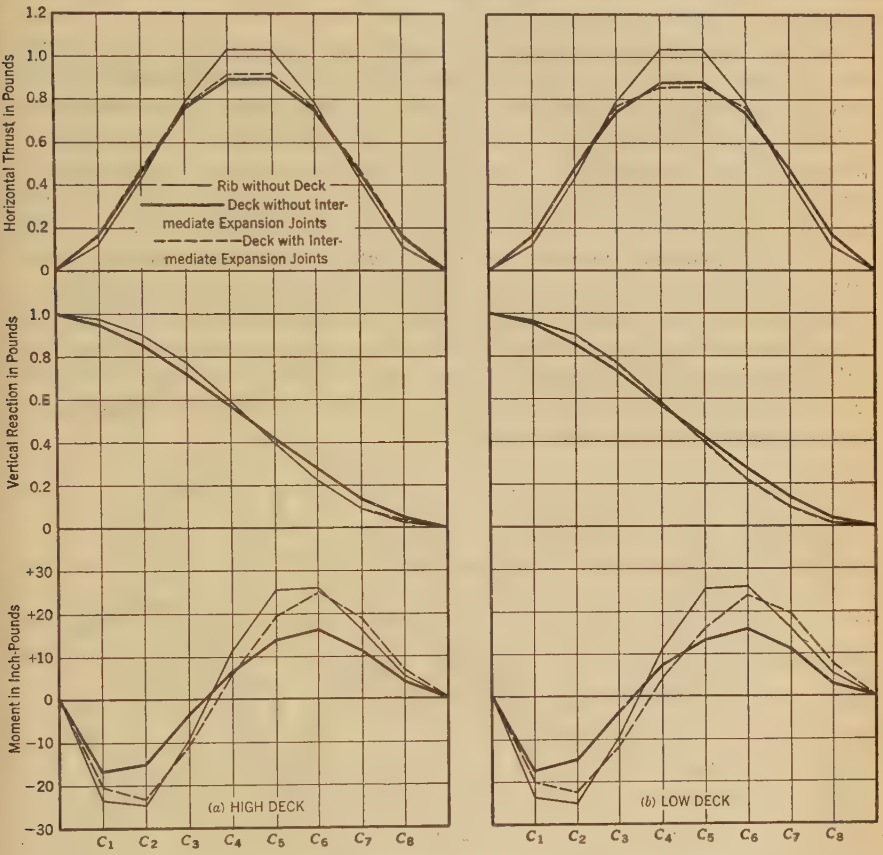


Fig. 11.—Influence Lines for Fixed-End Reactions; Various Types of Structures.

is about the same for a structure having a high deck, without expansion joints, as it is for one having a low deck, without expansion joints.

The stress at the springing of a single span is governed almost entirely by the fixed-end moment. It is, therefore, much less for the structure having a deck without expansion joints than it is for the other two structures. For the single span having expansion joints in the deck, the weak section is at Section C_3. The influence lines for stress at this section for various types of single-span structures, are given in Fig. 12.

The fact that the deck reduces the moment at the ends where its strengthening of the rib is the least, is of special interest in considering the load-carrying capacity of a rib and deck designed to function as a Vierendeel truss.

Influence of Flexure of Piers on Stress in Arches.—Figs. 8 to 10, inclusive, containing the influence lines for the reactions of the piers on the center span, show that the flexure of the piers greatly modifies the shape of the influence lines for moment and horizontal thrust but not for vertical reactions. The stress at the springing, possibly the weak section of an arch having a deck without intermediate expansion joints, is, approximately, a direct function of the moment, and the influence lines for the moment referred to, show that the live load compression at the springings will be greater at the intrados and less at the extrados for all five types of three-span structures on high piers than for similar single spans with fixed ends. However, the stress at this section is not critical for the structure tested.

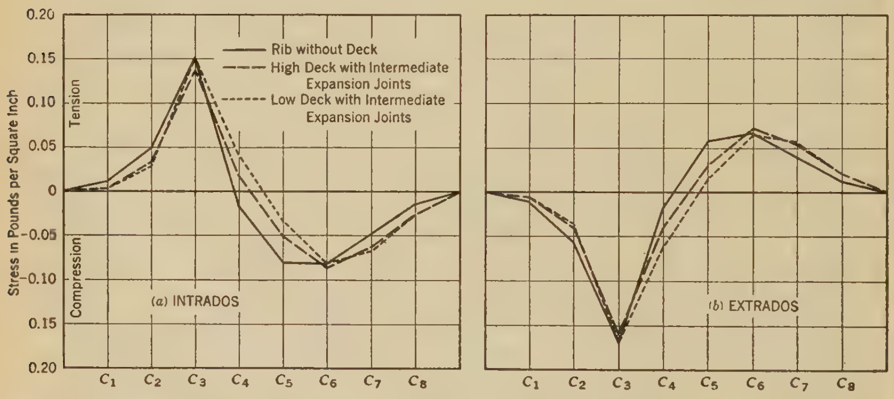


FIG. 12.—INFLUENCE LINES FOR STRESS AT C_3, FOR VARIOUS TYPES OF SINGLE SPANS WITH FIXED ENDS.

The moment at Section C_4 (the weak section of a three-span structure consisting of a rib without deck) is a function of all three components of the reactions at the springing, and the effect of the change in moment is offset by the change in the horizontal and vertical reactions. The influence lines for stress at Section C_4 for three-span structures having a rib without deck and having pier heights of 20 ft, 15 ft, 10 ft, and 0 ft (single span with fixed ends) are given in Fig. 13. These diagrams show that, even if the flexure of the pier greatly affects the reactions at the springings, the effect of the pier height on the stress at Section C_4 is much less. The stress at Point C_4 due to the design live load is 36% greater for a three-span structure having 20-ft piers than it is at Section C_3 for a single span with fixed ends. The stress at the same point due to the total design load, dead load plus live load, is 13% greater for a three-span structure having 20-ft piers than the maximum stress in a single span with fixed ends. All the values used in this comparison were determined by the elastic theory and are based on a value

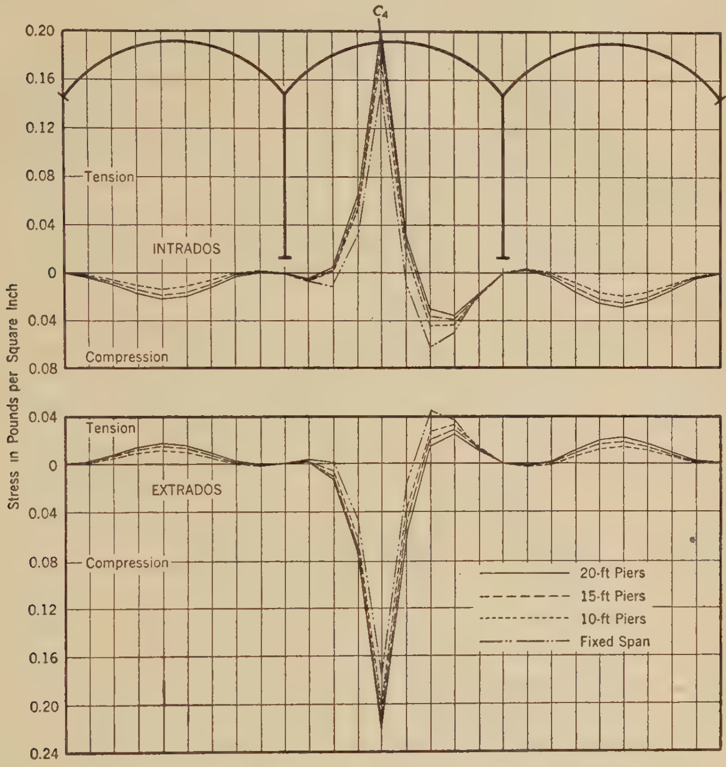


FIG. 13.—INFLUENCE DIAGRAM FOR STRESS AT LOAD POINT C_4.

of E for the concrete of 3 333 000 lb per sq in. It was assumed, furthermore, that concrete takes tension and that the abutments and the bases of piers are fixed.

The moment at Section C_3, the weak section of a structure having a deck with intermediate expansion joints, is a function of all three components of the reaction at the springing. The influence lines for stress at Section C_3 for a three-span structure having a high deck with intermediate expansion joints and pier heights of 20 ft, 15 ft, 10 ft, and 0 ft (single span with fixed ends), are given in Fig. 14(a). These influence lines were determined from an algebraic analysis based on experimentally determined elastic constants. The stress due to the design live load is somewhat greater (145 lb per

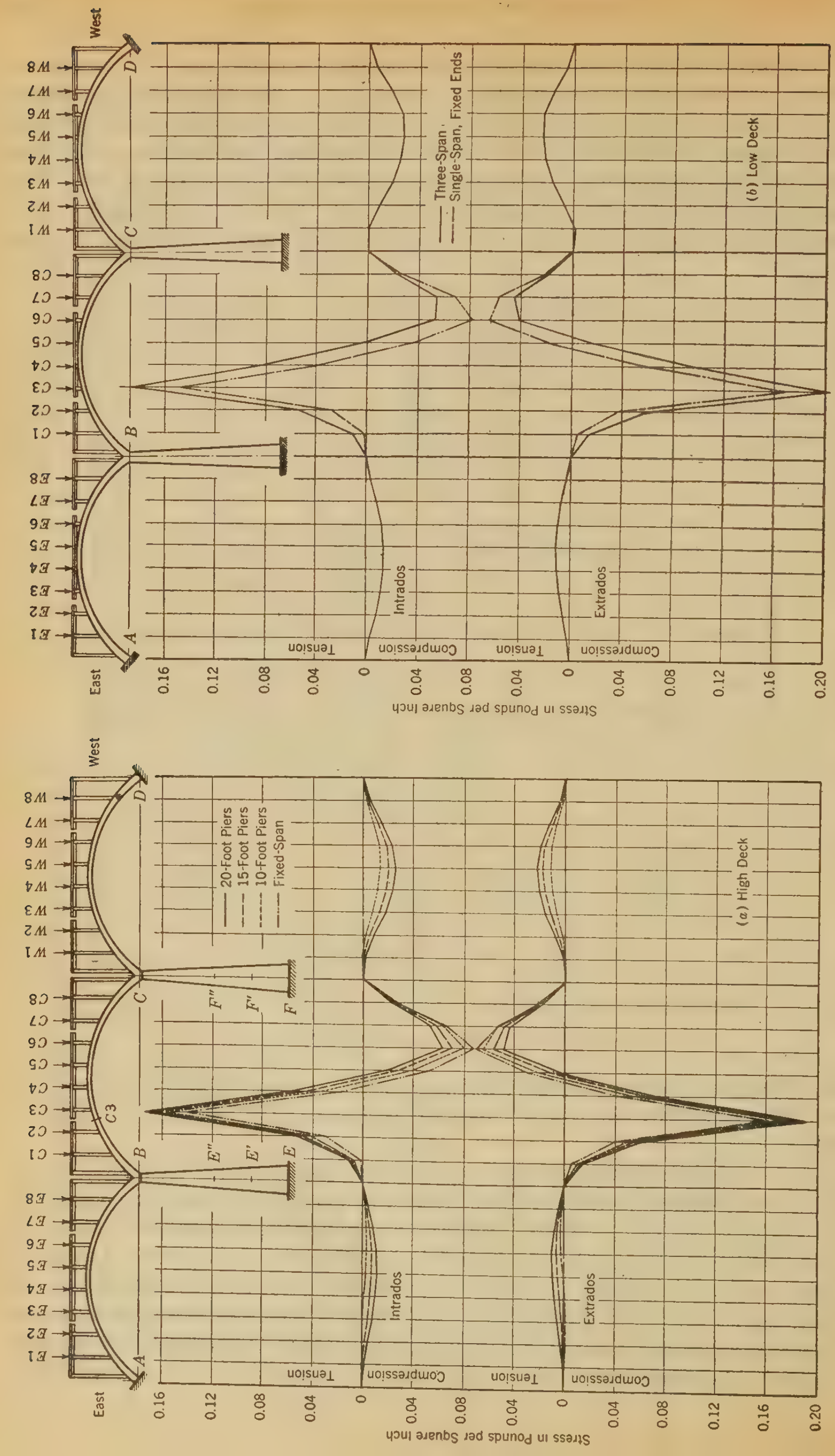


FIG. 14.—INFLUENCE LINES FOR STRESS AT C_3, WITH INTERMEDIATE EXPANSION JOINTS (FROM ALGEBRAIC ANALYSIS).

sq in.) for the structure with a high deck having 20-ft piers than for a similar single-span structure with fixed ends. This increase is 28% of the stress due to live load and 14% of the stress due to dead load plus live load in a single span with fixed ends.

The influence lines for stress at Section C_3 for a three-span structure having a low deck with intermediate expansion joints and pier heights of 20 ft and 0 ft (single span with fixed ends), are given in Fig. 14(*b*). The stress due to the design live load is 173 lb per sq in. greater for the three-span structure having 20-ft piers than for a similar single span with fixed ends. This is 31% of the stress due to the live load and 16% of the stress due to the live load plus dead load.

For a composite structure consisting of a rib and deck without intermediate expansion joints, the moment over the central portion of the span is greater than for a single span with fixed ends; but the composite structure consisting of the rib and deck has a much greater moment-resisting capacity than a rib without deck and the presence of a large moment over the central portion of the span is more than offset by the strengthening effect of the deck.

Effect of Variations in the Modulus of Elasticity of the Concrete on Stress in the Rib.—In the study made to determine the effect of a variation in the modulus of elasticity of the concrete on the stress in a three-span arch series having decks, the influence lines for stress were determined for two structures which had the same dimensions, but which were assumed to have been made of material having different physical properties. For the first structure, the elastic constants were the same for all spans, but for the second structure, while the elastic constants for the center span were the same as for the first structure, those for the end spans were only two-thirds as great. This comparison was made for structures having a high deck with and without intermediate expansion joints, the pier height being 20 ft in both instances. The assumed variations in the physical properties did not greatly change the influence ordinates and it is believed that any probable variation in the modulus of elasticity of the concrete will not seriously affect the live load stresses. However, because of the peculiar action of a multiple-span arch on high piers, the effect of variations in the properties of the material on the dead load stresses is not easily determined.

For the three-span structure consisting of a rib without deck, the weak section is at Section C_4. For a homogeneous symmetrical structure, the dead load stress should have the same value at Sections E_4, E_5, C_4, C_5, W_4, and W_5. The most dependable experimental method of obtaining the dead load stress is to determine the magnitude of the thrust from the measured reactions and the position of the thrust from the measured strain at the intrados and extrados. The consistent relation between the pressure centers located from the measured strain and the thrust line located by the elastic theory, shown in Fig. 15, appears to justify the conclusion that the dead load stresses in this structure did not differ greatly from those in a similar single-span arch subjected to the same dead load.

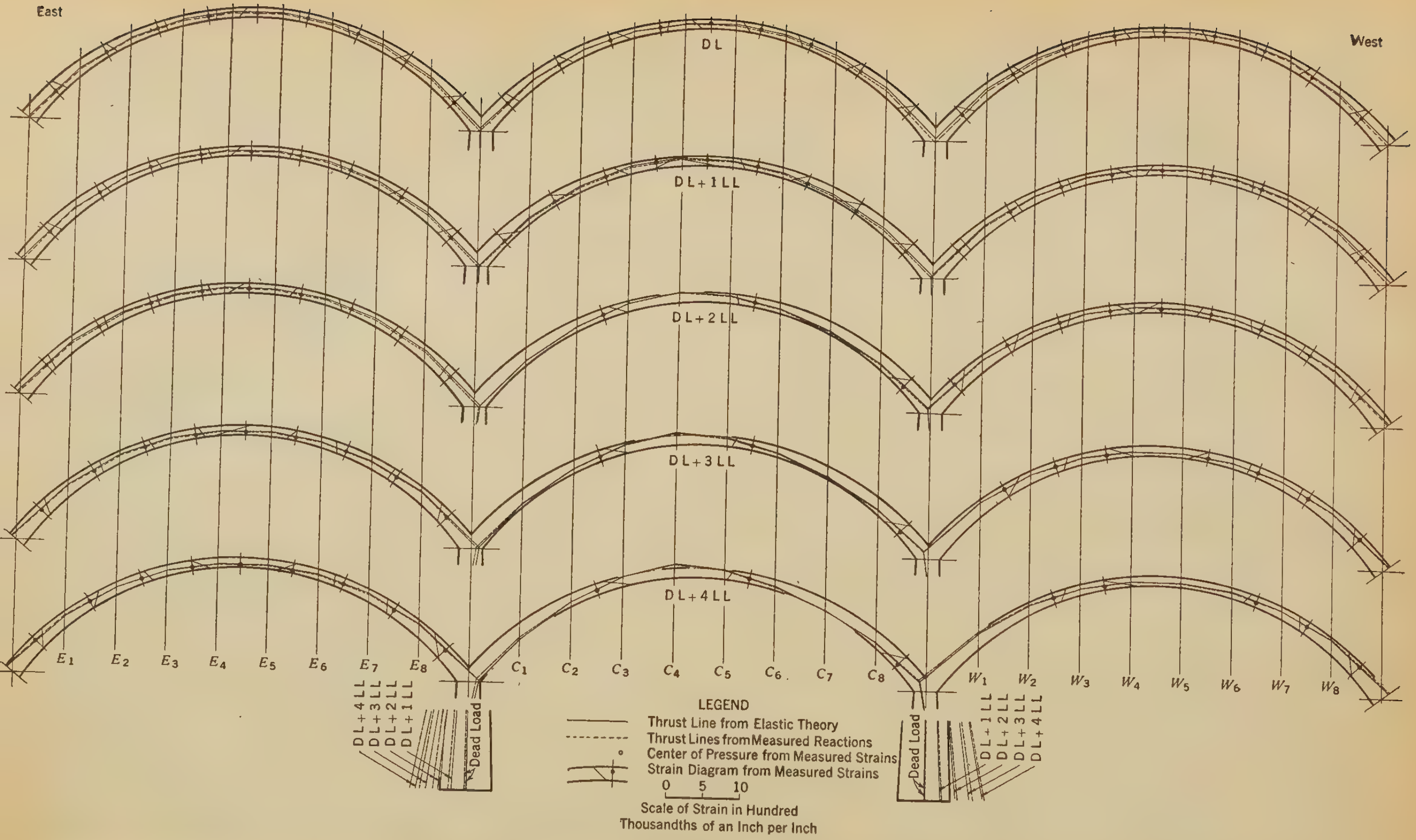

East
West
DL
DL+1LL
DL+2LL
DL+3LL
DL+4LL
E_1 E_2 E_3 E_4 E_5 E_6 E_7 E_8
C_1 C_2 C_3 C_4 C_5 C_6 C_7 C_8
W_1 W_2 W_3 W_4 W_5 W_6 W_7 W_8
DL+4LL
DL+3LL
DL+2LL
DL+1LL
Dead Load
Dead Load
DL+1LL
DL+2LL
DL+3LL
DL+4LL
LEGEND
Thrust Line from Elastic Theory
Thrust Lines from Measured Reactions
Center of Pressure from Measured Strains
Strain Diagram from Measured Strains
0 5 10
Scale of Strain in Hundred Thousandths of an Inch per Inch

FIG. 15.—THRUST LINES; TEST TO FAILURE (20-FOOT PIERS).

For the structures having a deck without intermediate expansion joints the critical stress is at the intrados of Section B of Span BC. The dead load stress should be the same at the springings of all spans. The stresses at these sections, determined from the measured thrust the line of action of which was located from the measured strain, are given in Table 1. The maximum and average values for the structure having a high deck are — 338 lb per sq in. and — 232 lb per sq in., respectively, and the maximum and average values for the structure having a low deck are — 198 lb per sq in. and — 155 lb per sq in.

TABLE 1.—DEAD-LOAD STRESSES DETERMINED FROM THE THRUST. THREE-SPAN STRUCTURE HAVING A DECK WITHOUT INTERMEDIATE EXPANSION JOINTS

	STRESS AT INTRADOS AT SECTION:						
	A of Span AB	B of Span AB	B of Span BC	C of Span BC	C of Span CD	D of Span CD	Average
High deck.....	—287	—267	—176	—110	—215	—338	—232
Low deck......	—181	—198	—137	—122	—137	—154	—155

For the structures having a deck with intermediate expansion joints the critical stress is at the extrados of Section C_3. The thrust line and centers of pressure for these structures are shown in Fig. 16. The dead load stresses at the sections corresponding to Section C_3, determined from the measured thrust the line of action of which was located by the measured strain, are given in Table 2. The maximum and average values for the structure having a high deck are — 782 lb per sq in. and — 647 lb per sq in., respectively, and the corresponding values for a structure having a low deck are — 755 lb per sq in. and — 646 lb per sq in., respectively.

TABLE 2.—DEAD-LOAD STRESSES DETERMINED FROM THE THRUST. THREE-SPAN STRUCTURE HAVING A DECK WITH INTERMEDIATE EXPANSION JOINTS

	STRESS AT EXTRADOS AT SECTION:						
	E_3	E_6	C_3	C_6	W_3	W_6	Average
High deck.....	—711	—437	—782	—606	—638	—707	—647
Low deck......	—560	—755	—610	—685	—626	—637	—646

The data given in the preceding paragraphs have value only as interesting information pertaining to a limited number of individual structures and cannot be made the basis of any general conclusions relative to the effect of variations in the material on the dead load stresses in a multiple-span arch bridge. They do indicate, however, the possibility of the random occurrence of a dead load stress at some section 100 to 150 lb per sq in. greater than the dead load stress at the corresponding section in a similar single-span arch subjected to the same load; but it is not clear that this excess in the dead load stress is due to variations in the properties of the concrete. An extended study of this subject is suggested. The structures used in this study should consist of a rib without deck so as to be readily susceptible of algebraic analysis.

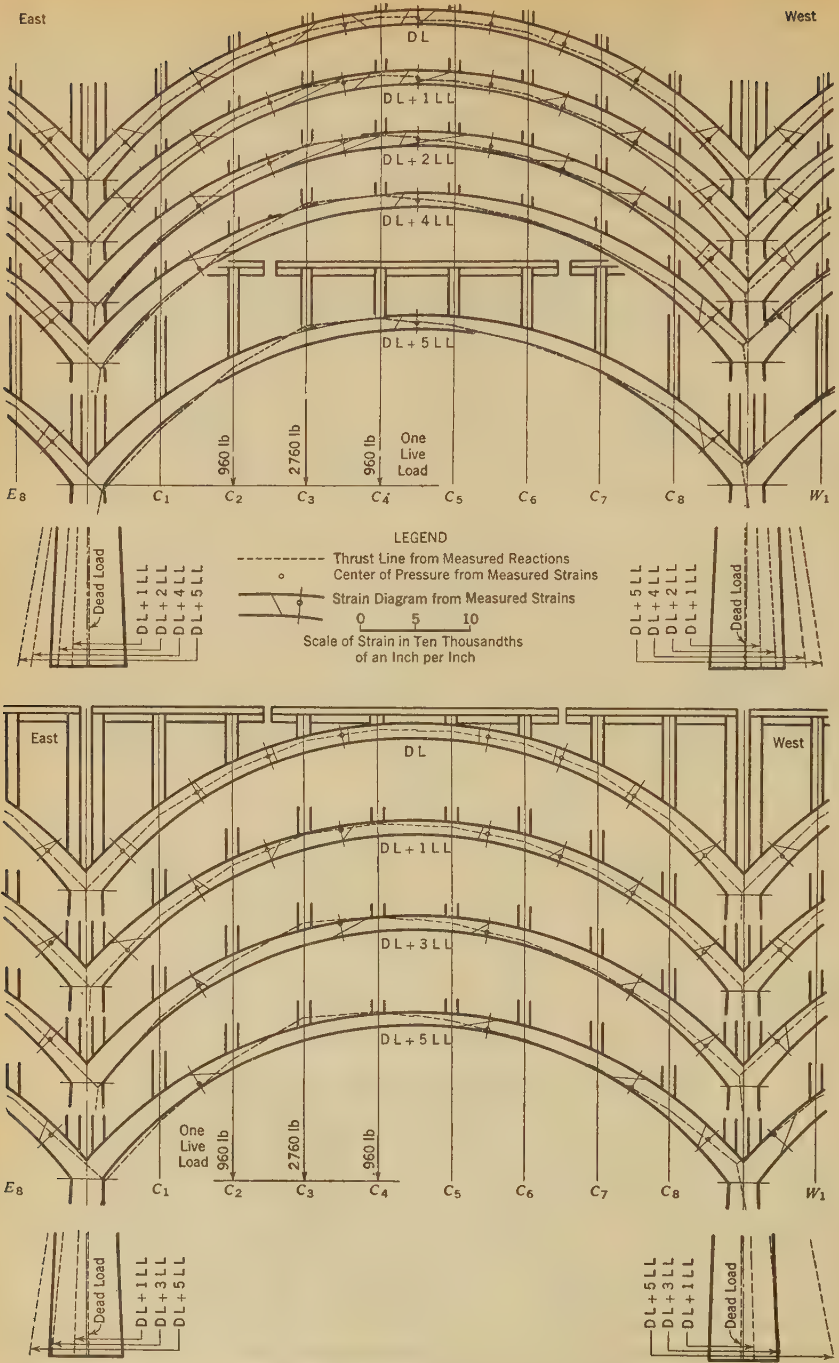

East
West
DL
DL + 1 LL
DL + 2 LL
DL + 4 LL
DL + 5 LL
960 lb
2760 lb
960 lb
One Live Load
E_8
C_1
C_2
C_3
C_4
C_5
C_6
C_7
C_8
W_1
Dead Load
DL + 1 LL
DL + 2 LL
DL + 4 LL
DL + 5 LL
LEGEND
Thrust Line from Measured Reactions
Center of Pressure from Measured Strains
Strain Diagram from Measured Strains
0
5
10
Scale of Strain in Ten Thousandths of an Inch per Inch
East
West
DL
DL + 1 LL
DL + 3 LL
DL + 5 LL
One Live Load
960 lb
2760 lb
960 lb
E_8
C_1
C_2
C_3
C_4
C_5
C_6
C_7
C_8
W_1
Dead Load
DL + 1 LL
DL + 3 LL
DL + 5 LL

FIG. 16.—THRUST LINES FOR CAPACITY LOAD; MAXIMUM STRESS AT C_3; WITH INTERMEDIATE EXPANSION JOINTS.

and the modulus of elasticity of the concrete should be assigned different values in various parts of the structure. Algebraic analyses of these structures should establish the probable effect of the variations in modulus of elasticity of the concrete on the dead-load stresses in a multiple-span arch series.

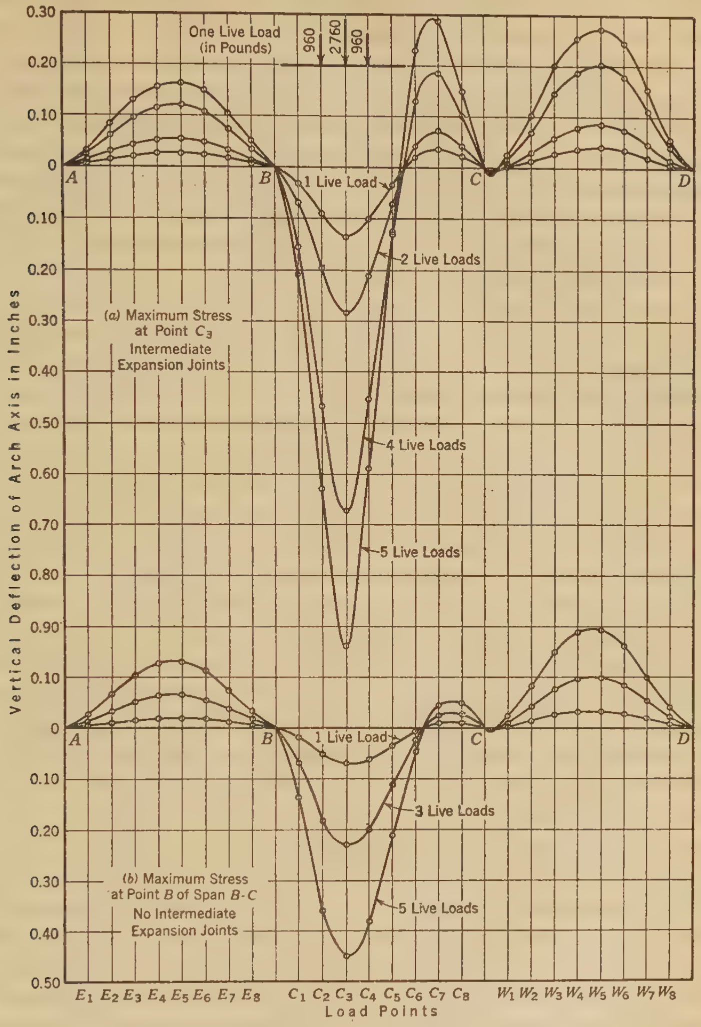


FIG. 17.—VERTICAL DEFLECTION OF ARCH AXIS; CAPACITY TEST FOR MAXIMUM STRESS.

Effect of Intermediate Expansion Joints.—The desirability of using intermediate expansion joints in the deck of an arch depends on whether movement will take place if they are provided and also on the effect they will have on the strength of the structure. The tests indicate that the ratio of movement at an expansion joint to the change in span is 0.25 and 0.11 for the high-deck and the low-deck structures, respectively. If expansion joints are not provided the restraint of the deck will produce a deflection of the spandrel columns commensurate with the movement that would take place if the joints were provided. The deflection diagrams of Fig. 17 show that the structure without intermediate expansion joints is stiffer than the one with joints.

The capacity tests of the structure without intermediate expansion joints were not carried to failure and the relative strength of structures with and without expansion joints is not known definitely. Certain facts, however, that bear upon this question are quite clearly established: (1) For the structure without expansion joints, "5 L. L." in addition to the "D. L." produced nothing more than hair-cracks in the rib, and there were no cracks in the spandrel columns or deck large enough to indicate that failure was impending; (2) the section where the deflection of the pier causes the greatest increase in the stress is near the center of the span where the strengthening effect of the deck is the greatest; (3) although the thrust line for the structure as a whole falls outside the rib over the central part of the center arch, as shown in Fig. 18, the strain diagrams show little, if any, tension at the section where failure of the rib is most likely to occur, indicating that the thrust taken by the rib does not go much, if any, outside the kern. All these facts are indicative of a considerable strengthening effect of a deck without expansion joints at the weak sections of a three-span series of arches on high piers, and this strengthening effect is largely eliminated by the intermediate expansion joints.

For the structure with a high deck, the temperature moment at the springing is 28% greater without intermediate expansion joints than with them. For the structure with a low deck the difference is nearly zero. For the structure without intermediate expansion joints the temperature moment on the middle of the structure is resisted by the rib and deck acting integrally. For the structure with intermediate expansion joints the line of action of the temperature thrust passes so near to the rib in the panel under the expansion joint that the temperature moment is very small.

Unit Stresses Due to Design Load.—The unit stresses in a number of structures due to design load are compared in Table 3. The stresses in the structure having a rib without deck were determined by the elastic theory. The dead load stresses in the three-span structures having a deck were determined from the measured thrust the line of action of which was obtained from the measured strain. The live load stress for these structures was determined by the algebraic method based on experimental constants. The dead load stresses for the single span with fixed ends were determined by the elastic theory, on the basis that the rib carries the dead load unrestrained by the deck. The dead load is shown in Fig. 3 and is the same for all structures. The live load consists of a uniform moving load equivalent to a panel load of 960 lb, with a single concentration of 1 800 lb to be superimposed on the distributed load. For each structure the live load is distributed so as to produce the maximum compression at the critical section of that structure. The critical sections are listed in Column (3), Table 3, and the live load for

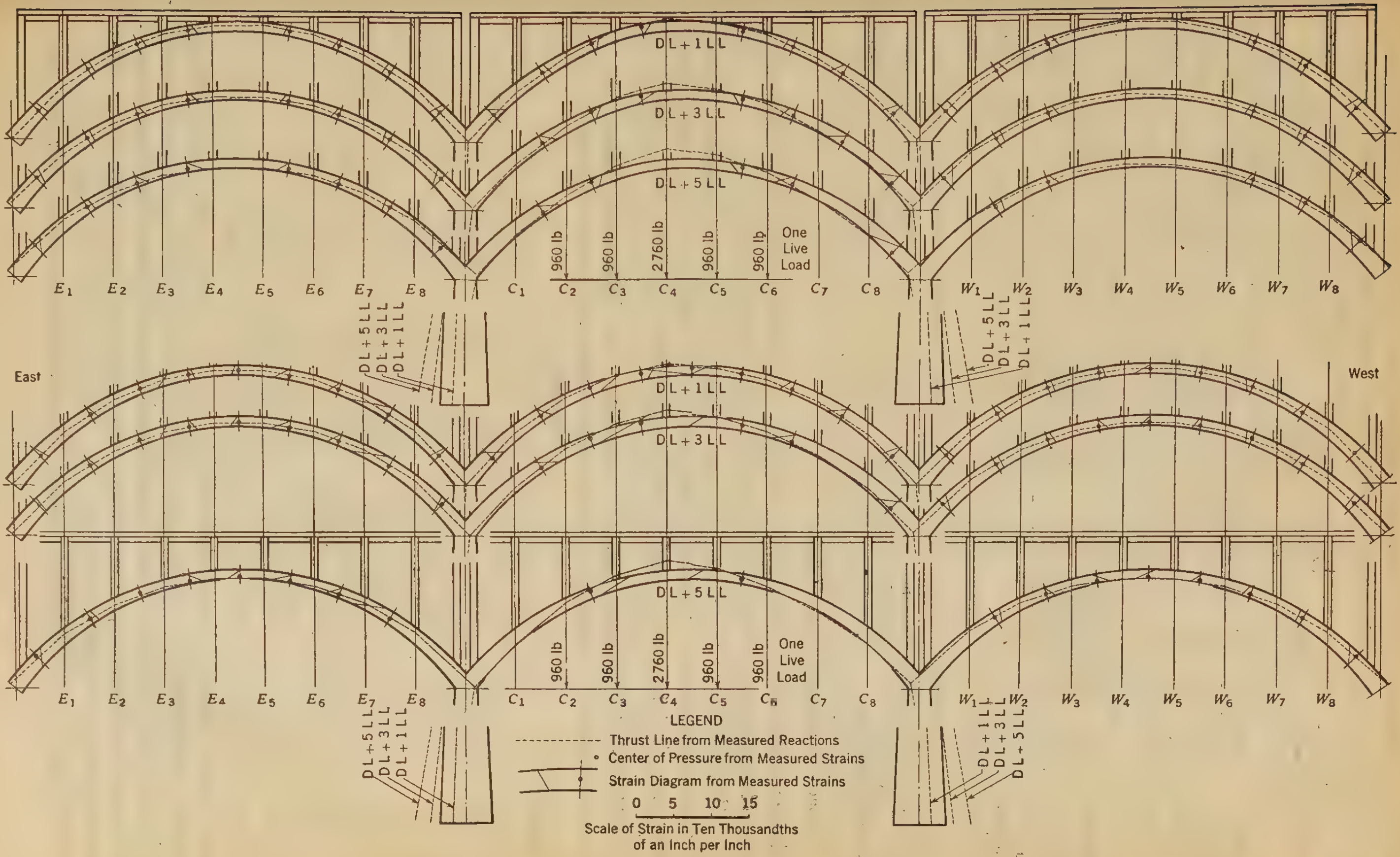

D L + 1 L L
D L + 3 L L
D L + 5 L L
960 lb
960 lb
2760 lb
960 lb
960 lb
One Live Load
E1 E2 E3 E4 E5 E6 E7 E8
C1 C2 C3 C4 C5 C6 C7 C8
W1 W2 W3 W4 W5 W6 W7 W8
D L + 5 L L
D L + 3 L L
D L + 1 L L
East
West
LEGEND
Thrust Line from Measured Reactions
Center of Pressure from Measured Strains
Strain Diagram from Measured Strains
0 5 10 15
Scale of Strain in Ten Thousandths of an Inch per Inch

FIG. 18.—THRUST LINES FOR CAPACITY LOAD; MAXIMUM STRESS AT C_4; WITHOUT INTERMEDIATE EXPANSION JOINTS.

TABLE 3.—MAXIMUM STRESSES FOR VARIOUS STRUCTURES DUE TO DESIGN LOAD

Structure	Pier height, in feet	Critical section	Live* Load No.	Stress, in Pounds per Square Inch					
				Dead Load		Live Load		Dead Load + Live Load	
(1)	(2)	(3)	(4)	Intrados (5)	Extrados (6)	Intrados (7)	Extrados (8)	Intrados (9)	Extrados (10)
Rib without deck	20	C_4	1	—142	—500	......	—736	......	—1 236
	15	C_4	2	—142	—500	......	—701	......	—1 201
	10	C_4	3	—142	—500	......	—665	......	—1 155
	Fixed end	3	4	— 92	—546	......	—543	......	—1 089
HIGH DECK									
Without expansion joints	20	*B* of Span *BC*	5	—287	—193	—334	......	—621	....
	15	*B* of Span *BC*	6	—251	—231	—310	......	—561	....
	10	*B* of Span *BC*	7	—192	—290	—276	......	—468	....
	Fixed end	East springing	8	—128	—342	......	—262	......	—604
With expansion joints	20	C_3	9	+152	—782	......	—665	......	—1 447
	10	C_3	10	+177	—809	......	—584	......	—1 393
	Fixed end	3	11	— 92	—546	......	—520	......	—1 066
LOW DECK									
Without expansion joints	20	*B* of Span *BC*	12	—137	—342	......	......	—480	....
	Fixed end	East springing	13	—128	—342	......	—264	......	—606
With expansion joints	20	C_3	14	+ 14	—610	......	—732	......	—1 342
	Fixed end	3	15	— 92	—546	......	—559	......	—1 105

* See Table 4 for description of these loads.

maximum stress at that section is referred to by the number in Column (4). The live load corresponding to this number is given in Table 4.

For the structure with a high deck and with intermediate expansion joints, the compression at the extrados of the section at Section C_3, in pounds per square inch, is 1 447, 1 393, and 1 066 for structures having pier heights, in

TABLE 4.—DISTRIBUTION OF LIVE LOAD FOR MAXIMUM STRESS

Live Load No.	Sections							
	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
1	368	953	960	2 760	938	296	0	0
2	300	939	960	2 760	919	240	0	0
3	198	898	960	2 760	881	169	0	0
4	960	960	2 760	647	18	0	0	0
5	960	2 760	960	960	956	401	0	0
6	960	2 760	960	960	880	169	0	0
7	960	2 760	960	960	474	0	0	0
8	0	0	363	952	960	2 760	960	960
9	960	960	2 760	959	433	0	0	0
10	960	960	2 760	917	236	0	0	0
11	960	960	2 760	868	152	0	0	0
12	960	2 760	960	960	960	703	34	0
13	0	0	376	954	960	2 760	960	960
14	960	960	2 760	960	713	39	0	0
15	960	960	2 760	941	308	0	0	0

feet, of 20, 10, and 0. A stress of 1 447 lb per sq in. for the three-span series compared with 1 066 lb per sq in. for a single span with fixed ends is of interest; but it should be noted that a considerable part of this difference occurs in the dead load stress. These values are comparable with 1 236, 1 155, and 1 089 for similar structures having a rib without deck. In this case, also, the difference is in the dead load stress. For the structure with a low deck the dead load stresses are smaller and the live load stresses are greater than for the structure with a high deck, but the differences do not exceed possible experimental errors, and the two structures appear to have about the same load-carrying capacity.

The stresses for the structure having a deck without intermediate expansion joints are for the section at the east springing and are very low. Whether danger of failure for these structures is greater over the central part than at the east springing was not definitely determined, but there was nothing to indicate that it is. However, failure in the superstructure followed by failure of the rib is possible and the stress at the springing is not a reliable indication of the likelihood of failure in the deck. Each of the structures having a deck without intermediate expansion joints carried the design dead load plus five times the design live load, the live load being distributed for maximum moment about the axis of the rib on a section through C_4, and there was no indication that failure was impending in either the deck or the rib.

Load-Carrying Capacity of Various Structures.—The results of the capacity tests are very re-assuring. All structures were tested at a pier height of 20 ft. Both structures having a deck without intermediate expansion joints carried "5 L. L." in addition to the "D. L." without indication of impending failure, both when loaded for failure at the springing and also for failure at Section C_4. The structure having a rib without deck carried "4.5 L. L.", and both structures having a deck with intermediate expansion joints carried a little more than "6 L. L.", in addition to the design "D. L." This was true even when the computed stress due to the design load, given in Table 3, was higher than specifications usually allow.

The compressive stress at failure for the three-span structure consisting of a rib without deck was 3 800 lb per sq in., comparable with the strength of the concrete in the control specimens of 3 310 lb per sq in. The compressive stress at failure for the structure having a high deck with intermediate expansion joints, based on measured reactions, was 3 200 lb per sq in., comparable with a strength of the concrete, as determined by control cylinders of 3 412 lb per sq in. The compressive stress at failure for the structure having a low deck was 4 400 lb per sq in., comparable with a strength of the concrete as determined by control cylinders of 4 281 lb per sq in.

The location of the thrust line in an arch is particularly indicative of the ease with which the structure is carrying its load, because flexure in the rib is very destructive. Fig. 15 shows the thrust line at the various load increments of the capacity test of the structure consisting of a rib without deck. The full line represents the position of the thrust line from the elastic theory and the broken line from the measured reactions. The agreement between the two positions is remarkably close, especially at the heavy loads. The rib contained many cracks at three live loads and some of them had attained a width of a few hundredths of an inch at four live loads; and yet, the accuracy of the elastic theory based on the assumption that concrete takes tension, does not seem to be seriously impaired. However, this is really only another way of saying what was stated previously, that varying the elastic constants for a span does not greatly alter the stress distribution. The centers of pressure (the centers of gravity of the trapezoid of strain corrected for the reinforcement steel), fall on the thrust lines with remarkable consistency. The character of the failure of the rib of the structure having a high deck with intermediate expansion joints, is shown in Fig. 19.

Relative Merits of High and Low Decks.—The stresses in the rib of the various structures due to the design load, given in Table 3, indicate that there is not much difference between the load-carrying capacities of the ribs of structures having high and low decks. The temperature moment at the springing, for structures having intermediate expansion joints, is about 64% greater for the high deck than it is for the low deck. For structures without intermediate expansion joints the difference is about 31 per cent.

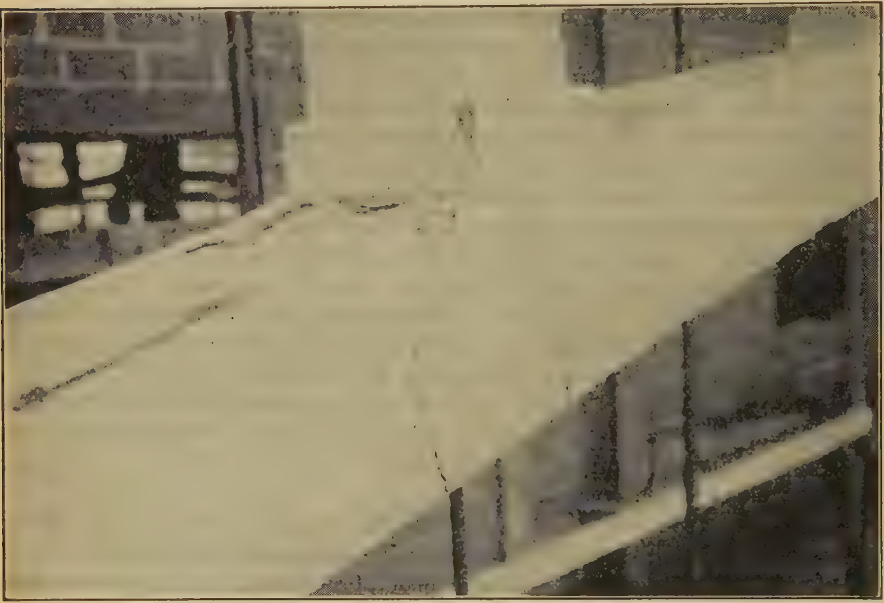


FIG. 19.—FAILURE OF RIB AT C_3. HIGH DECK WITH INTERMEDIATE EXPANSION JOINTS.

The low deck is integral with the rib at the crown, and the probability of a shear failure in the spandrel columns due to gravity loads is less than for a structure with the deck a considerable distance above the rib at the crown. The movement at the intermediate expansion joints due to a change in span is very much less for the low deck than it is for the high deck, making the need for expansion joints less for the former than for the latter structure.

SUMMARY OF RESULTS

Many tests have been made in connection with this investigation and only a small part of the experimental data have been presented in this paper. All the data have been analyzed, however, and this analysis appears to justify the following conclusions relative to the behavior of a three-span arch series on high piers:

For a Structure Consisting of a Rib without Deck.—

(1) The elastic theory based on the usual assumptions gives values for the moment, thrust, and shear at various sections, that agree with the measured values within the tolerance of the tests.

(2) In analyzing a multiple-span arch series, the moment of inertia of a section may be computed on the basis that the concrete takes tension.

(3) Considerable cracking of the arch rib does not greatly alter the position or magnitude of the thrust.

(4) The maximum unit compression due to design load was 13% greater for a three-span series on 20-ft piers than for a similar single span with fixed ends.

For a Structure Having a Deck.—

(5) The influence ordinates for fixed-end reactions is quite different for a structure having a deck than for one consisting of a rib without a deck. The effect of the deck is to reduce the moment at the springing, where it is all resisted by the rib, and to increase the moment over the middle of the span where the deck acts with the rib.

(6) The reactions at the springing can be determined by an algebraic method, using experimentally determined fixed-end reactions and elastic constants for the arch spans. The constants may be obtained from tests of single-span concrete structures or of celluloid models. A considerable variation in the elastic constants will not seriously affect the resulting stresses if the constants are statically consistent, but the fixed-end reactions must be determined accurately if the analysis is used to determine the dead load stresses.

(7) A deck without intermediate expansion joints increases the stiffness and the moment-resisting capacity of the central part of the structure. Intermediate expansion joints reduce both these effects.

(8) The ratio of the movement at each expansion to a change in span is about 0.25 for the structure having a high deck and 0.11 for that having a low deck.

(9) For the structure having a high deck with intermediate expansion joints, the unit compression due to design load was 36% greater for the three-span structure on 20-ft piers than for a single span with fixed ends. For the structure having a low deck with intermediate expansion joints, the difference was 21 per cent. A considerable part of this difference was due to the difference in the dead load stress, a function which would not have been influenced by the flexure of the piers if the specimen had been a symmetrical structure of homogeneous material, symmetrically loaded.

(10) There is considerable evidence indicating that the dead load stress in a multiple-span structure may exceed the corresponding stress in a similar single span having fixed ends.

For All Structures Tested.—

(11) The concrete in the arch developed approximately the same unit stress as the same concrete in 6 by 12-in. control cylinders.

ACKNOWLEDGMENTS

The tests described in this paper are a part of the investigation resulting from the co-operative agreement entered into by the Engineering Experiment Station of the University of Illinois, of which the late Milo S. Ketchum, M. Am. Soc. C. E., was the Director, and the United States Bureau of Public Roads Thomas H. MacDonald, Chief of Bureau. The tests were planned by the writer in consultation with Messrs. E. F. Kelley, Chief of Division of Tests, and A. L. Gemeny, Senior Structural Engineer, both of the U. S. Bureau of Public Roads, and with Professor George E. Beggs, E. H. Harder, A. C. Janni, and Professor Clyde T. Morris, members and Chairman, respectively, of the Special Committee of the Society on Concrete and Reinforced Concrete Arches. The experimental work was done by Ralph W. Kluge, Jun. Am. Soc. C. E., Special Research Assistant in Civil Engineering, assisted by Messrs. F. B. Metterhausen, F. P. Thomas, G. E. Jewett, Jun. Am. Soc. C. E., B. C. Schmeltzer, B. S. Tucker, R. Evans, and W. M. Weidemeyer, all working under the supervision of the writer. The computations involving the experimental data were made by Professor E. C. Grafton, of the Armour Institute of Technology, John N. Pirok, Jun. Am. Soc. C. E., Research Graduate Assistant in Civil Engineering, and Nathan M. Newmark, Jun. Am. Soc. C. E., Graduate Student in Civil Engineering.

The tests were made in the Materials Testing Laboratory of the University of Illinois. The direct expenses of the tests were paid from funds provided by: U. S. Bureau of Public Roads, Engineering Foundation, Universal Atlas Cement Company, Illinois Steel Company, American Bridge Company, Jones and Laughlin Steel Company, Interstate Sand and Gravel Company, Lincoln Sand and Gravel Company, Neal Sand and Gravel Company, Fairbanks, Morse and Company, and the Society.

DISCUSSION

C. B. McCullough,[4] M. Am. Soc. C. E. (by letter).—This presentation of laboratory tests of multiple-span reinforced concrete arch bridges is not only logical and coherent, but also gives evidence of an extraordinary degree of care and precision throughout the research. The section entitled "Description of Specimens and Apparatus," which deals with testing apparatus and equipment, has a value that extends beyond the scope of these particular tests, pointing the way to methods for further experimentation along the line of stress distribution in structures that are highly indeterminate and complex. The method for simulating or reproducing foundation fixity at various pier heights is especially ingenious. A similar comment may also be made in reference to the method of measuring rotations.

The scope of the investigation includes three phases of structural analysis concerning which there appears to be a dearth of published data: (1) An experimental verification of the elastic theory as applied to multi-span arches; (2) a determination of the approximate degree to which a variation in the elastic properties of the materials (or a variation in elastic constants due to any other condition) operates to modify stress distribution and to introduce error in the analysis; and (3), the effect of superstructure restraint and a study of the function of expansion joints. All these are questions that have appeared for some time to merit further investigation and study, and the results of Professor Wilson's researches furnish data which hitherto have been needed badly by every engineer dealing with the design of this structural type.

The influence lines indicated in Fig. 8 check the elastic theory with a degree of accuracy that is reassuring, and the two methods, one based upon pier-top displacements and the other on measured reactions, also check in a satisfying manner. The same comment may be made in reference to Figs. 9 and 10 wherein influence lines have been developed by five different methods.

The analysis of a multi-span arch series, utilizing experimentally determined elastic constants, appears to present far-reaching possibilities. Professor Wilson refers to a method developed under his supervision by Donald E. Larson, Jun. Am. Soc. C. E.,[3] in 1930, but does not go into this question in any great detail. The importance of this phase of the research merits more consideration. It is to be hoped that in closing his paper Professor Wilson will find it possible to present this matter more completely.

The entire research appears to be grouped about two major inquiries—the first having to do with the effect of elastic piers in multi-span arch construction, and the second pertaining to the influence of the deck and spandrel structure.

In reference to the former (the effect of elastic piers of varying heights), an inspection of Figs. 13 and 14 discloses the fact that the general effect of increasing the height of elastic piers is to reduce the horizontal thrust, to reduce moments at or near the pier skewbacks, and to increase stress near the center of the span. In this connection an analysis was made in the writer's office a few years ago embracing a three-span arch structure on elastic piers. Because of certain location studies involved, and as a matter of general interest, analyses were made of three designs, each comprising a series of three 150-ft arches and differing only in the matter of pier heights. Design No. 1 involved a pier height of 45.8 ft; Design No. 2, a pier height of 15.8 ft, and Design No. 3, a pier height of 95.8 ft.

---

[4] Bridge Engr., State Highway Dept., Salem, Ore.

The influence lines for the moment and thrust at three points on Span 1, for each of these three designs, are plotted in Fig 20. These influence lines naturally exhibit the same tendencies as those noted in Professor Wilson's paper; that is, an increase in pier flexibility is accompanied by a decrease in thrust, a decrease in skewback moment at the pier, and a general increase in stress throughout the region adjacent to the crown. It is of interest, however, to note the marked effect of loads in adjacent spans, especially for the two designs involving relatively high piers. Professor Wilson's researches indicate an increase in stress as pier heights are increased, amounting to 13% for the three-span series on 20-ft piers, when compared with a single span of like dimension having fixed ends. When the effect of the deck and spandrel structure is taken into consideration this increase amounts to 17 per cent. A study of the influence lines given in Fig. 20 indicates that a stress increase considerably greater than the foregoing may be expected if the effect of loading in adjacent spans is considered; and, while this fact need cause no serious concern in view of the improbability of simultaneous loading in all spans of a series in such position as to cause maximum stress, such a condition should always be investigated, especially for extreme pier heights.

The second phase of Professor Wilson's paper has to do with the effect of the supported deck and spandrel structure on arch rib strains and stresses. In general, his researches disclose a reduction in horizontal thrust and in bending moment at the springing as a result of the placement of an integrally supported spandrel structure. When expansion joints are inserted in the deck, the same tendencies are exhibited, but to a much less degree. In this connection, certain observations in reference to the effect upon arch rib strains of a spandrel structure and deck, even with the utmost degree of articulation possible, were made by the writer in connection with the operation of decentering the Rogue River Bridge, in Southern Oregon, in 1932. A full report of a co-operative research on this subject has been published for limited distribution by the U. S. Bureau of Public Roads and the Oregon State Highway Commission.[5]

This bridge was constructed after the Freyssinet system[6] which, as applied in this case, consists in the complete severance of the arch rib at the crown, and the introduction at the severed section of a system of hydraulic jacks by which the arch may be raised from its falsework support and adjusted to any

---

[5] "Application of Freyssinet Method of Concrete Arch Construction," by A. L. Gemeny and C. B. McCullough, M. Am. Soc. C. E., *Technical Bulletin No. 2*, Oregon State Highway Comm., 1933.

[6] *Civil Engineering*, February, 1932, p. 91.

predetermined elastic state. The arch thus handled is converted into two statically determinate cantilever segments, which render it possible to study stress distribution under varying conditions with a considerable degree of accuracy.

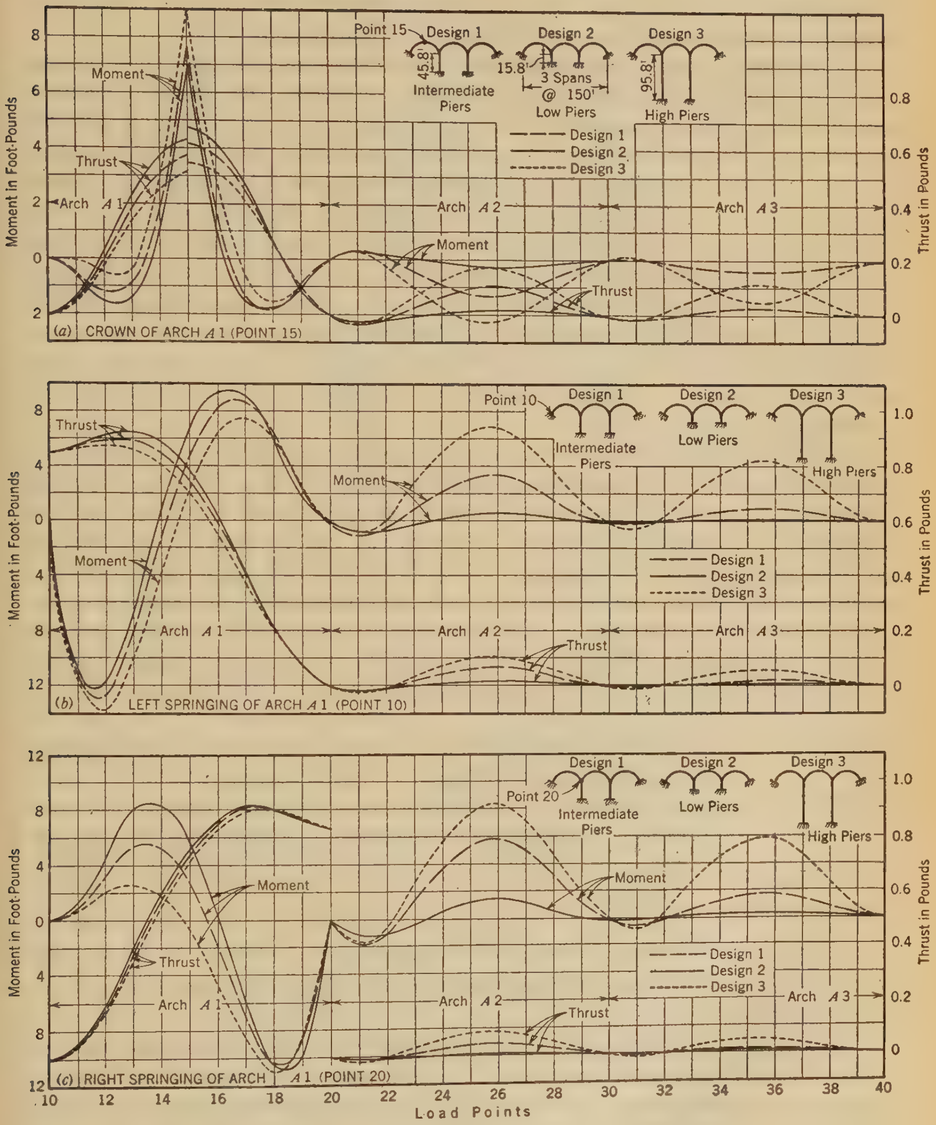


FIG. 20.—COMPARISON OF MOMENTS AND THRUSTS.

The articulation thus attempted[7] was about as thorough as can ordinarily be achieved under construction conditions. In other words, the degree of restraint exercised by the spandrel structure was apparently reduced to as nearly a minimum as is consistent with construction practice. Nevertheless, the observed crown movement was less than one-third of the theoretical for an unrestrained rib. For example, at a total crown-thrust value of 840 000 lb, the computed crown spread for an unrestrained rib amounts to 1.60 in. as against an average observed value of slightly less than 0.50 in.

---

[7] *Proceedings*, Am. Concrete Inst., October, 1932, p. 57.

In connection with the decentering operations, and as part of a general research program inaugurated in connection therewith, electric telemeters of the McCollom-Peters cartridge type[8] were installed, at the extrados and the intrados, at five points along the axis of the arch rib, these telemeters being located at each springing, at each quarter-point, and at or near the crown. The telemeter forces, before decentering pressures had been reached, were found to be much less than the applied or jacking force, indicating that during this stage of the operation a considerable part of the horizontal thrust was being taken by the centering; however, the observations indicated that after the theoretical decentering thrust had been reached, the rib was carrying the full horizontal thrust at the crown section. This is exactly as it should be since the supported spandrel structure could have no effect on crown stress.

Table 5 indicates the percentage of theoretical thrust observed from the telemeters at the various skewbacks and also at the various quarter-points for the second runs. An inspection of these data discloses the fact that the quarter-point stresses are quite uniform. In each case about 75% of the theoretical thrust is being carried by the arch rib proper while 25% is being carried by the superstructure. At the skewbacks the conditions are quite different, the percentage of theoretical thrust carried by the arch rib proper varying from 48 to 81. The variation in skewback thrusts points strongly to the fact that the sliding joints over the pier columns did not function as complete articulations and that the degree of restraint in each case was quite variable.

TABLE 5.—PERCENTAGE OF THEORETICAL THRUST OBSERVED FROM TELEMETERS (HORIZONTAL JACKING FORCE, 1 000 000 POUNDS).

Span No.	Percentage at quarter-points	Percentage at skewbacks	Span No.	Percentage at quarter-points	Percentage at skewbacks
1	68	67	4	72	76
2	71	48	6	74	72
3	78	81	7	74	52

Therefore, it would appear that, notwithstanding the care exercised to free the arch rib of spandrel restraint to the maximum possible extent, this restraint was great enough to reduce the horizontal crown strains to a value less than one-third the corresponding theoretical values for an unrestrained or free rib, and that, furthermore, the spandrel structure absorbed about 25% of the thrust at the quarter-points and an amount at the skewback which was highly variable because of the varying degree of restraint exercised by the sliding expansion joints over the pier columns.

---

[8] "A New Electric Telemeter," by Burton McCollom and O. S. Peters, *Technologic Paper No. 247,* National Bureau of Standards, U. S. Dept. of Commerce.

Another interesting fact in reference to the behavior of the expansion joints and spandrel structure articulations is indicated in Fig. 21 wherein are plotted the quarter-point moments as determined from telemeter readings against applied crown thrust. Between Readings 107 and 111 it will be noted that the positive moments gradually increase as crown thrusts are increased. Between Readings 111 and 113, however, a decrease of more than 100 000 lb in applied crown thrust produces no change in moment. Below this point, however, as crown pressures are slacked off, quarter-point moments decrease in accordance with theory. The same characteristic lag is indicated at the bottom of the graph between Readings 101 and 107 and also between Readings 116 and 119. Reading 97 (not shown) is at zero moment and thrust.

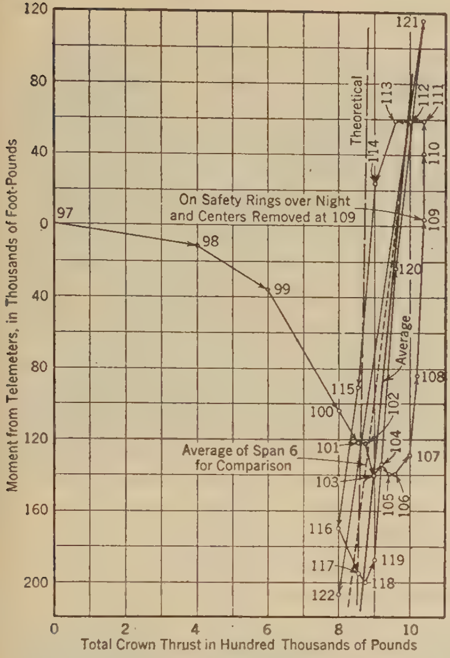
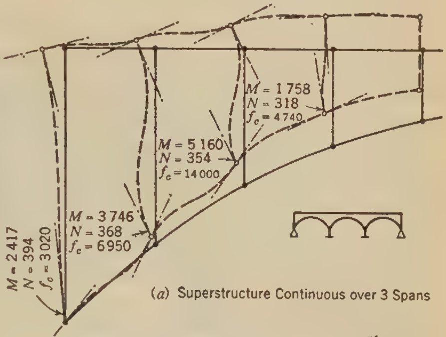
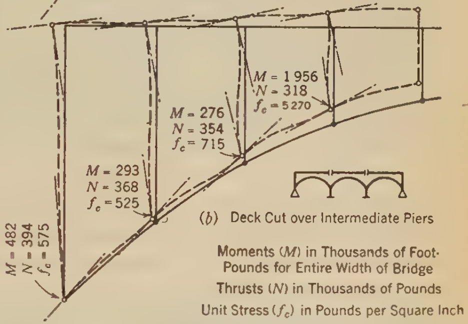


FIG. 21.—TOTAL CROWN THRUST, SPAN NO. 7.



(a) Superstructure Continuous over 3 Spans



(b) Deck Cut over Intermediate Piers

Moments (M) in Thousands of Foot-Pounds for Entire Width of Bridge
Thrusts (N) in Thousands of Pounds
Unit Stress (f_c) in Pounds per Square Inch

FIG. 22.—MECHANICAL ANALYSIS FOR TEMPERATURE STRESSES IN END SPAN.

The same characteristic at the top and bottom of all graphs indicating the relationship between applied crown thrust and arch rib forces and strains at various points persisted throughout the entire test. At first, it was thought that this phenomenon might be explained on the theory of friction in the jacks, but careful calibration and recalibration through a series of test cycles failed to disclose any appreciable jack friction. The only other hypothesis upon which this phenomenon can be explained is that of a frictional resistance to a reversal of motion in the articulations and expansion joints. As long as movement persisted in one direction, superstructure restraint appeared to be more or less uniform and fixed. Whenever motion was reversed, however, there appeared to be sufficient static friction in the articulations to retain the arch in a fixed position, and a considerable increase or decrease in crown thrusts appeared necessary in order to break loose the articulations and permit movement in the reverse direction.

All these results point inevitably to the conclusion that spandrel structure restraint is present to a certain extent even in the most carefully articulated designs; and to the further fact that sliding joints are likely to introduce strain conditions of an exceedingly variant character.

The Rogue River tests were conducted under construction conditions, and certain sources of error were inevitably introduced; however, the persistence, throughout the entire tests, of the results noted herein leaves little room for doubt as to the general tendencies exhibited by the spandrel structure in question. As a result of these researches, it may be logically concluded that the only methods by which the stress distribution in an arch carrying a supported spandrel structure and deck can be determined and controlled accurately, are: (1) By means of a system of articulations much more complete and positive than those ordinarily used heretofore; (2) by means of a system of stress and strain control in the field such as was adopted at the Rogue River Bridge; and (3) by the adoption of a design without expansion joints and a method of calculation that takes into consideration the complete effect of the spandrel structure. In view of the foregoing conclusions, the researches reported by Professor Wilson become exceedingly important.

In connection with the utilization of a spandrel structure without intermediate expansion joints, a word of warning should be sounded against a procedure which carries this idea too far. Fig. 22 is the result of a temperature stress analysis made in the writer's office in 1932. Fig. 22 (*a*) indicates the moments, thrusts, and fiber stresses at the base of spandrel columns supported by a series of three arch spans, no expansion joints being placed throughout any part of the spandrel structure from end to end. Fig. 22 (*b*) indicates the marked reduction in stress at corresponding points when the deck is severed over the intermediate piers. Even in this latter case the stress at the base of the short column exceeds the ultimate strength of the material, indicating the need for a re-design of the spandrel columns or the provision for further articulation.

All in all, the problem presented is rather complex, and the need for the utmost refinement in analyses of large and important structures becomes increasingly apparent as the problem is further investigated. Structural designers have been altogether too prone to neglect spandrel structure effects and to assume a freedom of arch movement under varying stress conditions which was never realized in actual service. The result in many instances is quite apparent in the formation of cracks at column bases, the opening of construction joints, spalling, and disintegration around expansion joints, and other like conditions. Because of the relief afforded by plastic flow and a redistribution of stress, such conditions may not be sufficient to cause structural failure but, nevertheless, they militate against service life and increase maintenance expense.

Carroll L. Mann, Jr.,[9] Esq. (by letter).—Following parallel with the investigation of concrete arches at the University of Illinois and during the first half of 1933, the writer, working under the direction of George E. Beggs, M. Am. Soc. C. E., built and analyzed by the deformeter gauge method,[10] a precise celluloid model of a multiple-span arch bridge.[11]

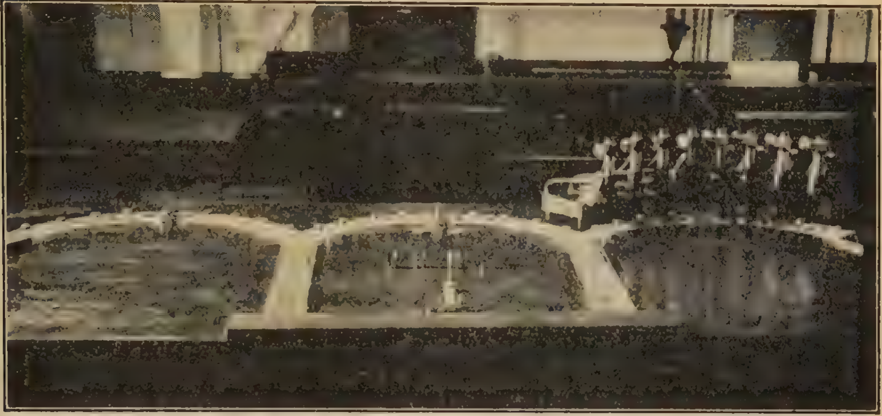


Fig. 23.—Celluloid Model of the Three-Span Concrete Arch Ribs on Slender Piers.

The word, "precise", as applied to these models, has a significant meaning. Throughout the entire analysis of the model various experiments were performed on seemingly unimportant details to determine the best methods

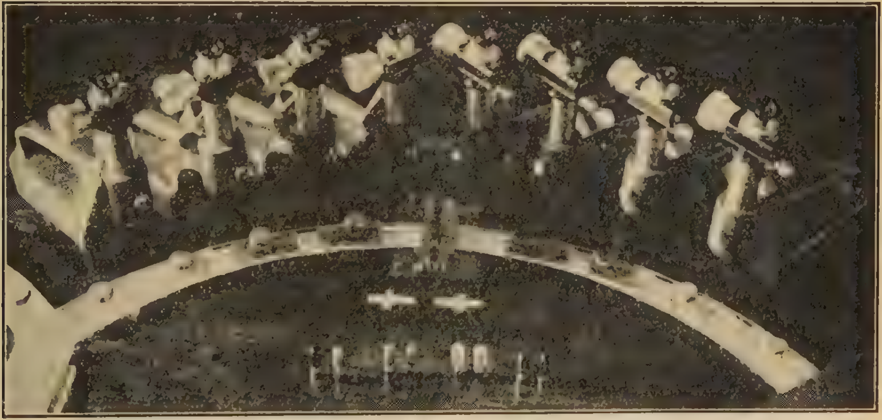


Fig. 24.—End Span of the Three-Span Celluloid Model, Showing Testing Instruments.

---

[9] Insp., Lock and Dam No. 3, Fayetteville, N. C., Dist. Office, U. S. Corps of Engrs., Wilmington, N. C.

[10] For explanation of the method, see *Transactions*, Am. Soc. C. E., Vol. 88 (1925), pp. 1208–1230; also, McCullough and Thayer, "Elastic Arch Bridges", Chapter VII.

[11] "Model Analysis of Multiple-Span Concrete Arch Bridge", by Carroll L. Mann, Jr., Thesis presented to Princeton Univ. in partial fulfillment of the requirement for the degree of Civil Engineer.

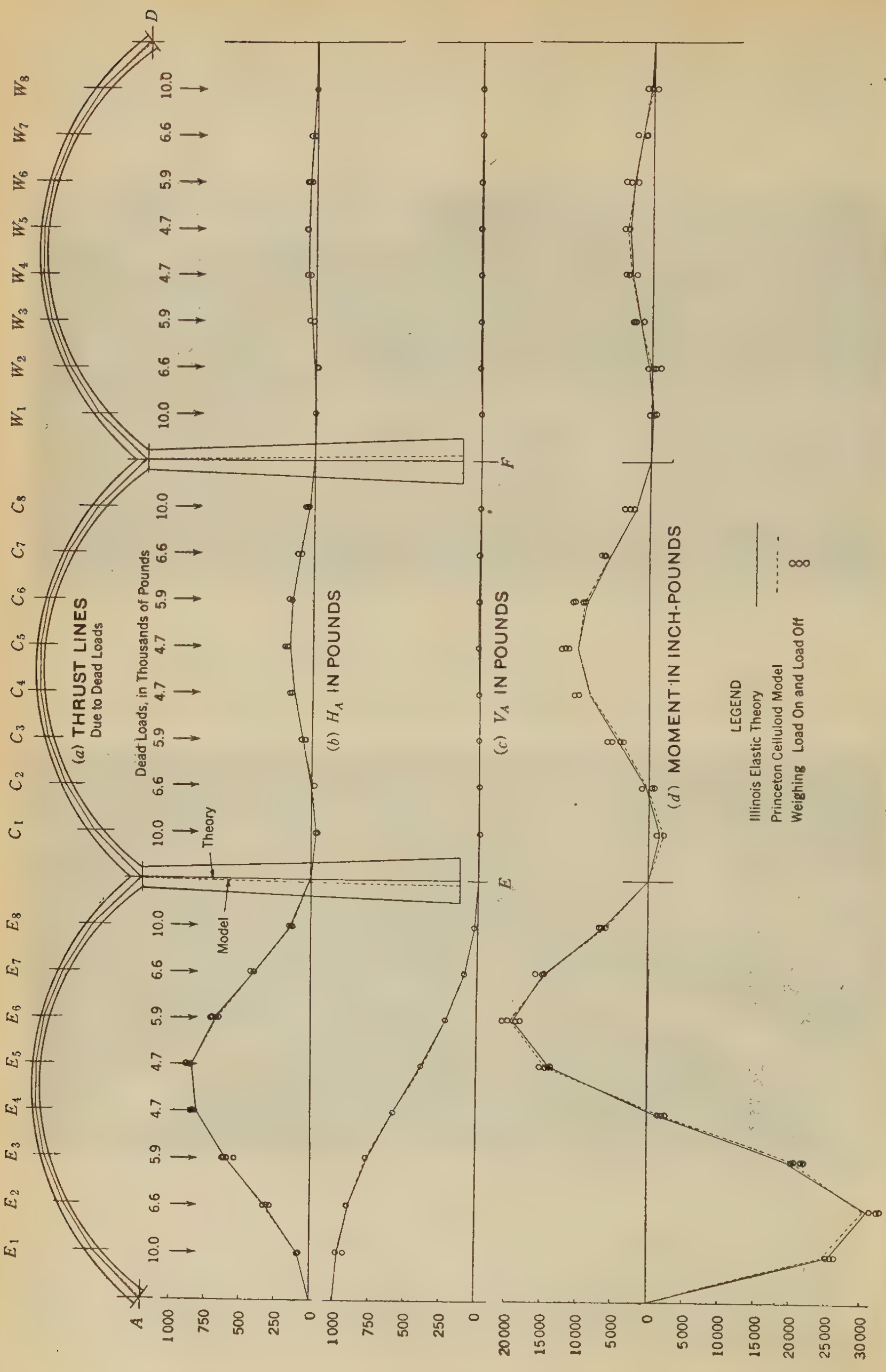


FIG. 25.—INFLUENCE LINES FOR REACTIONS AT A DUE TO 1 000-POUND UNIT LOAD.

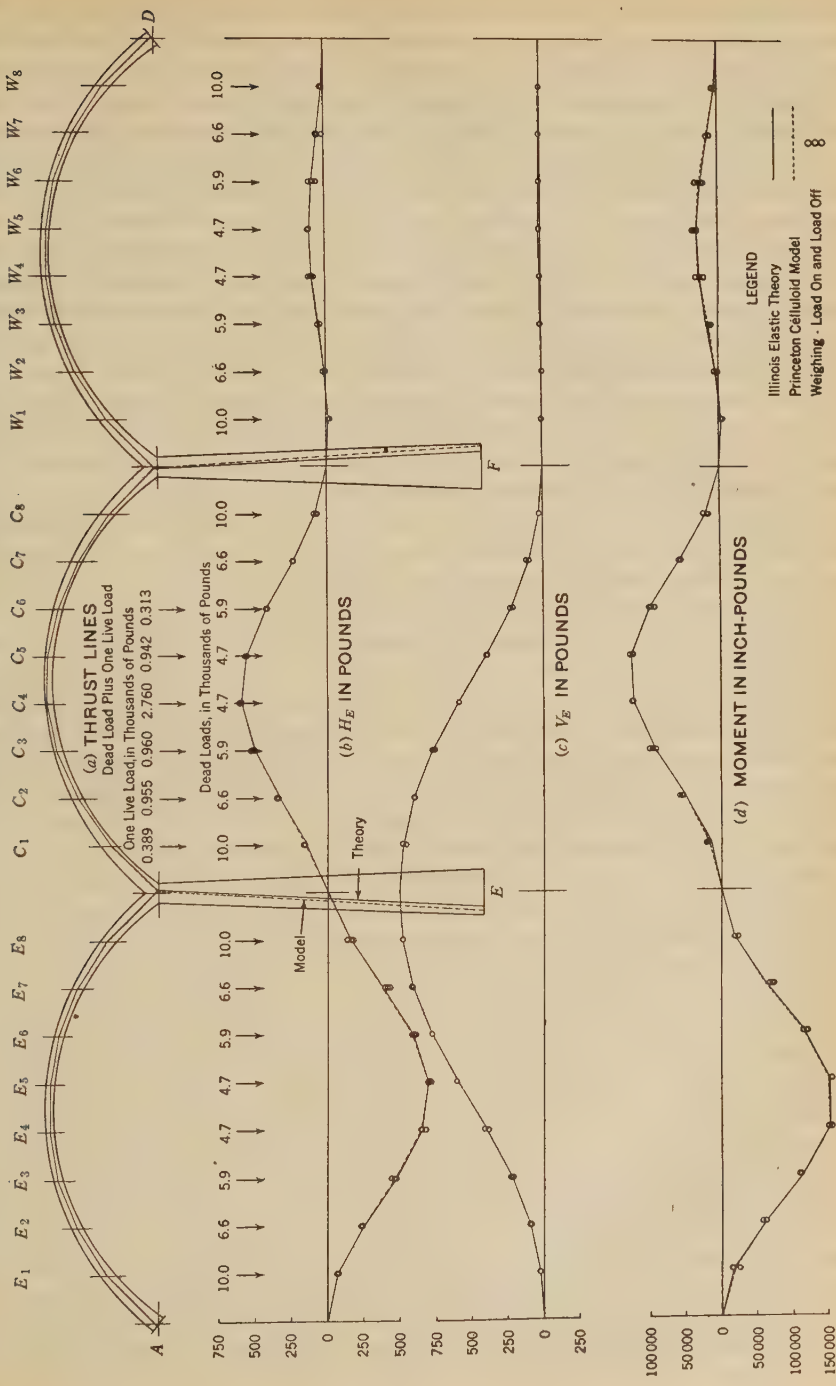


FIG. 26.—INFLUENCE LINES FOR REACTIONS AT *E* DUE TO 1 000-POUND UNIT LOAD.

of construction and operation. The work, therefore, served a secondary purpose in that the results of these observations were recorded as specifications for the best possible technique of precise model construction and operation.

Model of Three-Span Arch Bridge.—The continuous concrete structure, the model of which is shown in Fig. 23, has three arch ribs each of 27-ft span, 6.75-ft rise, and is supported by two abutments and two 20-ft piers. Both the abutments and the two pier bases are assumed as fixed, and, consequently, the structure is indeterminate in the ninth degree. For the analysis of this model, a floating gauge was attached and carefully oriented at the crown section of each of the three ribs and the components of crown reactions were determined at these sections as the 1 000-lb vertical unit load was assumed to move across the structure. There were eight load points on each concrete arch rib, spaced 3 ft apart along the horizontal.

With the components of the forces experimentally determined at each of the three crown sections, the structure became statically determinate. These forces were then transferred to the abutments and pier bases by means of statics and the terminal reaction influence lines so derived.

A clearer detail of the right-hand span of the multiple-arch model, including the microscopes, the floating gauge, and the anchorage of the arch terminal between steel plates, is shown in Fig. 24.

Comparison of Experimental and Theoretical Results.—Influence lines for horizontal, vertical, and moment reactions at the left abutment, *A*, of the three-span structure, for a 1 000-lb unit load, are shown in Fig. 25. The circles represent the results obtained for the concrete structure by weighing the reaction components on very precise lever scales. The thrust lines for dead load, as computed both from the model influence values and by theory, are shown in Fig. 25(*a*). This structure was shaped so that the thrust line for dead load might follow closely the center line of the arch ribs and piers, passing precisely through the centers of the arch crowns and the terminals. In the theoretical analysis, rib-shortening was taken into account, but not pier-shortening. It will be noted that thrust lines determined by model and by theory are for the most part almost identical in the arches, separating a negligible amount at the end portions. The two lines deviate slightly down the piers, the theory following the center line and the model deviating from it. The influence lines for reactions at the left base, *E*, as determined by the three methods, are shown in Fig. 26. The thrust lines as computed from model and theory for dead load plus one live load are plotted in Fig. 26(*a*). In general, the deviation of the model thrust lines from the theoretical thrust lines is small, both for dead load and for dead load plus one live load.

If Figs. 25 and 26 are compared with Fig. 15 of the paper, it will be noted that the thrust lines determined from the measured reactions on the prototype diverge from the theoretical thrust lines in the same direction as the thrust lines found from the model test. In no case are the deviations from elastic theory of practical importance; but, nevertheless, the agreement of the test of the prototype and of the model in this divergence of the position of thrust lines from the theory may be significant; and it excites some interest.

Conclusion.—The small variations between results obtained by the theory, by the model, and by the test of the concrete structure are not of practical importance. The differences may be due in part to experimental errors, but undoubtedly some of them are chargeable to assumptions made in the theory. The theory necessarily includes more assumptions than the model, in which Nature itself works automatically, with few limitations.

M. Hirschthal,[12] M. Am. Soc. C. E. (by letter).—Of special interest are the effects, described in this paper, of expansion joints in the deck of the spandrel construction on the resistance of the arch ring or rib. There is no doubt that a continuous spandrel system extending from pier to pier, fixed to the arch at its various points, will increase the resistance of the arch rib or ring, to the various stresses to which it is subjected. This is natural, since the structure then becomes a braced arch or a Vierendeel truss with a curved lower chord and laterally supported top chord.

The difficulty lies in the necessity of designing each of the joints of this rigidly connected frame to resist the distortions due to the movement of the arch, particularly under the action of temperature stresses. With a drop in temperature the crown of the arch falls while the haunches rise, which action is transmitted to the deck through the spandrel columns or walls, causing distortions at their junction with the deck. These distortions increase greatly with long arch spans and high spandrel systems and require heavy sections to resist them.

The object in providing intermediate expansion joints in decks, therefore, is not simply to guard against the effect of temperature changes longitudinally, but to eliminate the stresses induced in the deck by the distortion of the arch ring or rib due to changes in temperature in that member. By allowing for free movement in the deck construction for this condition, it is not necessary to design for distortions due to this cause, but to select the proper position for the location of the joints.

There is further danger in the absence of intermediate expansion joints in long span arches, and it lies in the fact that, particularly in railroad bridges, there is superimposed over the open-spandrel construction a heavy parapet as a guard against possible derailment, which then becomes a continuous girder spanning from pier to pier and subject to the foregoing distortions.

The writer wishes to cite an interesting experience in this connection: Between 1908 and 1911, the Delaware, Lackawanna, and Western Railroad Company constructed a cut-off line between Hopatcong, N. J., and Slateford, Pa., in the course of which two concrete arch viaducts were constructed—one over Paulins Kill, at Hainesburg, N, J., and the other over the Delaware River, at Portland, Pa. Both these viaducts were built without expansion joints, except at the piers, the floor systems being of arch construction so that the sections at the junction with the spandrels are quite heavy.

In 1912, preliminary to the design of the proposed structures on the second cut-off line between Clarks Summit and Hallstead, Pa., an examination was made of the parapets of the aforementioned two viaducts to determine whether intermediate expansion joints should be provided in the new structures. These older structures had been in operation a sufficient length of time to have been subjected to several cycles of temperature changes at that time. The examination disclosed vertical cracks in the parapets, particularly in those of the Delaware River Viaduct, possibly due to the fact that the arches had considerably less rise in proportion to span. Although the cracks were not serious, they indicated the effects of the omission of intermediate expansion joints, and it was decided to provide them in the then proposed structures—the Martin's Creek and Tunkhannock Viaducts. These two viaducts have been in operation for more than twenty years, and the expansion joints have functioned so as to eliminate this difficulty.

---

[12] Concrete Engr., D. L. & W. R. R., Hoboken, N. J.

Another interesting phenomenon was observed in connection with the effect of temperature stresses on arch bridges with solid spandrel walls retaining the fill. These arches were of smaller spans, 20 to 45 ft, some of which were constructed early in the progress of the first cut-off line. Before any live load was placed on these arch bridges, the spandrel walls were found to have vertical or nearly vertical cracks at each haunch which, it was decided, were due to the distortion of the arch ring resulting from temperature changes, the high section over the haunch being subjected to greater deformation due to its height. The effect being similar to shear cracks in a beam, it was decided to place shear reinforcement in the spandrel walls in subsequent structures with the result that these cracks have not appeared. Strangely, this difficulty was not experienced with the flat over-head highway arches—probably due to the fact that the height at the haunch is much smaller and the section of the arch ring much lighter. This fact is borne out by the author's conclusion in connection with the relative effects on high and low spandrel construction.

Another point to which the writer would call attention is in connection with Professor Wilson's preliminary statement (see "Description of Specimens and Apparatus: Specimens") that "the weight of the specimen was not great enough to produce a dead-load stress commensurate with the dead-load stresses encountered in the design of a full-sized bridge."

This is significant and emphasizes the care necessary in drawing conclusions from model test results and applying them to full-sized structures. The problem is intricate when the material in both model and prototype is identical; it is more so when the models are of a material having elastic properties entirely different from those of the structure; and it is particularly serious when the structure is an arch and temperature effects are to be considered. In this type of structure, the thrust and moment due to temperature change are proportional to the cube of the depth of arch section and inversely as the square of a function of the rise so that no direct relationship exists between a reduced scale structure and one of full size.

Attention is called to the summary of conclusions, showing an appreciable increase in stress in multiple-arch spans on slender piers over that in arches of single spans with fixed abutments. This should be a warning in the design of multiple-span arch structures of the so-called "rigid frame" type when computing dead load stresses in its members.

It is needless to state that the author has contributed exceedingly valuable information in a field where it is most urgently needed, and his painstaking attention to all the considerations involved cannot be praised too highly. It is to be hoped that similar tests on skew arches will be undertaken to provide the necessary information for such structures.

Wilbur M. Wilson,[13] M. Am. Soc. C. E. (by letter).—In the absence of any considerable disagreement with the interpretation of the data, little appears to be needed in the way of a closure.

Mr. McCullough has mentioned the desirability of a more complete presentation of the combined algebraic-and-experimental method of analyzing multiple-span arch series. This method is presented in the Final Report of the Special Committee on Concrete and Reinforced Concrete Arches.[14]

Quite properly, Mr. McCullough calls attention to the possibility of an increase in stress at the springing of one arch due to live load on adjacent arches. The live load used in the tests is for highway bridges and the single heavy concentration overshadows the smaller distributed load. For a railway bridge, the load on spans adjacent to the one being considered might produce considerable stress thereby increasing the difference between the stress on a multiple-span structure and that in a similar single span. A point that the writer neglected to mention, is the fact that, although the negative live load moment at the springing of the middle span is reduced by the flexure of the pier, the positive moment at that section and the negative moment at the abutments are increased.

Mr. Mann's analysis of a three-span arch series by means of an elastic model is of interest, showing as it does the agreement that may be obtained between values derived from tests of models and from an algebraic analysis when sufficient experience has been acquired to assure skill and accuracy with both methods. However, the experience of the writer is that for a three-span structure consisting of ribs without deck, which are readily susceptible of algebraic analysis, less time is required and the results are more acceptable if the structure is analyzed by the elastic theory than by the use of an elastic model. In the analysis of a three-span structure with a deck, the spans of which are not readily susceptible of algebraic analysis, an experimental analysis involving the use of elastic models appears to be the most acceptable; and comparisons such as Mr. Mann and others have made would appear to indicate that the results of such an analysis are dependable, providing the tests are properly planned and care is exercised to prevent experimentation errors. Whether the model analysis is better accomplished by tests of a model of the entire structure or by tests of a model of a single span, using the experimental data in the combined algebraic-and-experimental method is, to a certain extent, a matter of personal preference. The writer prefers the latter, and for the following reasons.

---

[13] Research Prof., Structural Eng., Univ. of Illinois, Urbana, Ill.

[14] This report is included in this volume (Vol. 100), of *Transactions*.

The difference in the time required to make the model—between a single-span and a three-span model—is more than that required to make the algebraic analysis. The results of the algebraic analysis are statically consistent, which is not always true with the results of an all-experimental method. With the combined algebraic-and-experimental method a considerable error in the experimental constants produces only a small error in the stress in the rib computed from the constants.

The three-span structure with the low deck, shown in Fig. 2(*g*), was analyzed by the use of an elastic model, and the results agree closely with those obtained from tests of the concrete specimens.[15] The differences between the results obtained by the two methods are possibly no greater than the differences that might exist between the behavior of two concrete arch bridges built to the same drawings and specifications.

Mr. Hirschthal directs attention to the influence upon the rib of the action of the spandrel columns and deck, and particularly the influence of expansion joints in the deck. These are important questions and questions for which there are as yet no definite and final answers. The early impression was that the interaction of the rib and deck was an asset since it appeared to increase the load-carrying capacity of the structure. More recently attention has been directed to the detrimental effect of the interaction which, although increasing the load-carrying capacity of the rib, may overstress the spandrel columns and deck. Possibly, the final solution (particularly for extremely long-span arches) will be to develop a design having such a relation of rib to superstructure that it will be possible to take advantage of the increase in the load-carrying capacity resulting from interaction without injuriously overstressing the spandrel columns and deck. This is desirable, not so much because of the direct saving in material, but because of the reduction in the dead load, which constitutes almost the entire load in a long-span, reinforced concrete arch bridge. This is an important hoped for development of the future. If the tests, which are the subject of the paper, have contributed to this solution, or if they have demonstrated the desirability of such a design, the investigation will have been profitable.

Mr. Hirschthal relates his experiences with the concrete arches of the Delaware, Lackawanna, and Western Railroad Company and states that cracks occurred in the parapets of two bridges built without intermediate expansion joints and did not occur in the parapets of two other bridges having intermediate expansion joints.

The writer would distinguish between expansion joints in the deck and in the parapet. The horizontal movement that would occur if intermediate expansion joints were provided increases with the distance above the rib. The top of the parapet is a considerable distance above the rib at the crown for these bridges. The cracking of the parapet could have been prevented by providing intermediate expansion joints in it whether or not there were corresponding expansion joints in the deck.

---

[15] See *Bulletin 270*. Univ. of Illinois Eng. Experiment Station, Urbana, Ill.

If the arch barrel, or rib, for an open spandrel arch is extremely heavy so as to overwhelm the deck, as it was for the Delaware River Bridge,[16] referred to by Mr. Hirschthal, the stiffer the deck the more likely it is to crack if intermediate expansion joints are not provided. (Mr. Hirschthal kindly furnished the writer with blue-prints showing the details of the bridges referred to by him.) The deck for all but the Delaware River Bridge was a considerable distance above the rib at the crown, and the bridges fall in the class for which the tests by the writer[17] show that a considerable horizontal movement would occur if intermediate expansion joints were provided. The deck for the Delaware River Bridge was high enough to be not integral with rib at the crown although not so high as for the other bridges. Without having sufficient knowledge as to the interaction between the rib and deck to form a final opinion, it appears to the writer that there are features of this bridge that make it particularly liable to crack if intermediate expansion joints are not provided. Although the deck is not especially high, the parapet is high enough so that some horizontal movement would have taken place at intermediate expansion joints if they had been provided. The arch barrel is extremely heavy. The deck spans are short and, therefore, so stiff that they cannot conform to the flexure of the rib without cracking and yet are not strong enough to prevent this flexure. The situation is further aggravated by the fact that the section of the deck is less at the center of the deck span than at the transverse spandrel walls.

It is possible that the need for intermediate expansion joints in the deck of an arch bridge is not entirely a question of the position of the deck relative to the rib, although that is a vital relation, and both tests and a visualization of the action of the structure indicate that the horizontal movement that would occur if expansion joints were provided is a function of the height of the deck above the rib. The relative stiffness of the deck, spandrel columns, and rib is also important as they probably affect the vertical and angular motion that would occur if expansion joints were provided; they also affect the stress that will result if the movement is prevented or reduced by making the deck continuous.

The studies made to determine the movement at expansion joints[17] were limited to horizontal movement, the opening and closing of the joint. In view of the questions raised by Mr. Hirschthal, further studies to determine the relative vertical and angular movement of the two ends of the deck that meet at an expansion joint, appear to be desirable. This can be done in an acceptable manner with elastic models by methods similar to those used in studying the horizontal movement.

Another question pertaining to the design of multiple-span arch bridges which the writer believes should obtain more attention than it receives, at least in the literature on arches, is the distribution of the vertical distance from the bases of the piers to the top of the roadway. The elevation of the roadway is usually fixed by the approaches, and that of the bases of the piers by the foundation conditions. The difference between these two elevations is usually fixed within narrow limits, but the total vertical distance can be divided into three parts at the discretion of the designer. These parts are: (1) The height of the pier; (2) the rise of the arch; and (3) the distance of the deck above the rib at the crown.

---

[16] *Engineering News*, Vol. 62, No. 26, p. 713.

[17] *Bulletin 226* and *Bulletin 270*, Univ. of Illinois Eng. Experiment Station.

In so far as the limited number of tests show, for a particular rib and a particular deck, the height of the deck above the rib does not affect the load-carrying capacity of the structure, although it does affect somewhat the stresses resulting from temperature changes. Increasing the height of the deck above the crown of the rib also increases the horizontal movement that would take place at intermediate expansion joints if they are provided. This is compensated for, at least in part, by the greater flexibility of the longer spandrel columns. Therefore, it would appear that, structurally, the high deck has no appreciable advantage over the low deck, or the reverse.

If the total available vertical distance is small, the natural design is to make the piers as low as the required flood channel will permit and make the deck so low as to be integral with the rib at the crown, thereby obtaining a maximum rise for the arch rib. If the total vertical distance is ample, or possibly excessive, the best distribution is not so easily apparent.

Increasing the flexibility of the piers of a multiple-span series increases the live-load stress in the rib, but a considerable increase in the flexibility causes a relatively small increase in the stress. Moreover, increasing the flexibility of the piers has a lesser effect on a rib with a large, than on one with a small, rise ratio. Increasing the height of the pier increases the flood channel. Increasing the rise ratio of the arch decreases the horizontal thrust on the piers due to load and decreases the temperature stress in the rib. It would appear, therefore, that there is no advantage in separating the deck from the rib at the crown as the vertical distance can be better disposed of by increasing the height of the pier and increasing the rise ratio of the arch rib. The use of a multiple-span series of flat arches on high piers would appear to be undesirable.

Mr. Hirschthal emphasizes the care necessary in drawing conclusions from tests of a small structure. The writer wishes to repeat this caution not only in applying the results obtained from a small structure to the design of a large structure, but also in drawing any general conclusions from a limited number of tests. Although the investigation included several tests, the results have little statistical value. In general, they are of value because they indicate how the structure functions rather than because of the absolute quantitative values obtained. Those interested are urged to study the detailed reports[18] and make their own interpretation of the data rather than depend entirely upon the writer's interpretation.

---

[18] *Bulletin 269* and *Bulletin 270*, Univ. of Illinois Eng. Experiment Station.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 1901

RENEWAL OF MITER-GATE BEARINGS, PANAMA CANAL

BY CLINTON MORSE,[1] JUN. AM. SOC. C. E.

WITH DISCUSSION BY MESSRS. E. S. RANDOLPH, AND CLINTON MORSE.

SYNOPSIS

The equipment and methods of raising and moving twelve miter-gate leaves of the Panama Canal, at Miraflores, are described in this paper. Considerable wear on the vertical bearing surfaces made replacement advisable. The quoin-bearings could be made accessible only by raising the leaves clear of the pintles and moving them from the wall. The largest leaves are 82 ft high and have an estimated gross weight of 790 tons.

Comparisons are made between the experimental equipment and the methods used at Pedro Miguel, in 1929, and the present system. Improvements in technique, suggested by experience gained in raising twenty-two leaves during three over-haul periods, are emphasized.

INTRODUCTION

The Panama Canal was opened to commercial traffic on August 15, 1914. Three lifts are required to reach the summit level, which is normally 85 ft above sea level. Twin chambers, each 110 ft wide, with a usable length of 1 000 ft, permit simultaneous northbound and southbound lockages. A diagram of the Miraflores Locks is shown in Fig. 1. At the time of their construction the lock-gates were by far the largest and heaviest that had ever been built. They contained a number of new features that have been incorporated in later designs. An unusual device was the use of removable nickel-steel plates for the miter and quoin-bearings shown in Fig. 2.

The leaves pivot on hemi-spherical nickel-steel pintles, 16 in. in diameter (Fig. 3). On the gates at the sea end of the lower chambers at Miraflores

---

NOTE.—Published in January, 1934, *Proceedings*.

[1] Jun. Engr., The Panama Canal, Balboa Heights, Canal Zone. Now Associate Member, Am. Soc. C. E.

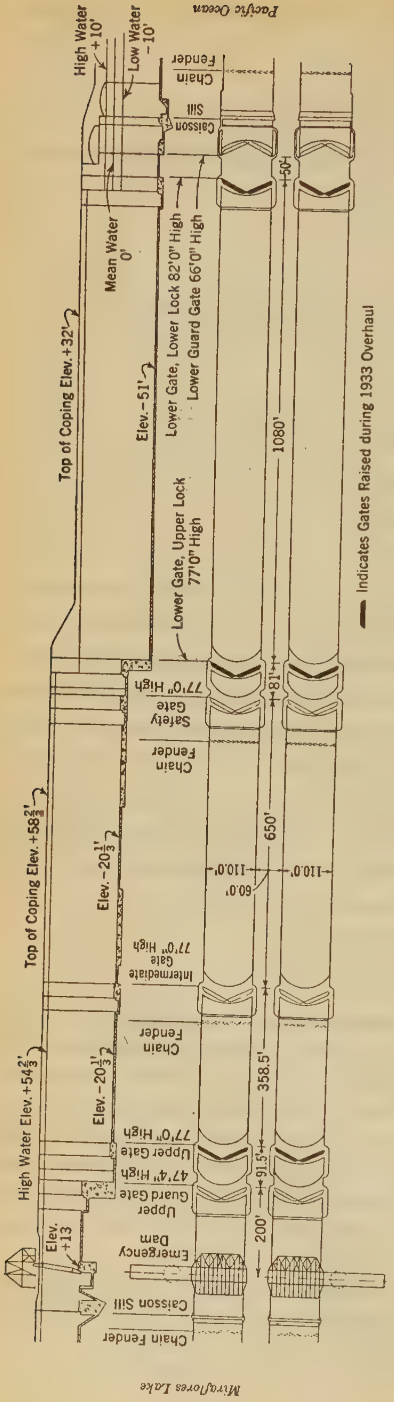

Pacific Ocean
High Water +10'
Low Water -10'
Chain Fender
Caisson Sill
Mean Water 0'
Top of Coping Elev. +32'
Elev. -51'
Lower Gate, Lower Lock 82'0" High
Lower Guard Gate 66'0" High
50'
1080'
Lower Gate, Upper Lock 77'0" High
Indicates Gates Raised during 1933 Overhaul
77'0" High
Safety Gate
81'
Chain Fender
Top of Coping Elev. +58½'
Elev. -20⅓'
650'
110.0'
60.0'
110.0'
Intermediate Gate
77'0" High
Chain Fender
358.5'
High Water Elev. +54⅔'
Elev. -20⅓'
77'0" High
Upper Gate
47'4" High
Upper Guard Gate
91.5'
Elev. +13
Emergency Dam
200'
Caisson Sill
Chain Fender
Miraflores Lake

FIG. 1.—MIRAFLORES LOCKS.

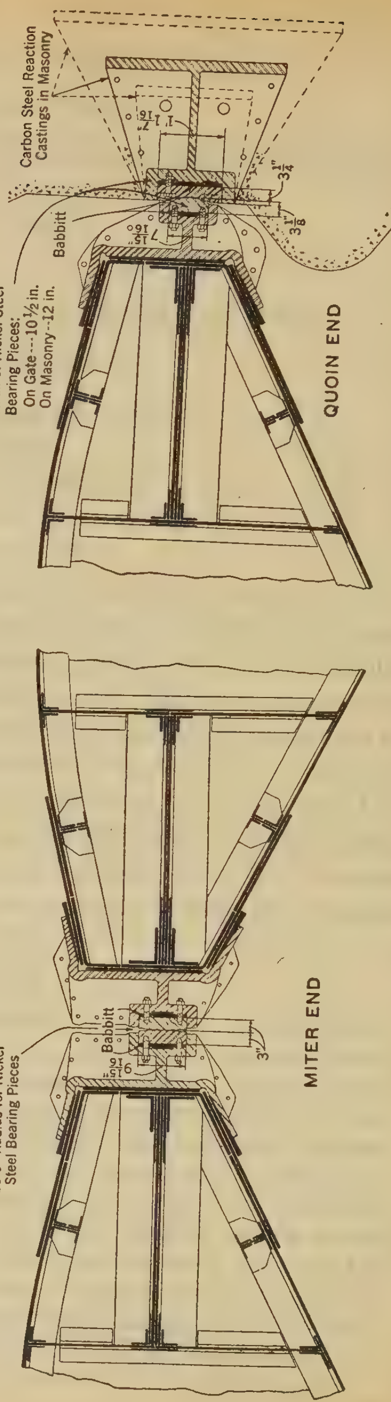

Carbon Steel Reaction Castings in Masonry
1 7/16"
3 1/4"
Babbitt
1 15/16"
3 1/8"
Radii of Nickel Steel Bearing Pieces:
On Gate--10½ in.
On Masonry--12 in.
QUOIN END
16'0" Radius for Nickel Steel Bearing Pieces
Babbitt
9 15/16"
3"
MITER END

FIG. 2.—SECTION SHOWING MITER AND QUOIN BEARING-PLATES

and on the lower guard-gates at Gatun, the pintle bushings—twelve in all—are of carbon steel; the others are of manganese bronze. Each leaf is held at the top, by a pin connection, to a V-shaped, vanadium-steel yoke which, in turn, is keyed to anchorages deeply embedded in the concrete walls.

The major dimensions of the gates (of which there are 92) are given in Fig. 4. Their gross weight is 426 to 790 tons each. The design and erection of these gates have been described[2] concisely by Henry Goldmark, M. Am. Soc. C. E., who was in direct charge of the work.

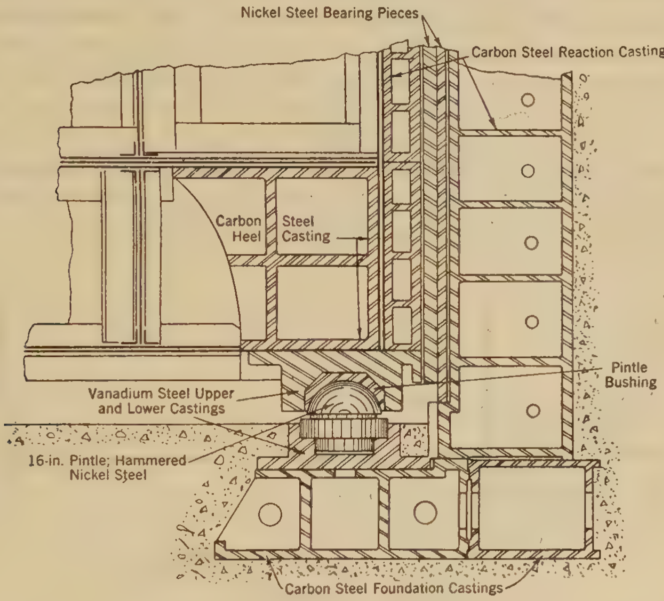
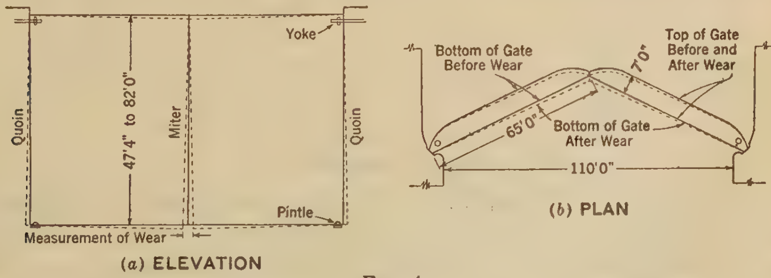

Nickel Steel Bearing Pieces
Carbon Steel Reaction Castings
Carbon Heel
Steel Casting
Pintle Bushing
Vanadium Steel Upper and Lower Castings
16-in. Pintle; Hammered Nickel Steel
Carbon Steel Foundation Castings

FIG. 3.—PINTLE AND QUOIN-BEARINGS.

The locks are overhauled during the dry season, which starts about January 1 and ends early in May. The first over-haul periods involved mostly repainting the gates and replacing the wooden sills damaged by the teredo.


Yoke
Quoin
47'4" to 82'0"
Miter
Quoin
Pintle
Measurement of Wear
Bottom of Gate Before Wear
Top of Gate Before and After Wear
7'0"
65'0"
Bottom of Gate After Wear
110'0"

(*a*) ELEVATION

(*b*) PLAN

FIG. 4.

In the period between 1924 and 1929, extensive repairs were made to the rising stem and the cylindrical valves.[3]

---

[2] "The Panama Canal," *Transactions,* International Eng. Congress, Vol. II, pp. 87-129.
[3] "Lock Valves," Manuals of Eng. Practice, No. 3, Am. Soc. C. E., pp. 54-71.

Present policy requires that each lock be given a general overhauling every four years, the periods alternating between the Atlantic and Pacific locks. Thus, one chamber of the canal is out of service, for about three months during the dry season, once every two years.

Effects of Worn Bearings

The wear on the bearing-plates was first measured during the over-haul at Gatun in 1924. The original sill clearance was $\frac{1}{2}$ in. at the miter end, tapering to zero at the quoin. Inspection showed that the gates rested against the fixed sills and did not meet at the miter point. The measurements were made in an effort to discover the cause of this condition. The total wear on the six bearing surfaces of a pair of leaves was taken to be the space the leaves stood apart at the bottom of the miter when the top plates were in contact. In Fig. 4(*a*) the full lines show the original position of the gate and the dotted lines the position after the bearings had become worn. Measurements made again in 1927 showed the the rate of wear was about $\frac{1}{32}$ in. per yr, but that it varied, depending on the severity of use.

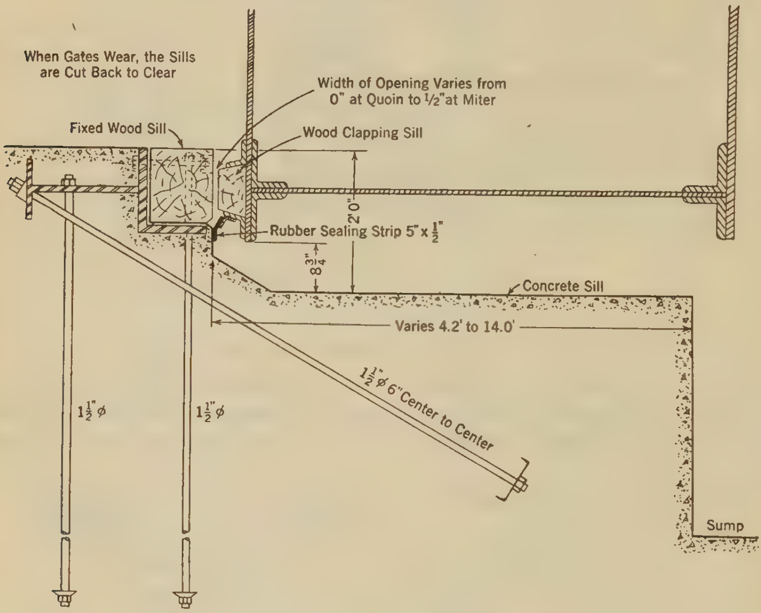

When Gates Wear, the Sills are Cut Back to Clear
Width of Opening Varies from 0" at Quoin to 1/2" at Miter
Fixed Wood Sill
Wood Clapping Sill
2'0"
Rubber Sealing Strip 5" x 1/2"
8 3/4"
Concrete Sill
Varies 4.2' to 14.0'
1 1/2" ø 6" Center to Center
1 1/2" ø
1 1/2" ø
Sump

Fig. 5.—Section Through Sill.

The gates act as a horizontal three-hinged arch when they are in position against a head of water. Wear on the vertical bearing surfaces causes rib-shortening and a consequent down-stream movement of the miter point. The movement down stream is 1.10 in. per in. of wear. It is necessary to trim the wood sills periodically (Fig. 5) in order to restore the original clearance.

The result of this trimming has been to increase the tendency of the leaves to warp when in operation. The top of the miter, which showed no wear, remained in its original position, whereas the bottom was forced down stream until the bearing-plates made contact; this caused the leaves to bend along a diagonal line extending from the top of the miter to the pintle. The distortion of the gates is indicated in Fig. 4 (*b*).

The objectionable results of the wear on the bearing surfaces made it obvious that they should be renewed, beginning with the main operating gates and those showing the greatest wear. While the nominal life of the bearing-plates is twenty years, increasing traffic would shorten their life. Twenty-two years would be required to renew all the bearing surfaces if eight leaves were repaired every two years. Even this plan would require that twelve leaves be repaired during one over-haul.

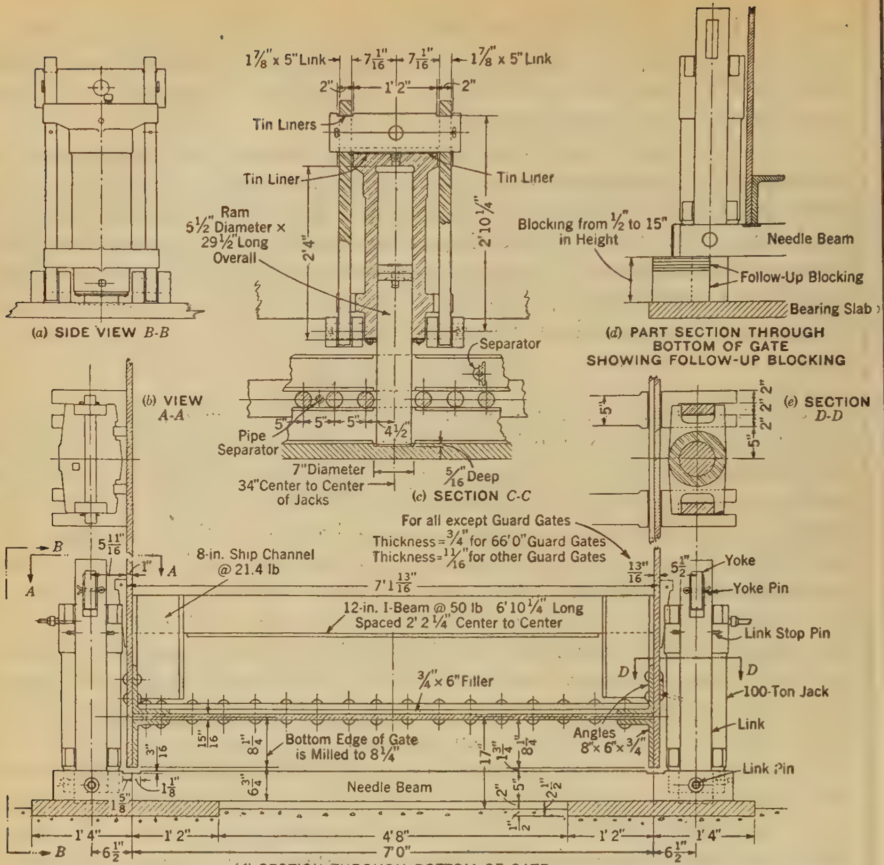
THE TRIAL RAISING

The only way in which the quoin-bearing could be made accessible was to raise the leaf from the pintle and then move it from the wall. Since no attempt had ever been made to raise and move leaves of this size before, the first attempt was largely an experiment to determine the proper equipment and methods to be used for renewing all the bearings. Economy demanded, and the short dry season made imperative, the design of standard equipment and methods which function with production-line efficiency. A proposal to float the gates out of the chamber and to repair them in dry dock was abandoned and E. S. Randolph, M. Am. Soc. C. E., developed a method of raising the leaves within the chamber by hydraulic jacks connected to needle-beams under the leaf, in such a way that the leaves are raised only enough to clear the pintle and then are rolled from the wall.

The first test of this equipment was made in 1929 at Pedro Miguel, when two leaves were raised and replaced successfully. Desirable minor changes in the equipment and mode of procedure were obvious at once. Two years later, eight leaves were raised at Gatun. The speed and efficiency with which the equipment functioned during this over-haul removed all doubts as to the practicability of the adopted design. Additional equipment was purchased and twelve leaves (see Fig. 1) were raised and replaced at Miraflores between January 3 and April 15, 1933.

GENERAL DESCRIPTION OF THE RAISING EQUIPMENT

The small clearance under the leaf (see Fig. 5) made it necessary to place the twelve jacks (grouped in four sets of three jacks each) on the sides. Each set is equi-distant from the transverse center line of the leaf, and the jacks are set with the rams down, bearing against steel slabs grouted in place. The needle-beams under the leaf are arranged in pairs, one beam on each side of a jack; a yoke across the top (closed end) of the cylinder passes through the slotted ends of link bars on each side of the jack; the lower end of each link bar fits between the forked end of a needle-beam; a pin through the fork and link bar completes the connection; and the weight of the leaf is carried on its flange-like bottom edge. Details of the jack-and-needle-beam assembly are shown in Fig. 6, and a typical set of three jacks is shown in Fig. 7.



(a) SIDE VIEW B-B

(b) VIEW A-A

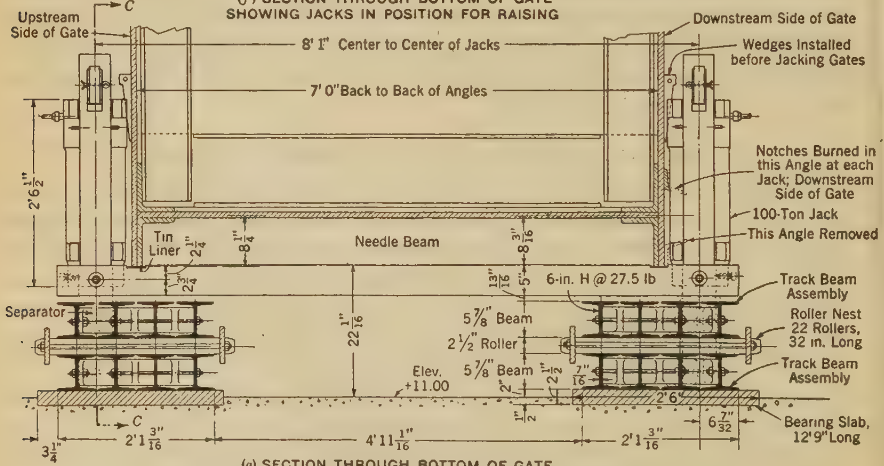
(c) SECTION C-C

(d) PART SECTION THROUGH BOTTOM OF GATE SHOWING FOLLOW-UP BLOCKING

(e) SECTION D-D

(f) SECTION THROUGH BOTTOM OF GATE SHOWING JACKS IN POSITION FOR RAISING

1⅞" x 5" Link
7 1/16"
7 1/16"
1⅞" x 5" Link
2"
1' 2"
2"
Tin Liners
Tin Liner
Tin Liner
Ram 5½" Diameter × 29½" Long Overall
2' 4"
2' 10¼"
Separator
Pipe Separator
5"
5"
5"
4½"
7" Diameter
34" Center to Center of Jacks
5/16" Deep
Blocking from ½" to 15" in Height
Needle Beam
Follow-Up Blocking
Bearing Slab
5"
2"
2"
2"
5"
For all except Guard Gates
Thickness = ¾" for 66' 0" Guard Gates
Thickness = 1 1/16" for other Guard Gates
B
A
5 11/16"
1"
A
8-in. Ship Channel @ 21.4 lb
7' 1 13/16"
13/16"
5½"
Yoke
Yoke Pin
12-in. I-Beam @ 50 lb 6' 10¼" Long Spaced 2' 2¼" Center to Center
Link Stop Pin
D
D
¾" x 6" Filler
100-Ton Jack
Link
15/16"
8¼"
3/16"
Bottom Edge of Gate is Milled to 8¼"
17/16"
1¾"
8¼"
Angles 8" x 6" x ¾"
Link Pin
1⅛"
6¾"
Needle Beam
2"
5"
2½"
1 5/8"
½"
1' 4"
1' 2"
4' 8"
1' 2"
1' 4"
B
6½"
7' 0"
6½"



(g) SECTION THROUGH BOTTOM OF GATE SHOWING JACKS EXTENDED AND ROLLER NESTS IN POSITION

C
Upstream Side of Gate
Downstream Side of Gate
8' 1" Center to Center of Jacks
Wedges Installed before Jacking Gates
7' 0" Back to Back of Angles
2' 6½"
Notches Burned in this Angle at each Jack; Downstream Side of Gate
100-Ton Jack
This Angle Removed
Tin Liner
2¼"
8¼"
Needle Beam
8 3/16"
3¼"
13/16"
5"
6-in. H @ 27.5 lb
Track Beam Assembly
Separator
22 1/16"
5⅞" Beam
2½" Roller
Roller Nest 22 Rollers, 32 in. Long
Elev. +11.00
5⅞" Beam
½"
2"
2"
7/16"
Track Beam Assembly
2' 6"
6 7/32"
½"
Bearing Slab, 12' 9" Long
C
3¼"
2' 1 3/16"
4' 11 1/16"
2' 1 3/16"

FIG. 6.—ARRANGEMENT OF JACKS

A mixture of switch oil and transformer oil is the hydraulic medium used to operate the jacks, and pressure is supplied by an electrically driven pump mounted on a detachable platform on the down-stream side of the leaf. The twelve jacks are interconnected to this pumping unit.

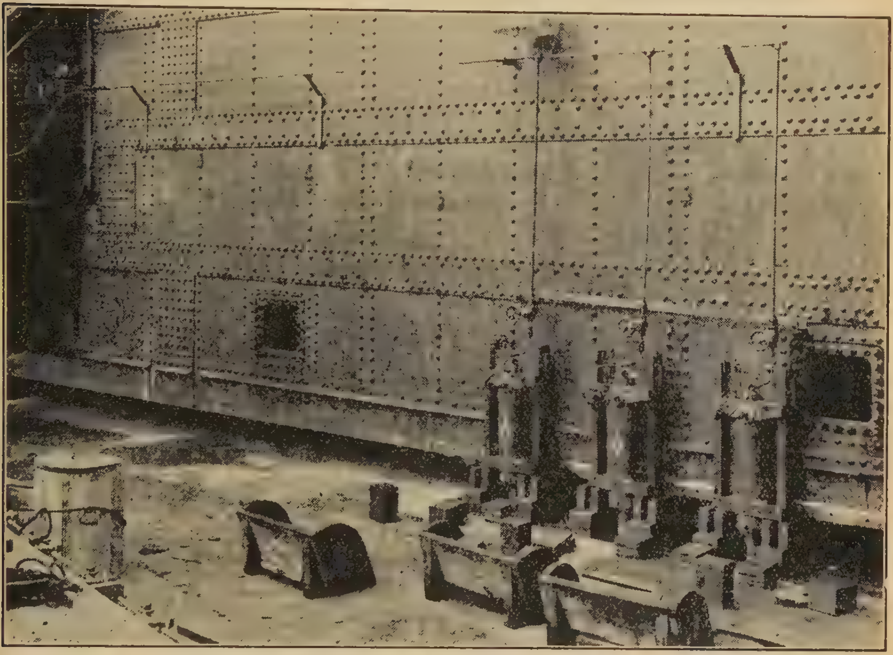


FIG. 7.—VIEW OF JACKS AND PUMP PLATFORM

A set of upper and lower roller tracks and a train of rollers is provided at each group of jacks. The needle-beams bear across the top of the upper tracks, with the lower tracks resting on the bearing slabs and separated from the upper tracks by the roller train. The track and roller assembly is designed to permit 4 ft of travel. Fig. 6(*c*) and Fig. 6 (*g*) show the leaf raised and the tracks and rollers in place. Overturning is prevented by a structural steel guide-frame and four steel cables anchored in the lock-walls.

Some of the principal unit stresses (in pounds per square inch) applying to this problem are, as follows:

	Normal	Maximum
Bearing between bottom of leaf and needle-beam	7 050	10 575
Extreme Fiber Stress in Needle-Beam When:		
Lifted by jacks	20 600	30 900
Resting on shims	26 000	40 350
Resting on tracks	8 000	12 000
Shearing stress in pins	8 300	12 450
Bearing in pins	15 700	23 550
Bending in pins	30 000	45 000

	Normal	Maximum
Tension in link at pin-hole	13 000	19 500
Bearing at top of slot	21 100	30 150
Double shear below pin-hole	8 290	12 425
Extreme fiber stress in yoke	18 850	28 275
Shear in yoke	6 870	10 505
Bearing on $2\frac{1}{2}$-in. rollers, in pounds per linear inch	(1 000)	......
Compression of web of H-beams in roller nests	3 140	4 710
Bearing on masonry under edge of bearing slabs when load is carried on follow-up blocking	400	600
Maximum bending stress in rams due to all lateral loads	.....	9 220
Oil working pressure in jacks	5 600	8 400

In the foregoing tabulation normal stresses are those caused by all parts functioning perfectly with an even distribution of the load; maximum stresses are those derived by the conditions of design. Due to the high unit stresses anticipated and the lack of clearance for enlarging the sections, the yokes, links, link-pins, and needle-beams were made of forged nickel-manganese steel.

Details of Lifting and Rolling Equipment

Jack Cylinders.—The jack cylinders are medium grade steel castings, made according to Federal Specifications No. 170, with the dimensions as shown in Fig. 6(*c*).

The bleeder valves shown in Fig. 7 consist of a standard $\frac{1}{2}$-in. pipe plug drilled and tapped for a $\frac{1}{4}$-in. needle-pointed spindle and a small packing nut. An oil column made up of a glass tube and $\frac{1}{4}$-in. brass pipe fittings is screwed into the side of the plug. The needle-point controls the outlet and allows the mixture of oil and air to pass up through the vertical glass tube and thence through a downward curving spout. A bucket is held under the spout to catch the oil expelled with the air. The absence of air bubbles passing through the glass tube gives an accurate indication as to when all the air has been expelled.

A gag screw, placed near the open end of the cylinder, can be tightened against a Tobin bronze gag plug to prevent the ram from falling out during handling. A felt wiping ring is fastened around the open end of the cylinder to prevent small particles of grit from working up between the cylinder and the ram.

Jack Rams.—The rams (see Fig. 6(*c*)) are of rolled carbon-steel bar stock (Society of Automotive Engineers Standard No. 2315) ground to a diameter 0.008 in. less than the bore of the cylinder, and case-hardened. No attempt was made to hold the diameters of the rams and cylinders to an exact dimension, but each ram was required to fit its own cylinder with the specified clearance.

The original jacks were provided with rams 0.005 in. smaller than the bore and were not case-hardened. The cylinder did not have the felt wiping ring and, consequently, small particles of grit entered when the jacks were lowered. When one of these small particles was caught it would raise a small sliver of metal on the ram. As the ram was pushed out, this sliver would gradually roll into a ball and "freeze" the jack. The jacks functioned perfectly at Miraflores and not a single case of scored rams or cylinders was evident.

Pumping System.—Pressure is supplied by a vertical triplex pump having a $\frac{1}{2}$-in. bore and a 4-in. stroke. The driving force is a 5-hp motor acting through a reduction gear giving a speed of 37 rpm. The unit is compactly mounted on a roofed platform which is hooked to the down-stream side of the leaf (a part of this platform is visible in Fig. 7). The high-pressure lines lead through calibrated gauges and flexible couplings to pipe lines, fastened to the leaf and leading to each end; thence they lead through the leaf to the up-stream side. Tee connections and short lengths of pipe lead from the main pressure line on the leaf to stop-valves, and thence through flexible couplings to the individual jacks. The stop-valves at the jacks (see Fig. 7) are normally open and are used only in testing or for cutting out individual jacks in case of their failure.

The pipes on the platform are arranged so that all valves are within reach of one man. The miter end, which is the lighter, tends to raise faster than the quoin so that it is necessary to throttle, slightly, the valves controlling the miter end. A spirit-level fastened to the gate enables the operator to raise both ends at the same rate, approximately 0.3 in. per min. The jacks are interconnected so that no jack can carry more than one-twelfth of the total load.

The high-pressure line is an extra heavy steel pipe, $\frac{1}{2}$ in. in inside diameter, with forged fittings and especially built valves, all of which were subjected to a pressure inspection test before acceptance. The threads were machine-cut on the job, in order to insure a perfect fit and a thick solution of shellac was found to be the most satisfactory medium for sealing the threaded joints. The flexible couplings are of annealed copper tubing, $\frac{1}{4}$ in. in inside diameter, with brass nipples soldered to the ends. Annealed copper washers were used in all couplings and unions because they had proved more satisfactory than either leather or tin, and little trouble was experienced with leaking connections.

One pump platform and one set of jacks serves two leaves. The jacks are uncoupled and assembled on the other leaf of the pair as soon as one leaf has been raised, but the pump platform is left in place until the leaf has been pushed from the wall. In transferring the platform to the other leaf, it is necessary to uncouple only the flexible joint at the edge and thus leave the fixed piping in place. The original pumping unit was placed on the floor of the sump on the up-stream side of the gates. The maze of pipes interfered considerably with the assembly of the rolling equipment. The present unit is compact, out of the way, and functions perfectly.

Shims.—Banca tin shims were used at all points of bearing in the lifting system, to overcome any uneven bearing and to insure an even distribution of the load by crushing. They were placed between the jacks and the yokes, between the yokes and the link bars, and between the needle-beams and the gate. Their effectiveness was thoroughly demonstrated, some being crushed to one-half their original thickness, while others were only slightly dented.

Banca tin was selected after a number of compression tests had also been made on solid and perforated specimens of aluminum, copper, lead, and type metal.

Roller Tracks.—The roller tracks are centered under the edge of the leaf, which necessitates assembling them around the extended jack rams. To permit this assembly, the inside flanges of the outer beams and the outside flanges of the inner beams are cut back to the web in three places. The upper and lower tracks are alike except for the location of these holes for the jack rams. Hand holes are cut through the webs of the outer beams at each jack location. These holes are necessary to permit the removal and replacement of the rocker disks under the rams. The three inner beams are bolted together in a unit so that the outer beams can be slipped into place quickly. The four upper tracks are made to move as a unit by connecting the up-stream and down-stream tracks to each other, and then connecting the quoin and miter ends by two H-beams.

The original tracks were not provided with hand holes and were only loosely bolted together. The difficulty experienced in assembling the original tracks was overcome by bolting the inner beams together and providing separate bolts for the outer beams.

Bearing-Blocks.—For the lifting and rolling operations the leaves are fixed in a partly open position, so as to allow sufficient room for scaffolds on the miter ends of the leaves after they had been pushed from the wall. In this position, each leaf partly overhangs the sump. A pair of bearing-blocks (made up of plates and angles) was provided under the miter end of each leaf to overcome the difference in elevation. One block in each pair has one corner cut off diagonally, and is slightly shorter, so that it may be placed parallel to the line of rolling and against the arched sill. A leveling screw is placed at each corner of the blocks making it a simple matter to bring them quickly and easily to grade. Punch-marks in the top plates indicate the longitudinal and transverse center lines of the jacks. The larger block of the pair weighs 14 980 lb; the smaller, 12 630 lb.

The original blocks were steel boxes set in place and filled with concrete, which made them so heavy that they were extremely difficult to remove with the available equipment. These old blocks were "scrapped" and the new all-steel blocks are easily placed in approximate position, later to be removed, by two electric cranes.

Bearing Slabs.—A rolled-steel bearing slab, (see Fig. 6(*g*)) is placed at each group of jacks. Three counterbores (Fig. 6(*c*)) are cut in the top of each slab to serve as recesses for the rocker disks under the jack rams. Eight leveling screws, one on each corner, and four along the center line, serve to keep the slab level while it is being grouted in place. The center lines of the jacks are indicated by punch-marks. The two slabs at the quoin end rest directly on the concrete sill; and the up-stream miter slab is supported entirely on the bearing-block, but the down-stream slab bears partly on the bearing-block and partly on the sill.

Guide-Frame and Cables.—The guide-frame shown in Fig. 8 is provided as a safety measure against possible overturning. It is designed in accordance with the specifications for steel railway bridges of the American Railway Engineering Association. The wall end of the frame is pinned to a casting that is held in place by a large jack-bolt and a 4-in. pin through a corbel of the leaf recess cover. The other end, equipped with rollers, rests on the leaf and is provided with a 4-ft slot to permit the leaf to travel as it is pushed from the wall. A casting that holds a spherical-headed pin is bolted to the top center of the leaf. This pin is provided with a square nut, made in halves and bolted together around the spherical head. The nut travels in the guide-frame slot when the leaf is pushed from the wall.

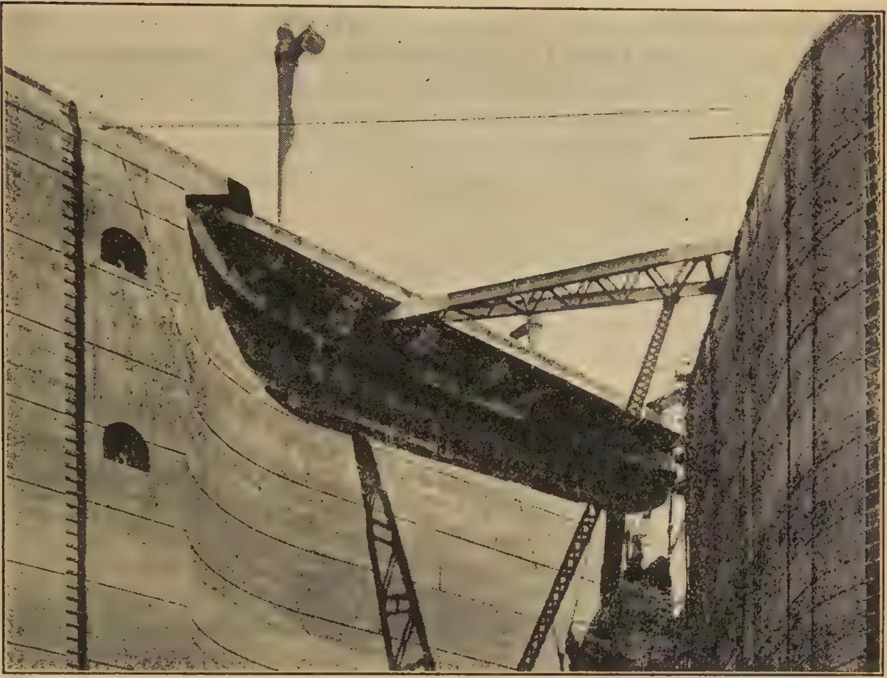


FIG. 8.—GUIDE-FRAME TO PREVENT OVERTURNING.

Four $1\frac{1}{4}$-in., galvanized, extra strong, plow-steel cables are fastened to the leaf with turnbuckles and anchored in the lock walls, two up stream and two down stream (see Fig. 8). These cables are used to correct slight lateral deflections during raising, lowering, and moving, the guide-frame being depended upon in case of emergency.

Pusher Jacks.—The leaves are pushed from the wall by two hydraulic jacks, one on each side of the leaf, which are suspended from the H-beams that connect the upper miter tracks to the upper quoin tracks. These jacks are interconnected, and pressure is supplied by the same pumping unit used in raising the leaf. The jacks, with rams permitting a 4-ft stroke, roll the leaf the required distance with one continuous movement.

The jack cylinder is a 5-ft section of 3-in., double, extra heavy, lap-welded, steel pipe. It is closed at one end by a plug and a bearing head welded in place. The bore is 2.425 in. in diameter with a solid steel ram, 2.410 in. in diameter and 4 ft 10½ in. long. A small hole drilled through the cylinder wall releases the pressure when the ram has traveled the necessary 4 ft. The jacks used during the Pedro Miguel and Gatun over-hauls had only a 12-in. stroke; pressure was supplied by a hand-pump and considerable time was lost in compressing and resetting the jacks after each stroke.

Timber Shoring.—The leaf was protected against lateral movement at the bottom, due to wind load and to unbalanced conditions, by timber shoring at the ends of the leaf.

Preparations for Raising the Leaves

Layout on the Lock-Walls.—The layout on top of the lock-walls for the location of the cable and guide-frame anchors (see Fig. 8) was made six months before the over-haul in order to allow the anchors to be completely set and grouted in place before beginning the actual work. The line and position of each anchor were referenced carefully to brass plugs grouted into the concrete and center-punched. This preliminary work was done largely by the operating forces in the wet season and during lulls in traffic through the Canal. Very few extra men were required, and no valuable time was lost at the beginning of the dry season.

Other preparatory work included: (*a*) Testing the jacks and pump unit; (*b*) distributing the equipment to its proper place on the lock-wall; (*c*) erecting and stocking temporary tool sheds and time-keeping offices; (*d*) installing field telephones at convenient points along the lock-wall; and (*e*) assembling such equipment as could be done at this time.

The equipment was distributed in an orderly manner by assigning a letter and a color to each pair of leaves. Each piece of equipment was stenciled with its proper mark as soon as it was received. Two electric cranes, converted from towing locomotives, and capable of ascending or descending the ramps, were used for much of this distribution work.

Layout on the Chamber Floor.—As soon as the weather indicated the beginning of the dry season, a floating caisson was placed in position and sunk at low tide. The water remaining in the chambers (approximately 10 ft in the upper lock and 41 ft in the lower lock) was pumped out by four electrically driven centrifugal pumps in the caisson. While the chambers were being unwatered, elevators were placed in position and small jib cranes were erected on the tops of the leaves at the miter and quoin ends.

Work began in the upper chamber as soon as it was dry. The sills were cleaned of mud and the water was pumped out of the sumps. The clapping-sill on the leaf and its lower retaining angle (see Fig. 5) were removed and the bottom and lower part of the leaf scraped and cleaned.

The down-stream face of the leaf served as the base line for the location of the bearing-blocks and slabs. The correct position of the base line was located by swinging the gate to the position fixed for the jacking and rolling operations. This position is determined by a direct measurement from a brass plug that indicates the center line of the guide-frame anchor, to the center of the spherical-headed pin on the gate. The position of the leaf is checked carefully because a mistake in this measurement would delay the work seriously. With the leaf in the correct position, the base line is established on the floor of the chamber and the leaf is then swung to the full open (against the wall) position. With the base line established, brass plugs are set on the longitudinal and transverse jack center lines for reference points. The area to be chipped under the bearing-blocks and slabs is next marked out, and the location of each jack and needle-beam indicated by punch-marks on the leaf.

Certain rivets must be burned out to permit passing the high-pressure piping through the leaf and to bolt the pump platform hangers (Fig. 7) in place. Sections of the sill angle are burned back to allow the jacks to be placed close to the face of the leaf (see Fig. 6(*g*)). The sections of the sill to be "burned" are marked with white paint; those cut from the angle are saved and later welded into place.

The bottom edge of the leaf is milled at each needle-beam location to insure a perfect bearing between the beams and leaf. A horizontal reference line, for the milling machine, is marked on the leaf from a bench-mark established in the sump.

The bearing slabs (Fig. 6(*g*)) are set with their tops only slightly above the level of the concrete sill, making it necessary to chip a recess in the sill for each of the quoin slabs. Because the depths of the sumps vary in each lock, it is necessary to build a concrete mat under the bearing-blocks in order to bring them to their proper elevation. As soon as the leaf layout has been completed, workers begin chipping the areas under the blocks and slabs. After the blocks have been placed in correct position and elevation, the mat is formed by pouring cement mortar between the bottom of the block and the floor of the sump. The mortar consists of 1 bag of cement to 2 sacks of sand, with $2\frac{1}{2}$ lb of calcium chloride for each bag of cement. Three vertical 2-in. pipes through the blocks acted as vents and prevented the formation of air pockets.

The four bearing slabs were carefully leveled (the permissible variation being $\frac{1}{64}$ in.) and checked before and after they were grouted in place. The correct position of the slabs and blocks was easily established by bringing the punch-marks on them to piano wires stretched between the reference plugs.

Assembling and Testing Equipment.—As soon as the slabs were set and the milling was completed, the leaf was swung to the jacking position and the clearance between the milled bottom of the leaf and the slabs was carefully checked. A difference of $\frac{1}{16}$ in. in level would put undue stress in the equipment with the possibility of failure due to: (1) Crushing the edge of the leaf; (2) bending a needle-beam; or (3) crushing the webs of the roller-track beams. The few slight differences were corrected by spot-welding thin steel shims to the bottom of the leaf (see Fig. 6(g)). The needle-beams were then placed and connected to the jacks, the pump platform was hung in place and connected to the fixed piping, and the timber shoring set in place. The guide-frame was next placed in position, the cables were attached, and the leaf-operating strut was removed. At this point the jacking equipment was ready for test.

Each pair of jacks on the same needle-beam was tested separately, the pressure being raised gradually to 105 tons per jack, which was equal to a 50% over-load. After this test, each group of three jacks (Fig. 7) was tested to the same pressure, during which levels were taken on the slab under the jacks. Occasionally, a deflection of $\frac{1}{64}$ in. would be recorded with a load of 315 tons on the slab, but in most cases no deflection could be observed. Any leaking connections were tightened and the valve stems repacked if necessary.

The next step was to pull the yoke-pin at the top of the leaf. The jacks were used to lift the miter end slightly which allowed the pin to slip out easily. Then the wedges that key the yoke to the anchorages were removed; some, being very tight, were dislodged with difficulty. Several methods were tried, but the most satisfactory proved to be the following: A pneumatic paving breaker with a square-end bit was held on the wedges until the vibrations had broken the rust film between them, and heavy driving wedges and follow-up blocking were then placed back of the key wedges. The driving wedges were struck with the hammer of a minature pile-driving rig operated by the jib crane on the quoin end of the leaf. The necessary effect was secured by allowing a 150-lb weight to fall approximately 6 ft inside a 4-in. pipe placed over the head of the driving wedge. Ten or twelve blows of this hammer were usually sufficient to start the key wedges. While these wedges were being driven, two-thirds of the weight of the leaf was carried by the jacks and the remainder by follow-up blocking under the ends of the needle-beams. A sling was placed around the yoke and the leaf was ready for raising.

Raising the Leaves

With the standard equipment and the experience gained on previous overhauls, raising and rolling the leaves at Miraflores was largely a matter of routine. The end of each beam was blocked up during raising and lowering so that if the oil pressure failed the leaf would not be dropped more than $\frac{1}{4}$ in. One machinist and two helpers were stationed at each group of jacks while the supervisor assumed a position directly under the leaf so that each group was easily visible. The lifting was started with $\frac{3}{4}$ in. of blocking under the beams, and as the leaf was raised, the blocking was changed at a whistle signal from the supervisor. When each machinist had signaled a clearance of slightly more than $\frac{1}{4}$ in., the whistle was blown and the four groups changed the blocks together, the $\frac{1}{4}$-in. blocks being removed and a $\frac{1}{2}$-in. block put in place, making a total of 1 in. At the next whistle, the $\frac{1}{4}$-in.

block was replaced until another ½-in. block could be added. This procedure was continued until ten ½-in. blocks were in place. A solid 5-in. block was then substituted, one block at a time, for the set of ½-in. blocks. The leaf was then lowered, the pressure released, the wedges back of the jacks were removed, and the jacks reseated by hitting them lightly with a block of wood. A slight pressure was then put on the rams and the aligning wedges were replaced. The raising was continued as before until a total of 15 in. of blocking was in place, thus making a net lift of 14¼ in.

The re-alignment of the jacks was important because they are subjected to severe lateral thrusts if their axes of travel are not parallel. The average pressure required to lift the 82-ft leaves was 70 tons per jack (5 900 lb per sq in.), or a total of 840 tons, including the weight of the equipment and friction in the jacks. Fig. 7 shows the leaf partly raised and the follow-up blocking in place.

During the raising, rolling, and lowering operations, the lateral movement was checked with a transit on a line parallel to the down-stream face of the leaf. Observations were taken on targets clamped to webs of the reaction castings and perpendicular to the face. The targets consisted of 6-in. sections of a folding rule fastened to an arm welded to a C-clamp and deflections of $\frac{1}{32}$ in. could be read easily. The unbalanced loads were controlled by proper tension in the stay cables; a signal from the transitman would indicate the direction of any lateral movement to a machinist stationed on the guide-frame who, in turn, would direct helpers, one at each turnbuckle, as to the proper tension in any cable. In this manner, the guide-frame was kept free for any emergency.

Rolling the Leaf

With the leaf raised to its full height, the weight was carried partly by the jacks and partly by the follow-up blocking. The upper and lower groups of inner H-beams and the rollers were next installed. The full weight was then taken entirely by the jacks; the follow-up blocking was removed, the outer H-beams were bolted on, the guide-bars placed over the ends of the rollers, and the beams, tying the upper tracks together, bolted in place.

To make certain that the gate would move away from the wall on the predetermined line, it was essential that the rollers be parallel and their axes normal to that line. The front and rear rollers of each train were omitted and aligning, or dummy, rollers inserted in their place. These were long enough (9 ft 8 in.) to extend through the up-stream and down-stream guide-bars of the roller train, thus insuring that the rollers in opposite trains were parallel. The axes were made normal to the line of rolling by bringing the up-stream and down-stream ends of the dummy equi-distant from the punch-marks which indicate the transverse center lines of the jacks. The dummy rollers were then withdrawn and the regular rollers inserted. Then the leaf was landed on the tracks and preparations were made for rolling it.

With the leaf resting on the roller assembly, the lifting jacks were removed, compressed, and placed near the other leaf of the pair. The pusher jacks were hung in place (see Fig. 9), and connected to the pumping unit, and the timber shoring at the ends of the leaf was removed. Wood-blocking, consisting of 12-in. lengths of 6 by 6-in. timber, was then placed between the lower miter-end tracks and a striking point on the H-beams. These blocks were removed, one by one, as the leaf moved from the wall, thus providing a safety measure against the possibility of the leaf running away.

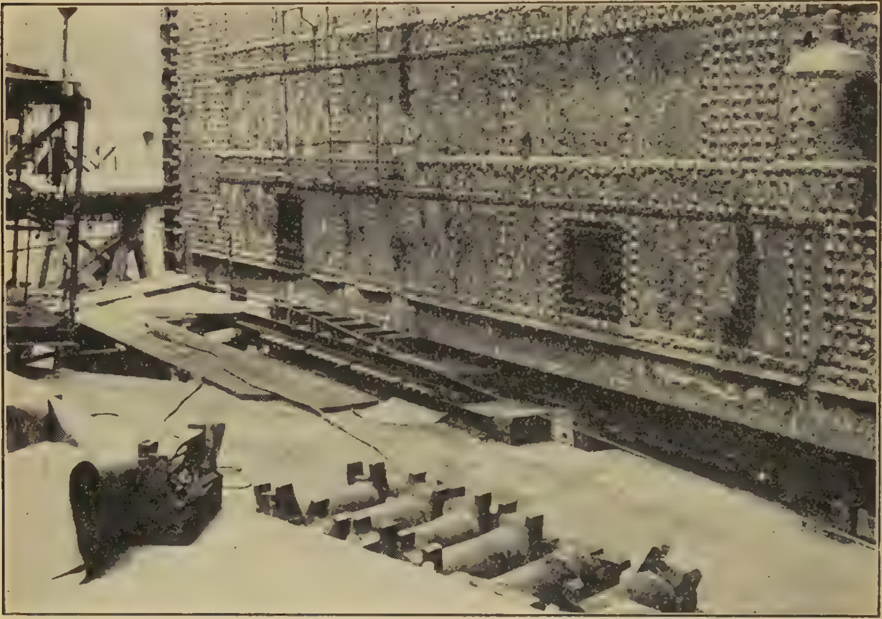


Fig. 9.—Down-Stream Miter End of Leaf Resting on Roller Truck.

The stay cables were carefully regulated and were arranged so that the arc of travel was divided: that is, when the leaf was 2 ft from the wall, the cables on the opposite sides of the leaf at each end were in line with each other and normal to the axis of the leaf. When the leaf had been rolled the required 4 ft, it was securely blocked in place.

The average pressure required to start a leaf was 5 tons per jack, or 10 tons in all; with the leaf rolling, the pressure dropped to $3\frac{1}{2}$ tons per jack. As an indication of the reduction in time required for raising and rolling, the following comparison is of interest:

	Raising time	Lowering time
Pedro Miguel, 1929	2 hr 55 min	37 min
Miraflores, 1933	1 hr 10 min	10 min

The leaves were rolled back and lowered in the same manner, but in reverse order. During lowering, they were landed on the follow-up blocking every $\frac{1}{2}$ in. of the last 5 in. of lowering. This was necessary because the jacks had to be reseated frequently as the leaf moved slightly away from the wall in settling over the new pintle. Fig. 9 shows the down-stream miter end resting on the roller tracks and the pusher jacks in place for moving a leaf back to the wall.

Removal of Old Bearings

Pintle and Bushing.—The pintles and pintle bushings were removed from their sockets by the use of a cast-steel jacking ring. Two concentric rows of twelve equally spaced holes were drilled around this ring; the outer row was tapped for 1¼-in. bolts, used as jacking-screws; and the inner row of $\frac{15}{16}$-in. holes was provided for use in removing the bushing.

This ring was placed over, and welded to, the pintle; two horizontal 1-in. dowels, passed through the ring and into dowel holes in the pintle, helped to hold it in place. Turning the jack-screws against the lower pintle casting (Fig. 3) started the pintle and raised it so that it could be lifted out with the jib crane.

Although the bushings are a shrink fit in the upper casting, no difficulty was found in removing them. The following method was the most efficient: Twelve equally spaced holes were drilled and tapped into the lower edge of the bushing. Short bolts, ⅞ in. in diameter, passed through the ring and set into these holes held the ring in place. The jack-screws in the outer circle bore against the upper casting (see Fig. 3). A hole was then drilled through the top of the bushing and tapped for a 1¼-in. pipe nipple. Flexible tubing was led from this fitting to a hand-operated hydraulic pump. Oil forced between the bushing and casting under a pressure of 600 to 800 lb per sq in. was usually sufficient to start the bushing. The pumping was stopped when the leakage around the bushing became excessive and the removal was completed with the ring and jack-screws.

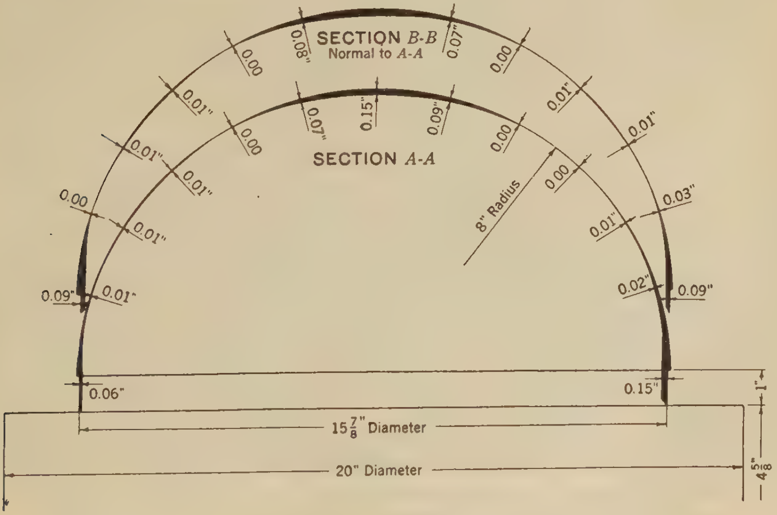


Fig. 10.—Section Showing Typical Wear on Pintle.

The upper pintle castings were examined carefully for evidence of movement or shearing of bolts. The loose castings were tightened by removing the old fastenings and reaming the holes for bolts and rivets of greater diameter. The old pintles and bushings were marked with the leaf number, and lines were marked on them to indicate planes parallel and perpendicular to the lock-wall. Later, the amount of wear on each pintle and bushing was measured along these lines. Figs. 10 and 11 show typical wear on the pintle and bushing, respectively, in the twenty years, 1913 to 1933. It is estimated that the gates were operated 71 600 times during this period.

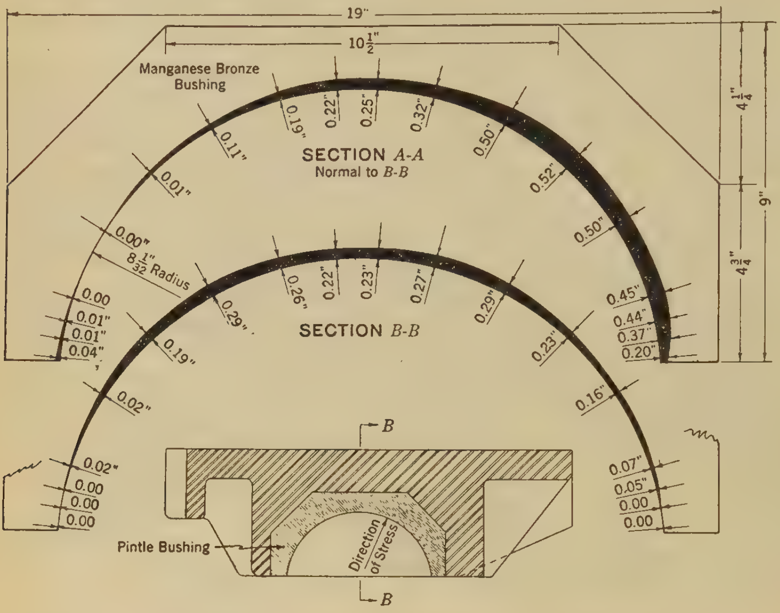

19"
$10\frac{1}{2}$"
Manganese Bronze Bushing
$4\frac{1}{4}$"
9"
$4\frac{3}{4}$"
SECTION *A-A*
Normal to *B-B*
$8\frac{1}{32}$" Radius
0.00
0.01"
0.01"
0.04"
0.00"
0.01"
0.11"
0.19"
0.22"
0.25"
0.32"
0.50"
0.52"
0.50"
0.45"
0.44"
0.37"
0.20"
SECTION *B-B*
0.00
0.00
0.00
0.02"
0.02"
0.19"
0.29"
0.26"
0.22"
0.23"
0.27"
0.29"
0.23"
0.16"
0.07"
0.05"
0.00
0.00
B
B
Pintle Bushing
Direction of Stress

FIG. 11.—SECTION SHOWING TYPICAL WEAR ON PINTLE BUSHING.

The new bushing is provided with five oil grooves and is prevented from turning by three vertical ¾-in. pins inserted in dowel-holes, one-half drilled in the casting and the other half in the bushing. The new pintle (actually an old pintle re-machined) was not placed in its socket until the leaf was ready to roll back, and then the socket was heavily red-leaded and the ball thoroughly greased.

Bearing-Plates.—The removal of the bearing-plates was one of the most difficult features of the first renewal in 1929. This involved: (1) Strongbacks bolted to the bearing-plates; (2) jack-screws through the strongbacks and bearing against the reaction castings; (3) jack-bolts placed between the ends of the stud-bolts and the castings; and (4) hand-driven wedges to remove the plates. It was a slow and laborious method. Details of plate fastenings are shown in Fig. 2.

The bearing-plates on the gates at Miraflores were removed by a vanadium steel wedge 13 ft long, with a taper of ⅜ in. per ft, which had an offset head keyed to a special driving head in an air-actuated pile-hammer. The wedge and hammer are shown in Fig. 12. By keying the wedge to the hammer, one crane handled both, while another crane handled the plates as they were taken off. The offset on the wedge and driving head permitted the hammer to slide down the face of the reaction castings as the wedge was driven behind the bearing-plates.

In order to start the wedge, it was first necessary to jack out the top of each bearing piece at least $\frac{1}{8}$ in. The holes in the face of the plates which were filled with babbitt metal were also drilled out. A novel means of starting the plates was tried on the last leaf repaired. A heavy pipe plug was drilled to receive a firing-pin and the end slightly counterbored to hold a 12-gauge shotgun shell. The wadding and pellets were removed and the shell was filled with $9\frac{3}{4}$ grains of smokeless powder. The plug holding the shell was screwed into the top hole in the face of each bearing-plate and the firing-pin hit with a swinging weight. The explosion was sufficient to blow the plate about $\frac{3}{4}$ in. away from the babbitt-metal backing.

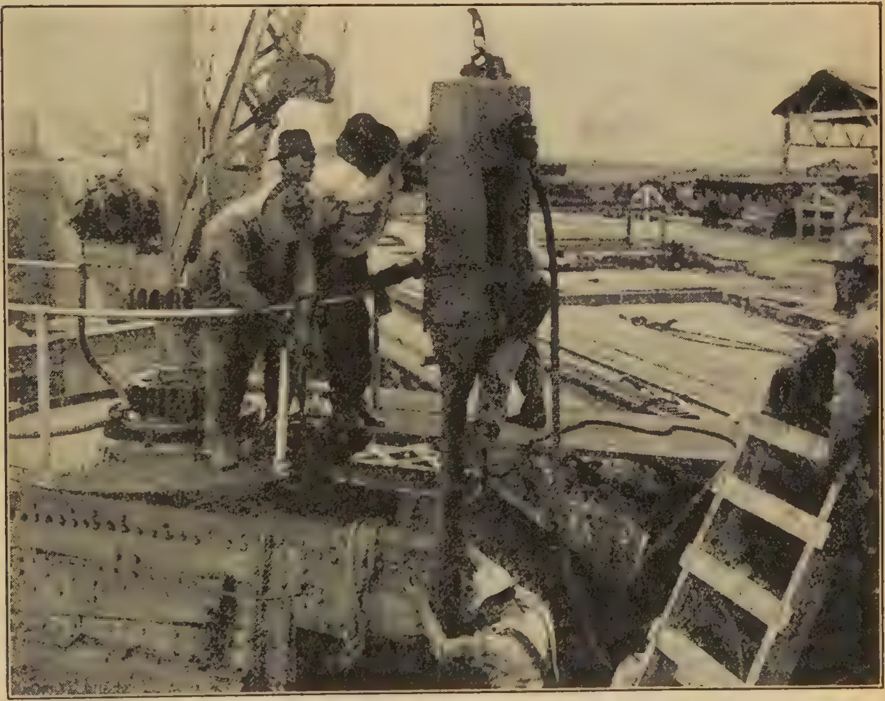
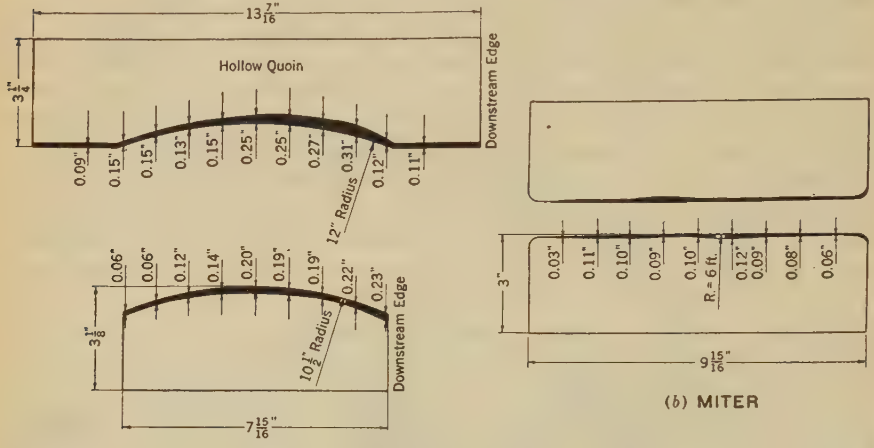


FIG. 12.—VANADIUM STEEL WEDGE AND HAMMER USED TO REMOVE BEARING-PLATES AT MIRAFLORES

The wedge and hammer were not used in removing the bearing-plates in the wall (hollow quoins) because of the possibility of loosening the reaction castings set in the concrete wall. Heavy strongbacks, made of a 4 by 4-in. bar, drilled and tapped for two capstan head jacking-bolts, were most satisfactory for removing these plates. They were provided with two expanding-jaw clamps, centered to fit into the counterbored nut recesses of the hollow quoin-plates. Although this method was slow, the results were entirely satisfactory. Each bearing-plate was marked and the wear measured at a later date. Fig. 13 shows typical wear on the bearing-plates.



(b) MITER

(a) QUOIN

FIG. 13.—TYPICAL WEAR ON QUOIN-PLATES.

Old Babbitt Metal.—The babbitt metal back of the bearing-plates was removed, the castings were cleaned, and the burrs formed by the original caulking removed by grinding. The new bearing pieces were then loosely bolted in place, preparatory to their final setting.

The babbitt metal is held in place by dovetailed grooves making it necessary to cut it out of the castings. A pneumatic paving breaker with a special cutting-bit proved very efficient in removing it. The bit was a blacksmith forging of T-section drawn to a point and the upstanding stem ground to a knife-edge. With the bit started between the babbitt metal and the casting, it was possible to drive this cutting tool down the full height of the leaf. No one was allowed near the leaf and the babbitt metal was allowed to fall to the chamber floor as it was cut from the casting.

The bearing-plates were removed and reset from double-decked scaffolds which were raised and lowered by the jib cranes. One scaffold, shaped to fit around the reaction castings, was sufficient for the miter end, but three were required at the quoin end; one shaped to fit between the gate and the wall, and one each for the up-stream and down-stream sides when the leaves had been rolled back. The scaffolds were suspended by chains hung from the ends of an H-beam which was fastened to the top of the top reaction casting and extended over the sides of the leaf. The scaffolds could be held at any desired elevation by passing pins through chain-guides on the scaffold and the chains.

Locomotive axles, quills, and car-wheels and lengths of valve stems, which had been salvaged from obsolete equipment, were utilized in building the jib cranes. These cranes, each with a capacity of 3 000 lb, at a radius of 13 ft, could be swung in a full circle and were equipped with a single-drum air hoist. The lower portion of one of them may be seen in Fig. 12.

Adjustment of New Bearing-Plates

The new bearing-plates (the majority of which were old plates salvaged from the Gatun repairs and re-machined) were set in essentially the same manner as in the original construction. The attempt was made to set them within 0.003 in. of their correct position, but no error greater than 0.005 in. was allowed. The plates were set in the following order: Hollow quoins, quoin-plates, and miter-plates.

Hollow Quoins.—The hollow quoins were set by gauges from a vertical piano wire. The correct position of this reference line was established with the aid of a device for centering pintles designed by engineers on the Panama Canal and consists of two parts: The cross-bar center (Fig. 14), which was set in the pintle socket; and the adjustable slide (Fig. 15), which was supported by a bent channel secured in the yoke casting. The slide was drilled with three accurately centered 0.033-in. holes, with a tensioning screw provided near each hole. One hole was at the pintle center; one, 6 in. from the face of the hollow quoins; and the other, 12 in. from the same face.

The steps in setting the device follow: (1) The cross-bar center, or "spider," was centered and clamped in the pintle socket; (2) the lower end of a No. 13 piano wire was fastened in the center of the spider and the upper end passed through the pintle center of the adjustable slide; (3) the piano wire was carefully plumbed, bottom to top, with a transit equipped with a striding level, and it was adjusted at the top by moving the slide; (4) the transit was brought to the top of the lock-wall and set up on a line normal to the chamber axis by transiting-in on the line established by the pintle center of the opposite leaf and the pintle center of the adjustable slide; (5) the center lines on the uprights at each end of the slide were brought to the transit line by revolving the slide around the center wire (this movement controlled the position of the projecting arm of the slide on which are the holes for the reference wires); (6) the reference wire was passed downward through the hole, 12 in. from the face of the plates, and fastened to the end of a projecting arm of the spider; (7) this line was plumbed, from the top to the bottom, and securely fastened (this wire is adjusted at the end of the projecting arm); and (8) as soon as the correct position of the pintle center on the movable slide had been established, reference punch-marks were placed at several points on the yoke casting. These marks were used later in bringing the yoke to its correct position.

Although the hollow quoins may be set from either the 6-in. or the 12-in. wire, the latter was found to give more accurate results. The plates are set with gauges made of sectors of stainless steel, $\frac{1}{16}$ in. thick, and have the arc of the sector cut to the radius of the hollow quoins. Scratch-marks on the gauge indicate the edges of the concave face of the plates and the position of the reference line. The gauge is held horizontally between the face of the plates and the reference wire, and the plates are turned, by temporary set screws on the center line and the permanent stud-bolts along the edges (see Fig. 14), until the center scratch-mark coincides with the reference wire. Due to inaccuracies in the original setting of the reaction castings, it was impossible to set certain plates correctly without first planing the edges.

With the hollow quoins correctly adjusted, the babbitt metal backing was poured (as described later), the set screws were removed, the edges of the plates caulked, the nut recesses filled with concrete, and the plates thoroughly cleaned. The leaf was then rolled back and lowered. The task of keying the yoke and hinging the leaf was carefully supervised and inspected. The jacking equipment was then removed, the operating strut connected, and the leaf was free to swing.

Quoin-Plates.—The quoin-plates were set (by jacking them against the hollow quoins) with the leaf in a position 1 in. up stream from the true miter point. This setting provided a slightly closer contact between the quoin-bearings when the leaf was exactly mitered. The line of contact was centered on the hollow quoins, and the plates were then forced (with short jack-bolts between the ends of the plate studs and the reaction castings) against the hollow quoins until it was impossible to push a 0.005-in. feeler between the faces of the plates. The jack-bolts were essentially a $\frac{3}{4}$-in. hexagonal rod with one end tapped for a conical-pointed nut. The ends of the studs were countersunk to prevent the jacking nut from slipping off. In setting these quoin-plates, it was absolutely essential for the men on opposite sides of the leaf to work together, and they were provided with portable telephones for easy communication. To set the quoin-plates in the exceedingly cramped working space required some of the most arduous work of the entire over-haul. In order to pour the babbitt metal behind the plates, it was necessary to swing the leaf to the full open (against the wall) position.

Miter-Plates.—The miter-plates were set early in the morning, as follows: (1) Both leaves of the pair were brought to the miter point; (2) the top and bottom plates were set so that the thickness of the babbitt metal was the same on each; (3) the leaves were opened partly and a reference wire was stretched from top to bottom and 2 in. from the face of the plates on one leaf; (4) the plates were set by a 2-in. gauge from the reference wire, and the babbitt metal was poured; (5) at the same hour of a following morning, the leaves were mitered again; (6) the bottom plates of the second leaf were brought in contact with those of the first leaf and the top of the top plates was separated by a $\frac{1}{32}$-in. filler; and (7) the plates were set and babbitted. The effect of this setting was to form a V-shaped opening, $\frac{1}{32}$ in. wide at the top, which served to compensate slightly for the effects of temperature expansion and wear.

Pouring Babbitt Metal.—After each set of bearing-plates had been adjusted, the spaces behind the plates were filled with babbitt metal; this work of pouring the molten metal from the small and swinging scaffolds was the most dangerous operation of the over-haul.

The babbitt metal was heated in a double-ended, oil-burning, brick-lined, steel box furnace, placed on the chamber floor. Two openings were provided

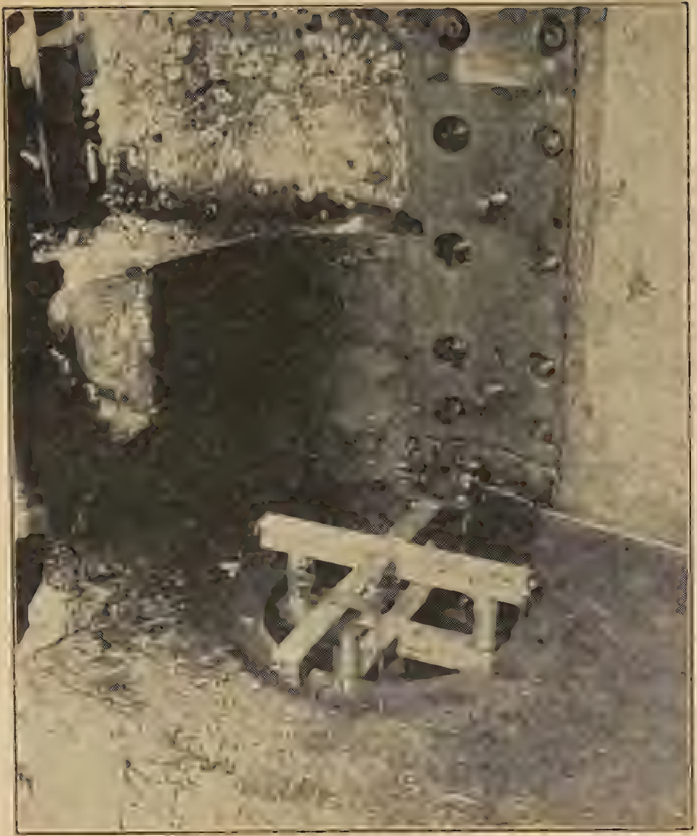


Fig. 14.—Cross-Bar of Pintle Centering Device Set in Pintle Socket.

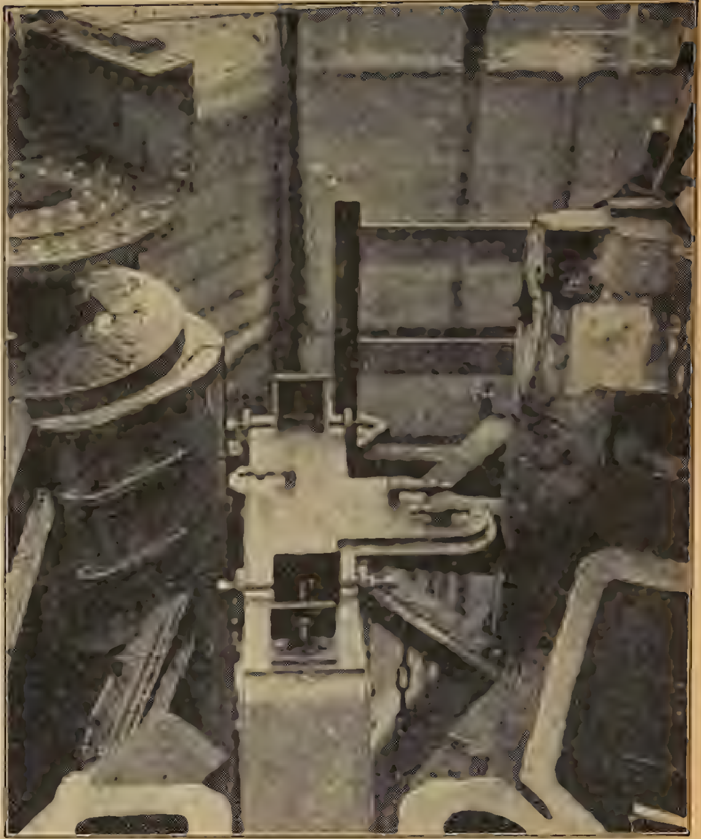


Fig. 15.—Adjustable Slide for Pintle Centering Device, Supported by a Bent Channel Secured in a Yoke Casting.

in the top for the pots, the compartment under each pot being fired with separate air-oil burners, with the smoke flue in the center of the furnace.

The spaces behind the plates were heated by compressed air to dry out any moisture and to prevent the molten metal from cooling too rapidly. The air circulated through the coils in the flue and was conducted through a 1-in. pipe to another vertical pipe near the edge of the plates. The vertical pipe extended about 25 ft above the bottom plate and was provided with branches leading to tapped holes in the face of the bearing-plates. The lower plates were heated for about two hours before the workmen began pouring. Experience showed that it was necessary to pre-heat only the lower two or three plates rather than the entire height, as was done on the previous over-hauls. The heat from each pour was sufficient to increase the temperature, in the plates directly above, to a higher degree than was possible with hot air.

The metal was heated until a dry wooden stick thrust into it would ignite instantly. Four men were required on the scaffold during the pouring. A helper poured a small quantity of kerosene into the funnel just previous to pouring, and he was provided with a supply of wet clay for stopping leaks. The signalman and the crane operator were equipped with telephone head-sets in order to minimize the danger of a misunderstood hand signal.

When the plates had cooled, the asbestos caulking was removed, and the nuts on the studs were tightened to take up the shrinkage of the babbitt metal. The edges of the plates were then caulked with lead, and lead plugs driven into the holes in the face of the plates. Subsequent inspection showed that the plates had been set in so accurately that no grinding of the faces was necessary.

Changes to Original Gate Design

Automatic Pintle-Oiling Device.—In the original design no provision was made for oiling the pintle. The new bushings are provided with five oil grooves leading to an oil passage drilled through the bushing and upper pintle casting. An extra heavy 1-in. brass tube leads from this oil passage to an oil pump on top of the leaf, which is placed so that the plunger is actuated by the leaf-operating strut when the leaf approaches the miter position.

It is a question whether the life of the bearing-plates is increased by oiling the pintle. The bushing must yield with the wear on the quoin-plates in order that the leaves may close to secure the necessary arch effect when there is a head of water behind them. However, with oil on the pintles the leaves swing more smoothly.

Concrete Sills.—The fixed wooden sills (Fig. 5) were replaced with concrete sills because the wooden ones were subject to the attacks of the teredo and require expensive replacement. The wooden clapping sills on the gate were increased in thickness to allow for the additional trimming. Two rubber pads, 1½ in. thick, were set 1 in. into the face of the clapping sill near the miter end to prevent the leaves from slamming too forcibly against the concrete sill.

Sealing Bead on Bearing-Plates.—To secure a truly water-tight bearing, projecting beads of babbitt metal were poured into small dovetailed grooves in the faces of the miter and quoin bearing-plates. This was the last operation before the water was let into the chamber. At the miter end, two grooves were located 2 in. each side of the center line of the plates on one leaf only. At the quoin end, one groove was placed 2⅝ in. down stream from the center of the quoin-plates on each leaf.

General Observations

This paper describes the various operations in reference to one leaf only. It should be understood, however, that three complete sets of equipment were provided, and that the bearings were renewed simultaneously on six leaves in each chamber. The work was done continuously by three 8-hr shifts, six days per week. Numerous flood-lights provided ample illumination for all night work with the exception of raising or lowering the leaves, which was done only in daylight.

Each group of leaves was in direct charge of a foreman, with a general supervisor responsible for the three groups. Engineers and inspectors on each shift were responsible for the correct layout, proper use of the equipment, and the final adjustment of the new bearings.

The following non-related comments are worthy of attention: (*a*) Extensometer readings, taken during the gate-raising, gave results that checked closely with the computed stresses; (*b*) the average cost per leaf (labor and material), was $14 340.77; and (*c*) at the conclusion of the Miraflores overhaul, the equipment was repainted, re-marked, and shipped to Gatun for storage until the next over-haul on those locks.

The equipment and methods for renewing the bearings on the Panama Canal gates have become practically standardized, and only very minor changes in detail are probable.

Acknowledgment

The plant and equipment for overhauling the gates were originally designed by E. S. Randolph, M. Am. Soc. C. E., and many of the improvements are due to his recommendations, made at the conclusion of the trial raising at Pedro Miguel. The unit stresses, as given in this paper, and other supporting data, are taken from Mr. Randolph's report to the Assistant Engineer of Maintenance of The Panama Canal, in 1929. The writer, as Junior Engineer, was in charge of engineering and inspection on one of the 8-hour shifts.

DISCUSSION

E. S. RANDOLPH,[4] M. AM. SOC. C. E. (by letter).—The subject has been so well covered in this paper that there is little comment to offer. The first, or experimental, repairs conducted under the technical supervision of the writer, were reported in 1930.[5] It is gratifying to the engineer of the original equipment that much of it was still in use in 1933, after being previously used in 1929 and 1931. An important event in the 1931 operation was the test that was run as a preliminary drill and exercise for the apparatus and crew before attempting to lift a gate. Had not this test been run, there would have been serious trouble.

The preliminary test was made with all equipment for one gate assembled and a load of structural material instead of the gate. The test load was much lighter than the gate. It was noted that the travel of the jacks was at uneven rates. Examination showed scoring of the rams and cylinders in seven of the twelve jacks; one had a groove $\frac{3}{8}$ in. wide and $\frac{3}{32}$ in. deep; furthermore, foundry sand and some metal chips and cuttings from drilling and tapping operations were found in the cylinders of the jacks. New rams of case-hardened material were made; the cylinders were thoroughly cleaned; and felt wiping washers were provided at the end of each cylinder so as to fit tightly against the rams. The improved jacks were finally assembled in the machine shop and kept in a locker until used; not the least trouble from scoring was experienced thereafter.

Another preliminary test was made by placing two jacks with rams in opposition. Yokes and links, as provided for regular operation, were used, and additional double links completed the tension members. The jacks were then loaded to 110 tons of 2 000 lb without any signs of distress.

While being constructed, more clearance under the gates would have facilitated their removal; and wider sills would have made special foundations for the jacks unnecessary. The gates were erected in a raised position, moved against the lock-wall, lowered over the pintle bearing, and yokes were fitted to the top. While being erected and moved, each leaf was supported on two steel falsework frames, one on each side. These frames were supported on wedges for lowering the leaves, and the wedges rested on roller nests for moving the gates to the walls. Consideration was given to re-assembling these frames for repairing the gates with jacks instead of wedges, but this was found to be too laborious for the work intended. A description of the gates and their erection was published[6] in book form in 1916.

CLINTON MORSE,[7] ASSOC. M. AM. SOC. C. E. (by letter).—An item that was not clearly emphasized in the paper was the low cost of the gate-raising equipment. The total cost to date (1935) of the three sets of equipment is

---

[4] Madden Dam, Canal Zone.

[5] *The Military Engineer*, May-June, 1930.

[6] "The Panama Canal," by George W. Goethals, M. Am. Soc. C. E., Vol. II, New York, McGraw-Hill Pub. Co., 1916.

[7] Jun. Engr., The Panama Canal, Balboa Heights, Canal Zone.

$151 656.94. This amount includes the cost of such minor changes as were considered necessary in the interests of safety and economy. Since the equipment will be used in raising all the leaves, 92 in number, it is proper to pro-rate this cost against each leaf, making a cost per leaf of $1 648.44.

The writer wishes to express his appreciation of Mr. Randolph's comments concerning the preliminary drill and tests of equipment which were made previous to the 1929 gate-raising.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 1902

WATER-POWER DEVELOPMENT OF THE ST. LAWRENCE RIVER[1]

BY DANIEL W. MEAD,[2] HON. M. AM. SOC. C. E.

WITH DISCUSSION BY MESSRS. T. H. HOGG, THERON M. RIPLEY, W. S. LEE, FRANK E. BONNER, RUFUS W. PUTNAM, JAMES W. RICKEY, WALTER M. SMITH, J. W. BEARDSLEY, L. F. HARZA, ADOLPH J. ACKERMAN, AND DANIEL W. MEAD.

SYNOPSIS

An analysis of the proposed official plans for the development of water power along the International Section of the St. Lawrence River, is contained in this paper. Since the Wooten-Bowden Report in 1921 on the development of the proposed St. Lawrence Waterway, there has been controversy between the United States and Canadian members of the various engineering boards that have made reports on the development of the International Section, in regard to the relative merits of a single-stage and a two-stage plan of development of the water-power resources. The American engineers have consistently advocated a single-stage plan, and the Canadian engineers have consistently advocated a two-stage plan.

In December, 1931, the Joint Board of Engineers (United States and Canada), consisting of an entirely new membership of the United States Section and of the original members of the Canadian Section, was reconvened and in March, 1932, was instructed to reach an agreement on a plan for early execution. On April 9, 1932, the Joint Board submitted the "Reconvened" Report on which the treaty now under consideration is based.

If this analysis is correct, the value of the two-stage plan will be nearly $95 000 000 less than the alternate Joint Board plan for a single-stage development at Barnhart Island, and at least $154 000 000 less than the New

---

[1] Presented at the Joint Meeting of the Power Division of the Society with the American Institute of Electrical Engineers and the Hydraulics Division of American Society of Mechanical Engineers, Chicago, Ill., June 29, 1933, and published in August, 1933, *Proceedings*.

[2] Prof. Emeritus, Hydr. and San. Eng., Univ. of Wisconsin; Cons. Engr., Madison, Wis.

York single-stage plan. The advantages of the various plans are not adequately discussed in the report of the Joint Board, and no reference is made in the Joint Board Reconvened Report as to why the two-stage plan is recommended. The reasons set forth in the Conference Report (Canadian) seem entirely inadequate to offset the aforementioned disadvantages. If the reports and this analysis is correct, it will be a very serious mistake and a misfortune to both the United States and Canada, in the writer's opinion, if the treaty now pending is ratified on the basis of the Joint Board Reconvened Report.

INTRODUCTION

With his limited knowledge of the International Rapids Section of the St. Lawrence River, the writer, ordinarily, would not have presumed to discuss this important question. As a disinterested engineer familiar with water-power development, he was requested by the Power Division of the Society to examine the various reports on this subject and to analyze the comparative values of the principal plans on the basis of these reports.

A proper report should be so prepared that it will give any intelligent engineer, familiar with the character of the work considered, fair understanding of the reasons for the recommendations made in the report. As an American citizen, and especially as an hydraulic engineer interested in water-power development, the writer believes that it is his right to know, in the case at hand, the reasons for the recommendations, on the basis of which an important treaty is expected to be consummated, and to judge of the validity of such recommendations from the information furnished in these reports.

If his interpretation of the conditions is faulty or if his conclusions are in error, he believes that he is entitled to have such conditions explained and such errors corrected. His mind is entirely open on the entire subject, but he believes that the people of the United States and Canada who are asked to spend hundreds of millions of dollars on this project are entitled to something better than an opinion, no matter from how distinguished a source, and to consider this project on the broader economic basis on which the writer has attempted to examine it.

The investigation leading to the treaty, now (1933) before the United States Congress, began with the work of Col. W. P. Wooten, Corps of Engineers, U. S. A., M. Am. Soc. C. E., and Mr. W. A. Bowden, Canadian Government Engineer, in 1920–21. The results of their investigation were outlined in their report to the International Joint Commission, dated June 24, 1921.[8]

This Board was so restricted in time that it was unable to go into the subject in great detail, but submitted a set of recommendations on which its members were able to agree, and pointed out certain others on which they differed. This report was considered by the International Joint Commission in 1921, together with certain alternate suggestions, submitted by or on behalf of the Hydro-Electric Power Commission of Canada, The New York and Ontario Power Company, Hugh L. Cooper and Company, of New York City, and Mr. S. M. Lovelace, of Montreal, Que., Canada.

---

[8] See Senate Doc. No. 179, 67th Cong., 2d Session.

The Commission accepted the plans of the Engineering Board as a basis for the recommended improvements of the St. Lawrence River with the suggestion that both the plans of the Engineering Board and the alternate plans should be referred to a larger board for final determination.[4]

In 1924 a Joint Board of Engineers was appointed, consisting of six members, three from the United States and three from Canada. This Board was asked to review the Wooten-Bowden Report and to extend its inquiries to certain additional matters relevant to proposals made by the International Joint Commission in its report dated December 19, 1921. The studies of this Joint Board extended over several years, and its report was submitted in 1926. Most of the engineering questions were cleared up by this report, and a large number of questions of fact and opinion which were referred to it, were definitely answered, but the Board failed to agree on the best method of developing power and navigation in the International Rapids Section.[5]

The Canadian Section favored a two-dam plan (see Fig. 1 (*A*)) with the upper dam at Ogden Island, and the lower near the foot of the Rapids. The American Section, however, favored a single dam (see Fig. 1(*B*)), near the foot of the Rapids. Both Sections agreed that both plans were feasible from an engineering standpoint, and suggested that the choice between them be determined by constructing agencies at a later date; they also agreed that the two-dam plan would cost about $30 000 000 more than the single-dam plan.

After several years, in which the subject was studied further, a Conference Board of Engineers was appointed in Canada. This Board recommended moving the upper dam in the two-dam plan from Ogden Island to Crysler Island, farther down the river.[6] By this change the difference in first cost between the one-stage and the two-stage plans was increased to about $40 000 000. The change in location also resulted in raising the proposed water level along the Iroquois-Morrisburg Section to such height that the problem of protecting this section, which had been in controversy before, became substantially the same in the two plans proposed.

In 1930, the State of New York established a Commission which was instructed to report on "a comprehensive plan for the development and operation of the water power resources of the State on the St. Lawrence River. * * *" This Commission reported[7] under date of January 15, 1931, and its engineers advised:

"A modification of the one-dam plan which eliminates the unusual serious hazards of the previous plans and makes possible the development of a somewhat increased amount of power. The estimated saving in first cost under

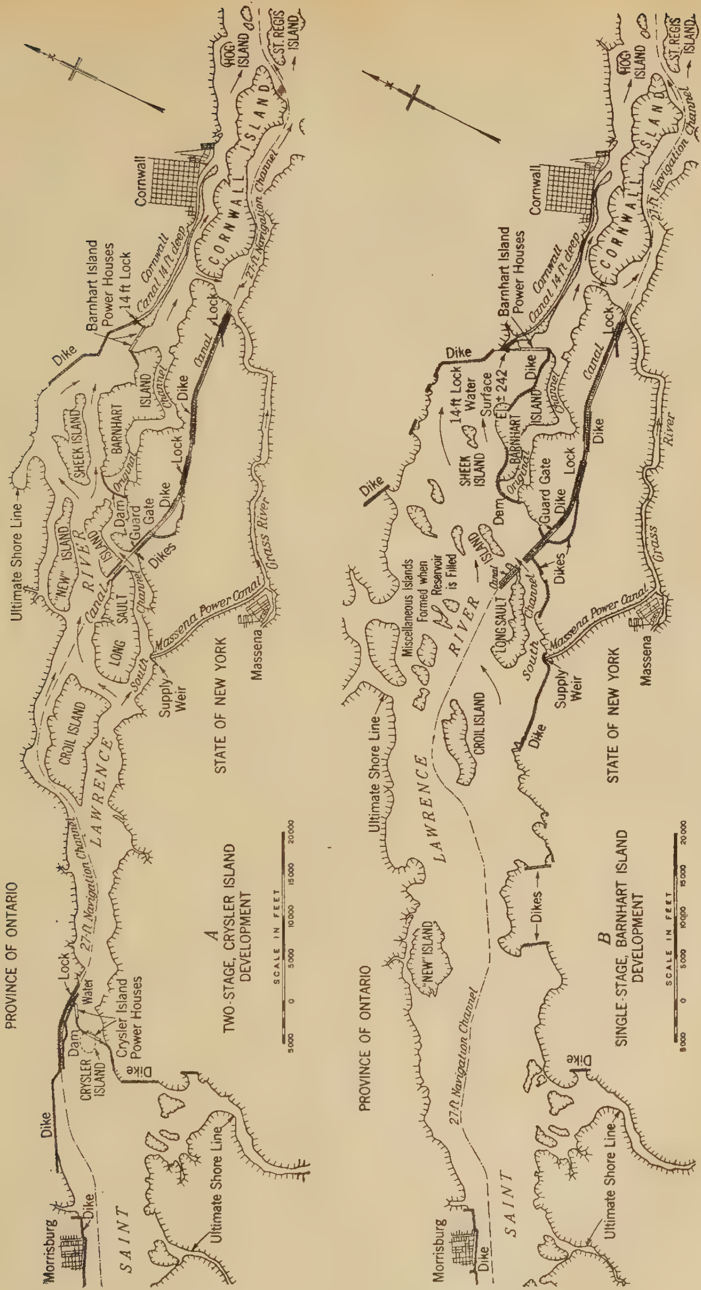
---

[4] See Senate Doc. No. 114, 67th Cong., 2d Session.

[5] See Rept. of Joint Board of Engrs., with Appendix, dated November 16, 1926, Ottawa, Ont., Canada, 1927.

[6] See Rept. of Conference of Canadian Engrs. on the International Rapids Section, with Appendix and plates, dated December 30, 1929, Ottawa, Ont., Canada, 1930.

[7] See Rept. of the St. Lawrence Power Development Comm., dated January 15, 1931, Albany, N. Y.


PROVINCE OF ONTARIO
Ultimate Shore Line
Morrisburg
Dike
SAINT
LAWRENCE
RIVER
27-ft. Navigation Channel
Lock
Water
Dam
CRYSLER ISLAND
Crysler Island Power Houses
CROIL ISLAND
LONG SAULT ISLAND
"NEW" ISLAND
SHEEK ISLAND
BARNHART ISLAND
Original Channel
South Channel
Canal
Dam
Guard Gate
Dikes
Supply Weir
Massena Power Canal
Massena
Grass River
Barnhart Island Power Houses
14 ft Lock
Cornwall Canal 14 ft. deep
Cornwall
CORNWALL ISLAND
HOG ISLAND
ST. REGIS ISLAND
STATE OF NEW YORK
SCALE IN FEET
5000 0 5000 10000 15000 20000
Miscellaneous Islands Formed when Reservoir is Filled
14-ft Lock
Water Surface El. ± 242

A
TWO-STAGE, CRYSLER ISLAND DEVELOPMENT

B
SINGLE-STAGE, BARNHART ISLAND DEVELOPMENT

FIG. 1.—COMPARISON OF TWO-DAM AND SINGLE-DAM PLANS FOR DEVELOPING ST. LAWRENCE RIVER.

this plan as compared with the two-stage Crysler Island plan was increased further to about $70 000 000.

"In December, 1931, the Joint Board of Engineers was instructed to meet and report on a mutually satisfactory plan for the improvement of the International Rapids Section.[8]

"In March, 1932, the Board was instructed to reach an agreement on a plan for early execution that would provide effectively for navigation and power requirements."[9]

It should be noted that the Canadian members of the Joint Board of Engineers were appointed in 1924 and consisted of Mr. Duncan W. McLachlan, of the Department of Railways and Canals, of Ottawa, Olivier O. Lefebvre, M. Am. Soc. C. E., Chief Engineer, Quebec Streams Commission, of Montreal, and Charles H. Mitchell, M. Am. Soc. C. E., of Toronto. They have continued unchanged from the time of their appointment to the present (1933). The original United States members of the Board consisted of the late Lt.-Gen. Edgar Jadwin, U. S. A., M. Am. Soc. C. E. (then Chief of Engineers, U. S. A.), Col. William Kelly, Corps of Engineers, U. S. A., M. Am. Soc. C. E., and Brig.-Gen. G. B. Pillsbury, U. S. A., M. Am. Soc. C. E., (then Colonel, Corps of Engineers, U. S. A.); while the United States members of the Reconvened Joint Board of 1930 consisted of Col. H. B. Ferguson, Corps of Engineers, U. S. A., *Chairman,* Col. E. M. Markham, Corps of Engineers, U. S. A., M. Am. Soc. C. E., and General Pillsbury. In June, 1930, Maj. Eugene Reybold, Corps of Engineers, U. S. A., was detailed to replace General Pillsbury.

It is unfortunate that the same members of the Board of United States Engineers were not employed continuously on this project. New board members appointed to co-operate with others who have served for many years on a project as involved as that of the navigation and power development of the International Section of the St. Lawrence River, are at a great disadvantage. The various plans that might be adopted are so numerous and the possible method of effecting the same results are so involved that only long and continued contact with the problem will insure the familiarity necessary to secure the best results. Such conditions, together with the instructions under which the Board was working, necessitated coming to a conclusion on a "mutually satisfactory plan" which could "provide effectively for navigation and power requirements" and did not involve either the "best" or the "most economical" development for navigation or for power.

It is also unfortunate that the economic desirability of river and harbor improvements is usually determined by legislative bodies or by non-engineering fact-finding boards. The engineering boards appointed to plan such improvements are seldom expected to look into the economic phases of such problems in their broader aspects. First cost, while always of importance, must yield to various questions of safety, expedience, and influence. Apparently, for the foregoing or for similar reasons, none of the engineering reports considered, except that of the New York Board (which is confined principally to the question of water-power development), discusses the projects involved from an economic standpoint either as to navigation or water-power development, except as to the items of first cost.

---

[8] See Rept. of Joint Board of Engrs. (Reconvened), p. 4, Paragraph 16.

[9] *Loc. cit.,* p. 4, Paragraph 7.

As navigation works are seldom if ever constructed from the avails of bond issues, but from annual or occasional appropriations, the matter of interest during construction is almost never considered. In the case at hand, it is expected that the water-power development will be undertaken by the State of New York and the Province of Ontario, Canada, and will involve bond issues and a considerable expense of interest during construction. Other economic considerations, such as cost of maintenance, operation, depreciation, amortization, etc., also become of importance quite equal perhaps to the matter of first cost. It would seem, therefore, that those interested in this project, which involves so great an expense and which is of such great importance to thousands of citizens of both the United States and Canada, are entitled to some knowledge of the economic relations of the project recommended in the "Reconvened" Report to other plants which seem equally or more economically feasible.

The writer has analyzed the problem of water-power development on the International Section of the St. Lawrence River on the basis of the plans and estimates presented in the various reports and on the assumption that such plans are adequate for the purpose and that such estimates are reasonably correct, or at least fairly comparative. He has found certain discrepancies between some of the detailed estimates and the summaries, but such differences are of minor importance and in no material way do they affect the ultimate results of the analyses. The costs, estimated on a higher market than prevails at present (1933), are believed to be fairly comparative; any attempt to correct prices or details to meet the writer's ideas would involve confusion and would destroy the values of the analyses and comparisons made.

The writer has confined his discussion to the three most important plans proposed in the several official reports that have been issued by various boards of engineers appointed by the United States, Canada, and New York State to report on the St. Lawrence Waterway and the incidental water power to be developed in the International Section of the river. So far as the plans and estimates of cost are concerned, they are taken unmodified from the official reports or, if modified, the reasons for such changes are discussed.

In this paper the writer has analyzed the comparative advantage of three official plans, as follows: First, the two-stage Reconvened Plan; second, the single-stage Conference (Barnhart Island) Plan; and third, the single-stage New York (Massena Point) Plan.

The plan referred to in the "Reconvened" Report on which the treaty under consideration is based, is the two-stage plan advocated by the Canadian engineers and apparently accepted by the United States engineers, not on its merits, but because the engineers were instructed to agree by a certain date on a feasible plan for power development and navigation (see Fig. 1(*A*)).

Concerning the plan, the following statement[10] was made by President Hoover: "The cost is slightly more (for the two-stage development) but the Canadian officials have felt that the two-stage development is desirable for many reasons * * *."

---

[10] Treaty, p. 11.

TABLE 1.—COMPARATIVE DATA RELATING TO THE THREE PLANS ANALYZED

Item No.	Description	(A) Two-stage, Reconvened Plan		(B) Single-stage, Conference Plan		(C) Single-stage, New York Plan	
	(a) ANALYSIS OF ESTIMATED HEAD (IN FEET)						
1	Season	Winter*	Summer*	Winter*	Summer*	Winter†	Summer†
2	Crysler Island	19.6	24.2	......	......	......	....
3	Barnhart Island	56.4	60.4	76.4	86.1	......	....
4	Massena Point	......	......	......	......	84	88
5	Total (Items Nos. 2, 3, and 4)	76.0	84.6	76.4	86.1	84	88
6	Decrease in winter head	8.6	......	9.7	......	4	....
	(b) ANALYSIS OF ESTIMATED POWER (IN HORSE-POWER YEARS)						
7	Primary	1 596 000		1 604 400		1 764 000	
8	Secondary	294 100		314 200		228 000	
9	Total (Items Nos. 7 and 8)	1 890 100		1 918 600		1 992 000	
10	Excess over two-stage plan	....		28 500		101 900	
	(c) COMPARATIVE ESTIMATED COSTS (27-FOOT NAVIGATION; NO INTEREST DURING CONSTRUCTION)						
11	Primarily for power	$132 452 000		$87 481 000		$81 150 000	
12	Common to power and navigation	108 102 000		117 047 000		101 495 000	
13	Solely for navigation	34 188 000		27 741 000		20 773 000	
14	For 27-ft. channel	0		0		807 000	
15	Sub-total (Items Nos. 11, 12, 13, and 14)	$274 742 000		$232 269 000		$204 225 000	
16	To enlarge river section 5 000 sq. ft. between Ogden and Lotus Island (not included in two-stage estimate)	0		4 149 000		....	
17	Total official estimates (Items Nos. 15 and 16	$274 742 000		$236 418 000		$204 225 000	
18	Excess cost, two-stage plan	....		$38 324 000		$70 517 000	
	(d) HORSE-POWER CAPACITY OF PROPOSED EQUIPMENT						
19	Capacity	2 199 960		2 287 700		2 200 000	
20	Excess over Plan C	....		—87 700		....	
21	Deficiency, compared with Plan C	+40		....		....	
22	Equalized total (Item No. 19 minus Item No. 20 or plus Item No. 21)	2 200 000		2 200 000		2 200 000	
	(e) OFFICIAL ESTIMATES OF FIRST COST (WORKS SOLELY FOR POWER)						
	Upper Pool, Crysler Island; Depreciable:						
23	Substructures	$18 987 070		....		....	
24	Superstructure and machinery	30 611 680		....		....	
25	Upper Pool, Crysler Island: Not depreciable	5 906 250		....		....	
	One-Half of Cost Common to Power and Navigation:						
26	Depreciable	6 763 500		....		....	
27	Non-depreciable	31 154 500		....		....	
28	Sub-total (Items Nos. 23 to 27)	$93 423 000		....		....	
	Lower Pool, Barnhart Island; Depreciable:						
29	Substructure	$21 311 870		$37 819 000		$27 526 500	
30	Superstructure and machinery	43 249 020		49 662 000		47 179 100	
31	Lower Pool, Barnhart Island; Not depreciable	12 386 070		4 149 000		6 444 400	
	One-Half of Cost Common to Power and Navigation:						
32	Depreciable	5 795 700		11 119 500		14 926 500	
33	Non-depreciable	10 337 340		47 404 000		35 821 000	
34	Sub-total (Items Nos. 29 to 33)	$93 080 000		$150 153 500		$131 897 500	
35	Official totals (Item No. 28 plus Item No. 34)	$186 503 000		$150 153 500		$131 897 500	

* Conference Rept., pp. 10 and 11.
† New York Rept., p. 45.

TABLE 1.—(*Continued*)

Item No.	Description	(A) Two-stage, Reconvened Plan	(B) Single-stage, Conference Plan	(C) Single-stage, New York Plan
	(*f*) ANNUAL EXPENSE			
36	Five per cent. interest on investment (5% of Item No. 35)	$9 325 150	$7 507 675	$6 594 875
37	Salaries	633 000	367 000	367 000
38	Station material and supplies	252 000	144 000	144 000
39	Maintenance and repairs to structures (0.125%)	158 398	123 251	112 040
40	Liability insurance (11.5% of annual payroll)	72 795	42 205	42 205
41	Management	500 000	500 000	500 000
42	Depreciation (1.75% of depreciable property)	2 217 580	1 725 509	1 568 562
43	Amortization (1% of capital cost, Item No. 35)	1 865 030	1 501 535	1 318 975
44	Total annual expense (Items Nos. Nos. 36 to 43)	$15 023 953	$11 911 175	$10 647 657
	(*g*) COMPARISON OF POWER PLANT COSTS BASED ON OFFICIAL ESTIMATES (ITEM NO. 35)			
45	Estimated total power (horse-power years; see Item No. 9)	1 890 100	1 918 600	1 992 000
46	Excess over two-stage plan	....	28 500	101 900
47	Estimated capital cost per primary horse-power year $\left\{\frac{\text{Item No. 35}}{\text{Item No. 7}}\right\}$	$116.96	$93.59	$74.77
48	Estimated capital cost per total horse-power year $\left\{\frac{\text{Item No. 35}}{\text{Item No. 9}}\right\}$	98.67	78.26	66.21
49	Difference in favor of Plans B and C (capital cost per total horse-power year)	....	20.41	32.46
50	Total income with primary power at $15 and secondary power at $10	26 881 000	27 208 000	28 740 000
51	Total annual expense (Item No. 44)	15 023 953	11 911 175	10 647 657
52	Net income	$11 857 047	$15 296 825	$18 032 343
53	Excess annual income of Plans B and C over Plan A	....	3 439 778	6 235 296
54	Difference, capitalized at 5%	....	68 795 560	124 705 920
	(*h*) PARTLY CORRECTED CAPITAL COST			
55	Official estimate of capital cost (Item No. 35)	$186 503 000	$150 153 500	$131 897 500
56	Additional cost of 5 000 sq. ft. of excavation	4 149 000	....	....
57	Additional 40 hp. deficient capacity	1 600	....	....
58	Deduct 87 700 hp. excess capacity	....	3 508 000	....
59	Corrected estimate of capital cost	$190 653 600	$146 645 500	$131 897 500
	(*i*) ANNUAL EXPENSE BASED ON PARTLY CORRECTED CAPITAL COST			
60	Five per cent. interest on investment (Item No. 59)	$9 532 680	$7 332 275	$10 647 657‡
61	Salaries	633 000	367 000	....
62	Materials and supplies	252 000	144 000	....
63	Maintenance and repairs (structures, 0.125%)	158 400	118 866	§
64	Liability insurance (11.5% of annual payroll)	72 795	42 205	....
65	Management	500 000	500 000	....
66	Depreciation (1.75%)	2 217 608	1 664 119	....
67	Amortization (1% of capital cost)	1 906 536	1 466 455	....
68	Total annual expense	$15 273 019	$11 634 920	$10 647 657

‡ See Item No. 44.

§ No correction.

TABLE 1.—(*Continued*)

Item No.	Description	(A) Two-stage, Reconvened Plan	(B) Single-stage, Conference Plan	(C) Single-stage, New York Plan
	(*j*) Comparison of Power Plant Cost Based on Partly Corrected Estimate (Item No. 59)			
69	Corrected installed capacity (Item No. 22)...........................	$2 200 000	$2 200 000	$2 200 000
70	Total estimated power (horse-power years (Item No. 9))...............	1 890 100	1 918 600	1 992 000
71	Excess power (horse-power years) Plans B and C over Plan A............	....	28 500	101 900
72	Capital cost per horse-power year $\left\{\frac{\text{Item No. 59}}{\text{Item No. 9}}\right\}$	100.87	76.43	66.21
73	Difference in favor of Plans B and C...	....	24.44	34.66
74	Net annual income (Item No. 50 minus Item No. 68).....................	11 608 000	15 573 080	18 092 343
75	Excess net income, Plans B and C over Plan A..........................	....	3 965 080	6 484 343
76	Excess income (Item No. 75) capitalized at 5%.........................	....	79 301 600	129 686 860
	(*k*) Additional Corrections of Official Estimates (for Head)			
77	Increased head, in feet...............	—1.3 ‖	+1.5 ¶	....
78	Estimated change in primary power (horse-power years)...............	—27 300	+31 500	....
79	Estimated change in secondary power (horse-power years)...............	—2 400	—23 600	....
80	Estimated change in total power (horse-power years).....................	—29 700	+7 900	....
81	Change in annual income............	—$433 500	+$236 500	....
	(*l*) Comparison of Plans Based on Corrected Estimates and Heads (See Item No. 59)			
82	Corrected installed capacity (Item No. 22)...........................	2 200 000	2 200 000	2 200 000
83	Primary horse-power years...........	1 568 700	1 635 900	1 764 000
84	Secondary horse-power years..........	291 700	290 600	228 000
85	Corrected total horse-power years (Item No. 70 minus Item No. 80).........	1 860 400	1 926 500	1 992 000
86	Excess horse-power years, Plans B and C over Plan A..................	....	66 100	131 600
87	Estimated cost per primary horse-power year $\left\{\frac{\text{Item No. 59}}{\text{Item No. 83}}\right\}$	$121.54	$89.64	$74.77
88	Estimated cost per total horse-power year $\left\{\frac{\text{Item No. 59}}{\text{Item No. 85}}\right\}$	102.48	76.12	66.21
89	Difference in favor of Plans B and C in cost per total horse-power year......	....	26.36	36.27
90	Corrected total income with primary power at $15 and secondary power at $10..............................	26 447 500**	27 444 500††	28 740 000
91	Correct total annual expense (Item No. 68)........................	15 273 019	11 634 920	10 647 657
92	Net income (Items Nos. 90 minus Item No. 91)........................	$11 174 481	$15 809 580	$18 092 343
93	Difference in net income..............	....	$4 635 099	$6 917 862
94	Difference capitalized at 5%..........	....	92 701 980	138 357 240
	(*m*) First Cost with Respect to Both Power and Navigation (27-Foot Navigation; No Interest Charges)			
95	Works for power, plus one-half work for navigation and power (Item No. 35)	$186 503 000	$150 153 500	$131 897 500
96	Excess of Plan A over Plans B and C...	....	(36 349 500)	(54 605 500)
97	Works for navigation plus one-half work for navigation and power...........	88 239 000	86 264 500	72 327 500
98	Excess of Plan A over Plans B and C..	....	(1 974 500)	(15 911 500)
99	Total first cost (Items No. 95 plus Item No. 97).........................	$274 742 000	$236 418 000	$204 225 000
100	Total excess of Plan A over Plan B and Plan C..........................	....	($38 324 000)	($70 517 000)

‖ Twelve months.
¶ Three winter months.
** Reduced for loss by power; see Table 1 (*k*).
†† Increased for extra winter power; see Table 1 (*k*).

An examination of the official reports,[11] shows that the slight difference in first cost of the two-stage plan over the single-stage (Barnhart Island) plan is $38 324 000 (see Fig 1(*B*)).

Further consideration also shows that the two-stage plan is: (1) Deficient in output and, therefore, in earning capacity; and (2) more expensive in cost of operation. The consideration of these two items makes the relative cost of the two-stage plan much greater than is represented by the first cost.

TABLE 2.—OPERATING CHARGES (ONE PLANT, CANADA OR UNITED STATES) FROM NEW YORK REPORT, APPENDIX A.

Salaries	Single-stage		Extra for two-stage		Total two-stage
Direct Operation Staff:					
Chief engineer (1)	$12 000		....		....
Electrical engineer (1)	8 000		....		....
Hydraulic engineer (1)	8 000		....		....
		$28 000	....		....
Draftsmen (4)	$12 000		(2) $6 000		....
Accountant (1)	4 000		(1) 4 000		....
Clerks (4)	8 000		(2) 4 000		....
Telephone operators (3)	3 000		(3) 3 000		....
		27 000		$17 000	....
		$55 000			
Power-House Operation:					
Station superintendent (1)	$6 000		$6 000		....
Assistant superintendents (3)	12 000		12 000		....
Dispatchers (3)	11 000		11 000		....
Group chiefs (7)	17 500		(6) 15 000		....
Operators (14)	28 000		(12) 24 000		....
		$74 500		$68 000	....
Outside Operation:					
Regular and extra help on dam	$30 000		$25 000		....
Sub-station	6 000		6 000		....
Oil and waste	4 000		3 000		....
		$40 000		$34 000	....
Maintenance and Repairs:					
Chief electrician (1)	$3 000		$3 000		....
Master mechanic (1)	3 000		3 000		....
Mechanics (4)	8 000		8 000		....
		$14 000		$14 000	....
Total payroll (one plant)		$183 500		$133 000	....
Total payroll two-stage (four plants): 2 ($183 500+133 000)	....		....		$633 000
Materials and Supplies: Regular and extraordinary at $3 000 per unit		$72 000		$54 000	....
Regular and extraordinary at $3 000 per unit for two-stage (four plants) 2 ($72 000+$54 000)	....		....		$252 000
Total for single plant		$255 500		$187 000	....
Total for two-plant		$511 000		$374 000	....
Total for four-plant (two-stage) ($511 000+$374 000)	....		....		$885 000
Estimated annual operating cost	New York Plan (Single-Stage)				$511 000
	Conference Plan (Single-Stage)				511 000
	Reconvened Plan (Two-Stage)				885 000

Fig. 2 is a map of Plan *C*, the single-stage, Massena Point development, and Fig. 3 shows the profiles of all three plans, for comparison.

CONTROLLING PHYSICAL CONDITIONS

In all computations in the various reports involving stream flow it has been assumed that the summer flow (for nine months of each year) will be

---

[11] Reconvened Rept., p. 9, and Conference Rept., p. 11.

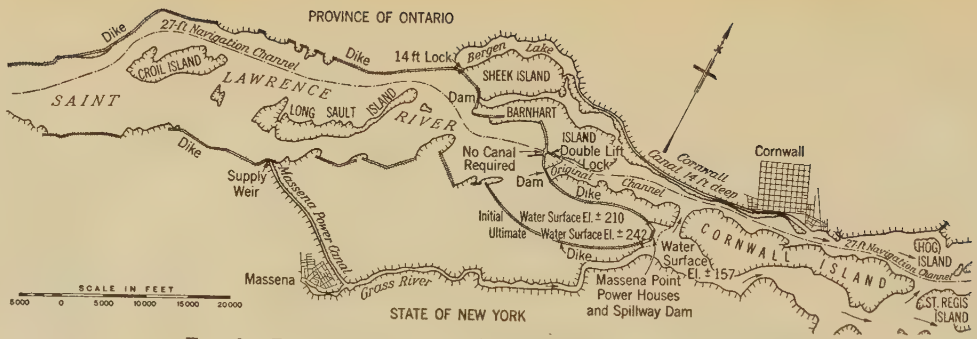

PROVINCE OF ONTARIO
Dike
27-ft Navigation Channel
Dike
14 ft Lock
Bergen Lake
SHEEK ISLAND
CROIL ISLAND
LAWRENCE
SAINT
LONG SAULT ISLAND
RIVER
Dam
BARNHART ISLAND
Double Lift Lock
No Canal Required
Dam
Original Channel
Dike
Cornwall Canal 14 ft deep
Cornwall
Dike
Supply Weir
Massena Power Canal
Initial Water Surface El. ± 210
Ultimate Water Surface El. ± 242
Dike
Water Surface El. ± 157
CORNWALL ISLAND
27-ft Navigation Channel
(HOG) ISLAND
ST. REGIS ISLAND
Massena
Grass River
STATE OF NEW YORK
Massena Point Power Houses and Spillway Dam
SCALE IN FEET
5000 0 5000 10000 15000 20000

FIG. 2.—PLAN C; SINGLE-STAGE MASSENA POINT DEVELOPMENT.

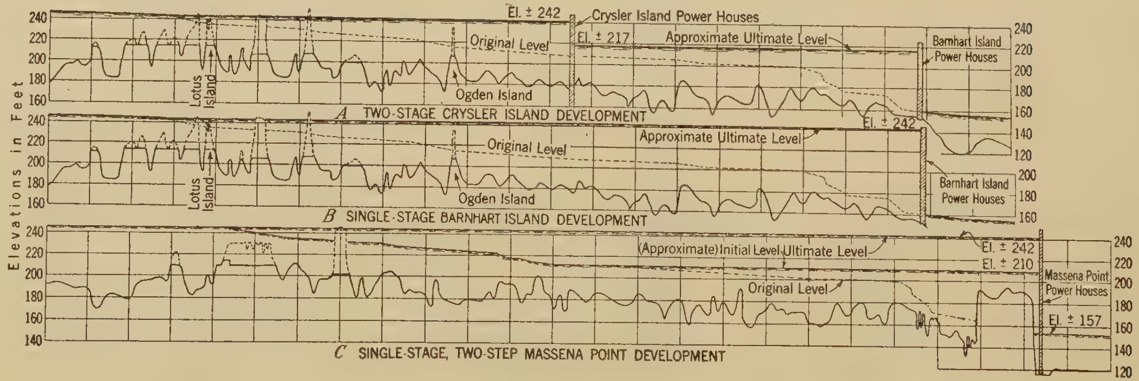


FIG. 3.—COMPARATIVE PROFILES, PLANS A, B, AND C.

235 000 cu ft per sec, and that during the three months of winter the flow will be reduced to 210 000 cu ft per sec.

The estimated head available in the three plans considered is shown in Table 1(*a*) and the horse-power years calculated on the basis of the stream flow are given in Table 1(*b*). Table 1(*c*) shows a summary of official estimates of the total cost of the three combined navigation and water power projects.[12]

In neither the Reconvened Report nor in the Conference Report does the writer find any estimates of operating costs. Therefore, he has based his estimates (see Table 2) on the New York Report, Appendix A, and while he would personally change these estimates in some respects, he believes that the official figures used are comparatively correct and suitable for this analysis.

FIRST ANALYSIS

In the first analysis the writer used the official figures as given in the official reports, without exception, taking operating charges from the New York Report. The cost figures for each water-power project are made up of: (1) The official estimate of the cost of works to be used solely for power; and (2) one-half the official estimate of the cost of works common to navigation and power.

While it is possible that some other proportion of this cost of the work common to navigation and power may be charged to the cost of power, such change in cost will apply to all plans and will not seriously affect the comparisons made.

In Table 1(*e*) are given the official estimates of the cost of the plans subdivided into depreciable and non-depreciable elements for the purpose of calculating the annual expense for each plan. In Table 1(*f*) are given estimates of the annual expense, of each of the three plans, made on the basis used in the New York Report, except that annual depreciation is taken at $1\frac{3}{4}\%$ of the value of depreciable property and annual amortization is calculated on the basis of 1% of the total capital cost.

In Table 1(*g*) are given; (*a*) The capital cost (Item No. 35), installed capacity (Item No. 19), and the estimated horse-power years output (Item No. 45); (*b*) the capital cost per horse-power year (Item No. 48); (*c*) the annual earnings based on $15 per hp year for firm power and $10 per hp year for secondary power (Item No. 50); (*d*) the excess capital costs per horse-power year of the two-stage plan over the single-stage plans (Item No. 49); (*e*) the earning capacity of each plant (Item No. 52); (*f*) the excess earning capacity of the single-stage plans over the two-stage plan (Item No. 53); and (*g*) the capitalized value of the excess earning capacities (Item No. 54).

Based on the unchanged estimates of the official reports, Table 1(*g*) shows that when the full financial conditions of power development are considered the relative cost of the two-stage plan is: (*a*) Almost $69 000 000 more than the single-stage Conference Plan; and (*b*) almost $125 000 000 more than the New York Plan.

---

[12] Reconvened Rept., p. 9; Conference Rept., p. 11; and, New York Rept., p. 50, respectively.

SECOND ANALYSIS

In the first analysis official estimates have been used so as to make the comparisons as far as possible independent of the writer's interpretations of these reports.

There are, however, certain estimates in these reports which the writer believes to be clearly in error, and which should be corrected to make the plans truly comparative:

(*a*) In the Conference Plan, $4 149 000 of capital cost is included to secure 5 000 sq ft more cross-section in the area between Lotus Island and Morrisburg.[13]

The writer understands that the Canadian engineers consider 95 000 sq ft as entirely adequate in this section for the two-stage plan, but that the single-stage plan will require a further increased section due to the manner of its operation, its pool fluctuations in winter, and certain ice conditions which might change the gradient so as to reduce the section between Ogden and Lotus Islands sufficiently to increase velocities above the limit necessary to permit the formation of surface ice and prevent the formation of frazil ice.

With his limited knowledge of the local conditions, the writer is unable to concede the probability of any such occurrence. In the two-stage plan the fall to Crysler Island must be equal to the head necessary to produce the flow through the section.

At Crysler Island a certain head is utilized, and a head of from 1.3 to 1.5 ft will be lost in passing the water through the power-house and turbines. In the same way a head sufficient to cause the flow through the lower section will be required from the Crysler Island tail-water to the power-house at Barnhart Island.

If the Crysler Island power-house is eliminated there will be an increase of 25 ft in depth for the 18-mile reach from the Barnhart Island plant to Crysler Island, thus reducing the head necessary to create the flow in the lower reach of the river, and eliminating also the head lost in the two-stage power-house at that point. Certainly with any reasonable operation the tendency should be to increase rather than to decrease the depth of water in the reaches above Crysler Island. Truly, ice troubles may follow mismanagement with either plan, but would seem more likely to occur with the two-stage plan and its exceedingly limited pondage. With proper design and proper operation, trouble with frazil ice will be confined to the early period of winter just before surface ice is formed, and should be far less with the single-stage plan at or near Barnhart Island with its extra depth and consequent lower velocities. If there is any valid reason for including this extra expense in the single-stage and not in the two-stage development, it is not obvious. This amount, therefore, is added to the cost of the two-stage development.

---

[13] See Conference Rept., p. 5, Paragraph 16 (*b*).

(*b*) In the estimate of installed horse-power of machines, the single-stage Conference Plan is penalized by adding 87 740 hp in machine equipment above that used in the two-stage plan, and 87 700 hp more than is provided in the New York Plan.

This extra capacity may be desirable for the equipment of the plant in question but if so there would be a necessity for increased capacity in the two-stage and New York plans. For the purpose of comparison, the equipment of the plants is equalized on the basis of a total capacity of 2 200 000 hp.

The estimated cost of installation per horse-power is shown in Table I of the Joint Report (page 281) and is segregated as follows:

Item 1 (*b*) Power-house substructure........	$24 000 000
Item 1 (*e*) Tail-race excavation............	3 799 000
Item 1 (*g*) Superstructure and machinery....	53 571 000
Total	$81 370 000
Overhead, 12.5% (see Joint Report, page 287).	10 171 000
Total cost of 2 287 000 installed horse-power..	$91 541 000
Approximate cost per horse-power........	$40

The cost of the 40 hp additional equipment, which is recognized as quite insignificant, is added therefore to the two-stage plan, and the cost of 87 700 hp extra equipment (at $40 per hp) is deducted from the cost of the single-stage Conference Plan to make the plans fairly comparative The foregoing corrections to the official estimates is shown in Table 1(*h*).

The annual expense involved is calculated in Table 1(*i*) on the basis of data in Table 1(*h*). The comparison of the three plants based on these partly corrected capital costs is shown in Table 1(*j*), which also shows that, based on the estimates corrected for the two errors mentioned, the cost of the two-stage plan is: (*a*) $79 000 000 more than the single-stage Conference Plan; and (*b*) almost $130 000 000 more than the New York Plan.

THIRD ANALYSIS

In the second analysis, official estimates have been corrected for certain errors which seem to the writer beyond reasonable question. In this third analysis, these same errors are corrected and certain other errors are considered which the writer thinks should also be taken into account. These errors are:

(*a*) It seems certain that the extra loss in head due to the use of the water through two plants over the loss due to the use in the single plant of the single-stage plan has not been allowed for in the official estimates of the power which can be generated in the two-stage plan. A correction for this loss in the quantity of power and in the resulting net income, therefore, has been deducted in the estimates of power for the two-stage plan (see Table 1(*k*)).

(*b*) It seems equally clear that an error has been made in the estimate of the winter head in the single-stage Conference Plan. This error has been estimated by the writer at 1.5 ft (although it would probably be greater than this amount) and a correction has been made in the estimated output of power and corresponding net income for the single-stage Conference Plan (see Table 1(*k*)).

Based on the four corrections discussed, the three plans are compared in Table 1(*l*). These data show that the relative cost of the two-stage plan is: (*a*) Nearly $93 000 000 greater than the single-stage Conference Plan; and (*b*) more than $138 000 000 greater than the New York Plan.

In the three analyses previously discussed the writer has considered only the estimated cost of the water-power development, including in such cost one-half the estimated cost of works common to navigation and power. It will be noted from Table 1(*m*) that, in addition to the difference shown in the cost of power plans, there are also differences in the cost of the navigation works plus one-half the cost of works common to navigation and power, which must be added to the differences previously shown, in order to ascertain the entire difference in capitalized values.

Taking the excess cost of navigation into account (Table 1(*m*)), as well as the capitalized costs of the power plants, the difference in the complete plans for power and navigation will be as shown in Table 3. In considering the data in this table, it should be noted that in none of the reports (with the exception of the New York Report) has interest during construction been considered; and that without definite plans of comparative construction procedure no such estimates are practicable. If interest during construction were considered, as well as depreciation and amortization of navigation works, the difference shown in Table 3 would be considerably increased.

TABLE 3.—DIFFERENCE IN VALUE OF PLANS, BASED ON CAPITALIZED NET INCOME AS PREVIOUSLY DETERMINED, CORRECTED FOR DIFFERENCE IN NAVIGATION COSTS

Item No.	Analytical comparison	DEFICIENCY IN VALUE OF TWO-STAGE PLAN COMPARED WITH:	
		Single-Stage Conference Plan (B)	Single-Stage New York Plan (C)
1	Based on official estimates (Table 1 (*g*))	$68 795 560	$124 705 920
2	(*a*) Extra cost of navigation work (Table 1 (*m*))	1 974 500	15 911 500
3	(*b*) Total difference (Items Nos. 1 and 2)	$70 770 060	$140 617 420
4	Estimates corrected for excavation and extra capacity (Table 1 (*j*))	$79 301 600	$129 686 860
5	(*a*) Extra for navigation work	1 974 500	15 911 500
6	(*b*) Total difference (Items Nos. 4 and 5)	$81 276 100	$145 598 360
7	Based on corrected estimates and heads (Table 1 (*l*))	$92 701 980	$138 357 240
8	(*a*) Extra for navigation work	1 974 500	15 911 500
9	(*b*) Total difference (Items Nos. 7 and 8)	$94 676 480	$154 268 740

It will justly be claimed that such analyses as have been made are not conclusive because there may be elements of greater importance than relative financial comparisons.

One claim of advantage for the two-stage plan is that it can be developed in two steps while the single-stage Barnhart Island development must be built in one step, and the development charge on the latter during the time necessary to develop a market may exceed the difference in first cost, $38 324 000. (See Table 1(*c*), Plan *B*.)

The Joint Board of Engineers may not have found it feasible to develop the single-stage Barnhart Island plan in two steps, but its members evidently did not consider the fact that the single-stage plan at Massena Point could be developed in two steps. This matter is discussed in the New York Report (pages 46-47), but is here tabulated (see Table 4), in order to show the comparison of the two two-step plans.

TABLE 4.—INITIAL DEVELOPMENT TWO-STAGE CRYSLER ISLAND PLAN WITH INITIAL SINGLE-STAGE NEW YORK PLAN

Item No.	Comparison	Crysler Island Plan (A)	New York Plan (C)
1	Head, in feet	24	50
2	Proposed installation, horse-power	593 000	660 000
3	Available horse-power years for sale	504 000	600 000
4	Capital cost (exclusive of navigation)	$93 423 000	$82 502 000
5	Excess in cost of Plan A over Plan C	....	$10 921 000
6	Capital cost per horse-power year available	$185	$138
7	Capital cost advantage of Plan C over Plan A per horse-power year	....	$47
8	Capital cost, navigation works	$46 137 000	$887 000
9	Excess cost of navigation works, Plan A over Plan C	....	$45 250 000
	FIRST STEP (14-FOOT NAVIGATION)		
10	Power works	$93 423 000	$82 502 000
11	Navigation works	46 137 000	887 000
12	Sub-total	$139 560 000	$83 389 000
13	Difference	$56 171 000	
	SECOND STEP (27-FOOT NAVIGATION)		
14	Power works	$93 080 000	$42 796 000
15	Navigation works	42 102 000	78 040 000
16	Sub-total	$135 182 000	$120 836 000
17	Total	$274 742 000	$204 225 000
18	Difference in total first cost	$70 517 000	
19	Capitalized difference (Table 3)	$154 268 740	
	SALEABLE HORSE-POWER YEARS		
20	First step	504 000	600 000
21	Difference	96 000	
22	Second step	1 386 100	1 392 000
23	Total	1 890 100	1 992 000
24	Difference	101 900	

It is evident from Table 4 that the two-stage plan has a distinct disadvantage in comparison with the New York Plan, because Items Nos. 5, 7, 9, 13, 18, 19, 21, and 24 are uniformly to the advantage of the New York Plan as to both the first and second step.

Other reasons for favoring the two-stage plan are given in the Conference Report (see page 8) as follows:

> "It will be noted, by reference to the estimates given, that the single-stage project apparently costs about $38 000 000 less than the double-stage project. The difference in cost is compensated for by the earlier date that power may be made available from the double-stage project, the extent of the economic losses to the country from the flooding of additional lands, as well as the prime considerations mentioned in paragraph 9 of the Report."

(*a*) It has already been shown that while the two-stage plan will cost over $38 000 000 more than the single-stage Barnhart Island Plan and over $70 500 000 more than the single-stage Massena Point Plan, the comparative cost of the two-stage plan based on the official estimate will be about $70 770 060, and probably about $94 676 480 more than the single-stage Barnhart Island Plan, and about $140 617 420, and probably about $154 268 740, more than the Massena Point Plan. (See Table 3.)

(*b*) That the difference in cost of the two-stage plan will be in any degree compensated for by the earlier date at which power may be available is more than questionable.

The time of completion of both the first step of the two-stage plan at Crysler Island and the first step of the Massena Point Plan is estimated as five years.

The Crysler Island plans require coffer-dams aggregating about 9 000 ft in length, to be built in water up to 40 ft in depth, across the St. Lawrence River channel, and maintained during long winters with heavy ice running, and are likely to involve many contingencies and consequent delays.

The spillway and power-house at Massena Point will be founded in excavation on land "at a safe distance from the water's edge" and will be constructed "in the dry," thus avoiding the hazards of river construction. Certainly the chances in favor of early completion are much in favor of the Massena Plan.

(*c*) "The economic losses to the country caused by the flooding of additional lands" mentioned in the foregoing quotation, involves 5 760 acres extra in the United States and 9 600 acres extra in Canada, or a total of 15 360 acres, the value of which at any reasonably assumed price is entirely insignificant when the greater advantages of the single-stage development are considered.

In the Conference Report (page 4, paragraph 9), it is stated:

> "If, as an alternative, the International Rapids Section be improved by a single-stage project then control of flow must be secured by operation of gates and turbines at a power plant which will develop about 2 000 000 horse power. This is not regarded favorably by the down river interests, since this single plant would serve a large and populous territory both in Canada and the United States, and operation restrictions would be difficult to impose in view of the fact that it would be a dominant interest in the territory served. Again, the additional flooding of land and the extent of the embankments, along with some adverse effects at the end of the navigation season, make the single stage project undesirable."

(*a*) It is understood that in any plan the primary control of the flow of the river will be accomplished by means of control works at Galop Island. That such control is anticipated for the two-stage plan is shown by the "Reconvened" Report (page 5, paragraph 19 (*d*)) which refers to:

> "A free open channel south of Galop Island for navigation, together with a diversion channel through Galop Island capable of discharge control in the interest of both navigation and power."

Such works are also applicable to any other plan. Secondary control must take place at the power-house and spillway under any plan of development. In all cases the river must flow in its normal volume either through the turbines or sluice-gates, or both, as occasion may require. The capacity of the plant will have little bearing on the matter of control as in each case the entire river flow must be balanced between the turbines and sluice-gate discharge at all times, and such control is not influenced by the head available below the structure. This control can be accomplished by automatically controlled sluice-gates as recommended by the New York Commission.[14]

(*b*) If the proposed works are operated so as to assure normal flow of the river, "down river interests" can have no valid complaints.

It is scarcely possible that any "dominant interest" of "the populous territory" of the United States and Canada will be able to affect operating restrictions injuriously to the paramount object of navigation. If the power authorities of the State of New York and the Province of Ontario can be influenced to ignore navigation interests in favor of power users, it is safe to assume that the Federal Authorities of the United States and Canada can be relied upon to enforce all necessary restrictions.

It can scarcely be demonstrated that complicating the situation by increasing the number of power plants, which in any case must be operated in parallel, would simplify operation, management, or control.

Safety of Montreal and Communities Adjacent to Embankments

While the safety of the City of Montreal and the communities adjacent to the embankments is not specially discussed in the reports, it is incidentally referred to as a matter of importance, which it certainly is.

In all plans herein considered the normal pool level above Crysler Island will be at Elevation 242. In either of the single-stage plans, considered, the water above the proposed dam will be raised to this same elevation, which will necessarily involve longer and higher embankments.

The Barnhart Island single-stage plan will require about 20 miles of embankment below Crysler Island, as compared with about 4 miles in the two-stage plan. The depth of water against the embankment in any single-stage plan will generally be less than 20 ft, and much of it will be less than 10 ft. The maximum depth for a short distance is 40 ft. The large quantities of suitable material which must be excavated for navigation purposes, and the exceptionally large cross-section for the embankment proposed by the Joint Board gives an assurance of safety the degree of which can be increased to almost any limit desired at very reasonable cost. (The embankments shown on Plate 5 of the Conference Report have a top width of 50 ft; a water slope of 4 to 1, heavily riprapped above normal water level; a land slope of 5 to 1, with a very heavy rock-fill toe and a free-board of 20 ft.)

---

[14] See New York Rept., p. 44.

As to the safety of Montreal, it should be noted that if any of the dams proposed in the single-stage plan could be removed instantaneously, the water in Montreal Harbor would be raised no more than a few inches. As the level of water has frequently been raised at Montreal from 10 to 15 ft, or more, by ice jams, there can be no question of trouble of this nature.

CONCLUSION

In conclusion, the writer would say that if the reports and estimates considered are correct, there can be no question that the single-stage Barnhart Island Plan, is at least $70 000 000 greater in value than the two-stage Crysler Island Plan, and that the single-stage New York Massena Point Plan exceeds the two-stage plan in value by at least $140 000 000. (See Table 3, Item No. 3.)

The New York Plan, which apparently received no consideration by the Joint Board, seems to be not only much less expensive in first cost, but also much less hazardous in construction, and to be an improvement from a navigation standpoint by eliminating one lock, greatly reducing the necessary length of canal.

The writer can see no valid reasons to justify the two-stage plan which can outweigh the millions of dollars of value in favor of other plans. If the reports are in error or if the reasons for advising the two-stage plan cannot be made manifest, the writer believes that this project should be further and completely investigated before the United States and Canada are asked to make the great permanent investment involved by this project.

DISCUSSION

T. H. HOGG,[15] M. AM. SOC. C. E. (by letter).—Before considering the paper under discussion, the writer wishes to state that in his opinion the choice of a single-stage or a double-stage plan cannot be settled satisfactorily solely on engineering or economic grounds. He would like to make his standpoint clear in regard to the entire question of the proper means of developing the International Reach of the St. Lawrence River. That part of the river under discussion forms the boundary between the United States and Canada. New York State, on the American side, and the Province of Ontario, on the Canadian side, border on the river, and by virtue of their position and ownership of the water power incidental thereto, are vitally interested in the proposed development. It is obvious, therefore, that National, State, and Provincial opinions must be given consideration. For these reasons the question of the relative suitability of a single-stage as against a two-stage plan of development is not a simple one. With these conditions in mind, a treaty, between the United States and Canada, has been negotiated, signed, and now (1933) awaits ratification.

General.—Because of the magnitude of the undertaking and its international aspects, the development of the St. Lawrence Waterway has occasioned the most widespread interest and discussion on the part of the general public and those interests particularly affected. From an engineering standpoint, it has been the subject of wide discussion, since it has occupied attention in one form or another for more than three decades. At the turn of the century, the Board of Deep Waterways issued an exhaustive report on the navigation features. Following the publication of this report, the Long Sault Development Company and the allied Canadian company—the St. Lawrence Power Company—made studies of the power possibilities of the Long Sault Rapids and secured a charter to develop power at that point; but owing to the fact that its charter was found to be unconstitutional, the Company did not go on with the work.

In 1918, the Hydro-Electric Power Commission of Ontario began its studies, which have been carried on practically continuously since that date. In 1920, the International Joint Commission referred the question of improving the International Section of the St. Lawrence River to an International Engineering Board. This was followed by investigations on the part of various private interests and by New York State. It will be realized, therefore, that many plans must have been studied and discarded. This, in all likelihood, accounts for what may appear to be lack of sufficient detail in the later reports.

Naturally, when dealing with such a project, there must be differences of opinion as to the proper method of handling the problem presented, and, for these reasons, it is evident that the determination of the proper course to follow cannot be based on purely engineering grounds. Neither, when it comes to the economic side, can the intangible values (which occupy positions of widely varying importance in the view of those studying the problem), be lightly brushed aside.

---

[15] Chf. Hydr. Engr., Hydro-Elec. Power Comm. of Ontario, Toronto, Ont., Canada.

In considering the entire question of the development of the St. Lawrence River, the fact that it is an international river cannot be ignored. Each country must deal with the problems that arise, in the way best suited to itself. What may appear as an obvious solution of a domestic question to one may not be at all suitable when dealt with by the other. These matters only add to the difficulty of securing a proper solution of the problem.

In his very interesting paper, the author has given a general outline of the latest studies, and has indicated quite clearly that he does not agree with the report of the Joint Board of Engineers. He also shows a marked preference for the plan of development presented by the Engineering Board of the St. Lawrence Power Development Commission for the State of New York, and uses it as a basis for comparison with other plans.

Discussion.—The project referred to as Plan *A*—the "Two-Stage Reconvened Plan"—is the Crysler Island Two-Stage Project, with the upper dam and power-houses at Crysler Island and the lower dam and power-houses respectively, at the upper and lower ends of Barnhart Island (see Fig. 1(*A*)). Plan *B*—the "Single-Stage Conference Plan" (Fig. 1(*B*))—is essentially the Barnhart Island, Single-Stage, 242 Project presented in the report of the Joint Board of Engineers. It was presented subsequently, with minor changes, in a report by the Conference of Canadian Engineers, a board made up of the Canadian members of the Joint Board and two engineers representing the Province of Ontario. Plan *C*—the "Single-Stage Massena Plan" (Fig. 2)—is that project which was reported on favorably by the Engineering Board of the New York State St. Lawrence Power Development Commission.

A comparison of the relative merits of a single-stage as against a two-stage project involves questions other than those of engineering import, and, since the author has indicated a certain preference, it is proposed to deal with it in the following way: First, examine the two single-stage methods of development; and, second, discuss the reasons which led up to accepting the two-stage plan and its relative merits compared with other proposed plans.

Professor Mead has undertaken a comparison of the various plans on the basis of the estimated costs. It is of prime importance, therefore, that each estimate be on the same basis. In so far as Plans *A* and *B* are concerned, they are strictly comparable, since great care was exercised to insure that the estimates of the Conference and of the Joint Board were on the same basis. In respect to the estimates of these two projects and that of the Massena Point project (Plan *C*), there is good reason for taking exception to the comparison.

Capital Cost.—In Table 1(*c*), the author shows that the total capital costs of Plans *B* and *C* are $236 418 000 and $204 225 000, respectively, indicating apparently a balance in favor of the latter, of $32 193 000. An examination of the detailed estimates of the two projects discloses that the estimated costs of comparable items and unit prices are not on the same basis. It is found that $15 403 000 should be added to the estimate for Plan *C* for land damages, rehabilitation, etc., on the Canadian side, $1 250 000 for contingencies on "land damages", and $12 477 000 for power-house machinery and equipment, etc.

If the unit prices for work to be done in the same or similar locations are equalized, the estimate should be increased by $5 006 000, and a further sum of $11 340 000 should be added to cover a supply weir at the head of the Massena Canal, the excavation of an additional 5 000 sq ft in the channels between Lotus and Ogden Islands, and to provide for a double flight of locks. These additions bring the total estimated cost of Plan *C* to $249 701 000, as against $236 418 000 for Plan *B*. It is not contended that the equipment cannot be purchased or the work cannot be done for the prices given, but the foregoing adjustments are required in order that the estimates of all three plans may be on the same basis.

Head Developed.—With regard to the head that may be developed under Plan *C*, the official report states (page 5), that "the head will vary therefore from 84 to 88 feet, and is expected to average 85 feet". This variation in head corresponds to the fluctuation in tail-water level due to the variation in discharge, and does not take into account the fluctuation in the head-water levels due to the same cause. For a flow of 235 000 cu ft per sec, and a regulated Lake Ontario level of 246.32, the head-water level at Massena Point would be 242.15. Due to the fact that the total flow of the river will be discharged into the pool at the south end of Polly's Gut, and considering the channel improvements provided in the estimates, the water level will be from 0.8 ft to 1.5 ft higher than under natural conditions, giving a tail-water level of 156.0 and a corresponding head of 86.15 ft, instead of the 88 ft used by the author in computing the power output for open-water conditions.

The head that may be secured under winter conditions should also be corrected. The loss in head above the plant in winter, due to increased slope, will be as great as that in Plan *B*, and would amount to 4.3 ft. In addition, there will be a loss, due to higher tail-water levels. In an ordinary winter, the water level below Polly's Gut averages between 10 and 15 ft above normal water level. The flow of ice will be reduced, but during the latter part of December, or early January (depending on the season), some packing will occur between the Lake St. Francis ice sheet and Polly's Gut, as this channel will always be open. Under these circumstances, a rise of from 5 to 6 ft is to be expected. The head at the Massena Point plant, corresponding to the winter flow of 210 000 cu ft per sec, would be 76.65 ft.

Power Output.—In reference to the output that may be expected from the plants, the writer is unable to agree with the figures used by Professor Mead. Primary power is continuous power and, therefore. is limited by the winter head and flow. Secondary power is that which is available in addition to primary power during the remainder of the year. Expressing the output in horse-power years, based on a winter flow of 210 000 cu ft per sec (three months), and a summer flow of 235 000 cu ft per sec (nine months), and using the corrected heads, it will be seen from Table 5(*b*) that the total horse-power years available are reduced for each of the plans, but that the greatest change is for the Massena Point project (Plan *C*). Instead of an output of 1 992 000 hp-yr (Table 1(*b*)), it should be 1 852 605 hp-yr, a reduction of 139 395 hp-yr (see Table 5(*b*), Item No. 9). The output for Plans *A* and *B* should be corrected by deducting 67 380 hp-yr and 68 110 hp-yr, respectively, from the figures given in Table 1 (*b*), Item No. 8.

Hazards of Construction.—The elimination of the hazards of construction in the Massena Point project, when compared with the projects proposed by the Joint Board and the Conference, has been stressed by its proponents and also by the author. An examination of the results of borings at the site proposed for the Massena Point power-houses and spillway shows that the over-burden varies in depth from 65 to 120 ft. To open such an excavation in the materials to be encountered, with adjacent water levels 65 ft above the rock-surface elevation, is certainly not less hazardous than that involved at the sites at the foot of Barnhart Island, where the over-burden is a maximum of 50 ft in depth and varies down to a few feet, and the unwatering is a very minor problem.

The main dam above Hawkin's Point, proposed in Plan *C*, is a rock-fill structure with a blanket of sluiced material on the up-stream side. No provision is made for diverting any of the flow while the structure is being built in the main channel for a water level above the dam lower than Elevation 200. Above this elevation the water would be diverted over the sluice-ways at Massena Point. The cribs carrying the construction trestle must be placed in water 35 to 40 ft in depth, in velocities up to 15 ft per sec, with a flow of from 215 000 to 280 000 cu ft per sec; and, since it is not proposed to clear off the rock surface, there is likelihood of underscouring. Not only is this proposal very hazardous, but there is great uncertainty as to the quantity of material required in the blanket because no one can foretell what slope the materials will take when placed under water by sluicing methods.

The location of the concrete dam proposed in Plan *B* is such that at least 70% of it can be built in still water, a great part of which is quite shallow, and where the rock is lying at a depth varying from 15 to 25 ft below the water surface. By building this section of the dam up to the surface of the river bed, the remainder can then be put in by diverting a large part of the flow, after which the entire dam can be raised a section at a time to its final elevation. This proposal is considered by those intimately acquainted with the river, as much less hazardous than the one suggested by the proponents of Plan *C*.

Summary of Comparisons of Single-Stage Plans.—To summarize, then, a comparison of the single-stage plans shows that, after the estimates are made strictly comparable, the capital cost of Plan B is $13 283 000 less than that of Plan *C*. Correcting for slope and loss in the tail-race, the summer heads on the two plants for a flow of 235 000 cu ft per sec would be 86.1 and 86.2 ft, respectively, and, in winter, for a flow of 215 000 cu ft per sec, the heads would be 76.4 and 76.65 ft, respectively. In respect to hazard, Plan *B* is more free from that objection than Plan *C*, and the quantities, especially in the dam, are capable of being estimated much more closely. From the Canadian standpoint it is more satisfactory because the Canadian power-house can be placed on Canadian territory.

In the report of the Board of Engineers, New York State, it is stated that "they [the Board] do not feel that so strong a position can be taken as to the superiority of the Massena Point one-stage plan over other single-stage plans." Apparently, the plan was finally presented for consideration, not because its authors were convinced of its overwhelming superiority, but as an alternative which might help in a final determination of a satisfactory scheme of development. The report states:

> "It is appreciated that the choice of the exact location of the dams and power houses must be determined along with other negotiations which, of necessity, remain to be made with the Canadian interests."

Comparison of Plan A with Plans B and C.—In considering the two-stage Crysler Island plan contained in the "Reconvened Report," it is pointed out that to state "the Canadian engineers have consistently advocated a two-stage plan of developing the International Rapids Section," is erroneous. In the first report the Canadian engineers were not convinced it was desirable or feasible, on account of ice conditions, to raise the pool level at Barnhart Island to Elevation 242.0, but they favored a single-stage plan with a lower level, namely, 232.0, with a control dam at Ogden Island. In presenting the Ogden Island two-stage, as an alternative to the single-stage plan, the Canadian engineers were endeavoring to meet the wishes of the power interests, to make available for power as much of the total fall in that section of the river as possible.

Following the 1921 report, both the Hydro-Electric Power Commission of Ontario and the Department of Railways and Canals (which organization secured all the field data on the Canadian side and carried on the office investigations for the Canadian representative) made further intensive studies of the winter conditions on the river. The viewpoint of the Canadian Government officials was that, with the open-water reaches that would exist under any single-stage plan studied to that time, serious ice jams were certain to occur, similar to those conditions which have been observed from Weever's Point up stream under natural conditions. The down-river interests looked on these possibilities with great apprehension; they considered that it would inevitably lead to disaster and insisted that in any plan of development their interests must be amply safeguarded.

Ice Conditions.—It is emphasized that the seriousness of the ice problem in relation to the development of the St. Lawrence River cannot be too greatly stressed. Under natural conditions there is a stretch of river, more than fifty miles long, in which ice in its various forms—board, frazil, anchor, and slush—is formed on a colossal scale. This accumulating under the ice sheet at the upper end of Lake St. Francis has caused shoves and packing to occur all the way from Summerstown up stream to Lock 21, and on numerous occasions it has resulted in raising the water level at the foot of Polly's Gut and the lower end of Barnhart Island from 15 to 25 ft above normal level. Jams have formed below Weever's Point, which have caused the ice to pack up stream to Morrisburg, resulting in a rise in water level of 12 to 15 ft at the latter point. Study of the river during the winter season has engendered a wholesome respect for the problem of providing safe winter operating conditions.

Reasons for Crysler Island Two-Stage Plan.—In order that the objections of the down-river interests might be met, the Canadian officials proposed a control dam at Ogden Island, so that the flow could be throttled if a jam began to form. This led to a demand on the part of the power interests of Ontario that the control dam should be designed to develop power.

To raise the pool level to Elevation 242.0, without disturbing present lake levels, requires very heavy channel improvements from the head of Galops Rapids down stream. Coupled with the studies of winter conditions, it was considered that the only safe way to deal with the situation was to enlarge the channels until an ice cover could be secured, and to make provision for the control of the flow to prevent ice gorging, since the velocities could not be reduced in the entire reach to the point where ice would form in place, as the formation of an ice cover is dependent to a considerable extent upon the packing of ice carried in from the open reaches above.

While these studies were being made, the communities on the Canadian side of the International Rapids Section of the St. Lawrence River were deeply interested and, through their parliamentary representatives, discussed the effect to them of the different methods of development proposed; so there was built up, through various organizations, a tangible and strong public opinion for the adoption of a two-stage plan. The expressed public opinion finally had to be given serious consideration. The grounds for the stand taken by these communities were as follows.

The north shore of the St. Lawrence River constitutes the oldest settled section of the Province of Ontario, and, in many cases, the land is still held and occupied by descendants of the original holders. This area was known as "the Front," from which the back townships were settled; and the river towns and villages were, and still constitute, the marketing and trading centers for a distance half-way to the Ottawa River. The objection of these communities to disturbance is strenuous and continuous, so much so that it has come to be recognized by both the Provincial and Dominion Governments; strengthened by the objections of the down-river interests, it is a real and insurmountable obstacle to securing the single-stage plan. It may be pointed out that Plan *A*, the two-stage plan, has also been objected to on the score of disturbance, and while there are some grounds for these objections, still the single-stage plan would result in disturbance to, and the elimination of, the entire tier of communities from above Cornwall to the head of Galops Rapids, a distance of forty miles; in the two-stage plan, on the other hand, the disturbance would be confined to one-half that distance and would not extend any appreciable distance back from the river.

In a two-stage plan, with the upper plant at Ogden Island taking the place of a control dam at that point, an ice cover could only be secured below the plant by very heavy work in the channels as far down stream as Crysler Island for any water-surface elevation below 225.0. After numerous borings had been made to determine the elevation of the rock surface, it was decided that the proper position for the upper plant in a two-stage plan was at Crysler Island. This was the reason Plan *A* was finally proposed by the Canadian engineers; and the Joint Board agreed that, "all things considered, the scheme was feasible from an engineering standpoint."

The foregoing is a brief outline of the steps followed by the Canadian engineers in reaching the position as to the desirability of a two-stage plan, and indicates the engineering problems.

Table 5(*a*) gives a comparison of the three plans for the heads which would obtain under winter and summer conditions. In regard to the power produced under the three plans of development, each is expressed in horse-power years. With the corrected heads and using 85% efficiency in each case, Table 5(*b*) gives a comparison of the power output under the three methods of development.

Comparison of Plans of Development on Basis of Capital Cost.—Table 5(*c*) gives a comparison of the three plans on the basis of capital cost after they have been placed on a comparative basis by corrections to the unit prices and other items, as outlined in a preceding paragraph. The corrections to the estimates of Plan *C* do not entirely adjust the differences. When studying the plan of the New York State Board after the report was presented, the layout shown therein was accepted, but the quantities were taken off and estimates were prepared in the same way as for Plans *A* and *B*. It was then found that the capital cost was $273 822 000 as compared with $236 418 000 for Plan *B*. In view of this, it was considered that Plan *C* was distinctly inferior to other single-stage plans that had already been studied.

In Table 5(*d*), the costs are subdivided in order that the annual expenses may be calculated. The values for Plan *C* are corrected in accordance with the revised capital costs for that project. Table 5(*e*) shows the estimated annual expenses which would be incurred in the operation of the three projects. These are based on the experience which the Hydro-Electric Power Commission of Ontario has had in operating its plants. The rates used are neither the highest nor the lowest, but are considered to represent, fairly, the operation costs of these plants. Item No. 28, Table 5, includes management, engineering staff, superintendence, record staff, stenographers, telephone operators, operating payroll and supplies, maintenance and repairs, liability insurance, and the operation and maintenace of the dams.

The estimated cost of maintaining and operating the plants is given in Table 6.

A comparison of the power plant costs of the three plans, based on corrected capital costs and equalized installed capacity is given in

TABLE 5.—WRITER'S COMPARATIVE DATA PERTAINING TO PLANS *A*, *B*, AND *C*

Item No.	Description	Plan *A*; Two-Stage, Crysler Island, "Reconvened" Plan		Plan *B*; Single-Stage, Barnhart Island, Conference Plan		Plan *C*; Single-Stage, Massena Point, New York Plan	
(1)	(2)	Winter (3)	Summer (4)	Winter (5)	Summer (6)	Winter (7)	Summer (8)
	(*a*) COMPARATIVE HEADS, IN FEET (SEE TABLE 1 (*a*))						
1	Project elevations (see Fig. 3)	217	217	242	242	242	242
2	Head at Crysler Island	19.6	24.2	....	....	....	....
3	Head at Barnhart Island	56.4	60.4	76.4	86.1	....	....
4	Head at Massena Point	....	....	....	....	76.65	86.15
5	Total head	76.0	84.6	76.4	86.1	76.65	86.15
6	Reduction in head in winter	8.6	....	9.7	....	9.5	....
	(*b*) POWER OUTPUT (SEE TABLE 1 (*b*)), IN HORSE-POWER YEARS						
7	Primary power	1 539 125		1 547 225		1 552 290	
8	Secondary power	283 595		303 265		300 315	
9	Total power	1 822 720		1 850 490		1 852 605	
10	Excess power, Plans B and C, over Plan A	....		27 770		29 885	
	(*c*) COMPARATIVE ESTIMATED COSTS (NO INTEREST DURING CONSTRUCTION; 27-FOOT NAVIGATION (SEE TABLE 1 (*c*))						
11	Works primarily for power	$132 452 000		$87 481 000		$93 627 000	
12	Works common to power and navigation	108 102 000		121 196 000		129 094 000	
13	Works solely for navigation	34 188 000		27 741 000		26 982 000	
14	Total	$274 742 000		$236 418 000		$249 703 000	
15	Excess cost of Plan A over Plans B and C	....		$38 324 000		$25 039 000	
	(*d*) COSTS SUBDIVIDED TO COMPUTE ANNUAL EXPENSES (SEE TABLE 1 (*e*)).						
	Upper Pool; Power Works:						
16	Depreciable	$49 736 000		....		....	
17	Non-depreciable	5 769 000		....		....	
	Upper Pool; One-Half Common Works:						
18	Depreciable	6 764 000		....		....	
19	Non-depreciable	31 154 000		....		....	
20	Sub-total (Items Nos. 16 to 19)	$93 423 000		....		....	
	Lower Pool; Power Works:						
21	Depreciable	$64 695 000		$78 719 000		$86 216 000	
22	Non-depreciable	12 252 000		7 717 000		7 411 000	
	Lower Pool; One-Half Common Works:						
23	Depreciable	7 093 000		11 336 000		15 180 000	
24	Non-depreciable	9 040 000		49 262 000		49 367 000	
25	Sub-total (Items Nos. 21 to 24)	$ 93 080 000		$147 034 000		$158 174 000	
26	Total (Items Nos. 20 and 25)	$186 503 000		$147 034 000		$158 174 000	
	(*e*) ESTIMATED ANNUAL EXPENSE (SEE TABLE 1 (*f*) AND TABLE 5 (*d*) (ITEM NO. 26))						
27	Interest at 5%	$9 325 150		$7 351 700		$7 908 000	
28	Maintenance and operation	1 598 000		1 579 000		1 579 000	
29	Depreciation (1.25%)	1 603 600		1 138 750		1 267 450	
30	Sinking fund (1.053%)	1 963 875		1 559 270		1 665 570	
31	Total annual costs (Item Nos. 27 to 30)	$14 490 625		$11 628 720		$12 420 020	
32	Difference between cost of Plan A and Plans B and C, respectively	....		$2 861 905		$2 070 605	
	(*f*) COMPARISON OF POWER PLANT COSTS BASED ON CORRECTED CAPITAL COST AND EQUALIZED INSTALLED CAPACITY						
33	Estimated capital cost for power	$186 503 000		$147 034 000		$156 099 000	
34	Equalized installed capacity	2 200 000		2 200 000		2 200 000	
35	Total cost of output (horse-power years)	1 822 720		1 850 490		1 852 605	
36	Excess output, Plans B and C over Plan A	....		27 770		29 885	
37	Capital cost per horse-power year: $\left\{\frac{\text{Item No. 33}}{\text{Item No. 35}}\right\}$	102.37		79.40		84.26	
38	Excess in favor of Plans B and C (cost per horse-power year)	....		22.97		18.11	
39	Total income	25 922 800		26 241 000		26 287 500	
40	Total annual charges	14 490 600		11 628 700		12 420 000	
41	Net annual income (Item No. 39 minus Item No. 40)	11 432 200		14 612 300		13 867 500	
42	Excess annual incomes, Plans B and C over Plan A	....		3 180 100		2 435 300	
43	Difference capitalized at 5%	....		63 602 000		48 706 000	

Table 5(*f*). This is parallel to the author's comparisons presented in Table 1(*l*). If the differences between the net income from the single-stage plans (*B* and *C*) and Plan *A* are capitalized at 5%, the results compare with those computed by Professor Mead, as follows:

Reference	Plan *B*	Plan *C*
Table 5(*f*), Item No. 43.......	$63 602 000	$48 706 000
Table 1(*l*), Item No. 94.......	$92 701 980	$138 357 240

TABLE 6—ESTIMATED OPERATING COSTS (SEE TABLE 2)

Item No.	Description	CRYSLER ISLAND TWO-STAGE 217 PROJECT				BARNHART ISLAND SINGLE-STAGE 242 PROJECT		MASSENA POINT SINGLE-STAGE PROJECT	
		Upper Plant		Lower Plant					
		Operation	Maintenance	Operation	Maintenance	Operation	Maintenance	Operation	Maintenance
1	Operating staff (engineering)..........	....	....	$30 000	....	$30 000	....	$30 000	....
2	Superintendent, assistant superintendent, record staff, stenographers, telephone operators, etc........	....	....	30 000	....	30 000	....	30 000	....
3	Operation payroll and supplies..........	$52 000	....	238 000	....	280 500	....	280 500	....
4	Maintenance and repairs.............	....	$102 000	....	$272 000	....	$374 000	....	$374 000
5	Total.........	$52 000	$102 000	$298 000	$272 000	$340 500	$374 000	$340 000	$374 000
6	Total operation and maintenance......	$154 000		$570 000		$714 500		$714 500	
7	Total operation and maintenance: two-stage, four plants..	....		1 448 000		....		....	
8	Total operation and maintenance: one-stage, two plants..	....		....		1 429 000		1 429 000	
9	Management........	....		150 000		150 000		150 000	
10	Total.........	$154 000		$1 598 000		$1 579 000		$1 579 000	

It is confidently asserted that in so far as Plan *C* is concerned, the tremendous difference in capital value presented in Table 3 would vanish entirely if, as was indicated in an earlier paragraph, the plan were placed on the same basis as the others as regards safety, freedom from operation hazard, and navigation facilities, and if the questionable proposal of placing a rock-fill structure in a river of the nature of the St. Lawrence and at the point proposed were eliminated.

Reasons for Not Accepting Author's Set-Up.—For the reasons given in the discussion of capital costs, no further reference has been made to Professor Mead's "First Analysis" of the three plans, but what is considered to be the correct comparison is given in Tables 5 and 6. It is proposed now to give the reasons for not accepting the corrections made by the author in his second and third analyses. Before doing so, attention is called to the fact that the Canadian organizations studying the problem have been living with the river for a number of years, and that practically the same personnel has been engaged on the work throughout. While these organizations have been working independently, they have conferred and discussed the problems freely. Furthermore, the Hydro-Electric Power Commission of Ontario, has discussed these questions with organizations from the United States side of the line, and has furnished, freely, whatever information they might desire. On the United States side, while the personnel has not been so continuously on the problem, the United States Government organization (the Army Engineers, and, through them, the Lake Survey) has been engaged on the river more or less continuously for a number of years.

In contrast to these conditions, the New York State Engineering Board reported in a little more than three months after its organization and, as a Board, never had the opportunity to study winter conditions, first hand, which is probably the greatest problem of this development. Surely then, its plan should scarcely be used as a measuring stick to judge the other methods of development from the point of view of cost, etc. The author has acknowledged that, in making the analysis, he has been handicapped by his lack of knowledge concerning local conditions, and his difficulties are fully appreciated. The writer wishes to emphasize his belief that the plan recommended, and on which the treaty between the United States and Canada has been negotiated, has been arrived at after careful and long study, and, all things considered, represents the best solution of the problem.

Necessity for Increased Cross-Section in the Single-Stage Plan.—In his capital cost comparisons (Table 1(*c*)), Professor Mead has added the sum of $4 149 000 to Plan *A* to provide an additional 5 000 ft to the cross-sectional area in the reach between Lotus and Ogden Islands, similar to that in Plan *B*. This addition is not considered necessary. In explanation, the additional cross-sectional area was considered necessary in the single-stage plan to reduce the danger from ice gorging. This danger is greater under the single-stage plan for the following reasons: The control dam at the head of the Galops Rapids is situated in an artificial channel and provides only partial control. Due to changes in atmospheric pressure, surges are set up on Lake Ontario which will cause a rise in gauge height of 2 to 3 ft in a few hours at Prescott, or at the head of Galops Rapids, resulting in a relatively material increase in slope in the pool above the power plant, with a consequent increase in velocity and flow.

Occurring at a time when the ice on the pool is forming, as these surges frequently do, could result in the formation of an ice gorge, with serious results to the power output during the remainder of the winter season. These surges can be anticipated, and in Plan *A* there is time, by closing the sluice-gates in the upper dam, to raise the pool level and counteract the increased slope; and, at the same time, by transferring part of the load to the lower plant, it is possible to reduce or eliminate any reduction in power output during the period that the flow is being manipulated. In the single-stage plan, with its much larger pool, it would be impossible, in most cases, to reduce the slope in time to meet the emergency. It was considered that the extra cross-sectional area would provide the additional safeguard to reduce this hazard.

Heads at Crysler and Barnhart Islands.—It is not considered that the head on the Crysler Island plant should be reduced by from 1.3 to 1.5 ft (Table 1(*k*)), any more than the heads on the plants at Barnhart Island or Massena Point. The loss through the plant is taken care of in the efficiency used in computing the power output. The necessity of providing an extra 5 000 sq ft in the channels below Lotus Island has been explained. The apparent excess loss in head of 1.5 ft on the Barnhart Island plant in winter (Table 1(*k*)), is due to the extra accumulation of ice in the tail-race in any single-stage plan.

Power Costs, Barnhart Island.—If the installed capacity of the plants at Barnhart Island and Massena Point is equalized, the cost of the Barnhart Island development should only be reduced by the cost of the electrical equipment, since there would be no reduction in the spacing of the units, or in the size of the turbines, to take care of the 87 700 hp, considering the number and size of the units. This reduction in cost would not amount to more than $1 045 000.

Navigation Costs.—With reference to the navigation costs, the estimates in the Massena Point plan provide for a single flight of locks on Barnhart Island; the fact that the tonnage capacity was not the same in each case, was not considered. In both the other plans, the capacity of the locks is 16 000 000 tons, while that of the single-flight locks in Plan *C* is only 12 000 000 tons. In order that the capacity may be increased, the estimates were increased to provide for double-flight locks.

Comparison of Initial Developments.—The author has compared the upper development of the two-stage plan with the first-stage of the Massena Point project (Table 4), and, while of interest, it is not expected now that the need will arise for such a program of construction. In considering the relative output of the Crysler Island plant and the Massena Point initial development, it is contended that all the 600 000 hp available from the latter plant will be prime power; in other words, the head of 50 ft will be available throughout the year. For an elevation of 210 at Massena Point, the upper end of the pool would be about at Morrisburg. With the natural slope in the river above that point, there would be twenty miles of open water from which ice would be carried down into the pool during the entire winter. This would certainly result in an increased slope and reduced head-water level. The tail-water level would be increased by the accumulation of ice in the channel below Polly's Gut.

It is estimated that during the latter part of December, and during January, February, and the early part of March, the head will be reduced by 20 to 25 ft in an ordinary winter, so that the net head on the plant would be between 25 and 30 ft, instead of the 50 ft assumed in Table 4. The net head on the Crysler Island initial development—owing to the fact that natural levels would obtain below the plant—would be from 28 to 30 ft. The power output of each plant is also incorrect. The available continuous power from the Crysler Island plant would be 568 000 hp, and from the Massena Point plant, 506 000 hp, or 62 000 hp less than from the two-stage Crysler Island plant.

The capital cost of the initial development of the Crysler Island plant is given as $93 423 000, or $185 per available hp, in Table 4. This capital cost includes one-half the cost of works common to power and navigation, amounting to $37 918 000. A large part of this sum is chargeable to channel improvements, which are as valuable to the Barnhart Island plant as to the Crysler Island plant. It has been considered as fair that only one-third the cost of channel improvements above the plant should be charged against the upper plant, and the remaining two-thirds against the lower plant. This division is on the basis of installed capacity, approximately. The capital cost, therefore, of the Crysler Island plant, as given in Table 4, should be reduced by $14 360 000.

The total capital cost of providing 25-ft navigation in the upper pool is charged against the initial development of the two-stage plan, although Table 4 includes the sub-title, "First Step (14-Foot Navigation)", while, in the Massena Point plan, only the cost of providing a lock for the continuance of the present 14-ft navigation is included. The cost of providing similar navigation facilities in Plan *A* is included in the estimates of the cost of works common to navigation and power for the upper pool, and amounts to $1 205 000. The total shown in Table 4 to cover the cost of navigation, therefore, should be omitted. With these corrections, the capital cost of power for the Crysler Island initial development becomes $79 063 000, and that of the initial Massena Point development becomes $89 284 000, and the corresponding cost per available continuous horse-power would be $139.25 and $176.50, respectively.

In Table 3, Professor Mead summarizes his comparison of the three plans, and the conclusion he reaches is that Plan *C* is from $140 600 000 to $154 268 000 better than Plan *A*. It has been shown that these comparisons do not give the proper picture because compared with Plan *B*, when the difference in income is capitalized, the balance against the two-stage plan is $63 600 000 (see Table 5(*f*), a fact that has always been known and taken into consideration. In so far as Plan *C* is concerned, instead of being superior to Plan *B* by $60 000 000 (Table 3), on the basis of capitalized earnings, it is inferior by about $14 900 000, leaving out of consideration the fact that from an engineering standpoint its proponents were not at all certain that it was superior to Plan *B*, and that there is good reason to doubt that it is as free from construction hazard.

The author has made reference to the probable effect on the water level in Montreal Harbor if, by some mischance, the dikes were breached. His statement that, in such an event, the level would only be raised by a few inches is undoubtedly correct; but he has lost sight of the fact that in Quebec, above Montreal, there are industrial centers and areas that are not in the fortunate position of Montreal. In the event of such a disaster, they would suffer great damage, and there is the possibility that a new channel might be created for the St. Lawrence south of the new Beauharnois Power Canal. These communities cannot help but be apprehensive, and their apprehension takes the form of serious objection through their political representatives.

Conclusion.—In conclusion, it is pointed out that the comparison drawn, using Plan *C* as a basis, is incorrect, because the plans as presented in the official reports are not comparable.

Revising the estimates of the official report by using the same unit prices and adding certain items that are omitted, but not altering the quantities, would bring them more nearly in line with those of Plans *A* and *B*. The capital cost of Plan *C* would then be (Table 5, Item No. 14) $249 703 000, instead of $204 225 000 used by the author in Table 1(*c*). If the estimates of quantities and costs are prepared in the same way as those of Plans *A* and *B*, the capital cost would be $273 822 000.

As between Plans *C* and *B*, it has been shown that when placed on the same basis, Plan *B* is distinctly superior as regards total capital cost, cost of power, revenue, annual charges, freedom from construction and operation hazard, and capacity and superiority of navigation works. If objection is taken to placing the American power-house on Barnhart Island, it is quite possible, with little increase in cost, to place a concrete dam below the head of Barnhart Island, alter the navigation route by leaving the river above Robinson's Bay, build twin-flight locks at the mouth of Grass River, and place the American power-house on the main shore below Hawkin's Point, where the rock is much higher than at the proposed Massena Point site.

Considering hazards of construction, a careful analysis of the proposed methods of construction and a knowledge of the physical conditions, the hazards in the two-stage plan are not as great as those to be expected in the Massena Point plan. The capital cost is practically the same when the estimates are prepared on the same basis, and, in addition, there are those advantages in its favor which are associated with reduced disturbance and satisfied communities below the developments.

A comparison of Plan *A* and Plan *B* indicates that, both from the standpoint of capital cost and the annual costs which would be chargeable against power, the latter is preferable, although both plans are sound from the standpoint of the engineering problems involved and from the broad economic viewpoint. After years of study of all aspects of the development of the International Rapids Section of the St. Lawrence River—during the course of which it has been abundantly evident that there are a great many factors of other than an engineering, or a purely economic, character which have had a profound influence upon the problem, and to which as a consequence it was necessary to give due weight—Canadian engineers have reached the conclusion that the two-stage Crysler Island plan is the most suitable method of meeting, satisfactorily, all the varying conditions involved.

THERON M. RIPLEY,[16] M. AM. SOC. C. E. (by letter).—In considering this paper the reader is left in almost as confused a state of mind as has been the condition in the past from the reading of the various estimates put out by the Government engineers and other engineering organizations. Moreover, all these various reports and papers are devoted principally to the estimated cost of the construction of one or other of the various plans, and little or nothing is said about their economic justification. There has come to the writer's desk only one of these numerous reports that contained much information on this question of economics and in which the statement seemed to be based on estimated figures of costs of power delivered where there might be some probability of its use.

---

[16] Cons. Engr., Buffalo, N. Y.

The "Report of the St. Lawrence Power Development Commission; Submitted to the Governor and the Legislature of the State of New York, January 15, 1931," contains an estimate of the cost of steam generated in, and of St. Lawrence power transmitted to, New York and up-State cities. It contains, also, extensive tabulations of the cost of electricity now furnished to many up-State municipalities and to New York City.

The Marketing Board, after submitting the aforementioned estimates and tabulations, completes its report to the Commission with these words:

> "A further source of electric development appears in railway electrification. It seems probable that within a ten-year period enough of the railroads of the State will be electrified to require 1 000 000 000 kilowatt-hours per year.
>
> "In view of the past increases in power required and in view of the special developments that are likely to take place, we conclude there will be an ample market for St. Lawrence power and that there will be sufficient continuous load to take the entire St. Lawrence output available to New York within a brief period after the completion of the plant."

There is no doubt but that contracts for a large portion or all of the St. Lawrence power could be secured providing said contracts were on a basis of guaranteed delivery with the resources of the State of New York behind such guaranty.

With the knowledge that the report on the New York Plan was made by a Commission appointed by a Governor who was known to be sympathetic to the proposition of the State developing its water powers, and, further, in view of the fact that the Commission was careful to confine itself to the realm of possibilities rather than probabilities, one is still left in doubt as to whether the findings should be accepted as conclusive economically or whether there were other political and social reasons for the conclusions stated.

In other words, the question is: Can electricity be furnished to "possible" users for a less expenditure of time and money than by the development and transmission of St. Lawrence power?

During the three decades that this problem has been a subject of widest discussion the cost of producting electric power by steam has undergone radical changes in the following major lines: (*a*) The development of steam turbines; (*b*) the development of the mechanically fed boiler (which is best illustrated by the change from a grimy man with a coal scoop to the white-collar man in front of an instrument board); (*c*) the construction of the high-speed generator with a capacity of 100 000 hp; and (*d*) the transmission of the power at voltages never dreamed of at the beginning of the Twentieth Century. In spite of all these changes, power development on the rapids of the St. Lawrence still holds a position in the public conscience which the writer believes is due in a large measure to the magnitude of the undertaking.

In Tables 1 and 2, Professor Mead shows operating charges and net income. The former is based on a capacity factor of 100% and, therefore, the power would have to be used at the point of generation continuously throughout the year. Justly, the author was not concerned with the economics of power use; nevertheless, this question must be a main issue at all times. It is believed that no one is so optimistic as to believe that 1 000 000 hp, even if available, would be used along the banks of the St. Lawrence River in New York State within any time that can be predicated. The moment this power is put on the transmission line its quantity at the receiving end diminishes and its price rises.

Relative to whether the Reconvened Plan, the Conference Plan, or the New York Plan should be the one constructed, it would be interesting to learn the estimated value of this power when distributed to those parts of the Eastern United States that would be able to utilize it, and the cost of generating steam power in the same communities. In making such an estimate the class of use would be a major consideration and a conservative load factor would have to be assumed for the estimate to be fair either to the St. Lawrence power or to the steam power.

The writer understands that steam-electric power can be produced to-day in the principal cities of New York State for a fraction of a cent per kilowatt-hour and that mechanical and electrical engineers admit that further economies are possible; in fact, they state that such economies will come. If steam-electric power can be put on the low-tension bars for 3 or 4 mills per kw-hr, it may be difficult for St. Lawrence power to compete with it over any long-distance transmission and with a normal load factor.

W. S. LEE,[17] M. AM. SOC. C. E. (by letter).—This paper is extremely pertinent and interesting to the American as well as to the Canadian Engineering Fraternity. The subject is very important and should not be criticized in an unfriendly manner by either one side or the other; the entire Engineering Profession should enter into this discussion in a spirit of determining the best possible development that can be made. This development forms part of the boundary between two great friendly nations, and they will always be mutually interested in it and will have friendly interest in one another. American engineers have too many friends in Canada and *vice versa* for this matter to be discussed in any other way except for the best interests of all concerned; therefore, any discussion that would tend to enlighten the situation before the public as well as the Engineering Profession, aiming toward the development of the best possible plan in this project is, of course, very pleasing to every one.

---

[17] Vice-Pres. and Chf. Engr., Duke Power Co., Charlotte, N. C. Mr. Lee died on March 24, 1934.

If this project is carefully studied (as it should be) and properly developed it will be a great acomplishment. If it is poorly developed and mistakes are made, such mistakes will stand out always, and the work will be a poor presentation of the ability of American and Canadian engineers. The writer urges the Engineering Profession to join in a most careful study of the best laid plans, in order to decide on the most economical and enduring construction, so that it may be not only a great advantage to the two countries, but the embodiment of good engineering.

Frank E. Bonner,[18] M. Am. Soc. C. E. (by letter).—Proposals for the utilization of the St. Lawrence River present several complex questions. None is more important to the people of the United States than that relating to the merit of the single-stage plan as compared with the two-stage plan for improvement of the International Section of the river for the combined purposes of navigation and power production. Professor Mead's paper reviews this issue with admirable sagacity and candor.

The subject is of timely interest and searching examination by the Engineering Profession constitutes a high public duty. Several years ago, while dealing with various plans presented to the Federal Power Commission for the development of the St. Lawrence, the writer became keenly impressed with the probable danger of wasteful and extravagant improvement should the program be guided by professional politicians rather than by competent engineers. It is true, of course, that some other elements must be considered, but, primarily, the entire problem is one of engineering in its broadest aspects.

If the St. Lawrence project is to be launched in the near future by joint effort of the two nations, it is vital that the partnership be based on complete and sympathetic recognition of the rights and problems of each. In order to reach equitable agreement, concessions should be expected on both sides, but in so far as humanly possible these concessions must be evenly balanced or the proposed project will fail for want of public support in one country or the other.

Preliminary negotiations of the past decade (1923–33) have developed that the chief item of disagreement concerns the single-stage plan advocated by representatives of the United States *versus* the two-stage plan preferred by the Canadians. The precipitate acceptance of the Canadian contention in 1932 without adequate explanation naturally provokes a question as to whether the surrender might have been influenced by some temporary political expediency, or whether compensating advantages have been granted the United States in other respects. A satisfactory showing that the proposed treaty does not impose inequitable burdens on the United States should be a necessary prerequisite to unconditional ratification by the Senate.

Professor Mead's analysis clearly demonstrates that the single-stage development is far more favorable than the other from the standpoint of the United States. The original cost will be about $40 000 000 less than the cost of the two-stage plan while at the same time power production is increased, operation costs are reduced, and navigation is served fully as well. The modified single-stage plan proposed by the State of New York offers promise of even larger advantages, but because of uncertainty as to whether the estimates are strictly comparable, definite appraisal cannot be made with assurance. However, in view of the high standing of the engineers responsible for the New York plan, it may safely be assumed that the single-stage plan considered by the Joint Board can be further improved with consequent widening of the margin of advantage over the two-stage plan.

---

[18] Cons. Engr., Piedmont, Calif.

In the face of these established facts it is difficult to understand the Canadian preference for the two-stage plan. The reasons advanced seem rather vague and indefinite. Apparently, they relate mainly to local prejudice against marginal flowage and fear by some interests of aggravating ice conditions along the lower river. Although sentiment favoring the two-stage plan may be of sufficient weight to justify preference for the more costly development by Canada, it is not evident that the actuating factors are of such a character as to warrant the United States assuming a greatly increased burden.

If early development of the St. Lawrence is more urgently needed by the economic requirements of the United States it might be desirable to undertake more than a proportionate share of the obligations in order to expedite completion. Apparently, however, such is not the case. Available information shows no pressing need for proceeding with the project at present in so far as the United States is concerned. Doubtless, it will be a desirable improvement eventually, but under present conditions the enterprise is more likely to be a liability than an asset.

The project will provide only two things—a new transportation facility and a huge block of electric energy. These will be useful only to the extent that they may either supply requirements which cannot be met by present equipment or furnish service at a sufficiently lower cost to displace existing competitive facilities to the general public benefit. Active agitation for the St. Lawrence Waterway began during the post-war period, when existing transportation systems were heavily overloaded; but the situation has completely changed and there is now a great surplus of transportation. The same is true of electric power. Under the circumstances, the St. Lawrence project will have to meet the test of competition, and construction will be justified only when it can serve much more economically than the facilities it will displace.

The navigation project involves considerably more of course than the items included in Professor Mead's summary, which is confined to the International Rapids Section. The many ramifications of the entire navigation enterprise are too complex for more than incidental attention in connection with the present discussion. It should be observed, however, that an outlay of several hundred million dollars will be required of the United States, exclusive of extensive harbor improvements, and comprehensive analysis[19] shows that if tolls are placed on the traffic, as they ought to be, sufficient to cover the interest, maintenance, and operation charges, the venture offers little hope of reducing present charges for transportation between Lake ports and the Atlantic seaboard.

---

[19] "The St. Lawrence Navigation and Power Project," pub. by the Brookings Inst., 1929.

It is realized that Professor Mead's analysis relating to cost and net income of power production was intended merely for the purpose of comparing the relative values of the different development plans, but it is possible that the method of presentation may convey some misapprehension regarding the margin of profit possible from this feature of the project and may thus lead to the conclusion that the United States can afford to deal generously with Canada.

The available output will be subject to considerable seasonal variation. Due to the combined effect of low water and reduced head during the winter months, the production will be lowest at the time of highest demand in the electric market. Under the circumstances the continuous primary power will be represented by the winter capability of the generating plant.

Under natural conditions the winter discharge from Lake Ontario occasionally diminishes to about 175 000 sec-ft but, with the moderate regulation planned, the flow will seldom fall below 190 000 sec-ft. Discharge of 200 000 sec-ft may be maintained approximately 90% of the time and for 64% of the time 210 000 sec-ft would be available (50-yr period, 1881–1930).[20] For this same period a flow of 235 000 sec-ft would be available about 48% of the time. In some years this would extend through 10 months of the year, but during dry cycles the discharge may be considerably less for as long as 33 consecutive months. During the period, 1923–1927, the controlled flow would exceed 225 000 sec-ft in only 11 of the 60 months. In view of the irregularity of the higher flows it is clear that part of the potential power is of very poor quality and of doubtful usefulness.

Taking 76.4 ft as the average winter head (Barnhart Island Plan) and the discharge at 190 000 sec-ft, the dependable primary power will amount to 1 402 158 hp. The secondary power of good quality will be represented by that excess which may be produced from discharges up to 210 000 sec-ft. This block may be estimated as follows. Let T_1, T_2, and T_3 represent the proportional time elements, respectively, of (*a*) summer power (9 months), from discharges of 200 000 to 210 000 sec-ft; (*b*) the winter power (3 months), from discharges of 190 000 to 200 000 sec-ft; and (*c*) the winter power from discharges of 200 000 to 210 000 sec-ft. Then, from the foregoing time-flow percentages: $T_1 = 0.75 - \dfrac{(0.75 - 0.64)\,\dfrac{0.75 - 0.64}{0.90 - 0.64}}{2} = 0.72673$;

$T_2 = 0.25\,\dfrac{1.00 + 0.90}{2} = 0.2375$; and $T_3 = \dfrac{(0.90 - 0.75)\,\dfrac{0.90 - 0.75}{0.90 - 0.64}}{2}$ $= 0.04327$.

---

[20] Plate 19, of Appendix B (revised to November, 1930), Joint Board Rept. of 1926.

The good quality secondary power from discharges up to 210 000 sec-ft equals [86.1 (200 000 × 0.75 + 10 000 T_1) + 76.4 (190 000 × 0.25 + 10 000 T_2 + 10 000 T_3)] × 0.096594 − 1 402 158 = (13 540 714.73 + 3 843 508.28) 0.096594 − 1 402 158 = 277 054 hp (average). Similarly, the poor quality secondary output produced from discharges ranging from 210 000 to 235 000 sec-ft may be computed, as follows: 25 000 × 86.1 $\times \frac{(0.48 + 0.64)}{2} \times 0.096594$ = 116 429 hp (average).

The estimated average annual production is summarized in Table 7.

TABLE 7.—AVERAGE ANNUAL POWER PRODUCTION.

Class	Horse-power	Kilowatts	Kilowatt-hours
Primary	1 402 158	1 046 010	9 163 000 000
Secondary-good quality	277 054	206 682	1 810 500 000
Secondary-poor quality	116 429	86 856	760 900 000
Total	1 795 641	1 339 548	11 734 400 000
United States' share (50%)	....	....	5 867 200 000

The construction cost of the works chargeable to power, as adjusted by Professor Mead for an installation of 2 200 000 hp at Barnhart Island, amounts to $146 645 500 (Item No. 59, Table 1). This is based on the assumption that the cost of the dams, dikes, channel excavation, etc., useful and necessary for both navigation and power, will be apportioned equally, 50% to each. Power may be called upon to bear a heavier share of the burden, but for present purposes the more optimistic proration will be adopted. Interest during construction amounting to 12½% (5 years at 5% × ½), or $18 330 700, will bring the total outlay to $164 976 200 or, say, $165 000 000, by the time the plant is ready for operation. The investment of the United States in the construction, therefore, may be taken as $82 500 000.

The annual cost of the power development facilities of the United States estimated on a commercial basis for comparison with competitive sources is, as follows:

Interest ($82 500 000 @ 5%)	$4 125 000
Amortization (40 yr @ 5%; factor, 0.0083)	684 750
Depreciation ($50 000 000, 33 yr; factor, 0.0125)	625 000
Operation and maintenance (60 cents per hp installed)	660 000
General expenses, legal and management (15 cents per hp installed)	165 000
Federal license fee (90% capacity 669 000 hp @ 25 cents)	167 400
New York State charges @ 1.5%	1 237 500
Total	$7 664 650

The foregoing financial set-up assumes the development undertaken by some form of a public corporation probably financed by revenue bonds and with obligation to pay the customary Federal charges and State fees equivalent to taxes.

An important additional item of cost in a large development of this kind arises from the idle investment during the years immediately succeeding completion while the market is being built up to the full capacity of the project. Contracts for the Boulder Canyon power were negotiated on a basis of a 4-yr absorption period, but it is now evident that a considerably longer time will be required for the growth and adjustments of the tributary markets to utilize the available output. In the report of the New York State Commission eight years was assumed as the absorption period[21] for the St. Lawrence power. On account of possible savings by means of progressive partial installation it seems reasonable to assume five years and estimate the added capital charge to the project by taking one-half the annual costs for such period. This amounts to \$19 161 625 and increases the total investment at the time the plant reaches full operation to \$101 662 000. Interest and amortization on the added capital charges increase the annual costs to a total of \$8 781 800.

One-half the primary output that will be available to the United States amounts to 4 581 500 000 kw-hr. This presumes discontinuing the diversion of approximately 20 000 sec-ft which supplies the existing Massena Plant and instead serving equivalent energy amounting to about 80 000 hp of peak and 400 000 000 kw-hr per year from the main plant. The primary output available for disposal is thus decreased to 4 181 500 000 kw-hr. At the same time, by crediting proportionate operation, maintenance, and other appropriate costs amounting to \$207 800, the annual cost of the generating project is reduced to \$8 574 000.

Forecast of the probable revenue from the sale of power opens up a complicated subject which can be treated here in broad outline only. Obviously, the secondary power, if utilized at all, will have to be sold at very low rates for seasonal industrial purposes in the immediate locality of the plant. Under present conditions, such demand would be negligible, but eventually substantial fractions of these irregular energy blocks might be salvaged by new industries attracted by low-cost power. The share of the United States of the secondary output of good quality averages 905 250 000 kw-hr, and of the poor quality blocks, 380 450 000 kw-hr. Disposal of 70% of the former at 1.0 mill and 50% of the latter at 0.5 mill would probably represent about the maximum expectation. The gross return on such basis would average about \$725 000 annually.

The primary power production will have to be marketed at high load factor by wholesaling the large industries induced to migrate to the river, or by transmission to the distant metropolitan load centers. The Boston, Mass., and the Newark, N. J., Districts probably offer the most promising prospects. Transmission in either case would be about 325 miles. New York

---

[21] Rept. of the St. Lawrence Power Development Comm. (State of New York). January 15, 1931, p. 59.

City could be reached with about the same length of line, but in competition with local steam-generated energy the value of the imported power would be considerably less due to the costly underground cable facilities required for access to the city sub-stations. Present dimensions and characteristics of the load of the Boston and Newark Districts might cause some question about the capacity of these markets to utilize the St. Lawrence primary energy as base load at high load factor about ten years hence. It is recognized that the reputations of most authoritative prognosticators relative to the probable growth of electric load have suffered heavily during the past few years, but nevertheless the assumption is made that in due course the Boston and Newark Districts, with taps serving some of the secondary load centers en route, could each absorb about one-half the St. Lawrence primary energy at 85% utilization factor and with minimum load factor of 80 per cent. This combination imposes peaks about 6% in excess of the average primary output, but it is considered that the resulting fluctuation of discharge would be within the permissible limits during the non-navigation season and, under the more severe requirements for constant flow during the summer, the variation of output could be accomplished without seriously affecting the characteristics or value of the secondary power. Without going into details, it may be assumed that the cost of generation by local steam plants at 80% load factor will be approximately 4½ mills per kw-hr as developed by the report of the New York State Commission.[22] This rate, therefore, represents the maximum market value of the imported power delivered at load centers.

For transmission of the primary output of 4 181 500 000 kw-hr (477 340 average kw) with a utilization factor of 85%, load factor of 80%, and transmission efficiency of 87% (line, 9.5%; step-down, 1%; and synchronous condensers, 2.5%), the necessary peak capacity at delivery terminals will total 441 241 kw. This will require four 220-kv circuits (two each to Boston and Newark), with delivery capacity of 110 000 kw each. At $25 000 per circuit-mile, including right of way, the lines will cost $32 500 000. Terminal sub-stations with synchronous condensers at $16 per kw and sectionalizing and switching stations will cost $7 500 000 more, thus making the total investment in transmission facilities $40 000 000. The annual cost on a 9% basis (interest, 5%; amortization, 1%; and depreciation, operation, and maintenance, 3%) will amount to $3 600 000, not including taxes which, if estimated at 1.5% of the investment, would increase the total annual cost to $4 200 000.

Steam stand-by, equivalent to 50% of the delivery capacity, or 220 000 kw, will be necessary to assure reliability of service comparable with local production. If estimated at $80 per kw the investment will be $17 600 000 and the annual cost, exclusive of taxes, $2 436 500 (interest, 5%; amortization, 1%; depreciation, 3%; operation and maintenance, $2.25 per kw; general expense, 50 cents per kw; and, stand-by fuel, 0.25 ton @ $4.50 per kw). Taxes at 1.5% will increase the annual stand-by cost to $2 700 500.

---

[22] Rept. of St. Lawrence Power Development Comm. (State of New York), January 15, 1931, pp. 80-82.

The cost of the delivered energy amounting to 3 092 219 250 kw-hr (4 181 500 000 × 85% utilization factor × 87% transmission efficiency) is summarized by the following tabulation:

	Including taxes	Without taxes on transmission facilities
Production cost ($8 574 000 less $725 000 return from secondary)	$7 849 000	$7 849 000
Transmission and sub-station cost..	4 200 000	3 600 000
Stand-by cost.....................	2 700 500	2 436 500
Total delivery cost.........	$14 749 500	$13 885 500
Cost per kilowatt-hour delivered, in mills	4.77	4.49

It is apparent from these general figures that in competition with equivalent local steam-plant generation, at approximately 4½ mills, the introduction of St. Lawrence power into the large metropolitan districts fails to provide a very favorable possibility even though the transmission lines, sub-stations, and stand-by plants should be relieved of taxes. In any event, it will be clear that there is no hope of obtaining any large profits or extensive public benefits from the enterprise.

The alternative of endeavoring to market the output in the immediate vicinity of the generating plant by building up large electro-chemical and electro-metallurgical works, although largely speculative, seems to offer a somewhat beter chance of accomplishment. The development of such a project, however, will require time, a vast expenditure of money, and the closing of industries in other sections. The feasibility will depend largely on the cheapness of the power rates. In order to attract large industrial load, it is doubtful whether the average rate can be higher than 2¼ mills per kw-hr ($14.70 per average hp). Disposal of the primary output at such rate, with 85% utilization factor, would produce revenue of $7 997 119. This would cover the generating cost by only a slender margin after taking credit for the possible return of $725 000 from secondary power.

From the foregoing, it will be evident that the situation offers no assurance of power revenue in excess of that needed to meet the bare carrying charges. Substantial increase in the capital outlay as involved with the adoption of the two-stage plan can easily thrust the project from the border line of financial feasibility into the outer zone of economic impractibility. Hence, the importance of this question of development plan to the United States. In Canada, the situation is different on account of large load centers being close to the power site. The extra cost of about $40 000 000 for the two-stage development is chargeable mostly to the power facilities and would raise production costs approximately 25 per cent. With such an increase, the marketability of the output would be extremely doubtful. Under the circumstances, it would appear that if the two-stage plan is adopted, Canada, in all fairness, should be willing to assume the additional cost.

Rufus W. Putnam,[23] M. Am. Soc. C. E. (by letter).—Reference is made in this paper to a few of the navigation aspects of the power developments under consideration; furthermore, in the tables showing comparative costs, items appear for costs of navigation features embodied in the three plans. This discussion proposes to set forth the writer's views on the relative merits of these plans for water-power development as affecting the navigation interests involved.

Factors affecting the utilization of the waterway by vessels include channel dimensions and curvature, cross-currents, ice conditions, and lockages. Table 8 sets forth some of these factors.

TABLE 8.—Factors Affecting Utilization of the Waterway by Vessels

	Plan *A* (Crysler)	Plan *B* (Barnhart)	Plan *C* (New York)
Length of canal, in miles	9	8	0
Number of locks	3	2	2*

* In flight.

Channels.—As to channel widths, depths, and curvature, the same specifications have been followed in each plan and, therefore, no differences exist. Plan *A* and Plan *B* are about equal as to length of restricted channel, but they have a disadvantage compared with Plan *C*. A ship will require about an hour more to navigate the channels available with either the Crysler two-stage project, or the Barnhart single-stage project, than it will with the New York single-stage project. Assuming the ultimate capacity provided for in Plans *A* and *B*, as 24 000 000 tons per year, with average cargoes of 5 000 tons per vessel, the vessel-hours lost would be 4 800. At a cost of $30 per hr the annual loss becomes $144 000, which, capitalized at 5%, represents a capital advantage in favor of Plan *C* of $7 200 000. This represents a minimum loss based on capacity tonnage for Plans *A* or *B*, for if smaller average cargoes are assumed the number of vessel-hours lost increases proportionately, but the hourly vessel cost does not decrease in like proportion.

Lockages.—Plan *A* will require three lockages per trip per vessel, while Plans *B* and *C* will require only two. It is customary to allow an average of 1 hr per lockage, even though many passages are made in much less time. The records at the locks at Sault Ste. Marie bear out this conclusion.

The New York Plan (*C*) has two locks in flight, while the locks provided by Plans *A* and *B* are separated sufficiently to have no added influence on time of transit. The Joint Board of Engineers estimates the annual capacity of a single dock development at 24 000 000 tons, but places that of an improvement involving a set of flight locks at 16 000 000 tons per year. The average time of 1 hr per transit through a single lock, therefore, becomes 1½ hr for each lock in a set of flight locks. For comparative purposes, therefore, lockage times are as follows: Plan *A*, 3 hr; Plan *B*, 2 hr; and, Plan *C*, 3 hr.

Using the same assumptions as to size and costs of vessels and annual capacity of the waterway as before, Plan *B* has a capitalized advantage of $7 200 000 as regards Plans *A* and *C*. Combining the capitalized value of lock delays with those involved in channel transit, it appears that Plan *A* entails a loss of $7 200 000 as compared with Plans *B* and *C*, while Plans *B* and *C* are of equal value in these respects.

---

[23] Director, Chicago Regional Port Comm., Chicago, Ill.

Capacity.—A further consideration involved is that of the ultimate capacity of the three projects which is 24 000 000 tons per year for Plans *A* and *B*, and 16 000 000 tons per year for projects involving flight locks, according to the report by the Joint Board of Engineers. If Plan *C* is followed, it will be necessary to install a parallel flight of locks in order to obtain the same capacity as that afforded by Plans *A* and *B*. This will involve an expense of between $7 000 000 and $8 000 000. As far as ice conditions and cross-currents are concerned, it does not appear from any of the plans available that any difference exists between Plans *A*, *B*, and *C*.

Summary.—The writer considers that, from the point of view of the interests of navigation alone, Plan *B* is superior to both Plans *A* and *C* which are of about equal ultimate value.

JAMES W. RICKEY,[24] M. AM. SOC. C. E. (by letter).—Engineers are under a moral obligation to advise the public and to assist in forming an enlightened public opinion on such matters as the development of water power along the International Section of the St. Lawrence River, which they are particularly qualified to discuss.

The author's analysis points unmistakably to the conclusion that the development of this section of the river in a single-stage plan is superior in every engineering aspect to its development in a two-stage plan. This raises the question as to why the two-stage plan was incorporated in the Treaty, now (November, 1933) awaiting ratification. A study of the various reports apparently provides the answer, namely, that an agreement between the Canadian and the American Sections of the Joint Board of Engineers must be reached before any development can be undertaken, and that the Canadian Section would agree to the two-stage plan and no other, notwithstanding the less satisfactory navigation facilities, the reduced power output, and the greatly increased cost of the two-stage plan.

It is vitally necessary that the people of both countries and their responsible representatives be fully acquainted with all the facts before adopting a definite plan for the public development of this great national resource. The facts clearly made evident by the paper are that, when using the same unit prices in both estimates of cost, the single-stage Plan *B* will be worth about $94 000 000 more and will cost about $38 000 000 less than the two-stage Plan *A*; and, that when similar comparison is made between Plan *C* and Plan *A* the corresponding sums will be very much greater. (See Table 3, Item No. 9, and Table 1, Item No. 18.)

The author is to be congratulated highly upon his clear, unbiased analysis of the facts set forth in the various published Government reports. The paper and the discussion incident thereto should be of great assistance in securing the early execution of this project and in accordance with the best interests of all concerned.

---

[24] Chf. Hydr. Engr., Aluminum Co. of America, Pittsburgh, Pa.

Walter M. Smith,[25] M. Am. Soc. C. E. (by letter).—The canals at the International Section of the St. Lawrence River, are somewhat analogous to the Sault Ste Marie Canal, although the class of boats passing through it will be quite different. As about 60% of the freight carried through the "Soo" Canal is iron ore, a type of vessel not sufficiently strong for ocean going, has been developed for this traffic. Therefore, conclusions drawn from traffic through the "Soo" Canal can only be applied in a very general manner to the International Section of the St. Lawrence River.

Table 9 shows statistics of traffic through the "Soo" Canal from which some general conclusions may be drawn. It will be seen from Table 9(*a*), Item No. 11, for example, that in 1929 in moderately heavy traffic the speed in the canal was about 2 miles per hr, whereas in Table 9(*b*), Item No. 23, it is shown that in maximum traffic through each lock it was from 0.8 mile to 1.4 miles per hr. Thus, it will be seen that as traffic increases the speed is cut down and all vessels lose more time in transit. The canal at the "Soo" is 300 ft wide against 200 ft proposed for the canal in Professor Mead's study, and yet the engineers in charge claim that it is not sufficiently wide for two-way traffic. It will be seen that the average size of vessel in the

TABLE 9.—Statistics for Sault Ste Marie Canal

Item No.	Description	Names of Locks			Totals
		Poe (3)	Davis (4)	Sabin (5)	
(1)	(2)				(6)
	(*a*) 1929 Statistics				
1	Number of lockages....................	3 852	3 910	3 790	11 553
2	Number of vessels....................	4 856	5 632	5 721	16 209
3	Total tonnage....................	14 688 835	36 037 355	50 590 508	101 316 698
4	Average tonnage per lockage............	3 800	9 200	13 300	8 800
5	Average tonnage per vessel............	3 000	6 400	8 800	6 250
6	Average number of lockages per day.....	10.5	10.7	10.0	31.7
7	Average time per lockage, in seconds.....	1 370	1 996	2 244	....
8	Lift, in feet (approximately)............	21	21	21	....
9	Average time in canal, in minutes.......	....	....	....	70
10	Length of canal, including lockage, in miles	....	....	....	1.75
11	Speed in canal, in miles per hour........	1.94	1.96	2.26	....
	(*b*) Statistics for Maximum Year at Each Lock				
12	Year....................	1906	1918	1923	....
13	Number of lockages....................	5 845	4 676	3 691	....
14	Number of vessels....................	9 968	7 259	6 675	....
15	Vessel interval, in minutes..............	52	72	80	....
16	Total tonnage....................	43 083 490	60 946 348	55 591 159	....
17	Average tonnage per lockage............	7 400	13 000	15 000	....
18	Average tonnage per vessel............	4 300	8 400	8 400	....
19	Average lockage per day................	16	13	10	....
20	Average time per lockage, in seconds.....	2 071	2 268	2 754	....
21	Average time through the canal, including lockage, in minutes..............	153	91	111	....
22	Speed in canal, in miles per hour........	0.8	1.40	1.10	....
23	Average Tonnage per Year, for:				
24	20 years....................	25 854 000	....	....	....
25	18 years....................	27 480 000	34 600 000	....	....
26	16 years....................	31 420 000	38 437 000	45 070 000	....
27	5 years....................	35 592 000	56 635 000	49 467 000	....
28	1 year....................	43 083 000	60 946 000	55 591 000	....

---

[25] Chf. Engr., State Dept. of Public Works and Bldgs., Div. of Waterways, Chicago, Ill.

"Soo" Canal in 1929 was 6 250 dead weight tons (Table 9(*a*), Item No. 5, Column (6)); therefore, this size of vessel will be used for a comparison in this study.

The Poe Lock at the "Soo" Canal is the same length as the proposed locks on the canal being discussed, but is 100 ft wide instead of 80 ft as is proposed for the locks in question. It will be seen from Table 9(*b*) (Item No. 16, Column (3)) that 43 083 490 tons of freight passed through the Poe Lock in 1906, but it will be noticed that there was an average of 1.7 vessels per lockage, and making a correction for an average of only a single vessel of average size per lockage would give a traffic of about 26 000 000 tons per year, which is double the estimate of the Joint Board of Engineers for the capacity through the locks in question, as long as they are single locks. It will also be seen that if there had been open-water approaches to the Poe Lock the capacity would be much higher because the average time per lockage was only 34½ min, which would give 42 lockages per day mathematically possible, whereas the number was only 16, or about 40% of capacity. It will also be seen that it took 2 hr to navigate the 1¾ miles of canal, or three and one-half times as long as to go through the lock.

The data for the Davis and Sabin Locks are not comparable with the proposed locks as they are 1 300 ft long, although they are the same width (80 ft) as the proposed locks. On account of the great increase in traffic in the "Soo" Canal these locks were built to accommodate two of the largest lake steamers at the same time. It is interesting, however, as showing what capacity may be reached in locking. No other canal in existence has ever passed more than 60% of the traffic of one of these locks.

It will be noted that the great traffic through the Poe Lock was in 1906 long before either of the other two locks were built, the Davis Lock having been put in commission in 1914 and the Sabin Lock in 1919. After the completion of these two locks the passenger traffic was turned over to the Poe Lock and the two large locks turned over to freight traffic almost exclusively. Therefore, the 14 684 835, shown for the Poe Lock in 1929, is registered tonnage; in all other cases, it is dead weight tonnage.

It will be noted that as the tonnage fell from 43 000 000 to 14 700 000 the locking time decreased from 34.5 to 22.8 min, and the speed in the canal increased from 0.8 mile to 1.94 miles per hr. This shows vividly the bottleneck effect of any narrow canal in slowing down traffic as the volume of traffic increases.

Assume that passing places will be made in the proposed canal sufficiently close together for a vessel to pass from one to the other in the same time as it takes to go through the lock. In this case the mathematically possible maximum traffic would be 1 vessel every 25 min through the lock; but every vessel will be waiting one-half the time throughout its passage through the canal.

Assume next the existence of an ideal condition of a series of standard-sized vessels, 6 250 tons, moving with clock-like precision, one going through the lock every 25 min. This would give 58 lockages per day, or a capacity of 58 × 6 250 = 362 500 tons per day, or 132 000 000 tons per year. Assume also that it is possible to attain 33% of this tonnage as was done at the Poe Lock in 1906. It will then be seen that it is not the lock that is the bottle-neck, but the canal. At the Poe Lock with a canal 300 ft wide, only an average speed of 0.8 mile per hr was attained in the canal; therefore, assume a speed of 1 mile per hr in the canal. The upper canal at Crysler Island is 2.5 miles long and the lower one at Barnhart Island, 7.5 miles long; a total length of 10 miles. Therefore, it will take a vessel 11 hr, including the time through lockages, to get through the canal.

In the Massena Point scheme there is a proposed channel, 450 ft wide, leading up to the lock down stream for 2 miles, and open-lake navigation on the up-stream side so that vessels can come and go at open-river navigation speed except for the 2 miles, where the speed will have to be reduced somewhat, although vessels can pass at moderate speed. To pass through the flight of two locks having a lift equal to the three locks in the other scheme will require about 1 hr. This would give 24 lockages per day, or 24 × 6 250 = 150 000 tons per day, or 54 750 000 tons per year. As there is no bottle-neck canal here, these locks can certainly pass more than the canals and locks in the Crysler Island Plan. The vessels approaching these locks could certainly run 10 miles per hr in the open river and at least 8 miles per hr in the 450-ft channel; therefore, it will require only 2 hr to go 10 miles, including the lockage. This will be a saving in time of 9 hr.

It costs about $64 per hr to operate a vessel of 6 250 tons; thus, the extra expense for each vessel in the Crysler Island Plan will be $64 × 9 = $576 and 19 vessels will be $576 × 19 = $10 944 per day, or $10 944 × 365 = $3 994 560 per year. This amount capitalized at 5% gives $79 891 200. Therefore, under the foregoing assumptions, other things being equal, an additional expenditure of about $80 000 000 would be justified on the Massena Point Plan over and above what the Crysler Island Plan would cost for navigation alone.

In studying the advisability of building a sea-level or lock canal in 1906, the Board of Consulting Engineers for the Panama Canal estimated that the time it would require a vessel of the size used herein would be from 8.6 hr for 10 vessels per day, to 11.1 hr for 30 vessels per day, to pass through the sea-level canal, with passing places 2.5 miles apart. The same size of vessel could pass through the 85-ft level lock canal, including the six locks, in 9.5 hr for 10 vessels, and 10.0 hr for 30 vessels, per day. This was a speed of from 7.7 to 7.1 miles per hr in the lock canal, allowing 3 hr for lockage, and an average speed of from 5.7 to 4.5 miles for 10 vessels per day per hour, with an average of 30 vessels per day, for the sea-level canal with passing places 2.5 miles apart. The dimensions assumed for the two canals are given in Table 10.

It will be noticed that even with only 12.4 of the 49.7 miles of the lock canal, with a bottom width of less than 500 ft, the speed of vessels, with 10 vessels per day, including lockages, was 7.7 miles per hr, and for 30 vessels, 7.1 miles per hr, in the canal.

The high speed of vessels assumed for the lock canal at Panama is explained by the fact that 75% of the canal is 500 ft, or more, and 50%, 1 000 ft, or more, wide. Therefore, 50% of the distance is practically open-lake navigation. In channels 500 ft wide, vessels of the size herein assumed can pass each other at full speed. With channels 300 ft wide they can pass at reduced speed without stopping; but at 200 ft, or less, in width one vessel will have to stop and the other greatly reduce its speed.

TABLE 10.—Dimensions of Sea-Level and Lock Canals Proposed for Panama Canal

Item No. (1)	Bottom width, in feet (2)	Length of sea-level canal, in miles (3)	Length of 85-ft level canal, in miles (4)
1	150	21.1	....
2	200	18.8	4.7
3	300	3.9	7.7
4	340	0.3	....
5	350	0.5	....
6	500	....	8.2
7	800	....	3.8
8	1 000	....	19.5
9	Unlimited	4.7	5.8
10	Total....	49.3	49.7

Even if it is assumed that the vessels passing through the canals in the Crysler Island Plan can average 2 miles per hr, including time lost at meeting places, it will take each vessel about 6.5 hr to pass through the 10 miles of canal and locks, or 4.5 hr more than in the Massena Point Plan. This is as favorable an assumption as can be made and even under these conditions the time lost will be valued at about $2 000 000, and capitalized at 5%, about $40 000 000.

When the traffic is increased sufficiently to require an enlargement of the canals and locks the capacity of the Massena Point Project can be doubled approximately by building an additional flight of locks alongside the other at a cost of about 70% of the cost of the original flight of locks and appurtenances. As the cost of the original flight of locks and other work solely for navigation is estimated at $21 138 000, this increase will be $21 138 000 × 70% = $14 796 600. To increase the capacity of the Crysler Island Project in the same proportion will require an increase of probably nearer 65%, and as the estimate of cost of work purely for navigation is $34 138 000, the increase in cost would be $34 138 000 × 65% = $22 189 700, or an increase of $7 393 100.

It is evident from the foregoing analysis that, other things being equal, the Massena Point Plan is far preferable to the Crysler Island Project from a standpoint of economy. In the Barnhart Island Project there is a moderate gain over the Crysler Island Project, but as the length is about the same as that of the lower canal in the Crysler Island Project, the gain would be only about 25% as great as for the Massena Point Project.

J. W. Beardsley,[26] M. Am. Soc. C. E. (by letter).—The author has presented a critical study of the three St. Lawrence River projects submitted to him for comparison, and has arranged his finding in exceptionally convenient form for ready reference. He states that his knowledge of the rapids of the river is limited. He implies that there are phases of the problem other than the engineering that should be considered. He also implies a compliment to the Canadian Government on account of its policy of re-appointing the members of its Commission, and, possibly, a criticism of the United States in not following a similar policy. If these implications had been clearly stated, the writer would have agreed heartily.

The report of the Deep Waterways Commission (1900)[27] relates specifically to navigation problems and estimates of cost for channels, 21 and 30 ft deep. In that report, the writer discusses the geology of this section.[28] Broadly, the rapid St. Lawrence River flows along a ridge and the sluggish Grass River in a valley, 50 ft lower and 3 miles distant.

In one case while inspecting wash drill borings, the writer noted that the overlying boulder clay was being penetrated with difficulty. The drill broke through it, and the following 70 ft in blue clay was drilled in about 30 min, or as rapidly as connections could be made. Similar formation was found at Massena Point and along the southeast shore of Cornwall Island. It was this formation that caused the failure of one of the piers during the construction of the Ontario and Western Railway Bridge. The foundations were on a thin layer of boulder clay through which the increased velocity of the stream cut down to the clay formation. Boulder clay, found previously when making sub-surface explorations, had been discovered resting on rock in place. To find it resting on a considerable body of blue clay was a new experience to the writer—a condition that may be explained if secondary clay deposits from a prior glacier had been bridged by the ice-cap of a subsequent glacier.

In another case, the formation of frazil in the forebay was creating an anchor-ice barrier in the tail-race, which reduced the head by one-half. During past years the northern water-power companies have had considerable trouble in maintaining a free tail-race under such conditions—an important item when continuous electrical service is financially necessary.

These facts have been stated to indicate that at least several years of critical observation of the St. Lawrence is needed to estimate properly the effects of ice-jams, anchor ice, frazil, water-stages, etc., and to counteract them. In general, the northeastern part of the United States is well provided with electrical energy. New electrical devices for household use are appearing almost daily, and the various systems are constantly soliciting consumers for surplus power.

A circle described about Massena Point as a center, with a radius of about 300 miles, will include Boston, Mass., New York, N. Y., and Buffalo, N. Y. The principal systems with their subsidiaries operating in New York State are the Niagara Hudson, the Associated Gas and Electric, and the New York

---

[26] Cons. Engr., Syracuse, N. Y.

[27] H. R. Doc. No. 149, 56th Congress, 2d Session.

[28] *Loc cit.*, Appendix 11, p. 398.

Edison. Of these three systems, the Niagara Hudson serves the greater area (and probably the greater population, excluding New York City) of the State and more definitely the northern half, including the area adjacent to the St. Lawrence River. It is interconnected with systems serving contiguous territory. A few facts regarding that system will illustrate the points the writer wishes to make.

More than 60% of the total capacity of the Niagara Hudson Company, (1 695 000 kw), is in water-power plants. The load factor on the day of maximum demand in 1932 was 78 per cent. The number of farms estimated within the franchise area is 74 374 (the total in the State was 159 806 in 1930); the number of farm consumers is 27 184, or 38% of the farms in the franchise area. The annual report to stockholders (1932) shows an average farm rate, in cents per kilowatt-hour, as follows: 1929 = 7.384; 1930 = 6.764; 1931 = 6.229; and 1932 = 6.059. (The average rate for residence customers was similarly reduced from 5.028 cents per kw-hr in 1929 to 4.658 cents in 1932). Due to unfavorable location, or greater distance from existing transmission and distribution lines, the rate for many of the remaining farms wishing electrical service would be considerably higher even if no charge were made for the energy supplied.

In 1932, the number of shareholders of common stock was 98 300, and dividends were paid to the amount of $10 471 878, but reductions have been made in the 1933 dividends. It seems reasonable to assume that many consumers of all classes are holders of common stock, and that any sharp competition or undue volume of surplus energy will cause a considerable reduction in dividends, offsetting any practical reduction in rates. It is generally conceded that even if power was furnished free, the reduction in rates in many cases would be nominal because of the total costs of distribution.

The proposed development of the St. Lawrence River in the near future would throw into a field, already amply supplied, nearly 2 000 000 hp of energy. Can practical use be made of it without the serious depreciation of existing plants? A complete plan should show how this block of power will be marketed at a benefit to the entire State and community.

The sum of the cost of Plans *A*, *B*, and *C* given in Table 1, Item No. 35 (or Item No. 85), divided by the sum of primary and secondary horse-power, Table 1, Item No. 9 (or Item No. 59), gives an average development of about $80. In view of the continuing improvements in steam power with its advantages of location favorable for transportation, markets, labor, and fuel, this average cost is not financially attractive.

On the St. Lawrence River the development of power and of navigation are closely related and interdependent. When navigation is possible for vessels of about 25-ft draft, the benefits to Canada and to several of the Northern Central States will be large. The effect on the Barge Canal and the industries of New York State and New England will doubtless be harmful. In any case they will require broad re-adjustment.

There is no doubt about the ultimate development of the St. Lawrence. However, present conditions throughout the nation, with large electrical projects under way on the Colorado River, the Tennessee River, and in the Columbia River Basin, suggest at least that the St. Lawrence development can be postponed advantageously to both countries. Reduced utility rates is a popular slogan—a vote getter. Electrical appliances are no longer luxuries in American homes. If the excessive rates and unfair practices charged[29] against some companies are true, they can be regulated by equitable laws far more economically for the average citizen than by throwing a huge block of power, without definite plans for its utilization, into an area now provided with surplus power. The writer has made no study of conditions in Canada. If they are similar to those in the United States, the safest plan would be to delay construction until the profitable use of the additional power arises.

It is an enticing vision to plan great power developments, interconnected throughout the nation, technically and financially operated at reasonable profits, furnishing energy to consumers at fair rates, reducing drudgery, and making more profitable the increasing hours of leisure. Such a plan seems to be the goal of at least a few of existing operating and holding companies. Some have exploited the public until faith is lost in all. Will plants operated by the Government reach the desired goal? Does the history of Muscle Shoals prove it? The possibilities of the St. Lawrence project will remain "available assets without depreciation" until utilized.

Conclusion.—The writer's conclusion is a repetition of the last sentence of Professor Mead's "Synopsis," with a modification of his final words, namely, it will be a serious mistake and a misfortune to both the United States and Canada, if the treaty now pending is ratified on the basis of the construction in the near future of works controlling the major hydro-electric power of the St. Lawrence River.

L. F. HARZA,[30] M. AM. SOC. C. E. (by letter).—The author has done a great service to the taxpayers and power users of both Canada and the United States by his analysis of the proposed official plans for developing water power along the International Section of the St. Lawrence River. Since he has had no previous connection with any of the St. Lawrence River investigations, his analysis is unbiased by preconceived ideas. On the other hand, the analysis may have fallen into error if the unit prices or other basic assumptions of Plan *C* are not comparable with Plans *A* and *B*, a question regarding which a difference of opinion exists.

The excess cost and its equivalent, of the two-project plan over either of the single-project plans, is so great, even after allowing for possible error, that exceedingly momentous reasons of absolute vital concern to the success of the project would need to be advanced to justify the extra cost and equivalent capitalized excess operating expense.

It is important to realize that the best navigation facilities, in combination with the best power development, are of paramount importance to Canada and the United States. Although power development is secondary in general to navigation, it is the prime requisite to Ontario and New York. The "best plan" is that which provides, with maximum economy, the most satisfactory combined navigation and power facilities, giving due consideration to: Physical conditions, capital charges, cost of operation, maintenance, renewals and repairs of structures, navigation, amount of primary power developable, reliability and cost of power, etc.

---

[29] *Atlantic Monthly*, July, 1933.

[30] Cons. Engr. and Pres., Harza Eng. Co., Chicago, Ill.

Navigation.—It would seem that navigation merits far greater consideration than it has received in the published reports. Under Plan *A* the ships must pass through about 10 miles of canals and 3 locks as against 8 miles of canals and 2 locks in Plan *B* and only 2 locks—there being no canals—in Plan *C*. For fair comparison, it would be necessary to add to the cost of Plan *A* the capitalized value of the lost time per year of all vessels expected to navigate this waterway. Competent authority has estimated that 3 hr more, or 6 hr per round trip, is required to navigate the 10 miles of canal and 3 single locks of Plan *A*, as compared with the double lock without canal of Plan *C*. If anticipated traffic in this waterway is to be realized, the capitalized value of this time might well be a deciding factor in favor of Plan *C*.

Arguments Against the Single-Stage Development.—In the last paragraph of his paper, the author states that he can see no valid reasons to justify the two-stage plan which can outweigh the millions of dollars of value in favor of other plans. It would seem proper, therefore, to analyze the arguments advanced against a single-stage development.

The following objection is stated[31] in the Conference Report:

> "If, as an alternative, the International Rapids Section be improved by a single-stage project, then control of flow must be secured by operation of gates and turbines at a power plant which will develop about 2 000 000 horsepower. This is not regarded favorably by the down river interests, since this single plant would serve a large and populous territory both in Canada and the United States, and operation restrictions would be difficult to impose in view of the fact that it would be a dominant interest in the territory served. Again, the additional flooding of land [15 360 acres, of which 9 600 acres are in Canada] and the extent of the embankments, along with some adverse effects at the end of the navigation season, make the single-stage project undesirable."

The "control of flow," as noted in the preceding paragraph, was referred[32] to by a member of the Canadian Section, when testifying before the Canadian Special Committee on the St. Lawrence River development:

> "We insist that the control of the flow from Lake Ontario be not connected with a power plant having a capacity of several million horsepower. The ideal plan would be the construction of a control dam for the flow only. But as this plan would involve the loss of a certain quantity of power which may become very valuable, and which has an important value to-day, we agreed to connect this dam with a hydroelectric plant of much less proportions, and which can be operated as a basic plant with a uniform load 24 hours a day. * * * For this reason we favor concentration at two points—'the two-stage development.' The control dam for Lake Ontario would be at Crysler Island, a few miles below the town of Morrisburg."

---

[31] See Conference Report, 1929, p. 4, paragraph 9.

[32] See First Annual Rept. of the Power Authority of the State of New York, p. 23.

The safety of the City of Montreal is referred[33] to in the Treaty as follows:

> "The Canadian project of a 2-stage development in the International Section has been adopted instead of the original American project of a single-stage development. The cost is slightly more, but the Canadian officials have felt that the 2-stage development is desirable for many reasons, amongst others for the complete assurance of the safety of the City of Montreal."

In analyzing the foregoing arguments against the single-stage development, it is enlightening to consider carefully the 2 000 000 hp. partly completed (fall of 1932), all-Canadian, single-stage development on the St. Lawrence River near Beauharnois, Que., and much closer to (in fact only about 28 miles above) Montreal. In order to make this development efficient, economical, and in accordance with the latest and best engineering practice, practically every feature of general arrangement was incorporated in it that is so strongly objected to in a single-stage international development, with the result that it stands for the "best plan" from the standpoint of engineering design, of capital costs, of sustaining charges, and of cheapness of product to the Canadian power consumer.

The 83-ft fall between Lake St. Francis and Lake St. Louis is developed at Beauharnois[34] by a single-stage project, where the "control of flow must be secured by the operation of gates and turbines at a power plant that will develop about 2 000 000 hp." and hence "a dominant interest in the territory served." The power house is at the end of a canal, 15 miles long, embanked by dikes on both sides throughout its entire length. The entire normal flow of the river will be used to carry the ultimate load and, when part load is carried, the surplus water will be passed through the sluicegates yet to be built across the main river channel at a distance of more than 15 miles from the power house.

Since the general physical conditions obtaining at Beauharnois are substantially the same as those obtaining in the International Section of the river (except in respect to loss of land by submergence), it would seem that the Beauharnois development stands in mute refutation of the foregoing arguments against a single-stage development under Plan *B* or Plan *C*.

Adoption of the Two-Stage Plan.—The Reconvened Report forms a part of the proposed Treaty between the United States and Canada, now (November, 1933) awaiting ratification. Upon this Treaty depends the joint expenditure of $543 000 000,[35] of which sum, $274 742 000 is allocated to the two-stage development, termed Plan *A* in the paper.

For many years prior to the issuance of the Joint Board's Reconvened Report, dated April 9, 1932, it was known that a wide difference of opinion existed between the Canadian and American Sections as to the relative merits of the two-stage and single-stage plans of development. It was also known that most members of the several American Sections of the Joint Board were strongly opposed to the two-stage plan and that many arguments advanced in support of such plan were not considered by those Sections to be of sufficient importance to justify the excess cost from a standpoint of sound engineering and economic reasoning rather than sentiment.

---

[33] "Great Lakes-St. Lawrence Deep Waterway Treaty," *Publication No. 347*, Dept. of State, 1932, p. 11.

[34] "Beauharnois Development of the St. Lawrence River," by W. S. Lee, M. Am. Soc. C. E., *Electrical Engineering*, June, 1933, p. 377.

[35] "Great Lakes-St. Lawrence Deep Waterway Treaty," *Publication No. 347*, Dept. of State, 1932, p. 10.

It seems evident that the two-stage plan was adopted for the simple and single reason that a one-stage plan could not be agreed upon under the Board's instructions to report on "a mutually satisfactory plan for the improvement of the International Rapids Section.[36] The author's analysis would indicate that it represents an unfortunate international compromise, apparently not the best and most economical engineering solution of the problem.

Suggested Procedure.—The great difference of opinion revealed by the author as to the merit of the two-stage *versus* the single-stage plans, raises considerable doubt as to the wisdom of the ratification of the Treaty as it now reads. It would seem that a speedy and satisfactory determination of the "best plan" will be possible only after a final, critical analysis is made of all available data pertinent to Plans *A, B,* and *C,* by an International Board, comprising all new members from both countries without preconceived or previously advocated or supported ideas to defend. Before such a board the members of previous boards should appear as witnesses, not as principals.

Unquestionably, either country has the right to prefer and to insist upon the adoption of a plan which is more expensive than the "best plan;" but in doing so, the right of the taxpayer and the ultimate consumer to be correctly informed as to the extra financial burdens he must carry, should be considered.

If a fair international compromise is to be reached, it would seem that the country which prefers the "best plan" should not be penalized by being forced to carry the extra costs imposed by a less economical plan. Therefore, if for any reason of sentiment or otherwise either country should prefer a more expensive plan, the extra capital cost and the extra operating expense should be determined and stated. In this way, both countries will understand in advance what extra cost is incurred because of the adoption of the more expensive plan. Such extra cost could be met by proper allocation of cost between the two countries or proper division of the power output.

The International Boundary should be adjusted, if necessary, so that the Canadian power house will be on the Canadian shore, regardless of the adopted plan. Such adjustment of boundary could also well equalize the loss of agricultural land by the two countries. Only the southern part of Canada is climatically adapted to intensive diversified agriculture and it is quite natural for Canadians to resent any disproportionate sacrifice of land in this region.

Under such procedure, the public mind will be assured of the superiority and fairness of the adopted plan, the opposition to this development will largely disappear, and the citizens of both countries will give to this great project the whole-hearted support it merits.

---

[36] See Rept. of Joint Board of Engineers (Reconvened), p. 3, paragraph 6.

ADOLPH J. ACKERMAN,[37] ASSOC. M. AM. SOC. C. E. (by letter).—The early history of the St. Lawrence Waterway project seems to be developing with surprising similarity to the early events in the planning of the Panama Canal. The general features of both projects quite obviously run along parallel lines: They are the two most important artificial navigation routes in the Western hemisphere; the estimated cost to complete the Canal when the United States took possession was about the same as the present estimated cost of the Waterway; engineering and construction problems are of similar magnitude and beyond ordinary conception; matters of policy in each project are of international importance; both are of vital concern to the same industry, the railroads, and where the Waterway at present has its chief controversy in the single-stage *versus* the two-stage plan, the Canal was debated in highest circles on the feature of the sea-level type *versus* the lock type. It is a matter of record that a majority of the Board created in 1905 to study the problem reported in favor of the sea-level canal. Subsequently, however, the Senate, giving heed to the Minority Report, passed a bill providing for the lock canal.

The present-day visitor to Panama, who heretofore has more or less taken the Canal "for granted," is at once impressed by the profound logic of its general plan. Such approval, in the end, is the supreme mark of engineering achievement and of unselfish administration of a public trust. It is, obviously, the basis on which Professor Mead has founded his analyses in calling attention to apparent deficiencies in the plans now being advocated in the Treaty negotiations for the Waterway.

At this time it seems likely that the parallelism of the St. Lawrence Waterway and the Panama Canal may be continued. It remains to be seen to what extent the "minority opinion" will receive adequate consideration or will influence future acts.

An unusual opportunity exists on the St. Lawrence Waterway project to use an engineering model to a greater extent than heretofore has ever been attempted. The importance of preliminary investigations by means of models has received general recognition in the United States only in the past few years. One of the greatest current examples is found in the Mississippi flood-control problem where engineering policies and traditions which have stood inviolate for fifty years or more, are boldly being replaced by a new policy of scientific planning on hydraulic models. It has been ably stated[38] that all these changes of policy have long had advocates. They have come to fruition now because of engineering direction that does not fear to experiment. Boldness of engineering initiative buttressed by experimental research has been the inspiration of the new planning. This spirit of research and of willingness to put its conclusions to actual trial deserves express recognition. It has prevailed in every rank of the Engineering Profession.

It is the responsibility of the Engineering Profession to earn this sort of recognition in planning the St. Lawrence Waterway. There are numerous problems of operating conditions and their effect on navigation, ice movements, erosion, river control, and construction procedure and hazards, which may be clearly understood by the few engineers who have devoted the greater part of their lives to a study of the St. Lawrence River, but it is important that their opinions and deductions be studied in the most scientific manner, and compared with any new findings, before becoming rigid policies.

---

[37] Constr. Plant Engr., Tennessee Val. Authority, Knoxville, Tenn.

[38] Editorial, *Engineering News-Record*, June 22, 1933, p. 819.

In the past decade engineers have adopted the science of models to a point where practically all important water-power projects are designed after observing the hydraulic performance of the principal elements on models. However, very often the general features had become established sufficiently to prevent the adoption of major improvements indicated by the model. It is safe to predict that hydraulic models of the St. Lawrence project will eventually be built. By building a comprehensive model as the first step, on which the various proposed plans may be studied in sufficient detail to demonstrate all conditions of operation, navigation, and construction procedure, the project is assured of the soundest basis of planning, and the best answer is given to those clamoring for immediate action.

Not only will disagreement among engineers be dispelled and individual definitions of the term, "hazard," give way to the most logical plan, but plans and sketches will be replaced by a physical picture which is at once understood by public officials, representatives, and the public at large. Such a model can serve as a valuable educational medium among the people directly concerned with the project, and, in place of inferences of political pressure or inadequate educational effort, its scientific operation will direct confidence toward the engineers and officials charged with the responsibility of creating the best and most economical St. Lawrence Waterway.

DANIEL W. MEAD,[39] HON. M. AM. SOC. C. E. (by letter).—The writer is pleased with Mr. Hogg's frank elucidation of the Canadian point of view in regard to the proposed St. Lawrence power development. He is in agreement with the views expressed far more than the discussion would seem to indicate. For example, he agrees with the first statement that "* * * the choice of a single-stage or a double-stage plan cannot be settled satisfactorily solely on engineering or economic grounds." This statement is verified by the official treaty lately before the Senate for consideration, which by no stretch of the imagination can be considered as the best engineering or economic solution.

The writer also agrees with Mr. Hogg that "* * * the fact that it [the St. Lawrence] is an international river cannot be ignored." While, in general, governmental activities must perhaps be based on "political" economy instead of on economics, the difficulties become greatly accentuated when a project becomes international and increase probably somewhat faster than the square of the number of Governments mutually interested.

These difficulties practically vanish, however, if between two nations the administration of one of them is so anxious for the project to materialize that it is not only ready to submit to dictation as to the plans for the development and its cost, but also to surrender, forever, sovereignty over non-international waters lying entirely within its own borders, and, finally, to limit arbitrarily the use of such waters by its own people.

---

[39] Prof. Emeritus, Hydr. and San. Eng., Univ. of Wisconsin; Cons. Engr., Madison, Wis.

Canada and the Province of Ontario are to be congratulated on their intelligent handling of this entire project. To reiterate Mr. Hogg's statement (under the heading, "Reasons for Not Accepting Author's Set-Up"), "* * * the Canadian organizations studying the problem have been living with the river for a number of years, and * * * practically the same personnel has been engaged in the work throughout." The Canadian and Provincial organizations worked independently, but conferred and discussed the problem fully. They found the single-stage development of greater economic value, but such a project involved the raising of the water of the St. Lawrence for about forty miles up the river above the projected dam site. The Canadian inhabitants along this stretch of river apparently became alarmed at the proposal to increase the height of the water, which involved (to them) unknown danger, real or imaginary. Therefore, they appealed to their Parliamentary representatives, and apparently succeeded in creating so strong a public opinion as to render this plan politically impossible.

With the Crysler Island Two-Stage Plan, only the upper twenty miles of the St. Lawrence is to be raised, and this to a less degree; so that while the people along this reach also objected, the ill effect was not considered so apparent and their influence was not sufficient to block the demand of the Ontario Hydro-Electric Commission for the maximum amount of power which could be developed "all things considered." Having fully decided what they could and would do, the Canadian members of the Joint Board were prepared to meet the members of the Joint Board from the United States.

The policy of the United States in regard to the study of this project has been quite different. It has been the continuing policy of the U. S. Government, as far as the Corps of Engineers is concerned, to place an engineering officer on a Federal job for a limited time only, and about the time that he becomes really useful to send him to some other and often quite different type of project. On the power project of the St. Lawrence International Section, for example, there seems to have been no continuing individual service of the American representation; each of the American Sections of the several Joint Boards that have considered this subject, has consisted in general of entirely different engineering officers and, hence, these Sections have been at a great disadvantage and more or less dependent on the Canadian Section and the information furnished by it.

The arbitrary position of the Canadian Section of the Joint Board, and the substantial agreement of the Power Authority of the State of New York and its engineers with the writer's conclusions, is best described by reference to the Second Annual Report of the Power Authority of the State of New York for the year ending December 31, 1932 (pp. 17–18).

Referring to the complications introduced in the negotiations between the U. S. Department of State and the Power Authority by the question of the engineering plans for the development of the International Rapids Section, the report states:

"* * * the Power Authority early in the negotiations directed its engineers to make a thorough study of all the plans that had been proposed by the engineers of the United States and Canada.

"They reported that the plan prepared for the St. Lawrence Power Development Commission of New York by its Advisory Board of Engineers headed by the late Major General Edgar Jadwin was the most economical and efficient, from the standpoint of both power and navigation, that had been proposed.

"The Trustees therefore formally presented this plan to the State Department with a recommendation that it be given serious consideration by the Joint Board of Engineers which had been reconvened by the United States and Canada in connection with the treaty negotiations. The Power Authority was subsequently informed that the American members of the Joint Board strongly favored this single stage plan but that the Canadian members were insisting upon a two-stage plan with the upper dam at Crysler Island."

According to this report, the objections of the Power Authority to the Crysler Island Plan were laid before the State Department on January 6, 1932. Being strongly impressed, the representatives of the Federal Administration arranged for a joint conference with the representatives of Canada on January 16, 1932. At this conference the engineers of the Power Authority proposed a modified two-stage plan, with a main dam at Massena Point, and a control dam (at which no power could be developed) at Galop Island. It was hoped that this compromise might be acceptable because it was similar to a plan that had originally been preferred by the Canadian engineers and that was abandoned only because the United States Engineers objected to the relatively slight loss of power involved.

The Canadians rejected this plan, however, as well as all proposals for single-stage development and, to quote from the report:

"When it became clear that a treaty could not be negotiated that did not provide for a two-stage plan, the Power Authority adopted the position that, whatever plan might be agreed upon between the United States and Canada, the power development in which New York was primarily interested should not be required to pay more than the power project would cost if constructed in accordance with the most economical plan."

When the foregoing conditions are considered in the light of the American Section's final instructions from Washington (see paper, under the heading, "Introduction"), and the rejection by the Canadian Section of any plan other than the Crysler Island Two-Stage Plan, the findings of the Joint Board of Engineers (Reconvened) that the two-stage plan "is practicable and feasible from an engineering point of view," seems the only one possible under the circumstances.

In some matters the writer is obliged to differ with Mr. Hogg. His statement (under the heading, "General") that "he [the writer] also shows a marked preference" for the New York Massena Point plan, conveys an impression that the writer was in favor of the plan. The writer knew nothing about the proposed New York plan until, in February or March, 1932, when he began a study of the various reports, referred to in the paper. The writer has no preference with respect to the different plans other than that which resulted from a critical analysis of those reports. He does not doubt that Mr. Hogg believes that the estimates of the New York Commission's report are on an entirely different basis from the two Canadian estimates of the two-stage and the single-stage plans, *A* and *B*, and that the corrections which he suggests remove the Massena Point plan entirely from reasonable consideration. On the other hand he cannot believe that the late Maj-Gen. Edgar Jadwin, U. S. A., M. Am. Soc. C. E., the distinguished Chairman of the New York Commission (who, also, at one time served as Chairman of one of the American Sections of the Joint Board and who apparently was well informed on the subject) and the other distinguished civilian engineers who served with him on the New York Board, would furnish their client with an underestimate of more than $26 000 000.

As already explained, the writer is unable to answer specifically the questions raised by Mr. Hogg, and regrets that those civilian engineers who, being sufficiently well informed on this subject to have furnished an effective answer, preferred not to do so.

In the second paragraph under the heading, "General", Mr. Hogg explains what "accounts for what may appear to be lack of sufficient detail in the later reports." This lack is serious concerning not only the power development but the entire St. Lawrence project. Volumes have been written for and against the project, much of which has been in the nature of propaganda. There is—as far as the writer is aware—no place for the interested to turn to get an unquestioned fair and unbiased discussion or undisputed facts showing the desirability of the project, on which an intelligent opinion can be based.

Answering the comments of Mr. Ripley the writer would say that he was assigned the question of a comparison of the various plans of the proposed hydraulic power developments of the International Section of the St. Lawrence River and has not even attempted to discuss the economics of power transmission and use. Fortunately, this matter is covered at some length in the discussion of Mr. Bonner. Neither did the writer attempt to discuss the still greater and more complex question of the St. Lawrence Waterway in its broader significance. The latter question is so complicated that, apparently, little attempt has been made to justify the construction of this project which seems to have become largely a political matter. A fairly complete discussion of this subject appears in "The Great Lakes and the St. Lawrence Ship Channel," by R. S. MacElwee, Affiliate, Am. Soc. C. E., and A. H. Ritter, published in 1921. Based on an apparently impartial examination of the data available, and on a personal examination of the route by Mr. MacElwee, the authors of this volume conclude: (1) That both the waterway and the power development will be very profitable to the citizens of the United States and Canada; (2) that the saving on the shipping of grain will amount to from 8 to 10 cents per bushel; and (3) that great savings will be made on practically all that is produced within the area tributary to the Great Lakes. To quote from the book:

> "This saving will amount annually to approximately the entire cost of the improvement required to admit ocean vessels into the Lakes. * * * The water power developed will not only afford a revenue which will maintain the improvement and amortize the full cost within a reasonable period but it will be the direct means of creating an industrial development which in itself will provide a large traffic on the waterway. * * *"

On the other hand, the most thorough discussion of both the navigation and power questions with which the writer is familiar, is contained in the book entitled, "The St. Lawrence Navigation and Power Project," by Messrs. Moulton, Morgan, Lee, and others, published in 1929. This book is apparently disinterested and is based on the investigation of a number of people extending over several years. Their findings are:

First.—That the saving to the farmer will be 4 cents per bushel on the shipment of wheat and rye (which they estimate constitutes more than 50% of the probable total traffic) from Duluth, Minn., to Montreal, Que., Canada; that is, to reduce the costs from 9 cents per bushel (which was its 1929 price) to 5 cents per bushel over the St. Lawrence Waterway, will cost the taxpayer 11 cents per bushel, or involve a total cost directly and indirectly of 16 cents per bushel.

Second.—That while the power may be economical in connection with electro-chemical or electro-metallurgical industries situated on the river near the power plant, it will be uneconomical if transmitted to the New York Metropolitan, or to the New England, Districts.

Even if this opinion is correct, the project is apparently as sound as most of the Government navigation-power projects now under way. The economics of this problem are greatly involved and many opinions concerning them are diametrically opposed. In the writer's opinion this project ultimately will be carried to completion. If, in general, the internal navigation projects of the United States are at all warranted, this project which opens up a great territory of the United States and Canada to ocean navigation should ultimately be of great importance. It seems unfortunate that the nations involved have failed to investigate the economics of the problem to the fullest possible extent, but such investigations are so rare in Government undertakings that its absence in this case should not be surprising.

The writer appreciates the discussions of Messrs. Putnam, Smith, and Harza on the effects of these different power developments on navigation. This is a subject with which the writer is unfamiliar and the conclusions of these engineers indicate that the estimate, in the paper, of the greater value of the Massena Point Plant is not overdrawn.

The writer agrees with Mr. Ackerman that experimental models are a sound basis for determining the results which will be obtained from new hydraulic design and may well be utilized in some of the new problems in the proposed development of the St. Lawrence River. The difficulties involved in the question under consideration are not so much technical, however, as they are political. There is no question in the mind of the writer that the best plan can be made perfectly safe at an expense which, as compared with other plans, would fully warrant its adoption. The matter of convincing the electorate as to the questions of safety and advisability is entirely different. These are problems which quite possibly can not be demonstrated to the satisfaction of all concerned.

Canadian authorities recognize the fact that it is the part of wisdom to delay this project until the present depression is over. The following extract from an editorial in the *Canadian Engineer*[40] gives the point of view of at least one Canadian official:

> "Last week further statements were made to the public regarding the St. Lawrence project. Thus C. Alfred Maguire, one of the Commissioners of the Ontario Hydro-Electric Power Commission, speaking regarding the Hydro-Power situation at Oakville, is reported as saying: 'I think Providence was with the Commission and the people of Ontario when they were not able to get the St. Lawrence at the particular time, because had we been able to get the St. Lawrence scheme completed we should have had $8 000 000 deficit every year. Instead of that the Commission got the private interests to build plants and give us the power from Quebec. * * *' Such a view expressed by one who is in close touch with these matters and who thus draws special attention to what might have proved a great economic loss, emphasizes the need for fuller economic consideration especially under the rapid and pronounced changes that characterize national and international trade relationships."

The writer agrees with Mr. Lee and Mr. Rickey that this important subject is a pertinent and proper subject for the discussion of engineers of both Canada and the United States and is worthy of the most careful, fair, and honest consideration.

---

[40] See Editorial, "Development in St. Lawrence Project," December 12, 1933.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 1903

LOSS OF HEAD IN ACTIVATED SLUDGE AERATION CHANNELS

By Darwin Wadsworth Townsend, M. Am. Soc. C. E.[1]

With Discussion by Messrs. H. L. Thackwell, Henry R. King, M. H. Klegerman, and Darwin Wadsworth Townsend.

Synopsis

This paper involves the consideration of no new hydraulic theory or formulas. The problem implied in the title includes an analysis of flow conditions that differ from those encountered in the usual channel problems, because the introduction of diffused air tends to retard velocity.

The presentation is semi-hypothetical, and the various functional values expressed are, strictly speaking, relative only. As such they are susceptible to direct application and use on the basis of any value of n that may be selected. The assumption is made that a roughness coefficient of $n = 0.013$, in Kutter's formulas, is reasonably applicable for all practical purposes to channels constructed in a manner similar to that described herein, and for unaerated sewage.

The purpose of the studies was to determine the extent to which flow is retarded when diffused compressed air is introduced into a flowing mixture of sewage and activated sludge, the total suspended matters in the mixture varying inappreciably from a normal value of 3 000 ppm. Numerous previous experiments have led the writer to conclude that this solids content does not change the hydraulic properties of the mixture from those of water alone.

The tests indicate definitely that: (*a*) Resistance to flow increases because of diffused air; (*b*) as the velocity of flow decreases, the resistance increases; and (*c*) the use of higher values of n is necessary because of increased resis-

---

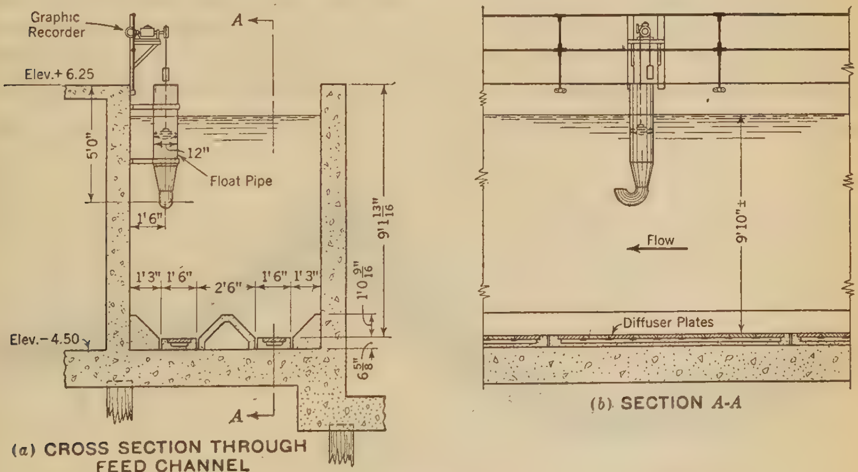
Note.—Published in January, 1934, *Proceedings*.

[1] Cons. Engr. (Consoer, Townsend, Older and Quinlan, Inc.), Milwaukee, Wis.

tance. The subject has been treated sufficiently herein to enable readers to comprehend the nature of the problem and the applicability of the results to problems of design.

Sanitary engineers are familiar with the fact that the activated sludge process of sewage treatment involves primarily the injection and diffusion of relatively large volumes of compressed air into the liquid sewage undergoing treatment, and that the countless ascending streams of tiny air bubbles injected at the tank or channel bottom offer resistance to the forward movement of the flowing sewage. This latter fact is obvious to one who has observed the surface appearance of an aeration tank or an aerated channel in which may be seen innumerable and continuously changing cross-currents, counter-currents, and swirls—all of which are indicative of resistance to the flowing stream.

It was because of this condition and the realization of its significance in hydraulic design, complicated by this flow-disturbance factor, that the writer undertook to determine the extent to which flow was retarded when diffused compressed air was introduced in the feed channels of the existing sewage treatment plant at Milwaukee, Wis. It is worthy of note that the selection of $n = 0.020$ in Kutter's formulas, was considered ultra-conservative in 1928 when the design of the present plant was begun, and that subsequent operation proved conclusively that this coefficient was inadequate.



(a) CROSS SECTION THROUGH FEED CHANNEL

(b) SECTION A-A

FIG. 1.—GRAPHIC RECORDER IN FEED CHANNEL.

The intent of this paper is not to formulate basic laws applicable to hydraulic properties of water or sewage, mechanically impregnated with diffused air. As a matter of interest, however, it may be assumed that: (1) Water impregnated or saturated with air has a specific gravity less than water alone; (2) because of this probable fact, it possesses hydraulic properties differing from those of plain water; and (3) if Assumptions (1) and (2) are correct, the hydraulic laws governing the flow of water would need to be modified somewhat, in order to classify this particular medium properly. However, in this paper, the subject must be treated from the viewpoint of an increased roughness coefficient, assuming the flowing medium to possess the hydraulic properties of water alone.

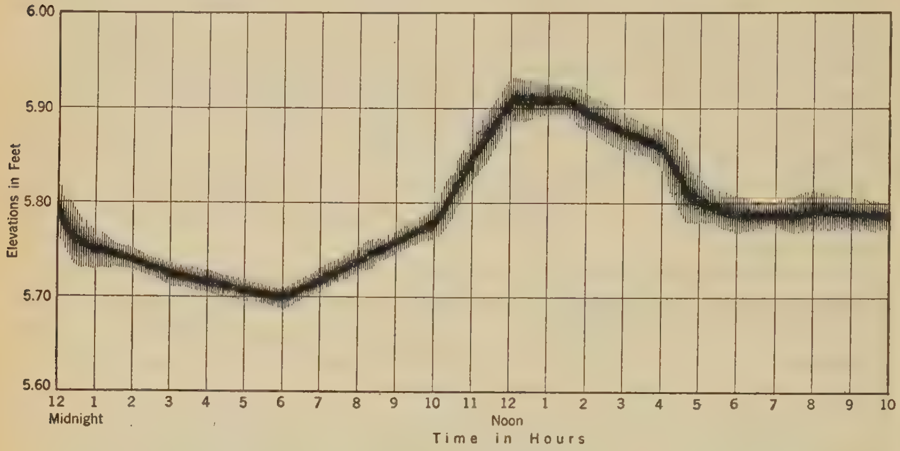


FIG. 2.—HEAD LEVEL CHART IN FEED CHANNEL, STATION NO. 1, JULY 8, 1928.

For the purpose of determining the roughness coefficient applicable to use in Kutter's formulas, two head level recording stations were established, 123.34 ft apart, in a section of an aerated-sewage feed channel where the cross-section was uniform. In this section, and under the conditions of the

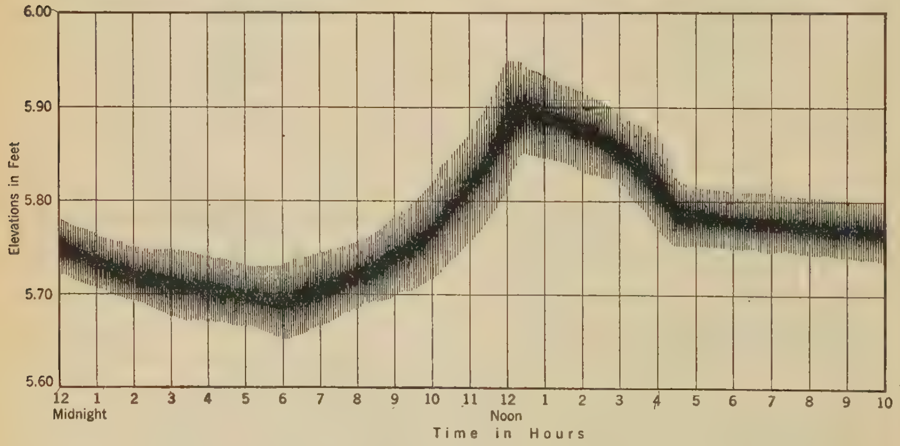


FIG. 3.—HEAD LEVEL CHART IN FEED CHANNEL, STATION NO. 2, JULY 8, 1928.

test, the cross-section area was 75.39 sq ft, the wetted perimeter was 27.47 ft, and the mean hydraulic radius was 2.774 ft. The floats for operating the recording instruments were mounted in 12-in. pipes with upturned entrances, 6 in. in diameter, at the bottom, to exclude the ascending streams of air bubbles, as much as possible.

The type of float-chamber finally adopted (see Fig. 1) was one of several under consideration. The representative reproduction shown in Fig. 2, of one of the many graphic records taken at the up-stream recording station, demonstrates the degree of quiescence. It will be noted that the pulsations in the float compartment were practically continuous and that they varied in vertical movement between 0.02 ft and 0.05 ft. The records at the down-stream station, a graphic representation of which is shown in Fig. 3, indicated pulsations of greater magnitude than those at the up-stream station because of an obviously greater degree of diffused air disturbance at this point. The vertical movement of these continuous pulsations may be seen to vary between 0.06 ft and 0.09 ft.

The channel in which the stations were established contained a pair of practically continuous rows of diffuser plates (see Fig. 1), no two of which could be assumed to possess identical characteristics of porosity and air discharge.

The rate at which air bubbles blend, or coalesce, however, has a more important effect on pulsations. Presumably, the coalescence factor assumes greater prominence and greater irregularity as the velocity of flow increases. The velocities observed during the tests ranged between 0.9 ft per sec and 1.3 ft per sec, computed on the basis of the wet cross-section and Venturi meter readings of sewage flow.

From Figs. 2 and 3 one might conclude (with logical justification, perhaps) that the flow-disturbance factor inherent in the particular flows under discussion is not susceptible to precise analysis; the writer claims only that it is reasonably applicable as an approximation in practical design problems. It is concluded without hesitancy, however, that final results stated in the paper are sufficiently on the side of safety to warrant confidence in their subsequent use.

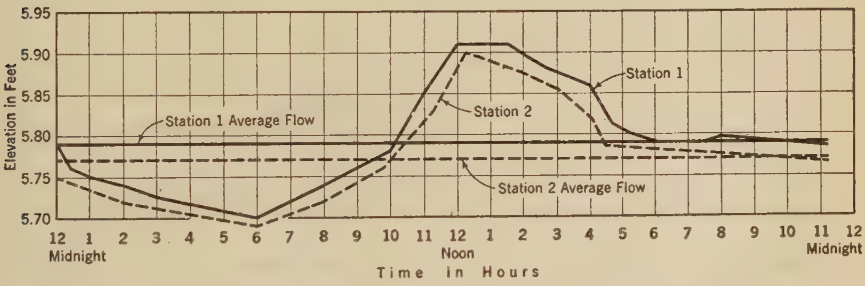


FIG. 4.—LIQUOR-LEVEL ELEVATIONS IN FEED CHANNEL, JULY 8, 1928.

A representative daily record of liquid levels taken at the two stations is presented in Fig. 4, the curves being values averaged from those shown in Figs. 2 and 3. It is significant and unusual, perhaps, that the total loss of frictional head between the two stations is much less (particularly when considering the record at the down-stream station) than the range in pulsations, the average being about 0.02 ft, and the pulsations ranging between 0.06 ft and 0.09 ft. The average curves in Fig. 4 compare favorably and consistently, however, with many similar records taken daily over a period of three weeks at the two stations. The average friction loss between these stations varied only slightly under comparable flows, generally about 0.02 ft to 0.03 ft.

A fact established quite definitely in connection with the loss of head determinations was that the coefficient of roughness (n-values) increased as the velocity of flow decreased. The approximate trend is indicated by the upper part of Curve 1, Fig. 5, which was constructed from liquid-level observations made in July, 1928, on the ridge-and-furrow aeration system. The value of n is 0.035 at 1.3 ft per sec and 0.0425 at 0.9 ft per sec, which indicates an increase in n of about 25% for a reduction in velocity of about 30%; and, *vice versa,* a decrease in n of 18% for an increase in velocity of 44%, considering only the two terminal points of the curve, as shown.

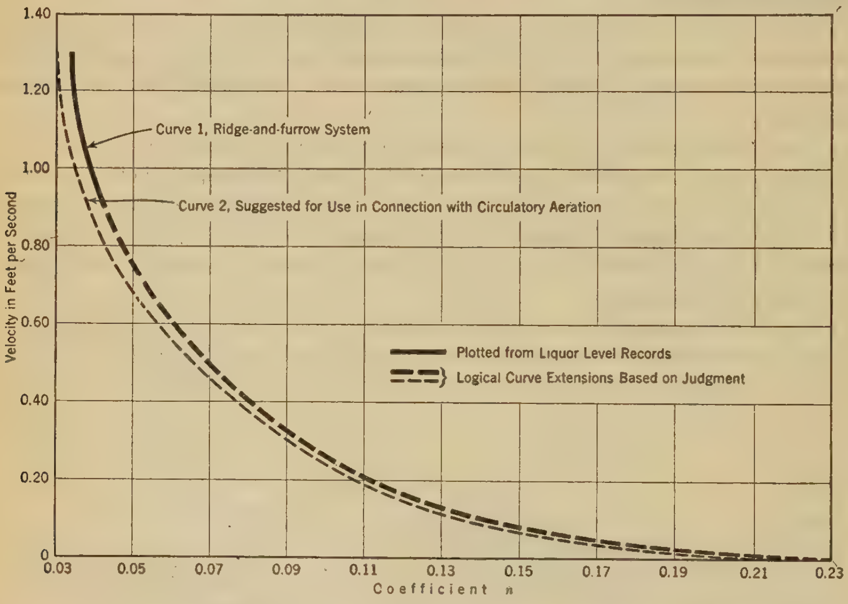


FIG. 5.—RELATIONSHIP BETWEEN VALUE OF n AND FLOW VELOCITIES IN AERATED FEED CHANNEL (WITH ESTIMATED EXTENSION BELOW VELOCITY OF 0.90 FOOT PER SECOND).

A hypothetical cause-and-effect theory may be ascribed to this varying roughness coefficient which, from the standpoint of practical merit and application, may possibly possess little to warrant any elaborate exploitation. Certain factors of influence appear to be sufficiently well defined, however, at least to warrant mention. Upon analysis, it becomes apparent that the hydraulic properties of a channel cross-section may be assumed to be changed artificially through the continuous presence in the flowing medium of the countless ascending streams of air bubbles, the degree of change depending primarily on the rate of diffused air injection. This statement is made with particular reference to an artificially and obviously decreased cross-section and the resultant increased wetted perimeter, ascribing to an air-bubble surface the same, or nearly the same, value of unit wetted perimeter as that which applies to the wetted surface of the channel structure itself. On the basis of this hypothesis, it is obvious that the "air"-wetted perimeter would be increased theoretically in proportion to the rate of diffused air injection, as previously noted, and, as a consequence, hydraulic radii would decrease in somewhat the same proportion, ignoring for the time being the extent of the fixed wetted surface of the channel structure and its definitely known influence upon the hydraulic properties of the channel.

Because of the tendency of the air bubbles to coalesce the degree and irregularity of which may vary with both the velocity of stream flow and rate of air injection, it may readily be seen how impractical of strictly theoretical application are certain factors which, apparently, are rather clearly defined and understood. For example, if there were no coalescence, and if the almost continuous streams of air bubbles could be assumed to rise in truly vertical streams substantially unchanged as to size and position, that part of the wetted perimeter herein ascribed to injected air could possibly be assumed to vary in direct proportion to the rate at which air was injected and to the total bubble surface exposed to the liquid in which the bubbles are contained. Furthermore, air bubbles are relatively small immediately after they are ejected from the pores of a diffuser plate (ordinarily $\frac{1}{16}$ in., or less, in diameter); frequent observations made in connection with the diffusion of air into glass cylinders containing clear water, indicate that for a distance of a foot or more immediately above the surface of the diffuser plate practically the entire space appears to be occupied by diffused air. This, in turn, indicates that a substantial part of the lower zone of the cross-section is appreciably restricted as to availability for water displacement, and, consequently, to the forward movement of the flowing medium. As the bubbles reach the upper zones they increase appreciably and become more irregular as to size, which is a direct result of air expansion caused by the decreased water pressure and, more particularly, by coalescence.

During the period in which liquid-level records were being obtained, air was forced into the flowing admixture of sewage and activated sludge at a constant rate of 1 cu ft of free air per diffuser plate per min, and upon a strictly assumed theoretical basis the air-bubble resistance to flow was constant. However, inasmuch as resistance to flow, or the roughness coefficient, has been shown to increase as the velocity of flow decreased, it follows that a decrease in roughness coefficient with a fixed rate of air injection, may be attributable to an increased degree of coalescence as a direct result of increased velocity.

Table 1 has been compiled in order to show more clearly the possible extent to which the hydraulic properties of a channel are subject to assumed modification due to the air-injection feature. The values are based on: (*a*) The velocities that obtained during the period in which records were taken (0.9 ft to 1.3 ft per sec); and (*b*) the friction head per linear foot of channel, using $n = 0.013$ as a basis (see Table 1, Columns (1) and (2)).

Values of n and C, shown in Columns (3) and (4), were computed. Column (5) contains relative velocity reductions as a result of increased n-values over and above $n = 0.013$; in other words, the velocity values shown in this column are those which. if used in connection with values in Column (3), would produce the same friction heads as those shown in Column (2), and are translations of values of Column (3) into terms of relative velocity.

Furthermore, translations on the foregoing basis for relative mean radii, *MR*, and wetted perimeters, are shown in Table 1, Columns (6) and (7), respectively. These data indicate the degree to which the two hydraulic functions may be assumed to be altered, on the hypothesis that the wetted perimeter may be increased and the hydraulic radius decreased, due to the presence of vertically rising streams of air bubbles which are assumed to offer flow resistance comparable to that of a surface with a fixed wetted perimeter. Column (8) and (9) (Table 1) indicate the same as Columns (6) and (7), but are expressed in terms of percentage decrease and percentage increase in the functional values noted.

TABLE 1.—RELATIVE VALUES OF AERATED CHANNEL PROPERTIES, USING AS A BASIS AN UNAERATED CHANNEL OF CONSTANT CROSS-SECTION AND THE VALUE OF $n = 0.013$

(Slide-Rule Computations)

Velocity, in feet per second (1)	Friction head per foot for $n = 0.013$ (2)	Value of n of curves shown on Fig. 5 (3)	Value of C for n of curves shown on Fig. 5 (4)	Velocity for friction head corresponding to n on curve (5)	Value of mean radii corresponding to the velocity at $n = 0.013$ (6)	Wet perimeter corresponding to mean radii at $n = 0.013$ (7)	Percentage decrease of mean radii at $n = 0.013$ (8)	Percentage increase of wet perimeter at $n = 0.013$ (9)	Percentage increase of velocity at $n = 0.013$ (10)
1.40	0.0000392	......	.....	....	....	....	....	....	....
1.30	0.0000338	0.0348	49.8	0.481	0.66	114.2	416	416	270
1.20	0.0000288	0.0351	49.5	0.442	0.65	116.0	423	423	272
1.10	0.0000242	0.0363	48.6	0.396	0.63	119.5	436	436	278
1.00	0.0000200	0.0388	47.0	0.350	0.61	123.5	451	451	286
0.90	0.0000162	0.0424	44.6	0.298	0.565	133.5	486	486	302
0.80	0.0000128	......	.....	....	....	....	....	....	....

In Column (10) (Table 1), is noted in percentage the extent to which the velocities would be accelerated if the resistance causing the increased n-values were removed, and $n = 0.013$ were substituted.

Much more of possible interest might be presented relative to: (1) The cause and effect of the air injection; (2) the resulting retarding current disturbance factor; and (3) its influence on hydraulic problems of design. Nevertheless, it is hoped that this discussion has been in sufficient detail to solicit appreciation of, and to direct attention toward, the significance attributable to the flow characteristics of sewage impregnated with air.

In the system of aeration using circulatory flow, relatively few diffuser plates are needed as compared with the so-called ridge-and-furrow system in use at the plant in Milwaukee. It is the writer's judgment that less retardation of flow might be anticipated in connection with that system. In Fig. 5, Curve 2 has been plotted arbitrarily and is suggested for use as noted in connection with circulatory aeration.

The reasons for anticipating lower *n*-values are as follows (and it is to be clearly understood that these reasons are formulated and offered in the absence of supporting data other than that of attempted logical analysis): In visualizing the movements of sewage admixed with activated sludge undergoing aeration where diffuser plates are numerous, as in the ridge-and-furrow system, there is in the general proximity of each diffuser plate a complete induced cycle of ascending and descending currents, the total length of ascent and descent traversed being approximately equivalent to twice the depth of the overlying liquid. Obviously, then, the more numerous the diffuser plates are the more numerous will be the current cycles, the greater will be the total distance traversed in a given time, and the greater will be the velocity of travel.

As a result of the higher and more frequent upward and downward current velocities, greater resistance to forward or horizontal flow might reasonably, and obviously, be expected. Otherwise stated, the angle of the resultant of the horizontal and vertical flow components referred to the horizontal might be expected to increase appreciably as the vertical velocities increase, the velocity of horizontal flow remaining constant.

In the strictly circulatory or spiral aeration system whether in a tank or sizable channel, fewer diffuser plates are used and, consequently, fewer ascending and descending flow-retarding currents are present. In this system in which the diffuser plates are located along one side of a tank or channel, a circulatory flow around the entire periphery is induced as a result of the injection of air. This creates a sufficient head at the liquid surface over the diffusers to set up the circulatory flow from which the system derives its name.

Little or no well-defined current other than a relatively slow forward movement is present in the "core" of the liquid, or in that part somewhat removed from the well-defined peripheral currents. Therefore, in the absence of currents in these zones (which, in area, comprise the major part of the total cross-section) there is little or no resistance to forward flow, the lateral restricting currents in the forward flow existing in the general proximity of the sides and bottom of the tank and at the liquid surface.

The range of channel velocity herein noted (0.9 ft to 1.3 ft per sec), is that in which activated sludge particles would ordinarily be precipitated to the bottom of a channel, in a liquid sewage medium. The diffusion of air into the mass obviously is made necessary when the range of flow is great and when there is insufficient head to produce higher and self-cleansing velocities.

Thus far, the paper has treated mostly upon flow conditions in supply channels, on the basis of the records mentioned. In aeration tanks where exceedingly low velocities occur (little in excess of 0.02 ft to 0.03 ft per sec) a curve showing n-values plotted against velocity would be expected to approach the horizontal closely as the velocity nears zero. To investigate this phase a base rate of curvature was established from readings between $v = 0.90$ and $v = 1.30$ ft per sec, and, from this, Curve 1 (Fig. 5) was extended toward values of low velocities. This projection showed as an asymptote to the horizontal zero axis; for example, a value of $n = 0.20$ was found to correspond with a velocity of only 0.02 ft per sec. This chance projection confirming logical expectations is of considerable interest.

The projection of this curve is of considerable interest, since besides showing working n-values at various velocities its horizontal tangential point, falling in the immediate vicinity of zero velocity as it does, lends additional confidence to the manner and accuracy with which the liquid-level records were taken and the subsequent computations made. A lower curve has also been constructed for use in connection with circulatory aeration flows.

In closing, the writer wishes to acknowledge with thanks the assistance of William Landsiedel, Assoc. M. Am. Soc. C. E., who, under the writer's direction, prepared and arranged the data in the various diagrams.

DISCUSSION

H. L. Thackwell,[2] Assoc. M. Am. Soc. C. E. (by letter).—It is important to select the correct coefficient of *n* in Kutter's formula, when solving for velocities in open-channel flow, with the conditions of the regimen varying from tabulated values, as registered and advocated by various experimenters and observers. This fact is emphasized in Mr. Townsend's paper.

Present field practice in hydraulics is stressing non-uniform and varied flow in open channels, rather than the rudimentary notions of uniform movement. Striking cases of turbulent flow occur in sewage disposal works. Every form of non-uniform movement may be observed in modern sewage disposal plants, namely, the hydraulic jump as used for mixing chemicals and for aerating purposes; spiral movements in certain types of aerating tanks; wave motion in distributing launders for filters; baffled flow in settling tanks; whirls and eddies obtained in various ways by agitating mechanisms; and diversion of flow directions in conduits, diversion boxes, and over-step aerators.

For some time the writer has felt that the hydraulics of sewage works has been neglected, and that more attention has been given to process treatment than to the mechanics and hydraulics of the subject. The Ganguillet and Kutter formula, for determining the value of *c* in the older Chezy formula, still has a common acceptance and specific meaning to the average engineer trained in America. With this formula, if the correct coefficient of roughness, *n,* is chosen for the channel, it is possible to obtain the coefficient, *c,* in Chezy's formula, and if the measurements of cross-section area, slope, and wetted perimeter are arrived at accurately, an average value of *v* may be obtained. It is doubtful whether slope determinations influence, or are necessary in obtaining, a correct value of *n* in the Kutter formula, which was primarily intended for uniform flow conditions.

The author has made observations on the velocities of an open channel having a very turbulent flow, due to the presence of rising air bubbles (resulting in a consequent swirling movement, both vertically and horizontally, and in a positive and negative direction), and he has computed a corresponding value of the coefficient, *n,* to fit such conditions. These observations range from 0.9 ft to 1.3 ft per sec. and, when such velocities are plotted with the coefficient, *n,* Mr. Townsend presents a curve which shows, within these limits, that the value of *n* apparently increases with lowering velocities.

Since the observed depths and varying cross-sectional areas are not given, it is presumed that the full wetted cross-section of the channel was used to obtain the mean velocity in each case, rather than the net cross-sectional area which might have been obtained by subtracting some bottom layer of air and water, where the jet action of the stream of air bubbles presents a definite barrier to forward horizontal flow. The trend of the curve thus plotted was ascertained and extended to zero-velocity limits, where the coefficient, *n,* was 0.23.

---

[2] Cons. and Const. Engr., Longview, Tex.

With the Chezy formula, as used in this case, the resistance to motion is proportional to the second power of the mean velocity, which is undoubtedly true for all velocities above the critical velocity. At this low critical point, however, the resistance may be proportional to the first power of the velocity.[3] The natural inference is that it would be fictitious accuracy to extend a trend curve beyond the limits of its governing equation. If the percentage of cross-sectional area, uninfluenced by jet action, had been used as a net cross-section, there is reason to believe that the coefficient, n, would not have increased with a decreasing velocity, but would have been nearly the same for all values of v. The difficulty would be to determine this net area uninfluenced by jet action, and yet supporting floating bubbles of air. An air-bubble surface (that is, freely floating and moving with the current of water), would not enter into the calculation of the wetted perimeter, any more than a log floating with a stream current would be considered in obtaining the wetted perimeter of a channel. If the bubble of air were fixed in position, the wetted circumference of such a bubble should be calculated and added to the wetted perimeter of the channel itself.

Before discussing the author's "cause and effect" theory in regard to the varying roughness coefficient, it will be necessary to obtain a better knowledge of the mechanics of the air bubble itself.

In June, 1932, the writer became interested in the subject of the mechanics of the air bubble and the laws governing it, and after searching literature on the subject, was surprised to find that very little experimental data had been published in the past on the subject of air induction into sewage or water. Mr. H. S. Allen made experiments on "The Motion of a Sphere in a Viscous Fluid," and published[4] his findings in 1900. That part of the subject-matter dealing with air bubbles was devoted to studying the velocity of very small bubbles rising in water. These bubbles were of such a small size (all less than $\frac{75}{100}$ mm), that the experiments and resulting data were not comparable to the larger air bubbles used for agitating water and sewage. Accordingly, the writer decided to make some simple experiments to obtain some definite first-hand information as to the velocity of the bubble rising in water, the size and shape of such bubbles, and their path of travel in ascending to the surface. The apparatus that was selected for the tests (see Figs. 6 and 7) consisted of a 5-ft glass cylinder with an inside diameter of 2 in., and having one end swaged and the other open. Both ends were fitted with rubber corks, the one at the swaged end had a single opening for a capillary tube insert, and the cylindrical end had two small tubes inserted in the stopper.

In one of these openings a small glass tube, 5 ft long, was inserted, having the submerged end fused down to a small J-shaped jet of very small bore. This tube was arranged so that it could be supported at any depth of submergence within the cylinder, thus acting as an air inductor into the sewage. The cylinder was supported vertically, by means of clamps, to the edge of a table. The other opening at the top of the stopper was connected by rubber tubing to a glass Y-tube, one arm of the Y-branch being connected by rubber hose to a mercury manometer, and the other to a vacuum suction pump. The lower end of the 2-in. cylinder was connected by tube and hose to another mercury manometer (see Fig. 6). The 2-in. cylinder was nearly filled with water. The J-inductor tube was next set in position, so that the jet opening was 4 ft below the surface of the water. The mercury gauges were set in position, and the lower gauge read 10.2 cm of mercury. The vacuum pump was turned on until the suction pressure occurring above the surface of the water was 10.2 cm of mercury; air was then drawn into the water immediately from the atmosphere, and liberated in the form of a stream of bubbles into the water column. Observations were made on the size of the bubbles, their shape and behavior of travel in rising to the surface, and the velocity of ascent.

---

[3] "Hydraulics and Its Applications," by A. H. Gibson, p. 295.

[4] *Philosophical Magazine*, September, 1900.

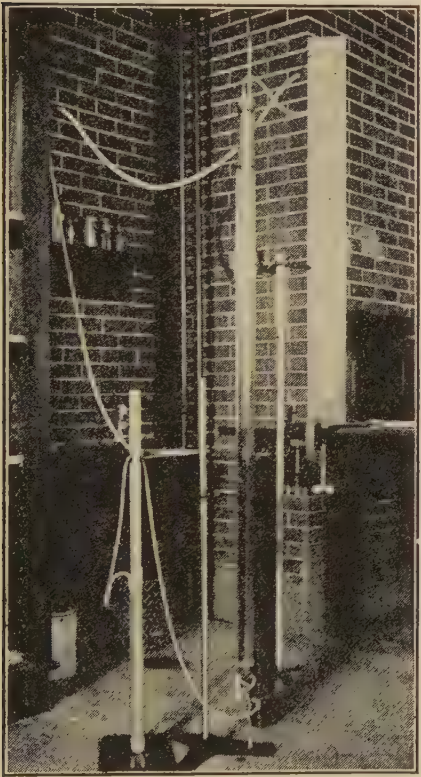


FIG. 6.—APPARATUS SHOWING RISING AIR BUBBLES.

FIG. 7.—AIR-JET MOUTHPIECE SHOWING ACTION AGAINST WATER.

Observations were also made by forcing air in through the inductor pipe by direct compression, rather than by suction, as by the vacuum process. As soon as air bubbles filled the cylinder in a continuous stream, the surface of the still water rose to a new level; this higher elevation was due to the displacement of air volume, and slightly due to a velocity head, built up by the rising bubbles. Measurements were made of the depth of submergence, the rise in water level due to air volume, the velocity at fractional and whole distances of the bubble ascent, and the manometer readings. The total depth of unaerated water was 141 cm and the gauges were kept at a constant pressure of 10.2 cm of mercury. A number of series of identical observations was made with slight variations of submergence. One set of these observations is given in Table 2 with the following supplementary comments:

TABLE 2.—OBSERVATIONS ON BUBBLE PHENOMENA

Item No.	Depth of water, in centimeters (1)	Manometer on water supply, in centimeters of mercury (2)	Manometer on air supply, in centimeters of mercury (3)	Rise of water level due to air volume, in centimeters (4)	Percentage, volume of air in column (5)	Velocity of bubble ascent, in feet per second (6)	Percentage of submergence (7)	Power product, Column (5) by Column (7) (8)
1	119	+10.3	—10.2	7.5	5.92	0.78	84.5	5.00
2	100	+10.2	—10.2	9.0	8.25	1.09	71.0	5.86
3	85	+10.2	—10.2	9.7	10.25	1.31	60.2	6.16
4	70	+10.2	—10.2	9.2	11.6	1.45	49.8	5.79
5	55	+10.3	—10.3	8.0±	12.6	1.68	39.02	4.93

Item No. 1.—At this stage the bubbles were pulsating slightly. Their diameters were from $\frac{1}{4}$ to $\frac{1}{2}$ in.

Item No. 2.—This was the beginning of marked turbulent flow, with bubbles coalescing.

Item No. 3.—Turbulence increased at this point so that large bubbles dominated the smaller ones.

Item No. 4.—Turbulence still further increased, so that small bubbles were held down by the larger ones.

Item No. 5.—Violently turbulent flow, such that the operator was unable to measure the velocity except by extending the velocity curves.

From the data collected in several series, curves were constructed, as shown in Fig. 8, and from these curves the following formulas are derived (approximately straight-line variations on the various series of data):

$$Y = 0.15\,(100 - X) + 4 \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

and,

$$V = 0.0192\,(100 - X) + 0.5 \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

in which, X = percentage of submergence (expressed as whole number); Y = percentage of air in a column of rising air; and V = vertical velocity, in feet per second, of rising bubbles.

By plotting the power product (Column (8), Table 2) against submergence (Column (7), Table 2) a maximum of aeration is indicated at 60% submergence (see Fig. 8, Curve C), with the provision that the air pressure is constant at the point of induction, this case being analogous to a constant

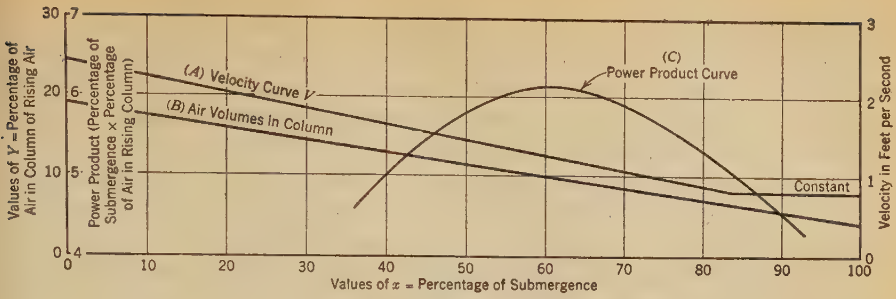


Fig. 8.

pressure blower connected to a variable-depth intake. From these data, and by observing the behavior of rising bubbles in the aerating cylinder, the following conclusions were drawn:

(1) Aeration by induction of compressed air into sewage or water may be steady, pulsating, or turbulent;

(2) With steady or pulsating conditions, the percentage of air in the cross-section is less than 7 by volume; and with all bubbles larger than a critical size (about 1 mm), the upward vertical velocity is constant and approximates 0.8 ft per sec (furthermore, this velocity is non-accelerating after the air has started to float freely as an individual bubble);

(3) With turbulent flow the upward velocity is non-accelerating, and varies in almost direct proportion to the percentage of air in the rising column (this velocity may vary from 0.8 ft per sec to 2 ft per sec, which upper limit is reached when the air volume of displacement is 16% of the water volume and at 22% submergence, provided the air pressure is constant at the point of induction);

(4) Bubbles have the same velocity in rising through water, whether they have been introduced by means of positive pressure, or by negative pressure (this deduction, of course, agrees with theory);

(5) The percentage of air injected is in direct proportion to the percentage of submergence, when the air is inducted at a constant pressure, and with the same type of injector orifice;

(6) The maximum inducted air remaining in the water at any one time, occurs when there is a submergence of 60%, and at this depth the aeration appears to be the most efficient (see Curve *A*, Fig. 8);

(7) The specific gravity of air mixed with water is in direct proportion to the volume of admixture; but when bubbles begin to appear and confined water is displaced upward, the specific gravity may be taken as that of water, provided no allowance has been made for the added depth of water column (temperature and atmospheric pressure not being considered);

(8) Air bubbles have a certain critical size in flotation, at which volume the shape changes from circular to lense or oblate spheroid; and,

(9) The heights of influence of jet action vary directly with the submergence, with the volume of air, and with the diffuser medium.

Referring to Conclusion (8), the sizes of bubbles are not proportional to the openings in the injector orifice, since a stream of air is first propelled and accelerated upward, which, on meeting resistance from the water, breaks up into bubbles of various sizes; on free flotation, each bubble changes in shape from a circle to a deformed oblate spheroid.

Each spheroid presents its largest surface area normally to the downward thrust of water, which thrust is directed toward the center of buoyancy of the bubble. Due to the elastic properties of the bubble surface, the resultant thrust falls outside the center of buoyancy, which, in turn, causes a tilting of the bubble with a consequent rising at an angle. This side slip is soon checked by resultant thrusts toward the opposite side of the center of buoyancy, and a corresponding shift occurs in bubble travel in the opposite direction. In this manner, the air floats upward with a zigzag movement and has a practically constant deviation from the vertical. Since the frictional resistances are proportional to the square of the velocity and directly to the exposed area the velocity soon becomes constant, and the volume remains nearly constant, due to the total compressive pressures acting against the surface area; these pressures are a summation of hydrostatic pressure and of velocity friction head. Apparently, these forces act automatically, and the size and velocity of the bubble remain practically the same throughout its travel range. Coalescence is the main cause of increased bubble size, but this factor will not cause increased velocities, or accelerating velocities, provided the proportion of entrapped air to water volume remains the same.

Referring to Conclusion (9), the minimum distance above the orifice, before the air bubble is freely floating and outside the influence of jet velocity, is about 6 in., and the maximum distance likely to occur in any ordinary orifice with various volumes and free flotation is 18 in.

The writer is aware of the fact that the foregoing observations are not conclusive, because the experiments were not sufficiently extensive and did not include the factors of temperature, humidity of air within the bubble of air itself, the atmospheric pressure, and the effect of interference within a cylinder of small caliber (2 in.). There was also a certain lack of precision and refinement in the measurements, the velocity being timed by stop-watch. However, the observations gave a fair idea of what was actually occurring and cleared up some points which, at first, seemed paradoxical; that is, that the bubbles do not have an accelerated velocity, and that the bubble does not increase in volume on rising through a lowered water pressure.

Returning now to the original discussion of the flow of sewage through an aerating launder, it is proper to conclude that the resistance to flow is caused by the following factors:

(*a*) The natural coefficient of friction of the sides and bottom of the channel itself;

(*b*) The resistance offered, mainly near the bottom of the channel, by the jet action or barrage of air particles which are traveling upward and floating horizontally with the current;

(*c*) The resistance due to cross-flow, or displacement flow under each rising bubble (this resistance will increase in direct proportion to the percentage of volume of air within the cross-section); and,

(*d*) The resistance due to vertical flow, upward and downward currents, which occur because unbalanced pressure columns have a higher or lower density caused by contained air. (The actual distance traveled by a particle of sewage is several times greater than the measured horizontal distance of flow).

Referring to Resistance (*c*), the path of the bubbles, traveling upward and floating sidewise, is measured by vector analysis. The higher the channel velocity, the more bubbles will remain in the sewage at any one time, on account of the longer travel distance before bursting at the surface. The lower the channel velocity, the greater is the resistance offered by the barrage at the bottom, since the air-jet has less deflection and such jet influence is continued for a longer time interval; however, there will be fewer air bubbles in the sewage on account of shorter travel in both time and distance.

A resultant of Resistances (*a*) to (*d*) will certainly slow down the velocity very appreciably. The fact remains, however, that the cross-section has been diminished far more by the influence of the bottom jet action in the case of low-velocity flow (having a constant volume of air per square foot of bottom area) than in the case of high-velocity flow. The agitation in the case of low flow should be greater than in the case of high flow, due to the increase of air to sewage ratio.

It would seem fairer to allow a deduction in area on account of jet action, rather than to show that a low velocity increases the coefficient of roughness, *n*, in Kutter's formula which, on the face of it, is contrary to theory.

In conclusion, the writer is grateful for this interesting paper by Mr. Townsend and for the light it has thrown on the subject of the hydraulics of aeration channels which, heretofore, has been much neglected.

It is to be regretted that the full data of the experiments were not published, such as the varying wetted cross-sections at different rates of measured flow. More experiments on this subject will be required before an accurate forecasting of the roughness coefficient, *n*, may be made on sewage aeration channels.

HENRY R. KING,[5] ESQ. (by letter).—This is the first publication of tests on the loss of head in aerated channels carrying activated sludge, that has come to the writer's attention. Such tests aid in the selection of economical channel sizes and are particularly helpful in the design of an aerated influent channel for a large battery of tanks, so that an equal division of the flow may be assured among the tank units with a minimum amount of regulation of the inlet gates.

The scarcity of data on the subject can be attributed to several reasons. The activated sludge process is relatively new; large plants are few; and, consequently, there is a lack of data on many special problems of design. The head losses occasioned by aeration constitute only a small part of those occurring in a plant as a whole. Of major importance are the losses at tank inlets and exits, through meters and bends, in passing over weirs, and in the conduits between the several stages involved in the complete treatment of the sewage.

---

[5] Asst. Civ. Engr., The Sanitary Dist. of Chicago, Chicago, Ill.

The designer may be 100% in error in estimating the additional friction of flow due to aeration, but the hydraulic capacity of the plant will be approximately as originally intended. Consequently, the need of experiment may not have been fully appreciated. If, however, it was, the fact remains that the aeration channels in an activated sludge plant are not as a rule very suitable for experiments. The velocities are low, the distances are short through which uniform flow obtains, and, consequently, the head losses are so slight as to make precise measurement difficult, inasmuch as measurements are required where the flow is disturbed by numerous rising air bubbles and where the variation in the porosity of the diffuser plates may result in a greater disturbance near one gauge than at another. However, the writer believes that tests can be made to secure worth while results.

It would have been of interest if Mr. Townsend had given a description of the procedure followed in making the tests at Milwaukee. Perhaps the floats would have been less disturbed by pulsations if the upturned entrances of the pipe connections to the float-tubes had been lower down in the channel than shown in Fig. 1. The weight of the liquid on the bottom of the channel remains constant regardless of the volume of entrained air. At an elevation some distance above the bottom, as the volume of air in the mixture increases, a greater proportion of the weight of the liquid is lifted above this elevation and, as a consequence, the pressure here increases. Therefore, one would presume that the nearer the entrances of the piezometer piping to the bottom of the channel the less the magnitude of the pulsations in the float-tubes, due to variations in the volume of air in the mixture.

The following procedure is suggested for making tests that involve relatively small losses, such as Mr. Townsend had at Milwaukee (0.02 to 0.03 ft). Care should first be taken to determine the inherent errors in the gauges and, before each test, they should be checked with the channel standing quiescent at the normal depth of flow. Then, the air should be turned on at the rate to be used during the test and the effect of aeration at zero velocity noted at the gauges. The flow should then be started and maintained at a constant rate for some time while the losses are recorded first with the air on and then with the air off. The difference in head loss for these two conditions of flow of course, would be that due to aeration.

In 1931, at the North Side Sewage Treatment Works, of The Sanitary District of Chicago, head-loss tests were made on a return sludge aeration channel, 4 ft wide, with vertical walls. On the bottom, on the center line, was one longitudinal row of diffuser plates (12 by 12 in.), in boxes, 18 in. wide, from which the bottom sloped upward at 45° to meet the channel walls. The return sludge flowed at depths between 4 and 5 ft above the plates. The hydraulic radius was approximately 1.4 ft and the rate of aeration, 1.4 cu ft of free air per min per lin ft of channel. The test was made over a length of 410 ft, in which the flow increased by five equal increments, each introduced from two pipes at right angles to the longitudinal axis of the channel about 6 in. above the surface and at intervals of about 70 ft. For 20 000 000 gal per day the velocity varied from 0 to 2.25 ft per sec, and the surface drop for the entire distance was 0.42 ft. Making allowances for head losses due to the falls of incoming sludge, in accordance with the method devised by Julian Hinds, M. Am. Soc. C. E., for side channel spillways,[6] the value of n in the Kutter formula was computed at 0.034, or more than twice the value for the same sludge in a closed conduit. This friction coefficient seemed high. To determine whether this was due to the use of air, another test was made for a total quantity of 20 000 000 gal per day. Stable flow was maintained while measurements were repeated several times, with the air alternately on and off.

The results showed an average surface drop of 0.42 ft with aeration and 0.41 ft without it. This test indicated that aeration had but slight effect on head loss in this particular case. Hence, the high loss must have been caused by the obstructions presented within the channel itself through the presence of the air pipes and the disturbances caused by the downward discharge of the pairs of incoming sludge pipes. Although the channel was far from ideal for the purpose of determining accurately the effect of aeration on head loss, the writer is inclined to believe, from the results obtained, that the effect may be relatively small when compared to the losses that occur without the presence of aeration.

M. H. KLEGERMAN,[7] ASSOC. M. AM. SOC. C. E. (by letter).—Further evidence that engineers engaged in the design and operation of hydraulic works are seeking and finding information concerning hydraulic coefficients, either to re-assure themselves of the work of the early investigators, or to find applications for new conditions, is presented by the tests performed at Milwaukee, Wis., and described in this paper.

Many of the empirical data upon which design to-day is based, had their beginning, as is generally known, in the middle of the Eighteenth Century (Brahms, 1753; Brahms and Chezy, author of the well-known formula, $V = c\sqrt{RS}$, in 1775), and many of the coefficients often accepted by engineers at the present time, without much concern as to their limitations or applicability to modern materials, were developed by Ganguillet and Kutter and others, in the latter part of the Nineteenth Century. In the course of the translations of the work of Ganguillet and Kutter, additional important contributions were made to the hydraulic literature on the subject of frictional coefficients by the various translators. Probably an outstanding contribution since the beginning of the Twentieth Century, can be found in the work of Williams and Hazen, in the development of their formula, now so well known, for the determination of flow in pipes, together with the publication of their hydraulic tables presenting frictional coefficients and head losses.

In the water-works field particularly, the comparatively recent introduction of new material (bituminous and cement coatings for steel and cast-iron pipes, used both for protection against corrosion and for increasing the carrying capacity by providing a smoother interior surface; the greater use of concrete pipe; and the recent introduction in the United States of such new pipe-manufacturing methods and materials as centrifugally spun cast-iron pipes, modern steel and concrete pipe, and pipe of a cement and asbestos compound) has created keen interest among engineers as to the question of coefficients of flow and their interpretation as to capacity.

---

[6] "Side Channel Spillways," by Julian Hinds, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 89 (1926), p. 881.

[7] Asst. Engr., Alexander Potter, New York, N. Y.

The technical literature of other countries, particularly Great Britain, also discloses that the relatively recent departure in pipe lining from the original practice of merely dipping cast-iron or steel pipes in a coal-tar and linseed-oil preparation, is directing considerable interest among European engineers to the necessity of obtaining a formula for the discharge of these new types of pipe. It is the opinion that without such a formula it is not feasible with certainty to watch the rate at which discharge is diminishing, since no common datum is available on which to base later results.

More information, too, is now sought not only on carrying capacities of conduits of these materials when new, but also on the relative ability to maintain initial carrying capacities and their rates of reduction, under various conditions of usage.

While engineers who have the facilities and opportunities at hand for making these tests are engaged upon such work, it would seem highly desirable to extend it into an examination of conditions of flow in which not only water is concerned, but in which water is in an intimate physical combination with other substances, such as air, in the case of aeration channels, or sludge, in the case of sludge force mains. Furthermore, conditions about sewage treatment plants present many cases of varied or unsteady flow, even where sewage alone is concerned. Instances of test data for head losses under such conditions, are not many, and an organized attempt to obtain and correlate mass data of this kind would prove of value to designing engineers.

The tests described by Mr. Townsend indicate that head losses occur to an extent probably not before fully realized, and must be considered in the design of channels of the type described. It is unfortunate that the tests had to be made on a comparatively short length of channel (the length being limited only to 123.34 ft), because, as stated in the paper, the range in pulsation of the recorders was from three to four and one-half times the loss in head over the section. In view of this condition, it is difficult, immediately, to accept the conclusion that the "final results stated in the paper are sufficiently on the side of safety to warrant confidence in their subsequent use." The likelihood of finding aeration channels of considerable length for testing purposes, is probably very limited, however.

It is noted that the value of the slope used for computations in Kutter's formula, was that of the water surface as determined from the depth recorders established at each station. In an open channel of the type described, it is likely that disturbances introduced by the injection of compressed air, atmospheric disturbances, irregularities of the channel and of the diffuser plates, etc., may have resulted in different depths of flow and, consequently, in different velocities at each of the observation stations. In that event the slope of the energy gradient, instead of the water surface, would result, of course, in more accurate determinations. The actual effect upon the value of n as so determined, however, can not be known without the information required to compute the energy gradients at each station.

If other tests of this nature should be undertaken, it would be of value to know what the temperature conditions and atmospheric pressures are during the time of test, as these conditions may influence air-bubble coalescence which, as the author described, has an important bearing on the resistance to flow. In open channels of the type used in the Milwaukee test, some resistance to flow must certainly be created by wind resistance at the water surface exposed to the atmosphere. In tests performed upon large sewers flowing partly full, the writer has found that values of n are somewhat higher (due, it is believed, to "air resistance" at the surface) than when flowing full or nearly full. Observations on the direction and intensity of wind during tests, therefore, might be of some value in comparisons of test data.

"Before-and after" tests—that is, determinations of head loss with and without aeration—are important in order that the effect of the latter may be fully evaluated. The author has apparently obtained such information, as he gives the value of $n = 0.013$ for the channel with non-aerated sewage, although it is not clear whether this value has been determined from tests with diffuser plates in position, or whether it is an assumed condition.

Information as to surface and bottom velocities, as well as on mean velocities and their inter-relation, would perhaps help determine to what extent the flow varied from "free-water" flow, and, also, in what sections of depth the greatest disturbances occur.

The varying value of n with varying velocities, as reported by the author (that is, $n = 0.03$ at a velocity of 1.3 ft per sec, with $n = 0.23$ as velocity approaches zero), indicates, of course, that the condition of resistance to flow as denoted by n, is "internal" in Nature; that is, it more nearly represents a state of the contained liquid under varying conditions, than a condition of the channel itself, with which Kutter's n is usually associated. While the author brings out the fact that for the purpose of his paper, the subject is treated from the viewpoint of an increased roughness coefficient, assuming the flowing medium to possess the hydraulic properties of water alone, it would appear of interest and along the methods often adopted in estimating transition losses, etc., to learn whether or not head losses for a flow of an air and sewage mixture could not be more practically expressed in terms of "percentage of loss of head for clear water." Expressed in these terms, there may be the further advantage of eliminating the use of a varying coefficient.

There is another explanation, however, that might be made in connection with the author's conclusion that the coefficient of roughness, n, increases as the velocity decreases.

By reference to Fig. 1, it will be observed that the channel in which the tests were performed is composed of two different materials, namely, the sides of concrete and the bottom of a combination of concrete and diffuser plates.

In other words, the bottom and sides of the channel are of different degrees of roughness; the roughness coefficients for the channel bottom and sides have different values.

Robert E. Horton, M. Am. Soc. C. E., refers[8] to this condition as "composite" roughness. According to Mr. Horton,

> "The value of Kutter's or Manning's n applicable to the sides of such a channel may be designated as n_s and that applicable to the bottom as n_b. The value of n derived from experiments on such a channel may be designated the 'equivalent' roughness. If a channel had a degree of roughness corresponding to n throughout its entire cross-section, it would give the same discharge and velocity as the actual or experimental channel at the same depth and slope."

Mr. Horton further established the fact that a value of equivalent roughness (assuming, of course, constant "internal" characteristics of the flowing liquid) derived from experiments on a given channel is really applicable to a single depth. In other words, if the roughness of the bottom is not the same as that of the channel sides, the equivalent value of n varies with the depth.

A formula suggested by Mr. Horton for determining the value of n applicable to channels with two degrees of roughness, is as follows:

$$n = \left(\frac{n_b^{\frac{3}{2}} + 2\,z\,n_s^{\frac{3}{2}}}{1 + 2\,z} \right)^{\frac{2}{3}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

in which, z = ratio of depth of flow in channel to bottom width.

As is evident from the foregoing, the composite n will vary with z, so that if $z = 0$, $n = n_b$, and n approaches n_s as z approaches infinity.

In the absence of more specific observational test data, it is not possible to state whether the results obtained by Mr. Townsend would agree fully with Equation (3). The original observations may contain sufficient data so as to make possible the computation of n_b from this equation (assuming n_s to be equal to 0.013.) Having computed n_b for a given value of z and n, it would then be a simple matter to substitute in Equation (3) the value of n_b thus obtained, and with a different observed value of z, compute the value of the composite n. Comparison of the value of n thus computed with that reported for a specific velocity, might establish whether or not the relation implied by the Horton formula holds in the case of aeration channels.

The author is to be commended for his work and method of presenting results. Of particular interest is the presentation of the "cause and effect" theories. His efforts will undoubtedly result in directing attention to this phase of sewage flow and should encourage others to investigate and publish their results for the benefit of all engaged in the design of sewage works.

DARWIN WADSWORTH TOWNSEND,[9] M. AM. SOC. C. E. (by letter).—Contributions to the subject-matter of his paper by Messrs. Thackwell, King, and Klegerman are gratefully acknowledged by the writer. Commenting generally upon these discussions, attention might well be directed again to the velocity-retarding effect attributable to air-bubble induction and the subsequent coalescence encountered in practice, as differentiated from the assumed hypothesis based upon laboratory observations made in connection with air bubbles rising through clear and quiescent water contained in glass cylinders.

---

[8] *Engineering News-Record*, November 30, 1933.

[9] Cons. Engr. (Consoer, Townsend, Older and Quinlan, Inc.) Milwaukee, Wis.

That reduced cross-sectional area obtains due to the continuous presence of diffused air throughout the liquid medium, there can be little doubt. There also can be little doubt that the continuous and irregular swirling of an aerated mass results in cross-currents, up-and-down currents, and diametrically opposed currents, which would vary, continuously, the degree and character of the flow in a cross-section. For this obvious reason the writer elected not to attempt to state increased frictional resistance in terms based upon reduced cross-section and consequent increased velocity. In the writer's judgment a roughness or retarding factor, most prominent in character, is present in the flowing liquid mass.

The air bubbles contained in the aerated mass do not merely float along in the stream current in an undisturbed state as a log of wood might be expected to float along on the surface of a stream; they are decidedly and irregularly active in their continuous ascent, through expansion and coalescence, and the combination of both. If small logs instead of air bubbles were liberated in a continuous stream from a channel bottom, the flow-retarding effect would be quite dissimilar to that which might be expected from the same logs floating horizontally in the current.

It is true that the friction loss per unit of aerated channel length may be relatively small under conditions of low velocity. These losses on the other hand may be relatively large under conditions of high velocities. Furthermore, the accumulation of many relatively small friction loss increments may, and do, represent a substantial percentage of the total friction head in the design of a sizable activated sludge plant.

The conditions that obtain in a flowing aerated stream do not lend themselves ideally to head-recording accuracy. This fact the writer attempted to make clear in his discussion relative to pulsations; the results he obtained, and those encountered in practice, are sufficiently accurate to warrant their subsequent use with safety in problems of design.

It is obvious why the *n*-value increases as flow velocity decreases. If the upward or vertical velocity of the air-bubble stream, for instance, were greater than that of the horizontal liquid flow velocity, a curve indicating the degree of resistance would more nearly approach the vertical than would be the case if the velocities were applied in the reverse order. In other words, the same general theory of head loss and velocity resultant which applies to two horizontally flowing streams uniting with each other at an angle, applies in a somewhat modified degree in the case of the problem under discussion. In both cases it becomes necessary to build up sufficient head to overcome the resistance encountered. The trend curve (Fig. 5), which was based somewhat upon chance projection is the best illustration of the relationship between *n*-values and velocities.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 1904

WILLIOT EQUATIONS FOR STATICALLY INDETERMINATE STRUCTURES

IN COMBINATION WITH MOMENT EQUATIONS IN TERMS OF ANGULAR DISPLACEMENTS

BY CHARLES A. ELLIS,[1] M. AM. SOC. C. E.

WITH DISCUSSION BY MESSRS. RALPH A. TUDOR, LEON S. MOISSEIFF, A. A. EREMIN, AND CHARLES A. ELLIS

SYNOPSIS

If the Williot diagram served only as the graphic solution for the deflections of a truss, it would still hold a prominent place among the analytic methods used by the designing engineer; but its usefulness is by no means so circumscribed. When combined with Maxwell's theorem of reciprocal displacements it is the quickest solution for statically indeterminate reactions in continuous spans, two-hinged arches, and similar structures. It also serves in determining the angular displacements of members in secondary stress analysis.

Another inherent quality in the Williot diagram recently discovered should be of considerable assistance in stress analysis. So far as known, it is a new idea in the theory of structures. Like many other inventions, necessity was the mother of this one. In studying a large and unusual structure, the writer encountered a problem in stress distribution that would not yield to an exact or even to a satisfactory solution because of the lack of sufficient equations, until he discovered them in the geometric properties of the Williot diagram.

This paper recites the circumstances leading up to this dilemma, describes the development of the Williot strain equations (Equations (13) to (21), inclusive, and Equations (29), (30), (36), and (37)), and shows how they were combined with moment equations expressed in terms of angular displacements to meet the situation.

NOTE.—Published in January, 1934, *Proceedings*.

[1] Prof. of Structural Eng., Purdue Univ., Lafayette, Ind.

Certain abnormal structures, such as some types of towers for suspension bridges, do not lend themselves readily to solution for statically indeterminate stresses. Special methods are found necessary. This paper describes the expedient of expressing moments in terms of angular displacements as the vehicle by which such problems may be carried to a solution. The expressions for moments at the ends of the member, *IK* (Fig. 1), for example, are usually written in the form:

$$\left.\begin{aligned} M_{ik} &= \frac{2\,EI}{l_{ik}}\,(2\,\phi_i + \phi_k - 3\,\psi_{ik}) \\ M_{ki} &= \frac{2\,EI}{l_{ik}}\,(2\,\phi_k + \phi_i - 3\,\psi_{ik}) \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots(1)$$

in which, E is the modulus of elasticity, and I represents the constant moment of inertia. When I is not constant throughout but has the magnitude, I_{ik}, for the upper half and I_{ki} for the lower half, and if $I_{ki} = nI_{ik}$, the expressions may be written, as follows:

$$\left.\begin{aligned} M_{ik} &= \frac{8\,n\,EI_{ik}}{l_{ik}\,(n^2 + 14\,n + 1)}\,[(n + 7)\,\phi_i + (2\,n + 2)\,\phi_k - (3\,n + 9)\,\psi_{ik}] \\ M_{ki} &= \frac{8\,n\,EI_{ik}}{l_{ik}\,(n^2 + 14\,n + 1)}\,[(7\,n + 1)\,\phi_k + (2\,n + 2)\phi_i - (9\,n + 3)\,\psi_{ik}] \end{aligned}\right\} \ldots(2)$$

Whatever the variations in I, the moments may still be expressed as functions of the angles, ϕ and ψ. Thus,

$$\left.\begin{aligned} M_{ik} &= a\,\phi_i + b\,\phi_k - c\,\psi_{ik} \\ M_{ki} &= d\,\phi_k + e\,\phi_i - f\,\psi_{ik} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

the coefficients, a to f, being expressed numerically when the variations in I are known.

Since the writing of these equations requires known moments of inertia, the application of this method postulates a designed structure. If the structure is statically indeterminate, in order to compute the approximate stresses required for designing, it is necessary to formulate a tentative design by making certain assumptions as to the probable behavior of the structure, which will render it statically determinate. This tentative design will contain all the errors made in the assumptions. It is desirable, therefore, that the assumptions closely approximate the actual conditions. This is not a difficult matter for the experienced engineer when dealing with normal structures.

Other structures, however, may be so novel or peculiar in type or so extraordinary in size, or both, that they do not come in this category. The writer found this to be true when designing the towers for a suspension bridge, given to him in the form shown in Figs. 2 and 3. This pattern of wind bracing makes the structure many times statically indeterminate. It may not be a good example of economic or efficient bracing, but it illustrates the discovery made while designing it.

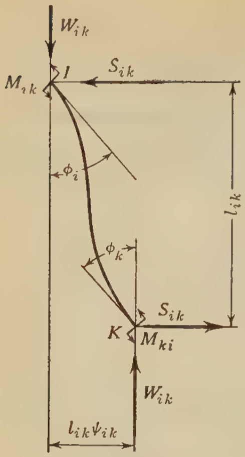

W_{ik}
M_{ik}
I
S_{ik}
ϕ_i
ϕ_k
l_{ik}
S_{ik}
K
M_{ki}
W_{ik}
$l_{ik}\psi_{ik}$

Fig. 1

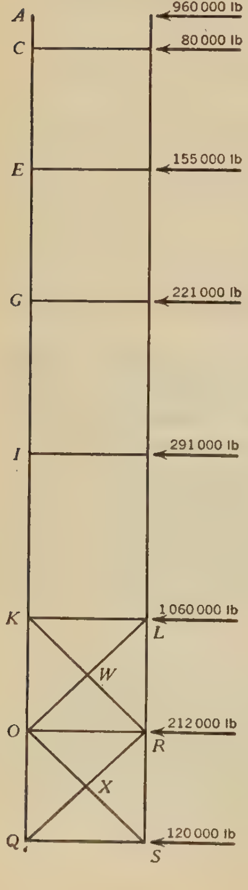

A
C
E
G
I
K
L
W
O
R
X
Q
S
960 000 lb
80 000 lb
155 000 lb
221 000 lb
291 000 lb
1 060 000 lb
212 000 lb
120 000 lb

FIG. 2.

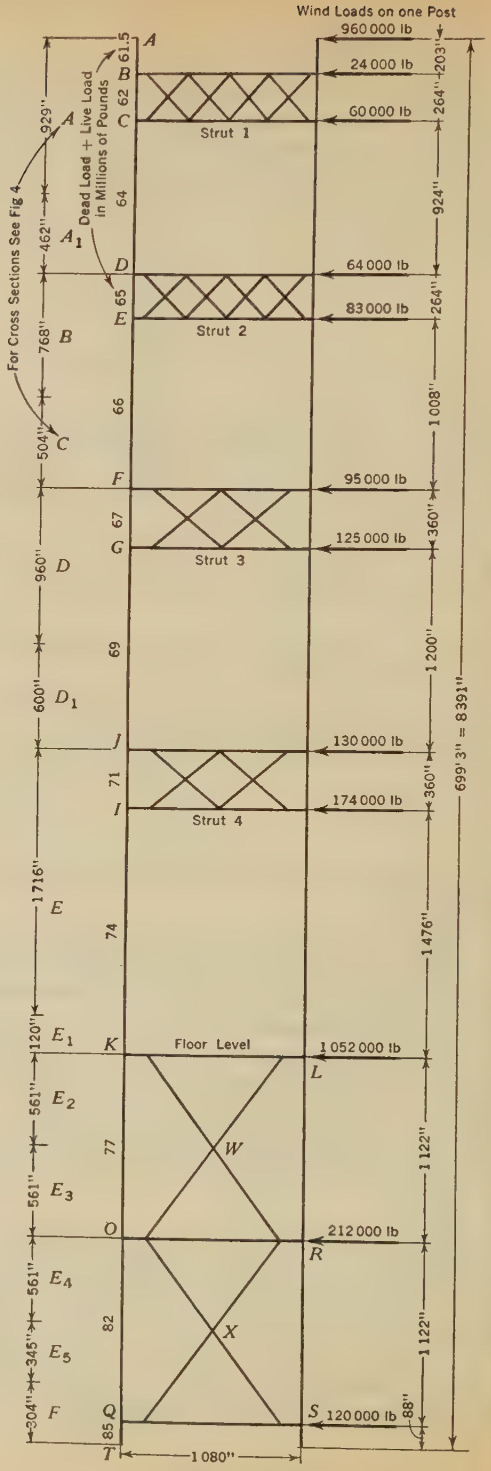

Wind Loads on one Post
960 000 lb
24 000 lb
60 000 lb
64 000 lb
83 000 lb
95 000 lb
125 000 lb
130 000 lb
174 000 lb
1 052 000 lb
212 000 lb
120 000 lb
203"
264"
924"
264"
1 008"
360"
1 200"
360"
1 476"
1 122"
1 122"
88"
699' 3" = 8 391"
Dead Load + Live Load in Millions of Pounds
61.5
62
64
65
66
67
69
71
74
77
82
85
For Cross Sections See Fig 4
929"
462"
768"
504"
960"
600"
1 716"
120"
561"
561"
561"
345"
304"
A
A₁
B
C
D
D₁
E
E₁
E₂
E₃
E₄
E₅
F
A
B
C
D
E
F
G
J
I
K
L
O
R
Q
S
T
W
X
Strut 1
Strut 2
Strut 3
Strut 4
Floor Level
1 080"

FIG. 3

The axial wind stresses and moments in the post members above the floor may be rendered statically determinate by assuming a point of contraflexure at the middle of each post member; or, in other words, by assuming that the moments at the two ends of each member are equal. The sum of these two moments for given loads is constant, or very nearly so, whatever the design and, furthermore, in the interest of economy, they should be nearly equal; but designing the horizontal struts to accomplish this end is not so simple a problem.

Again, if this were an ordinary structure, the wind stresses in the diagonals below the floor could be rendered statically determinate without appreciable error, by assuming that the horizontal shear was resisted entirely and equally by the two web systems. It is not an ordinary structure, however; the wind shear above the floor must be resisted entirely by shears and bending moments in the posts. The post members above the floor, therefore, must be very stiff; and those below the floor must be similar in size for practical purposes. Because of this extra stiffness, the posts participate with the bracing in resisting the shear to such an extent that the assumption of of all shear being resisted by the diagonals becomes a gross error. This paper will show how, in the first approximation, this assumption, for lack of a better one, was made for a tentative design, and how by a new process it was eliminated in the third approximation.

The design began and proceeded in a series of four approximations followed finally by a review. Each approximation included a computation of stresses based on certain assumptions, followed by a tentative design. As one approximation followed another the assumptions were either corrected or eliminated so that each tentative design approached more closely to the final design. This process was continued until all assumptions except one were eliminated. Since the external forces acting on each post were the same, their elastic curves were assumed to be identical. This assumption postulates equal bending moments and shears, equal and opposite axial wind stresses in the two posts, and equal values of ϕ and ψ for corresponding joints and members.

First Approximation

The towers were tentatively outlined as shown in Fig. 2. A reasonable share of the allowed unit stress was reserved for axial wind stress and bending stresses of all kinds, and the remainder was allocated to axial dead and live load stresses. Cross-sectional areas were tentatively determined and the cellular tower sections (Fig. 4) were laid out, giving due regard to slenderness ratios. The tower wind loads were computed and the additional wind loads from the cables and suspended structures were included at the top and at the floor level, *KL*, respectively. These loads, as shown in Figs. 2 and 3, are for one post only; they are the same for both posts.

The wind stresses in the diagonals were computed on the assumption that the two web systems resisted all the horizontal wind shear equally divided. After the erection and riveting of the two lower panels, the diagonals must participate with the posts in supporting whatever vertical dead and live loads are added to the posts thereafter.

Let p = the dead and live load unit stress in the post; p', the participation unit stress in the diagonal; and θ, the angle, OKW (Fig. 2). Then, $p' = p \cos^2 \theta$. This dead and live load participation stress was estimated, thus determining the allowable unit wind stress, and the diagonals were designed accordingly. The member, KL, was designed to meet the requirements of a floor-beam, and the two lower horizontal members were designed as nominal members free from primary stress. Except for the horizontal struts above the floor the first tentative design was complete.

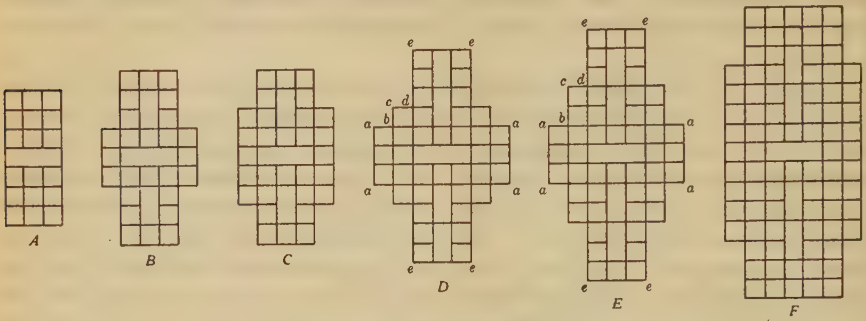


Fig. 4.—Typical Cross-Sections of Tower (See Fig. 3 for Locations).

Second Approximation

In the next computation, the transverse wind stresses and moments in the posts were rendered statically determinate by assuming a point of contraflexure at the mid-height of each panel. Analysis of the cables and suspended structure showed that the tower must submit to a longitudinal deflection at the top equivalent to 17 in. in either direction. The longitudinal moment of inertia of each post member was computed. The section of the post changed at each panel point. A parabola tangent to the center line at the base of the post and having an abscissa of 17 in. at the top, was assumed for the elastic curve. The moment of the eccentric dead, live, and wind loads was computed at each panel point and combined with the moment of a horizontal force, H, acting at the top. An expression for the deflection at the top was developed by the area-moment method and equated to 17 in. With this equation solved for H, the total longitudinal moment was computed for each panel point.

The axial dead, live, and wind stresses and the transverse and longitudinal moments were now approximately known. The resulting unit stresses at cross-sections above and below each panel point were computed for critical points in each cross-section, and the first tentative design was thus revised.

In this revision further attention was given to the number and arrangement of cells in the various cross-sections, and to the area and kind of metal to be used in each. These studies resulted in the use of six typical cross-sections, as shown in Fig. 4, the locations of which are given in Fig. 3. The saddle casting governed the size of Section A, and the minimum allowed radius of gyration for the lower half of the post determined the size and shape of Sections E to E_5, which are alike, except for the thickness of metal in the faces, *aa* and *ee* (Fig. 4). A minimum allowed thickness of metal fixed the minimum area of metal in any given section, and this fact greatly influenced the grade of steel used. The intermediate Sections B, C, and D, were finally chosen so that silicon steel was required only in Sections A, B, and C. In the remaining sections carbon steel with minimum thickness of plates ($\frac{7}{8}$ in.) and angles ($\frac{5}{8}$ in.) was used throughout, except in the extreme faces, *aa* and *ee*. By limiting the variations to these extreme faces, the transverse moments of inertia could be varied considerably by a relatively small change in area, and caused little change in the longitudinal moment of inertia, and *vice versa*. Later developments in the design proved that the sections were economic and well chosen.

THIRD APPROXIMATION

For the purpose of investigating the transverse elastic curve the structure was divided into two parts at a point just above the member, KL. The wind strains were computed from stresses as previously determined in the members of the lower part and a Williot diagram was drawn, assuming Q and S (Fig. 2) as fixed in position. Transverse moments of inertia were computed and the moments at the ends of each member expressed in the form of Equations (1) to (3), except that ψ for each member was given the numerical value determined from the Williot diagram. The wind forces on the structure are shown acting from right to left. The moments of these forces above and about any section are counter-clockwise and in this paper are called positive, hence the counter-clockwise couples, M, and the counter clockwise rotations, ϕ and ψ, are positive.

The equations, $\Sigma M = 0$, were written for Joints K, W, O, X, and Q. Since ϕ has the same value for corresponding joints; that is, $\phi_l = \phi_k$, etc., only the two diagonal members on the left of the center line of the tower need be considered in the equations, $\Sigma M = 0$, for Joints W and X. In the equation for Joint K, M_{ki} was not expressed in terms of ϕ_i and ϕ_k, but was equated to the known quantity found at the beginning of the second approximation, for by so doing ϕ_i was eliminated from the equation. The five equations were solved and the angles, ϕ, determined.

In the treatment of the upper portion, M_{ik} and M_{ki} were expressed in terms of ϕ_i, ϕ_k, and ψ_{ik}, and each was equated to its approximate value as determined in the second approximation. The known quantity obtained in the analysis of the lower portion was substituted for ϕ_k and the two equations were solved for ϕ_i and ψ_{ik}. The remaining members of the post were similarly treated, and the quantities, ϕ, for each joint and, ψ, for each member determined. From the known values of ψ the transverse moments of the eccentric dead and live loads were computed, and the sum, M_{ak}, of these moments from A to K was determined. Since the wind stresses were tensile in one post and compressive in the other, their eccentric moments balanced each other and, therefore, were not included.

Heretofore all computations had been based on the outline sketch of Fig. 2. A drawing to scale was now made of the transverse view. The horizontal struts above the floor were assigned vertical dimensions, giving a proper architectural balance to the structure, as outlined in Fig. 3. When the bracing below the floor was added, it was evident that, for good detailing, the diagonals should intersect the horizontals at the inside faces of the posts rather than on their center lines. These eccentric connections had also a theoretical advantage, since the resulting eccentric moments reduced the lateral deflection of the tower. The member, QS, was raised to a proper distance from the base and a new post member, QT, entered the problem.

The computations for this second tentative design had been made on the assumption that the shear in each panel below the floor was resisted entirely by the web system. The process by which this assumption was eliminated, namely, by analyzing the wind shear distribution between the posts and bracing, will now be explained. Throughout the analysis the following symbols with identification subscripts will be used. All dimensions are expressed in inches; all loads and stresses, in pounds; and all moments, in inch-pounds:

b = distance center to center of posts.
W = dead and live load stress in any post member.
H = horizontal shear in any post member, or the horizontal component of stress in any diagonal below the floor.
V = vertical component of stress in any diagonal below the floor, or the vertical shear in any strut above the floor.
P = axial wind stress in any member.
l = length of any member.
$\Delta = \dfrac{Pl}{AE}$ = the wind strain in any member.
S = one-half the total wind shear in any panel.
M_{ak} = sum of the transverse eccentric moments in one post from A to K.
M_W = moments of wind forces on one post above and about W.
M_X = moments of wind forces on one post above and about X.
M_T = moments of wind forces on one post about the center line of the tower at the base.

From Fig. 5,

$$H_{ko} + H_{kw} = S_{ko} \ldots\ldots\ldots\ldots (4)$$

also,

$$M_{ko} + M_{ok} + l_{ko}\,\psi_{ko}\,W_{ko} + l_{ko}\,H_{ko} = 0 \ldots\ldots\ldots\ldots (5)$$

or,

$$M_{ko} + M_{ok} + l_{ko}\,\psi_{ko}\,W_{ko} = l_{ko}\,(H_{kw} - S_{ko}) \ldots\ldots\ldots\ldots (6)$$

The quantities, M_{ko} and M_{ok}, may be expressed in terms of ϕ_k, ϕ_o, and ψ_{ko}, while l_{ko}, W_{ko}, and S_{ko} are known; hence, H_{kw} may be expressed in terms of ϕ_k, ϕ_o, and ψ_{ko}. As H_{kw} is the horizontal component of the stress in KW and KWR, the strains in these members, Δ_{kw} and Δ_{kwr}, may be expressed in terms of ϕ_k, ϕ_o, and ψ_{ko}. Likewise, Δ_{ox} and Δ_{oxs} may be expressed in terms of ϕ_o, ϕ_q, and ψ_{oq}.

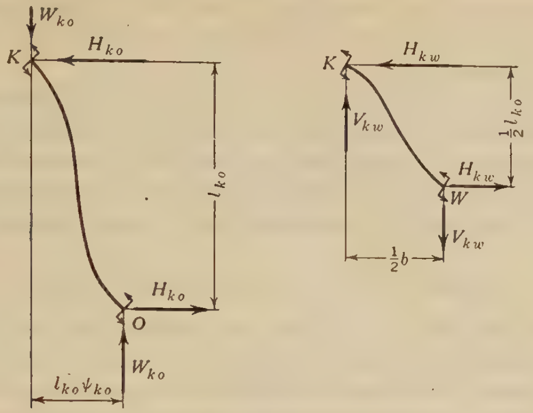


FIG. 5.

The stress in KO may be found by balancing the moments about W and observing that the moment at the middle of the member, KO, is $\frac{M_{ok} - M_{ko}}{2}$, whence,

$$\frac{b}{2} P_{ko} = M_W + M_{ak} + \frac{l_{ko}}{2} \psi_{ko} W_{ok} + \frac{M_{ok} - M_{ko}}{2} \quad \ldots\ldots\ldots\ldots(7)$$

from which, Δ_{ko} may be expressed in terms of ϕ_k, ϕ_o, and ψ_{ko}. Also,

$$\frac{b}{2} P_{oq} = M_X + M_{ak} + l_{ko} \psi_{ko} W_{ko} + \frac{l_{oq}}{2} \psi_{oq} W_{oq} + \frac{M_{qo} - M_{oq}}{2} \quad \ldots. (8)$$

from which, Δ_{oq} may be expressed in terms of ϕ_o, ϕ_q, ψ_{ko}, and ψ_{oq}. Likewise,

$$\frac{b}{2} P_{qt} = M_T + M_{ak} + l_{ko} \psi_{ko} W_{ko} + l_{oq} \psi_{oq} W_{oq} + l_{qt} \psi_{qt} W_{qt} + M_{tq} \quad \ldots. (9)$$

from which, Δ_{qt} may be expressed in terms of ϕ_q, ψ_{ko}, ψ_{oq}, and ψ_{qt}.

The equations, $\Sigma M = 0$, were written for Joints K, W, O, X, and Q; also, the equation,

$$M_{tq} + M_{qt} + l_{qt} \psi_{qt} W_{qt} + l_{qt} S_{qt} = 0 \quad \ldots\ldots\ldots\ldots\ldots(10)$$

a total of six equations.

A moment term, M_k, to account for the eccentric diagonal connection was included in the equation, $\Sigma M = 0$, for Joint K, as follows: Let a equal one-half the width of the post at K, then $M_k = aV_{kw}$. From Fig. 5:

$$M_{kw} + M_{wk} + \frac{1}{2} l_{ko} H_{kw} = \frac{1}{2} b\, V_{kw} \ldots\ldots\ldots\ldots\ldots(11)$$

whence M_k may be expressed in terms of ϕ_k, ϕ_w, and ψ_{kw}. The eccentric moments at O and Q are treated in a similar manner.

In the equation for Joint K a correction is made in the value of M_{ki}, which was previously determined as indicated in the first paragraph of the second approximation by assuming a point of contraflexure at the mid-height of the panel, or, in other words, by making $M_{ik} = M_{ki}$ in the equation, $M_{ik} + M_{ki} + l_{ik} S_{ik} = 0$.

After the value of M_{ki} had thus been obtained, the panel height, l_{ik}, was changed by assigning a depth to Strut 4; and also an approximate value for ψ_{ik} was determined, as described in the third paragraph of this third approximation. Therefore, the term, $l_{ik} \psi_{ik} W_{ik}$, representing the moment of the eccentric dead and live loads, may now enter the computations. Hence, from Fig. 1,

$$M_{ik} + M_{ki} = -l_{ik} S_{ik} - l_{ik} \psi_{ik} W_{ik} \quad \text{.............} (12)$$

From Equation (12) it is obvious that the sum, $M_{ik} + M_{ki}$, is statically determinate whatever the design, except for the quantity, ψ_{ik}, which is so comparatively small that any variation in its magnitude will have slight effect on the sum. For example, the magnitude of ψ_{ik} was found to be approximately 0.0017; hence,

$$\begin{aligned} -l_{ik} S_{ik} &= - \quad 1\,476 \times 1\,715\,000 = -\ 2\,531\,000\,000 \text{ in-lb} \\ -l_{ik} \psi_{ik} W_{ik} &= -1\,476 \times 0.0017 \times 74\,000\,000 = -\ 186\,000\,000 \text{ in-lb} \\ \hline M_{ik} + M_{ki} &= -\ 2\,717\,000\,000 \text{ in-lb} \end{aligned}$$

Instead of dividing this sum equally it was decided to divide it proportionately to the section moduli of the post at I and K, so that the resulting unit stresses would be equal, with the following result;

$$\begin{aligned} M_{ik} &= -\ 1\,291\,000\,000 \text{ in-lb} \\ M_{ki} &= -\ 1\,426\,000\,000 \text{ in-lb} \\ \hline M_{ik} + M_{ki} &= -\ 2\,717\,000\,000 \text{ in-lb} \end{aligned}$$

The magnitude of M_{ki} was thus arbitrarily fixed at $-1\,426\,000\,000$ in-lb, the design was made to conform thereto, and any variation in the sum of the two moments on account of later changes in ψ_{ik} was absorbed, therefore, in M_{ik}. The term, M_{ki}, is expressed numerically in the equation, $\Sigma M = 0$, for Joint K; therefore, the quantities, ϕ_i and ψ_{ik}, do not appear in the equation.

The post is assumed to be completely restrained at T; that is, $\phi_t = 0$. Hence, the unknowns appearing in the six equations are a ϕ for each of five joints and a ψ for each of ten members, making fifteen unknowns. Nine additional equations are necessary therefore, for a solution. These nine equations may be obtained from the geometrical properties of Fig. 6. This is a Williot diagram as ordinarily understood with certain basic variations. Usually such a diagram can be made only when the strains corresponding to Δ_{qx} and the other heavy lines on Fig. 6 are known. Actually, the lengths of these lines are unknown; but their directions are known. Hence, the Williot diagram can be sketched and from its geometrical properties relations can be set up between the various strains in the form of equations.

Letting θ (Fig. 6) indicate the angle that the diagonal makes with the vertical, the nine required equations will then be as follows:

$$l_{qx}\,\psi_{qx} = \Delta_{qt}\operatorname{cosec}\theta + \Delta_{qx}\cot\theta \quad\ldots\ldots\ldots\ldots(13)$$

$$l_{ox}\,\psi_{ox} = (\Delta_{qt} + \Delta_{oq})\operatorname{cosec}\theta + \Delta_{ox}\cot\theta \quad\ldots\ldots\ldots\ldots(14)$$

$$l_{ow}\,\psi_{ow} = (\Delta_{qt} + \Delta_{oq})\operatorname{cosec}\theta + \Delta_{ow}\cot\theta \quad\ldots\ldots\ldots\ldots(15)$$

$$l_{kw}\,\psi_{kw} = (\Delta_{qt} + \Delta_{oq} + \Delta_{ko})\operatorname{cosec}\theta + \Delta_{kw}\cot\theta \quad\ldots\ldots\ldots\ldots(16)$$

$$l_{oq}\,\psi_{oq} = (2\,\Delta_{qt} + \Delta_{oq})\cot\theta + \Delta_{oxs}\operatorname{cosec}\theta \quad\ldots\ldots\ldots\ldots(17)$$

$$l_{ko}\,\psi_{ko} = (2\,\Delta_{qt} + 2\,\Delta_{oq} + \Delta_{ko})\cot\theta + \Delta_{kwr}\operatorname{cosec}\theta \ldots\ldots\ldots\ldots(18)$$

$$b\,\psi_{qs} = 2\,\Delta_{qt} \ldots\ldots\ldots\ldots(19)$$

$$b\,\psi_{or} = 2\,(\Delta_{qt} + \Delta_{oq}) \quad\ldots\ldots\ldots\ldots(20)$$

$$b\,\psi_{kl} = 2\,(\Delta_{qt} + \Delta_{oq} + \Delta_{ko}) \quad\ldots\ldots\ldots\ldots(21)$$

The expression previously derived for each Δ, in terms of ϕ and ψ, may now be substituted in these nine equations, and the fifteen equations solved for the fifteen unknown quantities, ϕ and ψ.

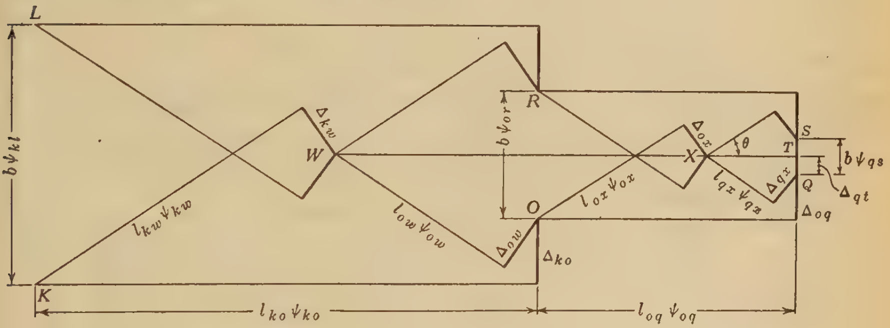


FIG. 6.

In this particular design, the diagonals in the two panels had equal areas since the wind shear differed but slightly in the two panels, one-half the shear being 2 767 000 lb in the upper panel and 2 979 000 lb in the lower panel. When the equations had been solved and the stresses, moments, and shears computed, it was found that the diagonal, *KWR*, was carrying a shear of 3 162 000 lb and resisting not only the wind shear but also the 395 000-lb shear in the post, *KO*, as well; while the diagonal, *OXS*, was carrying a shear of only 1 363 000 lb, the remaining 1 616 000 lb being resisted by the post, *OQ*. The moment, M_{ko}, was nearly double the moment, M_{ok}, while the moment, M_{oq}, was only about one-tenth the moment, M_{qo}, and a point of contraflexure still remained in this member. The stress in *KWR* was considerably more than double the stress in *OXS*.

These results were rather astonishing at first sight, but a numerical check showed that the moments, shears, and stresses balanced for each joint and section. The computations clearly indicated that a better balance in shear distribution should be sought and that strain rather than strength should be the governing factor in the design of the web system. Shifting from one system of bracing to another in the middle of a structure is not unlike changing commanders in the midst of a battle or swapping horses in the middle of a stream. Radical changes in the diagonals and minor changes in the posts were necessary before a satisfactory balance in shear distribution was secured, each set of changes being followed by corresponding revisions in the fifteen equations which were then solved for the fifteen quantities, ϕ and ψ.

In the treatment of the upper portion, M_{ki} was expressed in terms of ϕ_k, ϕ_i, and ψ_{ik}, and equated to its fixed magnitude. In Equation (12) the known magnitude of M_{ki} was substituted and M_{ik} expressed in terms of ϕ_k, ϕ_i, and ψ_{ik}. The quantity, ϕ_k, was known, having been determined from the computations made for the lower portion after the last revision mentioned in the preceding paragraph. The two equations were solved for ϕ_i and ψ_{ik} and the moment, M_{ik}, was computed. The sum, $M_{jg} + M_{gj}$, was then determined from an equation similar to Equation (12), the sum divided proportionately to the transverse section moduli of the post at J and G, and a definite magnitude fixed for M_{jg}.

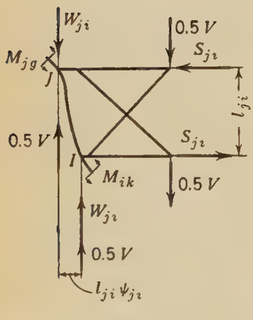


FIG. 7.

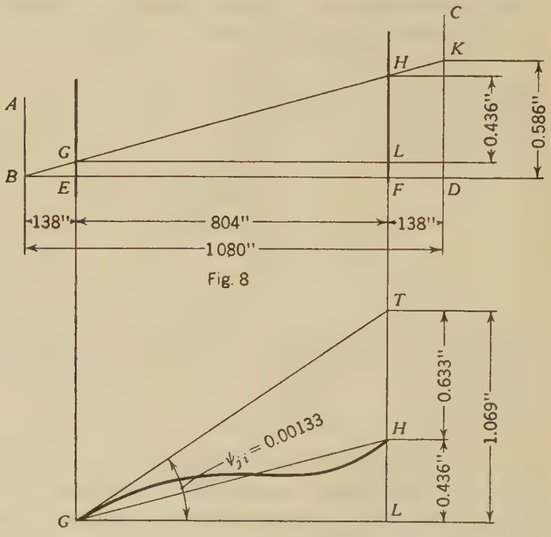


Fig. 8

FIG. 9.

If a point of contraflexure is assumed at the middle of Strut 4 then, from Fig. 7, if V is the vertical shear in the strut,

$$M_{ik} + M_{jg} + l_{ji}\,\psi_{ji}\,W_{ji} + l_{ji}\,S_{ji} = \frac{1}{2}\,bV \ldots\ldots\ldots\ldots (22)$$

All quantities in the first member of Equation (22) are known except ψ_{ji}, which in the first approximation of the strut may be assumed equal to ϕ_i and the shear, V, determined.

An interesting problem arises at this point. The following quantities are known: $V = 5\,138\,000$ lb; $\psi_{ji} = \phi_i = 0.00133$; $\Delta_{it} = \Delta_{ik} + \Delta_{ko} + \Delta_{oq} + \Delta_{qt} = 0.293$ in.; and $P_{ik} = 13\,269\,000$ lb. The strut must be designed so that, when rotated through the angle, ψ_{ji}, and then distorted by the shear, V, the elevations of the ends of the bottom chord will be consistent with the strains, Δ_{it}, in the posts below. In Fig. 8, let AB and CD represent the center lines of the posts; GE and HF, their inside faces; and EF the center line of the bottom chord of the strut—all in their unstrained positions. The strain in each post is $\Delta_{it} = 0.293$ in., shortening one post and lengthening the other so that the difference in elevations of B and D after distortion is $0.293 \times 2 = 0.586$ in. If this difference is represented by KD, then the difference in elevations of the two ends of the bottom chord is,

$$HL = \frac{804}{1080} \times 0.586 = 0.436 \ldots\ldots\ldots\ldots\ldots\ldots (23)$$

In Fig. 9, GT is the tangent to the elastic curve of the bottom chord making the angle, ψ_{ji}, with the horizontal, GL, hence,

$$TL = 804\,\psi_{ji} = 1.069 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (24)$$

whence,

$$TH = 1.069 - 0.436 = 0.633 \ldots\ldots\ldots\ldots\ldots\ldots (25)$$

The distance, TH, is the vertical deflection of the bottom chord to be accounted for by strains in the members of the strut. The problem, therefore, is to find the areas of the members which will cause this deflection when the members are stressed by a shear, $V = 5\,138\,000$ lb. The stresses, S, in terms of V, divided equally between the two web systems are given in Fig. 10.

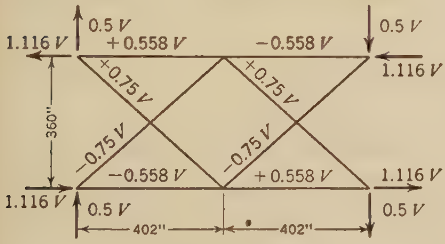

0.5 V
1.116 V
+0.558 V
−0.558 V
0.5 V
1.116 V
×0.75 V
×0.75 V
−0.75 V
−0.75 V
360"
−0.558 V
+0.558 V
1.116 V
1.116 V
0.5 V
0.5 V
402"
402"

FIG. 10.

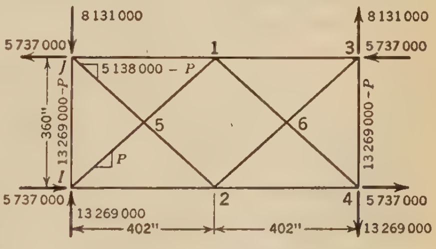

8 131 000
8 131 000
5 737 000
1
3
5 737 000
J
5 138 000 − P
360"
13 269 000 − P
5
6
13 269 000 − P
P
I
5 737 000
2
4
5 737 000
13 269 000
402"
402"
13 269 000

FIG. 11.

Let A = the area of all the members, then,

$$T\,H = \sum \frac{S\,u\,l}{A\,E} = \frac{0.000\,059\,V}{A} = \frac{303}{A} = 0.633 \ldots\ldots\ldots (26)$$

whence, $A = 478$ sq in.

This area gives low unit stresses—about 6 000 lb for the chords and 8 000 lb for the webs—which indicates that small strains or stiffness rather than strength governs the design. Since stiffness is required, the members themselves as well as the strut as a whole should be designed with this end in view, by concentrating as much metal in the extreme fibers or flanges as is consistent with good design.

The distribution of the shear between the two web systems, *I*-1-4 and *J*-2-3 (Fig. 11) will now be investigated. Let *P* represent the vertical component of stress in *I*-1, and 5 138 000 − *P*, the vertical component in *J*-2. The stresses, *S*, and the strains, Δ, multiplied by *E*, are given in Table 1.

TABLE 1.—Wind Stresses and Strains Multiplied by *E* in Strut 4, Third Approximation

Member		A	$\frac{l}{A}$	S	$E\,\Delta = \frac{Sl}{A}$
I-2.......	402	478	0.841	5 737 000 − 1.116 *P*	4 825 000 − 0.939 *P*
2-4.......	402	478	0.841	5 737 000 − 1.116 *P*	4 825 000 − 0.939 *P*
J-1.......	402	478	0.841	1.116 *P*	0.939 *P*
1-3.......	402	478	0.841	1.116 *P*	0.939 *P*
I-1.......	540	*A*	*a*	1.5 *P*	1.5 *a P*
1-4.......	540	*A*	*a*	1.5 *P*	1.5 *a P*
J-2.......	540	478	1.13	7 702 000 − 1.1499 *P*	8 701 000 − 1.693 *P*
2-3.......	540	478	1.13	7 702 000 − 1.1499 *P*	8 701 000 − 1.693 *P*
J-*I*.......	360	7 220	0.0498	13 269 000 − *P*	662 000 − 0.0498 *P*
3-4.......	360	7 220	0.0498	13 269 000 − *P*	662 000 − 0.0498 *P*

The post members, *J*-*I* and 3-4, have an area of 7 220 sq in. The areas of all strut members are 478 sq in., except the diagonals, *I*-1 and 1-4, which are assigned an unknown area, *A*, and an unknown value of $\frac{l}{A} = a$.

In Fig. 9, let $T\,H = d =$ the vertical deflection of Joint 4, due to strains in the members, then,

$$d = 804\,\psi_{ji} - 0.436 \quad \text{(27)}$$

or,

$$E\,d = 29\,000\,000\,(804\,\psi_{ji} - 0.436) = 23\,316\,000\,000\,\psi_{ji} - 12\,655\,000 \quad \text{(28)}$$

Two general expressions for *d* in terms of the strains may be obtained from the Williot diagram, Fig. 12. Joint *I* is fixed in position; Member *J*-*I*, fixed in a vertical direction; and the vertical deflection of Joint 4 is *I*-4. If $\theta =$ the angle, 1-*I*-2, then,

$$E\,d = E\,(\Delta_{j2} + \Delta_{23})\,\text{cosec}\,\theta + E\,(\Delta_{i2} + \Delta_{24})\cot\theta + E\,(\Delta_{ji} + \Delta_{34}) \quad \text{(29)}$$

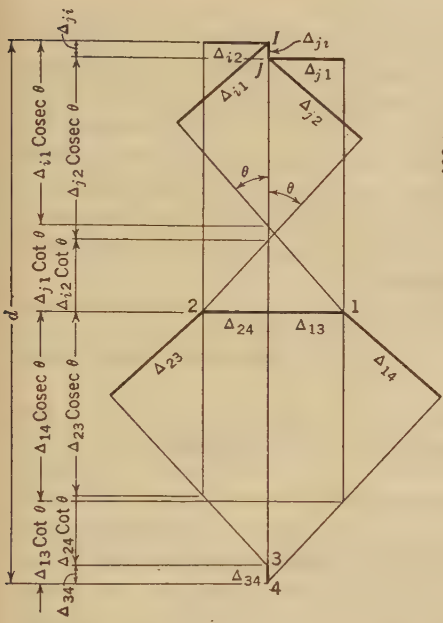
also,

$$E\,d = E\,(\Delta_{i1} + \Delta_{14})\,\text{cosec}\,\theta + E\,(\Delta_{j1} + \Delta_{13})\cot\theta \quad \text{(30)}$$

Equating the two expressions for *E d* in Equations (28) and (29), substituting the expressions for the strains, Δ, from Table 1, and solving,

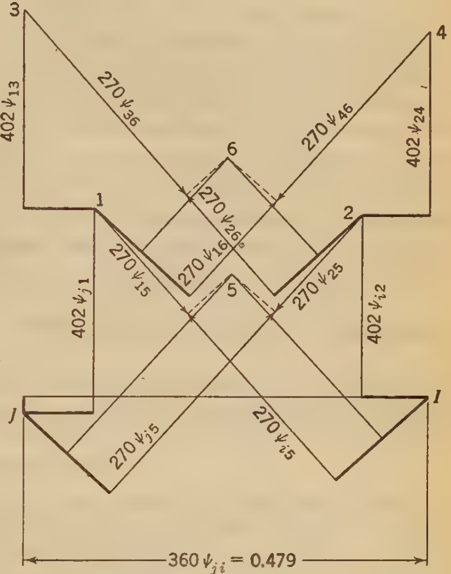
$$P = -\,3\,205\,000\,000\,\psi_{ji} + 6\,989\,000 \quad \text{(31)}$$

A Williot diagram, Fig. 13, may be drawn to determine an approximate value of ψ for each strut member. The stresses are computed on the assumption that the shear of 5 138 000 lb is resisted equally by the two web systems. The strains are determined therefrom by assuming 478 sq in. as the area of all members. Joint I is fixed in position and Member J-I is rotated through the angle, $\psi_{ji} = 0.00133$; whence the horizontal distance between I and J is $360\ \psi_{ji} = 0.479$ in. Then the vertical distance, 4-I, which corresponds to $H\ L$ in Fig. 9, scales approximately 0.436 in. In scaling the quantities, $l\psi$, they are found to be approximately symmetrical about the center line of the strut and the values of ψ are thus assumed. Exact instead of approximate values of ψ for the strut members could have been included in the computations by simply sketching Fig. 13 and developing therefrom the expression for each ψ in terms of ψ_{ji} and the strains, Δ, as given in Table 1. The expedient used, however, saved considerable labor at a small sacrifice in accuracy.



Δ_{ji}
I
Δ_{ji}
Δ_{i2}
J
Δ_{j1}
Δ_{i1}
Δ_{j2}
θ
θ
Δ_{i1} Cosec θ
Δ_{j2} Cosec θ
Δ_{j1} Cot θ
Δ_{i2} Cot θ
d
2
1
Δ_{24}
Δ_{13}
Δ_{23}
Δ_{14}
Δ_{14} Cosec θ
Δ_{23} Cosec θ
Δ_{13} Cot θ
Δ_{24} Cot θ
3
Δ_{34}
4
Δ_{34}

FIG. 12.



3
4
$402\psi_{13}$
$270\psi_{36}$
$270\psi_{46}$
$402\psi_{24}$
6
1
2
$270\psi_{26}$
$270\psi_{16}$
$270\psi_{15}$
$270\psi_{25}$
5
$402\psi_{j1}$
$402\psi_{i2}$
J
I
$270\psi_{j5}$
$270\psi_{i5}$
$360\psi_{ji} = 0.479$

FIG. 13.

The moment at the end of each member in the left half of the strut is expressed in terms of ϕ and ψ and the known value of ψ is substituted. The equations, $\Sigma M = 0$, are written for Joints I, 1, 2, and 5. At Joints 1 and 2, ΣM includes only the two members on the left of the center line. In the equation for Joint I there will be a term, M_i, to account for the eccentric connection of the diagonal, thus,

$$M_i = 138\ P = 138\ (-\ 3\,205\,000\,000\ \psi_{ji} + 6\,989\,000) \ldots\ldots. (32)$$

A fifth equation is written,

$$M_{ji} + M_{ij} + l_{ji} S_{ji} + l_{ji} \psi_{ji} W_{ji} - 402V = 0 \ldots\ldots (33)$$

Since ϕ_i is known, the five equations may be solved for ϕ_j, ϕ_1, ϕ_2, ϕ_6, and ψ_{ji}. Then the first four terms in the equation,

$$M_{ji} + M_{j6} + M_j + M_{j1} + M_{jg} = 0 \ldots\ldots\ldots\ldots\ldots (34)$$

may be computed and M_{jg} determined.

The magnitude of M_{jg} thus found differed considerably with the desired value previously assigned, since in approximating the shear, V, from Equation (22), ψ_{ji} was assumed equal to ϕ_i and no account was taken of the moments at the ends of the strut members at J and I. The value of ψ_{ji} was corrected in Equations (22), (24), and (25); the end moments of the four strut members were included in Equation (22); and a revised value of the shear, V, was obtained. When the computations had been revised a satisfactory agreement existed between the derived and assigned magnitudes of M_{jg} and no further revision was necessary.

The value of P, determined from Equation (31), was substituted in the expressions for $E\Delta$ in Table 1 and $a = \dfrac{l}{A}$ was determined from Equation (30), from which the required area of the diagonals, I-1 and 1-4, was found. The remaining struts were treated in a similar manner.

FOURTH APPROXIMATION

In the analysis of Strut 1 no variation in the shear, V, could be made, since $V = P_{cd}$. When the analysis had been completed the moment, M_{ba}, did not check with the moment of the known forces above B. This was accounted for by the fact that in Equations (7), (8), and (9) the eccentric moments, M_{ak}, were taken from a rough approximation to the elastic curve and no account was made of the moments at the ends of the sixteen strut members. The equations were corrected accordingly and the computations revised throughout.

The corrections for the lower portion were quite simple and, because no changes in design were necessary, were confined to the numerical terms in the equations affected. Revisions in design were necessary only in Struts 1 and 2. The value of M_{ba} was then in close agreement with the known value.

The expedient of assigning definite magnitudes to the moments M_{ki}, M_{jg}, M_{fe}, and M_{dc}, and making the design conform thereto greatly reduced the amount of work. Any set of computations and revisions was then confined to a relatively small part of the structure.

Further attention was now given to the longitudinal moments. In the second approximation a parabola was assumed for the elastic curve, tangent at the base with a deflection of 17 in. at the top. In the fourth approximation, x_1 and x_2 were taken as the deflections at K and F, and the deflections at all other points where there was a change in section were expressed as functions of the unknowns, x_1 and x_2. Expressions in terms of x_1, x_2, and H were developed by the area-moment method for the deflections at K, F, and the top, and were equated to x_1, x_2 and 17 in., respectively. The equations were solved, the remaining deflections determined, and the longitudinal moments computed. In the second approximation the deflections at F and K were 7.922 and 1.313 in., respectively, as computed from the parabola. In the analysis just described the deflections were found to be 7.631 and 1.175 in., respectively.

REVIEW OF THE DESIGN

Since the design had been made by fixing arbitrarily the magnitude of the moments, M_{ki}, M_{jg}, M_{fe}, and M_{dc}, and designing the structure accordingly, it was decided to make an independent set of computations, beginning in this case at the top. The following outline gives the gist of the method in greatly condensed form. The brief details should help other designers to a direct, time-saving solution.

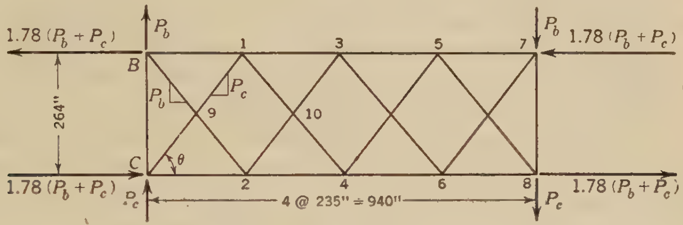


FIG. 14.

TABLE 2.—WIND STRESSES AND STRAINS MULTIPLIED BY E IN STRUT 1, REVIEW COMPUTATIONS

Member	l	A	$\frac{l}{A}$	S	$E\Delta = \frac{Sl}{A}$
C-2	235	452.5	0.519	$1.78\ P_b + 0.89\ P_c$	$0.925\ P_b + 0.462\ P_c$
2-4	235	452.5	0.519	$0.89\ P_c$	$0.462\ P_c$
4-6	235	452.5	0.519	$0.89\ P_c$	$0.462\ P_c$
6-8	235	452.5	0.519	$1.78\ P_b + 0.89\ P_c$	$0.925\ P_b + 0.462\ P_c$
B-1	235	452.5	0.519	$0.89\ P_b + 1.78\ P_c$	$0.462\ P_b + 0.925\ P_c$
1-3	235	452.5	0.519	$0.89\ P_b$	$0.462\ P_b$
3-5	235	452.5	0.519	$0.89\ P_b$	$0.462\ P_b$
5-7	235	452.5	0.519	$0.89\ P_b + 1.78\ P_c$	$0.462\ P_b + 0.925\ P_c$
C-1	353	463	0.763	$1.339\ P_c$	$1.022\ P_c$
1-4	353	463	0.763	$1.339\ P_c$	$1.022\ P_c$
4-5	353	463	0.763	$1.339\ P_c$	$1.022\ P_c$
5-8	353	463	0.763	$1.339\ P_c$	$1.022\ P_c$
B-2	353	452.5	0.781	$1.339\ P_b$	$1.046\ P_b$
2-3	353	452.5	0.781	$1.339\ P_b$	$1.046\ P_b$
3-6	353	452.5	0.781	$1.339\ P_b$	$1.046\ P_b$
6-7	353	452.5	0.781	$1.339\ P_b$	$1.046\ P_b$
B-C	264	3 895	0.068	P_b	$0.068\ P_b$
7-8	264	3 895	0.068	P_b	$0.068\ P_b$

Let P_b and P_c represent the vertical components of the stress in the diagonals of Strut 1, as shown in Fig. 14. The computations of stresses and strains were made as in Table 2. Let d = the vertical deflection of Joint 8 due to strains, and Δ_{ct} = strain in the post from C to T. Then,

$$d = 940\ \psi_{bc} - \left(2\ \Delta_{ct} \times \frac{940}{1080}\right)$$

or,

$$E\ d = 27\,260\,000\,000\ \psi_{bc} - 50\,481\,000\ \Delta_{ct} \ldots\ldots\ldots\ldots (35)$$

The following equations were derived from the geometry of a Williot diagram drawn similar to Fig. 12:

$$E\ d = 4\ E\ \Delta_{b2} \operatorname{cosec} \theta + E\ (4\Delta_{c2} + 2\ \Delta_{b1} + 2\ \Delta_{13}) \cot \theta$$
$$+ E\ (\Delta_{bc} + \Delta_{78}) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (36)$$

$$E\ d = 4\ E\ \Delta_{c1} \operatorname{cosec} \theta + E\ (4\ \Delta_{b1} + 2\ \Delta_{c2} + 2\ \Delta_{24}) \cot \theta \ldots . (37)$$

From Equations (36) and (37) P_b and P_c were expressed in terms of ψ_{bc} and Δ_{ct}. Eleven equations were written—one each for M_{ab} and M_{ba}; one each for Joints B, C, 1, 2, 3, 4, 9, and 10; and one for $M_{bc} + M_{cb}$. These eleven equations contained fourteen unknown quantities, ϕ_a, ϕ_b, ϕ_c, ϕ_d, ϕ_1, ϕ_2, ϕ_3, ϕ_4, ϕ_9, ϕ_{10}, ψ_{ab}, ψ_{bc}, ψ_{cd}, and Δ_{ct}. The quantities, ψ, for the strut members were not treated as unknowns; their magnitudes were obtained from the Williot diagram drawn similar to Fig. 13 after the final revision in the design had been made. Five of these fourteen unknowns, ϕ_c, ϕ_d, ψ_{bc}, ψ_{de}, and Δ_{ct}, appear in later equations. The remaining nine were eliminated from the eleven equations, leaving two equations containing the five unknowns, ϕ_c, ϕ_d, ψ_{bc}, ψ_{de}, and Δ_{ct}.

A similar treatment was given to Strut 2 which is 888 in. long. Then, as before,

$$d = 888\ \psi_{de} - \left(2\ \Delta_{ct} \times \frac{888}{1080}\right)$$

Two equations were developed from the geometry of a Williot diagram and a third equation was written as follows: $\Delta_{et} = \Delta_{ct} - \Delta_{cd} - \Delta_{de}$. From these three equations the vertical components in the diagonals, P_d, and P_e, were expressed in terms of ψ_{bc}, ψ_{de}, and Δ_{ct}. Eight equations, $\Sigma M = 0$, were written for the eight joints, one for $M_{cd} + M_{dc}$ and one for $M_{de} + M_{ed}$. These equations, with the two brought down from Strut 1, gave twelve equations from which nine unknowns were eliminated, leaving three equations having the six unknowns, ϕ_e, ϕ_f, ψ_{bc}, ψ_{de}, ψ_{ef}, and Δ_{ct}.

Strut 3 was similarly treated, resulting in four equations having seven unknowns, ϕ_g, ϕ_j, ψ_{bc}, ψ_{de}, ψ_{fg}, and Δ_{ct}. The analysis of Strut 4 gave five equations with eight unknowns, ϕ_i, ϕ_k, ψ_{bc}, ψ_{de}, ψ_{fg}, ψ_{ji}, ψ_{ik}, and Δ_{ct}.

An equation was now developed by equating two independent expressions for Δ_{kt} as follows:

$$\Delta_{kt} = \Delta_{ct} - (\Delta_{cd} + \Delta_{de} + \Delta_{ef} + \Delta_{fg} + \Delta_{gj} + \Delta_{ji} + \Delta_{ik})$$
$$= \Delta_{ko} + \Delta_{oq} + \Delta_{qt}$$

From these six equations, ψ_{bc}, ψ_{de}, ψ_{fg}, ψ_{ji}, and Δ_{ct}, were eliminated, leaving one equation having the following unknowns: ϕ_i, ϕ_k, ϕ_o, ϕ_q, ψ_{ik}, ψ_{ko}, ψ_{oq}, and ψ_{qt}. An equation in terms of ϕ_i, ϕ_k, and ψ_{ik}, was written from $M_{ik} + M_{ki}$. The lower portion was treated in the same way as in the fourth approximation, except that M_{ki} was expressed in terms of ϕ_i, ϕ_k, and ψ_{ik} instead of as a known quantity. This introduced two additional unknowns, ϕ_i and ψ_{ik}, into the equations for the lower portion, but the upper portion, in turn, supplied the two additional equations necessary for the solution.

All assumptions made in the approximate solutions have been eliminated except that of identical elastic curves for the two posts. Two possible exceptions to this statement may be claimed. One exception is the assumption of numerical terms in Equations (7), (8), and (9) for the sum of the moments, M_s, in the sixteen strut members and for the sum of the eccentric moments, M_{ak}. The error involved is the difference between the magnitudes appearing in the equations and the magnitudes derived therefrom.

For example, in the final review of the problem, the magnitudes used in the equations were $M_{ak} = +$ 665 000 000 in-lb and $M_s = -$ 239 000 000 in-lb, as determined from the fourth approximation. When the review was completed the magnitudes were found to be $M_{ak} = +$ 663 000 000 in-lb and $M_s = -$ 235 000 000 in-lb. In any given set of computations, M_{ak} and M_s, are, respectively, the same in all three Equations (7), (8), and (9); and when these are combined with either $M_W = +$ 9 707 000 000 in-lb in Equation (7), or with $M_X = +$ 12 930 000 000 in-lb in Equation (8), or with $M_T = +$ 14 874 000 000 in-lb in Equation (9), the errors are too small for consideration.

The other exception is the assumption that the post bases are in complete restraint against rotation, or $\phi_t = 0$. In order to ascertain the total possible range of moments, shears, and stresses, a review was also made on the assumption that the base was free to rotate by making $M_{tq} = 0$ and treating ϕ_t as an unknown quantity. The chief differences in the results of these two assumptions were confined to the part below the floor. For the fixed base, the moments in the posts and the stress in the upper diagonals were larger, and the stress in the lower diagonals smaller, than for the free base. The transverse deflection at the top was 11.14 in. for the fixed assumption and 11.31 in. for the free assumption.

CONCLUSION

It seems unnecessary to make extended comments or to draw specific conclusions from this expository rather than argumentative monograph, further than to emphasize the simplicity of the process by which the Williot equations are developed. The facility of their application, however, will vary according to the nature and degree of complexity of the problem in hand. There is, of course, an analogy between the Williot deflection equations and the energy or elastic equations derived from equating external to internal work. When confronted with the analysis of any statistically indeterminate structure, the analyst will solve the problem by the method with which he is the most familiar, whether it is the elastic, the deflection, or any other method, provided the particular method adopted ends in fruition. All methods, however, are not equally efficient. In the present case the completed analysis of the towers may be checked by equating internal and external work after the problem has been solved and the deflections are known; but the problem cannot be solved by this method except possibly by an almost endless procession of "cut and try" guesses as to the displacements of all joints in the structure; whereas by the method of expressing moments in terms of angular displacements, the displacements may be treated as unknowns and a solution obtained with the help of the Williot strain equations.

It may be that this method is known to others, but during his years of academic and professional experience the writer has never encountered a presentation of it, and probably never would have discovered the method had not necessity given it birth.

DISCUSSION

Ralph A. Tudor,[2] Assoc. M. Am. Soc. C. E. (by letter).—The complete and accurate analysis of a large suspension bridge tower is an unusual problem and one much more difficult than at first might appear. It is essentially a tall slender column braced in one direction only and subject to enormous vertical loads at its top, which are more often than not acting eccentrically. In addition, horizontal wind loads must be considered and only by comparison with the great vertical loads do they seem small.

The analysis of the tower legs for supporting the vertical loads and for the necessary bending longitudinal with the bridge has been more or less standardized, as described in Mr. Ellis' paper. However, the effect of the participation of the bracing with the legs in supporting the vertical loads, the effect of the transverse wind loads on all parts of the tower, and the magnitude of secondary stresses have not often been considered in detail. The method presented by Mr. Ellis, and his use of the geometry of a Williot diagram covers this phase of the problem and should give results very near the truth. Of great importance is the fact that the method considers, simultaneously, primary

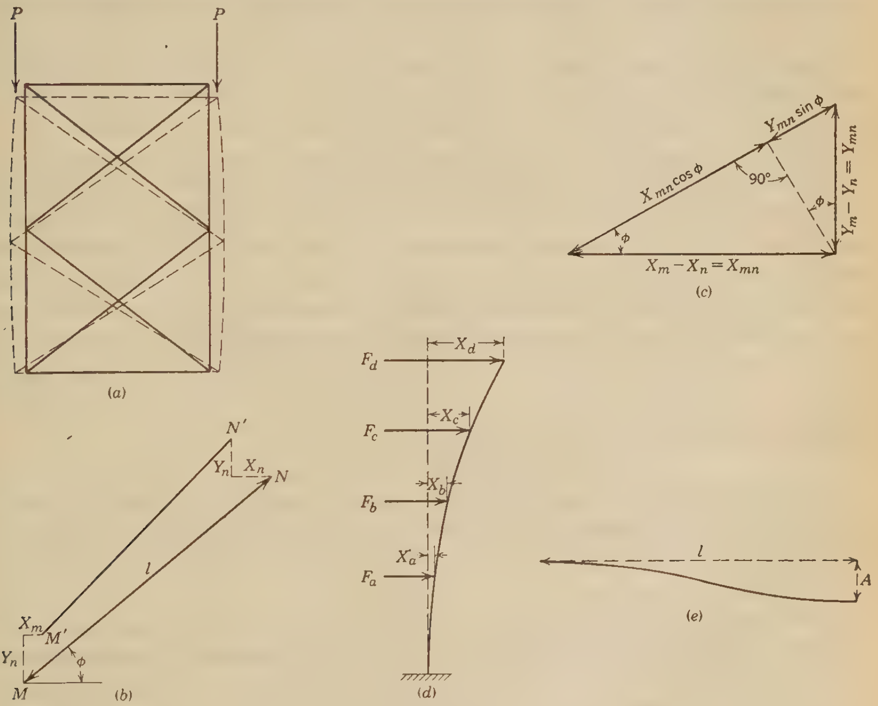


Fig. 15.

---

[2] Senior Designing Engr., Bridges, Div. of Highways, State Dept. of Public Works, San Francisco, Calif.

(including participation) and secondary stresses. In this manner, allowance is made for the full effect of one on the other without subsequent corrections.

It is unusually interesting that, while Mr. Ellis was finding necessity "the mother of invention," the analysis of another suspension bridge tower was proceeding elsewhere. The same vexing problems were recognized, and a solution was effected which is fundamentally very similar to that already presented. The tower differs in architectural detail in that it is braced throughout its entire length, except through the roadway portal, by a system of double diagonals.

Referring to Fig. 15(*a*), it can be seen that a portion of the tower fabricated as shown by the solid lines will be deformed into some such shape as that shown by the dotted lines when the loads, *P,* are applied. The elastic shortening of the legs under load will decrease the vertical length of the panels and will require a reduction in the length of the diagonals. This action will place stress in the diagonals and the horizontal component of this stress will be resisted by bending in the tower legs or stress in any horizontal that may exist. Any external horizontal load will create a new stress balance.

When the problem was first encountered it seemed that any solution must be based on the elastic features of the structure, and it was on this premise that a method was developed. The unknowns were the horizontal and vertical (X and Y) components of the movements of the joints. All stresses were expressed in terms of these unknowns and equations of stress balance set up.

Refer to Fig. 15(*b*) and consider any member, such as MN. The unknown components of the movement of Joint M are X_m and Y_m, and those of Joint N are X_n and Y_n. The horizontal component of the length of the member will change by an amount $(X_m - X_n) = X_{mn}$, and the vertical component by an amount $(Y_m - Y_n) = Y_{mn}$. If the slope of the member, ϕ, is assumed to remain unchanged (which is essentially true), the change in length of the member, MN, will be, $\delta_{mn} = X_{mn} \cos \phi + Y_{mn} \sin \phi$ (see Fig. 15(*c*)). The axial stress in the member will then be $\left(\frac{A\,E\,\delta}{l}\right)_{mn}$.

In this manner the axial stress in all members may be expressed in terms of the unknown joint movements. Since joint movements are a feature of the Williot diagram, the method being described is quite similar to that developed by Mr. Ellis.

If all joints were pin-connected and bending of members did not exist, the structure could be solved by equating the sum of the horizontal forces entering each joint to zero ($\Sigma H = 0$), and treating the vertical forces similarly ($\Sigma V = 0$). This would give two equations for each joint, which would equal the number of unknowns and permit a solution.

However, in examining the structure it can be seen that bending in the legs will materially influence the joint displacements and must be fully provided for in the analysis. On the other hand, for the particular tower considered, the web members are comparatively flexible, and their bending will not appreciably effect the distortion of the structure as a whole. Bearing this in mind, equations for $\Sigma V = 0$ may be stated for all joints and for $\Sigma H = 0$ for those joints not along the tower legs.

In stating the remaining equations, the tower legs are considered to be vertical cantilevers fixed at the base. At each joint along the leg a horizontal force of unknown magnitude is applied. This force is simply the algebraic sum of the horizontal components of the axial stresses carried by the members entering the joint, plus any externally applied load. Thus, in Fig. 15(*d*), the leg is acted upon by the forces, F_a, F_b, F_c, and F_d, and the joints are deflected by the amounts, X_a, X_b, X_c, and X_d. Then,

$$\begin{aligned} X_a &= M_{aa} F_a + M_{ab} F_b + M_{ac} F_c + M_{ad} F_d \\ X_b &= M_{ba} F_a + M_{bb} F_b + M_{bc} F_c + M_{bd} F_d \\ X_c &= M_{ca} F_a + M_{cb} F_b + M_{cc} F_c + M_{cd} F_d \\ X_d &= M_{da} F_a + M_{db} F_b + M_{dc} F_c + M_{dd} F_d \end{aligned}$$

The M-values are functions of the tower dimensions. By this means the remaining equations are provided, and their solution is only a matter of careful perseverance.

With the magnitude of the joint movements determined for a given condition of external loads, the axial stresses are computed by $\frac{A\,E\,\delta}{l}$. Bending stresses are determined by distributing the fixed end moments in accordance with the Cross method. The fixed end moments (see Fig. 15(*c*)), are equal to, $M = \frac{6\,E\,I\,\Delta}{l^2}$.

This method has been used to solve other frames in which bending of the members is an important item.

Leon S. Moisseiff,[3] M. Am. Soc. C. E. (by letter).—This paper is a contribution of the first order to the methods available to engineers for determining the strains and stresses in elastic frames subjected to considerable deformation and departure from their original geometric form. The author has extended the utilization of the device known as the Williot diagram to the rotation of the members of a stiff frame at any joint. He has opened thereby an approach for the investigator which in many instances will facilitate his labors and in others will afford the only possible path to a closely accurate determination of the behavior of a stiff frame under loads.

Mr. Ellis relates how, encountering the problem of a high tower subjected to large vertical and horizontal forces and designed as a stiff frame, he developed the presented solution. The writer believes that it is pertinent to fix here the co-ordinates of this case in time and space. The Golden Gate Bridge, now under construction (1934), crosses the entrance of San Francisco Bay by a suspended structure with a main span of 4 200 ft and side spans of 1 125 ft. This enormous span, the longest of any bridge, as a matter of design and clearance, requires high towers. They were made about 700 ft in height above the masonry pier and 746 ft above mean high water. The width of the bridge centers of towers, trusses, and cables is 90 ft, and the magnitude of the towers is such that about 21 000 tons of steel are required for one tower.

---

[3] Cons. Engr., New York, N. Y.

The location of the bridge over the Golden Gate, the entrance to the largest harbor on the Pacific Coast, in a landscape of rare scenic beauty, demanded a structure of chaste and majestic appearance. It was thought that by avoiding a multiplicity of diagonals for the bracing of the towers and leaving the cross-sectional view of the towers as free from obstructions as possible, a satisfactory solution would be attained. This led to the adoption of a stiff frame design with diagonals below the roadway and horizontal braces above it. The appearance of the northern tower more than realizes the anticipation of the designers.

This is the "flesh-and-blood" story of the skeleton tower which the author encountered and to which he refers. The method and procedure that he developed during his work on the design of the towers are the subject of his paper. It should properly be added that the final proportioning of the towers differs somewhat in cross-sectional areas from those given in the paper. In general, the procedure and method described were followed in the final design of the Golden Gate Bridge towers, on which the author has done excellent work.

The procedure in the final computations was as follows: For transverse wind loads all stresses in the tower were computed simultaneously; the final unknowns were expressed as the rotation angles of the joints and the rotations of the members.

The only simplification introduced in the analysis was in the treatment of the four horizontal struts above the roadway as units. It was made by neglecting the effect of the secondary stresses in the members of the struts on the bending moments in the tower shafts. The shear transmitted through the struts was expressed as a function of the unkown rotation angles of the joints and the end member (tower shaft). It was thus made possible to reduce the number of the final simultaneous equations to thirty, the bottom horizontal member being designed to carry no stress. One tower shaft has eleven joints; the diagonal bracing below the floor has two more; and, there are eleven members in the shaft and six in the diagonal bracing. As each of these represents an unknown, a total of thirty unknowns result. To include the omitted effect of the secondary stresses in the members of the struts in the procedure would have rendered the simultaneous solution nearly impossible. The two upper top struts alone would have added thirty-four additional final equations.

The secondary stresses in the members of the struts were computed subsequently from the primary stresses, and it was found that the resulting rotation angles at the ends were close to those derived from the solution of the thirty simultaneous equations. It may be added here that the top strut was analyzed subsequently by the same procedure as the tower. This was done in the computations for the tower model mentioned later, as an additional check of the original assumptions.

After the solution of the thirty simultaneous equations, the rotation angles of all joints and members of the shaft were known, as well as the rotation of the end members and end joints of the struts. These values, belonging to the top strut, were then introduced as known quantities, and it was thus possible to compute all stresses in the strut simultaneously by establishing and solving seventeen equations. The results proved that the transferred shear assumed in the original equations was closely correct and that the resulting stresses in the struts were also in agreement with the original computations.

The computations of the stresses and the design of the Golden Gate Bridge which, of course, included that of the towers, have been checked in the writer's office.

Considering the importance and size of the towers and the fact that novel methods of analysis had to be developed for more accurate computations of their stresses, it was decided to verify the design and computations by tests of a model which should simulate, as nearly as possible, the structure and assure elastic behavior within the limits of test loads and observations.

A model of stainless steel was fabricated which was almost an exact replica of the actual tower to the ratio of 1 to 56 in linear scale. The height of the model was 12.5 ft. The vertical load on top of the model was 38 000 lb, corresponding to the vertical load on top of the actual tower of about 120 000 000 lb.

The model was fabricated under the direction of George E. Beggs, M. Am. Soc. C. E., and the tests were conducted under his charge at Princeton University. The model tower was simultaneously loaded both vertically and laterally in proportion to the wind load on the actual tower. A most complete set of observations was made by strain-gauges. The results obtained from the observations on the model were in full agreement with the results of the theory used in the design of the tower.

It will be of interest to know that when the first design of the towers was made, practically for purposes of estimate, it was necessary to determine the lateral deflection of the tower tops. This was computed both by the work method and the slope method. It happened that, by each of these methods, a lateral displacement of about 6 in. was found. Upon closer examination of the problem it became apparent that the deflection found by the work method is due to the axial deformation of the tower shafts only and that the deflection found by the slope method is due to the flexure of the shafts only. Therefore, the two results, the one by the work method and the other by the slope method, should be added in order to obtain the approximate result of displacement. To this again should be added the effect of the eccentricity of the tower loads while so displaced and which will cause additional displacement. The total sum of the lateral displacement thus arrived at was about 17 in. It was then realized that a correct solution could only be obtained by developing a simultaneous integral theory. This has been done by the author. It should be stated here that the lateral deflection of the tower tops is 12.6 in. for the tower as built.

The discussion leads to a comparison of the stiff frame, braced horizontally, with the common diagonally braced frame. It is usually assumed in every-day practice that a diagonally braced frame is statically determinate and that its stresses are therefore definite and computed with little effort. Any one who has given some thought to the problem will admit, of course, that this is not so. In order to have a statically determinate structure, all joints must be hinged and frictionless. Structures are not built in this manner. The frame structures as built are many times statically indeterminate and the correct determination of their stresses is highly complicated.

The assumption of frictionless joints, however, is so prevalent because it is the basis of elementary class-room teaching. This is a necessary result of the evolution of engineering science. Humanity, in its desire to understand the action of natural forces and to forge tools to determine the quantitative character of their action, had to resort to simplifying assumptions. It was an enormous step forward when the conception of the hinged, frictionless frame of rigid members was evolved. To this simplified theory is due the great advance in the design of structures and bridges. For the smaller structures subjected to moderate loads the theory is sufficient. The larger bridges built with higher strength steels and subjected to greater forces and deformations demand a deeper analysis which should correspond more closely to the actual behavior of the structures.

From the point of view of a simultaneous solution a frame with diagonal bracing is more complicated and more laborious to compute than one with horizontal struts. Thus, the Golden Gate Bridge tower, as designed, required thirty equations and, if braced with diagonals, would have required thirty-five equations. The towers of the San Francisco-Oakland Bay Bridge did, in fact, require thirty-five equations.

In connection with the simultaneous analysis of frames and trusses it is well to point out that the separation of strains and stresses caused by external forces, into primary and secondary, is artificial and incorrect. Unless the frame is treated in its entirety erroneous results are frequently obtained. In some cases, even the direction of the stress is in error. There is an evident interaction of the members of a frame and only a simultaneous solution can approach their actual behavior.

A. A. Eremin,[4] Assoc. M. Am. Soc. C. E. (by letter).—An original method of computing the stresses in a suspension bridge tower is developed in this paper, including consideration of rigid connections at the joints.

A number of analytical and semi-graphical methods for designing frames have been proposed, the relative merits of which depend upon the error introduced, the simplicity of the operations, and the time saved. Mr. Ellis has developed equations for a direct relation between the angular displacements at the joints and the external forces. However, these equations are complicated and numerous. To compute the angular displacements in the simple symmetrical frame (Fig. 2) below Joints *K* and *L*, fifteen simultaneous equations must be solved. This is a difficult task even with a computing machine.

---

[4] Assoc. Bridge Designing Engr., Div. of Highways, State Dept. of Public Works, Sacramento, Calif.

Furthermore, in the Williot strain diagram, a geometrical relation between small deformations and long normal lines is sometimes difficult to express. Especially is this true in the case of a diagram for an arch frame. Stresses in a frame with rigid joints are generally computed in two operations: The primary stresses are determined under the assumption that the members of the frame are hinged at the joints; and the secondary stresses are computed to express an effect of the rigidities at the joints and eccentricities of the connections on the primary stresses.

Computation of the secondary stresses is based on the convenient formula for a change in the angle due to changes of lengths in all three members in any triangle, thus:

$$E\,\delta\alpha = (S_3 - S_2)\cot\beta + (S_3 - S_1)\cot\gamma \qquad (38)$$

and for checking

$$\delta\alpha + \delta\gamma + \delta\beta = 0 \qquad (39)$$

The simultaneous equations for computation of the angular displacements developed by Equation (38) may be arranged so that the arithmetical error in solving these equations will not be accumulating. Furthermore, any part of a frame may be analyzed separately.

However, Mr. Ellis has stated correctly that the facility of application of the Williot equations will vary according to the nature and degree of complexity of the problem in hand. The author is to be complimented for this interesting contribution to the theory of design of statically indeterminate structures.

CHARLES A. ELLIS,[5] M. AM. SOC. C. E. (by letter).—Thanks are due to those who have discussed this paper, for their kindly and constructive comments.

Mr. Eremin mentions that the equations are complicated and numerous. It is a fact that all mathematical expressions appear more or less complicated on first sight. The equations certainly are numerous; but the extraordinary structure rather than the method of analysis is responsible for the number.

Mr. Moisseiff has stated, in effect, that this new method provides the only possible path to a closely accurate determination of the behavior of a stiff frame under loads, especially when the frame is subjected to considerable deformation and departure from its original geometric shape. He has commented on the artificiality and incorrectness of solving for primary and secondary stresses separately. Mr. Tudor has also pointed out the great importance of "the fact that the method considers, simultaneously, primary

---

[5] Prof. of Structural Eng., Purdue Univ., Lafayette, Ind.

(including participation) and secondary stresses." These points were not sufficiently emphasized in the paper and a further word may be clarifying.

The diagonal members below the floor (Fig. 3) were originally designed (see "First Approximation") on the commonly accepted assumption that they would resist the total horizontal wind shear in each panel. The degree to which this assumption proved to be in error (more than 100%), because of extraordinary stiffness in the vertical posts and their consequent capacity to resist shear, is indicated in the "Third Approximation" below Fig. 6.

The discovery and first application of the Williot equations came about, as follows: The transverse elastic curve was investigated as described in the first three paragraphs of the "Third Approximation." An ordinary Williot diagram for the lower portion was drawn after strains had been computed from approximate stresses and the quantities, ψ, were determined therefrom. Equations $\Sigma M = 0$, were written and solved; and the moments, M, at the ends of each member were computed. With M_{ko} and M_{ok} approximately known, it was possible to compute the shear, H_{ko}, in the member, KO, from Equation (5). It was obvious at once that this shear was a very considerable quantity, much too large to be neglected in assuming that the diagonals resisted the total horizontal wind shear. Repeated attempts by trial and error were made to estimate the shear distribution between the posts and diagonals. Each attempt included a revised set of stresses and strains, and a revised Williot diagram, etc. These attempts seemed to be getting nowhere, either because the writer's guesses went wide of the mark or because of the numerous factors in the problem.

After repeated drawings of the Williot diagram from revised guesses as to shear distribution, it became obvious that each ψ could be expressed algebraically in terms of the strains in the members, as shown in Equations (13) to (21); so, instead of determining the quantities, ψ, from a Williot diagram drawn from assumed strains, the question arose "Why not express each ψ in terms of unknown strains?" It was clear that the value of ψ_{oq} and ψ_{ko} had much greater influence on shear distribution than values of ψ for any or all other members. Hence, in the first application of the Williot equations, ψ_{oq} and ψ_{ko} were expressed as in Equations (17) and (18), and approximate numerical values derived from the preceding Williot diagram were assigned to the remaining ψ's. Equations (4) to (9) show how the expressions for stresses, P, and resulting strains, Δ, were developed. The result of this solution, since it contained Equation (4), determined the shear distribution, and revealed the glaring error originally made in assuming that the diagonals resisted all the external shear. It was then decided to "go the whole route" and use all equations—(13) to (21)—thereby eliminating all assumptions possible. This procedure was taken after major revisions had been made in the design of the diagonals and minor changes in the posts. The great value of the Williot equations was at once apparent, and they were used later to good effect in the analysis of the upper part of the structure.

Mr. Moisseiff's discussion of the labor involved in the solution of a structure with diagonal bracing compared with one having horizontal struts, should be qualified somewhat. If the two diagonals in any panel are removed and one horizontal strut is substituted, the number of members is reduced; and if the horizontal member is of the plate girder type and if no consideration is given to its depth, except its moment of inertia, then Mr. Moisseiff's statement, that a structure with diagonal bracing is more complicated and more laborious to compute, is correct.

On the other hand, when the horizontal member is a truss 30 ft deep having very large chord and web members, a thorough analysis becomes much more complicated because, although two diagonals have been omitted in each panel, two post members have been added, together with a truss composed of numerous members. When the magnitude and novelty of the structure, and the consequent uncharted field of structural analysis with little or no precedent to follow, were considered, it was deemed wise to apply as thorough an analysis as possible. It was for these reasons that the Williot equations were applied also to the part above the floor.

Of course, the usual assumption that the diagonals resist the total shear in a panel would not result in such gross error in an ordinary truss ; but it is pertinent to emphasize that any method (Manderla's included) of computing secondary stresses in a truss, which fails to take account of the influence of secondary stress upon primary stress, can no longer be considered as "most exact."

The valuable and scholarly paper entitled "Secondary Stresses in Bridges,"[6] by C. B. von Abo, Assoc. M. Am. Soc. C. E., describes nearly all methods of secondary stress analysis known at that time, and gives solutions by the various methods, using the truss and load as shown in Fig. 16(a). The vertical shear in Panel 3–5 (Fig. 16(a)), for example, is 500 lb. The analysis shows that deformation imposes moments at the ends of the Members 2–4 and 3–5; consequently, these members are resisting transverse shear. These shears should be

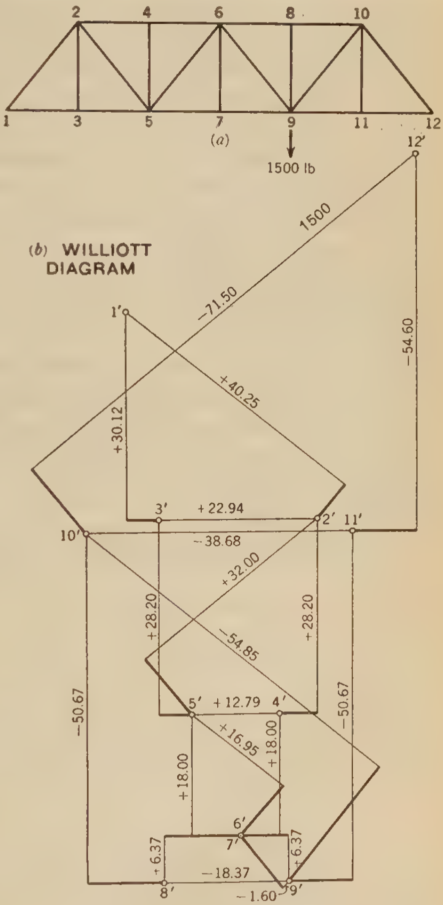


FIG. 16.—MOHR'S SEMI-GRAPHICAL METHOD OF ANALYZING SECONDARY STRESSES.

---

[6] *Transactions*, Am. Soc. C. E., Vol. 89 (1926), p. 69.

considered with the vertical component of the stress (primary and secondary) in Member 2–5, when balancing the total external shear of 500 lb in the panel. The true vertical component of the stress in this diagonal is not 500 lb, and any computation of secondary stress made on this assumption will be in error. The same conclusion applies to all members. The error here is small and of academic significance only, but Manderla's method can no longer stand as the model for accuracy. The analysis by the simultaneous solution is a closer approach to accuracy and will take on practical value as the chord members become relatively stiffer and relieve the diagonals in transferring shear from one end of the panel to the other.

The primary and secondary stresses in the truss analyzed by Mr. von Abo may be solved simultaneously by using the Williot equations with Mohr's semi-graphical method as follows: In the Williot diagram (Fig. 16(*b*)) let the strains in the members be represented by Δ_{12}, Δ_{23}, etc. The quantities, ρ (and hence ψ), can be expressed, by the Williot equations, in terms of the Δ's and the thirty-three equations written. Twelve equations, $\Sigma M = 0$, can be written, one for each joint. They will contain twelve unknowns, ϕ, and twenty-one unknowns, Δ. The twenty-one additional equations may be derived from statics, since the transverse shear in any member may be expressed as a function of the end moments of that member. A solution of these equations will give the Δ's from which the stresses can be determined.

The use of the Williot equations is by no means limited to structures of the type presented in the paper, or to solutions in connection with moment equations in terms of angular displacements. The writer has shown elsewhere[7] how Williot equations may be used to advantage in solving the stresses in a simple truss having four redundant members. Solutions by the well-known methods of work and least work are included to show the relative amount of labor involved in each of the three methods. An illustration is also given, showing how the Williot equations, had they been available, might have been used to good effect in the stress analysis of the towers of the George Washington Bridge.

---

[7] *Engineering News-Record*, April 26, 1934.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

TRANSACTIONS

Paper No. 1905

RAINFALL STUDIES FOR NEW YORK, N. Y.

By S. D. BLEICH,[1] ESQ.

WITH DISCUSSION BY MESSRS. H. ALDEN FOSTER, J. J. SLADE, JR., CHARLES W. SHERMAN, MERRILL M. BERNARD, CLIFFORD SEAVER, JOSE GARCIA MONTES, JR., RHODES E. RULE, AND S. D. BLEICH.

SYNOPSIS

The object of this paper is to collect rainfall data, arrange them in order of intensities, and then derive formulas and graphs which may be used for purposes of sewer design and investigations. Statistics of the heavier rainfalls have been culled from the records of the Central Park gauge in New York, N. Y., covering the period from 1869 to 1930, inclusive. The intensities for various durations are then listed in order of their magnitudes and several types of precipitation formulas are investigated. By comparison of the results the modified exponential type proves most nearly to accord with the observed data. With suggested constants this is recommended for use in New York City and, when properly adjusted as to values of coefficients, to other localities as well.

For the proper design of storm sewers it is essential to know fairly definitely the frequency at which the severe rainfall intensities occur for different intervals of time. The rate of rainfall is one of the factors used in estimating run-off in the determination of sewer capacities and the choice of its value will be governed by the frequency. The most severe intensities of rain are not generally used in obtaining the capacities of sewers, as the cost of such sewer sizes would be excessive and, in fact, unwarranted because of the rarity of these storms.

Severe rainfalls are listed in terms of the frequency of their occurrence. Further, it is necessary to classify them by their duration, by their rate or intensity corresponding to the duration, and by the frequency that such rates or intensities have been equalled or exceeded.

NOTE.—Published in February, 1934, *Proceedings*.

[1] Asst. Div. Engr., Board of Transportation, New York, N. Y.

Rainfalls are designated by referring to the frequencies at which certain intensities occur for various periods. For instance, a "2-year" storm is one in which the relation between rate of rainfall and corresponding duration has been equalled or exceeded on an average of once in two years for the entire recorded period.

RAINFALL DATA AND THEIR USES

It is apparent that the first task is to collect the basic data. This, in many respects, is the most difficult part of the work as the records for any one rain-gauge are enormous. Those from the Government Observatory in Central Park, New York City, which extend over 62 years, from 1869 to 1930, inclusive, and form the subject of this paper, required a great deal of patience and painstaking labor in tabulating the data as correctly as possible.[2] The earlier records do not give as complete details for short-time intensities as the more recent ones, and this must be borne in mind in interpreting them.

A summary in Table 1 shows the limitations found or assumed in collecting the data, while the storm records themselves are listed in Table 2. It is evident that there is no regularity in the recurrence of severe intensities for any period. In order, therefore, to obtain working rules, it is desirable to arrange all intensities in a regular sequence starting with the heaviest and proceeding to the lowest. This has been done in Table 3.

TABLE 1.—SCOPE OF RAINFALL TABULATION FOR CENTRAL PARK GAUGE, NEW YORK CITY, 1869 TO 1930, INCLUSIVE

Period, in minutes	Maximum observed intensity, in inches per hour	Minimum intensity considered, in inches per hour *
5	9.12	3.00
10	9.06	1.92
15	8.44	1.92
30	5.50	0.91
60	3.31	0.90
120	1.85	0.90

* Except years in which the maximum is less than the minimum given.

Since the record covers 62 years, the frequency (second column), corresponding to the first number, is $\frac{1}{62}$, or 0.016 times per year. A "5-year" storm lies between Serial Nos. 12 and 13, as one-fifth of 62 is 12.4.

Summarized from Table 3, the character of the various kinds of storms are as given in Table 4. Where the serial number is a fraction the rainfall intensities have been interpolated.

As the data in Table 3 are limited, and known as "random samples," any one sample may not conform to the general trend of the aggregate samples. This is readily indicated by the irregular lines when the data are plotted on cross-section paper.

The human mind strives to secure order in the relations of physical measurements and considers that the apparent irregularity is due to errors

---

[2] Data collected by Mr. O. Hufeland, *Engineering News*, August 31 and September 7, 1916, covering the period, 1869 to 1913, inclusive, supplemented by published records of the U. S. Weather Bureau. Gaps in these data indicate that such maximum rainfall intensities were not available.

TABLE 2.—Basic Chronological Records of Heavy Rainfalls, Central Park Gauge, New York City, 1869 to 1930. Intensity of Rainfall, in Inches per Hour, for Durations Given

Year	Date	Duration, in Minutes					
		5	10	15	30	60	120
1869	6/21	.....	2.10	.....	1.20	.....	.....
	8/13	5.88	3.12	2.24	.....	.....	.....
	8/15	5.88	2.22	.....	.....	.....	.....
1870	6/27	.....	2.22	2.00	1.52	.....	.....
	7/28	4.32	2.70	.....	.....	.....	.....
1871	6/7	4.20	3.54	3.12	1.82	.....	.....
	7/16	3.00	3.00	2.88	2.04	1.30	.....
	8/16	.....	.....	2.00	.....	.....	.....
	9/26	.....	2.28	2.28	1.62	0.97	.....
	10/11	6.48	4.38	3.60	2.20	1.14	.....
	10/26	3.24	2.94	2.88	1.82	.....	.....
1872	7/25	.....	.....	.....	1.14	1.00	.....
1873	7/5	3.96	3.00	2.48	1.54	.....	.....
	7/27	3.24	2.28	1.92	1.40	.....	.....
	8/21	3.00	2.46	.....	1.30	1.00	.....
1874	7/10	3.00	2.22	.....	1.14	0.98	.....
	7/12	4.20	3.00	2.40	1.34	.....	.....
1875	6/22	3.48	2.10	.....	.....	.....	.....
	7/26	5.00	5.00	4.56	2.54	1.27	.....
	8/18	3.72	3.30	2.64	1.84	1.25	.....
1876	5/26	.....	2.70	.....	.....	.....	.....
	8/16	3.36	3.24	3.00	2.28	1.09	.....
1877	6/21	.....	.....	.....	1.00	.....	.....
	7/3	3.36	2.58	2.00	.....	.....	.....
	7/17	3.20	3.18	3.12	2.00	1.09	.....
	8/14	.....	.....	2.00	1.40	.....	.....
	8/25	.....	2.58	.....	.....	.....	.....
	10/4	.....	2.00	2.00	1.80	1.50	1.08
1878	6/21	.....	.....	.....	1.36	1.00	.....
	7/10	.....	2.16	.....	.....	.....	.....
	7/21	.....	2.10	.....	.....	.....	.....
	7/31	.....	.....	.....	1.48	.....	.....
	8/1	.....	2.22	2.16	1.72	1.12	.....
	8/5	9.00	8.22	8.00	5.22	2.66	.....
	9/4	3.60	2.88	2.08	1.72	1.50	1.10
	11/27	.....	.....	.....	1.00	.....	.....
1879	8/25	3.00	2.40	2.00	1.08	.....	.....
1880	7/12	6.00	4.74	3.44	2.04	1.17	.....
	7/27	4.32	3.00	2.04	.....	.....	.....
	8/10	3.00	.....	.....	.....	.....	.....
	8/20	5.40	2.88	1.96	.....	.....	.....
1881	5/22	5.04	4.68	4.68	2.40	1.20	.....
	7/4	4.20	2.58	1.92	.....	.....	.....
1881	8/7	3.36	2.28	.....	.....	.....	.....
1882	6/15	4.00	.....	.....	.....	.....	.....
	6/26	.....	.....	1.92	.....	.....	.....
	9/11	.....	2.10	1.92	1.62	1.10	.....
	9/20	3.96	2.76	2.08	.....	.....	.....
	9/22	3.00	2.82	2.68	2.22	1.34	.....
	9/23	3.00	3.00	2.64	2.30	1.74	1.31
1883	5/21	3.00	2.52	2.32	1.86	1.51	.....
	6/6	4.44	.....	.....	.....	.....	.....
	7/13	5.16	3.48	2.84	1.56	.....	.....
1884	6/25	.....	2.04	1.92	1.56	1.00	.....
	7/4	3.00	.....	.....	.....	.....	.....
	7/12	3.00	2.28	.....	1.10	.....	.....
	8/5	4.20	3.18	2.48	1.36	.....	.....
	8/22	3.72	3.54	3.24	2.70	1.51	.....
	8/30	3.00	2.88	2.72	1.90	1.01	.....
1885	6/5	3.00	2.16	.....	.....	.....	.....
	7/29	3.48	2.10	.....	.....	.....	.....
	8/3	3.60	3.12	2.56	1.82	1.00	.....
1886	11/18	3.36	.....	.....	.....	.....	.....
1887	5/31	3.36	3.18	2.52	2.22	1.22	.....
	6/17	3.36	.....	.....	.....	.....	.....
	6/20	4.20	2.58	1.84	1.08	.....	.....
	6/22	3.12	2.52	2.04	1.40	1.08	.....
	8/18	4.92	2.58	.....	.....	.....	.....
	8/21	3.60	2.10	.....	.....	.....	.....
	8/22	.....	.....	.....	1.00	0.92	.....
	8/24	3.00	2.16	.....	.....	.....	.....
1888	5/28	.....	.....	.....	1.20	.....	.....
	5/29	.....	.....	1.60	.....	.....	.....
	6/26	3.30	2.82	2.48	1.78	.....	.....
	7/19	3.00	2.46	2.08	1.12	.....	.....
	8/4	.....	2.52	2.40	2.20	1.45	1.02
	8/21	2.76	2.40	2.28	1.62	1.21	.....
	9/8	.....	.....	.....	1.54	.....	.....
	9/20	3.00	3.00	.....	1.76	1.21	.....
1889	6/11	.....	1.98	.....	.....	.....	.....
	6/15	.....	.....	.....	1.10	.....	.....
	7/27	.....	.....	.....	1.20	.....	.....
	7/30	3.60	2.40	2.20	1.36	1.00	.....
	7/31	3.00	2.10	.....	.....	.....	.....
	8/2	3.00	.....	.....	.....	.....	.....
	9/17	3.24	2.70	2.24	.....	.....	.....
1890	6/6	3.60	3.48	2.48	1.50	.....	.....
	7/17	.....	2.28	.....	.....	.....	.....
	8/22	3.48	2.40	1.96	1.20	.....	.....
	9/16	2.88	2.04	.....	.....	.....	.....
	9/17	.....	.....	.....	1.00	.....	.....
1891	4/23	3.12	.....	.....	.....	.....	.....
	6/17	2.88	2.28	.....	1.00	.....	.....
	6/23	3.00	2.64	2.32	1.20	.....	.....
	8/23	.....	2.28	.....	1.22	1.04	.....
	9/5	.....	2.00	.....	1.10	.....	.....
1892	6/9	3.00	.....	.....	.....	.....	.....
	7/3	7.44	4.44	3.04	1.54	1.13	.....
	8/9	.....	.....	.....	1.06	.....	.....
	9/13	3.72	2.70	2.18	1.12	.....	.....
1893	6/6	3.84	3.60	2.44	1.28	1.08	.....
	7/26	3.84	2.40	.....	.....	.....	.....
	8/19	5.04	3.72	2.76	1.74	1.07	.....
	8/20	6.00	5.10	4.80	3.30	1.68	0.92
	8/23	.....	.....	.....	1.20	1.05	.....
	9/7	.....	2.10	.....	.....	.....	.....
	10/27	3.00	1.98	.....	1.08	.....	.....
1894	7/29	.....	2.40	.....	.....	.....	.....
	8/3	4.80	2.70	.....	1.00	.....	.....
	9/8	3.60	2.70	2.84	1.78	1.03	.....
	9/16	3.60	2.28	.....	.....	.....	.....
1895	6/5	5.64	3.72	2.60	1.38	.....	.....
	8/4	.....	.....	.....	1.16	.....	.....
	8/6	6.35	3.96	3.12	.....	.....	.....
	8/31	3.96	3.42	2.80	1.64	.....	.....
1896	5/28	.....	.....	.....	1.04	.....	.....
	6/10	4.32	2.64	2.00	.....	.....	.....
	7/6	4.20	2.70	2.64	2.12	1.85	.....
	7/15	3.60	2.00	.....	1.04	.....	.....
	7/20	3.00	2.10	.....	1.22	.....	.....
	7/22	3.24	2.04	2.00	1.20	0.92	.....
	7/30	.....	2.40	.....	.....	.....	.....
	9/13	3.90	3.90	2.92	1.70	0.94	.....
	9/15	3.60	2.04	.....	.....	.....	.....
	9/18	4.68	2.64	.....	.....	.....	.....
1897	5/24	5.52	3.72	3.24	2.20	1.18	.....
	6/9	2.88	2.10	.....	1.28	1.02	.....
	7/21	3.00	.....	.....	.....	.....	.....
	7/23	3.24	2.34	2.34	1.40	1.06	.....

TABLE 2.—(*Continued*)

Year	Date	DURATION, IN MINUTES					
		5	10	15	30	60	120
1897	7/28	.....	.....	.....	1.00	.....	.....
	8/23	3.00	3.00	2.76	2.22	1.74	0.96
	9/ 1	4.20	3.00	2.32	1.44	.....	.....
1898	4/24	3.60	2.28	.....	.....	.....	.....
	7/ 4	4.20	3.00	2.00	.....	.....	.....
1899	6/15	.....	.....	.....	1.62	.....	.....
	6/28	3.00	.....	.....	.....	.....	.....
	7/12	4.20	3.18	2.16	.....	.....	.....
	8/10	5.28	2.76	1.96	1.18	.....	.....
	9/ 1	.....	.....	.....	1.42	1.20	0.90
	9/ 3	4.80	3.00	2.80	1.70	.....	.....
1900	5/19	.....	2.00	1.92	1.58	0.95	.....
	8/12	3.00	.....	.....	.....	.....	.....
	8/26	3.84	3.30	3.20	2.30	.....	.....
1901	7/ 5	3.48	2.88	2.72	2.10	1.50	1.32
	7/ 6	4.32	3.66	3.00	1.54	.....	.....
	7/31	6.98	6.00	4.20	2.16	1.10	.....
	8/24	.....	.....	.....	1.54	1.08	.....
1902	6/21	.....	.....	.....	1.00	.....	.....
	7/20	.....	2.10	.....	.....	.....	.....
	7/28	6.00	5.16	4.60	4.08	3.02	.....
	8/ 5	6.00	5.16	4.52	2.90	1.61	.....
	8/19	.....	2.50	2.40	.....	.....	.....
1903	2/28	4.08	.....	.....	.....	.....	.....
	6/14	.....	2.00	2.00	1.42	.....	.....
	6/21	3.12	2.28	.....	.....	.....	.....
	6/29	2.88	2.70	2.60	1.82	1.55	1.05
	7/ 2	.....	2.40	.....	.....	.....	.....
	7/22	3.60	2.88	2.00	1.04	.....	.....
	9/16	.....	.....	.....	1.06	1.03	.....
	9/17	.....	1.98	.....	.....	.....	.....
	10/ 8	3.00	2.40	2.40	2.00	1.44	.....
	10/ 9	.....	.....	.....	1.50	1.35	1.21
1904	5/30	.....	2.10	.....	.....	.....	.....
	7/11	.....	.....	.....	1.00	.....	.....
	7/28	3.24	.....	.....	.....	.....	.....
	8/ 4	3.00	.....	.....	.....	.....	.....
	8/19	3.72	3.54	3.28	2.76	1.72	1.07
	9/14	.....	2.40	1.92	1.26	.....	.....
1905	6/22	.....	2.46	2.20	1.96	1.14	.....
	8/13	.....	2.70	2.44	1.40	1.11	.....
	10/19	.....	2.16	2.08	.....	.....	.....
1906	5/ 5	3.72	3.12	2.20	.....	.....	.....
	5/27	5.04	3.42	2.72	.....	.....	.....
	7/10	.....	2.04	.....	1.40	.....	.....
	7/17	3.24	3.12	2.44	.....	.....	.....
1906	9/13	3.00	.....	.....	.....	.....	.....
1907	8/19	3.00	.....	.....	.....	.....	.....
	8/24	.....	.....	.....	1.30	1.15	.....
	9/11	3.00	3.00	2.56	1.52	.....	.....
1908	7/14	4.80	3.30	2.60	2.20	1.16	.....
	9/28	.....	.....	.....	1.10	.....	.....
1909	6/28	.....	2.40	2.20	1.30	.....	.....
	8/ 4	.....	.....	.....	1.32	0.96	.....
	8/13	3.00	2.20	2.10	1.20	.....	.....
1911	3/27	3.24	1.98	.....	.....	.....	.....
	6/10	.....	2.52	2.40	.....	.....	.....
	6/11	3.00	2.60	2.40	1.54	1.28	.....
	7/20	3.84	3.60	2.60	1.34	.....	.....
	8/15	.....	2.40	2.08	1.56	.....	.....
	8/24	.....	.....	.....	1.36	1.08	.....
	10/19	.....	2.00	2.00	1.34	1.00	.....
1912	2/26	4.20	3.00	2.24	1.34	.....	.....
	5/30	6.72	4.92	3.68	2.00	1.03	.....
	7/21	4.80	3.48	2.76	1.78	0.96	.....
1913	6/20	4.32	2.52	.....	1.60	0.99	.....
	7/24	3.24	2.58	2.28	1.18	1.09	.....
	7/28	6.24	5.94	4.88	2.98	2.74	1.56
	8/ 7	.....	2.28	2.04	1.46	.....	.....
	9/ 4	7.20	6.90	6.36	5.24	3.31	1.85
	10/ 1	6.72	4.68	4.04	3.12	2.03	1.67
1914	7/ 2	.....	.....	.....	1.08	.....	.....
	8/21	3.12	2.52	2.44	1.84	0.98	.....
1915	6/15	3.24	2.64	2.00	1.24	.....	.....
	6/30	4.08	3.60	3.12	2.34	1.45	.....
	7/ 5	6.24	5.22	3.84	2.12	1.11	.....
	8/ 4	.....	2.04	1.96	1.68	1.24	1.05
1916	6/17	3.36	3.06	2.80	1.48	1.09	.....
	7/26	3.60	2.64	2.08	1.48	0.94	.....
1917	6/14	3.24	2.70	2.32	1.88	1.12	1.10
	9/ 1	.....	.....	.....	0.90	.....	.....
	12/18	.....	.....	.....	0.96	0.95	.....
1918	5/21	3.00	2.64	2.44	1.74	1.00	.....
	6/ 7	3.60	2.76	2.04	1.44	1.33	.....
	7/30	.....	.....	.....	1.10	.....	.....
1919	6/20	6.96	5.28	4.04	2.28	1.25	.....
	6/27	.....	.....	.....	1.16	.....	.....
	7/26	4.80	3.60	3.16	2.24	1.33	.....
	8/ 1	.....	.....	.....	1.08	.....	.....
	8/ 8	.....	.....	.....	1.16	.....	.....
	9/23	.....	.....	.....	1.70	1.39	1.02
1920	7/ 3	3.96	2.46	.....	1.16	1.09	.....
1920	7/24	4.56	3.60	3.40	2.00	1.17	1.02
	8/11	3.60	2.64	.....	1.06	.....	.....
1921	8/ 7	.....	2.10	.....	.....	.....	.....
	9/30	5.04	3.30	2.48	1.68	1.16	.....
1922	7/ 1	3.00	2.88	2.40	1.92	1.14	.....
	7/ 3	4.56	3.72	3.32	1.96	1.09	.....
	7/18	3.36	2.76	2.20	2.00	1.11	.....
1923	6/ 7	3.00	2.16	.....	.....	.....	.....
1924	5/24	.....	1.98	.....	.....	.....	.....
	8/26	.....	.....	.....	1.12	1.02	.....
1925	6/16	4.32	3.18	2.48	1.52	.....	.....
	6/25	4.80	3.90	3.36	1.72	.....	.....
	7/31	.....	.....	.....	1.00	.....	.....
	8/ 9	3.36	2.70	1.92	0.98	.....	.....
1926	7/25	6.36	5.22	4.88	2.80	1.52	.....
	8/12	4.56	4.20	3.16	2.14	2.14	1.22
	8/14	.....	.....	.....	1.02	.....	.....
	9/ 6	.....	1.92	.....	1.24	.....	.....
1927	5/23	5.64	3.42	2.44	1.28	.....	.....
	6/26	.....	.....	.....	1.32	.....	.....
	7/ 7	3.60	2.40	2.08	1.38	0.91	.....
	7/23	3.84	2.70	2.20	2.00	1.15	.....
	8/ 1	6.84	4.80	3.92	3.24	1.79	0.96
	8/ 8	3.00	2.76	2.76	1.68	1.20	.....
	8/14	.....	.....	.....	1.10	.....	.....
	10/13	3.60	3.30	2.60	1.86	1.19	.....
1928	6/ 9	3.96	3.54	2.48	1.38	.....	.....
	6/19	.....	.....	.....	1.20	0.95	.....
	6/24	.....	.....	.....	1.06	.....	.....
	7/ 6	.....	.....	.....	1.26	.....	.....
	7/12	.....	2.40	.....	1.22	.....	.....
	7/20	3.84	4.14	2.96	.....	.....	.....
	8/10	3.84	2.88	2.76	1.42	.....	.....
	10/19	7.56	3.78	2.52	1.36	.....	.....
1929	2/26	.....	.....	.....	1.06	.....	.....
	3/23	3.12	2.76	2.76	1.68	1.30	.....
	5/12	3.96	2.04	.....	0.98	.....	.....
	5/24	3.00	2.70	2.32	1.22	.....	.....
	6/28	.....	.....	.....	1.10	.....	.....
1930	6/26	.....	1.92	.....	0.94	.....	.....
	7/ 3	9.12	9.06	8.44	5.50	2.77	1.39
	7/24	3.96	3.42	2.48	1.40	.....	.....
	7/29	5.16	4.20	3.32	2.80	1.40	.....
	8/16	5.28	3.00	.....	.....	.....	.....
	8/23	.....	.....	.....	1.32	1.09	.....

TABLE 3.—RAINFALLS IN ORDER OF MAGNITUDE, CENTRAL PARK GAUGE, NEW YORK CITY, 1869 TO 1930. INTENSITY OF RAINFALL, IN INCHES PER HOUR, FOR DURATIONS GIVEN

Serial No.	Average number per year	Duration, in Minutes						Serial No.	Average number per year	Duration, in Minutes				
		5	10	15	30	60	120			5	10	15	30	60
1.......	0.016	9.12	9.06	8.44	5.50	3.31	1.85	32......	0.516	5.04	3.60	3.12	2.12	1.30
2.......	0.032	9.00	8.22	8.00	5.24	3.02	1.67	33......	0.532	5.04	3.60	3.04	2.12	1.30
3.......	0.048	7.56	6.90	6.40	5.22	2.77	1.56	34......	0.548	5.00	3.60	3.00	2.04	1.28
4.......	0.064	7.44	6.00	4.88	4.09	2.74	1.39	35......	0.565	4.92	3.60	3.00	2.04	1.27
5.......	0.080	7.20	5.94	4.88	3.30	2.66	1.32	36......	0.581	4.80	3.60	2.96	2.00	1.25
6.......	0.097	6.98	5.28	4.80	3.24	2.14	1.31	37......	0.597	4.80	3.54	2.92	2.00	1.25
7.......	0.113	6.96	5.28	4.68	3.12	2.03	1.22	38......	0.613	4.80	3.54	2.88	2.00	1.24
8.......	0.129	6.84	5.22	4.60	2.98	1.85	1.21	39......	0.628	4.80	3.54	2.88	2.00	1.22
9.......	0.145	6.72	5.22	4.56	2.90	1.79	1.10	40......	0.645	4.80	3.54	2.84	2.00	1.21
10.......	0.160	6.60	5.16	4.52	2.80	1.74	1.10	41......	0.661	4.68	3.48	2.80	2.00	1.21
11.......	0.176	6.48	5.16	4.20	2.80	1.74	1.08	42......	0.677	4.56	3.48	2.80	1.96	1.20
12.......	0.193	6.36	5.10	4.12	2.76	1.72	1.07	43......	0.694	4.56	3.42	2.80	1.96	1.20
13.......	0.210	6.35	5.00	4.04	2.70	1.68	1.05	44......	0.709	4.56	3.42	2.76	1.92	1.20
14.......	0.226	6.30	4.80	4.04	2.54	1.61	1.05	45......	0.726	4.44	3.42	2.76	1.90	1.19
15.......	0.242	6.24	4.74	3.92	2.40	1.55	1.02	46......	0.743	4.32	3.42	2.76	1.88	1.18
16.......	0.258	6.00	4.68	3.84	2.34	1.53	1.02	47......	0.758	4.32	3.30	2.76	1.86	1.17
17.......	0.274	5.00	4.68	3.60	2.30	1.52	0.96	48......	0.774	4.32	3.30	2.76	1.86	1.17
18.......	0.290	6.00	4.44	3.44	2.30	1.51	0.96	49......	0.791	4.32	3.30	2.72	1.84	1.16
19.......	0.306	6.00	4.38	3.40	2.28	1.51	0.96	50......	0.806	4.32	3.30	2.72	1.84	1.16
20.......	0.322	5.88	4.20	3.36	2.28	1.50	0.92	51......	0.823	4.32	3.30	2.72	1.82	1.15
21.......	0.338	5.88	4.20	3.32	2.24	1.50	0.90	52......	0.839	4.20	3.24	2.68	1.82	1.15
22.......	0.355	5.64	4.14	3.32	2.22	1.50	....	53......	0.856	4.20	3.18	2.64	1.82	1.14
23.......	0.371	5.64	3.96	3.28	2.22	1.45	....	54......	0.871	4.20	3.18	2.64	1.82	1.14
24.......	0.387	5.52	3.90	3.24	2.22	1.45	....	55......	0.886	4.20	3.18	2.64	1.80	1.14
25.......	0.404	5.40	3.90	3.24	2.20	1.44	....	56......	0.902	4.20	3.18	2.64	1.78	1.13
26.......	0.419	5.28	3.78	3.20	2.20	1.40	....	57......	0.918	4.20	3.18	2.60	1.78	1.12
27.......	0.436	5.28	3.72	3.16	2.20	1.39	....	58......	0.936	4.20	3.12	2.60	1.78	1.12
28.......	0.452	5.16	3.72	3.16	2.20	1.35	....	59......	0.953	4.20	3.12	2.60	1.76	1.11
29.......	0.468	5.16	3.72	3.12	2.16	1.34	....	60......	0.968	4.20	3.12	2.60	1.74	1.11
30.......	0.484	5.04	3.72	3.12	2.16	1.33	....	61......	0.984	4.14	3.12	2.60	1.74	1.11
31.......	0.500	5.04	3.66	3.12	2.14	1.33	....	62......	1.00	4.08	3.06	2.56	1.72	1.10

TABLE 3.—(*Continued*)

Serial No.	Average number per year	Duration, in Minutes				
		5	10	15	30	60
63	1.02	4.08	3.00	2.56	1.72	1.10
64	1.03	4.00	3.00	2.52	1.72	1.09
65	1.05	3.96	3.00	2.52	1.70	1.09
66	1.07	3.96	3.00	2.48	1.70	1.09
67	1.08	3.96	3.00	2.48	1.70	1.09
68	1.10	3.96	3.00	2.48	1.68	1.09
69	1.11	3.96	3.00	2.48	1.68	1.09
70	1.13	3.96	3.00	2.48	1.68	1.09
71	1.15	3.96	3.00	2.48	1.68	1.08
72	1.16	3.90	3.00	2.48	1.64	1.08
73	1.18	3.84	3.00	2.48	1.62	1.08
74	1.19	3.84	3.00	2.44	1.62	1.08
75	1.21	3.84	3.00	2.44	1.62	1.07
76	1.23	3.84	2.94	2.44	1.62	1.06
77	1.24	3.84	2.88	2.44	1.62	1.05
78	1.26	3.84	2.88	2.44	1.60	1.04
79	1.27	3.72	2.88	2.44	1.58	1.03
80	1.29	3.72	2.88	2.40	1.56	1.03
81	1.31	3.72	2.88	2.40	1.56	1.02
82	1.32	3.72	2.88	2.40	1.54	1.02
83	1.34	3.72	2.88	2.40	1.54	1.01
84	1.35	3.60	2.82	2.40	1.54	1.00
85	1.37	3.60	2.82	2.40	1.54	1.00
86	1.39	3.60	2.76	2.40	1.54	1.00
87	1.40	3.60	2.76	2.40	1.54	1.00
88	1.42	3.60	2.76	2.34	1.52	1.00
89	1.44	3.60	2.76	2.32	1.52	1.00
90	1.45	3.60	2.76	2.32	1.52	1.00
91	1.47	3.60	2.76	2.32	1.50	1.00
92	1.48	3.60	2.70	2.32	1.50	0.99
93	1.50	3.60	2.70	2.32	1.48	0.98
94	1.52	3.60	2.70	2.28	1.48	0.98
95	1.53	3.60	2.70	2.28	1.48	0.97
96	1.55	3.60	2.70	2.28	1.46	0.96
97	1.56	3.60	2.70	2.28	1.44	0.96
98	1.58	3.60	2.70	2.24	1.44	0.95
99	1.60	3.48	2.70	2.24	1.42	0.95
100	1.62	3.48	2.70	2.20	1.42	0.95
101	1.63	3.48	2.70	2.20	1.42	0.94
102	1.65	3.48	2.70	2.20	1.40	0.94
103	1.67	3.36	2.70	2.20	1.40	0.92
104	1.69	3.36	2.70	2.20	1.40	0.92
105	1.70	3.36	2.64	2.20	1.40	0.91
106	1.71	3.36	2.64	2.18	1.40	....
107	1.73	3.36	2.64	2.16	1.40	....
108	1.74	3.36	2.64	2.16	1.40	....
109	1.76	3.36	2.64	2.16	1.38	....
110	1.78	3.36	2.64	2.10	1.38	....
111	1.79	3.36	2.64	2.08	1.38	....
112	1.81	3.36	2.60	2.08	1.36	....
113	1.82	3.36	2.58	2.08	1.36	....
114	1.84	3.30	2.58	2.08	1.36	....
115	1.86	3.24	2.58	2.08	1.36	....
116	1.87	3.24	2.58	2.08	1.36	....
117	1.89	3.24	2.58	2.08	1.34	....
118	1.91	3.24	2.58	2.04	1.34	....
119	1.92	3.24	2.52	2.04	1.34	....
120	1.94	3.24	2.52	2.04	1.34	....
121	1.95	3.24	2.52	2.04	1.32	....
122	1.97	3.24	2.52	2.00	1.32	....
123	1.98	3.24	2.52	2.00	1.32	....
124	2.00	3.24	2.52	2.00	1.30	....

TABLE 3.—(*Continued*)

Serial No.	Average number per year	DURATION, IN MINUTES				Serial No.	Average number per year	DURATION, IN MINUTES			Serial No.	Average number per year	DURATION, IN MINUTES	
		5	10	15	30			5	10	30			10	30
125	2.01	3.24	2.50	2.00	1.30	156	2.51	3.00	2.28	1.12	187	3.02	2.04	1.00
126	2.03	3.20	2.48	2.00	1.30	157	2.53	3.00	2.28	1.12	188	3.04	2.04	1.00
127	2.05	3.12	2.46	2.00	1.28	158	2.55	3.00	2.28	1.10	189	3.06	2.04	1.00
128	2.07	3.12	2.46	2.00	1.28	159	2.57	3.00	2.22	1.10	190	3.07	2.04	1.00
129	2.08	3.12	2.46	2.00	1.28	160	2.58	3.00	2.22	1.10	191	3.08	2.00	0.98
130	2.10	3.12	2.46	2.00	1.26	161	2.59	3.00	2.22	1.10	192	3.10	2.00	0.98
131	2.12	3.12	2.46	2.00	1.26	162	2.61	3.00	2.22	1.10	193	3.11	2.00	0.96
132	2.13	3.00	2.40	2.00	1.24	163	2.63	3.00	2.20	1.10	194	3.13	2.00	0.94
133	2.15	3.00	2.40	2.00	1.24	164	2.65	3.00	2.16	1.10	195	3.15	2.00	0.90
134	2.16	3.00	2.40	2.00	1.24	165	2.66	3.00	2.16	1.09	196	3.16	2.00	....
135	2.18	3.00	2.40	1.96	1.22	166	2.67	3.00	2.16	1.08	197	3.18	1.98	....
136	2.19	3.00	2.40	1.96	1.22	167	2.69	....	2.16	1.08	198	3.20	1.98	....
137	2.21	3.00	2.40	1.96	1.22	168	2.71	....	2.16	1.08	199	3.21	1.98	....
138	2.23	3.00	2.40	1.96	1.22	169	2.73	....	2.10	1.08	200	3.23	1.98	....
139	2.24	3.00	2.40	1.95	1.20	170	2.75	....	2.10	1.08	201	3.24	1.98	....
140	2.26	3.00	2.40	1.92	1.20	171	2.76	....	2.10	1.06	202	3.26	1.92	....
141	2.28	3.00	2.40	1.92	1.20	172	2.77	....	2.10	1.06	203	3.27	1.92	....
142	2.29	3.00	2.40	1.92	1.20	173	2.79	....	2.10	1.06				
143	2.31	3.00	2.40	1.92	1.20	174	2.81	....	2.10	1.06				
144	2.33	3.00	2.40	1.92	1.20	175	2.83	....	2.10	1.06				
145	2.34	3.00	2.40	1.92	1.20	176	2.84	....	2.10	1.04				
146	2.36	3.00	2.34	1.92	1.20	177	2.85	....	2.10	1.04				
147	2.37	3.00	2.28	1.92	1.20	178	2.87	....	2.10	1.04				
148	2.39	3.00	2.28	....	1.18	179	2.89	....	2.10	1.02				
149	2.41	3.00	2.28	....	1.16	180	2.91	....	2.10	1.00				
150	2.42	3.00	2.28	....	1.16	181	2.92	....	2.10	1.00				
151	2.44	3.00	2.28	....	1.16	182	2.93	....	2.10	1.00				
152	2.45	3.00	2.28	....	1.16	183	2.95	....	2.04	1.00				
153	2.47	3.00	2.28	....	1.14	184	2.97	....	2.04	1.00				
154	2.48	3.00	2.28	....	1.14	185	2.99	....	2.04	1.00				
155	2.50	3.00	2.28	....	1.12	186	3.00	....	2.04	1.00				

in the measurements, to the incompleteness of the observations, or to both. To overcome such apparent deficiencies resort is made to various methods. A smooth curve may be drawn which would average the plotted data, or an equation may be developed by methods used in statistics or biometrics. In this study the graphical and algebraic methods have been used.

The data in Table 3 may be treated in three ways:

1.—A relation may be obtained for the data taken along the horizontal lines showing a relation for each frequency between duration and rainfall intensity.

2.—A relation may be obtained for the data listed in vertical columns under each given duration. This will show a relation for each duration between the rainfall intensity and the frequency.

3.—A single relation may be obtained between the three factors of duration, rainfall intensity, and frequency, using all the data both horizontally and vertically in a single comprehensive calculation.

TABLE 4.—CONDENSED STORM DATA, CENTRAL PARK GAUGE, NEW YORK CITY, 1869 TO 1930, INCLUSIVE

Duration, in minutes	Serial No.......... 62 Times per year..... 1 Storm designation.. 1 yr	31 0.5 2 yr	12.4 0.2 5 yr	6.2 0.1 10 yr	2.48 0.04 25 yr	1.24 0.02 50 yr
5...........	4.08	5.04	6.36	6.98	8.31	9.09
10...........	3.06	3.66	5.06	5.28	7.59	8.86
15...........	2.56	3.12	4.09	4.78	7.23	8.33
30...........	1.72	2.14	2.74	3.22	5.23	5.44
60...........	1.10	1.33	1.70	2.12	2.90	3.24
120...........	....	....	1.06	1.29	1.62	1.81

It is more convenient for general use to obtain the first form of relationship between the duration and the rainfall intensity for the "1-year" and other storms. In the Borough of Manhattan at least the "10-year" storm would be used for sewer design, whereas for certain parts of the Borough of Queens, the formula or curve for a "2-year" storm would be satisfactory.

The second form of relationship between frequency and rainfall intensity will give for each duration an entire gamut of storms, including 1-year, 2-year, and others. The third form, combining into one formula a relation between the three variable factors of intensity, duration, and frequency, would graduate the data on a surface, whereas the previous forms graduate the factors on a series of lines without showing any regularity among the different lines. All three forms of relationship are exemplified in this paper.

RECIPROCAL FORMULA

In Greater New York there is a partiality for a reciprocal formula of the type:

$$R = \frac{A}{t + b} \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \quad (1)$$

giving the relation between t, the duration of the storm, in minutes, and R, the corresponding rainfall intensity, in inches per hour, equalled or exceeded once in a certain number, n, of years. Hence, Equation (1) would be the formula for a theoretical "n-year" storm. For a 1-year storm ($n = 1$) and, similarly, for other storms, A and b would have one set of fixed values, which must be obtained for that storm. If done by trial this would be a laborious hit-and-miss method; but by the method of least squares the data given in Table 3 are sufficient. Calculations of A and b may be simplified by the following consideration: If the logarithms of Equation (1) are taken, it becomes,

$$\log R = \log A - \log (t + b) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

This equation is linear between $\log R$ and $\log (t + b)$. If logarithmic ruled paper is used the R and t data will plot as an irregular line or curve for each character of storm. If, however, to each value of t a certain fixed value for b is added, the effect is to straighten the curve. That value of b is used which will make the transformed line most nearly straight and, at the same time, plot to an angle of 45° with the axes. The graphs for Equation (1) are shown on Fig. 1, using the data from Table 3. The transformed lines are not indicated.

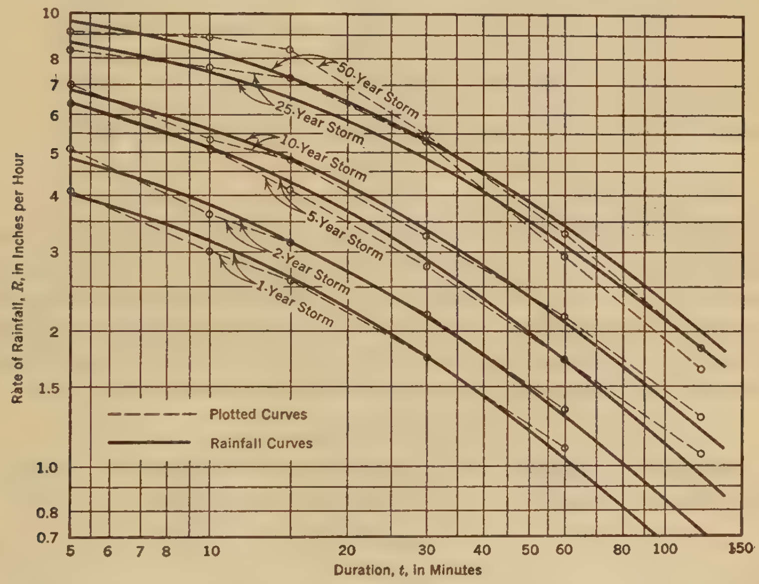


Fig. 1.—Rainfall Curves in Form, $R = \frac{A}{t + b}$. Data from Central Park Gauge, New York, N. Y., 1869 to 1930. Formulas for Various Storms Given in Table 5.

Now that b is known, A is computed from Equation (2) by the method of least squares as illustrated for a more complicated case in the Appendix.

Formulas of the type given in Equation (1) for the various storms are plotted in Fig. 1 and listed in Table 5. Comparisons are then made between the observed results and the results obtained from the formulas. The average difference between the observed intensity and the result from Equation (1) equals 0.182 in. per hr for the 10-year storm. Similar average differences for the various characters of storms have been computed as shown in Table 5.

The final average difference computed as described in the Appendix, is 0.303 which may be taken as a measure of the closeness with which formulas of the type of Equation (1), as illustrated in Fig. 1, fit the observations. This diagram shows that expressions similar to Equation (1) cannot be made to fit the data as accurately as is desired.

TABLE 5.—VARIATION OF RAINFALL CURVES BY RECIPROCAL FORMULA FROM OBSERVED DATA

Storms	Values for Rate R, Equation (1)	Average difference from observed data, in inches per hour
1-year	$\frac{76.4}{t + 14}$	0.075
2-year	$\frac{96.3}{t + 15}$	0.140
5-year	$\frac{128}{t + 15}$	0.099
10-year	$\frac{168}{t + 20}$	0.182
25-year	$\frac{261}{t + 25}$	0.404
50-year	$\frac{291}{t + 25}$	0.563
Final average difference	......	0.303

The formulas in Table 5 show the relationship between R and t for each frequency, n. If t is made, successively, 5 min, 10 min, and longer periods, in each of these formulas, a relation will be obtained between R and n. If these values, in turn, are plotted, an irregular line will be obtained, showing the necessity of further adjustment, as will be explained later.

EXPONENTAL FORMULA

In addition to Equation (1) a second type of formula for rainfall frequently used, is expressed in the following exponential form:

$$R = \frac{B}{t^c} \qquad (3)$$

in which, B and c are constants, having one set of values for a 1-year storm and a different set for a 2-year and other storms. Taking the log of Equation (3),

$$\log R = \log B - c \log t \qquad (4)$$

which is a straight line between log R and log t. This line may be obtained directly, whereas in Equation (1) a constant value had to be added to t to yield an approximate straight line. The tangent of the slope of the transformed lines represented by Equation (1) is the same for all storms, when their graphs are plotted with similar scales in both directions on logarithmic paper for the 1-year and other storms, all making an angle of 45° to the "log t" line. In Equation (4) the tangent of the slope is c, which differs for various storms. Therefore, the lines representing such storms on logarithmic paper, Fig. 2, will not be parallel. In this diagram, also, values of B and c have been determined by the method of least squares as given in the Appendix. The resulting forms of Equation (3) and the average deviations for the different storms are given in Table 6.

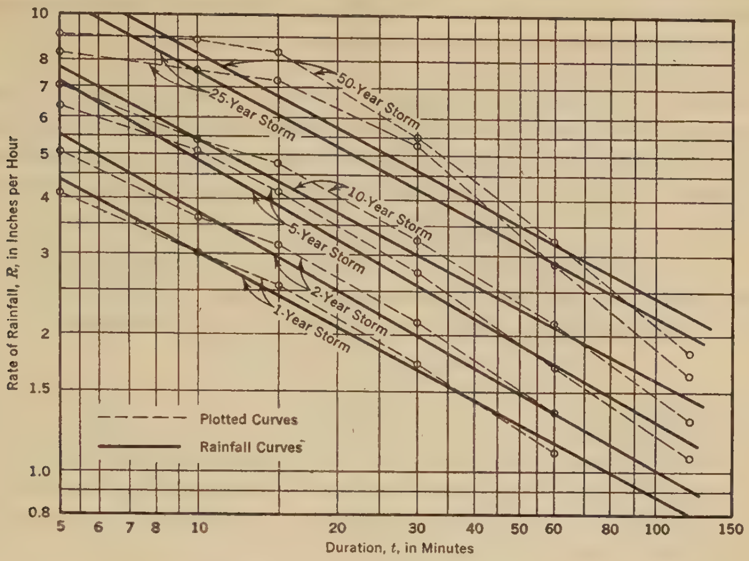


FIG. 2.—RAINFALL CURVES IN FORM, $R = \frac{B}{t^c}$. DATA FROM CENTRAL PARK GAUGE, 1869 TO 1930. FORMULAS FOR VARIOUS STORMS GIVEN IN TABLE 6.

TABLE 6.—VARIATION OF RAINFALL CURVES BY EXPONENTIAL FORMULA FROM OBSERVED DATA

Storm	Values for Rate R, Equation (3)	Average difference from observed data, in inches per hour
1-year	$\frac{10.11}{t^{0.529}}$	0.128
2-year	$\frac{12.38}{t^{0.53}}$	0.144
5-year	$\frac{18.1}{t^{0.576}}$	0.376
10-year	$\frac{18.1}{t^{0.53}}$	0.370
25-year	$\frac{24.8}{t^{0.526}}$	1.27
50-year	$\frac{28}{t^{0.53}}$	1.41
Final average difference	....	0.807

The first form of rainfall formula, the reciprocal type illustrated by Equation (1), has the defect that it requires the straight line between log R and log $(t + b)$ to be at a definite inclination of 45° downward to the right for all characters of storm, and, therefore, may not be made to conform to the observed data as closely as desired. This was actually found to be the case, as may be seen in Fig. 1. The second, or exponential, type, as represented by Equation (3) and Fig. 2, has the defect that it requires the relation between log R and log t to be straight. It also does not conform to the observed data.

That the reciprocal type fits the observed facts better than the exponential is apparent from inspection of the diagrams and also by comparing the final average differences between the observed and the theoretical results. The advantage is 0.303 for Equation (1) as against 0.807 for Equation (3).

MODIFIED EXPONENTIAL FORMULA

It at once suggests itself that a combination of Equations (1) and (3) may be made to conform to the observed data more closely than either one alone. In the most general form such an equation may be written:

$$R = \frac{C}{(t + d)^e} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

in which, C, d, and e are constants for each character of storm, the 1-year storm values differing from those of the 2-year and other storms. With this type of equation three constants have to be determined as compared with only two constants for Equations (1) and (3). Equation (5) may be written in logarithmic form as follows:

$$\log R = \log C - e \log (t + d) \quad \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

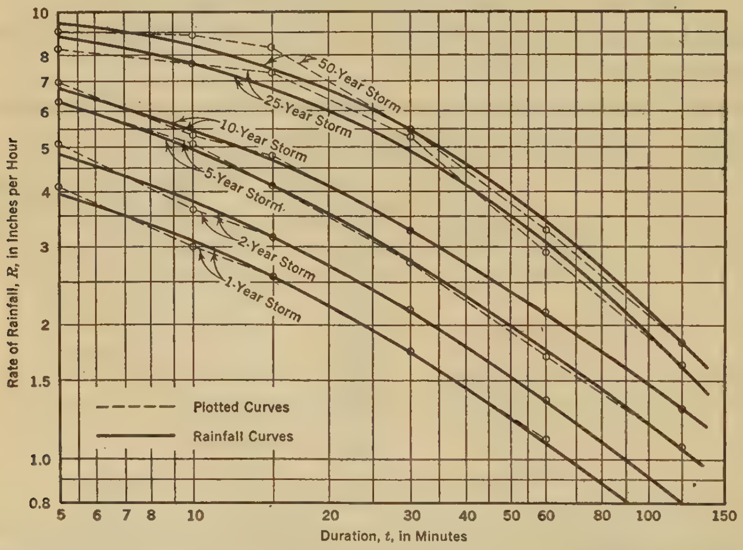


FIG. 3.—RAINFALL CURVES IN FORM, $R = \frac{C}{(t + d)^e}$. DATA FROM CENTRAL PARK GAUGE, 1869 TO 1930. FORMULAS FOR VARIOUS STORMS GIVEN IN TABLE 7.

This is not linear between log R and log t, but is linear between log R and log $(t + d)$. Therefore, if the various observed values are plotted on logarithmic paper (Fig. 3), the lines connecting them will be irregularly bent the same as in Fig. 1. When this bent line is transformed by adding a constant value, d, to each value of t, keeping the corresponding R the same, a differently bent line will be obtained. If the bending is increased, smaller values for d should be tried, and conversely until almost a straight line is obtained. This process is the same as was described in developing Fig. 1. The proper values of d which will make the transformed line almost straight can then be readily interpolated and checked by plotting.

After d has been obtained in this manner, e and C are computed by the method of least squares, as shown in detail for this case in the Appendix. The appropriate formulas for the various storm classifications and the resultant deviation of the plotted data from the curves for R (Fig. 3), are given in Table 7.

TABLE 7.—VARIATION OF RAINFALL CURVES BY MODIFIED EXPONENTIAL FORMULA FROM OBSERVED DATA

Storm	Values for Rate R, Equation (5)	Average difference from observed data, in inches
1-year..........................	$\frac{38.85}{(t+10)^{0.842}}$	0.057
2-year..........................	$\frac{41.62}{(t+9)^{0.818}}$	0.096
5-year..........................	$\frac{60.53}{(t+10)^{0.835}}$	0.061
10-year..........................	$\frac{63.75}{(t+12)^{0.795}}$	0.155
25-year..........................	$\frac{1\ 468}{(t+40)^{1.842}}$	0.329
50-year..........................	$\frac{4\ 201}{(t+50)^{1.518}}$	0.428
Final average difference..........	....	0.235

About 1921 the Bureau of Sewers of the City of New York developed various formulas by a trial-and-error method from the rainfall data, which formulas were plotted on ordinary cross-section paper. These formulas were

TABLE 8.—COMPARISON OF VARIOUS FORMULAS FOR RAINFALL, WITH OBSERVED DATA

Types of formulas	TWO-YEAR STORM		TEN-YEAR STORM	
	Formulas	Average difference from observed data, in inches per hour	Formulas	Average difference from observed data, in inches per hour
Old formulas....................	$\frac{120}{t+20}$	0.285	$\frac{150}{t+15}$	0.387
Reciprocal formulas (Fig. 1)........	$\frac{96.3}{t+15}$	0.140	$\frac{168}{t+20}$	0.182
Exponential formulas (Fig. 2).......	$\frac{12.38}{t^{0.53}}$	0.144	$\frac{18.1}{t^{0.53}}$	0.370
Modified exponential formulas (Fig. 3)	$\frac{41.62}{(t+9)^{0.818}}$	0.096	$\frac{63.75}{(t+12)^{0.795}}$	0.155

of the type given in Equation (1). It is interesting to give comparative results between these "old formulas" and the various formulas presented in this paper, as shown in Table 8. The average differences from the observed data are smallest for the formulas in Table 7, plotted in Fig. 3, and largest for the old formulas.

Relation Between Rainfall Intensity and Frequency

If the observed data in Table 3 are plotted for each duration given in the vertical columns, irregular lines will be obtained, showing the relation between the rainfall intensity, *R*, and the frequency, *n*, for the duration given. It is evident that there is as much justification for seeking order in the relation between *R* and *n* for each duration, *t*, as between *R* and *t* for each frequency, *n*, or character of storm that has been obtained in Figs. 1, 2, and 3. Studies have been made to obtain such relationship from Table 3.

The equation connecting *R*, rainfall intensity, and *n*, average number of times per year which it occurs or is exceeded, is:

$$R\,n^{h} = k \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

in which, *h* and *k* are constants for each duration, *t*. The various values of *h* and *k* are given in Fig. 4 with the curves and the observed data.

These formulas in Fig. 4 have been computed from the observed data by the method of least squares. The results will agree, in general, only approximately with those obtained from the formulas in Fig. 3.

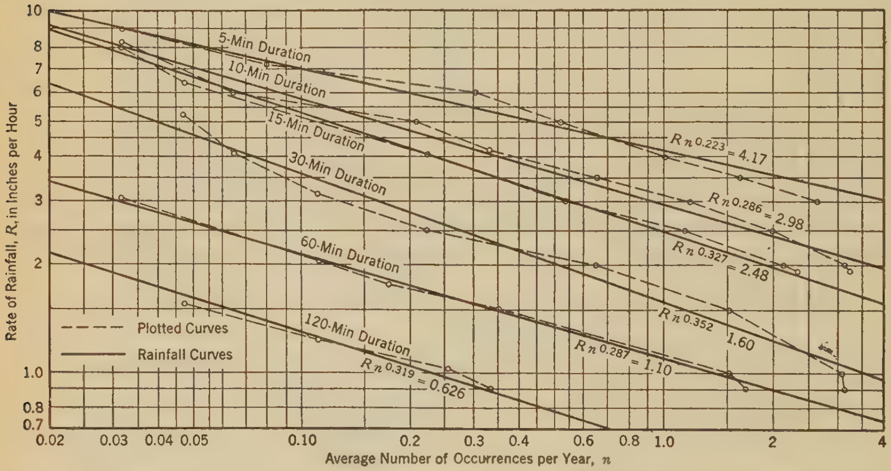


FIG. 4.—HEAVY RAINFALL FREQUENCIES IN FORM, $R\,n^{h} = k$. DATA FROM CENTRAL PARK GAUGE, NEW YORK, N. Y., FROM 1869 TO 1930.

It is desirable to harmonize all the data given in Table 3 showing a relation between *R*, *t*, and *n*. An approximate relation between the three factors has been obtained, as follows:

$$R\,n^{0.3} = \frac{42.5}{(t + 12)^{0.85}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

The logical refinement leading to the development of Equation (8) may be unnecessary in practice. It would probably be expedient to use the set of formulas which best fit the observations even if they do not harmonize with one another. Accordingly, it is recommended that the formulas in Table 7, and Fig. 3 be used in sewer investigations and design.

PROBABILITY METHOD

Rainfall records note certain facts which have already occurred. Like history they teach by example. If the conditions continue with little alteration, it is assumed that the past records may be used to anticipate what may occur in the future. Any regularity which may be perceived in past events may be assumed to recur in the future.

Various expressions have been given in this paper which tend to show such regularity by a method known as "curve fitting." Other methods can also be used in interpreting those records for future guidance. The best known are those used in biometrics and vital statistics, as developed by Karl Pearson and his school. They are offshoots of the theory of probability.

The duration curve is sometimes called the integral curve and gives the relative frequency that a certain magnitude has been equalled or exceeded. The magnitude may refer to average monthly or annual stream flow, total annual rainfall, or rainfall intensity for a 15-min duration. The frequency curve gives only the frequency of occurrence of a definite magnitude, showing, for instance, how many times an annual rainfall of 40 to 45 in. may be expected to occur in 100 years. The integral or duration formula is obtained from the frequency formula. Its expression is generally very complicated.

Various types of frequency curves or equations have been suggested. "Types I and III" appear to fit engineering data best. Type I applies to a range of records which are skewed and have both a finite minimum and a finite maximum value. Type III is used for records having a finite value in one direction only. From the manner in which the variations of the magnitude from the average occur in the recorded data the probable skew value is obtained by certain mathematical processes.

This probability method, applied to the data in Tables 2 and 3, shows that the Type III curve fits the data best. The coefficient of skew appears to vary from 1.56 to 3.02, with an average value of 2.22. The curves in Fig. 5 have been prepared for durations of 5, 10, 15, 30, 60 and 120 min, based on the Type III curve with a uniform coefficient of skew, $C\ S = 2.22$. From Fig. 5 it appears that once in 50 years a rainfall intensity of 7.40 in. per hr and more may occur for a 10-min duration; 4.55 in. per hr and more for 30 min; and 2.80 in. per hr and more for 60 min.

A comparison between the results obtained from Figs. 3 and 5 is given in Table 9. An inspection of Table 9 indicates that the modified exponential method gives results more in accord with the actual data than the probability method.

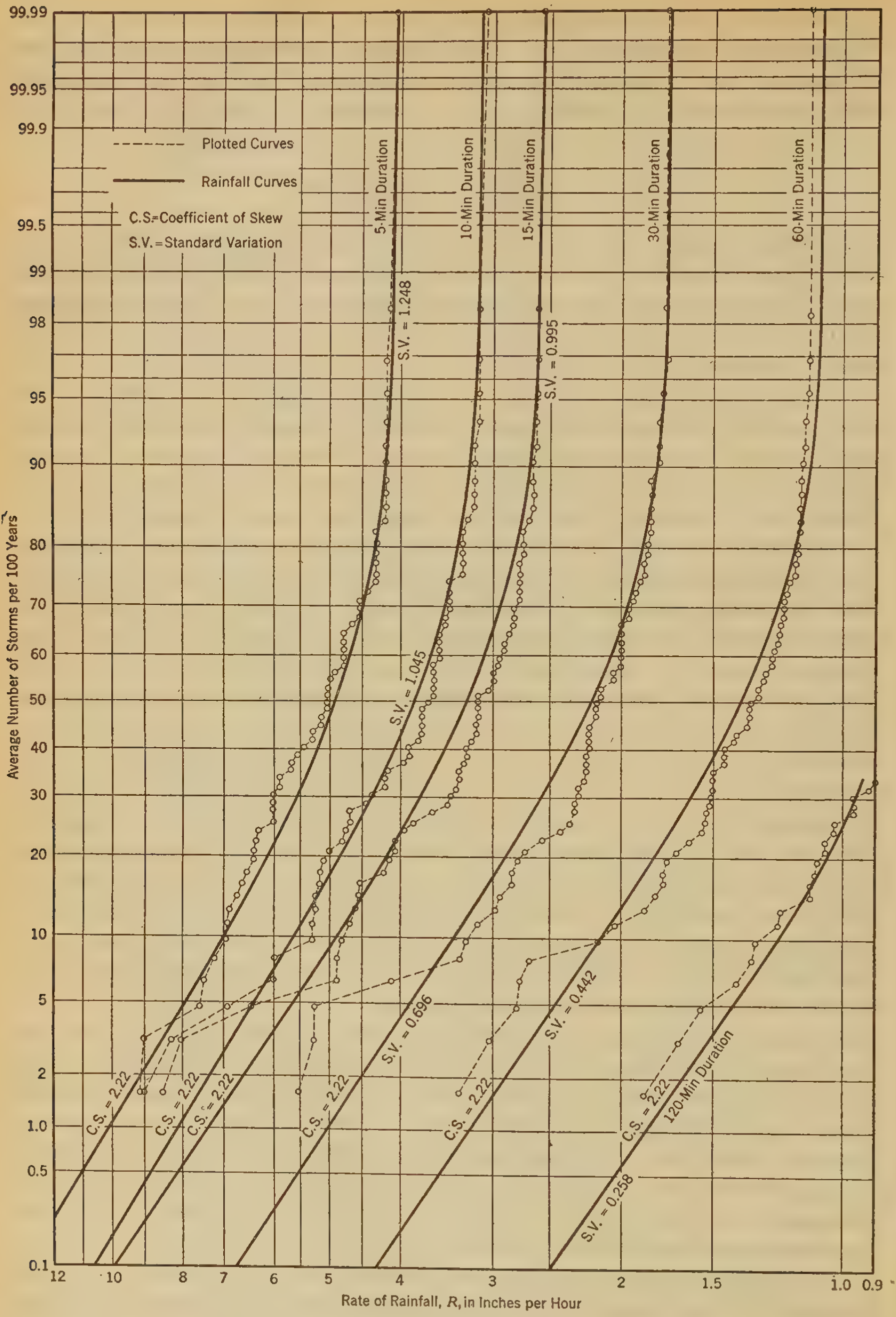


Fig. 5.—Heavy Rainfall Frequencies. Data from Central Park Gauge, New York, N. Y., 1869 to 1930. Curves by Probability Method.

TABLE 9.—RAINFALL INTENSITIES BY MODIFIED EXPONENTIAL AND PROBABILITY METHODS COMPARED WITH ACTUAL INTENSITIES

Duration, in minutes	Frequency, in years	n	Modified exponential method, Fig. 3, in inches per hour	Probability method, Fig. 5, in inches per hour	Actual intensities, in inches per hour
10	Once in 50.	0.02	8.20	7.40	8.86
30	Once in 50.	0.02	5.50	4.55	5.44
60	Once in 50.	0.02	3.40	2.80	3.24
10	Once in 10.	0.10	5.40	5.50	5.28
30	Once in 10.	0.10	3.25	3.38	3.22
60	Once in 10.	0.10	2.10	2.12	2.12
10	Once in 2.	0.50	3.79	3.81	3.66
30	Once in 2.	0.50	2.12	2.20	2.14
60	Once in 2.	0.50	1.33	1.38	1.33

CONCLUSION

Various types of formulas have been developed showing the relations between the rainfall intensities, durations, and frequencies of occurrence. The results, however, do not equally fit the recorded storms, and the single unified formula, Equation (8), does not appear to be sufficiently close to the observations to warrant its use for all kinds of storms. The six formulas given in Table 7 and plotted in Fig. 3 answer most accurately the requirements of the engineer, either for computing storm sewers or for making investigations of the flooding of buildings or of underground subways in New York City. It is also believed that this paper gives in ample detail the procedure to be used in obtaining similar formulas for locations other than New York City.

ACKNOWLEDGMENT

The paper was prepared in the course of work in the Sewer and Drainage Sub-Division of the Designing Office of the Board of Transportation, New York City. The collection of the data and the laborious calculations were made by William A. O'Leary, Assoc. M. Am. Soc. C. E. Acknowledgment is made to C. E. Conover, Division Engineer of the Designs Division, A. I. Raisman, Chief Designing Engineer, Members, Am. Soc. C. E., and Robert Ridgway, Past-President and Hon. M., Am. Soc. C. E., Consulting Engineer, of the Board of Transportation, all of whom made the work of preparing this paper possible.

APPENDIX

DERIVATION OF FORMULA FOR 10-YEAR STORM IN THE FORM, $R = \dfrac{C}{(t + d)^e}$

This Appendix explains the calculation of the modified exponential formula, Equation (5), for a 10-year storm; the calculations of the reciprocal formula, Equation (1), and the exponential formula, Equation (2), are similar. The data from Table 3 for a 10-year storm are plotted on logarithmic ruled paper shown as a broken line. By adding, however, a constant, 12, to t the graph will become very nearly straight. With Equation (5), the slope of the line is not restricted to 45° as required by Equation (1). The equation of this line may be found by the method of least squares. In this case: $R = \dfrac{C}{(t + 12)^e}$. Taking logarithms of both sides,

$$\log R = \log C - e \log (t + 12)$$

and using observed values of R and t,

$$\log R - \log C + e \log (t + 12) = D$$

in which, D is a small number. Then,

$$\Sigma\, [\log R - \log C + e \log (t + 12)]^2 = \Sigma\, D^2 = M$$

For M to be a minimum,

$$\frac{\partial M}{\partial \log C} = 0$$

$$\frac{\partial M}{\partial e} = 0$$

and,

$$\frac{\partial M}{\partial \log C} = -2\, \Sigma\, [\log R - \log C + e \log (t + 12)] = 0$$

that is,

$$\Sigma \log R = n \log C - e\, \Sigma \log (t + 12) \ldots \text{(1st Normal Equation)}$$

in which, n is the number of observations, which is equal to 6 in Table 10. Also,

$$\frac{\partial M}{\partial e} = 2\, \Sigma \left\{ [\log R - \log C + e \log (t + 12)] \times \log (t + 12) \right\} = 0$$

or,

$$\Sigma\, [\log R \times \log (t + 12)] = \log C\, \Sigma\, (t + 12)$$
$$- e\, \Sigma\, [\log (t + 12)]^2 \ldots\ldots\ldots \text{(2d Normal Equation)}$$

TABLE 10.—CALCULATION OF FACTORS IN NORMAL EQUATIONS.

t (1)	$t + 12$ (2)	$\log (t + 12)$ (3)	$[\log (t + 12)]^2$ (4)	Observed R (5)	$\log R$ (6)	$\log R \times \log (t + 12)$ (7)	Computed R (8)	Difference (9)	Difference² (10)
5........	17	1.2304	1.5139	6.98	0.8439	1.0383	6.69	—0.29	0.0841
10........	22	1.3424	1.8020	5.28	0.7226	0.9700	5.46	+0.18	0.0324
15........	27	1.4314	2.0489	4.78	0.6794	0.9725	4.62	—0.16	0.0256
30........	42	1.6232	2.6348	3.22	0.5079	0.8244	3.26	+0.04	0.0016
60........	72	1.8573	3.4495	2.12	0.3263	0.6060	2.13	+0.01	0.0001
120.......	132	2.1206	4.4969	1.29	0.1106	0.2345	1.32	+0.03	0.0009
Sums......	....	9.6053	15.9460	......	3.1907	4.6457	......	......	0.1447
Average difference	....	....	....	......	......	....	......	......	0.155

The data and computations from the observed values of t are listed most conveniently in the form given in Table 10. From this is found:

$$\text{Average difference} = \sqrt{\frac{0.1447}{6}} = 0.155$$

Completing the normal equations for the summations in Table 10,

$$3.1907 = 6 \log C - e \times 9.6053$$

and,

$$4.6457 = 9.6053 \log C - e \times 15.9460$$

Solving, $e = 0.795$ and $C = 63.75$. Therefore,

$$R = \frac{63.75}{(t + 12)^{0.795}}$$

in which, R = rate of rainfall, in inches per hour, and t = duration, in minutes.

DISCUSSION

H. Alden Foster,[3] M. Am. Soc. C. E. (by letter).—The author has analyzed the rainfall data by probability methods, based on the frequency curves of Karl Pearson. As no details of this analysis are presented, it may be helpful to engineers who are unfamiliar with these methods to state that the subject was treated, in 1924, by the writer[4] and, in 1927, by R. D. Goodrich, M. Am. Soc. C. E.[5]

In applying the frequency curve method, the author states that "the coefficient of skew appears to vary from 1.56 to 3.02, with an average value of 2.22." Fig. 5 was plotted, using the average coefficient of skew, 2.22, for all the curves. It is apparent from an inspection of Fig. 5 that this average value is not suitable for all the curves. It is not surprising, therefore, that the probability method used by Mr. Bleich gives results that differ from the actual data more than those obtained by means of the exponential method (Fig. 3, or Equation (5)).

The author states that "it is desirable to harmonize all the data given in Table 3 showing a relation between *R*, *t*, and *n*," and submits Equation (8) as a summary of these data. In his "Conclusion" he states that "Equation (8) does not appear to be sufficiently close to the observations to warrant its use for all kinds of storms." A generalized empirical formula is a dangerous tool to use in any case, unless its limitations are well understood. It would seem unwise, therefore, to publish a new rainfall formula that is admittedly unreliable even when applied to its basic data.

The rainfall records given in Tables 2 and 3 are well prepared, and should be of value to the profession. The formulas in Table 7 should also be useful as a guide for engineers studying problems of storm run-off in this part of the United States.

J. J. Slade,[6] Jr., Esq. (by letter).—An interesting and important correlation between the duration and intensities of New York storms is developed in this paper. The author has spared no labor in comparing the results obtained by the use of various methods of analysis, and he finds that his formula gives a better representation of the observed data than others that he considers.

One question is left open, however, which merits consideration: What are the values that any formula is expected to give? The observed data represent a set of events that have occurred in the past and that almost certainly will never occur again in the form once observed. The fact that an empirical formula gives, closely, the values of an observed set of events does not mean that it will fit closely a future set. It is not surprising that the author finds that the values obtained from his modified exponential formula agree more closely with observed values than do those obtained by the probability method, since he has determined the constants in his formula by the method of least squares to give just such an agreement.

---

[3] Res. Engr., Parsons, Klapp, Brinckerhoff & Douglas, North Platte, Nebr.

[4] "Theoretical Frequency Curves and Their Application to Engineering Problems," by H. Alden Foster, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 142.

[5] A graphical method for analyzing similar data is given in "Straight Line Plotting of Skew Frequency Data," by R. D. Goodrich, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 91 (1927), p. 1.

[6] Asst. Prof. of Eng. Mechanics, Rutgers Univ., New Brunswick, N. J.

Mr. Bleich regards the probability method as an alternate in the solution of this problem. The method is not alternative, however, but rather complementary to any method used. The correlation required in this particular problem is between members of a small set culled from a large group of events. Such a local set cannot be treated independently; it belongs to the whole group, and it is only as a part of the whole group that one may make assertions about it.

The probability method gives information about the entire group of events. For instance, it involves all the available information regarding the 30-min duration intensities, and then it indicates how, in the long run, these intensities will be distributed. It does not fit the record locally; it fits it as a whole. The past 62-yr record deviates from the probability curve, and so will the future record; but the probability curve furnishes a norm about which any record of the phenomenon in question will fluctuate.

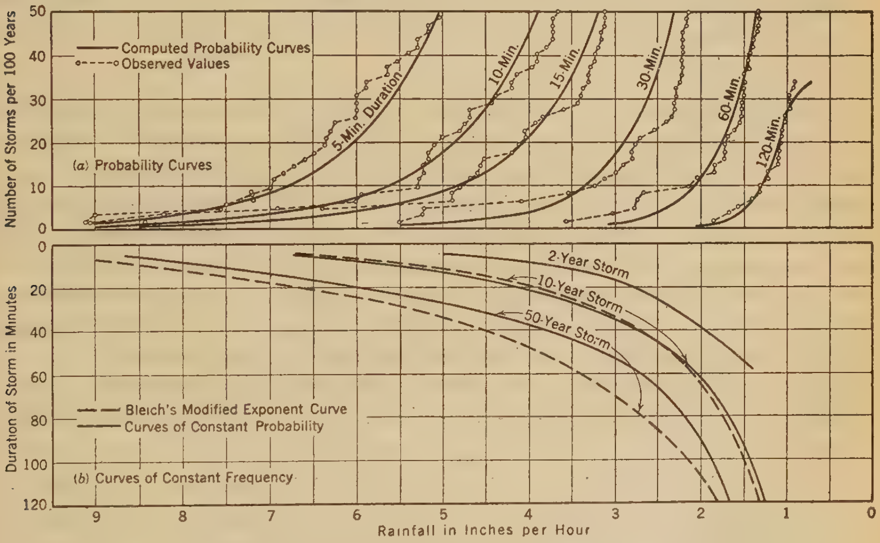


FIG. 6.—CURVES SHOWING THAT, LOCALLY, ALL DURATION CURVES DEVIATE FROM THE PROBABLE NORM IN MUCH THE SAME MANNER.

A glance at Fig. 6 shows that, locally, all the duration curves deviate from the probable norm in much the same manner. That the deviations should be so uniform is not surprising when one remembers that the grouping of the durations into the various classes is to a great extent artificial, and that these various classes really blend into each other. On the other hand, this uniform deviation may be taken to mean that the actual distribution of intensities is not the one given by the probable norm, but rather by some other curve which should fit the observed data more closely—only the observations of the next few hundred years will give an answer to this question. However, it is likely that this deviation is characteristic of the current meteorological period more than of the long-term distribution of intensities.

If these contentions are correct, the constants of the modified exponential formula should be computed, not from the observed data, but from the modified values given by the probability method, because the determination of the constants directly from the data makes use of only a few isolated observations. It does not in any way take into account the distribution of other intensities, whereas the values as adjusted by the probability method result from taking into account the entire observed set.

Fig. 6(*a*) gives the probability curves for the various durations carried out to a frequency of 50 storms in 100 yr. These curves are the same as those in Fig 5. The slight modifications result from the fact that the writer has used the true coefficient of skew for each curve instead of the author's constant value of 2.22. Fig 6(*b*) gives three curves of constant frequency as obtained from the curves of Fig 6(*a*), together with two of Mr. Bleich's modified exponential curves. It is seen that the 50-yr storm curve would be greatly modified by the suggestion made in this discussion. The curves have been plotted on ordinary co-ordinate paper so that they may be seen in their true relation to each other.

CHARLES W. SHERMAN,[7] M. AM. SOC. C. E. (by letter).—The writer has been greatly disappointed in Mr. Bleich's paper. He cannot but feel that the author has failed to analyze thoroughly the records of the automatic rain-gauge, which has been longest in use of any in the United States, and to derive results that will be of real value. It is especially discouraging to find that, apparently, Mr. Bleich has not even utilized all the data published[8] by O. Hufeland in 1916, and that, for years later than 1913, he has contented himself with the incomplete data published by the U. S. Weather Bureau, instead of compiling adequate data from the original chart records.

Under the circumstances, the writer will content himself with a brief statement of the principal points in which he feels that Mr. Bleich's paper is open to criticism:

1.—There is no description of the gauge and its exposure, and there is no reference to any other source from which such information can be obtained. Neither is there any comparison of records from this gauge and other gauges in the vicinity to indicate the accuracy of the record.

2.—No records of precipitation for periods between 30 and 60 min, or between 60 and 120 min, are given, nor are there any for durations exceeding 120 min.

3.—There is no indication that the author has given any consideration to what the writer has called the "Extended Duration Principle."[9] It is quite possible that a considerably larger number of records of storms of comparative long duration than the author has utilized, would have been obtained by the application of this principle.

---

[7] Cons. Engr. (Metcalf & Eddy), Boston, Mass.

[8] *Engineering News*, 1916, pp. 76, and 594.

[9] *Transactions*, Am. Soc. C. E., Vol. 95 (1931), p. 953.

4.—It does not appear logical that the constant, C, in Equation (5), should vary from 38.85 for storms of 1-yr frequency, to 4 201 for storms of 50-yr frequency; nor that the exponent, e, in the same formula should vary between 0.795 and 1.513.

In his own studies of rainfall intensity and frequency at Boston, Mass., the writer found[10] that C varied according to a mathematical law from 16.0 for 1-yr frequency to 45.9 for 50-yr frequency; and that d and e were constant. Similar results, although with other values of the constants, were found[11] for Detroit, Mich., by Milton F. Wagnitz, M. Am. Soc. C. E., and Lewis C. Wilcoxen, Assoc. M. Am. Soc. C. E.

Equation (8) is similar in form to that used by the writer and by Messrs. Wagnitz and Wilcoxen. Apparently, Mr. Bleich has not much confidence in it; but the writer suspects that if the analysis of the records had been more detailed, as suggested in the preceding Comments (2) and (3), considerable justification for Equation (8), or one similar to it, would have been found.

A comparison of this formlula with those devised for Detroit, and for Boston, is as follows:

Place	Formula	Derived by:
New York, N. Y.	$Rn^{0.3} = \frac{42.5}{(t + 12)^{0.85}}$	. . . . Bleich
Detroit, Mich.	$Rn^{0.263} = \frac{37.6}{(t + 8)^{0.855}}$	. . . . . Wagnitz and Wilcoxen
Boston, Mass.	$Rn^{0.27} = \frac{16.0}{(t + 7)^{0.7}}$	. . . . Sherman

5.—Mr. Bleich has made no mention of previous papers on rainfall intensity as indicated by the Central Park gauge, except that of Mr. Hufeland.* The data and discussions in the reports of the Committee on Rainfall and Run-Off of the Municipal Engineers of the City of New York, published in 1913 and 1922, are worthy of consideration and comment, as are also comments[12] by the late Kenneth Allen, M. Am. Soc. C. E., in 1921, and again in 1922.[13] Mr. Allen's formulas are of the same general form as those of Mr. Bleich, as given in Table 7; but the constants given by Mr. Allen differ from those presented by Mr. Bleich much more than would be expected as the result of including ten years of additional records. It would seem that some statement regarding the earlier studies and explanation for the different results obtained would be in order.

---

[10] *Transactions*, Am. Soc. C. E., Vol. 95 (1931), p. 955.

[11] *Loc. cit.*, p. 965.

[12] *Engineering News-Record*, 1921, Vol. 86, p. 588.

[13] *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), p. 119.

MERRILL M. BERNARD,[14] M. AM. SOC. C. E. (by letter).—This paper is to be commended as a presentation of an excellent procedure in the treatment of rainfall intensity data. Mr. Bleich has accomplished the task of fitting curves, accurately, to plotted points and developing the mathematical expression for the curves; but can the formulas for the 25-year storm and the 50-year storm be accepted as the permanent relationship of rainfall rate and duration? Will the next 63 years of record reproduce the same curve group? When the station has reached 80 years of record, will not several of the present maximum values remain unexceeded? Furthermore, is it not probable that the next 40 years of the record will contribute a number of values which will fall within the unusual interval between the author's 10-year and 25-year storm, thereby affecting the position of the higher frequency curves? What is the climatological cycle, in years, necessary to be spanned by a single station record, in order to make that record a dependable basis for the upper frequency values? How accurately does the composite record fulfill the purpose of the single station record?

The writer believes that the formula types, reciprocal and exponential, serve two distinct purposes, the former more accurately representing the comparatively short periods of continuous downpour and the latter the average of intermittent rainfall for the longer durations. His opinion has been that, because of a paucity in single-station records of adequate length, covering duration periods of less than 1 hr or 2 hr, the Meyer curve group,[15] properly interpolated geographically, could be accepted as the best available expression of rainfall intensity for such duration periods. That New York City could have adopted them without material error is shown subsequently.

The rainfall intensity formula for New York City, taken from the writer's charts,[16] and designed for durations of from 120 min to 4 days, is,

$$R_a = \frac{28\ F^{0.22}}{t^{0.75}} \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots (9)$$

in which, F is the frequency, in years, and t, the duration of rainfall exceeding 1 hr, in minutes. To conform with Mr. Bleich's notation the writer's i is given as R_a, the average rainfall intensity, in inches per hour.

It should be noted that Equation (9) is developed through researches which have utilized the theory that a composite record of several stations within a limited area may be taken as a single-station record having a length equal to the sum of their station-years.

If the formula is modified by the inclusion of an additive factor in its denominator, it can be brought into a general form that expresses rainfall rate in terms of any duration, from 5 min to several days, and for any frequency. This factor seems to be a function of frequency, and expressing it thus, the formula for New York City becomes,

---

[14] Cons. Engr., Crowley, La.

[15] "Elements of Hydrology," by Adolph F. Meyer, M. Am. Soc. C. E., Second Edition, pp. 197–198.

[16] *Transactions*, Am. Soc. C. E., Vol. 96, (1932), p. 618, Figs. 18, 19, and 20.

$$R_a = \frac{28\ F^{0.22}}{\left(t + \dfrac{9}{F^{0.20}}\right)^{0.75}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

Fig. 7 is a comparison of the resulting curve-group and the plotted curves of Mr. Bleich's study.

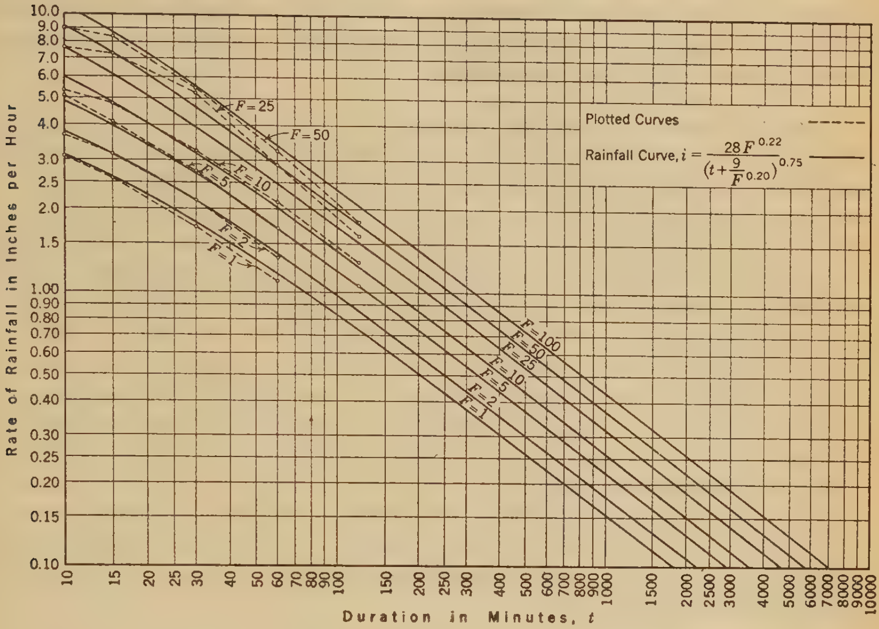


FIG. 7.

Table 11 gives the average deviation, in percentage, of the computed values from the observed values for the 1-year, 2-year, 5-year, and 10-year storms, and for durations of 5 to 120 min, for Mr. Bleich's modified exponential formulas, the writer's formula for New York City, and the Meyer formulas, in which the factors are given the average of the Group 2 and Group 3 values.[17]

TABLE 11.—COMPARISON OF AVERAGE DEVIATION FROM OBSERVED VALUES FOR THREE TYPES OF RAINFALL INTENSITY FORMULAS

Storm	AVERAGE DEVIATION FROM OBSERVED VALUES, PERCENTAGE		
	Author's modified exponential formulas	Equation (10)	Meyer's reciprocal formulas
1-year..................	1.50	3.39	7.26
2-year..................	2.27	2.21	7.03
5-year..................	1.49	1.51	4.07
10-year.................	2.29	5.79	3.40

---

[17] "Elements of Hydrology," Second Edition, p. 166.

The writer has sacrificed little in curve and point agreement to maintain the general form of his equation. A more general form suggesting itself is,

$$R_a = \frac{K\,F^x}{\left(t + \dfrac{B}{F^x}\right)^n} \quad\text{.......................(11)}$$

in which, K and B are coefficients and x and n are exponents, all depending for value on locality. Such a formula would permit the use of the rainfall intensity charts for all duration periods.

CLIFFORD SEAVER,[18] ASSOC. M. AM. SOC. C. E. (by letter).—A valuable service has been performed in bringing the analysis of Central Park rainfall records up to date. The preparation of a paper of this character involves a vast amount of labor, which has apparently been conscientiously done, and, except for a few doubtful figures in Table 2, there is reason to believe that the resulting curves are a close approximation of the available data. The logarithmic method of analysis is particularly interesting. Table 2 eliminates many inconsistencies in previously published data, but there are still inconsistent intensities in the storms of August 15, 1869, and August 16, 1876. In 1912 and 1913, two sets of data are available; that is, Mr. Hufeland's data and Weather Bureau data. The author has used Mr. Hufeland's data in 1912, and, except for the storm of July 1, 1913, the Weather Bureau data in 1913 and subsequent years.

In 1921, engineers of the Board of Estimate and Apportionment of the City of New York computed a set of rainfall curves based on Central Park gaugings through 1920. The curves which would be used in sewer design had the following equations:

15-Year Storm:

$$R = \frac{47}{(t + 8)^{0.75}} \quad\text{.......................(12)}$$

10-Year Storm:

$$R = \frac{43}{(t + 8)^{0.75}} \quad\text{.......................(13)}$$

5-Year Storm:

$$R = \frac{25}{(t + 4)^{0.67}} \quad\text{.......................(14)}$$

It was the intention to use the equation for the 15-year storm for the most highly developed areas, and those for the 10-year and 5-year storms for partly developed territory. The writer had no part in preparing these curves and does not know how much mathematical refinement was used in fitting them to the data, but it is interesting to note that Equations (12), (13), and (14) are of the "modified exponential type" (Equation (5)) found by the author to fit his data best.

---

[18] Asst. Engr., Board of Estimate and Apportionment, City of New York, New York, N. Y.

For a given frequency, the curves developed by the Board of Estimate and Apportionment are somewhat lighter than those proposed by the author, the Board's 10-year storm being approximately equal to the author's 5-year storm. This is due, primarily, to a different conception of frequency, rather than to any marked change in the data since 1920. The Board's engineers defined a 5-year storm as one which, over a 50-year period, would not be equalled during 40, or exceeded during 41, of those years. They placed no limits on the number or intensity of the cloudbursts that might be expected during the remaining nine years.

With this conception of frequency, it is a little difficult to define a 1-year storm, and, for this reason, the writer is inclined to favor the author's definition as being of more general application.

The use of a set of rainfall curves, each curve applicable to a different degree of development, never found much favor with the engineers of the Borough Sewer Bureaus. At the present time (1934), each Borough of the City of New York has its own rainfall curve, selected by its own engineers, and applicable to all parts of that Borough.

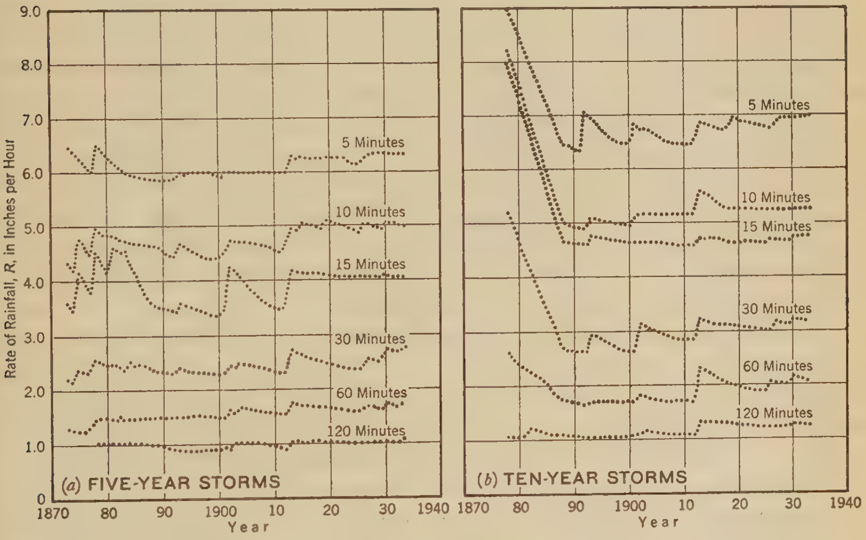


FIG. 8.—RAINFALL CURVES SHOWING YEARLY CHANGES IN DATA.

A fact sometimes forgotten by those using rainfall curves is that they are of a temporary nature and must be revised from time to time, if they are to remain reasonably accurate in view of all existing data. To illustrate this point, the writer has prepared two diagrams, which might be described as "life histories" of the author's 5 and 10-year rainfall curves. Fig. 8(*a*) shows yearly changes in the data for the author's 5-year storm, assuming them to be computed at the end of each calendar year, and Fig. 8(*b*) is the corresponding diagram for the 10-year storm; the curve labels are the durations.

During early years, the curves are affected by the shortness of the record and the unusually severe storm of 1878. Most of the curves reached their low point in the decade, 1890-1900, and, since then, have shown an irregular upward trend which may still be in progress, although the curves have been reasonably stable since 1913. If extended each year, a set of curves of this kind can be kept up to date with little labor, and will give prompt notice of any great changes in the data, which would require a revision of the rainfall curve.

In 1910, one would have said that a reliable 5-year rainfall curve could be obtained from a 42-year record; yet, if such a curve had been computed in that year, it would have been badly in need of revision by 1913, as changes in the data were nearly as great as the difference between the author's 5-year and 10-year curves. Fig. 8 shows that the author's curves, computed from the 1930 data, are still essentially correct, as changes in the data during the last three years have been small.

In his "Synopsis," the author recommends the modified exponential type of equation "for use in New York City, and, when properly adjusted as to values of coefficients, to other localities as well." If $e \doteq 1$, in Equation (5), a "reciprocal" equation is obtained, and, if $d = 0$, the result is an "exponential" equation. Equations (1) and (3), therefore, are merely special cases of the general type embodied in Equation (5). Of course, this type of equation can be adapted to a considerable range of data, but there is nothing in Mr. Bleich's paper to show that the author has ever attempted to fit his equations to rainfall data outside New York City. It is conceivable that rainfall data for some cities might plot in a form similar to the data for the author's 25-year or 50-year storms. The equations for these storms do not fit the data very closely, and it is probable that, for data of this character, a more complicated equation would be required. Mr. Bleich's recommendation, therefore, should be accepted with some reservations.

The author has proposed a set of rainfall curves for New York City. It is not entirely clear whether he means Greater New York, or merely the Borough of Manhattan. If the former, there is some question whether rainfall curves deduced from Central Park records would not be too severe for some of the outlying sections. It is a peculiar fact, but local conditions seem to exist, that make some parts of the city wetter than others.

A Committee on Sewer Capacity, authorized by the Board of Estimate and Apportionment of the City of New York in 1932, secured rainfall records on an experimental area in the Borough of Brooklyn from 1925 to 1929 and on an area in the Borough of Manhattan from 1927 to 1931. Records were taken continuously from May to November of each year, using three rain-gauges on each area.

The Manhattan area is in the Grand Central section of the Borough, 1 mile south of Central Park, and extends from about the middle of the Island to the East River. The Brooklyn area is in the East New York section of the Borough, 7 miles southeast of the Manhattan area and 4½ miles from the East River. In studying the records obtained on the two areas, the writer was impressed by the relatively greater intensity of the storms gauged on the Manhattan area, and, to obtain some concrete evidence on this point, made two comparative tabulations.

TABLE 12.—COMPARISON OF AVERAGE RAINFALL INTENSITIES, IN INCHES PER HOUR

Year	Location	Number of storms	DURATION, IN MINUTES						
			5	10	15	20	30	60	120
(a) BROOKLYN AND MANHATTAN EXPERIMENTAL AREAS									
1927........	Brooklyn...........	9	1.54	1.27	....	0.97	0.85	....	.....
	Manhattan.........	6	2.91	2.52	....	1.90	1.56	....	.....
1928........	Brooklyn...........	13	1.32	1.08	....	0.85	0.72	....	.....
	Manhattan.........	16	2.77	2.17	....	1.42	1.07	....	.....
1929........	Brooklyn...........	7	1.25	0.94	....	0.73	0.56	....	.....
	Manhattan.........	7	2.38	1.91	....	1.32	0.95	....	.....
1927–29.....	Brooklyn...........	29	1.37	1.10	....	0.85	0.72	....	.....
	Manhattan.........	29	2.70	2.18	....	1.50	1.14	....	.....
1927–29.....	Ratio: $\frac{\text{Brooklyn}}{\text{Manhattan}}$		0.51	0.50	....	0.57	0.63	....	.....
(b) BROOKLYN GAUGE NO. 3, AND CENTRAL PARK GAUGE									
1925........	Gauge No. 3........	2	1.98	1.54	1.13	....	0.64	0.40	0.23
	Central Park........	....	3.00	2.58	2.20	....	1.22	0.70	0.38
1926........	Gauge No. 3........	6	2.98	2.36	1.82	....	1.29	0.94	0.64
	Central Park........	....	2.58	2.09	1.69	....	1.15	0.87	0.56
1927........	Gauge No. 3........	7	2.15	1.61	1.35	....	1.00	0.71	0.48
	Central Park........	....	2.42	1.87	1.66	....	1.23	0.80	0.53
1928........	Gauge No. 3........	6	1.65	1.35	1.20	....	1.05	0.82	0.54
	Central Park........	....	3.30	2.06	1.71	....	1.09	0.66	0.43
1929........	Gauge No. 3........	2	1.05	0.80	0.71	....	0.52	0.69	0.42
	Central Park........	....	2.34	1.74	1.52	....	0.93	0.54	0.36
1925–29.....	Gauge No. 3........	23	2.13	1.66	1.36	....	1.02	0.77	0.51
	Central Park........	....	2.73	2.03	1.71	....	1.15	0.76	0.49
1925–29.....	Ratio: $\frac{\text{Gauge No. 3}}{\text{Central Park}}$		0.78	0.82	0.80	....	0.89	1.01	1.04

Table 12(*a*) gives the average yearly intensities for different durations in periods when gauging was in progress on both areas. Besides storms gauged on both areas the table includes a number of storms local to Brooklyn and others local to Manhattan. The total number of storms (29) happens to be the same on each area, but the yearly distribution varies. Many light storms, which would have no bearing on sewer design, were not analyzed and, therefore, are not included in Table 12(*a*). The average intensities for the entire period show that, for durations up to 30 min, the Manhattan storms had nearly twice the intensity of the Brooklyn storms.

Table 12(*b*) gives the average yearly intensities for different durations of identical storms gauged at Central Park and at the Committee's Rain-Gauge No. 3—one of the Brooklyn gauges, located 8½ miles southeast of Central Park and 5 miles from the East River. Storms local to Brooklyn, or local to

Manhattan, are excluded, and Table 12(*b*), therefore, includes a considerable number of "easterly" storms, which were more likely to be gauged at both points. Table 12(*b*) includes the storm of September 6, 1926, which was of moderate intensity at Central Park, but at Gauge No. 3 was a phenomenal double-peaked storm, which, for the 120-min duration, exceeded anything ever gauged at Central Park. The 5-year averages show lighter Brooklyn intensities up to a duration of 30 min and nearly equal intensities for longer durations.

Table 12 indicates that rainfall curves for Manhattan, and for interior parts of Brooklyn, might have somewhat different shapes. The curves would be identical for the 60-min and 120-min durations, but the Brooklyn curve would have lighter intensities for the shorter durations. Of course, the table does not cover a long enough period to make the evidence of much value, but it is of some significance that, in Table 12(*a*), the lighter Brooklyn intensities are in evidence during each of the three years considered and for all four durations. The writer is willing to hazard a guess that the heavier Manhattan storms are caused by the proximity of the Hudson and East Rivers.

To summarize the viewpoint herein expressed: Rainfall curves with frequencies of from 1 year to 10 years, represented by the author's modified exponential equations, are endorsed as being close approximations of existing Central Park rainfall data. It should be recognized, however, that these curves are of a temporary nature and may need revision at any time.

Table 12 suggests that these curves may be too severe for the design of branch sewer systems in interior sections of the Borough of Brooklyn and similarly located territory in the Borough of Queens; but, unfortunately, there are no long-term rainfall records in this territory to form the basis for a rainfall curve. It is probable that branch sewer systems in this territory, designed by the author's curves, would have a large factor of safety.

While the modified exponential type of equation, doubtless, can be adapted to rainfall data in many cities, as previously stated there is nothing in the paper to show that it is of universal application. The writer has been helped in his study of this paper by the clear and concise manner in which the subject-matter has been presented.

JOSE GARCIA MONTES, JR.,[19] ESQ. (by letter).—The writer ventures to suggest that the part of Mr. Bleich's analysis, assuming formulas of the type of Equation (1), could have been improved by plotting on semi-hyperbolic paper.[20] When so plotted the data in Table 4, however, indicate that the intensities from different storms do not follow the law defined by Equation (1); nor, in fact, do corresponding intensities for a single storm follow that law. If there is any general law that applies to these data, the writer believes that it will conform more closely to a duration type of curve than to any other type so far proposed.

---

[19] Havana, Cuba.

[20] *Transactions*, Am. Soc. C. E., Vol. LXXXIV (1921), p. 211.

The consolidation of records of different types of storms according to present general practice in analyzing frequency of rainfall intensities, and the practice of assuming intensities corresponding to different types, as homogeneous records, is not justified. From a practical standpoint when it is desired to determine exactly the frequency of occurrence of given intensities in different time elements, such consolidation may perhaps be permissible; but what the writer considers unjustified is the practice of representing the inter-relationship of various factors by means of mathematical equations, because the best of mathematical formulas can be only gross averages of the actual relationship. The fact that data plotted on arithmetical paper appear to follow the equations that are actually assumed for the particular case, is due probably to the effect of averaging intensities from different records, many of which, individually, might conform to quite different types of curves.

Judged as an empirical equation, Mr. Bleich's formula should be valuable to engineers practicing in and around New York City, who can make use of the data in designing storm sewers more intelligently. Based on local records covering a period of 61 years, the application of such an analysis is certainly justified. Moreover, this record is now of sufficient length to permit a detailed analysis of original values to learn, if possible, the true laws that underlie rainfall duration relations, if such laws exist. The classification of these data according to types and an independent analysis of data that fall in each class may possibly disclose interesting truths that have universal significance.

In analyzing such data statistically, the fact that it is not homogeneous is important. The rainfall probabilities applying to short duration periods must not be studied in the same class as those segregated in longer time units. The two classes of data follow different laws, those for very short time periods tending to be more stable than those for longer periods. If frequencies of the different classes of data were approximately the same, there would be some justification for consolidating the records. The frequencies are not the same, however; for example, in tropical countries, the electric storm is by far the most frequent and, therefore, these control the data pertaining to storms of short duration.

Generally, storms are assumed to fall in two broad classes: (*a*) High-pressure storms including electric storms; and (*b*) cyclones. In the temperate zones the first type yields most of the high rainfall intensities—up to a duration of 2 hr. In the tropics, on the other hand, at least near the Caribbean Sea, the most severe storms are mostly hurricanes or storms in the cyclone class. The latter yield the highest intensities for any duration. If now a record containing both types of intense storms is consolidated, the extreme items, such as those occurring once or twice in 50 years, would be automatically weighted to appear in the same order as storms of 15-year or 20-year occurrence. In the tropics, electric storms of a genuine catastrophic nature occur as rarely as two or three times in 50 years, whereas hurricanes and cyclones occur on the average of eight to ten times in 50 years. Consequently, the intensities and the frequency of occurrence, as well as the duration of these two classes of storms, cannot very well be consolidated. They constitute heterogeneous data that should not be analyzed in one group.

While the writer is not familiar with conditions in the United States from personal experience, his studies lead him to believe that heterogeneity of data may influence the result of analyzing intensities from records that include severe electric storms as well as storms of a cyclonic nature. A multiple electric storm (that is, two or more storms in rapid succession, or that overlap each other), yields a rainfall intensity for periods of 3 hr, or more, that is not at all comparable, by means of probability analyses, with normal cyclonic storms analyzed in the same way. As in the case of comparing West Indian hurricanes with severe electric storms, the exceptionally severe rainfall that accompanies a cyclone is less than the total rainfall produced by a recurring electric storm of 2 to 6 hr duration. In the North, the cyclonic type of storm may yield severe rainfall intensities for periods greater than 6 or 8 hr.

It would appear, then, that rainfall durations between 5 min and 6 hr, and especially between 15 min and 6 hr, are markedly inconsistent when analyzed by probability methods and are even more inconsistent when analyzed mathematically, for the reason that the record contains data of basically different characteristics. A careful classification of the data according to the class in which it belongs will show the trend much clearer, but as yet no definite law can be stated with confidence. Probably some obscure disturbing factors affect the results. Statistical analysis of data carefully segregated for the purpose in view will probably yield the kind of information that can be generalized.

A detailed record of rainfall intensities covering a period of 61 years is unique. Some of the information possible to be gleaned from it, particularly that applicable to New York City, is ably described in this valuable paper by Mr. Bleich; but information of much greater importance can be gained by a thorough classification analysis. In its entirety, information available as to rainfall intensities is quite meager. It is judged that the City of New York can well afford such a detailed study, or at least the expense of publishing the complete data available to date, thus affording an opportunity for independent investigators to extend the analysis further. Other records are available in the United States, which cover periods of nearly 50 years, and a study of two such records, in conjunction with that of Central Park described in this paper, should prove extremely valuable in a correlation analysis.

Rainfall intensity is important to agriculture, as one of the underlying factors affecting water losses. It is also of major importance in determining the yield of water-sheds for water supply and irrigation. Because of the great labor involved in analyzing intensity data, few efforts have been made, except in the sewerage field. With the data now accummulated, it is time to stress the necessity of such analysis in other fields.

Rhodes E. Rule,[21] Assoc. M. Am. Soc. C. E. (by letter).—The results of an analysis of rainfall records are presented in this paper, in which empirical formulas are derived and compared with the results of a parallel study by the probability method. The author finds that the "modified exponential method gives results more in accord with the actual data than the probability method." He adopted the "Type III" curve for use in his application of the probability method, but does not state why it was used except that it "fits the data best". Since goodness of fit was the sole criterion, it should be noted that a better fitting curve could have been obtained by (1) grouping the data; (2) fitting a separate curve to each group; and (3) taking the summation of all such curves to make the final frequency curve. Such a method would be cumbersome, and perhaps difficult to defend on rational grounds but, neverthless, it would be the best fitting curve. Carrying the argument a step further the data could have been plotted in the histogram form, and the fit would be perfect. Does this demonstrate that the data in their unsmoothed form are better than any theoretical smoothing of them?

---

[21] Civ. Engr. (Rule & Rule), Los Angeles, Calif.

Of course, this is not true and the fallacy of the argument lies in the fact that goodness of fit is not the sole test. In the final analysis the propriety of using a given type of curve must rest on a rational demonstration that the data follow some law which is exemplified by curves of the type adopted. Unfortunately, such a demonstration has not been made for the several kinds of curves in use for estimating hydrologic frequencies.

The literature of the application of probability methods to engineering data is practically confined to demonstrations of mathematical methods of curve fitting. The philosophy of the subject has been neglected, and the mathematical nicety of the familiar methods of treating skew frequency curves has too often been mistaken for a soundness of reason for their application to the problems at hand.

That the Pearson curves are not very satisfactory for use in representing hydrologic probabilities is indicated by several systematic departures of the curves from the data. When the same order of departure recurs in a large majority of cases investigation into the underlying causes is indicated as desirable and use of the curves should be made only with full recognition of their shortcomings.

Probably the most important of these systematic departures, and the only one the writer will attempt to discuss, is that of the extremely high values. A considerable number of long-term rainfall and run-off records contain more extremely high values than are indicated as probable by Pearson curves fitted to the data. An example of this is seen in Fig. 5. The 10-min duration curve indicates that an intensity of 9 in. per hr should occur, theoretically, 0.45 times in 100 yr, or once in 220 yr, whereas the actual record was once in 62 yr. The 15-min duration curve shows an intensity of 8 in. per hr to have a theoretical frequency of about 0.55 times in 100 yr, or once in 180 yr. Actually, this intensity was reached twice in the 62-yr period, or once in 32 yr, almost six times the theoretical frequency.

This same kind of departure may be noted in many widely scattered records, strongly indicating that major storms are controlled by a law of their own. If this is the case, the data of hydrology are heterogeneous and do not lend themselves to treatment by Pearson's constructions which are limited by him to homogeneous material.[22]

Even if heterogeneous it might be possible to group such data into sets of essentially homogeneous material, each of which might be treated by probability methods. If entirely heterogeneous some such treatment as that favored by the author would seem to be the most satisfactory. To the extent that his method gives frequencies for the higher intensities more nearly approaching those of the record, it is certainly to be preferred over present probability methods in studies in which the extreme values are sought to be determined within safe limits.

The writer clings to the idea that data of this kind can be handled properly by probability methods, but evidence continues to accumulate to cast doubt on the accuracy of present methods. The author's paper is an addition to this evidence. Surely it would be carrying faith in a method to an extreme to accept a theory that the probable frequency of occurrence of an event is only once in 180 yr when the event has actually occurred twice in 62 yr.

The author deserves the thanks of the profession for an interesting and understandable presentation of a study which contains much valuable information.

S. D. BLEICH,[23] ESQ. (by letter).—The remarks by Mr. Sherman bring out the fact that considerable work could be done to advantage on rainfall studies. While hydrologists appreciate the importance of such work, it is impossible to convince municipal authorities of its desirability. The criticisms made by Mr. Sherman are discussed in the order presented by him:

(1) The published records of the U. S. Weather Bureau Station in Central Park, New York City, were used. The records of the U. S. Weather Bureau Station at Battery Place, New York City, were studied independently, but this work has not yet (1934) been completed. The separate records of the two stations were not compared with each other.

(2) The Weather Bureau publishes the maximum rates of precipitation for 5, 10, 15, 30, 60, and 120 min, and for 24 hr. In the design of sewers for which purpose the formulas were primarily computed, the first six periods mentioned are of importance. Periods longer than 2 hr would be useful for water-sheds considerably greater than those contributing to sewers in New York City. For such areas, other formulas have been devised.[24] It is believed that the formulas given are sufficient for the use of all periods between 5 and 120 min, inclusive, and it would probably be safe to extend them to periods slightly longer than 2 hr.

(3) As stated, the printed records of the Weather Bureau were used and compared with the printed records of the Committee on Rainfall and Run-

---

[22] "Mathematical Contributions to the Theory of Evolution," by Karl Pearson. Draper's Company Research Memoirs, Biometric Series II, XIV, p. 4.

[23] Asst. Div. Engr., Board of Transportation, New York, N. Y.

[24] "Formulas for Rainfall Intensities of Long Duration", by Merrill M. Bernard, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 592; as well as "Frequency and Intensity of Excessive Rainfalls at Boston, Massachusetts", by Charles W. Sherman, M. Am. Soc. C. E., *Loc. cit.*, Vol. 95 (1931), p. 951.

Off of the Municipal Engineers of the City of New York and with those collected by Mr. Hufeland, and only where discrepancies appeared were the original records consulted.

(4) All the formulas given in the paper are empirical and were designed to fit the given data as closely as possible with the form of equations used. Mathematical niceties and preconceived ideas of the values of the constants and exponents were avoided. The formula devised by Mr. Sherman, as well as the writer's unified Equation (8), may satisfy the æsthetic feeling, but would be difficult to justify logically. It is evident that, by not restricting the values of the constants, formulas would be obtained which will give results that correspond most nearly to the data, as is amply shown by the paper. It is for this reason that Equation (8) was not considered as good as the group of formulas given under Equation (5).

Rain storms are extremely complicated phenomena and are not of a single type. The conditions which are the principal underlying causes of rainfall intensities that occur as often as once a year probably differ from those causing the very severe intensities that occur only rarely, say, once in 15 yr, or in 50 yr. It is not to be expected that the same formula, or the same constants, will express all types of storms equally well.

(5) The rainfall records published in the *Journal* of the Municipal Engineers of the City of New York, as well as Mr. Hufeland's paper, were consulted, and were compared with the records of the U. S. Weather Bureau and only where there were discrepancies were the original records consulted.

(6) The formulas of the late Kenneth Allen, M. Am. Soc. C. E., were given careful consideration. They were designed in a different manner, however, from the writer's formulas; consequently, the results must be interpreted differently. For each duration Mr. Allen selected the heaviest rainfall that occurred in each year and neglected all the other intensities, even though some of them might be heavier than the severest rainfall intensity that occurred in other years. The results given by Mr. Allen's formulas[25] are described by him as giving "the probability percentage of years in which the given intensity of rainfall continuing over a given number of minutes will not be exceeded", whereas the writer's formulas, grouped together, give the frequency or the number of times in a given period of years that a rainfall intensity for a given duration, in minutes, has been equalled or exceeded.

Mr. Montes states that the storms embraced in the study are of a fixed character, some being due to one set of causes and others to entirely different causes. The writer admits his inability to segregate the storms in their proper classification so as to be able to treat each homogeneous set separately. Each kind of storm would probably have its own distinctive law and formula. The writer's problem has been a simpler one, namely, to obtain, as accurately as possible, the aggregate frequency of rain intensities for different durations for all types of storms.

Mr. Montes' discussion is very instructive and brings out important facts which are likely to be forgotten while the investigator is absorbed with the arduous calculations. The study deserves the time and efforts of a few qualified hydrologists, and large cities would find the expense for conducting this research comparatively small and the results very profitable in planning municipal improvements.

---

[25] *Engineering News-Record*, April 7, 1921, p. 588.

Use of the duration type of curve was attempted, as illustrated in the paper, but the result was not altogether satisfactory. Mathematical formulas corresponding to such curves are quite unwieldy.

Professor Slade appears to be of the opinion that the probability method is the proper one for handling rainfall data. The difficulty with the method is that it cannot be justified rationally, particularly when a different coefficient of skew is used for each of the durations.

While it is ingenious, the probability method seems to be artificial and has only the appearance of mathematical logic. To use the values obtained by the skew probability method rather than the actual data in computing the modified exponential formula would result in values that would depart materially from the recorded values which no amount of mathematics could justify. If the physical laws were known, only then would there be good reason for using whatever calculations are necessary with the recorded data in order to obtain the numerical values for the constants in the formula which expresses the physical laws.

In Equation (11) Mr. Bernard presents a uniform expression embracing rainfall intensity, duration, and frequency, which probably could be made to fit the recorded data better than the writer's unified Equation (8), as the former contains five parameters which are to be determined, and the latter only four. For this very reason and also because of the expression proposed for the formula, it would be very laborious to obtain the values of the five parameters which would give results corresponding closely to the given data.

Mr. Seaver has taken considerable pains to check Table 2 giving the chronological records of heavy rainfall intensities at the Central Park gauge, in New York City. He has also made instructive studies which clearly indicate that the results given by rainfall formulas will vary with the number of years of observations, thereby giving rise to the conjecture that the present results may not be final, but will have to be modified with the accumulation of additional data in the future. This is probably true, but, in view of the long period already covered by the Central Park gauge, it is not probable that the results will be varied materially by subsequent studies.

By means of Table 12, Mr. Seaver demonstrates clearly that two locations, only a few miles apart, may have materially different characteristics of rainfall intensities. While only a few years are covered by this comparison, it is probable that extended studies will confirm the truth of this finding. It is important that definite knowledge of this variation be obtained, which can only be accomplished by locating a large number of rain-gauges at several points in each of the Boroughs of Greater New York City, to be accompanied by an exhaustive study of all the records, in order to learn the specific rainfall intensities and frequencies for each location. This would enable sewers to be designed so as to serve properly the areas where they are built, resulting in adequate drainage facilities without waste and in economy of construction which would more than offset the cost of such extended investigation.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 1906

FLEXIBLE "FIRST-STORY" CONSTRUCTION FOR EARTHQUAKE RESISTANCE

BY NORMAN B. GREEN,[1] ESQ.

WITH DISCUSSION BY MESSRS. LEE H. JOHNSON, EDWARD J. BEDNARSKI, MERIT P. WHITE AND PAUL L. KARTZKE, HOWARD G. SMITS, H. A. WILLIAMS, AND NORMAN B. GREEN.

SYNOPSIS

A method of analyzing multiple-story, steel-frame buildings to resist earthquakes, is presented in this paper. The theory is based on a flexible first story, without attempting to draw definite conclusions regarding the relative practicability of this type of construction. The conclusions to be drawn from the evidence presented are summarized at the end of the paper.

The problem of constructing high buildings of twenty or thirty stories so that they shall be capable of resisting earthquakes, has brought forth the suggestion that they be provided with a flexible story, upon which the structure shall swing. The original idea was that the earth motion would be absorbed in this flexible story, so that the upper part of the building need not be designed to resist a lateral earthquake force. Actually, such a structure is an elastic vibrating system and its maximum lateral deflection must be determined. Many high buildings on the Pacific Coast are stated to be of this flexible construction. However, no one knows how they will actually behave during an earthquake, since with one or two exceptions no attempt has been made at a dynamic design. In the exceptional cases, recourse was had to a dynamic model tested on the shaking-table. If this type of construction is to be of practical value, it is imperative that some convenient design method be developed, since every engineer does not have access to a shaking-table and, moreover, the experimental method is both costly and time-consuming.

NOTE.—Published in February, 1934, *Proceedings*.

[1] Structural Engr., San Francisco, Calif.

On a first approach to the problem it would appear that, by reason of the great irregularity of the earth motion (which is demonstrated by nearly any seismograph record), a mathematical solution is impossible. The designer, however, is not concerned with this motion of the earth as recorded by a seismograph, but he is interested in the acceleration and period of acceleration (that is, the time interval between two maxima of acceleration), since it is this change of velocity that provides the impulse to deflect an engineering structure and to set it in vibration. It is only recently that instruments have been available for measuring this acceleration, and an examination of the few available records of this kind taken during an earthquake show clearly that an accelerograph gives a much more regular swing than a seismograph recording the same earthquake. This regularity of swing and the fact of a nearly constant period of acceleration, can be best demonstrated by an examination of the accelerogram shown in Fig. 1, which is

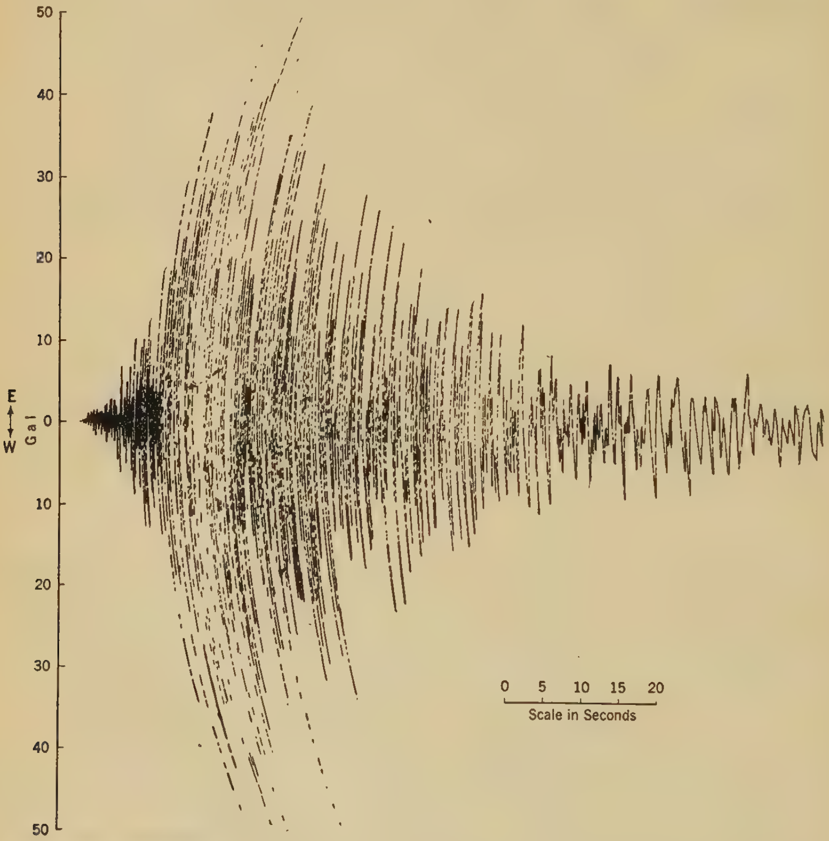


FIG. 1.—ACCELEROGRAM OF SEMI-DESTRUCTIVE EARTHQUAKE, SEPTEMBER 21, 1931, NORTHWEST OF TOKYO, JAPAN.

the record of a semi-destructive earthquake that occurred near Tokyo, Japan, in September, 1931.

The late Dr. K. Suyehiro concluded[2] that in all earthquakes the period of acceleration in a particular locality is confined within a very narrow range and that the motions of an earthquake that cause the predominant accelerations are due to the habitual motions peculiar to the ground. It was found that the prevailing periods for earthquakes in Japan were generally from 0.7 to 0.9 sec on alluvial ground and from 0.3 to 0.4 sec on well-consolidated ground.

A further examination of Fig. 1 will show that the acceleration varies directly with the time. This fact is particularly apparent in the right-hand part of the diagram. It is also apparent that the maximum acceleration is by no means uniform, since a full swing may be followed immediately by a much shorter swing, or by one that is unbalanced; in fact, any combination seems to be possible.

With these facts established, it is feasible to construct an artificial acceleration diagram and to compute what effect the corresponding earth motion will have upon the relatively simple type of elastic structure, as represented by a building with a flexible base. In order that the method shall be general in its application, it must embrace the case of an irregularly varying maximum acceleration as well as the possibility of a varying period of acceleration. This requirement necessitates that the equations of motion be written separately for each swing and that they be solved successively, using the terminal velocity and deflection for one swing as the initial ones for the succeeding swing. This is the method that will now be developed. It may be visualized as a kind of mathematical shaking-table, in which the artificial acceleration diagram has its counterpart in the cam, which is cut so as to impart to the table a motion that simulates a given earthquake.

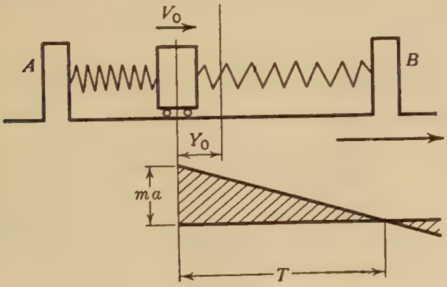


FIG. 2.

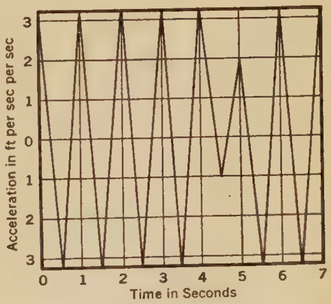


FIG. 3.—ACCELERATION DIAGRAM.

Consider a weight that rests upon rollers and is connected by two springs to Supports A and B, as indicated in Fig. 2. Assume that this weight is given an initial deflection, Y_0, and velocity, V_0, and that the axis of coordinates is taken through the position of static equilibrium. Assume furthermore that Supports A and B are accelerated toward the right, starting with an acceleration, a, which decreases as the ordinate of a straight line, to a zero acceleration at the expiration of a time, T, and continues a negative acceleration, as indicated in the lower diagram of Fig. 2. The ensuing motion of the supports, A and B, represents one swing of the artificial earthquake, and the motion of the weight can be determined, using the following notation:

---

[2] *Proceedings*, Am. Soc. C. E., May, 1932, Part 2.

W = weight of vibrating body, in pounds;
g = acceleration of gravity = 32.2 ft per sec per sec;
m = mass of vibrating body = $\dfrac{W}{g}$
a = initial acceleration of ground at the beginning of the swing, in feet per second per second;
e = elastic constant of vibrating system = $\dfrac{\text{Lateral force}}{\text{Deflection}}$;
t = independent variable, time, in seconds;
T = elapsed time from initial ground acceleration to zero acceleration;
y = deflection of the body from its position at rest, in feet;
Y_0 = initial deflection at $t = 0$;
V_0 = initial velocity at $t = 0$, in feet per second;
f = maximum deflection during one swing;
t_m = elapsed time from $t = 0$ to the maximum deflection;
v = velocity at any time, t;
v_n = velocity at the end of the nth swing;
y_n = deflection at the end of the nth swing; and,
t_n = elapsed time from the beginning to the end of the nth swing.

If positive forces, deflections, accelerations, and velocities are each taken in the direction of the initial force, $m\,a$, the differential equation of motion is expressed as:

$$m \frac{d^2y}{dt^2} = m\,a\left(1 - \frac{t}{T}\right) - e\,y \qquad \text{..................(1)}$$

and the general solution of this linear equation is:

$$y = C_1 \cos \sqrt{\frac{e}{m}}\, t + C_2 \sin \sqrt{\frac{e}{m}}\, t - \frac{m\,at}{Te} + \frac{m\,a}{e} \qquad \text{...........(2)}$$

in which C_1 and C_2 are constants of integration, that can be determined from the condition that when $t = 0$, $\dfrac{dy}{dt} = V_0$ and $y = Y_0$. For simplicity, let $A = \dfrac{e}{m}$; $M = \sqrt{A}\left(Y_0 - \dfrac{m\,a}{e}\right)$; $N = \left(V_0 + \dfrac{m\,a}{T\,e}\right)$; and $P = \dfrac{m\,a}{T\,e}$; then, after evaluating the constants:

$$y = \frac{M}{\sqrt{A}} \cos \sqrt{A}\, t + \frac{N}{\sqrt{A}} \sin \sqrt{A}\, t - P\,(t - T) \qquad \text{.........(3)}$$

and by differentiating again;

$$\frac{dy}{dt} = v = -M \sin \sqrt{A}\, t + N \cos \sqrt{A}\, t - P \qquad \text{..........(4)}$$

The value of t_m may be obtained by setting Equation (4) equal to zero and solving for t, whence:

$$t_m = \frac{1}{\sqrt{A}} \tan^{-1}\left(\frac{M\,N \pm P\sqrt{N^2 + M^2 - P^2}}{M^2 - P^2}\right) \qquad \text{..........(5)}$$

and if the quantity in the parenthesis is set equal to k:

$$t_m = \frac{1}{\sqrt{A}} \tan^{-1} k \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

The maximum deflection is obtained by substituting the value, t_m, into Equation (3), whence:

$$f = \frac{M}{\sqrt{A}} \cos (\tan^{-1} k) + \frac{N}{\sqrt{A}} \sin (\tan^{-1} k) - \frac{P}{\sqrt{A}} (\tan^{-1} k) + PT \quad ..(7)$$

As a special case of Equation (3), suppose that $a = 0$ and $V_0 = 0$, in which case this equation reduces to:

$$y = Y_0 \cos \sqrt{A}\, t \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

which represents simple harmonic motion. By equating the first derivative to zero and solving for t:

$$t = 2\pi \sqrt{\frac{m}{e}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

which is the period of free vibration of the structure.

As another special case let T equal infinity, which simply means that the acceleration, a, is constant and that Equation (7) then reduces to:

$$f = -\frac{m\,a}{e} (\cos \tan^{-1} 0 - 1) = -\frac{m\,a}{e} (\pm\ 1 - 1) \quad \ldots\ldots\ldots (10)$$

The minus sign must be used to get the maximum deflection, since the plus sign gives $f = 0$, which is the minimum deflection. Therefore:

$$f = \frac{2\,m\,a}{e} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (11)$$

but $\frac{m\,a}{e}$ represents the static deflection produced by a constant acceleration, a, and it is apparent, therefore, that with a suddenly applied acceleration which is in the nature of an impact, the maximum deflection produced is twice the static deflection. In order that this deflection can be developed, however, it is necessary that the earth motion be of sufficient duration in one direction; but since the effect is entirely independent of the stiffness of the structure (that is, of the value of e), a very stiff structure would require a very short motion. While the earthquake acceleration, of course, is not uniform, it may be considered so for a very short period of time or when the swing is very long.

A glance at Fig. 1 will show that the assumption of an instantly applied acceleration may not be so far from the truth, since the accelerogram starts its major motion with a long swing. This same abrupt start of the major motion is indicated very clearly on many seismograms. To illustrate the sudden start of a violent earthquake in the epicentral region, Dr. Suyehiro gives the evidence of an eye-witness that copper coins in a tin can with a lid, threw off the lid and leaped out, leaving the can standing, at the instant the earthquake began. This effect would require a downward acceleration greater than gravity, with a motion of several inches of amplitude.

The evidence of eye-witnesses to the Long Beach, Calif., earthquake in 1933 also describes a very abrupt beginning of the primary shock. An examination of the California Garage, in Long Beach, which is a three-story reinforced concrete structure, gives clear evidence that the shock began as a sudden jolt or jerk toward the southeast. The columns were shattered at the top in compression on the southeast side, with a corresponding tension crack near the floor on the same side. On the opposite side there was a slight spalling at the floor line and a tension crack near the ceiling. In other words, this injury indicates destructive deformation in one direction only, which may be attributed to a heavy impact effect in the direction of the first swing.

It has been assumed in the foregoing analysis that the entire vibrating mass is concentrated at the point of elastic support, and that the mass is rigid. As regards the first of these assumptions, it is true that the mass of the building structure towers far above the base columns; however, the only uncertainty that this fact introduces, arises from the added horizontal motion that the upper part of the structure may sustain, due to tipping as a unit on the base columns. Calculation shows that for the building presently to be described, this horizontal motion of the top, due to direct column stress, is only about 2% that due to column flexure. The second assumption cannot be true, of course, for an actual building; however, it is the relative stiffness that is of importance and the upper part of the structure can be made very rigid relative to the base, by the use of concrete bracing walls or some other equally efficient construction.

By the use of Equations (3), (4), (6), and (7), the motion of the elastic structure can be completely determined for any number of swings and for any variation of the maximum acceleration and the period of acceleration. For the first swing, Y_0 and V_0 are zero and, having determined the values of Y_1 and V_1, these are used with reversed sign as Y_0 and V_0 for the second swing; the process being repeated indefinitely. This step-by-step solution presents a clear picture of what is happening to the structure and enables the designer to vary the nature of the earth motion as the computations progress, so as to subject the structure to the most unfavorable conditions. These computations may be carried out on a 10-in. slide-rule, but this should be carefully read to at least three significant figures, and four where possible, since the results involve small differences. It is necessary to work accurately, as the results for a given swing affect all succeeding swings.

The computation of f is rather laborious, but it need only be computed for those swings in which y_n is large, since one of these swings will contain the greatest deflection of the entire series. In this connection, it should also be noted that when y_n and v_n are of the same sign, y_n is actually the greatest deflection in that swing, since in this case t_m will be found to be greater than t_n, which simply means that the computed value of f occurs outside the range of the swing. The positive value of k should be used, care being exercised to select that value which gives the maximum deflection within the assumed range of the swing.

The design of the base columns, as well as the determination of their lateral deflection, requires the use of a formula for combined direct stress and bending since, with the large deflections attained, the eccentric moment is of great importance. For the case of a column fixed at the upper end and hinged at the base, the deflection, d, may be expressed as:

$$d = \frac{P}{W}\left(\frac{\tan a\,L}{a} - L\right) \qquad (12)$$

in which, L is the length of the column that carries a direct load, W, and a lateral load, P, and $a = \sqrt{\frac{W}{E\,I}}$, in which, E and I are the modulus of elasticity and the moment of inertia, respectively. The value of d in Equation (12) is used in computing the stiffness factor and may also be used to compute the maximum column stress, S, from the familiar relation:

$$S = \frac{W}{A} + \frac{(PL + Wd)\,c}{I} \qquad (13)$$

in which, A is the area of the column section, and c is the distance from the neutral axis to the extreme fiber.

It should be noted that the column deflection from Equation (12), is a function of the load on the column; but it is also directly proportional to the lateral force, P, so that e is a constant as has been assumed in the analysis.

Application of the Analysis

As an example of the method of computation, consider the case of a 20-story steel-frame building of the office type, with four 20-ft panels in one side and eight 18-ft panels along the other side. Suppose that the story height is 12 ft, except for the first story which has been taken as 24 ft. For a building of this type the weight per cubic foot will be about 17 lb when a 24-lb live load is included, and 15 lb when the live load is neglected. The former value has been used in designing the columns for vertical load, while in computing the mass of the structure the latter value has been used. The first-story columns are assumed as hinged at the base and with hinged second-floor beam connections, in order to increase their flexibility. These columns are designed to resist a wind pressure of 20 lb, with an over-stress of 50%, in which case the lateral deflection from Equation (12) is 0.194 ft and, using this deflection value, the stiffness factor, e, is 3 760 000 and the period of vibration from Equation (9) is 3.58 sec.

The acceleration diagram of Fig. 3 has been constructed, assuming a constant period of 1 sec and a maximum acceleration of 3.2 ft per sec per sec. This diagram represents a total elapsed time of 7.0 sec and contains seven complete vibrations or impulse cycles. It has been constructed so as to produce unfavorable conditions of deflection, but, nevertheless, it is believed to represent a perfectly possible situation.

Computations for this set of conditions are shown in Table 1. It will be noted that the maximum deflection of 0.445 ft occurs in the fourteenth swing, and if this is compared with the maximum allowable deflection of 0.194, the columns are over-stressed by more than 100 per cent.

TABLE 1.—COMPUTATIONS

Swing	T	a	INITIAL CONDITIONS		M	N	P	MAXIMUM DEFLECTION			FINAL CONDITIONS		$\tan^{-1} k$
			Y_0	V_0				k	f	x_m	Y_n	V_n	
1...	0.25	3.2	0	0	−1.823	+4.150	4.150	....	....	.....	+0.120	−0.097	......
2...	0.25	3.2	−0.120	+0.097	−2.033	+4.247	4.150	....	....	.....	+0.082	+0.124	......
3...	0.25	3.2	+0.082	−0.124	−1.970	+4.026	4.150	....	....	.....	+0.009	−0.065	......
4...	0.25	3.2	+0.009	+0.065	−1.840	+4.215	4.150	....	....	.....	+0.143	−0.036	......
5...	0.25	3.2	−0.143	+0.036	−2.073	+4.186	4.150	....	....	.....	+0.043	+0 120	.....
6...	0.25	3.2	−0.043	−0.120	−1.900	+4.030	4.150	....	....	.....	+0.037	−0.120	...
7...	0.25	3.2	−0.037	+0.120	−1.890	+4.270	4.150	....	....	.....	+0.145	+0.035	.....
8...	0.25	3.2	−0.145	−0.035	−2.08	+4.115	4.150	....	....	...	+0.011	+0.080	.....
9...	0.38	3.2	−0.011	−0.080	−1.843	+2.650	2.730	....	....	.....	+0.162	+0.381	......
10...	0.167	1.0	−0.162	−0.381	−0.854	+1.562	1.943	....	....	.....	−0.272	−0.287	......
11...	0.193	2.0	+0.272	+0.287	−0.662	+3.647	3.360	....	....	.....	+0.328	−0.521	......
12...	0.25	3.2	−0.328	+0.521	−2.400	+4.671	4.150	....	....	.....	+0.134	+0.675	......
13...	0.25	3.2	−0.134	−0.675	−2.060	+3.475	4.150	....	....	.....	−0.263	−0.345	......
14...	0.25	3.2	+0.263	+0.345	−1.362	+4.495	4.150	+0.988	+0.445	0.443	+0.435	−0.232	44.66

CONCLUSIONS

In this paper the purpose is simply to present the method of analysis, and no effort has been made to draw definite conclusions regarding the practicability of the flexible-base type of building construction. Certain tentative conclusions may be drawn, however, from the preceding computations and from similar computations that have been made for other sets of conditions. These may be summarized briefly as follows:

1.—The period of acceleration is as important as the acceleration itself, in its effect on the deflection of the structure. A period of 0.7 sec instead of 1.0 sec, cuts the deflections roughly in half.

2.—The impact effect of an acceleration suddenly applied to a flexible structure of this type has little or no effect on the deflection.

3.—An irregular swing gives greater deflections than a uniform and rhythmic swing, and within the range of probable maximum periods of acceleration there is no evidence of resonance if the free period of vibration of the structure is kept above 2.0 or 2.5 sec.

4.—The maximum deflection produced in a structure by a given ground motion, is not much affected by considerable changes in the period of free vibration of the structure. For this reason, the best measure of the earthquake-resistant qualities of a given building of this type is simply the maximum horizontal deflection that it can sustain without exceeding allowable stresses in the columns.

5.—It is likely to be found difficult to provide sufficient lateral flexibility in this type of structure if the most unfavorable period and other conditions of acceleration are assumed. Long base columns will have to be provided, probably with some form of hinged base, in which case the steel that must be added in these columns and in the restraining beams to which they frame at their top, in order to provide a reasonable resistance to wind pressure, will be found to be a large and important item.

DISCUSSION

LEE H. JOHNSON,[3] ESQ. (by letter).—This is a helpful and thought-provoking paper on a subject of some controversy among structural engineers to-day. It is gratifying to note that the author points out that, in determining the effects of a vibration on a structure, the period of the acceleration is just as important as the acceleration itself. It should be emphasized even more strongly that the effects of a vibratory phenomenon, upon any object cannot be determined by considering only the acceleration. This fact seems to have been obscured by an imperfect understanding of the relation between acceleration, frequency, and amplitude of a vibration. Considering the simplest of periodic motions, a simple harmonic motion of the form,

$$y = D \sin \omega t \dots\dots\dots\dots\dots\dots\dots\dots (14)$$

in which, D = maximum amplitude of vibration, and ω = a constant angular velocity = $2 \pi F$, in which F = frequency by vibration. The acceleration is given by,

$$\frac{d^2y}{d t^2} = - D \omega^2 \sin \omega t = - D F^2 4 \pi^2 \sin \omega t \dots\dots\dots (15)$$

from which it is seen that the acceleration is a function of the amplitude, D, times the square of the frequency, F, and that it is, therefore, dependent upon both quantities. It follows from this that the acceleration varies as $D F^2$, and that two vibrations, differing widely in character and in their effect on a structure, may yet have the same acceleration. A vibration with a very high frequency and infinitesimal amplitude can perfectly well have the same acceleration as a vibration of relatively low frequency and appreciable amplitude, the product, $D F^2$, being the same in each case. However, the effect of a vibration of the former type on a building will, ordinarily, be negligible, while the effect of a vibration of the latter type, such as an earthquake, can be disastrous. If the frequency decreases still more to infinitesimal proportions and the amplitude becomes very great, the acceleration still remaining unchanged, a building would probably suffer no ill effects, although it would be moved a considerable distance from its position of rest. Apparently, this last case does not occur in practice.

Hence, it is necessary to know at least two of these quantities, acceleration, frequency, and amplitude, not only in order to find the effect of a vibration on a structure, but, what is still more fundamental, in order to describe the nature of the vibration itself. Although such a simple relation between these quantities, as the preceding, may not exist in the case of more complex, non-harmonic vibrations, still the quantities are related and are not independent.

The author's mathematical analysis calls for some remarks. To understand these it is well to have the fundamental concepts clearly in mind. A simple harmonic motion of amplitude, D, and frequency, F, may be represented by the motion of the projection upon a fixed line of a vector of magnitude, D, rotating at constant angular velocity, as shown in Fig. 4. In this case, $y = D \cos \omega t$.

---

[3] Asst., Harvard Eng. School, Harvard Univ., Cambridge, Mass.

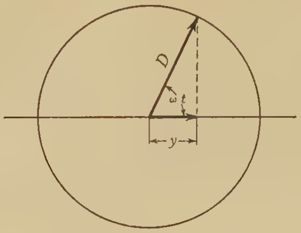


FIG. 4.

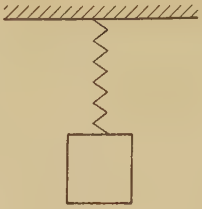


FIG. 5.

Examples of systems that give simple harmonic motion are a mass suspended on a perfectly elastic spring attached to a rigid support (Fig. 5) and a mass resting on frictionless rollers between perfectly elastic springs attached to rigid supports (Fig. 6). The masses are assumed to be non-elastic and the springs are assumed to be weightless.

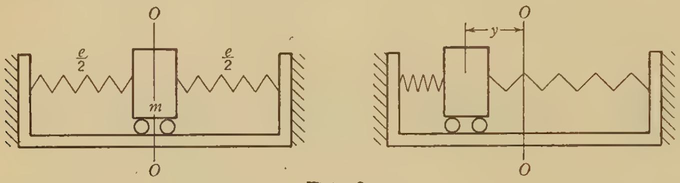


FIG. 6.

The author uses a system of the type shown in Fig. 6 as being analogous to a building supported on flexible columns with its upper stories constituting a rigid mass. The columns act as springs between the mass and the ground. Considering this system, the differential equation for the motion of the mass, m, is:

$$m \frac{d^2y}{dt^2} = -ey \quad \text{.........................(16)}$$

which means that at any instant the inertia force of the mass must be equal and opposite to the restoring force of the springs, an application of one of the basic laws of dynamic equilibrium. Writing the equation in the form:

$$\frac{d^2y}{dt^2} = -\frac{e}{m} y \quad \text{.........................(17)}$$

is equivalent to stating, simply, that the second derivative of a function is proportional to the function itself, and since sine and cosine functions are of this type, a solution may be assumed, of the form, $y = B_1 \cos \omega t + B_2 \sin \omega t$, in which, B_1, B_2, and ω are unknown. Substituting this in the differential Equation (17):

$$-B_1 \omega^2 \cos \omega t - B_2 \omega^2 \sin \omega t = -\frac{e}{m} (B_1 \cos \omega t + B_2 \sin \omega t)$$

whence, $\omega^2 = \frac{e}{m}$, $\omega = \sqrt{\frac{e}{m}}$, and $F = \frac{1}{2\,\pi}\sqrt{\frac{e}{m}}$, and the solution becomes:

$$y = B_1 \cos \sqrt{\frac{e}{m}}\, t + B_2 \sin \sqrt{\frac{e}{m}}\, t \quad \ldots\ldots\ldots\ldots\ldots\ldots (18)$$

The constants, B_1 and B_2, are determined from the initial conditions of the problem considered. For example, if at the start of the vibration when $t = 0$, the initial amplitude and velocity are given as $y = y_0$, and $\frac{dy}{dt} = 0$ then, substituting for $t = 0$ in Equation (18): $y_0 = B_1 \times 1 + B_2 \times 0$, whence, $B_1 = y_0$. Now,

$$\frac{d\,y}{d\,t} = -\, B_1 \sqrt{\frac{e}{m}} \sin \sqrt{\frac{e}{m}}\, t + B_2 \sqrt{\frac{e}{m}} \cos \sqrt{\frac{e}{m}}\, t$$

and, for $t = 0$, $0 = -\,B_1 \sqrt{\frac{e}{m}} \times 0 + B_2 \sqrt{\frac{e}{m}} \times 1$; whence, $B_2 = 0$. The final solution for this case is, therefore,

$$y = y_0 \cos \sqrt{\frac{e}{m}}\, t \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (19)$$

This is a simple harmonic motion of amplitude, y_0, and frequency, $\frac{1}{2\,\pi}\sqrt{\frac{e}{m}}$.

Next consider the case in which the supports are not fixed but execute a definite motion of their own, say, $x = x_0 \sin \omega t$ (see Fig. 7). Writing the differential equation of motion of the mass with respect to its position of rest:

$$m\,\frac{d^2 y}{d\,t^2} = -\, e\,(y + x) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (20)$$

However, the author seems to be interested primarily in the relative deflection of the top and bottom of the first-story columns in a building undergoing vibration which, in his mass and spring analogy, corresponds to the deflection of the mass from the moving support, rather than to the deflection from the position of rest. To find the relative motion of the mass with respect to the moving support it is seen from Fig. 7 that the movement relative to the support is $y + x$. Designating this motion by z: $z = y + x$, or $y = z - x$.

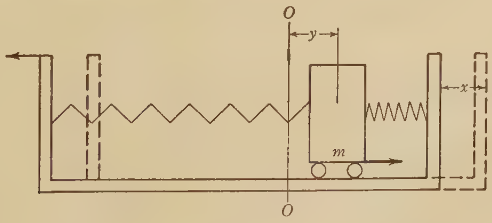


FIG. 7.

Substituting for y in Equation (20): $\frac{m\,d^2\,(z - x)}{d\,t^2} = -\, e\, z$, or,

$$m\,\frac{d^2 z}{d t^2} = m\,\frac{d^2 x}{d t^2} - e\,z \,\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (21)$$

which is the differential equation for the relative motion between the mass and the support. This corresponds to Equation (1) of the author's paper, except that where, in Equation (21), $\frac{d^2 x}{d t^2} = - x_0\,\omega^2 \sin \omega t$, he has chosen a function, $\frac{d^2 x}{d t^2} = a\left(1 = \frac{t}{T}\right)$, apparently representing the acceleration of an earthquake more closely than simple harmonic motion. The author's notation for y seems to be at fault. As the writer has shown, Equation (1) gives the motion of the mass relative to the support, which is what the author desires; but y in Equation (1), corresponding to z in the writer's Equation (21), is defined to be the deflection from the position of rest, instead of the relative deflection from the support.

It is to be noted that the natural frequency, or frequency of free vibration, of the system is obtained directly in solving the differential equation of motion and is, for the case considered, $F = \frac{1}{2\pi}\sqrt{\frac{e}{m}} = \frac{1}{2\pi}\sqrt{A}$, using the author's notation for A. The period of free vibration is simply, $T = \frac{1}{F} = 2\pi\sqrt{\frac{m}{e}} = 2\pi\,\frac{1}{\sqrt{A}}$.

It is not clear what the author had in mind in differentiating Equation (8) of his paper, and in setting the first derivative equal to zero in order to get the period of free vibration. From what has been shown herein, ω, corresponding to $\sqrt{A}$ in the author's Equation (8), is determined directly in solving the differential equation; and T is derived from the fundamental relations, $\omega = 2\pi F$ and $T = \frac{1}{F} = \frac{2\pi}{\omega}$.

Differentiating Equation (8), setting the resultant equation equal to zero, and solving for t, merely gives those values of t for which the velocity of the mass is zero: $\frac{dy}{dt} = 0 = - Y_0 \sqrt{A} \sin \sqrt{A}\, t$, whence, t must be such that $\sin \sqrt{A}\, t = 0$. Therefore, $t = \frac{n\pi}{\sqrt{A}} = n\pi\sqrt{\frac{m}{e}}$, in which, $n = 0, 1, 2, 3$, etc., since the sine of any integral value of π is zero. It is true that the natural period will be the time elapsing between two successive zero points of the velocity; that is, $T = 2\pi\sqrt{\frac{m}{e}}$, but the same would be true for zero points of the amplitude, and it is unnecessary to differentiate at all.

The author's conclusion that the maximum deflection produced in a structure by a given ground motion is not much affected by considerable changes in the period of free vibration of the structure is true only if the range of the periods of free vibration is sufficiently far above the period of the ground motion to avoid all possibilities of resonance.

Until more has been learned about the dynamics of building frames, the writer feels that the assumption that the upper stories of a building with flexible first-story columns constitute a rigid mass is a doubtful one. Simple tests made by the writer on models of various story heights show that the elasticity of the upper stories has a decided effect on the periods of free vibration, as is to be expected from purely theoretical considerations, even when the first-story columns are very flexible with respect to the upper story columns.

The writer believes that Mr. Green should be commended for presenting a method for finding the effect of any arbitrary non-harmonic vibration upon an idealized flexible building. To date, chiefly simple harmonic motions have been used in practical calculations on structures.

EDWARD J. BEDNARSKI,[4] ASSOC. M. AM. SOC. C. E. (by letter).—The author has raised an important question in the analysis of structures, namely, the influence of the connection between the structure and the foundation. He could have chosen several hypothetical variations of Fig. 2 to analyze a structure as, for example, a fixed connection resisting the forces acting in any direction and any moment created by these forces; a hinged connection that can not resist the moment and can take only the forces passing through the center of the hinge; and a connection on rollers or balls that may resist any moment, but takes only the forces directed normal to the plane of rolling.

In the case analyzed in the paper the connection is assumed of this latter type with elastic buffers preventing the structure from being carried away under occasional lateral pressure (see Fig. 2). The author could also have used the arrangement shown in Fig. 8, in which elastic springs are replaced by hydraulic "buffers" interconnected by a pipe and a dash-pot arrangement. This may be considered as an aperiodic connection if elastic expansion of the pipes and the cylinders is assumed to be negligible. Of course, an absolutely elastic or absolutely aperiodic arrangement does not exist.

Connections of this kind may be likened to electric transformers in which the primary and secondary coils are connected with different degrees of looseness, in an electrical sense. In a very loose connection, this transformer will oscillate only at a certain specific frequency; that is, the natural frequency of the coupled circuit. No other frequency would create an oscillatory current.

Theoretically, a structure resting on balls or rollers, devoid of friction, does not re-act to the vibratory movement of an earthquake in a horizontal plane. In practice, however, there may be a possibility of strains due to transferred vibrations. The more flexible the first story of the structure the less will be the lateral forces imparted to it; and the less elastic its connection with the foundation, the less will be the danger of resonant vibrations being set up in it.

---

[4] Structural Engr., San Francisco, Calif.

A connection between the structure and the foundation can not be made plastic and flexible by introducing comparatively slender columns in the first story. The use of a material with a very small elastic modulus and a relatively great plasticity between the structure and the foundation may serve the purpose in a practical sense, if the total shear resistance of the insulating material is great enough to prevent the structure from being moved by the wind pressure. The value of such shear resistance is the critical, maximum, lateral force that could ever be imparted to the structure by the earthquake without the occurrence of resonance in the horizontal plane. Common examples that illustrate this point are the old trick of snatching a napkin from beneath a number of full tumblers without spilling the contents, and the bullet passing through a pane of glass, making a clean round hole in it. The success of the tricks depends entirely on the velocity of movement, on the friction between the napkin and the tumblers, and on the shearing resistance of glass.

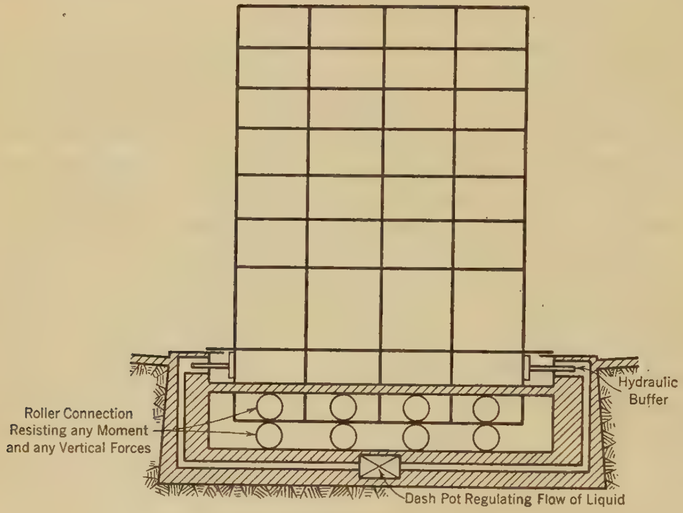


FIG. 8.

In his study[5] of the theory of sound, Sir John Rayleigh introduces quite an extensive analysis to determine the natural period of vibration for a single bar, fixed at one end and free at the other. In his investigation, Mr. Green introduces a formula for the period of vibration of a structure consisting of many bars, connected in many different ways, and supported by a system of columns, hinged at one end and fixed elastically at the other. In the light of this comparison, the formula offered in this paper seems too simple to be true. For one thing, the author has ignored the questions of modes of vibration, the time elapsed between complete rest and maximum acceleration, and internal friction.

---

[5] "The Theory of Sound," by John William Strutt Rayleigh, Second Edition, 1894–96. Vol. I, pp. 255–305.

Equation (1) is the well-known differential equation of forced oscillatory motion, with $m\,a\left(1 - \frac{t}{T}\right)$ substituted for $F \cos n\,t$, in which, F is an external oscillatory force corresponding to $m\,a$. For a constant amplitude this movement may be expressed as:

$$\frac{8\,m\,a}{\pi^2}\left\{\cos(n\,t) + \frac{1}{3^2}\cos(3\,n\,t) + \frac{1}{5^2}\cos(5\,n\,t) + \ldots + \frac{1}{(2\,q+1)^2}\cos\left[(2\,q+1)\,n\,t\right]\right\} \ldots\ldots\ldots\ldots(22)$$

in which, $n = \frac{2\,\pi}{4\,T}$ and q approaches ∞, $2\,q + 1$ being an odd number.

The actual force imparted, at the very beginning, to the structure supported by a flexible connection will be always within the limits of 0 and $m\,a$ for absolutely flexible connection and for absolutely rigid connection. This depends on the value of e, the rate at which the value of a changes, as well as the final value of a.

From the accelerogram, Fig. 1, it seems that the amplitude of the acceleration decreases uniformly from its maximum, to an almost negligible, value, within 55 sec. This value of the amplitude may be expressed as $a\,(1 - s\,t)$, in which, s = a decrement coefficient. For $t = 55$ sec, $1 - 55\,s = 0$; or $s = \frac{1}{55}$. Then the differential Equation (1) will take the form,

$$\frac{m\,d^2\,y}{d\,t^2} + r\,\frac{dy}{dt} + e\,y = \frac{8\,c\,m\,a}{\pi^2}(1 - s\,t)\left\{\cos(n\,t) + \frac{1}{3^2}\cos(3\,n\,t) + \frac{1}{5^2}\cos(5\,n\,t) \ldots + \frac{1}{(2\,q+1)^2}\cos\left[(2\,q+1)\,n\,t\right]\right\} \ldots\ldots(23)$$

This, the writer believes, is a more general formula for the problem. The correction factor, c, which is introduced to allow for the flexibility of the connection, is not a constant, of course; but it may be considered constant between certain limits. Likewise, the damping factor, r, is introduced to allow for the influence of internal frictional resistance.

When the connection is constructed so that the maximum force which can be imparted to the structure at the beginning is known beforehand the only danger lies in the possibility of synchronization of the vibration of the foundation with the vibration of the structure itself. It may be solved by the use of Equation (23), in which, $e = n^2_o\,m$ (in which, $n_o = \frac{2\,\pi}{t_o}$ and t_o is the time of the natural period, determined independently), and if the solution is not too complicated. In the latter case the check to determine the possibility of resonance may be made by the method introduced[6] by R. R. Martel, M. Am. Soc. C. E.

---

[6] "The Dynamic Behavior of Some Simple Bents Subjected to Established Harmonic Motion," by R. R. Martel, M. Am. Soc. C. E., *Proceedings*, World Eng. Congress, Tokyo, Japan, 1929, Vol. VIII, pp. 213–217.

The author deserves credit for introducing a step-by-step method of solving this problem. The interval of 7 sec chosen in the numerical example ("Application of Analysis") compared with 3.58 sec of the natural period for the structure includes less than two periods and it is not quite clear why a greater maximum than $f = 0.445$ ft would not occur. Were the computations extended a little further showing that the curve of maximum values of f is descending after 7 sec, the assertion regarding the maximum deflection would carry more weight.

Merit P. White,[7] Esq., and Paul L. Kartzke,[8] Esq. (by letter).—Although investigations have been carried on for several years at various laboratories in the United States and in Japan, there is still considerable to be learned in regard to the effect of earthquakes on structures. The author's assumptions that seismic acceleration is a linear function of time and that the initial shock is sudden seem as reasonable as any others that might be made for his analysis. Actually, the final results will be practically independent of either assumption. However, the evidence of an eye-witness of one of the Japanese earthquakes that copper coins in a can were thrown out at the first shock is not proof of a sudden initial acceleration unless the can happened to be so connected to the ground as to follow the ground motion.

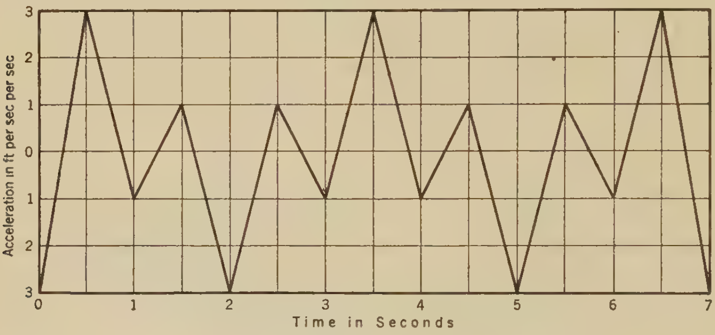


Fig. 9.—Acceleration Diagram

The author's use of plus and minus signs is rather confusing. His values of Y_n and V_n at the end of each half-second period (Table 1) apparently are considered positive if in the direction of the acceleration at the beginning of that period, and are reversed in sign when they become initial conditions (Y_0, V_0) for the next period. It would seem more logical to give the plus sign to deflections and velocities to the right, and the minus sign to those to the left. Incidentally, if the second integral of the author's acceleration curve is taken it will be found that the base of the building is displaced about 20 in. assuming it to start from rest.

Fig. 9 shows an acceleration diagram somewhat similar to Fig. 3, except for the presence of a rather long period acceleration ($3\frac{1}{2}$ sec), in addition to the shorter waves. This acceleration pattern was applied to Mr. Green's building. The computations, based on the following equations,

---

[7] Asst. in Eng., California Inst. of Technology, Pasadena, Calif.

[8] Student, California Inst. of Technology, Pasadena, Calif.

$$Y_n = 0.639\ Y_o + 0.4385\ V_o + 0.202\ \frac{a}{T} - 0.117\ a \quad \ldots\ldots\ldots (24)$$

and,

$$V_n = -\ 1.350\ Y_o + 0.6392\ V_o + 0.117\ \frac{a}{T} - 0.4385\ a \quad \ldots\ldots\ldots (25)$$

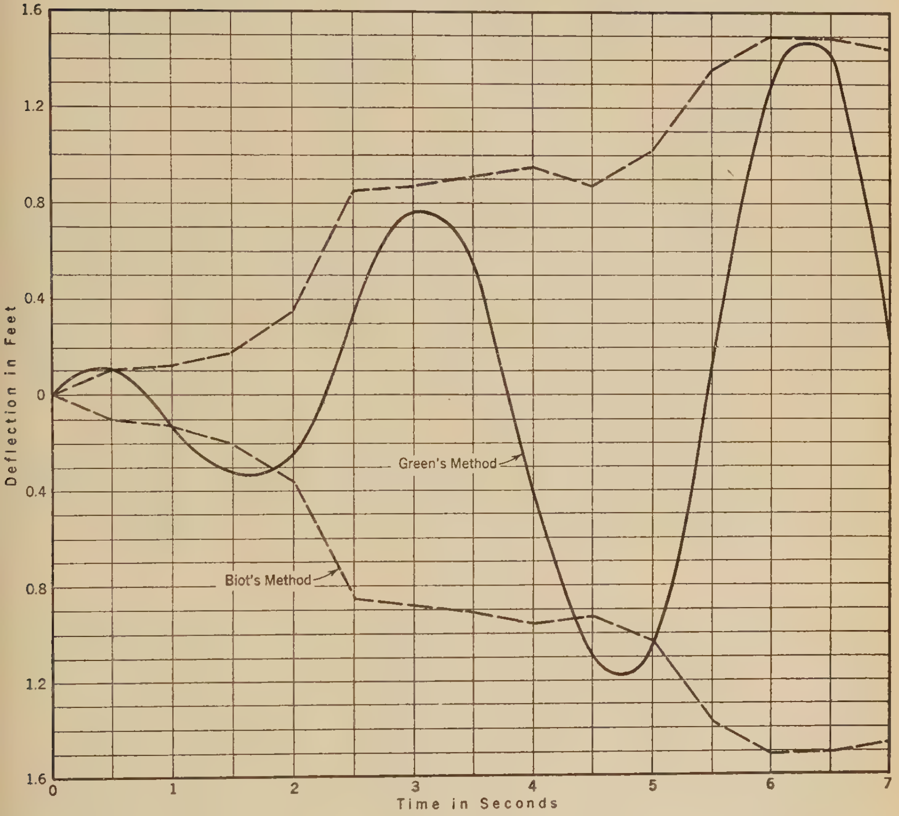


FIG. 10

yielded the deflections plotted in Fig. 10. The maximum deflection is about 1.5 ft, which is more than six times the deflection obtained by Mr. Green. Probably the next swing would be still greater.

Objection may be taken to the writers' use of a long-period acceleration, particularly in view of Mr. Green's reference to Dr. Suyehiro's conclusion that in all earthquakes the period of acceleration in a particular locality is confined to a very narrow range. An examination of the Long Beach, Vernon, and Los Angeles, Calif., records of the Long Beach earthquake of March, 1933, shows[9] the presence of waves of relatively long period. It is probable that these caused a great part of the damage to buildings with a long free period.

As an independent check on the foregoing results a calculation was made using a method of analysis[10] developed by Dr. M. Biot. This analysis was developed for a building with any number of flexible stories, in which all the stories above the first have the same flexibility. Very briefly, this method consists of considering the actual ground displacement curve, applying appropriate constants which depend on the characteristics of the building, and, by using graphical integration, obtaining an envelope (limiting curve) which will give the limits of the actual displacement curve of the floor for which calculations are made. For the flexible first-story building the equation of this envelope becomes:

$$u^2_0 = \left[\int g(t) \ \sin\ 2\ \pi\ \nu_0\ t\ dt \right]^2 + \left[\int g(t)\ \cos\ 2\ \pi\ \nu_0\ t\ dt \right]^2 \ldots . \ (26)$$

in which, $g(t)$ is the ground displacement curve and ν_0, the fundamental frequency of the building. The results obtained are shown in Fig. 10. Dr. Biot's method of analysis has been checked by comparison with actual deflections of a model upon a shaking-table. The agreement is quite satisfactory.

The deflections obtained in this example are obviously excessive. Furthermore, it can be seen that a careful choice of an acceleration pattern will give a similar result for any given structure. The results obtained from Mr. Green's method (or any similar method) will depend on the original choice of acceleration pattern. If one could be sure that all earthquakes in a given region would be alike, this method would be very valuable. Unfortunately, this is not the case and, hence, this method of analysis, as it stands, may be misleading.

HOWARD G. SMITS,[11] ESQ. (by letter).—The analysis developed by Mr. Green is interesting; it is quite a new thought as applied to the problem of duplicating, mathematically, such complex vibrations, as earthquake oscillations. Undoubtedly, complex earth motions can be approximated by this method of description more closely than by the usual periodic sine curve or cosine curve expressions.

It is to be noted, however, that the same general method of utilizing unequal maximum accelerations may be used with the customary harmonic theory. Substituting Fig. 11 for Mr. Green's acceleration diagram (Fig. 3), it will be seen that, in general, the periods and accelerations are the same in the two cases with the exception that with the curved diagram the acceleration is zero when $t = 0$. At any given time the ground is moving with simple harmonic motion, such that the acceleration of the ground is $a \sin \frac{\pi}{2\,T}\, t$.

---

[9] *Engineering News-Record*, June 22, 1933.

[10] *Proceedings*, National Academy of Science, February, 1933.

[11] Chf. Designing Engr. for Oliver G. Bowen, Los Angeles, Calif.

Using Mr. Green's nomenclature the differential equation for the horizontal deflection of the tops of the columns from their bases is:

$$m \frac{d^2y}{dt^2} = m\, a \sin \frac{\pi}{2\,T}\, t - e\, y \quad \ldots\ldots\ldots\ldots\ldots\ldots (27)$$

and the general solution of this equation is:

$$y = C_3 \sin \sqrt{\frac{e}{m}}\, t + C_4 \cos \sqrt{\frac{e}{m}}\, t + \frac{a}{\dfrac{e}{m} - \dfrac{\pi_2}{4\,T^2}} \sin \frac{\pi}{2\,T}\, t \quad \ldots (28)$$

in which, a = maximum acceleration, and T = time for acceleration to decrease from the maximum to zero (one-fourth the full period of oscillation).

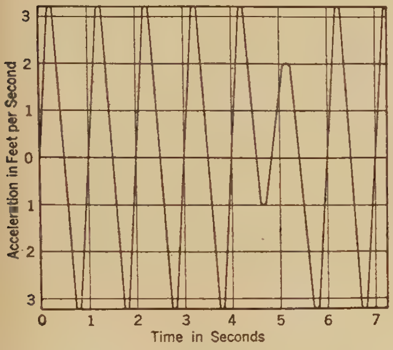


FIG. 11.—ACCELERATION DIAGRAM.

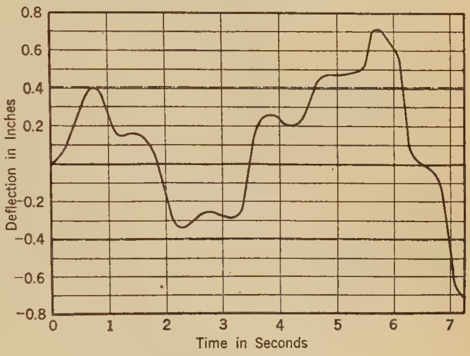


FIG. 12.—DEFLECTION DIAGRAM.

Two constants of integration, C_3 and C_4, are involved which are similar to C_1 and C_2 in Mr. Green's analysis. These constants are evaluated from the initial conditions, namely, when $t = 0$, $\frac{dy}{dt} = V_0$; $y = Y_0$; and,

$$Y_0 = C_4 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (29)$$

and,

$$V_0 = C_3 \sqrt{\frac{e}{m}} + \frac{\dfrac{a\,\pi}{2\,T}}{\dfrac{e}{m} - \dfrac{\pi^2}{4\,T^2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (30)$$

Let $\sqrt{\frac{e}{m}} = Q$, $\frac{\pi}{2\,T} = R$, and $Q^2 - R^2 = b$. Then, solving for C_3 and C_4 and substituting in Equation (28):

$$y = \left(\frac{V_0}{Q} - \frac{a\,R}{b\,Q}\right) \sin Q\,t + Y_0 \cos Q\,t + \frac{a}{b} \sin R\,t \quad \ldots\ldots (31)$$

and,

$$\frac{dy}{dt} = v = \left(V_0 - \frac{a\,R}{b}\right) \cos Q\,t - Q\; Y_0 \sin Q\,t + \frac{a\,R}{b} \cos R\,t \quad \ldots (32)$$

The maximum deflection may be taken as that given by Equation (31) at the time, t, that the ground acceleration is a maximum. The error of this assumption is slight except in the region of resonance; that is, when the period of the ground approaches the free period of the columns.

Mr. Green might have included damping in his discussion. Damping, which is caused by the absorption of energy by the building, may be taken as proportional to the first power of the velocity. If p is taken as the proportionality constant, Equation (1) becomes:

$$m\,\frac{d^2y}{dt^2} + p\,\frac{dy}{dt} = m\,a\left(1 - \frac{t}{T}\right) - e\,y \quad \ldots\ldots\ldots\ldots(33)$$

and the general solution of this linear equation is:

$$y = e^{-j}\left[C_5 \sin \sqrt{\frac{e}{m} - \left(\frac{p}{2\,m}\right)^2} + C_6 \cos \sqrt{\frac{e}{m} - \left(\frac{p}{2\,m}\right)^2}\,\right] + \frac{m\,a}{e}\left[1 + \frac{p}{e\,T} - \frac{t}{T}\right] \quad \ldots\ldots\ldots\ldots(34)$$

in which $j = \frac{p}{2\,m}\,t$; and C_5 and C_6 are the constants of integration and are determined by the initial conditions. Actually, damping is an important consideration; but it is not pertinent to the conclusions of this discussion.

Equations (31) and (32) may be applied in much the same manner as Equations (3), (4) and (5). Table 2 shows the computations for this set of conditions. The horizontal deflection of the tops of the columns from their bases is shown as it varies with time in Fig. 12.

TABLE 2.—COMPUTATIONS

Duration from beginning of motion, t_a	Duration of uniform periodic motion, t_p	T	a	INITIAL CONDITIONS		R	b	FINAL CONDITIONS	
				Y_0	V_0			Y_n	V_n
4.50	4.50	0.250	3.2	0	0	6.29	—36.4	+0.312	+0.525
4.83	0.33	0.165	1.0	+0.312	+0.525	10.40	—105.0	+0.453	+0.121
5.50	0.67	0.335	2.0	+0.453	+0.121	4.70	—19.0	+0.502	—0.392
7.25	1.75	0.250	3.2	+0.502	—0.392	6.29	—36.4	—0.718	—0.157

Conclusions.—If the ground had continued to oscillate with the same motion with which it began, the maximum possible deflection would have been 0.402 in. (indicated by the broken lines in Fig. 12). The introduction of this slight variation of the ground has increased the maximum possible deflection approximately 79 per cent. This is the most important point brought out by this discussion, because it shows that a very slight variation of steady ground motion, opportunely injected, causes a deflection much in excess of that indicated by the "magnification factor" often used in vibration philosophy.

One should use caution in applying these theories in analyzing multi-story buildings, even if the first story is flexible, because the equations under discussion do not apply. While the top stories may be considerably stiffer than the first story, this does not preclude the possibility of vibration in the top stories—and, in some cases, resonance in these upper stories at a higher frequency than that with which the base or first story is vibrating. These vibrations of the upper stories materially affect the motions of the first story. The conclusions given are applicable only to a one-story bent and not to a multi-story building.

H. A. Williams,[12] Assoc. M. Am. Soc. C. E. (by letter).—The assumption is introduced in this paper that, if the first story of a building is made flexible, the remainder of the superstructure can be considered a rigid mass in so far as the dynamic analysis is concerned. The writer has made a number of shaking-table experiments in the Vibration Laboratory at Stanford University with two modifications of a dynamic model of a 17-story building (originally designed and built by Professor L. S. Jacobsen) to ascertain to what extent the author's assumption is valid.

For brevity in the discussion, the nomenclature will be given at once:

e = elastic or spring constant (the same as that used by the author); e_1 is the constant for the first story; e_2, that for the second story; etc.

$\gamma = \frac{e_2}{e_1}$ = ratio of flexibility.

T_n = fundamental period of vibration of model, in seconds, when the superstructure is not considered rigid. It is approximately equal to:

$$T_n = 2\,\pi\,\sqrt{\frac{\Sigma\,W\,\Delta^2}{g\,\Sigma\,W\,\Delta}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (35)$$

in which, W is the weight of one floor of the model, in pounds, and Δ is the horizontal deflection of that floor from the position of static equilibrium when the entire model is subjected to a horizontal force equal to the pull of gravity.[13]

T_r = fundamental period of model, in seconds, when superstructure is considered rigid. It is equal to:

$$T_r = 2\,\pi\,\sqrt{\frac{m}{e_1}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (36)$$

in which, m is the mass of the entire superstructure.

T_t = period of the shaking-table, in seconds.

The first model was dynamically similar, as far as shear distortions are concerned, to a symmetrical 17-story steel frame office building of conventional design. The 8-story model was dynamically similar to the same building with the upper nine stories removed. Frictional effects in the models were not dynamically similar to those in the actual buildings. The weight per floor for Floors 2 to 13, inclusive, was 2 650 500 lb. The next three floors averaged approximately 2 500 000 lb, and the roof, 2 120 000 lb. The elastic constant, e_2, was 57 000 000 lb per in.; the constant, e_{16}, was 27 000 000 lb per in. The intermediate constants varied approximately as a straight line between these two values. The approximate ratio of height to width was 3 to 2 for the 17-story building and 2 to 3 for the 8-story building.

---

[12] Instructor in Civ. Eng., Stanford Univ., Stanford University, Calif.

[13] "Vibration Problems in Engineering", by S. Timoshenko, p. 63.

The shaking-table on which the model was placed, weighed more than $3\frac{1}{2}$ tons. Hence, the motion of the model did not affect the motion of the table, which was anchored at one end by a set of springs, in such a manner that it would be set in motion by dropping a pendulum against a bumper spring on the other end. After the impact, the table had a damped harmonic motion, as shown by the lower curve in Fig. 13. The period of of motion was changed for some of the tests by changing the anchor springs. The amplitude was varied by changing the height of the pendulum drop.

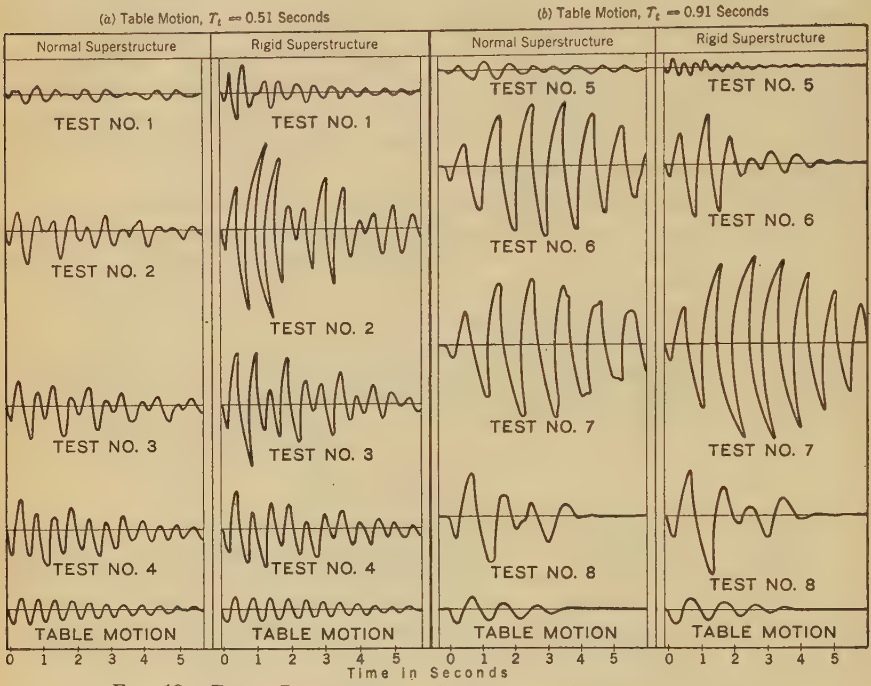


FIG. 13.—TESTS RESULTS FOR A MODEL OF A 17-STORY BUILDING.

The apparatus was arranged so that one record giving the relative motion of the second floor of the model with respect to the table and another giving the motion of the table with respect to the laboratory floor, were obtained simultaneously. In terms of an actual building, these records correspond to the relative motion of the second floor with respect to the ground and the absolute motion of the ground, respectively. In the test records, the table amplitude should be divided by 2 when compared quantitatively to the model amplitude.

For Test No. 1, the value of e_1 for the model corresponded to an elastic constant of 60 800 000 lb per in. in the actual building. Hence, the flexibility ratio was $\gamma = \frac{e_2}{e_1} = 0.95$ (see Table 3(*a*)). This ratio of flexibility would correspond to that found in some conventional buildings of this height.

The computed fundamental period of the model for this value of γ was $T_n = 0.75$ sec. The computed period of the second mode of vibration was approximately 0.25 sec. The table was then given the motion indicated by the record at the bottom of Fig. 13, and the corresponding motion of the second floor of the model was recorded with the superstructure in the normal condition of flexibility. The superstructure of the model was then braced so that it acted as a rigid mass, and the same test was repeated. The motion of the second floor for this condition is shown by the right-hand record for Test No. 1.

TABLE 3.—COMPUTED PERIODS OF VIBRATION OF BUILDING MODELS

17-STORY BUILDING (SEE FIG. 13)						8-STORY BUILDING				
Test No.	Ratio of flexibility, γ	Periods of Vibration				Test No.	Periods of Vibration			
		Normal Structure		Rigid Structure			Normal Structure		Rigid Structure	
		T_n, in seconds	Ratio, $\frac{T_n}{T_t}$	T_r, in seconds	Ratio, $\frac{T_r}{T_t}$		T_n, in seconds	Ratio, $\frac{T_n}{T_t}$	T_r, in seconds	Ratio, $\frac{T_r}{T_t}$
(a) PERIOD OF TABLE MOTION, T_t, EQUALS 0.51 SECONDS										
1......	0.95	0.75	1.47	0.30	0.59	9.....	0.32	0.63	0.18	0.35
2......	6.10	0.94	1.84	0.67	1.31	10.....	0.56	1.10	0.43	0.84
3......	9.60	1.07	2.10	0.85	1.67	11.....	0.60	1.18	0.55	1.08
4......	38.80	1.82	3.57	1.65	3.23	12.....	1.10	2.16	1.00	1.96
(b) PERIOD OF TABLE MOTION, T_t, EQUALS 0.91 SECONDS										
5......	0.95	0.75	0.82	0.30	0.33	13.....	0.32	0.35	0.18	0.20
6......	6.10	0.94	1.03	0.67	0.74	14.....	0.56	0.62	0.43	0.47
7......	9.60	1.07	1.18	0.85	0.93	15.....	0.60	0.66	0.55	0.60
8......	38.80	1.82	2.00	1.65	1.81	16.....	1.10	1.21	1.00	1.10

The first story was then made more flexible and the same procedure was followed for Test No. 2, etc. Since the table motion was the same for all tests in Fig. 13(*a*), it is shown only once, at the bottom.

Repetition of the same tests for the 17-story model, using a different table motion, gave the results shown in Fig. 13(*b*). The upper nine stories were then removed and the same procedure was followed for the 8-story model. It should be noted that an 8-story building with such large elastic constants would be quite stiff for the height involved.

The test records in Fig. 13 indicate that the second-floor motions with the two types of superstructure are not alike for this particular 17-story building when γ is less than approximately 40; and, even then, they are not exactly alike. The second mode of vibration whan γ was 38.8 was approximately 0.36 sec. It is quite probable that, if the table motion happened to have the same period as the second mode of vibration of the model, the favorable comparisons obtained in Tests Nos. 4 and 8, Fig. 13, would not hold.

The test records for the 8-story model are not shown, but they also give little evidence in favor of assuming a rigid superstructure in the 8-story building when γ is much less than 40.

Tests Nos. 1 to 4 might lead one to conclude that the assumption of a rigid superstructure is on the side of safety, because all the curves on the right-hand side of Fig. 13(*a*) have larger amplitudes than those on the left-hand side. Tests Nos. 6, 10, and 15 indicated that such a conclusion is erroneous. In general, if the ratio, $\frac{T_n}{T_t}$, is approximately 1.00 (exact resonance for the fundamental mode of vibration) and, at the same time, is closer to unity than $\frac{T_r}{T_t}$, the amplitude of the second-floor motion for the normal superstructure will be larger than when the superstructure is assumed to be rigid.

The experiments have covered a comparatively narrow range. Definite conclusions applying to all types of structures cannot be drawn from these few tests, or from any theoretical step-by-step method of analysis. In general, it would appear that if the elastic constants and floor weights are of the order of those given herein, there is little reason to believe that the author's method of analysis should apply to buildings higher than eight or ten stories unless the flexibility ratio, γ, is at least 40. This ratio can probably be somewhat less for buildings of fewer stories if the superstructure is fairly rigid. It probably should be increased if the building exceeds fifteen stories.

The writer understands that Mr. Green has in mind flexibility ratios as large as 50 or 60. The period of 3.58 sec for the building which he used in his illustration would indicate as much. The practicability of using a flexibility ratio of even 40, of course, is open to debate. Observation of the model on the shaking-table makes one dubious of the success of such a building. A small movement of the ground can set up a rather large relative motion of the structure even though the system is considerably removed from the resonance condition. Hence, certain practical difficulties must be overcome to prevent damage to the flexible story from every slight earth shock.

Some engineers believe it is practical to construct the first-story walls and ceiling so that the motion of the superstructure will result in no damage to the flexible story. If such a building has a flexibility ratio of 40, or more, its behavior in wind storms would have to be determined. It has also been suggested that hollow tile walls, for instance, could be installed in the flexible story to prevent damage from small ground motions and also to stiffen the building against gusts of wind. In a severe quake, these walls would soon shatter and allow the flexible story to come into full play. Until the tile walls are badly shattered, it is quite possible that vibrations will be set up in the superstructure and, also, that the elastic constant for the flexible story will be non-linear. These factors should be borne in mind when interpreting any mathematical analysis of such a building.

If it is granted that large flexibility ratios are practicable, Mr. Green's method of analysis, when used by one thoroughly familiar with the dynamic principles involved, should give some indication of the action of the building in an earthquake. Obviously, any quantitative results of such an analysis should be accepted with strong reservations. Even though the designer is skilful in simulating probable earth motions, there are several important factors which he would usually neglect because of lack of time or lack of reliable information.

For example, if a building is tall and narrow, direct stress in the columns and load-bearing walls will result in a certain amount of flexibility in the superstructure. This factor can be included when computing the elastic constants. If it is neglected, the flexibility ratio will be too high. Moreover, the rotation of the ground at the foundation line affects the motion of any building, the effect being dependent on the width and height of the building and on the nature of the ground below the foundation line. Another factor is damping friction which decreases the amplitude of motion. Little is known at present about the magnitude of these two factors as far as buildings are concerned.

Norman B. Green,[14] Esq. (by letter).—The active interest in this subject, as evidenced by the discussion, is gratifying. One of the main objects of the paper was to elicit ideas and criticism from other engineers, since the fundamental idea involved is relatively new.

The flexible first story, perhaps, may be considered as an expedient for rendering the structure susceptible of dynamic analysis. The only other alternative is to utilize the so-called "rigid" type of design, in which the entire building is assumed to move with the ground and is subjected to the ground acceleration. This assumption is only permissible if the free vibration period of the structure is very short, which limits this type of construction to low buildings having some very stiff form of bracing, such as masonry walls.

The entire analysis of the building with a flexible base devolves upon the assumption that the upper part of the structure may be considered as a rigid mass. This assumption has been called in question by Mr. Johnson and Mr. Smits. The writer believes that the question is not as to whether the assumption is valid, but rather within what limits it is valid. One extreme would be a wide building of two stories, having concrete second-story walls with a small window area, supported on long first-story steel columns. In this case there seems to be no question but that the assumption is valid. On the other hand, a narrow building of thirty stories, having a large window area, would offer considerable uncertainty. The problem presented by the assumption is not susceptible of theoretical analysis, and the writer is therefore greatly indebted to Mr. Williams for his experimental investigation along this line. Reference to Fig. 13 of Mr. William's discussion will show that the displacement curves for the normal and rigid structure for Tests Nos. 4 and 8, are very closely the same. This favorable result was secured with a ratio of second-story stiffness to first-story stiffness of only 38.8. The writer believes that a much greater stiffness ratio than this can easily be secured.

---

[14] Structural Engr., San Francisco, Calif.

Consider, for example, the 20-story steel frame building described by the writer and to which he applied the analysis. The effective height of the flexible first story is 36 ft, which gives an elastic constant of 3 760 000. If it is assumed that the earth motion is parallel to the short dimension of the building, the lateral load acting on the upper or "rigid" part is resisted by the two end walls, each consisting of four 20-ft bays with an over-all width of 82 ft. If there are two 4 by 7-ft windows in each bay, the window area will be 23% of the total, which is a fair average condition for buildings of this type. As an approximation it may be assumed that the lateral distortion in any one story is produced by the shear and bending deflection of the individual piers between the windows, these being taken as fixed at the window head and sill. On this basis, with a 10-in. concrete wall, the elastic constant for one story is 840 000 000 and the ratio of this constant to that for the first story is 223. This computation neglects that part of the story distortion that arises from the bending of the spandrels and the consequent rotation of the ends of the piers. Inasmuch as the spandrels are relatively very deep and short and since about one-half the total deflection of the average pier is due to shear, it is not likely that the error involved is large.

The writer is indebted to Messrs. White and Kartzke for their analysis of the 20-story building, utilizing the acceleration diagram of Fig. 9, in which a condition of resonance is approached. The long waves of this diagram have an acceleration period, not of $3\frac{1}{2}$ sec as they state, but of 3 sec—so that complete resonance with the building having a free period of 3.58 sec is not attained. Even so, an examination of Fig. 10 shows a rapid increase in the building deflection. The important point demonstrated is that only one or two long swings of a period fairly close to the period of the building, are necessary to produce destructive deflections.

Messrs. White and Kartzke are certainly correct in stating that a careful choice of an acceleration pattern will lead to destructive deflections for any given structure; but it is also possible to conceive of a type of gravity loading that will cause the failure of any structure. The occurrence of an acceleration pattern of the character of Fig. 9, having a period of 3 sec and an acceleration of 10% gravity, is not probable in the light of present knowledge of the subject. Referring to the accelerograph records of the Long Beach earthquake of March, 1933,[15] it will be found that there is a well-marked association of long-period waves with low accelerations and short-period waves with high accelerations. The longest waves recorded had a period of about 2.5 sec, with a maximum acceleration of from 1% to 4% gravity; the highest recorded accelerations of from 20 to 30% of gravity, on the other hand, were associated with short periods of from 0.7 to 0.3 sec. If this proves to be characteristic of all earthquakes, the design of so-called "rigid" structures offers serious difficulties, since to be consistent very high accelerations and seismic coefficients should be applied. On the other hand, the flexible-base structure is relatively immune to the effects of these very high short-period accelerations.

---

[15] *Engineering News-Record*, June 22, 1933.

As demonstrated by Messrs. White and Kartzke the acceleration diagram of Fig. 3 shows a total ground displacement of about 20 in. The ground motion is periodic up to the end of the fourth second, with an amplitude of motion of 1.6 in. At this point the equilibrium is disturbed by the introduction of an irregularity in the acceleration diagram, so that the ground motion thereafter is of varying velocity, in a constant direction. Obviously, this same diagram could not be extended indefinitely, since it would result in an unlimited ground displacement; however, the progressively increasing ground movement can be halted at any point by the insertion of another appropriate irregularity into the diagram. This simply illustrates the desirability of integrating each acceleration diagram before it is employed in the analysis of a structure. This can be done quite easily by making use of the fact that the algebraic sum of the areas enclosed between the acceleration diagram and the axis of zero acceleration up to any time, is equal to the velocity at that time; the algebraic sum of the areas enclosed between the velocity diagram and the axis of zero velocity gives the displacement during that period of time. The areas can readily be determined by plotting the acceleration diagram and the velocity diagram on cross-section paper.

In this connection the important point is that introduced in the discussion by Mr. Smits, namely, that the insertion of a slight irregularity in the acceleration diagram, which institutes a progressive ground motion in one direction, produces large deflections of the building. No such result can be revealed by the conventional investigation in which only a simple harmonic motion of the ground is considered. The effect is entirely independent of resonance.

The question may arise as to whether any such sustained motion of the ground in one direction is possible. This type of motion cannot be recorded during the progress of the earthquake by any existing seismograph or displacement meter; however, precise surveys made before and after the San Francisco earthquake of 1906[16] indicate large permanent displacements. This is illustrated by Fig. 14 which has been plotted from the results of these surveys. A survey line crossing the main fault at right angles (and which was straight prior to the earthquake) was subsequently found to be broken at the fault, the west half having been displaced 3.6 m toward the north and the east half, 2.3 m toward the south. This lateral displacement of the ground progressively decreased each way from the fault line, as indicated on the diagram, so that a large area of land suffered a permanent shear distortion.

Mr. Smits has extended the writer's equations to include the effect of damping, making this proportional to the relative velocity. Since the lateral load is intended to be resisted in the flexible story by means of steel columns alone and without the help of exterior walls or interior partitions, it does not appear that there would be any considerable damping as long as the stress in these columns is kept within the elastic limit of the material. Further application of the analysis to flexible-base buildings might indicate, however, the desirability of introducing some form of artificial damping, so as to limit the effects of possible resonance with the ground motion. Perhaps, this could be done by locating the flexible story below the ground, with the base of the rigid superstructure at the first-floor level, so that damping devices could be introduced at this point to re-act against the earth. With this position of the flexible story, all walls and partitions above the ground level could be made rigid and the practical difficulties mentioned by Mr. Williams, attendant upon securing a flexible story in the superstructure, would be eliminated. In this case, the flexible story would also meet the theoretical requirement of a strictly constant value of the elastic factor, e, so that there is no uncertainty as to the starting conditions. As stated by Mr. Williams, this will not be the case if it is necessary to rupture first-story walls and partitions before the lateral load is carried by the steel columns in the flexible story.

---

[16] Rept. of the California State Earthquake Investigation Comm., Vol. 11.

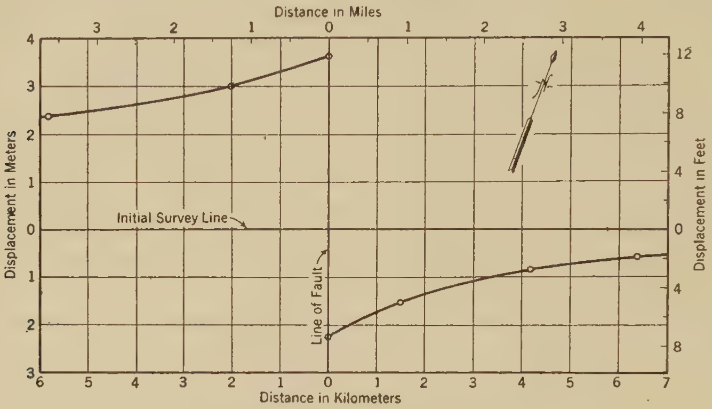


FIG. 14.—PERMANENT GROUND DISPLACEMENTS IN THE SAN FRANCISCO EARTHQUAKE OF 1906

Mr. Johnson states that the effects of a vibratory phenomenon upon any object cannot be determined by considering only the acceleration. This is not true for non-harmonic vibration of the type the writer has utilized in the analysis, since in this case the acceleration may be established entirely independently of both the amplitude and the frequency. Given an arbitrary acceleration diagram, the first integration establishes the velocity and the second determines the displacements or ground motion. Two constants of integration are introduced, which are known if the initial conditions of velocity and displacement are known. The motion is then completely determined. The statement referred to is correct in the case of simple harmonic motion, the fundamental definition of which establishes a relation between acceleration, amplitude, and frequency.

The discussion by Mr. Johnson mentions an uncertainty regarding the writer's notation for y. This quantity, y, is defined as the deflection of the body from its position at rest; that is, at rest relative to the support.

In other words the variable, y, is relative displacement. It would be a clearer statement of the dynamic relation, perhaps, if the writer's Equation (1) had been written in the form:

$$\frac{d^2 y}{d\,t^2} = a\left(1 - \frac{t}{T}\right) - \frac{e\,y}{m} \quad \text{.....................(37)}$$

which is simply an algebraic relation between three accelerations, considered as vector quantities. It states that the acceleration of the mass relative to the support is equal to the absolute acceleration of the support minus the relative acceleration impressed on the mass by the spring. The conventional form of equation representing a forced vibration involves two variables: (1) The absolute displacement of the support; and (2) the absolute displacement of the vibrating mass. The difference between these two variables is the displacement of the mass relative to the support. The writer considers it simpler and more direct in this case to introduce one dependent variable only, which is the relative displacement that it is desired to determine.

The idealized system shown in Fig. 2, in which the vibrating mass is assumed to move only in a straight line in the plane of the paper, has one degree of freedom and only one mode of vibration. The natural period of this vibration is correctly given by Equation (9), which Mr. Bednarski believes to be too simple to be true. It is this very simplicity of dynamic analysis which recommends the flexible-base building. On the other hand the dynamic analysis of a bar fixed at one end and free at the other, cited by Mr. Bednarski, involves harmonics and is a much more involved problem. Such a cantilever beam is similar, in general, to a high building that is flexible throughout its height, and well illustrates the uncertainties and difficulties attendant upon any dynamic analysis of such a structure.

An actual flexible-base building has two degrees of freedom for motions of translation; however, the columns should be arranged so as to give the same elastic constant about either axis, so that its natural frequency will be the same for vibrations in either direction. In case the center of mass of the structure does not coincide with the center of elastic resistance, a couple is introduced, which will produce rotatory vibrations superimposed upon those of translation. This case is outside the scope of the writer's paper.

No mention has heretofore been made of the seismic forces acting on the upper or "rigid" parts of the structure; but these forces must be considered in designing the bracing system, since the requisite strength as well as stiffness must be secured. The acceleration of the upper part of the building is produced by the horizontal reaction of the top of the base columns and, therefore, it will be a maximum when the deflection of the base columns is a maximum. Then, if a_m is the maximum acceleration to which the upper part of the building is subjected:

$$a_m = \frac{e \times \text{maximum column deflection}}{m} \quad \text{...............(38)}$$

Consider, for example, the 20-story building referred to under the heading "Application of the Analysis" in the paper. The mass of the rigid part of this building is 1 220 000; the elastic constant, e, is 3 760 000; and, from

Table 1, the maximum column deflection is 0.445 ft. Substituting these values in Equation (38), the acceleration is found to be 1.37 ft per sec per sec. The maximum deflection that occurs during the first 4 sec of periodic ground motion, is 0.145 ft which gives to the building superstructure an acceleration of 0.45 ft per sec per sec. This is only 14% of the full ground acceleration and illustrates the large reduction in earthquake stresses that may be produced by a flexible story when the ground motion is periodic and even if it is of fairly long period.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 1907

INVESTIGATION OF WEB BUCKLING IN STEEL BEAMS

By Inge Lyse,[1] M. Am. Soc. C. E., and
H. J. Godfrey,[2] Esq.

With Discussion by Messrs. R. L. Moore and E. C. Hartmann,
and Inge Lyse and H. J. Godfrey

Synopsis

The results of an investigation on the web failure of structural steel beams are reported in this paper. Tests were made on rolled sections as well as on sections made from plates by means of electric arc welding. The depth-thickness ratio of the web of the beams varied considerably; however, all the beams gave indication of initial failure due to shear. At the initial failure of the beam (its yield point), the computed maximum horizontal shearing stress in the web was found to correspond very well with the shearing yield-point stress of the material. The conclusion is drawn that the shearing stresses developed in the web, rather than buckling, determine the usefulness of a beam for depth-thickness ratios of 70 or less. The trend of the relationship indicated that buckling of the web might occur at a depth-thickness ratio of approximately 80.

Introduction

Since the degree of web buckling in beams and girders has long been a moot question, and since experimental data are scarce, an investigation of this subject was an urgent need. A theoretical study[3] was presented by Professor S. Timoshenko in 1933, but as no experimental results were offered

---

Note.—Published in February, 1934, *Proceedings*.

[1] Research Associate Prof. of Eng. Materials, in Chg., Fritz Eng. Laboratory, Eng., Lehigh Univ., Bethlehem, Pa.

[2] Former Research Fellow in Civ. Eng., Lehigh Univ., Bethlehem, Pa.

[3] "Stability of Plate Girders Subjected to Bending," by S. Timoshenko, Preliminary Publication, International Assoc. for Bridge and Structural Eng., Paris, 1932, p. 129; abstracted in *Civil Engineering*, August, 1933, p. 456.

in support of this theory, the question remained unsolved. In that paper Professor Timoshenko presents such a complete discussion of earlier experimental results that the inclusion of a bibliography in this paper is obviated.

For the purpose of securing reliable experimental information, a cooperative investigation was undertaken by the Bethlehem Steel Company and the Fritz Engineering Laboratory of Lehigh University, Bethlehem, Pa. All the beams where designed and furnished by the Bethlehem Steel Company and tested at the Fritz Engineering Laboratory. Special acknowledgment is made to Jonathan Jones and C. H. Mercer, Members, Am. Soc. C. E., Chief Engineer, and Consulting Engineer, respectively, of the McClintic-Marshall Corporation, and to V. E. Ellstrom, Manager of Sales Engineering, Bethlehem Steel Company, for their valuable assistance in carrying out the investigation.

Since the usefulness of a beam is determined by the maximum load it can sustain without excessive deflection, the determination of its yield point becomes the most important factor in the testing. Emphasis was placed, therefore, upon securing the actual yield-point strength instead of the ultimate. The ultimate load has little significance beyond the fact that it is a measure of the toughness of the beam after it has lost its usefulness. In the study of the data the yield-point strength of the beam was used as the criterion for its load-carrying capacity.

PROGRAM

In order to obtain a suitable method of testing for use in the major investigation, a preliminary series of tests was run. This series included four Bethlehem B12–28 rolled sections, two of which had free ends; the remaining two had steel plates welded to the end sections to prevent failure by end twisting. A 1-ft sample section of each beam was furnished for the preparation of test coupons, on which the tensile and shearing properties of the material were determined.

Two groups of beams, designed to secure a definite failure in the web, constituted the major investigation. The first group consisted of five beams, three of which were made by welding steel plates together; the remaining two were rolled sections reinforced with cover-plates welded to the flanges. The second group consisted of five beams, all of which were welded sections. Since the design formulas make the depth-thickness ratio of the web the criterion for the working stresses, beams having high ratios were included in these tests. The highest $\frac{h}{t}$ ratio for rolled sections was about 55, and for welded sections about 70. (In this paper, h represents the clear distance between the flanges, and t, the thickness of the web.)

PRELIMINARY INVESTIGATION

Information relating to the four beams tested in the preliminary investigation is presented in Table 1(*a*). It is noted that these beams had

TABLE 1.—TEST RESULTS, INVESTIGATION OF WEB BUCKLING

Date of test, 1932 (1)	Beam No. (2)	Net web area, ht, in square inches (3)	Web ratio, $\frac{h}{t}$ (4)	Moment of inertia, I, in inches⁴ (5)	Section modulus, S, in inches³ (6)	Reaction, V, in pounds (7)	STRESSES, IN POUNDS PER SQUARE INCH						Reaction, in pounds (14)
							$\frac{V}{ht}$ (8)	$\frac{Vay}{It}$ (9)	$f=\frac{M}{S}$ (10)	Shear in web specimens (11)	Tension in web specimens (12)	Tension in flange specimens (13)	
(a) PRELIMINARY INVESTIGATION ON 12-INCH 28-POUND I-BEAMS (B 12-28)													
April 1........	1	3.06	40.2	205.5	34.2	55 000	17 950	18 550	29 000	23 650*	....	37 340* 58 810†	60 000†
April 5........	2	3.06	40.2	205.5	34.2	60 000	19 600	20 250	31 600	22 500* 50 750†	....	37 970* 59 260†	64 500†
May 27........	3	3.02	40.8	200.0	33.33	65 000	21 550	22 400	35 100	22 300* 50 200†	40 800* 59 370†	39 950* 58 150†	70 750†
July 1........	4	3.02	40.8	200.0	33.33	65 600	21 700	22 600	35 400	24 500* 50 600†	39 550* 57 800†	....	72 225†
(b) MAJOR INVESTIGATION, GROUP *A*													
June 15........	*WB*- 1	3.47	56.5	1 938	226	77 500	22 300	20 400	14 700	22 000* 53 500†	43 250* 62 000†	....	109 250‖
May 20........	*WB*- 2	3.57	54.9	1 955	228	95 000	26 600	24 400	17 900	24 500* 51 300†	47 810* 60 900†	....	127 800‖
June 16........	*WB*- 3	4.36	58.9	2 419	254	115 000	26 300	24 700	21 700	26 470* 51 600†	49 600* 61 425†	....	139 000‖
June 30........	*WB*- 4	12.41	54.7	3 342‡ 12 850§	240‡ 832§	280 000	22 500	24 200‡ 21 130§	20 200	22 000* 53 400†	43 500* 61 600†	....	298 800‖
June 16........	*WB*- 5	8.15	52.2	1 560‡ 6 922§	141‡ 550§	210 000	25 800	25 700‡ 23 250§	21 150	29 200* 55 630†	51 300* 66 650†	....	225 000‖
(c) MAJOR INVESTIGATION, GROUP *B*													
November 30...	*WB*- 6	4.40	70.0	2 866	278.5	78 800	17 900	16 800	14 100	17 450* 46 900†	33 080* 46 800†	....	96 300‖
December 1....	*WB*- 7	3.88	60.6	2 216	241.8	74 100	19 100	17 800	13 800	18 600* 45 850†	33 700* 48 400†	....	95 050‖
December 2....	*WB*- 8	4.10	59.7	2 326	249.2	71 550	17 500	16 250	12 930	19 920* 46 250†	29 680* 46 080†	....	99 750‖
December 1....	*WB*- 9	3.12	50.0	1 523	196.5	59 450	19 050	17 250	12 100	18 480* 44 800†	30 280* 46 980†	....	92 000‖
December 2....	*WB*-10	3.14	49.4	1 520	196.5	60 900	19 380	17 580	12 400	18 400* 46 250†	30 270* 47 150†	....	94 000‖

* Yield-point stress. † Ultimate. ‡ At section having no cover-plates. § At section having cover-plates. ‖ Maximum.

an $\frac{h}{t}$ ratio of about 40. They were tested in the 300 000-lb screw-power machine, the load being applied at the rate of 0.05 in. per min. The sizes of all beams were measured with micrometers and were found to differ slightly from those listed in handbooks. The properties based on the actual dimensions are given in Table 1(*a*).

The beams were supported by a roller at one end and by a spherical bearing-block at the other end. The load was applied through a roller and a spherical bearing-block at the quarter-points of the span. At that side of the center line where a roller was used as a support, a spherical bearing-block was used for the application of the load, and *vice versa*, as shown in Fig. 1. Lateral deflections of the web were measured by dial gauges placed on both sides of the web along a line connecting the loading point and the support. A movement of 0.0001 in. could be read directly by these gauges. On the first beam tested (No. 1, Table 1(*a*)), the gauges were supported by a frame attached to the table of the machine. This arrangement was not satisfactory, however, due to the relative movement between the beam and the table. For the remaining three beams the frame holding the gauges was clamped directly to the flanges of the beams. Vertical deflections were also observed on the two beams that had plates welded to the end sections.

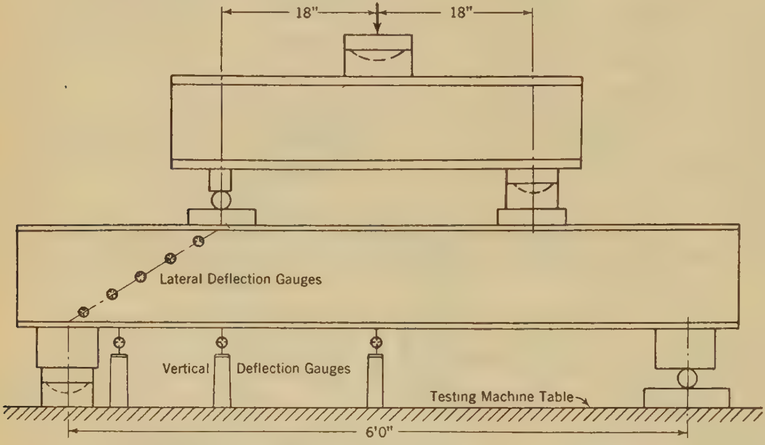


FIG. 1.—LOADING ARRANGEMENT FOR PRELIMINARY INVESTIGATION

In the first free-ended beam the center of the web was $\frac{3}{16}$-in. off center with respect to the top flange. The web of this beam began to scale at a load of 110 000 lb, which was taken as the yield point. However, it continued to take load until a total of 120 000 lb had been applied. At this load one end twisted sidewise. The eccentricity of the top flange may have contributed to an earlier twisting than would otherwise have occurred.

The greatest lateral web deflection occurred in Beam No. 2, Table 1(*a*), near the top flange where, at a load of 110 000 lb, the deflection was about

0.075 in. To a load of 60 000 lb the increase in lateral deflection was nearly constant for each increment of load. Beyond 60 000 lb the rate of increase in lateral deflection became greater for each additional increment. Strain-gauge measurements were also taken at points shown in Fig. 2, by means of a 2-in. strain-gauge. The compressive strains obtained are shown in Fig 3. It is noted that at a load of 110 000 lb none of the gauge lines showed strain near the yield-point strain of the material in the web, but there is a tendency for the rate to increase. The beam first began to scale at a load of 120 000 lb which was taken as the yield point. The beam continued to take load until a maximum of 129 000 lb was reached, at which time the ends twisted sidewise.

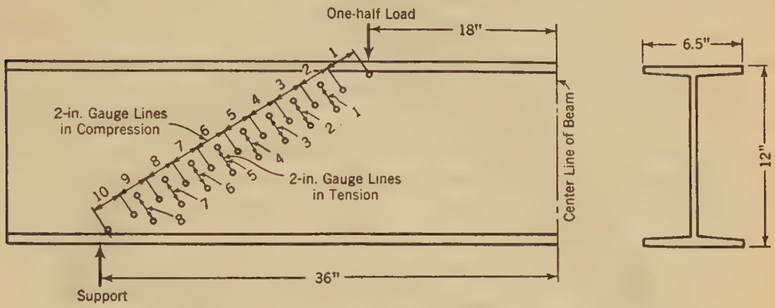

One-half Load
18"
6.5"
2-in. Gauge Lines in Compression
2-in. Gauge Lines in Tension
Center Line of Beam
12"
36"
Support
1
2
3
4
5
6
7
8
9
10

FIG. 2.—LOCATION OF STRAIN-GAUGE POINTS ALONG WEB

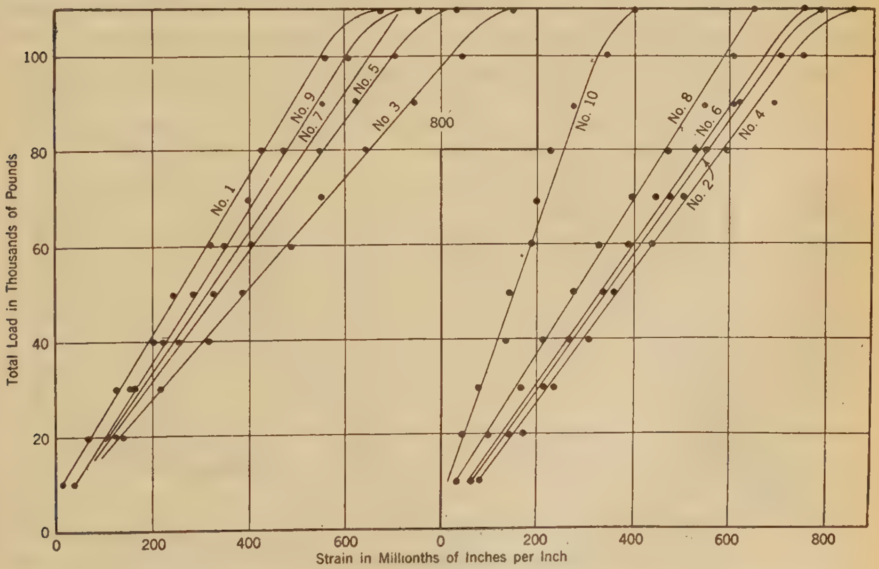


FIG. 3.—AVERAGE STRAIN IN COMPRESSION ALONG LINE CONNECTING LOADING POINT AND SUPPORT

Table 1(*a*) shows that the test coupons gave shearing yield-point stress in the web of 23 650 and 22 500 lb per sq in., respectively, for Beams Nos. 1

and 2. The average yield point and ultimate strength in tension, as determined by tests on the coupons from the flanges of these beams, were 37 700 and 59 000 lb per sq in. The shearing yield-point stress was thus about 64% of the yield-point stress in tension. The computed maximum shearing stresses in the web were 18 550 and 20 250 lb per sq in. at the yield point of the two beams. The maximum fiber stresses in the flanges were 29 000 and 31 600 lb per sq in. at the yield point of the beams. These data show that the yield point of the material had not been reached, either in shear or in tension, at the load at which scaling occurred. Furthermore,

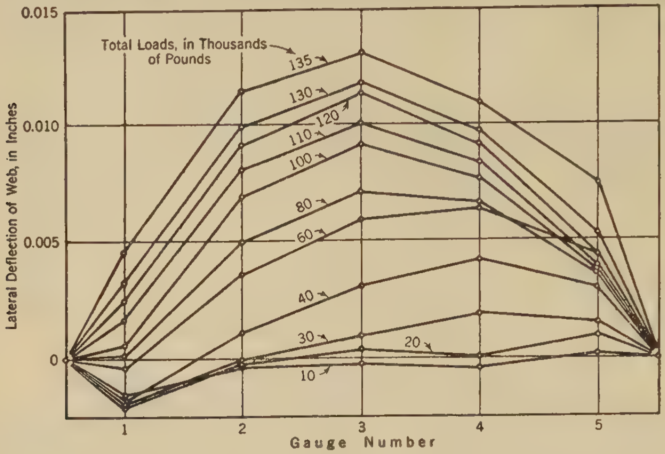


Fig. 4.—Lateral Deflection of Web of Beam No. 3, Table 1

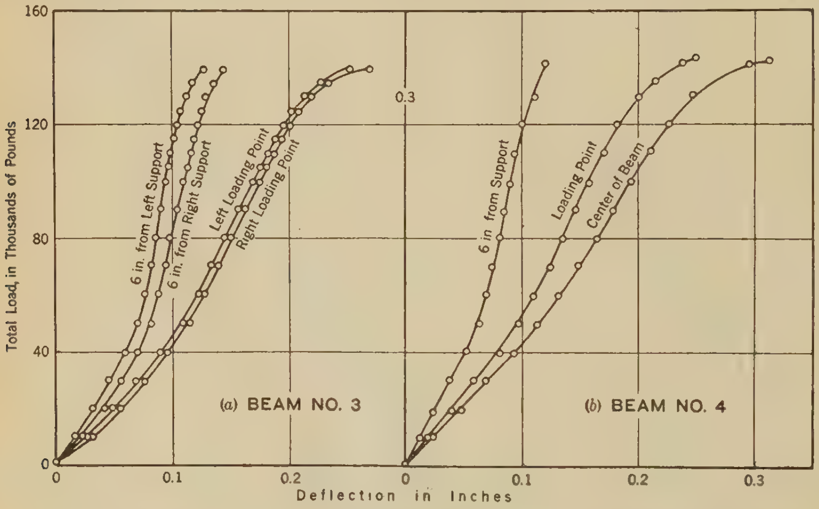


Fig. 5.—Deflection Curves for Beams Nos. 3 and 4, Table 1 (Ultimate Load, Beam No. 3, 141 500 Pounds; Beam No. 4, 144 450 Pounds)

the failure was due to end twisting instead of web buckling or shear. In order to prevent end twisting steel plates were welded to the ends of the remaining beams.

Both vertical and lateral deflections were observed for the beams with end plates. These beams were whitewashed before the testing so that strain lines could be observed more readily. The loading arrangement is shown in Fig. 1. The lateral web deflections for Beam No. 3, shown in Fig. 4, were similar in shape to those for Beam No. 4. However, the lateral deflection of Beam No. 4 became less after a load of 50 000 lb had been applied, whereas in Beam No. 3 the deflection increased throughout the test. It should be noted that at a load of 135 000 lb, the lateral web deflection of Beam No. 3 was only 0.013 in. The similarity of the vertical deflection curves was also very marked, as shown in Fig. 5. The yield point for these two beams was taken as the point at which the slope of the tangent to the deflection curve was twice as large as the slope of the straight part of the curve. The loads at the yield points were 130 000 and 131 000 lb.

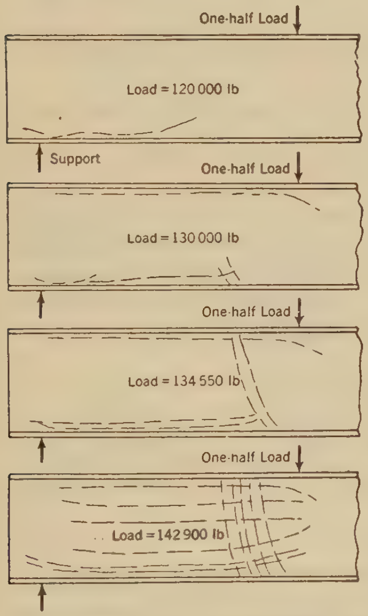

One-half Load
Load = 120 000 lb
Support
One-half Load
Load = 130 000 lb
One-half Load
Load = 134 550 lb
One-half Load
Load = 142 900 lb

FIG. 6.—FLAKING OF WHITEWASH ON BEAM NO. 4

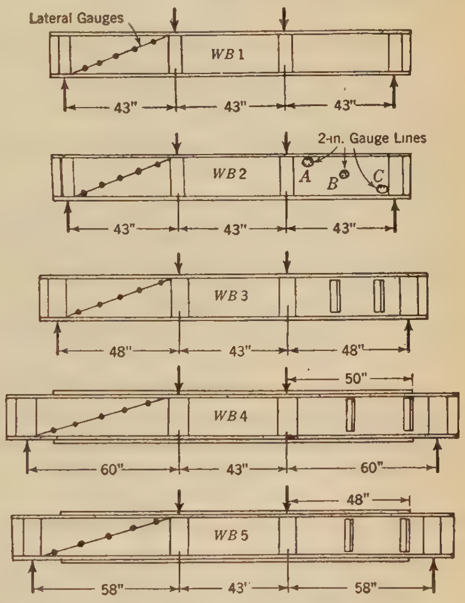

Lateral Gauges
WB 1
43"
43"
43"
2-in. Gauge Lines
WB 2
A
B
C
43"
43"
43"
WB 3
48"
43"
48"
WB 4
50"
60"
43"
60"
WB 5
48"
58"
43"
58"

FIG. 7.—MAKE-UP AND LOADING ARRANGEMENT FOR MAJOR INVESTIGATION, GROUP A

The beams continued to take load beyond the yield point until maximum loads of 141 500 and 144 450 lb were reached, at which time, end twisting occurred in a manner similar to Beams Nos. 1 and 2.

At a load of about 110 000 lb, the first strain lines appeared on both beams in the form of horizontal lines, at the root of the web near the support. With increased load, more strain lines appeared over the support and also below the loading point. At loads of 138 500 and 134 500 lb, respectively, for Beams Nos. 3 and 4, the yielding was so great that it produced a drop in the lever of the testing machine. Further increase in the load produced vertical strain lines in the web between the support and the loading point. All these lines were evidently due to shear. An illustration of the progressive formation of the strain lines is shown in Fig. 6.

Computed maximum shearing stresses in the web at the yield point, were found to be 22 400 and 22 600 lb per sq in. for Beams Nos. 3 and 4. These agree closely with the yield-point stresses in shear (22 300 and 24 500 lb per sq in.) obtained on the test coupons. At the yield point of the beams the maximum fiber stresses in the flanges were 35 100 and 35 400 lb per sq in. Since the tensile yield-point stresses of the coupons were 40 800 and 39 550 lb per sq in., the yield point of the beam was not caused by the flexural stresses. If the flexural stresses were computed for loads corresponding to the drop of the beam, they would still be less than the yield-point stress of the material. It may be concluded, therefore, that the yield point of these beams was determined by the yield point in shear of the material in the web.

It was demonstrated that no web buckling appeared for $\frac{h}{t}$ ratios of 40. In the major series the $\frac{h}{t}$ ratios were considerably greater than 40, and it was deemed advisable to restrain the ends of the beams in order to prevent end twisting.

Tension specimens made from the outer edge of the flange usually were of higher strength than those made from the center of the flange. Tension specimens were also made from the web, and were found to be uniform for all four beams. Shearing strengths of the material in the web were obtained on slotted plate specimens tested in a tension machine.[4]

MAJOR INVESTIGATION, GROUP *A*

Group *A* (see Table 1(*b*)) consisted of five beams, three of which were all welded sections and two were reinforced rolled sections. The welded beams (*WB*-1, *WB*-2, and *WB*-3) were made of $\frac{1}{4}$-in. tank plates for the web, and $1\frac{1}{2}$-in. plates for the flanges; these had $\frac{h}{t}$ ratios of 56.6, 54.9, and 58.9, respectively. All the beams had reinforcement plate stiffeners at the loading points and supports. Steel plates were welded to each end section to prevent end twisting. In addition, Beam *WB*-3 had two angle stiffeners on each side of the web in one of the panels between the support and the loading point. The make-up and the loading arrangement for these beams are shown in Fig. 7. Both vertical deflections of the beam and lateral web deflections were observed during the testing of these beams. The location of the gauges was similar to that of the beams in the preliminary tests. The beam designated in Table 1(*b*) as *WB*-2 was not whitewashed, and strain-gauge observations were taken with a 2-in. gauge length at points of the web, as indicated in Fig. 7. All other beams in this group were whitewashed and had no strain-gauge observations. The properties of the beams based on their actual dimensions, and the results of the tests are given in Table 1(*b*).

---

[4] Description of these tests is given in a paper entitled "Shearing Properties and Poisson's Ratio of Structural and Alloy Steels," by Inge Lyse, M. Am. Soc. C. E., and H. J. Godfrey, *Proceedings*, Am. Soc. for Testing Materials, Pt. II, p. 274, 1933.

For Beam *WB*-1 the first local strain lines appeared under the loading point near the junction of the web and the top flange, at a load of only 50 000 lb. At a load of 125 000 lb horizontal and vertical strain lines appeared in the panel between the loading point and the support. The size and number of strain lines increased with an increase in load. The yield point at 155 000 lb was determined from the deflection curve in the same manner as that used in the preliminary investigation. The beam continued to take load until a maximum of 218 500 lb was attained and, at this point, the load fell off, accompanied by a gradual sagging of the beam.

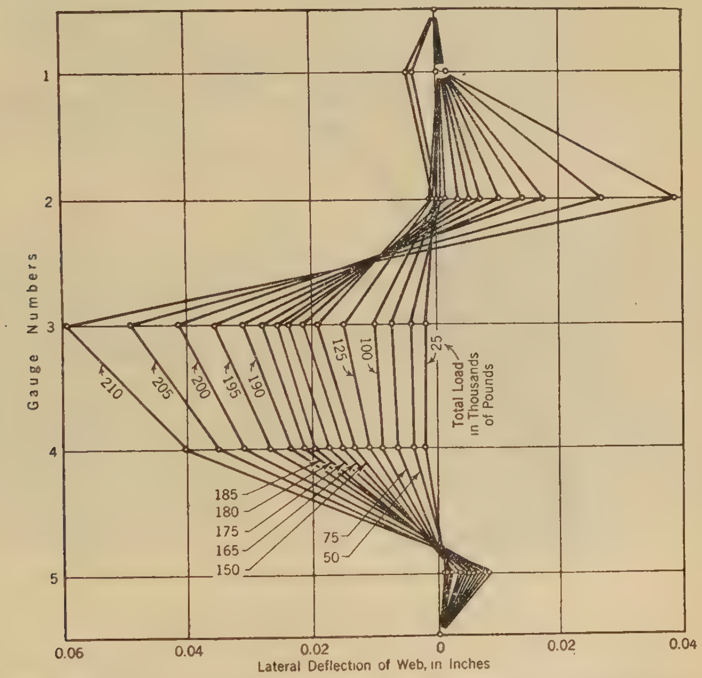


FIG. 8.—LATERAL DEFLECTION OF WEB OF 17-INCH WELDED BEAM *WB*-1, TABLE 1(*b*) (ULTIMATE LOAD, 218 500 POUNDS)

Lateral web deflections for Beam *WB*-1 are given in Fig. 8. It is noted that the center of the web deflected in opposite directions to the deflections at the top and bottom. The web deflection at the yield point of the beam was only 0.02 in., indicating that buckling was not the cause of yielding. The vertical deflections of this beam are given in Fig. 9. It is seen that a fairly sharp increase in the rate of deflection took place at a load of

155 000 lb. This load produced a computed maximum shearing stress of 20 400 lb per sq in. The coupon gave a shearing yield-point stress of 22 000 lb per sq in., which does not differ greatly from the computed shearing stress at the yield point of the beam.

Fig. 10 shows the appearance of Beam *WB*-1 (Table 1(*b*)) at the yield-point load. It is to be noted that the horizontal and vertical strain lines are predominating. Furthermore, a number of local strain lines are grouped along the welds. After the beam had reached the maximum load, the buckling of the web could easily be seen.

Beam *WB*-2 was tested in a manner similar to Beam *WB*-1, except that strain-gauge observations were also taken. The location of the observation points is indicated in Fig. 7. The first strain lines on the web appeared at a load of 68 000 lb while the yield point, as determined from the deflection curve, was found to be at a load of 190 000 lb. The vertical deflections are shown in Fig. 11. The beam continued to take load beyond the yield point and reached a maximum of 255 600 lb, at which time a slight buckle could be seen in one of the web panels. The computed maximum shearing stress at the yield point of the beam was 24 400 lb per sq in. This compared very well with the shearing yield-point stress of 24 500 lb per sq in., obtained on the coupons. The strains obtained by means of the 2-in. gauge are plotted in Fig. 12. It is noted that certain gauge lines showed strains approaching the yield-point strains at a load of 175 000 lb.

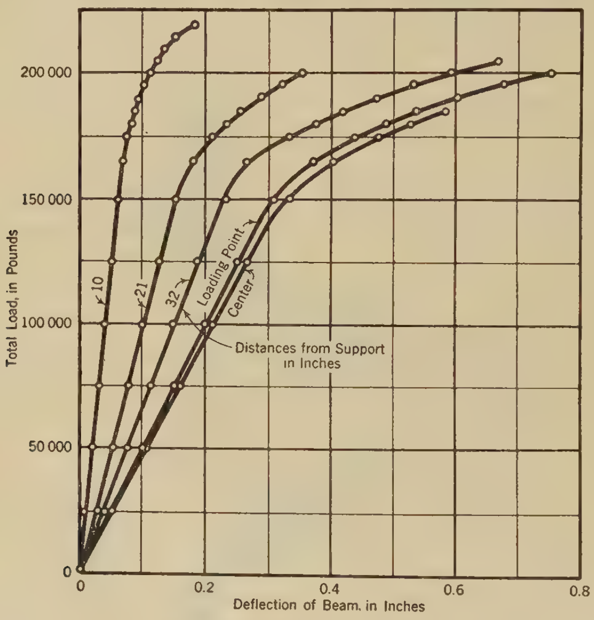


FIG. 9.—DEFLECTION OF 17-INCH WELDED BEAM *WB*-1, TABLE 1(*b*) (ULTIMATE LOAD, 218 500 POUNDS)

Welded Beam *WB*-3 was tested in the same manner as Beam *WB*-1. The first local strain lines appeared at a load of 38 000 lb. After a load of 50 000 lb had been applied the load was released to 1 000 lb and a set reading observed. This was also done after every following increment of loading up to the maximum load. The extent to which the lateral web deflections became set increased considerably after the yield point of the beam had been reached. The vertical deflections in Fig. 13, showed only very small sets at low loads. The yield point of the beam was reached at a load of 230 000 lb, and the vertical permanent set increased considerably after this load had been exceeded. The computed maximum shearing stress at the yield point of the beam was 24 700 lb per sq in. This compares favorably with a shearing yield-point stress of 26 470 lb per sq in. for the coupons. While there is no indication of web buckling at the yield point of the beam, definite web buckling appeared at a maximum load of 278 000 lb. Welded beams proved very satisfactory and showed no indication of distress in the fillet welds.

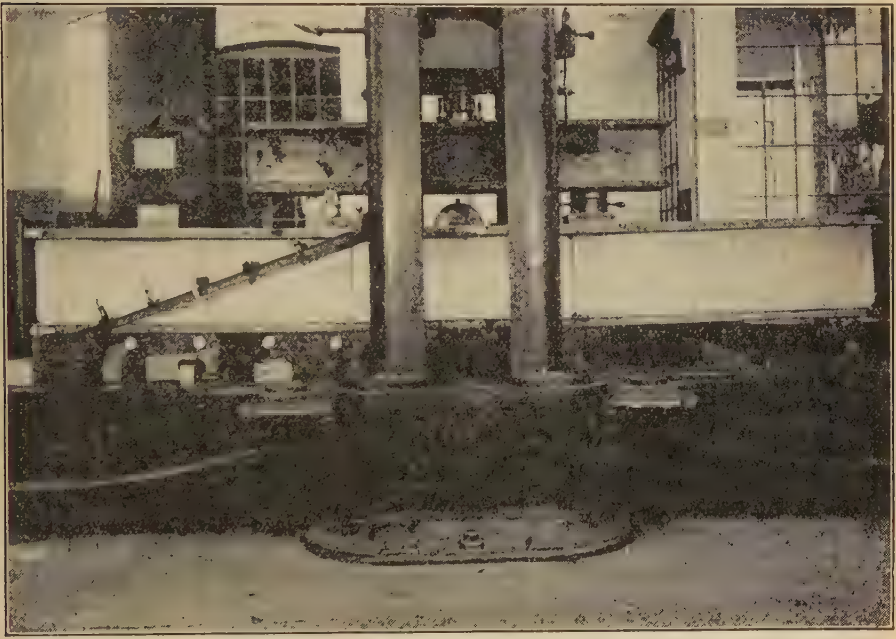


Fig. 10.—Welded Beam, *WB*-1, Showing Web Failure Due to Shear

Beam *WB*-4 was a rolled Bethlehem B28-91 section, the make-up of which is shown in Fig. 7. This beam had plate stiffeners welded to the web at points of support and loading and also had riveted angle stiffeners in one of the end panels. In order to prevent flexural failure, cover-plates that extended to within a short distance of the supports, were welded to the flanges.

Due to the size of this beam, it was necessary to use the 800 000-lb testing machine. As the load was applied the entire beam deflected sidewise to some extent until at a load of 475 000 lb that end of the beam which was supported by the spherical bearing-block twisted. The deflections showed that up to the load at which twisting occurred there was no indication of yielding.

Since the twisting of one end of the beam evidently was due to the loading arrangement, it was deemed advisable to re-test this beam under more favorable conditions. Consequently, lateral restraint for the ends of the beam, in the form of channels bolted to the supporting beam, was used in the re-testing. The friction between the upper flange of the beam and the channels was kept at a minimum by the use of rollers, which were also used at both supports in the re-testing. The vertical deflections obtained during the re-testing are shown in Fig. 14, from which the yield point of the beam was determined at the load of 560 000 lb. Set readings were observed at a load of 25 000 lb, and it was noted that the lateral set was very small in comparison to the lateral web deflection. This may be due either to the better restraint on the ends of the beam by the use of channels, or to the substitution of the roller for the spherical bearing-block. The beam began to twist at a load of 325 000 lb. This rotation continued with the increase in load until at the maximum of 597 600 lb the beam

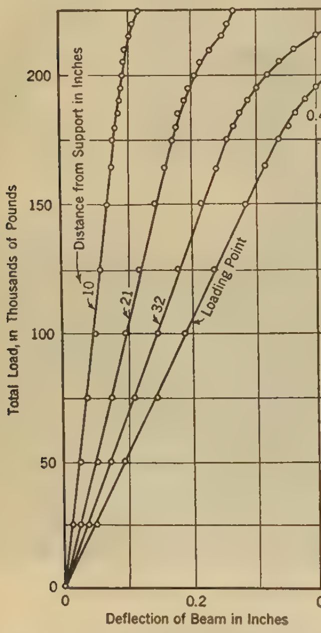


Fig. 11.—Deflection of 17-Inch Welded Beam *WB*-2 (Ultimate Load, 255 600 Pounds).

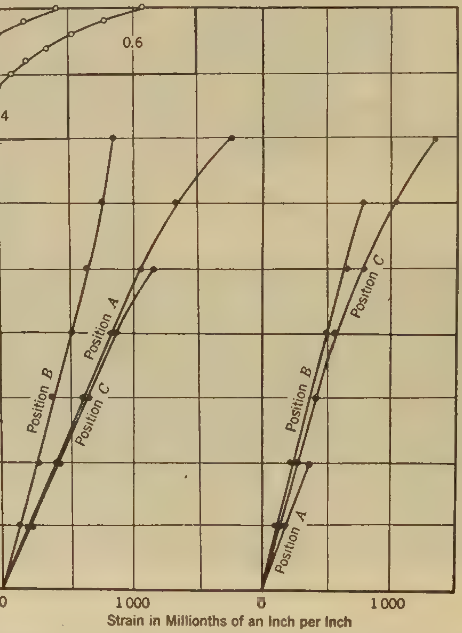


Fig. 12.—Average Tension and Compression Strains on Web of Beam *WB*-2

twisted so much that the entire loading rig was out of position. At the maximum load, the web did not scale except on the stiffeners and along the junction between the web and the flange.

At the yield-point load the computed maximum shearing stress in the web was 24 200 lb per sq in. in the section between the support and the cover-plates on the flanges, and 21 130 lb per sq in. in the section having cover-plates. The yield-point stress in shear obtained from tests of coupons was 22 000 lb per sq in. For this beam also, the shearing yield-point stress obtained from the material agreed very well with the shearing stress computed for the web at the yield point of the beam.

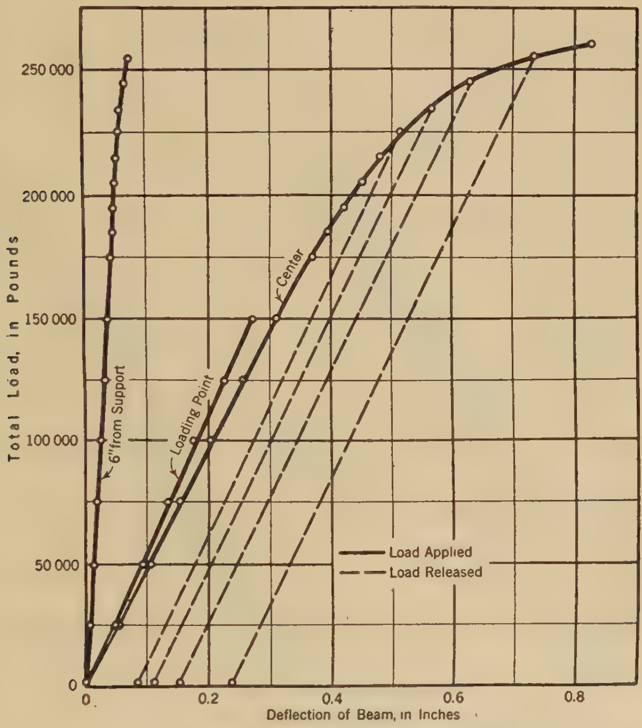


FIG. 13.—DEFLECTION OF 19-INCH WELDED BEAM *WB*-3 (ULTIMATE LOAD, 278 000 POUNDS)

Beam *WB*-5 was a 22-in., 62-lb I-beam reinforced with plate stiffeners and cover-plates, as shown in Fig. 7. The first appearance of scaling on the web was discovered at a load of 50 000 lb. As the load was increased, the scaling increased, especially at the stiffeners. The strain lines on the web were primarily horizontal, indicating shearing stress. Since the scaling had no relation to the yield point of the beam, it may have been caused by high internal strains in the material. The vertical deflections and sets are given in Fig. 15, from which the yield point of the beam was determined at a load of 420 000 lb. The lateral deflections increased until a load of 275 000 lb was reached. For greater loads there was a tendency for the web to return to its original position. The beam continued to take load until a maximum of 450 000 lb was reached, at which time it continued to deflect vertically without any increase in the load. It showed no indication of web buckling, even at the maximum load when most of the whitewash had flaked off in the parts between the support and the loading point.

At the section of the web between the support and the cover-plate, the computed maximum shearing stress at the yield point of the beam was 25 700 lb per sq in. and 23 250 lb per sq in. in the section having cover-plates. These values are somewhat less than the shearing yield-point stress of 29 200 lb per sq in. for the coupons, which was considerably greater than any of the values obtained from coupons of the other beams. The latter showed shearing yield-point stresses between 22 000 and 26 470 lb per sq in. It seems, therefore, that the value of 29 200 lb per sq in, may be somewhat in error. For Beams *WB*-1 to *WB*-4 the ratio between the yield point in shear and that in tension varied between 0.506 and 0.534. However, the

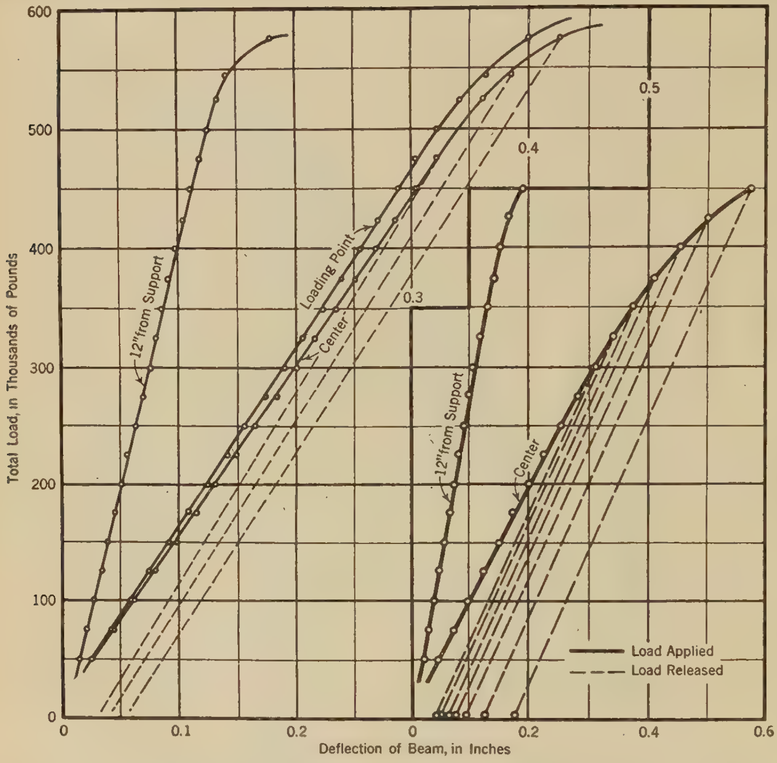


FIG. 14.—DEFLECTION OF A REINFORCED B-22-INCH, 62-POUND, BETHLEHEM I-BEAM, *WB*-5 (ULTIMATE LOAD, 450 000 POUNDS)

FIG. 15.—DEFLECTION OF A REINFORCED B-22-INCH, 62-POUND, BETHLEHEM I-BEAM, *WB*-5 (ULTIMATE LOAD, 450 000 POUNDS)

ratio obtained for Beam *WB*-5 was 0.569, which is considerably more than the other ratios. Granting that the yield-point stress of the coupons was in error, Beam *WB*-5 also showed a fair agreement between the computed maximum shearing stress at the yield point of the beam and the shearing yield-point stress of the material.

As seen from Table 1(*b*), the computed maximum fiber stress in the flanges at the yield point of the beams in this group was only 21 700 lb per sq in. This is so much less than the yield point of the material, which had for a minimum 43 500 lb per sq in., that it is quite evident that the beams did not yield in flexure.

MAJOR INVESTIGATION, GROUP *B*

Group *B* consisted of five welded beams (*WB*-6 to *WB*-10, Table 1(*c*)), all of which were whitewashed before being tested in the 300 000-lb machine. The load was applied to the center of the beam through a spherical bearing-block, and rollers were used at both supports. To prevent end twisting, plates were welded to each end section of the beams, as was done in Group *A*. Plate stiffeners were welded on the web at the supports and loading points so that local failure would not occur. The make-up and loading arrangement of these beams are shown in Fig. 16. Vertical and lateral deflections were observed, and strain-gauge readings were taken on various parts of the beams by a 10-in. Whittemore strain-gauge.

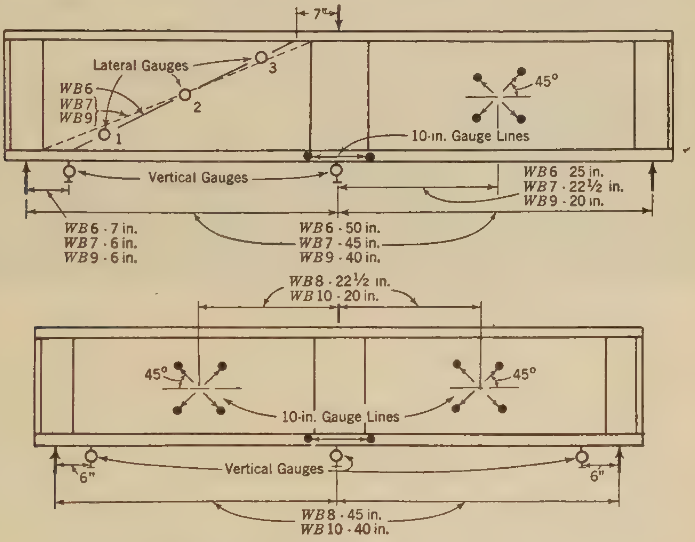

7"
Lateral Gauges
3
2
1
WB6
WB7
WB9
45°
10-in. Gauge Lines
Vertical Gauges
WB6 25 in.
WB7 · 22½ in.
WB9 · 20 in.
WB6 · 7 in.
WB7 · 6 in.
WB9 · 6 in.
WB6 · 50 in.
WB7 · 45 in.
WB9 · 40 in.
WB8 · 22½ in.
WB10 · 20 in.
45°
10-in. Gauge Lines
45°
Vertical Gauges
6"
6"
WB8 · 45 in.
WB10 · 40 in.

FIG. 16.—MAKE-UP AND LOADING ARRANGEMENT FOR MAJOR INVESTIGATION, GROUP *B* BEAMS

Table 1(*c*) gives the properties of these beams based on actual dimensions. It is to be noted that the $\frac{h}{t}$ ratios ranged from 49.4 to 70.0. The mild steel plates used for the webs were found to be very ductile for all the beams in this group. Consequently, the lever of the testing machine dropped decidedly at a load that was taken as the yield point of the beam.

Beam *WB*-6 had a web ratio of 70. The lateral gauges were placed along the web in one of the panels, and strain-gauge readings were taken in the center of the other panel. The strain-gauge lines were at 45° with the horizontal so that both compressive and tensile strains were measured. Strain-gauge readings were also taken in the center of the bottom flange, in order to determine the maximum flexural stresses developed. The position of the gauges and strain-gauge holes are shown in Fig. 16.

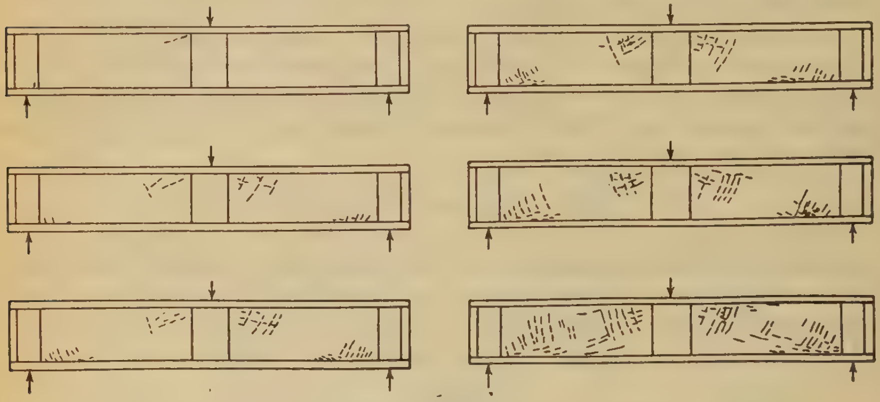


Fig. 17.—General Formation of Strain Lines on Web of Beams

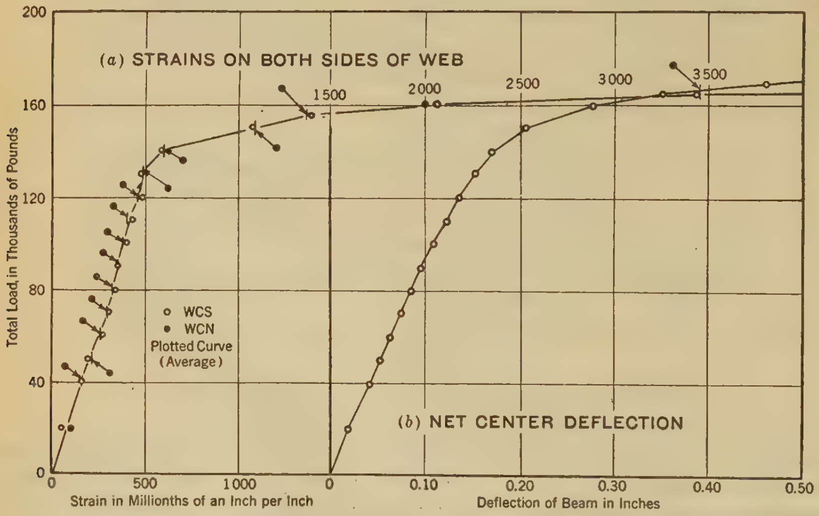


Fig. 18.—Compressive Strains and Net Center Deflections, Beam *WB*-6 (Ultimate Load, 192 600 Pounds)

Flaking occurred first at a load of 24 000 lb, in the form of vertical lines, at the top corner of the web below the loading point. As the load increased, approximately vertical and horizontal strain lines extended across the web. At a load of 50 000 lb, strain lines appeared in the web near one of the supports. A gradual increase in the strain lines followed an increase in load. The general formation of strain lines as they appeared on the beams is shown in Fig. 17.

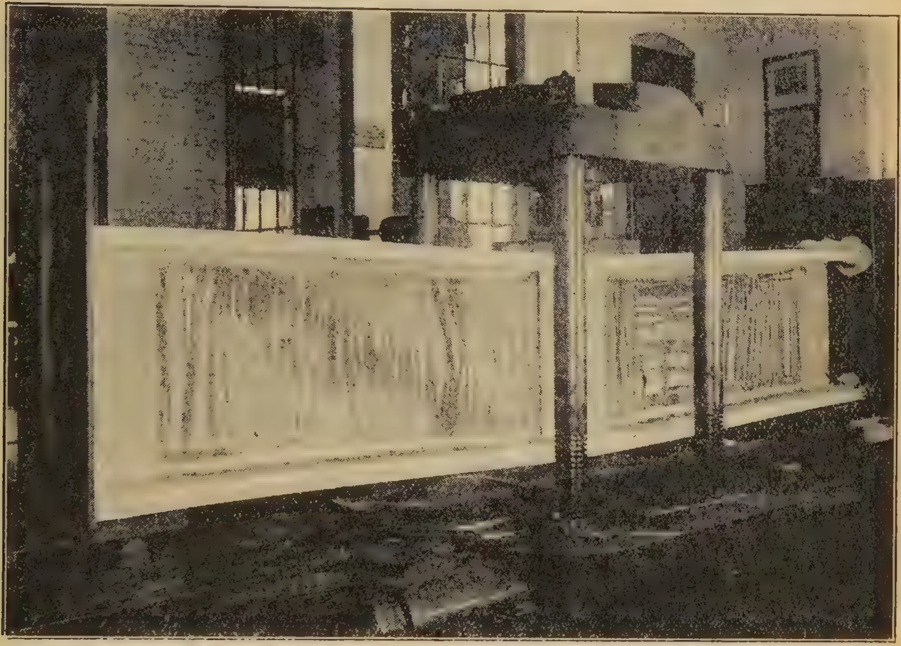


Fig. 19.—Appearance of Beam *WB*-6 at the Maximum Load of 192 600 Pounds

Net deflections (difference between the center and end deflections) of Beam *WB*-6 are shown in Fig. 18(*b*), from which it is noted that an increase in the rate of deflection occurred at a load of 140 000 lb. At a load of 150 000 lb, the rate of deflection increased very sharply, and a decided drop of the lever was observed at a load of 157 600 lb. No buckling was observed at this load, and the maximum lateral web deflection was only about 0.008 in. at a load of 150 000 lb; it decreased with further increase in load. The strains observed for the flange indicated that the stress in the flange was very low at the yield point of the beam. The tensile strains in the web increased regularly until a load of 120 000 lb was reached. At 140 000 lb there was a decided increase in the tension strains. The compressive strains in the web are shown in Fig. 18(*a*), in which the strains on both sides of the web have been plotted separately in order to bring out the buckling behavior. It was found that the strains on each side coincide almost exactly throughout the test, indicating that no buckling took place within the loads for which observations were taken. Had buckling occurred, the strains on one side of the web would have increased much faster than those on the opposite side, due to the bending effect.

Fig. 18(*a*) is, therefore, a good illustration of the fact that no buckling took place at the yield point of the beam. The beam continued to take load until a maximum of 192 600 lb was attained. With further motion of the head of the testing machine the web in one of the panels buckled considerably, as shown in Fig. 19. Table 1(*c*) shows a close agreement between the shearing stress in the web at the drop of the lever and the yield-point stress of the material, indicating that the yielding of the beam was due to the yielding of the material in shear. The computed maximum shearing stress at the yield point of the beam was 16 800 lb per sq in. This value agrees with the yield-point stress in shear of 17 450 lb per sq in., as determined by the test coupons. It is noted that the maximum shearing stress is less than the total shear divided by the net area of the web. This is due to the unusually thick flanges on these beams, which tend to increase the moment of inertia of the beam relatively more than the statical moment.

Beam *WB*-7, of which the loading arrangement and make-up are shown in Fig. 16, had a web ratio of 60.6. The first strain lines appeared on the web near the loading point at a load of 20 000 lb. Nearly vertical and horizontal strain lines continued to appear in the usual manner with an increase in the load. The net deflection curve for Beam *WB*-7 is shown in Fig. 20(*b*). At a load of approximately 130 000 lb, an increase in the rate of deflection was noted and, at a load of 148 200 lb, the lever dropped decidedly. The lateral deflection reached a maximum of about 0.025 in. at a load of 150 000 lb. A decided increase in the tension strains was noted at a load of 120 000 lb, while the flange strains showed no indication of yielding at the yield point of the beam. The compressive strains are shown

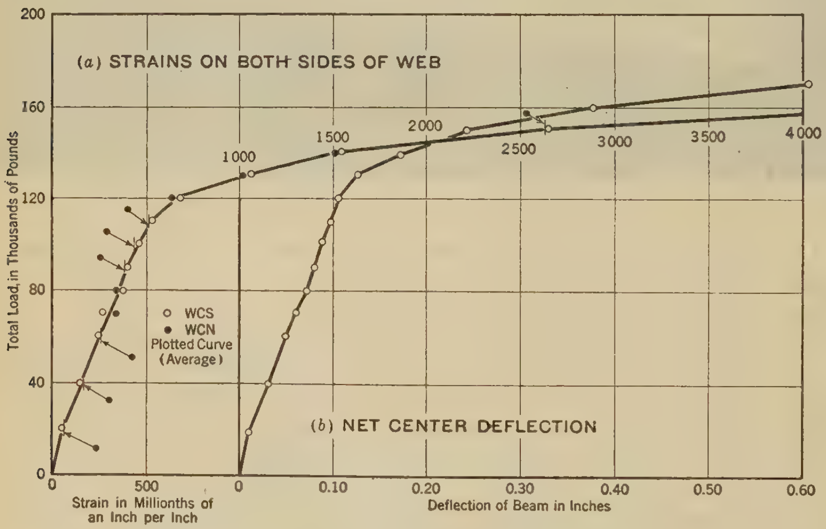


FIG. 20.—COMPRESSIVE STRAINS AND NET CENTER DEFLECTIONS, BEAM *WB*-7 (ULTIMATE LOAD, 190 100 POUNDS).

in Fig. 20(*a*). They were almost the same on both sides of the web throughout the test, indicating that no buckling occurred. A sharp increase in the compressive strains is noted at a load of 120 000 lb, which agrees with the behavior of the tensile strains. The beam continued to take load until one of the web panels began to buckle at a maximum load of 190 100 lb. The computed stresses are given in Table 1(*c*). The maximum shearing stress in the web at the drop of the lever was 17 800 lb per sq in., which agrees with the yield point in shear of 18 600 lb per sq in., as determined from the coupons.

Beam *WB*-8 was a companion of Beam *WB*-7 and had a web ratio of 59.7. Instead of lateral gauges being placed along one of the web panels, strain-gauge readings were taken in both panels to determine whether the load was evenly distributed on both sides of the loading point. The loading arrangement and make-up are shown in Fig. 16. This beam behaved similarly to Beam *WB*-7 throughout the loading, and the strain curves indicate a fairly even distribution of load. An increase in the rate of deflection occurred at a load of 130 000 lb, and a pronounced drop of the lever took place at a load of 143 100 lb. The tensile strains for both panels showed an increase in strain at a load of 120 000 lb. The compressive strains for both sides of the web also indicate that buckling did not take place within the range of loading for which observations were made. The beam continued to take load after the yield point was reached, until a buckle in both panels began at a maximum load of 199 500 lb. The results, as given in Table 1(*c*), show maximum shearing stress in the web of 16 250 lb per sq in. at the drop of the lever. This value is somewhat less than that obtained for Beam *WB*-7 and also less than the yield-point stress in shear as found by the test coupons.

Beam *WB*-9, the loading arrangement and make-up of which are shown in Fig. 16, had a web ratio of 50. Strain lines were visible on the web near the loading point at a load of 20 000 lb. Both vertical and horizontal strain lines appeared with an increase in load in the same manner as described for the other beams in this group. The deflection curves showed an increase in the rate of deflection at a load of 100 000 lb, and a decided drop of the lever took place at a load of 118 900 lb. A maximum lateral web deflection of 0.0045 in. was observed at a load of 130 000 lb, which indicates that buckling had not occurred up to that point. A decided increase in the web strains occurred at a load of 90 000 lb. The compression strains for both sides of the web indicate that at loads greater than the yield point of the beam they are slightly greater on the north side than on the south side. The maximum load carried was 184 000 lb, at which both web panels buckled. The results which are given in Table 1(*c*), show a good agreement between the shearing stresses in the web at the drop of the lever and the yield-point stress of the material. The maximum shearing stress was 17 250 lb per sq in., and the plate coupons gave 18 480 lb per sq in.

Beam *WB*-10 (Table 1(*c*)) was a companion to Beam *WB*-9. The first strain lines appeared on the web at a load of 20 000 lb near the loading point. Vertical and horizontal strain lines appeared in the usual manner as the load was increased. The deflection curve showed an increase in the rate of deflection at a load of about 120 000 lb, and a drop of the lever occurred at a load of 121 800 lb. Both tensile and compressive strains in the web showed an increase in the rate of strain at a load of 100 000 lb. The agreement of the strains on both sides of the web indicated that buckling did not occur. The beam continued to take load until a maximum of 188 000 lb had been reached. The recults, which are given in Table 1(*c*), show a good agreement between this beam and its companion, Beam *WB*-9. The computed shearing stresses at the drop of the lever check the yield-point stress in shear as found by the test coupons.

In all these beams the maximum stresses in the flanges at the drop of the lever were less than one-half the yield-point stress in tension of the material.

EFFECT OF $h:t$ RATIO

In order to study the relation between the $\frac{h}{t}$ ratio and the shearing stresses developed, Fig. 21 was prepared. The beams in Group *B* were the only ones that lent themselves to such a study. It is noted that the shearing stress at the drop of the lever of the testing machine was very nearly the same for all beams, regardless of their slenderness ratios. This indicates that up to an $\frac{h}{t}$ ratio of 70 the yield-point stress in shear of the web material determines the useful load-carrying capacity of the beam. For $\frac{h}{t}$ ratios of 70, or less, there seems to be no reason for designing beams on the basis of web buckling.

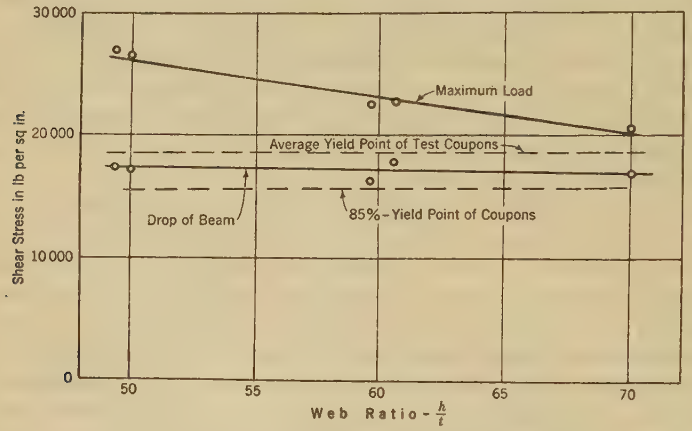


FIG. 21.—COMPUTED MAXIMUM SHEARING STRESS IN WEB OF BEAMS AND YIELD-POINT STRESS IN SHEAR OF TEST COUPONS

The shearing stress at maximum load is also shown in Fig. 21. The stress developed in the web is greater for beams having a low slenderness ratio than for beams having a high ratio. The maximum shearing stress decreased quite regularly with the increase in the $\frac{h}{t}$ ratios, indicating that the toughness of the beam decreased with the increase in the ratio.

If the web had buckled in any of the beams, the maximum load would have been equal to, or less than, the yield-point load. The trend of the relationship indicates that buckling of the web might occur at an $\frac{h}{t}$ ratio of about 80. This value compares very favorably with Professor Timoshenko's theoretical value of 90 for an assumed yield-point stress in shear of 20 000 lb per sq in. for structural steel.

SUMMARY

Although the number of beams tested in this investigation was too small on which to base final conclusions, the results obtained indicated that:

1.—The beams with free ends did not develop the full yield-point strength of the material, either in shear or in tension, due to failure in end twisting.

2.—At the yield point of all the beams which had plates welded to the ends, the computed maximum shearing stress in the web corresponded very well with the yield point in shear of the material.

3.—No beam showed any evidence of buckling at, or below, its yield point, indicating that with $\frac{h}{t}$ ratios up to 70 there is no danger of web buckling. Buckling may be expected to occur at $\frac{h}{t}$ ratios of 80 or more.

4.—The yield point of the beam, rather than the maximum load, should be used as a criterion for the factor of safety. In general, the average shearing stress in the web should be based on net area; that is, on $h\ t$, rather than on gross area, $D\ t$.

5.—The first appearance of strain lines had no relation to the yield point of the beam.

6.—The yield point of the beam was not affected by the $\frac{h}{t}$ ratio of the web. The maximum load, however, decreased with an increase in the $\frac{h}{t}$ ratio.

7.—The shearing yield-point stress of the material was the important factor for all the beams included in this investigation.

DISCUSSION

R. L. MOORE,[5] ESQ., and E. C. HARTMANN,[6] JUN. AM. SOC. C E. (by letter).—Web failures in beams may occur in any one of the following ways: (1) Yielding of the web material; (2) shear or diagonal compression buckling; and (3) buckling at points of load concentrations. The tests reported by Messrs. Lyse and Godfrey, on steel beams with depth-thickness ratios of 70 or less, involved principally the first type of failure. While their observations are a valuable contribution to present knowledge of web behavior, the investigation can scarcely be considered as one dealing with the interesting phenomena of web buckling and its related problems.

This discussion contains a brief description of a series of tests on aluminum alloy **I**-beams and girders involving web failures of each of the aforementioned types conducted at the laboratories where the writers are employed. Table 2 is a summary of the results obtained on several different sizes of rolled and extruded **I**-beams of the duralumin type of alloy known as 17S-T. The following are the nominal mechanical properties of this material: Tensile strength, 58 000 lb per sq in.; yield strength, 35 000 lb per sq in.; elongation in 2 in., 20%; modulus of elasticity, 10 000 000 lb per sq in.; shear strength, 35 000 lb per sq in.; and shear yield strength, 20 000 lb per sq in.

All the beams in Table 2 failed in the web. In Tests Nos. 1, 2, 7, 8, and 9, the web failed by buckling as a column over the reaction. In Test No. 10, failure occurred by buckling as a column under the load point. In Tests Nos. 3, 4, 5, and 6, failure occurred by yielding of the material, complicated in the cases of Tests Nos. 3 and 4 by some column buckling of the web. None of the foregoing failures resulted from elastic instability in the web, permanent set being encountered in every case. This is not surprising, however, since the depth-thickness ratios ranged from about 20 for the 5-in. **I**-beams to about 25 for the 12-in. **I**-beams. According to the theory of buckling of flat plates in shear, elastic buckling in Alloy 17S-T becomes critical for depth-thickness ratios of 52, or more.[7]

It is interesting to note that most of the failures occurred by column buckling at points of load concentrations. Failures of this type seem to occur when the average compressive stress exceeds the column strength of the web. In computing this stress it has been found that the load or reaction may be considered uniformly distributed over an area equal to:

$$a = t\ (b + h) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

in which, in addition to the notation of the paper, b = effective length of bearing-block, in inches. This area should be considered as symmetrically placed with respect to the load or reaction and should be altered in the case of the latter if the overhang of the beam is shorter than $\frac{1}{2}(b + h)$. In beams of normal proportions, without stiffeners, column buckling of the web at points of concentration is more likely to occur than any other type of web failure. Messrs. Lyse and Godfrey prevented this behavior, except in the preliminary tests, by means of web stiffeners, end plates, and suitable bearing-blocks at points of concentrated load and reaction.

---

[5] Research Engr., Aluminum Research Laboratories, Aluminum Co. of America, New Kensington, Pa.

[6] Research Engr., Aluminum Co. of America, New Kensington, Pa.

[7] "Strength of Materials," by S. Timoshenko.

TABLE 2.—RESULTS OF TESTS ON ROLLED AND EXTRUDED ALUMINUM ALLOY I-BEAMS.

Test No.	Nominal moment of inertia, in inches ^{4}	Depth of beam, in inches	Web thickness, in inches	Span, in inches	Distance from load to nearest reaction, in inches	Load at failure, in pounds	Average computed shear stress at failure, in pounds per square inch	Stiffeners
1	12.3	5	0.210	12	6	55 000	26 200	None
2	12.3	5	0.210	24	12	43 400	20 600	None
3	12.3	5	0.220	48	4	43 000	35 700	None
4	12.3	5	0.220	48	8	38 500	29 100	None
5	12.3	5	0.220	48	4	77 000	64 200	Load and reaction
6	12.3	5	0.220	48	8	47 900	36 200	Load and reaction
7	36.1	7	0.260	48	7	53 800	25 300	None
8	50.0	8	0.275	48	8	59 400	22 500	None
9	229.4	12	0.435	72	12	124 000	19 800	None
10	229.4	12	0.430	72	12	138 200	22 400	Reaction

Test No. 5 (Table 2) is an interesting example of what may be accomplished by the use of stiffeners in increasing the web strength of beams. A load corresponding to an average computed web shear of 64 200 lb per sq in., almost double the nominal shearing ultimate stress, was carried without producing complete failure in the beam. In Test No. 3, without stiffeners, the average stress at failure was only 35 700 lb per sq in. A similar comparison is found in Test No. 4 and Test No. 6, although the strengthening effect of the stiffeners was not nearly so pronounced. Fig. 22 is a view of Specimens Nos. 5 and 6 showing the shearing distortion possible without actual rupture of the metal.

Values of ultimate strength and corresponding average shearing stresses in the web have been given in Table 2. No values have been given for the yield points of the beams because the load deflection curves break over too gradually to permit the selection of values which are consistently satisfactory. Theoretically, beams will begin to yield when either the web or flange stresses first exceed the elastic range of the material, and since the ordinary formulas of mechanics are quite accurate within the elastic range, the load corresponding to the first yielding of a beam can be calculated quite definitely if the stress-strain relations of the material are known in tension, compression, and shear. It should be pointed out in this connection that aluminum alloys have no pronounced yield point such as mild steel and, for this reason, the yielding occurs very gradually and the point of first yield does not represent a distinct change of behavior of a beam under load. The ultimate load-carrying capacity of a beam, as determined experimentally, is generally a much more definite value to use in comparison of beams which do not fail by elastic buckling and one which, in the opinion of the writers, is of considerable importance to the designer.

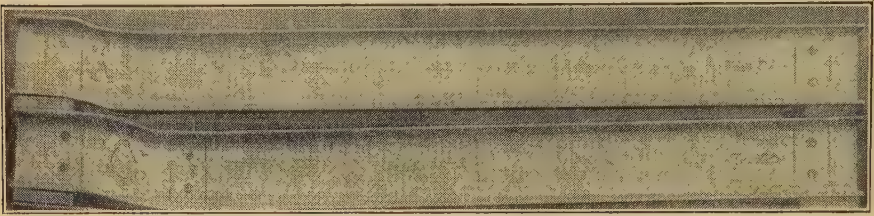


FIG. 22.—WEB FAILURES IN 5-INCH I-BEAMS (TESTS NOS. 5 AND 6, TABLE 2).

Figs. 23 and 24 show the distribution of stress in the 12-in. I-beam used in Test No. 10. The stress determinations were made by a 2-in. Berry strain-gauge, all measurements on the web being taken on rosettes (four gauge lines intersecting at a point) spaced on $2\frac{1}{2}$-in. centers over the area adjacent to the load and reaction point. All readings were taken on both sides of the web and averaged. In view of the recommendation of Messrs. Lyse and Godfrey that the average web shear be computed on the basis of the net area of the web (h t), attention is called to Fig. 24(b). The agreement there between the measured stresses and the average shear on the gross area (D t) is very

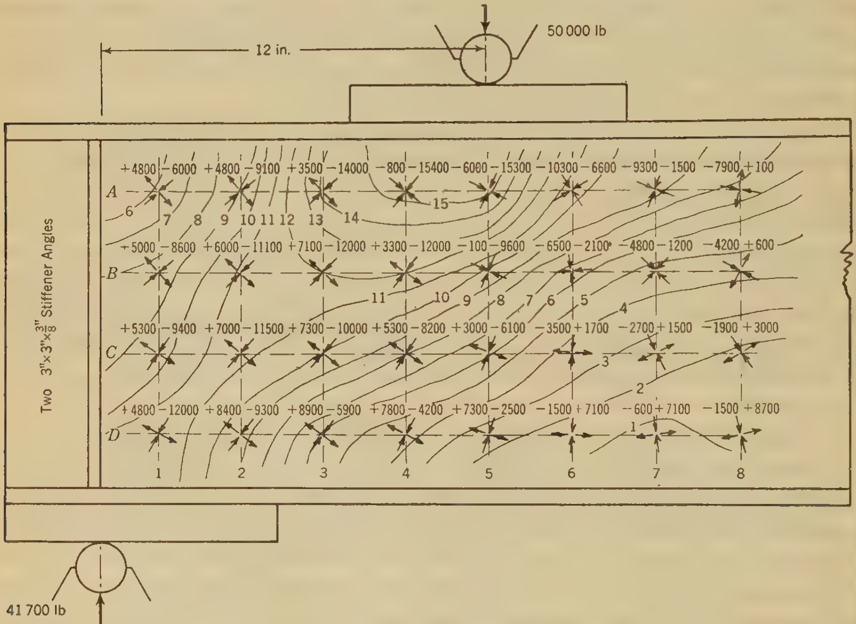
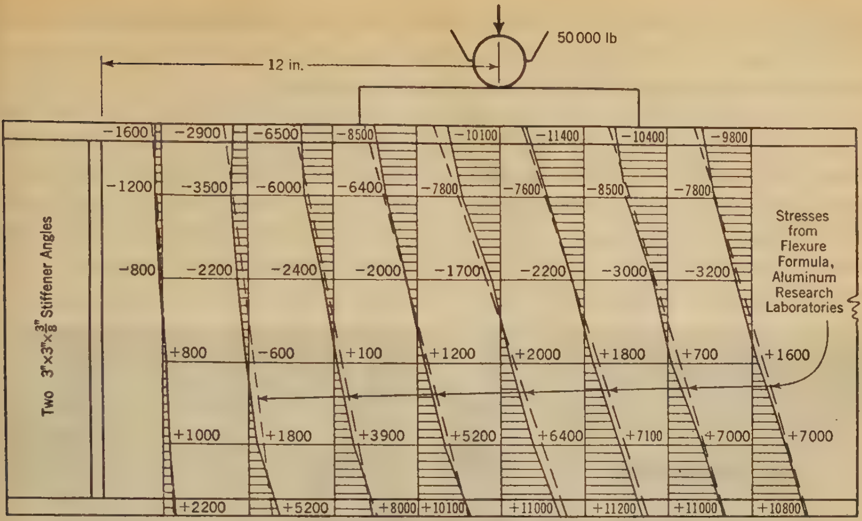
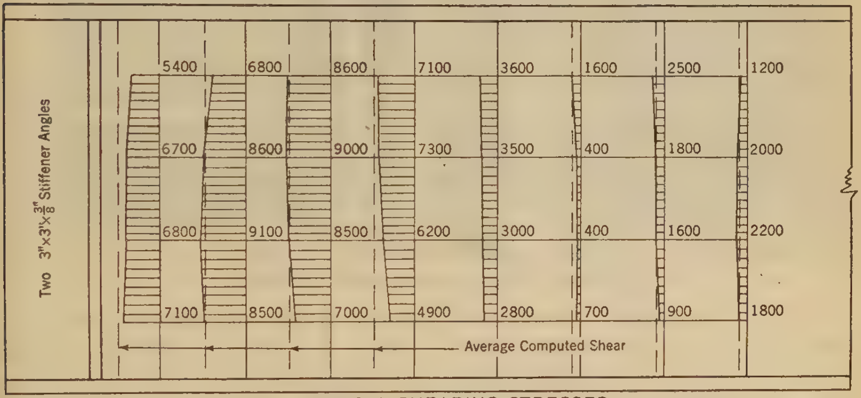

50000 lb
12 in.
Two 3"×3"×$\frac{3}{8}$" Stiffener Angles
+4800 −6000 +4800 −9100 +3500 −14000 −800 −15400 −6000 −15300 −10300 −6600 −9300 −1500 −7900 +100
A
6
7
8
9
10
11
12
13
14
15
+5000 −8600 +6000 −11100 +7100 −12000 +3300 −12000 −100 −9600 −6500 −2100 −4800 −1200 −4200 +600
B
11
10
9
8
7
6
5
4
+5300 −9400 +7000 −11500 +7300 −10000 +5300 −8200 +3000 −6100 −3500 +1700 −2700 +1500 −1900 +3000
C
3
2
+4800 −12000 +8400 −9300 +8900 −5900 +7800 −4200 +7300 −2500 −1500 +7100 −600 +7100 −1500 +8700
D
1
1
2
3
4
5
6
7
8
41700 lb

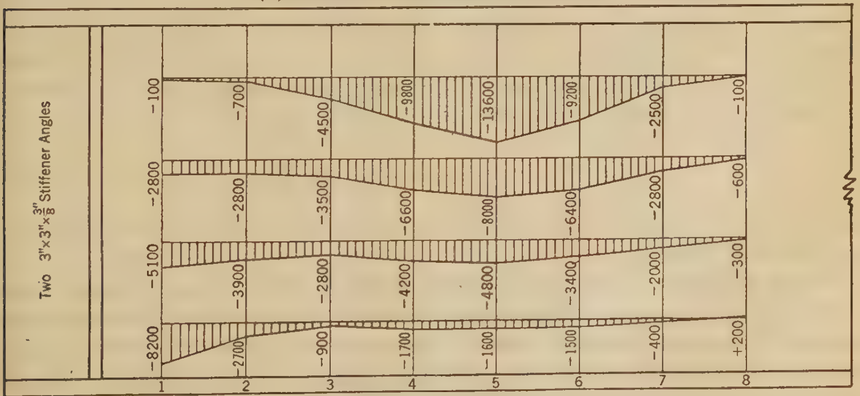
FIG. 23.—MEASURED PRINCIPAL STRESSES IN WEB WITH CONTOURS OF EQUAL COMPRESSIVE STRESS.


50 000 lb
12 in.
−1600
−2900
−6500
−8500
−10100
−11400
−10400
−9800
−1200
−3500
−6000
−6400
−7800
−7600
−8500
−7800
−800
−2200
−2400
−2000
−1700
−2200
−3000
−3200
+800
−600
+100
+1200
+2000
+1800
+700
+1600
+1000
+1800
+3900
+5200
+6400
+7100
+7000
+7000
+2200
+5200
+8000
+10100
+11000
+11200
+11000
+10800
Two 3"×3"×3/8" Stiffener Angles
Stresses from Flexure Formula, Aluminum Research Laboratories

(a) HORIZONTAL STRESSES


5400
6800
8600
7100
3600
1600
2500
1200
6700
8600
9000
7300
3500
400
1800
2000
6800
9100
8500
6200
3000
400
1600
2200
7100
8500
7000
4900
2800
700
900
1800
Average Computed Shear
Two 3"×3"×3/8" Stiffener Angles

(b) VERTICAL SHEARING STRESSES


−100
−700
−4500
−9800
−13600
−9200
−2500
−100
−2800
−2800
−3500
−6600
−8000
−6400
−2800
−600
−5100
−3900
−2800
−4200
−4800
−3400
−2000
−300
−8200
−2700
−900
−1700
−1600
−1500
−400
+200
1
2
3
4
5
6
7
8
Two 3"×3"×3/8" Stiffener Angles

(c) VERTICAL STRESSES

FIG. 24.—MEASURED STRESSES IN THE 12-INCH I-BEAM USED IN TEST NO. 10, TABLE 2 (SEE, ALSO, FIG. 23).

satisfactory. The use of net area seems even less justifiable when, as shown for the welded beams of the steel series, it gives average values greater than the maximum as computed by the rational formula.

Fig. 25 shows an aluminum alloy plate girder, 30 in. deep (24-in. clear depth) with a $\frac{1}{4}$-in. web, on which numerous tests have been made in connection with a study of elastic buckling phenomena. Strains and lateral deflections in the web were determined for different positions of load, sizes of bearing-block, and arrangements of vertical and horizontal stiffeners. While the results obtained are too comprehensive to be presented here in detail, it is believed a few observations will be of interest.

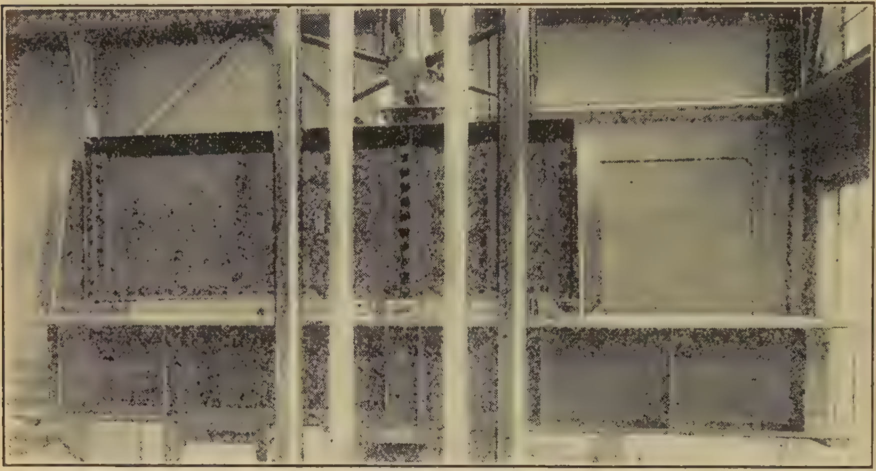


Fig. 25.—Test of Aluminum Alloy Plate Girder.

When no web stiffeners were used, failures of the web occurred by elastic buckling as a column directly under the load. It was found that the loading required to produce this type of failure could be predicted reasonably well by the same method outlined previously in connection with the discussion of Table 2. This same type of failure also occurred when stiffeners were used and the load was applied between the stiffeners. In such cases it was found, however, that in computing the average compressive stresses the load should be considered uniformly distributed over an area equal to:

$$t\,(b+h)\left[1+2\left(\frac{h}{s}\right)^2\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

in which, s = the clear distance between stiffeners, and the other terms are as previously defined.

When the web was properly stiffened at points of concentrated load, shear or diagonal compression buckling was obtained. Since the lateral deflections occurred gradually, however, almost from the first application of the load, it was difficult to select a point at which the buckling of the web might be said to be critical. In some instances, deflections of as much as $\frac{1}{4}$ in. were observed without permanent set. While this behavior made it impossible to obtain a precise experimental check on the theory of elastic buckling,[7] the writers believe the theory to be satisfactory for predicting loads at which the lateral deflections may become appreciable. It was quite evident, however, that the computed buckling strength of the web was not necessarily a criterion of its ultimate load-carrying capacity.

A double-web box girder of Alloy 17S-T was investigated for web buckling in much the same manner as previously described. The webs in this girder were 15 in. deep by $\frac{1}{16}$ in. thick, having a ratio of unsupported depth to thickness of 172. The results of the tests on this specimen tended to confirm those obtained on the plate girder. It was particularly noticeable in this case that the elastic buckling of the web was not a definite indication of the ultimate load-carrying capacity of the specimen. Even after relatively large lateral deflections were observed, the girder continued to carry higher loads with no evidence of permanent set or other distress. This is a point of considerable interest to designers because there is sometimes a tendency to over-emphasize the importance of elastic buckling. In structures in which the load-carrying capacity considerably exceeds the point of elastic buckling, the writers can see no reason why both factors should not be considered in design.

In concluding this discussion, the following points may be summarized:

1.—In normally proportioned beams, without stiffeners, column buckling of the web at points of load concentrations is more likely to occur than any other type of web failure.

2.—When stiffeners are used at points of concentrated load and reaction, the ultimate load-carrying capacity of a beam may exceed to a considerable extent the point at which yielding of the web material begins.

3.—Since there is no marked yield point in aluminum alloy beams the ultimate load-carrying capacity seems to be the most essential information to be obtained from beam tests in which no elastic buckling occurs.

4.—The measured distribution of stress in the web of a 12-in. I-beam indicates that the average shearing stress should be computed on the basis of gross area of the web rather than on net area, as recommended by Messrs. Lyse and Godfrey.

5.—The buckling of thin webs is a gradual rather than a sudden phenomenon so that it is difficult to obtain a precise experimental check on the theory of elastic buckling. However, the theory apparently provides a satisfactory means of determining the loads at which the lateral deflections may become appreciable.

INGE LYSE,[8] M. AM. SOC. C. E., AND H. J. GODFREY,[9] ESQ. (by letter).—The data on aluminum alloy beams presented by Messrs. Moore and Hartmann are valuable additions to the results obtained on steel beams by the writers. Unfortunately, the data on the aluminum beams give only the ultimate load with no indication of the load at which the beam would become useless due to excessive deflections. As already stated in the paper, the load at failure has no significance except that it is a measure of the toughness of a beam after it has passed its usefulness. In structural design, the load at which the deflection exceeds the maximum permitted by the usefulness of the building must be taken as the basis for estimating the factor of safety. Since all the aluminum alloy beams had depth-thickness ratios of only 25 and less, buckling of the web would not take place at stresses less than the yield-point stress of the material.

---

[8] Research Associate Prof. of, Eng. Materials, in Chg., Fritz Eng. Laboratory, Lehigh Univ., Bethlehem, Pa.

[9] Former Research Fellow in Civ. Eng., Lehigh Univ., Bethlehem, Pa.

No information is given as to the stress in the web at the yielding of the beam and, consequently, Messrs. Moore and Hartmann give little support to their criticism of the writers' recommendation that the average web shear be computed on the net area ($h\ t$) rather than on the gross area ($D\ t$). The observed stresses presented in Fig. 24(*b*) do not necessarily represent the stresses that cause yielding of the beam. H. M. Westergaard, M. Am. Soc. C. E., has shown that the concentration of shearing stress at the fillet between the web and the flange of the beam may become significant for beams having relatively small fillets. Fig. 21 also shows that the computed stresses at the yielding of the beam, instead of being greater (as they should be according to Messrs. Moore and Hartmann) are less than the yield-point stresses of the web material.

One-half the beams reported by Messrs. Moore and Hartmann failed by what has been termed web crippling. While web buckling is a failure produced by lateral instability of the web as a whole, web crippling is a local failure produced by excessive compressive stresses. An investigation of web crippling was also conducted at Lehigh University with the co-operation of the Bethlehem Steel Company and its subsidiary, the McClintic-Marshall Corporation, at the time of the tests of web buckling. The results were not included in the paper, in order to make a definite distinction between the two types of stress condition. They are presented herewith.

Web crippling is principally a local failure produced by excessive compressive stresses, and the problem of preventing its occurrence is most common in the design of structural beams supported on seat angles. The aim of the investigation was to establish the stress at which yielding due to web crippling took place.

The program included the testing of six 22-in., 58-lb, rolled, Bethlehem beams, four of which were cut from the same section and two from another section. The information for these beams which had an $\frac{h}{t}$ ratio of about 52, is given in Table 3. Three of the beams were designed to fail at the point of application of the load and the other three, at the supports. The nominal bearing lengths were 7 and 11 in. for the center failures, and $3\frac{1}{2}$ and $5\frac{1}{2}$ in. for the end failures.

In order to prevent failure due to end twisting, steel plates were welded to the bottom flange at each end of the beam. In each end plate, at a height even with the top flange, there was a slot which served as a guide for bars welded to the center of the top flange. This arrangement, as shown in Fig. 26, allowed the top flange to deflect vertically, but restrained it from moving laterally.

TABLE 3.—WEB CRIPPLING OF BEAMS WITH $\frac{h}{t} = 52$

Beam	Web thickness, in inches, t.	Bearing length, in inches, $A + 2N$	Bearing area, in square inches	Yield-point load, in pounds	Compressive yield-point stress, in pounds per square inch	Tensile yield-point stress of material, in pounds per square inch	Ultimate load, in pounds
CT-1.....	0.397	4.74	1.88	190 000	50 500	49 000	202 000
CT-2.....	0.397	8.24	3.27	160 000	49 000	49 000	223 000
CT-3.....	0.397	6.74	2.68	220 000*	41 000*	50 000	231 500*
CT-4.....	0.397	12.24	4.86	210 000*	43 200*	50 000	264 400*
CT-5.....	0.407	4.69	1.91	160 000	41 900	44 660	205 000
CT-6.....	0.407	8.17	3.33	150 000	45 000	44 660	209 500

* Doubtful, due to thin bearing plates.

All the beams were supported by rollers and loaded at the center of the span. A roller was also used for the application of the load for Beams Nos. 1 and 2, but a spherical bearing block was used for this purpose in the testing of the remaining four beams. Steel bearing plates of the proper length were used to transfer the load from the rollers and bearing-block to the beam. Vertical deflections and strain-gauge observations were taken on all beams. Except for Beam No. 1, all beams were whitewashed before testing.

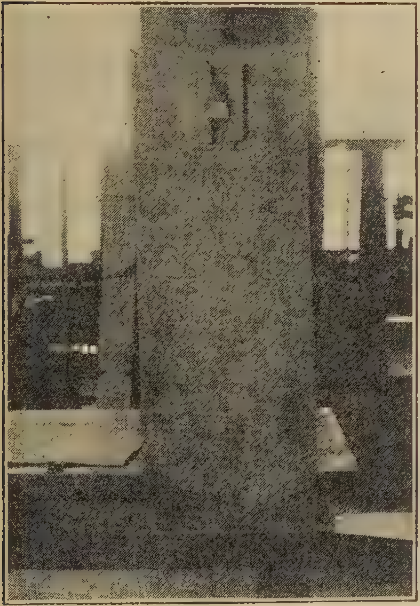


FIG. 26.—BEAM *CT*-5, SHOWING END PLATE FOR PREVENTION OF END TWISTING.

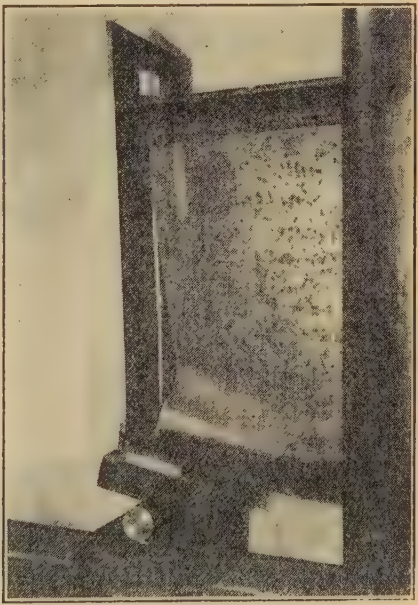


FIG. 27.—BEAM *CT*-1, SHOWING FAILURE AT THE SUPPORT DUE TO WEB CRIPPLING.

Strain lines in the web appeared at both supports at a load of 115 000 lb, and, at the center, at a load of 200 000 lb in Beam No. 1. At a load of 190 000 lb, the strains increased greatly, indicating that the actual yielding of the beam occurred. The strains observed at a point immediately above the flange indicate that the yield point of the material in the web was reached at considerably lower loads. However, this local yielding did not have any effect on the beam as a whole, which continued to take load until a maximum of 202 000 lb was reached, at which time the web crippled at the end, as shown in Fig. 27.

In computing the actual bearing length at the root of the web it was assumed that the stress was distributed through the flange on an angle of 45 degrees. The total bearing length thus equals the length of the bearing plate plus twice the thickness of the flange at the root of the web. The thickness of the flange was measured by micrometers at a point next to the fillet, as indicated in Table 3. The length of the bearing plate was $3\frac{1}{2}$ in. at the supports where the beam was designed to fail. The compressive stress at the root of the web at the yield point of the beam was 50 500 lb per sq in., which compares very well with the average tensile yield-point stress of 49 000 lb per sq in. for the web material. The full yield-point stress of the web was thus utilized in this beam.

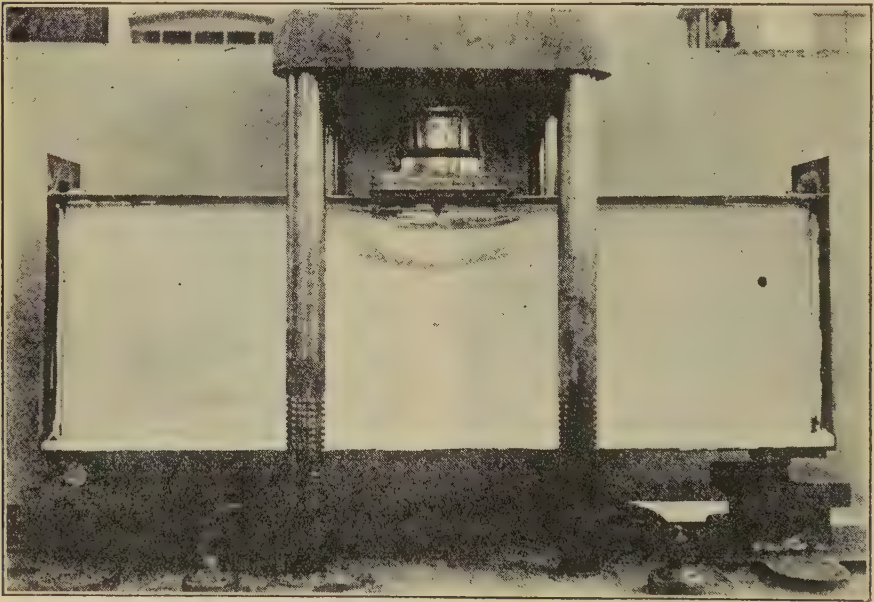


FIG. 28.—BEAM *CT*-4, SHOWING FAILURE AT THE CENTER DUE TO WEB CRIPPLING.

Beam No. 2 was designed to fail at the loading point where the length of the bearing plate was 7 in. Strain lines appeared at the supports at a load of 100 000 lb and at the loading point at 140 000 lb. Since the failure of this beam occurred at the loading point no definite indication of yielding could be obtained from the deflection curves. The strain-gauge results showed that the strains near the root of the web increased at a high rate above a load of 120 000 lb. The strains measured at the center of the web indicated a yielding at a load of 160 000 lb, which was taken as the yield-point load of the beam. The beam continued to take load until a maximum of 223 000 lb was reached, at which time the web crippled at the loading point.

At the yield point of the beam the maximum compressive stress at the root of the web was 49 000 lb per sq in., or the same as the tensile yield-point stress of the web material.

Beam No. 3 was designed to fail at the supports where the length of the bearing plates was $5\frac{1}{2}$ in. To prevent failure at the center, stiffeners were welded to both sides of the web at this point.

The first strain lines appeared in the web near the center at a load of 60 000 lb. The strain lines continued to form and, at a load of 110 000 lb, they appeared in the web at the supports. The observed strains indicated yielding at a load of 220 000 lb, which was taken as the yield point of the beam, while the deflection curves indicated yielding at a load of 230 000 lb. At the maximum load of 231 500 lb, the web crippled at one of the supports. The bearing plate used for this beam was too thin for effective distribution of the load from the roller to the flange of the beam and, consequently, contributed to a concentration of stress at the center of the plate, causing local failure at low average stress.

The maximum compressive stress at the root of the web was 41 000 lb per sq in. at the yield point of the beam. This is considerably less than the tensile yield-point stress of 50 000 lb per sq in. for the material in the web.

As Beam No. 4 was designed to fail at the center, stiffeners were welded to the web at the supports. The length of the bearing plate at the center of this beam was 11 in.

The first strain lines appeared in the web at the supports at a load of 90 000 lb and, at 120 000 lb, they appeared under the loading point. The observed strains indicated a yielding at a load of 210 000 lb which was taken as the yield-point load. The beam continued to take load until a maximum of 264 400 lb was reached, at which time the web crippled at the center of the beam, as shown in Fig. 28. Also, for this beam, the bearing plate was too thin for uniform distribution of the load.

At the yield-point load the maximum compressive stress at the root of the web was 43 200 lb per sq in., which is considerably less than the tensile yield-point stress of the web material.

Beam No. 5 was similar to Beam No. 1, having a bearing length of $3\frac{1}{2}$ in. at the supports where it was designed to fail. The first strain lines appeared in the web at the supports at a load of 60 000 lb, and at the loading point at a load of 130 000 lb. The vertical deflections, as well as the strains, showed that the beam yielded at a load of 160 000 lb. A slight crippling of the web at the support was noted at 190 000 lb, and, at a load of 205 000 lb, the web crippled.

At the yield-point load of 160 000 lb, the computed stress at the root of the web was 41 900 lb per sq in., which compares favorably with the yield-point stress of 44 660 lb per sq in. for the web material.

Beam No. 6 was similar to Beam No. 2, having a bearing length of 7 in. at the loading point. As this beam failed at the center, the vertical deflections indicated that there was no yielding of the beam as a whole. The strain curves, however, indicated a bending tendency in the web at a load of 80 000 lb. The rate of strains increased sharply at a load of 150 000 lb, which was taken as the yield point of the beam. The beam continued to take load until a maximum of 209 500 lb was reached and, at this load, the web crippled under the center.

At the yield-point load of the beam the maximum compressive stress at the root of the web was 45 000 lb per sq in., which is close to the yield-point stress of 44 660 lb per sq in. for the web material.

Both the flexural and shearing stresses were low in all these beams, so there was no indication of yielding due to these stresses.

Since only six beams were tested in this investigation, no general conclusions can be drawn. However, the results indicate that:

1.—The appearance of the first strain lines has no relation to the yielding of the beam as a whole, but is an indication of high local stress.

2.—The compressive stress at the root of the web at the yield point of the beam corresponds quite well with the tensile yield-point stress of the material for properly designed bearing plates.

3.—The yielding was caused by the compressive stresses at the root of the web.

4.—The following formula may be used in computing the compressive stress at the root of the web: $f_c = \frac{R}{t(A + 2N)}$, in which, f = compressive stress; R = load on bearing plate; t = thickness of web; A = length of bearing plate; and N = thickness of flange at edge of fillet.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 1908

ANALYSIS OF SHEET-PILE BULKHEADS

BY PAUL BAUMANN,[1] M. AM. SOC. C. E.

WITH DISCUSSION BY MESSRS. JACOB FELD, R. L. VAUGHN, M. A. DRUCKER, RAYMOND P. PENNOYER, D. P. KRYNINE, E. h. O. FRANZIUS, THEODOR VON KÁRMÁN, E. LOHMEYER, AND PAUL BAUMANN.

SYNOPSIS

The introduction of the deep-arch, steel, sheet-pile section in 1912 has gradually led to the construction of bulkheads of unprecedented height. During this development the designer has been confronted by serious difficulties because of the uncertainties as to the effective passive resistance of the ground and the interlock efficiency of the piles. Only since the end of the World War have large-scale experiments on these important questions been made in the United States and Europe, the outstanding ones on passive resistance of soil being those by Professor E. h. O. Franzius in the hydraulic laboratory of the Institute of Technology, Hanover, Germany.

The exploration of the phenomenon of passive resistance of soil still being in its infancy, the present design of bulkheads is based on the convenient assumption that passive resistance of soil may be represented by static forces that vary linearly with the depth, irrespective of the magnitude of movement of the wall. The elastic behavior of the soil is thereby ignored.

The object of this paper is: (1) To outline and to interpret the result of full-sized tests on a steel sheet-pile bulkhead, particularly in regard to the effective, passive resistance of sand and the interlock efficiency of deep-arch sheet-piling; (2) to offer suggestions pertaining to a possible stress distribution in the passive prism between the wall and the surface of rupture; and (3) to present a new theory of the stability of bulkheads, which takes into consideration the elasticity of both wall and soil and which treats the problem as statically indeterminate.

NOTE.—Published in March, 1934, *Proceedings*.

[1] Asst. Engr., in Chg., Mountain Div., Los Angeles County Flood Control Dist., Los Angeles, Calif.

INTRODUCTION

During the construction, in 1929-1930, of the Outer Harbor of the City of Long Beach, Calif., the steel, sheet-pile bulkhead failed in two places (see Fig. 1); that is, in 12 ft and 25 ft of water, more or less, respectively. As the City and the contractor disagreed on the cause of these failures, a suit was filed by the latter, which was tried in the Superior Court of Los Angeles County during the second half of 1932.

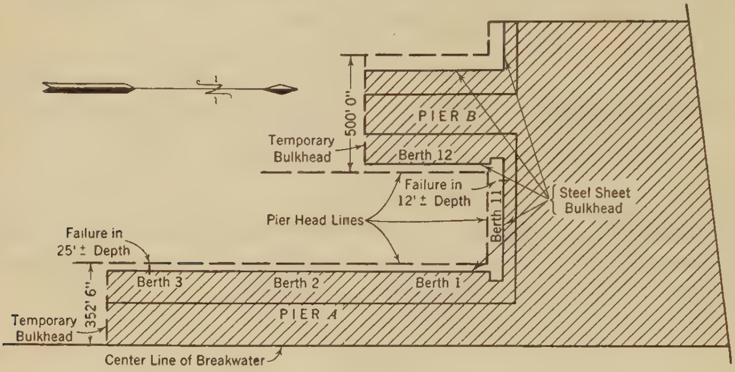


FIG. 1.

In substance the plaintiff claimed that the failure was due to faulty design and sea action on the toe slopes, while it was the defendant's contention that the failure was caused by faulty construction, particularly due to over-dredging the toe slopes and failure to place a preliminary fill behind the bulkhead.

At both places, the pushing out of the steel sheet-piling was directly attributable to the removal of material below the theoretical toe slope (Fig. 7) by whatever agency that caused it.

The toe slopes were undercut by a suction dredger as in ordinary side-slope work, which was a dangerous procedure in view of the importance of the toe material relative to the stability of the bulkheads.

On the other hand, a terrific storm occurred during the dredging operation, which demolished part of the break-water and caused considerable agitation in the harbor basin, and it was probably this evidence more than anything else that prompted a judgment substantially in favor of the contractor.

For the purpose of establishing facts in connection with this case the following tests were made: (1) Soil tests on samples from the fill area and from the channel; (2) tests to establish the depth of water standing in the fill area; (3) tests to establish the passive resistance of the fill; and (4) tests to establish the minimum interlock efficiency of steel sheet-piling.

SOIL TESTS

The soil tests were made in the City Laboratory in Long Beach, in accordance with the standards of the American Society for Testing Materials where such standards existed, and with methods adopted by the supervising engineers, Charles T. Leeds, M. Am. Soc. C. E., Frederick J. Converse, Assoc. M. Am. Soc. C. E., and the writer. These tests were concerned particularly with the determination of the angle of repose "in the dry" and under water.

For the determination of the angle of repose in the dry, a standard inverted 12-in. slump cone was set on a smooth steel plate and filled with dry material. The cone was then lifted slowly, permitting the material to escape at the bottom (smaller) opening and to form a cone. Without exception these cones were very regular, and the angle of repose differed little at various locations.

Three methods were investigated to determine the angle of repose under water. In this connection, a water-tight container, about 2 by 2 by 2 ft, with a glass front, was used.

With the first method, a dry cone was produced in the empty container which was then gradually flooded until it was completely submerged. Complete saturation was indicated when air bubbles no longer escaped, after which the water was siphoned off carefully and the angle of repose measured in at least three different places.

With the second method, saturated material was released through the large opening of a 6-in. slump cone into the filled container; the water was then siphoned off and the angle of repose measured as before.

Finally, with the third method, a 6-in. slump cone was set on the bottom of the empty container in an inverted position and was weighted down so as to make it water-tight at the bottom. The container was then filled to within $\frac{1}{2}$ in. from the top of the cone and the latter was filled with dry material. The cone was then raised slowly until all the material had escaped at the bottom and had formed a cone. When the water was siphoned off, the angle of repose was measured as before.

As the last method consistently gave smaller angles of repose than the first and second methods, only the results due to Method 3 were recorded.

Standing Water in the Fill Area.—For the purpose of establishing the standing water level, two holes, roughly 30 ft in diameter at the surface and with the bottom at mean lower low water, were excavated and the water-surface elevation was recorded three times daily for a period of several weeks. This work was done in the fall of 1930. The observations showed a very small variation of the water-surface elevation from + 3.50 at a distance of 50 to 75 ft from the bulkhead.

TESTS TO ESTABLISH THE PASSIVE RESISTANCE OF THE FILL

Test Equipment.—For the purpose of establishing the passive resistance of the fill, steel sheet-piling was driven in the fill area of Pier *B* (Fig. 1) so as to form a tank, as shown in Fig. 2.

The face of the tank, consisting of seven sheets, 28 ft long, and driven to a maximum of 12 ft into the ground, was free to deflect (fully 10 in.) within the yield of the copper seals. To each of three alternate sheets (see Fig. 2(*b*)), being convex to the outside, six $\frac{1}{2}$-in. threaded rods were brazed, 3 ft on centers, so as to have the bottom rods coincident with the ground surface. The copper seals were carried to a depth of 7 ft, more or less (minimum, 5 ft), below the surface.

At a vertical distance of 15 ft above the ground surface, four of the seven sheets forming the test wall were anchored to the back-wall by means of 1-in. rods, but walers were omitted to prevent interference with slipping along the interlock. As sea water was the medium that produced pressure, the interlocks of the test wall were made partly water-tight through oakum and mastic, the first being loosely placed, not caulked, and serving to keep the mastic from squeezing through the joints.

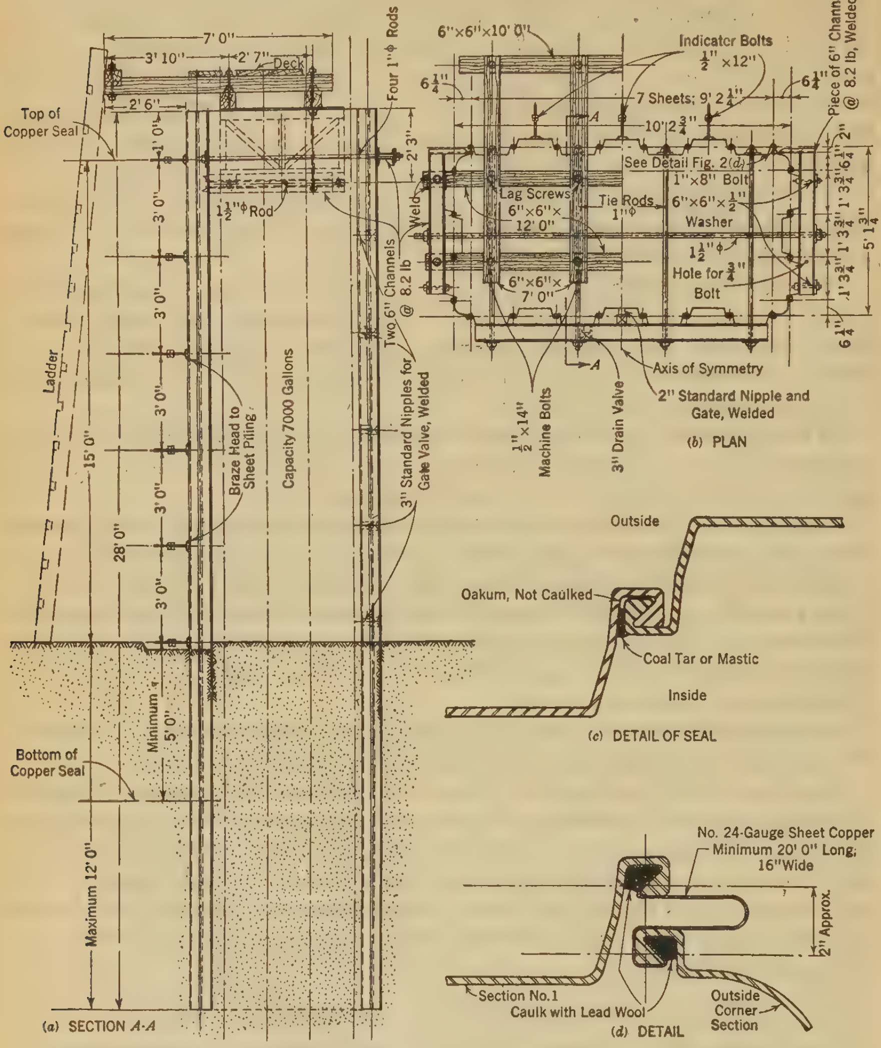

7' 0"
3' 10"
2' 7"
Deck
2' 6"
Top of Copper Seal
1' 0"
3' 0"
3' 0"
3' 0"
3' 0"
3' 0"
15' 0"
28' 0"
Ladder
1½"ϕ Rod
Braze Head to Sheet Piling
Capacity 7000 Gallons
Four 1"ϕ Rods
2' 3"
Weld
Two 6" Channels @ 8.2 lb
3" Standard Nipples for Gate Valve, Welded
Minimum 5' 0"
Bottom of Copper Seal
Maximum 12' 0"
(a) SECTION A-A
6"×6"×10' 0"
Indicator Bolts ½"×12"
6¼"
7 Sheets; 9' 2¼"
6¼"
Piece of 6" Channel @ 8.2 lb, Welded
A
10' 2¾"
See Detail Fig. 2(d)
1"×8" Bolt
Lag Screws
6"×6"×12' 0"
Tie Rods 1"ϕ
6"×6"×½" Washer
1½"ϕ
Hole for ¾" Bolt
6"×6"×7' 0"
2"
6¼"
1' 3¾"
1' 3¾"
1' 3¾"
6¼"
5' 1¾"
A
Axis of Symmetry
2" Standard Nipple and Gate, Welded
½"×14" Machine Bolts
3" Drain Valve
(b) PLAN
Outside
Oakum, Not Caulked
Coal Tar or Mastic
Inside
(c) DETAIL OF SEAL
No. 24-Gauge Sheet Copper Minimum 20' 0" Long; 16" Wide
2" Approx.
Section No. 1
Caulk with Lead Wool
Outside Corner Section
(d) DETAIL

FIG. 2.

Five 3-in. valves, 3 ft on centers, were welded to one of the sheets of the back-wall for the purpose of regulating the water surface during the test.

The deflections were measured by means of a transit set up 100 ft from the tank and sighting parallel to the test wall approximately flush with the free ends of the indicator rods. While the transit remained stationary, a nut was screwed on each indicator rod until its flat side was in line with the vertical hair of the transit. Each indicator nut was secured in its position by a lock nut. The distances on the indicator rods corresponding to the deflections were determined with a pair of dividers and were then sealed by means of a special rule reading to 0.01 in.

Prior to the test the fill in front and immediately adjacent to the tank was flooded with sea water and was kept wet during the tests. A test hole made with a 6-in. auger immediately before the final test showed the ground to be in a wet condition between the surface and the limit of capillary attraction which appeared to be about at Elevation + 4.50, or 1 ft above standing water.

Test Results.—The average sand in this location is believed to be closely represented by Sample *L* in Table 1; that is, the average specific weight was 2.70, and the percentage of voids, 50, which gives a weight per cubic foot of about 85 lb. The average moisture content in the sand was estimated at from 15 to 20% by weight, which means a 50% saturation, more or less. The effective weight of the material, offering passive resistance, was estimated at 100 lb per cu ft. The angle of internal friction (30°) was assumed to coincide with the angle of repose under water.

TABLE 1.—TESTS ON SAMPLES OF FILL MATERIAL

Sample	Weight per cubic foot, dry and rodded, in pounds	ANGLE OF REPOSE		Specific gravity	Percentage of voids
		Dry	Wet		
A.......	78.16	37°	30°, 29°, and 30°	2.70	50.72
B.......	101.56	35°	27°, 30° 30′, and 28°	2.70	39.75
C.......	102.42	35°	23°, 27°, and 26°	2.68	38.70
D.......	82.67	36°	30°, 30°, and 30°	2.72	51.25
E.......	78.05	37°	31°, 31°, and 31°	2.70	53.63
F.......	89.70	35°	27°, 30°, and 29°	2.70	46.71
G.......	76.52	39°	30°, 31°, and 30°	2.70	54.54
H.......	95.64	36°	30°, 28°, and 30°	2.70	43.20
I.......	89.85	36°	32°, 30°, and 32°	2.71	46.82
J.......	97.19	35°	31°, 34°, and 35°	2.60	40.05
K.......	87.65	36°	31°, 30°, and 31°	2.70	47.93
L.......	81.07	37°	31°, 30° 30′, and 31°	2.70	51.84
M.......	56.49	39°	35°, 36°, and 35°	2.62	65.41
N.......	75.79	37°	28°, 26°, and 27°	2.70	54.98

The active pressure below the ground surface is subject to estimation, since a theoretical analysis of such pressure is impossible, due particularly to the uncertainty of the hydrostatic pressure under the influence of percolation. The average fineness of the material at this location (Sample *L*, Fig. 3), permits a small percolation velocity, and the corresponding pressure on the sheet-piling was assumed to vary parabolically rather than linearly between the ground surface and the bottom of the sheet-piling, the latter coinciding approximately with standing water level. The parabolic distribution of pressure (in contradistinction to the usual assumpton of linear distribution with a zero lateral pressure at the bottom of the sheet-piling) is believed more nearly to represent the true condition due to the relatively short duration of the test. The influence on the stability of the structure of this assumption, as compared to the linear distribution, is small.

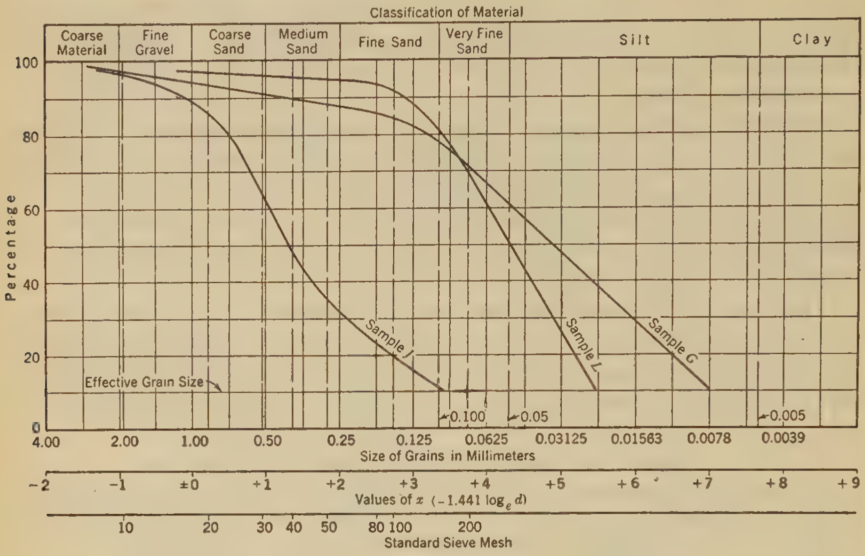


FIG. 3.

Under these assumptions the active forces are determined, the principal one being the water pressure between the upper support and the ground surface, which is known exactly. The active forces below the ground surface are not known exactly, but can be approximated with reasonable accuracy. The error that may be involved affects the ratio between the actual passive resistance and that based on Coulomb's formula, to a small degree.

DETERMINATION OF EFFECTIVENESS FACTOR OF PASSIVE RESISTANCE

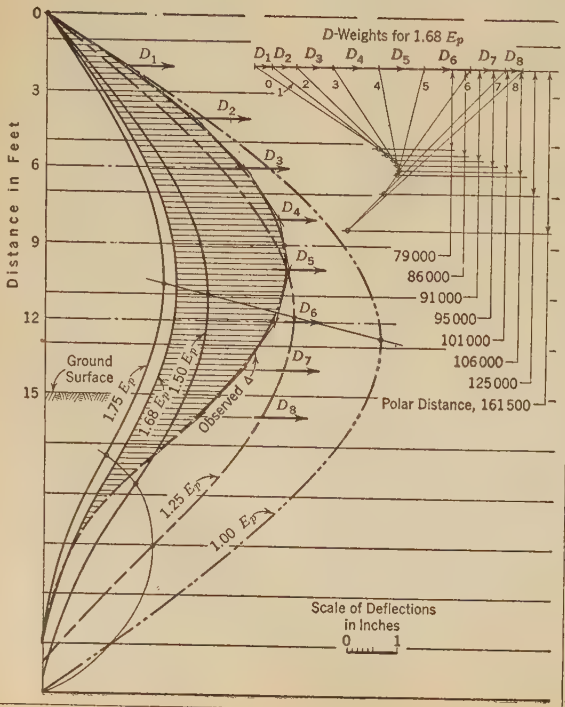
The equilibrium conditions were analyzed for passive resistances of 100, 125, 150, and 175%, respectively, of the one due to Coulomb's formula for a frictionless wall. The Coulomb formula is:

$$E_p = \frac{1}{2} \gamma t^2 \tan^2 \left(45 + \frac{\phi}{2}\right)$$

in which, E_p is the passive resistance of earth, in pounds; γ, the weight per cubic foot of sand; t, the penetration of sheet-piling, in feet; and ϕ, the angle of internal friction assumed to be equal to the angle of repose under water = 30 degrees.

The equilibrium conditions for each case were based on the interlocked section modulus, namely, $S = 9.30$ in.^{3} and a modulus of elasticity of $E = 29 \times 10^6$ lb per sq in. These elastic lines were finally plotted (see Table 2) to the same scale as the observed line, with the result that an effectiveness factor of 1.68 was found to represent the actual conditions most closely. The following values, per foot of wall, apply in the case of Table 2:

TABLE 2.—MOMENTS, DEFLECTIONS, AND EFFECTIVE SECTION MODULUS

Deflections	Moments, M, for $1.75E$, in foot-pounds	Unit Stress, $\frac{M}{S}$, in Pounds per Square Inch		Deflections, Δ, in Inches		Effective					Fiber stress f, in pounds per square inch
		Single sheet-pile	Interlocked sheet-piling	Observed	Computed	EI (percentage)	I (percentage)	E		S, in inches^3	
								(Percentage)	In pounds per square inch		
	3 200	9 200	4 130	0.68	0.33	39.50	40.00	99.00	28.75×10^6	3.72	10 350
	9 425	27 200	12 150	1.91	0.93	43.00	44.50	96.50	28.00×10^6	4.14	27 300
	15 000	43 250	19 350	3.00	1.44	45.50	50.00	91.25	26.50×10^6	4.65	38 700
	19 200	55 400	24 750	3.80	1.82	47.50	56.25	84.50	24.50×10^6	5.23	44 000
	21 600	62 350	27 900	4.16	2.03	50.50	62.25	81.00	23.50×10^6	5.80	44 700
	21 900	63 200	28 300	4.15	2.07	53.00	64.00	82.75	24.00×10^6	5.95	44 200
	19 200	55 400	24 750	3.74	1.93	62.50	67.25	93.00	27.00×10^6	6.28	36 700
	12 900	37 200	16 630	3.00	1.59	80.75	82.25	98.00	28.50×10^6	7.65	20 300
	3 300	9 525	4 260	......	1.16	(99)	(100)	(99)	28.75×10^6	(9.30)	(4 270)
	−8 220	23 700	10 600	......	0.70	......	......	......	......	......	......
	−16 200	46 750	20 900	......	0.33	......	......	......	......	......	......
	−17 700	51 100	22 800	......	0.09	......	......	......	......	......	......
	−8 220	23 700	10 600	......	−0.05	......	......	......	......	......	......
	±0	±0	±0	......	−0.09	......	......	......	......	......	......

Distance in Feet
0
3
6
9
12
15
D-Weights for 1.68 E_p
D_1 D_2 D_3 D_4 D_5 D_6 D_7 D_8
0 1 2 3 4 5 6 7 8
79 000
86 000
91 000
95 000
101 000
106 000
125 000
Polar Distance, 161 500
Ground Surface
1.75 E_p
1.68 E_p
1.50 E_p
Observed Δ
1.25 E_p
1.00 E_p
Scale of Deflections in Inches
0 1

For a single sheet-pile,

$$S_{x\text{-}x} = 4.17 \text{ in.}^3$$
$$I_{x\text{-}x} = 2.94 \times 4.17 = 12.25 \text{ in.}^4$$
$$EI_{x\text{-}x} = 12.25 \times 29 \times 10^6 = 355 \times 10^6 \text{ in}^2\text{-lb.}$$

For interlocked sheet-piling,

$$S_{n\text{-}n} = 9.30 \text{ in.}^3$$
$$I_{n\text{-}n} = 2.94 \times 9.30 = 27.35 \text{ in.}^4$$
$$EI_{n\text{-}n} = 27.35 \times 29 \times 10^6 = 793 \times 10^6 \text{ in}^2\text{-lb.}$$

The interlock efficiency—that is, the effective moment of inertia of the wall about the axis of symmetry—was found by interpolating the D-weights for 1.68 E_p and by determining the polar distances of the string polygon (Table 2), which coincides with the observed deflection curve. The effective index of rigidity, $E\,I$, in percentage of its maximum value for $E = 29 \times 10^6$ lb per sq in. and $I = 27.35$ in.^{4}, is found to be:

$$E\,I = \frac{\text{Polar distance} \times 100}{H_{\text{max.}}} \quad \text{.......................(1)}$$

and the effective section modulus was found by trial and error; that is, by determining corresponding E-values which, in turn, will check the stress-strain diagram of the average of seven samples as shown in Fig. 4. The moments for 1.68 E_p were not interpolated, but those for 1.75 E_p were used instead. As these latter moments are slightly smaller than the first ones, the result is in keeping with the purpose of the test, namely, to establish minimum values of interlock efficiency.

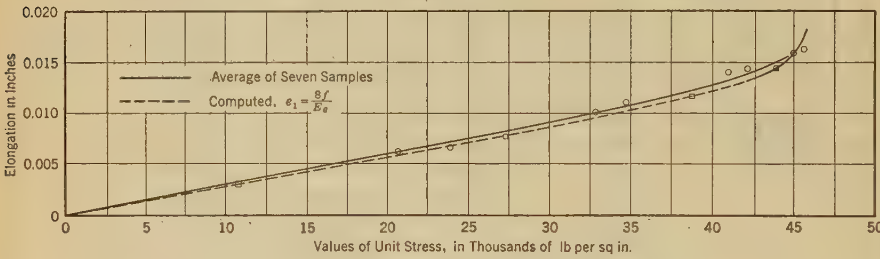


FIG. 4.

METHOD OF GRAPHICAL ANALYSIS

The method of analyzing equilibrium conditions for various effectiveness factors of passive resistance is shown in Fig. 5 for the single case of one and three-quarters times passive resistance due to Coulomb. In principle it is the method outlined by Dr. Lohmeyer[2], which shows decided refinements as compared to methods heretofore used in the statical analysis of bulkheads. These refinements are due particularly to the recognition of the effect of

---

[2] "Der Grundbau." von Brennecke-Lohmeyer. Fourth Edition. 1930. Vol. II. p. 77 *et seq.*

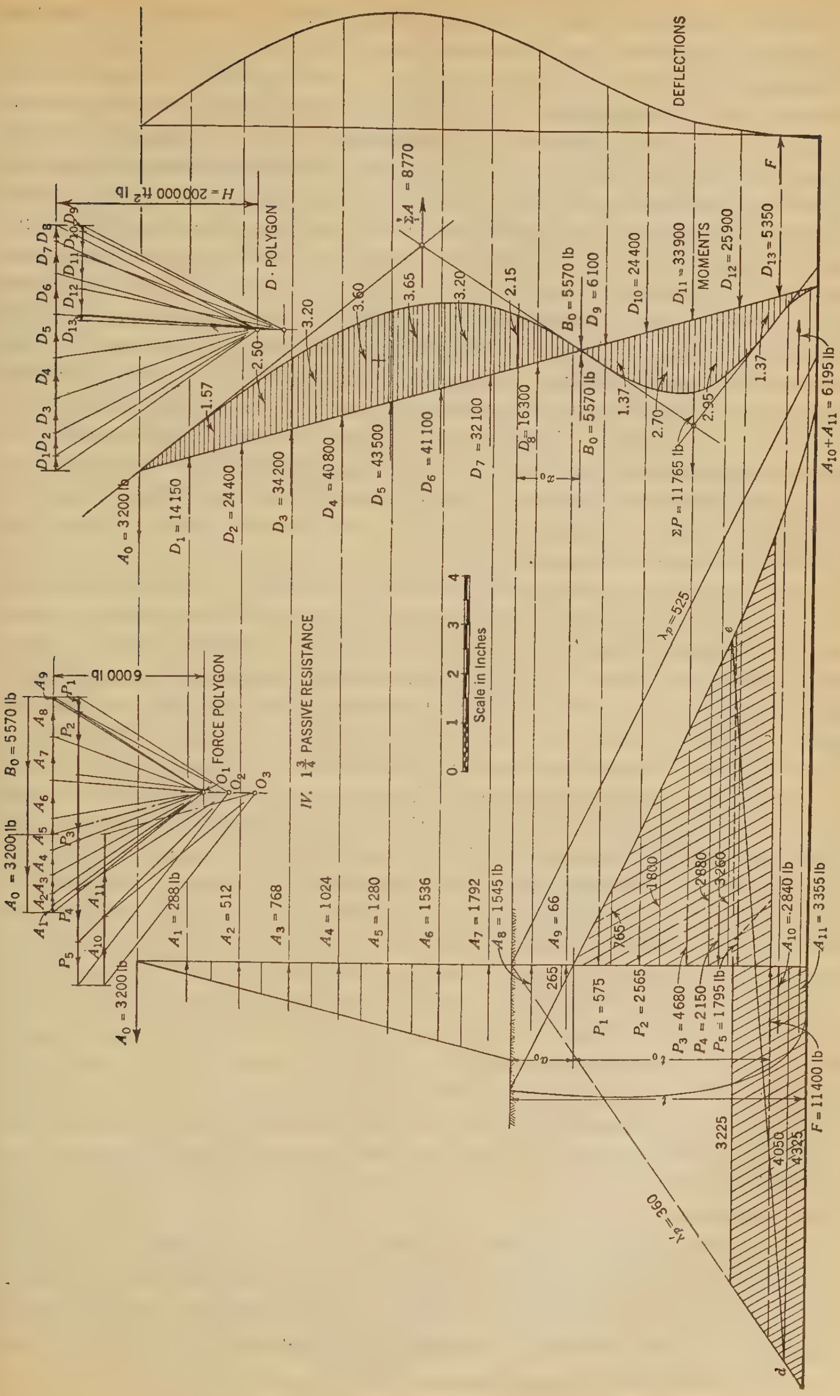

DEFLECTIONS
MOMENTS
D. POLYGON
H = 200 000 ft² lb
FORCE POLYGON
6000 lb
IV. 1¾ PASSIVE RESISTANCE
Scale in Inches
0 1 2 3 4
A₀ = 3200 lb
B₀ = 5570 lb
A₁ A₂ A₃ A₄ A₅ A₆ A₇ A₈ A₉ A₁₀ A₁₁
P₁ P₂ P₃ P₄ P₅
O₁ O₂ O₃
D₁ D₂ D₃ D₄ D₅ D₆ D₇ D₈ D₉ D₁₀ D₁₁ D₁₂ D₁₃
A₀ = 3200 lb
D₁ = 14150
D₂ = 24400
D₃ = 34200
D₄ = 40800
D₅ = 43500
D₆ = 41100
D₇ = 32100
D₈ = 16300
B₀ = 5570 lb
D₉ = 6100
D₁₀ = 24400
D₁₁ = 33900
D₁₂ = 25900
D₁₃ = 5350
F
1.57
2.50
3.20
3.60
3.65
3.20
2.15
1.37
2.70
2.95
1.37
ΣA = 8770
ΣP = 11765 lb
A₁₀+A₁₁ = 6195 lb
λₚ = 525
λₐ' = 360
e
d
a₀
t₀
x₀
A₀ = 3200 lb
A₁ = 288 lb
A₂ = 512
A₃ = 768
A₄ = 1024
A₅ = 1280
A₆ = 1536
A₇ = 1792
A₈ = 1545 lb
A₉ = 66
265
765
1800
2880
3260
P₁ = 575
P₂ = 2565
P₃ = 4680
P₄ = 2150
P₅ = 1795 lb
A₁₀ = -2840 lb
A₁₁ = 3355 lb
F = 11400 lb
3225
4050
4325

FIG. 5.

restraint in the ground, which has been comprehensively treated by Raymond P. Pennoyer, Assoc. M. Am. Soc. C. E.[3]

The passive resistance on the active side, that is, back of the wall, is of the same order of magnitude as the one in front and, in most cases, may be assumed to be equal to the latter as an upper limit. An opinion is extant[4] to the effect that the passive resistance on the active side is greatly reduced, due to friction between the soil and the wall, as compared to the passive resistance that would develop without wall friction. The writer does not agree with this viewpoint. Such a condition could exist only as long as no movement of the bottom part of the sheet-piling toward the back of the wall occurred; or, in other words, as long as the piles were subjected to active pressure. As soon as deflection begins and the bottom of the wall, or the part adjacent to it, moves backward (thereby displacing the particles) the forces getting into play are due to passive resistance, as on the other side of the wall. Any friction between the soil and the wall is augmenting this passive resistance, and the principal difference between the two cases is the angle of inclination of the sheet-piling in front of the wall, which (due to its "overhang") tends to increase the slope of the resistance vector, while back of the wall it tends to decrease it.

If the back requires a smaller allowance for passive resistance than the front of the wall, it is due to this difference in slope and the relative movement of the sheet-piling, which is governed by the distance of the point of rotation above the bottom. To produce a certain passive resistance at a certain depth requires a certain movement tending to displace the particles and thereby compress them. This movement is not entirely elastic, however; it is partly plastic. There is a certain lost motion, so to speak, that does not meet appreciable passive resistance. However, it must be remembered that due to the driving of the sheet-piles the adjacent ground is already compressed, due to a displacement which is equal to the thickness of the piles. The magnitude of this compression is reflected in the frictional resistance to both driving and pulling of the sheets. "Refusal" is due to this friction, which can only develop under compression; and so also is the enormous resistance to raising a pile after it has been driven to refusal.

Due to the forward deflection of the sheet-piling above the point of rotation, the ground stretches on the active side, and its density is thereby reduced, which would lead to a reduced, passive resistance. In the case of an ordinary bulkhead, however, the fill back of the sheet-pile wall acts as a surcharge and, in most cases, will more than compensate for the stretching. As to the test under consideration, the passive resistance behind the wall was assumed to be 70% or less of the passive resistance in front, because the surcharge was water.

The writer's opinion on this important phase of bulkhead design is strengthened by observations of the shape of sheets pulled at Berth 3, Pier *A* (Fig. 1), after the bulkhead had blown out with approximately 27 ft of fill behind it; and of the shape of sheets used in the test (Fig. 6). As to piles that were pulled after the failure at Berth 3 (Fig. 1), a statical analysis, for the original conditions, shows no restraint (Fig. 7), while the sheets, after pulling, showed decided counterflexure, which would have been impossible under the contention[4] that passive resistance on the active side is only 18% of that in front of the wall.

---

[3] "Design of Steel Sheet-Piling Bulkheads," *Civil Engineering*, Vol. 3, No. 11, November, 1933.

[4] "Der Grundbau" von Brennecke-Lohmeyer, Vol. II, p. 83.

Returning to the statical analysis, the next step will be to ascertain whether or not there is a restraint. For this purpose subtract the active from the passive pressure area and extend the resulting passive pressure area; that is, the λ_p–λ_a line indefinitely downward (Fig. 5(*a*)), divides the pressure area conveniently, and combines the corresponding active and passive forces in a force polygon. Then draw the moment diagram (Fig. 5(*b*)), which will be S-shaped, and extend it far enough downward to cover the entire known or probable penetration.

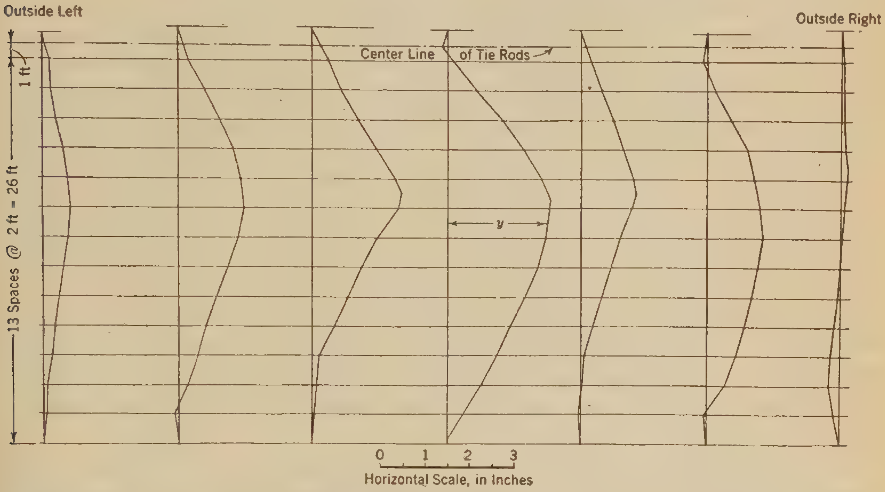


FIG. 6.—PERMANENT DEFORMATION OF TEST PILES FACING TEST WALL.

The minimum, theoretical penetration is then immediately found by means of a line through the upper support point and tangent to the moment line. The depth of the point of contact below the natural ground surface is then equal to the minimum theoretical penetration.

The next step will be to determine the practical minimum penetration, in which case the passive resistance at the bottom of the sheet-piling must be zero. At this penetration there is no restraint, but any additional penetration will cause part, and finally full, restraint.

In graphic statics, the three equilibrium conditions – $\Sigma H = 0$; $\Sigma V = 0$; and, $\Sigma M = 0$ – are satisfied if both the polygons close, the force polygon satisfying $\Sigma H = 0$ and $\Sigma V = 0$, and the string polygon being such that the sum of all the moments about an arbitrary point is zero ($\Sigma M = 0$).

If the penetration is given, as in this case, the problem is to find the most unfavorable load distribution for which equilibrium exists. If the penetration is such as to permit restraint to develop, the problem is again facilitated in using a single force, which, in this case, must go through the point of rotation of the sheet-piling; that is, the point some distance above the bottom which, during deflection, remains stationary. Force F (Fig. 5) again must be found by trial; it represents the resultant resistance of the cross-hatched area behind the sheet-piling. The final load distribution is found by deducting the cross-hatched resistance area on the active side from that on the passive side, and by drawing Line d'-e' so as to balance the differential areas above and below this line.

In Fig. 7 a section through the bulkhead at Berth 3 (Fig. 1) at the place of failure is shown. A 1-ft width of wall is considered. The method of analysis as previously outlined is used to ascertain the safety of the structure as originally designed. The analysis is based on the following properties, established by the foregoing tests and by supplementary tests on soil from the channel bottom; Angle of repose, dry, = 35° and, under water, = 30°; weight of dry material = 90 lb per cu ft; weight of material under water = 52 lb per cu ft; average voids = 40%; weight of sea water, W, = 64 lb per cu ft; level of water, standing in fill = Elevation + 3.50; effective lowest low water = Elevation − 1.50; and effective hydrostatic head = 5 ft.

The analysis is further based on the following assumptions: Surcharge on surface, $p_1 = 500$ lb per sq ft; the effectiveness factor of passive resistance of natural ground and a rough wall = 2; and the angle of internal friction of the sand is equal to the angle of repose and is constant, that is, independent of the depth.

The timber piles (not shown in Fig. 7), at least 16 in. in diameter, that support the relieving platform, take one-third the active earth pressure. The spacing of these piles in the first row, which is 2 ft 6 in. back of the steel sheet-piling, is 2 ft 11 in. (average) from center to center. This assumption is believed to be conservative, as nearly one-third the volume of the active prism is retained by the timber piling without the aid of arching.

The influence of the reinforced concrete encasement beam at the top of the sheet-piling was neglected, but the restraint of the sheet-piling due to the part that was encased in the cantilever end of the reinforced concrete relieving platform, was considered. The critical section of the platform has an over-all depth of 22 in. and the center line of the reinforcing steel, consisting of $\frac{3}{4}$-in. square bars, 8 in. on centers, is 19 in. from the top.

The load area due to earth pressure was determined by means of the relations, $\lambda_a = \gamma \tan^2\left(45 - \frac{\phi}{2}\right)$ and $e_a = \frac{\lambda_a p}{\gamma}$. The influence on the bulkhead of earth and surcharge above the relieving platform was included.

The minimum passive resistance, E_p, of the material below the theoretical toe slope was determined graphically by means of Culmann's E-line method at 20 400, or 64% of the minimum passive resistance for a level ground surface at Elevation — 24.50.

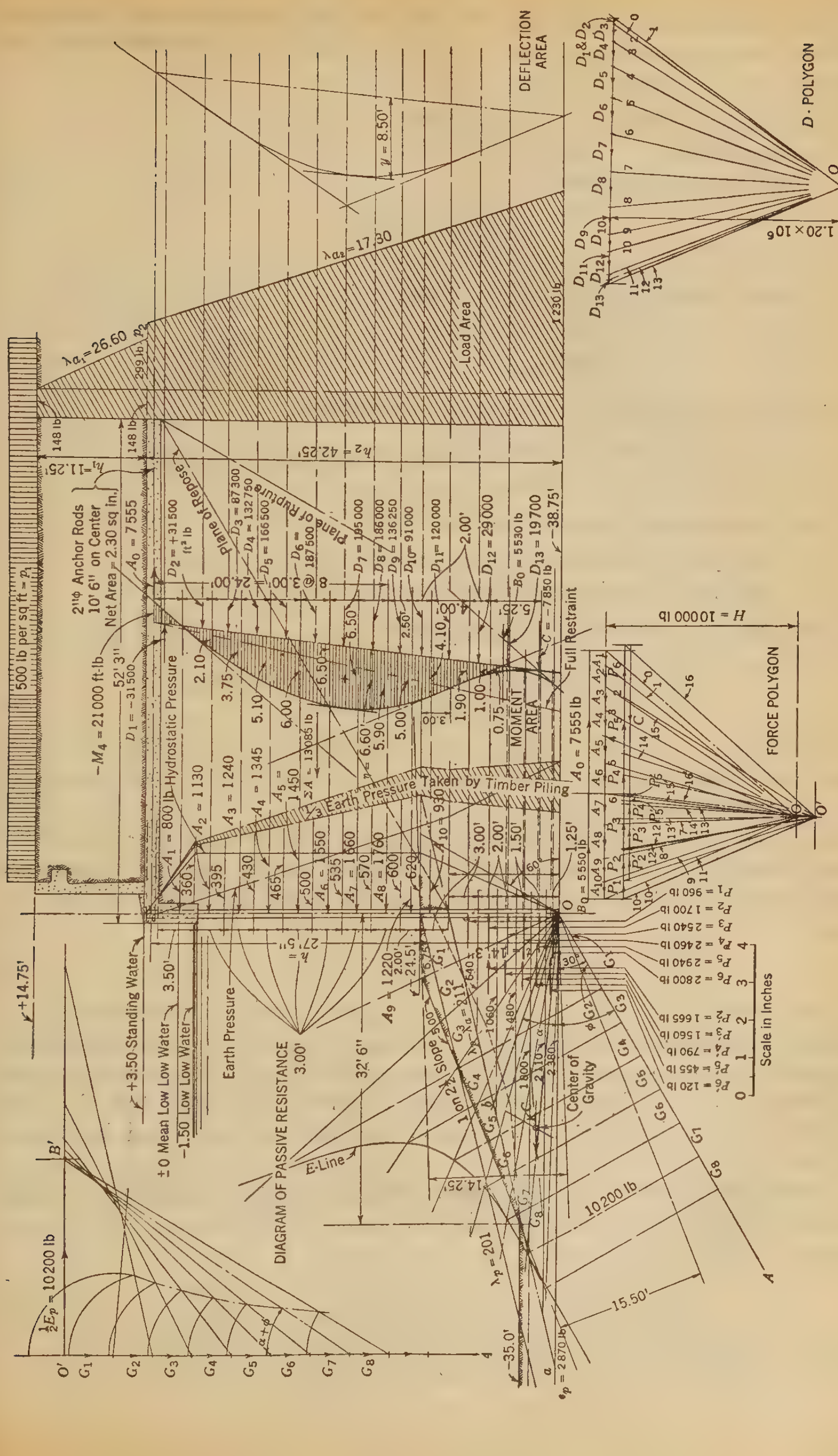

500 lb per sq ft = p_1
2"φ Anchor Rods
10' 6" on Center
Net Area = 2.30 sq in.
h_1 = 11.25'
h_2 = 42.25'
148 lb
299 lb
1230 lb
λa_1 = 26.60
λa_2 = 17.30
Load Area
DEFLECTION AREA
y = 8.50'
D-POLYGON
1.20×10⁶
FORCE POLYGON
H = 10000 lb
MOMENT AREA
Full Restraint
Plane of Repose
Plane of Rupture
Hydrostatic Pressure
Earth Pressure
⅔ Earth Pressure Taken by Timber Piling
$-M_4$ = 21 000 ft-lb
52' 3"
D_1 = −31 500
A_0 = 7 555
A_0 = 7555 lb
B_0 = 5 550 lb
B_0 = 5 530 lb
C = −7 850 lb
D_{13} = 19 700
D_{12} = 29 000
D_{11} = 120 000
D_{10} = 91 000
D_9 = 136 250
D_8 = 186 000
D_7 = 195 000
D_6 = 187 500
D_5 = 166 500
D_4 = 132 750
D_3 = 87 300
D_2 = +31 500 ft² lb
A_1 = 800 lb
A_2 = 1130
A_3 = 1240
A_4 = 1345
A_5 = 1450
A_6 = 1550
A_7 = 1660
A_8 = 1760
A_9 = 1220
A_{10} = 930
ΣA = 13085 lb
P_1 = 960 lb
P_2 = 1 700 lb
P_3 = 2 540 lb
P_4 = 2 460 lb
P_5 = 2 940 lb
P_6 = 2 800 lb
P_2' = 1 665 lb
P_3' = 1 560 lb
P_4' = 790 lb
P_5' = 455 lb
P_6' = 120 lb
Scale in Inches
+14.75'
+3.50 Standing Water
±0 Mean Low Low Water
−1.50 Low Low Water
3.50'
h = 27.5'
32' 6"
DIAGRAM OF PASSIVE RESISTANCE
3.00'
E-Line
Slope 5.00
1 on 2½
λa = 211
λ_p = 201
e_p = 2 870 lb
−35.0'
14.25'
15.50'
10200 lb
½E_p = 10200 lb
Center of Gravity
α+φ
O'
B'
A
G_1 G_2 G_3 G_4 G_5 G_6 G_7 G_8

FIG. 7.—EXAMPLE OF BULKHEAD ANALYSIS.

The analysis shows that the lower end of the bulkhead is not restrained and that it is a case of practical, minimum penetration. The findings may be summarized as follows:

Net passive resistance, $E_p = 13\,400$ lb
Lower reaction, $B_o = 5\,530$ lb
Safety against push-out,

$$S_p = \frac{13\,400}{5\,530} = 2.42$$

Average stress in anchor rod and bars,

$$f_a = \frac{10.50 \times 7\,555}{2.30 \times 20 \times 0.39} = 7\,850 \text{ lb per sq in.}$$

Maximum fiber stress in steel sheet-piling used,

$$f_s = \frac{6.60 \times 10\,000 \times 12}{25.86} = \pm\ 30\,600 \text{ lb per sq in.}$$

Stresses in critical concrete section:

Maximum compression in concrete = 460 lb per sq in.
Maximum tension in reinforcing steel = 17 400 lb per sq in.

Maximum deflection of steel sheet-piling,

$$\Delta\ (\text{max.}) = \frac{1.20 \times 10^6 \times 8.50 \times 12^3}{29 \times 10^6 \times 25.86 \times 5.687} = 4.13 \text{ in.}$$

The average of twenty-eight tests of specimens cut at random from the steel sheet-piling showed a yield point of 45 782 lb per sq in. and a breaking strength of 75 510 lb per sq in.

If the distance from the original ground surface at the center line of the sheet-piling to the zero point of the load area is termed a_o, and the distance from there to the bottom is t_o, then it may be shown[5] that the total penetration is,

$$t = a_o + 1.20\ t_o \qquad \text{(2)}$$

The distance, X_o (Fig. 7), which is the point of zero moment below the original ground surface, reaches a minimum value for full restraint, and the same is given at approximately 0.10 h_o, provided the angle of repose of the ground under water is at least 30 degrees. The height of the top of the fill above the natural ground at the center line of the sheet-piling is denoted by h_o.

From the equation for the restraining or negative moment and from Equation (2), a formula for the penetration, t_f, for full restraint is obtained, which when simplified reads,

$$t_f = 1.60\ a_o - 0.06\ h_o + 1.20\sqrt{\frac{6\ B_o}{2\ \lambda_p - \lambda_a}} \qquad \text{(3)}$$

---

[5] "Der Grundbau," von Brennecke-Lohmeyer, Vol. II, p. 85.

and, in which, in order to get t, in feet, a_0 and h_0 are introduced, in feet, B_0, in pounds, and,

$$\lambda_p = \gamma \tan^2 \left(45 + \frac{\phi}{2}\right) \quad \text{.....................(4)}$$

and,

$$\lambda_a = \gamma \tan^2 \left(45 - \frac{\phi}{2}\right) \quad \text{.....................(5)}$$

Equation (3) represents a valuable contribution to the art of bulkhead design, as it eliminates all the trial work previously described as far as the condition of full restraint is concerned. The deflections were determined graphically by Mohr's method; that is, by using the moment area as load area for a constant index of rigidity, $E\,I$.

From a standpoint of safety only, full restraint is always desirable, but the corresponding penetration is not necessarily the most economical one. The section modulus, S, of steel sheet-pile sections varies approximately with the square of the weight, G, per unit length. Thus,

$$\frac{S_1}{S_2} = \frac{G^2{}_1}{G^2{}_2}, \text{ or } \frac{G_1}{G_2} = \sqrt{\frac{S_1}{S_2}} \quad \text{.....................(6)}$$

On the other hand, the required section modulus, based on a limiting fiber stress, is directly proportional to the maximum moment. Therefore,

$$\frac{G_1}{G_2} = \sqrt{\frac{M_1}{M_2}} = \sqrt{\frac{\zeta_1}{\zeta_2}} \quad \text{.....................(7)}$$

if $M_1 = H_0\,\zeta_1$ and $M_2 = H_0\,\zeta_2$.

If the length of sheets in a wall with part restraint is l_1, and that for full restraint is l_2, the required theoretical steel weight per unit length of l_2 in percentage of l_1 is,

$$G = 100\,\frac{l_2}{l_1}\sqrt{\frac{\zeta_2}{\zeta_1}} \quad \text{.....................(8)}$$

which is not necessarily less than 100. The practical economy factor may differ from the theoretical because it is governed by the available sections. This is illustrated in the example shown in Fig. 7. The section used was such as to resist a maximum moment of 6.60 × 10 000 × 12 = 792 000 in-lb, and the total length of sheet-piling was 42 ft, of which, 27.75 ft is above the ground and 14.25 ft is penetration. To produce full restraint 19.40 ft of penetration would be required, which would make a total length of sheets of 27.75 + 19.40 = 47.15 ft, and which would reduce the maximum moment to $\frac{4.60}{6.60} \times 792\,000 = 552\,000$ in-lb.

To keep within the same stress, namely, 30 600 lb per sq in., the section modulus of the next lighter section would be insufficient making the use of the original section for both cases necessary. Instead of a saving, there would actually be a greater cost for total restraint, due to additional length.

It must be understood that the load distribution below the original ground surface, shown in Fig. 7, is merely an approximation of the actual distribution. The latter could easily be arrived at by mere examination and trial, but would be of no advantage from the standpoint of safety, because the triangular distribution as used in the analysis gives greater maximum moments and calls for greater penetration.

The bulkhead, as shown in Fig. 7, differs from the test wall (Fig. 2) in that the sheets are partly restrained at the top due to a reinforced concrete encasement and in that the ground in front of the bulkhead is dredged to a 1 on $2\frac{1}{2}$ slope, leaving a berm of about 6.75 ft.

In view of the fact that an effectiveness factor of 1.68 was established by the test in the artificial fill, the assumption of a factor of 2.00 for natural ground is believed to be safe even for sloping ground.

ANALYTICAL DETERMINATION OF INTERLOCK EFFICIENCY

The observed, elastic line is closely approximated by two hyperbolic spirals with a common point and tangent at the point of minimum radius of curvature; that is, maximum moment. This curve becomes faulty near the top support where it is of small importance, as the radius of curvature remains finite. However, it closely checks the elastic line in the vicinity of minimum radius of curvature.

The equation of the upper spiral going through the upper point of support is,

$$\sqrt{(u_1 - z)^2 + (v_1 + y)^2}\left\{\text{arc tan}\,\frac{v_1 + y}{u_1 - z} + \alpha_1\right\} = a \text{ ....(9)}$$

and the equation of the lower spiral, going through the observed point at the ground surface, is:

$$\sqrt{(u_2 + z)^2 + (v_2 + y)^2}\left\{\text{arc tan}\,\frac{v_2 + y}{u_2 + z} + \alpha_2\right\} = a \text{ ....(10)}$$

The constants, u_1, u_2, v_1, v_2, α_1, α_2, and a, are determined from the observed elastic line, and the tangent in any point may then be determined for the upper curve through,

$$\frac{dy}{dz} = \frac{a\,(u_1 - z) - (v_1 + y)\sqrt{(u_1 - z)^2 + (v_1 + y)^2}}{a\,(v_1 + y) + (u_1 + z)\sqrt{(u_1 - z)^2 + (v_1 + y)^2}} \text{ ..... (11)}$$

and through,

$$\frac{dy}{dz} = \frac{-\,a(u_2 + z) + (v_2 + y)\sqrt{(u_2 + z)^2 + (v_2 + y)^2}}{a\,(v_2 + y) + (u_2 + z)\sqrt{(u_2 + z)^2 + (v_2 + y)^2}} \text{ ......(12)}$$

for the lower curve.

Further, the radius of curvature for the upper curve is expressed as:

$$\rho_1 = \frac{\sqrt{(u_1 - z)^2 + (v_1 + y)^2}}{\sin^3\left\{\text{arc tan}\frac{a}{\sqrt{(u_1 - z)^2 + (v_1 + y)^2}}\right\}} \quad \ldots\ldots\ldots\ldots(13)$$

and for the lower curve,

$$\rho_2 = \frac{\sqrt{(u_2 + z)^2 + (v_2 + y)^2}}{\sin^3\left\{\text{arc tan}\frac{a}{\sqrt{(u_2 + z)^2 + (v_2 + y)^2}}\right\}} \quad \ldots\ldots\ldots\ldots(14)$$

The common point follows from $\rho_1 = \rho_2$, which is the case for $z = 10.17$ ft. As the moment, $M = A_0 z - \frac{64\ z^3}{6}$, must be a maximum at this point: $\frac{dM}{dz} = 0 = A_0 - 32\ z^2$; and $A_0 = 3\,310$ lb. This reaction checks very closely the one determined graphically for 1.68 E_p by interpolation.

Fig. 8 shows ρ as a function of z. The dotted line at the lower end indicates a correction, based on the boundary condition that ρ must become infinite for $z = 18 \pm$. A horizontal line through $z = 18$, therefore, should be asymptotic to the ρ-curve. It is believed that these corrected ρ-values are a better approximation of the actual condition, and it is for this reason that they were used in the computation of the interlock efficiency. The lower

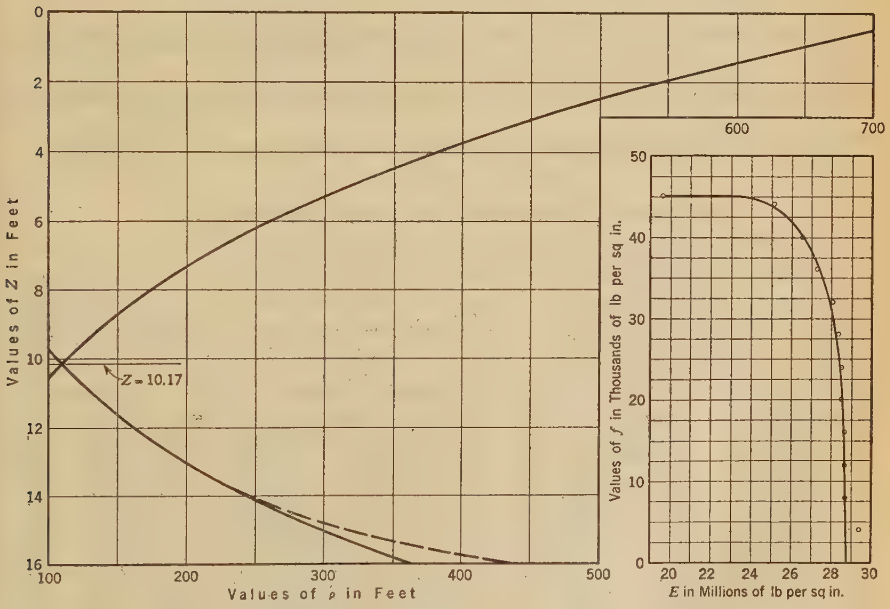


FIG. 8.—RADIUS OF CURVATURE.

FIG. 9.—RELATION BETWEEN f AND E.

spiral is subject to greater error than the upper one, because it does not go through the origin of co-ordinates and is governed by fewer check points.

In Fig. 9, the fiber stress, f, is shown as a function of the modulus of elasticity, E, based on the average of seven tests. Knowing ρ and M as functions of z, the next step is to find a relation between ρ and E, which follows from the simplified equation of the elastic line,

$$\frac{d^2y}{dz^2} = \frac{1}{\rho} = \frac{M}{EI} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (15)$$

Notwithstanding the fact that the test involves material deflections which are not negligible as compared to the over-all dimensions of the wall, the error in using the approximate term, $\frac{d^2y}{dz^2}$, for $\frac{1}{\rho}$, instead of $\frac{\frac{d^2y}{dz^2}}{\left[1+\left(\frac{dy}{dz}\right)^2\right]^{\frac{3}{2}}}$, is small since $\frac{dy}{dz}$ varies within a range of 0.07 which means that the maximum value of $\left(\frac{dy}{dz}\right)^2$ is about 0.005.

In the formula, $E = \frac{\rho M}{I} = \frac{\rho f}{e}$, e is the distance from the neutral axis. In other words, e is the distance from the axis of symmetry of the section to the extreme fiber, in this case, $e = 2\frac{15}{16}$ in. = 2.937 in., and $\frac{f}{E} = \frac{e}{\rho}$.

TABLE 3.—COMPUTATION OF INTERLOCK EFFICIENCY

z	z^3	$A_o\,z$	$10.67z^3$	M, in foot-pounds	EI, in foot²-pounds		f	E	I_e, in inches	S_e, in inches	Interlock efficiency (percentage)
							In pounds per square inch	In pounds per square inch			
0	0	0	0	0	751	......	......	......	......	....	......
1	1	3 310	10.67	3 299	639	2.172×10^6	10 680	28.74×10^6	10.88	3.72	40
2	8	6 620	85.50	6 534	540	3.53×10^6	13 020	28.72×10^6	17.70	6.05	65
3	27	9 930	288.50	9 641	453	4.37×10^6	15 475	28.66×10^6	21.96	7.43	80
4	64	13 240	683	12 557	380	4.77×10^6	18 425	28.59×10^6	24.02	8.18	88
5	125	16 550	1 335	15 215	316	4.81×10^6	22 050	28.49×10^6	24.32	8.27	89
6	216	19 860	2 308	17 552	262	4.60×10^6	26 420	28.29×10^6	23.42	8.00	86
7	343	23 170	3 663	19 507	214	4.19×10^6	31 870	27.89×10^6	21.63	7.35	79
8	512	26 480	5 465	21 015	175	3.68×10^6	37 780	27.10×10^6	19.60	6.70	72
9	729	29 790	7 785	22 005	141	3.105×10^6	43 250	24.95×10^6	17.95	6.14	66
10	1 000	33 100	10 670	22 430	114	2.557×10^6	45 600	21.25×10^6	17.35	5.86	63
11	1 331	36 410	14 220	22 190	132	2.93×10^6	44 450	23.98×10^6	17.60	5.95	64
12	1 728	39 720	18 450	21 270	162	3.446×10^6	40 200	26.55×10^6	18.68	6.32	68
13	2 197	43 030	23 470	19 560	200	3.91×10^6	33 880	27.68×10^6	20.35	6.88	74
14	2 746	46 340	29 300	17 040	248	4.225×10^6	27 830	28.20×10^6	21.60	7.35	79
15	3 375	49 650	36 040	13 610	320	4.355×10^6	21 760	28.48×10^6	22.05	7.53	81

In Table 3 the interlock efficiency and the effective section modulus are computed and the resulting fiber stresses are shown in Fig. 10. It is interesting to note that near the upper support the effective section modulus is smaller than the one for the single sheet which can only be interpreted as the effect of flattening out, due to the anchor pull and the lack of a waling and of lateral support.

The same effect probably accounts for the rapid drop of the interlock efficiency toward the point of minimum radius of curvature since the sheets did not have any lateral support, because the wall is short; they would tend to flatten due to lateral thrust. As the ground is approached and penetrated, the lateral freedom disappears rapidly and the interlock efficiency only shows the effect of longitudinal slippage. In fairness to the manufacturers of this steel sheet-piling, the writer has indicated a dotted line in Fig. 10, which he believes would be a close approximation of the actual interlock efficiency of a bulkhead of equal height as the one used in the test, but with waling at the tie-backs, lateral support, and, last but not least, back-filled with saturated earth instead of water.

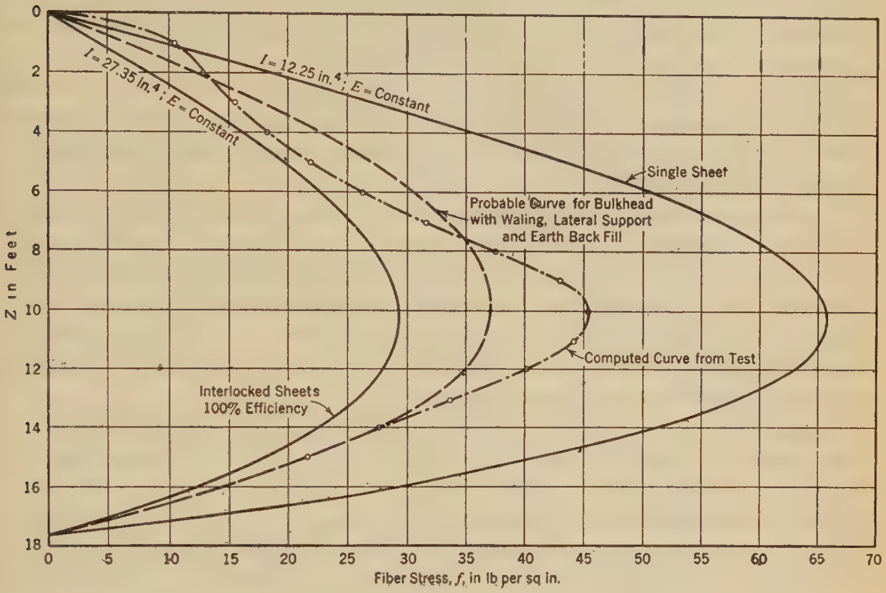


Fig. 10.—Fiber Stresses Computed from Test.

In the case of a sheet-pile wall driven in natural ground and held at the top by a concrete beam (Fig. 7), or otherwise, but so as positively to prevent slippage, the section modulus for the interlocked sheets as given by the manufacturer could be used without hesitancy.

Passive Resistance as a Function of Both z and y

In the foregoing, both active pressure and passive resistance of earth were treated as a statically determinate problem in accordance with the present theory. In other words, earth was treated like a fluid of smaller or greater unit weight than earth for active pressure and passive resistance, respectively.

Evidently, this is not the true condition, but is an approximation that may lead to satisfactory results for practical purposes if properly applied. In the case of active pressure on an unyielding wall, it is an exact interpretation, because then the problem is statically determinate.

The pressures for this condition are maximum and decrease as the wall begins to yield, due to a stretch in the grain structure; and, finally, they reach a minimum immediately before the grain structure collapses. This movement is only partly elastic (in its first stage) and quickly becomes irregular and too complex for mathematical analysis.

From an engineering standpoint, the most important feature is that the pressures are maxima for the static condition and that it is on the side of safety to consider the active pressure as remaining static, even if the retaining structure may yield.

Passive resistance, in contradistinction to active pressure, is more nearly a phenomenon which is approachable by the ordinary theory of elasticity, at least to the extent of a qualitative analysis, although it is by no means free of complexities. To overcome these obstacles which stubbornly defy an exact analysis, the writer knows no better solution than the one suggested[6] by Charles Terzaghi, M. Am. Soc. C. E., namely, to class earth mechanics as a descriptive rather than as an exact science.

Large-sized tests to establish the passive resistance of sand have been made recently in the hydraulic laboratory of the Institute of Technology of Hanover, Germany, under the direction of Professor O. Franzius.[7]

These tests were made in a box, 6.6 ft wide by 23 ft long by 6.6 ft high, and the sand used would be comparable to Sample *J* shown in Fig. 3. Friction between the sand and the wall was investigated, as well as the case of a frictionless wall, the latter being produced by letting the wall rise (exactly counterbalanced) under the influence of friction. Values of E_p for various heights of fill are shown as functions of y, which is the horizontal movement of the wall; and likewise for the coefficient of friction, tan δ, between the sand and the wall; the latter, however, is in too small a scale to be used quantitatively. In all cases the sand was filled in in thin layers and tamped. The outstanding result of all the tests is the fact that with or without wall friction, y is a steady function of E_p and that the coefficient of friction between sand and wall decreases with the depth.

The relation between the movement, y, of the wall and E_p is satisfactorily expressed by,

$$y = f_1\, E_p + f_2\, E^{(n_0)}{}_p \qquad (16)$$

in which, f_1 and f_2 are constants for one and the same height of fill $(z - h)$, and n_0 is an exponent which, according to Professor Terzaghi[8] is approximately 2, which checks the value found by Franzius for wet sand.

---

[6] "Erdbaumechanik auf bodenphysikalischer Grundlage," von Charles Terzaghi, Franz Deutike, 1925.

[7] *Der Bauingenieur*, 9 Jahrgang, 1928, Hefte 43 und 44.

[8] "Erdbaumechanik," by Charles Terzaghi, p. 332.

For $y = 0$ there is active (static) pressure only; that is, $E_p = E_0$. Therefore, $0 = f_1 + f_2 E_0$, and,

$$E_0 = -\frac{f_1}{f_2} = \frac{\gamma\, \xi_0\, (z-h)^2}{2} \quad \text{.......................(17)}$$

in which, ξ_0, according to Professor Terzaghi,[9] is of the order of magnitude of 0.42.

Introducing Equation (17) into Equation (16),

$$y = f_2\,(-E_0\, E_p + E_p^2) \quad \text{.......................(18)}$$

from which, f_2 may be determined for values of y and E_p obtained by test. The value, f_1, then follows from Equation (17) and, obviously, is negative.

Factors f_1 and f_2 can be expressed as steady functions of $(z - h)$, or, in general terms,

$$f_1 = -\frac{a}{(z-h)^m} \quad \text{.............................(19)}$$

and,

$$f_2 = \frac{b}{(z-h)^n} \quad \text{.............................(20)}$$

Solving Equation (16) for E_p and making $n_0 = 2$:

$$E_p = \frac{-f_1 + \sqrt{f_1^2 + 4\, f_2\, y}}{2\, f_2} \quad \text{..................(21)}$$

and, because $dE_p = e_p\, dz$, or $\dfrac{d\,E_p}{d\,z} = e_p$:

$$e_p = \left[f_2 \left(-\frac{d\,f_1}{d\,z} + \frac{f_1 \dfrac{d\,f_1}{d\,z} + 2\,y \dfrac{d\,f_2}{d\,z}}{\sqrt{f^2 + 4\, f_2 y}} \right) - \left(f_1 + \sqrt{f_1^2 + 4\, f_2\, y} \right) \frac{d\,f_2}{d\,z} \right] \frac{1}{2\, f_2^2} \quad \text{................(22)}$$

Introducing Equations (19) and (20), and differentiating:

$$e_p = \left\{ \frac{b}{(z-h)^n} \left[-\frac{a\,m}{(z-h)^{m+1}} + \frac{-\dfrac{a^2 m}{(z-h)^{2m+1}} - 2\,y\,\dfrac{b_n}{(z-h)^{n+1}}}{\sqrt{f_1^2 + 4\, f_2 y}} \right] + \left[\frac{a}{(z-h)^m} + \sqrt{f_1^2 + 4\, f_2 y} \right] \frac{b\,n}{(z-h)^{n+1}} \right\} \frac{(z-h)^{2n}}{2\, b^2} \quad \text{......(23)}$$

or,

$$e_p = \frac{a\,(n-m)\,(z-h)^{(n-m)-1}}{2\,b} \left\{ 1 + \frac{a}{R\,(z-h)^m} \right\} + \frac{n\,y}{(z-h)\,R} \quad \text{...(24)}$$

in which, $R = \sqrt{\dfrac{a^2}{(z-h)^{2m}} + \dfrac{4\,b\,y}{(z-h)^n}}$.

---

[9] "Erdbaumechanik," by Charles Terzaghi, pp. 187 and 195.

From the aforementioned tests by Professor Franzius and, particularly, for $y = 100$ mm $= 0.328$ ft; for wet sand and a rough wall, the following numerical values were found, based on the assumption that the passive resistance, E_p, as determined by test, is due to a width of 6.6 ft (2 m): $a = 10.00 \times 10^{-6}$; $b = 0.476 \times 10^{-6}$; $m = 0.535$; and, $n = 2.535$.

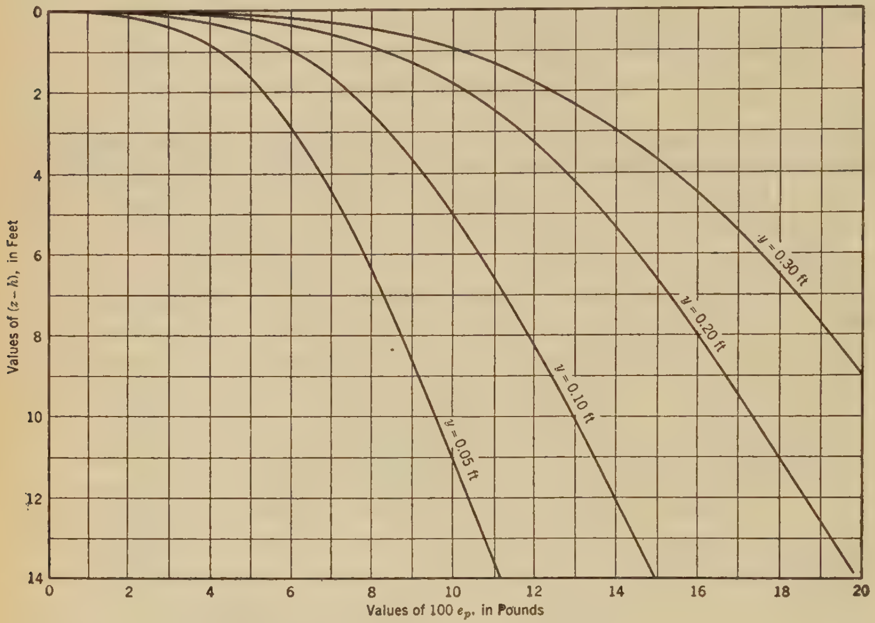


FIG. 11.—DISTRIBUTION OF SPECIFIC PASSIVE RESISTANCE, e_p.

The evaluation of Equation (24) is shown in Fig. 11. The first term in Equation (24) represents principally the influence of active pressure, and a variation of y only affects it through the root in the denominator. For $y = 0$, the term in the bracket becomes 2; then, $e_p = e_a$, and is due to active pressure only.

With sufficient accuracy, Equation (24) may be written in a simpler form, as follows:

$$e_p = \left(\gamma\, \xi_0 - 23 \sqrt[10]{y}\right)(z - h) + \frac{n y}{(z - h) R} \quad \ldots\ldots\ldots\ldots(25)$$

Remembering that $n - m = 2$, or $n = m + 2$, the last term may be written as follows:

$$\frac{n \sqrt{y}}{2 \sqrt{b}} \left\{ (z - h)^{0.5m} \left[1 + \frac{a^2}{4\,by}(z - h)^{2-m}\right]^{-\frac{1}{2}} \right\}$$

It now remains to determine whether the term, $\left[1 + \frac{a^2}{4\,by}(z - h)^{2-m}\right]^{-\frac{1}{2}}$, is convergent for such values of y as are of practical importance, thus:

$$\left[1 + \frac{a^2}{4\,by}(z-h)^{2-m}\right]^{-\frac{1}{2}} = 1 - \frac{a^2}{2 \times 4\,by}(z-h)^{2-m} + \frac{3}{8}\left(\frac{a^2}{4\,by}\right)^2 (z-h)^{4-2m}$$
$$- \frac{15}{48}\left(\frac{a^2}{4\,by}\right)^3 (z-h)^{6-3m} + \ldots$$

Introducing values for a and b, $\frac{a^2}{4\,b} = 0.0000525$, and:

$$\left[1 + \frac{a^2}{4\,by}(z-h)^{2-m}\right]^{-\frac{1}{2}} = 1 - \frac{0.0000525}{2\,y}(z-h)^{2-m}$$
$$+ \frac{3}{8}\left(\frac{0.0000525}{y}\right)^2 (z-h)^{4-2m} \;\ldots\ldots\ldots\ldots\ldots\ldots (26)$$

Movements of y much less than 0.01 ft are not believed to be of practical interest as they are likely to be partly plastic; their effect is small and difficult to observe. For $y = 0.01$, the right-hand side of Equation (26) becomes:

$$1 - 0.002625\,(z-h)^{2-m} + 0.000002582\,(z-h)^{4-2m} - \ldots$$

Multiplying by $(z-h)^{0.5m}$:

$$(z-h)^{0.5m} - 0.002625\,(z-h)^{v} + 0.000002582\,(z-h)^{2v} \ldots$$

in which $v = 2 - 0.5\,m$. For $(z-h) = 1$, the decrement due to the second and third terms is roughly ¼% of the first and for $(z-h) = 20$, it is about 14% of the first; which means that, with close approximation.

$$e_p = (\gamma\,\xi_0 - 23\,\sqrt[10]{y})\,(z-h) + \frac{n\,\sqrt{y}\,(z-h)^{0.5m}}{2\,\sqrt{b}} \;\ldots\ldots\ldots (27)$$

Equation (27) represents the general form. It may be further simplified in the case of driven sheet-pile bulkheads in quasi-level ground since, for $y = 0$, the active pressure is balanced, and the first term may be neglected. The specific passive resistance then becomes,

$$e_p = \frac{n\,\sqrt{y}\,(z-h)^{0.5m}}{2\,\sqrt{b}} \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (28)$$

The slope then becomes,

$$\frac{de_p}{dz} = \frac{nm\,\sqrt{y}}{4\,\sqrt{b}}\,(z-h)^{0.5\,m-1} \;\ldots\ldots\ldots\ldots\ldots\ldots (29)$$

and the curvature,

$$\frac{d^2e_p}{dz^2} = \frac{(m-2)\,nm\,\sqrt{y}}{8\,\sqrt{b}}\,(z-h)^{0.5\,m-2} \;\ldots\ldots\ldots\ldots (30)$$

The vertical, specific pressure due to the weight of the material alone at a depth, $(z - h)$, below the surface, is $\gamma\,(z - h)$, and the ratio, $e_p : \gamma\,(z - h)$, is, from Equation (27):

$$\frac{e_p}{\gamma\,(z-h)} = \xi = \xi_0 - \frac{23\sqrt[10]{y}}{\gamma} + \frac{n\sqrt{y}}{2\gamma\sqrt{b}}\,(z-h)^{0.5\,m-1} \quad \ldots\ldots(31)$$

which shows that ξ decreases with the depth.

According to the present theory,

$$\xi = \tan^2\left(45 + \frac{\phi}{2}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(32)$$

in which, ϕ is the angle of internal friction offering resistance to sliding along the surface of rupture. Solving for ϕ:

$$\phi = 2\ \mathrm{arc\,tan}\ \sqrt{\xi} - \frac{\pi}{2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(33)$$

The influence of wall friction is neglected. The feature of primary interest is the decrease of ϕ with the depth similar to δ and the increase with y. This checks qualitatively at least the results of experiments on active pressure by Jacob Feld,[10] M. Am. Soc. C. E., and Professor Terzaghi.[11]

If, as stated previously, wall friction is neglected and $\gamma\,(z - h)$ and e_p are assumed to be principal stresses (that is, horizontal and vertical shear are zero); if $(z - h)$ and η are the co-ordinates of a point of the surface of rupture; and, if ϵ is the angle between a differential length of the surface of rupture and the vertical:

$$\frac{d\eta}{dz} = -\tan\epsilon = -\sqrt{\xi} = -\sqrt{\xi_0 - \frac{23\sqrt[10]{y}}{\gamma} + \frac{n\sqrt{y}}{2\gamma\sqrt{b}}\,(z-h)^{0.5\,m-1}}$$

$$= -\left(\sqrt{\frac{n\sqrt{y}}{2\gamma\sqrt{b}}}\right)(z-h)^{0.25\,(m-2)}\left\{1 + \frac{\dfrac{\gamma\xi_0 - 23\sqrt[10]{y}}{\gamma}}{\dfrac{n\sqrt{y}}{2\gamma\sqrt{b}}}\,(z-h)^{1-0.5\,m}\right\}^{\frac{1}{2}}$$

$$= -c_1\,(z-h)^{0.25(m-2)}\left\{1 + c_2\,(z-h)^{1-0.5m}\right\}^{\frac{1}{2}} \ldots\ldots\ldots(34)$$

The term in the bracket equals $1 + \frac{1}{2}\,c_2\,(z - h)^{1-0.5m} - \frac{1}{8}\,c_2^2\,(z - h)^{2-m} + \ldots.$ For $y = 0.10$: Equation (34) yields $c_2 = 0.0407$; and $c_2^2 = 0.00165$; and for $y = 0.30$: $c_2 = 0.0216$; and $c_2^2 = 0.000466$. For $y = 0.10$ and $(z - h) = 1$, the term in the bracket yields $1 + 0.02035 - 0.000207 + \ldots.$ For $y = 0.30$ and $(z - h) = 1$: $1 + 0.0108 - 0.000058 + \ldots$; while for $y = 0.10$ and $(z - h) = 10$: $1 + 0.110 - 0.006$; and for $y = 0.30$ and $(z - h) = 10$: $1 + 0.058 - 0.0017 \ldots.$

---

[10] *Transactions*, Am. Soc. C. E., Vol. LXXXVI (1923), p. 1448.

[11] *Engineering*, June 13, 1930, p. 754 (Fig. 20).

Neglecting the third member:

$$\frac{d\eta}{dz} = -\left\{ c_1 (z - h)^{0.25\,(m-2)} + \frac{c_1 c_2}{2} (z - h)^{0.25\,(2-m)} \right\} \ldots\ldots (35)$$

Integrating, Equation (35) becomes (for $z = H$ and $\eta = 0$):

$$\eta = -\left\{ \frac{4\,c_1}{m+2} (z - h)^{0.25\,(m+2)} + \frac{2\,c_1 c_2}{6 - m} (z - h)^{0.25\,(6-m)} \right\} + C \ldots (36)$$

in which,

$$C = + \frac{4\,c_1}{m+2} (H - h)^{0.25\,(m+2)} + \frac{2\,c_1 c_2}{6 - m} (H - h)^{0.25\,(6-m)} \ldots\ldots (37)$$

and the equation for η is:

$$\eta = \frac{4\,c_1}{m+2} \left\{ (H - h)^{0.25\,(m+2)} - (z - h)^{0.25\,(m+2)} \right\}$$

$$+ \frac{2\,c_1 c_2}{6 - m} \left\{ (H - h)^{0.25\,(6-m)} - (z - h)^{0.25\,(6-m)} \right\} \ldots\ldots\ldots\ldots (38)$$

in which,

$$c_1 = \sqrt{\frac{n}{2\,\gamma \sqrt{\frac{b}{y}}}}, \text{ and } c_2 = 2\,\frac{(\gamma\,\xi_0 - 23\,\sqrt[10]{y}}{n} \sqrt{\frac{b}{y}}$$

The shape of this surface of rupture indicates that the premise on which the derivation of its formula (Equation (38)) is based—that e_p and $\gamma\,(z - h)$, are principal stresses—is erroneous. In other words, for a rough wall, shear is active along horizontal and vertical planes.

For a frictionless wall (which, of course, never occurs in practice), the equation for the plane of rupture should read:

$$\eta = A + B\,(z - h) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (39)$$

in which, A, from observation, is $\dfrac{H - h}{\tan\left(45 - \dfrac{\phi}{2}\right)}$, and,

$$B = -\frac{A}{H - h} = -\frac{1}{\tan\left(45 - \frac{\phi}{2}\right)} \ldots\ldots\ldots\ldots\ldots (40)$$

Equation (39) then reads:

$$\eta = \frac{1}{\tan\left(45 - \frac{\phi}{2}\right)} \left\{ (H - h) - (z - h) \right\} \ldots\ldots\ldots\ldots (41)$$

Comparing this with Equation (38): $\dfrac{1}{\tan\left(45 - \dfrac{\phi}{2}\right)} = 2\,c_1\left(\dfrac{2}{m+2} + \dfrac{c_2}{6-m}\right)$; $\dfrac{m+2}{4} = 1$; and, $\dfrac{6-m}{4} = 1$. For $m = 2$, $\dfrac{1}{\tan\left(45 - \dfrac{\phi}{2}\right)} = c_1\left(1 + \dfrac{c_2}{2}\right)$.

For $\phi = 30°$, therefore, $\dfrac{1}{\tan\left(45 - \dfrac{\phi}{2}\right)} = 1.735$, and, as before, c_1 and c_2 for corresponding values of y are:

y	c_1	c_2
0.01	1.355	0.1394
0.10	2.405	0.0407
0.20	2.865	0.024
0.30	3.170	0.0216

Plotting these values, $c_1\left(1 + \dfrac{c_2}{2}\right) = 1.735$ for $y = 0.02$ ft (see Fig. 12). On the other hand, for $y = 0.40$ ft, which would correspond to a movement approaching the upper, practical limit, the term, $c_1\left(1 + \dfrac{c_2}{2}\right) = 3.45$, which would indicate an angle of internal friction, $\phi = 59$ degrees.

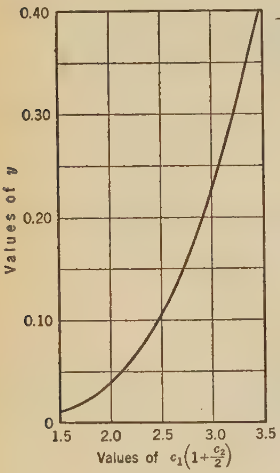


FIG. 12.

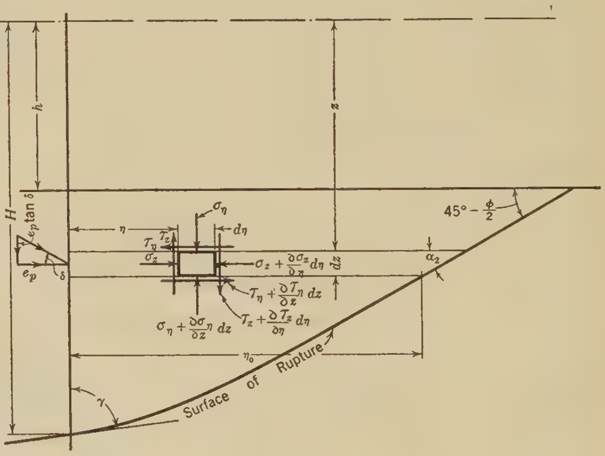

h
H
z
$e_p \tan\delta$
e_p
δ
η
$d\eta$
σ_η
τ_z
τ_η
σ_z
$\sigma_z + \frac{\partial\sigma_z}{\partial\eta}d\eta$
dz
$\tau_\eta + \frac{\partial\tau_\eta}{\partial z} dz$
$\tau_z + \frac{\partial\tau_z}{\partial\eta} d\eta$
$\sigma_\eta + \frac{\partial\sigma_\eta}{\partial z} dz$
$45° - \frac{\phi}{2}$
α_2
η_0
γ
Surface of Rupture

FIG. 13.

Introducing $m = 2$ in Equation (27),

$$e_p = \left\{ \left(\gamma\, \xi_0 - 23 \sqrt[10]{y}\right) + \frac{n_0 \sqrt{y}}{2 \sqrt{b_0}} \right\} (z - h) \ldots\ldots\ldots\ldots (42)$$

which means that for a constant value of y the distribution of the specific, passive resistance becomes linear. The term in the bracket corresponds to $\lambda_p = \gamma \tan^2 \left(45 + \frac{\phi}{2}\right)$, and ϕ, which is now independent of z, could be determined as soon as n_0 and b_0 are obtained from tests.

It is evident from the foregoing that the friction between the sand and the wall, which in practice is always present to a greater or less degree, is responsible for most of the complexities in the phenomenon of passive resistance.

POSSIBLE STRESS DISTRIBUTION IN PASSIVE PRISM FOR $\delta \neq 0$ AND EQUATION OF SURFACE OF RUPTURE

Assuming a parallelopipedon (Fig. 13), acted upon by both normal and tangential forces, the conditions of equilibrium may be expressed as follows:

$$\frac{-\partial \sigma_z}{\partial \eta} + \frac{\partial \tau_\eta}{\partial z} = 0 \ldots\ldots\ldots\ldots (43)$$

$$\frac{-\partial \sigma_\eta}{\partial z} + \frac{\partial \tau_z}{\partial \eta} + \gamma = 0 \ldots\ldots\ldots\ldots (44)$$

and,

$$\tau_\eta + \tau_z = 0 \ldots\ldots\ldots\ldots (45)$$

For the shear stress, τ_z,

$$\tau_z\, dz = e_p \tan \delta\, dz \ldots\ldots\ldots\ldots (46)$$

or,

$$\tau_z = e_p \tan \delta \ldots\ldots\ldots\ldots (47)$$

in which, $\tan \delta = f_1(z)$. Then, $\frac{\partial \tau_z}{\partial \eta} = 0$; and, because $\sigma_\eta = \gamma (z - h)$:

$$\frac{\partial \sigma_\eta}{\partial z} = \gamma \ldots\ldots\ldots\ldots (48)$$

which satisfies Equation (44).

From Equations (43) and (45):

$$\frac{\partial \sigma_z}{\partial \eta} = \frac{\partial \tau_\eta}{\partial z} = -\left\{ \frac{d e_p}{dz} \tan \delta + e_p \frac{d (\tan \delta)}{dz} \right\} = -f_2(z)$$

Integrating: $\sigma_z = -f_2(z) [\eta]_0^{(\eta o)} + C.$

Factor $\eta = 0$ when $\sigma_z = e_p$; therefore, $C = e_p$ and,

$$\sigma_z = -f_2(z) [\eta]_0^{(\eta o)} + e_p \ldots\ldots\ldots\ldots (49)$$

The unit stresses then are:

$$\sigma_z = e_p - f_2\,(z)\,[\eta]_0^{(\eta_0)} \quad \ldots\ldots (50)$$

$$\sigma_\eta = \gamma\,(z - h) \quad \ldots\ldots (51)$$

and,

$$\tau_\eta = -\,\tau_z = e_p \tan\delta = \pm\,\tau \quad \ldots\ldots (52)$$

The principal normal stresses are:

$$\sigma_{\text{max.}} = \frac{\sigma_\eta + \sigma_z}{2} + \tfrac{1}{2}\sqrt{4\,\tau^2 + (\sigma_\eta - \sigma_z)^2} \quad \ldots\ldots (53)$$

and,

$$\sigma_{\text{min.}} = \frac{\sigma_\eta + \sigma_z}{2} - \tfrac{1}{2}\sqrt{4\,\tau^2 + (\sigma_\eta - \sigma_z)^2} \quad \ldots\ldots (54)$$

The direction of these principal stresses follows from,

$$\tan 2\alpha_1 = \frac{2\,e_p \tan\delta}{e_p - f_2\,(z)\,[\eta]_0^{(\eta_0)} - \gamma\,(z - h)} \quad \ldots\ldots (55)$$

The principal shear stresses are.

$$\tau_{\text{max.}} = +\,\tfrac{1}{2}\sqrt{4\,\tau^2 + (\sigma_\eta - \sigma_z)^2}$$

and,

$$\tau_{\text{min.}} = -\,\tfrac{1}{2}\sqrt{4\,\tau^2 + (\sigma_\eta - \sigma_z)^2}$$

and the direction in which they occur follows from:

$$\tan 2\alpha_2 = \frac{\gamma\,(z - h) - e_p + f_2\,(z)\,[\eta]_0^{(\eta_0)}}{2\,e_p \tan\delta} \quad \ldots\ldots (56)$$

From the condition that the slope of the surface of rupture and the direction of maximum shear must coincide:

$$\tan 2\alpha_2 = \tan 2\,(90 - \epsilon) - \frac{2\,\dfrac{dz}{d\eta}}{1 - \left(\dfrac{dz}{d\eta}\right)^2} \quad \ldots\ldots (57)$$

and a differential equation of the surface of rupture is obtained as follows:

$$+\left[\left(\frac{d\eta}{dz}\right)^2 - 1\right]\left[\gamma\,(z - h) - e_p + f_2\,(z)\,\eta\right] - \frac{4\,d\eta}{dz}\,e_p \tan\delta = 0 \quad ..(58)$$

The writer is not aware of a general solution of Equation (58). Using $\tan\delta =$ constant as a first approximation, Equation (56) gives plausible values of α_2 except near the ground surface.

The surface of rupture could be expressed satisfactorily by an hyperbolic spiral with its polar axis inclined under an angle of $45 - \dfrac{\phi}{2}$ to the horizontal and its pole in front of and above the bottom of the wall. The normal distance of the polar axis from the foot of the wall would then be a function of the wall friction and, for a frictionless wall ($\tan \delta = 0$), this distance would be zero.

Dr. Alfred Streck derives[12] an equation for E_p which is based on the curved shape of a surface of rupture in its lower part and on friction between the sand and the wall. The upper part of the surface of rupture, based on observation, is assumed to be straight, and the dividing point between the straight and the curved part is found by drawing a straight line at an angle of $45 - \frac{\phi}{2}$ to the horizontal, through the top of the wall. The tangent to the surface of rupture at the bottom of the wall is determined by means of Winkler's stress ellipse; that is, by the relation,

$$\sin (2\beta + \delta) = \frac{\sin \delta}{\sin \phi} \quad \text{......................}(59)$$

in which, β = the angle between the wall and the major axis of the ellipse; δ, the angle of friction between the sand and the wall; and ϕ, the angle of internal friction of sand.

Dr. Streck shows the result of his research work in a diagram from which may be obtained the effectiveness factor, μ, if ϕ and δ are known. He also shows values of μ for $\phi = 35°$ and δ = variable, based on the assumption of a plane surface of rupture to the effect that such an assumption is not on the side of safety, particularly for $\delta > 15$ degrees.

Up to $\delta = 15°$ the maximum error is about 12%; that is, $\mu = 5.50$ for a curved surface and 6.20 for a plane surface. The passive resistance for a wall of a height, h, then is: $E_p = \frac{1}{2} \gamma \mu h^2$.

THE DESIGN OF BULKHEADS, A STATICALLY INDETERMINATE PROBLEM

Despite the refinements suggested by Brennecke-Lohmeyer, as previously described, the present theory of bulkhead design is defective in assuming passive resistance to be static. In other words, it is based on the assumption that soil with a certain characteristic offers a fixed passive resistance, irrespective of the movement of the wall.

The fallacy of this premise becomes quite evident if two sheet-pile walls are compared, having equal lengths and widths above and below ground, the section moduli of which are considerably different and are driven in exactly the same soil.

According to the present method of design, the elastic lines of the two walls will differ due to the different section moduli only, and the passive resistance will remain constant. The only adjustment required to satisfy this condition is in changing the polar distance of the D-polygon by the ratio of the two section moduli. However, from the foregoing, it is evident that passive resistance is a function of both horizontal movement and depth and that it varies much more with the former than with the latter.

---

[12] "Beitrag zur Frage des Erdwiderstandes," von Alfred Streck, *Der Bauingenieur*, 1926, Hefte 1 und 2.

The differential equation of the elastic line below the original ground surface for a triangular active load is:

$$EI\,\frac{d^4y}{dz^4} = \lambda_a\,(z-h) - \frac{n\sqrt{y}}{2\sqrt{b}}\,(z-h)^{0.5m} = a - p \;\ldots\ldots(60)$$

the solution of which is very involved indeed. The only boundary conditions known are $y = 0$ and $\frac{dy}{dz} = 0$ for $z = \infty$. The elastic line thus is comparable with a wave which is gradually diminishing as the depth below the ground increases.

Statically, a sheet-pile bulkhead is a beam, simply supported at or near the upper end and on an infinite number of elastic supports below the ground surface.

If y_0 denotes the yield of the soil at a depth, $z - h$, due to a unit load, then the yield due to a load, p, is $y_0 p$, and if the lengthening of the tie-rod due to a unit force is C_0, then the lengthening due to a force, A_0, is $C_0\,A_0$. The yield, y_0, follows from $e_p = 1$; namely, from Equation (28),

$$y_0 = \frac{4\,b}{n^2\,(z-h)^m} \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots(61)$$

and,

$$C_0 = \frac{L}{E\,A} \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(62)$$

in which, L = the length of the tie-rod; and, A, the cross-sectional area of the rod. As these forces grow from zero to their full value during deformation, the general equation of the total work of deformation, or the stored-up energy per linear width of wall is:

$$A = \tfrac{1}{2}\,C_0\,A_0^2 + \tfrac{1}{2}\int_h^\infty \tfrac{1}{3}\,y_0\,p^3\,dz + \int_0^\infty \tfrac{1}{2}\,\frac{M^2}{EI}\,dz \;\ldots\ldots\ldots\;(63)$$

in which,

$$M = A_0\,z - \int_{x=0}^{x=h} a\,(z-x)\,dx + \int_{x=h}^{x=z} p\,(z-x)\,dx \;\ldots\ldots(64)$$

and a = the active pressure. As the amplitude of the wave, representing the elastic line, diminishes rapidly below the ground surface, the boundary conditions, $y = 0$ and $\frac{dy}{dz} = 0$, will be satisfied closely enough for practical purposes at a depth, $z = H$, and Equation (63) becomes finite.

For equilibrium:

$$A_0 - \int_0^h a\,dz + \int_h^H p\,dz = 0 \;\ldots\ldots\ldots\ldots\ldots\ldots(65)$$

and,

$$-\int_0^h a\,z\,dz + \int_h^H p\,z\,dz = 0 \ldots\ldots\ldots\ldots\ldots\ldots(66)$$

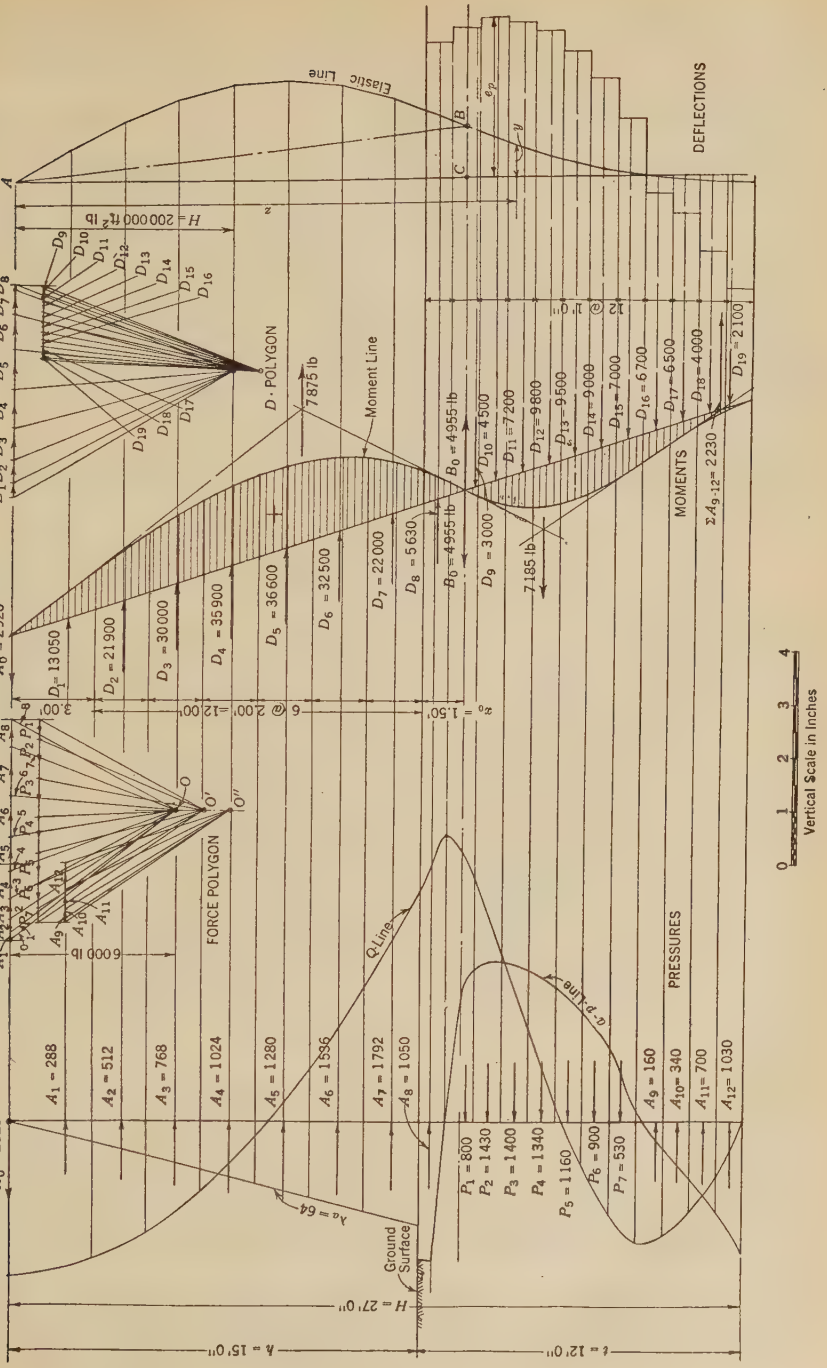

DEFLECTIONS
Elastic Line
A
B
C
e_p
y
z
$H = 200\,000$ ft² lb
D · POLYGON
D_1 D_2 D_3 D_4 D_5 D_6 D_7 D_8
D_9 D_{10} D_{11} D_{12} D_{13} D_{14} D_{15} D_{16} D_{17} D_{18} D_{19}
Moment Line
MOMENTS
7875 lb
7185 lb
$A_0 = 2920$
$D_1 = 13\,050$
$D_2 = 21\,900$
$D_3 = 30\,000$
$D_4 = 35\,900$
$D_5 = 36\,600$
$D_6 = 32\,500$
$D_7 = 22\,000$
$D_8 = 5\,630$
$B_0 = 4955$ lb
$B_0 = 4955$ lb
$D_9 = 3000$
$D_{10} = 4500$
$D_{11} = 7200$
$D_{12} = 9800$
$D_{13} = 9500$
$D_{14} = 9000$
$D_{15} = 7000$
$D_{16} = 6700$
$D_{17} = 6500$
$D_{18} = 4000$
$D_{19} = 2100$
$\Sigma A_{9\text{-}12} = 2230$
3.00'
6 @ 2.00' = 12.00'
$x_0 = 1.50'$
9 10 11 12
FORCE POLYGON
O
O'
O''
6000 lb
A_1 A_2 A_3 A_4 A_5 A_6 A_7 A_8
A_9 A_{10} A_{11} A_{12}
P_1 P_2 P_3 P_4 P_5 P_6 P_7
Q-Line
q-p-Line
PRESSURES
$A_0 = 2920$
$A_1 = 288$
$A_2 = 512$
$A_3 = 768$
$A_4 = 1024$
$A_5 = 1280$
$A_6 = 1536$
$A_7 = 1792$
$A_8 = 1050$
$A_9 = 160$
$A_{10} = 340$
$A_{11} = 700$
$A_{12} = 1030$
$P_1 = 800$
$P_2 = 1430$
$P_3 = 1400$
$P_4 = 1340$
$P_5 = 1160$
$P_6 = 900$
$P_7 = 530$
$\lambda_a = 64$
Ground Surface
$H = 27'0''$
$h = 15'0''$
$t = 12'0''$
Vertical Scale in Inches
0 1 2 3 4

FIG. 14.

The problem must be divided into the part above and the part below the ground. If p is positive, it denotes passive resistance on the passive side, and if it is negative it denotes passive resistance on the active side. Factor a is a given function of z.

Introducing Equations (64) and (65) into Equation (63), differentiating by p, and making $\frac{dA}{dp} = 0$, satisfies the condition that the statically indeterminate forces must make the work of deformation a minimum. The solution then consists in assuming p as a function of z and in varying it until all conditions are satisfied, which is a tedious task indeed.

An exact analytical solution is impossible if the active loading is not a steady function of z, which is often the case due: (*a*) To the influence of hydrostatic pressure; (*b*) to the influence of non-uniform material; or, (*c*) to the influence of fill above a relieving platform (see Fig. 7).

On the other hand, a graphical solution hereafter presented is always possible regardless of the load distribution. In this graphical method of analysis,

$$e_p = \frac{n\sqrt{y}}{2\sqrt{b}}(z - h)^{0.5m} \quad \text{..................(67)}$$

is used instead of $e_p = \lambda_p (z - h)$. It is best explained by means of an example, as shown in Fig. 14, in which it is applied to the test bulkhead. The active loading is the same as before and so is the method of determining the moment line due to active loads above the ground surface. To base the analysis as closely as possible on the actual conditions, the top 6 in. of the resisting ground was not included, due to the fact that during the test the corresponding material had to be removed for the purpose of reading deflections of the bottom indicator rods. Full active pressure then extends to 6 in. below the ground surface.

A line of closure is then assumed, which fixes the distance, x_0, below the ground surface. In this connection it may be said that due to the true distribution of passive resistance (Fig. 11), the distance, x_0, for the ground with an angle of internal friction of 30°, or more, is smaller than would follow if it was based on $e_p = \lambda_p (z - h)$, and, in the case of a wall fixed at the bottom, $x_0 \leqq 0.05\, h$.

Next, the elastic line is drawn for the simple beam with the span, $h + x_0$, in the same manner as previously described. The horizontal distances between this line and Line *A–B* are the horizontal deflections of the simple beam.

The ground is then divided in horizontal strips, for convenience, 1 ft in height, and the distance, *B–C*, is assumed, which determines the yield of the ground and with it the passive resistance between this point and the surface.

The corresponding force assumed to apply at the center of the first 1-ft strip is found from a diagram (Fig. 15), showing a relation of e_p as a function of z and y and by plotting the differential force, $\lambda_a - e_p$, in the force polygon, a new ray is obtained for the extension of the moment line. In turn, the increment of D thereby determined is plotted in the D-polygon, and the corresponding ray serves to extend the elastic line to the center of the next strip below. This procedure is repeated until the moment diagram is closed.

Two or three trials may be required; that is, two or three assumptions of x_0 and the distance, B–C, may have to be made, to close the moment diagram. However, an erroneous assumption will promptly be detected without the necessity of carrying the moment or the elastic line down very far.

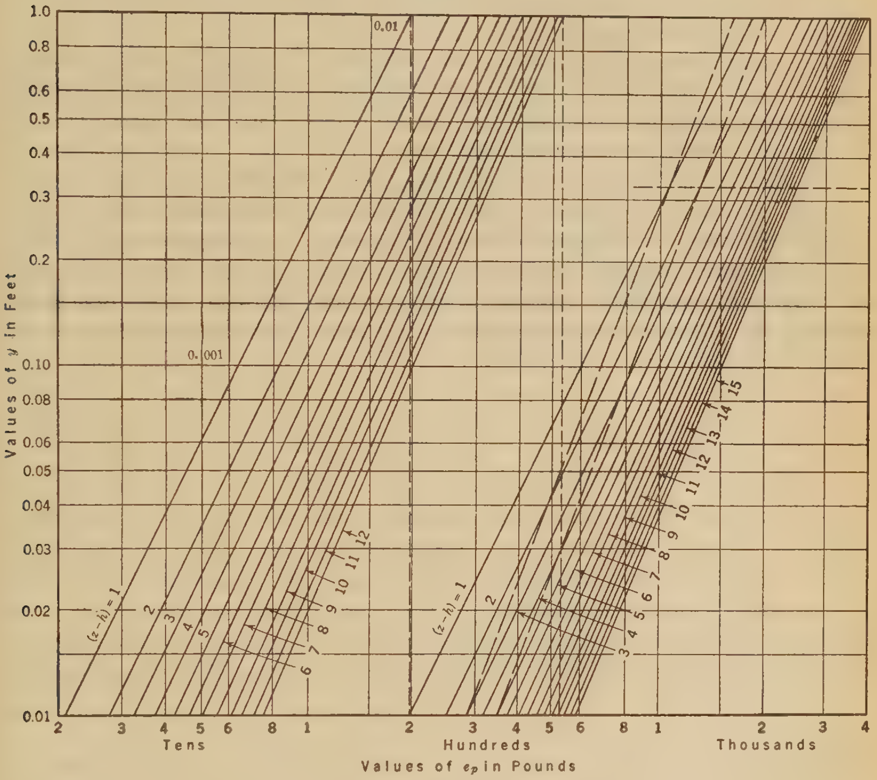


FIG. 15.—RELATION BETWEEN e_p AND y FOR CORRESPONDING VALUES OF $z - h$

As to the example (Fig. 13), it was necessary to apply a factor of 2.50 to the values of e_p to establish equilibrium. If the writer was correct in interpreting the test results of Professor Franzius as being due to a width of wall of 6.6 ft (2 m) rather than of unit width, this would mean that the passive resistance, e_p, of the hydraulic fill in the Outer Harbor at Long Beach, which was about one year old, was as much as 400% of the fill as used in the laboratory tests; but should the test results refer to a unit width, the hydraulic fill would then be about twice as effective as the laboratory fill, and the latter would compare closely with the former without wall friction.

It is interesting to note that the top reaction, A_0, as determined by this new method of analysis, checks the one obtained from extensometer readings on the tie-rods within 1 per cent.

CONCLUSIONS

From the foregoing studies, the following conclusions can be drawn:

1.—The angle of internal friction, ϕ, of granular soil resisting the pressure of a rough wall, and the angle of friction, δ, between the wall and the soil decrease with the depth and increase with the path of the wall.

2.—The angle, ϕ, approaches 90° at the surface (solid state) and zero (liquid state) with increasing depth.

3.—The distribution of the specific resistance, e_p, is not linear as heretofore assumed. It is more nearly trapezoidal than triangular, and the greater specific resistance near the surface raises the point of counterflexure ($M = 0$) and creates a better moment distribution.

4.—The intrinsic physical properties of granular soil, that is, the constants, b, m, and n, in Equation (27), should be determined by test.

5.—The interlock efficiency of steel sheet-piling is over-estimated at 100% unless slippage along the interlock is positively prevented. In the design of ordinary bulkheads with walers and anchors near the top and a non-liquid back-fill, a 75% efficiency, as advocated by American manufacturers, should not be exceeded.

DISCUSSION

Jacob Feld,[13] M. Am. Soc. C. E. (by letter).—Because of the omission of a list of symbols and definitions, this paper is difficult to read and, therefore, much of its value will never be brought to the attention of many engineers who would profit by a careful study of Mr. Baumann's methods of analysis. Furthermore, it would be easier to follow the author's mathematics if the source of the equations were given.

The object of the paper is threefold—and the writer's opinion of each section, in summary, is as follows:

1.—The description of tests made to determine effective passive resistance of sand and of interlock efficiency of sheet-piling is so detailed, for a clear description, that even a re-reading leaves one without a picture of what was intended or what was determined. The results obtained are open to serious criticism, as described more fully hereafter.

2.—The suggestion of a possible stress distribution in the passive prism (or soil-volume resisting pressure) in as far as it applies to the particular problem of the failure of certain sheet-piling at Long Beach, Calif., is based on the results of the tests, and is so full of additional assumptions, that it is not convincing. In so far as it deals with the general problem, keeping in mind the assumptions made, it is a real contribution to the study of soil mechanics.

3.—The new theory of the stability of bulkheads is an ingenious mathematical analysis.

A test is described in which deformations of seven interlocked sheet-piles, embedded 12 ft in sand and acted upon by a hydrostratic pressure of 15 ft of water, were measured. The set of sheets is described as free to deflect without restraint from the side walls of the bin holding the water. The walls of the bin also consisted of sheet-piles driven to the same depth as the test sheets. The purpose of the test was to simulate the condition of a sheet-pile bulkhead which had failed.

There can be no question as to the total value of the acting pressure, on all the sheets. However, the proximity of the rear of the bin to the tested face (about 5 ft), seriously affected the capacity of the soil to resist pressure. This was most serious near the side walls, where these walls, also acted upon by the water pressure, caused resisting stresses in the soil which was supposed to sustain the test piles. As a result, the end piles were forced to throw part of their load toward the center piles which, in addition, were not provided with a full passive prism. An inspection of Fig. 6 showing permanent deformation in the test sheets certainly bears out the foregoing statements. The loads taken by the various sheets were not equal. The analyst makes the assumption that each sheet sustained the same acting pressure. When a permanent deformation of about 2.5 in. remains after a measured maximum deformation of 4.16 in., one wonders what right the author has to use the elastic stress relation to convert strains into stress.

---

[13] Cons. Engr., New York, N. Y.

Some data for the evaluation of the parameters, unit weight, and internal friction required for the interpretation, are listed. However, the following assumptions are made in converting the measured deflections of the sheet-piling to a diagram showing the amount and sign of the pressures acting on the sheet-pile for its full height (Fig. 5):

(*a*) The angle of internal friction equals the angle of natural slope. Some difficulty was found in measuring even the angle of natural slope in the immersed or saturated state, and no attempt was made to determine the coefficient of internal friction.

(*b*) The weight of the average sand was 85 lb per cu ft with 15 to 20% moisture by weight. The great range of values in Table 1 shows a non-homogeneous fill.

(*c*) The angle of internal friction was assumed to be 30° under water.

(*d*) The active pressure of the soil inside the sheet-pile enclosure was assumed to vary parabolically; the active pressure is assumed to act at the same time as passive resistance (see Fig. 5). If the soil was saturated throughout the test, there is no question as to the amount or variation of active pressure, except as the bin action of the enclosure affects the pressure of the sand; the water pressure is definite. Active pressure and passive resistance may act on the same sheet on opposite sides, but not on the same side. The latter is equivalent to assuming motion of the sheet in two opposite directions at the same time.

(*e*) No attempt was made to evaluate the effect of water seeping from the bottom of the test bin.

(*f*) Deflections were computed for several values of a multiplier inserted into the Coulomb formula for passive resistance, and the curves were plotted. A value of 1.68 for the multiplier is arrived at, because that deflection curve, plotted for the full depth of the sheet-pile (27 ft), is considered nearest the deflection curve plotted from the observed deflections. Since the measured deflections were for the top 15 ft only, and the amount of deflections measured is about twice that computed, the real reason for this choice of factor is far from self-evident.

(*g*) Following a discussion of different assumptions as to the maximum value of passive resistance in front of, and in back of, the test wall, and the effect of wall friction and wall deformation on these values, there appears the statement (see, "Method of Graphic Analysis"): "As to the test under consideration, the passive resistance behind the wall was assumed to be 70% or less of the passive resistance in front, because the surcharge was water." Since there was no surcharge in front, why the reduction? A reduction was required for the effect of the rear wall, not for lack of earth surcharge.

The results of the test analysis are next applied to a proof that the sheet-piling bulkhead which failed should have failed. In addition to the foregoing assumptions, it should be pointed out that the bulkhead had a concrete relieving platform resting on timber piles (assumed by the author to take one-third the active earth pressure); and that the "effectiveness ratio" or multiplier of the Coulomb formula equals 2. A method of trial wedges is used to determine the wedge of minimum resistance, assuming such wedge to start at the bottom of the sheet-piling (Fig. 7). If this assumption is true, there cannot be a balancing force at the heel of the pile, and equilibrium cannot exist. The writer cannot see any justification for applying the results of the test to the analysis of the bulkhead; nor is the computed maximum deflection of 4.13 in. convincing. With a yield point of 45 782 lb per sq in., and a maximum stress of 30 000 lb per sq in., there should not be a large amount of permanent set; and, yet, the author states that the shape of the sheets pulled at the bulkhead that had failed agreed with that of the test sheets (Fig. 6) and these had a permanent set of 2.5 in. after a maximum deflection of 4.16 in.

The "Analytical Determination of Interlock Efficiency" is based on approximating a curve the equation of which is known, to the plotted shape of the test piles. The results are subject to the same criticism as that which applies to the test. In addition, the method used is only applicable to deformations within elastic limits, and the measured deformations certainly exceeded that limit.

The suggested theory of stability is a good form of attack on the problem. However, the author fails to point out that it only applies within very small limits, practically within elastic deformations of the soil structure. In stating that: "In the foregoing, * * * earth was treated like a fluid of smaller or greater unit weight than earth for active and passive resistance, respectively," the author falls into an improper use of the word "fluid." A fundamental law of fluids can be re-worded into the requirement that at all depths active pressure is equal and opposite to the passive resistance to pressure.

Before accepting the author's formulas quantitatively, the assumptions must be further checked by experiment. Equation (16) involves the assumption that the passive resistance to pressure varies inversely with an indefinite power of the depth and with the square root of the deflection. Equation (18) shows a relation from which the unknowns may be determined if the lateral resistance at various movements can be measured. However, the value at zero movement must first be measured, this being called the active pressure. The writer is convinced that at zero movement there is no method for measuring the pressure, either active or passive.

Before using the results obtained by Franzius, it should be carefully noted that the wall used in his tests was moved laterally against the soil, that the maximum pressure did not occur at the beginning of movement, but only after the body of the fill was crushed. Such complete lateral movement does not occur in the case of sheet-piling. When the wall is pushed into the fill, the total resistance measured is the sum of (*a*) the passive resistance of the earth to pressure; (*b*) the resistance to the crushing of adjacent grains; and (*c*) the resistance caused by physical displacement.

If no friction exists along the wall (a hypothetical case) the last item disappears, because the soil mass merely adjusts itself by moving upward, just as a liquid would. If friction does exist, a resistance greater than the Coulomb formula would indicate, is measured. Incidentally, the Coulomb formula does not involve the assumption that the amount of movement is large enough to induce any resistance except the passive resistance of the weight of that mass in back of the wall which will give the minimum resistance.

For the benefit of those who do not have ready access to the original papers by Franzius, a summary of results obtained have been made available[14] by the late John R. Freeman, Past-President and Hon. M. Am. Soc. C. E. The conclusions listed in Mr. Freeman's book do not agree with some of the assumptions made in this paper.

The writer cannot see how Equation (25) is derived. For deformations within elastic bounds, a term containing the tenth root of the deformation certainly cannot be very considerable in magnitude.

Following Equation (33) the author states that his analysis shows a decrease in the coefficients of internal friction and of friction between sand and wall, with an increase in depth and an increase in wall movement, and also refers to the writer's experiments,[15] for qualitative agreement. Before completing this discussion, the writer tried to find such agreement in his own report and found just the opposite. In his experimental work he found no appreciable variation in either coefficient with either depth or amount of wall movement. Experimental work on the variation of the coefficient of internal friction with applied load has consistently shown no relationship.

The writer agrees that the friction along the wall is responsible for most of the complexities in the phenomenon of passive resistance (see text following Equation (42)). A clearer picture can perhaps be drawn by considering the resistance prism as being acted upon by a drag equal to the friction along the wall. This drag is dissipated through the fill, and causes a curvature in the surface of the rupture.

In view of the foregoing remarks, the writer cannot agree with the conclusions of the paper.

R. L. VAUGHN,[16] M. AM. SOC. C. E. (by letter).—The treatment given in Fig. 7 represents an advanced method of analyzing a bulkhead, which takes into account the restraining action of the ground and the relieving action of the piles back of the bulkhead. It recognizes, as well, the fact that the passive resistance of the soil will be some value greater than that given by the unmodified Coulomb formula. The author himself states that the theory upon which this investigation is based is defective, and devotes much of the remainder of the paper to a demonstration of how hopelessly involved the discussion becomes when an effort is made to develop a better theory.

This is an excellent paper, giving the results of painstaking research and investigation; it presents much information not heretofore generally available in the English language. The writer has found some parts of the mathematical treatment confusing and difficult to follow, and ventures to suggest that the addition of a table of notation would be an improvement.

---

[14] Hydraulic Laboratory Practice," by John R. Freeman, pp. 599–603.
[15] *Transactions*, Am. Soc. C. E., Vol. LXXXVI (1923), p. 1448.
[16] San Francisco, Calif.

Perhaps a discussion of a theory of design should not be extended to include a scrutiny of the assumptions as to facts, but most engineers will agree that in any design such assumptions should conform to reasonable expectations. The author presents Fig. 7 as an analysis of the bulkhead at Long Beach, Calif. It becomes in order, therefore, to comment on the asserted physical conditions there shown, as well as to review the admitted defects in theory.

While it is generally referred to as the Coulomb formula, it is understood that the expression, $p = w \tan^2 (45° - \frac{1}{2}\phi)$, is not the exact form originally developed by Coulomb. For many years, however, this formula has been the basis for most discussions concerning earth pressures. The symbol, ϕ, was originally used to designate the angle of repose of a cohesionless granular material, and it has been so utilized in many standard texts. Numerous tables of angles of repose of various materials have been published. However, when materials are not cohesionless the term, "angle of repose," becomes meaningless, since the surfaces they will assume, will not be planes and will vary with the height of bank being observed. It has long been recognized that ϕ should be regarded as the angle of internal resistance rather than as the angle of repose. When the internal unit resistance or shear was considered to be the sum of the true friction plus a constant due to cohesion, it was possible to form a conception of ϕ by considering that it was that angle the tangent of which was the shear divided by the normal pressure. At present, it is recognized: (1) That the friction in a given material may not be constant for different depths; (2) that the cohesion varies with the moisture content and probably with the pressure; (3) that the surface of rupture is not a plane, but is dependent on the values of cohesion and friction; and (4) that, in addition to being dependent upon other things, the horizontal pressure against a wall is also some function of the motion of the wall. Recent mathematical discussions have attempted to take all these factors into account and still retain ϕ. Under these circumstances it is difficult to form any conception of ϕ, and certainly it is far from being the angle of repose. The best definition for this angle at present would be that angle which, when used in the Coulomb formula, will give the correct answer. It is suggested that confusion would be avoided if ϕ was reserved for its original meaning and if some other symbol was used in modern formulas.

The method followed by the author to determine the value of ϕ is open to two objections. It has never been proved that the surface angle of repose has any direct relation to the internal pressure, and there is no reason to think that surfaces of repose of samples 6 in. or 1 ft high will be the same as the angle of repose for a bank 20 ft high.

Permanent deformations exhibited by the piling from the test wall indicate that there must have been a considerable degree of restraint at the corners of the tank. This restraint could not be attributed to the copper seals, which appear to have sufficient flexibility, and must have been due to an arching action against the unloaded ground each side of that against which pressure was directly exerted. Any such restraint would cause the wall to appear stiffer than was actually the case and would result in a correspondingly high computed value of the interlock efficiency. This circumstance, together with the fact that the curve assumed by the piling below the surface of the ground must be estimated, would tend to cast some doubt on the experimental determination of the interlock efficiency and also on that of the passive resistance. A further uncertainty is introduced through the arbitrary assumption of the relation between the passive resistance back of the wall and the same resistance in front of the wall.

After having derived a value for the passive resistance in the test case by a process involving assumptions as to the passive resistance back of the wall, assumptions as to the water pressure in the ground, both in front of and back of the wall, and a derived value of the stiffness of the sheet-piling, the author then derives from the passive resistance thus obtained an "effectiveness factor" to be applied to the Coulomb formula. Aside from the question as to whether or not the passive resistance in the test case has been correctly determined, and aside from the question of using for ϕ values obtained from observations on the surfaces of small samples, the writer knows of no warrant that the same or any other effectiveness factor would be proper for the conditions shown in Fig. 7. If the observations have been made with any approach to accuracy, there must be something wrong about a theory that culminates in a formula the answer to which must be multiplied by a factor of 1.68 or by 2 in order to get the correct result.

Leading authorities now agree that one fault with the Coulomb formula lies in its neglect to include the effect of friction against the face of the bulkhead. This friction only becomes effective apparently when there is an appreciable movement of the wall outward. Equations (16) to (58), inclusive, are presented to show the effect of such friction and, in them, the rather new theory is introduced that ϕ is a function of the displacement of the wall.

That the Coulomb formula, with the addition of an "effectiveness factor" may give questionable results, is evident from the author's actual application of it in Fig. 7. From tests on an experimental box, as described in the paper, it was determined that the effectiveness factor should be 1.68 for this material when it had been deposited in a fill, and for the (assumedly) undisturbed material in the toe slopes it was estimated that a factor of 2 would be suitable. Using that factor, a passive resistance of 20 400 lb per lin ft of wall is obtained for the toe slope at Berth 3. Where the "net passive resistance" of 13 400 lb (referred to in the summary of findings preceding Equation (2)) came from, is not clear, but the former value is the one given in Fig. 7. A rough check on the credibility of this quantity may be obtained by considering resistance against sliding along a plane joining the bottom of the sheet-piles to the nearest point in the bottom of the channel.

The elevation of the bottom of the channel is at − 36.0 (at least, it was agreed on this work to pay for excavation to that depth). The elevation of the bottom of the sheet-piles is at − 38.75; the difference in elevation between the two points is, therefore, 2.75 ft. The horizontal distance between points is 32.5 ft, and the slope is, therefore, 2.75 ÷ 32.5 = 0.0847. Using the author's value of 52 lb per cu ft for the weight of the material, submerged, a simple calculation shows that the submerged weight of 1 lin ft of the ideal toe slope at this point is 14 600 lb. Since the prism must be shoved up an adverse grade, the actual weight should be multiplied by the factor, 1.0847, giving an effective weight against sliding of 15 835 lb. Now, if the author's value for the passive resistance of this prism is correct, the coefficient of this material against sliding on itself must be 20 400 ÷ 15 835 = 1.29. In the construction of hydraulic-fill dams a coefficient of 0.35 is considered safe, although observations have shown that, at times, the coefficient may reach a value of as much as 0.75. Any conclusion to the effect that a fine sand saturated with water can oppose a resistance to sliding equal to one and one-quarter times its own weight seems neither conservative nor reliable.

The author states that American manufacturers advocate the use of 75% of the interlocked section modulus for steel sheet-piles. As far as the writer has been able to learn no such recommendation has been made by American manufacturers except with the qualification that, if such a modulus is to be used, some positive means to prevent slipping along the interlock must be adopted before any load is applied against the piling.

To sum up the objections to the theory upon which the analysis given in Fig. 7 is based: It is assumed that ϕ is a constant and equal to the observed angle of a small sample, whereas there is reason to believe that neither assumption is true. It is assumed that the surface of rupture is a plane, whereas it is generally admitted that such will not be the case. The method takes no cognizance of the effects of friction on the piling. The manner in which the passive resistance is obtained gives a result which, for this class of material, is almost impossibly high. The use of an interlocked section modulus is questionable in the absence of definite and positive means for preventing slippage along the interlock. The author points out many of these objections in his paper.

Turning now to the assumptions of fact. These are two in number: (1) The assumption that the toe slopes would be undisturbed and should retain their theoretical dimensions; and (2) the assumption that the elevation of the water back of the wall would be + 3.5, which is practically mean sea level.

The author states that the toe slopes had been undercut and that in view of the importance of the slopes this was a dangerous procedure. The dredging was conducted in the only practicable manner by which the required channel could be excavated with a cutter type of pipe-line dredge. On the other hand, the design was seriously defective in that the bulkhead had been placed within 32.5 ft of a channel 36.0 ft deep, and that the maximum penetration of the piling was only 2.75 ft below the contract depth in the channel. Especially in view of the unstable and easily erodible character of the material at Long Beach it is not too much to state that any method of excavation would have been dangerous under the circumstances.

The manner in which an hydraulic dredge excavates is not widely known outside of dredging circles and some description is necessary. The entire machine is rotated bodily about a fixed point at the stern, while on successive swings the cutter is lowered to suitable depths, thus removing the material in horizontal layers. There is no method of control by which the depth can be varied with any degree of accuracy as the dredger swings from side to side; that is, the cutter cannot be slid down or up a slope. A floating machine, more than 200 ft long, weighing several thousand tons, controlled by wires, and often operating in rough water, cannot be manipulated with extreme precision; but it is unquestionable that, by a proper adjustment of the width of swing to the depth of cutter, the latter could be placed within a very few feet of the theoretical side-slope line on each swing. At first sight, it would be reasonable to believe that in this manner a practically perfect side slope could be excavated.

However, while the dredge can be swung to any desired width sidewise, it cannot be moved ahead, and as the cutter is lowered on successive swings a nearly vertical bank, circular in plan, is left ahead of the machine. If the material would stand in such a vertical bank until the dredge could be advanced to a position for the next series of swings, there would be no difficulty. Almost invariably, however, the bank will cave down suddenly in a large mass when the cutter has reached some point near the bottom of the cut. Much of this material will fall into a position back of the path of the cutter and cannot be reached. Likewise, some of the material will fall on to the side slopes which have just been formed, will tear loose more or less of the material which has been left there and, with that addition, will also slide down into the cut and come to rest at some point where it cannot be reached. For this reason it is usually found that if an attempt is made to dig the side slopes exactly to line, the result will be a large and unexpected amount of apparent over-dredging there. If the width of swing is narrowed in an attempt to let the material take its own slope, temporarily the slopes will stand at an unnaturally steep angle, only to cave down later and shoal up the channel after the dredge has passed.

One method of overcoming the effect of caving would be to traverse the entire length of channel repeatedly, removing one layer of material at each trip. The ordinary pipe-line dredge is not adapted to such an operation and if it were attempted the output would be greatly reduced and the cost of dredging would be correspondingly increased. No owner would be willing to meet the increased cost and no contractor who contemplated this procedure would ever get any work.

On the other hand, if an attempt is made to finish the channel on the first trip an excessive amount of over-dredging on both sides and bottom is required, as, otherwise, shoaling will occur, which will necessitate a clean-up operation. Consequently, a compromise is reached.

At least one clean-up trip cannot be avoided (except where the cuts are very light) and, therefore, it is just as well to leave enough material to be worth while on the second trip. On the first passage the dredge is swung out over the side slopes and most of the material above the slope line is removed. This operation is necessarily in a series of steps which will be partly above and partly below the slope line. An effort is made to proportion the steps so that the material which remains above the slope line, plus whatever is expected to cave from the bank ahead, will just about fill the holes below the slope line. No effort is made to bring the bottom down to grade; about 4 or 5 ft of material is left for the clean-up.

On the clean-up trip no further dredging is done on the side slopes except at the toe; in fact, the cutter is never raised more than a foot or two above the bottom at any time on this trip. Dredging along the edges of the channel will again disturb the material in the slopes and, at some delayed time, at least some additional material will slide down. If the clean-up is restricted to the specified channel any material that slides down will fall into it, shoaling it along the edges and making another clean-up necessary. It is the custom, therefore, to swing the cutter a short distance over the line in order to provide a space into which such additional materials may come to rest without encroaching into the required channel. The slopes, when of certain materials, may not reach a condition of final equilibrium for weeks or even months after the clean-up has been made. It is not possible, therefore, to determine with any degree of exactitude, by means of soundings, just how much extra width must be dug; it is necessary to guess at it. If the clean-up is carried too wide there will be much excess excavation for which no payment is received. If the swings are kept too close to the line another clean-up will be necessary. At Long Beach the records showed that, following progress surveys, it was found necessary to return and clean up various portions of the edges of the channel as often as three or four times, in order to obtain the required depth along the lines. Such operations cannot be conducted without some additional disturbance of the material in the slopes.

This stepping of the slopes and slight overcutting at the toe is the undercutting process to which the author refers. There is nothing precise about it, but it is the best that can be done. During an experience extending over a period of twenty-one years the writer has never known a job where this process, or some variation of it, was not used. In that time, he has not seen even one job on which some surplus material had not been removed from the slopes.

In view of the uncertain behavior of all earth, and in view of the unavoidable limitations of the hydraulic dredging process, it is the writer's opinion that it would be most unwise for a designer to rely upon obtaining an undisturbed side slope of specified dimensions along any channel.

Where the purpose of a bulkhead is to retain the fill back of an apron wharf, the temptation is great to place the bulkhead close to the channel line, thereby reducing the width and cost of the wharf. Observation and experience indicate that in most materials it is dangerous to depend upon the embankment above the level of the bottom of the channel for much support unless the horizontal distance between the bulkhead and the channel is at least one and one-half times the vertical distance from top of fill to bottom of channel. Even with such proportions, the field engineer should be prepared to reinforce the embankment with rock at the first sign of weakness.

The second assumption of fact has to do with the water pressure to which the wall was subjected. The method used to determine the water level would have been satisfactory if this had been merely an investigation to determine the stability of the bulkhead at the time the observations were taken, but such was not the case. This analysis was undertaken to determine whether or not the original design had been adequate. The actual water load which the bulkhead would be unavoidably called upon to resist during the construction period is far in excess of that corresponding to the assumed water level. This is the most serious error in the analysis. Even had all assumptions as to section modulus, active pressures, and passive resistances, been fully realized, the bulkhead would have failed if the fill had been placed by the hydraulic method, which is the one specified in the contract. Since, undoubtedly, unsuspected water pressures have been at the bottom of many bulkhead troubles, and since the pressures that may arise in an hydraulic fill have been but little discussed and are not widely understood, it follows that this topic is worthy of some consideration.

During the progress of the trial to which the author refers an effort was made to justify the assumed water level on the grounds that a preliminary embankment should have been placed behind the bulkhead and that this, together with the relieving platform, would have prevented any higher level against the wall. In fact, even if suitable material for the purpose had been available, the preliminary embankment would have been of only limited value in this case, while the relieving platform made things much worse instead of better.

There is a tradition in the dredging business that the best thing to do with a weak bulkhead is to pump an embankment behind it before impounding any head of water. The writer has not encountered a completely satisfactory explanation of the manner in which this procedure relieves the pressure against the bulkhead; and it appears that it may not always do so.

The usually accepted explanations are: (*a*) That the materials first deposited have time to settle, consolidate, and develop cohesion; (*b*) that the presence of the embankment will prevent the crowding of any semi-liquid fill against the bulkhead; (*c*) that the natural grading process due to the action of the water leaves the heaviest particles, presumably those with the greatest coefficient of internal resistance, against the bulkhead; and, finally, (*d*) that the embankment "keeps the water away from the bulkhead." Except for the last one these explanations appear rather reasonable.

When discharging at Long Beach, the dredge was pumping at the rate of about 50 cu ft per sec, of which only about 7% was solid material; the remainder was water. When an embankment is pumped against a bulkhead, together with such a surplus of water, it is impossible to believe that the material is not completely saturated, which means, of course, that the water level in the soil will rise to the top of the bulkhead. It follows, therefore (according to the commonly accepted theory), that at the time the embankment is being placed against the structure the latter must be withstanding a full head of water. However, bulkheads have been used, successfully, which obviously could not withstand such a head of water, let alone an additional pressure due to earth. A question naturally arises as to how such a state of affairs could be possible.

The answer to this seeming paradox lies partly in the character of the bulkheads which have been used and partly in a rather obscure phase of the behavior of ground-waters. Imagine, first, a high conical pile of earth resting on a flat impervious surface. Suppose this earth to be saturated and to be kept saturated by a constant addition of water at the top of the cone. According to the theory of water pressures in saturated earth, it might be supposed that the water pressure at the base, immediately under the apex, would be that due to a head of water equal to the height of the cone. Such would not be the case, however, because the water would be free to flow downward and outward through the earth. Any flow of water through the earth results in a relatively great loss of head. The true variation in pressure throughout the cone involves some complicated considerations, but it is obvious that flow would take place and, therefore, full static head would not exist.

Suppose that the cone is divided vertically and that one-half of it is replaced by an impermeable bulkhead. From considerations of symmetry it is evident that the pressure against the bulkhead would be that which had existed along the same plane in the cone. This now represents the condition that exists when pumping is first started to place an embankment back of a wall. As the fill accumulates, the end of the pipe, the apex of the cone, is gradually shifted along the wall until finally a complete embankment has been placed. As the pipe end is shifted there follows beneath it a triangular area of water pressure, the intensity of which varies from a maximum vertically under the pipe to zero at the edge of the cone of saturation. It is evident that the total area of bulkhead subjected to water pressure at one time is limited, so that some support may be gained from adjoining parts of the wall. It is also evident that at no point is there a pressure due to the full height of water above it. In the actual case, conditions are further complicated by the circumstance that the surface supporting the embankment is also permeable, thus permitting a direct downward flow and a further reduction in water pressure. If the supporting ground happens to be saturated, the variation in pressure throughout this ground-water also becomes complicated.

If the bulkhead, instead of being impermeable, has numerous leaks through it there will be an additional and very substantial reduction in pressure corresponding to the increased velocity in the water of saturation occasioned by the escape of water through the leaks. Until recently nearly all bulkheads encountered in the dredging business have been wooden ones which were anything but tight. It has been in connection with such leaky bulkheads that the embankment technique has been developed. This may be a most important point. Some modern types of bulkhead should be nearly impermeable, and it is likely that such structures will not experience the degree of relief from water pressure through the use of an embankment which past experience with leaky bulkheads would indicate. All designers of impermeable bulkheads for use in connection with hydraulic fills should give the subject of water pressure serious consideration.

It is thus seen that, provided no water is impounded, it is possible to pump an embankment back of a wall and yet avoid a full hydrostatic head equal to the height of saturated earth. After the embankment is completed a different set of conditions arise. In the great majority of cases it is necessary to impound water to the full height of the fill before the latter can be completed. At this time, if the bulkhead is water-tight and the embankment placed behind it is permeable, the impounded water will find its own height against the bulkhead, and, as far as water pressure is concerned, the embankment is of no benefit. If the reverse condition obtains, the water can run out through the bulkhead faster than it can flow through the embankment, and there may be little, if any, water pressure against the wall.

The bulkhead at Long Beach belonged to the former class; as designed it should have been practically impermeable, while the sheet-piles had sufficient penetration to impede, greatly, any underflow. The only material available for an embankment was a sand that was relatively permeable. Under the most ideal performance the embankment would have been of limited benefit, and it would have been impossible to avoid the accumulation of substantial heads against some parts of the structure. Certainly, the water would have risen far above mean sea level, the elevation used by the author in his analysis.

In the design, Berths 1, 2, 3, and 12 had been provided with a concrete relieving platform more than 50 ft wide, supported on a veritable forest of piles. The presence of this platform made it impossible to place an effective embankment against the bulkhead. Furthermore, the platform had numerous small openings in it, sufficient to have transmitted, freely, pressure from any ground-water that might have accumulated above it; and it is inconceivable that a water-tight fill could have been deposited against the under side of the platform by any means whatever; there must have been numerous spaces at this point of a capacity sufficient to permit the unobstructed flow of water from points back of the platform to the face of the bulkhead and to have permitted the transmission of pressures through the water filling such spaces.

Relieving platforms are supposed to do what their name implies, relieve pressure against the bulkhead. It seems proper to point out that under some circumstances they may defeat their own purpose. In the present instance, for illustration, suppose that, first, the space under the platform had been filled as well as possible by means of pumping or sluicing, and that then an impermeable fill had been deposited on top of the platform by any suitable means. This produces as good an embankment behind the bulkhead as could be obtained by any reasonable and practicable method. The final step would then be to fill the remainder of the area by pumping. To do this it is necessary to raise the ground-water to the top of the fill. There would be nothing to prevent water finding its way along the under side of the platform to the face of the bulkhead and transmitting full hydrostatic pressure to that point. In the imagined illustration, the platform would be cutting off the pressure due to a fill of about 12 ft, undoubtedly, but it would be letting in the pressure due to 12 ft of water which is much worse. It is suggested, therefore, that in the case of hydraulic fills, relieving platforms may have a most dubious value.

At present, there is a prospect that some rather high débris dams may be built in California. If this should come to pass it is to be hoped that devices for measuring pressures (both of water and of materials) at various depths and at various time intervals will be incorporated in them. If the variety of débris impounded should cover a wide range in different places, then after sufficient observations had been taken it might be possible to evolve an empirical method of determining earth and water pressures which would be, at the same time, reasonably simple and fairly reliable.

M. A. DRUCKER,[17] ESQ. (by letter.)—The experiment on full-sized steel sheet-piling, under field conditions approximating those to which actual structures are subject, should prove of great value when designing this rather recent type of construction. It is of special importance at the present time, since the assumptions upon which designs are based, have recently undergone a radical change. At about the time when, as the author states, high bulkhead walls began to be constructed, the depths of sheeting embedment and the stresses in anchor and sheeting were determined by considering an active pressure greater than, and a passive resistance less than, the theoretical values obtained for them. Professor Wolmar Fellenius[18] recommended that the active pressures be taken as 25% greater, and the passive resistance as 25% less, than their theoretical values. This is quite different from the assumption that the actual passive resistance is twice the theoretical.

The author is quite right in pointing out that the present theory of bulkhead design is defective in that it does not take into consideration the compression of the soil due to the movement of the wall. By taking the effect of this compression into account he obtains a passive resistance, as illustrated by Fig. 14, which is quite at variance with that obtained by following the Brennecke-Lohmeyer method, as illustrated by Fig. 5. Mr. Baumann's method satisfies the main conditions necessary to be met when using the latter theory. These conditions are: (*a*) The moments about any point on the sheeting must balance, and their sum be zero about the bottom; and (*b*) the sum of the loads on one side of the sheeting must equal the sum of the loads on the other side. With these conditions as a basis, the design is then determined by means of a graphical method.

At first glance this method appears to be highly scientific. On analysis, however, it becomes apparent that, in effect, it is based primarily on an arbitrarily assumed point of zero moment below the ground surface which, in turn, determines the passive resistance distribution for the entire depth of embedment. The fact that the moment diagram, based on this passive resistance distribution, shows that the moment is zero at about the same distance below the ground surface, as originally assumed, can be taken as a check on the arithmetical or graphical work, but not as a proof of the validity of the assumption.

---

[17] Structural Designer with Board of Transportation, New York, N. Y.

[18] "Calculation of Docks and Bulkheads Walls," *Engineering Record*, September 20. 1913.

In accordance with this method the depth of embedment, t, is obtained from Equation (2), in which, a is the distance from the ground surface to an assumed point of zero moment, and t_0 is equal to $\sqrt{\dfrac{6\,B_0}{2\,p_p - p_a}}$, as will be shown subsequently. The symbols under the radical denote the following: p_p is the theoretical passive soil resistance; p_a, the unit active pressure increment below the ground surface; and B_0, the reaction at the point of assumed zero moment, then,

$$t = a + 1.2 \times \sqrt{\frac{6\,B_0}{2\,p_p - p_a}} \quad \text{..................} (68)$$

Pennoyer's formula[19] for the necessary depth of embedment is,

$$t = 1.1\left(a + \sqrt{\frac{6\,B_0}{2\,p_p - p_a}}\right) \quad \text{..................} (69)$$

The value of t, obtained by using Equation (68), is somewhat larger than that obtained from Equation (69). It may be noted that the latter gives the same depth of embedment as would be obtained by simply balancing moments, of all pressures, about the anchor rod, using a passive resistance equal to the theoretical value instead of twice that value.

The following analysis will show that the assumption of zero bending moment at the distance, a, below the original ground surface, or bottom of channel, also determines the passive resistance distribution below Point C, Fig. 16. Furthermore, the only significance of this point is that the active pressure per unit area is equal to the unit passive resistance.

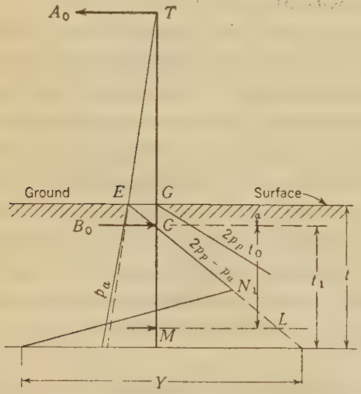


FIG. 16.

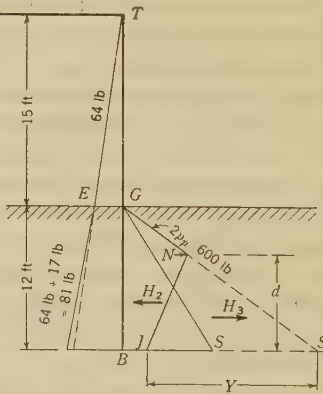


FIG. 17.

---

[19] "Design of Steel Sheet-Piling Bulkheads," by Raymond P. Pennoyer, Assoc. M. Am. Soc. C. E., *Civil Engineering*, November, 1933, p. 615.

In Fig. 16, t designates the total depth of embedment; $t_1 = t - a$; and t_0 is that distance below Point C that would make the moment about Point M, of the pressure triangle, MCL, equal to the moment of B_0 about that point, or $B_0 t_0 = (2\, p_p - p_a) \times \frac{t^3_0}{6}$, from which,

$$t_0 = \sqrt{\frac{6\, B_0}{2\, p_p - p_a}} \qquad (70)$$

It also follows that,

$$B_0 = \frac{t^2_0}{6} (2\, p_p - p_a) \qquad (71)$$

As the moment at Point C is assumed to be zero, the part of the sheeting designated by t_1 may be treated as a cantilever sheeting of this length, subject to a force, B_0, at the top, and embedded in a soil the passive resistance of which is $(2\, p_p - p_a)$. For such a case the passive resistance distribution may be readily determined from the formula,

$$Y = \frac{[(2\, p_p - p_a)\, t^2_1 - 2\, B_0]^2}{(2\, p_p - p_a)\, t^3_1 - 6\, B_0\, t_1} \qquad (72)$$

which is based on having the horizontal forces, and their moments about any point, balance.

Substituting in Equation (72) the value of B_0 as given by Equation (71) and, also, substituting $\frac{t_1}{1.2}$ for t_0, the value of Y is found to be $1.93\, t_1 \times (2\, p_p - p_a)$, and,

$$\overline{JB} = 0.93\, t_1\, (2\, p_p - p_a) \qquad (73)$$

If d represents the distance of the point, N_1, above B, then, also, as obtained for cantilever sheeting,

$$d = \frac{(2\, p_p - p_a)\, t^2_1 - 2\, B_0}{Y} \qquad (74)$$

By making the substitutions indicated,

$$d = 0.40\, t_1 \qquad (75)$$

If the depth, t, is obtained from Equation (69), then, by making substitutions similar to the foregoing, $\overline{JB}$ and d were found to have the following values:

$$\overline{JB} = 1.60\, t_1\, (2\, p_p - p_a) \qquad (76)$$

and,

$$d = 0.28\, t_1 \qquad (77)$$

Although Equations (68) and (69) are based on the same, or on new, principles of bulkhead design, a comparison of the values of JB, as obtained from Equations (73) and (76)—as well as the values of d from Equations

(75) and (77)—show that the pressure distributions are quite different despite the fact that each of the values will satisfy the conditions for such design.

It will now be shown that for any depth of embedment, t, which is greater than that required for the assumed actual unit passive soil resistance, any number of passive resistance distributions may be obtained by varying the position of Point N (Fig. 17), and yet satisfy Conditions (a) and (b). On this basis, however, the moment may not be zero at a distance, a, below the ground surface. The writer has seen no good reason given, however, and none is apparent, why the moment should be zero at that distance below the ground surface. For this analysis, the pressures, length of sheeting, and depth of embedment will be taken the same as given for the author's experiment (see Fig. 17).

The total pressure on the sheeting due to water is $64 \times \frac{27^2}{2} = 23\,300$ lb. The active pressure due to the soil, which has an effective weight of 52 lb, will be taken as 17 lb per sq ft per ft of depth, giving a total pressure of $17 \times \frac{(12)^2}{2} = 1\,220$ lb. The sum of these pressures is 24 520 lb. The sum of their moments about the point, T, is $23\,300 \times \frac{2}{3} \times 27 + 1\,220 (15 + \frac{2}{3} \times 12)$, or 448 000 ft-lb. If it were assumed that these pressures are resisted at the bottom of the sheeting by a uniformly varying pressure of a total magnitude, H_2, represented by BGS (Fig. 17) then $H_2 = \frac{448\,000}{15 + 0.67 \times 12} = 19\,500$ lb, giving a required unit passive resistance equal to $\frac{19\,500 \times 2}{(12)^2} = 270$ lb. The moment of H_2 about Point $B = 19\,500 \times 4 = 78\,000$ ft-lb. For this condition, then, taking moments about B, $27 A_0 = 23\,300 \times 9 + 1\,220 \times 4 - 78\,000 = 215\,000 - 78\,000$, which equals 137 000 ft-lb, and $A_0 = 5\,070$ lb. This stress in the anchor rod, per foot of wall, is considerably greater than the several values obtained by the author. It is probably much larger than would exist for the case of a steel sheeting. It was obtained here for the purpose of indicating what the anchor stress would be in case the actual passive resistance were not larger than the theoretical value. This large stress may also be taken as an indication of what the anchor stress might be in the case of a rigid bulkhead which condition would be approached if a concrete wall were used. While it seems reasonable to consider that steel sheeting has some flexibility, it does not seem likely that it could be sufficiently strong to withstand the pressure it is subject to and yet be flexible enough to exert a large back-kick at the bottom of the sheeting at the same time that it also exerts a large pressure at the front of the sheeting only a short distance above the bottom, as shown in Fig. 5.

However, assuming that the actual passive resistance is equal to twice the theoretical value, and that the sheeting has some flexibility, Conditions (a) and (b), previously stated, may be satisfied for any number of passive resistance distributions at the front of the wall for the same depth of embedment. These distributions may be obtained by assuming Point N (Fig. 17) at different distances above Point B.

After assuming the position of Point N, the next step is to determine the value of the negative pressure triangle, JNS_2, represented by H_3, which is that part of the pressure, BGS_2, which does not need to be called into action. The magnitude of this pressure may be obtained by taking moments about Point T; and, after obtaining H_3, the value of A_0 may be obtained by taking moments about Point B, so that, for a passive resistance equal to $2\,p_p$, or 600 lb, the pressure triangle, BGS_2, which will be referred to as H_4, equals $600 \times \frac{(12)^2}{2} = 43\,200$ lb, and its moment about Point T is $43\,200\,(15 + 0.67 \times 12) = 995\,000$ ft-lb. This moment, minus that of active pressures about the same point, would be equal to the moment of H_3 about that point; or, $H_3\left(27 - \frac{d}{3}\right) = 995\,000 - 448\,000 = 547\,000$ ft-lb. For balancing moments about Point T, then, the magnitude of H_3 would depend on d, giving,

$$H_3 = \frac{547\,000}{27 - \frac{d}{3}} \quad \text{.......................} \quad (78)$$

Similarly, the stress in the anchor rod, A_0, would depend on the value of d and H_3, and could be obtained by taking moments about B. The moment of H_4 about $B = 600 \times \frac{(12)^3}{6} = 172\,800$ ft-lb, while the sum of the moments of the active pressures, about this point, is 215 000 ft-lb, as previously obtained. Therefore, $27\,A_0 = 215\,000 - 172\,800 + H_3 \times \frac{d}{3} = 42\,200 + H_3 \times \frac{d}{3}$, or,

$$A_0 = \frac{42\,200 + H_3 \times \frac{d}{3}}{27} \quad \text{....................} \quad (79)$$

Furthermore, if $\overline{JS_2} = Y$, then,

$$Y = \frac{2\,H_3}{d} \quad \text{.........................} \quad (80)$$

and,

$$\overline{BJ} = 7\,200 - Y \text{.........................} (81)$$

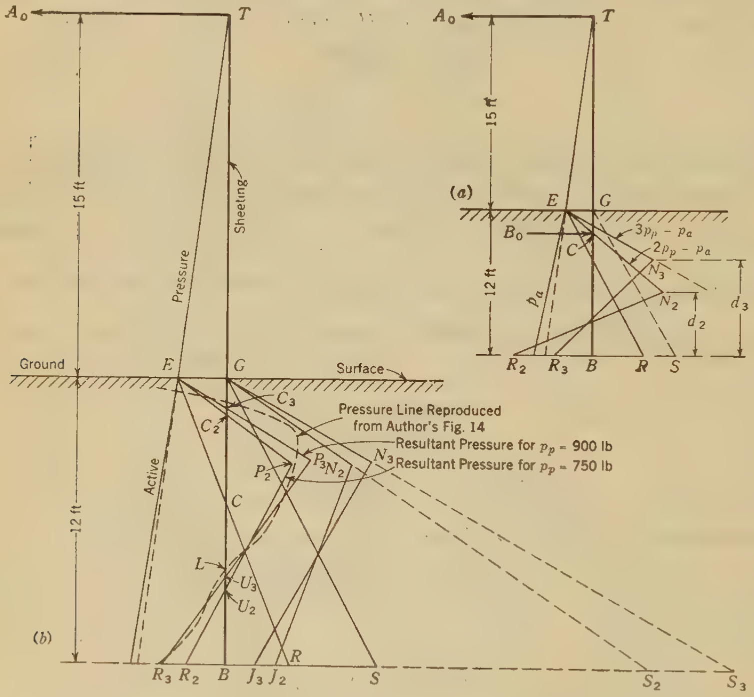
To balance horizontal pressures, $H_4 + A_0 - H_3 = 24\,520$ lb, or,

$$H_3 - A_0 = H_4 - 24\,520 = 43\,200 - 24\,520 = 18\,680 \text{ lb.....} (82)$$

Table 4 gives the values, obtained by means of Equation (78) to Equation (82), for several distances of d arbitrarily assumed. The closeness of the values in Column (7) to 18 680, given by Equation (82), shows that the horizontal pressures balance for all values of *d*. The moments balance, of course, because the values of H_3 and A_0 in Table 4 (Columns (4) and (6)), were obtained on that basis; so that, while the Brennecke-Lohmeyer method,[20] which is essentially the same as the Blum method,[21] gives a passive resistance distribution that checks mathematically, other possible distributions may be obtained which check likewise. In fact, by assuming a large passive resistance for the soil, and a small value for *d*, it would be possible to satisfy Conditions (*a*) and (*b*) and to obtain not only a small tensile anchor stress, but even a result

TABLE 4.—COMPUTATION OF EQUATION (78) TO EQUATION (82)

Distance, *d*, in feet (1)	Equation (78)			Equation (79)		Equation (82) (7)	Equation (80) (8)	Equation (81) (9)
	$\frac{1}{3}d$ (2)	$27-\frac{1}{3}d$ (3)	H_3 (4)	$\frac{1}{3}H_3 d$ (5)	A_0 (6)			
3........	1	26	21 000	21 000	2 340	18 660	14 000	—6 800
6........	2	25	21 850	43 700	3 180	18 670	7 280	— 80
9........	3	24	22 800	68 400	4 100	18 700	5 070	+2 130
12........	4	23	23 750	95 000	5 070	18 680	3 960	+3 240



A_0
T
15 ft
(a)
E
G
B_0
C
$3p_p - p_a$
$2p_p - p_a$
N_3
N_2
p_a
12 ft
d_2
d_3
R_2
R_3
B
R
S
A_0
T
15 ft
Sheeting
Pressure
Ground
E
G
Surface
C_3
C_2
Pressure Line Reproduced from Author's Fig. 14
Resultant Pressure for p_p = 900 lb
Resultant Pressure for p_p = 750 lb
P_3
N_2
N_3
P_2
Active
C
12 ft
L
U_3
U_2
(b)
R
R_3
R_2
B
J_3
J_2
S
S_2
S_3

FIG. 18.

---

[20] *Die Bautechnik*, No. 5, 1930.

[21] Carnegie Steel Sheet-Piling, Insert B.

indicating that the rod is in compression. This would be an absurdity, of course, because even if the sheeting could act as a cantilever, it would deflect away from the soil at the top of the wall.

That it is irrational to assume zero moment at Point C, Fig. 18(a), may be shown in another way. For a passive resistance of $2\,p_p = 600$ lb and an active pressure below the ground surface = 81 lb, $a = \frac{960\ (= EG)}{600 - 81} = \frac{960}{519} = 1.85$ ft. Therefore, $t_1 = 12.0 - 1.85 = 10.15$ ft.

Treating the part, TC, as a simple span and taking moments about Point T, B_0 was found to be 5 100 lb, and, after substituting values in Equation (72), BR_2 was found to be 2 780 lb, and from Equation (74), d_2 was found to be 5.4 ft; also, A_0 was found to be 3 020 lb. for this condition.

Similarly, for an assumed actual passive resistance of $3\,p_p$, or 900 lb, the following values were obtained: $a = 1.17$ ft; $B_0 = 4\,990$ lb; $BR_3 = 1\,470$ lb; $d_3 = 8.32$ ft; and $A_0 = 2\,780$ lb.

For the purpose of comparison, some of the foregoing values, as well as the corresponding ones previously obtained for a required passive resistance of 270 lb represented by Triangle BGS are given in Table 5.

TABLE 5.—COMPARISON OF STRESSES

Assumed passive resistance, in pounds (1)	PRESSURE AT BOTTOM OF SHEETING		Anchor stress, A_0, in pounds (4)
	Distance (see Fig. 18(a)) (2)	Pressure, in pounds (3)	
270	BR	+1 310	5 070
600	BR_2	—2 780	3 020
900	BR_3	—1 470	2 780

It would be reasonable to expect that the values in any column of Table 5 should vary approximately as the assumed passive resistance, or, at least, they should vary consistently. As the values for the passive resistance of 270 lb were obtained by simply balancing moments about Points T and B, which may be termed the "ordinary method," in which no arbitrary assumptions are involved, these values may be taken as the basis for comparing the others. It is seen from Table 5 that for a passive resistance of 600 lb, the pressure at the bottom of the sheeting is 4 090 lb less than for a passive resistance of 270 lb, while for a resistance of 900 lb, this pressure is 1 310 lb greater than for the resistance of 600 lb. Thus, these pressures do not even vary in the same direction although those for the 600 and 900-lb resistances were obtained by the same method. From Column (3), Table 5, and Fig. 18(a), it may be inferred that the higher the assumed passive resistance the nearer will the pressure at the bottom of the sheeting approach that for the passive resistance of 270 lb. This shows clearly that the assumption of zero moment at the distance, a, below the ground surface is not reasonable. This may also be seen by comparing the values in Column (4), Table 5. The difference between the A_0 values for a passive resistance of 600 lb and that for 270 lb is about eight times as great as that between the values for 900 and 600 lb.

In view of such inconsistent results, obtained by assuming zero moment at the point where a line drawn through Point E and representing the assumed passive resistance, minus the active pressure, reaches the sheeting line, TB, and based on analyses which will not be gone into here, the writer suggests that the formula,

$$d = t\,(0.65 + 0.02\,r) \qquad (83)$$

will give approximately accurate passive resistance distribution for any assumed unit actual passive resistance.

In Equation (83), d is the distance from the bottom of the sheeting to the point, N, and r is the ratio of the assumed unit passive resistance to that unit resistance for which a uniformly varying pressure from Point G to Point B would balance the moments about Point T. Generally, the latter resistance is the theoretical passive resistance of the soil.

In Fig. 14, Mr. Baumann gives the pressures, moments, and deflections based on his analysis of passive resistance as indicated in Fig. 11. In the paragraph preceding "Conclusions," the author states that he found it necessary to apply a factor of 2.5 for the passive resistance of the soil, in order to establish equilibrium. This factor would give a passive resistance of 2.5 × 300, or, 750 lb. As the unit passive resistance required for a uniformly varying resistance for the entire depth of embedment was found by the writer to be 270 lb for a passive resistance of 750 lb, $r = \frac{750}{270} = 2.78$. Substituting this value in Equation (83), d is found to be $0.705\,t$, or 8.45 ft.

As previously found, the sum of the active pressures is equal to 24 520 lb; the sum of their moments about Point T equals 448 000 ft-lb; and the sum of their moments about Point B equals 215 000 ft-lb. With GS_2, Fig. 18(b), varying at the rate of 750 lb per ft of depth, and H_3 representing the pressure, $J_2\,N_2\,S_2$, that will have to be deducted in order to balance moments about the point, T, then, $H_3\left(27 - \frac{8.45}{3}\right) = (2.78 - 1) \times 448\,000 = 800\,000 = 24.18\,H_3$, and, $H_3 = 33\,100$ lb; from which $\overline{J_2\,S_2} = \frac{33\,100 \times 2}{8.45} = 7\,830$ lb.

As $\overline{BS_2} = 750 \times 12 = 9\,000$ lb, $\overline{BJ_2} = 1\,170$ lb. Furthermore, the value of A_0, for this condition, was found to be 3 420 lb.

The resulting pressure distribution, after subtracting the active pressure from the passive resistance, is $\overline{TEC_2} + \overline{C_2\,P_2\,U_2} + \overline{U_2\,R_2\,B}$, as shown on Fig. 18($b$). The author's pressure line, from Fig. 14, is reproduced as a broken line. This pressure distribution is higher and also comes to the left of that just described. This is due to the fact that, in accordance with Mr. Baumann's analysis, as shown in Fig. 11, the passive resistance line is a convex curve instead of having a linear variation.

To approach more nearly the author's resulting pressure line, and still assume a linear variation for the passive resistance, a resistance factor of 3 was assumed, or a value of 900 lb per sq ft. On this basis, $r = \frac{900}{270} = 3.33$, and $d = 8.6$ ft, giving a resultant pressure distribution, $\overline{TEC_3} + \overline{C_3P_3U_3} + \overline{U_3R_3B}$. This approaches more closely the resultant pressure distribution shown by the broken line. The value of A_0 for this condition was found to be 2 980 lb which compares very closely with Mr. Baumann's value of 2 920 lb. It may be noted that the pressure at the bottom of the sheeting decreases consistently as the value of r increases.

While the author states that the compression on the soil depends on the movement of the sheeting, it appears that he overlooked this point when obtaining the deflection curves on the diagram to the left of Table 2. For the curve marked, $e_p = 1.00$, for instance, the shift at the bottom of the sheeting is indicated to be zero. For such a unit resistance, the passive resistance distribution would be the same as *BGS* in Fig. 18 (*b*), and in order to develop the pressure, *BS*, the sheeting would have to move forward at the bottom.

The method of designing sheeting by considering the resultant pressures—that is, the passive resistance minus the active pressure—involves an element of risk in that the actual pressures may be overlooked. This statement is prompted by the manner in which the author obtains the factor of safety against the wall being pushed out for the case illustrated by Fig. 7. In the text preceding Equation (2), he finds this factor of safety to be 2.42. The writer, however, found this factor of safety to be 1.64 in a manner outlined as follows:

Sum of A_1 to A_9, inclusive (Fig. 7), in pounds	= 12 155
Active pressure, in pounds, below ground surface $= \frac{14.25}{2}(620 + 480)$	= 7 850
Total active pressure, in pounds	= 20 005
Anchor stress, in pounds, is given as	= 7 555
Pressure, in pounds, to be resisted at the bottom of the sheeting	= 12 450
Total passive resistance, in pounds, $= 201 \times \frac{(14.25)^2}{2}$	= 20 400
Factor of safety $= \frac{20\,400}{12\,450}$	= 1.64

As indicated, this factor of safety (1.64) was obtained on the basis of the passive resistance being 201 lb, which is twice the theoretical value for this case.

Raymond P. Pennoyer,[22] Assoc. M. Am. Soc. C. E. (by letter).—An ingenious treatment of an involved subject is described in this paper which clearly indicates careful research, and which contains valuable contributions toward the logical design of sheet-pile bulkheads.

---

[22] Structural Engineer, Carnegie Steel Co., Pittsburgh, Pa.

Although the writer questions whether Mr. Baumann's accurate development of the passive resistance of soil can as yet be made of practical value, in view of the present-day uncertain knowledge of the other essential properties, and although he differs very seriously with the conclusion that 75% of the interlocked value of a sheet-pile can be assumed, his paper has shown methods whereby some of the problems encountered in design can be solved along sound engineering lines.

One example that has been of great assistance to the writer is the application of Culmann's *E*-line method to the passive resistance value of a berm.

It is regrettable that Mr. Baumann did not elaborate his test apparatus, indicated in Fig. 2, and that he did not take more deflection observations under various loadings, because the conclusions reached by his excellent treatment of the theory involved would then have been much more convincing. A steel frame, as nearly rigid as possible, around the enclosure, with the steel sheet-piles bearing against it, would have eliminated the necessity of assuming the supporting value of the soil, and the problem would have been reduced to a study of a simply supported beam with a fixed support at one end. If, after observing deflections, the frame had been removed, the supporting value of the soil could then have been determined accurately from another set of loads and deflection readings.

To add to the uncertain value of the test, only one set of deflection readings was secured, and that only after the steel was stressed beyond its elastic limit. This is clearly indicated in Fig. 6, showing the permanent "set" of the steel after the test. Mr. Baumann does not state whether the measurements were taken before or after the sheet-piles were pulled, or whether the piles were examined for straightness before the test. The fact that they all showed different amounts of "set," illustrates the erratic behavior of steel when stressed beyond its elastic limit.

If several sets of deflection readings had been taken with the water at increasing elevations within the enclosure, so that the steel was loaded within its elastic limit, and if the results were then examined, a conclusive study would have resulted. The writer, however, ventures a statement that, as the load increased on the sheet-piling and slippage progressed in the interlocks, values of the interlocked section modulus under the different loads would have been at such variance that no conclusions as to what could actually be assumed for the interlocked value would be possible.

Mr. Baumann is to be complimented on the ingenious and unique procedure whereby he has determined the actual section modulus of the bulkhead from the observed and calculated deflections. However, if the elastic limit of the steel had not been exceeded, the same results could have been secured by a simple calculation, using the known value of 29×10^6 for the modulus of elasticity of steel. The author's attempt to plot a curve giving values for the modulus of elasticity of steel, indicated in his Figs. 4 and 9, as a result of only seven tests, renders the conclusions of doubtful value. Another serious fact that has been overlooked is that steel follows no constant law for the ratio of stress to strain beyond the elastic limit. and the fundamental calculations in the paper are based on steel always having the same strain at the same stress. Steel, when stressed beyond its elastic limit, is most erratic in its behavior. That tests made from similar steels do not then show the same strain for equal stress, is well illustrated in the Final Report[23] of the Special Committee on Steel Columns and Struts. The modulus of elasticity of the steel in the author's Fig. 4 leaves the straight line at about 37 000 lb per sq in. If Fig. 4 were constructed to a larger scale beyond this point, the writer believes it would be seen that the curve shown could not have been drawn from accurate application of test values.

The writer further questions the value of the final figures in view of the accuracy that is necessary in the intermediate calculations when these values are founded on graphical readings and interpolations, with special reference to a variable modulus of elasticity of steel which has been assigned a value to an accuracy of 10 000 units by graphics.

The actual longitudinal shear along the interlocks of the sheet-piles in the test at the tie-rod, where it is maximum, can be calculated from the following equation:

$$s = \frac{S\,M_s}{I} \quad \text{..........................} (84)$$

in which, s = longitudinal shear, in pounds per linear inch; S = horizontal shear, in pounds, = 2 920 lb per ft (Fig. 14 of the paper) = 3 833 lb for a 15¾-in width of pile; M_s = moment of resistance per foot of width = area (square inches) × distance from center of interlock to the neutral axis of the sheet-pile = 7.9 × 1.7 = 13.43; and, I = interlocked moment of inertia = 27.35 in.^{2}

The actual shear is found to be about 1 880 lb per lin in. This shear must be resisted by friction at the point investigated because, if the shear exceeds the friction value at any point, slippage will occur. Any claim that a sheet-pile interlock sufficiently free to drive, no matter what its design, will, after driving (and without welding or otherwise positively preventing slippage), develop sufficient friction in itself to eliminate slippage when the load is applied, cannot be substantiated.

From a study of the figures derived, it is difficult for the writer to understand what has justified Mr. Baumann's conclusion that 75% of the full interlocked value of a sheet-pile can be assumed invariably. The only certain value which can be assigned to a sheet-pile in which the interlocks are on what must be the neutral axis in assuming an interlocked value, is that of the single, non-interlocked pile, when one considers such variables as conditions of loading, straightness, lengths of the sheet-piles, and unavoidable irregularities in the interlocks. The sheet-piles might even hold to one value during the short period of a test, but there is no guaranty that the interlocks will not slip later and that the value will be reduced to that which approximates the non-interlocked single pile. The interlocked value, or any portion thereof, should not be used, unless the interlocks are welded or longitudinal slippage is positively prevented in some other manner. There is danger of overstressing the steel in assuming a section modulus greater than that of the sheet-piling.

---

[23] *Transactions*, Am. Soc. C. E., Vol. LXXXIII (1919–1920), p. 1630, Pl. XXXV.

The writer emphatically denies that American manufacturers advocate the use of 75% of the interlocked value of a sheet-pile, as stated in Conclusion 5 of the paper. On the contrary, American manufacturers have always been consistent in advocating only the use of the single, non-interlocked value of a sheet-pile, unless the possibility of longitudinal slippage is positively eliminated.

Penetration.—The author does not explain the derivation of Equation (3) for the calculation of the penetration of the piling which develops the full constraint of the earth. This equation may be compared with a similar one derived by Dr. H. Blum and the writer in another connection.[24] Its derivation is here illustrated in Fig. 19(*a*). The passive resistance of the soil is treated as linear. The method eliminates any necessity for approximating the relative passive value of the soil on the front and the back of the wall.

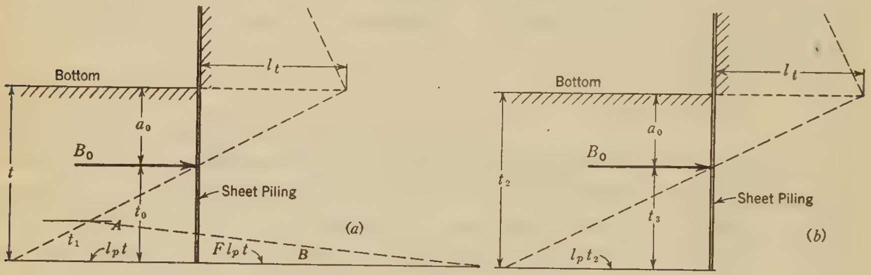


FIG. 19.

With Mr. Baumann's notation let: $\lambda_p - \lambda_a$ = net increment of the passive resistance value of the earth, as developed by the Coulomb formula, e_t = active pressure at the bottom, in pounds per square foot; and B_0 = equivalent reaction of the equivalent simple beam on the bottom support, in pounds per foot of width as determined graphically.

Doubling the value of the passive resistance as derived by the Coulomb formula, the location of the support of the equivalent beam below the bottom is, in feet:

$$a_0 = \frac{e_t}{2\ (\lambda_p - \lambda_a)} \qquad (85)$$

Furthermore,

$$\text{Area } A = \frac{(2\ \lambda_p - \lambda_a)\ t_0^2}{2} \qquad (86)$$

and,

$$\text{Area } B = \frac{(2\ \lambda_p - \lambda_a)\ t_0\ (I + F)\ t_1}{2} \qquad (87)$$

---

[24] *Civil Engineering*, November, 1934, Equation (9), p. 618.

in which, F = the ratio of the passive resistance value of the soil on the right or fill side of the wall to the left or outside. Equating pressure loads:

$$B_0 = \frac{(2\,\lambda_p - \lambda_a)\,t^2_0}{2} - \left[\frac{(2\,\lambda_p - \lambda_a)\,t_0\,(I + F)\,t_1}{2}\right] \ldots\ldots (88)$$

and,

$$t_1 = \frac{(2\,\lambda_p - \lambda_a)\,t^2_0 - 2\,B_0}{(2\,\lambda_p - \lambda_a)\,t_0\,(I + F)} \ldots\ldots (89)$$

Equating moments around the bottom of the pile:

$$B_0\,t_0 - \left[\frac{(2\,\lambda_p - \lambda_a)\,t^3_0}{6} + \frac{(2\,\lambda_p - \lambda_a)\,t_0\,(I + F)\,(t^2_1)}{6}\right] = 0 \ldots (90)$$

Substituting the value of t_1 from Equation (89):

$$t^4_0 - \left[\frac{B_0\,t^2_0}{2\,\lambda_p - \lambda_a}\right]\left[\frac{6\,(I + F) - 4}{F}\right] - \frac{4\,B^2_0}{(2\,\lambda_p - \lambda_a)^2\,F} = 0 \ldots (91)$$

For simplification, the loading as illustrated in Fig. 19(*a*) can be approximated as in Fig. 19(*b*). Equating moments:

$$B_0\,t_3 = \frac{(2\,\lambda_p - \lambda_a)\,t^3_3}{6}$$

and,

$$t_3 = \sqrt{\frac{6\,B_0}{2\,\lambda_p - \lambda_a}} \ldots\ldots (92)$$

Let the ratio of t_0 (Fig. 19(*a*)) to t_3 (Fig. 19(*b*)), equal a factor, $K_2 = \frac{t_0}{t_3}$.

Then,

$$t_0 = K_2\,\sqrt{\frac{6\,B_0}{2\,\lambda_p - \lambda_a}} \ldots\ldots (93)$$

and Equation (91) can be written,

$$K^4_2 - K^2_2\left[\frac{1 + 3\,F}{3\,F}\right] - \frac{1}{9\,F} = 0 \ldots\ldots (94)$$

For any value of F (Fig. 19(*a*)), the value of $K_2 = \frac{t_0}{t_3}$ can be calculated from Equation (94).

Equation (94) indicates that the ratio, K_2, is entirely independent of the soil conditions, including the magnitude of B_0. Furthermore, by increasing the ratio of F, the ratio, K_2, approaches unity. Since, even under the most unfavorable circumstances, the value of F can never be less than 1.0, the depth to drive the piling will never be more than 1.2 times that calculated from the approximate loading condition, as illustrated in Fig. 19(*b*). Normally, the factor, 1.1, applies, but if the soil confined behind the steel sheet-pile is heavy fill, with a high active lateral pressure and a low passive value, this factor may rise to 1.2.

The final result for the depth to drive the sheet-piling in order to utilize the full constraining action of the earth from Equation (92) is:

$$t = K_2 \left[a_0 + \sqrt{\frac{6\ B_0}{2\ \lambda_p - \lambda_a}} \right] \ldots\ldots\ldots\ldots\ldots\ldots (95)$$

Equation (95) corresponds to Equation (3). Using $h_0 = 15$; $\lambda_p - \lambda_a = 525$ lb per sq ft (see Fig. 5); $a_0 = 1.8$ ft; $B_0 = 5\,570$ lb; and $K_2 = 1.2$; the penetration, t, is found to be 10.9 ft by Equation (95), and 10.6 ft by Equation (3). The results are equal for all practical purposes. A useful check on Equations (84) to (95) is illustrated in Fig. 6 of the paper, indicating that constraint was actually present at 10 to 11 ft of penetration.

Mr. Baumann's analysis of the theoretical distribution of the passive resistance of the soil and its application to the test is most unique, interesting, and thorough. The writer extends congratulations on bringing this study to its probable ultimate of mathematical accuracy. However, judgment is reserved as to the feasible practical application to the wide variety of conditions found in practice.

It has long been recognized that the supporting resistance of the earth was not represented by a straight line, but was nearer that developed by Mr. Baumann. However, the straight-line method of calculation, as used in Figs. 5, 7, and 19(*a*), will continue to be sufficiently accurate at least until the greater inaccuracies of the basic earth-load assumptions are removed.

Three factors are necessary in Equation (67), all of which depend on the character of the soil, and must be determined by test; and, furthermore, such factors might be variable even in closely adjacent locations. The writer is not prepared to state whether such tests can be made of practical value when it comes to the study of stratified, undisturbed, compacted, natural soil at considerable depths.

He is not convinced that the passive value of a soil will increase continuously, no matter to what extent lateral deflection or movement proceeds, as is inferred in Fig. 15 and in the paragraph containing Equations (61) and (62). Compression of the soil, when the sheet-piling is driven, very probably causes sufficient lateral movement to develop as much passive resistance as is safe to assume. Application of the theory is further complicated when earth of more than one type, or a combination of earth and water, is confined behind the bulkhead, or when the sheet-piling is driven into soils having strata of different values.

Therefore, the writer recommends the use of the straight-line procedure for the calculation of the penetration and the corresponding bending moment. This method more closely approximates the true constraining action of the earth at the bottom of a sheet-pile than it is possible at present to approximate the active lateral earth pressures on a sheet-pile.

D. P. KRYNINE,[25] M. AM. SOC. C. E. (by letter).—Few engineers have analyzed the behavior of sheet-piling mathematically. Credit should be given to Dr. H. Blum who practically was the first to make such a comprehensive analysis; next was Dr. E. Lohmeyer.[2] A short history of the work of both Dr. Blum and Dr. Lohmeyer has been given by the writer elsewhere.[26] The paper by Mr. Baumann is a valuable contribution in this field.

---

[25] Research Assoc. in Soil Mechanics, Dept. of Civ. Eng., Yale Univ., New Haven, Conn.

For some reason the ingenious experiment represented in Fig. 2 was never duplicated and checked and, consequently, Mr. Baumann was obliged to base his analysis on a considerable number of additional assumptions. As far as some of the latter are concerned, the writer's opinion differs markedly from that of Mr. Baumann.

Conception of Passive Resistance.—If a rigid loaded plate is placed on the upper horizontal surface of a perfectly elastic body, it sinks as a result of elastic deformations, which disappear when the load is removed. In the case of excessive load, plastic deformations may take place, and the body may fail. The cycle of deformations in this case is: (1) Elastic; and (2) plastic. If an actual soil mass is loaded in the aforementioned manner, the settlement begins at once, even under a relatively light load; and it disappears only partly when the load is released. This does not mean that one part of the deformation is plastic, and another elastic; the explanation is that the non-reversible part of the settlement under a relatively light load is due mostly to a simple downward motion of particles which try to penetrate into the interstices between others. The cycle of deformations in this case is: (1) Simple motion of particles, which may be accompanied by partial elastic action; (2) elastic deformations; and, finally, (3) plastic deformations. The supporting power of the soil is due mostly to its shearing resistance, and not so much to its capacity to resist compression stresses. Actually, if there were no shearing resistance at all, the soil would not be capable of supporting any load. This may be easily understood from the following analogy: Water is practically incompressible, but its bearing power is zero because it lacks shearing resistance entirely.

If a vertical wall is pushed into a sand mass, the succession of deformations is analogous to that of the preceding case: First, a simple motion of particles; next, elastic deformations; and, finally, plastic deformations. The passive resistance of the sand is its capacity to stand shearing stresses close to the end of Stage (2). Obviously, the stressed condition of the "passive prism" is more complicated than simple shear and consists also in some compression and other phenomena which are rather difficult to specify. A noteworthy feature is the stretching of a narrow zone along the shearing surface, which may be observed in the laboratory if one wall of the experimental box is made of glass. In the case of wet experimental sand, change of the moisture content in that zone may be easily observed by the naked eye. In the initial stage, (1), the passive resistance practically does not act. In the case of excessive horizontal load, the sand mass may fail along a shearing surface as indicated in Fig. 20.

In the Blum-Lohmeyer method (Fig. 5) the passive resistance is assumed to act from the very beginning of the experiment, which the writer has shown in the preceding remarks to be untrue. In experiments such as those of

---

[26] "Comments on Failures in Sheet-Piling", by D. P. Krynine, M. Am. Soc. C. E., *Civil Engineering*, Vol. 3, No. 10 (October, 1933), p. 582.

Professor Franzius,[7] the passive resistance begins to act earlier than in natural sands, but at any rate later than is assumed in the Blum-Lohmeyer method. Professor Franzius' experiments were made on tamped sand, and his experimental data should be applied with certain care.

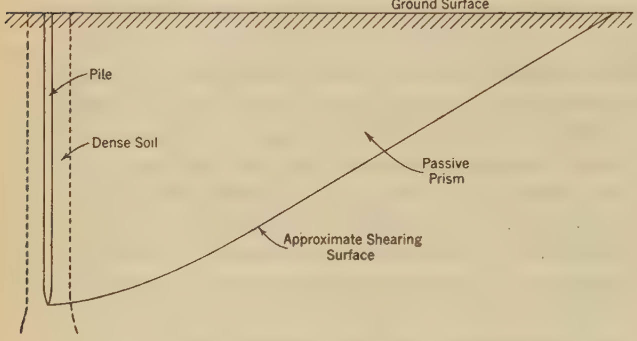


FIG. 20.

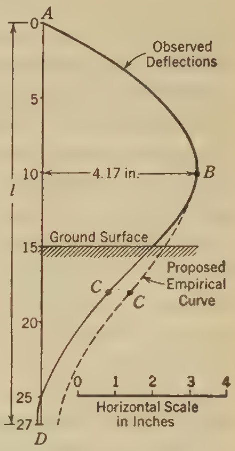


FIG. 21.

Compression of the Soil Close to Sheet-Piling.—Under the heading "Method of Graphical Analysis", Mr. Baumann expresses his belief that the ground adjacent to a bulkhead is compressed when sheet-piles are driven. This is perfectly true, but this circumstance cannot balance the initial "lost motion" of particles described previously. Actually, although the displacements of the adjacent soil which occur in pile-driving, is equal to the thickness of the sheet-piles, the compressed zone is rather restricted and forms a kind of dense "envelope" around a pile or a sheet-pile (see Fig. 20). Several years ago the following experiment was made in the writer's laboratory. A pile model, 20 in. long, and somewhat more than 3 in. in diameter, was driven under field conditions into clay; then a cross-section through the center of the pile, 10 in. below its tip, was made, using sharp shovels and knives. One-half the earth mass was removed and another one-half extracted in the form of a monolith. The surface of that monolith was carefully studied optically by Dr. G. I. Pokrowski, using the reflectometer.[27] The light-dispersing capacity of the soil depends on its density; and it was found that at a distance of about two or three diameters of the pile the light-dispersing capacity and, hence, the density of the soil, were more or less constant and increased sensibly in the proximity of the pile. Thus, it was concluded that the width of the "envelope" in question, is rather insignificant. As far as the present case is concerned, this should mean that the process of sheet-pile bending was resisted by the dense soil very slightly, if at all. Fig. 20 shows diagrammatically the relative insignificance of that "envelope", as compared with the passive prism.

---

[27] "Soil Investigations in 1928-1929", by G. I. Pokrowski and N. V. Laletine (in Russian), p. 65.

Observed Deflections; Equation of the Elastic Line.—According to the preceding considerations, a part of the bending occurred when there was no passive resistance; and another part when that resistance was already acting. Perhaps this explains why the observed deflection ordinates were twice those of the computed ordinates (Table 2).

Equations (9) to (15) correspond to the fitting of empirical curves to the experimental deflections. Mr. Baumann states (under the heading, "Analytical Determination of Interlock Efficiency",) that, "the observed elastic line is closely approximated by two hyperbolic spirals with a common point and tangent at the point of minimum radius of curvature; that is, maximum moment." Furthermore, he states that the upper spiral (Distance *AB,* Fig. 21), becomes faulty near the top support, and that the lower spiral does not satisfy the boundary condition, namely, that at Point *C,* where z is about 18 ft., the radius of curvature should be infinite. Thus, both spirals are faulty, but Mr. Baumann believes that his empirical curve "closely checks the elastic line in the vicinity of minimum radius of curvature". Certainly this is true; but the shape of that part of the experimental curve is such that many other curves may fit it. Particularly, was the choice of spirals incorrect since a hyperbolic spiral winds steadily about its pole and never flattens so as to have an inflection point, such as Point *C,* Fig. 21. An empirical curve should fit the entire elastic line, and not only a part of it. For instance, the following curve, although far from being ideal, may serve that purpose better:

$$\Delta = k \left[2 \tanh \frac{\pi z}{l} - \tanh \frac{\pi}{l} (z + \beta) - \tanh \frac{\pi}{l} (z - \beta) \right] \ldots (96)$$

in which, Δ = deflection, in inches; z = vertical distance, in feet, from the upper support, *A* (origin of co-ordinates); l = total length of the sheet-pile, equal to $28 - 1 = 27$ ft; and k and β = numerical parameters.

Should the curve of Equation (96) possess a maximum of $\Delta = 4.17$ in. at the point, $z = 0.375\ l = 10.1$ ft, the values of the parameters would be $k = 3.29$ and $\beta = 16.3$. In Fig. 21, the observed deflection curve is shown by a heavy solid line, and its probable extension by a light solid line. The empirical curve practically coincides with the observed deflection line between the points, *A* and *B,* and its lower part is shown by a dotted line. The value of Δ is not zero at $z = l$; but the curve would go exactly through the toe of the bulkhead if the z-axis were turned slightly. The curve of Equation (96) is used in electrical engineering for determining dynamic action of one coil of an electrical machine on another.[28]

Interpretation of the Permanent Deformations of the Test Piles.—Fig. 2 indicates that there were seven test piles facing the test wall. The deflections were measured on three outside sheet-piles, as shown in Fig. 2(*b*). Nothing is known about the differential deflection; hence, it should be assumed that the deflections on all the piles were uniform within some reasonable limits, and that the curve of observed deflections as shown in Table 2, represents an average.

---

[28] "Electromagnetic Problems in Electrical Engineering", by B. Hague, 1929, p. 204 *et seq.*

The passive resistance, however, was not the same for each of those piles; it was greater for the outside piles and less for the center pile. This follows from the definition of passive resistance, which is shearing resistance of the soil along a certain surface. Such a shearing resistance, corresponding to the center of the tank, lay in a plane normal to the test wall (a two-dimensional problem); while, for the outside piles, the shearing surfaces could extend freely not only in front of the tank, but sidewise as well. The greater resisting force prevented the outside piles from excessive plastic bending in vertical planes, as is clearly seen in Fig. 6. This phenomenon agrees fully with a statement which was already advanced elsewhere[29], namely, an isolated pile is stronger against overturning than the same pile placed in a row with others.

However, Fig. 6 also shows that all the piles underwent plastic deformations; hence, the applicability of elastic theories, by Mr. Baumann's method, is not fully justified. Plastic condition may even have started at the very beginning of the experiment owing to hammer blows, when the piles were driven to refusal; it was probably only emphasized during the experiment.

If there were no interlocking action, the center pile would deflect during the experiment more than the others, but because of the rigidity of the entire system in horizontal planes, it could not do so. A striking analogy as to the influence of rigidity may be found in the settlement of structures. On certain soils a non-rigid structure (as, for example, an earth embankment) settles more in the middle than on the edges, while a rigid foundation slab settles evenly. Reverting to the given case, it should be stated that the elastic condition apparently was never destroyed in the interlocking joints; therefore, upon removal of the loads, the interlocking joints returned to the original shape, so far as the plastic deformations permitted. In other words, the joints acted as heavy elastic springs connected by weaker central parts of the piles.

Consider a wall of, say, ten panels, each consisting of seven piles, exactly as shown in Fig. 2. Any panel of this wall (including those on the outside) would be weaker than the experimental wall in question, since the "three-dimensional" passive resistance would act only at the ends of that wall. Therefore, in practice, the data obtained in the experiment of Mr. Baumann must be applied with care.

Stress Distribution in Passive Prism.—The problem of passive resistance may be reduced to that of a vertical loaded plate acting on a soil mass bounded from above by a horizontal plane. This is a somewhat complicated particular case of the general problem, when a loaded plate acts on the horizontal surface of an infinite soil mass. It is necessary to solve the general problem for both non-plastic and plastic soils, after which it will be easier to deal with particular cases; and that general problem has not yet been definitely solved in soil mechanics.

The Proposed Method of Design. — As Mr. Baumann rightly states, the method of designing bulkheads proposed by Blum-Lohmeyer is defective in assuming the passive resistance to be static. The writer believes that the Blum-Lohmeyer method is undoubtedly a commendable step forward in comparison with old, quite empirical methods. It has little, if any, connection with the study of physical properties of soils, however, and, therefore, is rather unacceptable from the point of view of steadily developing soil mechanics. The method in question should be considered as a temporary one, which sooner or later will be abandoned; therefore, all complicated constructions (as, for instance, those shown in Fig. 7), should not impress the designer unduly. Mr. Baumann proves beyond question that passive resistance depends on both the depth of a given point and the movement of the wall. In Fig. 14 a fine geometric construction is given. It is to be regretted, however, that the author did not use modern soil mechanics to its fullest extent. For instance, his opinion as to plastic soils remains completely unknown; and such soils are widely dealt with in bulkhead construction.

---

[29] "Resistance of Sheet-Piling to Overturning", by D. P. Krynine, M. Am. Soc. C. E., and R. W. Abbett, Assoc. M. Am. Soc. C. E., *Civil Engineering*, Vol. 3, No. 5 (May, 1933), p. 263 *et seq.*

Conclusion.—This discussion is not to be considered as a criticism of Mr. Baumann's work. On the contrary, he should be highly commended for his effort, patience, and accuracy in observations and computations. His work is a valuable contribution to the study of bulkheads, more fruitful than any other existing; and if incontestable conclusions cannot be reached as yet, this is easily explained by the extreme difficulty of dealing with the subject.

DR. ING. E. h. O. FRANZIUS,[30] (by letter).—That present design methods are incomplete has been correctly emphasized in this paper. Time and time again even engineers of note resort to irrational methods as soon as they encounter difficulties in the design of bulkheads. One of the most favored "tricks" has been the assumption of restraint at the bottom of the sheet-piling, without an analysis of the forces producing such restraint. As long as there is no proof that such forces occur, the introduction of a restraint is equivalent to the appearance of a *Deus ex machina,* who is expected to deliver the designer from embarrassment. The fact that bulkheads designed in this manner have not failed is not due to the quality of the design, but to the fact that the active forces are smaller than assumed, owing to cohesion.

An attempt is made in the paper to determine the forces that act on a sheet-pile wall from the observed deflections and, finally, a new method of analysis is presented. This method of analysis is an ingenious, progressive step, and will materially advance designers in this field of engineering in their struggle for cognition.

Evidently, the principal objective of the tests was to observe the elastic line of a sheet-pile wall under load and to determine from this line the forces causing it. As it was not possible to measure the earth pressures directly, it was necessary to determine them through calculation in such a way as to satisfy the observed deflections. In doing so two errors did occur which are of decisive importance in the evaluation of these forces:

1.—In computing the active earth pressure in the tank by means of Coulomb's formula, the author neglects the friction on the side walls. The writer's experiments with large earth-pressure testing apparatus in the Laboratory for Foundation and Hydraulic Tests of the Institute of Technology, of Hanover, Germany, showed this side-wall friction to be of decisive importance. In some cases the active earth pressure was reduced to one-half, due to this side-wall friction. In fact, the latter is equivalent to a lateral anchorage of the sliding wedge. Furthermore, the distance between the front and the back wall of the test tank is too small for the active sliding wedge to develop fully. Both these effects result in a reduction of the active pressure. On the other hand, the sand contains some silt which tends to increase the active pressure beyond the value obtained by Coulomb's formula. It is not possible, therefore, to determine the effective, actual pressure without direct and exact test.

---

[30] Prof., Inst. of Technology, Hanover, Germany.

In his tests with two large testing apparatus, the writer found it necessary to lubricate the side walls with soap paste after first covering them with several layers of paper. The side-wall friction was reduced thereby to about 10% of its original value so as to render its effect on the total earth pressure quite small.

A further uncertainty is inherent in the angle of internal friction of soil under water. The author conducted three tests to determine this angle, two of which gave 31° and the third, 30 degrees. As the latter test called for the greatest movement of sand particles, prior to finding their equilibrium position, the writer would consider the greatest angle (that is, 31°) as closer to the true one. However, the difference is too small to be of decisive influence, although combined with other values it may assume a certain significance.

That the angle of 31° for the material under water is much smaller than the angle for the material "in the dry" is due to silt in the sand. The writer's tests showed that the angle of friction of clean sand under water is exactly the same as in dry sand.

2.—If these uncertainties affecting the active pressure were of slight importance the actual forces due to passive resistance in front of the wall could be found with fair approximation. The earth-resistance values obtained by the writer in his tests at Hanover (and which the author uses as a basis of his derivations) are due to a wall width of 2 m (6.56 ft), which means that the resistance values found to establish equilibrium according to the author's new method of analysis would be about four times as great. The values as measured by the writer are for clean, granular sand with practically no cohesion. On the other hand, the soil tested by the author had ingredients which, conditions permitting, will produce cohesion, although with the material under water the cohesion would be almost negligible. However, as the soil used in the author's tests has constants that differ from that used by the writer, an immediate comparison is impossible. One conclusion is certain, however, the earth resistance values as determined by the author are not due entirely to the prism directly in front of the wall; they are also due to the wedges that fan out laterally beyond a line through the side walls. The writer has had the influence of these side wedges investigated through tests on anchor-plates.[31] The top of these anchor-plates was some distance below the ground surface. The tests showed that the passive resistance was more than twice the value reported by Krey and more than three times that based on the Coulomb theory.

The same must be true of the author's tests. The influence of side friction is great. If the test wall had been twice or three times as wide as it actually was, the values of passive resistance would have been quite different because of the reduction in the influence of side friction to one-half and one-third, respectively. The relative magnitude of side friction and passive resistance can only be determined through tests with specially constructed apparatus.

It is much to the writer's regret to conclude that the passive resistance values, as established by the author's test, do not truly represent the passive resistance value of that soil per unit width, of a wall of infinite width. This is also reflected in the permanent deformation of the sheet-piles (Fig. 6), as the outside piles show smaller deflections than the inside ones, due to the influence of the lateral fanning out of the resisting wedge.

As the deflections below the ground surface could not be measured, a considerable part of the elastic line and, in fact, the part subjected to the greatest loads, was not established. The results to date merit an extension of these tests by the installation of such apparatus in laboratories as will permit measuring the passive resistance of soil directly.

The derivation of a relation between the specific earth resistance, e_p, the path of the wall, y, and the depth of fill $(z-h)$, and the mathematical treatment of the decrease of the angle of internal friction, ϕ, and of the angle of friction between wall and earth, δ, with increasing depth and with an increase of the path of the wall, give conclusive insight into the stress distribution of the passive prism. This holds strictly true, however, only for homogeneous material. The investigations should be extended so as to include the combined action of several kinds of earth deposited in layers as is generally the case in Nature. Whether or not it will be possible to represent such conditions in a simple form usable for practical designs, is somewhat doubtful at this time, due to the abundance of highly involved phenomena which are encountered in the more exact analysis of passive resistance of earth.

More easily comprehensible and better arranged for practical use are the author's derivations of the shape of the surfaces of rupture based on approximate functions and the method of considering the sheet-pile bulkhead as a statically indeterminate structure. This method will lend itself to practical use if it is possible to establish the aforementioned relations (the "resistance-path" curves), between passive resistance, path of wall, and height of wall for those kinds of earth that will be encountered in connection with bulkhead construction.

By means of tests, or by computation, it would then be possible to plot diagrams such as Fig. 15 for all the ground conditions that normally will be encountered. This would remove all obstacles to the practical application of the author's new theory of the analysis of bulkheads.

---

[31] "Erdwiderstand auf Ankerplatten", von Dr. Ing. Wilhelm Buchholz, Pre-Print from the Yearbook of the Hafenbautechnische Gesellschaft, Vol. 12, 1930–31.

Naturally, the new method of analysis proposed in the paper is entirely independent of the magnitude of the forces as far as theory is concerned. This method of design will always give better results than the present method for one and the same system of forces; but just because this new method is valuable, the true set-up of forces should be determined more accurately, and to accomplish this the continuation and extension of the author's tests, or perhaps the introduction of new tests, is necessary.

THEODOR VON KÁRMÁN,[32] M. AM. SOC. C. E. (by letter).—The analysis of sheet-pile bulkheads in this paper is a comprehensive presentation of the problem of designing sheet-piles loaded with active or passive earth pressure. In some respects this analysis constitutes a desirable extension of the known methods; for example, (*a*) Mr. Baumann, fortunately, has hit upon the idea of introducing reliable values for the passive earth pressure which has been measured experimentally; and (*b*) he takes into account the influence, on the load distribution, of the yielding of the soil under the passive earth pressure. After formulating the problem in the proper way, Mr. Baumann develops a method of calculation analogous to that used in the case of elastically supported beams, applying the theorem of least work. This method represents progress.

Attention is called to Equation (63), in which it appears that the term corresponding to the work stored in the soil by the passive earth pressure, is incorrect. Mr. Baumann calculates the yield, y_0, due to unit load and makes the work equal to $\frac{1}{2}\, y_0\, p^2$, in which, p is the actual value of the load. The application of this procedure seems to be restricted to the case of a linear relation between yield and load. However, in Mr. Baumann's analysis, the load, p, is connected with the yield, y, by a non-linear relation, $p = C \sqrt{y}$, in which, C is a given function of the depth $(z - h)$. In this case, the term to be varied should be written, $\int_0^p y\, dp = \int_0^p \frac{p^2}{C^2}\, dp = \frac{1}{3}\, \frac{p^3}{C^2}$, whereas Mr. Baumann writes, $y_0 = \frac{1}{C^2}$, and uses the expression, $\frac{1}{2}\, y_0\, p^2 = \frac{1}{2}\, \frac{p^2}{C^2}$, for the work, A.

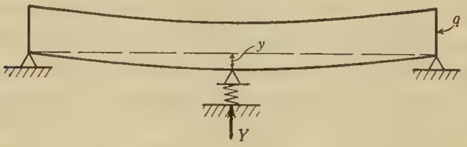


FIG. 22

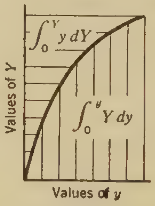


FIG. 23.

---

[32] Prof. and Director, Aeronautical Laboratory, California Inst. of Technology, Pasadena, Calif.

It might be useful to explain the difference between the two viewpoints by a simple example. Assume that a beam is supported by two fixed supports at its two ends and has a "yielding" support in the center (see Fig. 22). The law between the reaction force, Y, and the "yield," y (that is, the deflection of the beam), is given by the general relation, $y = f(Y)$. The beam is loaded by the uniform load, q; denoting the span by l, the deflection in the center, without taking into account the yielding support, would be:

$$y_1 = \frac{5}{384} \frac{q\,l^4}{I\,E} \quad \text{.......................(97)}$$

The deflection corresponding to the force, Y, alone would be:

$$y_2 = -\frac{1}{48} \frac{Y\,l^3}{I\,E} \quad \text{.......................(98)}$$

Hence, the total deflection, $y = y_1 + y_2$, is equal to:

$$y = \frac{5}{384} \frac{q\,l^4}{I\,E} - \frac{1}{48} \frac{Y\,l^3}{I\,E} \quad \text{...................(99)}$$

and the value of the reaction, Y, is given by the equation:

$$\frac{5}{384} \frac{q\,l^4}{I\,E} - \frac{1}{48} \frac{Y\,l^3}{I\,E} = f\,(Y) \quad \text{................(100)}$$

The same problem can be solved by the theorem of least work. The elastic work stored in the beam is:

$$A_1 = \tfrac{1}{2} \int_0^l \frac{M^2}{I\,E}\, dx \quad \text{.....................(101)}$$

in which, M is the total moment due to the load, p, and the statically unknown reaction, Y. It is known (or, it can be shown by simple calculation) that $-\frac{d\,A_1}{d\,Y}$ is equal to the deflection, y, at the point of application of the statically unknown force, Y. Therefore, Equation (100) which determines Y, can be written in the form:

$$-\frac{d\,A_1}{d\,Y} = f\,(Y) \quad \text{.......................(102)}$$

Making $\int_0^Y f\,(Y)\; dY = A_2$, Equation (102) is equivalent to the statement:

$$\frac{d}{d\,Y}\,(A_1 + A_2) = 0 \quad \text{.....................(103)}$$

or, $A_1 + A_2 =$ minimum.

The quantity, $A_2 = \int_0^Y f\,(Y)\; dY$, is sometimes termed the "complementary work" ("Ergaenzungsarbeit"). Its physical interpretation is obvious from Fig. 23. It represents the difference between the product, $Y\,y$, and the actual work stored in the yielding support, $\int_0^Y Y\,dy$. It is easily seen that in the case of the linear law:

$$\int_0^Y y\,dY = \int_0^Y Y\,dy = \tfrac{1}{2}\,y\,Y \;\ldots\ldots\ldots\ldots\ldots(104)$$

Hence, in this case, the value of the reaction, Y, is determined by the minimum of the total work; that is, of the sum of the work stored in the beam and that stored in the elastic support.

It is obvious from this example that Mr. Baumann should introduce the term denoting complementary work in his Equation (63). However, it is believed that this minor inconsistency does not materially detract from the value of his new analysis of the sheet-pile bulkhead problem.

Dr. Ing. E. Lohmeyer[33] (by letter).—The calculations pertaining to the tests described in this paper are based on the method developed by Dr. Ing. Blum,[34] and first published by the writer[35] in 1930. The tests are very instructive, and Mr. Baumann's extremely thorough and expert mathematical study indicates particularly the direction for further scientific treatment of the extraordinarily complicated problems of the passive resistance of soil. It is to be regretted that additional tests with varied loads, could not have been run. Nevertheless, the data presented confirm the usefulness of Blum's method, which the writer recommended for the calculation of bulkheads several years ago, for the reason that, since it takes account of restraint at the bottom, it approaches essentially closer to actual conditions than any other method of calculation used to that time. Mr. Baumann's experiments result in a somewhat different distribution of the passive resistance, the most important results, however (the position of the "substituted" support and the depth of penetration) remain substantially unaltered. Henceforth, it will not be necessary to base such calculations on curved surfaces of rupture, as developed by Mr. Baumann, because present knowledge of the action of soil is so incomplete that the more precise, but more elaborate, calculation is not worth the trouble.

The writer endorsed the Blum method as a particularly short rapid one, which makes it possible to gain a quick understanding of the influence of the fundamental premises concerning soil properties, the internal friction of the soil, water levels, etc. Changes in these premises within the possible (and also within the probable) allowable limits nearly always alter the results of the calculation to a greater degree than the method of calculation no matter how carefully that may be applied. Nevertheless, the scientific value of the more precise calculation for the development of earth-pressure investigations as proposed by the author remains unchallenged. In special cases of unusually large bulkheads it will be used without question, particularly when the action of the soil is known beyond doubt, which means when the soil lacks cohesion.

---

[33] Oberbaudirektor i. R., Berlin, Germany.

[34] "Einspannungsverhaeltnisse bei Bohlenwerken", von Dr. Ing. Blum, Berlin, 1931, Wilhelm Ernst & Sohn.

[35] "Der Grundbau", von Brennecke-Lohmeyer, 4. Aufl., pp. 61–92, Berlin, 1930, Wilhelm Ernst & Sohn.

In any case, one cannot be too conservative in assigning a numerical value to the passive resistance of a toe slope under water in front of the bulkhead; if there is any doubt one should consider that resistance as zero. However, the writer does not wish to enlarge on this question; he would rather confine himself to one phase—the problem of the section modulus of the interlocking steel sheet-pile discussed in the paper.

When drawing his conclusions from the deflections measured during the tests in regard to the section modulus of the sheet-piling, Mr. Baumann made a mistake, which the writer himself has also made in the past.[36] In Table 2 the proportion of the moments of inertia, I, is exactly equal to the section modulus. Correspondingly, in Table 3, the section modulus, S_e, has been computed from the moments of inertia, I_e, according to the equation:

$$S_e = \frac{I_e}{e} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (105)$$

by entering e (a constant) as the distance of the extreme fiber from the axis of symmetry of the section.

The concept of the moment of resistance of a beam originates with the general equation:

$$f = \frac{M\,e}{I} = \frac{M}{S} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (106)$$

in which, f = the stress in the extreme fiber of the cross-section; M, the bending moment resisted by the beam; I, the moment of inertia of the cross-section of the beam in relation to the gravity axis; and, e, the distance of the extreme fiber from the neutral axis.

In the case of simple bending the neutral axis coincides with the gravity axis and e is constant. If the beam is subjected to an axial force in its longitudinal direction, however, the neutral axis and the gravity axis no longer coincide, and e varies in proportion to the bending moment and the longitudinal force.

Consider a deep, arched, sheet-pile wall with interlocks, in the axis of the wall, set so tightly that they may be considered as a unit. In this case, the piles are subjected to simple bending only, the wall bends around the gravity axis, n - n, the neutral axis coincides with the gravity axis, and the stress is as indicated in Fig. 24(*a*). If there is no friction or "jamming" in the interlock to oppose the reciprocal shifting of the piles—or, in other words, if there is no shearing force transmitted in the interlock—so that piles can slide in it without resistance, then the individual piles are likewise subjected to simple bending. In that case, however, they deflect around their own axes of gravity, x - x (Fig. 24), the neutral axis for each pile lies in this axis of gravity, and the resulting stress diagram is as shown in Fig. 24(*b*). For the calculation of the stress, I and e must be entered in Equation (106) in relation to the axis, x - x.

---

[36] *Bautechnik*, 1927, pp. 26, 49, and 73.

The distribution of stress varies between the two boundary cases illustrated by Fig. 24(*a*) and Fig. 24(*b*), respectively. It depends on the amount of the shearing force which the interlocks of one pile can transmit to the other. As long as the shearing force created in the interlock by the load cannot be entirely absorbed, one cannot base the calculation of the extreme fiber stress upon the wall as a unit. The designer must begin with the individual pile that is subjected to its respective share of the bending moment resulting from the total load, and to two longitudinal forces that are caused by the transmission of the shearing force of the neighboring pile in the interlock. If this shearing force equals zero, the individual piles are subjected to a pure bending stress, and the neutral axis lies in the axis of gravity of the individual pile, x - x. If the shearing stress increases, the compression or tension stress from the shearing force is added to the pure bending stress, and the neutral axis moves from the position, x - x, parallel to the wall axis, n - n (as demonstrated in Fig. 24(*c*)) which it reaches, if the interlock is capable of taking care of the full shearing stress. The extreme fiber stresses are to be calculated according to Equation (106) by inserting I in relation to the axis of gravity of the individual piles, and e as the distance of the extreme fiber from the neutral axis. This means that e is variable if the shearing stress is between zero and the full value, and in that case it cannot be assumed as a constant, as Mr. Baumann has done.

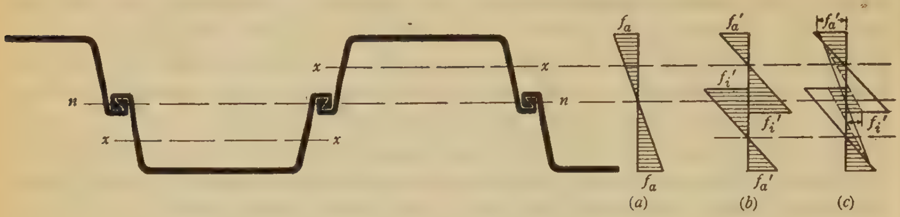


FIG. 24

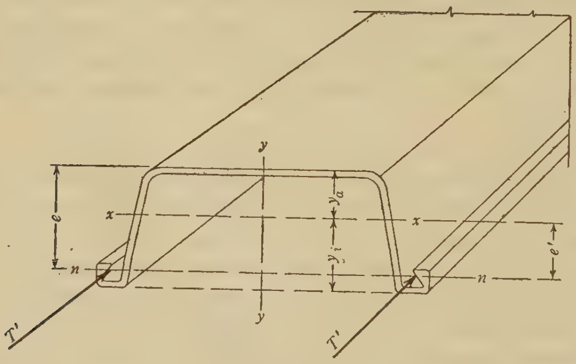


FIG. 25

The position of the neutral axis depends on the magnitude of the transmitted shearing stress. In the following analyses, its distance, y_x, from the axis of gravity of the individual pile is to be calculated in its relation to the magnitude of the shearing stress. For this purpose one of the piles is considered as being removed from the wall (Fig. 25) and subjected to its share, M', of the bending moment resulting from the load on the wall and to a shearing stress, T', which acts in each interlock. In addition, the following symbols will be used:

I_x = moment of inertia of the individual pile in relation to its axis of gravity, $x - x$;
I_n = moment of inertia of the individual pile in relation to the wall axis, $n - n$;
A_f = cross-sectional area of the individual pile;
e = distance of the extreme fiber from the wall axis, $n - n$;
e' = distance of the axis of gravity, $x - x$, of the individual pile from the wall axis, $n - n$;
y = distance of a random point in the cross-section, A_f, from the axis of gravity, $x - x$, of the individual pile (positive downward);
y_x = distance of the neutral axis from the axis of gravity, $x - x$, of the individual pile;
y_a = distance of the extreme fiber opposite the interlock side of the individual pile, from the axis of gravity, $x - x$;
y_i = distance of the extreme fiber on the interlock side of the individual pile from the axis of gravity, $x - x$;
T' = the shearing force transmitted to each interlock of the examined pile from the neighboring pile;
T = the maximum value of the shearing force in the interlock that develops if the interlock is capable of absorbing the entire shearing force caused by the load;
f'_x = tension at any given point in the cross-section, A_f;
f'_a = extreme fiber stress on the outside of the individual pile (at a distance, $y_a = -[e - e']$, from the gravity axis, $x - x$);
f'_i = extreme fiber stress of the interlock side of the pile (at a distance, y_i, from the gravity axis, $x - x$);
f_a = extreme fiber stress on the outside of the wall with full development of the shearing stress;
f_i = extreme fiber stress on the interlock side of the pile with full development of the shearing stress;
δ' = deflection of the wall at a given point with partial development of the shearing stress in the interlock;
δ = deflection of the wall at a given point with full development of the shearing stress in the interlock; and,
E = modulus of elasticity.

According to Fig. 25:

$$f'_x = \frac{(M' + 2\,T'\,e')\,y}{I_x} + \frac{2\,T'}{A_f} \qquad \text{.............. (107)}$$

The equation for the neutral axis is obtained by substituting $f_x' = 0$ in Equation (107); thus:

$$y_x = -\frac{2\,T'\,I_x}{A_f\,(M' + 2\,T'\,e')} \qquad \text{.................. (108)}$$

and,

$$T' = -\frac{M' A_f y_x}{2(I_x + A_f e' y_x)} \qquad \text{(109)}$$

Substituting the value of T' in Equation (109), in Equation (107):

$$f'_x = \left(M' - M' \frac{A_f e' y_x}{I_x + A_f e' y_x}\right)\frac{y}{I_x} - M' \frac{y_x}{I_x + A_f e' y_x} \qquad \text{(110)}$$

or,

$$f'_x = \frac{y - y_x}{I_x + A_f e' y_x} M' \qquad \text{(111)}$$

For the extreme case (see Fig. 24(*b*)) in which no shearing force is transmitted in the interlock (that is, for $T' = 0$), the neutral axis coincides with the gravity axis, x - x, of the individual pile, and $y_x = 0$. The extreme fiber stresses of the pile in this case are expressed by Equations (107) and (110), thus: On the outside of the pile, with $y = -y_a = -(e - e')$:

$$f'_a = -\frac{M'(e - e')}{I_x} \qquad \text{(112)}$$

and on the interlock side of the pile, with $y = y_i$:

$$f_i = +\frac{M' y_i}{I_x} \qquad \text{(113)}$$

For the extreme case (see Fig. 24(*a*)), in which the shearing force resulting from the load is fully taken up in the interlock (that is, for $T' = T$), the neutral axis coincides with the axis of the wall, n - n; and $y_x = e'$. Then, the shearing force according to Equation (109) is,

$$T = -\frac{M' A_f e'}{2(I_x + A_f (e')^2)} \qquad \text{(114)}$$

and,

$$T = -\frac{M' A_f e'}{2 I_n} \qquad \text{(115)}$$

and according to Equation (111), the extreme fiber stress on the outside of the pile, with $y = -(e - e')$:

$$f_a = -M' \frac{e}{I_x + A_f (e')^2} \qquad \text{(116)}$$

or,

$$f_a = -\frac{M' e}{I_n} \qquad \text{(117)}$$

The value for f_a in Equation (117) also results directly from Equation (106). The magnitude of the extreme fiber stress, f_i, on the interlock side of the pile is insignificant.

The deflection at a given point follows the law:

$$\delta_x = \frac{1}{E\,I} \int M\,x\,dx \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (118)$$

in which, the quantity, $\int M\,x\,dx$, represents the statical moment of the moment area in relation to the vertical through the support. The moment area of the individual pile consists of the pile's share, M', of the bending moment from the load and of the moment resulting from the transmitted shearing force, T'. If the value, T', from Equation (109) is inserted:

$$\delta' = \frac{1}{E\,I_x} \int (M' + 2\,T'\,e')\,x\,dx$$

$$= \frac{1}{E\,I_x} \int M' \left(1 - \frac{A_f\,e'\,y_x}{I_x + A_f\,e'\,y_x}\right) x\,dx \quad \ldots\ldots\ldots\ldots (119)$$

or,

$$\delta' = \frac{1}{E} \int \frac{M'\,x}{I_x + A_f\,e'\,y_x}\,dx \quad \ldots\ldots\ldots\ldots\ldots\ldots (120)$$

One may assume with sufficient approximation that, for equal loads, the share of the shearing force absorption, and, consequently, the distance, y_x, from the gravity axis, x - x, is equal within the boundaries of the moment area, so that:

$$\delta' = \frac{1}{E\,(I_x + A_f\,e'\,y_x)} \int M'\,x\,dx \quad \ldots\ldots\ldots\ldots (121)$$

If the shearing force is fully absorbed in the interlock, $y_x = e'$, and, hence, the deflection,

$$\delta = \frac{1}{E\,(I_x + A_f\,(e')^2)} \int M'\,x\,dx = \frac{1}{E\,I_n} \int M'\,x\,dx \quad \ldots\ldots (122)$$

The value for δ in Equation (122) also results directly from Equation (118).

With the assistance of the foregoing equations, one can determine for each value of the shearing force acting in the interlock of the piles the relevant extreme fiber stresses and deflections in proportion to the acting extreme fiber stresses and deflections in case of complete absorption of the shearing force.

For this purpose one introduces the following proportions, which result for the shearing force from Equations (109) and (114), for the extreme fiber stresses from Equations (110) and (116), and for the deflections from Equations (121) and (122); thus:

$$\frac{T'}{T} = \frac{y_x}{e'} \times \frac{I_x + A_f\,(e')^2}{I_x + A_f\,e'\,y_x} \quad \ldots\ldots\ldots\ldots\ldots\ldots (123)$$

$$\frac{f'_x}{f_a} = -\frac{y - y_x}{e} \times \frac{I_x + A_f\,(e')^2}{I_x + A_f\,e'\,y_x} \quad \ldots\ldots\ldots\ldots\ldots (124)$$

and,

$$\frac{\delta'}{\delta} = \frac{I_x + A_f\,(e')^2}{I_x + A_f\,e'\,y_x} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(125)$$

For instance, for a certain sheet-pile section, the following standard characteristics are published: $I_x = 2\,097$ cm^4; $A_f = 58.6$ cm^2; $e = 13.5$ cm; $e' = 7.34$ cm; $y_t = 9.49$ cm; and, $y_a = -6.16$ cm.

Substituting these values, the expression common to Equations (123) to (125) will be:

$$\frac{I_x + A_f\,(e')^2}{I_x + A_f\,e'\,y_x} = \frac{2\,097 + 58.6 \times 7.34^2}{2\,097 + 58.6 \times 7.34 \times y_x}$$

$$= \frac{5\,253}{2\,097 + 430.1\,y_x} = \frac{12.213}{4.876 + y_x}$$

and, according to Equations (123) to (125):

$$\frac{T'}{T} = \frac{12.213 \times y_x}{7.34\,(4.876 + y_x)} = \frac{1.664\,y_x}{4.876 + y_x} \quad \ldots\ldots\ldots\ldots(126)$$

$$\frac{f'_a}{f_a} = \frac{6.16 + y_x}{13.5} \times \frac{12.213}{4.876 + y_x} = \frac{0.905\,(6.16 + y_x)}{4.876 + y_x} \quad \ldots\ldots(127)$$

$$\frac{f'_t}{f_a} = \frac{0.905\,(9.49 - y_x)}{4.876 + y_x} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(128)$$

and,

$$\frac{\delta'}{\delta} = \frac{12.213}{4.876 + y_x} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(129)$$

These values were calculated for different distances, y_x, of the neutral axis from the axis of gravity of the individual pile, and the results plotted in Fig. 26(*a*), beginning with the position of the neutral axis; that is, for the different magnitudes of y_x. For each value of y_x one can easily begin with the share of the shearing force corresponding to this position (which is absorbed by the interlock) and the increase of the extreme fiber stresses and deflections as compared to the condition where the shearing force is fully transmitted in the interlock. This process has been repeated in Fig. 26(*b*), except that the values of y_x have been so distorted that the plotted values of $\frac{T'}{T}$ appear as a straight line. Consequently, the curves for the stresses and deflections also become straight and the stresses and the deflections increase uniformly with the decrease of the shearing force transmitted in the interlock.

If the interlock cannot absorb the shearing force fully, the extreme fiber stresses and the deflections increase with the diminishing transmission of shear. Compared to the maximum stress, which appears with the shear transmission in the interlock, the extreme fiber stress, f'_a, of the outer fiber grows slowly with an equal load, but with decreasing shear transmission. The

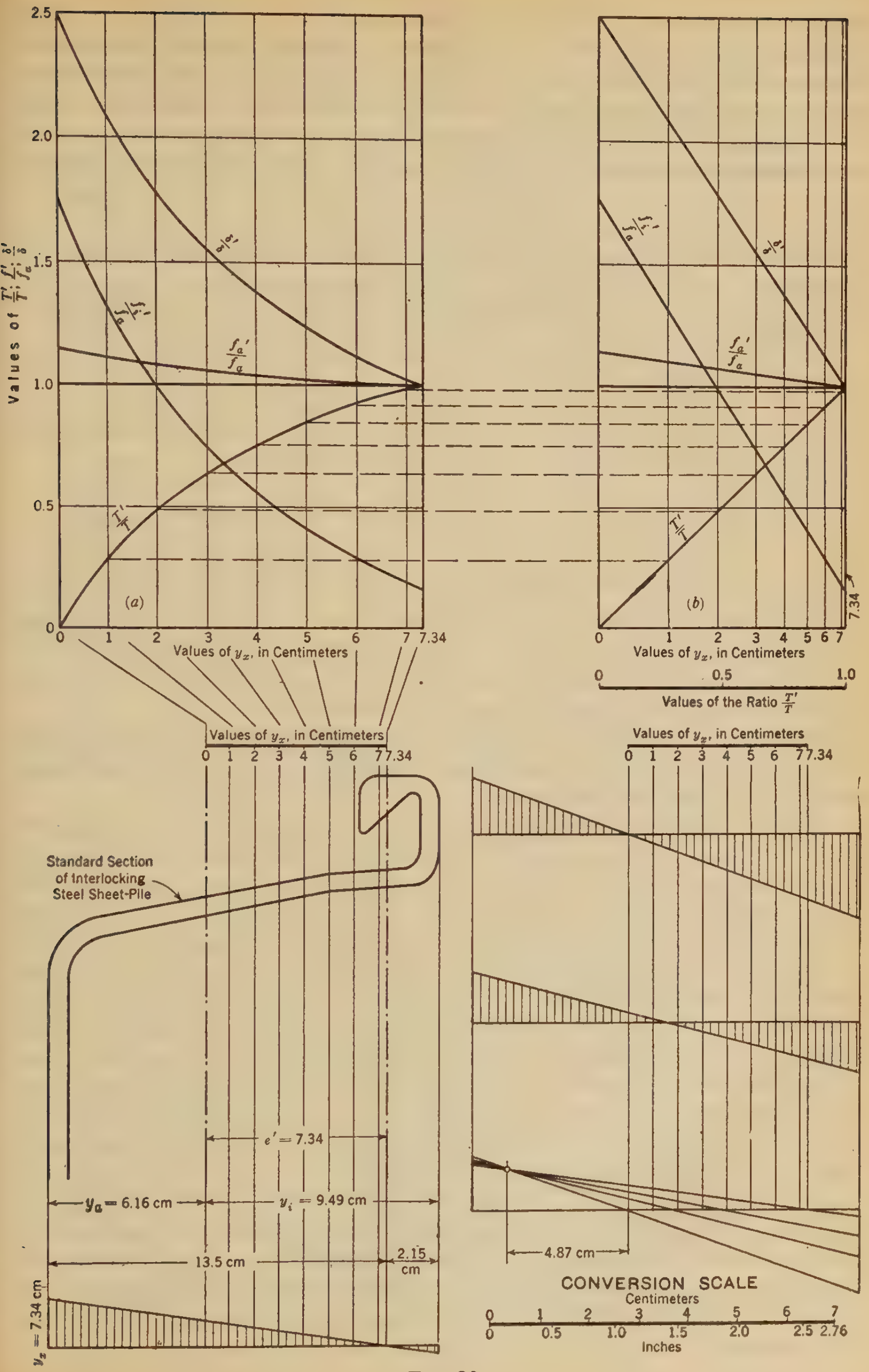

2.5
2.0
1.5
1.0
0.5
0
Values of $\frac{T'}{T}$; $\frac{f_i'}{f_a}$; $\frac{\delta'}{\delta}$
$\frac{\delta'}{\delta}$
$\frac{f_i'}{f_a}$
$\frac{f_a'}{f_a}$
$\frac{T'}{T}$
(a)
(b)
0 1 2 3 4 5 6 7 7.34
Values of y_x, in Centimeters
0 0.5 1.0
Values of the Ratio $\frac{T'}{T}$
7.34
Standard Section of Interlocking Steel Sheet-Pile
$e' = 7.34$
$y_a = 6.16$ cm
$y_i = 9.49$ cm
13.5 cm
2.15 cm
$y_x = 7.34$ cm
4.87 cm
CONVERSION SCALE
Centimeters
0 1 2 3 4 5 6 7
0 0.5 1.0 1.5 2.0 2.5 2.76
Inches

FIG. 26

extreme fiber stress, f'_i, of the inner extreme fiber of the pile (on the interlock side) grows faster from its originally insignificant value ($0.16\ f_a$), and reaches, with $f'_i = f'_a = 1.08\ f_a$, the value of the outer extreme fiber stress, if the interlock absorbs only 0.42 of the full shearing force. Hence, $f'_i > f'_a$.

A summary of the values corresponding to the two boundary cases, and the conditions under which $T' = 0.42\ T$, and $f'_i = f'_a$, is given in Table 6.

TABLE 6.

Shearing force transmitted in interlock (1)	Deflection (2)	Outer extreme fiber stress (3)	Inner extreme fiber stress (interlock side) (4)	Equivalent moment of resistance (5)
$= T'$	$= \delta'$	$= f'_a$	$= f'_i$	$= S'_e$
$= 1.0\ T$	$= 1.00\ \delta$	$= f_a$	$= 0.16\ f_a$	$= S_e$
$= 0.42\ T$	$= 1.87\ \delta$	$= 1.08\ f_a$	$= 1.08\ f_a$	$= 0.926\ S_e$
$= 0$	$= 2.51\ \delta$	$= 1.15\ f_a$	....	$= 0.87\ S_e$
		......	$= 1.765\ f_a$	$= 0.567\ S_e$

In Column (5), Table 6, the "equivalent" section modulus, S'_e, is computed as proportional to the full "catalog value," S_e, relative to the axis of the wall. Introducing this section modulus, S'_e, in Equation (106), one arrives at the extreme fiber stress resulting from the load.

The foregoing values are valid for a particular type and size of standard sheet-pile. For other sizes of the same type, similar values apply, if anything, less favorably than the foregoing.

The aforementioned calculation permits a definite conclusion that the extreme fiber stress in the standard sheet-piling under discussion increases only insignificantly (about 8 to 10%), if the interlock can absorb about 0.4 to 0.5 of the shearing force created by the load. This is equivalent to a reduction of the section modulus from 91 to 93 per cent. If the interlock absorbs a still smaller share of the shearing force, the extreme fiber stress of the outer fiber does not increase considerably (only to about 115% of the original value) which would mean a total reduction of the section modulus to 87%, if no shearing force is transmitted. The extreme fiber stress of the inner fiber (in the interlock), however, increases more rapidly, if the shearing force drops below 0.4 to 0.5; but in this case it must be considered that for the protected extreme fiber of the interlock a higher stress appears permissible than for the extreme fiber at the back of the pile.

In conclusion, it may be stated that the question of the strength of steel sheet-piling (and, therefore, the question of the section modulus) is often over-estimated as compared with a careful examination of the penetration. One should always observe that, nearly without exception, failures occur, not because the wall is too weak, but because it has not been driven sufficiently deep.

PAUL BAUMANN,[37] M. AM. SOC. C. E. (by letter).—The use of high sheet-pile bulkheads is almost entirely limited to water-front engineering and particularly to harbor work. The increase in draft of ocean-going vessels has called for deeper slips and they, in turn, have called for higher bulkheads. This trend toward greater structural dimensions has been present in nearly all fields of engineering for the past two decades and bulkheads of unprecedented height are reasonably certain to be built in the near future. It is this consideration that prompted the writer to present his paper, besides the desire to deliver the theory on passive resistance from the principal obstruction to a deeper insight into the intrinsic behavior of soil, namely, the angle of repose or the angle of internal friction, conveniently but erroneously used as constant values; of substituting simple formulas for the specific, passive resistance in which the classical angle, ϕ, does not appear and the constants of which can only be correctly ascertained by test; and last, but not least, of preventing serious error in analyzing sheet-pile bulkheads without taking into account the relative flexibility of sheet-piling and the relative yield of the ground.

The response which this paper has met is very encouraging indeed. The writer is much indebted to the discussers who, in a truly professional spirit, have contributed so generously to its academic and practical value. Generally, they seem to agree that a step toward a more exact solution of sheet-pile problems is quite timely.

As all the symbols used in the paper were defined when and where they appear, a list of notations was omitted by the writer, due to lack of space, and it is much to his regret that this omission seems to have caused undue difficulties to some of the discussers in studying the paper. The same holds true of definitions of mathematical deductions which were minimized for the same reason, although as far as the writer is aware, none of the substitutions or transformations, exact or approximate, was made without a brief explanation.

Those who failed to study the paper in all its details inevitably found it difficult to separate the results of: (*a*) The test bulkhead analysis; (*b*) the outer harbor bulkhead analysis; (*c*) the laboratory tests of Professor Franzius; (*d*) the writer's deductions, based on these laboratory tests (including the true distribution of passive resistance, the possible stress distribution in the passive prism, and the differential equation of the surface of rupture); (*e*) the new theory for the design of sheet-pile bulkheads; and, (*f*) the application of this theory to the test bulkhead.

Mr. Feld believes that the end piles in the test wall took less load than the inside piles because of active pressure on the end walls causing an outward movement which, in turn, engaged a passive prism and thereby deprived the earth in front of the end piles of some of its resistance. This is contrary to the opinions of Professor Franzius and Professor Krynine who conclude that the end piles had more support in the ground than the other piles because of the wedges that "fan out" laterally beyond a vertical plane through the side walls. In principle, the writer agrees with Professor Franzius and Professor Krynine so far as an addition to, rather than a deduction from, the support of the end piles is concerned. However, Mr. Feld is correct in concluding from Fig. 6 that the end piles took less load than the inside piles; but this was due to fast percolation between the test wall and the side walls, owing to the short penetration of the copper seals. This short penetration actually caused a blow-out shortly after the full head was reached and was kept constant until the deflection was measured, which put the test results, both as to passive resistance and interlock efficiency, on the side of safety.

---

[37] Asst. Engr., in Chg., Mountain Div., Los Angeles County Flood Control Dist., Los Angeles, Calif.

Naturally, such discrepancies are not desirable, but under the circumstances it was quite impossible to prevent them, because, this was a field test, hurriedly prepared in connection with a lawsuit and not a carefully prepared laboratory experiment. It is hoped that this statement will answer objections made by Messrs. Feld, Vaughn, Pennoyer, Franzius, and Krynine in regard to shortcomings of the test bulkhead.

In his paper[38] on "Lateral Earth Pressures," Mr. Feld states: "It should be noted that ϕ' [angle of friction between earth and wall] decreases as the height of the fill increases * * *." Furthermore, he consistently found the center of pressure to be above the third-point of the height of the fill, indicating non-linear (parabolic) distribution of pressure and, therefore, a variable angle of internal friction. Nevertheless, he cannot agree with any of the writer's conclusions.

In discussing the use of the angle of repose instead of the angle of internal friction, Messrs. Feld and Vaughn seem to overlook the fact that the angle of repose of the material under water was used. This angle of repose is substantially the same as the angle of internal friction under water as capillary action or cohesion is eliminated thereby. The determination of the angle of internal friction of the material in the dry, as suggested by Mr. Feld, is probably subject at least to as much error as the determination of the angle of internal friction of the material under water, as arrived at by the writer.

In stating that the writer's new theory "only applies within very small limits, practically within elastic deformations of the soil structure," Mr. Feld evidently was misled by the writer's use of the wording, "elasticity of both wall and soil," which was used in its descriptive rather than in its physical meaning. The theory applies fully within the limits of the tests through which the constant, b, m, and n, are determined, whereas the practical limits naturally are governed by a safety factor. Thus, if the tests should show that a wall movement of y in. is necessary to cause failure of the soil structure along the surface of rupture, only a fraction of y would be allowed in practice. As the deflection diagrams (that is, the elastic lines) can be drawn to such scales as will greatly magnify the distance, y, the smallness of these movements is of no concern.

The writer agrees with Mr. Feld that there is no method for measuring passive resistance for zero movement, because there is none, but he does not agree as far as active pressure is concerned, provided zero is considered from a standpoint of testing (that is, relative pressure as against absolute pressure).

---

[38] *Transactions*, Am. Soc. C. E., Vol. LXXXVI (1923), p. 1470.

Mr. Vaughn gives an excellent treatise on dredging. However, his claim that relieving platforms are not effective is not supported by accounts of European harbor developments where this type was originated and has since been used extensively. It is true though that to make relieving platforms effective, considerable care is required, particularly during the filling through the holes. The grain diameter of the fill material behind the bulkhead at Berth 3 at Long Beach was generally less than 0.1 mm and, therefore, was of such size as to lend itself to a "quick" condition and in this state substantially acted as a liquid. Relieving platforms are predicated on stable fills of reasonable density and permeability which is physically impossible to accomplish within the period of construction with material of such fineness.

Mr. Vaughn believes that there is cohesion in soil under water, but at the same time he admits that when dredged vertically the bank does not stand up any length of time, but collapses just like granular soil. The answer is that there is no appreciable cohesion in soil under water. He compares resistance factors in passive prisms with sliding factors in hydraulic-fill dams and seems to be under the impression that they should be the same. The former is expressed by the term, $\tan\left(45 + \frac{\phi}{2}\right)$, and the latter by $\tan \phi$, which cannot be equal. Probably, he was thinking of active pressure, in which case $\tan \phi = \tan\left(45 - \frac{\phi}{2}\right)$ for $\phi = 30$ degrees. He has overlooked the fact that the passive resistance of a prism is always greater than its weight as long as the bottom of the wall is below the lowest ground surface. For level ground and a frictionless wall, the ratio, $\frac{E_p}{G} = \tan\left(45 + \frac{\phi}{2}\right) = 1.73$ for $\phi = 30°$, whereas the analysis shown in Fig. 7 gave $\frac{E_p}{G} = \frac{20\,400}{14\,650} = 1.40$ for a sloping ground and a rough wall, which is consistent. Thus, before casting doubt on the validity of this latter factor Mr. Vaughn should disprove the validity of the classical theory of passive resistance of earth.

Mr. Drucker refers to the use of the factor, 2, which was applied to the Coulomb formula and describes this procedure as an assumption that the actual passive resistance is twice the theoretical. This is incorrect. The fact is that the actual passive resistance for a rough wall was assumed to be twice the theoretical for a frictionless wall, according to Coulomb's formula. The analysis of the test bulkhead shows an effectiveness factor of 1.68 in spite of the weakening of the passive prism through percolation at the ends, as mentioned before, and a reduction in wall friction in addition.

Mr. Drucker refers to Equation (69) which is from a paper[19] by R. P. Pennoyer, Assoc. M. Am. Soc. C. E., and shows correctly, by mathematical deductions, that the points of zero moment and zero load do not necessarily coincide. The writer fully agrees with Mr. Drucker and always did agree as may be seen from Equations (2) and (3) and the intervening text. Throughout all the analyses the distance, a_0, between the ground surface and the point of zero load was kept apart from the distance, X_0, between the ground surface and the point of zero moment. The term, 0.06 h_0, in Equation (3) is, in fact, nothing but the approximate equivalent of 0.60 X_0.

Thus, Equation (69) and all of Mr. Pennoyer's deductions, referred to by Mr. Drucker, are not general, but are based on the special case, $a_0 = X_0$.

Mr. Drucker's Equation (83) is a fair approximation for *d*, the exact equation, as derived by the writer, being:

$$d = \frac{0.235\, t^5_1 - 0.532\, t^4_1\, (a_0 - X_0) - 0.69\, t^3_1\, (a_0 - X_0)^2}{0.59\, t^4_1 - 1.59\, t^2_1\, (a_0 - X_0)^2 + (a_0 - X_0)^4}$$
$$+ \frac{0.437\, t^2_1\, (a_0 - X_0)^3 + \frac{t_1}{2}\, (a_0 - X_0)^4 + \frac{1}{3}\, (a_0 - X_0)^5}{0.59\, t^4_1 - 1.59\, t^2_1\, (a_0 - X_0)^2 + (a_0 - X_0)^4} \quad \ldots\ldots(130)$$

Mr. Drucker doubts the scientific background of the writer's new method of analysis because he believes it to be "based primarily on an arbitrarily assumed point of zero moment below the ground surface which, in turn, determines the passive resistance distribution for the entire depth of embedment." Indeed, if such were the case, the method would be not only unscientific, but worthless. However, such is not the case, and the point of zero moment is, in fact, the final answer in the problem. In the paragraphs following Equation (67) the writer made evident, that the line of closure of the moment diagram is assumed, subject to correction, and that it may serve as an aid to the designer to know that the true point of zero moment is generally closer to the surface than 0.10 h_0 due to the parabolic distribution of the passive resistance, at least as far as steel sheet-piling is concerned.

It is hoped that the analysis of a high bulkhead, presented subsequently herein will serve to clear up most of Mr. Drucker's questions and to disprove Dr. Lohmeyer's contention that the new method of analysis is substantially the same as that of Dr. Blum. The two initial assumptions in the new method (namely, the line of closure in the moment diagram and the line of reference in the deflection diagram) are subject to correction until the one and only position of each line is found which will satisfy the equilibrium conditions. For a given active loading and specific passive resistance, $e_p = f\ (z, y)$, there is one and only one correct position for these lines, and the corresponding load and moment distributions follow automatically and are not subject to arbitration. In spite of painstaking efforts, Mr. Drucker did not succeed in approximating the load distribution as obtained by the new theory. This would be particularly evident if his pressure lines in Fig. 18 had been drawn to a point 6 in. below the ground surface as they should have been.

Contrary to the writer, Mr. Drucker computes the factor of safety against "push-out" (Fig. 7) on the basis of total, rather than differential, loading.

In other words, he assumes that the criterion of this safety is governed by the ratio of ultimate and existing, active loading. This may prove advantageous in certain cases. However, if the active loading is definite (as it is generally), it would seem to the writer that the more logical answer to the question of safety against "push-out," is to determine how many times the existing, passive resistance could be reduced before failure would occur. This is the way the writer arrived at his factor of 2.42.

Both Mr. Vaughn and Mr. Pennoyer take exception to the writer's statement that 75% interlock efficiency was advocated by American manufacturers. Regretfully, the writer admits that he has never had the pleasure of discussing this question with American manufacturers. However, he heard this statement during interviews with sales representatives of American manufacturers, and he wishes to apologize for having taken it for granted.

In answer to Mr. Pennoyer's query, the writer wishes to emphasize that Equation (3) is quoted from "Der Grundbau."[8] This should explain the subsequent statement in the paper regarding its value. The same applies to Mr. Pennoyer's Equation (95), as was outlined before, namely, that it applies to one special case only; that is, $a_o = X_o$. Contrary to Mr. Pennoyer's claim, the writer does not advocate that, invariably, the designer should assume a 75% interlock efficiency; he actually points out what requisites are necessary to justify such an assumption.

The writer agrees with Dr. Lohmeyer that failures of steel sheet-pile bulkheads designed on a basis of 100% interlock efficiency, as has been customary in many places in the world, were due to "push-out" rather than to failure of the piles. As to his doubts concerning the justification of a new theory, Mr. Pennoyer is referred to the comparative analyses introduced subsequently herein.

Mr. Pennoyer and Dr. Lohmeyer are correct in their claim that the determination of the interlock efficiency and of the maximum stresses is approximate only and of qualitative rather than of quantitative value.

Dr. Lohmeyer's exact analysis of stresses and deflections under the influence of slippage in the interlock is a real contribution to research in this field. Whether or not the solution is as complete from a physical standpoint as from a mathematical standpoint remains to be shown by tests. It is quite possible that the occurrence of opposite stresses at points in contact in the interlocks has an influence which is not reflected in the analysis. As may be seen from Fig. 10 the writer was aware of the fact that the stress limits are due to zero friction (single sheet) and zero slippage (interlocked sheets), but he was not aware of the great deviation from a linear relation between the two, as brought out by Dr. Lohmeyer.

Several years ago the writer approached the analysis of the effect of clearance in the interlock on the basis of compound beam action (differential curvature), but found the influence on the stresses quite small. This differential curvature causes a small restraining effect at or near the supports which tends to reduce the field moment. For a compound beam on two supports, acted upon by a uniform load, the formula of the moment due to differential curvature, to be superimposed on the moment for a simple beam, is,

$$\Delta M \doteq \frac{2.655\, I\, e'\, f^2}{e''\, E} \left(\frac{20\, X^4}{l^4} - \frac{40\, X^3}{l^3} + \frac{18\, X^2}{l^2} + \frac{2\, X}{l} - 1 \right)$$

in which l = the span length; e' = distance, in inches, to outside web fiber; e'' = distance, in inches, to center of interlock from axis of gravity of single sheet; and, f = allowable fiber stress, in pounds per square inch. If interlocked sheet-piling is used as a horizontal beam on two supports, and the loading is resting on the upper sheets only, a condition of practically zero interlock efficiency may be produced which, although of theoretical interest, is in no way related to the actual conditions of bulkheads.

The most important feature brought out by Dr. Lohmeyer is the relatively small increment in fiber stress with a relatively great decrement in shear transmission in the interlock. Thus, in using the ratio between the actual outside fiber stress and the theoretical outside fiber stress for zero slippage as interlock efficiency the latter would show higher values, based on Dr. Lohmeyer's exact method, than those based on the writer's approximate method.

With the loading, moment distribution, and deflections known, the interlock efficiency can be determined directly by Dr. Lohmeyer's method, provided the sheets are not stressed beyond the elastic limit.

Professor Krynine's discussion displays an intimate knowledge of soil mechanics and deals, in a fundamental and constructive manner, with the essentials only. There is no doubt but that Equation (96) furnishes a better and more logical approximation of the elastic line than the hyperbolic spiral used by the writer and, due to its simplicity, would be well suited for introduction in the analytical solution as outlined in Equations (61) to (66), inclusive.

The zone of greater density next to a pile driven into a granular mass, so far as its dimension normal to the axis of the pile is concerned, is governed entirely by the thickness of the pile and the density of the material. If a granular mass of high density is produced by suitable rolling or tamping at the proper water content or, if it is due to consolidation under load in such a manner as to cause its particles to assume the closest position possible, its behavior will be quite different from a plastic mass such as clay of the same density. A pile driven into such a granular mass would engage an envelope which would be many times as great as that for the same pile driven into clay. This is clearly demonstrated in a simple way by checking the penetration of a plasticity needle, as outlined by R. R. Proctor, Assoc. M. Am. Soc. C. E.,[39] in granular soils and clays. It may be found that with granular material the size of the cylindrical container has a considerable influence even if its diameter is many times the diameter of the needle, whereas with clay the particles seem to be able to slip by each other due to their flaky shape without any appreciable influence on the particles a few needle diameters away. This classification of soil by means of a plasticity needle appears very promising indeed, and it is quite possible that before long the needle will obviate elaborate and costly tests for the determination of soil constants.

---

[39] "Fundamental Principles of Soil Compaction," *Engineering News-Record*, August 31, September 7, 21, and 28, 1933.

If the outside piles had had more support in the ground due to the sidewise extension of the passive prism (which would have been the case had excessive percolation been prevented through the extension of the copper seals to the bottom of the sheet-piling), the inner piles would have thrown some of their load on these outer piles and the result would have been a more uniform deflection and a more uniform, permanent set. However, evidence of percolation appeared under a head of 12 ft—that is, for a gradient of a little more than unity—and it increased rapidly as the maximum head of 15 ft was reached. This feature had been foreseen and was considered desirable for the purpose of securing results on the side of safety.

As a check on the conclusion regarding the decrease of the angle of internal friction with the depth, the writer determined the constants, m, n, and b, in Equation (27) from Professor Franzius' experiments on a frictionless wall as follows: $m = 1.20$; $n = 3.20$; and $b = 1.37 \times 10^{-6}$, which are the values used in the following example of bulkhead analysis.

The angle of internal friction, as a function of z for $y = 0.30$ ft, is shown in Fig. 27 which was plotted from Equation (33).

The equation of the surface of rupture for $y = 0.30$ ft was derived from the differential equation, $\frac{d\eta}{dz} = \sqrt{\xi}$, namely,

$$\eta = 3.47\left((H - h)^{0.80} - (z - h)^{0.80}\right) + 0.0325\left((H - h)^{1.20} - (z + h)^{1.20}\right). \quad (131)$$

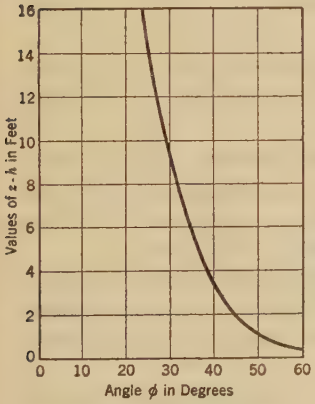


FIG. 27.—ANGLE OF INTERNAL FRICTION, ϕ, FOR $y = 0.30$ FEET.

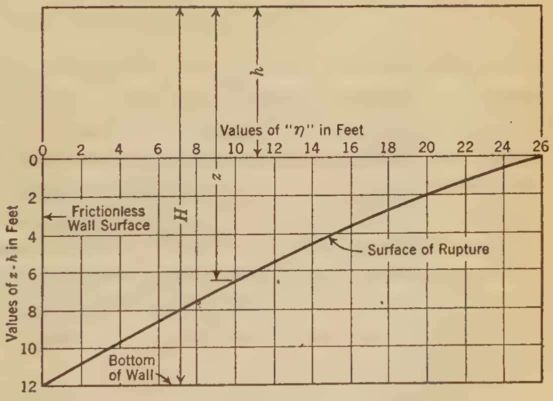


FIG. 28.—SURFACE OF RUPTURE FOR FRICTIONLESS WALL.

which is shown graphically in Fig. 28. It is interesting to note that the theoretical surface of rupture is nearly straight in spite of a variable angle, ϕ. It is slightly convex upward—that is, opposite the surface of rupture for a rough wall. The writer wonders if the straight surfaces generally found by tests on frictionless walls may not be due to the fact that the walls are never quite frictionless and that the slight friction which cannot be eliminated, is just enough to overcome the upward convexity.

In an interesting and convincing manner, Professor von Kármán points out that $\frac{1}{3}\, y_0\, p^3$ should be introduced in the second term of Equation (63) instead of $\frac{1}{2}\, y_0\, p^2$. The writer is appreciative of this correction which has been made as suggested.

Illustrative Example.—The question raised by several discussers as to whether the writer's new theory is justified, is best answered by means of an example. In Figs. 29, 30, and 31, the analyses of a bulkhead are shown, on the assumption that either steel or reinforced concrete sheet-piling is to be used, having the properties as shown in Table 7.

TABLE 7.—PROPERTIES OF SHEET-PILING; ILLUSTRATIVE EXAMPLE

Material	Moment of inertia, I, in inches ^{4}	Section modulus, S, in inches ^{3}	Modulus of elasticity, E, in pounds per square inch	Product, $E\,I$, in inch^2-pound units
Steel................	673.28	77.50	29×10^6	$19\ 560 \times 10^6$
Concrete............	27 000	1 800	3×10^6	$81\ 000 \times 10^6$

For purposes of comparison the bulkhead is analyzed on the basis of the Blum-Lohmeyer as well as the writer's method and the following soil properties are used: For the Blum-Lohmeyer: $\phi = 30°$; $\gamma = 100$ lb; and, the effectiveness factor is 2; and for the writer's method: (constants computed from Franzius' test for a frictionless wall):

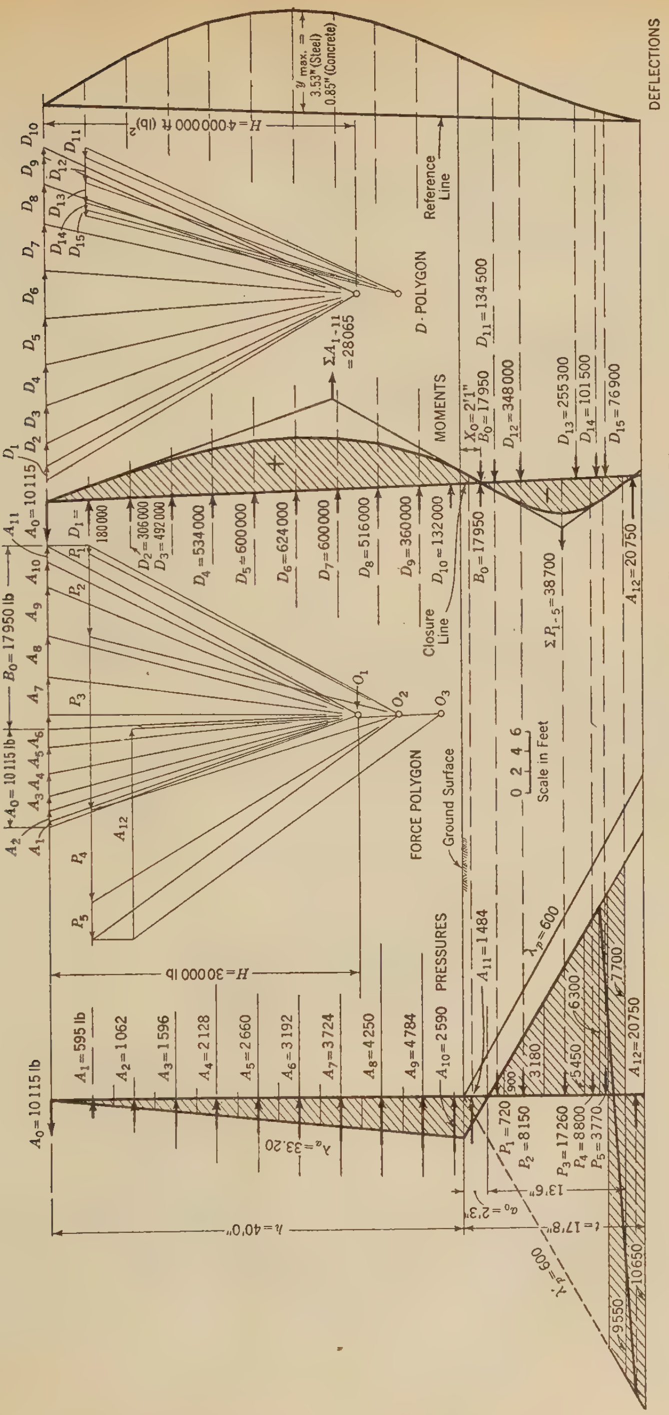
$$e_p = \frac{160\sqrt{y}}{\sqrt{1.37 \times 10^{-6}}}\,(z - h)^{0.60} \quad\ldots\ldots\ldots\ldots\ldots\ldots(132)$$

and the effectiveness factor is 2.5. The factor, 2.5, was used to make the two methods of analyses directly comparable. Values of e_p in terms of the path, y, and the depth, $z - h$, are shown in Fig. 32.

The results of the analysis of the steel and the concrete sheet-pile bulkheads, respectively, by the Blum-Lohmeyer method are analogous except for the deflections, which differ in inverted proportion to the product, $E\,I$, of the two sections (see Table 7). This analysis is shown in Fig. 29.

In contradistinction, the writer's method gives markedly different results for the two types of bulkhead. Fig. 30 shows the analysis of the steel sheet-pile bulkhead, whereas the analysis of the reinforced concrete sheet-pile bulkhead is shown in Fig. 31.

The comparative results are shown in Table 8. They confirm the writer's prediction that the present method of analysis (that is, the Blum-Lohmeyer method) cannot hold true at the same time for relatively flexible, and relatively rigid, bulkheads. The Blum-Lohmeyer method gives results which are in close agreement with those of the writer so far as a relatively flexible bulkhead (steel sheet-piling) is concerned; but it proves to be dangerously in error if applied to a relatively rigid bulkhead (reinforced concrete sheet-piling). Indeed this error is such as to result in a shortage of penetration, necessary



DEFLECTIONS
y max. = 3.53" (Steel) 0.85" (Concrete)
H = 4 000 000 ft (lb)²
Reference Line
D-POLYGON
D_1 D_2 D_3 D_4 D_5 D_6 D_7 D_8 D_9 D_{10} D_{11} D_{12} D_{13} D_{14} D_{15}
ΣA_{1-11} = 28065
MOMENTS
X_0 = 2'1"
B_0 = 17950
D_{11} = 134 500
D_{12} = 348 000
D_{13} = 255 300
D_{14} = 101 500
D_{15} = 76 900
A_0 = 10115
A_{11}
D_1 = 180 000
D_2 = 306 000
D_3 = 492 000
D_4 = 534 000
D_5 = 600 000
D_6 = 624 000
D_7 = 600 000
D_8 = 516 000
D_9 = 360 000
D_{10} = 132 000
Closure Line
B_0 = 17950
ΣP_{1-5} = 38700
A_{12} = 20750
A_2 A_1 A_3 A_4 A_5 A_6 A_7 A_8 A_9 A_{10}
A_0 = 10 115 lb
B_0 = 17 950 lb
P_1 P_2 P_3 P_4 P_5
A_{12}
O_1 O_2 O_3
FORCE POLYGON
Ground Surface
0 2 4 6
Scale in Feet
H = 30 000 lb
A_0 = 10 115 lb
A_1 = 595 lb
A_2 = 1 062
A_3 = 1 596
A_4 = 2 128
A_5 = 2 660
A_6 = 3 192
A_7 = 3 724
A_8 = 4 250
A_9 = 4 784
A_{10} = 2 590
PRESSURES
A_{11} = 1484
λ_p = 600
λ_a = 33.20
h = 40.0"
a_0 = 2.3'
13.6"
t = 17.8"
λ'_p = 600
P_1 = 720
P_2 = 8150
P_3 = 17 260
P_4 = 8800
P_5 = 3770
900
3180
5450
6300
7700
A_{12} = 20750
9550
10650

FIG. 29.—ANALYSIS OF A BULKHEAD BY BLUM-LOHMEYER METHOD.

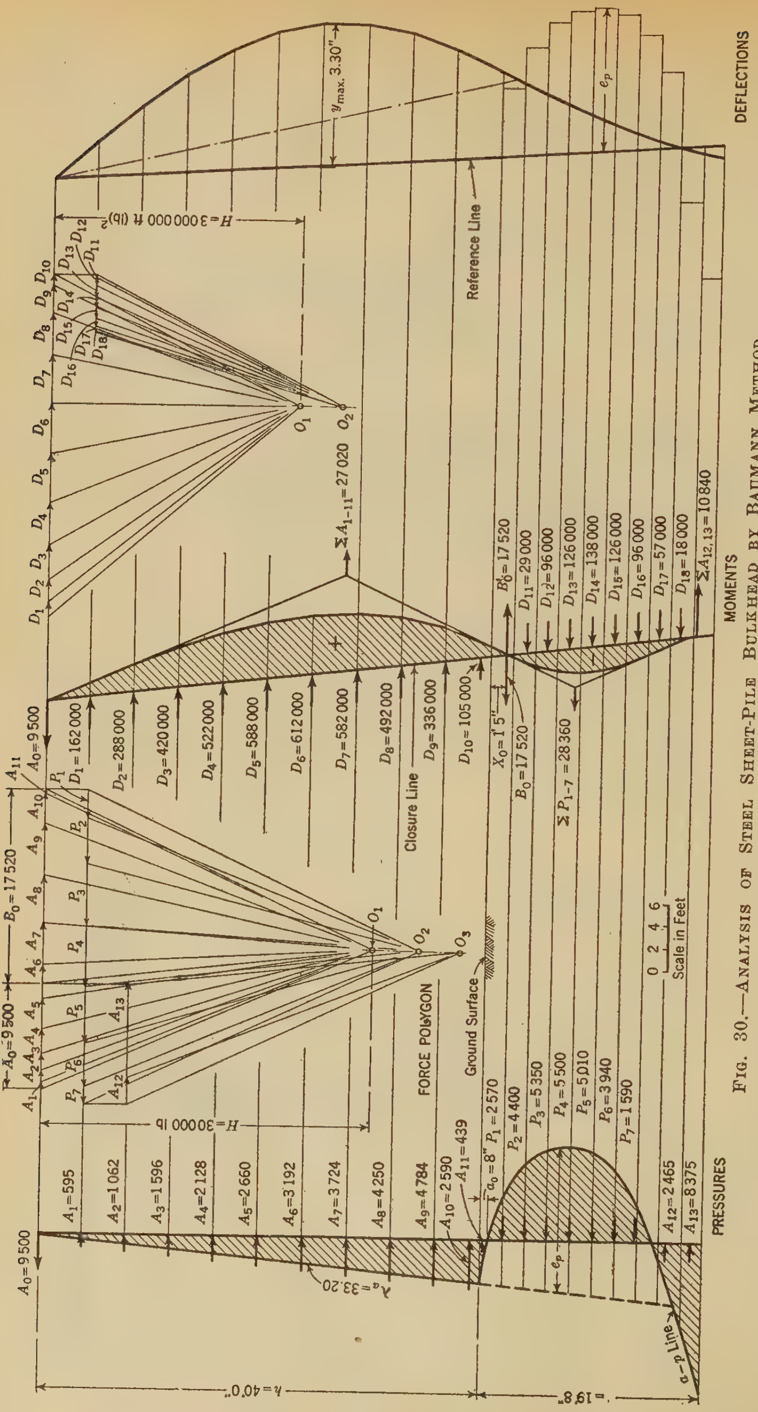

DEFLECTIONS
$y_{max.}$ 3.30"
e_p
Reference Line
$H = 3000000$ ft (lb)²
D_1 D_2 D_3 D_4 D_5 D_6 D_7 D_8 D_9 D_{10}
D_{11} D_{12} D_{13} D_{14} D_{15} D_{16} D_{17} D_{18}
O_1 O_2
$\Sigma A_{1-11} = 27020$
MOMENTS
$A_0 = 9500$
$D_1 = 162000$
$D_2 = 288000$
$D_3 = 420000$
$D_4 = 522000$
$D_5 = 588000$
$D_6 = 612000$
$D_7 = 582000$
$D_8 = 492000$
$D_9 = 336000$
$D_{10} = 105000$
Closure Line
$X_0 = 1'5"$
$B_0 = 17520$
$B_0' = 17520$
$D_{11} = 29000$
$D_{12} = 96000$
$D_{13} = 126000$
$D_{14} = 138000$
$D_{15} = 126000$
$D_{16} = 96000$
$D_{17} = 57000$
$D_{18} = 18000$
$\Sigma A_{12,13} = 10840$
$\Sigma P_{1-7} = 28360$
$A_0 = 9500$
$B_0 = 17520$
A_1 A_2 A_3 A_4 A_5 A_6 A_7 A_8 A_9 A_{10} A_{11} A_{12} A_{13}
P_1 P_2 P_3 P_4 P_5 P_6 P_7
O_1 O_2 O_3
$H = 30000$ lb
FORCE POLYGON
Ground Surface
0 2 4 6
Scale in Feet
PRESSURES
$A_0 = 9500$
$A_1 = 595$
$A_2 = 1062$
$A_3 = 1596$
$A_4 = 2128$
$A_5 = 2660$
$A_6 = 3192$
$A_7 = 3724$
$A_8 = 4250$
$A_9 = 4784$
$A_{10} = 2590$
$A_{11} = 439$
$a_0 = 8"$
$P_1 = 2570$
$P_2 = 4400$
$P_3 = 5350$
$P_4 = 5500$
$P_5 = 5010$
$P_6 = 3940$
$P_7 = 1590$
$A_{12} = 2465$
$A_{13} = 8375$
$\lambda_a = 33.20$
e_p
$a-p$ Line
$h = 40.0"$
$= 19.8"$

FIG. 30.—ANALYSIS OF STEEL SHEET-PILE BULKHEAD BY BAUMANN METHOD.

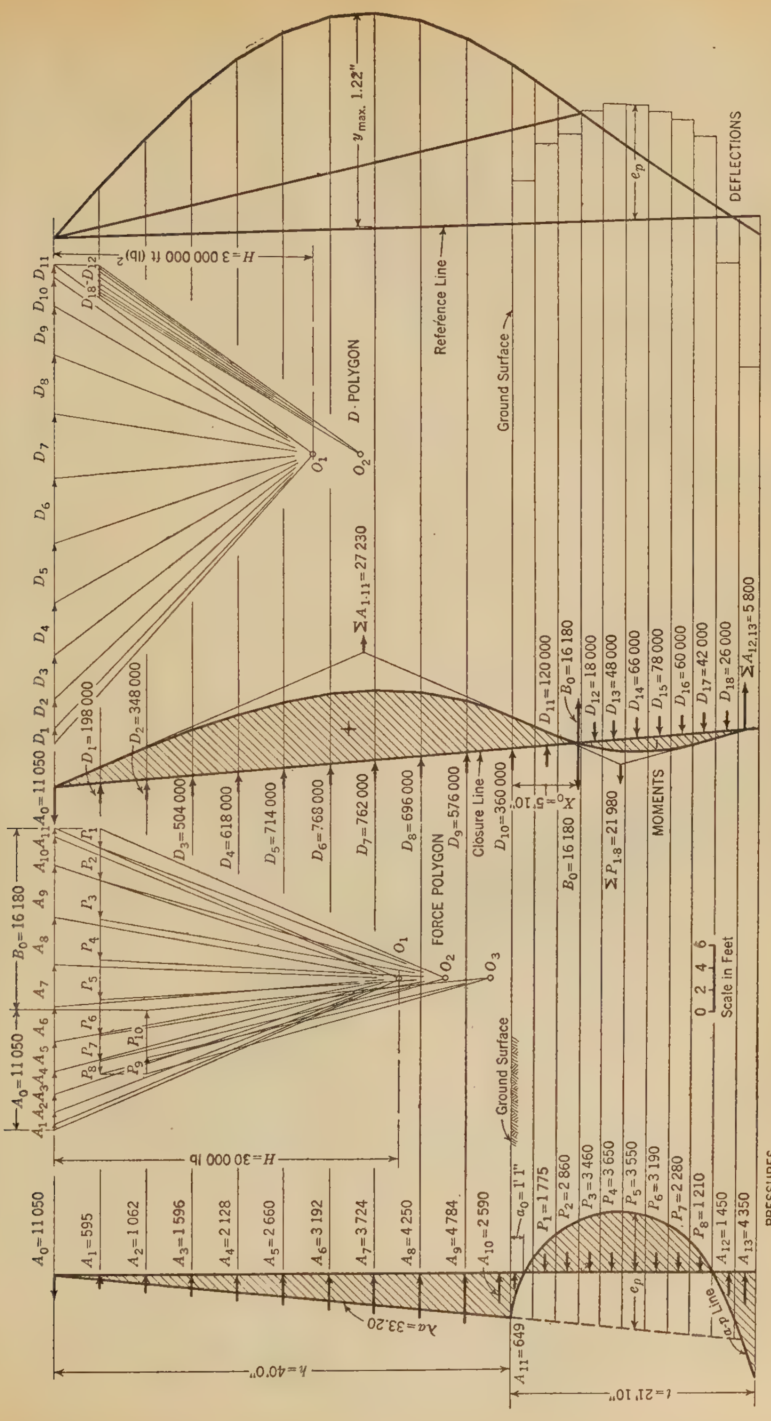

DEFLECTIONS
$y_{max.}$ 1.22"
e_p
Reference Line
H=3 000 000 ft (lb)^2
D_1 D_2 D_3 D_4 D_5 D_6 D_7 D_8 D_9 D_{10} D_{11}
D_{18}-D_{12}
O_1
O_2
D · POLYGON
Ground Surface
$\Sigma A_{1\text{-}11}$=27 230
D_1=198 000
D_2=348 000
D_3=504 000
D_4=618 000
D_5=714 000
D_6=768 000
D_7=762 000
D_8=696 000
D_9=576 000
D_{10}=360 000
D_{11}=120 000
B_0=16 180
D_{12}=18 000
D_{13}=48 000
D_{14}=66 000
D_{15}=78 000
D_{16}=60 000
D_{17}=42 000
D_{18}=26 000
$\Sigma A_{12,13}$=5 800
x_0=5' 10"
B_0=16 180
$\Sigma P_{1\text{-}8}$=21 980
MOMENTS
Closure Line
FORCE POLYGON
A_0=11 050
B_0=16 180
A_0=11 050
A_1 A_2 A_3 A_4 A_5 A_6 A_7 A_8 A_9 A_{10} A_{11} A_0
P_1 P_2 P_3 P_4 P_5 P_6 P_7 P_8 P_9 P_{10}
O_1
O_2
O_3
H=30 000 lb
Ground Surface
0 2 4 6
Scale in Feet
A_0=11 050
A_1=595
A_2=1 062
A_3=1 596
A_4=2 128
A_5=2 660
A_6=3 192
A_7=3 724
A_8=4 250
A_9=4 784
A_{10}=2 590
a_0=1' 1"
P_1=1 775
P_2=2 860
P_3=3 460
P_4=3 650
P_5=3 550
P_6=3 190
P_7=2 280
P_8=1 210
A_{12}=1 450
A_{13}=4 350
PRESSURES
λa=33.20
e_p
a-p Line
A_{11}=649
h=40.0"
t=21' 10"

FIG. 31.—ANALYSIS OF CONCRETE SHEET-PILE BULKHEAD BY BAUMANN METHOD.

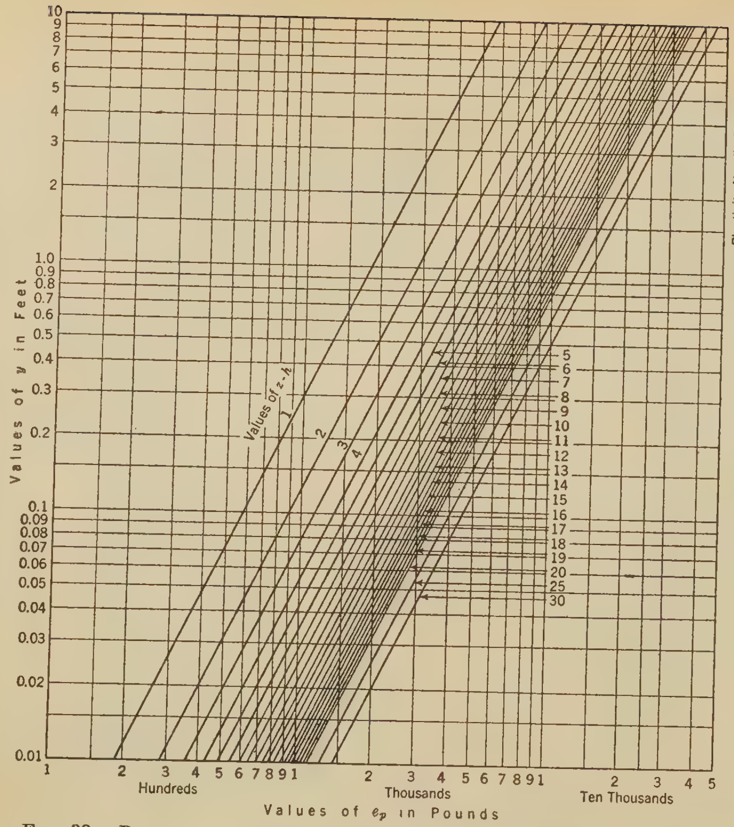


FIG. 32.—RELATION BETWEEN e_p AND y FOR CORRESPONDING VALUES $(z - h)$ BASED ON FRICTIONLESS WALL.

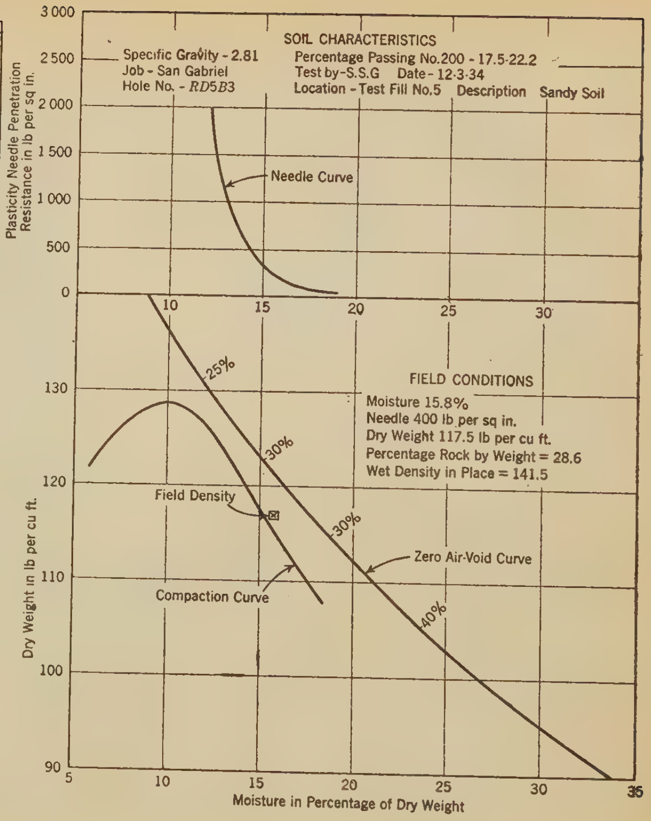


FIG. 33.—EXAMPLE OF SOIL TEST DIAGRAM

for equilibrium, of nearly 20%; in a maximum moment and a maximum fiber stress which are about 80% of the actual ones; in a maximum deflection which is about 70% of the actual one; and in a distance, X_0, of the point of zero moment below the ground surface which is about 40% of the actual one. This is a feature of interest from a standpoint of both safety and economy.

TABLE 8.—COMPARISON OF RESULTS; ILLUSTRATIVE EXAMPLE

Method of analysis	STEEL SHEET-PILES				REINFORCED CONCRETE SHEET-PILES			
	Depth of penetration, t, in feet	Variable, horizontal distance, y_{max}, in inches	Moment, M_{max}, in foot-pounds	Fiber stress, f_{max}, in pounds per square inch	Depth of penetration, t, in feet	Variable, horizontal distance, y_{max}, in inches	Moment, M_{max}, in foot-pounds	Fiber stress, f_{max}, in pounds per square inch
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Blum-Lohmeyer....	17.7	3.50	159 300	24 700	17.7	0.85	159 300	885
Writer......	19.7	3.37	156 000	24 200	21.8	1.22	192 000	1 070

In brief, the writer's analyses show that for one and the same active loading and for one and the same soil, offering passive resistance, a relatively flexible sheet-pile wall requires less penetration than a relatively rigid sheet-pile wall for the same degree of restraint and safety against "push-out."

In his constructive discussion, Professor Franzius suggests that passive resistance tests should be made on a great variety of soil samples and the results plotted in a manner similar to that shown in Fig. 15 for the purpose of facilitating the use of the writer's method in practice. The writer would like to add that sieve tests as well as compaction and needle tests should be made with the same material and the results plotted as described by Mr. Proctor and as shown in Fig. 33. The intrinsic properties of soil are reflected in these diagrams, and it is the writer's belief that a direct relation between needle readings and passive resistance would result.

The writer is indebted to Mr. Fred Schulhof for valuable assistance in the work involved in the comparative analyses.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 1909

SAND MIXTURES AND SAND MOVEMENT IN FLUVIAL MODELS

BY HANS KRAMER,[1] M. AM. SOC. C. E.

WITH DISCUSSION BY MESSRS. JOHN LEIGHLY, PAUL W. THOMPSON, GERARD H. MATTHES, R. H. KEAYS, F. T. MAVIS, V. V. TCHIKOFF, MORROUGH P. O'BRIEN AND BRUCE D. RINDLAUB, HERBERT D. VOGEL, JOSEPH B. TIFFANY, JR., AND CARL E. BENTZEL, LORENZ G. STRAUB, AND HANS KRAMER.

SYNOPSIS

The manner in which solid matter is transported by running water (leaving out of consideration material that is transported in chemical solution or by flotation), may be classified in two ways, as follows:

1.—Traction, whereby solid material moves essentially by rolling, sliding, or saltation along the bed of a watercourse, motion being produced by the action of water flowing along the inclined plane of the bed.

2.—Suspension, whereby solid material does not come in contact with the bed for an appreciable time, motion being produced by upward velocity components of the current.

Strictly speaking, these two modes of transportation grade into each other and are not sharply distinguishable. For practical purposes, however, they may be considered separately. In general, the settlement of suspended material plays an important rôle in the building up of banks, but in the absence of large natural or artificial reservoir areas in the river course it is practically a negligible factor in the transformation of the bed, the predominating factor that determines bed configuration being that of transportation by traction. This paper deals entirely with the phenomenon of traction. It is an abridged translation of a thesis entitled "Modellgeschiebe und Schleppkraft," submitted by the writer and accepted by the Technical University of Dresden, Germany, in partial fulfillment of the requirements for the degree

---

NOTE.—Published in April, 1934, *Proceedings*.

[1] Capt., Corps of Engrs., U. S. Army; Dist. Engr., U. S. Engr. Office, Tucumcari, N. Mex.

of Doctor of Engineering.[2] The original thesis and an unabridged translation, are on file in the Engineering Societies Library, New York, N. Y.

The most comprehensive experimental investigation and diagnosis of the phenomenon of traction that has been conducted was the work of Gilbert[2a] who, like other earlier American investigators, attempted to establish a basic relation between traction and velocity of the current. European investigators, following the theories of du Buat[3] and du Boys,[4] have developed a simple relation between traction and the slope and depth, which is adopted as the basis of this paper. In May, 1932, Leighly[5] presented an American treatise based on the theory of tractive force. Professor Leighly's contribution is valuable and timely; it confirms the results and conclusions presented herein in many ways and helpfully broadens the application of the theory of tractive force to channels of finite width and variable depth.

The researches of Gilbert and of European hydraulicians are reviewed in this paper and are augumented by the results of experiments performed by the writer in 1931 under the auspices of the Prussian Experiment Institute for Hydraulic Engineering and Shipbuilding, at Berlin, hereafter referred to as the P. E. I. Although the motive and conduct of the writer's investigations are based primarily upon problems of laboratory experimentation with fluvial models having movable beds, the fundamental principles of traction involved therein are of general application. The following discussion resulting from these studies is divided into two main parts. The first part covers the experiments and their immediate results; its field of interest is limited to the conduct of laboratory experiments involving open channels with movable beds. The second part deals with the theory of transportation by traction, and develops empirically a rational formula for the computation of the critical tractive force required to produce movement of bed load, that is, erosion; it is of broader application to the problems of river engineering.

PART I.—BED-LOAD MIXTURES FOR FLUVIAL MODELS

INTRODUCTION

Experience in European hydraulic laboratories[6] has shown that in the present state of experimental science strict compliance with the laws of similitude must be waived in fluvial models in which bed movement is of predominating importance. The departures from strict similarity are due essentially to the following causes:

---

[2] *Bulletin 9*, Preussischen Versuchsanstalt für Wasserbau und Schiffbau, Berlin, Germany, March, 1932.

[2a] "The Transportation of Débris by Running Water," by G. K. Gilbert, *Professional Paper No. 86*, U. S. Geological Survey, Washington, 1914.

[3] "Principes d'Hydraulique," par E. du Buat, Paris, 1786. (Quoted from Schoklitsch.)

[4] "Le Rhone et les Rivières à Lit Affouillable," par P. du Boys, *Annales des Ponts et Chaussées*, 1879, II.

[5] "Toward a Theory of the Morphologic Significance of Turbulence in the Flow of Water in Streams," by John B. Leighly, Univ. of California Publications in Geography, Vol. 6, No. 1, pp. 1 to 22.

[6] "Hydraulic Laboratory Practice," edited by the late John R. Freeman, Past-President and Hon. M. Am. Soc. C. E., Am. Soc. Mech. Engrs., N. Y., 1929.

(1) Model bed-load material cannot be made geometrically similar to natural bed material, since a scale reduction of the latter would give a powder-like substance unsuitable for model use. Therefore, model "geschiebe" is selected for its experimental properties rather than for considerations of similarity.

(2) The slope of the model is chosen to suit the model bed material. This results in an arbitrary departure from strict similarity.

(3) In some cases, although not in all, the two departures from similitude mentioned in Causes (1) and (2) must be supplemented by an additional artifice. This consists of a distortion of scales by the use of a depth scale differing from the scale of length and width. Depth distortion is principally necessary for the avoidance of model flow unlike that occurring in Nature; for example, laminar or shooting flow in the model where turbulent or streaming flow obtains in Nature.

The conditions cited are of extreme importance in laboratory experiments with river models. The practical consequence is that the kind of bed material suitable for use in a given model must usually be determined and tested in preliminary experiments. Such preliminary experiments are costly and time-consuming. Detailed experiments of this kind were conducted by Engels[7] in 1931 at the Research Institute for Hydraulics and Water Power, at Munich, Germany, and also by Rehbock in the River Hydraulics Laboratory, at Karlsruhe, Germany, in connection with his Rhine experiments.

MOTIVE, SCOPE, AND PURPOSE OF EXPERIMENTS

In a similar manner the preliminary bed-load investigation has been a usual step at the P. E. I. in almost every new problem in which hydraulic characteristics present new model conditions. The systematic experiments herein described were conducted in an effort to acquire information that would eliminate these recurring difficulties. The period from October, 1931, to January, 1932, was required for their completion.

The scope and purpose of the experiments may be stated as follows: It is comparatively easy to find a material suitable for the production of high-water bed formations in which the bed load is generally in motion only during the higher stages. The present study has been confined, therefore, to the determination of suitable bed material for the modeling of rivers in which the "geschiebe" is in motion at stages as low as mean low water. (That condition of similitude which requires that model "geschiebe" be in motion or at rest at stages equivalent to those for like conditions in Nature is termed in hydraulic experimental science, "the 'geschiebe' movement limit"). An additional requirement is that no excessive riffles shall form when maximum flow occurs in the model.

---

[7] "Grossmodellversuche über das Verhalten eines geschiebeführenden gewundenen Wasserlaufes unter der Einwirkung wechselnder Wasserstände und verschiedenartiger Eindeichungen," von H. Engels, *Wasserkraft und Wasserwirtschaft, Nos. 3 and 4*, 1932; see, also, "Large-Scale Experiments in River Hydraulics," by H. Engels and Hans Kramer, Assoc. M. Am. Soc. C. E., *Civil Engineering*, Vol. 2, No. 11, November, 1932.

With these requirements as a basis, the general problem which results may be outlined more specifically as follows:

Assumptions: Given a stream having bed-load movement at all stages down to low water and maintaining, within required limits, turbulent (and not laminar), streaming (and not shooting), flow; a model fulfilling these conditions is to be constructed to scales of 1:50, 1:100, 1:150, and 1:200.

Question: What "geschiebe" mixture curve (or gradation curve) should be selected to meet these conditions?

Required: (*a*) By computation: Limiting model slopes to meet the upper and lower criteria for flow; and (*b*) by experiment: Suitable mixture curve for use of quartz sand.

Beyond the immediate practical purpose of the experiments lay the hope, as in all research, of achieving a systematic formulation of the observed phenomena. This investigation forms Part II of this paper.

The foregoing outline indicates that these experiments apply directly to laboratory practice and are not immediately applicable to Nature. Nevertheless, this paper has indirect practical value in improving and facilitating laboratory practice, which in this particular field is fraught with the greatest difficulties. However, the aforementioned limitations on the scope of these experiments must be kept constantly in mind.

PROGRAM OF EXPERIMENTS

Three series of experiments (I, II, and III), were performed, each with a different mixture of sand, each series number corresponding to the number assigned to the sand used during that series. During the course of each series, the following conditions were varied between certain limits: (*a*) Slopes of the bed and water surface; and (*b*) depth of water in the flume.

The sands consisted of well-rounded quartz particles, and had a unit weight of 2.70 grams per cu cm. The percentage composition is shown in the gradation curves in Fig. 1. Descriptions of the sands tested are, as follows:

Series I.—Pit sand from which grains greater than 5.00 mm in diameter had been sifted.

Series II.—Sand from the same source from which grains greater than 1.77 mm in diameter had been sifted.

Series III.—The same pit sand, with grains greater than 5.00 mm and less than 0.385 mm in diameter sifted out.

Each mixture of sand was tested with four bottom slopes: 1:400, 1:600, 1:800, and 1:1 000. The water-surface slope was maintained constantly equal to the bottom slope.

Each experiment was started with very shallow depth of flow, which was increased gradually to permit determination of the point at which bed-load movement began. With further increasing depths, observations of the movement of bed load and the formation of sand riffles were made. The depth was then observed at which the upper limit of usefulness of the mixture, as determined by the appearance of disturbing sand riffles, was reached. The flow was then reduced in order to determine the cessation of bed-load movement. The depth of flow was regulated by varying the discharge, at the same time regulating the tail-gate, to maintain the required constant slope.

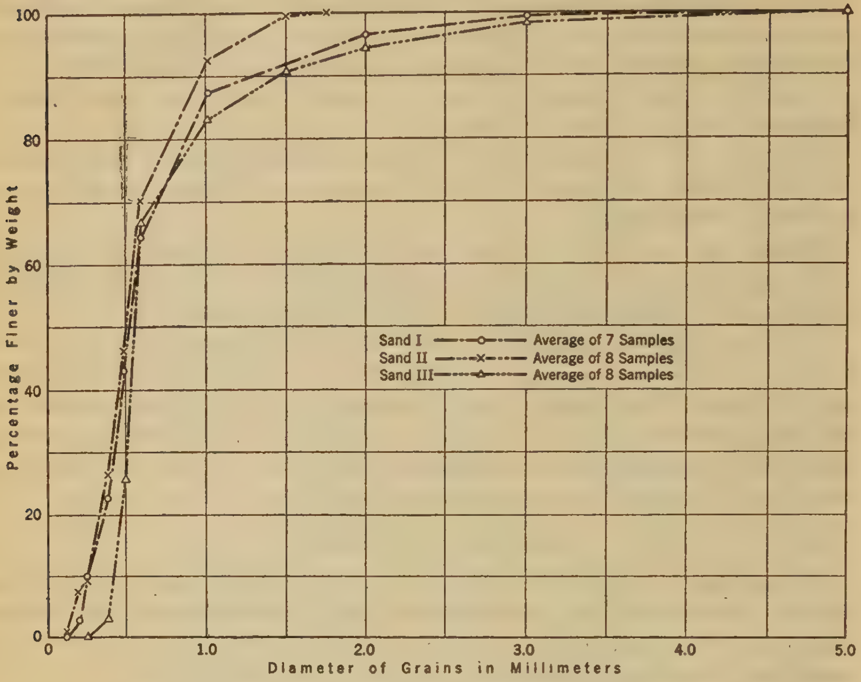


FIG. 1.—GRADATION CURVES FOR EXPERIMENTAL SAND MIXTURES

EXPERIMENTAL SET-UP AND MEASURING DEVICES

(*a*).—*Flume and Water Circuit.*—An open flume, 14 m (45.93 ft) long and of rectangular cross-section, 80.7 cm (31¾ in.) wide by 30 cm (12 in.) deep, was used for these experiments. The vertical interior walls of the flume were artificially roughened with grains of sand, 2 to 3 mm in diameter, sprinkled on a coating of wet oil paint to which the grains adhered after the paint dried. Approximately 90 such grains adhered to every 10 sq cm of wall surface. This artificial roughness is shown in Fig. 2.

The cycle of the water used in these experiments was as follows: (1) A centrifugal pump raised the water from a reservoir to an overhead tank which was equipped with a long rectangular weir designed to maintain a uniform head; (2) from the overhead tank the water flowed through a pipe equipped with a valve, and through the baffled stilling chamber into the flume, at the lower end of which was an adjustable tail-gate; and (3) from the flume, the water flowed into a weir box and over a measuring weir back into the pump reservoir. The set-up is shown in Fig. 3.

(*b*).—*Bed Construction.*—The bed of the flume was prepared and constructed as follows: Along the axis of the bottom of the flume, at 1-m (39⅜-in.)

intervals, 12 d finishing nails were driven to a given grade as determined by precise leveling. Damp sand was then placed in the flume and by means of wooden strips worked to the grade indicated by the nails. The strips were then removed, and the surface of the sand was carefully smoothed with a trowel. The thickness of the layer of sand was approximately 7 cm (2¾ in.). At the upper and lower ends of the flume the sand layer was abutted by a block of concrete which tapered off gradually to the bottom of the flume. Immediately below the upper abutment, the bottom of the flume was covered with coarse gravel for a distance of about 40 cm (15¾ in.), forming a transition reach.

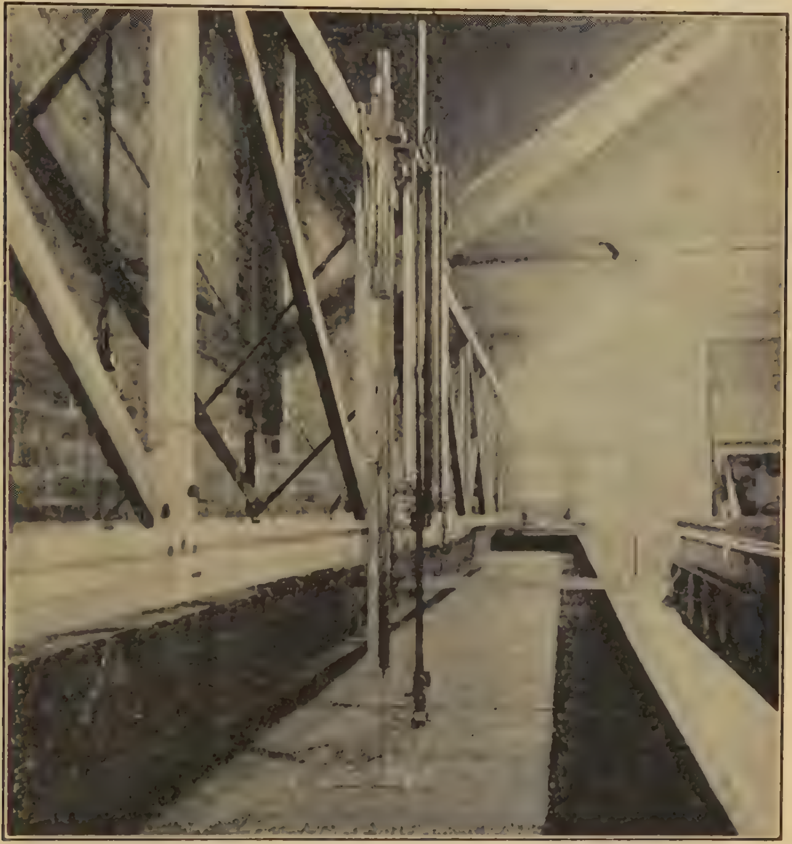


FIG. 2.—NEWLY CONSTRUCTED BED READY FOR EXPERIMENT

Each time the slope was changed, the nails ("grade stakes") were reset. The remaining sand was either thoroughly mixed with fresh sand of like composition or entirely removed and replaced by sand of different composition leveled off as previously described.

(c).—Water Surface.—At 2-m (6.6-ft) intervals along the axis of the flume seven holes had been bored through the solid wooden base and connected by

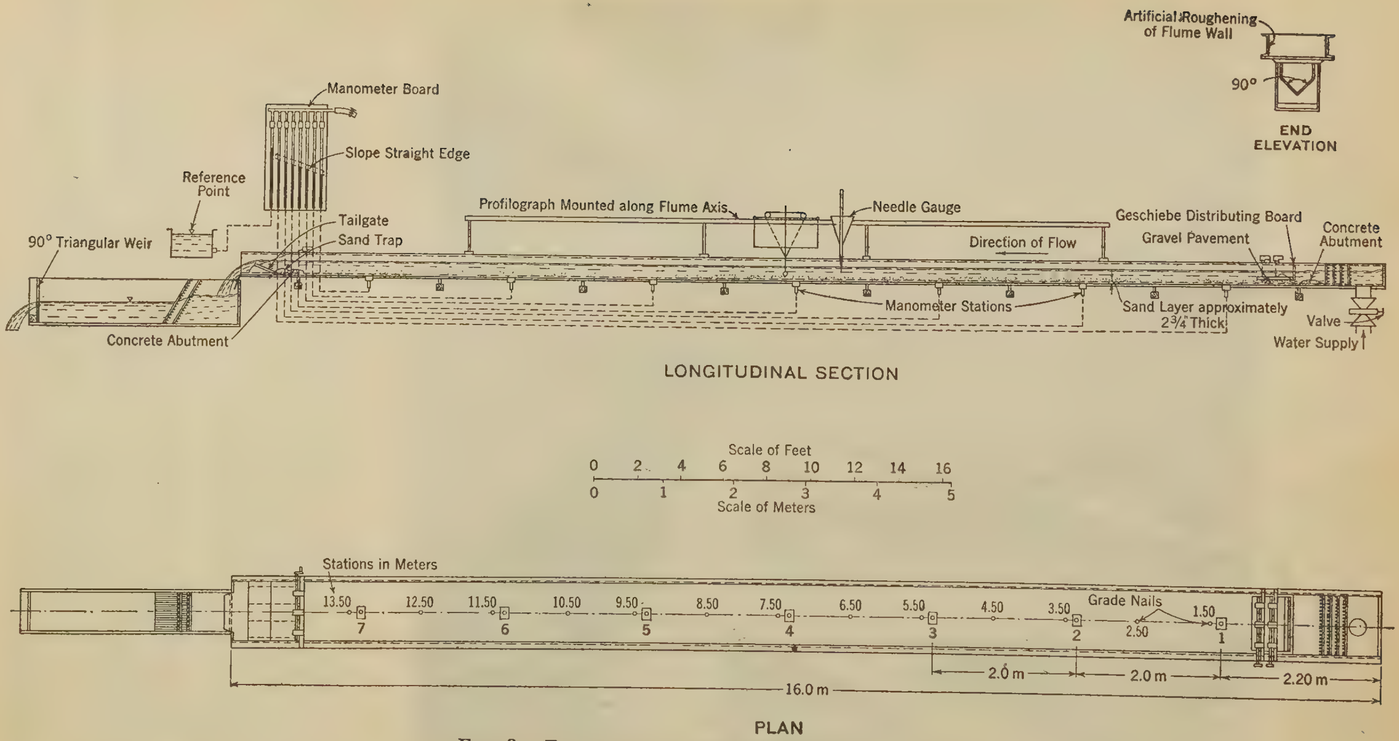

Artificial Roughening of Flume Wall
90°
END ELEVATION
Manometer Board
Slope Straight Edge
Reference Point
Tailgate
Sand Trap
90° Triangular Weir
Concrete Abutment
Profilograph Mounted along Flume Axis
Needle Gauge
Geschiebe Distributing Board
Gravel Pavement
Concrete Abutment
Direction of Flow
Manometer Stations
Sand Layer approximately 2¾" Thick
Valve
Water Supply
LONGITUDINAL SECTION
Scale of Feet
0 2 4 6 8 10 12 14 16
0 1 2 3 4 5
Scale of Meters
Stations in Meters
13.50
12.50
11.50
10.50
9.50
8.50
7.50
6.50
5.50
4.50
3.50
2.50
1.50
7
6
5
4
3
2
1
Grade Nails
2.0 m
2.0 m
2.20 m
16.0 m
PLAN

FIG. 3.—FLUME FOR "GESCHIEBE" EXPERIMENTS

lead pipes and rubber hose to manometer tubes. These manometer stations were numbered from 1 to 7, reading down stream. The tubes were formed into a battery, equipped with a backboard of millimeter cross-section paper with mirror background, and placed conveniently near the lower end of the flume. Since the manometer tubes were mounted at equal intervals, it was possible, by means of an adjustable celluloid straight-edge, to check the slope visually. An eighth manometer tube was similarly installed and connected with a vessel securely fixed to the wall of the building. The water surface of this vessel served as a zero point for the other manometer tubes. All tubes were connected at their upper ends; they were air-tight so that their meniscuses could be brought to any convenient height by means of a suction pump. The water surface was maintained parallel to the bottom slope by regulating the tail-gate, which was operated directly by the slope observer at the battery of manometer tubes. Readings were taken only after stable flow conditions had become established.

(d).—Bed Slopes.—The methods of determining the bed slopes and corresponding water depths were improved as the experiments progressed. The arrangements used for this purpose were, in chronological order, as follows:

(1) For Experiment I (Slopes, 1:400, 1:600, 1:800, and 1:1 000): At Manometer Stations Nos. 2, 4, and 6,—that is, at 4-m intervals—measuring racks were mounted across the flume with a needle-gauge attached to each over the center line of the flume. The zero of these gauges was determined by observations on a still-water surface in the flume. When depth measurements were to be taken, a 13 by 18-cm ($5\frac{1}{8}$ by $7\frac{1}{16}$-in.) plate of glass, 1.5 cm ($\frac{19}{32}$ in). thick was placed carefully on the sand bed beneath the needle-point. After the water had been drained off or stilled, the depth was measured by lowering the needle-gauge to the glass plate. This procedure determined depths at three points at 4-m intervals along the axis of the flume.

(2) For Experiment II (Slope, 1:1 000); In this case the three gauging devices at Stations Nos. 2, 4, and 6 (Fig. 3), were supplemented by two similar installations at the intermediate points, Stations Nos. 3 and 5. Measurements on the bed were made again with the aid of glass plates, thus determining depths at five points at 2-m intervals along the axis of the flume.

(3) For Experiment II (Slope, 1:800), and All Subsequent Experiments: The five racks and needle-gauges were dismantled and replaced by a profilograph bridge, 9 m (29.5 ft) long, mounted parallel to the axis of the flume. This device recorded, directly, the actual depth against a 10:1 reduction in length. At every interruption and drainage of the model, this procedure gave the complete longitudinal profile between Stations Nos. 2 and 6. The zero of the bridge was determined from a still-water surface by means of a needle-gauge attached to the upper edge of the bridge. The profilograph proved so successful that it was used on all subsequent experiments. Fig. 4 shows the arrangement.

Throughout the conduct of these experiments, a condition of steady, uniform flow was sought. However, since water flowing in an open channel (especially in a channel with a movable bed) follows natural laws and not the will of the observer, this condition is not always perfectly attainable. It was important, therefore, to adopt a procedure that would offset, or at least minimize, the influence of unavoidable deviations from uniform flow. To this end, the comparatively long reach between Stations Nos. 2 and 6 was used for observations, and in every case the mean of the slope between these stations was considered in conjunction with the mean depth over the same reach. It was believed that the advantage of this long reach, with its tendency to neutralize any local irregularities in slope or velocity, would outweigh the disadvantage of comparatively short entrance and exit sections.

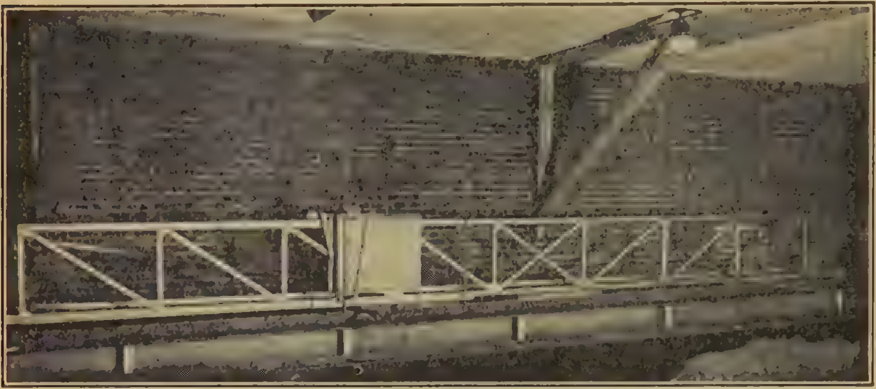


FIG. 4.—VIEW OF PROFILOGRAPH

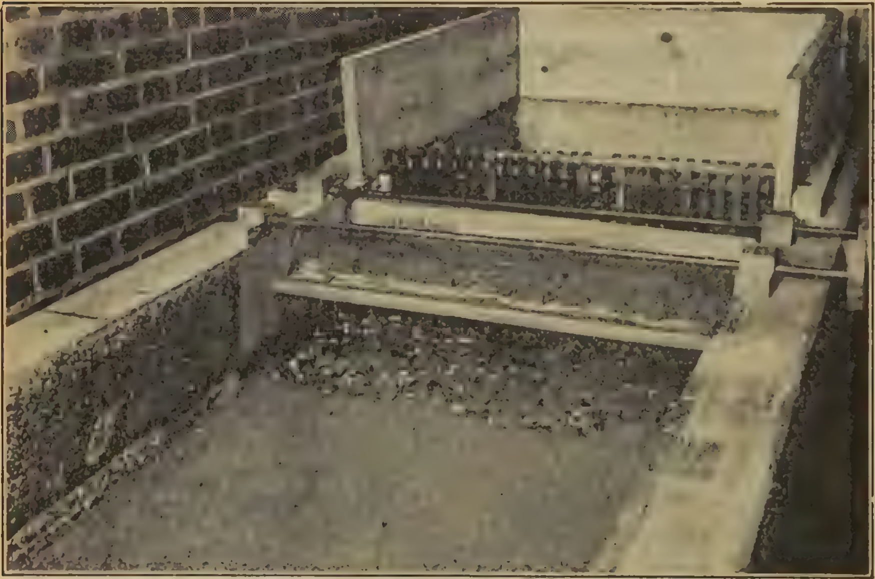


FIG. 5.—HEAD OF FLUME SHOWING DISTRIBUTING BOARD FOR INTRODUCING "GESCHIEBE"

(e).—Discharge.—Below the discharge end of the flume was a 90° triangular weir, which had been carefully calibrated. (The calibration agreed closely with the common formula, $q = 0.014\ h^{2.5}$, in which, q is measured in liters and h in centimeters.) The head, h, was measured with a needle-gauge at a point 1.2 m (3 ft 11¼ in.) above the weir.

The regulation of the discharge or stage was accomplished by means of the valve described in Paragraph (*a*). In making changes in stage, the valve was always manipulated very slowly in order to avoid disturbances from sudden changes of flow.

(f).—Water Temperature.—The temperature of the water was determined by a thermometer suspended under water in the weir box.

(g).—Quantities of Bed-Load Material.—The quantities introduced and discharged were measured in a tin vessel with a capacity of approximately 1 liter (0.26 gal). The material to be introduced was spread in a moist condition on an adjustable distributing board (see Fig. 5) at the upper end of the flume. This distributing board also served as a float for stilling the inflowing water. The sand was brought uniformly to a 1-cm (⅜-in.) slit at the upper edge of the board whence it seeped slowly into the water and was carried away. This method of feeding the "geschiebe," carefully done by an experienced assistant, considerably reduced the disturbances usually caused by sudden introductions of the material. To forestall a possible down-stream movement of riffles caused by unavoidable irregularities in introducing the sand, small local disturbances in the sand bed occurring near the upper end of the flume were smoothed out gently with a small trowel.

As a result of the increased section, the space between the tail-gate and the lower concrete abutment served as a trap for the sand carried from the flume. The trap was roughly calibrated so that the volume of "geschiebe" could be read directly. By adjusting the inflow into the flume to the observed rate of deposit in the sand-trap, a balance was soon reached.

(h).—Sand Analysis.—The composition of the sand mixtures used was determined by the usual mechanical means of sifting with standard sieves. All the sieve analyses were run by means of a mechanical sifting machine, and samples were taken and analyzed during all stages of the experiments.

(i).—Surface Velocities.—The surface velocities of the water along the axis of the flume were determined by timing, with a stop-watch, the passage of small paper surface floats over a known distance.

(j).—Type of Flow—Laminar or Turbulent.—Through the introduction of potassium permanganate crystals, the type of flow at low stages and low velocities was determined from the characteristic appearance of colored stream lines.

DEFINITIONS

Terms and expressions used in these experiments to designate various stages of bed-load movement and riffle formation are defined with a view to eliminating, as far as possible, the involuntary subjective judgment of the observer. Although primarily intended for laboratory use, these definitions may also, with certain reservations, be applied to Nature.

(a).—Bed-Load Movement.—Various writers, such as Schoklitsch[8] and Schaffernak,[9] have already pointed out the difficulty of establishing concise conceptions and clear definitions of "geschiebe" movement. The following are arranged in stages where dividing lines, however, are not clearly definable; they apply to graded sand mixtures:

1.—"None" refers to that condition in which absolutely no particles are in motion.

2.—"Weak" movement indicates that a few or several of the smallest sand particles are in motion, in isolated spots, and in countable numbers. By "countable" is meant that by confining the field of observation to, say, 1 sq cm (0.155 sq in.), the particles in motion can be counted by the observer.

3.—"Medium" movement is used for that condition in which grains of mean diameter are in motion in numbers too large to be countable. Such movement is no longer local in character. It is not yet strong enough to affect bed configuration and does not result in transportation of an appreciable quantity of material.

4.—"General" means that condition in which sand grains up to and including the largest are in motion. Since in these experiments, as in general laboratory practice, sand grains larger than a certain size have been sifted out, this designation has a definite significance. This movement is also not local but general. It is sufficiently vigorous to change the bed configuration, although at lower stages this action takes place only slowly. There is an appreciable quantity of material transported with this condition of movement. In this paper this condition is termed the "lower limit" of usefulness of the sand.

No higher stages of movement are designated, although increasing intensity might be based upon the quantity of bed-load movement. Intermediate stages are denoted by plus or minus signs. Based upon the definitions of the various stages of movement, the "critical tractive force," T_0, can be defined as that tractive force which brings about general movement of the mixture; that is, the force that transports the largest particles present.

(b).—Riffles.—If it is difficult to define the foregoing stages of bed-load movement, the task of defining various stages of riffles seems almost hopeless. The requirements given under "Motive, Scope, and Purpose of the Experiments," call for no excessive riffles up to maximum high water. Investigators are faced immediately with the fact—confirmed by these experiments as well as those of other investigators, such as Gilbert,[26] Blasius,[10] and Owens[11]—that riffles appear at the lowest stages at which general bed-load movement occurs. Although the riffles appearing in the early stages of the experiment may be qualitatively like those in Nature, their size must be kept within very modest

---

[8] "Geschiebebewegung in Flüssen und an Stauwerken," von A. Schoklitsch, Vienna, 1926.

[9] "Die Ausbildung von Gleichgewichtsprofilen in geraden Flusstrecken mit Geschiebebett," von F. Schaffernak, *Mitteilungen* der Versuchsanstalt für Wasserbau im Ministerium für öffentliche Arbeiten, Vienna, 1916.

[10] "Über die Abhängigheit der Formen der Riffeln und Geschiebebänke vom Gefälle," von H. Blasius, *Zeitschrift für Bauwesen*, 1912.

[11] "The Settlement and Transport of Sand in Water," by J. S. Owens, *Engineering*, 1912.

limits lest the usefulness of the model be adversely affected. In Nature, riffles do not visibly alter the smoothness of the flow line; the same requirement must apply to riffles in a model.

Another fact connected with the term, "excessive riffles," is that decreasing stages or decreasing tractive force at constant slope have a tendency to flatten out those riffles formed at higher—although not excessively high—stages. This tendency also corresponds to Nature. The effectiveness of this tendency in the model is dependent upon the initial (that is, maximum) riffle formation, as well as upon the duration and magnitude of the tractive force of the reduced flow.

In this sense, riffles could be termed "tolerable" which, at reduced flow, would be smoothed out within a reasonable time. "Reasonable time" is dependent for definition on the time scale of the model. Since in practice it is impossible to maintain a strict adherence to Froude's law of similitude (in which the model time scale is equal to the square root of the horizontal scale), the time scale for river model experiments is selected from prior experience and tested in preliminary experiments. Under the usual operating conditions the hydrograph representing a year's flow in Nature is made to correspond with a working day in the model. Should this period (1 yr) be insufficient to reproduce results in Nature properly, the run is repeated rather than spread over two working days. The flood stage will thus usually extend over a maximum of 1 hr, and the mean water on the average from 4 to 6 hr. To insure continuous use of the model it is inefficient and undesirable to resort

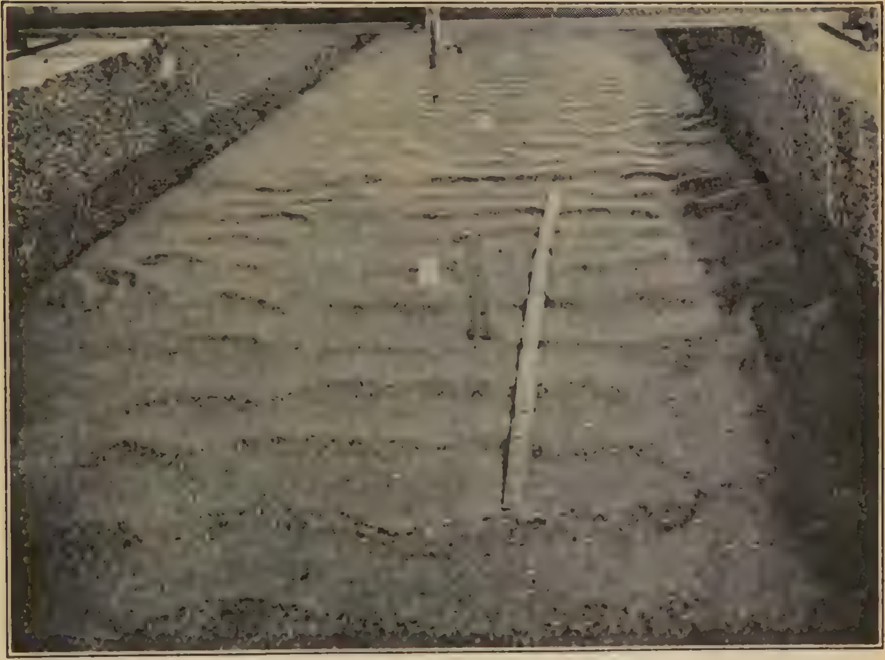


FIG. 6.—EXCESSIVE RIFFLES

to manual smoothing or troweling of riffles. It is necessary, therefore, that riffles formed at high water and flood-stage flows be smoothed out by the current at subsequent lower stages of flow. In these experiments, this property of the sand, or of the riffles, was subjected to special duration tests for each experimental series.

Another manner of characterizing the size of "tolerable" riffles would be by measuring their vertical height. Although, in general, river experiments involving models with movable beds can produce at the present stage of experimental science only qualitative results, the error introduced by the presence of riffles should be kept preferably within from 5 to 8% with a maximum of 10 per cent. Accordingly, the permissible depth of non-disturbing riffles may be taken as from 5 to 8% of the depth of water forming them. During the course of these experiments the riffle depths were measured at first and, later, as a result of practice, were estimated by eye.

Based upon the foregoing consideration, "excessive riffles" may be defined as those having a depth greater than 8% of the depth of flow which formed them within a 1-hr run and which subsequent lower stages failed to remove satisfactorily. This condition is termed herein as the "upper limit" of usefulness of the sand mixture. An example of these excessive riffles is shown in Fig. 6. Rehbock[12] and Rohringer[13] have described similar conditions.

Other definitions are: "Medium" (that is still tolerable) riffles are from 3 to 5% of the depth of flow that forms them; "weak" riffles are less than 3% of the depth of flow that forms them; and, "smooth bed" denotes that there are no visible riffle formations whatever. Figs. 2, 6, 7, and 8 show various bed conditions or riffle stages.

The location of the upper limit, as arrived at by direct observation, may be checked by computation. This check is derived from the fact that Chezy's coefficient *C*, initially increases with rising water and increasing bed-load movement, but upon reaching a critical stage in bed development, or "riffle roughness," begins to decrease. This reversal in the *C*-value appears to take place at the upper limit. At this stage of knowledge, however, the point of reversal cannot be computed beforehand. It is possible to compute the point of reversal only from the data of a series of observations and thereby check the more subjective visual determination.

TABULATION OF EXPERIMENTAL DATA

The experimental data are presented in Table 1, with the following designations and dimensions: The subordinate table headings are designations of experiments, with the following system of notation: The principal experiment series, which also serve as indices to the classifications of the sand mixtures, are designated by the Roman numerals, I, II, and III. Within each series, each run is designated by referring to the slope used for that run; for example, Table 1 (*f*), Series II, Slope 1:600 refers to Experiment Series II (using Sand Mixture II), with a bed slope of 1:600.

---

[12] "Bettausbildung, Abfluss und Geschiebebewegung bei Wasserläufen," von Th. Rehbock, *Zeitschrift* der Deutschen Geologischen Gesellschaft, No. 81, 1929.

[13] "Report on Study of Hydrotechnical Questions by Means of Laboratory Researches on Reduced Scale Models: Comparison of the Results of Such Researches with Those of Direct Observation of the Natural Phenomena, with a View to Ascertaining How Far the Law of Similitude Is True," by A. Rohringer, XV Inter. Cong. of Navigation, Venice, 1931.

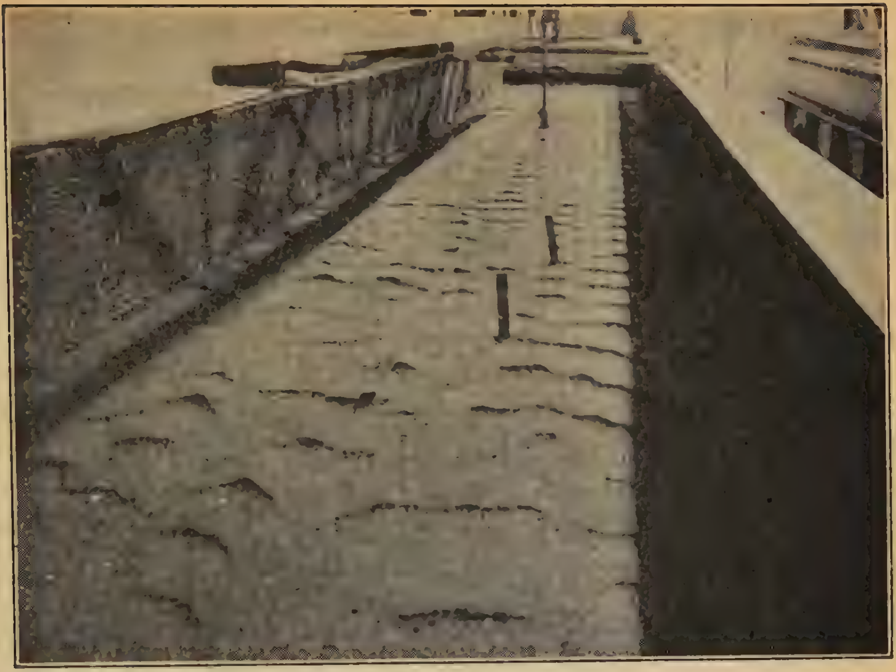


FIG. 7.—MEDIUM RIFFLES PRODUCED IN 40 MINUTES BY FLOW

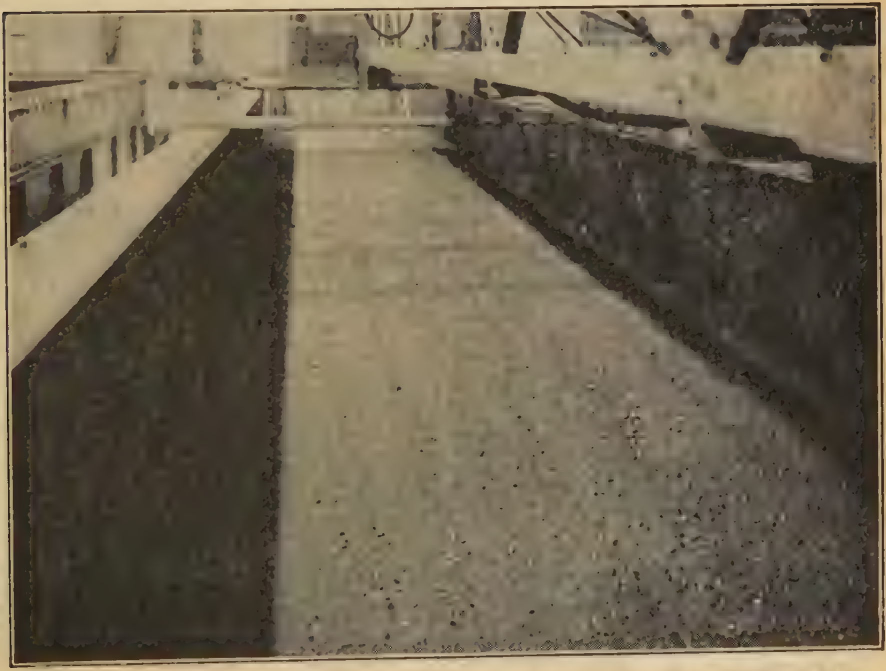


FIG. 8.—SMOOTH BED AFTER RIFFLES SHOWN IN FIG. 7 HAD BEEN WASHED OUT

TABLE 1.—OBSERVED EXPERIMENTAL DATA AND EVALUATION (TURBULENT FLOW)

Reading No. (1)	Duration of flow, in minutes (2)	Discharge, Q, in liters per second (3)	Slope, S, of water surface (4)	Mean depth, d, of water, in centimeters (5)	Surface velocity, v, in centimeters per second (6)	Area of cross-section, A, in square centimeters (7)	Wetted perimeter of cross-section, p, in centimeters (8)	Hydraulic radius, R, in centimeters (9)	Mean velocity, V_m, in centimeters per second (10)	Chezy coefficient, C (11)	Tractive force, T, in grams per square meter (12)	Coefficiency of kinematic viscosity, ν, in square centimeters per second (13)	Reynolds' number, R (14)	Velocity of wave, in centimeters per second (15)	Bed movement (16)	Riffles (17)
(*a*) SERIES I, SLOPE, 1:400																
1	20	3.19	0.00246	1.65	....	133.	84.00	1.58	24.0	38.4	40.6	0.0125	3 000	40.3	Weak	Smooth bed
2	20	0.97	0.00244	0.94	21.8	76.	82.58	0.92	12.8	26.9	22.9	0.0125	935	30.4	None	“ “
3	25	1.66	0.00238	1.18	23.4	95.2	83.06	1.15	17.5	33.4	28.1	0.0124	1 615	34.1	“	“ “
4	30	2.79	0.00240	1.57	29.0	126.5	83.84	1.51	22.1	36.7	37.7	0.0124	2 860	39.3	Weak	“ “
5	25	4.80	0.00240	1.78	....	143.5	84.26	1.70	33.5	52.5	42.7	0.0125	4 560	41.9	Medium	“ “
5*a**	....	.....	**0.00250**	**2.02**	....	.....	.....	**1.94**	**35.4**	**50.7**	**50.9**	**0.0125**	**5 500**	**44.8**	**Lower Limit**	
6	25	7.08	0.00250	2.36	42.0	190.5	85.42	2.23	37.2	49.7	59.0	0.0125	6 630	48.1	General	Weak riffles
7	25	10.8	0.00242	3.17	48.8	256.	87.04	2.94	42.2	50.5	76.6	0.0125	9 920	55.8	“	“ “
8	25	12.2	0.00255	3.56	....	287.	87.82	3.27	42.6	46.7	90.8	0.0127	10 950	58.3	“	Medium “
9	25	19.5	0.00262	4.85	....	391.	90.40	4.33	49.9	46.9	127.	0.0127	17 100	69.0	“	“ “
9*a**	....	.....	**0.00250**	**5.30**	....	.....	.....	**4.68**	**50.7**	**46.8**	**132.5**	**0.0127**	**18 700**	**72.1**	**Upper Limit**	
10	20	25.0	0.00255	5.86	....	473.	92.42	5.12	52.9	46.3	149.	0.0127	21 400	76.0	General	Excessive riffles
11	25	32.0	0.00255	7.01	69.5	565.	94.72	5.96	56.6	42.4	178.5	0.0127	26 600	83.0	“	“
12	15	34.0	0.00265	7.69	70.2	620.	96.08	6.45	54.9	42.1	203.5	0.0127	27 900	87.0	“	“
(*b*) SERIES I, SLOPE, 1:600																
1	15	2.60	0.00160	1.67	25.8	134.5	84.04	1.60	19.4	38.3	26.8	0.0126	2 450	40.5	None	Smooth bed
2	20	2.69	0.00165	1.95	30.1	157.	84.50	1.86	17.2	21.0	32.2	0.0125	2 550	43.8	Weak	“ “
3	20	5.65	0.00173	2.54	35.3	205.	85.78	2.39	27.6	42.8	43.9	0.0125	5 260	50.1	“	“ “
4	20	7.4	0.00163	3.00	38.7	242.	86.70	2.79	30.6	45.4	48.9	0.0125	6 800	54.4	Medium (+)	“ “
4*a**	....	.....	**0.00167**	**3.10**	....	.....	.....	**2.88**	**31.9**	**46.0**	**51.6**	**0.0125**	**7 350**	**55.2**	**Lower Limit**	
5	15	9.7	0.00163	3.49	....	281.5	87.68	3.21	34.5	47.6	56.9	0.0125	8 820	58.6	General	Weak riffles
6	30	13.2	0.00163	4.10	....	331.	88.90	3.72	39.8	51.1	66.8	0.0125	11 800	63.5	“	“ “
7	30	16.5	0.00176	4.70	....	379.	90.10	4.20	43.6	50.7	82.6	0.0126	14 470	68.0	“	Medium (—)
8	22	21.0	0.00163	5.57	....	449.	91.84	4.89	46.8	52.5	90.9	0.0126	18 100	74.0	“	Medium
8*a**	....	.....	**0.00167**	**6.14**	....	.....	.....	**5.33**	**51.4**	**54.5**	**102.3**	**0.0126**	**21 700**	**77.9**	**Upper Limit**	
9	20	28.4	0.00166	6.57	....	530.	93.84	5.65	53.6	55.9	108.	0.0126	23 900	80.4	General	Excessive riffles
10	16	33.5	0.00165	7.62	....	615.	95.94	6.41	54.5	53.1	125.8	0.0126	27 700	86.6	“	“ “
11	18	37.1	0.00168	8.30	....	669.	97.30	6.87	55.5	47.1	139.5	0.0126	30 200	90.4	“	“ “
12	19	2.53	0.00168	1.78	23.5	143.5	84.26	1.70	17.7	32.2	29.9	0.0126	2 380	41.9	Weak	“ “
(*c*) SERIES I, SLOPE, 1:800																
1	29	3.15	0.00126	1.86	26.3	150.	84.42	1.78	21.0	46.2	23.4	0.0126	2 970	42.8	None	Smooth bed
2	21	4.7	0.00126	2.33	31.1	188.	85.36	2.20	25.0	49.5	29.4	0.0127	4 330	47.9	Weak	“ “
3	25	6.55	0.00126	2.73	35.6	220.	86.16	2.55	29.8	54.8	34.4	0.0127	5 990	51.9	Weak (+)	“ “
4	19	8.25	0.00126	3.16	....	255.	87.02	2.93	32.4	55.6	39.8	0.0127	7 500	55.7	Medium	“ “
5	20	10.2	0.00124	3.68	....	296.5	88.06	3.37	34.5	53.4	45.7	0.0127	9 150	60.2	Medium (+)	“ “
6	21	11.3	0.00126	3.97	....	320.	88.64	3.61	35.4	52.5	50.0	0.0127	10 050	62.5	General (—)	“ “
6*a**	....	.....	**0.00125**	**4.12**	....	.....	.....	**3.74**	**36.4**	**53.3**	**51.5**	**0.0127**	**10 700**	**63.8**	**Lower Limit**	
7	19	13.2	0.00128	4.27	....	344.	89.24	3.85	38.4	54.8	54.5	0.0127	11 600	64.8	General	Weak riffles
8	19	16.4	0.00125	4.82	48.1	388.	90.34	4.30	42.3	57.7	60.2	0.0127	14 350	68.8	“	“ “
9	35	21.0	0.00125	5.48	54.8	441.	91.66	4.81	47.7	61.5	68.5	0.0126	18 150	73.4	“	Medium “
9*a**	....	.....	**0.00125**	**6.80**	**57.5**	.....	.....	**5.82**	**46.3**	**54.3**	**85.0**	**0.0126**	**21 300**	**81.9**	**Upper Limit**	
10	32	26.5	0.00129	7.22	58.8	581.	95.14	6.11	45.6	51.0	93.2	0.0126	22 050	84.4	General	Excessive riffles
11	27	33.7	0.00132	8.19	....	660.	97.06	6.80	51.0	53.8	108.	0.0125	27 750	89.6	“	“
12	300	12.6	0.00126	4.47	....	360.	89.64	4.01	35.0	49.2	56.3	0.0124	11 250	66.3	“	Washing out of riffles
13	15	9.0	0.00123	3.57	43.7	287.5	87.84	3.28	31.3	49.4	43.7	0.0124	8 250	58.3	General (—)	
14	12	4.5	0.00125	2.45	29.7	197.	85.60	2.30	22.9	42.7	30.6	0.0124	4 250	49.1	Weak	
15	13	3.2	0.00125	2.01	26.2	162.	84.72	1.91	19.8	40.4	25.1	0.0124	3 050	44.4	None	Weak riffles “ “

* Interpolated.

TABLE 1.—(*Continued*)

Reading No. (1)	Duration of flow, in minutes (2)	Discharge, Q, in liters per second (3)	Slope, S, of water surface (4)	Mean depth, d, of water, in centimeters (5)	Surface velocity, v, in centimeters per second (6)	Area of cross-section, A, in square centimeters (7)	Wetted perimeter of cross-section, p, in centimeters (8)	Hydraulic radius, R, in centimeters (9)	Mean velocity, V_m, in centimeters per second (10)	Chezy coefficient, C (11)	Tractive force, T, in grams per square meter (12)	Coefficient of kinematic viscosity, ν, in square centimeters per second (13)	Reynolds' number, R (14)	Velocity of wave, in centimeters per second (15)	Bed movement (16)	Riffles (17)
(*d*) Series I, Slope, 1:1 000																
1	20	5.5	0.00101	2.69	30.7	217.	86.08	2.52	25.4	50.2	27.2	0.0125	5 120	51.4	Weak	Smooth bed
2	15	6.6	0.00098	3.06	32.4	246.	86.82	2.83	26.8	50.9	29.8	0.0124	6 110	54.9	"	" "
3	17	7.9	0.00101	3.34	34.9	269.	87.38	3.08	29.4	52.7	33.7	0.0124	7 290	57.2	"	" "
4	20	9.4	0.001	3.71	36.4	299.	88.12	3.39	31.4	53.8	37.1	0.0124	8 600	60.5	Medium (—)	" "
5	22	11.25	0.001	4.14	39.2	334.	88.98	3.75	33.7	55.0	41.4	0.0124	10 200	63.8	Medium	" "
6	25	13.8	0.00098	4.72	....	381.	90.14	4.23	36.2	56.3	46.1	0.0124	12 350	68.1	Medium (+)	Weak riffles
7	17	16.0	0.00101	5.08	....	410.	90.86	4.52	39.1	57.9	51.3	0.0124	14 200	70.6	General (—)	" "
7*a**	....	.....	**0.001**	**5.30**	....	.....	......	**4.68**	**40.2**	**58.8**	**53.0**	**0.0124**	**15 100**	**72.3**	**Lower Limit**	
8	50	18.4	0.00105	5.37	48.0	433.	91.44	4.74	42.5	60.1	56.5	0.0124	16 200	72.6	General	Weak riffles
9	24	22.0	0.001	6.04	....	486.	92.78	5.24	45.3	62.5	60.4	0.0124	19 200	77.0	"	Medium (—) riffles
10	23	23.0	0.00103	6.87	....	554.	94.44	5.86	41.6	53.7	70.5	0.0124	19 700	82.1	"	"
11	25	28.0	0.00105	7.32	....	590.	95.34	6.19	47.5	58.8	76.9	0.0123	23 800	84.9	"	Medium riffles
12	22	30.5	0.00098	8.02	....	646.	96.74	6.68	47.3	58.5	78.3	0.0123	25 650	88.9	"	Medium (+) riffles
12*a**	....	.....	**0.001**	**8.10**	....	.....	......	**6.75**	**48.1**	**58.5**	**81.0**	**0.0123**	**26 400**	**89.2**	**Upper Limit**	
13	22	35.0	0.00099	8.71	....	703.	98.12	7.16	49.8	59.3	86.3	0.0123	28 900	92.5	General	Excessive riffles
14	60	36.7	0.001	10.16	63.0	820.	101.02	8.12	44.8	49.8	101.6	0.0124	29 250	99.9	"	"
15	45	14.7	0.00104	5.45	39.2	439.	91.60	4.80	33.5	47.3	56.7	0.0124	12 950	73.2	"	"
16	15	10.8	0.00099	4.73	....	382.	90.16	4.24	28.3	43.7	46.8	0.0124	9 700	68.2	General (—)	"
17	20	4.15	0.00098	2.74	23.1	221.	86.18	2.56	18.8	37.5	26.7	0.0124	3 880	51.9	Weak	"
(*e*) Series II, Slope, 1:400																
1	19	2.2	0.00218	1.18	27.3	95.3	83.06	1.15	23.1	46.6	25.7	0.0124	2 150	34.1	Weak	Smooth bed
2	16	2.75	0.00228	1.29	29.6	104.	83.28	1.25	26.4	49.3	29.4	0.0124	2 670	35.6	"	" "
3	15	3.4	0.0025	1.39	32.1	112.	83.48	1.35	30.3	52.0	34.8	0.0124	3 310	37.0	Medium	" "
4	20	4.35	0.00244	1.56	35.9	126.	83.82	1.51	34.5	56.9	38.1	0.0123	4 230	39.2	General (—)	" "
4*a**	....	.....	**0.0025**	**1.64**	**36.8**	.....	.....	**1.58**	**35.3**	**56.1**	**41.0**	**0.0123**	**4 530**	**40.2**	**Lower Limit**	
5	18	6.05	0.0025	1.98	39.8	160.	84.66	1.89	37.8	55.0	49.5	0.0123	5 810	44.2	General	Smooth bed
6	16	7.95	0.00251	2.38	43.9	192.	85.46	2.25	41.5	55.1	59.8	0.0124	7 560	48.4	"	Weak riffles
7	21	8.9	0.00253	2.63	44.7	212.	85.96	2.47	42.0	53.1	66.5	0.0124	8 350	50.9	"	Medium riffles
8	60	5.2	0.00248	1.88	34.3	151.5	84.46	1.80	34.3	51.4	46.6	0.0124	4 950	43.0	"	Medium (—) riffles
9	20	7.5	0.00251	2.17	42.1	175.	85.04	2.06	42.9	59.6	54.5	0.0123	7 160	46.3	"	Medium riffles
10	20	9.3	0.0025	2.67	44.7	215.	86.04	2.50	43.3	54.8	66.7	0.0123	8 760	51.4	"	"
10*a**	....	.....	**0.0025**	**3.14**	**45.5**	.....	......	**2.92**	**42.5**	**49.7**	**75.9**	**0.0123**	**10 000**	**55.6**	**Upper Limit**	
11	20	10.8	0.00254	3.17	45.9	256.	87.04	2.94	42.2	48.8	80.5	0.0123	10 000	55.9	General	Excessive riffles
12	14	9.1	0.00239	2.86	42.7	231.	86.42	2.68	39.4	49.2	68.5	0.0123	8 520	53.1	"	"
13	16	7.05	0.0025	2.38	37.2	192.	85.46	2.25	36.7	48.8	59.5	0.0123	6 730	48.4	"	"
14	45	6.6	0.0025	2.49	35.2	201.	85.68	2.35	32.9	43.0	62.3	0.0123	6 300	49.5	"	"
15	100	5.55	0.00249	2.20	29.2	177.5	85.10	2.09	31.2	43.1	54.8	0.0122	5 350	46.5	"	"
16	17	4.55	0.00253	1.90	27.6	153.	84.50	1.81	29.7	43.8	48.0	0.0122	4 400	43.3	"	"
17	18	3.38	0.0025	1.60	25.8	129.	83.90	1.54	26.2	42.1	40.0	0.0120	3 370	39.7	General (—)	"
18	18	2.78	0.0025	1.47	24.7	118.5	83.64	1.42	18.7	31.3	36.8	0.0120	2 210	38.1	Medium (—)	"

* Interpolated.

TABLE 1.—(*Continued*)

Reading No. (1)	Duration of flow, in minutes (2)	Discharge, Q, in liters per second (3)	Slope, S, of water surface (4)	Mean depth, d, of water, in centimeters (5)	Surface velocity, v, in centimeters per second (6)	Area of cross-section, A, in square centimeters (7)	Wetted perimeter of cross-section, p, in centimeters (8)	Hydraulic radius, R, in centimeters (9)	Mean velocity, V_m, in centimeters per second (10)	Chezy coefficient, C (11)	Tractive force, T, in grams per square meter (12)	Coefficient of kinematic viscosity, ν, in square centimeters per second (13)	Reynolds' number, R (14)	Velocity of wave, in centimeters per second (15)	Bed movement (16)	Riffles (17)
(*f*) SERIES II, SLOPE, 1:600																
1	18	1.70	0.00165	1.24	22.9	100.	83.18	1.20	17.0	38.2	20.5	0.0128	1 600	34.9	None	Smooth bec
2	21	3.08	0.00166	1.63	28.4	131.8	83.96	1.57	23.4	45.9	27.1	0.0128	2 870	40.0	Weak	" "
3	13	3.65	0.00168	1.78	31.0	144.	84.26	1.71	25.3	47.2	29.9	0.0128	3 380	41.8	"	" "
4	15	4.15	0.00168	2.16	34.7	174.5	85.02	2.06	23.8	40.5	36.3	0.1127	3 860	46.1	Medium	" "
5	19	6.1	0.00166	2.44	36.0	197.	85.58	2.30	31.0	50.1	40.5	0.0127	5 620	49.0	General(—)	" "
5*a**	....	.....	**0.00167**	**2.50**	**36.6**	.....	.....	**2.36**	**31.0**	**49.4**	**41.7**	**0.0127**	**5 760**	**49.6**	**Lower**	**Limit**
6	21	6.6	0.00168	2.63	37.7	212.	85.96	2.46	31.1	48.4	44.2	0.0127	6 040	50.9	General	Smooth bed
7	18	9.3	0.00169	3.19	43.1	258.	87.08	2.96	36.0	50.8	53.9	0.0127	8 400	56.0	"	" "
8	13	11.1	0.00166	3.52	45.0	284.	87.74	3.24	39.1	53.3	58.5	0.0127	10 000	58.9	"	Weak riffles
9	20	12.96	0.00166	3.68	45.7	297.	88.06	3.38	43.6	58.2	61.0	0.0127	11 600	60.2	"	Medium "
9*a**	....	.....	**0.00167**	**4.25**	**47.4**	.....	.....	**3.84**	**42.8**	**53.5**	**70.9**	**0.0127**	**13 000**	**64.8**	**Upper**	**Limit**
10	21	16.46	0.00166	4.86	49.1	393.	90.42	4.35	41.9	49.2	80.7	0.0126	14 400	69.2	General	Excessive riffles
11	15	16.77	0.00169	5.14	47.0	415.	90.98	4.56	40.4	46.0	86.8	0.0126	14 600	71.1	"	"
12	13	8.31	0.00166	3.59	33.0	290.	87.88	3.31	28.7	38.7	59.5	0.0126	7 530	59.4	"	"
13	14	5.31	0.00165	2.94	26.5	238.	86.58	2.75	22.3	33.1	48.5	0.0125	4 910	53.6	General(—)	"
14	15	3.96	0.00168	2.47	22.4	200.	85.64	2.34	19.8	31.5	41.5	0.0125	3 700	49.3	Weak	"
(*g*) SERIES II, SLOPE, 1:800																
1	30	2.06	0.00125	1.41	23.9	113.5	83.52	1.34	18.2	44.4	17.6	0.0124	1 960	37.2	None	Smooth bed
2	22	3.7	0.00125	1.91	28.5	154.	84.52	1.82	24.0	50.3	23.9	0.0124	3 520	43.3	Weak	" "
3	16	4.8	0.00125	2.24	32.0	180.5	85.18	2.12	26.6	51.6	28.0	0.0124	4 525	47.0	Weak (+)	" "
4	24	6.55	0.00125	2.70	36.0	218.	86.10	2.53	30.0	53.3	33.7	0.0124	6 100	51.5	Medium (+)	" "
4*a**	....	.....	**0.00125**	**2.86**	**37.0**	.....	.....	**2.63**	**31.6**	**54.6**	**35.8**	**0.0124**	**6 820**	**53.1**	**Lower**	**Limit**
5	18	8.1	0.00125	3.03	37.9	244.5	86.76	2.82	33.1	55.8	37.9	0.0124	7 500	54.6	General	Weak riffles
6	23	10.5	0.00125	3.64	42.3	293.5	87.98	3.34	35.8	55.4	45.5	0.0124	9 600	59.8	"	" "
7	36	12.5	0.00128	4.01	44.2	323.5	88.72	3.65	38.7	56.6	51.0	0.0124	11 350	62.8	"	Medium "
7*a**	....	.....	**0.00125**	**4.16**	....	.....	.....	**3.77**	**38.1**	**55.5**	**52.0**	**0.0124**	**11 600**	**64.0**	**Upper**	**Limit**
8	35	12.8	0.00125	4.20	....	339.	89.10	3.81	37.8	54.7	52.5	0.0124	11 600	64.3	General	Excessive riffles
9	75	15.0	0.00128	5.53	....	446.	91.76	4.86	33.6	42.6	70.5	0.0124	13 200	73.8	"	" "
10	18	6.49	0.00125	3.62	....	292.	87.94	3.32	22.2	34.4	45.3	0.0124	5 960	59.7	Medium	" "
11	22	4.55	0.00126	3.03	23.0	244.5	86.76	2.82	18.6	31.2	38.3	0.0124	4 240	54.6	Weak	" "
(*h*) SERIES II, SLOPE, 1:1 000																
1	17	6.1	0.00099	2.09	25.2	168.5	84.88	1.99	36.2	82.3	20.7	0.0125	5 750	45.3	None	Smooth bed
2	15	6.5	0.001	2.51	29.2	202.	86.02	2.35	32.2	66.4	25.1	0.0125	6 050	49.6	Weak	" "
3	16	7.4	0.00098	3.06	34.1	246.	86.82	2.84	30.1	57.2	29.8	0.0126	6 770	54.9	Medium (—)	" "
4	17	9.6	0.001	3.60	37.9	290.	87.90	3.31	33.1	57.5	36.0	0.0126	8 690	59.5	General(—)	" "
4*a**	....	.....	**0.001**	**3.72**	....	.....	.....	**3.40**	**33.6**	**57.5**	**37.2**	**0.0126**	**9 040**	**60.5**	**Lower**	**Limit**
5	13	11.0	0.001	3.96	....	319.	88.62	3.60	34.5	57.5	39.6	0.0126	9 830	62.5	General	Smooth bed
6	12	13.8	0.001	4.62	....	373.	89.94	4.15	37.1	57.6	46.2	0.0126	12 200	67.5	"	" "
7	18	16.5	0.001	5.21	....	420.	91.12	4.61	39.3	57.9	52.1	0.0126	14 350	71.6	"	Weak riffles
8	25	20.6	0.001	6.05	....	487.5	92.80	5.26	42.3	58.4	60.5	0.0128	17 400	77.0	"	Medium "
8*a**	....	.....	**0.001**	**6.33**	....	.....	.....	**5.47**	**42.1**	**57.0**	**63.3**	**0.0128**	**18 000**	**79.0**	**Upper**	**Limit**
9	30	23.2	0.001	6.90	....	556.	94.50	5.90	41.8	54.4	69.0	0.0128	19 300	82.5	General	Excessive riffles

* Interpolated.

TABLE 1.—(*Continued*)

Reading No. (1)	Duration of flow, in minutes (2)	Discharge, Q, in liters per second (3)	Slope, S, of water surface (4)	Mean depth, d, of water, in centimeters (5)	Surface velocity, v, in centimeters per second (6)	Area of cross-section, A, in square centimeters (7)	Wetted perimeter of cross-section, p, in centimeters (8)	Hydraulic radius, R, in centimeters (9)	Mean velocity, V_m, in centimeters per second (10)	Chezy coefficient, C (11)	Tractive force, T, in grams per square meter (12)	Coefficient of kinematic viscosity, ν, in square centimeters per second (13)	Reynolds' number, R (14)	Velocity of wave, in centimeters per second (15)	Bed movement (16)	Riffles (17)
(*i*) SERIES III, SLOPE, 1:400																
1	14	1.33	0.0025	0.77	23.5	62.2	82.24	0.76	21.4	49.6	19.3	0.0127	1 280	27.5	None	Smooth bed
2	17	2.35	0.0025	1.14	28.2	92.0	82.98	1.11	25.6	48.5	28.5	0.0127	2 240	33.5	Weak	" "
3	13	2.89	0.0025	1.31	30.5	106.	83.32	1.27	27.3	48.4	32.7	0.0126	2 740	36.0	"	" "
4	13	3.2	0.0025	1.41	31.8	114.	83.52	1.37	28.1	48.1	35.2	0.0126	3 060	37.2	Medium (—)	" "
5	18	3.34	0.0025	1.43	32.2	115.5	83.56	1.38	28.9	49.1	35.7	0.0126	3 180	37.5	Medium	" "
6	17	3.85	0.0025	1.57	34.0	127.	83.87	1.52	30.3	49.1	39.2	0.0126	3 670	39.3	Medium (+)	" "
7	16	4.25	0.0025	1.68	35.4	136.	84.06	1.62	31.3	49.2	42.0	0.0126	4 020	40.6	General (—)	" "
8	18	4.98	0.00251	1.93	37.0	156.	84.56	1.84	32.0	47.1	48.4	0.0126	4 670	43.6	"	" "
8*a**	....	.....	**0.0025**	**2.01**	**37.7**	.....	.....	**1.91**	**32.5**	**47.0**	**50.2**	**0.0126**	**4 930**	**44.5**	**Lower Limit**	
9	12	5.85	0.0025	2.15	39.0	174.	85.00	2.05	33.6	46.9	53.8	0.0126	5 480	46.0	General	Smooth bed
10	18	6.3	0.0025	2.28	39.4	184.	85.26	2.16	34.3	46.6	57.0	0.0126	5 900	47.4	"	" "
11	80	6.3	0.00240	2.27	40.1	183.	85.24	2.15	34.4	47.9	54.5	0.0125	5 870	47.2	"	" "
12	23	8.25	0.00248	2.74	43.5	222.	86.18	2.60	37.2	46.3	68.0	0.0125	7 720	51.9	"	Weak riffles
13	15	10.0	0.0025	3.12	46.0	252.	86.94	2.90	39.7	46.5	78.0	0.0125	9 200	55.5	"	" "
14	16	11.8	0.0025	3.45	48.2	279.	87.60	3.19	42.4	47.4	86.3	0.0125	10 800	58.2	"	" "
15	34	12.9	0.0025	3.71	51.5	299.	88.12	3.39	43.2	46.9	92.8	0.0125	11 700	60.5	"	Medium (—) riffles
16	13	6.45	0.0025	2.33	39.6	188.	85.36	2.20	34.3	46.2	58.3	0.0125	6 020	47.8	"	Smooth bed
17	22	14.4	0.0025	4.03	51.2	325.	88.76	3.66	44.4	46.3	100.8	0.0125	13 000	63.0	"	Medium (—) riffles
18	17	16.5	0.00248	4.35	54.8	351.	89.40	3.93	47.1	47.7	108.0	0.0125	14 800	65.5	"	Medium riffles
19	22	10.35	0.00253	3.21	47.0	259.	87.11	2.98	40.0	46.1	81.1	0.0125	9 520	56.3	"	Smoothbed
20	23	19.3	0.00253	4.85	57.5	392.	90.40	4.34	49.3	47.2	122.5	0.0125	17 100	69.0	"	Medium riffles
20*a**	....	.....	**0.0025**	**5.19**	**59.5**	.....	.....	**4.60**	**50.9**	**47.4**	**129.8**	**0.0125**	**18 700**	**71.5**	**Upper Limit**	
21	45	23.3	0.00249	5.51	61.5	445.	91.72	4.86	52.4	47.6	137.0	0.0125	20 300	73.9	General	Excessive riffles
(*j*) SERIES III, SLOPE, 1:600																
1	12	4.21	0.00167	1.85	30.3	149.	84.40	1.77	28.3	52.2	30.9	0.0127	3 940	42.6	Weak	Smooth bed
2	16	5.10	0.00167	2.15	34.1	173.	85.00	2.04	29.5	50.6	35.5	0.0127	4 730	46.0	Weak (+)	" "
3	13	6.20	0.00167	2.45	36.5	198.	85.60	2.31	31.3	50.4	41.0	0.0127	5 700	49.1	Medium (+)	" "
4	13	7.05	0.00167	2.67	38.1	218.	86.04	2.53	32.3	49.7	44.6	0.0127	6 450	51.2	General (—)	" "
5	16	7.9	7.00167	2.88	39.2	232.	86.46	2.69	34.0	50.8	48.1	0.0127	7 200	53.3	"	" "
5*a**	....	.....	**0.00167**	**2.93**	**39.5**	.....	.....	**2.73**	**34.2**	**50.7**	**48.8**	**0.0127**	**7 360**	**53.7**	**Lower Limit**	
6	12	8.4	0.00167	3.01	40.1	243.	86.72	2.80	34.6	50.6	50.3	0.0127	7 660	54.5	General	Smooth bed
7	82	8.4	0.00167	3.04	40.1	245.	86.78	2.82	34.3	50.0	50.7	0.0126	7 680	54.7	"	" "
8	22	11.0	0.00167	3.65	43.7	294.	88.00	3.34	37.5	50.3	61.0	0.0126	9 900	60.0	"	Weak riffles
9	19	13.2	0.00169	4.11	46.0	332.	88.92	3.74	39.8	50.1	69.5	0.0126	11 800	63.6	"	" "
10	15	15.5	0.00167	4.56	49.4	368.	89.82	4.10	42.2	51.0	76.1	0.0126	13 700	67.0	"	" "
11	21	17.2	0.00167	5.00	50.0	404.	90.70	4.45	42.6	49.4	83.5	0.0126	15 050	70.2	"	Medium (—) riffles
12	17	11.3	0.00167	3.80	43.7	306.	88.30	3.46	36.9	48.5	63.4	0.0124	10 300	61.1	"	Smooth bed
13	35	18.0	0.00167	5.33	51.6	430.	91.36	4.71	41.9	47.3	89.0	0.0124	15 900	72.5	"	Medium (—) riffles
14	20	20.9	0.00167	5.77	54.0	465.	92.24	5.04	45.0	49.0	96.5	0.0124	18 300	75.3	"	"
15	22	22.1	0.00167	6.04	54.8	486.	92.78	5.25	45.5	48.7	100.9	0.0124	19 300	77.1	"	Medium riffles
16	23	24.0	0.00167	6.54	56.7	519.	93.78	5.54	46.4	48.2	109.0	0.0124	20 700	80.3	"	"
16*a**	....	.....	**0.00167**	**6.58**	**57.4**	.....	.....	**5.66**	**47.0**	**48.4**	**109.7**	0.0124	**21 500**	**80.5**	**Upper Limit**	
17	15	26.7	0.00165	6.78	59.3	546.	94.26	5.80	48.9	50.0	111.9	0.0124	22 900	81.6	General	Excessive riffles
18	45	11.2	0.00167	3.81	43.7	307.	88.32	3.48	36.5	47.9	63.6	0.0124	10 250	61.2	"	Weak riffles
19	91	11.0	0.00167	3.80	43.7	306.	88.80	3.46	36.0	47.4	63.4	0.0123	10 120	61.1	"	Smoothbed
20	12	8.4	0.00168	3.23	40.6	261.	87.16	3.00	32.2	45.3	54.3	0.0123	7 840	56.5	"	" "
21	11	7.2	0.00174	2.88	37.9	232.	86.46	2.69	31.1	45.4	50.1	0.0123	6 800	53.3	General(—)	" "
22	10	5.93	0.00171	2.57	35.7	207.	85.84	2.42	28.6	44.4	44.0	0.0123	5 610	50.2	medium	" "

* Interpolated.

TABLE 1.—(*Continued*)

Reading No. (1)	Duration of flow, in minutes (2)	Discharge, Q, in liters per second (3)	Slope, S, of water surface (4)	Mean depth, d, of water, in centimeters (5)	Surface velocity, v, in centimeters per second (6)	Area of cross-section, A, in square centimeters (7)	Wetted perimeter of cross-section, p, in centimeters (8)	Hydraulic radius, R, in centimeters (9)	Mean velocity, V_m, in centimeters per second (10)	Chezy coefficient, C (11)	Tractive force, T, in grams per square meter (12)	Coefficient of kinematic viscosity, ν, in square centimeters per second (13)	Reynolds' number, R (14)	Velocity of wave, in centimeters per second (15)	Bed movement (16)	Riffles (17)
(*k*) Series III, Slope, 1:800																
1	10	5.8	0.00125	2.54	33.2	205.	85.78	2.39	28.3	51.7	31.8	0.0130	5 180	50.0	Weak	Smooth bed
2	12	7.9	0.00125	3.07	36.7	247.	86.84	2.85	32.0	53.6	38.3	0.0130	7 000	55.0	Medium (—)	" "
3	17	9.5	0.00125	3.40	39.6	274.	87.50	3.13	34.7	55.4	42.5	0.0130	8 350	57.9	Medium (+)	" "
4	11	10.75	0.00125	3.67	41.5	296.	88.04	3.36	36.3	56.0	45.9	0.0130	9 350	61.0	General(—)	" "
4*a**	....	.....	**0.00125**	**3.75**	**41.8**	.....	.....	**3.43**	**36.7**	**56.1**	**46.9**	**0.0130**	**9 650**	**60.8**	**Lower Limit**	
5	14	11.85	0.00125	3.91	42.5	315.	88.52	3.55	37.6	56.4	48.9	0.0130	10 200	62.0	General	Smooth bed
6	36	13.5	0.00126	4.34	45.4	350.	89.38	3.92	38.6	55.0	54.6	0.0130	11 600	65.4	"	" "
7	58	14.75	0.00125	4.63	47.0	373.	89.96	4.15	39.6	55.0	58.0	0.0127	12 900	67.5	"	Weak riffles
8	15	16.6	0.00125	5.07	48.1	409.	90.84	4.50	40.5	53.9	63.5	0.0127	14 300	70.5	"	" "
9	16	18.2	0.00125	5.48	50.2	442.	91.66	4.82	41.2	53.1	68.5	0.0127	15 650	73.4	"	Weak (+) riffles
10	19	20.9	0.00125	5.96	52.2	481.	92.62	5.19	43.5	54.0	74.5	0.0127	17 800	76.5	"	Medium (—) riffles
11	15	23.2	0.00125	6.39	53.7	515.	93.48	5.51	45.0	54.1	80.0	0.0127	19 500	79.1	"	Medium riffles
12	40	26.4	0.00125	6.91	56.3	557.	94.52	5.90	47.4	55.2	86.5	0.0127	22 000	82.5	" "	" "
13	20	26.4	0.00125	7.02	56.3	566.	94.74	5.97	46.6	54.0	87.8	0.0126	22 000	83.1	"	"
14	30	23.1	0.00125	6.60	53.7	532.	93.90	5.67	43.5	51.6	82.5	0.0126	19 500	80.6	"	Medium (—) riffles
15	30	21.0	0.00125	6.26	52.0	505.	93.22	5.42	41.6	50.5	78.3	0.0126	17 900	78.5		
16	30	18.4	0.00125	5.69	49.6	459.	92.08	4.88	40.1	51.4	71.1	0.0126	15 500	74.8	"	Weak riffles
17	80	16.5	0.00125	5.29	47.3	427.	91.28	4.68	38.6	50.5	66.1	0.0124	14 500	72.1	"	Smooth bed
18	35	14.9	0.00125	4.84	45.1	390.	90.38	4.32	38.2	52.0	60.5	0.0124	13 300	69.0	"	" "
19	15	13.7	0.00125	4.60	44.4	371.	89.90	4.12	36.9	51.4	57.5	0.0124	12 300	67.4	"	Weak riffles
20	12	11.85	0.00125	4.22	42.5	340.	89.14	3.82	34.9	50.5	52.8	0.0124	10 800	64.5	General(—)	" "
21	10	10.6	0.00125	3.93	40.4	317.	88.56	3.58	33.5	50.1	49.1	0.0124	9 700	62.2	Medium	" "
22	11	7.6	0.00125	3.22	36.7	260.	87.14	2.98	29.2	46.2	40.3	0.0124	7 030	56.4	Weak	Smooth bed
23	10	3.4	0.00125	2.02	27.3	163.	84.74	1.92	20.9	42.6	25.2	0.0124	3 240	44.6	None	" "
24	30	23.3	0.00125	6.70	53.7	540.	94.10	5.74	43.1	50.9	83.8	0.0124	20 000	81.2	General	Medium (—) riffles
25	30	26.6	0.00125	7.30	56.7	589.	95.30	6.19	45.2	51.4	91.4	0.0124	22 600	84.9	"	Medium riffles
26	30	28.0	0.00125	7.51	57.5	606.	95.72	6.34	46.2	51.9	94.0	0.0124	23 600	86.0	"	"
27	45	30.5	0.00125	7.92	58.8	639.	96.54	6.62	47.8	52.5	99.0	0.0124	25 500	88.4	"	Medium (+) riffles
27*a**	....	.....	**0.00125**	**8.11**	**59.1**	.....	.....	**6.75**	**47.9**	**52.1**	**101.4**	**0.0124**	**26 000**	**89.4**	**Upper Limit**	
28	80	33.7	0.00125	8.67	60.1	699.	98.04	7.14	48.2	51.0	108.4	0.0125	27 600	92.5	General	Excessive riffles
(*l*) Series III, Slope, 1:1 000																
1	11	4.9	0.001	2.53	29.3	204.	85.76	2.38	24.0	49.1	25.3	0.0126	4 550	50.0	None	Smooth bed
2	11	8.2	0.001	3.36	35.8	271.	87.42	3.10	30.3	54.4	33.6	0.0126	7 480	57.6	Weak	" "
3	12	10.4	0.001	3.92	38.3	316.	88.54	3.57	32.9	55.0	39.2	0.0126	9 320	62.1	Medium	" "
4	16	10.9	0.001	3.99	39.8	322.	88.68	3.64	33.9	56 2	39.9	0.0125	9 810	62.6	Medium (+)	" "
5	10	12.0	0.00105	4.25	40.6	343.	89.20	3.85	35.1	55.2	44.6	0.0125	10 760	64.7	General(—)	" "
6	20	13.9	0.001	4.76	42.3	384.	90.22	4.26	36.2	55.4	47.6	0.0125	12 300	68.5	General(—)	" "
6*a**	....	.....	**0.001**	**4.83**	**42.6**	.....	.....	**4.32**	**36.5**	**55.5**	**48.3**	**0.0125**	**12 600**	**69.0**	**Lower Limit**	
7	9	14.8	0.001	4.97	43.3	401.	90.64	4.43	37.0	55.5	49.7	0.0125	13 100	70.0	General	Smooth bed
8	14	17.5	0.001	5.62	45.5	454.	91.94	4.94	38.6	50.0	56.2	0.0125	15 200	74.5	"	" "
9	23	19.6	0.001	6.15	48.5	496.	93.00	5.34	39.5	54.1	61.5	0.0125	16 900	77.8	"	" "
10	21	22.6	0.001	6.69	49.7	540.	94.08	5.74	41.8	55.2	66.9	0.0125	19 200	81.1	"	Weak riffles
11	23	25.0	0.00101	6.86	53.0	554.	94.42	5.87	45.2	58.6	69.4	0.0125	21 200	82.2	"	" "
12	19	26.6	0.001	7.30	55.2	589.	95.30	6.18	45.2	57.5	73.0	0.0125	22 300	84.9	"	Medium (—) riffles
13	20	29.5	0.001	7.76	57.5	626.	96.22	6.51	47.1	58.4	77.6	0.0125	24 500	87.5	"	"
14	32	31.5	0.00105	8.34	58.9	673.	97.38	6.92	46.8	54.9	87.5	0.0125	26 000	90.6	"	"
15	65	33.75	0.00101	8.94	58.0	721.	98.58	7.32	46.8	54.4	89.4	0.0124	27 500	93.9	"	Medium riffles
16	25	35.5	0.001	9.26	59.7	747.	99.22	7.54	47.5	54.7	92.6	0.0124	28 800	95.5	"	"
16*a**	....	.....	**0.001**	**9.42**	**59.8**	.....	.....	**7.64**	**47.4**	**54.3**	**94.2**	**0.0124**	**29 200**	**96.4**	**Upper Limit**	
17	20	37.7	0.001	9.88	60.2	797.	10046	7.94	47.3	53.0	98.8	0.0124	30 300	98.6	General	Excessive riffles
18	52	23.1	0.00099	6.98	49.4	564.	94.66	5.96	41.0	53.4	69.1	0.0125	19 500	83.0	"	Weak riffles
19	21	15.06	0.001	5.20	42.8	419.	91.10	4.60	36.0	53.0	52.0	0.0125	13 300	71.5	Medium (+)	" "
20	16	12.0	0.001	4.46	39.0	360.	89.56	4.02	33.3	52.5	44.6	0.0124	10 760	66.4	Medium (—)	" "
21	14	9.5	0.001	3.96	35.8	319.	88.62	3.60	29.8	49.6	39.6	0.0124	8 600	62.5	Weak	" "
22	22	4.99	0.001	2.72	28.1	219.5	86.14	2.55	22.8	45.2	27.2	0.0124	4 670	51.8	None	" "

* Interpolated.

Column (2), the duration of flow, refers to the time interval between the given reading and the previous reading. Cessations and interruptions of the experiment are indicated by blank spaces between two lines of observations. The progress of the experiments can thus be read directly from the table.

In Column (3), the value, Q, is read from the weir calibration curve, using the measured head as argument. Note that 1 liter per sec is about 0.035 cu ft per sec.

The values of S in Column (4) were computed from Manometer Readings Nos. 2 and 6 as the mean slope between these stations. Column (5) contains the mean of the depths at Manometers Nos. 2, 3, 4, 5, and 6. The depth thus obtained was compared with the recorded profile and, if necessary, corrected for the total change of the mean bottom elevation for instances where the introduction and discharge of bed-load material were not satisfactorily in equilibrium. The values of surface velocity in Column (6) were from float observations. Note that 1 cm per sec is equal to 0.3937 in. per sec.

In Column (7), A, the cross-sectional area, is equal to the product of d in Column (5) and 80.7 cm, the constant width of the flume. Likewise, in Column (8), p, the wetted perimeter of the cross-section, is equal to the sum of $2d$ (from Column (5)) and the width of the flume (80.7 cm).

The artificial roughening of the walls of the flume permits the assumption that the roughness of the walls is about equal to that of the bed. This is confirmed when the resulting C-values (Column (11)) are compared with values of roughness given by Strickler.[14] In this manner the cumbersome, but not necessarily more reliable, computations involving a coefficient based on different degrees of roughness of walls and bed are avoided.

The hydraulic radius in Column (9) is equal to $\frac{A}{p} = \frac{\text{Column (7)}}{\text{Column (8)}}$; in Column (10), $V_m = 1\,000 \times \frac{Q}{A} = \left(1\,000 \times \frac{\text{Column (3)}}{\text{Column (7)}}\right)$; and the Chezy coefficient, C, in Column (11), is found by solving the formula:

$$C = \frac{V_m}{\sqrt{RS}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

the necessary values being taken from Columns (4), (9), and (10), Table 1.

The tractive force, T (see Column (12)) $= 10\,000\ S\ d$, values of S and d being taken from Columns (4) and (5). The theory of tractive force is presented in Part II of this paper. The coefficient of kinematic viscosity, ν (see Column 13)), is read from a viscosity temperature curve; Reynolds' number (see Column (14)), $\mathsf{R} = \frac{VR}{\nu}$, is the product of Columns (9) and (10) divided by Column (13). The wave velocity in Column (15) $= \sqrt{gd}$, in which, $g = 981$ cm per sec^2, and d is from Column (5).

---

[14] "Beitrag zur Frage der Geschwindigkeitsformel und der Rauhigkeitszahlen für Ströme, Kanale und geschlossene Leitungen," von A. Strickler, *Mitteilungen* des Amtes für Wasserwirtschaft, No. 16, Bern, 1913.

The locations of the lower and upper limits (that is, the beginning of general bed-load movement and the appearance of excessive riffles) are shown in Columns (16) and (17). The experimental procedure makes it improbable that any single reading will locate one of the critical limits exactly; a limit is more likely to be bracketed between two successive readings. Therefore, the critical value of the tractive force is interpolated between two readings, the exact location within the interval being estimated from the observations. The critical values thus determined are shown in Table 1 in bold-faced type.

TRANSFERABILITY OF EXPERIMENTAL RESULTS

The results of the experiments must be judged first by their transferability. Eisner's seven limits[15] that enter into consideration here are applicable for the transference from model to Nature, or, as in this case, from flume to model.

1.—The Concurrence Limit.—In river models using sand beds (that is, in "geschiebe" experiments) both gravity and friction exert determining influences on the phenomena of flow. Gravitational similitude is obtained when Froude's number, $F = \frac{V^2}{Rg}$, for model and Nature (or for a second model), has the same value. Frictional similitude is obtained when Reynolds' number, $\mathsf{R} = \frac{VR}{\nu}$, has the same value for both. With either one of these forces exerting a determining influence, the other may be neglected. In experimentation involving bed-load movement however, both forces have a pronounced effect. To satisfy simultaneously both mutually contradictory requirements, however, is impossible. A concurrence limit develops, then, that can not be overcome in practice. As a result of this difficulty, these experiments, as well as all others on bed-load movements, are not directly transferable. This largely justifies the usual conservative manner in which results of river experiments involving bed-load movement are judged, and is responsible for the acceptance of their qualitative but not of their quantitative results.

2.—The Cavitation Limit.—This limit does not enter into "geschiebe" experiments.

3.—The Critical Velocity for Appearance of Surface Waves.—The limit for surface waves is at 23 cm per sec (9 in. per sec). At lesser velocities these waves cannot occur due to surface tension.

Column (6), Table 1, shows that surface velocities at all ascertained limiting points are invariably greater than the given critical value of 23 cm per sec. It is also noteworthy that for a surface velocity less than the critical value no bed-load movement whatever took place, and that the first discernible movement occurred just after the critical surface velocity had been exceeded. The data contained in Schoklitsch's valuable compilation[16] of critical velocities as observed by du Buat, Telford, Blackwell, and others, and the unpublished findings from the comprehensive "geschiebe" experiments conducted by Krey, show substantially the same results in this respect as do the present experiments. Nevertheless, the existence of a basic connection between surface velocities (or surface waves) and "geschiebe" movement is a moot question.

---

[15] "Offene Gerinne," von F. Eisner (Handbuch der Experimentalphysik, Vol. IV), Wien, und Harms, Leipzig, 1932. (Printer's proof made available before publication through courtesy of the author.)

[16] Über Schleppkraft und Geschiebebewegung," von A. Schoklitsch, Leipzig und Berlin, 1914.

4.—The Laminar-Turbulent Limit.—This limit results from the requirement that Reynolds' number, R, should be greater than 5 000, to insure turbulent flow with a liberal safety factor. That this requirement was met by all the critical points obtained is shown by Column (14), Table 1.

5.—The Shooting-Streaming Flow Limit.—This limit may also be determined mathematically from the relation that for the rectangular cross-section used, the mean velocity must be less than the wave velocity, $\sqrt{gd}$ (Column (15), Table 1). That this condition was met for all critical points, and indeed for all observations, is shown by the direct comparison of the mean velocity, V (Column (10)) with the wave velocity, $\sqrt{gd}$ (Column (15)).

Moreover, with the existing slopes and C-values (Column (11)), shooting flow is certainly not to be anticipated. As Blasius[10] has shown in a similar manner under the assumption that $C = 45$ m$^{0.5}$ per sec and $R = d$ for infinite width, the transition from streaming to shooting flow can only occur when the slope approaches 1:200: V (mean) $= C\ \sqrt{RS} = C\ \sqrt{Sd}$; and, V (shooting) $= \sqrt{gd}$. At the limit, then, $45\ \sqrt{Sd} = \sqrt{9.81d}$; from which, $S = 0.0049 =$ (approx.) 1:200.

The slope of 1:200 is also to be considered, therefore, as the upper limit for the common expression, "moderate slope." This limit has its special significance, however, not only for the type of flow, but for "geschiebe" movement as well. The elaborate experiments made by Gilbert[2a] in 1907 to 1909, not only fixed the starting point for bed-load movement, but brought out two other pronounced critical stages in the process of movement, which have been confirmed by Forchheimer[17]: (*a*) Transition from ordinary riffles progressing down stream ("dunes"), to a completely smooth or washed out bed; and (*b*) transition from this smooth bed to long, rolling sand waves, progressing up stream ("anti-dunes").

For the work in hand, Gilbert's data have been converted into the metric system and correlated with the wave velocity, $\sqrt{gd}$. The resulting values are shown in Fig. 9. This shows that the first critical point is just above the critical wave velocity—the measurements, as Gilbert himself admitted, were not highly accurate—and that the second critical point is located in nearly constant relation to the first.

A purely mathematical analysis of riffle formations has been developed by Exner[18]. His able treatise however, is based, upon rather doubtful simplifying assumptions.

---

[17] "Hydraulik," von Ph. Forchheimer, Leipzig und Berlin, 1930.

[18] "Über die Wechselwirkung zwischen Wasser und Geschiebe in Flüssen," von F. M. Exner, *Sitzungsberichte* der Akademie der Wissenschaften in Wien, Section IIA, Vol. 134, Vienna, 1925.

6.—The Roughness Limit.—Since the walls of the experimental flume were roughened as in river model construction, a certain similarity in the roughness relation was obtained which is an important factor in the transferability of the results. This limit cannot be ignored even in flumes having movable beds[19].

7.—The "Geschiebe" Movement Limit.—The "geschiebe" movement limit, with which these experiments are basically concerned, requires that bed-load movement begin and end in the model at stages corresponding to those for like conditions in Nature. The determination of this limit was one of the objects of these experiments. From the results it may be tentatively concluded that the inception of "geschiebe" movement with ordinary quartz sand mixtures cannot be expected at surface velocities of less than 23 cm per sec (9 in. per sec).

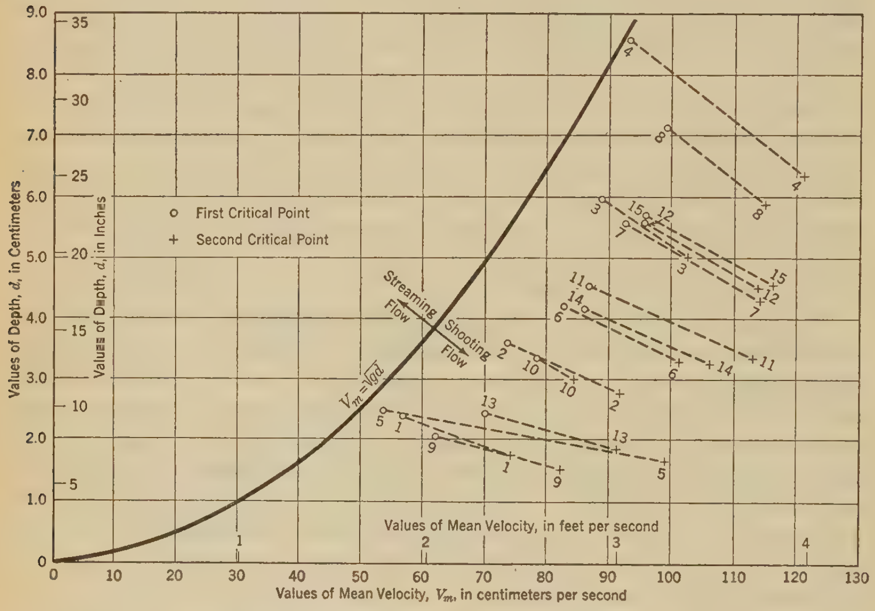


FIG. 9.—CRITICAL POINTS IN RIFFLE DEVELOPMENT

In addition to the aforementioned limiting conditions as given by Eisner,[15] there arises the question of whether the results obtained from studies in straight flumes of rectangular cross-section are readily transferable to river models with sinuous courses and irregular cross-section. Gilbert[2a] arrived at the conclusion, which is accepted in modern practice, that with equal width-depth ratios, the results of his principal experiments (with a straight flume) were, in general, quantitatively transferable to channels of different trace. With reference to the experiments here under discussion such transference is permissible to the degree that the following conditions are approached: (1) Uniform flow in model and flume; and (2) infinite width with respect to depth.

---

[19] "Wasserbauliche Strömungslehre," von P. Némenyi (Handbuch der physikalischen und technischen Mechanik, Auerbach und Hort, Vol. V), Leipzig, 1931.

RESULTS OF EXPERIMENTS

The three principal variables of these experiments—depth, slope, and sand mixture—form the criteria for the systematic study of the results. Except for special isolated cases, the study of velocity was purposely not undertaken because that is a derived and not an independent criterion, being a function of depth and slope; these are also the primary determinants for tractive force. Velocities computed from the observations are listed for reference in Table 1.

The Lower Limit

According to the definition given under "Experimental Set-Up and Measuring Devices," Paragraph (*a*), the critical tractive force is that which effects a general movement of the bed load up to and including its largest component particles.

1.—Depth-Slope Ratio for a Constant Sand Mixture.—Fig. 10 shows that the points for the lower limit of tractive force for any of the given sand mixtures lie, except for very moderate dispersion, on a straight line parallel to the x-axis. Since the ordinates in Fig. 10 represent tractive force ($T = \rho\ S\ d$), this straight-line relation may be described in the mathematical terms: $T_0 = \text{constant}$. This means that for the three sand types investigated, the lower limit of tractive force remains constant between the slopes, 1 : 400 and 1 : 1 000, and, therefore, is not determined by either d alone, or S alone, but is solely a constant product, $\rho\ S\ d$.

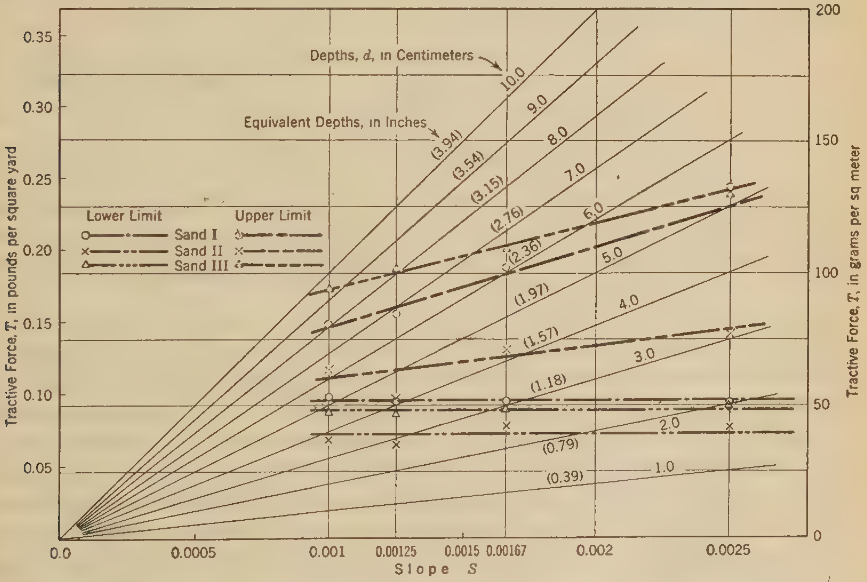


FIG. 10.—LOWER AND UPPER CRITICAL TRACTIVE FORCE IN RELATION TO SLOPE

The fact that identical "geschiebe" in open channels of moderate slope invariably have the same lower limit of tractive force has already been established from extensive observation by other experimenters, and is again confirmed by these experiments. Kreuter[20] has probably made the most exhaustive studies of this subject in Nature. Schaffernak's experiments[9] with a single sand mixture gave the same results. Schoklitsch's experiments[16] furnished verification with uni-granular sand. Gilbert's experiments,[2a] although lacking fully dependable slope measurements, agree substantially with this fact. Likewise, Krey's experiments, and other unpublished data derived by the P. E. I. from various bed-load experiments, show the same results.

In view of these repeated independent confirmations, it would appear justifiable to recognize the fact that these natural phenomena occur in accordance with a fixed law. It will be designated as the Law of Constant Critical Tractive Force.

Steiger's verification[21] of this law is of interest. Steiger states that:

> "The compilation of many observations of the action of revetments during high water reveals that the damage * * * is constant in those instances in which the product of the slope multiplied by the depth has a constant value."

2.—Dependence of Lower Critical Tractive Force on Properties of Bed-Load Mixture.—A study of the lower critical tractive force in its relation to the three sand mixtures tested (Fig. 10), will show T_0 for Sand I to have the greatest value and, for Sand II, the least. From the description under the heading, "Program of Experiments," of the method of preparing the three sand mixtures and from the sieve curves in Fig. 1, the three sand types may be roughly classified as follows (50% of the grains are larger and 50% are smaller than the median grain diameter):

Sand	Size of grains, in millimeters	Median diameter of grains, in millimeters
I	0.0 to 5.0	0.53
II	0.0 to 1.77	0.51
III	0.385 to 5.0	0.55

The comparison between Sand I and Sand II shows that the removal of the larger particles without change in the existing finer components decreases the lower critical tractive force. In view of the definition of the lower critical tractive force, this result is logical.

The lower critical tractive force for Sand III, which is Sand I with grains of less than 0.385 mm sifted out, is less than for Sand I. Although the total difference is small, a difference in that direction was observed in each experimental run (at each different slope) so that this result could scarcely be ascribed to accident or errors of measurement. By median grain size or any other criterion, Sand III certainly must be considered coarser than Sand I. Nevertheless, the coarser sand has the lower critical tractive force. This result is also in apparent contradiction to the usually accepted belief that the addition, or rather admixture, of finer sand particles increases the movability of a mixture. However, this assumption is not tenable when carried to the theoretical extreme. The addition of ever-increasing quantities of fine particles to a given mixture leads eventually to a condition in which the added material acts as a binder for the original components of the mixture and prevents the movement of the originally loose but now compacted particles. Assuming, further, that the material added is entirely lacking in cohesion—which is never quite true in reality—there is an increase in the area of friction surfaces the resistance of which the original grains must overcome in order to attain motion.

---

[20] "Beitrag zur Theorie der Geschiebeführung," von F. Kreuter, *Zeifschrift für Gewässerkunde*, 1898.

[21] Haltbarkeit der Uferbauten in bezug auf die Schleppkraft des fliessenden Wassers," von A. v. Steiger, *Schweizerische Bauzeitung*, 1918. (Quoted from review by K. Mayerhofer, *Osterreichische Wochenschrift für den Öffentlichen Baudienst*, 1919.)

du Boys[4] has given a mathematical analysis of the inter-relation of grain size and grain density (grain quantity)—in terms of area rather than in terms of volume—in its effects on the critical tractive force. He arrived at the perfectly logical conclusion that, all other factors remaining equal, the critical tractive force varies directly with the grain volume and with the number of grains per unit area.

The addition of finer material—as the creation of Sand I from Sand III might be imagined—had two results: A reduction of the mean grain volume; and a reduction of the total voids (increase in denseness of the mixture). The first result would increase the movability of the mixture; the second, would decrease it. The same conflicting action is also true when the process is reversed; that is, when adding coarser grains to a given mix. Then an increase in the mean grain volume and in the void space results, and opposing influences again affect the critical tractive force.

It is usually accepted in practice that the addition of finer particles to the sand mixture results in a decrease in the critical tractive force. This assumption is probably due to the fact that in the usual case not enough has been added to pass the point at which the reduction in the mean volume of the grains would be outweighed by the increase in denseness. As to Sand I and Sand III, it is evident that in Sand I the interesting condition was reached in which the influence of the decreasing total void (with tendency to increase the tractive force) was greater than that of the decrease in grain volume (with tendency to reduce the tractive force). In this instance, evidently, the condition of optimum movability, which is reached through gradual addition of fine particles to coarse mixtures, has been exceeded.

The relative effect of these two properties—grain volume and denseness—cannot be derived directly from these experiments. The answer to this question would require detailed and systematic experimentation in which denseness would have to be chosen as an additional variable. Gilbert's extensive research[2a] includes such experiments. He conducted systematic experiments on the movability of "geschiebe," first with eight different uni-granular sands, and then with artificial bi-granular mixtures composed of two of the primary sizes. Within each combination, mixtures of various proportions of the primary components were prepared, including 20%, 33%, 50%, 67%, and 80% of the finer grains. The relative movability of the various mixtures was determined by measurement of the load moved per unit time—all other factors being maintained constant throughout the experiment—and the percentage of voids was measured directly. The results of these experiments show: (*a*) That with the addition of fine to coarse grains, the movability of the mixture increases and reaches a maximum when the mixture contains from 60 to 90% (an average, then, of 75%) of the finer component; and the movability then decreases until it reaches that of the finer primary component; and (*b*) that the total void of the mixture is always smaller than that of the uni-granular mixture, and the mixture is densest (that is, it occupies the least volume), when it contains from 30 to 40% of the finer component.

From these observations Gilbert[2a] concludes that the optimum of movability does not coincide directly with the optimum of denseness, with which conclusion the results of these experiments agree.

In any case, the common practice of judging the relative movability of various mixtures only by the mean grain diameter or by any other function of the grain size, cannot be justified either theoretically or experimentally. Both denseness and grain size are essential factors in determining movability.

3.—Lower Limit for Commencement and Cessation of Movement.—As seen from the remarks in Column (16), Table 1, numerous observations were made during falling stages to determine the point at which bed-load movement ceased. The general result of the comparable observations is that the tractive force at which movement stops is from 0 to 20% (on the average, 12%) larger than that at which it begins. On the other hand, observations in Nature conducted by Krapf[22] and by Kreuter[23] show that the tractive force was measured to be 20% greater at the commencement of "geschiebe" movement than at its cessation, while Schoklitsch's experiments[16] with uni-granular sand showed "that this difference depends on the finer constituents of the sand and that for a clean uniform sand, the difference approaches zero."

The foregoing apparent contradictions certainly require explanation. In the first place, a comparison between critical tractive force observations during rising and falling stages can be made only by taking into consideration the existing roughness relations at the time of the observations and the laws of mechanics governing movement in the two cases. At equal depths and slopes and, therefore, at equal tractive forces, the relative roughness determines the rate of flow and the impulse of a given mass of water.

The commencement of bed-load movement in ordinary mixtures follows as a result of the removal or loosening of the finer particles which bind the mixture, whereby the coarser grains become more and more exposed until at last their static resistance to movement is unable longer to overcome the force of the flowing water. With the movement in progress, its continuation without any acceleration or deceleration, requires merely an exertion of force equal to the frictional resistance to movement, which is always less than that required to overcome the frictional resistance at rest. Considering the impulse of a given mass of water, therefore, the following relation must hold:

$$I_{0\ (\text{Beginning})} > I_{0\ (\text{End})}$$

or,

$$d_0\, u_0 > d'_0\, u'_0$$

---

[22] "Die Schwemmstoffführung des Rheins und anderer Gewässer," von Ph. Krapf, Osterreichische *Wochenschrift für den öffentlichen Baudienst*, 1919.

[23] "Der Flussbau," von F. Kreuter (Handbuch der Ingenieurwissenschaften, Pt. III, Vol. 6, Fifth Edition), Leipzig, 1921.

Thus, the conclusions of Krapf and Kreuter are justified under the assumption that, for like stages of movement, equal velocities existed at equal depths and slopes. This essential condition, however, is not mentioned specifically in their reports.

In Schoklitsch's experiments[16] the impulse difference probably could not have been measured, since the bed-load material, due to its special composition, possessed a low frictional resistance at rest, and in movement produced no noticeable changes in bed roughness or velocities.

In the present experiments, the commencement of movement occurred on an artificially constructed smooth bed. For the observations made after cessation of movement, the bottom surface had been disturbed by natural forces, resulting either in riffles or in the re-arrangement or sorting of the particles. As may be seen from the corresponding C-values in Column (11), Table 1, the bed in each case was relatively rougher at the time of cessation of movement than at its commencement. This difference in roughness resulted in the fact that, for the same depth of water—the slope being held constant—the water would flow through the flume at a lesser velocity during falling stages than during rising stages. The difference amounted to from 0 to about 25% (14% on the average). A simple computation shows that, in comparable observations of these experiments, the impulse was less at the cessation of bed-load movement than at its start, and amounted on the average to $(100\% - 14\%) \times (100\% + 12\%) = 96\%$ of the starting impulse. These experiments, therefore, are in accord with the physical laws involved.

If relative roughness or velocity is considered, the foregoing observations are found to be in accord with the law of impulse.

The Upper Limit

According to the definition under the heading, "Definitions," Paragraph (*b*), the upper critical tractive force is that which just precludes excessive riffles. In evaluating the results of the experiments it must be recognized, however, that the upper limit is much more difficult to establish than the lower limit; is not as distinct; and, because of the unavoidable subjective influences, it is not determinable with a very high degree of exactness. Nevertheless, the results are of qualitative value.

1.—Interdependence of Depth and Slope for a Given Sand Mixture.—Fig. 10 shows that the curve for the upper limit plotted against slope may be considered as a straight line, which, however, is inclined, and is not, as with the lower limit, parallel to the x-axis. Since the tangent of the angle of inclination is positive, this curve shows that the upper critical tractive force increases with the slope, or that at decreasing slopes the upper limit is reached with a lesser tractive force.

2.—Dependence on Mixture Properties.—A critical comparison of the curves of the upper limit of the three sand mixtures (Fig. 10) shows that Sand II has a pronouncedly lower, upper limit of tractive force than the other two sand mixtures. Sand II is composed of a finer and more nearly uniform mixture than Sands I and III. This result is identical with Krey's Elbe experiments[24] from which he drew the conclusions "that riffle formation depends on the uniformity of the grain size (mixture ratio) as well as on the absolute size of the larger grains."

A comparison of Sands I and III in this respect shows that Sand III has a generally higher upper limit than Sand I. Sand III has the same coarse grains as Sand I, but as a result of the sifting out of grains smaller than 0.385 mm is of considerably different composition in its finer elements (see Fig. 1). This indicates that the addition of finer particles to a given mixture would lower the upper limit, or that excessive riffles would occur at a lesser tractive force.

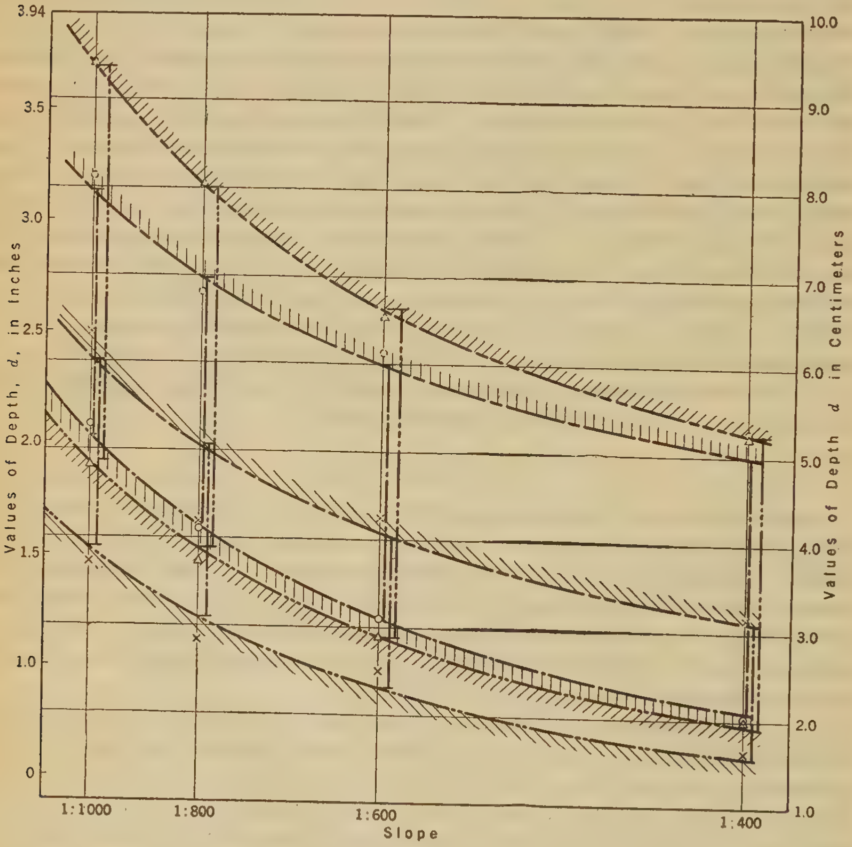


FIG. 11.—UTILIZABLE DEPTH IN RELATION TO SLOPE

The Zone of Usefulness

The immediate practical purpose of these experiments was, as outlined under "Motive, Scope, and Purpose of the Experiments," the determination of suitable mixtures for models of streams in which bed-load movement occurs even at low-water stages. Accordingly, the zone of usefulness of a mixture lies below the determined upper limit and above the determined lower limit. Since linear scales (length, width, and depth) are the first consideration in model design, it is desirable that the zone of usefulness be judged from the standpoint of depth. It should be re-emphasized, however, that the determination of the upper limit cannot be made with the same precision as the lower limit. From the ordinates in Fig. 11, the practical depth range can be determined for each of the sand mixtures which were investigated.

---

[24] "Elbeversuche," von H. Krey (Unpublished report on Elbe Experiments), Prussian Experiment Inst., Berlin.

1.—Depth-Slope Dependency of a Sand Mixture.—It follows from Fig. 11 that, for one and the same mixture, the magnitude of the practical depth range is sensibly constant for all slopes, and that, therefore, it is independent of the slope.

The simplified assumption (made in the discussion of the upper limit curve) that, within the limits of this experiment, this latter curve is an inclined straight line, makes the conclusion regarding the constancy of the practical depth range capable of mathematical proof. However, if the upper limit curve is a constant curve, such as would intersect the y-axis at the point, $T'_o = k_o$, and if it is asymptotic to the vertical line at the point, $S = S_{(critical)}$, it can also be proved mathematically that the zone of usefulness (that is, the ordinate segment between the upper limit curve and the lower limit curve, $T_o = k_o$), increases with increasing slopes.

For practical purposes, however, the assertion suffices that the slope has no decisive influence on the zone of usefulness but, other considerations being equal, the greater slope is preferable.

2.—Dependence on Mixture Properties.—Fig. 11 tends to show that the zone of usefulness for Sand II is confined to narrower limits than that of the other sand types. A comparison of Sand II with Sand I indicates that the sifting of the coarser components from the mixture, despite the consequent reduction of the lower critical tractive force, causes an even greater decrease in the usable depth range.

A comparison of Sands I and III shows that the zone of usefulness of the latter includes all that of the former, a fact which was brought out by the previous discussions on the upper and lower limits. The sifting out of the finer components (Sand III from Sand I) raises the position of the upper limit; but the influence on the lower limit is dependent on the changes in the void and mixture relations. According to these results, it appears entirely possible and probable that a mixture prepared by a less thorough sifting than was given Sand III would result in both a higher upper and lower lower limit than for Sand I. This procedure would be of practical value when, for instance, for a given depth scale, or rather a given low-water depth, the corresponding slope given in Fig. 11 for available mixtures proved undesirably low.

The *C*-Values

For convenience, the C-values of the Chezy formula, $V = C\sqrt{RS}$ derived from observed data within the zone of usefulness of each sand mixture and for each slope are compiled in Table 2. These data should be used only for rough preliminary computation in model design. In calculations for transference from a particular model to Nature, the model C-values used in the computations should be taken from the actual model.

TABLE 2.—C-Values Within the Zone of Usefulness (in Meters$^{0.5}$ per Second Units)

Sand	Slopes				
	1:400	1:600	1:800	1:1 000	Averages
I........	48.6	50.6	56.7	58.7	54.0
II........	54.4	52.7	55.6	57.7	55.0
III........	46.9	49.4	52.8	55.3	51.0
Average	50.0	51.0	55.0	57.0	53.0

Summary of Part I

From the results of these experiments, as checked by reference to the work of other investigators, the following conclusions, which may be applied directly to river models of comparable width-depth ratio, were drawn:

1.—One and the same bed-load material under the usual moderate slopes invariably gives the same value for the lower critical tractive force. This is the Law of Constant Critical Tractive Force.

2.—Grain size and denseness have definite opposing effects on the critical tractive force or bed-load mixtures. Therefore, it is not justifiable to judge the movability of a mixture only by its grain size.

3.—For a given sand mixture, the upper limit (and with it the occurrence of excessive riffles) varies directly with the slope.

4.—Well-graded coarse mixtures reduce the tendency toward excessive riffles.

5.—Based on requirements of laboratory technique, well-graded sand mixtures are preferable for river models. Removal of the finer components is sometimes advantageous.

PART II. — COMPUTATION OF CRITICAL TRACTIVE FORCE

General

An attempt will now be made to derive analytically a practical formula for computing the critical tractive force for a "geschiebe" mixture and to check it by existing experimental data.

The essential conditions to be fulfilled by a formula for the critical tractive force are:

(1) It must be dimensionally correct; therefore, it must give an equation for (tractive) force per unit area having the dimensions (weight × length^{-2}) on both sides.

(2) Its useful range must be as great as possible. This requires that it take into consideration, satisfactorily, all essential influences, and that it be applicable over the full range of values of the individual influences.

(3) The individual factors must be capable of definite determination by simple direct measurements or from reliable experimental data.

(4) Its structure must be simple.

THE TRACTIVE FORCE THEORY—DU BOYS' EQUATION FOR TRACTIVE FORCE

du Buat's[3] basic idea of expressing friction between running water and a river bed by the product of the slope and the weight of water overlying the bed, was expanded to a general theory by du Boys,[4] as a result of observations on the Rhône. According to Kreuter,[25] du Boys' theory is built up as follows: Assume a regular stream with uniform flow, the bed of which is in a state of equilibrium. The water surface is an inclined plane parallel to the stream bed. The slope = S, and the depth of water = d. At a given instant a prism of water with a bottom surface area of unity is resting on the bed. If the bed exerted no retarding influence whatever, the prism of water would slide down an inclined plane of the slope, S, thus increasing its kinetic energy. If m is the mass of an element of water and v the velocity of the same element, the increase in kinetic energy during the very short period of time, dt (neglecting differentials of higher order), would be:

$$\frac{m}{2}\left[(v + dv)^2 - v^2\right] = m\,v\,dv \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Since a state of equilibrium was presupposed, however, this energy is used in overcoming bed resistance. The acceleration is $\frac{dv}{dt} \doteq g\,S$. The distance covered in the time, dt, is $v\,dt$. The work of resistance, the product of the retarding force and the distance, is $m\,g\,S\,v\,dt$. The retarding force itself is $m\,g\,S$.

If, instead of the element, m, the entire mass of water, M, is introduced into the term, it is possible to state, with sufficient accuracy (since S is very small), that $M = \rho\,\frac{d}{g}$, in which, ρ represents the unit weight of water. Then the tractive force opposed to the bed resistance is, per unit of area:

$$T = \rho\,S\,d \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Equation (3) is known as du Boys' Equation of Tractive Force. Its application is subject to the following qualifications and limitations: It presupposes uniform flow of water and infinite width of channel. The expression does not apply to ideal, that is, frictionless, fluids but only to viscous fluids. It embraces both the action of gravity and that of friction. With the further assumption that friction at the free-water surface may be neglected and that the sine of the angle of slope may be considered, with sufficient accuracy, equal to its tangent, the expression is valid for all types of flow; that is, for laminar as well as for turbulent, and for shooting as well as streaming, flows.

---

[25] "Der Flussbau," von F. Kreuter (Handbuch der Ingenieurwissenschaften, Pt. III, Vol. 6, Fourth Edition), Leipzig, 1907.

FUNDAMENTALS OF FORMULATION

Physical Fundamentals

The determining factors for a practical equation for the critical tractive force, T_0 (or, better expressed, for a mixture's critical resistance to movement) may be written as follows: $T_0 = f$ (grain size, grain distribution, grain form, denseness, unit weight in water).

Of these factors, grain size and distribution are determinable by the usual sifting processes, testing enough samples to insure proper determination of the mean values. The geometrical shape of the grain is determinable through laborious measurements,[26] but may be considered constant for the common types of quartz sand, so that the form factor in this computation may be considered equal to unity. The void ratio of a mixture in a given condition is a measure of its denseness. The texture factor must be omitted from consideration since it cannot be evaluated; in a certain sense, however, it is already embraced in the grain size, grain distribution, and denseness factors.

The formal equation of the determining factors may now be simplified to:

$$T_0 = f\ (D)\ (e)\ (\rho_1 - \rho) \qquad (4)$$

in which, D is the grain diameter; e, the void ratio; ρ_1, the unit weight of the material (for quartz sand the average is 2.65 grams per cu cm); and ρ, the unit weight of water (usually, 1.00 gram per cu cm). (The factor, $(\rho_1 - \rho)$, would thus represent the effective density of the material.)

Practical Derivation of a New Formula

1.—Procedure.—Any attempt to derive in a theoretical, mathematical manner the constants for Equation (4) results in a tedious, cumbersome procedure involving the solution of several simultaneous equations, each expressing for a particular sand mix the relation between the factors of Equation (4). Furthermore, any equation derived in this manner would be subject to interpolation only in the narrow range covered by the types of sand used in its derivation. A more simple procedure will be adopted, involving an empirical method of development.

A guide for this purpose is suggested by the method that Hummel[27] introduced in 1930 for the determination of the strength of concrete from the gradation curve of the aggregate. He had before him the problem that neither the denseness alone nor the grain surface alone could be used as a sole criterion for the quality of the aggregate. Nevertheless, he succeeded in demonstrating, by means of an ingenious treatment, that, with all other factors remaining the same, the strength of concrete (or the quality of the aggregate) may be judged from the sieve analysis.

Apparently, it requires considerable imagination to perceive any connection between the strength of concrete and bed-load movability. Nevertheless, these entirely separate subjects have a certain common characteristic. The strength of concrete, as is known, depends on the void ratio as well as on the grain surface (size of aggregate). These two properties have opposing effects on the strength of concrete, just as on bed-load movability (which also varies with size and void ratio), in which both influences also have opposing effects.

---

[26] "Die Darstellung von Körnerformen und der Kornverteilung loser Massen sowie Gesetzmässigkeiten beim Werden von Schottern und Sanden," von R. Grengg, *Zeitschrift für Geschiebeforschung*, Vol. III, Berlin, 1927.

[27] "Die Auswertung von Siebanalysen und der Abram'sche Feinheitsmodul," von A. Hummel, *Zement*, No. 15, 1930.

Applying Hummel's procedure, the sieve analysis curves for all available experimental data were evaluated in the following manner: As in soil mechanics, the gradation curves were plotted to semi-logarithmic scale. The

TABLE 3.—COMPILATION OF EXPERIMENTAL DATA: CRITICAL TRACTIVE FORCE AND EVALUATION OF GRADATION CURVE

Description of sand	Classification of data	Observed critical tractive force, T_0, in grams per square meter	Specific gravity of sand, ρ_1 in grams per cubic centimeter	EVALUATION OF GRADATION CURVE									
				Semi-Logarithmic					Linear Plotting				
				A	A_A	A_B	$\frac{A_B}{A_A}$	$\frac{A(\rho_1-\rho)}{\frac{A_B}{A_A}}$	A	A_A	A_B	$\frac{A_B}{A_A}$	$\frac{A(\rho_1-\rho)}{\frac{A_B}{A_A}}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
(*a*) EXPERIMENTS BY WRITER													
I	(1)	52	2.70	73.4	46.2	27.2	0.589	214	35.3	26.0	9.3	0.358	168
II	(1)	39	2.70	67.4	42.5	24.9	0.586	196	27.9	19.1	8.8	0.461	103
III	(1)	49	2.70	80.8	47.7	33.1	0.694	198	40.0	28.3	11.7	0.414	164
(*b*) EXPERIMENTS BY F. SCHAFFERNAK													
A	(1)	75	2.65	115.8	64.1	51.7	0.805	237	76.8	49.1	27.7	0.564	225
(*c*) EXPERIMENTS BY A. SCHOKLITSCH													
B	(2)	544	2.6	181.4	90.7	90.7	1.00	290	326.0	163.0	163.0	1.00	521
C	(2)	263	2.6	160.8	80.4	80.4	1.00	257	202.5	101.2	101.2	1.00	324
D	(2)	113	2.6	135.4	67.7	67.7	1.00	217	112.8	56.4	56.4	1.00	180
E	(2)	44	2.6	109.4	54.7	54.7	1.00	175	62.0	31.0	31.0	1.00	99
F	(2)	28	2.6	96.2	48.1	48.1	1.00	154	45.8	22.9	22.9	1.00	74
(*d*) EXPERIMENTS BY H. KREY													
G	(3)	24	2.68	56.6	30.5	26.1	0.856	112	18.8	10.3	8.5	0.825	39
H	(3)	28	2.61	71.6	37.3	34.3	0.921	125	26.3	14.1	12.2	0.869	49
J	(3)	32	2.57	89.0	47.2	41.8	0.887	158	40.0	22.5	17.5	0.778	81
(*e*) EXPERIMENTS BY P. E. I.													
K	(4)	51	2.65	90.5	60.2	30.3	0.504	297	57.7	45.6	12.1	0.265	359
L	(4)	42	2.65	78.6	53.0	25.6	0.483	269	42.3	33.1	9.2	0.278	251
M	(1)	58	2.65	67.8	45.1	22.7	0.503	258	41.9	31.4	10.5	0.366	189
N	(1)	56	2.65	79.5	48.1	31.4	0.653	201	37.2	26.6	10.6	0.399	154
O	(3)	24	2.65	32.8	23.7	9.1	0.384	141	12.2	8.5	3.7	0.435	46
P	(1)	56	2.65	81.0	50.6	30.4	0.600	223	40.3	29.7	10.6	0.357	186
Q	(1)	50	2.65	77.1	47.3	29.8	0.617	206	34.3	24.1	10.2	0.424	134
(*f*) EXPERIMENTS BY H. ENGELS													
R	(3)	100	2.65	99.5	65.6	33.9	0.517	318	74.2	60.4	13.8	0.229	535
(*g*) EXPERIMENTS BY G. K. GILBERT													
S	(4)	39	2.69	75.7	39.1	36.6	0.935	137	28.8	15.2	13.6	0.896	54
T	(4)	61	2.69	127.3	67.0	60.3	0.900	240	95.3	55.6	39.7	0.714	225
U	(4)	295	2.69	156.5	79.8	76.7	0.962	275	185.5	99.4	86.1	0.867	361
V	(4)	490	2.69	171.5	87.9	83.6	0.953	304	264.8	145.6	119.2	0.820	547

scales used were, for the grain diameter, on the X-axis, log 0.1 to log 1.0 mm = 10 cm; and for weight percentages, on the Y-axis, 100% = 10 cm. The area, A, bounded by the sieve curve, the Y-axis, and the horizontal line at the point, $Y = 100\%$, was measured with a planimeter. The values thus obtained appear in Column (7), Table 3. A comparison of these values with the corresponding values of the critical tractive force as derived from the experiments (Table 3, Column (3)) already indicates some relationship. As a measure of movability, this A-value is inadequate, however, since gradation curves of entirely different form (that is, mixtures of entirely different character and properties of movability), could all have the same A-value. This value, therefore, which Hummel found entirely adequate for his purpose, is not in itself sufficient for judging the properties of the mixture from its sieve analysis.

Since the analysis of the curves in the manner described had already produced a clue, it seemed desirable to extend this method in order to discover any relationship that might exist. The A-value embraces the grain size of the mixture, but not the grain distribution or void ratio. Evidently, this latter property must be somehow expressible from the shape of the curve. It seemed logical and definite to make some sub-division of the A-area and to consider the relation between the sub-areas.

Following this line of thought, the A-area was divided into two arbitrary parts, A_A, the area above, and A_B, the area below, the abscissa at the point, $p = 50\%$ (Fig. 12). The partial areas were then measured with a planimeter for all available sieve curves and the ratio, $A_B : A_A$, was computed. The term,

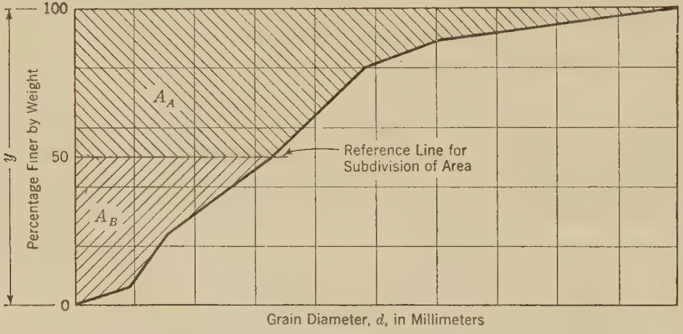

100
50
0
y
Percentage Finer by Weight
A_A
A_B
Reference Line for Subdivision of Area
Grain Diameter, d, in Millimeters

FIG. 12.—EXPLANATION OF EVALUATION OF GRADATION CURVE.

$\frac{A(\rho_1 - \rho)}{\frac{A_B}{A_A}}$, was then computed (see Table 3, Columns (5) to (9)) and plotted against the corresponding T_0 values. The resulting graph (which is not reproduced in this paper) showed an approximate straight-line relation, as follows:

$$T_0 = \frac{k\,A\,(\rho_1 - \rho)}{\frac{A_B}{A_A}} \qquad \text{(5)}$$

Following this encouraging result, a similar computation was made for linear, instead of semi-logarithmic, plots of the sieve curves shown in Figs. 1 and 13, representing the sands used in these experiments as well as those used by other experimenters. The resulting A, A_A, and A_B areas as well as the values of $\frac{A_B}{A_A}$ and $\frac{A(\rho_1-\rho)}{\frac{A_B}{A_A}}$, are compiled in Table 3, Columns (10) to (14). The graphical representation in Fig. 14 shows approximately the straight-line relation:

$$T_0 = k_1 \frac{A\,(\rho_1 - \rho)}{\frac{A_B}{A_A}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(6)$$

It must be noted that the points on Fig. 14 represent data from several sources, many from experiments not directly comparable with those described herein. Accordingly, the worth of these data is roughly classified (Table 3, Column (2)) as follows: (1) Observations directly comparable to the present experiments, or comparable values of the first order; and (2), (3), and (4) decreasingly comparable values. This classification is not meant as a disparagement of the experiments cited, but only as an aid in weighing and judging the dispersion of data. Although the line in Fig. 14 could perhaps be drawn differently, it gives a satisfactory mean of the eight highest rated points in the Comparison Class (1): I, II, III, *A*, *M*, *N*, *P*, and *Q*.

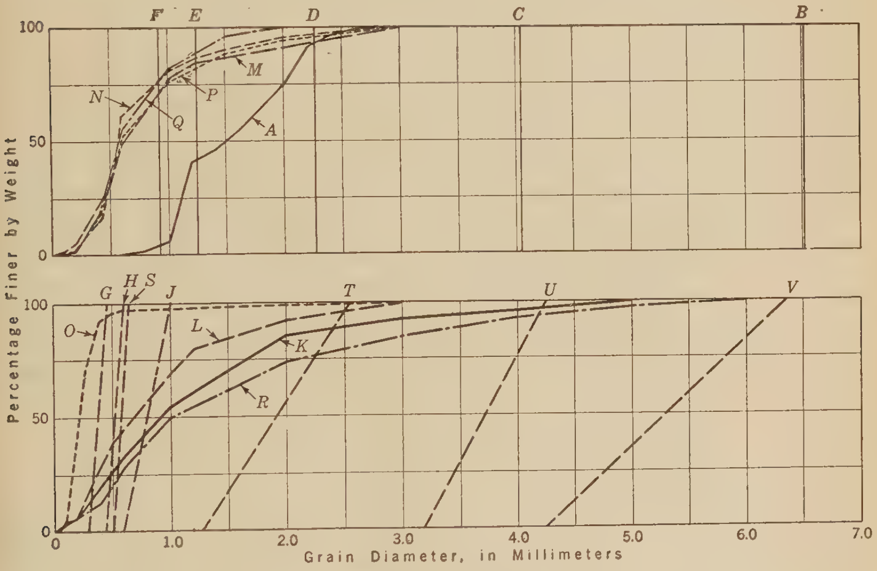


FIG. 13.—GRADATION CURVES FOR TABLE 3.

After careful consideration of the advantages and disadvantages of the linear and the semi-logarithmic methods of plotting the gradation curve, which were used in deriving Equations (6) and (5), respectively, it was decided to adopt the linear method and Equation (6) as the basis for further study.

The significance of the individual factors of Equation (6) requires detailed consideration. T_0, the critical tractive force, has been defined under the heading "Definitions." The factor, $(\rho_1 - \rho)$, is the unit weight of the sand in water. The area, A, can be expressed mathematically as: $\sum_{p=0\%}^{p=100\%} D \, \Delta p$, in which D is the grain diameter, and p, the percentage of finer material. The expression:

$$\frac{\sum_{p=0\%}^{p=100\%} D \, \Delta p}{\sum_{p=0\%}^{p=100\%} p} \qquad \text{(7)}$$

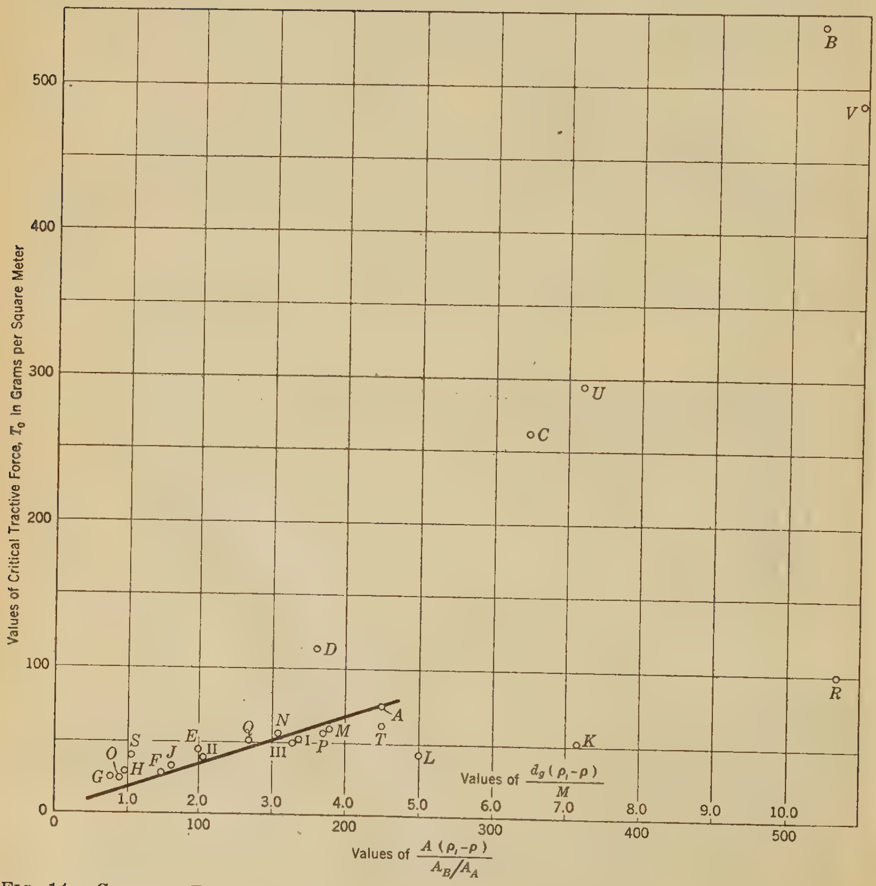


FIG. 14.—CRITICAL TRACTIVE FORCE IN RELATION TO EVALUATION OF GRADATION CURVES

gives the mean abscissa of the gradation curve of the A- area. The mean grain diameter, D (a designation first used by Krey in preference to a median value), is the grain size corresponding to this mean abscissa.

In a similar manner, $\frac{A_B}{A_A}$ can be expressed mathematically as:

$$\frac{\sum\limits_{p=0\,\%}^{p=a\,\%} D\,\Delta p}{\sum\limits_{p=a\,\%}^{p=100\,\%} D\,\Delta p} \quad \text{.........................(8)}$$

or, since the reference line in this evaluation is arbitrarily drawn through $p = 50\%$:

$$\frac{A_B}{A_A} = \frac{\sum\limits_{p=0\,\%}^{p=50\,\%} D\,\Delta p}{\sum\limits_{p=50\,\%}^{p=100\,\%} D\,\Delta p} = M \quad \text{..................(9)}$$

(In Equation (9), A is also equal to $A_B + A_A$). The relation, $\frac{A_B}{A_A}$, as indicated in Equation (9) is, to a certain extent, a measure of the gradation of the mixture. It will be designated as the "uniformity modulus," M. For perfectly uniform grains this modulus, with the reference line through the point, $p = 50\%$, has the value, unity; for mixtures, its value is always less than unity; and with increasing range of maximum and minimum grain sizes, the value approaches zero.

3.—The Ultimate Equation (10).—From the foregoing definitions, and with the value of the constant, k', selected from Fig. 13, Equation (6) can now be written:

$$T_0 = \frac{100}{6}\,\frac{D\,(\rho_1 - \rho)}{M} \quad \text{......................(10)}$$

In Equation (10), T_0 = the critical tractive force, in grams per square meter; D = the mean grain diameter, in millimeters; $(\rho_1 - \rho)$ = the unit weight of the material in water, in grams per cubic centimeter (effective density); and M is the uniformity modulus. The constant, $\frac{100}{6}$, was expressed deliberately as a fraction, since an expression in more precise form would not be justified by the accuracy of the basic data. It is to be noted that the values, D and M, which are derived from the gradation curves, are in no way dependent on the scales to which gradation curves may be plotted.

Equation (10), which is merely a special form of the general Equation (4), expresses in its structure the following relations: (a) The critical tractive force, T_0, is proportional to the unit weight of the material in water; (b) the critical tractive force is proportional to the grain size; and (*c*) the critical tractive force is inversely proportional to the void ratio; the void ratio is expressed by the uniformity modulus, M.

The foregoing summary shows that the formula, despite its unusual derivation, is nevertheless rational in its structure and conforms to proved theories. Since it was derived empirically and from limited data, its scope of application and probable accuracy are subject, of course, to obvious limitations. The range of the basic data indicates an accuracy of the results of the formula within ± 15 per cent. Considering the difficulties in the actual determination of the critical tractive force, this would seem admissible. By virtue of its structure, the formula is applicable to the quartz sands commonly used in laboratories, including gradations of moderate irregularity, containing grains up to 5 mm in diameter, and having a critical tractive force of as much as 80 grams per sq m. The final formula (Equation (10)), can be simplified for ordinary quartz sand of average unit weight of 2.65 grams per cu cm, to:

$$T_o = 27.5 \frac{D}{M} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (11)$$

Finally, Equation (10) satisfies the requirements noted under "Tractive Force Theory": (1) It is dimensionally correct; (2) it takes all determining influences into consideration; that is, it describes the process and is applicable to the sands commonly used in river models; (3) its individual factors are definite and readily determinable; and (4) its structure is simple.

Summary of Part II

In Part II the writer has endeavored to formulate in a practical, useful manner the commencement of bed-load movement. To this end, an empirical procedure successfully used in concrete design was adopted. By means of a further development of the procedure as it related to this problem, a simple relationship was derived empirically and expressed by Equation (10). In this case the influence of grain size is represented by the mean grain diameter, and the influence of void ratio by a uniformity modulus. These factors can be obtained either by planimetering the area above the gradation curve plotted to any linear scales, or by simple computations based on the curve or on the tabulation of the sieve analysis data. To a certain extent these factors indicate the characteristic properties of the mixture curve; at the same time, they embrace the entire curve. A critique of the derived formula in the light of existing theoretical knowledge and experimental facts confirms the rationality of its structure. However, due to limited basic data, the constant given for Equation (10) cannot be accepted as final, and definite limits for its present utilization are stated.

The considerations presented in Part II are nearly a chronological presentation of the line of thought followed. The reason for this form of treatment is, that where complex natural phenomena defy direct analysis of the fundamental laws governing a process, it is possible, nevertheless, to form logical inferences from individual observations. In any event, it is noteworthy that, in an allied field of applied science, a similar procedure has already given useful results while pure theory has thus far been unsuccessful.

Practical Applications

Studies of the type described herein can be applied advantageously to the solution of actual problems of river regulation, in which the objective, briefly summarized, is to attain a state of equilibrium between the force of flowing water and the resistance of the natural or artificially modified channel. In planning regulatory measures, a knowledge of the hydrographic characteristics of the stream, its properties of resistance, and the effects of structures is essential. The hydrographic data, such as slope, discharge, velocity, trace, cross-section, etc., can usually be obtained by measurement, but knowledge of the behavior of the bed material is usually lacking, despite the fact that it is one of the fundamental factors in design. Attempts have been made to acquire data on the movement of the bed material by direct observation, but as yet the results have been meager.

It is possible, however, to supplement these direct observations by an indirect analytical procedure, which has been proved sound and workable. This procedure can be outlined, as follows:

1.—Samples of bed material are taken from the stream in question, care being taken that the samples are representative.

2.—They are analyzed, and subjected to a flume test to determine: (*a*) Critical tractive force: Since the tractive force is the product of $\rho\, S\, d$, the determination of its critical value can be made by an arbitrary choice of d or S in the flume, and the determination of the corresponding value of the other factor at the time of the commencement of general movement; and (*b*) behavior of the bed-load material at various magnitudes of the tractive force. The flume analysis can be made either in a laboratory flume, or in an improvised flume at the project site.

3.—The data obtained can be utilized in predicting the behavior of the material in the actual stream, for which the slopes and mean depths at various stages are known.

In case it is not possible to make a flume test of the sample of bed-load material, the critical tractive force can be computed directly from the gradation curve by means of an empirical formula. The formula developed in Part II of this paper may serve the purpose, but until the reliability of such formulas is established, it would be preferable to determine the tractive resistance experimentally.

In addition to the direct utilization in engineering practice of experimental data pertaining to bed-load material, the availability of these data is of great value to the laboratory that is engaged in model experimentation. With a thorough knowledge of the behavior of bed load, it may be possible to achieve quantitative as well as qualitative results from studies of river models. In most cases it is not possible, or desirable, to use in the model study either the bed material taken from the actual stream, or a geometrically similar material. Flume tests of the prototype material and of the material arbitrarily chosen for use in the model can be made, and scalar relations determined both for the critical tractive force and for the load corresponding to various magnitudes of tractive force. With such movability ratios known, it is possible to place greater confidence in quantitative interpretations of river model studies.

The foregoing treatment indicates a manner of overcoming some important obstacles that impede the progress of the practical hydraulic engineer and of the experimenter. The hydraulic engineer can obtain indispensable basic bed-load data by the simple tests indicated. The understanding of the movability relations between natural and model bed load enables the experimenter to interpret model results quantitatively. It is to be hoped that the suggested systematic determination and use of fundamental bed-load data may contribute to the advance of science in the field of practical and experimental river hydraulics.

ACKNOWLEDGMENTS

Grateful acknowledgment is due the Prussian Experiment Institute for Hydraulic Engineering and Shipbuilding, Berlin, the co-operation of which made these experiments possible. Professor R. Seifert, Mr. B. Körner, the late Dr. F. Eisner, and others of the staff of the P. E. I., rendered invaluable aid during the conduct of the experimental program and the preparation of the original German thesis; Lieut. R. Whitaker, Corps of Engineers, U. S. Army, assisted in taking observations; and Professors O. Kirschmer and H. Heiser, of Dresden, Germany, constructively criticized the German manuscript.

Mr. C. R. Warndof prepared the preliminary draft of the English translation, and Lieut. Thomas A. Lane, Corps of Engineers, U. S. Army, Jun. Am. Soc. C. E., and Joseph B. Tiffany, Jr., Jun. Am. Soc. C. E., further reviewed and edited the English manuscript.

Finally, this work is a direct result of the appointment of the writer as Freeman Traveling Scholar by the Society. In grateful recognition of the personal encouragement of the late John R. Freeman, Past-President and Hon. M. Am. Soc. C. E., and of his generous and broad-visioned patronage of hydraulic research, this work is humbly dedicated to his memory.

DISCUSSION

JOHN LEIGHLY,[28] ESQ. (by letter).—The experimental data and other evidence presented in this paper serve to establish beyond a reasonable doubt the constancy of critical tractive force for a given sand mixture. The author's use of the du Boys equation in computing tractive force is a great improvement over procedures based on a supposed "bed velocity," and he deserves credit for again setting that venerable equation in a place of honor. Unfortunately, his experiments do not provide the definitive empirical critique of the equation that one would like to see. A possible objection to his method of computing tractive force is that he has used depth rather than hydraulic radius as a measure of the mass of water acting on a unit of bed area. In proceeding thus, however, he was evidently justified by the conditions of his experiments, in which the water had ratios of width to depth between 15:1 and 40:1. Recomputation of the critical tractive force in his observations, using hydraulic radius instead of depth, yields results that do not differ materially from his conclusions.

The ultimate result of Captain Kramer's investigation, and the one to be subjected to the closest scrutiny, is the final equation, which aims to express the mobility of a sand mixture in terms of its mechanical composition. Further work, both experimental and theoretical, must be done before a definitive formulation of this important relation may be had. The author's solution is ingenious; to gain practical ends, Gordian knots must be cut; and in the present instance some quality of the gradation curve is clearly the appropriate weapon. In testing the validity of his equation, Captain Kramer has had to use inhomogeneous data; but lack of homogeneity alone will not account for the wide scattering of plotted points in Fig. 14. It is worth while to examine the predicates of his final equation.

The distinctive term, $\frac{1}{M}$, in this equation, is the ratio, $A_A : A_B$, between the areas of the upper and lower divisions of that part of the field of co-ordinates that lies between the axis of ordinates and the cumulative gradation curve in Fig. 12. This ratio is used as a coefficient to the mean particle diameter of a sand, as a measure of the range of size of particle. The precise significance of the numerical value of the coefficient in any given instance is not at all immediately evident. Readers of this paper would be grateful for a description, supported by the citation of a large number of gradation curves, of just what the ratio proves as to the grain distribution of any sand. The following statements do not aim to provide such a discussion, but only to show the effect of applying the ratio to the results of the experimental measurements cited by the author.

Range of grain size is not always, as he implies under "3.—The Ultimate Equation (10)," the primary factor that determines the numerical value of the coefficient, $A_A : A_B$. If one considers gradation curves which (when plotted as are those in Fig. 13) appear as straight lines with their lower extremities at the origin of co-ordinates, the ratio, $A_A : A_B$, regardless of range of grain diameter, will always equal 3. The qualities measured by the departure of the numerical value of the ratio from 3 (in curves of a given range such that their lower extremities are at or near zero of the axis of abscissas) are those that affect the slope of the gradation curve in the several parts of the range: Concentration of grains in the vicinity of a modal diameter; and the location of the modal diameter within the range. The closer the modal diameter lies to the lower extremity of the range, and the greater the concentration of grains about the mode, the higher the value of $A_A : A_B$, and thus, by Captain Kramer's equation, the less the mobility of the sand. That proposition is doubtless worthy of discussion, but the experimental data scarcely suffice to prove or to disprove it. The important fact —a purely geometric one—to be noted is that with the same range of grain size from zero (or very nearly zero) upward, it is possible for the ratio to have any one of a large number of values ranging upward from slightly more than 1, depending in any particular sand on the modal diameter and on the deviations of the diameters of the component grains from the mode. The large possible range of numerical values of the ratio, and the uncertainty of the significance of its variations, make it a dangerous element in any equation designed to express mobility. The danger is always present that it may modify too strongly the essential term of any such equation, the mean diameter.

---

[28] Asst. Prof. of Geography, Univ. of California, Berkeley, Calif.

A second influence exerted on the numerical value of the ratio, $A_A : A_B$, that is independent of range of grain size, may be seen by comparing the gradation curves the lower extremities of which are near zero diameter with those of sands the fine constituents of which have been naturally or artificially removed. The values of the ratio in curves, *B* to *H* and *S* to *V*, in Fig. 13, are determined not so much by the range of grain diameter as by the scalar value of the diameter of the smallest particle in each sand. If a gradation curve having the slope of Curve *T* in the diagram be thought of as having a range of diameter from zero to 1.25 mm, the ratio, $A_A : A_B$, will have the numerical value, 3. Let the curve now move in the field of coordinates from left to right, the range remaining constant; as the mean diameter, and, with it, the maximum and minimum diameters, increase, the value of the ratio decreases and approaches 1. That is to imply that range of diameter is important when the smallest grains are very small, but as they become larger the significance of range decreases. That, again, is a proposition that can be discussed, but that rests on shaky evidence.

The objections urged in the foregoing two paragraphs may appear hypothetical; but the experimental data which the author cites show plainly that the use of his ratio actually distorts the fundamental relation obtaining between coarseness of sand and critical tractive force. For example: The mean diameters of the sands represented in Fig. 13 by the curves, *J* and *L*, are not greatly different; their ratio is about 1:1.1. The experimentally determined critical tractive forces of the two sands have a ratio of about 1:1.3. The quantities tabulated in Table 3, Column (14), however, that are intended to be proportional to critical tractive force, have a ratio of 1:3.1 for the two sands. This discrepancy is not the result of difference in range of diameter of particle. The ratio, $A_A : A_B$, has, for Curve *J*, a value of nearly 1.3 and for Curve *L*, 3.6; but if Curve *J* were moved to the left in the diagram until the finest constituents of the sand it represents became as fine as the finest in Sand *L*, the ratio would be 3, thus being nearly as great as for Curve *L*, although the range would be less than one-half that of Curve *L*. If the range of Curve *J* were then increased, the ratio would remain constant at 3, as long as the finest constituent particles had diameters of nearly zero. When Curve *J* assumed the same range as Curve *L*, the ratio would still be only five-sixths of the ratio for Curve *L*. As matters stand in Fig. 13, the diameter of the finest constituents of Sand *J*, not its total range of diameter, is what determines the numerical value given by evaluating Captain Kramer's final equation, and gives rise to the discrepancy between the computed indices of the critical tractive forces in Sands *J* and *L*. Evaluation of the equation without the ratio, or even naive judgment, on the simple basis of relative coarseness, would give results closer to the experimental determinations of critical tractive force than the full equation.

Comparison of the places occupied by Sands *C*, *K*, and *U* in Fig. 14 provides further adverse evidence; the ratio, $A_A : A_B$, gives to these three sands nearly the same index of critical tractive force, although Sands *C* and *U* are much coarser than Sand *K* and although the experimental data show that Sands *C* and *U*, as again would be expected from naive consideration of their coarseness as compared with that of Sand *K*, are far more resistant to traction than Sand *K*. The ratio separates sands that belong together, and brings together sands that differ widely when judged according to the primary criterion—mean diameter.

Finally, if the values of *A* in Column (10) Table 3, proportional to the mean diameters of the several sands, are plotted against the experimentally measured critical tractive forces of Column (3), the correlation, particularly for the coarser sands, is much better than that shown in Fig. 14; and the evidence of the coarser sands should be given more weight than that of the finer ones, since with them the errors of observation do not weigh so heavily in the results of computation.

It is altogether probable that an ideal equation expressing critical tractive force as a function of mechanical composition of sands would contain terms representing qualities other than mean diameter. Certainly, however, the derivation of these terms must begin from the variations seen in the results of experiment when mean diameter is held constant or is eliminated by computation, while range of grain size, modal diameter, and dispersion of grains through the range of diameter are varied. Furthermore, the term or terms included, in order to correct for these minor traits of mechanical composition, will be small as compared with the numerical expression of coarseness, which must remain the fundamental term in any such equation.

Paul W. Thompson,[29] Jun. Am. Soc. C. E. (by letter).—Among the important yet-to-be-solved problems that confront the small-scale-model experimentalist is that of simulating movement of stream-bed materials. In developing a practicable technique for the attack of this problem, Captain Kramer has performed a service valuable to the science of hydraulic model experimentation. That much of immediate value has resulted from the author's work is evidenced by the fact that his methods and reasoning are now being applied and extended in several of the leading laboratories of the United States. However, so difficult and complex is the problem of simulating the movement of stream-bed materials that despite the author's work the problem still seems all but impossible of solution. So much remains to be known that by comparison almost nothing is known.

Consider the actual bearing which the author's studies have on the problem of simulating movement of stream-bed materials. His experiments have to do with determining the critical tractive forces for certain sands and with analyzing these data to develop a formula for determining critical tractive forces for these and other sands. The "geschiebe" movement limit, quoted by the author, requires that bed-load movement begin and end in the model at stages corresponding to those for like conditions in Nature. If bed movement in the model is to be similar to that in Nature, the definition of this limit should be revised to require that, at all stages, bed-load movement in the model be similar to what it is at corresponding stages in Nature. To satisfy the limit as thus stated necessitates that cognizance be taken not of relative critical tractive forces, but of relative rates of movement of the bed loads. The distinction between critical tractive force and rate of movement is brought out by the author's definitions, enunciated in the case of critical tractive force, but implied only in the case of rate of movement. Thus, critical tractive force is that lowest value of slope × depth × unit weight of water which results in general movement of the sand in question; rate of movement is simply the total quantity of bed load moved through any cross-section per unit of time.

The important point to be noted is that the relations existing between the critical tractive forces of given sands are not the same as those existing between the rates of movement of those sands. This fact is demonstrated by data derived from experiments conducted at the U. S. Waterways Experiment Station. In another connection, Lieut. K. D. Nichols has given a table of the critical tractive forces for four sands studied in these experiments and a set of curves showing the rates at which the same sands move under varying tractive forces.[30] Although the critical tractive force for Sand *C* of the curves cited is about three times that for Sand *B*, the rate of movement for Sand *C* is much greater than for Sand *B* in most of the cases shown. The opposite of this fact is indicated for Sands *B* and *A*. If the curves for Sands *B* and *C* were continued to the left, they would cross; perhaps if they were continued to the right, they would cross again; and, in any event, it must be concluded that the relations between the critical tractive forces of these sands in no way defines the relations between the rates of movement of the sands. If only data concerning critical tractive forces are known, it is not warrantable to draw therefrom conclusions regarding rates of movement of the sands in question. Therefore, data concerning critical tractive forces throw little light on the matter of the "geschiebe" movement limit in the case where this limit is stated so as to give it practical bearing on the problem of the simulation of movement of stream-bed materials.

---

[29] Second Lieut., Corps of Engrs., U. S. Army, Omaha, Nebr.

[30] See pp. 160–161, Table 4 and Fig. 6.

The writer does not intend to convey the impression that, given data secured through flume tests on rates of movement of the sands in question, it follows that simulation of bed movement may be obtained. Too many other factors are present to complicate this problem. Until the movement of débris by natural streams is understood thoroughly, it will obviously be impossible to simulate such movement.

The law of the constant critical tractive force (the truth of which is essential to most of the "practical applications" listed by the author) is certainly true for the sands investigated through the ranges of values investigated. However, it is doubtful whether this law remains true for slopes and depths of the order of those in Nature. It is difficult to believe that a given depth-times-slope in a model stream which has a value for Reynolds number of, say, 10 000, will move the same limiting mixture of sand as a natural stream that has the same depth-times-slope, but which has a value of Reynolds number of, say, 1 000 000. Still more difficult is it to believe that the rate of movement of a sand would be the same under the conditions cited. Movability depends, quantitatively, on the type of movement. With a low value of Reynolds number, saltation would probably be relatively less important than with a high value of the number, and the rate of movement of the sand would certainly be affected thereby. It may be noted that divergencies in the value of Reynolds numbers of the order noted herein actually occur between model and Nature in most cases. This fact is evidenced by a consideration of the data in Lieut. Nichols' Table 4, previously cited.

The conclusions stated under "1.—The Concurrence Limit" must be questioned. It is stated therein that in river models using sand beds both gravity and friction exert determining influences on the phenomena of flow. It is stated that this fact is responsible for the acceptance, qualitatively but not quantitatively, of data secured from a movable bed model. The friction referred to in this limit is the internal friction due to the viscosity of the fluid, while the gravity referred to is the familiar force, $g \times$ weight. It is not apparent how the presence of a movable bed in a model would act to increase the importance of the frictional force over the force of gravity. As far as the relative importance of the frictional and gravity forces goes, open-channel models with fixed beds certainly enjoy no advantage over those with movable beds. However, a "concurrence limit" that does operate to forestall the acceptance quantitatively of data from a movable bed model derives from the fact that quantitative movability of sand defies simulation.

It must be emphasized that Captain Kramer's work is of far-reaching value. He has blazed a trail into fields which heretofore most experimentalists have feared to tread. His success in arriving at certain rational conclusions has encouraged other experimentalists to undertake the difficult problem of the simulation of the movement of stream-bed materials. Whether or not this problem is ever solved, Captain Kramer will be entitled to rank high among the pioneers who have advanced the science of hydraulic model experimentation in the United States.

GERARD H. MATTHES,[31] M. AM. SOC. C. E. (by letter).—The relation between the movement of the sedimentary load and the tractive force of flowing water is clearly shown in this paper. It should be illuminating to those members of the profession who are still struggling with the complex relations between transportation of sediment and current velocity. The tractive-force formula omits velocity from consideration and is of a simplicity that merits the attention of all engineers concerned with practical river problems. It applies as much to living streams as it does to laboratory models.

The opposing effects on bed-load movability of grain size and void ratio have always been difficult to visualize. The author's approach to ascertain these effects by following the method used by Hummel[27] in determining the strength of concrete from sieve analysis of the aggregate is ingenious and promising. It is hoped that this method will be further developed by experiments on other sand mixtures to the end that the constant in Equation (10) may be determined definitely.

As pointed out by Captain Kramer, his laboratory determinations of bed-load movement can be applied advantageously to actual river problems. In 1933, the writer undertook to apply the knowledge gained from the research work done in the field of tractive force by the author and others, together with the experimental work at the U. S. Waterways Experiment Station, to Mississippi River problems. The progress thus made in connection with important dredging operations is extemely encouraging, and has demonstrated that, in practice, the slope-depth relation is more satisfactory than any velocity formula. The relation is simple and is not concerned with either time, velocity, or friction factors, being merely a function of mass and gravity. It does not show how much or how fast bed-load materials are moved, but indicates what conditions must exist to keep particles of any given weight and size in motion. It makes clear that a deep river having a light gradient may exert as much tractive force along its bed as a shallow river on a steeper slope, regardless of the velocities in either one. Tractive force may be pictured, non-technically, as the forward sweeping force exerted along the river bottom by the weight of an imaginary column of water (in height equal to the depth of the river), in its movement down stream. This movement is caused by the fall of the river as indicated by its surface slope, and is independent of local inequalities in the river bed. The latter merely serve to shorten or to lengthen the column of water from point to point down stream.

---

[31] Prin. Engr., Office of Pres., Mississippi River Comm., Vicksburg, Miss.

It will be seen that tractive force necessarily varies from point to point in any one cross-section and also from point to point down stream.

The writer has found it convenient to use the following method for deriving the du Boys formula, the notations being the same as those used by the author, but with English measures instead of metric. Referring to Fig. 15,

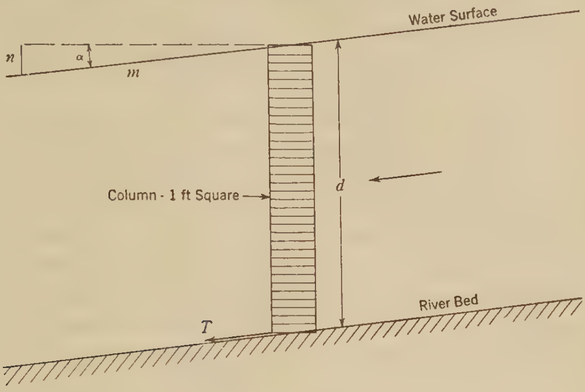


FIG. 15.

let T = the tractive force, in pounds per square foot, of river bed; d = the depth, in feet, equal to the height of a column of water, 1 sq ft in cross-section; ρ = the weight, in pounds, of 1 cu ft. of water; S = the water surface slope $= \frac{n}{m}$; α = the angle of slope; and g = the acceleration due to gravity, in feet per second.

Proceeding from the basic expression; Tractive force = mass × acceleration; mass $= \frac{\text{weight}}{g} = \frac{\rho\, d}{g}$; and, acceleration down an inclined plane $= g \sin \alpha = g \frac{n}{m} = g\, S$. Substituting these values, gives the author's Equation (3). It will be seen that g is canceled, thus eliminating all that relates to velocity and time.

The fact that neither the du Boys formula nor the Kramer formula affords any measure of the rate at which bed material is moved is no limitation to their utilization. In practice, it is seldom important to know how much bed load moves along the river bottom in a given time. The important question is under what conditions movement takes place and, conversely, when movement ceases. After all, the quantity of material of any given kind transported by a natural stream depends on many conditions wholly independent of the available energy for moving such material. This is true of the suspended load as well as of the bed load.

As stated in the paper, flume analysis of the bed-load materials of a stream, whether made in a hydraulic laboratory or in an improvised flume at the project site, will enable an engineer to determine the behavoir of these materials in the stream itself for the prevailing combinations of water surface slope and depth. This enables him to compute readily what minimum depth must be maintained for any given slope, in order that the channel may develop sufficient kinetic energy in the form of tractive effort to transport the heaviest as well as the lightest materials composing the bed load, and thus to prevent shoaling. In its simplest form, the problem, in practice, usually relates to discovering why a river persists in building bars in a particular locality at certain stages, and what should be done to correct this condition.

A study of prevailing slopes and depths, the tractive force values corresponding thereto, and the critical tractive force required to move the various types of bed-load materials, affords the most direct manner of attacking such a problem. In the case of the Lower Mississippi River, such a study has led to the tentative adoption of a value of 0.25 lb per sq ft of river-bed surface as the critical tractive force, T_0, necessary for imparting motion to the entire range of materials predominatingly found on the bed of the river, up to and including loose sub-angular gravel 1 in. in longest diameter, with a specific gravity of 2.65. This value of T_o is predicated on tractive force experiments conducted in a tilting flume at the U. S. Waterways Experiment Station. Table 4 gives combinations of depths and slopes that provide a tractive force equivalent to 0.25 lb per sq ft. Column (1) refers to the actual depth at the point of measurement and not to the average depth or hydraulic radius.

TABLE 4.—DEPTHS AND SLOPES NECESSARY TO PROVIDE A TRACTIVE FORCE OF 0.25 POUND PER SQUARE FOOT.

Depth, in feet (1)	Fall, in feet per mile (2)	Slope, S, in feet per foot (3)	Depth, in feet (1)	Fall, in feet per mile (2)	Slope, S, in feet per foot. (3)
25	0.84	0.00016	100	0.21	0.00004
50	0.42	0.00008	125	0.17	0.000032
75	0.28	0.000053	150	0.14	0.000027

In other words, any reach in the Lower Mississippi River the combinations of depth and slope of which equal or exceed those in Table 4, according to the du Boys formula, should be free from shoaling. The conditions actually found to exist in this river appear to confirm this claim. In the course of the study, extensive sampling of the materials composing the river bed and of the materials moving by traction was undertaken. Much headway has been made in securing representative samples but, so far, quantitative determination of the amount of bed load in motion, under any given conditions, has met with little success. While much remains to be done toward determining a rational practice on which to predicate plans for channel improvements by this method, the results obtained to date, in general, are satisfactory and appear to confirm the soundness of the slope-depth principle.

The statement by the author to the effect that tractive force applies only to mobile materials, cannot be over-emphasized. Any sedimentary materials that have been submerged long enough to have become compacted, are not readily moved by traction. This is especially true in the streams of the eastern half of the United States where compaction is an important factor, and less so in the streams of the Arid West the beds of which, by reason of being dry many months each year, suffer little compaction. Kinetic energy in forms other than tractive force are required to tear loose compacted sedimentary materials and set them in motion again, whether by traction or in suspended form. Tractive force, therefore, is not synonymous with all forms of erosive energy. However, it is erosive in its action as regards deposits of uncompacted, that is, mobile materials.

It seems incredible that the simple relation between tractive force and slope and depth, should have remained obscure for so long a period, considering that it has been known since 1786. Why so much time and effort have been devoted in trying to link sediment movement with velocity finds its answer in psychology. The forces at work in a natural stream carrying sediment, are not readily analyzed, as the water is too turbid to permit of making eye observations below the surface. The human eye, looking down upon the water surface, is powerless to see such a stream at work along its bottom and banks, the only places where physical changes are wrought; but it is ever beguiled by the ceaseless motion at its surface. This surface motion, both as to velocity and direction, affords no reliable index as to what is going on below the surface, which is all the eye can see. Nothing has been more natural than for Man, through the ages, to try to link all that takes place under a stream surface with the motion visible at the surface, and failing in this, to link it with motion measurable by him somewhere within the liquid prism.

In the last analysis, the movement of water, including that of its sedimentary load, is a matter of mass and energy. Velocity is not a force—it is not even a good measure of it in the case of flowing water. For this reason it has served as a poor approach to discovering the laws of sediment transportation and deposition. A century and a half of experimentation and research in transportation of débris by running water, in which have participated some of the foremost hydraulic engineers and scientists of the world, has yielded only one definite answer, namely, that there is no simple or direct relation between the velocity in a stream at any point in its cross-section and the movement of its sedimentary load, whether this movement is in suspended form or whether it is sliding, rolling, or saltation. The mathematical relations that have been evolved, whatever scientific merit they may possess, are entirely too cumbersome for use in every-day river engineering problems. Formulas in which bottom velocity is used as the criterion for bed-load movement appear promising, but so far have found little application because the river engineer, in practice, is never in a position to determine bottom velocities and directions of bottom currents quickly or with any degree of reliability. There is no part of any vertical velocity curve that is more difficult of determination than the bottom part, and this is true in a river as well as in a laboratory flume. By contrast, the tractive-force formula has been found exceptionally convenient for practical use. Water-surface slopes may be read from a suitable profile and need be approximate only, while depths may be obtained quickly with a sounding lead to the nearest foot. A simple tabulation such as Table 4, or a diagram, save computation, but even in the absence of such facilities, the computation is readily made in the field.

The paper is fittingly dedicated to the memory of the late Mr. John R. Freeman. From his association with Mr. Freeman in his earnest endeavors to foster hydraulic laboratory practice in the United States, the writer is moved to exclaim: Would that Mr. Freeman could have lived to have seen the fruits of his labors as exemplified by this paper and by the work done at the U. S. Waterways Experiment Station during the two years, 1933 and 1934.

R. H. KEAYS,[32] M. AM. SOC. C. E. (by letter).—Flood control of alluvial rivers and the accompanying problems of erosion and deposition have been studied extensively in Europe by the use of hydraulic models, and more recently in the United States. Much useful information has been obtained in this way; but running all through the literature on the subject doubt seems to be expressed as to their practical application to the prototypes.

It is to be inferred that, in problems of erosion and deposition, the results obtained are at best qualitative only. In some cases it is supposed that the phenomena observed in the model may be reversed in the prototype. The difficulties are well explained in various papers published by the Society.

What is needed now, after so many prototypes have been constructed on the basis of experiments with models, is information as to how these theoretical deductions check in practice. Whatever conclusions are drawn, it is to be noted that no extensive violations of the laws governing the natural flow of water in alluvial rivers are permissible. Some of these phenomena that might be called laws, may be mentioned offhand.

The first of these laws the writer conceives to be that, other things being equal, the slope of the river tends to be substantially constant at all points throughout its course. At first sight, there might seem to be many violations of this law, but closer examination will probably show that other things are not equal, or that recent geological changes have interfered with the river finding its true regimen.

The writer has seen this law stated a little differently, in that all river slopes tend to become flat as one goes down stream. This is usually true, of course, but in this case one speaks of rivers that continually increase in volume down stream by the addition of water from tributaries, and, therefore, other things are not equal.

On the other hand, the Nile is said to have a constant slope from Assuan to the sea. As is well known the Nile has no tributaries in this stretch, and there is even a loss from evaporation. The foregoing statement is particularly true up river from Cairo several hundred miles. The two branches of the river in the extensive delta have a somewhat greater slope.

There are also natural irregularities in the slope of river caused by the influence of large tributaries, especially those of steeper grade carrying much "geschiebe". Such material tends to deposit below the junction with the main river on account of the decrease in velocity. The main river then backs up enough to erode this material, thus flattening the slope up stream.

---

[32] New York, N. Y.

A second law might be stated by saying that all alluvial rivers tend to flow in a single channel. Islands are a temporary phenomenon from a long-time point of view. They are constantly being formed by cut-offs, but their back channels silt up again.

The writer scarcely dares to suggest a third law because it is such a controversial matter, namely, that all alluvial rivers tend to meander. In general, large rivers tend to form bigger meanders than small ones, but specifically each river is a law unto itself in this respect. Among factors governing the meanders may be mentioned the size of the river, the quantity and character of "geschiebe" carried, relative high and low-water stages, and frequency and regularity of floods. Particularly to be mentioned in this latter respect is the regular annual flood of the Nile as compared to the usual idea of excessive floods of varying intensity at long intervals.

The writer would suggest also that it might be stated as a law, that all alluvial rivers tend to silt up their beds during prolonged low-water periods and to flush them at time of flood. A flood coming after a long period of low-water flow is, therefore, more disastrous than it would be otherwise. On account of this tendency of rivers to silt up their beds during low water, and *vice versa,* it is difficult to establish any flow curves for a river that will indicate the flow merely by observing the stage. Such curves must be checked frequently.

When left to themselves, rivers continually change their channels. The concave beds are extended by erosion, thus lengthening the river and flattening the slope and decreasing the average velocity. Then, a cut-off is made which shortens the river again and restores the original slope.

For many years the policy of the Mississippi River Commission has been to prevent cut-offs, in consequence of which the river has been lengthened about 5 per cent. The writer does not recall, however, that any one has mentioned this fact in proposing new cut-offs in the Mississippi. It would be quite reasonable to suppose that the river might be shortened as much as it has been lengthened without causing any trouble.

F. T. MAVIS,[33] ASSOC. M. AM. SOC. C. E. (by letter).—Data presented in this paper should be helpful in the proportioning of river models with erodible beds. Table 1, showing in detail the data obtained in the tests, is a contribution to the study of movable bed models. The writer feels, however, that the sands used in the tests have been inadequately described by the gradation curves plotted on Cartesion co-ordinate paper in Fig. 1. Gradation curves plotted on semi-logarithmic paper, or on logarithmic-probability paper, give a more accurate picture of the gradation of sands over the entire range from fine to coarse-grained sizes.

The author recognizes three variables as affecting the results of his experiments, namely, depth, slope, and sand mixture. He states that "except for special isolated cases, the study of velocity was purposely not undertaken because that is a derived and not an independent criterion, being a function of depth and slope." Evidently, the author is tacitly making further assumptions: (1) That cross-currents are negligible; (2) that velocity distributions (that is, velocity-depth curves) are virtually identical at each cross-section of the channel; and (3) that the shape and specific gravity of sand particles are the same for each mixture. Apparently, he is using the Chezy formula, $V = C\sqrt{RS}$, to express the relationship between velocity, depth, and slope.

---

[33] Assoc. Prof., Mechanics and Hydraulics, The State Univ. of Iowa, and Assoc. Director in Charge of Laboratory, Iowa Inst. of Hydr. Research, Iowa City, Iowa.

If the coefficient, C, in the Chezy formula was actually a constant it would, of course, be independent of Reynolds' number, $\mathsf{R} = \frac{V\,d}{\nu}$. Experience seems to indicate that, for a given channel, C varies approximately as $\mathsf{R}^{\frac{1}{2m}}$, in which, $m = 1$ in the range of laminar flow, and $m = 4$ to 6, or more, in the range of turbulent flow. If the coefficient of kinematic viscosity, ν, is constant this leads at once to the well-known general expression:

$$V = k\,R^{ma+a}\,S^{ma} \quad\text{.......................}(12)$$

in which, $a = \frac{1}{2\,m - 1}$. If $m = 5$, and if k is a function of the roughness of the channel and the coefficient of kinematic viscosity,

$$V = k\,R^{\frac{2}{3}}\,S^{\frac{5}{9}} \quad\text{........................}(13)$$

Inasmuch as the slope in a model is equal to the vertical scale distortion factor times the corresponding slope in the prototype—if the model is constructed to scale—it will be evident that, with small error, Equation (13) can be replaced by the Manning formula:

$$V = \frac{1.486}{n}\,R^{\frac{2}{3}}\,S^{\frac{1}{2}} \quad\text{......................}(14)$$

in which, n is the roughness factor.

The writer computed values of Manning's n from the data given in Table 1 and plotted curves (of which Fig. 16 for Series I is an example) for each series, slope, and hydraulic radius. These curves seem to justify the following observations relative to the author's tests:

(1) Within the "zone of usefulness", as defined by the author, the median value of Manning's n for each series is, as follows:

Sand	Size of grains, in millimeters	Median value of Manning's n
I	0 to 5.00	0.024
II	0 to 1.77	0.022
III	0.385 to 5.00	0.025

One-half the number of observations in this zone deviate less than about 8% from these values of n.

(2) After riffles have formed, that is, above the "upper limit", Manning's *n* generally increases sharply to values of the order of 20% above values in the zone of usefulness.

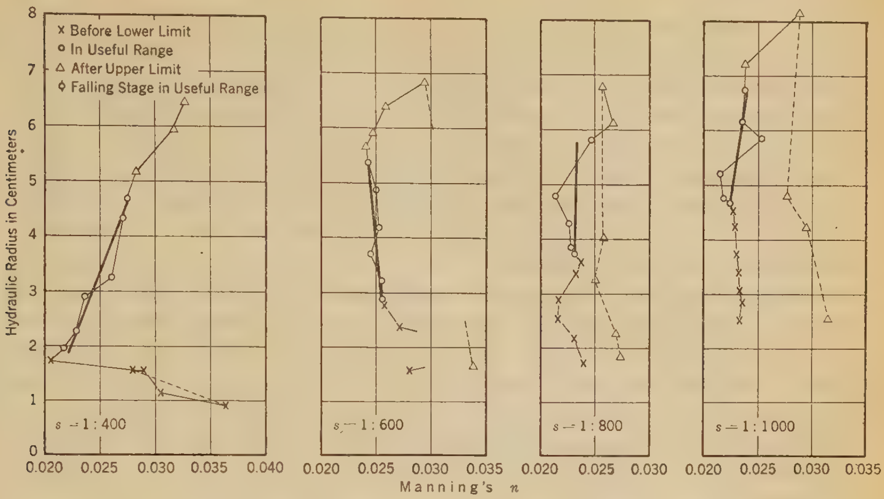


FIG. 16.—ROUGHNESS COEFFICIENTS FOR RIVER MODELS, SERIES I.

(3) Before general bed movement takes place in channels where the depth of flow is greater than 1.5 cm, values of Manning's *n* are essentially the same as, or somewhat less than, corresponding values in the zone of usefulness.

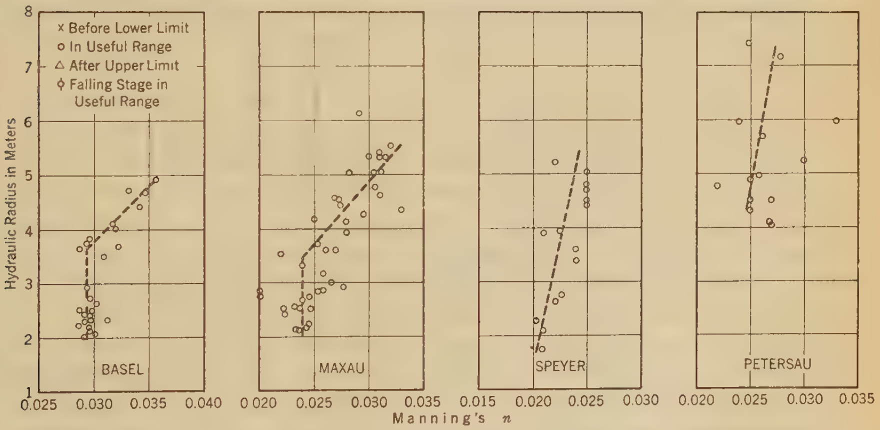


FIG. 17.—ROUGHNESS COEFFICIENTS FOR THE RHINE RIVER.

Fig. 17 was prepared from data collected by Heinrich Wittmann[34] in a study of the effect of correction works between Basel, Switzerland, and Mannheim, Germany, upon the bed-load movement of the River Rhine. These observations on the Rhine indicate that there is a general increase in the value of Manning's n with an increase in bed-load movement, and they would not appear to be at variance with the observations reported by the author in Table 1.

---

[34] "Der Einfluss der Korrektion des Rheins Zwischen Basel und Mannheim auf die Geschiebebewegung des Rheins", von Heinrich Wittmann, *Deutsche Wasserwirtschaft*, Heft 10, 11, 12. (Doctor's dissertation prepared under the direction of Prof. Rehbock and Dr. Boss, Karlsruhe, Germany.)

V. V. Tchikoff,[35] M. Am. Soc. C. E. (by letter).—A good example of laboratory investigation is offered in this excellent paper. There are serious doubts, however, as to the theory of tractive forces, or "drag" theory, itself, as well as the quantitative transferability of experimental results. The "drag" force is simply a component, parallel to the slope, of the weight of a vertical column of water of height, d, in contact with a unit area of bed of a channel with uniform flow as expressed by Equation (3). This formula is considered by the "drag" theory as a constant relation between the slope and the depth for the same bed-load material. This theory is not generally recognized. "The imperfection in this concept is evident, and it is clear that the phenomena as conceived by du Boys was a scheme somewhat lacking in the real course of things", states Professor Francesco Marzolo.[36] Professor Reynold considers the force exerted by the current on sand as the function of the velocities. Professor A. H. Gibson states that the erosive power of water varies as the square of its velocity.[37] This "drag" theory has not even been mentioned in many important books. The theory has the following limitations:

1.—It presupposes infinite width of channel and, according to Schoklitsch,[38] an additional coefficient is required for channels in which the ratio of width to depth is less than thirty. The "drag" theory does not take into consideration the type and nature of the cross-section of the channel.

2.—Schaffernak[39] considers the du Boys law only applicable for certain sizes of grain, namely, for homogeneous materials with a grain size up to about 10 mm.

3.—According to the author the slopes must be "usual moderate slopes", as he stated in establishing his law of constant critical tractive forces.

4.—The type of the flow does not affect the "drag" theory, and it is considered applicable for laminar, turbulent, shooting, or streaming flows.

These four limitations cover practically every phase of hydraulic phenomena of flow and are so serious, in fact, that they annul the entire theory. There are direct contradictions between the requirements for hydraulic similitude of channels and those fulfilling the "drag" theory. As the writer had occasion to state in another connection,[40] the first requirement for similitude of turbulent flow is the same kinetic factor. This is nothing but the fulfillment of Froude's law. The kinetic factor, or twice the ratio of kinetic energy to the hydraulic radius, gives only the physical and quantitative state of turbidity. Expressing the velocity through the kinetic factor:

$$V = \sqrt{g K_r R} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (15)$$

---

[35] Cons. Engr., New York, N. Y.

[36] "Hydraulic Laboratory Practice", p. 755.

[37] "Hydraulics and Its Application", Fourth Edition, p. 340.

[38] "Hydraulisches Rechnen", von Weyrauch und Strobel (1930), p. 79.

[39] "Hydraulic Laboratory Practice", p. 428.

[40] See p. 170, *et seq.*

Comparing with Chezy's equation:

$$SC^2 = gK_r \dots\dots\dots\dots\dots\dots\dots\dots (16)$$

in which, C = Chezy's coefficient and $K_r = \frac{V^2}{g\,R}$ = the kinetic factor. The right-hand part of Equation (16) is constant for the same hydraulically similar channels.

Equation (16) gives an important relation between the slope and elements of the cross-section. The same relation is expressed in Equation (3):

$$Sd = \text{a constant} \dots\dots\dots\dots\dots\dots\dots\dots (17)$$

To meet the requirements for hydraulic similitude and the "drag" theory, both Equations (16) and (17) must hold, or:

$$C = \text{constant} \times d \dots\dots\dots\dots\dots\dots\dots\dots (18)$$

This is the new and non-existent expression for Chezy's coefficient, C. The search for the relation between the slope and other elements in some stable alluvial channels which are hydraulically similar, led Dr. H. E. Hurst to the following equation for the Lower Chenab Irrigation Project[41]:

$$S = 116 \times 10^{-6} \left(1 + \frac{2.12}{R}\right) \dots\dots\dots\dots\dots (19)$$

If Equation (19), with $K_r = 3.98\%$ (its value for the Lower Chenab Irrigation Project,[42] see Table 5) is substituted in Equation (16):

$$C = \frac{105.4}{\sqrt{1 + \frac{2.12}{R}}} \dots\dots\dots\dots\dots\dots\dots\dots (20)$$

TABLE 5.—General Hydraulic Factors; British Indian Irrigation Projects

Name of irrigation project (1)	Numbers of observations (2)	Mean kinetic factor, K_r (percentages) (3)	Coefficient of hydraulic similitude, Y (4)	Hydraulic number, H (5)	Difference from mean (percentages) (6)	Tractive Force, in Grams per Square Meter: Maximum (7)	Minimum (8)
Upper Bari Doab......	22	4.15	7.20	35.3	—12.8	381	58
Lower Chenab........	35	3.98	8.86	44.4	+ 9.6	537	112
Godaveri West........	15	2.04	5.55	38.8	— 4.2	264	88
Delta................	....	....	....	....	....	....	....
Mean weighted....	....	....	....	40.5	0.0	....	....

This new type of equation for the Chezy coefficient is most correct compared with all existing formulas, such as those of Kutter, Bazin, Manning, etc. All of the latter are entirely different from Equation (18) expressing the "drag" theory. In other words, the channel fulfilling the "drag" law will not be hydraulically similar. (A formula of the same type as Equation (20) has been suggested by the late W. M. Penniman,[42] M. Am. Soc. C. E. Equation (20) has numerical coefficients corresponding to the coefficient of roughness, $n = 0.0225$.)

---

[41] *Minutes of Proceedings*, Inst. C. E., Vol. 229, Session 1929–30, Pt. I, p. 328.

[42] *Engineering News-Record*, May 10, 1934, p. 608.

The idea of the kinetic factor facilitates the judgment of the state of turbulency of flow. Table 6 gives the kinetic factor for the lower limit of tractive force and the adjacent readings (Column (4)) for Series II of the author's data. The kinetic factor varies from 34% for a slope of 1:1 000 to 80% for a slope of 1:400, being more constant for smaller slopes.

TABLE 6.—GENERAL HYDRAULIC FACTORS; SAND, SERIES II, FROM AUTHOR'S DATA

Slope	Reading number	Tractive force, in grams per square meter	Kinetic factor (percentages)	Coefficient of hydraulic similitude	Hydraulic number	Ratio of width to depth
1:400	4	38.1	80	1 263	1 413	....
	4a	41.0	81	....	....	49
	5	49.5	77	896	1 022	....
1:600	5	40.5	43	501	759	....
	5a	41.7	42	....	....	32
	6	44.2	40	443	703	....
1:800	4	33.7	36	405	675	....
	4a	35.8	39	....	....	28
	5	37.9	40	362	575	....
1:1 000	4	36	34	264	455	....
	4a	37.2	34	....	....	22
	5	39.6	34	235	405	....

Mr. Gerald Lacey[43] gives the approximate equation for the relation between his silting factor and the diameter of the predominant type of sand, which expressed through the kinetic factor, is:

$$D = 8.6\ K_r^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (21)$$

in which, D, is the diameter of the sand grains, in inches. Sand of Series II had the maximum diameter of 1.77 mm = 0.07 in. and a median diameter of 0.51 mm = 0.02 in. Substitution of these diameters into Equation (21) gives the corresponding value of the kinetic factors as 9.0 and 4.8%, respectively. Comparing this value of the kinetic factor with that of the flume in the paper, it is seen that the latter is several times greater. In other words, maintaining hydraulic similitude, the bed-load material in the prototype corresponding to the "geschiebe" in the flume, must be considerably coarser. Its size of grain, calculated according to the kinetic factor in the flume, will be, for $K_r = 0.80$, — $D = 5.5$ in., and for $K_r = 0.34$, — $D = 1$ in.

To give a concrete idea as to the value of the kinetic factor in canals. Table 7 is given, based on some alluvial canals studied by Fred C. Scobey.

---

[43] "Stable Channels in Alluvium", by Gerald Lacey, *Minutes of Proceedings*, Inst. C. E., Vol. 229, Pt. I, p. 283.

M. Am. Soc. C. E.[44] The last two lines of Table 7 give data on the Mississippi River and its model according to Herbert D. Vogel, Assoc. M. Am. Soc. C. E.[45]

TABLE 7.—GENERAL HYDRAULIC FACTORS; ALLUVIAL CANALS AND MISSISSIPPI RIVER

Reference No.	Name	Description of channel	Discharge per second, in feet	Hydraulic radius, in feet	Mean velocity per second, in feet	Hydraulic grade; 5×10^{-4}	Kinetic factor (percentage)	Coefficient of hydraulic similitude	Hydraulic number	Tractive force, in grams per square meter
123*a*	Bear River	Silt	109.56	1.86	2.36	2.7	9.30	17.0	56	160
146*a*	Logan and Hyde Park	Sand	23.55	1.40	1.08	1.5	2.59	8.6	53	101
168*a*	Louden	Clean sand	62.00	1.52	1.66	3.8	5.63	17.7	75	173
169*a*	Mesa lateral	Fine silt bed	40.32	1.66	1.47	2.6	4.04	8.8	44	116
204	Fullerton	Loose sand bed	15.72	1.03	1.14	4.93	3.92	14.4	73	173
170*a*	Rio Grande lateral	Gravel	146.60	1.42	3.86	22.0	32.6	51.2	90	980
210	Parley's Ditch lateral	Sand bed	4.44	0.66	1.20	11.7	6.78	15.5	60	264
201*a*	Rocky Ford	Loose sand bed	41.20	1.52	1.68	5.8	5.74	11.7	49	299
214*a*	Rio Grande lateral	Gravel	380	1.87	4.66	36.6	36.1	58.2	97	2 087
Mississippi River	....	....	1 125 000	51.00*	4.76	0.44	1.38	8.5	72	684
Vogel's model	....	....	0.0828	0.255*	0.355	10.56	1.53	5.0	40	82

* Mean depth.

Comparison of Tables 6 and 7 in relation to the kinetic factors and tractive forces shows:

1.—Only two cases (Nos. 170*a* and 214*a* in Table 7), the kinetic factor being 32.6 and 36.1, more or less approach the same value in the flume. There is gravel in both cases. This proves again that the bed-load material in the prototype must be considerably coarser in order to correspond to the same material in the flume.

2. —The tractive forces in Table 7 are higher in general than in the flume, but all channels are not scouring. Three instances are especially interesting. Two are referred to the same channel with a gravel bed, in which, $T = 980$ and 2 087 grams per sq m, and the third is the Mississippi River, with $T = 684$. All three values are much greater than the tractive forces in the paper.

The author suggests, as the most practical procedure, that the bottom silt taken from the prototype be tested in the flumes, but such "geschiebe" in a model flume will correspond to a much coarser material in the prototype. In designing the model of a channel with a different kinetic factor, it is necessary to take the corresponding bed-load material and, if Equation (21) is more or less correct, the ratio of the diameters of the "geschiebe" must be equal to the kinetic factor, squared.

---

[44] *Bulletin No. 194*, U. S. Dept. of Agriculture.

[45] "Geometric *Versus* Hydraulic Similitude", by Herbert D. Vogel, Assoc. M. Am. Soc. C. E., *Civil Engineering*, August, 1932, p. 471.

Columns (7) and (8) of Table 5 show the maximum and minimum values of tractive forces. These three stable alluvial channels, as already stated, are hydraulically similar. Each of them transports its own specific sand corresponding to its kinetic factor. The tractive forces vary greatly, usually having a greater value in the larger channels. The maximum values are again considerably larger than in the flume, but scouring has no place.

The author referred to Gilbert's conclusion that with equal width-depth ratios, the result of experiments with a flume are, in general, quantitively transferable to channels of different trace. This means that if the cross-sections of the model and the prototype are a rectangle, they are geometrically similar; but the cross-sections of models of hydraulically similar channels should be distorted[40, 45] and this distortion must be given consideration because the preservation of hydraulic similitude depends not only on the same kinetic factor, but also on the form of the cross-section. To express this dependence the writer suggests the following equation[40]:

$$Q = Y\,R^3 \qquad \text{(22)}$$

in which, Y is a constant for hydraulically similar channels and may be termed the "coefficient of hydraulic similitude".

Another interesting relation between the kinetic factor, K_r, and the coefficient, Y, is:

$$H = \frac{Y}{K_r^{0.5}} \qquad \text{(23)}$$

Of course, Y and K_r being constant, the coefficient, H, is constant, too, for the same hydraulically similar channels. However, this last coefficient (termed the "hydraulic number") presents an interesting parameter.

Table 5 gives the value of H for three British Indian irrigation projects in which the channels are hydraulically similar. The hydraulic numbers for these three different projects with stable alluvial channels are about the same (approximately 40.0). This naturally suggests the idea that it is quite probable that all stable alluvial channels have the same hydraulic number. Stable alluvial channels are understood to be non-silting and non-scouring, ideal channels with a uniform flow of water and a full silt load. The value of the hydraulic numbers is computed for channels given in all the tables. It is of special interest to note that Lieut. Vogel's model has the same exact value of hydraulic number (40), while all others in Table 7 have larger values. The value, 40, appears to be a limit. All channels in Table 7 are non-scouring, and if the value of the hydraulic number in the alluvial stable channel is less than 40, it may be assumed that they have scouring tendencies when the kinetic factor corresponds with the silt material.

In data concerning the scouring of some drainage channels observed by Charles E. Ramser,[46] M. Am. Soc. C. E. (Table 8), the hydraulic number changes from 14.9 to 22.4 (less than 40) and the author's data differ completely from Tables 5, 7, and 8.

---

[46] "Erosion and Silting of Dredged Ditches", by Charles E. Ramser, M. Am. Soc. C. E., *Technical Bulletin No. 184*, U. S. Dept. of Agriculture, June, 1930.

The phenomena of scouring does not occur in Nature as in a rectangular flume. First, it is known that the upper parts of the channel's banks, even under normal conditions, have a tendency to wash out, become round, and settle. The depths of water in the upper parts, and tractive forces, are less; but scouring occurs and settlement of washed material also occurs where the depth and tractive forces are greater.

TABLE 8.—GENERAL HYDRAULIC FACTORS; ERODING DRAINAGE CHANNELS

Stream No.	Observation No.	Name	Description of channels	Discharge per second, in cubic feet	Hydraulic radius, in feet	Mean velocity, in feet per second	Slope, $S \times 10^{-4}$	KINETIC FACTOR (PERCENTAGES)		COEFFICIENT OF HYDRAULIC SIMILITUDE		HYDRAULIC NUMBER		Tractive force, in grams per square meter
								Actual	Mean	Actual	Mean	Actual	Mean	
1	1*	Mud Creek, near Tupelo, Miss.	Alluvial, sandy, wax-like clay.	794	5.6	3.6	....	.....	.....	.....	.....	.....	.....	.....
	2			914	5.6	3.6	....	7.2	.....	5.2	.....	19.4	.....	.....
	3			1 221	6.5	4.0	....	7.6	.....	4.5	.....	16.1	.....	.....
	4			1 280	6.0	3.8	3.98	7.4	7.4	5.9	5.2	21.8	19.1	904
2	1*	Twenty-Mile Creek, near Baldwyn, Miss.	Wax-like clay, loam, containing considerable sand.	611	5.0	3.5	....	.....	.....	.....	.....	.....	.....	.....
	2			1 758	7.2	5.8	....	14.3	.....	4.7	.....	12.5	.....	.....
	3			2 402	7.4	4.7	11.6	7.4	10.8	4.7	4.7	17.3	14.9	2 996
3	1*	Chawappah Creek, near Shannon, Miss.	Sandy loam at top to wax-like clay at bottom.	582	4.9	3.5	....	.....	.....	.....	.....	.....	.....	.....
	2			1 354	6.6	4.35	....	7.6	.....	4.7	.....	17.2	.....	.....
	3			1 392	6.1	4.1	....	8.7	.....	6.1	.....	20.8	.....	.....
	4			1 456	5.9	4.0	9.29	8.4	8.3	7.1	6.0	24.4	20.8	1 910
11	1*	South Fork, Deer River, near Jackson, Tenn.	Firm, wax-like clay.	1 620	7.0	4.0	....	.....	.....	.....	.....	.....	.....	.....
	2			1 740	7.1	4.1	....	7.3	.....	4.9	.....	18.0	.....	.....
	3			2 342	8.2	4.5	5.15	7.8	7.5	4.2	4.5	15.2	16.6	1 388
13	1*	North Fork, Deer River, near Trenton, Tenn.	Alluvial silty loam at top, heavy silty clay at bottom.	883	5.3	3.9	....	.....	.....	.....	.....	.....	.....	.....
	2			1 512	6.6	4.6	....	9.8	.....	5.3	.....	16.8	.....	.....
	3			1 838	6.6	4.5	....	9.75	.....	6.4	.....	20.4	.....	.....
	4			2 144	7.0	4.8	5.02	10.1	9.9	6.3	6.0	19.6	18.9	1 176
15	1*	Sugar Creek, near Henderson, Tenn.	Light-yellow clay, very tenacious and much less easily eroded than soil of Stream No. 11.	345	3.7	3.2	....	.....	.....	.....	.....	.....	.....	.....
	2			400	3.8	3.3	....	8.7	.....	7.3	.....	24.8	.....	.....
	3			473	4.2	3.5	....	9.1	.....	6.4	.....	21.1	.....	.....
	4			513	4.3	3.55	8.38	9.1	9.0	6.4	6.7	21.3	22.4	1 232

* Original cross-sections at time of construction computed from dimensions of channels given in specifications. Mean values were computed without considering original cross-sections.

Scouring and silting on a bank do not depend on the depth of the water, but on its velocity. Increased velocity will scour the banks of the channel and make it wider. A deep narrow stream has a tendency to widen itself, but the shallow wide stream will deepen some part of the channel to become narrower by silting the remainder. In both cases the stream inclines to adjust itself in a stable channel. Two critical conditions for erosion and silting, in the writer's opinion, depend upon the condition that the kinetic factor must correspond to the transporting material and hydraulic number. For any particular kinetic factor and silt there will be two values of the hydraulic number between which the channel will be stable. For channels having a non-cemented or non-packed bed of the same silt, carried by fully charged water, there will be the same hydraulic number (equal probably to about 40). In the case of a settled bed the difference may be considerable. The cohesion between the particles has to be overcome. The finer the settled materials, the larger the molecular bonds and skin friction will be; but the gravity action increases with the size of the particles. Therefore, clay and sand both need higher velocities to cause erosion than the coarse silt, which gives minimum erosion-velocity conditions.[47]

There is no turbulent flow with parallel filaments of water. The distribution of velocity over the cross-section is the all-important factor. Cross-currents greatly affect the phenomena of silting and scouring. Velocities at any point are constantly changing. Scouring and silting depend on the distribution of velocity along the cross-section (especially at the bottom) and not so much on their absolute value as on their relative changes. If a model is built correctly these relative changes of velocities should be about the same as in the prototype. This explains why hydraulically similar, alluvial, channels are capable of transporting the same material.

In studying any phenomena of flow in channels by the method of models, the first requirement is the hydraulic similitude of flow. Failing to fulfill this requirement and basing conclusions on the doubtful "drag" theory, the author's type of experiments seems to differ from the actual condition in Nature; they may have only a qualitative meaning. No doubt, however, the author's work (especially his new formula for tractive forces based on the silt graduation curve) is very important in laboratory practice.

MORROUGH P. O'BRIEN,[48] ASSOC. M. AM. SOC. C. E., AND BRUCE D. RINDLAUB,[49] ESQ. (by letter).—The rate of movement of bed load in channels is known to increase rapidly with tractive force and, for this reason, a visual determination of the critical tractive force is probably sufficiently accurate for model work; but for establishing a general law, the more precise weighing method appears desirable. In this connection it should be mentioned that there is quantitative evidence indicating that each fraction of a mixture behaves approximately as it would if present alone, at least in the region of general movement. If this statement is true of a mixture acted upon by a tractive force too small to move the larger fractions, the critical tractive force for each fraction can be obtained approximately by analyzing the material caught in the weighing pan. For each sample, the tractive force acting is approximately the critical value for the largest particles caught.

The use of depth times slope as the tractive force is perhaps sufficiently accurate for model work; but it is not a sound procedure since some resistance to flow is exerted by the side walls of the channel. The average tractive force on the side walls and bottom is:

$$T = w\,R\,s \qquad (24)$$

in which, s is the slope of the energy gradient (not the slope of either the bottom or the water surface); and R is the hydraulic radius. The tractive force on the bottom exceeds the average value in most cases and approaches, but does not equal, $w\,D\,s$.

---

[47] "Problems in the Theory of River Engineering", by Herbert Chatley, Inst. C. E., Selected Engineering Papers No. 71, p. 17.

[48] Assoc. Prof. of Mech. Eng., Univ. of California, Berkeley, Calif.

[49] Berkeley, Calif.

In regard to the slope to be used in computing the tractive force, if sufficient data are not available to compute the slope of the energy gradient, the slope of the bottom is more nearly correct than that of the water surface, if the bottom is inclined downward in the direction of flow.

The author mentions a paper by Leighly[5] which describes a method of estimating the distribution of tractive force over the wetted perimeter of a channel. Another method that appears to be more sound can be developed from von Kármán's theory of turbulent flow near a flat boundary surface. The essence of the theory underlying this method is that the velocity distribution, at a distance from the boundary greater than the thickness of the laminar layer, depends only upon the shearing or tractive force within the fluid. The equation for the tractive force is:

$$T = \rho \left[\frac{c\,(u - u_0)}{\log_e \dfrac{z}{z_0}} \right]^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (25)$$

in which, ρ is the mass density; u and u_0 are the velocities at distances, z and z_0, from the boundary; and c is a coefficient found from experiment to have a value of 0.4. In order to apply Equation (25), velocities must have been measured at distances from the boundary which are small as compared with the dimensions of the cross-section. Application of this method involves the construction of velocity profiles along lines perpendicular to the bottom at each point.

An interesting possibility of this method is that it may be sufficiently precise to compute the value of the friction coefficient from the velocity distribution alone without measuring the slope of the energy gradient. The advantage of this method is that it would give the average value of n at one cross-section rather than the average over a finite reach.

Assuming that T has been computed for a number of points around the wetted perimeter, the total resistance per unit length of channel is, $\int_0^p T\,dp$, in which, p is the wetted perimeter, and dp is an infinitesimal length along the perimeter. The total resistance is obtained by measuring the area under the curve of T as a function of p. The resistance as computed by the Manning equation is, $w\ A\ \dfrac{v^2\ n^2}{(1.49)^2\ R^{\frac{4}{3}}}$. Equating the two expressions for the resistance, and solving for n,

$$n^2 = \frac{(1.49)^2\ R^{\frac{4}{3}} \int_0^p\ T\,dp}{w\ A\ v^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (26)$$

in which, A is the area of the cross-section, and v is the average velocity. Comparison of the value of n computed by this method with that obtained in the usual way will give a check on the validity of von Kármán's theory as applied to open channel flow.

Returning to the critical tractive force and its relationship to the characteristics of a sand mixture, the writers are not convinced that the modulus, *M,* brings about an improvement in the correlation of the existing data. An analysis[50] of the data of Gilbert and others indicated that the critical tractive force can be related to the median diameter alone, with the same or less deviation of individual experimental points, and the relationship is much simpler to apply.

No quantitative theory of bed movement yet advanced adequately represents all the factors involved and, for the present at least, the characteristics of each sand to be used in a movable-bed model must be determined independently. A general theory is needed, but much additional experimental work will be required to develop one that can be applied with confidence. In the meantime, engineers designing models should use present equations with caution.

HERBERT D. VOGEL,[51] ASSOC. M. AM. SOC. C. E. (by letter).—The work of Captain Kramer in Germany, as reported in his paper, has served the past few years as no inconsiderable aid to all experimenters interested in the performance of model streams. Prior to the release of his data much conjecturing was needfully resorted to each time it became necessary to design a movable-bed river model; more recently, it has been possible to approach each problem from a standpoint of reason and logic.

Two statements of the author appear worthy at this time of special repetition and emphasis: (1) "* * * strict compliance with the laws of similitude must be waived in fluvial models in which bed movement is of predominating importance;" and (2) "since it [Equation (10)] was derived empirically and from limited data, its scope of application and probable accuracy are subject, of course, to obvious limitations."

Referring to the first of these considerations it is evident that the requirements for adequate bed movement in a fluvial model are at variance with those indicated for preservation of strict Froudian relationships. This is true because for any fixed model depth it is generally necessary to apply steeper slopes in order to produce a movement of the bed material similar to that of Nature, whereas to secure a velocity scale equal to the square root of the vertical scale, a model must be provided, in nearly every case, with flatter slopes than would result from normal distortion. Similarly, a large degree of distortion is conducive to greater bed movement, whereas little or no distortion is desirable from the standpoint of satisfying the Froudian law. Thus, the experimenter is faced with contradictory requirements, and the only solution is by way of deciding from an estimate of the situation the line of attack to pursue, keeping in mind the mission to be accomplished. This is generally an easy task.

---

[50] "The Transportation of Bed-Load by Streams", by M. P. O'Brien, Assoc. M. Am. Soc. C. E., and B. D. Rindlaub, *Transactions*, Am. Geophysical Union, 1934.

[51] Lieut., Corps of Engrs., U. S. Army, The Command and General Staff School, Fort Leavenworth, Kans.

The second point emphasized by the author, may cause some doubt in the minds of a few as to the reliability of his uniformity modulus. This is relatively unimportant, however. because future investigations will serve to reveal better values for it from time to time. The important fact is that a start has been made in the right direction and a line thus indicated for further research.

Investigations are being conducted currently in different parts of the world to determine specifically the effects on movement resulting from changing the specific gravity or the mixture of the bed material. Tests are also being made to determine volume rates of movement and the effects of turbulence, roughness, etc., on movement. All these have followed from the original investigation conducted and reported upon by Captain Kramer.

JOSEPH B. TIFFANY, JR.,[52] JUN. AM. SOC. C. E., and CARL E. BENTZEL,[53] ESQ. (by letter).—The results of Captain Kramer's work in Germany have been of especial value to the Staff at the U. S. Waterways Experiment Station, at Vicksburg, Miss., where they have been used as the starting point for a series of investigations of bed-load movement. Nine sand mixtures were tested in a tilting flume, ranging in mean grain size from 0.205 mm (0.008 in.) to 4.077 mm (0.161 in.).

Each sand was tested at three different slopes, 0.0010, 0.0015, and 0.0020, closely following the range of values used by Captain Kramer. The technique adopted for the tests was also similar to that of the paper, the general practice being to set the tilting flume at the desired slope, mould the sand to this slope, and by adjustment of the tail-gate, to maintain the slope of the water surface at this same value throughout each run, thus insuring uniform flow. Each test was started with a small flow, usually in the range of laminar flow. After an equilibrium had been reached, and a complete set of observations made, the flow was increased slightly, the tail-gate was again adjusted, if necessary, to maintain the proper slope, and another set of observations was made. This procedure was repeated, increasing the flow by small increments, until the maximum capacity of the circulating system had been reached.

The results of these studies have been assembled by the writers[54] and, therefore, it is not deemed necessary to repeat this information in the present discussion. However, the writers do wish to point out a few limitations to Captain Kramer's methods which have not before been properly emphasized.

In the first place, it has not been found practicable by the writers to utilize the visual method of determining the commencement of general movement of bed-load. This difficulty was first encountered in the early stages of the studies, when water from the artificial lake was being used in the flume. This water was often too turbid, especially after rains, to allow observations at depths greater than 2 or 3 in. After the completion of, the tests on the first three sand mixtures, however, the flume was connected with a circulating clear-water system; but it still was found impossible to obtain consistent results from the visual method of determining general movement. In spite of the care with which the operators made their observations, the element of subjective judgment entered into the results, and it was frequently found that two or more skilled observers differed by as much as 100% in their individual selection of the point of general movement.

---

[52] Care, U. S. Waterways Experiment Station, Vicksburg, Miss.

[53] Research Asst., U. S. Waterways Experiment Station, Vicksburg, Miss.

[54] "Studies of River Bed Materials and Their Movement, with Special Reference to the Lower Mississippi River", *Paper 17,* U. S. Waterways Experiment Station.

This difficulty has been overcome by the adoption of a mechanical method of spotting general movement; that is, basing its determination primarily upon analyses of the sand in motion and comparison of these analyses with that of the original mixture.

Another difficulty arose from the definition of general movement, proposed by Captain Kramer, namely (see "Definitions: Bed-Load Movement" (Definition 4)), "that condition in which sand grains up to and including the largest are in motion". Implied throughout the author's paper is the assumption that the smallest materials are placed in motion first, the medium-sized grains next, and the largest grains last. From this, it follows that the movement of the largest grains is the real criterion for the condition of general movement. Correspondence with the author has verified the fact that he used the largest grains as his criterion for movement, assuming that when they were in motion, all sizes up to the largest were also in motion. However, the writers have found that for sands of mean grain size smaller than about 0.5 mm (0.0197 in.), the large grains moved first, and that if their movement were used as a measure for general movement, an erroneous value of critical tractive force would be obtained. This curious phenomenon has been noted independently by several observers, and has been checked by sieve analyses which have been made of the sand caught in the trap at the lower end of the flume.

This condition was not observed in the two largest sands tested, the mean grain sizes of which were 4.077 mm (0.161 in.) and 0.586 mm (0.023 in.), respectively. It was observed, however, for each of the other seven mixtures, all of which were of smaller mean grain size than the finest mixture tested by the author.

Fig. 18 illustrates this condition. The mean grain size of the material in motion (as determined from sieve analyses of the trapped materials) is compared with the mean grain size of the original mixture in Fig. 18(*a*) (see the horizontal dash-dot line). The average size of the material in motion in the first five runs in this test on Sand No. 4 was much larger than that of the original sand mixture. It was only after the appearance of riffles, at a much higher value of tractive force than the critical cited by Captain Kramer, that the materials in motion in this test became reasonably representative of the original mixture.

In Fig. 18(*c*), the curve for the rate of bed-load transportation illustrates another weakness in Captain Kramer's definition for critical tractive force. With this sand, and with all the seven finest mixtures tested, there was a gradually increasing rate of movement on a smooth bottom, up to the point where riffles began to form. At this point the bed suddenly became much rougher, the velocity decreased, and the depth for a given discharge had to be increased by raising the tail-gate, in order to maintain uniform flow. The result was an appreciable slowing down of the movement of the bed-load, with a complete cessation of progressive down-stream movement in the case of the finest mixtures. The author's value for critical tractive force, however, either from Equation (10), or from his visual method, locates his "general movement" on the smooth bed; and at higher values than his "critical", there may be almost no bed-load movement at all. This fact is well illustrated in Table 9. The value, d_0, in Fig. 18(c), is the depth at which "general movement" obtains on the smooth bottom, according to the author's method. Fig. 19 shows the values of critical tractive force plotted against the sand characteristics. The experimental value for Sand No. 4 is seen to be 0.0084 lb per sq ft, and the value as taken from the author's curve in Fig. 19 is 0.0070 lb per sq ft. For a slope of 0.0010, the latter tractive force is obtained at a depth of 0.112 ft. Fig. 18(c) shows that at this depth there was practically no movement of Sand No. 4.

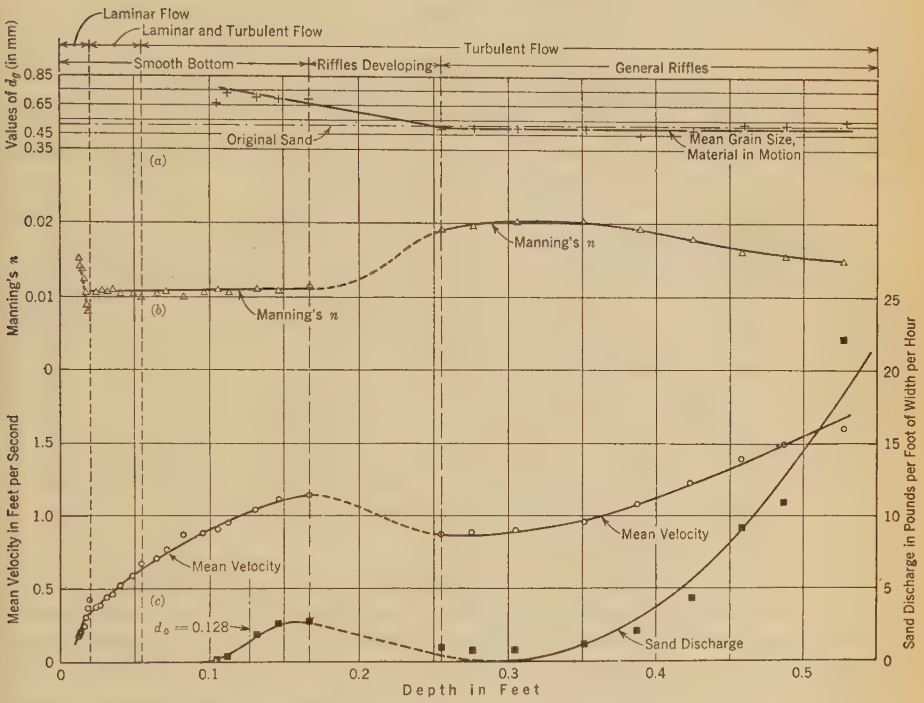


FIG. 18.—STUDY OF SAND NO. 4; SLOPE, 0.001.

In Fig. 19 the authority for the plotted points may be identified, as follows:

Points	Authority
1 to 8	U. S. Waterways Experiment Station
I to III	Hans Kramer
A	F. Schaffernak
B to *F*	A. Schoklitsch
G to *J*	H. Krey
K to *Q*	Prussian Experiment Institute for Hydraulic Engineering and Shipbuilding
R	H. Engels
S to *V*	G. K. Gilbert[2a]

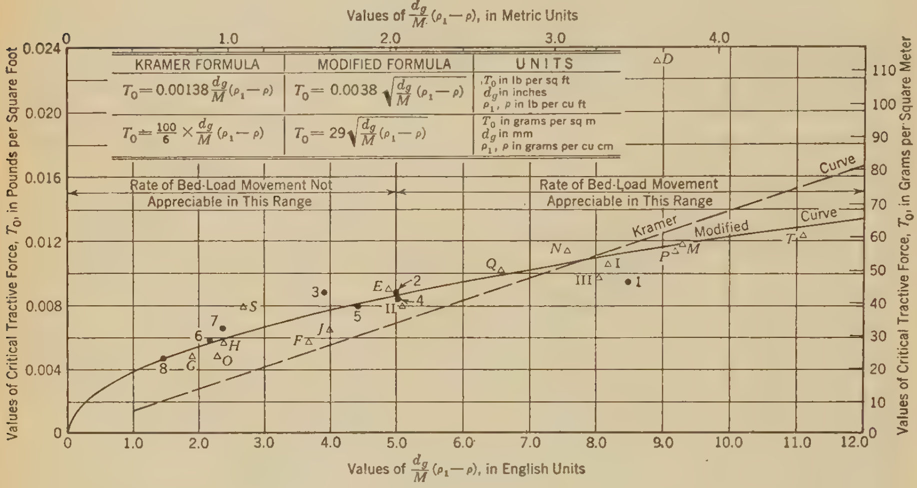


FIG. 19.

The values at the Experiment Station were obtained from an analysis of the data after the completion of the tests, and were selected to conform as closely as possible to Captain Kramer's definition. In their selection, due weight was given the visual classification of the movement noted during the test, the rate of movement, and the roughness value of the bed, and it is believed that they are consistent with the author's data. The following criticisms should be made of Fig. 19:

1.—A new curve, parabolic in shape, has been drawn to replace the straight line drawn by Captain Kramer through his points. The equation for the "modified" curve is (T_o, in pounds per square foot; D_g, in inches; and ρ_1 and ρ, in pounds per cubic foot):

$$T_o = 0.0038\sqrt{\frac{D_g}{M}(\rho_1 - \rho)} \qquad (27)$$

or (T_o, in grams per square meter; D_g, in millimeters; and ρ_1 and ρ, in grams per cubic centimeter),

$$T_o = 29\sqrt{\frac{D_g}{M}(\rho_1 - \rho)} \qquad (28)$$

which corresponds with the author's Equation (10) for the straight line. It must be noted that in Equations (27) and (28) the coefficient before the radical no longer is dimensionless, but has the units, $\frac{M^{\frac{1}{2}}}{L^{\frac{1}{2}}\,T}$ (using the mass-length-time system).

2.—It is believed that the part of the curve to the left of the abscissa value of 5.0 (English units) has no practical value, and should not be used for any practical design work, either in hydraulic models or in regulatory works for rivers. The reason for this limitation is that sand mixtures falling within this range will riffle to abnormal heights at values of tractive force higher than those indicated by the curve, and will cause a retardation, or even a cessation, of the rate of movement at tractive force values greater than the author's "critical". Table 9 illustrates this fact.

TABLE 9.—RETARDATION OF BED-LOAD MOVEMENT AT TRACTIVE FORCES ABOVE THE AUTHOR'S "CRITICAL"

Sand No.	MEAN GRAIN SIZE:		Uniformity modulus, M	Value of $\frac{D_g}{M}(\rho_1 - \rho)$ D_g, in inches, ρ_1, ρ, in pounds per cubic foot	CRITICAL TRACTIVE FORCE, IN POUNDS PER SQUARE FOOT			RATE OF MOVEMENT, IN POUNDS PER FOOT WIDTH PER HOUR (DRY WEIGHT)	
	Millimeters	Inches			Average experimental value	Value from author's curve	Value from modified curve	At critical tractive force	Least at tractive forces above critical
(a) UNITED STATES WATERWAYS EXPERIMENT STATION									
1	0.586	0.0230	0.280	8.50	0.0094	0.0117	0.0112	2.5±	1.5
2	0.541	0.0213	0.439	5.04	0.0088	0.0069	0.0086	1.0±	1.0±
3	0.525	0.0207	0.539	3.98	0.0088	0.0055	0.0076	1.0±	1.5
4	0.506	0.0199	0.406	5.04	0.0084	0.0070	0.0086	1.0±	0.04
5	0.483	0.0190	0.438	4.49	0.0080	0.0062	0.0080	1.0±	0.10
6	0.347	0.0137	0.643	2.20	0.0060	0.0030	0.0057	0.02±	0.0
7	0.310	0.0122	0.525	2.40	0.0066	0.0034	0.0060	0.10±	0.03
8	0.205	0.0081	0.560	1.50	0.0047	0.0021	0.0047	0.04±	0.0
9	4.077	0.1605	0.566	29.20	0.0580	0.0404	0.0205	1.0	1.0
(b) THE AUTHOR									
I	0.705	0.0278	0.358	8.27	0.0106	0.0114	0.0110	......	......
II	0.558	0.0220	0.461	5.06	0.0080	0.0070	0.0086	......	......
III	0.800	0.0315	0.414	8.06	0.0098	0.0112	0.0109	......	......

The experimental values of critical tractive force for Sands Nos. 1 and 2, Table 9, throw some light on the value of the uniformity modulus, *M*, which was proposed by Captain Kramer as a measure of voids ratio. These two sands differ by only 0.045 mm (0.0017 in.) in mean grain size, but their uniformity moduli are 0.280 and 0.439, respectively. In spite of this great difference in *M*-values, the critical tractive forces were nearly equal, being 0.0094 and 0.0088 lb per sq ft, respectively. Although this one fact is not sufficient evidence on which to accept or reject the use of the uniformity modulus, *M*, it does indicate that it might be more reasonable to give it less weight in the equation for critical tractive force.

A preliminary investigation has been made to determine whether the value of *M* is a reliable measure for the voids ratio. Nine mixtures were moulded in the tilting flume, separated from each other by wooden partitions, and allowed to stand under water for some hours under like conditions. Two samples were then taken from each mixture, and the mean grain size, uniformity modulus, and voids ratio for each were determined. Table 10 contains a summary of the results of these analyses, the values for voids ratio being the average of the two values determined.

TABLE 10.—Relation Between Uniformity Modulus (M) and Voids Ratio

Mean grain size, in millimeters	Uniformity modulus, M	Voids ratio
(a) United States Waterways Experiment Station		
0.566	0.30	0.61
0.668	0.33	0.63
0.557	0.42	0.68
0.505	0.42	0.71
0.349	0.43	0.73
0.547	0.44	0.65
0.216	0.53	0.85
3.761	0.54	0.65
0.240	0.58	0.86
(b) The Author		
0.705	0.36	0.62
0.800	0.41	0.67
0.558	0.46	0.63

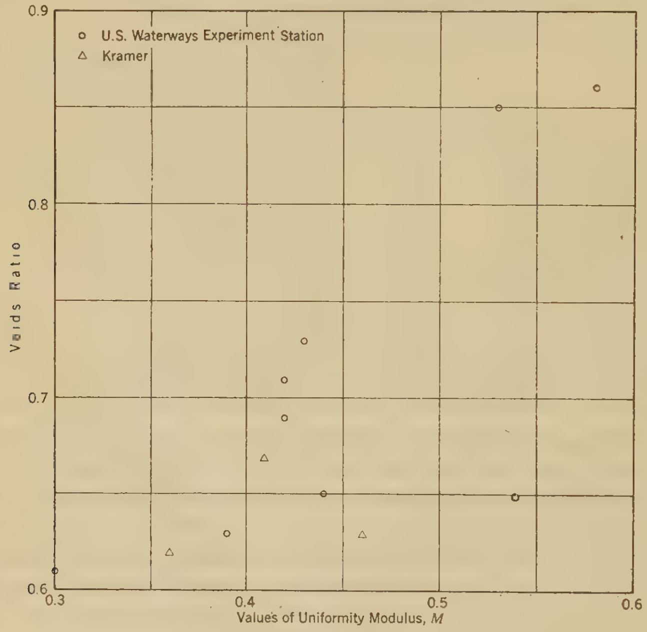


FIG. 20.

Table 10(b) contains values for Captain Kramer's sands, Nos. I, III, and II, respectively. These values were listed by him in his unabridged paper (record manuscript on file in Engineering Societies Library), for loose sand under water, the condition similar to that under which the nine samples were tested at the Waterways Experiment Station.

Plotting the values of M against corresponding values of voids ratio (see Fig. 20) indicates that while there may be a tendency for the one to vary with the other, the points are too scattered to show any consistent relationship.

In conclusion, it must be recognized that the author has given an impetus to the study of bed-load movement. It is gratifying to note that much thought is now being given to this subject.

LORENZ G. STRAUB,[55] ASSOC. M. AM. SOC. C. E. (by letter).—There is so much that is not known about the mechanics of sediment transportation in open channels that the amount known is negligible in comparison. Captain Kramer's experimental investigations and the analysis of the results of his own experiments and those of other investigators are, therefore, to be welcomed by the profession, particularly in the field of experimental design of erodible-bed streams.

It is doubtless mostly the lack of knowledge of the mechanics of bed-load movement (as well as suspended-load movement) which frequently leads engineers of long experience in the art of river regulation to voice the opinion that each river is a law unto itself—that the knowledge gained in the study of the behavior of one river is of little value in predicting the occurrences in another. Not infrequently experienced river engineers compare a river to a human personality whose "character and peculiarities must be understood in order to make him manageable." In truth, however, in the case of the inanimate river at least, the statements are warranted only because of the inadequate knowledge of the invariable, although complex, natural laws governing the occurrences.

In reviewing literature which has appeared during the past few decades on the subject of bed-load movement of rivers, one finds many statements which appear to be contradictory and lead to confusion. More careful scrutiny, however, shows that at least in some instances the apparently contradictory statements are reconcilable. A typical example concerns the transportability of bed load; one group of investigators, including the late G. K. Gilbert, of the U. S. Geological Survey, has analyzed experimental data on the basis of stream velocity. Another group led primarily by the du Boys method of approach has adopted expressions which include tractive force as a primary function in the relation for rate of bed-load movement. It may be shown by the use of suitable open-channel flow formulas as a connecting link that, when correctly formulated, the two types of equations must yield similar results.

At present, there is a definite gap between theoretical assumptions and actual occurrences. From a practical viewpoint this is unimportant provided the formulas theoretically derived give results corresponding to actual occurrences when suitable empirical coefficients are introduced. Thus, the du Boys equation assumes that a series of thin laminæ of sediment moves smoothly over one another over the entire stream bed. As a matter of fact, observation reveals that, except for unusual flow conditions, the sedimentary material moves down stream in riffles which roll over and over in a manner comparable to the movement of sand dunes. Just down stream of the miniature dune there is virtually no motion of sediment particles, such particles being rolled up the dune only to fall motionless at the down-stream or leeward side. This type of sediment movement is far different from that assumed in setting up the du Boys theory. Nevertheless, for sediment with certain size limits (ranging possibly from something less than 1 mm to about 4 or 5 mm in diameter) observations have shown that the equation gives satisfactory results with regard to the quantity of material moved, provided satisfactory empirical coefficients are used. Therefore, for all practical purposes, the use of the equation is justifiable even if the motion occurrences are different from those assumed in setting up the theory.

---

[55] Associate Prof. of Hydraulics, and Head, Hydraulics Div., Univ. of Minnesota, Experimental Eng. Laboratories, Minneapolis, Minn.

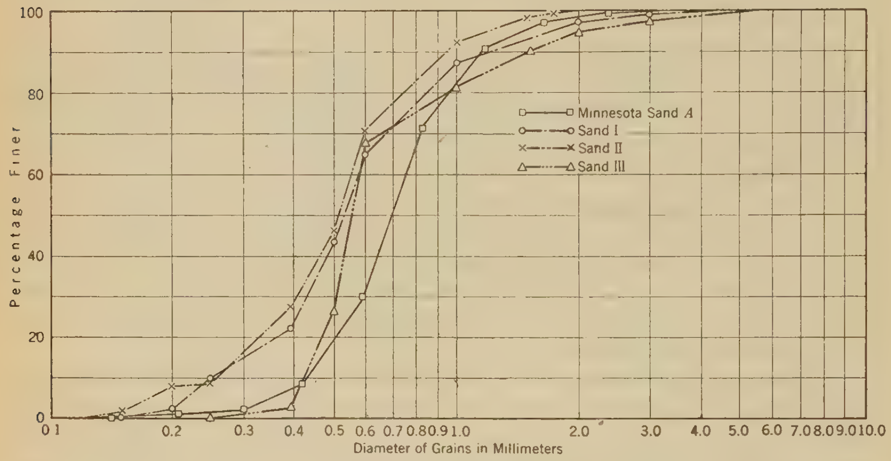


FIG. 21.—MECHANICAL COMPOSITION OF RIVER SEDIMENT USED IN STUDIES OF BED-LOAD MOVEMENT.

The author's Conclusion 1 concerning the "Law of Constant Critical Tractive Force," in which he states that "one and the same bed-load material under the usual moderate slopes invariably gives the same value for the lower critical tractive force," agrees very well with the writer's observations in laboratory experiments on a wide variety of river sands. However, there are a large number of investigators of note who have questioned the applicability of this law, particularly in the case of sediments made up of particles ranging within wide size limits. It will be found that the acceptance of this law makes possible the formulation of equations which will closely define the bed load for wide variations in flow conditions.

In connection with a series of experiments being conducted (1934) at the University of Minnesota, Minneapolis, Minn., careful observations are being made of the nature of riffle formation and movement for different types of sediment. A typical example is here presented; the type of sand, at least as far as average size is concerned, is similar to the types used by the author.

The material is typical river sand collected from the bed of one of the mid-Western streams. The mechanical composition is shown graphically by a semi-logarithmic plotting in Fig. 21, the author's Types I, II, and III, being superimposed for the purpose of comparison. Experiments are performed by maintaining the water-discharge constant and supplying bed-load material at the up-stream end of a channel at a given rate, the rate of feeding being maintained until complete equilibrium of all flow conditions is obtained in the alluvial channel. Fig. 22 shows the results of measurements for a typical set of runs in which the water discharge was maintained at 0.30 cu ft per sec per ft width of channel. Figs. 23, 24, and 25 show photographic records of the riffle formation on the stream bottom for three different flow conditions, and the corresponding water surfaces; the water discharge (0.20 cu ft per sec) is the same for the three cases. The plotted points are the results of individual experimental determinations. Because of the occurrence

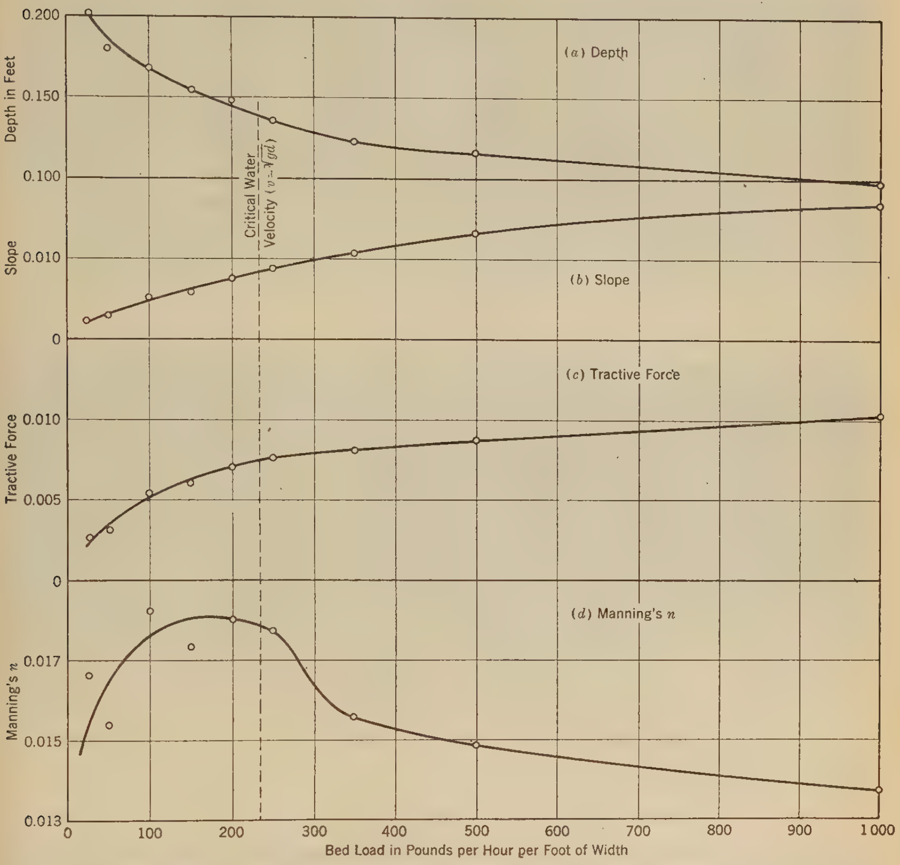
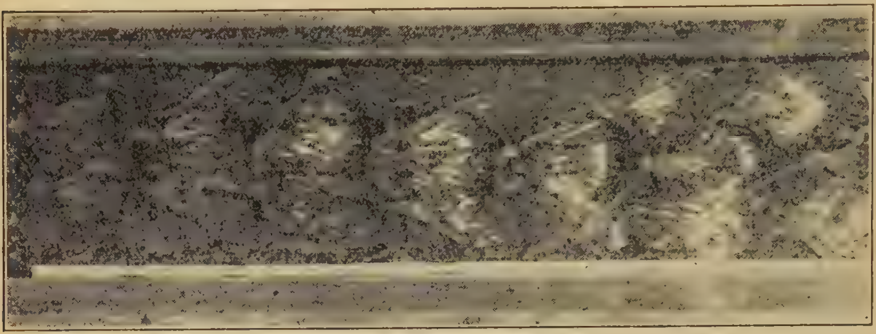
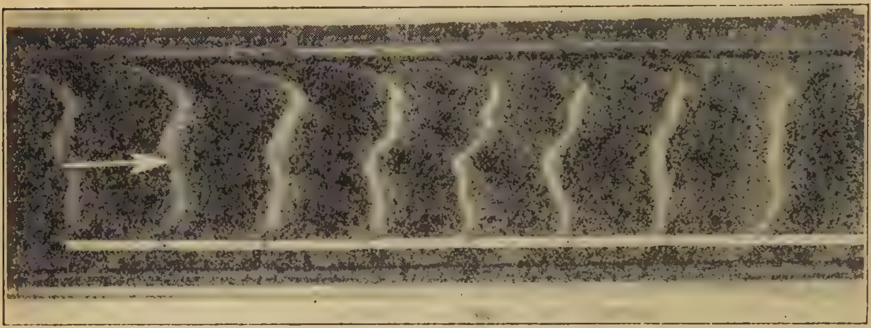


FIG. 22.—RESULTS OF MEASUREMENTS FOR TYPICAL SET OF EXPERIMENTS MADE TO DETERMINE CHARACTERISTICS OF BED-LOAD MOVEMENT. (DISCHARGE IS 0.30 CU FT PER SEC PER FOOT WIDTH OF CHANNEL FOR ALL MEASUREMENTS.)

of undulations in the elevation of the stream bottom some question arises as to the proper choice of depth. In the analysis presented in Fig. 22, the depth is taken as the average; that is, the distance from the water surface to the mean elevation of the bottom; or, roughly, to a point about one-half the height of the riffle.



(*a*) Water Surface.

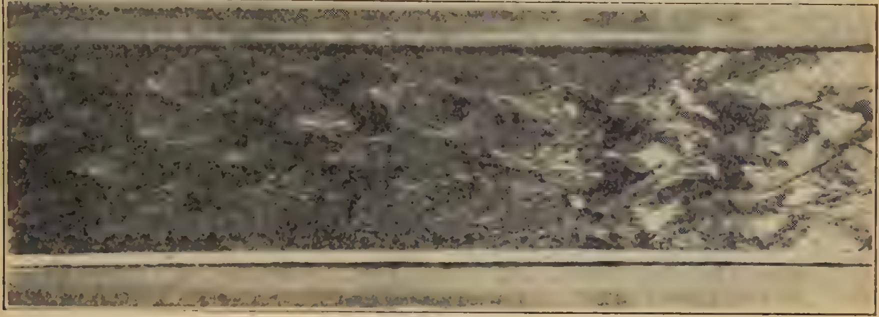


(*b*) Character of Stream Bed.

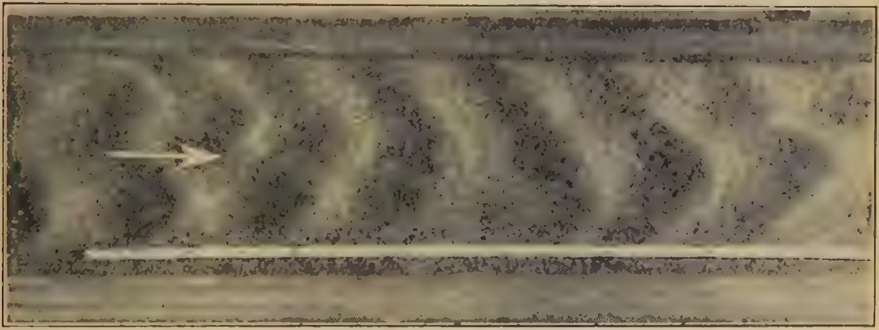
Fig. 23.—Photographic Record for a Channel in Which Velocity of Flow Is Near the Energy Critical. (Bed Load, 300 Lb. per Hr per Foot Breadth of Channel; Mean Height of Riffle, 0.048 Ft; Mean Length of Riffle, 0.53 Ft; and Slope of Gradient, 0.0124 Ft).

It is probable that this arbitrary choice has some influence on the variation in the computed values of *n* in Manning's formula. This seems evident from a comparison of the value of *n* and the mean height of the riffle presented in Fig. 26. Preliminary studies indicate that the depth should probably be taken as the distance from the water surface to a point practically at the elevation of the crest of the riffles, particularly when the riffles are definitely of dune shape, with a flat up-stream slope and a steep down-stream slope; in the case of smooth undulations such as those indicated by the photograph presented in Fig. 24(*b*), the depth is probably more nearly the distance from the water surface to a point somewhat below the crest of the riffle. More studies should be made to determine the correct relation for analytical treatment. In any case, since an arbitrary value is taken for the depth corresponding to the average depth of water, it will be found that the roughness factor, n, in Manning's formula, increases with the relative height of the riffles.

Apparently, the size and character of the riffles are definitely dependent on the mechanical composition of the sedimentary material and the magnitude of the traction force. However, there are a number of other influential criteria. Unquestionably, the so-called kineticity of the flow condition has considerable influence. Experiments seem to show that the height and sharpness of the riffle approach a maximum when the flow in the channel is near the energy critical (where the water velocity is equal to the wave velocity, $v = \sqrt{gd}$), as indicated by Fig. 23(b); for higher velocities the undulations are less rugged, and for still greater velocities relative to the energy critical the bed appears to become smoother until, finally, the riffles disappear entirely, leaving a smooth stream bed, as indicated by Fig. 25(b). Unfortunately, to the present time, the experimental data available do not permit definite conclusions concerning the relative influence of magnitude of traction force and degree of kineticity on the character of the riffle formation. Most experiments that have been performed by various in-



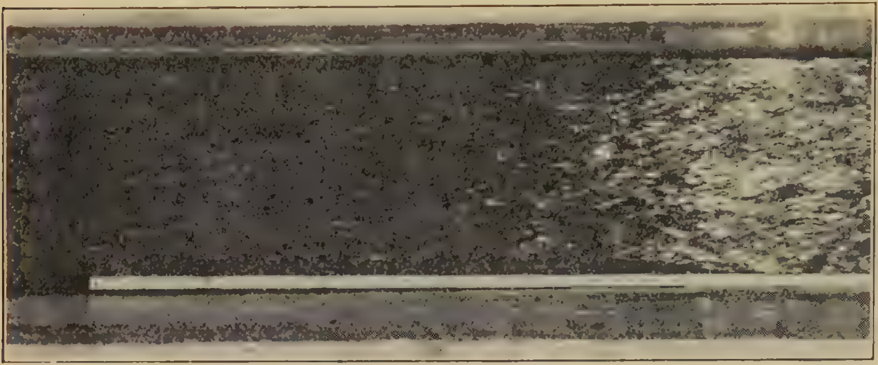
(a) Water Surface.



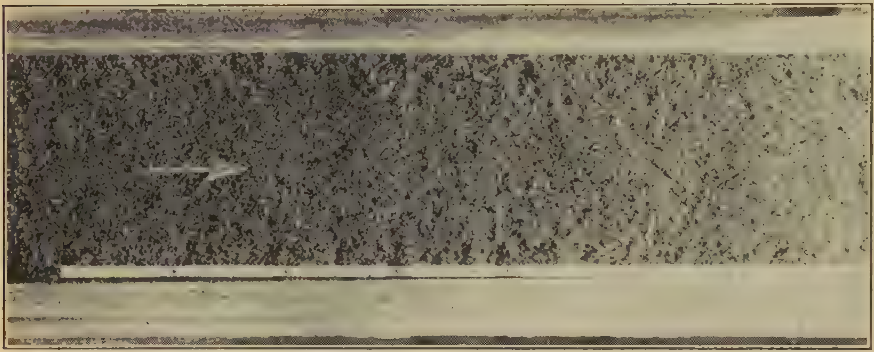
(b) Character of Stream Bed.

Fig. 24.—Photographic Record for a Channel in Which Velocity of Flow Is Considerably Beyond the Energy Critical. (Bed Load, 450 Lb per Hr. per Foot Breadth of Channel; Mean Height of Riffle, 0.047 Ft; Mean Length of Riffle, 0.61 Ft; and Slope of Gradient, 0.0143 Ft.)

vestigators to the present time have either been almost entirely for streaming flow, such as the author's, or for shooting flow, such as Gilbert's experiments, although some of the studies of the latter include flow conditions with the velocity less than that for the energy critical.



(*a*) WATER SURFACE.



(*b*) CHARACTER OF STREAM BED.

FIG. 25.—PHOTOGRAPHIC RECORD FOR A CHANNEL, IN WHICH VELOCITY OF FLOW IS GREATLY BEYOND THE ENERGY CRITICAL. (BED LOAD, 800 LB PER HR. PER FOOT BREADTH OF CHANNEL; NO RIFFLES OBSERVED; AND SLOPE OF GRADIENT, 0.0205 FT.)

The writer's observations indicate that the length of the riffles is much less subject to fluctuation with flow conditions than the height. However, much remains to be done to arrive at conclusions concerning the variation in riffle lengths. There is some indication that the length increases with the water discharge when the sediment load per unit breadth of channel remains constant.

An analysis of river hydraulics problems by means of erodible-bed models will emphasize the necessity of further knowledge concerning the mechanics of bed-load movement. Almost invariably the occurrence of riffles in the model is so greatly out of proportion to those which obtain in the prototype that the results of the studies have grave limitations in their applicability to the prediction of the results to be obtained from projected control measures.

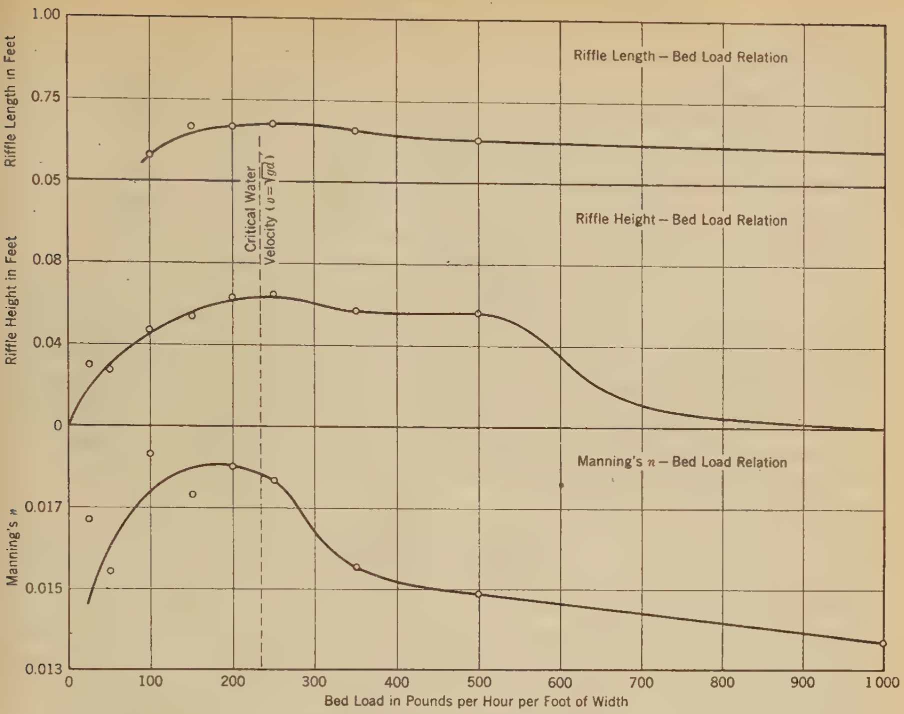


FIG. 26.—COMPARISON OF VALUES OF n IN MANNING'S FORMULA WITH MEAN HEIGHT OF RIFFLE FOR WATER DISCHARGE OF 0.030 CU FT PER SEC PER FOOT WIDTH OF CHANNEL AND VARIATION IN BED LOAD.

HANS KRAMER,[56] M. AM. SOC. C. E. (by letter).—The discussions submitted have proved most gratifying, in that they constitute a fairly complete cross-section of science, representing, as they do, the analytical, the experimental, and the applied branches. A number of the points raised by the discussers were covered in the original German manuscript and in the unabridged translation,[57] but unfortunately could not be included in the published paper because of space limitations.

The validity of the du Boys expression for tractive force was the subject of comment in all the discussions, except that by Mr. Keays, with Messrs. Leighly, Matthes, Tiffany and Bentzel, and Straub favoring its use, and Messrs. Thompson, Tchikoff, Mavis, and O'Brien and Rindlaub questioning its value.

Mr. Tchikoff is the most emphatic in his rejection of the du Boys concept, and lists four limitations which he believes to be so serious as to annul the entire theory. In answer to these four limitations, the following pertinent facts may be stated:

1.—In most streams the assumption of infinite width is reasonable. In the Mississippi River, for instance, with average widths of 3 000 to 6 000 ft and average depths of 10 to 50 ft, the width-depth ratio, for all practical purposes, can be considered infinite.

---

[56] Capt., Corps of Engrs., U. S. Army; Dist. Engr., U. S. Engr. Office, Tucumcari, N. Mex.

[57] On file at Engineering Societies Library, New York, N. Y.

2.—Successful applications of the du Boys expression have been made with materials much larger than 10 mm in grain sizes. Perhaps the most striking example is the experiment performed at the U. S. Waterways Experiment Station with regular tetrahedral blocks 10 in. and 5 in. on a side, in which an excellent verification of the du Boys expression was obtained.[58] Mr. Matthes in his discussion tells of successful experiments with gravel up to 1 in. in longest dimension, in which the du Boys theory was used.

3.—Most alluvial rivers have the "usual moderate slopes" necessary for the validity of the law of constant critical tractive force. The average slope of the Mississippi River in its lower reaches is less than 0.5 ft per mile, or 0.000095, which is several times less than the slopes used in the experiments.

4.—The fact that the type of flow—whether laminar, turbulent, shooting, or streaming—does not affect the theory is hardly a limitation. Rather, it demonstrates that the du Boys theory is sufficiently inclusive to cover all conditions of flow.

It is realized that there are some discrepancies between the theoretical assumptions of the du Boys expression and actual occurrences. As Professor Straub has pointed out, however, since the equation gives satisfactory results when the proper empirical coefficients are introduced, its use is justifiable even though the motion occurrences are somewhat different from those assumed in the theory.

Professor Leighly has analyzed the significance of the M-value, or uniformity modulus, as a measure of the denseness of a sand mixture. He believes, however, that M is given too much weight in the ultimate equation (Equation (10)), which expresses critical tractive force in terms of the characteristics of the gradation curve, and suggests that "the derivation of these terms must begin from the variations seen in the results of experiment when mean diameter is held constant or is eliminated by computation * * *." In this connection, it was pointed out in both the German original and in the unabridged translation that the scarcity of comparable data at the time these experiments were made precluded the possibility of this procedure, and made it necessary to make an assumption on the basis of existing data as to the weight which should be given to M. The plotting of the data led to the tentative conclusion that the first power of M provided a reasonable weight for this factor. Further studies of critical tractive force now (1935) being completed at various hydraulic laboratories should provide information from which a refinement of Equation (10), including a more nearly exact evaluation of the effect of voids ratio, can be made. At the present writing conclusive data are not available.

It is to be noted that Professor Leighly's analyses of several curves in Fig. 13 fail to take into account the classification of the comparability of the data given in Column (2) of Table 3. In particular, the data relating to Mixtures J and L are given weights, 3 and 4, respectively, in the classification; Mixtures *B* to *H* and *S* to *V*, likewise, are classified as of secondary or less importance, so far as their comparability with the basic experiments is concerned.

---

[58] "Investigations of Certain Proposed Methods of Bank and Embankment Protection." *Paper 12*, U. S. Waterways Experiment Station, July, 1933.

Lieut. Thompson believes that the rate of bed-load movement, rather than the critical tractive force, is the important factor to be considered in the regulation of alluvial streams. In contrast with his opinion is that expressed by Mr. Matthes, who emphasizes the importance of knowing the conditions under which movement commences and ceases. Lieut. Thompson also states that the relations between the critical tractive forces of given sands are not the same as those between their rates of movement, basing his statement on data secured by Lieut. K. D. Nichols at the U. S. Waterways Experiment Station.[30] The experiments cited by Lieut. Nichols are of questionable value because they were made with fairly fine sand mixtures, at relatively low tractive forces, where the influence of riffle formations tended to retard the movement of the finer mixtures. Later information secured at the Experiment Station[54] indicates (1) that at higher tractive forces, after the riffles have been smoothed out, the relations between the rates of movement are similar to those between the critical tractive forces; and (2) that the rate of movement can be very definitely related to the du Boys expression for tractive force.

Mr. Matthes' remarks are especially noteworthy, since they reflect the views of a practical engineer of wide experience in the field of river regulation. It must be kept in mind constantly that even though experimenters and scientists have not yet been able to discover all the laws governing "geschiebe" movement, practical engineers engaged in controlling navigable streams must make use of all the available information that is susceptible of direct application. Mr. Matthes shows that the du Boys expression provides a workable basis for the application of existent data to the problem of river regulation.

The need for a closer correlation of hydraulic model results with the corresponding natural phenomena is emphasized by Mr. Keays. The paper by Herbert D. Vogel, M. Am. Soc. C. E., entitled, "Hydraulic Laboratory Results and Their Verification in Nature,"[59] contains several definite comparisons of this type. As an extension of the information presented by Lieut. Vogel, the U. S. Waterways Experiment Station is actively collecting verification data from all the principal hydraulic laboratories in the United States and in Europe. These data, when assembled and published, should be of great value both to practical hydraulic engineers and to experimenters in the laboratories, in judging the reliability of model studies.

The question of Cartesian *versus* semi-logarithmic or logarithmic plotting of gradation curves is brought up by Professor Mavis. The semi-logarithmic method is undoubtedly preferable for clay and silt mixtures. In these studies, however, the Cartesian method had very definite advantages, where a planimeter was used to measure the areas involved in the calculations of A, A_A, A_B, and M. The use of semi-logarithmic paper would have presented an unnecessary complication. Further discussion of the relative merits of the two methods is contained in the unabridged translation of the original German manuscript.

---

[59] *Proceedings*, Am. Soc. C. E., January, 1935, pp. 57–73.

The three assumptions which Professor Mavis states were tacitly made are believed to be substantially true. Inasmuch as the experiments were performed with uniform flow in a straight flume, cross-currents were negligible, and the velocity distributions were virtually identical at all cross-sections. Furthermore, since the sand mixtures were from the same source, the shape and specific gravity of the particles were the same for all three mixtures. It is believed to be a fair assumption that the form-factor may be disregarded in considering ordinary quartz sands.

Curves for Chezy's C, similar to those presented by Professor Mavis for Manning's n in Fig. 16, were plotted in connection with the initial studies, but were not published. The computed C-values were presented in Table 1, Column (11). Professor Mavis' conversion of these values affords a valuable extension of the basic data.

In addition to his discussion of the validity of the du Boys theory, Mr. Tchikoff attacks the problem of "geschiebe" movement from the standpoint of the kinetic flow factor. He does not make clear his definition of silt, however, nor does he explain why he believes it necessary that the "silt" in the model should be the same as the material in Nature. As Lieut. Vogel has pointed out,[60] a model should usually be designed to fulfill but one mission. If bed-load movement is the controlling factor, the bed-load material in the model should be selected to give similarity of action. This similarity is dependent on the selection of the bed material and on the control of the depths and slopes in the model.

Mr. Tchikoff apparently assumes that a stream which is not scouring is inactive as far as bed-load movement is concerned. It is well known, however, that even in channel beds that maintain the same configuration from year to year there is considerable bed-load movement. The material carried out of any given reach is continuously being replaced by material from up stream, with a consequent stability of bed configuration.

It is believed that Mr. Tchikoff has presented an erroneous analysis of the process of bank scouring. He states that the upper parts of the bank are washed out and settle to the bottom. Actually, the upper bank caving is secondary, and usually results from the undermining of the under-burden by the higher tractive forces at the bottom.

An evaluation of tractive force based on the distribution of velocities through the cross-section is suggested by Professor O'Brien and Lieut. Rindlaub. This method is probably sound theoretically, but it entails practical difficulties which, thus far, have precluded its use. The accurate measurement of velocities near a movable stream bed or model bed is almost impossible. Any instrument that is placed close enough to the bed to measure the bottom velocities will induce local scouring, which, in turn, causes eddies and rollers which nullify the reliability of the measurement.

---

[60] See p. 183.

It should be noted, in connection with the statement by Professor O'Brien and Lieut. Rindlaub that side-wall friction has an appreciable resistance to flow, that the walls of the flume used in the experiments were roughened with grains of sand, sprinkled on a coating of wet oil paint to which the grains adhered after the paint dried. The resulting wall roughness was about equal to that of the bed.

Lieut. Vogel's discussion, although general in nature, is significant in view of his extensive experience as Director of the U. S. Waterways Experiment Station. His remarks serve to complement the other discussions, submitted by scientists and by practical river engineers.

Most of the criticisms voiced by Messrs. Tiffany and Bentzel arise from the fact that their experiments were made with sands in a lower range of grain sizes than that covered by the basic mixtures. An examination of their curve, Fig. 19, reveals the fact that their suggested lower limit of usefulness for the critical tractive force curve, at an abscissa value of 5.0 (English units), almost exactly coincides with the plotted value of Sand II, the smallest of the three sands tested. Moreover, all the eight highest rated points in Comparison Class (1) (see, Points I, II, III, *A, M, N, P,* and *Q,* in Fig. 14), which formed the principal basis for the critical tractive force curve, fall to the right of their suggested lower limit. In the range of fine mixtures the lower limitation advocated by Messrs. Tiffany and Bentzel is undoubtedly indicated. Such mixtures, however, are inherently unsuited for use in ordinary fluvial models because of their excessive riffling tendency.

The investigations of voids ratio made by Messrs. Tiffany and Bentzel demonstrate the fact that a refinement in the M-value is necessary before a general formula for critical tractive force can be developed. It is believed, however, that the basis for this refinement lies in the gradation curve of the sand mixture, and that a definite relationship between the gradation curve and the voids ratio can be discovered. With sufficient experimental data, the mathematical method suggested in the original paper, under the heading, "Practical Derivation of a New Formula: 1.—Procedure,"[61] should reveal this relationship. A more complete explanation of this procedure is contained in the unabridged translation.[57]

Professor Straub has presented some valuable supplementary information, covering a range of tractive force higher than that covered by the basic experiments. It is interesting to note that he has adopted the experimental technique used by Gilbert,[2a] wherein the discharge and rate of sand feeding are held constant, and the experiment is continued until an equilibrium of slope and depth of flow are obtained. Most experimenters, having found this equilibrium difficult to attain and difficult to recognize when it is reached, have preferred to hold the slope and discharge constant, and to adjust the rate of sand feeding to the rate at which the material is being caught in the sand-trap.

Although the knowledge of "geschiebe" movement is still inadequate, extensive research programs now (1935) being undertaken at the U. S. Waterways Experiment Station; at the Prussian Experiment Institute for Hydraulic Engineering and Shipbuilding, by Capt. Hugh J. Casey, Corps of Engineers, U. S. Army, M. Am. Soc. C. E.; at the University of Minnesota by Professor Straub; and at numerous other hydraulic laboratories, should result in definite progress in this science.

---

[61] See p. 830.

Acknowledgment.—These studies have been made more valuable because of the writer's privilege of maintaining a close personal contact with the staff of the U. S. Waterways Experiment Station, where investigations of a similar nature have been carried forward. Acknowledgment is due to Lieut. H. D. Vogel, Corps of Engineers, U. S. Army, Assoc. M. Am. Soc. C. E., former Director of the Station, to Lieut. F. H. Falkner, Corps of Engineers, U. S. Army, Jun. Am. Soc. C. E., the present Director, and to Joseph B. Tiffany, Jr., Jun. Am. Soc. C. E., Junior Engineer at the Station, who is in charge of "geschiebe" experiments there, and who has aided in the preparation of both the original paper and the closing discussion.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 1910

THE RESERVOIR AS A FLOOD-CONTROL STRUCTURE

By George R. Clemens,[1] Assoc. M. Am. Soc. C. E.

With Discussion by Messrs. C. Maxwell Stanley, F. Knapp, Charles S. Bennett, Gerard H. Matthes, and George R. Clemens.

Synopsis

This paper presents in an abbreviated form an analysis of the methods of obtaining flood control by means of reservoirs. It begins with a general definition of the flood-control problem and a classification of all possible methods of control. The detailed analysis deals only with the reservoir solution of the problem. For the purpose of bringing to the mind of the reader the various facts that are necessary to consider, a rough outline of the general procedure followed in approaching a specific flood-control problem is given. This may appear elementary to the experienced engineer, but should serve to catalog the various items that must be considered.

The design of a reservoir system is analyzed by discussing: (*a*) The method of operation to be selected; (*b*) testing one method for a specific reservoir; (*c*) transmission of reservoir reduction to area to be protected, with a detailed analysis of methods of determining valley storage effect; and (*d*) typical examples of three other methods of storage operation.

The operation of the reservoir for various other purposes, such as water power, irrigation, navigation, and water supply combined with flood control is next considered. This is followed by a discussion of the place of the reservoir in a system with other structures.

In conclusion, there is a summary of the considerations affecting reservoir control. Although most of the general principles are well known to engineers it is believed the paper serves a useful purpose in correlating and classifying methods and in presenting a number of new angles in the practical problem of determining the possible operation of a reservoir system.

Note.—Published in May, 1934, *Proceedings*.

[1] Senior Engr., Mississippi River Comm., Vicksburg, Miss.

INTRODUCTION

When rainfall occurs that provides more run-off than can be carried within the normal channels of existing streams a flood results. The excess water overflows the valley lands, invades developed areas, and causes destruction of property or, in extreme cases, loss of human life. There is no known method of regulating the rainfall itself. Nature alone controls the cycle from sea to sky to earth. Man's efforts are confined to attempts at guiding the water on that part of its course from earth back to the sea. The regulation of the waters that would cause floods presents the flood-control problem.

Many structures are built to control water: Waterways of proper depth and width provide arteries of transportation; water controlled and discharged through water-wheels provides power; water may be caught in storage basins and distributed for irrigation or water supply; and water that is an actual or potential source of damage or danger to property or to human life may be controlled to prevent floods. When this control of water involves improvement or reclamation of property not damaged in its present condition, the control is termed "reclamation," and when it involves the prevention of flood damage this is termed "flood control."

Historical.—The common law holds that flood water is a common enemy and that any individual may protect himself against it. From the beginning of recorded history this principle has been followed and a wide range of structures and methods has been developed for combating the "common enemy." Every country in the world has some flood problems. The Po River of Italy, the Seine of France, the Rhine of Germany, the Thames of England, the Yellow of China, and many others have absorbed the efforts of countless engineers in the struggle for their control. In the United States the most notable struggle has been that with the Mississippi River. As early as 1717 the settlers at New Orleans, La., began to protect themselves against overflow. The work has grown in magnitude until at the end of 1933 the Federal Government had spent $351 372 000 in an effort to control this great river. Many smaller but still important projects in this country have included the Sacramento River of California, the Arkansas River of Colorado, the Colorado River of Arizona and Nevada, and the Miami River of Ohio.

The individual farmer may cut a ditch to divert the flood water of a small creek from his field. He may build a small levee to prevent this flood water from invading his fields or buildings in the flats. However, when the area to be protected becomes of considerable size it has been the usual experience that the works of protection are of such magnitude as to be entirely beyond the means of the individual to finance or to construct. Thus, individuals have grouped themselves together in districts. The rights of the individual have become merged with those of the group, and works have been developed which provide the greatest benefit to the majority of the group. Different groups frequently have conflicting interests and still larger units are formed in an attempt to arrive at a solution which will be of greatest benefit to the greatest number. This grouping has grown to State-wide control in some cases and, in the case of the Mississippi River, to control by the Federal Government.

The most comprehensive flood-control investigations in recent years have been those conducted by the United States War Department under the Corps of Engineers, United States Army. These were largely an outgrowth of the great Mississippi River flood of 1927. For several years prior to that flood a comprehensive study of water resources was being planned by the Engineer Corps. This study was to cover navigation, flood control, water power, and irrigation. The 1927 Mississippi River flood emphasized the importance of these studies as relating to flood control and was largely responsible for the fact that the money for these studies was practically doubled and work undertaken on a much larger and more comprehensive scale than previously anticipated. Many of the studies have been completed and a wealth of information has been collected for practically all the important streams of the United States. General plans have been formulated for most streams and any agency confronted with the development of a flood-control plan should find itself with a large part of the groundwork completed and the necessary basic information available to proceed with the detailed development of any specific project, or the co-ordination of any series of projects.

Possible Methods of Flood Control.—Floods are rather common. In one form or another they are experienced by practically every individual and community. Consequently, the methods of protection against a flood causing damage are many and varied. The following general classification of methods is offered:

(*A*) Works to retain or regulate flood waters above the area where damage is caused: (1) Storage reservoirs, with controlled outlets operated in accordance with a pre-determined plan; and (2) retarding reservoirs, with uncontrolled operation of outlets designed so that their maximum flow will not exceed the channel capacity.

(*B*) Works to provide secondary channels through affected areas: (1) Improvement of secondary existing channels; (2) constructing side channels; (3) overbank diversions; and (4) outlets combining overbank and channel diversions.

(*C*) Works to increase the capacity of existing channels through affected areas: (1) Levee systems; (2) cut-offs; and (3) channel deepening and improvement.

This paper will be confined to an analysis of Group (*A*). Naturally, combinations of Groups (*A*), (*B*), and (*C*) are often feasible and desirable. This analysis will deal with the function of the reservoir—its limitations and advantages—and will discuss its place with regard to the whole, and the useful functions it may be expected to perform.

PRELIMINARY INVESTIGATION REQUIRED

The engineer is ordinarily called upon to design a flood-protection project after a disastrous flood has occurred. In some few cases potential flood damage is anticipated, and a project is designed and constructed prior to the occurrence of what might have been a major damaging flood. In any case the value of a project is the sum of money that those protected from flood damage will pay as representing the benefit occurring by reason of insured protection from future damage. To determine this value the area subject to overflow must be blocked out and the existing and probable future concentration of all values in various units of it must be determined. Study should be made of the manner in which floods have caused damage in the past; economic surveys should be made of the amount of such damage; and estimates should be prepared of the possible damage that might be caused by future floods. By considering all these factors an estimate may be reached of the probable value of future flood protection which could be used as a basis for assessing the cost against those benefited.

When a reservoir solution (Group (*A*)) is to be considered many more data concerning flood volumes must be obtained than for other plans. The reservoirs either store the flood water during the danger period (Group (*A*–1)), or so retard the flow during this period that it does not exceed the capacity of the existing channels (Group (*A*–2)). Both studies require the collection of many hydrological and hydraulic data.

OUTLINE OF PRELIMINARY INVESTIGATIONS FOR A TYPICAL STREAM

An outline of the data required and collected[2] for a consideration of the flood-control problems of a part of the Ouachita River, in Arkansas and Louisiana, in so far as they pertain to reservoir analysis, will best illustrate the procedure in approaching such a project. Fig. 1 is a map of the Ouachita River Basin. By field investigation it was found that the valley lands from Malvern, Ark., to Camden, Ark., have been damaged by floods. To determine the sum that might be spent economically for a flood-protection project an economic survey was made of past flood damage in this danger zone and an estimate was prepared of the value of the elimination of this damage to the centers of population and the farm lands in the valley.

Boundaries of the area to be protected having been located, the first problem of the investigation was to determine the capacity of the existing flood channel. As no field survey was available, a modified hydrographic survey was made which included: (*a*) The determination of river sections at critical points; (*b*) the determination of valley cross-sections at controlling points; (*c*) the low-water river-slope profile; (*d*) location of high-water marks, and determination of high-water profile; (*e*) elevation of controlling top banks; and (*f*) location and size of tributary streams.

Gauges were established and read at critical points along the main stream and on the principal tributaries. It was learned that old gauge records were available for a considerable period for Hot Springs, Malvern, Arkadelphia, and Camden, Ark. These records were collected, and all gauges were referenced to a common datum plane. Discharge measurements were made at each of these key gauge stations and the relation of stage to discharge was determined. In most cases this was plotted as a single curve, as shown in Fig. 2. At Camden, however, it was necessary to develop curves to correct for the variations of local slope as in Fig. 3. To use this type of curve successfully,

---

[2] Rept. on Ouachita River and Tributaries, Arkansas and Louisiana, submitted to Congress through official channels.

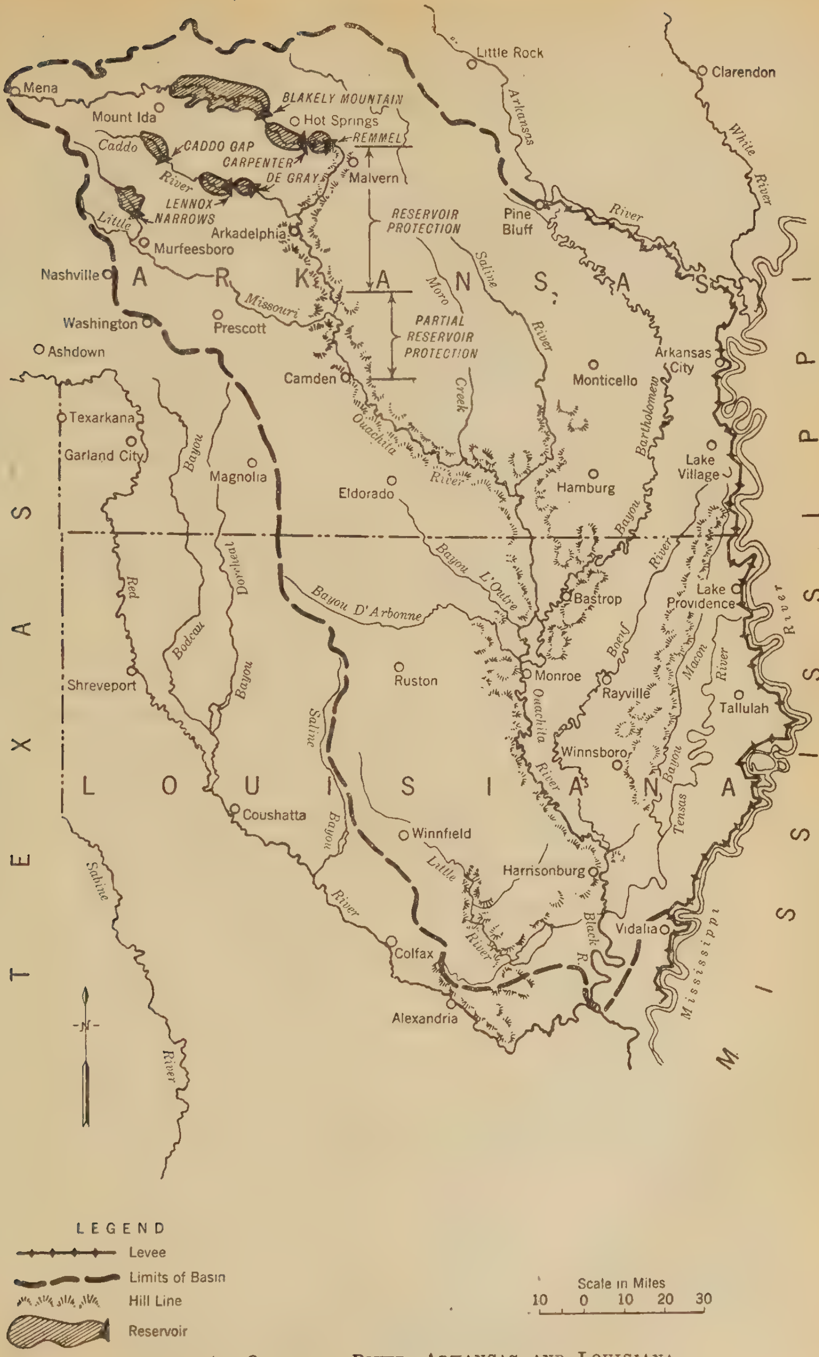

Mena
Mount Ida
Caddo
CADDO GAP
BLAKELY MOUNTAIN
Hot Springs
REMMEL
CARPENTER
DE GRAY
Malvern
River
LENNOX
NARROWS
Little
Arkadelphia
Murfeesboro
Nashville
Little Rock
Arkansas
Clarendon
White
River
Pine Bluff
River
RESERVOIR PROTECTION
PARTIAL RESERVOIR PROTECTION
A R K A N S A S
Missouri
Prescott
Washington
Ashdown
Moro
Saline
River
Camden
Creek
Monticello
Arkansas City
Bartholomew
Lake Village
Texarkana
Garland City
Bayou
Magnolia
Eldorado
Ouachita
River
Hamburg
Bayou
River
Bayou L'Outre
Dorcheat
Red
Bayou D'Arbonne
Bastrop
Lake Providence
Boeuf
Macon
River
River
Bodcau
Bayou
Shreveport
Ruston
Monroe
Rayville
Tallulah
Saline
Ouachita
River
Winnsboro
Bayou
Tensas
L O U I S I A N A
Coushatta
Bayou
Winnfield
Harrisonburg
Little
River
Black R.
Vidalia
Mississippi
Colfax
Alexandria
Sabine
River
T E X A S
M I S S I S S I P P I
N
LEGEND
Levee
Limits of Basin
Hill Line
Reservoir
Scale in Miles
10 0 10 20 30

FIG. 1.—OUACHITA RIVER, ARKANSAS AND LOUISIANA

accurate simultaneous readings at both gauges used in the slope measurements are required. The curve used depends on the slope, which is defined as the fall, in feet, from the U. S. Engineer Office gauge to the U. S. Weather Bureau gauge, both at Camden. All the old discharge records were collected and studied. Both daily rates of discharge and total volume in flood peaks were determined where possible. Gauge records were arranged so as to show the extent of both available gauge and discharge records (see Table 1).

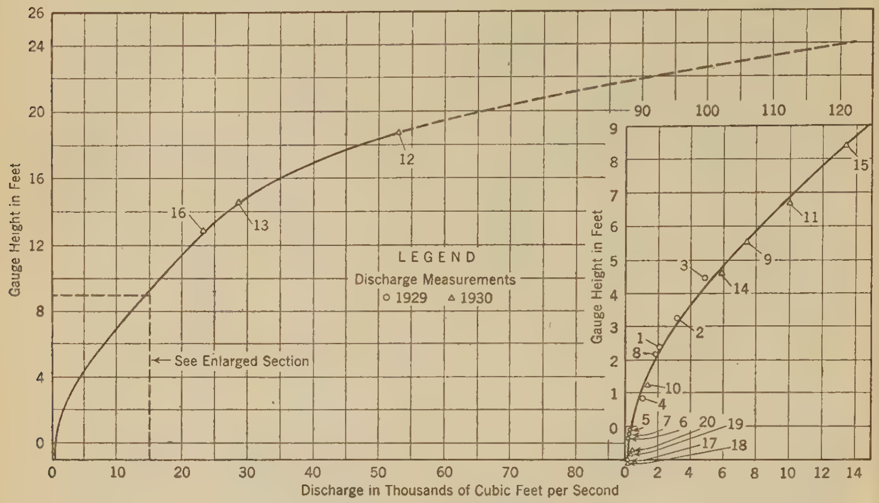


FIG. 2.—RATING CURVE, OUACHITA RIVER, AT ARKADELPHIA, ARKANSAS, 1929–1930.

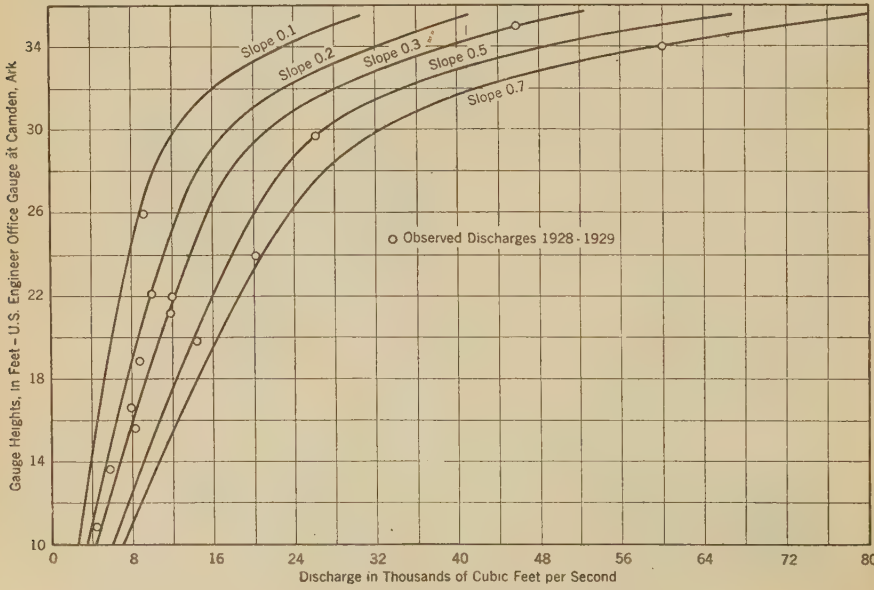
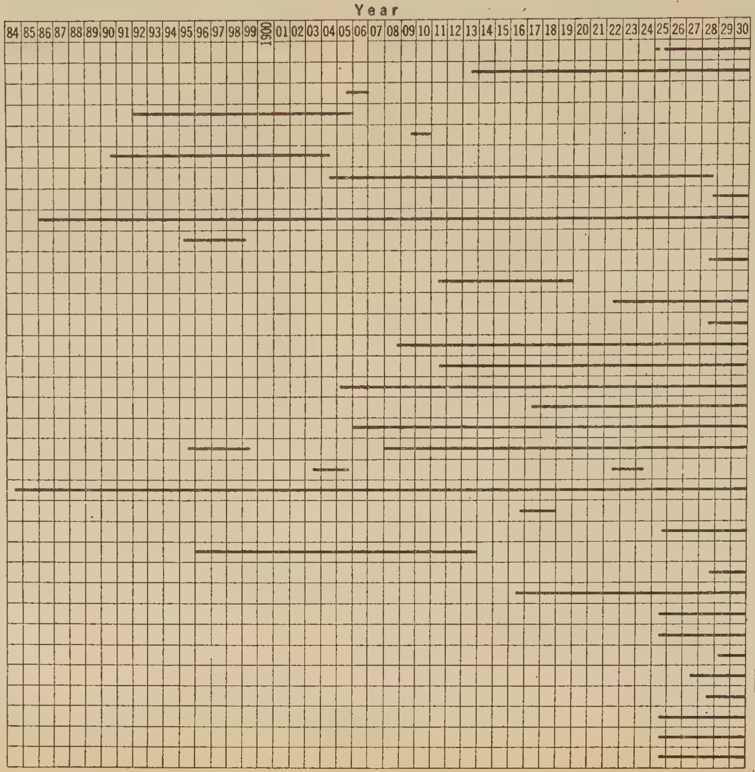


FIG. 3.—RATING CURVES, OUACHITA RIVER, FOR CAMDEN, ARKANSAS

TABLE 1.—GAUGE RECORDS, OUACHITA RIVER AND TRIBUTARIES

Item No.	Gauge (1)	River (2)	Year 	Maximum Record Date (3)	Maximum Record Feet (4)	Minimum Record Date (5)	Minimum Record Feet (6)
1	Acme, La.	Black		1927	....	9–18–25	6.9
2	Arkadelphia, Ark.	Ouachita		4–21–27	23.9	9–1–25	−0.7
3	Arkadelphia, Ark.	Ouachita		1–23–06	19.7	8–13–05	1.3
4	Black River Station, La.	Black		4–20–03	52.3	10–28–97	0.8
5	Blanco Springs, Ark.	Ouachita		4–4–10	1.80	12–6–10	1.0
6	Camden, Ark.	Ouachita		3–23–94	43.25	9–21–04	1.95
7	Camden, Ark.	Ouachita		1–6–07	42.9	8–17–24	1.1
8	Camden, Ark.	Ouachita		5–21–30	41.05	10–5–30	3.2
9	Camden, Ark.	Ouachita		4–25–27	41.0	8–5–29	3.3
10	Champagnolle, Ark.	Ouachita		3–25–97	33.3	9–25–96	0.0
11	Cold Spring Bar, Ark.	Ouachita		5–17–29	23.8	9–4–29	−0.08
12	Fletchers Landing, Ark.	Ouachita		4–16–12	82.9	12–10–12	47.4
13	Hot Springs, Ark.	Ouachita		5–16–23	43.9	9–30–28	4.5
14	Jonesville, La.	Black		6–15–29	52.9	8–28–30	6.5
15	Lock and Dam No. 2	Ouachita		5–18–27	67.2	11–24–22	7.6
16	Lock and Dam No. 3	Ouachita		5–15–27	57.0	11–22–14	1.1
17	Lock and Dam No. 4	Ouachita		5–4–27	48.9	11–21–16	−0.4
18	Lock and Dam No. 5	Ouachita		5–3–27	45.8	8–23–18	0.0
19	Lock and Dam No. 6	Ouachita		4–30–27	43.0	8–5–14	−2.2
20	Lock and Dam No. 8	Ouachita		4–27–27	38.9	12–6–12	0.0
21	Malvern, Ark.	Ouachita		5–9–03	20.2	12–4–04	0.1
22	Monroe, La.	Ouachita		5–4–27	48.7	10–1–97	0.0
23	Newport Landing, Ark.	Ouachita		......	.....	....	.....
24	Remmeldam, Ark.	Ouachita		1927	.....	....	.....
25	Riverton, La.	Ouachita		3–29–03	50.3	9–25–96	0.9
26	Beekman, La.	BayouBartholomew		5–21–30	23.6	10–7–29	1.2
27	Clayton, La.	Tensas		5–17–27	67.3	8–24–25	14.8
28	Delhi, La.	Bayou Macon		5–10–27	34.6	11–25–28	0.0
29	Farmerville, La.	Bayou D'Arbonne		5–19–27	34.5	9–6–25	0.7
30	Farmerville, La.	Bayou D'Arbonne		5–21–30	36.1	7–27–30	10.6
31	Murfreesboro, Ark.	Little Missouri		5–3–30	14.0	7–23–30	1.7
32	Warren, Ark.	Saline		5–19–30	25.9	8–29–29	1.8
33	Warren, Ark.	Saline		4–25–27	26.5	9–23–25	0.7
34	Girard, La.	Boeuf		5–12–30	29.4	6–10–26	0.7
35	Wilmot, Ark.	BayouBartholomew		5–10–27	25.9	9–27–29	1.1

Supplementary to Table 1, the following data are to be noted: Item No. 9, Column (4), a maximum reading of 46.0 ft was estimated on May 12, 1882; Item No. 12, Columns (4) and (6), elevations are referred to mean Gulf level; Item No. 23, Columns (3) to (6), the river overtopped this gauge in March, 1917, and April, 1918; it reached a point below the bottom of the gauge in December, 1916, and June, 1918; Item No. 24, Columns (4) to (6), an automatic gauge is installed at this point, due to the intermittent discharge caused by a power-house; Item No. 26, Column (4), on April 17, 1927, the occurrence of an estimated maximum reading of 26.75 ft is recorded; and, Item No. 29, Columns (3) and (4), the maximum gauge reading and the date are approximate.

In addition to the gauge and discharge records a study was made of all available rainfall records. For convenience in studying these records, a map showing the location of all hydrological stations was prepared. In many cases where run-off records were not available for all points for the greatest storms they were built up from these rainfall data.

By studying the rainfall and run-off records it was apparent that two or three great floods were the most severe that had been experienced. These were selected as test storms to be used in checking reservoir design. Isohyetal charts were quite valuable in studying these great storms. For the greatest storms (as determined by the isohyetal charts) rainfall run-off charts (see Fig. 4 and Table 2) enabled a study to be made of the total flood volume contributed by each tributary.

To use these greatest floods of record as design floods for a reservoir system it was further necessary to develop hydrographs of daily flow at a number of key points in the danger zone to be protected and at the sites of reservoirs selected for investigation. These hydrographs were determined from gauge readings and discharge data or by computations from them. In some cases hydrographs were developed from the rainfall run-off relations and in others by using the proportional drainage areas. For example, in the latter case, the discharge hydrograph for a dam site in Blakely Mountain is computed from the known hydrograph values for Hot Springs, Ark., by reducing readings in proportion to the relative drainage areas concerned. The area contributing to the gauge at Hot Springs being 1 420 sq miles and the area contributing to the gauge at the dam site, 1 190 sq miles, the readings are reduced in the ratio of 1 190 : 1 420, or 0.837.

If a hydrograph is plotted using a logarithmic scale for the discharge ordinates, the proportional drainage-area method becomes much simplified. Multiplication is then accomplished by addition and hydrographs of proportional areas can be determined from known hydrographs by marking off uniform distances above or below the known curve.

A field reconnaissance was made of the entire drainage basin to determine all possible reservoir sites. All available maps of sites that were considered promising were collected and examined. From the general data obtained by this reconnaissance and from existing maps a tentative system (see Fig. 1) was calculated to control the waters to the extent shown to be necessary by the run-off studies.

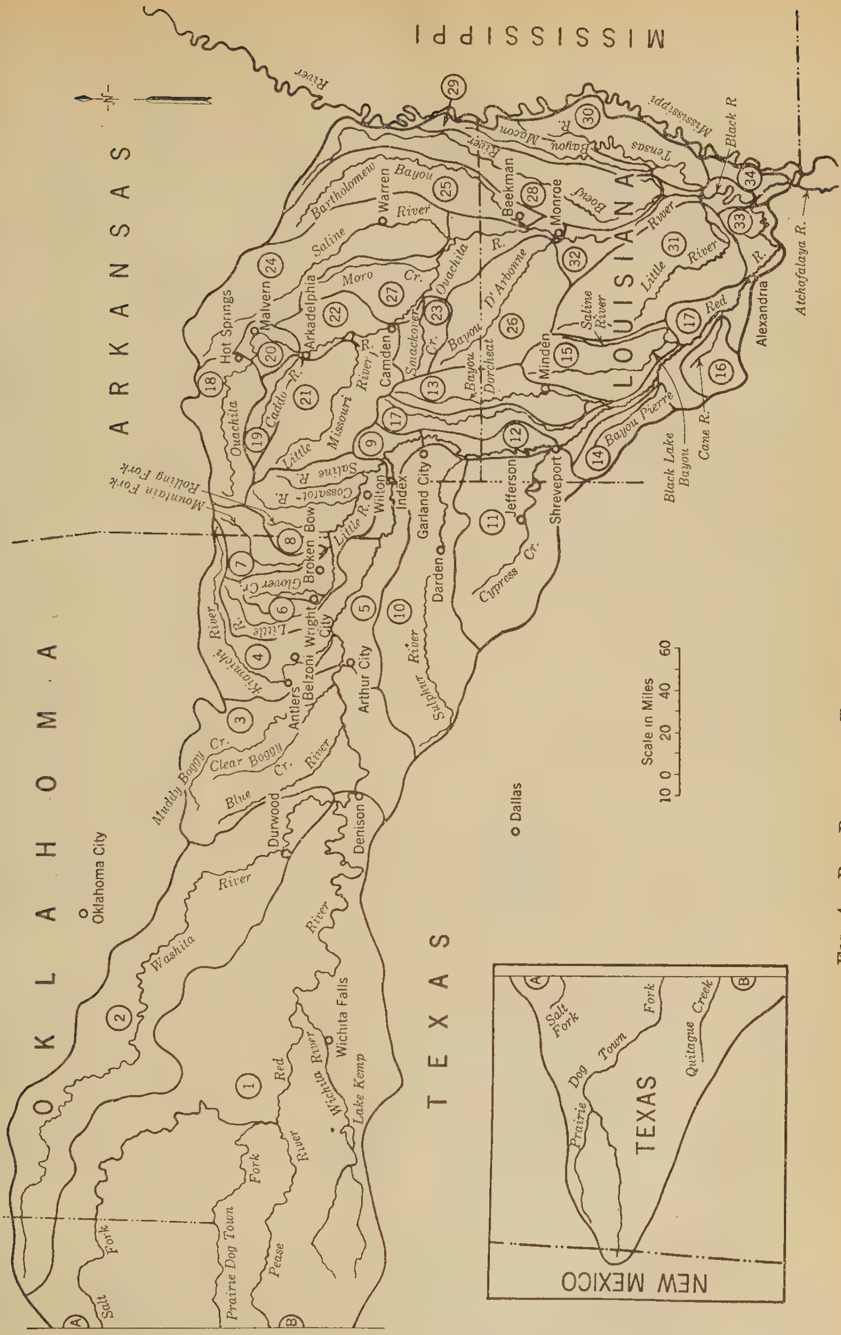

OKLAHOMA
ARKANSAS
LOUISIANA
MISSISSIPPI
TEXAS
N
Oklahoma City
Dallas
Scale in Miles
10 0 20 40 60
Mississippi River
Salt Fork
Prairie Dog Town Fork
Pease River
Red River
Wichita River
Wichita Falls
Lake Kemp
Washita River
Denison
Durwood
Blue River
Clear Boggy Cr.
Muddy Boggy Cr.
Antlers
Belzoni
Wright City
Arthur City
Kiamichi River
Little R.
Glover Cr.
Broken Bow
Mountain Fork
Rolling Fork
Cossatot R.
Saline R.
Little R.
Wilton
Index
Garland City
Darden
Sulphur River
Cypress Cr.
Jefferson
Shreveport
Ouachita
Caddo R.
Little Missouri River
Hot Springs
Malvern
Arkadelphia
Saline
Moro Cr.
Camden
Smackover Cr.
Ouachita R.
Bartholomew Bayou
Warren
Saline River
Bayou D'Arbonne
Bayou Dorcheat
Minden
Beekman
Monroe
Boeuf
Bayou Macon
Tensas
Mississippi
Black R.
Little River
Saline River
Red R.
Alexandria
Atchafalaya R.
Bayou Pierre
Black Lake Bayou
Cane R.
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
A
B
Salt Fork
Dog Town Fork
Prairie
Quitaque Creek
TEXAS
NEW MEXICO

FIG. 4.—RED RIVER AND TRIBUTARIES, ORIGIN OF 1927 FLOOD.

TABLE 2.—ORIGIN OF 1927 FLOOD, RED RIVER BASIN (SEE FIG. 4)

Name of area	Area No.	Area, in square miles	Rainfall, in acre-feet	Computed run-off, in acre-feet
Red River above Denison, Tex.	1	30 542	4 306 639	716 539
Washita River	2	7 786	2 397 083	532 152
Total, Red River above Denison	....	38 328	6 703 722	1 248 691
Drainage, Denison to Arthur City, Tex.	3	4 780	2 143 027	1 877 183
Total, Red River above Arthur City	....	43 108	8 846 749	3 125 874
Kiamichi River	4	1 726	887 362	859 942
Other drainage, Arthur City to Index	5	1 746	774 508	572 069
Total, Red River above Index	....	46 580	10 508 619	4 557 885
Little River above Wright City	6	704	436 900	383 030
Mt. Fork	7	812	438 300	384 258
Other drainage, Little River	8	2 618	1 356 100	....
Total, Little River above mouth	......	4 134	2 231 300	1 956 181
Other drainage, Index to Garland	9	423	272 923	218 061
Total, Red River above Garland City	......	51 137	13 012 842	6 732 127
Sulphur River	10	3 983	1 656 400	924 106
Cypress River	11	3 510	1 625 900	709 543
Other drainage, Garland City to Shreveport, La.	12	667	289 300	163 512
Total, Red River above Shreveport	......	59 297	16 584 442	8 529 288
Bayou Dorcheat	13	1 447	603 600	263 411
Bayou Pierre	14	785	322 000	140 521
Saline River and Black Lake Bayou	15	1 471	665 600	290 468
Cane River	16	628	363 600	158 675
Other drainage, Shreveport to Alexandria, La.	17	2 223	1 119 900	488 724
Total, Red River above Alexandria	......	65 851	19 659 142	9 871 087
Ouachita above Hot Springs, Ark.	18	1 304	1 020 591	882 490
Caddo River	19	492	300 000	259 410
Other drainage, Hot Springs to Arkadelphia, Ark.	20	515	365 000	240 827
Total, Ouachita above Arkadelphia	......	2 311	1 685 591	1 382 727
Little Missouri River	21	2 100	1 069 900	705 920
Other drainage, Arkadelphia to Camden, Ark.	22	980	589 200	....
Total, Ouachita above Camden	......	5 391	3 344 691	2 206 903
Smackover Creek	23	557	316 100	208 563
Saline River,	24	3 281	2 050 100	1 352 656
Bayou Bartholomew	25	1 912	988 000	474 141
Bayou D'Arbonne	26	2 348	1 150 800	502 209
Other drainage, Camden to Monroe, La.	27	2 188	1 147 400	757 055
Total, Ouachita above Monroe	......	15 677	8 997 091	5 501 527
Boeuf River	28	2 221	1 003 500	240 840
Bayou Macon	29	1 366	647 500	155 400
Tensas River	30	1 542	529 500	84 720
Little River	31	3 007	1 487 000	648 927
Other drainage, Monroe to mouth of Black River	32	973	370 344	122 995
Total, Ouachita above mouth of Black River	......	24 786	13 034 935	6 754 409
Other drainage, Alexandria to mouth of Black River	33	416	247 100	107 834
Mouth of Black River to head of Atchafalaya River	34	368	85 300	13 648
Total, Red River above head of Atchafalaya River	......	91 421	33 026 477	16 746 978

Field surveys of the system shown in Fig. 1 sufficient to prepare general estimates of cost and capacity were made where existing data were insufficient. From these surveys reservoir locations were determined; dams were designed in a general way; capacities were determined for various heights of dam; the area overflowed in the reservoir was estimated; and general estimates of cost for dam and reservoir were prepared. A project for protection from future floods may be designed with all the foregoing data available.

ANALYSIS OF THE DESIGN OF A RESERVOIR SYSTEM

Assume that an area subject to flood damage has been determined; that the extent of the damage for a possible future flood is such that a considerable sum may be expended profitably in insuring against such damage; that general studies indicate considerable low-cost reservoir storage available; that studies show that much of the flood water originates above the potential reservoir sites; and that it appears feasible to design a reservoir system, and to analyze its merits. Two general types of reservoir operation may be considered: The storage basin (Group (*A*–1)); and the retarding basin (Group (*A*–2)) mentioned previously.

Before any system is studied in detail it is necessary to decide on the measure of protection to be given. This may include protection against any future flood as great as any past floods of record, or some allowance may be made to provide for a greater future flood.

In analyzing a reservoir system particular care must be taken in considering past floods. The flood of highest crest very frequently has less volume above what might be termed a "danger stage" than some flood of smaller peak height. As the success of reservoir operation is dependent on the handling of the flood volume at the time it occurs, it is necessary to consider floods of various types. Daily hydrographs of stream flow become of great importance because the flood volume must be routed through the reservoir as it occurs and not as it might have averaged for monthly periods. Hydrographs of daily flow—at least, one covering a sharp short flood, and another a long flat flood—must be analyzed in detail. They should cover the range of experienced flows and should allow for future greater floods as may seem desirable. In some cases, hydrographs of only past floods may be considered, and the capacity of each reservoir made greater arbitrarily than that required to control these record floods in order to provide the factor of safety desired.

Having determined the degree of flood protection desired, the merits of each reservoir system should be considered and one or more methods of operation selected for detail study with the hydraulic data collected. This ordinarily means the selection of one or more design floods and the trial operation of the proposed system during such a flood.

Retarding Basin Operation.—The retarding basin method of operation (Group (*A*–2)) has gained considerable prominence in recent years largely due to its use on the Miami River, in Ohio.[8] It depends for success on the existing channel—or on an improved channel as is the case on the Miami River—being sufficient to provide for the run-off below the reservoirs and a considerable flow through the reservoirs as well. The reservoir outlet conduits are designed so that the maximum outflow, when added to the estimated local run-off below the dam sites, will not exceed the improved channel capacity. Spillways are provided so that excess flow can be released to protect the dams should the maximum stream flow exceed the designer's estimates. One of the greatest advantages of such a system is its automatic operation—no decision being required during a flood as to which gate to open and when to open it. On the other hand, it may have a limited benefit because it may so equate the flow of a tributary as to increase the flood peak of a main stream below it by changing the time of maximum contribution of the tributary.

---

[8] Technical Repts., Miami Conservancy Dist., Dayton, Ohio.

Considerable variation may be desirable in the size of retarding basin outlets. For an area with long moderate floods, small outlets may be desirable to give a minimum reservoir contribution to the stream below. A long duration of the flood increases the likelihood that the outflow may coincide with extreme local flows below the reservoir, and it makes a small rate desirable. For short extreme floods rather large outlets are ordinarily preferable to insure getting rid of the water as rapidly as possible. All types of intermediate designs are possible, depending on the particular flood problem.

Operation Methods for Storage Basins.—The methods of operating storage reservoirs (Group (A-1)) may be as follows: (1) All flow may be stored during the period determined by the rule for operation; (2) all flow may be stored except such quantities as are released uniformly: (3) all flow may be stored except a variable quantity released depending on the stream flow obtained from local areas below the reservoir; and (4) flow may be stored at a uniform rate, passing all excess flow during a calendar period.

Reservoir (1) is more nearly "fool-proof" in its operation than any other system. If based on a calendar period, or a period determined by pre-selected gauge heights, it is practically automatic except as to the release of stored waters, which must follow some predetermined rule. If the period of storage involves prediction of gauge heights to determine its time of functioning, the system becomes only as good as the prediction system.

A uniform release (Reservoir (2)) may be required by prior water rights below. It is really a minor modification of Reservoir (1). It requires somewhat less capacity to control a given flood, but the stream capacity down stream must be sufficient to contain the water released, in addition to the local inflow.

Irregular release of water (Reservoir (3)) is frequently an intricate and difficult problem. It involves knowledge of stream capacities down stream, and accurate determination of the quantity of local run-off that will reach all critical points at the same time as water released from storage. Some of this may not have fallen as rain at the time the water above the dam is released. Improper operation may result in damage suits, as many water-power operators well know. It is a complicated and uncertain method to analyze with respect to possible future floods and is usually only considered as a possible improvement in the nature of a factor of safety on a system considering the storage of all flood flows.

Reservoir (4) was considered in 1927 by the Reservoir Board for Mississippi River Flood Control.[4] This arrangement has little merit except in the case of a big system, such as the Mississippi, where a flood may come from many different sources. In the case of the Mississippi River, it has the advantage of giving a more or less dependable reduction of a definite quantity during a critical period which, if the storage system is large enough, would be sufficient to reduce the flood crest to the capacity of the existing levee system.

---

[4] Committee on Flood Control, Committee Doc. No. 2, 70th Cong., 1st Sess.

Storage systems (Group (*A*–1)) are necessary wherever the local run-off below the reservoir may require most of the available channel capacity. Reservoirs of this type have the advantage of being capable of controlling stream flow as desired. It is possible to reduce the flow from the reservoir at will. However, once it is stored, it is frequently a serious problem to determine just when and how to let it go.

Operating the Reservoir.—The storage or retarding of flow during the flood-danger period must be carefully analyzed before selecting the type of operation to adopt. Most streams have many rises or crests. Most of these rises do no damage to the valley. Only certain rises become high enough to cause damage. As the rain falls and gauge readings begin to rise the question looms up: "Will this be a damaging flood, and if so, how much of it can be passed safely down the river before the reservoir gates are closed?" In certain cases it is possible to store every flood at the reservoir as it occurs, but in most cases the storage capacity is limited either by physical features or cost, and it is necessary to permit all small floods to pass. In the case of the retarding basin, the size of outlets must be determined in the design. In the case of the storage basin not only must outlets be designed, but a rule must be selected for their operation.

It is of prime importance that the rule of operation for a storage system be determined in advance, and be such that it can be used progressively as the flood develops. It is very simple to design a reservoir system to control a flood peak that has occurred in the past, but foresight is limited during a flood. Certain information is obtained from gauges above the reservoirs and, in many cases, flows may be predicted for limited periods in advance. With these data, the rule of operation must be workable. The entire flood is not visible to the operater, and he must do his work on the basis of what he can see.

In cases where the period of operation can be based on definite stages at key gauges, its determination is quite simple. A study of past floods shows that all flow above a given stage must be stored or regulated. When this stage is reached the reservoirs are placed in operation.

Operation on a rule requiring prediction of gauge heights is frequently desirable. If all flows above a fixed stage are stored, large reservoir capacity is required. This may prove costly and it becomes desirable to utilize storage as nearly as possible to impound only that part of the flood volume that causes the damage. Water stored before the danger stage is of no value in reducing floods and merely fills space required by the flood water that will cause damage.

The most common method of predicting advance stages is by studying the relation between comparable gauges. Where no important tributary or no large storage basin is located between the gauges compared, this is a fairly satisfactory method. However, even when these conditions do not exist, changes in relation may occur from time to time due to changes in river regimen. Fig. 5 gives an illustration of a typical gauge-relation study for a case that has no complicating factors. This shows the relation between gauges at Columbus, Ky., and New Madrid, Mo., on the Mississippi River. With an experienced stage at the up-stream station the probable stage at the lower station may be read directly from the curve. In cases where it is necessary to extend past a tributary, the volumetric contribution of the tributary coinciding with the crest of the main stream may be estimated and applied as a correction to the main river gauge below. Similarly, an allowance may be made for a storage basin. The results are more or less approximate. On many streams fairly good results may be obtained. It is estimated that stages for Cairo, Ill., at the mouth of the Ohio River, may be predicted at least four or five days in advance and that stages at New Orleans, on the Mississippi River, may be predicted about two weeks or more in advance. Under conditions of small contributions of lower tributaries these periods may be lengthened somewhat. However, on many small streams, the forward limit of predictions may be a matter of hours.

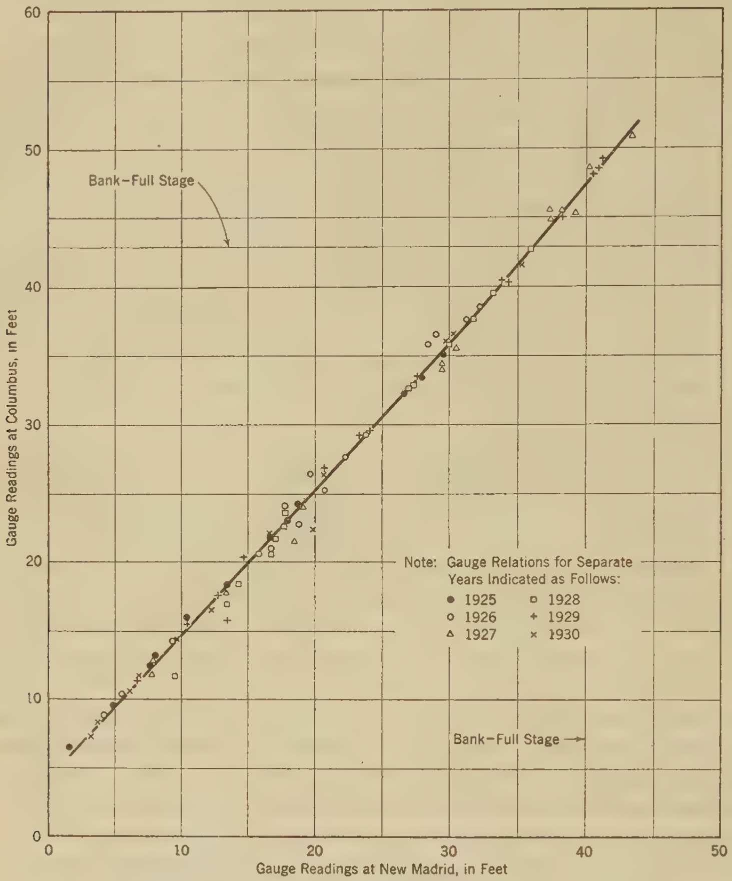


FIG. 5.—RELATION BETWEEN GAUGES, COLUMBUS, KENTUCKY, AND NEW MADRID, MISSOURI, MISSISSIPPI RIVER, 1925–1930

A single-key gauge at or near a critical area with a definite maximum allowable stage may be used as a basis for operating reservoirs within prediction distance of this gauge. For instance, gauge heights may be predicted at Cairo, Ill., five days in advance. If 50 ft is set as an allowable maximum at this gauge, all reservoirs within a water-flow range of five days may be operated to store their entire contribution at all times, when 50 ft, or more, seems probable. By allowing a few feet as a factor of safety (it is advisable to try to hold to 50 ft when, say, 53 ft will still do no damage), this system can be made fairly satisfactory. Undoubtedly, it gives effective use of close-in storage. A reservoir five days away, for example, will begin storing the day a gauge reading of 50 is predicted (predictions made five days ahead). The reservoir one day away would not begin storing until the day before a gauge reading of 50 ft was predicted. In this way the maximum quantity of harmless (less than 50 ft) water would be permitted to pass down stream and the near-by reservoirs would be held in readiness to make their storage most effective in impounding water only when it would cause damage.

An extension of the key-gauge method is sometimes possible on a large system. It may be found that a combination of certain maximum gauge heights on a group of tributaries will not give trouble. Any height in excess of that stated for any tributary may give trouble; therefore, a key gauge may be picked for each tributary and each one regulated so that its flow will not exceed its allowable maximum. By preventing any tributary from introducing a damaging quantity of water, the flow of the main river may be kept at less than its own allowable maximum at the danger zone.

It is sometimes found that a key gauge on the main stream may be selected and all flow that originates above reservoirs for a given stage at the key point, when something less than a higher stated crest is predicted, may be passed safely through the danger zone. For instance, if Arkansas City is the key gauge on the Mississippi River for the operation of reservoirs on the Red River that protect the Mississippi River against Red River floods, it is found by studying gauge records that when not more than 50 ft is predicted at Arkansas City, no water need be stored in Red River reservoirs; but if a gauge of more than 50 ft is predicted, storage should begin at 35 ft at all reservoirs 28 days from the mouth of Red River and at corresponding dates for closer reservoirs. Storage of all flow at reservoirs should continue until 48 ft is reached on a falling stage after the predicted crest. This method, checked against 40 or 50 years of hydrographs, was found to be reliable in all cases.

In many cases no satisfactory stage-predicting system can be developed. It is necessary then to store all flow above reservoirs for a calendar period and to equate all floods to a fixed maximum contribution of the stream during the remainder of the year and during the period in which a reservoir is emptying. This requires a large storage, and there is usually considerable unused storage in any single flood.

On some streams an historic study will show that all damaging floods of record have occurred during a comparatively short calendar period. Analysis of conditions causing these floods may show that weather conditions are such that, barring a change of climate, the same experience may be expected in the future. Under such conditions the rule of operation may be based on a specified operation during the dangerous calendar period.

Tests of Operation Method Selected.—Having selected the method or methods of reservoir operation considered best adapted to the solution of a particular flood problem, each method must be tested by determining its operation during the selected type floods of record and the design flood. In making these tests it is important that the operation be based only upon the information that would be available each day as the flood actually developed. Only the foresight of tested prediction can be considered.

A retarding-basin system of reservoirs (Group (*A*-2)) has been selected to demonstrate the foregoing points. One of the reservoirs in the system under consideration is near Antlers, Okla., on the Kiamichi River. Fig. 6 shows the storage for various elevations of water surface and the discharge through the reservoir outlets. A general summary of pertinent design data for a masonry dam, in this connection, is as follows:

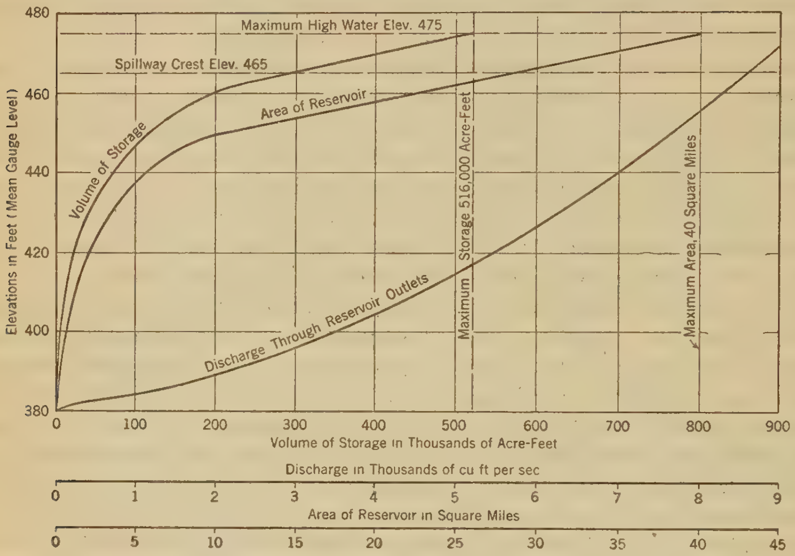


FIG. 6.—AREA AND CAPACITY CURVES, PROPOSED RESERVOIR, NEAR ANTLERS, OKLAHOMA

Drainage area above the reservoir, in square miles	1 400
Surface area of proposed reservoir, in acres	25 600
Storage capacity, in acre-feet	516 000
Height of dam above the valley, in feet	105
Length of dam at crest, in feet	950
Elevation, crest of dam, in feet	485
Elevation, crest of spillway, in feet	465
Cost of reservoir, in dollars per acre-feet of storage	$6.26

TABLE 3.—RESERVOIR OUTFLOW CURVE, ANTLERS RESERVOIR, KIAMICHI RIVER

Difference in water surface, reservoir and river below, in feet (1)	Computed discharge through conduits, in cubic feet per second (2)	Equivalent gauge height at Belzoni gauge, in feet (3)	Stage above low water required to pass conduit discharge (river below reservoir), in feet (4)	Water-surface elevation in reservoir to pass conduit discharge, in feet (mean Gulf level) (5)
0.5	660	6.1	2.6	383.1
1.0	940	6.7	3.2	384.2
2.0	1 340	7.4	3.9	385.9
3.0	1 660	8.0	4.5	387.5
5.0	2 180	9.0	5.5	390.5
7.0	2 600	9.6	6.1	393.1
10.0	3 120	10.6	7.1	397.1
15.0	3 880	11.8	8.3	403.3
20.0	4 480	12.8	9.3	409.3
30.0	5 560	14.6	11.1	421.1
45.0	6 840	16.5	13.0	438.0
60.0	8 000	18.5	15.0	455.0
80.0	9 260	20.3	16.8	476.8
100.0	10 440	21.8	18.3	498.3

Table 3 gives the information from which the outflow curve was determined. Fig. 7 is a hydrograph of the unmodified 1927 flood at the reservoir site. In Column (2), Table 3, ten conduits, 5 ft in diameter, were considered,

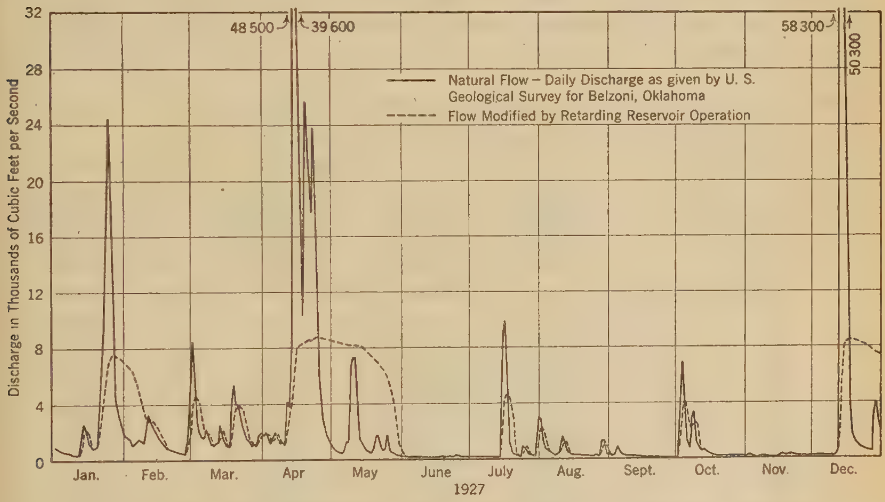


FIG. 7.—DISCHARGE HYDROGRAPH, KIAMICHI RIVER, AT ANTLERS RESERVOIR

and the discharge was computed by the Williams-Hazen formula, assuming entrance and exit losses equal to velocity head; that is,

$$H = H_f + H_0 \dots\dots\dots\dots\dots\dots\dots\dots (1)$$

With $H_f = L\,S$ and $H_0 = \frac{V^2}{2g}$:

$$V = 1.318\, C_1\, R^{0.63}\, S^{0.54} \dots\dots\dots\dots\dots\dots (2)$$

in which, H = the total head available (Column (1), Table 3); H_f, the head consumed in friction; H_0, the head loss at entrance and exit, assumed equal to velocity head; L, length of conduits; $S = \frac{H_f}{L}$; V, mean velocity in conduits; C_1, roughness coefficients of conduits; and R, hydraulic radius of conduits.

The Belzoni gauge referred to in Column (3), Table 3, is a regular station situated a short distance below the reservoir. Low water is 3.5 ft above zero on this gauge. In Column (4), it is assumed that the relative stage discharge is approximately the same below the reservoir as it is at the Belzoni gauge. Referring to Column (5), low water on the zero stage in the reservoir is 380 ft above mean Gulf level. The water surface in the reservoir is equal to Elevation 380 plus the river stage below the reservoir plus the difference in the water surface required to give the various discharges.

To determine the effect of the Antlers Reservoir on the 1927 flood (one of the large record floods), the inflows have been routed through the reservoir and the modified outflow has been determined. The routing may be accomplished by either analytical or graphical solution of the equation:

$$I = O + S \qquad \text{(3)}$$

in which, I = inflow; O = outflow; and S = storage. The modified outflow is shown on Fig. 7. Graphical solution is obtained by the same method as discussed subsequently, for studying valley storage. Table 4 gives the basic data used in plotting the curves for this particular routing. Different scale curves were plotted for the different ranges in storage to make the accuracy of the graphical work comparable to the accuracy of the flows being routed. Table 5 is an extract from the graphical routing. The curves used were similar to those in Fig. 8 used for the studies of valley storage effect by the "reach" reservoir method. The method of using the curves is discussed subsequently. Analytical routing requires only the use of storage and outflow curves with inflows. Table 6 gives the analytical routing of the same inflows as that cov-

TABLE 4.—DATA REQUIRED TO PLOT CURVES FOR GRAPHICAL RESERVOIR ROUTING, ANTLERS RESERVOIR, KIAMICHI RIVER

Water-surface elevation in reservoir, in feet above mean Gulf level	Reservoir storage, in cubic feet per second per day	Conduit discharge, in cubic feet per second	Base for storage indication equals storage minus one-half of outflow, in cubic feet per second per day	Storage indication equals storage plus one-half of outflow, in cubic feet per second per day
380	0	......	......	......
382	200	290	55	345
385	500	1 150	—75	1 075
388	800	1 800	—100	1 700
390	1 000	2 140	—70	2 070
395	1 850	2 860	420	3 280
400	3 000	3 500	1 250	4 750
410	6 500	4 500	4 250	8 750
420	12 500	5 450	9 775	15 225
430	21 500	6 250	18 375	24 625
440	37 000	7 000	33 500	40 500
450	60 000	7 650	56 175	63 825
460	98 500	8 300	94 350	102 650
465	150 000	8 600	145 700	154 300
475	258 000	9 200	253 400	262 600

TABLE 5.—GRAPHICAL RESERVOIR ROUTING, ANTLERS RESERVOIR, KIAMICHI RIVER

Date	Inflow, in cubic feet per second	Inflow sum, in cubic feet per second	Average inflow, in cubic feet per second	Accumulated storage, in cubic feet per second per day*	Outflow, in cubic feet per second†	Water-surface elevation in reservoir, in feet above mean Gulf level‡
1927:						
July 14	39	....	......	200	39	382.0
July 15	8 510	8 549	4 275	2 600	3 270	398.2
July 16	10 100	18 610	9 305	7 900	4 770	412.7
July 17	3 940	14 040	7 020	10 000	5 120	416.3
July 18	1 160	5 100	2 550	7 650	4 720	412.2
July 19	617	1 777	888	4 200	3 930	404.0
July 20	436	1 053	526	1 450	2 580	393.0
July 21	330	766	383	280	520	382.8
July 22	265	595	297	200	290	382.0
December 11	78	....	......	200	78	382.0
December 12	465	543	271	200	271	382.0
December 13	26 100	26 565	13 282	10 200	5 200	417.0
December 14	58 300	84 400	42 200	47 000	7 320	445.0
December 15	50 300	108 600	54 300	93 000	8 200	458.7
December 16	24 400	74 700	37 350	121 000	8 440	462.3
December 17	4 260	28 660	14 330	127 000	8 470	462.9
December 18	1 700	5 960	2 980	122 300	8 440	462.4
December 19	1 280	2 980	1 490	115 700	8 410	461.8

*As scaled from storage curve. † As scaled from outflow curve. ‡ After first day, elevation is determined by storage indication curve.

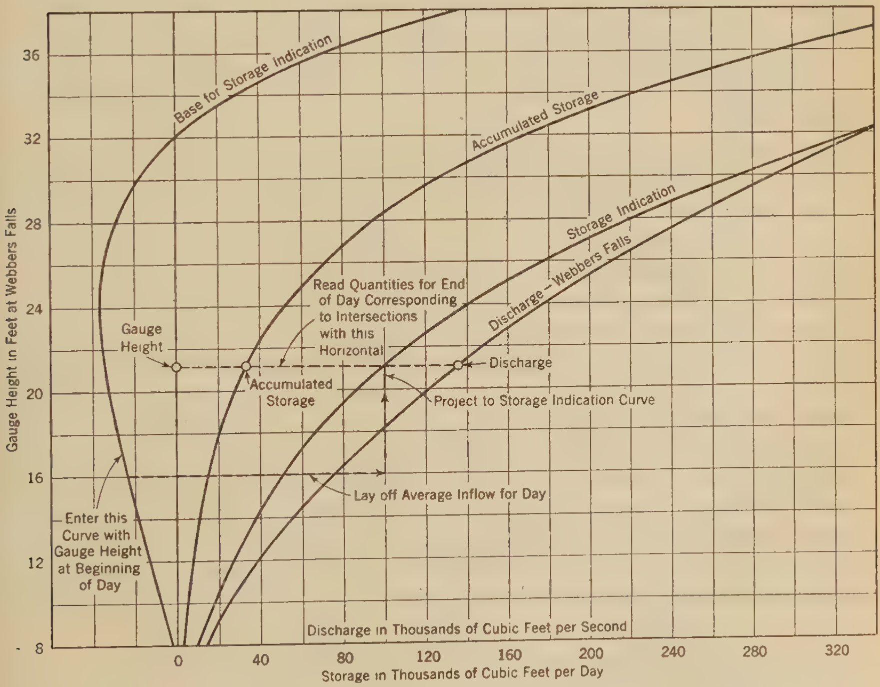


FIG. 8.—REACH RESERVOIR ROUTING, ARKANSAS RIVER, TULSA TO WEBBERS FALLS, OKLAHOMA

TABLE 6.—ANALYTICAL RESERVOIR ROUTING, ANTLERS RESERVOIR. KIAMICHI RIVER

Date	Inflow, in in cubic feet per second	Average inflow, in cubic feet per second	Storage change, in cubic feet per second per day	Accumulated storage, in cubic feet per second per day	Outflow, in cubic feet per second	Average outflow, in cubic feet per second	Water surface elevation in reservoir, in feet, mean Gulf level
1927:							
July 14	39			200	39		382.0
July 15	8 510	4 275	2 600	2 800	3 360	1 700	399.0
July 16	10 100	9 305	5 300	8 100	4 760	4 060	412.5
July 17	3 940	7 020	2 100	10 200	5 100	4 930	416.0
July 18	1 160	2 550	—2 400	7 800	4 680	4 890	411.5
July 19	617	888	—3 400	4 400	3 920	4 300	404.0
July 20	436	526	—2 700	1 700	2 600	3 260	393.0
July 21	330	383	—1 350	350	750	1 670	383.5
July 22	265	297	—150	200	290	520	382.0
December 11	78			200	78		382.0
December 12	465	271	30	230	380	229	382.3
December 13	26 100	13 282	10 470	10 700	5 220	2 800	417.5
December 14	58 300	42 200	36 000	46 700	7 310	6 265	444.7
December 15	50 300	54 300	46 600	93 300	8 200	7 755	458.8
December 16	24 400	37 350	29 000	122 300	8 440	8 320	462.4
December 17	4 260	14 330	5 900	128 200	8 480	8 460	463.0
December 18	1 700	2 980	—5 500	122 700	8 450	8 465	462.5
December 19	1 280	1 490	—7 000	115 700	8 410	8 430	461.8

ered by the graphical routing in Table 5. In addition to the outflow, the routing study gives the accumulated storage in the reservoir each day. For the Antlers Reservoir, the maximum storage required to control the 1927 flood was 393 000 acre-ft. The maximum available storage in the reservoir, as shown in Fig. 6, is 516 000 acre-feet. For this flood, therefore, the reservoir has about 25% excess capacity which might be considered a sufficient factor of safety.

By routing each of the selected record floods and the design flood through each reservoir, the maximum capacity required for each is determined. The record floods should include one of maximum volume during a flood season and one which produced the maximum flow rate (maximum stage of record). Other floods should be tested to make certain that no combination of flows would produce an accumulated storage in excess of the reservoir capacity. By taking the maximum storage required at each site for any of the floods tested, a system capacity is obtained ordinarily in excess of that required for the maximum recorded general storm over the basin and, consequently, would give protection against a flood more infrequent than experienced floods. In many cases this may be considered sufficient protection and this capacity may be selected for the preparation of cost estimates. If not, additional capacity may be allowed as considered necessary. In some cases it may be considered that the most severe record flood is of such remote probability that somewhat less protection is all that is justified. In such cases less capacity than the maximum required, as shown by the study of record floods, may be selected as desired.

Transmission of Reservoir Effect to Area to Be Protected.—Each reservoir in the system is analyzed as outlined. However, this does not give the controlled flood at the critical area unless perchance there is only one reservoir and the area is directly below it. In other cases, the controlled outflow of the reservoirs must be followed down the stream channels below, increased by the run-off below the dam, combined with the flow of other tributaries whether or not controlled by reservoirs, and the modifying effect of the channel or valley storage on the new flood wave determined. By comparing this wave with the unmodified flow the reduction at the area to be protected can be determined and the success of the project measured.

Determining the modified flood wave involves three major problems: (*A*) Time travel of reduced wave from reservoir to critical area; (*B*) run-off of local area below reservoirs; and (*C*) effect of "valley storage." By combining reservoir outflows, allowing for differences in time of travel, adding local run-off, and allowing for the effect of valley storage, controlled flow at key points in the critical area is determined.

The time of travel of flood crests (Problem (*A*)) is difficult to determine. It varies considerably depending on the height of the flood wave and its position relative to previous floods. In many cases the waves become merged with preceding crests, and it is impossible to identify any particular crest at the lower station. The most satisfactory method of determining time of travel is to plot the time between two stations as a function of stage. Even at best, considerable approximation is necessary to arrive at a schedule of time travel.

The run-off of local areas (Problem (*B*)) can be determined ordinarily by proportioning the drainage area to some adjacent area for which the run-off is known and computing run-off as proportional to drainage area. In some cases it is necessary to approximate run-off from rainfall records. This is not particularly satisfactory as daily flows must be estimated and rainfall run-off relations do not give particularly satisfactory daily flows.

The inflow to successive reaches of the main stream may be determined by adding reservoir outflows to local run-off (Problem (*C*)). If the channel or over-bank valley storage is fairly large these flows will be flattened or equated in passing through a given reach. Two methods of computing this flattening are presented. The second is in fact only a general approximation, but in many cases it is all that is possible. The many uncertainties affecting computations often make exact determination of valley storage effects impracticable, and allowances by judgment of the designing engineer are all that are feasible.

The two methods presented for computing valley storage effect might be termed: (*a*) Reach-reservoir method; and, (*b*) proportioning to hydrographs plotted from experience. Method (*a*) has been used in various forms at a number of U. S. Engineer Offices. Each office using the method has introduced some special modifications, but the general principles are approximately the same. In principle, this method considers the river length to be analyzed as being divided into a number of reaches, each of which is considered as a reservoir with a surface slope parallel to the mean water surface slope of the reach. The quantity of water stored in this reach for various gauge heights at the lower end is determined by computing the day-by-day storage from Equation (3).

If the daily inflow and outflow are known for a given flood, the daily storage changes may be computed from Equation (3). Summing the daily changes, the accumulated storage at successive dates may be computed and plotted as a function of the gauge height at the lower end of the reach (using the same gauge for which the discharge is also known). The plotted storages will follow approximately a uniform curve varying somewhat with rising and falling stages, but ordinarily sufficiently close so that an average curve may be drawn through them.

Having determined a storage curve for the reach for a known flood or series of floods and knowing the stage-discharge relation, other floods for which only inflows are known, may be routed through the reach, and its storage effect on them determined. In its simplest form this would consist of entering the reach with the inflows of the flood to be tested, modifying these flows by the storage, and determining the outflow. This routing can be accomplished graphically by curves such as those in Fig. 8. These consist of a storage curve; an outflow curve; a base for a storage indication curve (storage minus one-half the outflow); and, a storage indication curve (storage plus one-half the outflow).

Fig. 8 demonstrates the method of using these curves to obtain stage, outflow, and an accumulated storage at the end of each day for a known stage at the beginning of the day, and inflow during the day. To use the curves, beginning stages and beginning inflows are required. Mean values are obtained by averaging the inflow of one day with that of the following day. The graphical solution gives outflow at the end of the day corresponding to the stage at the same time. Averaging this outflow with that for the preceding day and adding the storage change, the total checks the inflow. The construction of the curves is such as to conform to these requirements.

In applying the reach-reservoir method, it is found that the inaccuracies of gauge and discharge data make certain modifications advisable. For instance, when the storage curve is plotted it only passes through the average of the points used for its construction. If a study is being made of the reduction that can be obtained on a certain flood of record by the operation of certain reservoirs to withhold known quantities during a flood period, what is called a differential routing study is made. First, the actual stages obtained during the unmodified flood wave are tabulated. Using these stages and the known outflows, the inflows as they would scale using the storage curve, are determined. Next, inflow reductions from the river reach above and from reservoirs in the reach are determined. These are subtracted from the adjusted inflows (those scaled using the storage curve) and reduced inflows are determined for use with the routing curves. These curves give reduced stages and outflows such as would result from the reach-reservoir operation. Entering the next reach, the outflow reductions are used. As these are differentials from the actual outflows computed, using inflows that checked the storage curve, they are more nearly correct than if the entire inflow and outflow were carried through the routing. An example of this differential routing is given in Table 7 for a part of the 1927 flood routed through the

TABLE 7.—Reach-Reservoir Method, Flood Routing, Arkansas River, Tulsa to Webber's Falls, Oklahoma, Valley Storage Study

Date	Daily reduction in flow at Tulsa (head of reach), in cubic feet per second (1)	Daily Reductions in Flow from: Oologah Reservoir on Verdigris River, in cubic feet per second (2)	Daily Reductions in Flow from: Pensacola, Markham, and Fort Gibson Reservoirs on Grand River, in cubic feet per second (3)	Total daily reduction in reach, in cubic feet per second (4)	Mean daily reduction, in cubic feet per second (5)	Mean natural daily inflow adjusted to storage curve, in cubic feet per second (6)	Mean modified daily inflow, in cubic feet per second (7)	Foot of Reach, Webber's Falls: Modified stage, in feet (8)	Foot of Reach, Webber's Falls: Modified daily outflow, in cubic feet per second (9)	Foot of Reach, Webber's Falls: Natural stage, in feet (10)	Foot of Reach, Webber's Falls: Natural daily outflow, in cubic feet per second (11)	Foot of Reach, Webber's Falls: Daily reduction in outflow, in cubic feet per second (12)
1926:												
April 10	1 200	....	....	1 200	600	125 000	124 400	20.2	126 600	20.2	126 600	....
April 11	2 100	....	43 000	45 100	22 500	130 000	107 500	18.8	107 700	20.7	133 300	25 600
April 12	2 300	....	78 000	80 300	62 700	174 000	111 300	19.4	116 400	23.8	179 600	63 200
April 13	19 200	12 900	109 000	141 100	110 700	244 000	133 300	21.1	139 100	27.3	241 100	102 000
April 14	31 600	17 800	181 000	230 400	185 700	354 000	168 300	23.4	173 400	31.6	330 800	157 400
April 15	18 700	32 800	210 000	261 500	245 900	387 000	141 100	21.4	143 300	33.1	365 400	222 100
April 16	6 400	29 100	197 000	232 500	247 000	343 000	96 000	17.3	89 600	32.5	350 500	260 900
April 17	3 500	35 200	185 000	223 700	228 600	297 000	68 400	14.7	61 400	31.0	317 000	255 600
April 18	1 500	55 000	188 000	244 500	234 100	307 000	72 900	16.2	76 900	30.8	313 000	236 100
April 19	800	39 800	166 000	206 600	225 500	312 000	86 500	17.4	90 800	30.9	315 000	224 200
April 20	7 800	38 300	132 000	178 100	192 300	273 000	80 700	16.3	78 000	29.6	287 700	209 700
April 21	7 600	25 600	129 000	162 200	170 100	260 000	89 900	17.9	96 500	28.7	268 900	172 400
April 22	4 700	27 400	32 000	64 100	113 200	275 000	161 800	23.4	173 400	29.1	277 300	103 900
April 23	2 100	23 200	....	25 300	44 700	244 000	199 300	25.1	200 800	27.9	252 600	51 800
April 24	1 800	14 600	....	16 400	20 800	194 000	173 200	23.7	178 000	25.2	202 600	24 600
April 25	600	10 500	....	11 100	13 800	164 000	150 200	22.1	153 500	23.1	168 600	15 100
April 26	600	....	....	600	5 800	147 000	141 200	21.4	143 300	21.8	149 100	5 800
April 27	......	....	....	....	300	143 000	142 700	21.6	146 200	21.6	146 200	....

reach of the Arkansas River from Tulsa to Webber's Falls, Okla. The inflow reduction from the reach above is entered in Column (1), Table 7, and the outflow reduction from the reach is given in Column (12). This method was used with considerable success by the Memphis (Tenn.), Engineer District in studying flood control of the White and Arkansas Rivers.

The other method of determining valley storage effect (Method (*b*)) is to plot a large number of flood-flow hydrographs for the gauges at the head and foot of a reach. It is usually found that one of these hydrographs closely approximates the hydrograph of inflows to the reach as modified by the reservoir system. By the simple expedient of modifying the computed outflow hydrograph from the reach to correspond to the effect obtained on the hydrograph based on experience, the approximate effect of valley storage is determined. This is admittedly a more or less approximate solution, but in many cases it is the only feasible method and gives fair results if the hydrograph used as a model is rather closely approximated by the hydrograph being analyzed.

After computing effects of valley storage and increase due to local run-off below reservoirs, the controlled flow at the key gauges at the overflow area is determined for the reservoir system being analyzed for each of the various test floods. By use of a stage-discharge relation at each of these gauges the stage as it would be controlled is determined. If each of the record floods and the design flood result in controlled stages below bank-full at the key points when routed through the selected retarding-basin reservoir system, the system will protect the danger area.

Any of the storage systems described may be tested in a manner similar to the retarding-basin system. Reservoirs are selected—design storms determined—and a rule of operation is adopted. Using the design inflow and following the selected rule of operation, the modified flow and the storage required are computed. If the modified flow results in gauge heights below the danger stage at the overflow area for each flood and if the storage required is less than the reservoir capacity, the system will be satisfactory.

Hydrographs of the natural and modified flow at the reservoir site are shown in Fig. 9 for a theoretical reservoir on the Gasconade River, at Rich Fountain, Mo. It has been assumed that this reservoir would operate to store all flow except a uniform release of 2 000 cu ft per sec for the period from February 15 to May 15. During the remainder of the year it would regulate the flow to a maximum of 10 000 cu ft per sec. Typical extracts for the operation during 1927 are shown in Table 8(*a*). During the period from February 15 to May 15 all flow in excess of 2 000 cu ft per sec has been stored. Several fairly large flood peaks have been held back and the flow of the tributary has been controlled to the fixed release value of 2 000 cu ft per sec. A storage of 1 467 100 acre-ft would have been required. The effect at a down-stream point would be determined by considering the inflow reductions as they would have been modified by valley storage. To continue the operation to regulate the flow for the remainder of the year to 10 000 cu ft per sec a maximum rate would have required a capacity of 1 694 560 acre-ft.

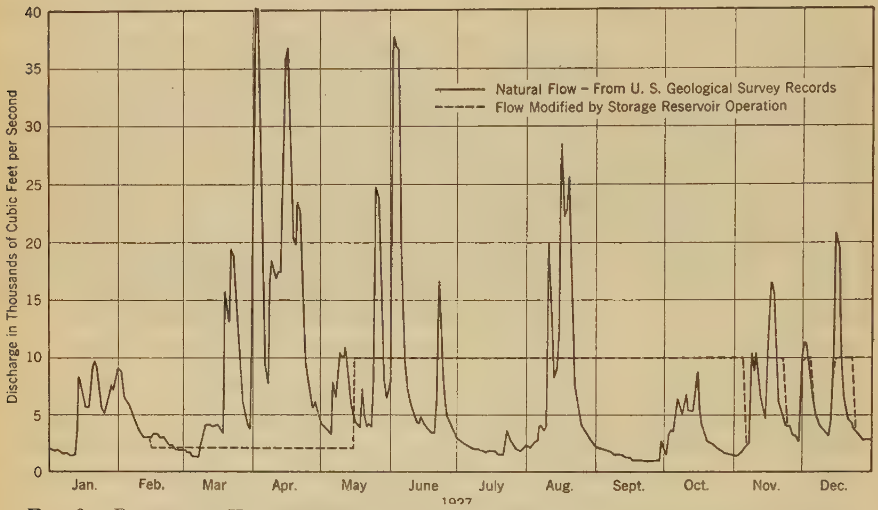


FIG. 9.—DISCHARGE HYDROGRAPH, GASCONADE RIVER, AT RICH FOUNTAIN, MISSOURI

Hydrographs of the natural and modified flow at the reservoir site are shown in Fig. 10 for a theoretical reservoir on the Kansas River, at Bonner

TABLE 8.—RESULTS OF STORAGE RESERVOIR OPERATION (GIVEN ONLY IN TYPICAL PERIODS)

Date	Inflow, in cubic feet per second	Storage, in cubic feet per second per day	Accumulated storage, in cubic feet per second per day	Outflow, in cubic feet per second
(*a*) GASCONADE RIVER AT RICH FOUNTAIN, MO., JANUARY 1 TO FEBRUARY 14, OUTFLOW EQUALS INFLOW				
1927:				
February 15	2 930	930	930	2 000
February 16	3 290	1 290	2 220	2 000
February 17	3 290	1 290	3 510	2 000
February 18	3 170	1 170	4 680	2 000
February 25	2 110	110	8 810	2 000
February 26	2 110	110	8 920	2 000
February 27	2 000	0	8 920	2 000
February 28	1 890	—110	8 810	2 000
March 18	3 290	1 290	24 690	2 000
March 19	15 700	13 700	38 390	2 000
March 20	14 500	12 500	50 890	2 000
March 21	13 100	11 100	61 990	2 000
May 13	9 160	7 160	726 360	2 000
May 14	6 160	4 160	730 520	2 000
May 15	5 030	3 030	733 550	2 000
May 16	4 380	—5 620	727 930	10 000
May 17	4 010	—5 990	721 940	10 000
May 18	3 770	—6 230	715 710	10 000
June 6	18 900	8 900	846 780	10 000
June 7	10 500	500	847 280*	10 000
June 8	7 780	—2 220	845 060	10 000
(*b*) KANSAS RIVER AT BONNER SPRINGS, KANS., JANUARY 1 TO FEBRUARY 28, OUTFLOW EQUALS INFLOW				
1927:				
March 1	2 500	2 500	2 500	0
March 2	2 660	2 660	5 160	0
March 3	2 660	2 660	7 820	0
March 31	3 180	3 180	94 240	0
April 1	28 700	28 700	122 940	0
April 2	47 200	42 060	165 000	5 140
April 3	26 100	5 000	170 000	21 000
April 4	17 000	5 000	175 000	12 000
May 29	5 880	5 000	450 000	880
May 30	5 150	5 000	455 000	150
May 31	4 670	4 670	459 670	0
June 1	4 010	—15 990	443 680	20 000
June 2	4 670	—15 330	428 350	20 000
June 25	20 800	800	461 400	20 000
June 26	20 400	400	461 800†	20 000
June 27	18 700	—1 300	460 500	20 000
June 28	16 800	—3 200	457 300	20 000
August 4	5 880	—14 120	18 320	20 000
August 5	7 150	—12 850	5 470	20 000
August 6	10 100	—5 470	0	15 570
August 7	8 200	0	0	8 200
(*c*) OSAGE RIVER AT BAGNELL, MO., JANUARY 1 TO MARCH 15, OUTFLOW EQUALS INFLOW				
1927:				
March 16	8 590	8 590	8 590	0
March 17	10 000	10 000	18 590	0
March 18	11 600	11 600	30 190	0
April 16	103 000	103 000	2 012 090	0
April 17	106 000	106 000	2 118 090	0
April 18	105 000	105 000	2 223 090	0
April 19	102 000	102 000	2 325 090	0
April 20	102 000	102 000	2 427 090	0
April 25	84 500	84 500	2 893 590	0
April 26	79 700	79 700	2 973 290‡	0
April 27	73 100	0	2 973 290	73 100
April 28	66 300	0	2 973 290	66 300
May 12	55 400	0	2 973 290	55 400
May 13	48 100	—26 900	2 946 390	75 000
May 14	37 900	—37 100	2 909 290	75 000
June 5	80 400	80 400	2 611 980	0
June 6	58 200	58 200	2 670 180	0
June 7	40 000	0	2 670 180	40 000
July 29	6 760	—68 240	55 450	75 000
July 30	6 000	—55 450	0	61 450
July 31	6 500	0	0	6 500

* Maximum storage equals 1 694 560 acre-ft.
† Maximum storage equals 923 600 acre-ft.
‡ Maximum storage equals 5 946 580 acre-ft.

Springs, Kans. It has been assumed that the reservoir would operate so as to store at a uniform rate of 5 000 cu ft per sec per day when such flow was available during the period from March 1 to June 1. If a deficiency from the schedule develops, this deficiency may be compensated by the first rise that will make it up. Table 8(*b*) shows typical extracts from the reservoir operation. The reservoir could be counted on to reduce flows at down-stream points by a uniform quantity of 5 000 cu ft per sec, except for the first month when the flow was less than this amount. A maximum capacity of 919 340 acre-ft would have been required to accomplish this between March 1 and June 1. It has further been assumed that during the remainder of the year, the reservoir would operate to control the maximum peak flow of the stream to 20 000 cu ft per sec. This operation would require a peak storage of 923 600 acre-ft.

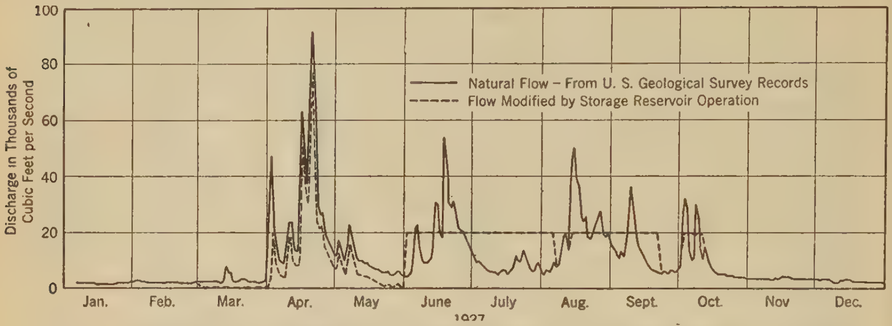


FIG. 10.—DISCHARGE HYDROGRAPH, KANSAS RIVER, AT BONNER SPRINGS, KANSAS

Hydrographs of the natural and modified flow at the reservoir site are shown in Fig. 11 for a theoretical reservoir on the Osage River, at Bagnell, Mo. This reservoir is assumed as being five days from Cairo, Ill., and operated to store all flow when 50 ft or more is predicted on the Cairo gauge. It is permitted to release water at a rate not to exceed 75 000 cu ft per sec, when 40 ft or less is predicted at Cairo. Extracts of the operation for 1927 are shown in Table 8(*c*). It is noted that the reservoir stored at a nearly uni-

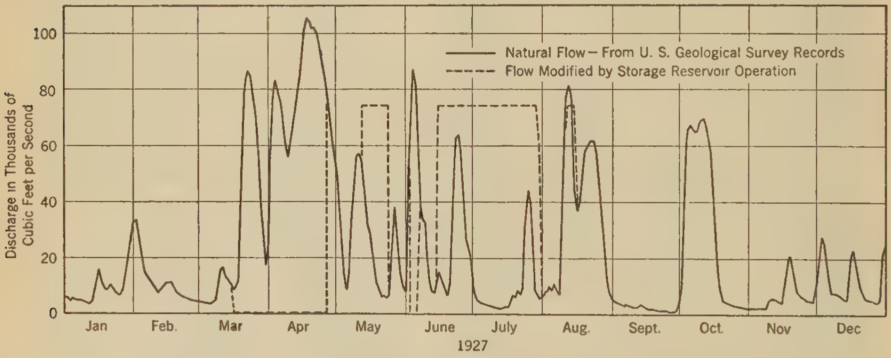


FIG. 11.—DISCHARGE HYDROGRAPH, OSAGE RIVER, AT BAGNELL, MISSOURI

form rate of about 100 000 cu ft per sec. for several days before, during, and after the Cairo crest. Even allowing for valley storage reduction this would mean practically 100 000 cu ft per sec could have been withheld from the Cairo flood crest by this reservoir. To have accomplished this would have required a storage of slightly less than 6 000 000 acre-ft. The storage required to accomplish the regulation for the remainder of the year would be less than this quantity.

Comparison of Systems.—The result that might be obtained by each of several different systems should be carefully compared and analyzed. The particular advantages of each should be weighed and balanced against the disadvantages. The cost of each system should be determined and its annual charges computed. The cost and difficulties of maintenance, operation, and renewal should be considered. By comparing costs (including annual charges) with the results obtained, the best system for any particular project may be selected. If the cost is less than the estimated value of protection, the plan may be justified, providing the cost of financing is not so great as to reverse this balance. To be adopted, however, not only must these conditions exist, but the plan must be cheaper than any other plan that will give equivalent satisfactory protection.

COMBINED USE OF RESERVOIRS

Reservoirs acting alone as flood-control structures have been treated thus far in this paper. Extension of the function of the reservoir to include other interests in combination with flood control is sometimes feasible.

General Classification.—The consideration of combined use of a reservoir for several purposes may be divided logically into two general classifications: (*A*) Combined-purpose storage, or joint use of the same storage space for different purposes; and (*B*) combined-interval storage, or separate storage units combined in one reservoir to reduce cost. Each class should be considered for: (*a*) Combined operation to provide flood control and to obtain development of water power; (*b*) flood control and irrigation; (*c*) flood control and domestic or industrial water supply; (*d*) flood control and navigation; and (*e*) combination of any three or all of the purposes mentioned in Items (*a*) to (*d*).

Flood Control and Water Power.—Usually, the possibility of obtaining water power from a reservoir constructed for flood protection (Class (*A-a*)) is considered hopefully by every organization planning a flood-control system. It seems to offer a solution to that perplexing problem of how to pay for the flood-protection plan. A water-power plant might furnish revenue to pay operating expenses and to retire bonds. However, detailed consideration ordinarily reveals many inherent difficulties to such a plan.

In an ordinary retarding-basin plan, where it is not possible to utilize the lower part of the reservoir as a retention basin or "head developer," there will be considerable periods when no head is developed at the dam and, consequently, no power. This would mean a large power-plant installation for only brief and uncertain periods of operation. Only a very general consideration is necessary to satisfy the engineer that no saving in cost can be obtained by including a power plant in such a plan.

For a storage system there would also be no head during periods of no storage. However, the lower part of the reservoir often develops considerable head for only a small percentage of the total capacity. By considering this lower part as below the draw-down limit and used only to develop head, it is possible to have a minimum head for power development. This storage volume is considered non-effective and is not included in determining the required capacity.

The demands of water power and flood control are more or less contrary. The power reservoir is filled as rapidly as possible during periods of high flow to insure a maximum yearly output. When the reservoir is full and high flows occur they are merely passed over the spillway. On the other hand, the flood-control reservoir is kept as low as possible, in order that the storage capacity may be available to store the peak of the abnormally high floods. A large power reservoir is frequently a very efficient reducer of small floods, but it may have little or no effect on large floods should they occur at the time of year when the reservoir is nearly full. A flood-control storage reservoir, even with a minimum draw-down limit, would permit of very irregular development of water power. There would be practically run-of-river operation through the minimum head during long periods of ordinary flow when it is desired to keep the reservoir low. There might be little or no development during the storage period, depending on the quantity of water that could be released under the method of operation adopted. During the emptying period after the flood, fairly efficient, but still variable, power development might be obtained.

In a very large interconnected power system, it is possible that the part-time water power developed at a flood-control dam might be absorbed efficiently. In such cases it takes a detailed and careful study to determine whether the power developed will pay the cost of plant installation. In many cases, the return would be less than the cost of the plant and in practically all cases at the present time the cost would be more than that of developing power by other methods. Future economic trends might change this balance and make development of plant under some such conditions economical.

The most important objection to the ordinary operation of a flood-control reservoir from the power standpoint is that of uncertainty of available power. The power plant must be capable of furnishing power on demand. It must have dependable capacity. Unless an installation can furnish power when required, other plant must be installed to meet the peak demands and the water-power plant loses a large part of its potential value. The flood-control reservoir operates only when there are floods (time of occurrence unknown); therefore, it cannot be depended on to reduce peak power plant installation. However, there are special cases where modified operation may correct this basic difficulty to some extent.

By sacrificing some measure of flood protection and power production, a combined use of the same storage is sometimes feasible. Floods can be expected to occur at certain seasons. Power demands for the locality have a certain seasonal variation. Operating schedules for plants in a system with other power producers might be arranged to permit the seasonal use of production from a water-power plant at the flood-control reservoir.

These factors may be combined to develop a rule curve for operating the reservoir to obtain the best possible use for both functions. Such a curve would consist of a schedule of elevations of desired water surface in the reservoir, set arbitrarily, and to be obtained throughout a year. For the conditions set forth in the preceding paragraph such a curve would provide that the reservoir be kept at a low elevation during the flood-producing period. As this period closes, increased elevations are permitted to provide increased head and storage for the low-water season. During the low-water season this storage is drawn down, regulating the power output and making certain that the reservoir is low at the beginning of the next high-water season. An interesting example of such operation is a stream in California that has both snow and rainfall run-off. A damaging flood only results from a combination of both snow and rainfall run-off. Therefore, a reservoir system may be designed to remain low until the snow run-off occurs and then either to store or to regulate the flow when it does occur, depending on whether or not it is combined with heavy rainfall. Enough storage is retained at the end of such a flood period to regulate the flow through the summer. This special case gives a fairly good operation particularly as the probable occurrence of storm rainfall is quite definitely limited to a rather short calendar period.

If storage capacity is available in excess of that required for flood-control purposes, combined interval storage may be considered (Class (*B-a*)). In effect, this usually is reduced to the task of determining the most economical power-plant installation and considering flood storage as a surcharge on the power reservoir. The flood surcharge on the power reservoir would consist of a certain zone of storage operated only in the interest of flood control. It might be operated to give complete protection for the maximum probable flood or to give protection from ordinary floods, depending on the economics involved. The operation of this zone would follow some one of the storage or retarding basin plans as discussed previously herein.

The power interval of this type of combined reservoir has a certain value in controlling or reducing floods that occur when normal power operation has the reservoir drawn down. The flood-control section also has some power value in that it furnishes additional flow and extra head during flood periods. Thus, a fair degree of mutual benefit is obtained, which adds to the combined value of the project. However, one important fact must always be considered in all combined projects; there must be some use for both functions. There must be a market for the power and there must be some damage hazard eliminated by providing flood protection. Until the value of both these uses exceeds the cost there can be no justification for construction even if the project may be physically feasible.

Flood Control and Irrigation.—In territory where rainfall is more or less deficient there is frequently agitation for a combination of irrigation and flood protection by means of reservoirs (Class (*A-b*)). Such rainfall as occurs frequently causes floods. Storage of these flood waters to be released during later periods of drought would seem to offer considerable promise of combined use of the same storage space. If there was any regularity in the quantity or occurrence of the flood flow this principle would work very well.

However, floods vary so much in volume and time of occurrence that, in most cases, it is either impossible to obtain assurance of flood protection by having the reservoir empty at the time of flood occurrence or to carry the flood water over a protracted period of drought for irrigation. If land is to be prepared for irrigation, and if crops are to be grown, the water supply must be dependable. This would require the release of stored water in accordance with a rule based on a period of low supply. If a period of high supply occurs with a sharp concentration in the form of a flood—it is quite possible that the reservoir would be full, or nearly full, and that the flood would not be greatly diminished.

The second type of combined operation (Class (*B-b*)) (designation of separate storage bands within the same reservoir) offers more possibilities. There is a certain overlapping of the use of each band. Floods occur in certain seasons and a moderate draw-down of the irrigation storage may be permitted during these periods. On the other hand, when the flood period is over, a certain percentage of the flood-storage zone may be utilized for carrying over flood water for irrigation use.

Generally, in territory where irrigation is desirable, storage space is not particularly expensive. Thus, a large storage volume may be obtained for combined use. In some cases of existing reservoirs, flood protection is obtained as a more or less incidental benefit. At Lake Kemp, an irrigation and water supply reservoir, near Wichita Falls, Tex., it was found too expensive to re-locate several highways running through the site under existing conditions. The lake level, therefore, is kept 15 ft or more below the spillway crest. This surcharge has a volume equal to most floods for the tributary if a bank-full capacity is discharged through the conduits in the dam during the flood. Since the completion of the dam several floods have been equated by this means to down-stream channel capacity. The principle of a surcharge for extreme flood protection in the case of an irrigation reservoir is worthy of considerable study wherever there is a concentration of value with potential flood damage below sufficient to justify a considerable expenditure for flood protection.

Flood Control and Water Supply.—The combination of flood protection with domestic or industrial water supply (Class (*A-c*)) is very similar to the combination with irrigation, except that it applies to a much wider territory. The main difficulty in all cases is in the development of dependable supply for domestic or industrial use without, at the same time, filling up the volume required for the flood-control plan.

As with irrigation it might almost be stated as axiomatic that no combined project is possible unless there is more storage than is required for flood control and unless a combined operation of separate storage zones is possible

(Class (*B-c*)). In effect, this means a flood surcharge on a water supply reservoir. In many cases this is physically feasible, but the funds that can be raised to construct the flood-control surcharges are not sufficient to pay the additional cost. Hence, the industry builds only for itself, or the town builds for a water supply only sufficient for its needs in the near future. Many good dam sites are occupied by structures very much smaller than is feasible or desirable from the standpoint of efficient use of the water resources of the stream on which they are located; but to build the higher dam would be costly, and although flood benefits are large at the time a flood is occurring, they are seemingly non-existent when it comes to raising funds to provide structures for their realization. It is not the purpose of this statement to convey the impression that surcharges on water supply or on irrigation reservoirs are generally desirable. They usually would cost more than the capitalized flood damage that they would eliminate, but justified or not, they are almost never built because of the difficulty of collecting the additional money required from those who might be protected. They do offer about the only practicable combined operation in general and would give fairly good protection in many cases.

Flood Control and Navigation.—The combination of reservoir storage for open-river navigation with flood control (Class (*A-d*) or Class (*B-d*)) is like that of irrigation and water supply. It offers even less possibilities than either of these. This is due to the fact that a relatively large increase of low-water flow is needed ordinarily to provide modern open-river navigation. This alone requires a large storage. This means that insufficient storage is left for flood control and that unsatisfactory results are obtained for either use.

There is one combined navigation and flood-control plan, however, that can be applied in some cases, and it always warrants consideration. If canalization is being considered on a relatively large stream where the dam sites are such that considerable excess storage in the form of a flood surcharge can be obtained at each pool without causing excessive damage in the valley, this type of combination may be feasible. By routing the flood volume through the several pools of the system, the outflow at the lower end may be regulated to give a considerably lower flood crest. This method in effect merely creates a stream with a large amount of valley storage to equate flood peaks. Obviously, it is predicated on potential navigation being sufficient to justify the cost of the lock and dam system, with flood control contributing only that amount necessary to pay for the surcharge on the various navigation dams.

Combined Operation in Interest of Three or More Functions.—The combination of three or more of the uses stated involves most of the difficulties covered in the discussion for combining any two uses. Under certain conditions the combination of three uses is better than two. Generally, this results by reason of the same use benefiting two of the three functions. A reservoir with a flood-control surcharge on a navigation dam may also permit the development of water power. An irrigation dam may be used to develop power by depending to some extent on the flood-control surcharge to furnish the extra water for power operation. Essentially, the engineering problems involved are similar to those outlined previously herein.

Summary.—In most combined projects the problem of distributing the cost and return is so complicated that it almost always requires some arbitrary allocation to the various functions, particularly if any Government agency is a party to the project. The practical difficulties encountered by an organization developed to manage an irrigation project that is combined with a water-power business have been well stated[5] by Charles C. Cragin, M. Am. Soc. C. E. This difficulty would apply to the operation of any combined project. Each function requires its special type of organization, and the development of a single organization to handle more than one function offers many difficulties. The most important consideration in arriving at a conclusion as to the justification for a combined project is that each function must have a value—a real value—that will justify contribution toward the cost of construction, mainenance, and operation of the project developed.

Reservoirs Combined with Other Structures

Flood-control projects may be developed by combining reservoirs with: (*A*) Side channels; (*B*) over-bank floodways; (*C*) channel improvements; (*D*) closure of outlets; and, (*E*) levees.

Although a physically possible combination, Combination (*A*) of reservoirs supplemented by side channels is seldom feasible economically. More or less duplicate structures are required to control the excess peak beyond the capacity of the main stream. Ordinarily, either the reservoir system or the side channel can be made large enough in itself at less cost than that of providing both systems of structures.

To combine reservoirs and over-bank floodways has the same objections as Combination (*A*). However, there may be conditions when it would be feasible. If the floodway can be reduced materially by controlling a part of the peak in reservoirs, this reduction may more than balance the reservoir cost. A variation of this might be a combination in which reservoirs and the natural stream could control all but a relatively small part of the infrequent high floods. This part might be permitted, on such rare occasions, to overflow through certain areas of low development in the overflow plain. This necessarily assumes the cost of such floodways to be relatively small. Under these circumstances, the combination is worthy of detailed analysis. If both the floodway and the reservoirs involve large expenditures, it may be expected that either one or the other, made large enough to control the entire excess flow, will be less expensive than any combination.

Natural or artificial obstructions frequently produce reaches of limited channel capacity in sections of overflow areas. In many cases these obstructions can be removed or can be compensated for at moderate cost to give much larger channel capacity (Combination (*C*)). This automatically reduces the extent of reservoir control necessary by permitting more flow through the reservoirs or a less area to be controlled by reservoirs above the danger zone.

---

[5] "Development of Hydro-Electric Power as an Aid to Irrigation," by Charles C. Cragin, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 98 (1933), p. 1240.

The Miami River[3] flood-control project illustrates this principle. The channel was improved through the reaches at Dayton, Ohio, to permit a controlled flow greater than normal. This increased the peak flow permissible below the reservoirs and reduced their size to such an extent as to make a feasible combination.

It is possible that channel improvements in the form of cut-offs may so increase the natural flow in a danger zone as to make a feasible combination with reservoirs. Cut-offs must be analyzed carefully, however, and exhaustive studies made as to their probable ultimate effect. In some cases their effects may be only temporary. This may or may not be a serious difficulty. In other cases, the improvement obtained may persist for many years.

On a small stream in Iowa a series of cut-offs was made to produce a straight channel with increased capacity. After fifteen years this stream shows a marked tendency to regain its sinuous course. However, in this time, the improved channel has controlled or reduced sufficient floods to pay for its cost. In other cases, cut-off shortening persists for long periods. Cut-offs were made on the Mississippi River, near the mouth of the Red River, in 1831 and 1848, respectively. The shortening effected by these cut-offs has been only slightly regained in about 100 years. A series of interesting experiments on cut-off effects[6] was conducted by the U. S. Waterways Experiment Station, Vicksburg, Miss., in 1933. A cut-off on the Mississippi River was opened by dredging at Diamond Point, Miss., about 12 miles below Vicksburg, in 1933.

On many rivers, outlets or low banks permit overflow into side basins long before the banks begin to overflow generally (Combination (*D*)). This is particularly true on alluvial rivers where the banks are higher than the back country. By closing the outlets the quantity to be carried in the main stream may be increased, but the stage at which overflow begins is also increased. By increasing the stage of general overflow, much less reservoir capacity is required than without the closure, and a feasible combination may result.

Combination (*E*), of reservoirs with levees, is quite similar to that of reservoirs and closure of outlets. The levees may merely close the low places in the bank and by a moderate expenditure so increase the channel capacity as to reduce materially the reservoir storage required. In some cases of streams partly protected by existing levee systems, moderate improvement of these systems in conjunction with reservoirs to take the high peaks may give a feasible combination. In general, levees and reservoirs offer possibilities when a moderate expenditure for levees will give a system controlling the long, moderate floods, or portions of floods, with reservoirs controlling the sharp flood crests that, with a relatively small volume, might cause failure of the levee system.

Summary.—The substitution of other structures in a reservoir plan is often feasible if moderate expenditure will eliminate a large percentage of reservoir storage. Expensive reservoirs on certain tributaries may be eliminated altogether if the channel capacity of the main stream can be increased to provide for the maximum probable flow from the tributary. In other cases, where a levee or floodway plan can control all but a relatively small portion of the flood crest, it may be found that a reservoir system is the least costly method of controlling such a peak.

---

[6] *Paper I,* U. S. Waterways Experiment Station, Mississippi River Comm., Vicksburg, Miss.

CONCLUSIONS

Several primary considerations control the feasibility of a reservoir plan for flood control. The first consideration might be termed physical feasibility, expressed by the question, are there reservoirs with sufficient capacity so located above the area to be protected as to control that portion of the flood waters that may cause damage? This includes the selection of physically feasible dam sites, reservoir basins, and their location at points below the collecting areas for run-off and above the area to be protected. Generally speaking, reservoir control is feasible, physically. There is a wide range in the cost of storage required, but there are few cases in which such storage is not physically available.

The second consideration might be termed "operating feasibility" as defined by the proposition: Given the storage required to control a flood crest is it within the realm of probability that the reservoir can be operated properly? A reservoir that has been placed above a town to be protected, and fails, may cause more damage than the flood which it was designed to prevent. Even if it does not fail as a structure, it may fail in its function of flood control if the method of operation is not workable as a flood develops. In this paper many methods of operation have been discussed. Each has its advantages and disadvantages and conditions under which it might operate satisfactorily. There are, of course, other methods of operation. The method to be used depends largely on the characteristics of the flood that may be expected. The mission of each system must be kept carefully in mind in each case and the incidental damages as well as benefits considered. If a tributary is being regulated to protect an area on a main stream, the extent to which this regulation may interfere with tributary protection must be balanced against the value of protection gained on the main stream.

It is obvious that any storage system has certain inherent objectionable features: It involves the manipulation of gates according to some rule of design; the designer may not be the operator; or the operator may be restricted by hysteria in the community as the flood develops. He may fail to pass as much flow through the dam as the design calls for, with the resulting probable overtopping of the reservoir if the designer's estimate of flood flow is reached. However, detailed studies of many conditions indicate that many areas now subject to flood damage might be protected by a reservoir system the operation of which appears feasible.

The last and perhaps most important consideration is economic feasibility. A system of reservoirs may be physically feasible and may operate perfectly; but its construction may be entirely unwarranted due to excessive cost or considerably greater cost for the same measure of protection than that obtained by other methods of protection. To obtain a favorable balance for a reservoir plan requires: (*a*) A high concentration of value in the overflow area to be protected, such as a large industrial plant, a town, a city, or some other unit subject to relatively high flood damage; (*b*) low cost of storage; (*c*) a relatively large channel capacity below the reservoir capable of accommodating all ordinary flows; (*d*) a relatively sharp short flood to be controlled; and, (*e*) the reservoir ordinarily located fairly close to the area to be protected.

In comparison with levee plans of protection, the reservoir has the advantage in controlling sharp floods, but it is at a disadvantage in controlling long flat-crested floods. In protecting farm lands in an overflow plain, a reservoir system frequently offers a much better solution of the local drainage problem in that it keeps stream levels low and permits run-off of local rainfall, whereas the levee system retains high stream levels and may interfere seriously with local drainage. However, there are few cases in which reservoir protection of farm lands can be justified economically.

Summary.—The reservoir has many disadvantages as a flood-control structure. It is difficult to locate and operate; nevertheless, the advantages of protecting an area by reducing the flood crest that reaches it are such that a reservoir solution should be analyzed carefully before any flood-control plan is adopted.

ACKNOWLEDGMENTS

The writer wishes to acknowledge the assistance of David W. Persons, Jun. M. Am. Soc. C. E., in preparing this paper. He also wishes to acknowledge the co-operation of District Engineers at Memphis, Tenn., and Vicksburg, Miss., and Brig.-Gen. H. B. Ferguson, U. S. Army, President of the Mississippi River Commission, in permitting the use of the files of the Commission as a source of data.

DISCUSSION

C. Maxwell Stanley,[7] Jun. Am. Soc. C. E. (by letter).—Much information of a basic nature is assembled in this paper concerning the function and the design of reservoirs as flood-control structures. A number of methods of operating reservoirs and, correspondingly, of design, are presented, together with outlines of various manners of combined reservoir usage for flood control and other purposes. The methods of operation, and, hence, of design, of reservoirs for flood control are numerous. The ingenious methods which have been and will be developed to solve the mass of routine and tedious computations involved in the study of a reservoir system are equally numerous.

This paper has been written with a background largely of the Mississippi River, which, as the author points out, is undoubtedly the "No. 1" flood-control problem of this country. The size and scope of so large a flood-control project, as that required in the case of the Mississippi, create many situations and points of view which are not strictly applicable to projects of lesser size and scope. As an instance, consider the numerous references, in the paper, to the use of selected maximum floods of record in designing the controlling structures. While a considerable reliance on past records of stream flow may be justified when the flow comes from large areas and diversified sources, as in the case of the Mississippi, an equal reliance on past stream-flow records may cause grave error with drainage areas of only a few hundred or a few thousand square miles. In such instances, careful investigation of the rainfall probabilities must be made, as a single, heavy downpour may result in a run-off much greater than anything that has been experienced in the past. From the results of such an investigation, assisted, of course, by data on flood discharges of the past, a design flood may be determined which will form a basis for the proportioning of the controlling structures. Past records cannot be used blindly on any occurrence as far beyond the control of Man as rainfall and run-off.

What would have been the standards of design of controlling structures and the criterion of flood discharge on the Mississippi to-day, had the floods of 1927 been delayed until 1937 by some combination of hydrological circumstances beyond the control of Man?

Furthermore, small drainage areas do not offer the opportunities of predicting and forecasting run-off and river stage which are possible on a system the size of the Mississippi where reasonably accurate predictions of conditions may be made days and, sometimes, weeks in advance. In contrast, the floods on the small areas may develop within 24 hr and may come without warning as a result of a peculiar combination of rainfall and run-off conditions. The likelihood of sudden and sharp floods tends to rule against the intricate and completed schedules of reservoir operation which may be designed and used on a larger system where more time is available and where there is a competent staff to carry out the work of predicting the flood and scheduling the operation of the reservoirs. Indeed, the probability of the sudden and sharp flood peak on the small drainage area seems to demand, in most cases, a structure the operation of which is automatic and is not dependent upon the human element. This tends to eliminate the reservoir with an outlet controlled by gates and to favor the use of the retarding type of reservoir, or the complete elimination of reservoirs in favor of levees, channel improvements, etc.

---

[7] Cons. Engr. (Young & Stanley, Inc.), Muscatine, Iowa.

Considerable controversy has raged between those advocating the use of reservoirs to limit the flood discharge and those advocating the use of structures, such as levees, floodways, etc., which increase the carrying capacity of the channel. Much of such controversy might be avoided by giving careful attention to the purpose of economical flood control.

The purpose of flood control, of course, is to prevent damage to property, and loss of life. This purpose is accomplished by constructing the necessary structures to assure that the maximum flood discharge at the point to be protected is within the carrying capacity of the channel through which the flood must pass. Obviously, this end may be obtained by decreasing the size of the flood peak, by increasing the channel capacity, or by a combination of these two methods.

The function of a reservoir is to decrease the flood peak passing a given point, or points, and the function of channel improvements, levees, floodways, etc., is to increase the carrying capacity of the channel. Theoretically, suitable protection might be obtained by the use of either reservoirs alone or by channel improvements alone, provided, of course, that suitable reservoir sites are available. The reservoirs must be sufficiently large to limit the flood flow to the capacity of the unimproved channel, or the channel must be of sufficient magnitude to provide a channel capacity capable of carrying the maximum unregulated flow. In many cases, one of these solutions may provide the most practical and most economical answer to the problem, but in many cases a combination of the two methods (that is, the use of both reservoirs and channel improvements) may provide a more economical solution.

The proportion in which they should be combined is largely an economic problem which must be solved for each project. As an example, assume the very simple hypothetical case shown in Fig. 12 of a flood-control project to protect a single city from a river of relatively small drainage area. The maximum flood discharge from this area is assumed to be 100 000 cu ft per sec. On the water-shed of this river, it is possible to develop a single reservoir that will regulate the flow from 70% of the drainage area. The existing channel through the city is capable of carrying 40 000 cu ft per sec, or 40% of the designed flood peak for the project without further improvements.

If reservoirs alone are used to hold the peak flood within the capacity of the existing channel, an expenditure of $10 000 000 may be required. If, on the other hand, the reservoir is not developed and flood protection is obtained solely by improvements to the channel, construction of the levees, etc., the cost to care for the maximum flood may be only $6 000 000. On the face of this, it would indicate that the use of channel improvements alone might be the most economical solution.

However, there are possibilities of combined development which will provide a more economical arrangement. Suppose, for instance, that the channel is improved to increase its carrying capacity to 50 000 cu ft per sec of the maximum flood peak for which the system is being designed and that the reservoir is developed to limit the flood flow in the channel at the city to 50 000 cu ft per sec, or 50% of the maximum flood. As 30% of the drainage area is unregulated, this will mean that the reservoir must be capable of reducing the flow from the area controlled to 20 000 cu ft per sec, thus reducing the discharge at the city to 50 000 cu ft per sec. With such a development the costs would be as follows (taken from curves of Fig. 13):

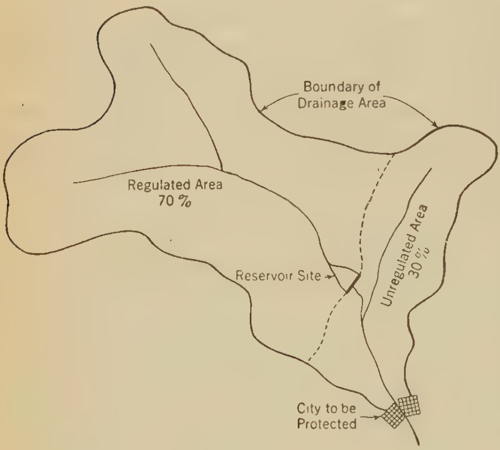


FIG. 12.

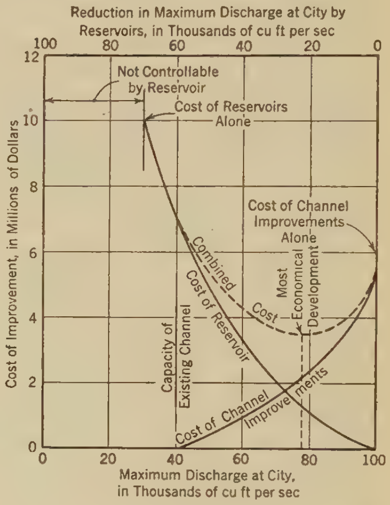


FIG. 13.

Cost of reservoir	$4 950 000
Cost of channel improvements	450 000
Total	$5 400 000

This represents a saving of $600 000 over protection by channel improvements alone.

A study of further combinations of reservoir development and channel improvements (made from the curves of Fig. 13) indicates that a minimum cost of flood control will be obtained with a channel capacity of 78% of the maximum flood peak, or 78 000 cu ft per sec, and a reservoir capacity sufficient to reduce the flow at the city to 78 000 cu ft per sec. Under this condition the reservoir must reduce the flow from the regulated area from

70 000 cu ft per sec to 48 000 cu ft per sec, or by 22 000 cu ft per sec. With this arrangement the costs are as follows:

Cost of reservoir	$1 300 000
Cost of channel improvement	2 200 000
Total	$3 500 000

This simple example is illustrative of the results that may be obtained by an intelligent combination of reservoirs and channel improvements. Naturally, the usual flood-control project will not be as simple as the hypothetical one considered herein, but the method illustrated is capable of application. Such methods were developed and used in the design of the Miami Conservancy District[8] which, of course, presented a much more complicated plan.

Reservoir and channel improvements both have their places in flood-control work; both have their advantages; and both have their limitations. The design of any given flood-control project should not be concerned solely with the relative merits of reservoirs alone or channel improvements alone, but should be concerned with obtaining the most economical plan, which may often be a combination of reservoirs and channel improvements.

F. Knapp,[9] Esq. (by letter).—In most practical cases the requirements of design for flood-control and water-power projects, or for modern open-river navigation, are of a conflicting nature. A water power plant is required to furnish firm power and dependable output and, therefore, should have a reservoir level as high as possible at all times. The level of the flood-control reservoir, on the other hand, should be kept as low as possible to maintain sufficient storage space to accommodate great floods. In the case of river navigation a certain minimum river stage must be maintained throughout the year. Combining these requirements results in heads for the power plant that vary between wide limits, with resulting inefficiency; at times of reservoir draw-down neither firm power nor dependable output can be guaranteed.

One possibility is to combine the functions of flood control and water power by operating the flood-control reservoir independently of the power plant by means of daily pumped storage. A certain firm head is developed by means of a maximum draw-down limit, in order to reduce the variations of head for the pumps, and the turbines, as much as possible. The power for the pumps is generated at night, by steam, thus increasing the load factor and hence decreasing the cost of the generated energy from these plants. Dependable firm power and output is being supplied by the pumped storage plant during only a few daylight hours. The pumped storage plant is thus operating as a peak load and stand-by plant. The combined effect of peak-load and base-load plants, working in parallel, is to supply the energy at more favorable conditions than could be obtained by steam plants alone.

---

[8] "Hydraulics of Miami Flood Control Project", by Sherman M. Woodward, M. Am. Soc. C. E., Technical Repts., Miami Conservancy Dist., Pt. 7, Chapter 12.

[9] Asst. Hydr. Engr., The São Paulo Tramway, Light & Power Co., Ltd., São Paulo. Brazil.

This solution, of course, is only possible when the flood-control reservoir is developed in connection, and in combination, with a large interconnected power system. A typical example of such a solution of outstanding importance is the Saale Reservoir, in Germany.[10] This development is of special interest in so far as it serves an additional purpose; namely, a provision for increasing the low-water levels of the River Elbe for modern open-river navigation. Such combined use of a single reservoir, or of several reservoirs, requires a combination of the interests of the three functions involved. In the case of the Saale Reservoir the governmental authorities utilize the useful storage throughout the year for navigation purposes. At certain times (generally well known in advance) water is stored to equalize flood waves. An extensive service for reporting river levels and weather conditions serves to predict the behavior of the reservoir and to accumulate experience during the first years. The operation of the reservoir by the Power Company in combination with the demands of the governmental authorities serves to simplify and to combine the conflicting interests.

CHARLES S. BENNETT,[11] M. AM. SOC. C. E. (by letter).—No apologies should be necessary, in connection with this paper, for the elementary cataloging of the steps to be followed; this outline procedure will no doubt have wide usage because of its clear and methodical presentation. The paper constitutes the most complete and practical outline of the method of developing a study of reservoir control of floods which has come to the writer's attention.

The outline of preliminary investigations for a typical stream brings to mind the difficulties encountered in attempting to obtain definite maximum run-off data for such studies. The existing records of rainfall and run-off for most areas are usually fragmentary, even in recent years. The current interest in studies relating to the control and distribution of run-off should emphasize the value of securing continuous and related rainfall and run-off data wherever possible. It seems to the writer that it might be possible for the existing Federal agencies to co-operate to the extent of arranging for more gauging stations at points on the various streams at which the records would tie in with a group of rainfall stations on the drainage area above. It would also be useful if the records would indicate the maximum flood flows at these stations, as well as mean daily flows.

It might be implied from the author's statement under the heading, "Retarding Basin Operation", that the works of the Miami Conservancy District utilize improved channels below the retarding dams for the entire river distance below the dams. The river channels were improved only through the cities below the dams, as the flood-control plan was designed to furnish complete protection to the cities only. Through the reaches of river intervening between these cities, no improvements were made, as the cost of such work would exceed the benefits to rural property. The benefits received by the lands below the dams, but not within the cities, are only the reduction of flood peaks due to the effect of the dams.

---

[10] "Die Saaletalsperre," von H. Kyser, *Elektrotechnische Zeitschrift*, Nos. 28 and 29, 1933.

[11] Engr., The Miami Conservancy Dist., Dayton, Ohio.

The retarding reservoir method of flood control is especially applicable to small streams for the local protection of lands in such areas, and such a system might have limited benefits to larger streams, such as the Ohio River or the Mississippi River, to which the controlled streams are tributary.

An advantage of the retarding-basin method of operation, from the economic standpoint, may be the fact that, in some cases, the agricultural lands within the basins need not be completely withdrawn from use, as is necessary in the case of storage basins. In the Miami Conservancy it has been possible to re-arrange the farm units in the basins and to relocate buildings on the higher sections of the new units, thus permitting continued agricultural use of the greater part of these areas. The lands originally purchased can be returned to private ownership, subject to the necessary flooding easements and restrictions. Of the 30 000 acres which were purchased by the Miami Conservancy District, approximately 20 000 acres have already been resold.

GERARD H. MATTHES,[12] M. AM. SOC. C. E. (by letter).—Reservoirs possess important potentialities as flood-control structures, as ably brought out by the author, but the significant fact remains that surprisingly few have been built for that purpose despite the urgent need for flood control that exists on many streams. In 1920 there were in the United States a paltry dozen reservoirs and retarding basins devoted exclusively to the control of floods. This number includes such minor units as the retarding basin on Dry River, near Watervliet, N. Y., and another, even smaller, on Paxton Creek, for the protection of Harrisburg, Pa. From estimates compiled by the writer in 1920 there were then in the world at large fewer than 100 flood-control structures of these two types. More than one-half of these were in Europe, 21 being in one river basin in Germany. Until a few years ago this paucity of flood-control reservoirs had not changed materially. In the United States, during the past six years, many reservoir projects for flood control have been surveyed and reported on, but to date (1935) few of these projects have reached the construction stage. The chief obstacle appears to be their unwarranted cost.

In general, a storage basin for the handling of large volumes of water ranks among the most costly of hydraulic structures, whatever may be its purpose. In hydro-electric development it is common for the dam and reservoir to represent the largest single item of investment cost, and the one upon which most often hinges the economic feasibility of the entire development. Experience teaches that reservoirs designed exclusively for flood control have been found less likely to be warranted on strict economic considerations, than have reservoirs designed for any other single purpose. There is a steadily growing tendency to build multiple-use reservoirs such that the cost may be prorated among two or more uses, often wholly unrelated. Flood control, which is often included in such combinations, is gaining steadily in importance and is emerging from the incidental by-product class in which it has largely figured in the past, into a well-defined reliable status. Boulder Dam, on the Colorado River, which is designed primarily for flood control and at which power will be generated, as well as water furnished for irrigation, domestic supply, and navigation purposes, is the outstanding example of the multiple-use type. Norris Dam, on the Clinch River, Tygart River Dam, on the head-waters of the Ohio River, and Fort Peck Dam on the Missouri River, are to serve not less than two purposes each. The flood-control reservoirs being built for the Los Angeles District will be operated so as to make available the impounded waters for irrigation and water supply. The reservoirs of the Muskingum Conservancy District will serve other purposes beside flood protection to the Muskingum Valley. Credit should be given the larger of the existing Western irrigation reservoirs which derive their water supply mainly from flood run-off, for having reduced flood flow more or less effectively although they have never been operated for that purpose.

---

[12] Prin. Engr., Office of Pres., Mississippi River Comm., Vicksburg, Miss.

The conclusion is inescapable that the multiple-use reservoir is destined to play an increasingly important rôle in the future. This is supported by the following considerations: (*a*) In the United States, to date, only densely populated, highly industrialized areas can afford the cost of reservoirs for the sole purpose of flood control; (*b*) costly impounding structures for the sole purpose of preventing flood damage to small towns and farm lands situated in natural flood-plains can rarely be justified on a strictly economic basis; (*c*) to waste flood waters immediately they have been impounded, as must be done at retarding basins or at reservoirs designed solely for flood control, is contrary to the modern concept of water resources utilization; and, (*d*) multiple use of reservoirs divides the burden of cost between two or more interests, and the increased benefits which result are more likely to place the undertaking on a sound economic basis.

Many of those who have written about the incompatibility of operating a reservoir, or system of reservoirs, for joint power development and flood control have overlooked the fact that this incompatibility resides not so much in the physical aspects of operation as in the divergence of interests which exists between corporate use and public conservation of water. The former is ever concerned with earning adequate returns to the investor; the latter with the greatest good to the public at large. There can be no unanimity of purpose, nor any "give and take," as between such divergent interests.

The author has done a great service in analyzing the various possibilities of the combined uses of reservoirs and by pointing out the respective shortcomings and merits of the various combinations. It should be emphasized, however, that even in the most advantageous physical set-ups outlined by him, absolute agreement between the interests sharing the storage space in a reservoir is essential. This agreement should amount to a unity of purpose and should take the form of unified control, if the best results are to be obtained because, in any combination, one usage may be called upon, at times, to sacrifice to the other. Conceivably, this may be unavoidable even where separate storage space is allotted to each usage. Unified control by a single interest, whether Federal, State, or other competent public agency, appears to be the only satisfactory solution whenever flood control is involved.

The practical operating difficulties of flood-control reservoirs have been much under-rated. This is attributable, in large measure, to the excellent performance of the flood-control system of the Miami Conservancy District, Ohio, which has caused a widespread but erroneous belief that the automatic retarding basin principle utilized there is applicable to any drainage basin where the topography is at all favorable toward retarding-basin construction. Primarily, it is adapted to relatively small water-sheds where flood run-off exhibits little variation in synchronization of flood crests from the various tributaries, and permits the development of a simple plan of operation. Once installed, this plan admits of no variation because the outlets are of fixed size. The operation of the retarding basins in the Miami Valley is singularly favored by the almost ideal crowfoot arrangement of the main tributaries. An important consideration which entered into the adoption of retarding basins there was the desire of the people of the Valley to preclude any possible use of the dams by power or other interests, whereby any of the flood-control features might later be sacrificed.

The retarding principle cannot be relied upon for satisfactory performance in large drainage basins, or in any flood-control system in which a large number of dams and reservoirs are required, or where the tributary system is at all complex. Automatic control of one tributary in such cases is likely to lead to unfavorable synchronization with flood crests from other tributaries similarly controlled, or not controlled, or with flood crests in the parent stream, as pointed out by the author. Other drawbacks of the retarding-basin principle are: Inflexibility in emergencies; steadily decreasing rate of outflow as the water level above the dam sinks; inability to flush out the river channel below for the purpose of removing sediment deposits; and impracticability of raising the height of the dam and the water level at some future time when loss of storage capacity due to sedimentation should make such a step desirable, without having to rebuild the fixed outlet to meet the increased head imposed on it.

As against these limitations gate control possesses many advantages, not the least of which is that a gate-controlled dam can be made to operate as a retarding dam, whenever this may be advantageous, by merely locking the gates at a predetermined height so as to pass water at the same rates of discharge as a dam provided with a retarding outlet. At other times such a dam permits of gate operation, and this, during the flood season and especially during emergencies, becomes of inestimable value. Gate control means virtually complete control. While gates can never be made as fool-proof as a fixed retarding outlet, and require replacement at intervals, it is entirely practicable to design them so as to prevent their being tampered with, short of the use of explosives.

Whatever may be the hazards of gate control with its implied reliance on human judgment and ingenuity, duly commented on by the author, the fact remains that there are many localities in the United States where no other type of reservoir control can safely be used. For instance, wherever major floods are caused by tropical storms the possible recurrence of which at 7-day or 8-day intervals must always be anticipated, reservoirs for flood control must be designed so as to permit being emptied completely on short notice in order to be available for the coming flood. Nothing short of complete control by means of ample gate capacity in the hands of experts can meet such a situation. At such times, it will be better to cause the rapid release of the impounded waters to overtop the river banks, thereby causing moderate inundation, than to permit the crest of the succeeding flood to go over the spillway and cause a disastrous inundation. This is choosing the lesser of two evils. A retarding basin, in such instance, would admit of no choice and might become a liability rather than an asset. The 7-day to 8-day interval between great floods is a matter of record on a number of streams along the South Atlantic Coast, and is equally likely to occur on the rivers of the Gulf Coast. Of course, the ideal plan would give reservoirs or retarding basins storage capacities sufficient to absorb the flood waters from more than one tropical storm, but this is seldom feasible for economic reasons.

Gate control is indispensable also in areas where cloudburst run-off must be coped with, especially where the flood-control reservoir controls a small drainage basin. Unless its operation is in competent hands, it may well happen that cloudburst run-off immediately following a major storm finds the reservoir full to the spillway level and must pass over the spillway. This cloudburst problem is peculiar to all streams in the Rocky Mountain region, to a large number of Appalachian streams, and to most of the streams in the arid regions. The retarding basins in the Miami Valley were not designed with either of these considerations in mind. The region is out of the path of tropical storms and cloudburst floods are not of great intensity there. The largest, at Englewood, if filled to near spillway level, requires 23 days to empty; two, similarly, would be emptied in from 7½ to 8 days, and the remaining two in 4½ days.

The flood-control system of the Miami Conservancy District is the most successful of its kind in the United States, but it is not a useful yardstick to apply indiscriminately to other flood problems. It would not be surprising if this system should be destined to remain in a class by itself. It never would have been economically feasible had it not been for the high valuation of property, great density of population, and the importance of the industries located in the flood zone. The 36 000 acres within the basins furnish protection to 80 000 acres of valley lands, a ratio of 1 acre of basin lands to about 2 acres protected, some of the farm land acreage being only partly protected. In addition to the five dams, cut-offs, channel dredging, and levees were required, bringing the net construction cost to nearly $30 000 000, no portion of which is covered by Federal grant or assistance.

The entire investment cost is being repaid by the interests benefited, about one-half by direct assessment on 77 000 pieces of property, and the remainder through tax levies for protection to public property of cities and counties. The 67 000 acres of farm lands collectively were assessed less than 5% of the total direct assessment levied. The remaining 13 000 acres of city and industrial property pay the other 95 per cent. As the farm lands are by no means second rate, these figures are significant in illustrating how little farm land can afford to be assessed on the strict benefit basis required under the Conservancy Act. It proves also that the remainder of the property in the flood zone was worthy of flood protection in perhaps an exceptional degree, as it shouldered successfully a heavy burden. There are few localities in the United States in need of flood protection to-day that could afford to finance a purely flood-control project on any such scale.

George R. Clemens,[13] Assoc. M. Am. Soc. C. E. (by letter).—A number of reservoir projects which involve flood control, either as a sole purpose or as an important one of several purposes, have been undertaken since this paper was prepared. Many other projects are being actively considered, and some will probably reach the construction stage in the near future. It has been extremely interesting to note that general consideration of the various types of projects fits readily into the classification outlined in the paper. Particularly is this true of the analysis of combined projects and the possibilities and limitations of combined use of the same reservoir for several purposes. The discussions of Mr. Knapp and Mr. Matthes have touched on combined use. Certain illustration of the original ideas presented may be helpful.

It was pointed out that, in general, combined use might be divided into two classes: (*A*) Combined purpose storage (combined use for more than one purpose of the same storage space); and (*B*) combined interval storage (a combination of several purposes by allotting portions of the reservoir to each separate use).

It was stated (and the discussion supports this viewpoint) that Use (*A*) is more difficult to obtain than Use (*B*). An example of each is presented for illustration.

On the Sacandaga River above Albany, N. Y., the Hudson River Regulating District has constructed a reservoir for the purpose of "regulating the flow of the Hudson and Sacandaga Rivers as required by the public welfare." This involves control of floods; increase of low-water flow in part of the New York State canals; increase of low-water flow in the interest of public health; and regulation of flow for water power. These several more or less conflicting requirements are met by use of water stored in a reservoir of 690 000 acre-ft capacity, operated by the District to comply as nearly as possible with the requirements of the law that created it. There is a large measure of power development of the 750 ft of fall in the Hudson and

---

[13] Senior Engr., Mississippi River Comm., Vicksburg, Miss.

Sacandaga Rivers from tide-water at Troy, N. Y., to the storage reservoir dam 80 miles up stream, as shown on the profile (Fig. 14). Obviously, there are times when each interest must sacrifice something to the other. According to data furnished by Edward H. Sargent, M. Am. Soc. C. E., Chief Engineer of the District, the results given in Table 9, were obtained in the years 1932, 1933, and 1934. The effect of operation during the year 1933, as reflected in the flows of the Hudson River at Spier Falls, N. Y., is shown in Fig. 15.

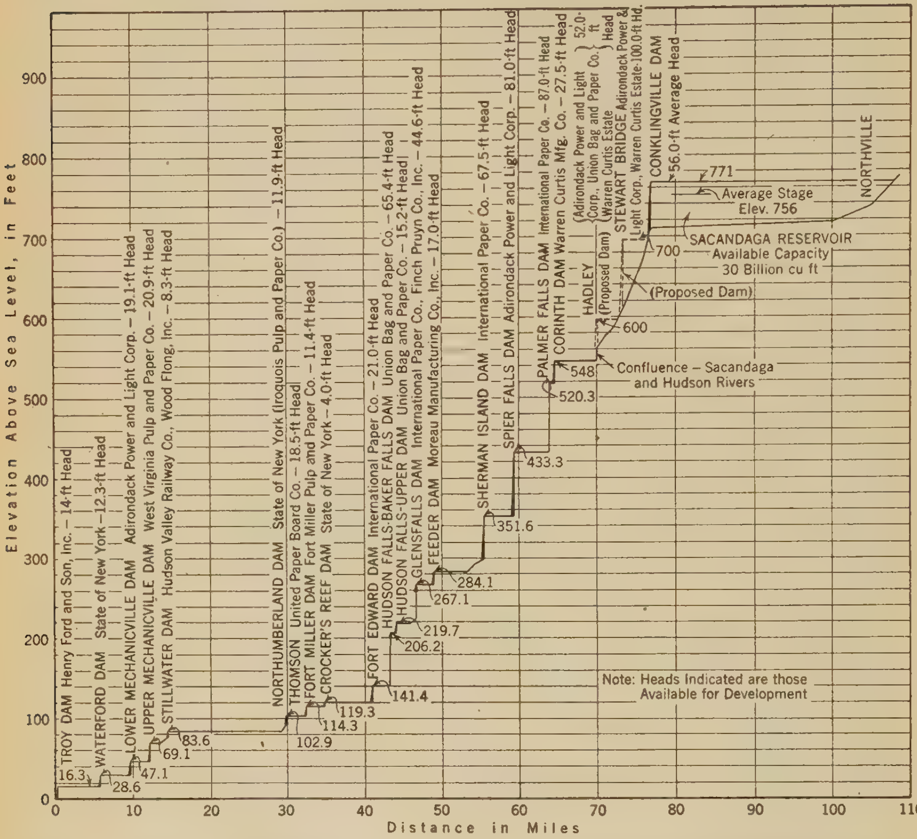


FIG. 14.—PROFILE OF THE HUDSON AND SACANDAGA RIVERS FROM TROY TO NORTHVILLE, N. Y.

Detail study of the operation charts reveals many interesting facts, but the most important is that, with a unified control, it is possible to obtain a joint use of storage in spite of the many conflicting demands. As can be noted from Table 9, floods are materially reduced and low-water flow is substantially increased. It is probable that the most important factor in obtaining the results with this combination reservoir is the unified control. This point was raised in Mr. Matthes' discussion and is believed to be very important to the success of large projects, particularly where it is sought to combine flood-control and water-power uses.

An example of the separate storage-space type of combination reservoir is found in Norris Dam under construction by the Tennessee Valley Authority on the Clinch River, a tributary of the Tennessee River. According to reports it is understood that the upper part of this reservoir comprising 1 400 000 acre-ft is to be reserved for flood-control storage, whereas the lower part of the storage is to regulate the flow for navigation and power development. Operation of this type of reservoir should be more simple than that of the Sacandaga Reservoir as there is a definite space allotted to each use. With intelligent operation it is possible, as pointed out in the basic paper, that some benefit may result to the separate purposes by overlapping the use of the storage space.

TABLE 9.—EFFECT OF OPERATING SACANDAGA RESERVOIR ON HUDSON RIVER AT SPIER FALLS, NEW YORK

(Units are in cubic feet per second)

Year	MAXIMUM FLOW		Decrease (daily average)	APPROXIMATE LOW		Increase (daily average)
	Without reservoir (computed) (2)	With reservoir in operation (3)		Without reservoir (computed) (5)	With reservoir in operation (6)	
(1)			(4)			(7)
1932	38 000	22 000	16 000	1 200	3 000	1 800
1933	51 000	27 000	24 000	600	2 800	2 200
1934	30 000	14 000	16 000	400	2 400	2 000

The writer appreciates the comment in discussion of his paper. The various statements by Mr. Bennett and Mr. Matthes regarding the Miami Conservancy District serve to amplify and extend the comments on that project in the paper. As stated by Mr. Stanley certain parts of the paper dealt with large streams, such as the Mississippi River, but it is believed that the general principles stated and classifications of projects outlined apply equally well to smaller systems. In fact, as both Mr. Bennett and Mr. Matthes point out, the retarding basin systems apply generally to smaller water-sheds and probably would not be satisfactory for large streams, such as the Mississippi. Control of outlets is quite generally a desirable feature even if the main purpose is to retard flood flow.

As brought out in Mr. Matthes' discussion the use of reservoirs for flood control alone has been limited It is also rather seldom that the cost of construction can be borne by the interests directly benefited. There are many indeterminate benefits however, to a reservoir system that at present are impossible to evaluate and assess. The general effects on business and transportation of the elimination of flood damage or even a flood hazard in any considerable area is something difficult to set down in dollars and cents. Employment of men on constructive work during periods of general unemployment has certain value. Prevention of interruption of business opera-

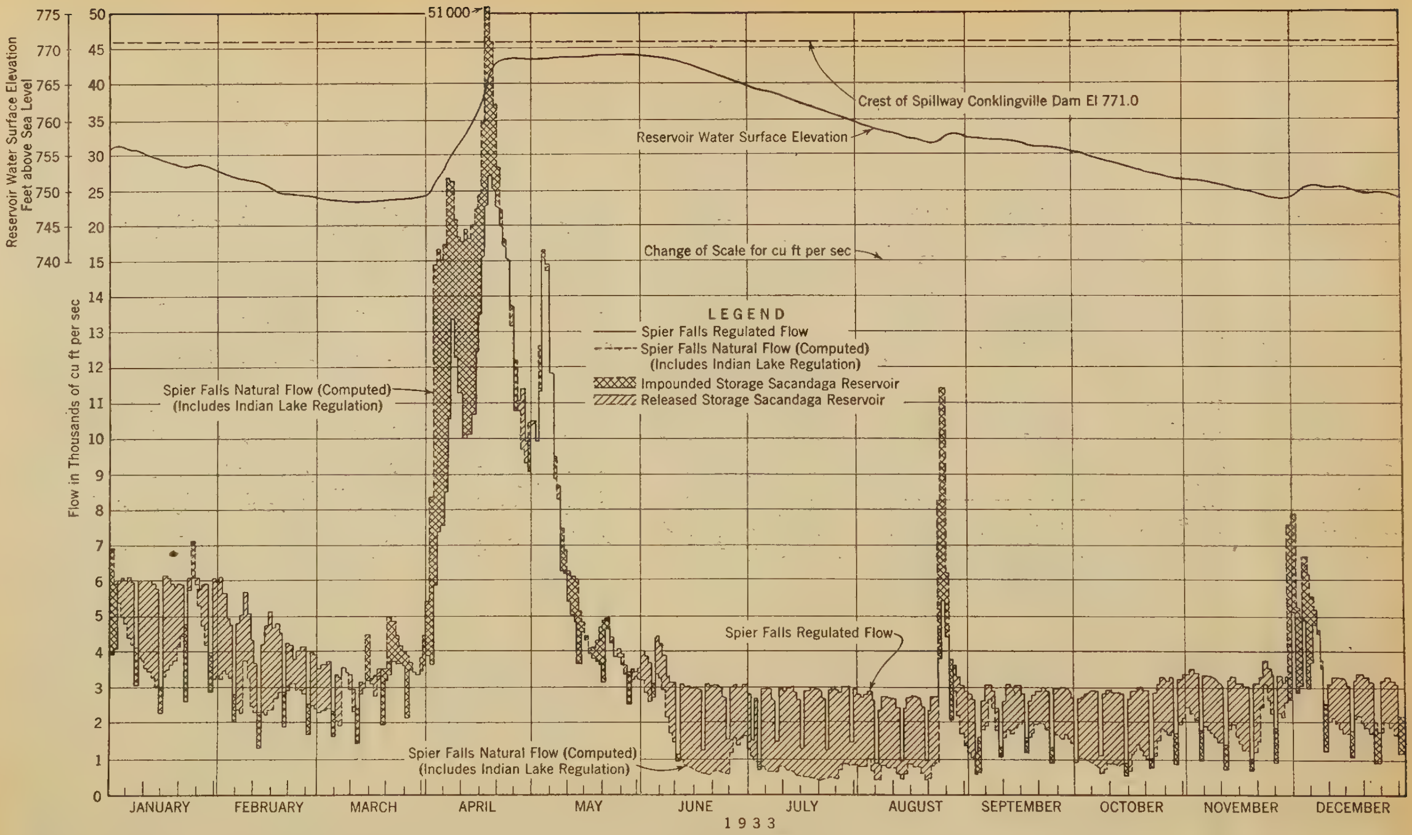


FIG. 15.—HYDROGRAPH OF HUDSON RIVER AT SPIER FALLS, N. Y., SHOWING EFFECT OF SACANDAGA RESERVOIR REGULATION FOR 1933 AND WATER SURFACE ELEVATION IN RESERVOIR.

tions, even where no so-called physical damage is sustained, has some community value. A plant that is forced to shut down and discontinue payrolls because of a flood causes a community loss even if there is no loss of property at the plant. As more flood-protection projects are constructed and the beneficial effects of existing projects become more thoroughly appreciated, it is probable that better methods of analyzing these economic losses will be developed. Methods will be devised for determining the value of benefits now classed as indefinite, indeterminate, or perhaps not even recognized. Such knowledge may have an important bearing on the planning of the flood-protection projects of the future.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 1911

STRESSES IN SPACE STRUCTURES

By F. H. Constant,[1] M. Am. Soc. C. E.

With Discussion by Messrs. William R. Osgood, L. E. Grinter, Charles M. Spofford, A. H. Finlay, and F. H. Constant.

Synopsis

The analysis of any but the simplest space structures is usually a tedious operation because of the three-dimensional nature of the problem. In the method herein presented most of these difficulties are avoided by substituting for the forces and stresses of the three-dimensional structure, a corresponding system of co-planar forces, the magnitudes of which may then be found by any of the well-known, simple processes ordinarily applied to the solution of planar structures. These co-planar forces, in turn, are readily translated back into the corresponding space stresses by dividing them by the cosine of the slope of the space member.

The method was originated by Professor Benjamin Mayor,[2] of the University of Lausanne, at Lausanne, Switzerland, but his presentation is based on the principles of the linear complex, a mathematical field little known to engineers and extremely difficult to follow. In this paper the writer has avoided the use of the complex and has based the theory on well-known principles of mechanics.

A system of forces is reducible at any point in space to a single resultant, R, and to a couple, S. Both may be represented by vectors acting at the point, making an angle, α, with each other. The vector, S, acts perpendicular to the plane of the couple, and by the usual convention, points in the direction of translation of a right-handed screw when rotated in the direction of the couple. For a clockwise couple, S will point toward the back of a watch and will be called positive.

Note.—Published in May, 1934, *Proceedings*.

[1] Prof. of Civ. Eng., Princeton Univ., Princeton, N. J.

[2] "Introduction à la Statique Graphique des Systèmes de l'Espace," par Benjamin Mayor, 1926.

If the point of reference is moved, R will remain invariable, but S, in general, changes in magnitude and slope. There is one line, the "central axis," parallel to R, in which the direction of S coincides with that of R; in other words, $\alpha = 0$ and both R and S_0 act along the central axis. (The particular value of S at the central axis is S_0.)

At any other point, distant d from the central axis, R still remains parallel to that axis, but S lies in the plane normal to d and makes the angle, α, with R represented by the relation,

$$\tan \alpha = \frac{R\,d}{S_0} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

When d is infinite, $\alpha = 90$ degrees. Every value of S has the common property that its component in the direction, R, is constant and equal to S_0, or,

$$S \cos \alpha = S_0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

In fact, every Vector S is tangent to a certain helix that is wrapped around a circular cylinder. The helix becomes steeper as the radius of the cylinder is less.

Besides the reduction to the screw type the system is also reducible to two resultants not lying generally in one plane. This reduction may be made in an infinite number of ways. The line of action of one of the vectors, F, may be given, and the other, F', may be required to lie in a plane having a given slope. The position of F' in this plane then becomes determinate and is said to be conjugate to F.

In Fig. 1, the line, $C\ A$, is the central axis which, for convenience, will always be assumed to be vertical. At some point, O, distant d from the line, $C\ A$, the system is equivalent to the vertical resultant, R, and a couple lying in some plane, $D\ E\ L\ G$, and represented by the normal vector, S. Assume, first, that one of the conjugate forces, $F = O\ J$, lies in the vertical plane containing $C\ A$, and that it intersects the latter at C. It is required to find its conjugate line in a horizontal plane. Through C draw a horizontal plane, $B\ C\ D\ E$. The line, $E\ D$, is the intersection of Planes $D\ E\ L\ G$ and $B\ C\ D\ E$. If Vector $O\ J$ is one of the conjugate forces its vertical component must be equal to R. Its horizontal component, $O\ H$, and an equal and opposite force, $E\ K$, acting along $E\ D$, will form a couple equal to S, because:

$$\overline{O\,H}\,l = \frac{R\,l}{\tan \phi} = \frac{R\,l\,d}{h} = \frac{S_0\,l \tan \alpha}{h} \text{ (from Equation (1))} = \frac{S\,l \cos \alpha \tan \alpha}{l \sin \alpha} = S$$

Therefore, $\overline{O\,J} = F$, and $\overline{E\,K} = F'$, are conjugate forces exactly equivalent to the original system.

Let b be the horizontal distance of F' from F_h. Then,

$$b = \frac{h}{\tan \alpha} = \frac{d \tan \phi}{\tan \alpha} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

From Equation (1), $\tan \alpha = \dfrac{R\,d}{S_0} = \dfrac{d}{a}\left(\text{when } a = \dfrac{S_0}{R}\right)$; whence,

$$b = a \tan \phi \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

The horizontal position of F' (or its projection) depends then only on $\frac{S_0}{R} = a$, and the slope of F. As nothing is known about the original (central) system, $\frac{S_0}{R} = a$ may be given any convenient value suitable for the scale of the drawing. As R is an independent parameter, ϕ may have any desired value, and if the vertical plane containing F is swung around $O\,I$ as an axis, the central axis will likewise swing about this axis and lie at some distance, d, from O consistent with the construction of Fig. 1. For each position and magnitude of F, there is a corresponding central system, but as the selection of a is arbitrary for each one, the same value of a may be used for all. Thus, all these systems are tied by the single relation, $\frac{S_0}{R} = a =$ a constant. For every force, F, then, acting at a given point, O, there exists a conjugate force, F', lying in a horizontal plane, the position of which may be determined at once by the use of Equation (4).

Let a system of forces acting at a joint of a space structure be designated by F_1, F_2, F_3, etc., and the corresponding conjugates by F'_1, F'_2, F'_3, etc. Each force and its conjugate are the resultants of a certain central system defined by a couple, S_0, and a resultant, R, acting along a central axis perpendicular to the plane containing F'. Vectors S_0 and R are different for each system, but in each one they satisfy the condition, $\frac{S_0}{R} = a =$ a constant. Moreover, all the central axes are parallel. Hence, all the forces, F_1, F_2, etc., and their conjugates, are reducible to a single central system defined by the relation,

$$\Sigma S_0 = a\,\Sigma R \qquad (5)$$

in which, ΣS_0 and ΣR are the sums of the individual vectors, S_0 and R, and a is any arbitrary constant. If the forces, F_1 F_2, etc., are in equilibrium, $\Sigma R = 0$; whence, from Equation (5), $\Sigma S_0 = 0$. Hence, there is no rotation in an horizontal plane. F_1, F_2, etc., being in equilibrium, cannot produce any rotation or translation in a horizontal plane. Neither, therefore, can the conjugates, F'_1 F'_2, etc. (or their projections upon a common horizontal plane). Hence, they, too, are in equilibrium. This is an important deduction, and the central principle upon which the method herein presented is based.

The bearing of this conclusion upon the solution of a space structure is now apparent. The forces meeting at a joint may be replaced by a system of non-concurrent, co-planar forces that are in equilibrium and the magnitudes of which may be found by the simple methods applicable to planar forces. Since the conjugate forces are parallel and equal to the horizontal components of the original stresses (but with opposite signs), the latter may be found by dividing each conjugate stress by the corresponding value of $\cos\phi$, and reversing the sign.

As determined from Equation (4), b apparently permits of two locations of F', namely, $E\,D$ on one side of $O\,A$, and $L\,G$ on the other. If, however,

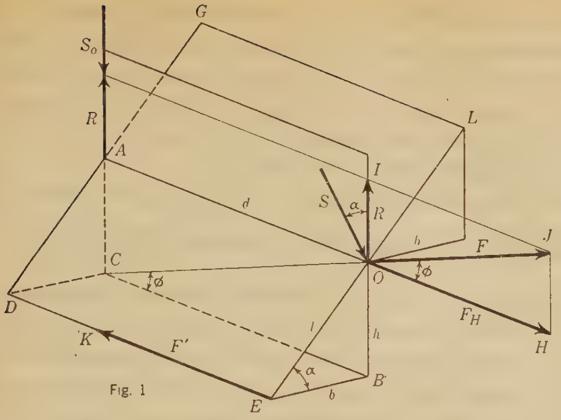

G
S₀
R
A
L
I
d
S
α
R
h
F
J
C
Ø
O
Ø
D
F_H
K
F′
l
h
H
α
B
b
E

Fig. 1

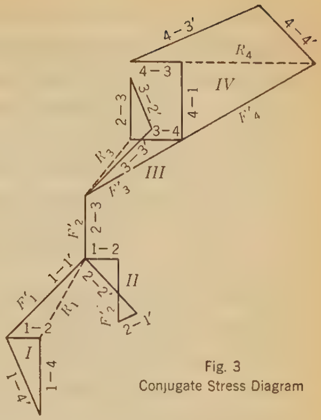

4−3′
4−4′
R₄
4−3
IV
2−3
3−2′
4−1
F′₄
3−4
R₃
3−3′
III
F′₃
2−3
F′₂
1−2
1−1′
II
2−2′
F′₁
R₁
F′₂
2−1′
1−2
I
1−4′
1−4

Fig. 3
Conjugate Stress Diagram

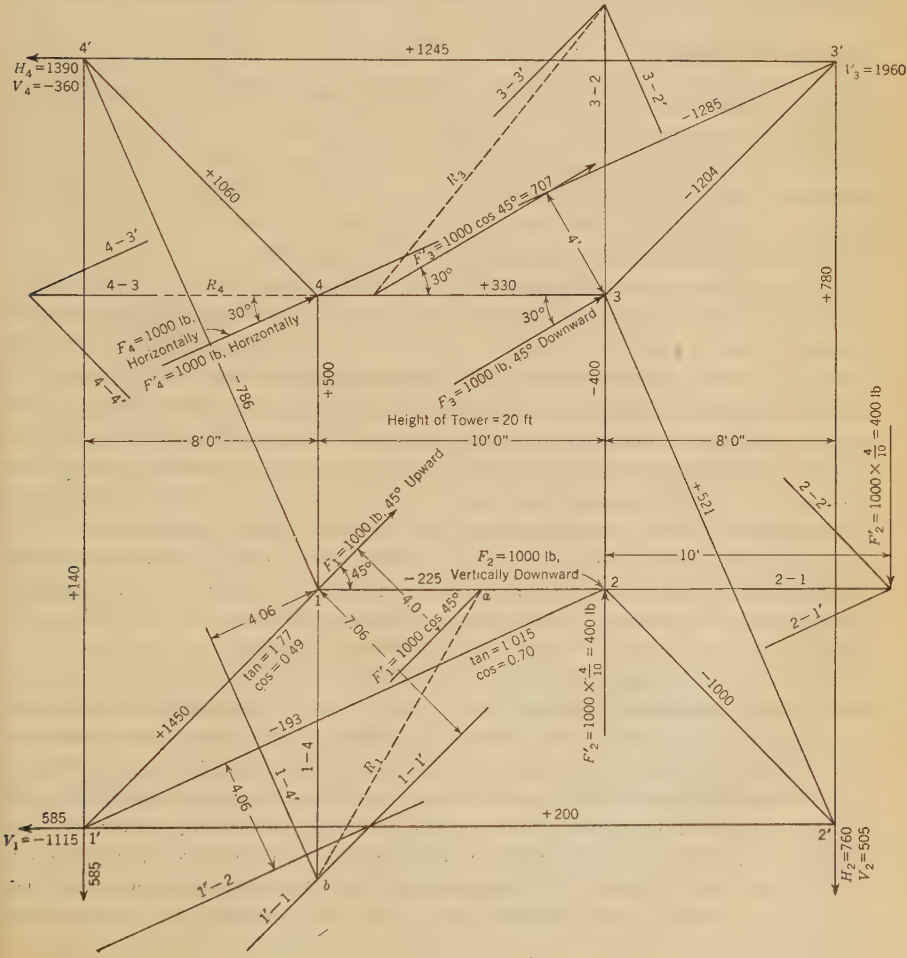

4′
+1245
3′
H₄ = 1390
V₄ = −360
V₃ = 1960
3−3′
3−2
3−2′
−1285
+1060
R₃
F′₃ = 1000 cos 45° = 707
−1204
4′
4−3′
4−3
R₄
4
30°
+330
3
+780
30°
F₄ = 1000 lb. Horizontally
30°
F′₄ = 1000 lb, Horizontally
F₃ = 1000 lb, 45° Downward
4−4′
−786
+500
−400
Height of Tower = 20 ft
8′ 0″
10′ 0″
8′ 0″
+521
F′₂ = 1000 × 4/10 = 400 lb
2−2′
F₁ = 1000 lb, 45° Upward
F₂ = 1000 lb, Vertically Downward
10′
+140
45°
−225
2
2−1
1
a
4.06
4.0
7.06
F′₁ = 1000 cos 45°
2−1′
tan = 1.77
cos = 0.49
tan = 1.015
cos = 0.70
F′₂ = 1000 × 4/10 = 400 lb
−1000
+1450
−193
1−4
R₁
1−1′
1−4′
4.06
+200
585
V₁ = −1115
1′
2′
H₂ = 760
V₂ = 505
585
1′−2
b
1′−1

Fig. 2.—Plan View of Tower

S, R, and F are directed as shown in Fig. 1, it is evident that F'_1 must act along $E\ D$ if its equal and opposite mate, $O\ H = F_h$, when combined with R, is to give a resultant acting in the direction, $O\ J$. Hence, the following rule:

Assuming S_0 and R to be positive and to act in the directions shown in Fig. 1, if the observer stands at O and looks along F as it ascends from the horizontal plane, the conjugate, F', will lie on his right.

This rule holds whether F acts away from or toward O, because when R is negative, S is likewise negative ($\frac{S_0}{R}$ always being assumed positive), and F' will still have the position shown in Fig. 1, but with its direction reversed.

If F is horizontal ($\phi = 0$), from Equation (5) $b = a \tan \phi = 0$, which indicates that the projections of F_h and F' coincide. If F is vertical ($\phi = 90°$), $F' = 0$, and b is ∞. Therefore, F' does not affect the conditions of translation of the conjugate forces; but it does produce a moment in the horizontal plane equal to S_0. In this case $R = F$, and from $\frac{S_0}{R} = a$, $S_0 = R\ a = F\ a =$ moment of F' about the central axis. Since F' is at infinity, this is equivalent to saying that it enters into the equations as a couple with a constant moment of $S_0 = F\ a$. Fig. 1 shows that when $F = R$ acts upward, the conjugate moment, $F\ a$, is clockwise. If F is nearly vertical, F' will fall beyond the limits of the drawing. In this case, F may be replaced by its vertical and horizontal components and treated as before.

The solution of the conjugate stresses is best effected by graphic or semigraphic methods. Only the horizontal projection of the structure is needed. Regarding each joint in turn as the center, O, draw the conjugate lines for all the external forces and members meeting at the joints. Those for horizontal forces pass through the joint and coincide with the corresponding lines in the plan view. The conjugates of the sloping members and forces will be parallel to their horizontal projections and may be located by the use of Equation (4). In a symmetrical dome or tower, there are relatively few different slopes, and most of the conjugate lines may be located by symmetry. The conjugate of a member will need to be drawn only once, as it will serve for the joints at both ends of that member. The steepest slope will fix the maximum practical value of a, unless it is more desirable, in the case of a very steep member, to work with its vertical and horizontal components.

When several members meeting at a joint of the space structure lie in one plane, their conjugates intersect in a point. If not more than one unknown force or stress lies without this plane, the value of its conjugate stress may be found by taking moments about the common intersection point of the remaining unknown conjugate stresses. When a joint is encountered at which more than two unknown stresses exist outside a plane containing the remainder, and the structure is apparently, but not actually, indeterminate, the well-known devices used in the analysis of space structures, such as the principle of the exchange of members, may likewise be applied to the solution of the conjugate stresses.

The conjugate stress, divided by the cosine of the slope, will give the space stress with its sense reversed. Changing its direction and applying it to the joint the stress is tension or compression according to whether it acts away from or toward the joint. To avoid this double reversal of sense the true direction of the horizontal components of the known forces may be used in the conjugate system, and the resulting conjugate will then have the correct sense. The rule for vertical forces will then read: When Force F acts upward, the conjugate moment, $F\ a$, will be counterclockwise, and will lead to the correct signs of the actual stresses.

The following simple example will suffice to illustrate all the foregoing points. The symmetrical four-legged tower in Fig. 2 is 20 ft high, with loads at the four upper joints, as follows: 1 000 lb at Joint 1, directed 45° inward and 45° upward; 1 000 lb vertical and downward at Joint 2; 1 000 lb at Joint 3, directed outward at 30° with Member 3-4 and 45° downward; and 1 000 lb horizontal at Joint 4 at 30° with Member 4-3. A value of $a = 4$ ft was chosen for the location of the conjugates. For the diagonal member, 1-4′, tan ϕ is 1.015, and since it slopes downward from the observer stationed at Joint 1, the conjugate lies on his left at a distance of $4 \times 1.015 = 4.06$ ft. Similarly, the conjugate for Member 1-1′ (tan $\phi = 1.77$) lies 7.06 ft to his left as he looks down the column, 1-1′. Both are parallel to the respective horizontal projections. The conjugates for all the other members are quickly drawn from symmetry, and those for the external forces at Joints 1, 3, and 4, are located in a similar manner and are given the directions of the horizontal components of the actual forces. The force at Joint 2, being vertical and downward, is equivalent to a conjugate couple, $1\,000 \times 4 = 4\,000$ ft-lb, acting in a clockwise direction.

A graphical solution, somewhat akin to a Maxwell diagram (Fig. 3) has been adopted, although, of course, all the conjugate stresses could just as readily have been found by the use of the three equations of equilibrium. The graphical solution of Joint 1, for example, depends on the fact that the resultant of two of the conjugate forces must pass through the point of intersection and must balance the resultant of the other three. Thus, the conjugates, F'_1, of the member, 1-2, meet at a and their resultant, R_1, must pass through b, the point of intersection of the conjugates of the members, 1-4′, 1-1′, and 1.4. The force triangle (Fig. 3) corresponding to the first three gives R_1, and the stress in Member 1-2. In similar manner, the other ring stresses are found, including that in Member 1-4. Then, returning to the forces meeting at the point, b, with R_1 and the stress in Member 1-4 known, the remaining two conjugates are obtained by completing the force polygon (Fig. 3) for the joint. The other joints are solved in a similar manner. At Joint 2, in order to take account of the clockwise moment of 4 000 ft-lb, two equal and opposite forces are introduced, one at the joint and the other at the intersection point of the remaining three conjugates. Since these points are 10 ft apart, the magnitude of the forces constituting the clockwise couple must be 400 lb.

At the base of the tower, Joint 1′ is fixed, Joint 4′ has no horizontal restraints, and Joints 2′ and 3′ are restrained in directions perpendicular to

Members 1′-2′ and 1′-4′, respectively. The vertical reactions are most quickly obtained by taking moments about the base joints of the known conjugate stresses of the column and diagonal meeting there. Thus, for V_1, taking moments about Joint 1′:

$$
\begin{array}{lr}
\text{Moment of column conjugate} = -707 \times 7.06 = & -4\,991 \\
\text{Moment of diagonal conjugate} = +135 \times 4.06 = & +548 \\
\hline
V_1 a \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots = & -4\,443 \\
V_1 = 4\,443 \div 4 \ldots\ldots\ldots\ldots\ldots\ldots = & -1\,111
\end{array}
$$

Since the moment, $V_1 a$, is counterclockwise V_1 acts upward.

The stresses in the bottom members and the horizontal reactions are obtained by a resolution of the conjugate stresses for the column and diagonal in the required directions, beginning with the free joint, 3′. The stresses and data for the solution of the structure are shown in the plan view (Fig. 2).

The advantages of the method increase with the complexity of the structure. If there are several cases of loading, the conjugates for the members remain unchanged, and only those for the external forces must be added in each case.

CONCLUSION

The use of articulated space structures has been limited in the United States, due possibly to the involved calculations or the more complicated character of the fabrication. There are many occasions, however, when they could be used to advantage. The method outlined herein should simplify the computation of the stresses considerably, and with the development of welding and other modern facilities in shop and field, such structures are likely to be used more frequently.

DISCUSSION

William R. Osgood,[3] Assoc. M. Am. Soc. C. E. (by letter).—Before reading this interesting paper it had never occurred to the writer that the equilibrium of a set of concurrent forces in space could be established by determining the conditions of equilibrium of a set of co-planar forces related to the set in space. Professor Constant is to be thanked for calling the attention of American engineers to the method.

The convenience of the method is so great that a study of the fundamentals from a slightly different point of view may be desirable. The following derivation, which seems simpler than Professor Constant's presentation, may be helpful to others in understanding the method.[4]

Let O in Fig. 1 represent the joint of a structure in space, and let F represent one of a system of forces, F_1, F_2, F_3, ..., F_n, acting at O. Each of these forces may be replaced by a horizontal force (reversed in the diagram):

$$F' = F \cos \phi \qquad (6)$$

anywhere in a vertical plane parallel to F, a vertical force,

$$R = F \sin \phi \qquad (7)$$

in a plane containing F and with the line of action at such a distance, d, from O that,

$$Rd = F' h \qquad (8)$$

and a horizontal couple,

$$S_0 = F' b \qquad (9)$$

By adjusting the values of d and S_0, the distances, h and b, respectively, may be taken at pleasure. If all the distances, h, measured vertically, are taken the same for all the forces of the given system, then F'_1, F'_2, F'_3, .., F'_n form a co-planar set; and by choosing, suitably, the distances, b_1, b_2, b_3, ..., b_n, measured horizontally perpendicular to F_1, F_2, F_2, .., F_n, respectively, it is possible to assure equilibrium of the co-planar set. Since the forces, F', are planar projections of the forces, F, which are in equilibrium, the resultant of the forces, F', cannot be a force; and if the condition,

$$\Sigma F' b = 0 \qquad (10)$$

can be satisfied, the resultant will not be a couple. Equations (9), (6), and (7) give,

$$b = \frac{S_0}{R} \tan \phi \qquad (11)$$

---

[3] Materials Testing Engr., National Bureau of Standards, Washington, D. C.

[4] See, also, "Graphische Statik räumlicher Kräftesysteme", von R. v. Mises, *Zeitschrift für Mathematik und Physik,* Vol. 64, 1917, p. 209.

and this value of b substituted in Equation (10) gives,

$$\Sigma F' \frac{S_0}{R} \tan \phi = 0 \qquad (12)$$

Now, if S_0 in each case is adjusted so that the ratio, $\frac{S_0}{R} = a =$ constant, for each force, and b is then taken as $b = a \tan \phi$ (Equation (4)), Equation (12) becomes,

$$a \Sigma F' \tan \phi = a \Sigma R = 0 \qquad (13)$$

Equation (13) is seen immediately to be satisfied since ΣR is the vertical component of the resultant of the given system of forces, $F_1, F_2, F_3, \ldots, F_n$, and these forces are in equilibrium. The set of forces, F'_1, F'_2, F'_3, ..., F'_n, obtained as indicated now form a co-planar, non-concurrent system in equilibrium which may be solved in any convenient manner. The forces, F, are then obtained from Equation (6).

L. E. Grinter,[5] Assoc. M. Am. Soc. C. E. (by letter).—An ingenious method for reducing the analysis of stresses in space structures to the problem of stress calculation in a plane is offered by Professor Constant. His explanation of this procedure is stated so clearly, in the simplest terms of mechanics, that one should have little difficulty in following the argument. The method should appeal particularly to those practicing engineers who, because of continual association with planar structures, prefer not to think in terms of stresses in space. The method possibly would have less appeal to the young engineer who has been thoroughly grounded in the study of stresses in three dimensions.

The problem of the analysis of a space structure cannot be solved by a standardized procedure in the manner of a simple roof truss or bridge truss. The explanation lies in the fact that there are many more types of space structures than there are types of planar structures. Of the determinate type alone there is the possibility of widely varying arrangements of reactions and of innumerable bar patterns giving rise to a pedestal, tower, dome, or envelope. Perhaps of greatest significance in this connection is the distinction between those types that may be analyzed by simple equations and those that require simultaneous equations.

Based on the foregoing distinction, the example analyzed by the author should be classed as a simple type of space structure because simultaneous equations are not involved. Referring to Fig. 2, one will note that at Joint 2, Members 1-2, 2-1′, and 2-2′ all lie in a single plane and, therefore, the component of the stress in Member 2-3 taken perpendicular to Plane 1′-2′-2-1 must equal the component of the 1 000-lb load perpendicular to the same plane, which shows the stress in Bar 2-3 to be—400 lb. At each of Joints 1, 2, 3, and 4, two such simple resolutions of forces make it possible to determine all stresses in the members of the top ring and in the sloping diagonals. With the stresses in the diagonals known, one can calculate the stresses in the members of the bottom ring and the values of the reactions by resolving forces in three directions at Joints 3′, 2′, 4′, and 1′, in succession. Simultaneous equations are not required.

---

[5] Prof., Structural Eng., Agri. and Mech. Coll. of Texas, College Station, Tex.

The writer is not certain exactly to what extent this procedure is simplified by the theory of conjugate forces presented by the author. Possibly the main advantage from the common viewpoint would be the reduction of the analysis to a problem in planar graphics as illustrated by Fig. 3. Maxwell's diagram is used so widely by practicing engineers and even by undergraduate students, that the analysis of the space structure by the use of a graphical procedure similar to the Maxwell diagram would be certain to prove attractive.

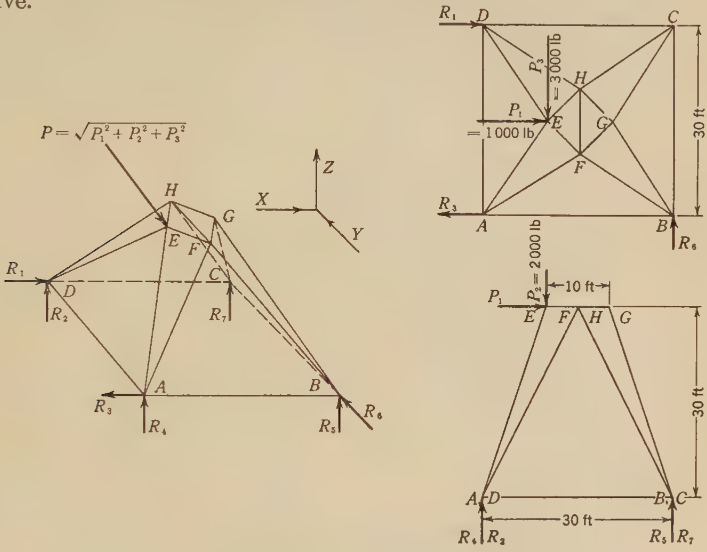


FIG. 4.—LOADS AND DIMENSIONS FOR PEDESTAL.

The writer has been interested in the study of space structures of a more complex type than that illustrated by Fig. 2. The space sketch of Fig. 4 shows a pedestal quite similar to the one analyzed by Professor Constant, but which is far more difficult to analyze by the procedure used by the writer. It will be noticed that there is no starting point for stress calculations at any joint of the upper ring, because at each joint there are only two unknown stresses that lie in a single plane. At the base it is possible to calculate directly the three horizontal reactions needed for stability, but the four vertical reactions are not determinable except in connection with the bar stresses.

A check upon the number of joints, bars, and reactions shows that there is a total of twenty-four reactions and bar stresses and eight joints. Since three equations of statics are available at each joint, the structure is statically determinate. A number of procedures might be adopted for the calculation of stresses, but the following seems to the writer to be as simple as any. The reactions will be calculated first, the value of R_6 from Fig. 4 being given the unknown value, V. The entire structure is treated as a free body; thus $\Sigma F_y = 0$; $R_6 - 3\,000 = 0$; $R_6 = +3\,000$; and, from $\Sigma M = 0$ about R_5 as an axis: $15\,000 - 60\,000 + 30\,R_1 = 0$, hence, $R_1 = +1\,500$ and $R_3 = -2\,500$. Assume that $R_5 = +V$, and then one obtains:

$$\Sigma M_{CD} = 0;\ 30\,000 + 90\,000 - 30\,V - 30\,R_4 = 0;\ R_4 = 4\,000 - V$$
$$\Sigma M_{BC} = 0;\ 30\,000 - 40\,000 + (120\,000 - 30\,V) + 30\,R_2 = 0;\ R_2 = V - 3\,666$$
$$\Sigma M_{AD} = 0;\ 30\,000 + 20\,000 - 30\,V - 30\,R_7 = 0;\ R_7 = 1\,666 - V$$
$$\Sigma F_z = 0;\ -2\,000 + V + (4\,000 - V) + (V - 3\,666) + (1\,666 - V) = 0 \text{ (Check)}$$

TABLE 1.—SOLUTION OF SIMULTANEOUS EQUATIONS

Equation	V	Number	DH	DE	DC	DA	CG	CB	BF	BA	Equations combined
(1)	1	—3666	$\frac{30}{35}$	$\frac{30}{35}$	......	......	......	......	......	......	(1)
(2)	......	1500	$\frac{15}{35}$	$\frac{10}{35}$	1	......	......	......	......	......	(2)
(3)	......	....	$-\frac{10}{35}$	$-\frac{15}{35}$	......	-1	......	......	......	......	(3)
(4)	-1	1666	$-\frac{30}{35}$	......	......	......	$\frac{30}{35}$	......	......	......	(4)
(5)	......	....	$\frac{15}{35}$	......	-1	......	$-\frac{10}{35}$	......	......	......	(5)
(6)	......	....	$\frac{10}{35}$	......	......	......	$-\frac{15}{35}$	-1	......	......	(6)
(7)	1	....	......	......	......	......	$-\frac{30}{35}$	......	$\frac{30}{35}$	......	(7)
(8)	......	....	......	......	......	......	$\frac{10}{35}$	......	$-\frac{15}{35}$	-1	(8)
(9)	......	3000	......	......	......	......	$-\frac{15}{35}$	1	$\frac{10}{35}$	......	(9)
(10)	......	1500	$\frac{30}{35}$	$\frac{10}{35}$	......	......	$-\frac{10}{35}$	......	......	......	(2) and (5)
(11)	......	3000	$\frac{10}{35}$	......	......	......	$-\frac{30}{35}$	......	$\frac{10}{35}$	......	(6) and (9)
(12)	1	-3666	$\frac{10}{35}$	......	......	-2		......	......	......	(1) and (3)
(13)	1	-8166	$-\frac{60}{35}$	......	......	......	$\frac{30}{35}$	......	......	......	(1) and (10)
(14)	1	....	......	......	......	......	$-\frac{10}{35}$	......	......	-2	(7) and (8)
(15)	1	-9000	$-\frac{30}{35}$	......	......	......	$\frac{60}{35}$	......	......	......	(7) and (11)
(16)	......	-7333	$-\frac{60}{35}$	......	......	......	$\frac{90}{35}$	......	......	......	(4) and (15)
(17)	......	1666	$-\frac{30}{35}$	......	......	......	$\frac{20}{35}$	......	......	-2	(4) and (14)
(18)	......	-6500	$-\frac{90}{35}$	......	......	......	$\frac{60}{35}$	......	......	......	(4) and (13)
(19)	......	-2000	$-\frac{20}{35}$	......	......	-2	$\frac{30}{35}$	......	......	......	(4) and (12)
(20)	......	-10666	......	......	......	......	$\frac{50}{35}$	......	......	4	(16) and (17)
(21)	......	-3000	......	......	......	......	$\frac{50}{35}$	......	......	......	(16) and (18)
(22)	......	-1333	......	......	......	6	......	......	......	......	(16) and (19)
Stresses	4433	....	-1133	233	-1083	222	2100	-1222	-3066	1916	....

From a study of the equations, $\Sigma F_z = 0$ at Joints F, G, and H, it becomes clear that the stresses in the following pairs of members are equal: AF and BF, BG and CG, and CH and DH. There are only eight unknown bar stresses and the unknown reaction, V, at Joints B, C, and D. The nine simultaneous equations that can be written for these joints are sufficient for the evaluation of these unknowns. The proper equations based upon the assumptions that all members are in tension and that the positive directions are upward, backward, and to the right, are given in the first nine lines of Table 1. The right-hand term of each equation is zero. Coefficients are determined from the projected lengths of the members. The solution of these simultaneous equations is accomplished by means of the combined equations given in Table 1, Lines 10 to 22, inclusive, whereas the final stresses are found in the last line of the table. With the stresses from Table 1 known, one can easily determine all other stresses in the pedestal as are shown in Fig. 5. These stresses, computed entirely by use of an 8-in. slide-rule, are not exact, but are quite accurate enough for design purposes.

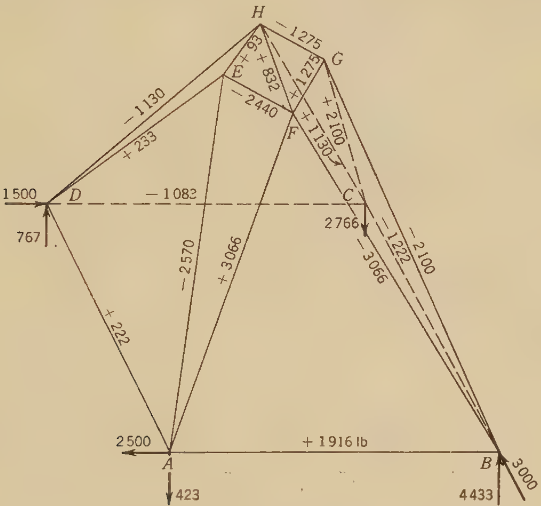


FIG. 5.—STRESSES IN MEMBERS OF PEDESTAL.

The analysis of this pedestal by the method of conjugate forces is not as simple as might be expected. Fig. 6 shows a plan view of the structure with the conjugate forces corresponding to Joint E in place, based on an arbitrarily selected value of 5 ft for the constant, a. Loads P_2 and P_3 from Fig. 4 have been combined into the force, F_2, in order to simplify the conjugate forces. Of course, the six conjugate forces shown form a system of planar forces in equilibrium; but, since there are four unknowns that intersect only in pairs, a direct solution of any one is impossible. Hence, simultaneous equations appear to be necessary.

Undoubtedly, one could introduce the conjugate forces for all other joints and then obtain a solution by simultaneous equations, but the writer is not certain as to just how this is best accomplished. In his own calculations, he was able to reduce the analysis to the solution of nine simultaneous equations involving the forces at Joints *B, C,* and *D* only. It is clear that a study of Joints *E, F, G,* and *H* would involve thirteen unknowns and would not permit of a solution by means of the twelve equations available. Hence, in any method it would seem necessary to tie in the analysis with the lower joints of the structure. Professor Constant would perform a service if he would be willing to present his own solution of this problem by the method of conjugate forces. Unless some device such as the use of substitute members can be found that will reduce the number of simultaneous equations to less than nine, the writer doubts that there would be any simpler analysis than the one herein presented. Any one who has attempted the solution of nine simultaneous equations by an ordinary slide-rule would welcome any method of reducing the number of simultaneous equations involved.

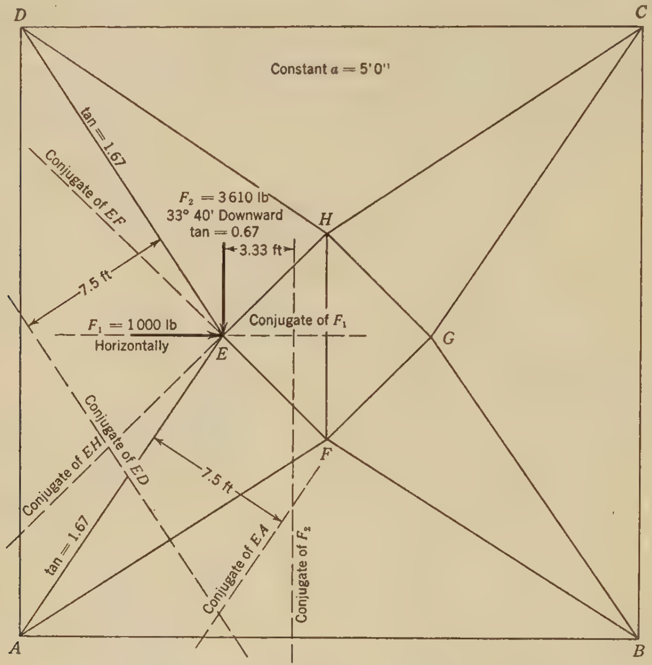

D
C
Constant a = 5'0"
tan = 1.67
Conjugate of EF
F_2 = 3610 lb
33° 40' Downward
tan = 0.67
3.33 ft
H
7.5 ft
F_1 = 1000 lb
Horizontally
Conjugate of F_1
G
E
Conjugate of ED
Conjugate of EH
F
7.5 ft
tan = 1.67
Conjugate of EA
Conjugate of F_2
A
B

FIG. 6.—PLAN OF PEDESTAL SHOWING CONJUGATES FOR JOINT *E*.

The pedestal of Fig. 4 was chosen by the writer for study, because he considers it a practical type of structure for the support of oil-fractionating towers and other such industrial apparatus. It offers advantages over the pedestal analyzed by the author in that all joints involve a symmetrical arrangement of members producing the maximum simplification of members and gusset-plates. There is also the advantage that all compression diagonals are of equal length and as short as possible. Because of the fact that only two members meeting at an upper joint, such as Joint *G*, lie in a single plane, the joint details of such a riveted structure would need careful study, but it is obvious that the pedestal could be constructed quite simply by welding. Where speed of erection is an important factor, the framed steel pedestal should undoubtedly be used in place of the circular concrete chimneys or rigid frame towers that have been widely used for the support of industrial equipment. The high wind stresses and large vertical loads involved could be resisted more economically by a framed pedestal of either the type analyzed by the author, or of the type suggested herein. The explanation is found in the fact that wind force is resisted by direct stress and that the flared pedestal can be given an adequate base without the need for long beam supports at the top. The fact that the major stresses are statically determinate is also an item of importance. Professor Constant has performed a real service by calling the attention of designing engineers to the fact that statically determinate space structures are available for their consideration and that the analysis of these structures need not be particularly complex.

CHARLES M. SPOFFORD,[6] M. AM. SOC. C. E. (by letter).—An interesting addition to the limited amount of material available in English relating to space frameworks, has been presented by Professor Constant, and the graphical method proposed by him is a convenient means of determining the stresses in such structures. The writer prefers analytical instead of graphical methods for determining stresses in most structures, including space frameworks, inasmuch as such methods are generally quicker and more precise. Moreover, when the computations are properly organized and directed they can be applied to the most complicated structures by inexperienced draftsmen and computers who can perform the necessary numerical work without knowing the underlying principles.

In his conclusions Professor Constant has under-estimated the prevalence of space frameworks in the United States. Common types of such structures are the combination of chords and lateral bracing in curved chord trusses and particularly in continuous bridges and arches, bridge-erection travelers, and water tank towers. The Schwedler dome, which is the most complicated of such structures, is not often built in the United States, although it has been used. Several important domes of this type, however, have been built elsewhere. Transmission towers also frequently require analysis as space frameworks.

It is comparatively simple to carry through the necessary computations by applying the following self-evident theorems[7] which, so far as the writer knows, were first formulated by himself:

Theorem (a): If several bars of any framework meet at a joint and all but one lie in the same plane, the component normal to this plane of the stress in that bar which does not lie in the plane will equal the algebraic sum of the components normal to the same plane of all the outer forces which may be applied at the joint under consideration.

---

[6] Hayward Prof. of Civ. Eng., Mass. Inst. Tech., Cambridge, Mass.; Cons. Engr. (Fay, Spofford & Thorndike), Boston, Mass.

[7] "Theory of Structures," by Charles M. Spofford, M. Am. Soc. C. E., Third Edition, McGraw-Hill Book Co., N. Y., p. 457.

Theorem (*b*): The moment of any force or bar stress acting in a given plane about an axis lying in that plane equals zero. It follows that the stress in any bar of a space framework may be determined by the method of moments in the same general manner as in the case of a planar structure; that is, if a section can be taken cutting a number of bars, all of which except bar "a" pass through or are parallel to a given axis, the stress in bar "a" may be found by applying $\Sigma M = 0$ about that axis of all the forces acting on the portion of the structure isolated by the section.

Theorem (*c*): At any joint at which no outer force is applied, and at which the stresses in all bars but two have been found to be zero, the stress in each of these two bars will also be zero, provided the two bars do not lie in the same straight line.

Theorems (*a*) and (*c*) are most useful since, when combined with the equations of statics, they permit the determination (without computations) of the bars in which the stress due to any given load is zero, thus simplifying the computations greatly.

Structures of the Schwedler dome type, chosen by the author for illustration, may or may not be statically determined depending on the pre-establishment of reaction directions. The writer has presented elsewhere[7] a method of investigating the statical condition of such structures. Engineers attempting to solve the ring stresses in one of these structures without first assuring himself that it is statically determined are certain to obtain inconsistent and disappointing results.

It would be helpful if Professor Constant would elaborate the method of conjugate forces to show its application to a Schwedler dome having more than one story and more than four sides.

A. H. Finlay,[8] Assoc. M. Am. Soc. C. E. (by letter).—The concise presentation of Professor Mayor's ingenious method of analysis of spatial stresses contained in Professor Constant's paper is extremely interesting and instructive—the more so because the author avoids the language of projective geometry. The idea of evaluating concurrent, non-co-planar forces by first evaluating a closely related set of non-concurrent, co-planar forces is novel and greatly facilitates analysis. The conventional method of analyzing space structures has too often been demonstrated using such simplified loadings that many of the real difficulties of a complete analysis of such structures have been concealed. As the author states, the advantages of the proposed method increase with the complexity of the structure and loading.

As a simplification, the central axes for all systems at any joint could be taken through the joint instead of at different arbitrary distances, *d*, from it, as in Fig. 1. If this is done all the conjugates at a joint will lie in the horizontal plane containing the joint instead of in horizontal planes at varying distances above or below it. There would then be no need to complicate the picture by introducing the couple, S, which, of course, is merely the vectorial sum of the couples, S_0 and $R(d)$; nor would there be any necessity to refer to the projections of the conjugates upon a common horizontal plane, as the author felt impelled to in the paragraph containing Equation (5). Lastly, as will be shown, this simplification permits a shorter and more easily followed derivation of the expression for the distance, b.

---

[8] Asst. Prof., Civ. Eng., Univ. of British Columbia, Vancouver, B. C., Canada.

The physical picture is simple: It is desired, for purposes of analysis, to combine each of the forces acting at a joint of the space structure with one other force so that in place of each of the original forces there will be a pair of forces equivalent to a system, composed, for convenience, of a vertical force, R, and a couple, S_o, acting in a horizontal plane containing the joint.

Fig. 7 illustrates the foregoing: $ABCD$ is a horizontal plane containing Joint O. Vector F is any force, bar stress, or load, acting at the joint. It is shown resolved into its vertical component, R, and its horizontal component, $F \cos \phi$. The additional force that must be combined with F so as to form a system composed of a vertical force, R, and a couple, S_o, in the plane, $ABCD$, must be F', which must be equal and opposite to $F \cos \phi$ and must act in the plane, $ABCD$, parallel to the horizontal component of F and distant b from it. The following relations exist, remembering that $S_o = R\ a$:

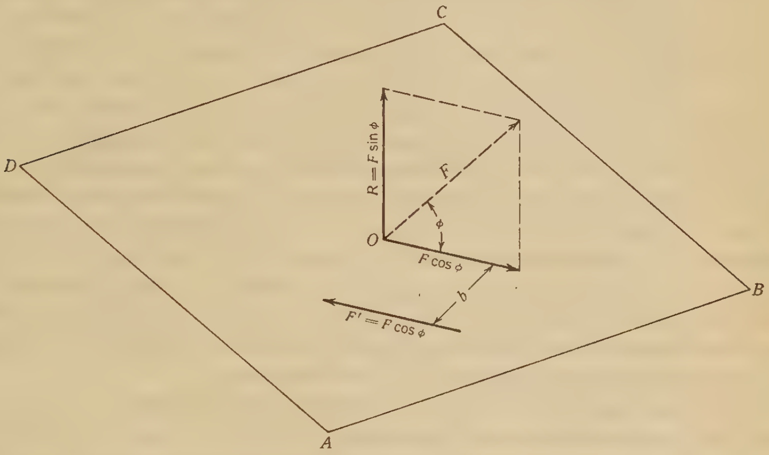


FIG. 7.

$$S_o = F'\ b = F \cos \phi\ b \qquad (14)$$

$$R\ a = F \sin \phi\ a \qquad (15)$$

and,

$$b = a \tan \phi \qquad (16)$$

Equation (16) fixes the line of action of the force, F'. The pair of forces, F and F', is equivalent to the system, R and S_o. Each of these systems ("screws" in the language of statics) is tied to the other systems at the joint, as has been stated by the author, by the fixed relation, $\frac{S_o}{R} = a$. This quantity is often referred to as the pitch of the screw; its value is fixed arbitrarily in the present case at the dictates of convenience, but must be the same, of course, for all systems or screws at the same joint. The author has demonstrated clearly that these additional co-planar forces, or conjugates, F' (of which there are as many for any joint as there may be loads and bar stresses), are themselves in equilibrium and upon this condition, of course, the whole method rests.

The sign convention adopted by the author, and its resulting rule for aiding in the location of the lines of action of the conjugates, are both clear and easily applied.

As Professor Constant states, the conjugates of forces that lie in the same plane intersect in a point. Possibly this is obvious, but a brief proof may not be out of place: Choose any two forces meeting at any joint. Their conjugates will intersect somewhere. If any other force acting through the joint lies in the plane of the first two, it may be resolved into components coinciding with their lines of action and these components, therefore, will have conjugates coinciding with the original conjugates, thus demonstrating the statement. Had the two original forces laid in the same vertical plane their conjugates would not have intersected, but would have been parallel. The full statement would appear to be that the conjugates of forces lying in the same plane will either intersect in a point or be parallel.

The example chosen by the author illustrates the method clearly without introducing complexities due to unusual bar arrangement. The loading is complex, and yet the solution is simple and rapid. Whether the vertical reactions are more easily computed, using the moments of the diagonal and column conjugates as suggested in the paper, or whether it is more convenient to find them by summing the vertical components of the diagonal and column stresses, is probably a matter of individual preference.

The method furnishes a graceful solution of a problem which in the ordinary way presents annoying difficulties. It is in no sense a trick method although at first glance it may so appear. The conjugate forces, which are so simply related to the desired bar stresses, are merely co-planar forces in equilibrium and, as such, are subject to the three simple equations, $\Sigma X = 0$, $\Sigma Y = 0$, and $\Sigma M = 0$, expressing this state. That these equations, susceptible as they are to graphical presentation, are more readily applied than $\Sigma X = 0$, $\Sigma Y = 0$, and $\Sigma Z = 0$, makes the method, in the writer's opinion, of real value, and Professor Constant is to be congratulated upon having made it available in English.

F. H. CONSTANT,[9] M. AM. SOC. C. E. (by letter).—One of the discussers, Mr. Osgood, has offered a simplified presentation of the derivation given in the paper. It may be simplified still further (avoiding reference to a distant axis) as follows: The force, F, is resolved into horizontal and vertical components:

$$F' = F \cos \phi \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (17)$$

and,

$$R = F \sin \phi \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (18)$$

---

[9] Prof. of Civ. Eng., Princeton Univ., Princeton, N. J.

both acting at O in Fig. 1. At some point in the horizontal plane through O let two equal and opposite forces act, each equal and parallel to F' and distant, b, from it. Then F is reducible to: (1) The vertical force, R, acting at O; (2) a horizontal (or conjugate) force, equal, parallel to, and acting in, the same direction as F', but distant, b, from it in the horizontal plane through O; and (3) a horizontal couple:

$$S_o = F' b \dots\dots\dots\dots\dots\dots\dots\dots (19)$$

in which, either S_o or b may be chosen at will. From Equations (17), (18), and (19),

$$b = \frac{S_o}{R} \tan \phi \dots\dots\dots\dots\dots\dots\dots\dots (20)$$

If a group of concurrent forces acting at O is in equilibrium, and if each force of the group is in like manner resolved, it is immediately clear that $\Sigma R = 0$, and the vector sum of the conjugate components is also zero. These latter are in equilibrium, then, against translation.

It can also be shown that the couples are in equilibrium, namely, $\Sigma S_o = 0$, provided S_o for each force, F, is properly selected. Let it be so chosen, in each case, that the ratio, $\frac{S_o}{R} = a =$ a constant. Then Equation (20) becomes identical with Equation (4). From Equations (4), (17), and (18),

$$\Sigma S_o = \Sigma F' b = a \Sigma F' \tan \phi = a \Sigma R = 0 \dots\dots\dots\dots (21)$$

Hence, the couples, S_o, are in equilibrium. Since the conjugates have numerically the same moment about O as the couples, but of opposite sign, they too are in equilibrium against rotation. The conjugates, therefore, constitute a system of non-current planar forces in equilibrium. Each conjugate is located by the equation, $b = a \tan \phi$, in which a is any arbitrarily chosen constant, expressed in feet. With this method of analysis the conjugates have the same direction as the horizontal components of the F-forces.

Professor Grinter's problem (Fig. 4) presents a good opportunity for demonstrating the relative simplicity of the conjugate method, although Professor Grinter has not done full justice to his own case. By the use of substitute members, the number of simultaneous equations could have been reduced to two, as is shown in the following solution by conjugate stresses (see Fig. 8).

Fig. 8(a) is a reproduction of Professor Grinter's Fig. 6, with all the necessary conjugates shown. The members, $E F$ and $H F$, are removed and substitute horizontal members, $F I$ and $D J$, inserted at joints where the analysis shows that they are obviously needed. Thus, as the joints are taken up in the order E, H, and G, the stress in $G F$ becomes known, and it is obvious that $F A$ and $F B$ cannot hold $G H$ in equilibrium. The substitute member, $F I$, therefore, is inserted. The procedure will show that none of the other joints offers any difficulty until D is reached, where a second substitute member, $D J$, is necessary. The solution involves three steps:

Fig. 8(*b*) shows the conjugate stresses due to the external force at *E* (the members, *E F* and *H F*, being removed); Fig. 8(*d*) shows the conjugate stresses due to a pair of equal and opposite unit loads, acting at *E* and *F* in the direction of *E F* (the external load and *H F* being removed); and, Fig. 8(*c*) shows the conjugate stresses due to a pair of equal and opposite unit loads, acting at *H* and *F* in the direction of *H F* (the external load and *E F* being removed). An inspection of these three polygons shows how simple and symmetrical they are, and how quickly the stresses may be written.

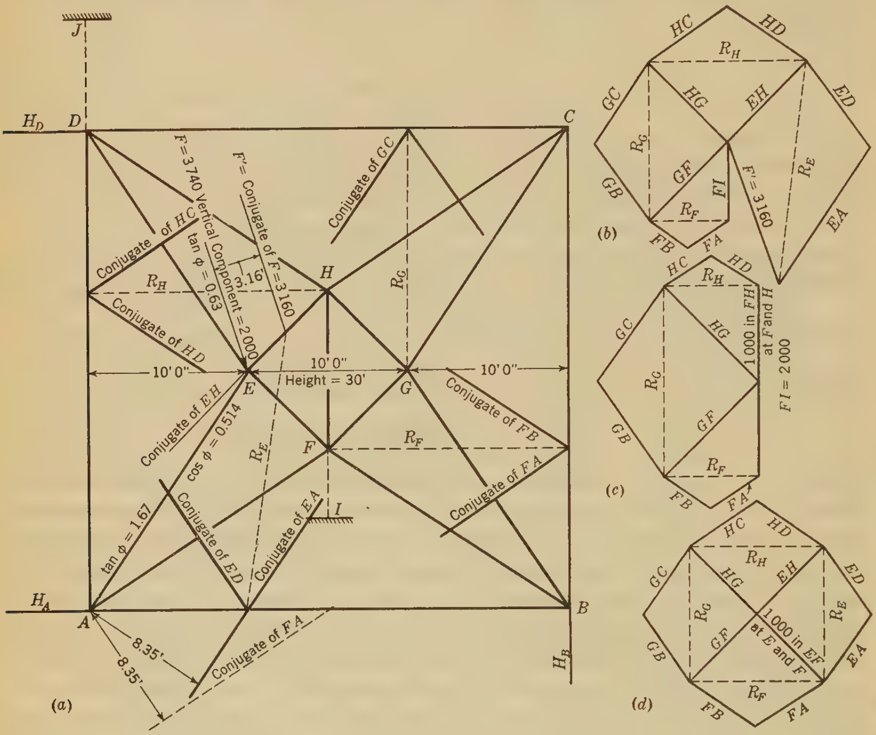

J
H_D
D
C
H
G
E
F
I
A
B
H_A
H_B
F = 3740 Vertical Component = 2000
tan φ = 0.63
F' = Conjugate of F = 3160
3.16'
Conjugate of GC
Conjugate of HC
Conjugate of HD
Conjugate of EH
Conjugate of ED
Conjugate of EA
Conjugate of FB
Conjugate of FA
R_H
R_G
R_E
R_F
10' 0"
Height = 30'
cos φ = 0.514
tan φ = 1.67
8.35'
(a)
HC
HD
GC
HG
EH
ED
GF
FI
F' = 3160
GB
FB
FA
EA
(b)
1 000 in FH at F and H
FI = 2000
(c)
1000 in EF at E and F
(d)

FIG. 8.

The stresses from these three cases, indicated by S_a, S_b, and S_c, respectively, are listed in Table 2. Those in the lower horizontal members and in the reactions, including those in the false member, *D J*, are included in Table 2, but are not shown on Fig. 8, as they are most easily obtained by analytic processes in the manner explained in the paper. Since the actual stress in *F I* and *D J* is zero, the following two equations can be written:

$$F\,I = 1\,650 - C \times 2\,000 = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (22)$$

and,

$$D\,J = 6\,324 + B \times 2\,356 - C \times 667 = 0 \ldots\ldots\ldots\ldots (23)$$

Whence, $C = +\ 0.825$, and $B = -\ 2.45$. The true conjugate stress in any member is then obtained from the general expression,

$$S = S_a - 2.45\ S_b + 0.825\ S_c \quad \ldots\ldots\ldots\ldots\ldots (24)$$

These values are entered in Column (5), Table 2, and the actual stresses, obtained by dividing each conjugate by cos ϕ, in Column (7). They check very closely with those computed by Professor Grinter (see Fig. 5).

TABLE 2.—STRESSES IN THE PEDESTAL IN FIG. 4 (SEE, ALSO, FIG. 8)

(Tensile stresses are positive; vertical reactions are positive when acting upward; and, horizontal reactions are positive when acting away from the joint)

Member (1)	S_a (2)	S_b (3)	S_c (4)	Conjugate force, F' (5)	Cosine ϕ (6)	Actual force, F (7)
E H...	—2 336	—1 000	....	+ 114	1.00	+ 114
H G...	+2 336	+1 000	—1 414	—1 280	1.00	—1 280
G F..	—2 336	—1 000	+1 414	+1 280	1.00	+1 280
F E...	....	+1 000	....	—2 450	1.00	—2 450
H F...	....	....	+1 000	+ 825	1.00	+ 825
E A...	—3 400	— 848	....	—1 322	0.514	—2 572
E D...	+2 200	+ 848	....	+ 122	0.514	+ 237
H D...	+1 980	+ 848	— 600	— 593	0.514	—1 154
H C...	—1 980	— 848	+ 600	+ 593	0.514	+1 154
G C..	—1 980	— 848	+1 200	+1 090	0.514	+2 120
G B..	+1 980	+ 848	—1 200	—1 090	0.514	—2 120
F B..	+1 000	+ 848	— 600	—1 575	0.514	—3 065
F A...	—1 000	— 848	+ 600	+1 575	0.514	+3 065
......	....	....	....	....	....	....
......	....	....	....	....	....	....
A D..	+3 391	+1 178	— 333	+ 230	1.00	+ 230
D C...	+2 750	+1 178	—1 167	—1 100	1.00	—1 100
C B...	+2 750	+1 178	—1 333	—1 236	1.00	—1 236
B A...	—1 933	—1 178	+1 167	+1 916	1.00	+1 916
H_A...	—4 656	—2 356	+1 667	+2 491	1.00	+2 491
H_D...	+5 622	+2 356	—1 667	—1 525	1.00	—1 525
H_B...	+4 956	+2 356	—2 667	—3 000	1.00	—3 000
V_A...	+7 348	+2 834	—1 000	....	......	— 420
V_B...	—4 976	—2 834	+3 000	....	......	+4 440
V_C...	+6 612	+2 834	—3 000	....	......	—2 800
V_D...	—6 980	—2 834	+1 000	....	......	+ 787
F I...	+1 650	....	—2 000	....	1.00	....
D J...	+6 324	+2 356	— 667	....	1.00	....

To study the different types of space structures and the proper arrangement of the reactions is interesting. Probably some types are distinctly more efficient and economical than others; but as yet very little research has been done in this field. German writers have simplified the equations used in the analytic solution of domes and towers, until the process is reduced almost to a mechanical one; and any one familiar with these processes will naturally regard them with favor. Professor Spofford was the first to present an adequate treatment of space structures in American literature.[7] In his discussion, he expresses a preference for analytical methods of stress determination, but the first choice which the designer of a space structure must make is whether to work with concurrent forces lying in three dimensions of space, or with non-concurrent forces lying in two dimensions. In the former case, he must adopt analytic methods of solution, the graphical method being entirely too cumbersome. With the latter method he may use either, as in the case of any planar structure. In the paper some of the stresses (the lower ring and the reactions) were obtained analytically by the use of the three static equations.

The writer purposely chose a simple form of dome in order to illustrate the principles, but the method applies equally well to domes or towers with more than four legs and with several stories. After the diagonal and post stresses of the upper story have been found, they become external forces applied at the top joints of the second story. The positions of their conjugates will already have been drawn, and the magnitudes of these conjugate stresses will be known. These conjugate stresses may be combined into a single resultant if desired. The solution of the second story then becomes a repetition of that of the first.

The writer appreciates Professor Finlay's gracious remarks and is grateful for his contribution to the simplification of the derivations. His analysis follows closely that of Mr. Osgood, which has already been covered in the earlier part of this closing discussion. No further simplification is possible, and it is probable that this derivation will be preferred to the more complex one of the paper. Professor Finlay's proof that the conjugates of forces that lie in the same plane must intersect in a point, is interesting and clear.

As the writer's sole aim was to make available in English a unique and valuable method of solving space structures, he gratefully acknowledges the contribution which each of the discussers has made in the furtherance of this purpose.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

TRANSACTIONS

Paper No. 1912

EXPERIMENTS WITH CONCRETE IN TORSION

By Paul Andersen[1], Assoc. M. Am. Soc. C. E.

With Discussion by Messrs. E. Mirabelli, Frank M. Russell, Leslie Turner, A. W. Fischer, H. J. Gilkey, A. A. Eremin, and Paul Andersen.

Synopsis

Torsional stresses are developed in a structural member if it is subjected to the action of couples that lie in planes perpendicular to its axis.

In the ordinary building frame, provision seldom is made for taking care of stresses of this nature, and current literature on the design of reinforced concrete structures contains little on the subject. The attitude of the designing engineer is to avoid torsional stresses rather than to take them into account; and as a rule they can be avoided quite successfully.

There are, nevertheless, a number of types of structural members in which twisting forces occur and such forces should not be left to take care of themselves. The most important case is that of longitudinal balcony girders that support cantilever beams. Exterior floor-beams will develop torsional stresses due to the deflection of adjacent loaded panels. A skew arch submitted to vertical load and a rigid-frame portal sustaining a horizontal wind pressure also present problems in torsional resistance of reinforced concrete.

This paper deals with the general behavior in torsion of plain and reinforced concrete; its resistance against failure; and various types of reinforcement that will increase its ultimate torsional strength.

Notation

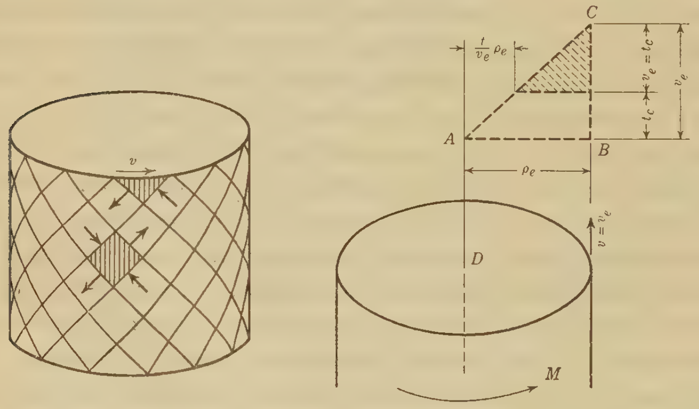
A summary of the notation introduced in this paper is presented herewith for the convenience of discussers:

a = one-half the side dimension of a square test prism;
b = spacing of bars measured on a horizontal cutting plane;

Note.—Published in May, 1934, *Proceedings*.

[1] Balboa Heights, Canal Zone.

c = compressive stress in test cylinders; as a subscript, c, denotes "concrete";
e = a subscript denoting "edge";
h = an influence ordinate of shearing stress;
n = ratio of moduli of elasticities;
s = a subscript denoting "steel".
t = tensile stress: t_c, permissible for concrete; t_{max}, maximum diagonal stress developed by a specimen without spirals;
v = shearing stress: v_e, along the edge of a cylinder produced by a twisting moment, M; v_ρ = shearing stress at a distance, ρ, from the axis of a cylinder; v_m = maximum shearing stress;
A = area: A_s, of steel bars;
E = modulus of elasticity: E', in shear;
F = a reinforcement efficiency factor that allows for surface tension taken by reinforcement;
K = a substitution factor; a unit stress;
M = moment; twisting moment;
N = number of bars, 45° to the axis, cut by a horizontal plane;
X = curve in Fig. 10;
Y = curve in Fig. 10;
θ = angle between two radii in a horizontal cutting plane;
ρ = radial distances in a horizontal cutting plane, measured from the axis of a cylinder; ρ_e, to the edge of a cylinder; ρ_s, to the steel reinforcement; ρ_c, to the point of excess permissible tension in the concrete;
ψ = angle of twist per unit length of cylinder.



(a) (b)

FIG. 1.

THEORETICAL CONSIDERATIONS

Torsion is a state of pure shear. In the case of a circular cylinder, of a homogeneous and isotropic material,

$$v = E' \rho \psi \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

in which, v is the shearing stress per unit area; E', the modulus of elasticity in shear; ρ, the distance from the axis of the cylinder; and ψ the angle of twist per unit length.

This state of pure shear is equivalent to tension in one direction and equal compression in the perpendicular direction. A rectangular element cut from the outer layer of a twisted shaft with its sides at 45° to the axis, therefore, will be subjected to the stresses shown in Fig 1(*a*), all of which are numerically equal to v. In the case of a material that is weaker in tension than in shear (such as concrete or cast iron) a crack along a helix inclined at 45° to the axis will occur when the ultimate tensile strength has been exceeded. It follows that the most effective reinforcement for a concrete member designed to resist failure in torsion is a spiral at an angle of 45° with the axis.

In the formulas that follow, the line of thought is analogous to that for the design of web reinforcement in a beam subjected to bending; namely, that sufficient reinforcement must be provided to resist tensile stresses in excess of the permissible tension for plain concrete. The line, AC, in Fig. 1(*b*), represents the variation in shearing stress along an arbitrary radius, AB. Let v_e be the shearing stress along the edge produced by a twisting moment, M, and let t_c denote the permissible tensile stress for plain concrete. The shaded triangle will then represent intensities of diagonal tension for which reinforcement must be provided.

Consider a small element at a distance, ρ, from the axis. The shearing force on this element in excess of the permissible stress is,

$$v = (v_\rho - t_c)\ \rho\, d\theta\, d\rho \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

The tensile and compressive components of this force are numerically equal. Each one equals,

$$v = (v_\rho - t_c)\ \rho\ d\theta\ d\rho\ \cos 45^\circ \ldots\ldots\ldots\ldots\ldots\ldots(3)$$

In accordance with the foregoing assumption, the sum of the moments with respect to the axis of the tensile components must equal the moment of the steel stresses with respect to the same axis, thus,

$$N\ t_s\ A_s\ \cos 45^\circ\ \rho_s = \int_0^{2\pi}\int_{\rho_c}^{\rho_e} (v_\rho - t_c)\ \cos^2 45^\circ\ \rho^2\ d\rho\ d\theta \ldots\ldots(4)$$

in which, N is the number of bars on an angle of 45° with the axis cut by a horizontal section; t_s, the allowable stress in steel bars; A_s, the cross-sectional area of a steel bar; ρ_s, the distance from Axis AD to the steel bars; v_ρ, the shearing stress at distance, ρ, from the axis; and $\rho_c = \frac{t_c\ \rho_e}{v_e}$ = a radial distance in Fig. 1(*b*).

In Equation (4) substitute:

$$v_\rho = \frac{\rho\ v_e}{\rho_e} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

and perform the integrations; thus:

$$N\ t_s\ A_s\ \rho_s = \frac{\pi\ \rho^3_e \sqrt{2}}{12\ v^3_e}\ (3\ v^4_e - 4\ v^3_e\ t_c + t^4_c) \ldots\ldots\ldots(6)$$

or,

$$\frac{6\sqrt{2}\ \ N\ t_s\ A_s\ \rho_s}{\pi\ \rho^3_e}\ v^3_e = (v_e - t_c)^2\ (3\ v^2_e + 2\ v_e\ t_c + t^2_c)\ \ldots\ldots(7)$$

If K is substituted for $\frac{\sqrt{2}\; N\, t_s\, A_s\, \rho_s}{\pi\, \rho^3_e}$, Equation (7) may be written,

$$6\, v^3_e\, K = (v_e - t_c)^2\, (3\, v^2_e + 2\, v_e\, t_c + t^2_c) \quad \text{.........(8)}$$

If the concrete is assumed as not taking any part of the shearing stresses when these exceed the permissible tension for plain concrete, the case is as shown in Fig. 2. The formulas corresponding to Equations (4) and (7) are, respectively:

$$N\, t_s\, A_s \cos 45^\circ\, \rho_s = \cos^2 45^\circ \int_0^{2\pi} d\theta \int_{\rho_c}^{\rho_e} v_\rho\, \rho^2\, d\rho \quad \text{.........(9)}$$

and,

$$2\, K = v_e - \frac{t^4_c}{v^3_e} \quad \text{.........(10)}$$

Equation (5) is identical for both cases.

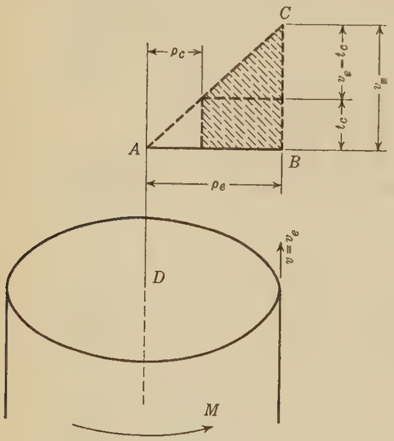


FIG. 2.

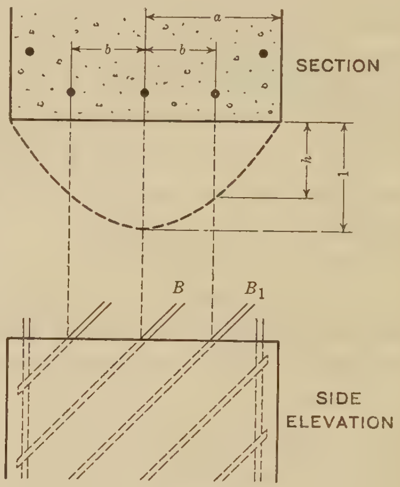


FIG. 3.

For a circular section, Equations (7) and (10) give the number of bars required if v_e and t_c are known; v_e is computed from the twisting moment, and t_c, as already stated, is the allowable tensile stress for the plain concrete. In designing a square section the equivalent circular section (that is, one that has the same torsional resistance) can be used.

The maximum shearing stress for a circular section with a radius, ρ_e, is

$$v_m = \frac{2\, M}{\pi\, \rho^3_e} \quad \text{.........(11)}$$

The maximum shearing stress for a square section with a length of side equal to 2 a is, approximately,

$$v_m = \frac{3\, M}{5\, a^3} \quad \text{.........(12)}$$

Equating Formulas (11) and (12); $a^3 = 0.943\ \rho^3_e$ and $\rho_e = 1.02\ a$. Hence, the equivalent circular section is approximately the round section with a radius equal to one-half the side of the square section, or the inscribed circular section.

Fig. 3 shows a square section subjected to torsion. The shearing stress varies along the edge approximately as a second degree parabola; therefore, the reinforcement is stressed more highly at the center than near the corners. If, therefore, the equivalent circular section is considered to be reinforced with the same number of rods, it is clear that it would show greater resistance, due to the fact that its reinforcement would be equally stressed and, therefore, better utilized. Consequently, if Equations (7) and (10) are used for a square section to express the surplus tension to be taken by the reinforcement, a factor, F (which is less than one) must be introduced. Therefore,

$$6\ K\ F\ v^3_e = (v_e - t_c)^2\ (3\ v^2_e + 2\ v_e\ t_c + t^2_c) \qquad (13)$$

or,

$$2\ K\ F = v_e - \frac{t^4_c}{v^3_e} \qquad (14)$$

In Equations (13) and (14), F is a reinforcement efficiency coefficient relating to the variation in tensile stress along the edge. The efficiency coefficient for the square section shown in Fig. 3 is,

$$F = \frac{4 + 8\ h}{12} = \frac{4 + 8\left(1 - \frac{b^2}{a^2}\right)}{12} \doteq 1 - \frac{2}{3} \times \frac{b^2}{a^2} \qquad (15)$$

PREVIOUS INVESTIGATIONS

Experiments with plain and reinforced concrete in torsion are recorded as early as 1904 at Stuttgart, Germany; in 1910 and 1911, by Professors Bach and O. Graf;[2] and, in 1920, by Professors Graf and E Mörsch, of the Engineering College of Stuttgart.[3] Further tests were made in 1922 by Messrs. Young, Sagar, and Hughes, of the University of Toronto, Toronto, Ont., Canada.[4]

APPARATUS AND TEST SPECIMENS

The testing, under torsion, of forty-eight specimens will be reported in this paper. The specimens were divided into six types as shown in Fig. 4, namely:

Type R: Six circular specimens of plain concrete; the control cylinder strength varied from 2 033 lb per sq in., to 5 535 lb per sq in.

Type B_1: Nine square specimens reinforced with ½-in. square corner rods; the control cylinder strength varied from 2 110 to 5 240 lb per sq in.

---

[2] "Deutscher Ausschuss für Eisenbeton," Heft 16, 1912, W. Ernst & Sohn, Berlin, Germany.

[3] "Der Eisenbetonbau," von E. Mörsch, Sixth Edition, Vol. I, Pt. II, Stuttgart, 1929, pp. 323–335.

[4] *Bulletin No. 3*, Univ. of Toronto, Univ. of Toronto Press, 1922, pp. 145–169.

Type B_2: Nine square specimens reinforced with $\frac{1}{2}$-in. square corner rods and $\frac{5}{32}$-in. round ties, spaced 6 in.; the control cylinder strength varied from 1 950 to 5 300 lb per sq in.

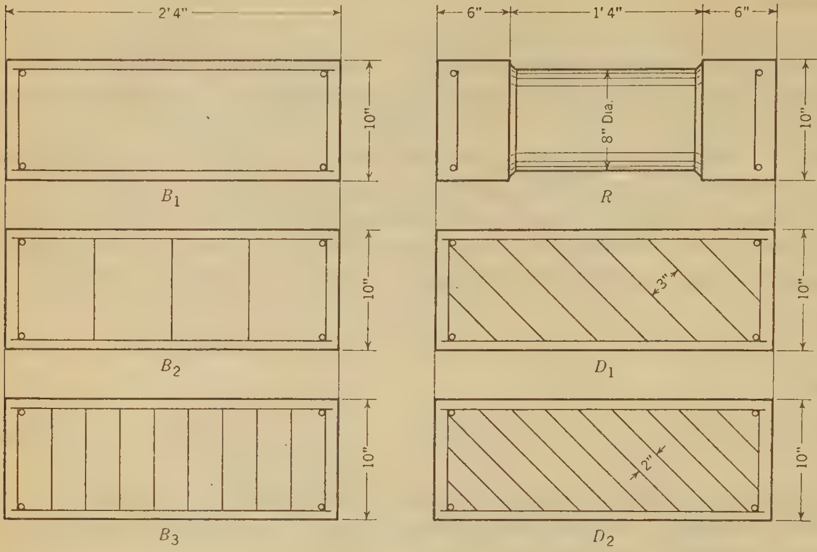


FIG. 4.

Type B_3: Nine square specimens reinforced with $\frac{1}{2}$-in. square corner rods and $\frac{5}{32}$-in. round ties spaced 3 in.; the control cylinder strength varied from 2 090 to 5 190 lb per sq in.

Type D_1: Six square specimens reinforced with $\frac{1}{2}$-in. square corner rods and $\frac{5}{32}$-in. round 45° spirals; the spacing of the spirals was 3 in., measured perpendicular to the 45° direction; the cylinder strength varied from 2 481 to 5 250 lb per sq in.

Type D_2: Nine square specimens reinforced with $\frac{1}{2}$-in. square corner rods and $\frac{5}{32}$-in. round 45° spirals; the spacing of the spirals was 2 in., measured normal to their direction; the cylinder strength varied from 1 950 to 5 300 lb per sq in.

The Portland cement used in manufacturing the concrete passed the standard specifications of the American Society for Testing Materials. Sieve analyses of the well-graded torpedo sand from the Wabash River, at Attica, Ind., showed a fineness modulus of 2.89 and a surface modulus of 18.5. The gravel, also from Attica, was graded from $\frac{1}{4}$ in. to 1 in. in size.

Concrete for all test specimens was machine-mixed for 4 min. The consistency is indicated by slumps of $4\frac{1}{2}$ to $7\frac{1}{2}$ in. for the various batches, with a mean value of 6 in., and an average flow (percentage increase in base diameter of flow cone) of 145. All specimens were cured moist for 28 days before testing.

The reinforcement consisted of hard-grade steel for corner rods and annealed black, soft-grade wire for spiraling and ties. It was subjected to the usual standard tensile tests, the results of which are given in Table 1.

As indicated in Fig. 4, the ends of the square specimens were not made larger than the test piece. However, in order to prevent local failures of the concrete at the ends, reinforcement in the form of $\frac{3}{4}$-in. square steel rods was embedded to furnish bearing against the structural steel grips of the testing machine. A standard torsion machine with a capacity of 230 000 in-lb was used.

TABLE 1.—TENSILE TESTS OF REINFORCEMENT

Size, in inches	Shape	Area, in square inches	Elastic limit, in pounds per square inch	Ultimate strength, in pounds per square inch	Percentage elongation in 8 in.
0.5	Square	0.25	78 800	134 800	7.5
0.1485	Round	0.01732	43 300	57 700	17.5

Fig. 5 shows the set-up for a square specimen. Strain-gauge readings in the 45° directions were taken on three sides of the test piece by a 2-in. Berry gauge. On the fourth side strains, also in the 45° directions, were recorded by two extensometers. Steel plugs for the extensometers as well as for the drilled strain-gauge holes were set and grouted in the concrete about two weeks before the day of testing. The angular distortion was measured as the difference in rotation between two sections 16 in. apart. To each section was attached a steel frame that made contact with the specimen in four points; the frame carried a horizontal steel bar. The angles of rotation of these bars were determined by a level-bar with attached screw micrometer.

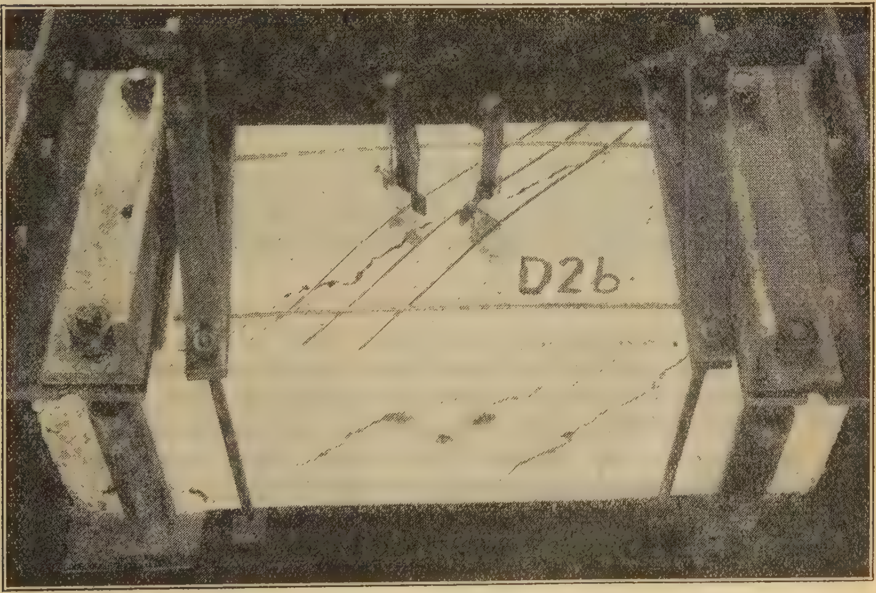


FIG. 5.—SET-UP FOR SQUARE SPECIMEN

The angle of twist was measured at intervals ranging from 5 000 to 15 000 in-lb; the interval was generally made smaller as the ultimate strength of the specimen was approached. Strain-gauge and extensometer readings were recorded at the same time as readings on the torque-indicating mechanism.

CONCRETE IN TORSION

Fig. 6 shows the stress-detrusion diagrams for three typical types of specimen, namely, R_1, plain concrete, low strength (ultimate, 250 lb per sq in.; control cylinder, 2 000 lb per sq in.); R_6, plain concrete, high strength (ultimate, 420 lb per sq in.; control cylinder, 5 200 lb per sq in.); and D_1, spirally reinforced concrete of high strength (ultimate, 750 lb per sq in.; control cylinder, 5 200 lb per sq in.).

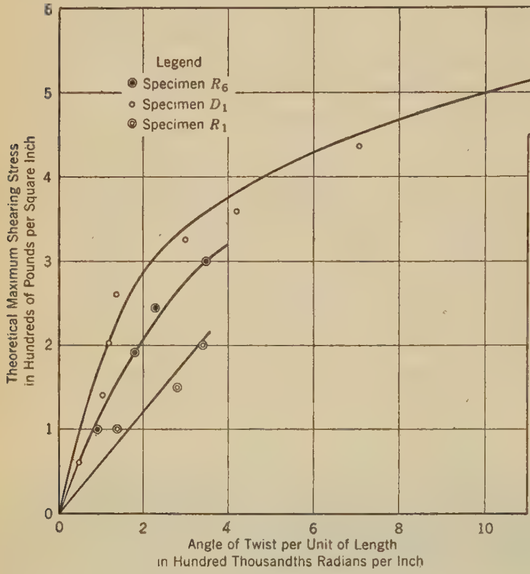


FIG. 6.

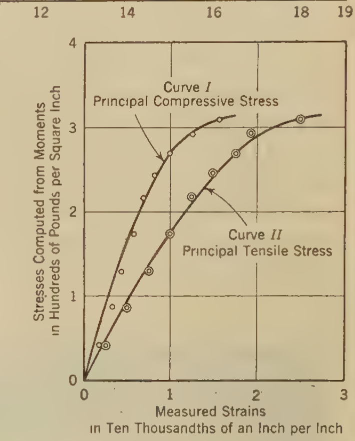


FIG. 7.

While the torque-twist curve for a plain concrete member is substantially straight, it is seen that in approaching the ultimate stress, the torque-twist curve of a spiral reinforced specimen shows a marked curvature and considerable twist, which indicates that the member is decidedly tough.

A noticeable difference was observed between the tensile and the compressive strains as recorded by the extensometers. Fig. 7 shows these strains plotted against stresses computed from the applied twisting moments. As the two principal stresses must be equal numerically, different strains can only be accounted for by different moduli of elasticity, or by differences in the value of Poisson's ratio, in tension and compression. This difference was more pronounced for low-strength, than for high-strength, concrete.

Table 2 gives the modulus of elasticity in shear for plain concrete of varying torsional strengths. These are secant moduli determined at about three-fourths the ultimate strength.

TABLE 2.—MODULUS OF ELASTICITY IN SHEAR

Compressive strength of control cylinder, in pounds per square inch	Ultimate torsional strength, in pounds per square inch	Modulus of elasticity in shear, in pounds per square inch
2 000	250	1 420 000
2 250	300	1 380 000
3 300	315	1 440 000
Unreliable	350	1 540 000
5 200	420	2 140 000

The state of stress of a concrete member in torsion resembles that in a web of a concrete beam in bending; that is, the governing stress is diagonal tension. Under torsional shear a round shaft of concrete, invariably, will fracture along a helix the angle of which is 45° with the axis. The fracture occurs where the tensile stress is a maximum, and indicates that concrete has a lower resistance to tension than to shear. Fig. 8 shows a typical torsion fracture of a round specimen.

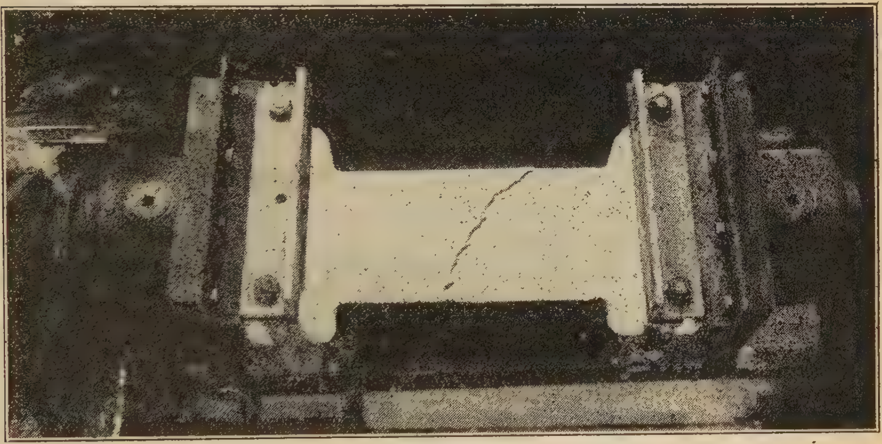


FIG. 8.—TYPICAL TORSION FRACTURE OF A ROUND SPECIMEN

Although the state of stress for a square section is more complicated than that for a circular section, a similar distribution occurs, and the square specimen develops fractures at angles with the axis of approximately 45 degrees. Fig. 5 shows a characteristic torsion failure for a square test piece.

ULTIMATE STRENGTH

The ultimate torsional moments were carefully recorded for each specimen. The corresponding maximum shearing stresses were computed by Equation (11) for a circular section with a radius, ρ_e; and by Equation (12) for a square section with a side equal to $2a$.

These stresses were plotted against the ultimate compressive strengths of the corresponding control cylinders in Fig. 9 and a straight line was fitted to each set by the method of least squares. Fig. 9 shows that the presence of corner rods adds to the torsional strength; that the addition of ties will further raise the ultimate strength; and, also, that the most effective type of reinforcement used in these tests is the 45° spiral.

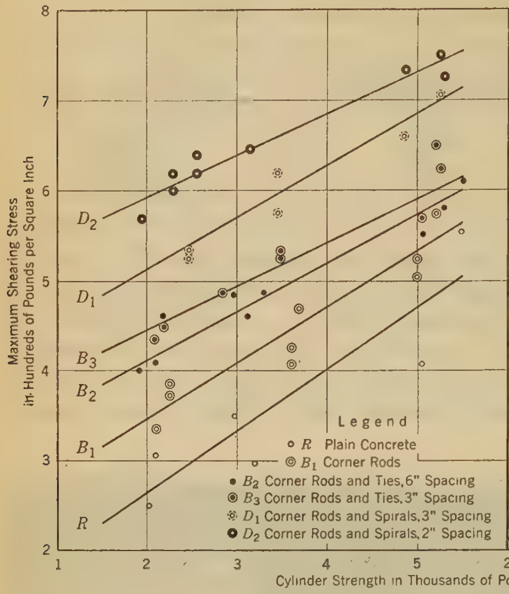


FIG. 9.

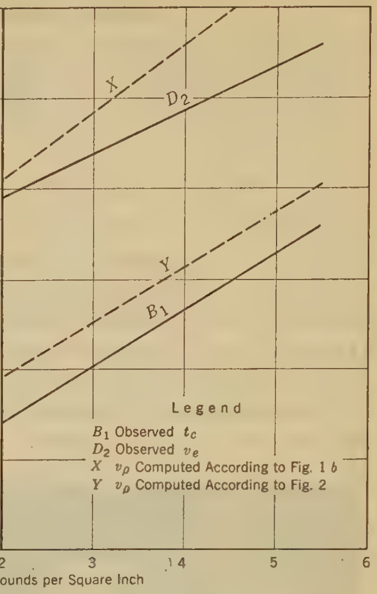


FIG. 10.

It is interesting to note that the quantity of steel wire per unit length of specimen is the same for Type B_3 and Type D_1. Thus, for Type B_3, $4 \times \frac{9.5}{3}$ = 12.7 in. of wire per in. of specimen; and, for Type D_1, $4 \times \frac{9.5 \times \sqrt{2}}{3 \times \sqrt{2}}$ = 12.7 in. of wire per in. of specimen.

By comparing the lines of Type B_3 and D_1, in Fig. 9, it is to be noted that the ultimate strength is considerably higher for the spiral reinforcement than for tie reinforcement.

In Fig. 10 a comparison has been made of the observed torsional strength of the specimens of Series D_2 (see Fig. 4), and the theoretical torsional strengths as computed by Equations (13) and (14).

Specimens B_1 were reinforced with four corner rods; Specimens D_2 had corner rods of the same size and additional close spirals. It seems natural to assume that the higher ultimate strength of Specimens D_2 is due entirely to the action of the spirals; or, in other words, that the differences between ordinates to Specimens D_2 and B_1 in Fig. 10 represent the quantities, $v_e - t_c$

(see Fig. 1 (*b*) and Fig. 2), in which, t_c is the maximum diagonal stress developed by a specimen without spirals, and v_e, the tensile stress sustained by a similar test piece having additional spiral reinforcement.

If it is desired to compute the ultimate strength, the quantity, $t_s A_s$, in Equations (13) and (14), equals the yield-point load in one wire and according to Table 1 is 750 lb.

The coefficient, F, can be calculated from Equation (15); thus,

$$F = 1 - \frac{2}{3} \times \frac{2.8^2}{5^2} = 0.791$$

Substituting these values, together with $\rho = 5$, $\rho_s = 4.5$, and $N = 9$, in Equations (13) and (14), gives,

$$519\, v^3_e = (v_e - t_c)^2\, (3\, v^2_e + 2\, v_e\, t_c + t^2_c) \quad\ldots\ldots\ldots\ldots(16)$$

or,

$$173 = v_e - \frac{t^4_c}{v^3_e} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(17)$$

in which, Equation (16) represents the case illustrated in Fig. 1(*b*) and Equation (17), the case indicated in Fig. 2.

If, in Equations (16) and (17), values of t_c are inserted corresponding to different cylinder strengths, c, as indicated by Line B_1 in Fig. 10, equations with one unknown, v_e, will result. These equations can be solved by trial. Corresponding values of t_c and v_e are given in Table 3.

TABLE 3.—VALUES OF t_c AND v_e CORRESPONDING TO DIFFERENT CYLINDER STRENGTHS

Values of c, in pounds per square inch	ACCORDING TO FIG. 1 (*b*)		ACCORDING TO FIG. 2	
	Tensile strength, t_c, for plain concrete	Shearing stress, v_e, along the edge of cylinder	Tensile strength, t_c, for plain concrete	Shearing stress, v_e, along the edge of cylinder
2 000	343	610	343	396
2 500	374	647	374	426
3 000	405	684	405	456
3 500	435	721	435	486
4 000	466	758	466	516
4 500	496	795	496	546
5 000	527	831	527	576

The lines connecting these points represent the theoretical torsional strengths of plain homogeneous members equivalent to specimens of Series D_2. As indicated in Fig. 10, the agreement between the observed v_e, and the value computed according to Fig. 1 (*b*), appears to be considerably better than that calculated according to Fig. 2.

CONCLUSIONS

Except for the German and Canadian experiments, already cited, few data have been published in regard to plain and reinforced concrete under torsion.

The conclusions that can be drawn from the experiments reported in this paper, and that more or less confirm the test results of these experiments, are:

1.—The modulus of elasticity in shear of concrete depends on its ultimate strength. High-strength concrete has a higher shearing modulus than low-strength concrete.

2.—Concrete in torsion fails in tension due to its low tensile strength. The safe working stress for concrete in torsion must be considerably less than its ultimate tensile strength; the factor of safety should not be less than 2.

3.—The addition of corner rods will increase torsional resistance. The writer believes, however, that this extra strength becomes proportionately less for larger sections; it should not be relied upon, therefore, in designing concrete members subjected to torsion.

4.—The addition of vertical web reinforcement will further increase the torsional resistance and can be considered in the design, provided this reinforcement forms completely closed ties such as are used in columns.

5.—The most effective type of reinforcement is the 45° spiral. It raises the ultimate torsional strength and adds considerably to the toughness of the member.

The deduction to be made from the experimental data presented in this paper is that the spiral reinforcement can be assumed to take practically all tensile stresses in excess of the ultimate tensile strength of the unreinforced concrete. If a suitable factor of safety is chosen, design formulas can be developed for structural concrete members on this basis (see Fig. 1(*b*)).

ACKNOWLEDGMENT

The experiments cited in this paper were performed in 1932 and 1933 in the Materials Testing Laboratory of the University of Illinois, Urbana, Ill., by Rex L. Brown, Assoc. M. Am. Soc. C. E., and the writer, under the direction of F. E. Richart, M. Am. Soc. C. E.

DISCUSSION

E. Mirabelli,[5] M. Am. Soc. C. E. (by letter).—An examination of Fig. 6 reveals that, up to the loading at which the plain concrete specimen fails, the action of a spirally reinforced square specimen is not much different from that of a cylindrical specimen made of plain concrete of the same strength. It may be inferred that reinforcement, when used in moderate quantity, does not take a prominent part in resisting torsion until the tensile strength of the concrete has been overcome. This conclusion is confirmed by test results discussed by Dr. E. Mörsch[3] which show that with spiral reinforcement initial cracks appear on a cylindrical specimen at a load that is only 15% greater than that which produces initial cracking (and simultaneous failure) on a plain concrete cylindrical specimen.

That spiral steel is not fully effective before the concrete yields, may be shown by the following analysis in which concrete is treated as an ideal material, no account is taken of the fact that its modulus of elasticity in tension differs from that in compression, and that stress is not exactly proportional to strain. The notation is the same as that used by the author.

In the spirally reinforced cylindrical member shown in Fig. 11, the steel stress due to torsion is uniform throughout and causes an elongation of the steel which, per unit length of cylinder, is,

$$\Delta_s = \frac{t_s}{E_s} \sqrt{2} \quad \dots\dots\dots\dots (18)$$

The steel deformation is due to twisting of the cylinder and the point, B, will move to B', making,

$$\overline{BB'} = 2 \frac{t_s}{E_s} \quad \dots\dots\dots\dots (19)$$

There will result an angular deformation which, per unit length of cylinder, will be,

$$\psi = \frac{2\, t_s}{E_s\, \rho_s} \quad \dots\dots\dots\dots (20)$$

Using Equation (1) and substituting $v_e = v_s \, \frac{\rho_e}{\rho_s}$, it is found that,

$$\psi = \frac{v_s}{E'_c\, \rho_s} \quad \dots\dots\dots\dots (21)$$

From Equations (20) and (21) it is found that,

$$v_s = 2 \frac{E'_c}{E_s} t_s \quad \dots\dots\dots\dots (22)$$

Equation (22) expresses the relation between the tension stress, t_s, in the spiral steel, and the torsional shearing stress, v_s, in the concrete adjacent to the steel, in terms of the modulus of elasticity, E_s, for steel and the modulus of rigidity, E'_c, for concrete. Assuming values of $E'_c = 1\,500\,000$ lb per sq in., and $E_s = 30\,000\,000$ lb per sq in., the relation becomes,

---

[5] Asst. Prof. of Structural Eng., Mass. Inst. Tech., Cambridge, Mass.

$$t_s = 10\, v_s \qquad (23)$$

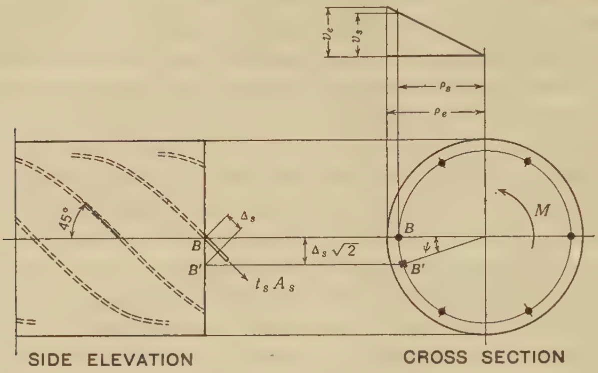


FIG. 11.

Neglecting the small reduction in concrete area due to the presence of steel, the part of the torsional resistance provided by the concrete is,

$$M_c = \frac{\pi}{2} \rho^3_e\, v_e \qquad (24)$$

and the contribution of the spiral steel is,

$$M_s = N A_s\, t_s \frac{1}{\sqrt{2}} \rho_s \qquad (25)$$

Assuming 1% of spiral steel and assuming $\rho_s = \frac{9}{10} \rho_e$, it follows that,

$$M_s = \frac{9\,\pi}{1\,000\,\sqrt{2}} \rho^3_e\, t_s \qquad (26)$$

and from Equations (23), (24), and (26), it is found that,

$$\frac{M_s}{M_c} = 0.1145 \qquad (27)$$

Thus, it is seen that the spiral steel provides about 10% of the total torsional resistance. The percentage resistance will increase in direct proportion to the percentage of steel supplied.

By a similar procedure it may be shown that, for a square member with 45° spiral reinforcement, before the concrete yields and re-distribution of stresses occurs, the relation between tension stress in the steel and shearing stress in the adjacent concrete is approximately,

$$v_s = 2.92 \frac{E'_c}{E_s} t_s \qquad (28)$$

Assuming the same values for E'_c and E_s as for the circular section,

$$t_s = 6.85\ v_s \dots\dots\dots\dots\dots\dots\dots\dots (29)$$

and for 1% spiral steel,

$$\frac{M_s}{M_c} = 0.0628 \dots\dots\dots\dots\dots\dots\dots\dots (30)$$

In this case the spiral steel provides about 6% of the total torsional resistance. Here, again, the percentage resistance will increase with the percentage of steel supplied.

The preceding anaylsis leads to a consideration of the deduction that (see "Conclusions"), "* * * the spiral reinforcement can be assumed to take practically all tensile stresses in excess of the ultimate tensile strength of the unreinforced concrete." This statement may be interpreted to mean that after the working stress in torsional shear for plain concrete has been exhausted, the remainder of the torsional moment, regardless of its magnitude, may be assigned to the spiral steel at a safe working stress. If the concrete working stress in torsional shear is taken as 150 lb per sq in., it follows from Equation (23) that, for a circular section, the corresponding steel stress is only 1 500 lb per sq in. Such a unit stress will result in a wasteful use of steel. If the steel is stressed to 18 000 lb per sq in., the adjacent concrete will tend to be stressed to about 1 800 lb per sq in., but will crack before this stress is reached, and only a small core of material will still be effective in diagonal tension.

It is reasonable to assume that the factor of safety to be used in design should be based on the loading that causes cracking rather than on that which produces ultimate failure even if, because of the presence of steel, the member shows considerable toughness and strength after cracking begins. Cracks are objectionable and, as indicated by Fig. 6, when the concrete begins to yield (at about one-half the ultimate strength of the member) there is a rapid increase in the rate of twist. From Table 2, it appears that a working stress of 150 lb per sq in. in torsion provides a factor of safety of about two for plain concrete, with an ultimate compressive strength of 2 250 lb. per sq in.

A safe, economical, and consistent method would be to design the member as if it were of plain concrete, using a limiting working stress of about 150 lb per sq in. (for 2 250-lb. concrete). For the purpose of increasing the factor of safety, enough spiral steel area may be provided (at 15 000 lb per sq in.) to resist the total design moment, assuming the concrete to have cracked and the steel to be acting alone. For a circular section, using $v_c = 150$ lb per sq in., $t_s = 15\,000$ lb per sq in., and $\rho_s = \frac{9}{10}\ \rho_c$, it is found that a total steel area of 0.79% of the total cross-section area is sufficient for this purpose. For a square section the corresponding figure is 0.66 per cent. If the torsional shear is 50 lb per sq in., or less, there will be a sufficient margin of safety without the use of reinforcement.

It may be noted that in a cylindrical member the maximum stress intensity occurs at the surface and is uniform throughout, whereas in a member of rectangular section the maximum stress occurs on a line along the middle of the long side. There is opportunity for considerable re-distribution of stress in a member of rectangular section before cracking occurs. For this reason a somewhat higher working stress intensity (about 200 lb per sq in. for 2 250-lb. concrete) might be used for a rectangular section than for a circular section.

Frank M. Russell,[6] Jun. Am. Soc. C. E. (by letter).—To the writer's knowledge, this paper is the first of its kind to be published on research investigations performed in the United States. It is very timely as the subject of torsion is becoming of more concern than in times past, because of the thoroughness and exactness to which modern designing is being extended.

When concrete is stressed to failure in torsion the outer fibers are stressed much more than the inner ones, due to their distance from the axis of the cylinder. The stress in this fiber, then, is not the true intensity of stress in the entire specimen. Upton has derived a mathematical analysis by which the true intensity of shearing stress above the elastic limit may be obtained.[7] In the particular case of a round shaft his results reduce to a constant correction factor of 0.75; that is, if a specimen is stressed beyond its elastic limit Equation (11) should read:

$$v_m = 0.75 \frac{2M}{\pi \rho^3_e} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (31)$$

Besides the previous investigations mentioned by the author, a more recent one, and one which covers the subject very thoroughly, was performed by Takenosuke Miyamoto, of Tokyo, Japan, in 1927.[8] Miyamoto even extended his investigations so far as to derive design formulas for reinforced concrete screw-piles. His work really continues and completes the experiments[3] which were started by Professor E. Mörsch, of Stuttgart, Germany. Miyamoto's tests include plain concrete cylinders, and cylinders reinforced, as follows: Spirals at 30°, 45°, and 60°; spirals at 45°, with axial rods; spirals at 60°, with axial rods; spirals at 30°, with rings; axial rods and rings; both inner and outer spirals at 45°; both outer and inner spirals at 45° with axial rods; and inner spirals supported by axial rods. He also gives a mathematical interpretation of all his results. He agrees with the author in that the 45° spiral is the strongest, and, further, that the 30° spiral is the weakest of the three and that the 60° spiral has a strength between that of the 45° and the 30° spirals. Miyamoto also agrees with the author that rings increase the torsional resistance more than axial rods. The specimens which he found to give the greatest resistance of all, however, were those reinforced with inner and outer spirals and axial rods.

---

[6] Boulder, Colo.

[7] "Johnson's Materials of Construction", Rewritten by Messrs. Withey and Aston, and Edited by F. E. Turneaure, Hon. M. Am. Soc. C. E., Seventh Edition, p. 21.

[8] "Torsion Tests with Plain and Reinforced Concrete Bodies."

The paper is quite complete, but more could be stated about concrete in torsion without reinforcement, in order to develop more knowledge concerning the behavior of plain concrete itself, as in the case of twist in a dam. Not from the standpoint of detracting from the subject of reinforcement, but from that of making the subject more complete, the writer wishes to state the results of his own research program on torsion of plain concrete.[9] The mixes used in this series of tests ranged in compressive strengths from 2 390 to 11 080 lb per sq in. In torsion these corresponding mixes tested 352 to 903 lb per sq in. Revised by Upton's correction factor for this case, the foregoing would mean that the true shearing stress at failure for these mixes would be 264 and 677 lb per sq in., respectively. A total of 100 cylinders, 12 in. long and 3 in. in diameter, were tested, using eight different mixes.

To obtain such a high strength concrete, the mixes were designed by the method first introduced by Inge Lyse,[10] M. Am. Soc. C. E. This method is based on the principle that the strength of concrete increases directly in proportion to the increase in cement content, provided the water and absolute volume remain constant. Four different mixes were adopted using cement-water (not water-cement) ratios of 2, 2.5, 3, and 3.5. Two sandstones were used as coarse aggregates, and for each of the four mixes of one there was a corresponding mix of the other. Both sandstones were of the Lyons formation, but one was red and the other white. They have different characteristics, however, the red sandstone having a specfic gravity of 2.50 and a percentage absorption of 1.5, and the white sandstone, a specific gravity of 2.56 and a percentage absorption of 1.0. Ideal Portland cement and a good local washed sand were used in the mortar. Table 4 shows the results of tests of a large percentage of the specimens all of which were cured under water and tested wet.

TABLE 4.—RESULTS OF TESTS*

Series†	Cement-water ratio, by weight	Water-cement ratio, by volume	Slump, in inches	AVERAGE COMPRESSIVE STRENGTH, IN POUNDS PER SQUARE INCH		ULTIMATE SHEAR STRENGTH, IN POUNDS PER SQUARE INCH		ULTIMATE SHEAR STRENGTH CORRECTED, IN POUNDS PER SQUARE INCH		MODULUS OF ELASTICITY, IN SHEAR, IN THOUSANDS OF POUNDS PER SQUARE INCH†	
				28 days	90 days	28 days	90 days	28 days	90 days	28 days	90 days
RD......	2.0	0.75	1.0	4 550	5 570	442	483	331	362	1 440	1 670
RE......	2.5	0.60	0.6	6 500	7 710	557	690	417	517	1 600	1 880
RF......	3.0	0.50	0.6	8 710	9 990	628	773	471	580	1 830	2 020
RG......	3.5	0.43	0.2	9 720	10 470	618	835	538	626	1 730	2 030
WD......	2.0	0.75	1.0	3 940	5 530	470	532	352	398	1 770	2 000
WE......	2.5	0.60	0.8	6 070	7 630	635	731	476	549	1 980	2 180
WF......	3.0	0.50	0.8	8 350	9 810	673	727	505	549	2 160	2 290
WG......	3.5	0.43	0.2	9 560	11 080	773	903	580	677	2 160	2 270

* All strengths and modulus of elasticity in shear are in pounds per square inch.
† *R* denotes coarse aggregate of red sandstone; *W* = white sandstone.

The writer's experiments agree perfectly with the author's conclusion which states that the modulus of elasticity in shear depends upon its ultimate strength, and that high strength concrete has a higher shearing modulus than low strength concrete. For strengths of concrete nearly twice that cited by the author, the writer found that the relationship was still a straight line. The white sandstone mixes tested somewhat higher than those of the red sandstone, but both showed straight-line relationships between ultimate strength and modulus of elasticity in shear, and were exactly parallel.

---

[9] "Torsional Elastic Properties of Higher Strength Portland Cement Concrete", by Frank M. Russell, Jun. Am. Soc. C. E.; submitted as a thesis to the Univ. of Colorado (1933) in partial fulfillment of the requirement for the degree of Master of Science.

[10] *Engineering News-Record*, Vol. 107, No. 19, November 5, 1931, pp. 723-724.

Several relationships were studied to see how closely torsional properties could be assumed to parallel compressive properties. The theory of Professor Lyse determines the relationship between cement-water ratio and ultimate strength in compression, but it does not mention whether the straight-line relationship holds true for torsion as well as compression. At earlier ages (7 days) the straight-line relationship holds. As the concrete ages the trends are the same, but a greater increase in strength occurs between cement-water ratios of 2 and 2.5 than for any cement ratio greater than 2.5.

On a construction project it is quite probable that a part of a structure may have intermittent loads applied before it is thoroughly cured. In such a case, it would be well to know just how this might affect a member subjected to torsional stresses. With nearly every set of the writer's tests a pair of specimens was tested to a load which was from 40 to 60% of the ultimate for that set. These pairs were tested at each 7, 28, and 90-day period, and restored to water after each test. In twelve of fourteen sets the modulus of elasticity in shear was higher due to partial loadings with the greatest difference at 28 days. There was no apparent effect on the ultimate strength due to partial loading.

Along this same line of investigation, tests were made to find the effect of running the load up twice before taking the third trial to failure. The partial loads varied from two-fifths to two-thirds of the ultimate strength. The results from this showed that the properties are not affected materially, even if the proportional elastic limit is somewhat exceeded.

Further investigation found a relationship to exist between the modulus of elasticity in compression and the modulus of elasticity in shear. While the curve shows a definite curvature for each sandstone mix, it is slightly different for each one. Fig. 12 shows the curves superimposed upon the same axes. The formula for the red sandstone curve is:

$$E' = 1.20 + 0.058E + 0.003E^2 + 0.002E^3 + 0.0007E^4 + 0.0001E^5 \quad \text{..(32)}$$

in which, E' and E are in millions of pounds per square inch. For the white sandstone, the formula is much simpler, being,

$$E' = 1.18 - 0.04E + 0.056E^2 \quad \text{................(33)}$$

The points plotted are the averages for each set of specimens tested. There is a possibility of one curve approximating all points plotted. It is quite probable that for other concrete mixes a similar relationship could be found, using this same principle. This is quite plausible from the fact that there is a definite relationship between modulus of elasticity and ultimate strength in compression, and between modulus of elasticity in shear and ultimate

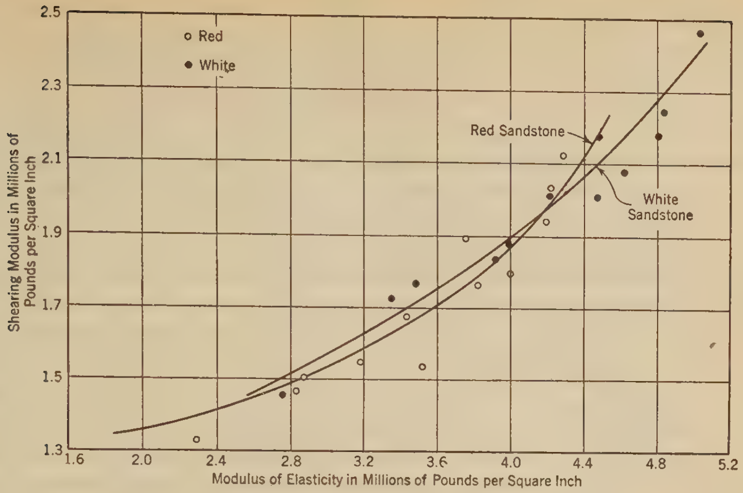


FIG. 12.—COMPOSITE OF ELASTIC CONSTANT CURVES.

strength in torsion. Then, with the same concrete, there should be a definite relationship between the two moduli. Such a curve would be a good basis for choosing a suitable factor of safety, as suggested in the author's "Conclusions".

LESLIE TURNER,[11] ESQ. (by letter).—A facet of concrete design as yet scarcely understood or appreciated, is opened for discussion in this paper. Doubtless, this topic has not been fully investigated owing to: (1) The complexity of the subject; and, (2) its lesser incidence and commercial importance vis-a-vis bending and direct stresses.

However, with more precision in design and with the more advanced types of structures, the effects of torsional resistance will be taken more and more into account, particularly when it becomes more widely appreciated that the influence of torsion on the factor of safety of reinforced concrete structures may be considerable. Consider, for example, the case of the simple slab spanning two directions on four walls; the torsional effects in thick slabs may be so large as to influence considerably the moment calculations. In all monolithic structures in which beams and slabs run into each other at right angles the torsional effect is always present, and is responsible to no mean degree for the additional security or factor of safety beyond the theoretical often found. At present, however, one cannot determine its amount with accuracy and precision, so that papers describing research on this subject are to be welcomed.

It is to be noted that the Tokyo experiments are not cited in the paper. These were carried out by M. Takenosuke about 1926 on ninety-six circular specimens 30 cm in diameter.[12] The results are in general accord with those of the author.

---

[11] Cons. Engr.; Technical Adviser, Trussed Concrete Steel Co., Ltd., London, England.

[12] "Torsional Strength of Reinforced Concrete", by Miyamoto Takenosuke, *Concrete and Constructional Engineering,* Vol. 22, 1927.

It is noted that Mr. Andersen has plotted the angle of twist against stress computed according to the conventional elastic theory; it would be interesting to have these angles plotted against torques. The reason for this is that the writer and Mr. V. C. Davies have found[13] the tensile stress-strain relation of plain concrete to be more nearly parabolic than straight, and may be expressed for the 1:2:4 concrete used, as: Stress, in pounds per square inch $= 244 - 450 \times 10^6 \, (0.000067 - \text{strain})^{1.5}$, or, in general terms,

$$f = t - C\,(D - d)^{1.5} \ldots\ldots\ldots\ldots\ldots\ldots (34)$$

in which, t = the maximum tensile stress; D, the maximum tensile strain; d, the strain at Stress f; and, C is a constant.

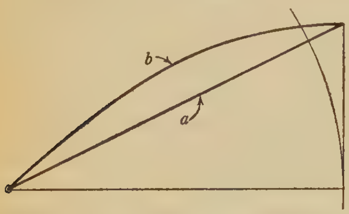


Fig. 13.

Thus, the stress distribution across a section follows Curve *b*, Fig. 13, rather than Curve *a*, the straight line. This is confirmed by Fig. 7, in which the tensile curve is parabolic in form. It differs from the writer's results, however, in the high value of the strains measured. The difference is more than would be accounted for by concrete mix, and it is difficult to explain. Contributory causes may be the shape of the section; the position at which the strain was measured; the influence of the reinforcement on the shrinkage of the concrete; and the fact that the tensile strain may be influenced by the presence of the diagonal compression in the torsion specimen when compared with a pure tension specimen. The values given in Table 2 for shear modulus agree quite well with the writer's results.

The foregoing disposes of the conventional elastic theory in relation to torsion and introduces what may be styled a hyper-elastic theory which modifies the formulas by which stresses are calculated for any particular section. The higher extensibility of concrete at high stresses, which follows from the parabolic tension curve, is a feature of considerable importance in torsion problems and explains another interesting discovery, namely, the lesser importance of the re-entrant angle in concrete sections, which, according to the conventional theory, is the vital spot. For most materials the stresses in the corner of a 90° angle, such as between the rib and tee of a beam, are infinite, whereas the writer's experiments have proved that this case does not hold for concrete although, of course, the addition of corner fillets does increase the strength of sections to certain degrees.

The writer did not make or test any circular sections and he is glad to note Mr. Andersen's confirmation of his own conclusion, that the circular section is sensibly the same as the exscribed square in torsional strength. On account of its larger area, of course, the square is slightly stronger than the circle.

The circular section with spiral reinforcement is the only one that lends itself to mathematical investigation. The basis for calculation for other sections is so obscure at present as to render purely theoretical treatment almost valueless. Furthermore, the practical difficulty in providing generally any but rectilinear reinforcement (longitudinal bars and links or ties), should not be overlooked, while the value of such reinforcement is still more difficult to assess theoretically. It is far less effective, of course, than spiral reinforcement.

---

[13] "Plain and Reinforced Concrete in Torsion", by L. Turner and V. C. Davies, *Selected Paper No. 165*, Inst. C. E., London, England, 1934.

The writer has preferred, therefore, to base his recommendations on the results of tests, with empirical formulas and rules for application to other and intermediate sections, shapes, and reinforcement. Rectangles, T-beams and L-beams have been considered, and, in this connection, it should be mentioned that the soap film analogy developed by Messrs. A. A. Griffith and G. I. Taylor,[14] has been very useful in pointing the way toward the solution of these problems. The question of torsion combined with bending and direct forces, of course, requires much further thought and research.

Mr. Andersen has developed some interesting formulas on the assumption that peripheral reinforcement is provided to take care of the tensile stresses in excess of the permissible stress for plain concrete. The question, however, is what is the permissible stress and what relationship should it bear to the ultimate to give the necessary factor of safety. In discussing Fig. 10 (see "Concrete in Torsion"), it is suggested that the concrete in a reinforced section can sustain a higher tensile stress than that in a plain section. While this may be "apparent", due, among other causes, to the reinforcement spreading incipient cracks, or to shrinkage causing stressing before torque is applied, it is difficult to accept different qualities for the same concrete.

The formulas could be adapted for the parabolic distribution, and then the factor of safety must be selected dependent upon the amount of external or visible cracking (if any) allowed. This would bring them into line with bending formulas where cracking is theoretically permitted in tension areas although, of course, it does not often develop, practically speaking. In other words, the cracks, if formed at all by the concrete yielding its stress to the steel, are so minute and dispersed as not to be noticeable.

The stress distribution along a side shown in Fig. 3 is the conventional form resulting from St. Venant's theory for elastic bodies, but it is more than doubtful that it applies to concrete except in the very early stages of stressing.

Finally, in welcoming heartily a paper on this somewhat involved subject, the author is to be thanked and congratulated on introducing new material and methods of attack and on presenting them so concisely.

A. W. Fischer,[15] Esq. (by letter).—The observed results obtained by the author are valuable when the member is subjected to pure torsional moment and the section is a square section. However, since practically all reinforced concrete members are not only subjected to torsion, but, at the same time, receive stresses due to bending moments, and since practically all of them are rectangular in section, the experiments in pure torsion on square sections do not help the designer for actual conditions that occur in most structural members. For that reason tests should be conducted to ascertain the combined torsional and diagonal shear on rectangular sections.

---

[14] "The Use of Soap Films in Solving Torsion Problems", *Proceedings*, Inst. Mech. Engrs., London, England, 1917, p. 787.

[15] Care, Pennsylvania Sugar Co., Philadelphia, Pa.

The author cites longitudinal balcony girders that support cantilever beams and exterior floor-beams. These two types certainly receive shear due to torsion and diagonal shear due to bending moments caused from vertical loads —and the depths of these members are generally from one and one-half to four times their width.

The writer does not recommend the use of inverted beams (that is, beams placed above the slab), but in some cases due to lighting conditions it is advisable to use them for the support of the outer edges of exterior slabs.

In such cases the top horizontal bars are quite essential; similar bars can be placed in balcony girders especially if the latter are designed as **T**-girders. The vertical tie stirrups must be placed as close to the outer and inner faces as permissible, and the wider the beams the more effective the stirrups will be.

If the stirrups are placed at 45° with the vertical they will be more effective, but for practical purposes it is better to place them vertically and use fairly small bars, closely spaced. Spirals will be more effective but from a practical standpoint they will not serve as well for the bottom tension steel and the top compression steel; nor can a spiral be flattened to suit a rectangular beam very easily, and, for that reason, vertical stirrups made in the form of a tie with ends having at least a hook of eight bar diameters will take care of the torsional and vertical shear in the most practical manner.

Although stirrups do not act until the beam begins to fail, if they are then brought into action so as to keep the beam from failing, they serve their purpose. A beam should be designed for the vertical and torsional shear, and the required number of vertical tie stirrups should be specified to take care of the combined shear.

H. J. Gilkey,[16] M. Am. Soc. C. E. (by letter).—To the citations in this paper, of representative illustrations of torsional stresses in concrete structures, might be added many others. Further enumeration seems unnecessary because the frequency of their occurrence and the need for more information on the torsional behavior of concrete must be apparent to any thoughtful concrete designer.

The scarcity of experimental evidence on torsional behavior is apparent from the paucity of references cited under the heading, "Previous Investigations". Moreover, every previous investigation noted was conducted outside the United States. Any available additional evidence should constitute a desirable supplement to that presented.

During 1928 and 1929 Fredrik Vogt, Assoc. M. Am. Soc. C. E., and the writer conducted torsional tests on plain concrete in connection with the extensive auxiliary test program which accompanied the construction and testing of models of the Stevenson Creek and Gibson Arch Dams at the University of Colorado, by the U. S. Bureau of Reclamation in co-operation with the Committee on Arch Dam Investigation of Engineering Foundation. Some of these data reached publication in May, 1934,[17] and, therefore, were not available, in published form, until after Mr. Andersen's paper was in print. The tests included no specimens reinforced against torsional stresses, but some of the results supplement Mr. Andersen's tests on plain concrete in an interesting manner.

---

[16] Prof. and Head of Dept. of Theoretical and Applied Mechanics, Iowa State Coll., Ames, Iowa.

[17] "Tests of Models of Arch Dams and Auxiliary Concrete Tests Conducted by the Bureau of Reclamation at the University of Colorado," by Engineering Foundation, Committee on Arch Dam Investigation, Sub-Committee on Model Tests, Vol. II (May, 1934).

In this discussion it is proposed to avoid repeating, as far as is reasonably possible, material available in the published report, by makng specific references to it when needed.

An arch dam is subjected to compressive, flexural, tensile, and shearing stresses. Torsion is, of course, a rotational shear. To interpret properly the test data obtained from the models, it was necessary to know the stress-strain properties of the material for each of these several types of stress. This involved, among other things, the determination of Poisson's ratio in compression, Young's modulus in compression, flexure, and tension, and the modulus of rigidity, or the corresponding shearing modulus. Thus, for two different concrete mixtures (that of the Stevenson Creek model and that of the Gibson model) there are available for direct comparison, results from all these several types of test. While the numbers of duplicate specimens are not large (except in the case of compression) the tests were well controlled and carefully executed, with the result that few values appear to be appreciably out of line.

When the Stevenson Creek tests were planned, it was expected that the testing of the model would be in the air-dry condition and it was necessary, therefore, to make the control program cover a range of cases which involved drying out in air for different lengths of time after various periods of moist curing. Later, it proved desirable to keep the model moist at all times up to and during the testing period, but the original varied curing program was followed through for the auxiliary model tests of Stevenson Creek Dam. This gives a wide range of strengths obtained by the variations in time and nature of curing and in the saturated and air-dry conditions at test. Thus, while the effect of varied curing conditions is only of secondary interest here, the different ages, curings, and test conditions do supply additional comparisons between torsional and other properties.

The test data appear in Table 5. It is to be noted that the details concerning curing and testing conditions (Column (2), Table 5) are given in Table 6. The torsional and flexural ultimate stresses in Column (4), Table 5, were computed by the elasticity formulas:

$$f = \frac{M\,c}{I} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(35)$$

and,

$$v = \frac{M\ \rho_e}{J}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(36)$$

in which, in addition to the notation in the paper, J = polar moment of inertia; c = distance from neutral axis to extreme fiber; and f = a unit fiber stress. Consequently, these values are approximate. Upton[18] developed a method for determining the true stresses for ultimate torsional and flexural loadings. The ultimate shearing stress at the extreme fibers in torsion reduces

---

[18] "Materials of Construction", by G. B. Upton, pp. 52 and 78; or Johnson's "Materials of Construction", rewritten by M. O. Withey and James Aston (1919), Fifth Edition, John Wiley & Sons, New York, N. Y., pp. 23 and 28.

TABLE 5.—Results of Tests in Compression, Flexure, Tension, and Torsion

Line No.	Total age (1)	Curing and testing condition* (2)	Kind of test (3)	Ultimate Strength, in Pounds per Square Inch: Approximate (4)	Ultimate Strength, in Pounds per Square Inch: Adjusted (by Upton's method) (5)	Modulus of elasticity, in millions of pounds per square inch (6)	Unit Deformation, in Ten Thousandths of an Inch per Inch, at: 50% of ultimate load (7)	Unit Deformation, in Ten Thousandths of an Inch per Inch, at: Ultimate load (8)
			(*a*) Series I*					
1	7 days	Standard	Compression	1 780	...	1.83	5.00	16.50
2	7 days	Standard	Flexure	401	267	2.67	0.75	2.30
3	7 days	Standard	Tension	234	...	2.10	0.34	0.78
4	7 days	Standard	Torsion	284	213	0.97	1.49	3.63
5	28 days	Standard	Compression	3 200	...	2.69	6.60	20.50
6	28 days	Standard	Flexure	570	380	2.91	1.04	2.56
7	28 days	Standard	Tension	275	...	2.65	0.48	1.10
8	28 days	Standard	Torsion	413	310	0.99	2.10	5.01
9	3 months	Standard	Compression	4 250	...	2.91	7.50	22.00
10	3 months	Standard	Flexure	652	435	3.80	0.86	2.18
11	3 months	Standard	Tension	329	...	3.20	0.44	1.04
12	3 months	Standard	Torsion	447	335	1.33	1.49	5.09
13	1 year	Standard	Compression	4 850	...	3.32	7.60	25.00
14	1 year	Standard	Flexure	631	421	3.03	1.05	2.57
15	1 year	Standard	Tension	333	...	3.05	0.55	1.42
16	1 year	Standard	Torsion	504	378	1.58	1.68	4.32
17	1 year	8W 4AD	Compression	5 086	...	2.68	10.10	34.80
18	1 year	8W 4AD	Flexure	850	567	2.90	1.47	3.24
19	1 year	8W 4AD	Tension	486	...	3.06	0.80	1.93
20	1 year	8W 4AD	Torsion	639	479	1.45	2.16	4.86
21	1 year	8W 4AW	Compression	3 710	...	2.68	7.20	26.60
22	1 year	8W 4AW	Flexure	720	480	2.83	1.28	2.85
23	1 year	8W 4AW	Tension	278	...	2.52	0.51	1.55
24	1 year	8W 4AW	Torsion	359	270	0.99	1.70	4.98
25	1 year	3W 9AD	Compression	4 026	...	2.44	8.70	30.60
26	1 year	3W 9AD	Flexure	675	450	2.84	1.24	2.78
27	1 year	3W 9AD	Tension	373	...	2.86	0.56	1.34
28	1 year	3W 9AD	Torsion	476	357	1.23	2.15	5.13
29	1 year	3W 9AW	Compression	3 226	...	2.53	6.70	23.60
30	1 year	3W 9AW	Flexure	520	347	2.40	1.11	3.28
31	1 year	3W 9AW	Tension	240	...	2.63	0.41	1.12
32	1 year	3W 9AW	Torsion	290	217	0.95	1.58	3.99
33	1 year	28W 11AD	Compression	3 542	...	2.25	9.70	26.80
34	1 year	28W 11AD	Flexure	624	416	2.44	1.08	2.93
35	1 year	28W 11AD	Tension	313	...	2.72	0.61	1.51
36	1 year	28W 11AD	Torsion	469	352	1.33	1.98	4.95
37	1 year	28W 11AW	Compression	2 866	...	2.30	6.20	22.80
38	1 year	28W 11AW	Flexure	449	300	2.15	1.04	2.66
39	1 year	28W 11AW	Tension	197	...	2.55	0.38	1.17
40	1 year	28W 11AW	Torsion	323	242	1.02	1.50	3.50
41	1 year	7W 12AD	Compression	3 096	...	2.25	7.20	23.70
42	1 year	7W 12AD	Flexure	632	421	2.68	1.25	2.61
43	1 year	7W 12AD	Tension	323	...	2.45	0.60	1.36
44	1 year	7W 12AD	Torsion	431	323	1.21	1.77	3.71
45	1 year	7W 12AW	Compression	2 462	...	2.23	5.70	20.50
46	1 year	7W 12AW	Flexure	438	292	2.27	0.97	1.96
47	1 year	7W 12AW	Tension	254	...	2.42	0.50	1.33
48	1 year	7W 12AW	Torsion	330	247	1.09	1.49	3.69
49	1 year	2W 12AD	Compression	2 470	...	2.04	6.00	20.90
50	1 year	2W 12AD	Flexure	540	360	2.05	1.32	2.92
51	1 year	2W 12AD	Tension	250	...	2.42	0.52	1.30
52	1 year	2W 12AD	Torsion	416	312	1.11	1.77	3.84
53	1 year	2W 12AW	Compression	2 026	...	1.94	5.50	17.70
54	1 year	2W 12AW	Flexure	416	277	2.02	1.03	2.17
55	1 year	2W 12AW	Tension	220	...	2.12	0.54	1.35
56	1 year	2W 12AW	Torsion	300	225	1.10	1.34	3.09

* See Table 6 for details of curing and testing conditions.

TABLE 5.—(*Continued*)

Line No.	Total age (1)	Curing and testing condition* (2)	Kind of test (3)	ULTIMATE STRENGTH, IN POUNDS PER SQUARE INCH		Modulus of elasticity, in millions of pounds per square inch (6)	UNIT DEFORMATION, IN TEN THOUSANDTHS OF AN INCH PER INCH, AT:	
				Approximate (4)	Adjusted (by Upton's method) (5)		50% of ultimate load (7)	Ultimate load (8)
				(*b*) SERIES II*				
57	7 days...	Standard...	Compression	1 995	...	2.27	4.70	19.20
58	7 days...	Standard...	Flexure.....	527	351	2.51	1.02	2.30
59	7 days...	Standard...	Tension....	201	...	....	....	.....
60	7 days...	Standard...	Torsion....	297	223	1.09	1.42	3.20
61	28 days...	Standard...	Compression	3 650	...	2.96	6.30	20.80
62	28 days...	Standard...	Flexure.....	571	381	3.21	0.91	2.10
63	28 days...	Standard...	Tension....	315	...	3.08	0.52	1.20
64	28 days...	Standard...	Torsion.....	467	350	1.49	1.45	3.40
65	5 months.	Standard...	Compression	4 768	...	3.40	7.10	22.40
66	5 months.	Standard...	Flexure.....	707	471	3.75	0.96	2.40
67	5 months.	Standard...	Tension....	420	...	3.99	0.55	1.10
68	5 months.	Standard...	Torsion....	605	454	2.32	1.84	4.00

* See Table 6 for details of curing and testing conditions.

to approximately three-fourths of that given by Equation (36), and in flexure the ultimate stress is two-thirds the modulus of rupture. These values are given in Column (5), Table 5.

Flexural unit deformations are computed from the formula:

$$e = \frac{6\ y\ d}{L^2} \qquad (37)$$

in which, e = unit deformation in extreme fibers at mid-span; y = measured deflection at mid-span; d = depth of beam; and L = clear span. Torsional unit deformations are computed from the formula:

$$e_s = \frac{\rho_e\ \psi_L}{L_g} \qquad (38)$$

in which, e_s = unit linear shearing displacement at the extreme fiber; ρ_e = radius of the circular torsion specimen; ψ_L = measured total twist, in radians, for length, L_g; and, L_g = gauge length over which detrusion or twist is measured.

TABLE 6.—DESCRIPTIONS OF AGES, CURINGS, AND TEST CONDITIONS

Item No.	Curing code number (1)	CURING TIME			Test condition (5)	Item No.	Curing code number (1)	CURING TIME			Test condition (5)
		Moist (2)	Air of laboratory (3)	Total (4)				Moist (2)	Air of laboratory (3)	Total (4)	
	(*a*) SERIES I										
1	Standard..	7 days..	....	7 days..	Wet	11	7W 12A D	7 days...	12 months*	1 year....	Dry
2	Standard..	28 days..	....	28 days..	Wet	12	7W 12AW	7 days...	12 months*	1 year....	Wet
3	Standard..	3 months	....	3 months	Wet	13	2W 12A D	2 days...	12 months*	1 year....	Dry
4	Standard..	1 year...	....	1 year...	Wet	14	2W 12AW	2 days...	12 months*	1 year....	Wet
5	8W 4AD..	8 months	4 months	1 year...	Dry						
6	8W 4AW..	8 months	4 months	1 year...	Wet		(*b*) SERIES II				
7	3W 9AD..	3 months	9 months	1 year...	Dry						
8	3W 9AW..	3 months	9 months	1 year...	Wet	15	Standard.	7 days..	....	7 days..	Wet
9	28W 11AD	28 days..	11 months	1 year...	Dry	16	Standard.	28 days..	....	28 days..	Wet
10	28W 11AW	28 days..	11 months	1 year...	Wet	17	Standard.	5 months	....	5 months	Wet

* Slightly less than 12 months.

Table 6 supplements Table 5 by supplying the details concerning curings, ages, and test conditions. Items Nos. 1 to 4, inclusive, and Items Nos. 15, 16, and 17, were tested after undergoing the standard moist curing for the periods given in Table 6. The remainder were cured in the air of the laboratory for the stated periods, after being cured moist. Items Nos. 6, 8, 10, 12, and 14, however, differ from the corresponding test, Items Nos. 5, 7, 9, 11, and 13, in that they were immersed for 24 hr prior to the test.

In Series I (Table 6(*a*)) the concrete mix was 1: 3.25 by weight; the water-cement ratio was 1.00 (7.5 gal per sack of cement); the slump was 8 in; and the aggregate was crushed granite screenings of $\frac{3}{8}$-in. maximum size. Moist curing was by immersion after removal of the moulds at two days. Specimens exposed to air at the age of two days were immersed for about an hour immediately after the removal of the moulds (see Items Nos. 13 and 14, Table 6). It was unsafe to handle the slender beams prior to an age of two days and it was deemed desirable that all moulds be removed at the same age.

In Series II (Table 6(*b*)) the concrete mix was 1: $2\frac{1}{2}$: 2 by weight; the water-cement ratio was 1.00 (7.5 gal per sack of cement); the slump was 7 in.; the fine aggregate was sand from 0 to No. 4 mesh; and the coarse aggregate was pea gravel graded from $\frac{3}{8}$ in. to $\frac{1}{2}$ in.

The specimens for both series were of the same kind, as follows: Compressive specimens were standard 3 by 6-in. cylinders, parallel tests being made on 6 by 12-in. and 2 by 4-in. cylinders. Flexural specimens were 3 by 3 by 40-in. beams loaded at the center, simply supported and tested on a 38-in. span; tensile specimens were 3 by 12-in. cylinders, the ends of which were clamped in spherically seated grips; and torsional specimens were of the same type as the tensile specimens. Each compressive result in Table 5 is from at least five tests on duplicate specimens. Other test results are from two or more duplicate specimens. The agreement in results obtained from companion specimens was satisfactory.

Table 7 contains several types of ratios useful in appraising the results of the tests. For example, in Table 7(*a*), are listed the ratios of ultimate strengths in compression, flexure, and tension to those in torsion. This includes both the apparent flexural and torsional ultimates and the actual strengths, as determined by Upton's method. Table 7(*b*) gives similar ratios for stiffn ss, the property of which the modulus of elasticity is a measure.

Table 7(*c*) shows the relationship of maximum linear unit deformations in the extreme fibers in torsion to those in compression, flexure, and tension. Ratios are given for the ultimate deformations (that is, those corresponding to the maximum load carried) and for deformations corresponding to 50% of the maximum load carried. The relationship at 50% of the ultimate load is the more significant value since it lies virtually within the elastic range of stress for the mixtures of these series,[19] and is more dependable than ratios based on strains which were observed while the specimen was deforming rapidly and at the point of fracture. Nevertheless, the uniformity of the

---

[19] See plotted stress-strain diagrams for compression, flexure, tension, and torsion, Rept. of Committee, Arch Dam Investigation, Engineering Foundation, Vol. II, pp. 476–477, 502–504, and 529–532.

TABLE 7.—COMPARISONS OF TORSIONAL PROPERTIES WITH THOSE IN COMPRESSION, FLEXURE, AND TENSION

Item No.*	(a) RATIOS OF COMPRESSIVE, FLEXURAL, AND TENSILE STRENGTHS TO THE TORSION STRENGTHS OF COMPANION SPECIMENS						(b) RATIOS OF STIFFNESS IN TORSION TO THAT IN COMPRESSION, FLEXURE, AND TENSION †			(c) RATIOS OF UNIT LINEAR DEFORMATIONS IN TORSION TO THOSE IN COMPRESSION, FLEXURE, AND TENSION					
	Nominal			Modified (Upton)						At 50% of Ultimate Load			At Ultimate Load		
	$\frac{\text{Compression}}{\text{torsion}}$ (1)	$\frac{\text{Flexure}}{\text{torsion}}$ (2)	$\frac{\text{Tension}}{\text{torsion}}$ (3)	$\frac{\text{Compression}}{\text{torsion}}$ (4)	$\frac{\text{Flexure}}{\text{torsion}}$ (5)	$\frac{\text{Tension}}{\text{torsion}}$ (6)	$\frac{\text{Torsion}}{\text{compression}}$ (7)	$\frac{\text{Torsion}}{\text{flexure}}$ (8)	$\frac{\text{Torsion}}{\text{tension}}$ (9)	$\frac{\text{Torsion}}{\text{compression}}$ (10)	$\frac{\text{Torsion}}{\text{flexure}}$ (11)	$\frac{\text{Torsion}}{\text{tension}}$ (12)	$\frac{\text{Torsion}}{\text{compression}}$ (13)	$\frac{\text{Torsion}}{\text{flexure}}$ (14)	$\frac{\text{Torsion}}{\text{tension}}$ (15)
(a) SERIES I															
1.....	6.28	1.42	0.83	8.36	1.26	1.10	0.53	0.36	0.46	0.30	2.00	4.40	0.22	1.58	4.65
2.....	7.75	1.38	0.67	10.30	1.23	0.89	0.38	0.34	0.37	0.32	2.01	4.38	0.24	1.95	4.57
3.....	9.50	1.46	0.74	12.70	1.30	0.98	0.46	0.35	0.42	0.20	1.73	3.39	0.23	2.32	4.88
4.....	9.65	1.25	0.66	12.85	1.12	0.89	0.48	0.52	0.52	0.22	1.60	3.05	0.17	1.68	3.04
5.....	7.97	1.33	0.76	10.65	1.19	1.02	0.54	0.50	0.47	0.21	1.47	2.70	0.14	1.50	2.52
6.....	10.37	2.01	0.78	13.75	1.78	1.03	0.37	0.35	0.40	0.23	1.33	3.33	0.19	1.75	3.20
7.....	8.45	1.42	0.78	11.28	1.26	1.04	0.51	0.44	0.43	0.25	1.73	3.85	0.17	1.84	3.82
8.....	11.10	1.79	0.83	14.90	1.60	1.11	0.38	0.40	0.36	0.24	1.42	3.88	0.17	1.21	3.54
9.....	7.55	1.33	0.67	10.05	1.18	0.89	0.59	0.55	0.49	0.20	1.82	3.24	0.19	1.68	3.27
10.....	8.90	1.39	0.61	11.90	1.24	0.82	0.44	0.48	0.40	0.24	1.44	3.95	0.15	1.31	2.98
11.....	7.20	1.47	0.75	9.60	1.30	1.00	0.54	0.45	0.49	0.25	1.42	2.95	0.16	1.88	2.73
12.....	7.48	1.33	0.77	10.00	1.18	1.03	0.49	0.48	0.45	0.26	1.53	2.98	0.18	1.88	2.76
13.....	5.95	1.30	0.60	7.95	1.15	0.80	0.54	0.54	0.46	0.30	1.34	3.40	0.18	1.31	2.94
14.....	6.78	1.39	0.73	9.02	1.23	0.98	0.57	0.55	0.52	0.24	1.30	2.48	0.17	1.42	2.29
(b) SERIES II															
15.....	6.72	1.77	0.68	8.95	1.57	0.90	0.48	0.43	....	0.30	1.38	....	0.17	1.39	....
16.....	7.80	1.23	0.68	10.50	1.09	0.91	0.51	0.47	0.48	0.23	1.58	2.78	0.16	1.62	2.83
17.....	7.82	1.17	0.70	10.50	1.04	0.93	0.68	0.62	0.58	0.26	1.92	3.35	0.18	1.66	3.64

* For details of curing time and testing conditions, see corresponding item numbers in Table 6.

† That is, ratio of modulus of rigidity to modulus of elasticity, or Young's modulus; or $\frac{E'}{E}$.

ratios at the ultimate, and the excellent agreement of trend with those at 50%, is interesting.

Table 8 supplies comparisons of the age-strength relationship for the several types of loading for all specimens that have been "standard cured".

TABLE 8.—COMPARISON OF THE AGE-STRENGTH RELATIONSHIPS FOR STANDARD CURING (RATES OF INCREASE AS PERCENTAGES OF 28-DAY STRENGTH)

Item No.	Age (1)	COMPRESSION		FLEXURE		TENSION		TORSION	
		Strength, in pounds per square inch (2)	Percentage of 28-day strength (3)	Strength, in pounds per square inch (4)	Percentage of 28-day strength (5)	Strength, in pounds per square inch (6)	Percentage of 28-day strength (7)	Strength, in pounds per square inch (8)	Percentage of 28-day strength (9)
(a) SERIES I									
1	7 days...	1 780	56	401	71	234	85	284	69
2	28 days....	3 200	100	570	100	275	100	413	100
3	3 months.	4 250	133	652	115	329	119	447	108
4	1 year....	4 850	152	631	111	333	121	504	122
(b) SERIES II.									
5	7 days...	1 995	55	527	93	201	64	297	64
6	28 days...	3 650	100	571	100	315	100	467	100
7	5 months.	4 768	131	707	123	420	133	605	127

Fig. 14 shows the torsional testing rig which was devised for these tests and the essential features of which were utilized for many subsequent torsional tests of the plaster-celite mixture used in the model of Boulder Dam and in Boulder Dam concrete researches conducted by the U. S. Bureau of Reclamation in its Denver, Colo., laboratories. In these tests it proved convenient to hook the rig on to the lever system of a hand-power beam testing machine which made it easy to apply known increments of load. Torque is applied entirely free of bending and excellent spiral fractures were obtained in all tests. The spiral fractures in these tests were longer than that shown in Fig. 8 of Mr. Andersen's paper. Any bending would tend to shorten the length of the fracture; or the difference may come from differences in the behavior of the respective concretes used. The angle of failure for Fig. 5 of the paper is more nearly similar to the breaks illustrated in Fig. 15.

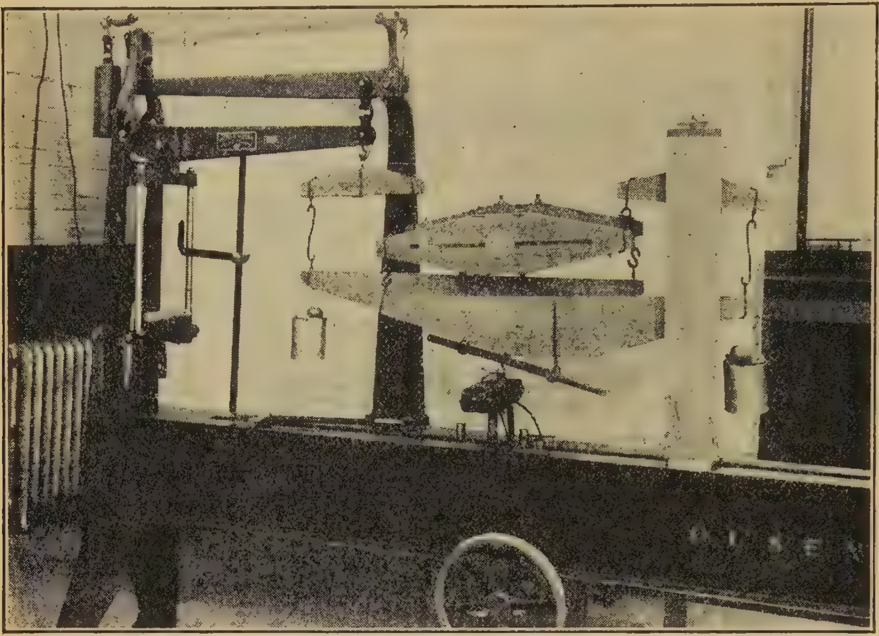


FIG. 14.—TORSIONAL TESTING RIG.

The latter view shows some typical fractures and more detail of the rig and the troptometer which was used for measurements of detrusions. Two breaks per specimen were usually obtained. After the first break the troptometer could not be used because of the shortened gauge length, but the second break gave excellent checks on the ultimate torsional strength of the specimen. Apparently, the strength of the unbroken part of the specimen was not affected by the first test since second tests gave strengths that were sometimes slightly greater, and sometimes slightly less, than those at the first test.[20]

---

[20] For a complete drawing of the rig and troptometer, see p. 455, of the Engineering Foundation, Arch Dam Investigation, Vol. II.

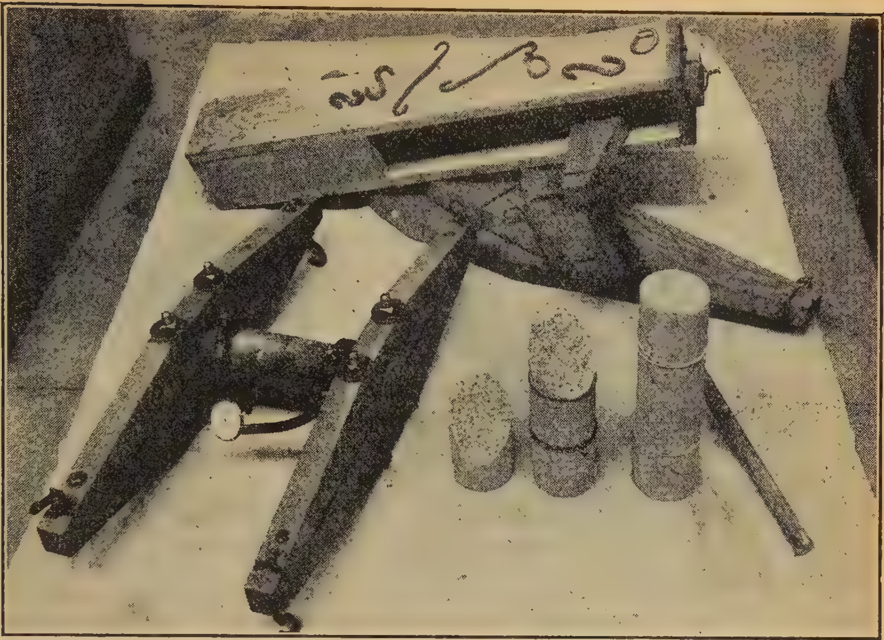


FIG. 15.—DETAIL OF TROPTOMETER, AND TYPICAL TORSION FAILURES.

Fig. 16 shows the three sizes of compressive specimens fully harnessed with longitudinal compressometers and lateral extensometers.[21] The compressometers were similar to those in use for years at the laboratories of the University of Illinois, Portland Cement Association, and others. However, their use on specimens as small as 3 by 6 in. and 2 by 4 in. had not been attempted elsewhere. The lateral extensometers were improvised for these tests and similar ones have since been devised by various laboratories.

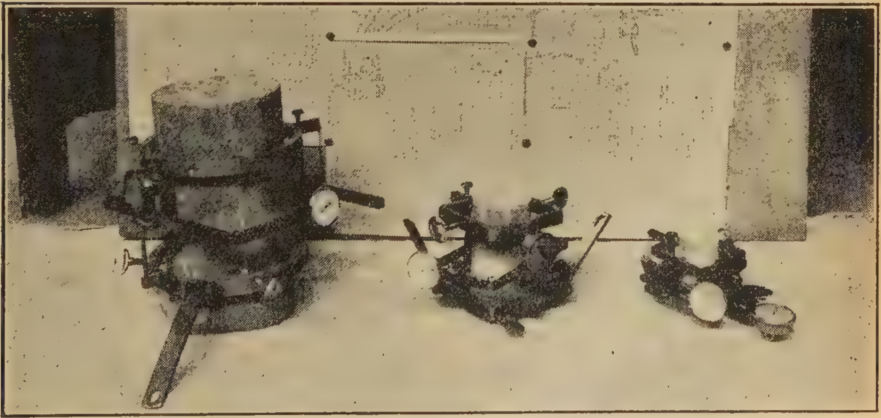


FIG. 16.—COMPRESSIVE SPECIMENS WITH LONGITUDINAL COMPRESSOMETER AND LATERAL EXTENSOMETERS.

---

[21] See, also, pp. 467 to 469, and pp. 472 and 473, of the Engineering Foundation, Arch Dam Investigation, Vol. II.

Fig. 17 shows the tensile grips, the tensile extensometer (the one with the "last-word" dials in the background), and a number of representative tensile breaks. In the foreground, and at the extreme right, are shown parts of another extensometer[22] not used in these tests. The tensile extensometers were developed for these tests and the tensile grips were patterned after grips previously used by the Portland Cement Association. After one test, with stress-strain data taken, several additional breaks with only the ultimate load observed could be obtained in the tensile tests. The strengths agreed well with those from the first break, as was true, also, for torsion.

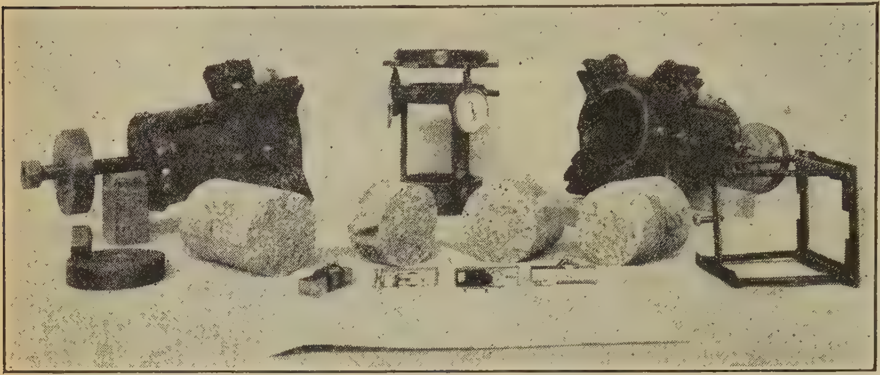


FIG. 17.—TENSILE GRIPS AND TENSILE EXTENSOMETER, WITH A NUMBER OF REPRESENTATIVE TENSILE BREAKS.

A study of Tables 7 and 8 appears to justify the following generalizations within the scope of these tests:

1.—From Table 7(*a*) it is evident that the apparent torsional ultimate strengths (Equation (36) for M_t at ultimate) can be expected to be from one-sixth to one-ninth the ultimate compressive strength, with one-eighth (say, $12\frac{1}{2}\%$) a fair average value. The true or modified torsional strength, as corrected by Upton's method, $v = \frac{3}{4}\,\frac{M_t\ \rho_e}{J}$ (see Table 5 and text description relating to it), will be about one-eleventh, or 9%, of the ultimate strength in compression.

2.—The apparent ultimate strength in torsion is about 70% of the apparent ultimate strength in flexure (Equation (35)), and the true torsional strength is about 80% of the true flexural strength $\left(f = \frac{2}{3}\,\frac{M\,c}{I}\right)$, as obtained by Upton's method.

3.—The apparent ultimate strength in torsion is about 25% greater than the observed tensile strength, but the true torsional strength is the same as the ultimate tensile strength as determined by test. The evidence on this score strongly supports Conclusion 2 of the paper. When bending is eliminated as fully as it was in the writer's tests and probably also in Mr. Andersen's torsional tests, it is possible that the torsion test gives the better measure of tensile strength. While the tensile grips themselves were spherically seated, it is still possible for some imperfection in alignment to occur at junctions of specimens with grips. Moreover, the tightening of the grips no doubt effects the cross-sections for a few inches from them. Only a small part of the material that lies in the path of torsional fracture can be located near a grip, whereas many of the tensile fractures were within the 1 in., or 2 in., of a grip. The excellent agreement between tensile strengths and corrected torsional strengths is re-assuring, however, and indicates no discrepancy of magnitude.

---

[22] See, also, pp. 452–453, 456–457, Engineering Foundation, Arch Dam Investigation, Vol. II.

4.—From Table 7(*b*) the ratio of torsional stiffness to stiffness in compression is between 0.40 and 0.50, with the mean value probably between 0.40 and 0.45. The values for flexure and tension are practically the same, which is in agreement with the general conclusion from various researches that the modulus of elasticity of concrete is practically the same in compression, flexure, and tension. For values of Poisson's ratio between 0.15 and 0.25, E' would lie between 0.435 E and 0.40 E, according to the formula,

$$E' = \frac{E}{2(1+\sigma)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (39)$$

in which, E' = the modulus of elasticity in shear (modulus of rigidity); E = modulus of elasticity; and σ = Poisson's ratio.[23]

Poisson's ratio was determined for all specimens tested in compression and was found to lie between 0.15 and 0.25, with about 0.20 as a fair average.[24] There apears to be no consistent variation in Poisson's ratio for any known variable (strength, richness of mixture, curing, or test condition, etc.) except a slight tendency to increase with the age at test under standard curing. This statement is based upon many test results from many different series and mixtures. The extreme range is from about 0.10 to slightly more than 0.30, the higher values having been from specimens several years old at test.

5.—From Table 7(*c*) the torsional linear unit displacement at the extreme fibers for 50% of the ultimate load is about 0.25 that in compression, 1.5 that in flexure, and more than three times that in tension. The ratios at ultimate load differ little from these values. In noting the fact that maximum torsional deformations exceed maximum tensile deformations in a ratio of virtually 3 to 1, it should be kept in mind that the average torsional deformation is much less than the maximum. The same thought applies likewise to the beam which finally fails progressively in tension, as does the torsional specimen.

6.—Table 5 shows wide variations in strengths, moduli of elasticities and deformations for the different ages, curings, and test conditions. It is extremely satisfying, therefore, to note that not one of these major variables appears to have any consistent effect upon the ratio of the torsional (or other) property to the corresponding property in compression, flexure, or tension. In other words, variations in mixture, materials, age, curing, and test condition effect the compressive, flexural, tensile, and torsional properties in essentially the same manner and to the same relative extent. (Support for the author's Conclusion 1 follows as a corollary of the writer's Conclusion 6.)

---

[23] See any textbook on Strength of Materials, such as that by James E. Boyd, Third Edition (1924), p. 58.

[24] Engineering Foundation, Arch Dam Investigation, Vol. II, pp. 476, 477, 487, 529, and 536.

7.—Data as to the relationship of age to strength for the standard curing condition are brought together in Table 8, in which the rates of increase of strengths with ages are expressed as percentages of the respective 28-day strengths. In general, the torsional behavior is very similar to that in flexure and tension. In all three of these respects, a higher percentage of the ultimate strength is developed in the early stages of the curing period than for compression. Not all the data for Series II support this statement, but other tests bear it out as far as tension and flexure are concerned.[25]

In this discussion no mention has been made of the relative toughness and resilience of the concretes in compression, flexure, tension, and torsion. The areas under the stress-strain diagrams are taken as measures of this property.[26]

Acknowledgment.—The varied data of this discussion were obtainable only because of the hearty support that was accorded the auxiliary test program by J. L. Savage and Ivan E. Houk, Members, Am. Soc. C. E., of the U. S. Bureau of Reclamation Staff, functioning under R. F. Walter, M. Am. Soc. C. E., Chief Engineer, and Elwood Mead, M. Am. Soc. C. E., Commissioner of Reclamation.

A. A. EREMIN,[27] ASSOC. M. AM. SOC. C. E. (by letter).—A new method of computing the shearing stresses distributed in plain concrete and reinforced concrete members sustaining torsion, is developed in this paper. In his tests the author studied the characteristics of members with square and round sections. In bridges and buildings, however, it occurs more often that the members sustaining torsion are of rectangular section. Torsion shearing stresses distributed in rectangular sections of reinforced concrete members may be analyzed by means of Equation (13) or Equation (14), assuming an equivalent round section.

The maximum torsion shearing stress along the center, of the wide side of a rectangular section of a plain concrete member, as developed by Professor Bach,[2] is,

$$v_m = \left(3 + \frac{2.6}{0.45 + \dfrac{B}{a}}\right) \frac{M}{8\,B\,a^2} \qquad (40)$$

---

[25] Strength-age graphs are shown for the same tests on pp. 461 and 526 of the Engineering Foundation, Arch Dam Investigation, Vol. II. On pp. 525 and 526, the torsional strength at 5 months should be 605 instead of 570. For the various curings and test conditions plotted graphs can be found on pp. 429, 438, 463, 464, and 465.

[26] Graphic comparisons are shown on pp. 496, 497, 539, and 540 of the Engineering Foundation, Arch Dam Investigation, Vol. II.

[27] Assoc. Bridge Designing Engr., Div. of Highways, State Dept. of Public Works, Sacramento, Calif.

in which, $2B$ = depth of the wide face; and $2a$ = width of the narrow face. From Equations (11) and (40), the radius of an equivalent round section may be determined, as follows:

$$\rho_e = \sqrt[3]{\frac{16\,B\,a^2}{\pi\left(3 + \dfrac{2.6}{0.45 + \dfrac{B}{a}}\right)}} \qquad \text{(41)}$$

In a square section, $2B = 2a$; therefore, from Equation (41), the radius of the equivalent round section for a square is $\rho_e = 1.02\,B$, or, approximately, a circle inscribed in the square, as shown by the author.

The efficiency coefficient for steel reinforcement at the wide and narrow faces of a rectangular member may be computed by Equation (15). The efficiency coefficient of steel along the narrow side of a rectangular section must be reduced by the ratio of the maximum shearing stress along the narrow side to the maximum shearing stress along the wide side, r.

The maximum torsional shearing stress along the center of the narrow side of a rectangular section is,

$$v_m = \left(3 + \frac{2.6}{0.45 + \dfrac{B}{a}}\right)\frac{M}{8\,a\,B^2} \qquad \text{(42)}$$

Therefore, from Equations (40) and (42), the ratio, r, is,

$$r = \frac{a}{B} \qquad \text{(43)}$$

Equations (40) and (42), for the maximum torsional shearing stresses, were derived by Professor Bach from tests of concrete members with rectangular sections for which the ratio, r, varied from square section proportions to $r = 0.5$. For more conclusive results further theoretical and experimental study is needed, of shear distribution due to torsion in the wider rectangular sections.

PAUL ANDERSEN,[28] ASSOC. M. AM. SOC. C. E. (by letter).—In closing the discussion of this paper the writer wishes to present, very briefly, some results of tests with concrete subjected to long-time torsional moments which, it is to be hoped, will throw additional light on the subject.

The aim of these experiments was to determine the rate of increase (if any) in shearing distortions under continued loading and the influence of aggregate and of different intensities of stress. The shape and general dimensions of the test pieces were those of Type *R* in Fig. 4, except that these flow specimens were made hollow with an inside diameter of 5 in. In such a section, the variation in magnitude of shearing stress will be comparatively small.

---

[28] Balboa Heights, Canal Zone.

In testing, the shafts were arranged in rows of four, and adjacent square parts were firmly bolted together by structural steel clamps. One end of this row was fixed against rotation, and a torsional moment was applied at the opposite end by a lever arm and a concrete block of known weight, as indicated in Fig. 18. The loaded end as well as the joints was supported on 2-in. steel balls, which were greased to allow rotation about the axis of the shaft. The method of measuring angular distortions was that previously explained in connection with Fig. 5.

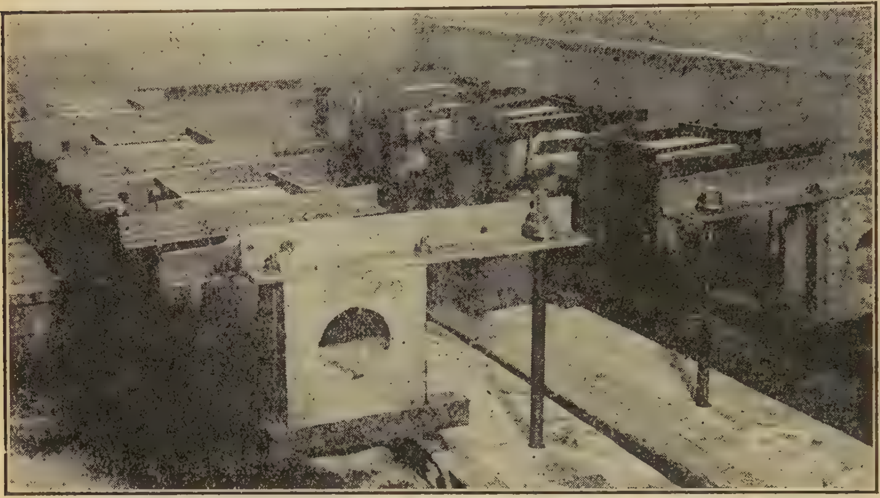


FIG. 18.—CONCRETE SPECIMENS SUSTAINING LONG-TIME TORSIONAL METHODS.

The recorded data from these tests showed that, due to shearing stresses, mortars and concretes flow in much the same manner as when subjected to compressive forces. The results obtained from two mortar specimens are plotted in Fig. 19. It should be noted that the stresses indicated are those midway between the inner and the outer surfaces, computed from the applied torsional moment. It appears from these tests that the sustained modulus of elasticity in shear is considerably lower than the immediate modulus, as

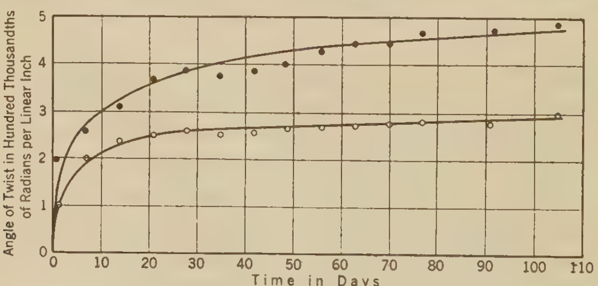


FIG. 19.—TIME SHEAR FLOW CURVES.

listed in Table 2. The writer concludes from this that the spiral reinforcement in a concrete member under torsion will actually provide a higher percentage of the total torsional resistance than Professor Mirabelli's discussion would indicate.

The writer apologizes for not including the Tokyo experiments[12] in the list of previous tests; recently, he has also read with much interest the paper by Messrs. Leslie Turner and V. C. Davies.[13] He agrees with Mr. Turner as to the difficulty of treating, mathematically, sections of rectangular, T, or L-shape. The empirical formulas referred to by Mr. Turner and fully explained in his paper[13] are interesting indeed.

Professor Gilkey has presented a valuable contribution to the discussion and the writer is pleased to note that the additional data appear to parallel his own findings. The test results presented by Mr. Russell are interesting, but the writer considers the empirical formulas, Equations (32) and (33), of doubtful value when compared to the established relationship,

$$E' = \frac{E}{2\,(1 + \sigma)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (44)$$

between the modulus of elasticity in compression and the modulus of elasticity in shear, σ being Poisson's ratio.

Properly, Mr. Fischer states that most concrete beams transmitting torsional moments are rectangular in section and that the writer's analysis, therefore, cannot be applied directly. In this connection, the writer is much interested in the possible extension of his theory, as presented by Mr. Eremin. Future investigations should be planned to verify Mr. Eremin's equations.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 1913

WAVE PRESSURES ON SEA-WALLS AND BREAKWATERS

By David A. Molitor[1], M. Am. Soc. C. E.

With Discussion by Messrs. Charles E. Fowler, Charles T. Leeds, A. A. Eremin, T. L. Condron and Chester L. Post, and David A. Molitor.

Synopsis

A method of evaluating wave pressures that may be expected in a given locality is presented in this paper and the writer applies the results of his observations to the investigation of structural stability, or design, of sea-walls and breakwaters. Since all factors contributing to the solution of the problems herein considered are only approximately knowable, no theoretical approach was deemed advisable. However, all available observational data were used in evolving a method of analysis.

Engineering structures should be designed with a certain margin of safety, yet not wastefully. In the past many structures, as first designed and built, required rebuilding after each severe storm until a satisfactory structure was finally developed. The design method herein presented should enable one to appraise his problem within reasonable limits, and arrive at a fairly economic solution, according well with experience on existing structures of enduring stability. The concluding remarks and two illustrative problems substantiate this claim.

Certain data on wind velocities are included because of the labor involved in collecting this information which is not readily obtainable from the United States Weather Bureau records. Data on weights of cribs and on coefficients of friction, so essential and important in design and stability investigations, are given because the elements of judgment and experience enter largely into their choice. The meager textbook information available may not even be applicable, or it may be misleading.

Note.—Published in May, 1934, *Proceedings*.

[1] Structural Engr., Procurement Div., Public Works Branch, Treasury Dept., Washington, D. C. (Formerly Prin. Designing Engr., Toronto Harbor Commrs., Toronto, Ont., Canada.)

Nature of the Problem

Sea-walls and breakwaters are exposed to the action of waves generated by high winds, sweeping over a considerable expanse of open water. The size of a wave for any particular locality depends on the velocity of the wind, duration of the storm, depth of water, and the greatest distance over which the wind can act, provided the water is of sufficient depth for wave formation.

Assuming a constant velocity and direction of wind, the height of a wave will increase from zero at the windward shore to some limiting value which the assumed wind velocity will just maintain. This distance from the windward shore to the wave is called the "fetch" of the wind. When the fetch is sufficiently great and the depth of water is ample, the wave will attain the limit of height before reaching the leeward shore, continuing with a constant height until some resistance is encountered in shallower water or against a structure of some kind.

As a wave increases in size according to the distance out from the windward shore, its length also increases and so does its velocity of propagation. When the balance is finally attained between the wind velocity and the wave dimensions, the velocity of the wave no longer increases, but may continue without reduction for long distances even after the wind has subsided.

Assuming for the present that the height of a wave may be estimated for a given fetch and wind velocity, as established for a given location where it is proposed to construct a sea-wall or a breakwater, the next step is to evaluate the probable mass and velocity of such a deep-water wave and, finally, to trace its retardation and diminished dimensions as it advances into shallower water previous to striking the wall where its moving mass is stopped. In case the wall is situated so that the maximum wave cannot strike normally, there will be further reduction in resolving the wave force normally to the face of the wall.

Having finally decided on the probable size of wave that may be expected to arrive in front of a wall, the next step is to estimate the height to which this wave will be piled up when completely obstructed, and then to decide on the height to which the wall is to be built. If the wall is not constructed to the full height of the completely obstructed wave, a portion of the wave will go over the top and the wave will be only partly obstructed.

When a wave is partly or completely stopped by a wall, a portion, or all, of its energy is converted into a dynamic force, which in turn represents the static equivalent of the expended energy. The wall structure must be capable of resisting this static equivalent without exceeding certain requirements of structural stability and safety.

The foregoing phases of the problem will next be examined in detail, to show that each element may be quantitatively evaluated in a logical sequence. It should be remembered, however, that all formulas used in this connection are based on observations which are in themselves quite approximate, while a theoretical basis is only remotely possible. The observational data herein cited were mostly those reported in 1904 by the late Capt. (afterward Lt.-Col.)

D. D. Gaillard, Corps of Engineers, U. S. Army,[2] supplemented by observations made by the writer in 1915, in connection with the harbor development, at Toronto, Ont., Canada.

WAVE HEIGHT, WIND VELOCITY, AND FETCH

For a given wind velocity, V, in miles per hour, and a fetch, D, in statute miles, the height, h, in feet, may be estimated for any wave with the aid of the following formulas:

For values of D greater than 20 miles:

$$h = 0.17 \sqrt{V D} \qquad (1)$$

and, for values of D less than 20 miles:

$$h = 0.17 \sqrt{V D} + 2.5 - \sqrt[4]{D} \qquad (2)$$

which are based on the formulas of Thomas Stevenson,[3] after introducing wind velocity as a variable, and using statute miles instead of nautical miles. These formulas apply only to inland lakes where the fetch is not likely to exceed the distance subjected to a violent wind in a single direction.

With a few exceptions ocean waves rarely exceed 45 ft in height, corresponding to a wind velocity of 75 miles per hr and a fetch of about 900 miles, which represents an unusual combination of circumstances. Ocean storms are generally more or less local, and usually cover not more than 50 to 100 miles. Since the oceans are of much greater extent than any possible storm, the fetch is restricted to the storm area, which may be anything from a few miles to, say, 900 miles, and no formula can be made to apply to such indeterminate conditions. Hence, the only reliable data relative to the height of ocean waves must be collected by direct observations for any given locality. An extensive collection of such observations was published by Captain Gaillard.[4]

LENGTH OF WAVE IN TERMS OF HEIGHT AND WIND VELOCITY

The highest wave has the minimum length ratio, and as the wind subsides the height diminishes and the length ratio increases. Fig. 1 gives certain dimensions for waves which will be used throughout this paper. The period, t, of a wave, is the time, in seconds, for the crest to travel the dis-

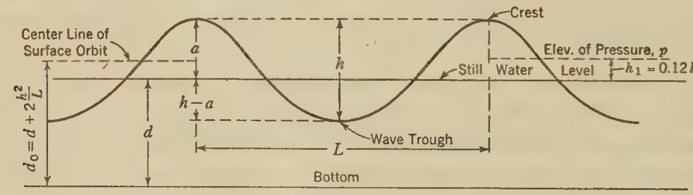


FIG. 1.

---

[2] "Wave Action in Relation to Engineering Structures," by Capt. D. D. Gaillard, Corps of Engrs., U. S. A., *Professional Papers No. 31*, U. S. Corps of Engrs., 1904.

[3] "Design and Construction of Harbours: A Treatise on Maritime Engineering," by Thomas Stevenson, Edition 2, Edinburgh, 1874.

[4] *Professional Papers No. 31*, U. S. Corps of Engrs., Table IX, p. 76.

tance, L, in feet, and $v = \frac{L}{t}$ is the velocity of wave propagation, in feet per second.

(a) The Ratio, $\frac{L}{h}$.—The length ratio, $\frac{L}{h}$, varies between rather wide limits depending on the velocity of the wind, V, the duration of a storm, and the depth of water in which the wave is formed. According to observations by Captain Gaillard in fresh and relatively shallow water, it varies between the limits, 9.1 and 15. A formula proposed herein, expresses this ratio in terms of wind velocity, V, as follows:

$$\frac{L}{h} = \frac{840}{V} \qquad (3)$$

Equation (3) gives the following values for various wind velocities:

V	$\frac{L}{h}$	V	$\frac{L}{h}$
30	28.0	55	15.3
35	24.7	60	14.0
40	21.0	65	12.9
45	18.7	70	12.0
50	16.8	75	11.2

For ocean waves, Dr. Gerhard Schott[5] gives the values listed in Table 1.

TABLE 1.—VALUES OF $\frac{L}{h}$ FOR OCEAN WAVES

Description	Beaufort scale	Wind velocity, in miles per hour	Wave length, L
Moderate wind	5	28	33 h
Strong wind	6 to 7	35	20 h
Storm	9	56	17 h

The ocean waves are thus seen to be relatively longer than waves on the Great Lakes, as might be expected.

(b) Height of Wave Above Still-Water Level.—Since the crest of a wave is above the still-water level and the trough is below that level, it is sometimes erroneously assumed that the still-water level represents the mean of the two. However, many observations by various investigators show the crest to be about $\frac{2}{3}$ h above the still-water level in deep water, and rising still higher in shallow water, a fact which must not be disregarded when examining the safety of a structure.

According to Captain Gaillard, the height of a wave crest above still-water level, before the wave breaks, is given by the following formulas:

For deep-water waves, with $d > 1.84\ h$,

$$a = \frac{h}{2} + \frac{h^2}{L} \qquad (4a)$$

---

[5] *Professional Papers No. 31,* U. S. Corps of Engrs., p. 87.

and for shallow-water waves, with $d < 1.84\ h$,

$$a = \frac{h}{2} + \frac{2\,h^2}{L} \qquad \text{(4b)}$$

For deep-water ocean waves, Rankine[6] gives the formula:

$$a = \frac{h}{2} + 0.785\,\frac{h^2}{L} \qquad \text{(4c)}$$

The term, still-water level, as used herein, represents a line (see Fig. 1) for which the sectional area of the wave ridge is equal to that of the wave hollow.

(c) Depth of Water in Which a Wave Breaks.—While waves may break in deep water due to the action of increasing wind and other causes, they always break when they reach water of insufficient depth. A knowledge of this minimum depth is necessary in determining the maximum wave that may be expected to arrive at a certain structure located in shallow water. Undoubtedly, the roughness of the bottom has much to do with this subject and it would seem that the depth in which a wave breaks is much less for a rough bottom than for a smooth sandy bottom.

For fifty-five observations at the Duluth Canal by Captain Gaillard, for waves from 7 to 13 ft in height and a sandy bottom sloping about 1:40, the waves broke in a depth equal to 1.72 h. Similar observations on Lake Superior near Presque Isle Point, Mich., and Grand Marais, Mich., with a rough bottom and waves from 6 to 9 ft high, the waves broke in a depth equal to 1.3 h. For ocean waves at St. Augustine, Fla., with a strong wind blowing in the direction of wave travel, the waves broke in a depth of 1.25 h.

Therefore, if d_1 = the depth in which a wave breaks, the relations listed in Table 2 may be accepted for storm waves with the wind in the direction of the wave travel.

TABLE 2.—DEPTHS OF WATER, d_1, IN WHICH A WAVE BREAKS

Wind velocity, V, in miles per hour	Depth of water, d_1, in which a wave breaks (in terms of h, the height of wave)	Body of water	Bottom condition
Gentle	1.34 h	Lake Superior	Sandy
26	1.42 h	Lake Superior	Sandy
30	1.30 h	Lake Superior	Rough
40	1.84 h	Lake Superior	Sandy
40	1.25 h	Ocean	Sandy

DYNAMIC PROPERTIES OF WAVES

To this point the paper has dealt with wave dimensions, and the relations existing between these and the actuating force. The dynamic properties, including the velocity of propagation, energy of a wave, wave pressure, and height of a completely obstructed wave will now be considered.

(a) Velocity of Propagation.—According to Rankine,[7] for long waves or for waves in water that is very shallow compared with the wave length, the velocity of propagation, v, is given by the formula:

$$v = \sqrt{g\,(d + \tfrac{3}{4}\,h)} = 5.68\,\sqrt{d + \tfrac{3}{4}\,h} \qquad \text{(5)}$$

---

[6] *Professional Papers No. 31*, U. S. Corps of Engrs., p. 37.

[7] *Loc. cit.*, p. 107.

in which, g = the acceleration due to gravity. According to Captain Gaillard, Equation (5) almost invariably gives results in excess of observed velocities. The theoretical velocity for deep-water waves with $d > \frac{L}{2}$, is:

$$v = \frac{L}{t} = \sqrt{\frac{g\,L}{2\,\pi}} = 2.26\,\sqrt{L} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

For shallow-water waves with $d < \frac{L}{2}$, Equation (6) becomes:

$$v = 2.26\,c\,\sqrt{L} \text{ when } \frac{d_0}{L} = \frac{d}{L} + 2\left(\frac{h}{L}\right)^2 \quad \ldots\ldots\ldots\ldots (7)$$

in which, $c = \sqrt{\frac{b_s}{a_s}}$, when $\frac{b_s}{a_s}$ expresses the ratio of the semi-minor to the semi-major axes of the surface orbits. Two other factors, β and μ, required in Equations (13) to (16), together with values of c, are given in Table 3.

TABLE 3.—VALUES OF c, β, AND μ, IN EQUATIONS (7), (13), (14), (15), AND (16).

Depth ratio, $\frac{d_0}{L}$	Values of c	Values of β,	Values of $\mu = c\,\beta$	Depth ratio, $\frac{d_0}{L}$	Values of c	Values of β	Values of $\mu = c\,\beta$
0.05.....	0.552	3.286	1.814	0.30	0.977	1.047	1.023
0.10.....	0.746	1.796	1.340	0.35	0.988	1.025	1.013
0.15.....	0.858	1.358	1.165	0.40	0.994	1.013	1.007
0.20.....	0.922	1.177	1.085	0.45	0.997	1.007	1.004
0.25.....	0.958	1.090	1.044	0.50	0.998	1.004	1.002

The wave length, L, and velocity, v, are materially altered when the depth gradually decreases toward a shoal or shore, and Captain Gaillard proposed an empirical formula based on a number of observations made at the Duluth Canal, where the waves entered the canal at a depth of 26 ft, while the portion of wave outside the canal entered shallow water, gradually shoaling to 3.3 ft. The formula is:

$$v_2 = 0.9\,v\,\sqrt[4]{\frac{d_2}{d}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

with a reduced height,

$$h_2 = h\,\sqrt{\frac{d_2}{d}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

in which, v is the velocity of the wave approaching from a depth, d, which should not be greater than $\frac{L}{2}$; and v_2 is the reduced velocity when the depth has shoaled to the lesser depth, d_2. A wave thus retarded will then have a reduced height, h_2.

In applying Equations (8) and (9) it is well to choose a depth of $\frac{L}{5}$ instead of $\frac{L}{2}$, because the dimensions of a deep-water wave are not appreciably altered until after the shoaling has become effective.

(b) The Energy of a Wave.—The theoretical energy, E, in foot-pounds, of a deep-water wave of length, L ft; height, h ft; and breadth of 1 ft, for fresh water, is given by:

$$E = 7.8\ L\ h^2 \left[1 - 4.935 \left(\frac{h}{L} \right)^2 \right] \ldots\ldots\ldots\ldots\ldots (10)$$

For salt water, the coeffecient, 7.8, becomes 8.0.

For shallow-water waves the energy decreases from 2 to 10% below the value given by Equation (10) and the appropriate formula becomes rather complicated. As no particular use is made of this formula, the matter will not be further investigated.

(c) Wave-Pressure Formulas.—Having enumerated the various steps in arriving at the probable dimensions of storm waves for a given locality, the important question confronting the designing engineer, is to appraise with some reasonable degree of accuracy, the force that such a wave develops in striking a proposed structure. The most satisfactory manner of attacking this problem is by direct measurements of the force expended by impinging waves on a solid structure carrying a series of self-registering dynamometers.

Dynamometer measurements were carried on for many years by Thomas Stevenson, at Skerryvore Rocks, in the Atlantic Ocean, in 1843 and 1844, and at Dunbar Harbor, in 1858.[8] Several attempts to measure wave pressures on the Great Lakes were made at various times, but without gaining much information of value. The experiments made, in 1901 to 1903, by Captain Gaillard on Lake Superior, stand as the most valuable contribution to this subject.

Before mentioning any experimental results, the following conclusions, arrived at by Captain Gaillard as a result of his work are, that: (1) The impact of a wave does not resemble that of a solid body; (2) the pressure indicated by the types of dynamometers heretofore used are due to dynamic action only; (3) the pressures mentioned in Conclusion (2) apparently conform to the hydrodynamic laws governing the action of a current flowing normally against a submerged plane; (4) a mass of water in air projected with a certain velocity against a plane surface can produce no greater pressure than would be caused by the steady flow against this surface of a jet of equal cross-section having the same velocity and striking at the same angle; (5) from Conclusions (1) to (4), it follows by inference, that a mass of water projected against a submerged plane surface of considerably smaller area than the cross-section of the mass, can produce no greater pressure than would be caused by the steady flow at the same velocity of a current against a submerged plane surface of equal area and similar to the first; and (6) as the most destructive waves act for an appreciable period, the pressures they exert can properly be measured by suitably constructed dynamometers.

---

[8] *Professional Papers No. 31,* U. S. Corps of Engrs., p. 145.

The formula for hydrodynamic pressure on a submerged plate (in pounds per square foot) according to Dubuat,[*] is:

$$p = kw \frac{v^2}{2g} \quad \dots\dots\dots\dots (11)$$

in which, v is the velocity of forward motion of the plate. The total velocity of the striking wave consists of the combined velocity of propagation, v, and the maximum orbital velocity, v_o, of a wave particle, so that Equation (11) should be modified to represent the maximum value (in pounds per square foot),

$$p_{\text{max.}} = \frac{k\,w}{2\,g} (v + v_o)^2 \quad \dots\dots\dots\dots (12)$$

in which, k is an empiric coefficient evaluated from Captain Gaillard's observations for the Great Lakes as 1.30 to 1.71 for winds from 30 to 70 miles per hr. For ocean storm waves, k may be taken as 1.8. For fresh-water waves, $w = 62.4$ lb per cu ft and for salt-water waves, $w = 64.4$ lb per cu ft, while the acceleration due to gravity is $g = 32.2$ sec-ft.

The velocity of propagation for waves formed in any depth of water is given by Equation (7). The maximum orbital velocity of a wave particle, similarly, is:

$$v_o = m\,c\,\sqrt{\frac{2\,\pi\,g}{L}} \quad \dots\dots\dots\dots (13)$$

in which, $m = \frac{\beta\,h}{2}$ = the semi-major axis of the elliptical orbit, and β is the ratio of the axes of the elliptical orbits. The numerical values of c and β, given in Table 3, render these expressions applicable to the solution of problems.

Introducing the numerical values of π and g, Equation (13) becomes,

$$v_o = \frac{c\,\beta\,h \times 14.22}{2\sqrt{L}} = 7.11\,\mu\,\frac{h}{\sqrt{L}} \quad \dots\dots\dots\dots (14)$$

and, with values of k, w, and g, substituted into Equation (12), the maximum unit wave pressure, in pounds per square foot, becomes:

$$p = \frac{k\,w}{2\,g} (v + v_o)^2 \quad \dots\dots\dots\dots (15)$$

in which, $v + v_o$ is the sum of Equations (7) and (13), or:

$$v + v_o = 2.26\,c\,\sqrt{L} + 7.11\,\mu\,\frac{h}{\sqrt{L}} \quad \dots\dots\dots\dots (16)$$

and $\frac{k\,w}{2\,g} = 1.71$ and 1.80, for fresh-water and for salt-water storm waves, respectively.

The values given for k are maximum for a 75-mile wind and may become as low as 1.3 for a 30-mile wind. When $\frac{d_o}{L}$ is equal to, or greater than, 0.5, $c = \mu = 1.00$, which is the condition for deep-water waves.

---

[*] *Professional Papers No. 31,* U. S. Corps of Engrs., p. 174.

Having determined the maximum pressure exerted by a striking wave, it is necessary to know the elevation above still-water level at which this maximum occurs, and the height to which a completely obstructed wave acts. The maximum wave pressure occurs at a height, h_1, above still-water level, the theoretical value of which is:

$$h_1 = 0.785 \frac{h^2}{L} \quad \text{(17)}$$

although better results are secured by:

$$h_1 = 0.12\, h \quad \text{(18)}$$

When a wave is completely obstructed by a vertical surface of sufficient height, the wave crest is raised to a height equal to $h_0 = 2\,a$ above the still-water level, and, at this height, the pressure becomes zero. With the foregoing data it is possible to design the entire wave-pressure area and thus evaluate the total force to be resisted by a proposed structure.

(d) Oblique Waves.—Waves striking a breakwater obliquely produce less pressure than those that strike normally. Using the relations indicated in Fig. 2, the wave pressure, p, from Equation (12), is reduced to a value,

$$p_n = p \sin^2 \alpha \quad \text{(19)}$$

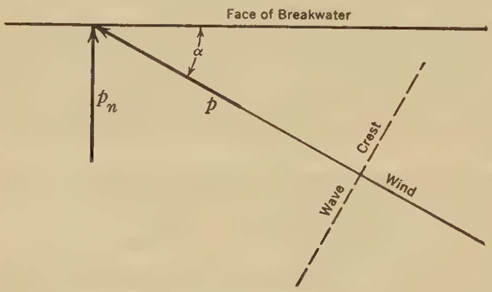


FIG. 2.

Observations made by the writer at Toronto, in 1915–1916, gave the following result: p = 1 460 lb per sq ft; p_n = 820 lb per sq ft; and α = 50 degrees. By Equation (19), p_n = 1 460 × 0.587 = 857 lb.

WAVE-PRESSURE OBSERVATIONS

(a) Wave-Pressure Observations on Lake Superior.—In 1901 and 1902 Captain Gaillard made numerous observations on Lake Superior which represent the best available data and will serve as a pattern for wave-pressure curves (see Table 4). It is regrettable that no accurate wind velocities were recorded, although it was stated that moderate wind, from 20 to 35 miles, produced these waves at Duluth, Minn. The pier carrying the dynamometers was not of sufficient height to obstruct the larger waves completely, so that for those higher than 13 ft the recorded pressures are for partly obstructed waves.

The coefficient, $\frac{k w}{2 g} = 1.3$, in Equation (12), was chosen because none of the waves listed in Table 4 was caused by extreme storms, while the possible maximum coefficient, 1.71, given for Equation (12) is more probable for winds from 70 to 75 miles per hr with shorter waves.

TABLE 4.—Wave-Pressure Measurements, Duluth Canal, Lake Superior

Date	Elevation, still-water level, in feet	Observed Maximum Wave Dimensions, Depth, d = 25 Feet			Maximum Dynamometer Readings, in Pounds per Square Foot			Computed		
					Elevations of Dynamometers					
		Height, h, in feet	Length, L, in feet	Velocity, v, in second-feet	+0.07 ft.	+3.74 ft.	+7.01 ft.	$\frac{d_0}{L}$, by Equation (7)	v_0, in second-feet	p, in pounds per square foot
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
1901:										
July 24	+1.7	12	150	24.2	250	1 150	1 030	0.180	7.8	1 330
Aug. 9	+1.9	12	130	24.2	370	1 190	780	0.210	8.0	1 348
Oct. 9	+1.9	13	150	29.6	0	1 615	1 260	0.180	8.4	1 876
Nov. 22	+1.4	14	150	27.2	0	1 605	1 605	0.184	9.0	1 705
Sept. 24	+1.9	16	250	33.2	1 630	2 255	2 050	0.110	9.3	2 344
					+7.04 ft.	+12.57 ft.	+16.18 ft.			
1902:										
Oct. 25	+1.7	16	200	30.0	1 755	1 335	0	0.138	9.5	2 026
Dec. 20	+1.7	16	210	31.0	1 700	1 430	515	0.131	9.4	2 120
Nov. 12	+1.7	18	250	32.0	2 370	2 195	1 370	0.110	10.5	2 344

In order to arrive at the total wave pressure to be resisted by a sea-wall or breakwater, it becomes necessary to design the complete wave-pressure curve from data previously calculated for a given wind velocity and fetch. This will determine the wave dimensions, h, L, and a, from which v, v_0, and p, may be evaluated. It now remains to find the total wave pressure for a completely or partly obstructed wave of these dimensions, and this must be approximated by patterning after some typical set of observations such as those shown in Fig. 3. For this case the fetch, D, is 260 miles, with a wind velocity, V, of 35 miles per hr. By Equation (1), $h = 0.17 \sqrt{35 \times 260} = 16.5$ ft. as compared with an observed value of h = 16 ft (see Fig. 3). Furthermore, $L = 210$ ft; $v = 31$ ft per sec; and $d = 25$ ft. Then, $h - a = 5.18$ ft; $h_0 = 2a = 21.64$ ft; $\frac{d_0}{L} = 0.13$; $c = 0.81$; $\mu = 1.22$; v_0 (by Equation (16)) $= 7.11 \times 1.22 \times \frac{16}{\sqrt{210}} = 9.57$ ft per sec. The pressure, p_0 (= 1 600 lb per sq ft at a point midway between Points B and C, Fig. 3), may be assumed equal to 0.72 p when designing a wave-pressure curve.

By selecting the maximum pressure values for a 16-ft wave from Table 4 and plotting them as a composite pressure curve, the section shown in Fig. 3 was obtained. The crest of this 16-ft wave determined by Equation (4b), for shallow water waves, was 10.8 ft above the still-water level, and this crest was raised to a height of 18.3 ft for the partly obstructed wave, with the trough at 5.18 ft below the still-water level.

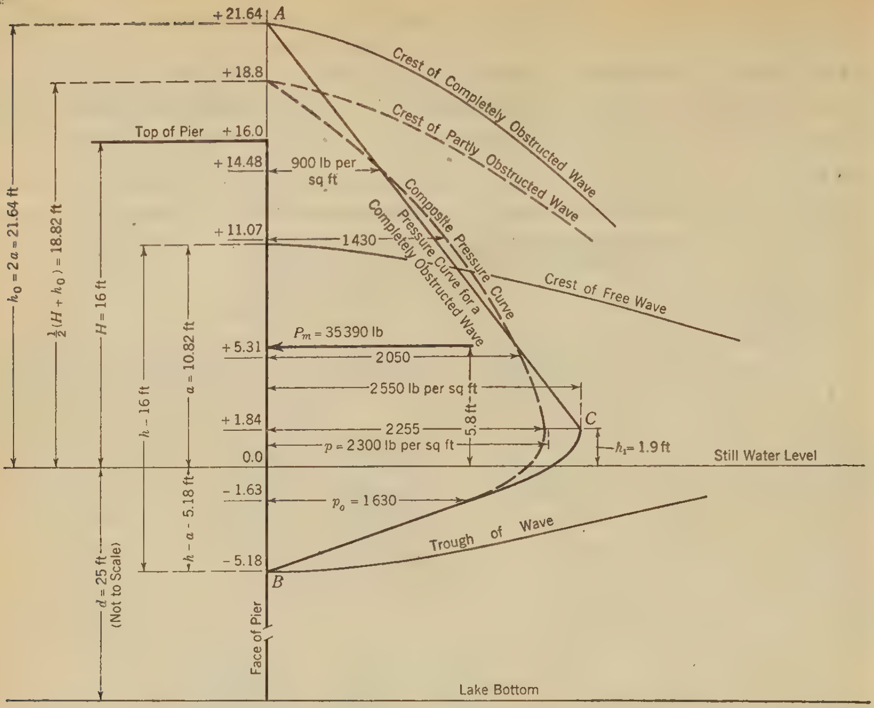

+21.64
A
Crest of Completely Obstructed Wave
+18.8
Crest of Partly Obstructed Wave
Top of Pier
+16.0
+14.48
900 lb per sq ft
Composite Pressure Curve
Completely Obstructed Wave
Pressure Curve for a
+11.07
1430
Crest of Free Wave
$h_0 = 2a = 21.64$ ft
$\frac{1}{2}(H + h_0) = 18.82$ ft
$H = 16$ ft
$a = 10.82$ ft
$h - 16$ ft
$P_m = 35\,390$ lb
+5.31
2050
2550 lb per sq ft
5.8 ft
+1.84
2255
C
$p = 2300$ lb per sq ft
$h_1 = 1.9$ ft
0.0
Still Water Level
-1.63
$p_0 = 1630$
$h - a - 5.18$ ft
Trough of Wave
-5.18
B
$d = 25$ ft (Not to Scale)
Face of Pier
Lake Bottom

FIG. 3.—A COMPOSITE WAVE PRESSURE CURVE BASED ON OBSERVATIONS FOR A 16-FOOT WAVE IN LAKE SUPERIOR, AT DULUTH, MINN.

Had the wave been completely obstructed, its crest would have reached a height, $h_0 = 2a = 21.6$ ft. The straight line, $A\ C$, in Fig. 3, is drawn to represent the total pressure of the completely obstructed wave with the maximum pressure at $h_1 = 0.12\ h = 1.9$ ft above the still-water level. (By Equation (15), $p = 1.4\ (31 + 9.57)^2 = 2\,300$ lb per sq ft.) The pressure curve is completed by drawing the curved line, $C\ B$. The total wave pressure for the completely obstructed wave is represented by the triangular area, $A\ B\ C = P_m = 35\,390$ lb, acting at the center of gravity for the pressure area which is 5.8 ft above still-water level. For the partly obstructed wave as observed, the total wave pressure, P, is computed as 33 340 lb, acting at 5.42 ft above the still-water level.

The composite wave pressure curve thus described, represents the plotting of the maximum pressures recorded, for the duration of a storm, on the several dynamometers located at different elevations. It is evident that this is not produced by any single wave, but that waves of various sizes contribute to produce the maximum pressures recorded. Hence, the total pressure area of such a curve is undoubtedly greater than the actual and thus includes a certain factor of safety.

(b) Wave-Pressure Observations on Lake Ontario.—The record herein presented was observed by the writer in connection with the sea-walls and

breakwater of the Harbor of Toronto, produced by a storm that began at 5:00 P. M., November 18, 1915. At about 7:00 A. M., on November 19, with an east wind, the maximum, V_m, of 42 miles per hr was attained. Dimensional wave measurements were made at 10:00 A. M., with the wind velocity at 30 miles per hr, after which it veered to the southeast and finally died in the south about 2:00 P. M.

The pressures were recorded on spring dynamometers mounted on a solid crib situated about 700 ft offshore in about 8 ft of water. At a distance of about 1 200 ft from the crib, the water deepened rapidly to about 40 ft. In front of the crib, three piles were driven at 150-ft centers forming an equilateral triangle and carrying gauge-boards for observing wave height, and the direction and velocity of wave crests. In Fig. 4(*a*) and Fig. 4(*b*), which show the results of these observations, the fetch, D, equals 156 miles from the east and 32 miles from the southeast. The wind velocity, V, was 40 miles per hr from the east, shifting to 30 miles per hr from the southeast, when the waves were at a maximum, and h_2 (Fig. 4(*b*)) = 5.5 ft; L = 118 ft; and v = 19.8 ft per sec.

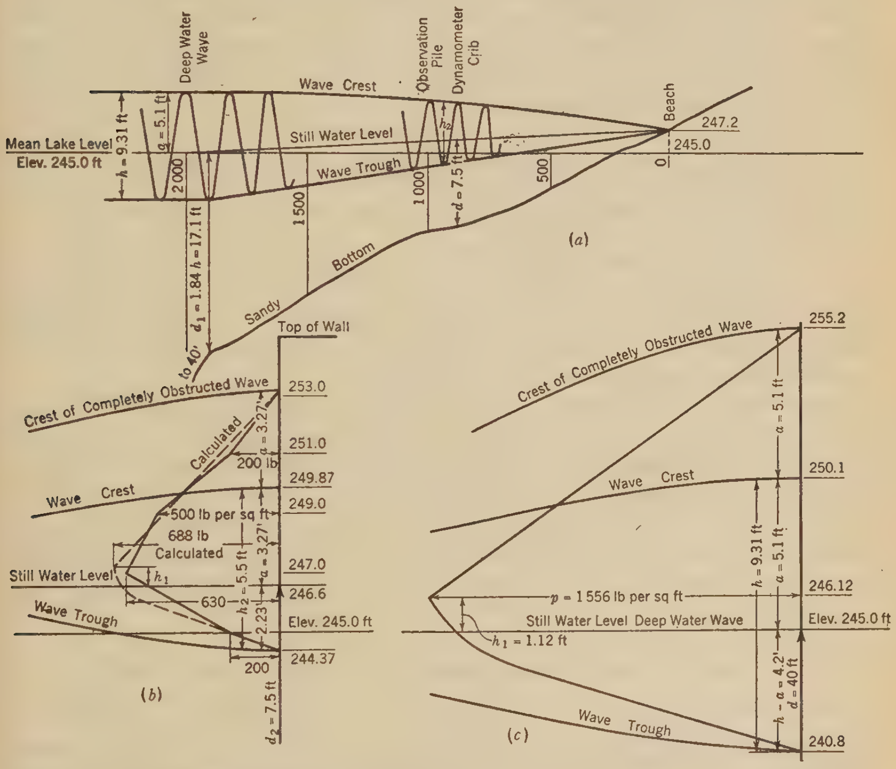

Deep Water Wave
Observation Pile
Dynamometer Crib
Wave Crest
Beach
247.2
245.0
Mean Lake Level
Elev. 245.0 ft
h = 9.31 ft
a = 5.1 ft
Still Water Level
Wave Trough
h_2
2000
1500
1000
500
0
d = 7.5 ft
(a)
Bottom
Sandy
d_1 = 1.84 h = 17.1 ft
to 40'
Top of Wall
Crest of Completely Obstructed Wave
253.0
Calculated
a = 3.27'
251.0
200 lb
249.87
Wave Crest
249.0
500 lb per sq ft
688 lb
Calculated
247.0
Still Water Level
h_1
h2 = 5.5 ft
a = 3.27'
246.6
630
Wave Trough
2.23'
Elev. 245.0 ft
244.37
200
(b)
d_2 = 7.5 ft
Crest of Completely Obstructed Wave
255.2
a = 5.1 ft
250.1
Wave Crest
h = 9.31 ft
a = 5.1 ft
246.12
p = 1 556 lb per sq ft
Still Water Level Deep Water Wave
Elev. 245.0 ft
h_1 = 1.12 ft
h − a = 4.2'
d = 40 ft
Wave Trough
(c)
240.8

FIG. 4.—WAVE OBSERVATIONS ON LAKE ONTARIO, AT TORONTO, ONT., CANADA, STORM OF NOVEMBER 19, 1915, USING SPRING DYNAMOMETER

By Equation (4) (with $C = 2$), $a = 3.27$ ft; $h_2 - a = 2.23$ ft; $h_1 = 0.12\ h_2$; $\frac{d_0}{L} = 0.068$; $c = 0.68$; and $\mu = 1.66$ (see Table 3). By Equations (7) and (14), $v_2 = 16.72$ and $v_0 = 6.28$, respectively. Consequently, $v_2 + v_0 = 23$ ft per sec, and, by Equation (15) (with $\frac{k\,w}{2\,g} = 1.3$), $p = 688$ lb per sq ft. Equation (8) gives $v_2 = 0.9\ v\ \sqrt[4]{\frac{7.5}{40}} = 17.45$ ft per sec. with a reduced height (according to Equation (9)) of $h_2 = 9.31\ \sqrt{\frac{7.5}{40}} = 4.05$ ft.

The deep-water wave dimensions in Fig. 4(*c*), were estimated for a wind velocity, V, of 30 miles per hr and a fetch, D, of 100 miles, representing something like an average condition prior to reaching the maximum. For this case, h (by Equation (1)) $= 9.31$ ft; $L = 21\ h = 195$ ft; $d_1 = 1.84\ h = 17.1$ ft; a (by Equation (4), with $C = 1$) $= 5.1$ ft; $h - a = 4.2$ ft; $h_1 = 0.12\ h = 1.12$ ft; $d = \frac{L}{5} = 40$ ft; $\frac{d_0}{L} = 0.22$; $c = 0.94$; and $\mu = 1.06$. By Equations (7) and (14), $v = 29.6$ and $v_0 = 5.0$, respectively. Consequently, $v + v_0 = 34.6$ ft per sec, and by Equation (15) (with $\frac{kw}{2\,g} = 1.3$), $p = 1\,556$ lb per sq ft. The still-water level was at Elevation 245.0 in deep water and began to rise when the depth was reduced to 17 ft where the waves broke, and finally reached Elevation 247.2 on the beach (Fig. 4(*a*)).

After entering shallow water, and producing the record shown by Fig. 4(*b*), the reduced wave had a height of 5.5 ft, a length of 118 ft, and a velocity of 19.8 ft per sec. These dimensions and pressures were checked by applying the foregoing equations to show the agreement between computed and observed values. It is thus seen that for a given condition of wind, fetch, and depth in which a proposed breakwater is to be built, the wave-pressure curve may be estimated with sufficient accuracy to test the stability of the proposed structure.

Stability of Breakwater Cribs Subjected to Wave Force

The foregoing parts of this paper present the several steps in evolving a wave-pressure curve from a given fetch and wind velocity, or, in other words, of evaluating the wave effect produced by a certain cause. The ultimate aim, however, is to design a breakwater crib of adequate dimensions to withstand, safely, the wave pressure to which it may be exposed during the most severe storms.

The complete solution of such problems will now be undertaken on the assumption that the exact location for a proposed structure and the maximum wind for the locality are known definitely. The examples chosen are the breakwater at Harbor Beach, Mich., and Magann's Pier at Toronto, both of which have withstood many severe storms since 1900 without showing any marked structural weakness. Hence, if the method of analysis herein given leads to findings in conformity with the actual behavior of these structures, the speculative features involved in the method will have been reduced to a negligible quantity. In this connection the wind roses in Fig. 5 and the weight data in Table 5 will be of value. The wind roses (Fig. 5) show maximum wind velocities for the eight cardinal points and the dates on which they occurred. These data were obtained from the officials of the U. S. Weather Bureau at the several stations. All the wind velocities are based on readings from three-cup anemometers.

TABLE 5.—Weight Data for Designing Breakwater Cribs

Material	Weight, in Pounds per Cubic Foot		Coefficients of Friction	
	In air	In water	Materials	Coefficient
Granite blocks	170	107.5	All surfaces of masonry or brickwork in contact	0.65 to 0.70
Limestone blocks	166	103.5	Stone or brickwork on moist unctuous clay	0.30
Limestone rip-rap (40% voids)	100	62.5	Stone on bed-rock, dry	0.70
Concrete, plain	148	85.5	Concrete blocks on well wetted concrete floor	0.70
Concrete, reinforced	152	89.5	Concrete blocks on rubble base, submerged	0.65
Timber, green, as shipped	35	27.5*	Rubble-filled cribs on rubble mound, submerged	0.90
Timber, wet (including bolts)	50	12.5*		
Average timber cribs, filled with limestone	92	50	Rubble-filled cribs on bed-rock submerged	0.60

* Uplift.

Problem 1.—Stability Investigation of Harbor Beach Breakwater (See Fig. 6).—The fetch, D, for Harbor Beach Breakwater, scaled from a map of Lake Huron, is found to be 132 miles north, or 92 miles northeast. The latter fetch is about normal to the breakwater. The maximum wind velocity from the north is taken as 60 miles per hr.

Since waves usually swing more or less normally to the shore as the water becomes shallower, the larger wave is assumed to arrive normally in front of the wall. Hence, the deep-water wave by Equation (1) has $h = 15.13$ ft and by Equation (3), $L = 14.0\ h = 212$ ft. The height of this deep-water wave above still-water level by Equation (4a) is $a = 8.65$ ft. making $h - a = 6.48$. The deep-water wave will break when the depth becomes $d_1 = 1.84\ h = 27.84$ ft. Since $d = 28$ ft, the wave will strike the breakwater before breaking, and hence without appreciable reduction due to shoaling effect. The problem is thus simplified.

By Equation (7), $\frac{d_0}{L} = \frac{28}{212} + 2\left(\frac{15.13}{212}\right)^2 = 0.142$, for which Table 3 gives $c = 0.829$ and $\mu = 1.21$, by interpolation. Equation (16) now gives $v + v_0 = 36.2$ ft per sec, and the maximum unit wave pressure by Equation (15) (with $\frac{kw}{2g} = 1.7$) becomes $p = 2\,230$ lb per sq ft, acting at $h_1 = 0.12\ h = 1.82$ ft above still-water level. Then, make $p_0 = 0.72\ p = 1\,600$ lb per sq ft,

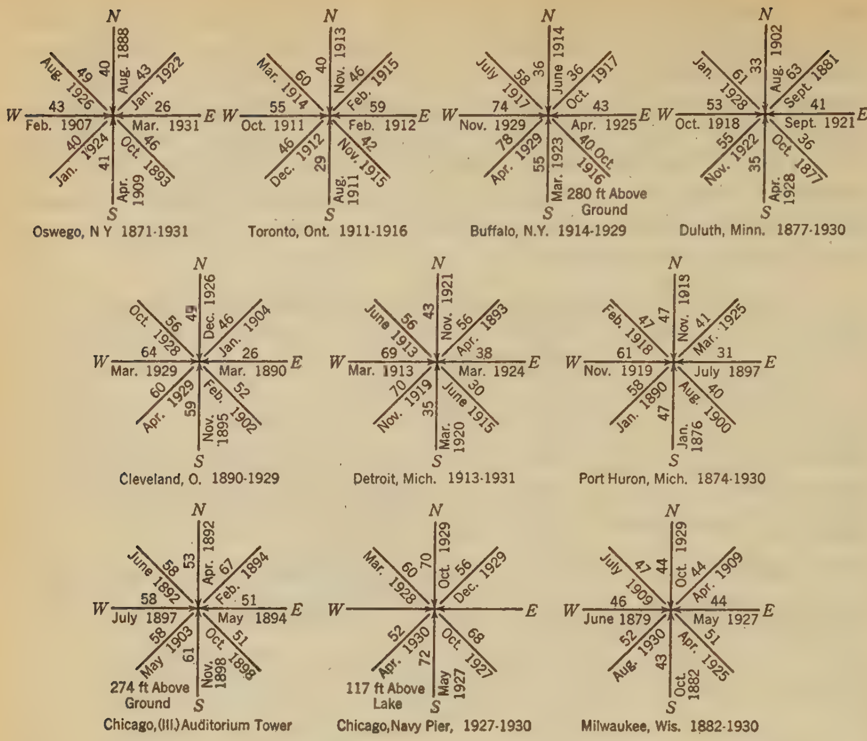


FIG. 5.—WIND RECORDS FOR THE GREAT LAKES

located midway between the wave trough and the maximum pressure ordinate. These data determine the complete wave-pressure curve shown in Fig. 6, which curve requires only slight modification for the portion of wave that is not obstructed by the superstructure.

The total effective wave pressure is thus found as $P = 27\,424$ lb per lin ft of crib and acts 31.3 ft above the bottom. The weight of the crib, allowing for buoyancy, is calculated as $G = 95\,150$ lb per lin ft, which combined with P gives the resultant, R, acting on the base and producing a maximum unit pressure, $f = \frac{2 \times 95\,150}{3 \times 9.8} = 6\,470$ lb per sq ft at the heel of the crib. No account was taken of the water on top of the superstructure.

The safety factor against overturning is then, $\frac{95\,150 \times 18.82}{27\,424 \times 31.3} = 2.08$, and against sliding, $\frac{95\,150 \times 0.5}{27\,424} = 1.74$. Referring again to Fig. 6, it will be noticed that the wave force, P, was combined directly with the weight, G, in testing the safety of the structure. This would be correct in testing the stability of the crib, provided the entire wave impact is imparted simultaneously to all parts of the structure.

However, since the superstructure does not attain its full height at the face of the crib, but slopes gradually for a distance of 18 ft, and then presents another surface to resist the upper portion of the wave, it must be apparent that the full wave force is not expended simultaneously, nor with the pressure assumed in the analysis. The wave could not be built up to its full height until after it had advanced to the last obstruction 1 sec after striking the first. In other words, the total wave energy is transferred to the crib during a greater period of time than was assumed in designing the wave-pressure curve, and, hence, the shock is in reality less severe than the calculation shows.

This example illustrates how the wave pressure may be rendered less severe simply by adopting a superstructure design that does not obstruct the wave in its entirety on a vertical surface, by allowing the wave to build up gradually. However, this may develop some difficulties in the design of the concrete superstructure, and no doubt will cause a larger proportion of the wave to jump over the wall. The back-wash may also be retarded to such an extent as to meet the next oncoming wave at an unfavorable time. The wave force striking the high center wall will subject the latter to a moment of about 9 600 ft-lb, which might cause the mass concrete deck to break unless steel reinforcement is used.

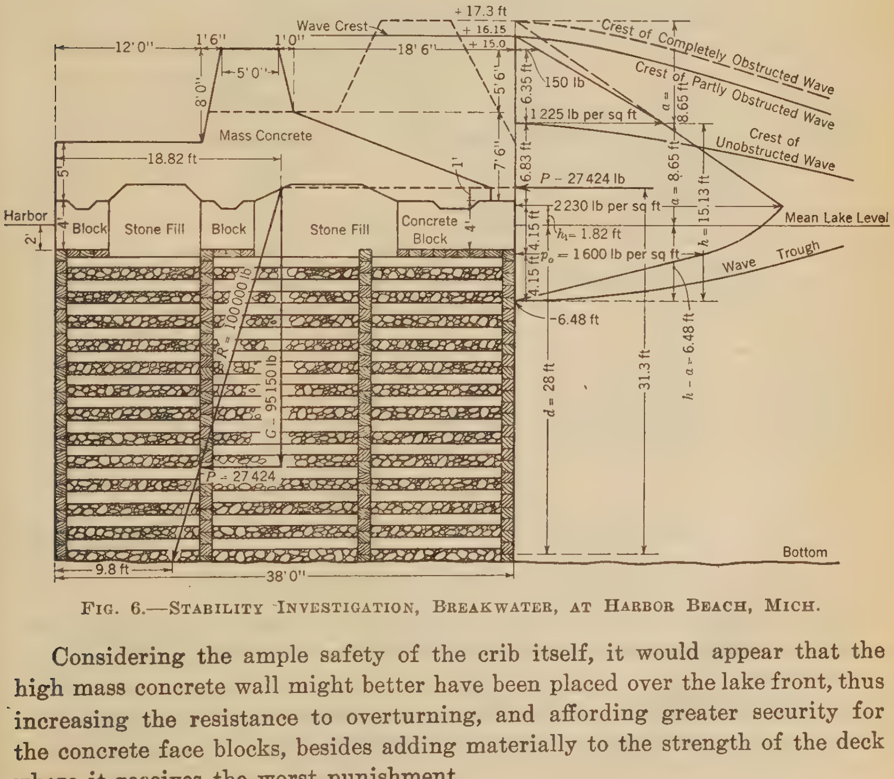

+ 17.3 ft
Wave Crest
+ 16.15
+ 15.0
12' 0''
1' 6''
1' 0''
18' 6''
5' 0''
8' 0''
5' 6''
6.35 ft
150 lb
Crest of Completely Obstructed Wave
Crest of Partly Obstructed Wave
a = 8.65 ft
1 225 lb per sq ft
Mass Concrete
18.82 ft
5'
1'
7' 6''
6.83 ft
Crest of Unobstructed Wave
P – 27 424 lb
a = 8.65 ft
h = 15.13 ft
2 230 lb per sq ft
Mean Lake Level
Harbor
4'
2'
Block
Stone Fill
Block
Stone Fill
Concrete Block
4'
4.15 ft
h_1 = 1.82 ft
p_o = 1 600 lb per sq ft
4.15 ft
Wave Trough
-6.48 ft
R = 100000 lb
G – 95 150 lb
d = 28 ft
31.3 ft
h – a = 6.48 ft
P = 27 424
Bottom
9.8 ft
38' 0''

FIG. 6.—STABILITY INVESTIGATION, BREAKWATER, AT HARBOR BEACH, MICH.

Considering the ample safety of the crib itself, it would appear that the high mass concrete wall might better have been placed over the lake front, thus increasing the resistance to overturning, and affording greater security for the concrete face blocks, besides adding materially to the strength of the deck where it receives the worst punishment.

In designing a breakwater superstructure, much depends on the purpose it is to serve. It may be necessary to obstruct oncoming waves completely, especially if ships are intended to moor on the harbor side. In other cases, it may not matter how much water is thrown over the wall as long as the wave force is rendered harmless. The superstructure of the Harbor Beach Breakwater, if intended to protect ships while tied on the harbor side, might have been designed as indicated by a dotted outline in Fig. 6, so as to obstruct more completely the oncoming waves, at the same time to add resisting moment to the crib and increased strength to the deck. The use of some reinforcement in connection with these superstructure designs is very desirable.

It is also necessary to provide "breathing" openings through the concrete deck to allow relief for the air pressure formed in the body of the crib. Wave pressure will also set up hydrostatic pressure on the inside of the cribs, thus exerting a bursting effect on every part of the structure, all of which must not be overlooked.

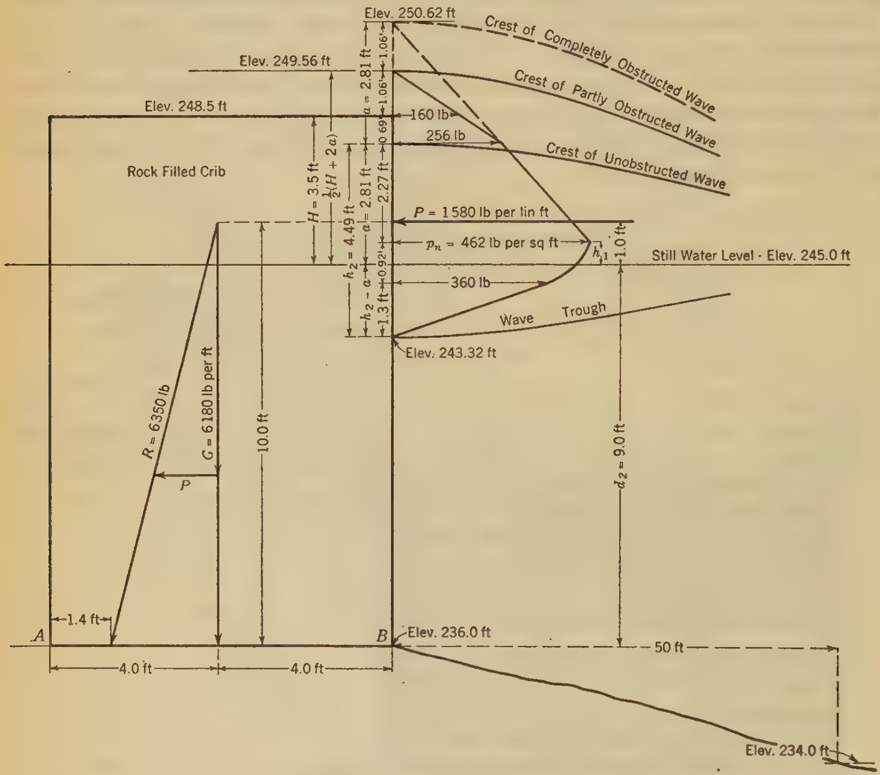

Elev. 250.62 ft
Crest of Completely Obstructed Wave
Elev. 249.56 ft
1.06'
a = 2.81 ft
1.06'
Crest of Partly Obstructed Wave
Elev. 248.5 ft
160 lb
0.69'
256 lb
Crest of Unobstructed Wave
Rock Filled Crib
H = 3.5 ft
$\frac{1}{2}(H + 2a)$
h_2 = 4.49 ft
a = 2.81 ft
2.27 ft
P = 1580 lb per lin ft
p_n = 462 lb per sq ft
h_1
1.0 ft
Still Water Level · Elev. 245.0 ft
0.92'
$h_2 - a$
1.3 ft
360 lb
Wave Trough
Elev. 243.32 ft
R = 6350 lb
G = 6180 lb per ft
10.0 ft
d_2 = 9.0 ft
P
1.4 ft
A
B
Elev. 236.0 ft
50 ft
4.0 ft
4.0 ft
Elev. 234.0 ft

FIG. 7.—MAGANN'S PIER, TORONTO, ONT., CANADA

Problem 2.—Stability Investigation of Magann's Pier.—This pier (see Fig. 7) is located between Jameson Street and Dowling Avenue, in Toronto, in 9 ft of water, deepening to 11 ft at a distance of 50 ft offshore. The worst conditions exist for a southeast wind with a fetch, $D = 30$ miles, making an angle, $\alpha = 68°$, with the lake face of the pier. A wind velocity of 55 miles per hr was taken as a possible maximum.

This problem involves all the complications of an inshore sea-wall, and differs from the previous case in that the deep-water wave will encounter shallow water before striking the pier, besides approaching the pier obliquely instead of normally. Also, the pier is not high enough to stop completely the oncoming wave. Hence, the deep-water wave suffers a reduction in height, is only partly obstructed, and the final reduced pressure acts only by the amount of its normal component. The stability investigation for the fetch and wind velocity previously noted, may proceed as follows: For $D = 30$ miles, $V = 55$ miles per hr, the deep-water wave, by Equation (1), will have a height, $h = 0.17 \sqrt{V D} = 6.90$ ft, and by Equation (2) find $L = 15.3\, h = 106$ ft.

The depth in which a deep-water wave may continue without appreciable effect on its height and length may be taken as $d = \frac{L}{5} = 21$ ft. Then, Equation (8) gives $\frac{d_0}{L} = \frac{d}{L} + 2\left(\frac{h}{L}\right)^2 = \frac{21}{106} + 2\left(\frac{6.9}{106}\right)^2 = 0.33$ and $c = 0.98$, from which, $v = 2.26\, c\sqrt{L} = 22.8$ sec-ft. With these wave dimensions and velocity, v, as a deep-water condition, the reduction in size and velocity due to shallow water may be found.

According to Equation (5) the deep-water wave will break when the depth is reduced to $d_1 = 1.84\, h = 12.7$ ft, and just before striking the pier, the depth is further reduced to $d_2 = 9.0$ ft, causing the initial velocity, v, to be retarded to a value, v_2, according to Equation (9). Thus, $v_2 = 0.9\, v\sqrt[4]{\frac{d_2}{d}} = 16.53$ sec-ft, and the wave thus retarded is reduced to a height, $h_2 = h\sqrt{\frac{d^2}{d}} = 6.9\sqrt{\frac{9.0}{21}} = 4.49$ ft, with a length, $L = 15.3\, h_2 = 68.7$ ft. The height of this reduced wave above still-water level, by Equation (4b), will be, $a = \frac{h_2}{2} + 2\,\frac{h^2_2}{L} = \frac{4.49}{2} + 2\left(\frac{4.49}{68.7}\right)^2 = 2.81$ ft, and $h_2 - a = 1.68$ ft. From Equation (8), find $\frac{d_0}{L} = \frac{9}{68.7} + 2\left(\frac{4.49}{68.7}\right)^2 = 0.261$, for which Table 1 gives $\mu = 1.04$. Then, from Equation (12), find $v_2 + v_0 = 16.53 + 7.11 \times 1.04 \times \frac{4.49}{\sqrt{68.7}} = 20.5$ sec-ft.

The maximum unit wave pressure then becomes (by Equation (15), with $\frac{k\,w}{2\,g} = 1.30$): $p = 546$ lb per sq ft, acting at $h_1 = 0.12\, h_2 = 0.54$ ft above still-water level. This pressure resolved normally to the pier as in Equation (19), becomes $p_n = 546 \times 0.846 = 462$ lb per sq ft. From the foregoing data the final reduced pressure curve, shown in Fig. 7, was obtained, giving the total wave pressure, $P = 1\,580$ lb per lin ft of pier. The weight of the crib, allowing for buoyancy, is $G = 6\,180$ lb per ft, which, combined with P, gives the resultant, R, acting on the basal plane, $A\ B$. The maximum unit pressure at the heel of the crib is, $f = \frac{2 \times 6\,180}{3 \times 1.4} = 2\,950$ lb per sq ft. Similar to Example 1, the factor of safety against overturning is, $\frac{6\,180 \times 4}{1\,580 \times 10} = 1.57$; and, against sliding (assuming a coefficient of friction of 0.5), is, $\frac{6\,180 \times 0.5}{1\,580} = 1.95$.

CONCLUDING REMARKS

In closing this subject the writer wishes to emphasize that stability investigations of this character, or designs based on similar calculations, must be regarded as more or less speculative; yet the methods herein presented for the first time, were developed after much thought and study, and are considered as accurate as the subject warrants.

This method of analysis was applied to existing breakwater structures at Presque Isle, Ontonagon, Grand Marais, Agate Bay, Portage Lake, and Marquette, all on Lake Superior. The breakwater at Buffalo, N. Y., comprising several designs with varied exposures, was also analyzed. In each case, the results of the analysis agreed closely with the behavior exhibited by these structures during many years of service.

The western breakwater at Toronto, built in 1915 and 1916, according to designs that were severely criticized at the time as being structurally inadequate, showed a factor of safety of about 4 against both overturning and sliding, according to an analysis made in 1915 by the writer. This breakwater, covering a length of about 3½ miles, has weathered all storms for the eighteen years since its construction without manifesting the slightest signs of weakness, again demonstrating the reliability of results obtainable with the method herein described.

DISCUSSION

Charles E. Fowler,[10] Esq. (by letter).—The detailed solution of wave pressures on engineering structures is very important and timely, and should lead to more careful designing of them. The use of a factor of safety of 2 is usually ample, as is used in other foundation work, but where the fundamental data are not complete or reliable, a larger factor should be used. The writer has found that Captain Gaillard's formulas[2] are not always representative of the actual facts encountered. Thus, in the Gulf hurricane of 1915, waves higher than 12 ft were generated by the fetch of about 20 miles across Lake Borgne, in Louisiana, and in a similar case at Corpus Christi, Tex., where about the same fetch generated similar waves which destroyed everything in front of them.

The waves of oscillation which occur out in the ocean have little destructive force except as they are encountered by ships plowing through them. When waves reach shallow water, however, they begin to break, and become the destructive waves of translation with which the engineer is concerned in the design of shore structures. Another factor is presented in such cases by storm tides, which in the case of the 1915 hurricane reached 9 ft at Rigolets, La., and 13 ft at the west end of Lake Pontchartrain. Such tides raise the elevation of the point of application of the force of the waves, and thus greatly increase their overturning effect.

The actual maximum wind velocities are usually not easily obtained, but, of course, they are a vital factor of design. Those given in the paper are certainly much lower than those that often occur. For example, in the 1915 hurricane, the maximum velocity reached 140 miles per hr, and while this was only in gusts the sustained velocity was well over 100 miles per hr. The wind velocities on the Pacific Coast, from south of the Columbia River north to Cape Flattery, often reached 90 miles per hr, and this was the velocity used by the writer in designing the foundation for the Tacoma, Wash., smelter chimney, which was about 600 ft high and which rose to a height of about 700 ft above Puget Sound.

The application of the wave force is at a point above the surface of the water, as shown by the curves accompanying the paper, and these agree well with the curves developed in Italy, and for the harbor work at Halifax, N. S., Canada.[11]

The greatest errors occur in the actual design of the structures built to resist the waves. A vertical surface must resist the full force of the waves, and a sloping surface which has less effective force to resist, often results in other serious problems to be met. In such cases the waves may dash into the air to a height of from 50 to 100 ft, and the tons of water falling from such a height may strike the top of a wall with enough force to destroy (eventually) a masonry wall of concrete, or the fill or structures back of the wall. The curved face of the Galveston sea-wall was designed to throw the water back into the Gulf, which it would do if it was high enough, but a much more scientific method might have been adopted. When the Seattle tide-flats were filled, the front of the fill was protected by a brush bulkhead, 30 ft high, with the butts of the fir brush from 2 to 4 in. in diameter outward, and on a 1:1 slope. In the most severe storms, with a wind velocity of more than 60 miles per hr, the waves would dash against these brush butts, break up completely, and fall back "dead" into the water. The concrete sea wall at North Beach, San Francisco, Calif., had a rough curved exterior which accomplished the same result.[12]

---

[10] Cons. Engr., Lansing, Mich.

[11] "World Ports and Harbor Data", by Charles E. Fowler.

Such designs are only another application of the design of the hydraulic jumps used to kill the force of the water issuing from the impounding or flood-control dams at Dayton, Ohio. The design of other structures to resist wind and wave action should be studied to reduce the exposed surface. In one large bridge structure it was found possible to reduce the exposed surface which had to oppose both wind and waves by a full 50%, by using circular compression members, eye-bar tension members, and lattice floorbeams and stringers. The savage tribes discovered these methods from practical experience as witnessed by the teepee of the American Indians, and in the circular and domed houses of Asiatic towns built to resist the winds and sand storms which blow with great violence.

The mathematical solutions presented in the paper are worthy of the authority presenting them, but too often the young or inexperienced engineer becomes lost in a maze of mathematical formulas, and fails to sense the best practical application of them. Models may be used in laboratory channels or wind tunnels to discover the best forms and as a means of avoiding costly errors. Each problem has its peculiar features, and the solution is often hampered by the materials available, but the best design can always be reached by a careful study of all the features and conditions involved.

CHARLES T. LEEDS,[13] M. AM. SOC. C. E. (by letter).—In bringing into small compass, and placing in usable form, a considerable part of the material on wave pressures available in many different sources, the author has performed a valuable service. As stated by Mr. Molitor, a valuable amount of observational data were collected by Lt.-Col. Gaillard who also contributed a most valuable discussion of the many phases of this problem, such as to render his book[2] one of the most important available references on this subject. Mr. Molitor has done a service in condensing and simplifying the great mass of material there available, and he has also added valuable data from his own experience. His brief statement of certain of the formulas compiled or deduced by Colonel Gaillard, and his demonstration of their application to specific examples should be of assistance to those not thoroughly familiar with this branch of engineering.

---

[12] "Ideals of Engineering Architecture", by Charles E. Fowler.

[13] Maj., Corps of Engrs., U. S. Army (*Retired*); Cons. Engr. (Quinton, Code & Hill-Leeds & Barnard), Los Angeles, Calif.

The mathematical statements would be more satisfying if the supporting reasons were more clearly stated. For instance, the formulas for wave height in terms of fetch, as deduced by Thomas Stevenson, are:

For $D > 30$ miles:

$$h = 1.5\sqrt{D} \tag{20}$$

and for $D < 30$ miles:

$$h = 1.5\sqrt{D} + 2.5 - \sqrt[4]{D} \tag{21}$$

In the paper, these formulas are changed (see Equations (1) and (2)), and the author states that these changes were made "after introducing wind velocity as a variable and using statute miles instead of nautical miles"; but he gives no supporting reasons or observations either for the assumed relation between wind velocity and wave height or for changing the 30-mile fetch used by Stevenson to 20 miles. It is evident that Equations (1) and (2) will give smaller theoretical wave heights than the Stevenson formulas (Equations (20) and (21)), in the case of any wind velocity less than 78 miles per hr.

The statement that "these formulas apply only to inland lakes" is not consistent with the fact that Stevenson's formulas were shown to be correct when he analyzed waves breaking on the shore of the North Sea in Scotland, of which the fetch varied from 1 mile to 165 miles.

Under the heading, "Wave Height, Wind Velocity, and Fetch," the author states: "Ocean storms are generally more or less local, and usually cover not more than 50 to 100 miles." Then he states "the fetch is restricted to the storm area, which may be anything from a few miles to, say, 900 miles." These two statements appear somewhat contradictory.

When one considers the numerous physical factors of uncertain variability which may affect the dimensions of waves, it seems questionable whether any formula can be constructed by which their heights can be computed reliably. In describing certain of his observations at Port Elizabeth, South Africa, Mr. William Shield states[14] that the greatest wind velocities recorded during the three gales observed were, respectively, 60, 68, and 58 miles per hr. On the other hand, the greatest height attained by the waves on the three occasions were, respectively, 21 ft, 13 ft, and 10 ft; the fetch, depth of water, etc., were practically the same in all cases.

This apparent anomaly, he attributes to a difference in the range of the respective gales or a variation in the direction of their course (possibly far from land); or to a combination of both causes. Hence, Mr. Shield deduced that even in gales of considerable duration, a given fetch, wind velocity, and depth of water will not always produce waves of uniform height because such height depends greatly on the varying conditions of the gales producing the waves, both as regards their extent and duration in one direction.

It sometimes happens that the line of longest fetch is not the direction from which the most severe winds blow. Furthermore, the veering of the wind into a direction from which the fetch is small may sometimes produce a wheeling around of the waves, such as to create heights greatly in excess of those to be anticipated from the formula.

---

[14] "Principles and Practice of Harbour Construction," by William Shield, p. 38.

Indeed, waves of destructive proportions are not always due to high winds occurring in the vicinity. In several instances within recent years, heavy ground swells have occurred along the Southern California coast with a moderate amount of resultant damage. There was little or no local wind at the time, and the only explanation seems to be the occurrence of distant storms in some part of the Pacific Ocean from which no report was obtainable.

The author truly states that the only reliable data relative to the height of ocean waves must be collected by direct observations for any given locality. This should be strongly emphasized and as there usually are few scientific observations available, the reliability of the observers must be carefully weighed.

Careful consideration must also be given to the under-water contours in the vicinity, and to the seaward or lakeward of the work proposed, as these contours may have a marked effect, both on the height and on the strength of the oncoming waves. The original design for the rubble-mound breakwater at San Luis Obispo, Calif., called for the construction of a part of its length along the crest of a submerged reef. Before its completion severe storms hammered down and drove bayward a considerable quantity of rock from the part of the breakwater that had then been constructed along the reef. When work was resumed, the following spring, the damaged part and the unfinished remainder of the breakwater were re-located a short distance back from the crest of the reef. It was found, as expected, that the wave impact on the breakwater was thereby greatly reduced.

In referring to his Equation (12) for the maximum value of the hydrodynamic pressure, Mr. Molitor states that "k is an empiric coefficient evaluated from Colonel Gaillard's observations for the Great Lakes as 1.30 to 1.71 * * *. For ocean storm waves, k may be taken as 1.8." Referring to his observations at St. Augustine, Fla., Colonel Gaillard states[15] that only in one case out of 107 observations did k reach its maximum possible theoretical limit of 2.0; whereas, in three cases, it equalled or exceeded 1.8. Consequently, he declared that k *may* have a possible value as great as 2.0.

The statement in the paper that "all factors contributing to the solution of the problem herein considered [the design of sea-walls and breakwaters] are only approximately knowable," might well be further emphasized. There is too great a tendency on the part of laymen and even of inexperienced engineers to expect wave-resistant structures to be designed with the exactness and economy of land structures, such as buildings and bridges in which the forces to be resisted can be determined with a high degree of accuracy. Moreover, there are forces and contingencies which only experience can foresee and which are likely to be overlooked by even the best theorists.

An example of this fact occurred some years ago when certain plans for a concrete caisson breakwater were being reviewed by the writer. As drawn, they were pleasing in appearance and apparently showed a high degree of economy. After careful analysis, however, it was necessary to recommend the rejection of these plans, because the assumption as to the force of the waves was inadequate and because other assumptions showed a lack of practical experience on the part of the designer. Based on textbook formulas only, the design was adequate, but subsequent occurrences have demonstrated that if the breakwater had been built as designed, it would have failed.

---

[15] *Professional Papers No. 31*, U. S. Corps of Engrs., p. 177.

In another instance, concrete caissons for a breakwater were actually constructed, and the first of them was floated into approximate position near its final location and then grounded temporarily on the bottom. As far as the writer knows, the caisson was well constructed, but inadequate attention had been given to the design of the breakwater foundations and to the method of placing. While the caisson was temporarily grounded on the sandy bottom, the ocean surge produced a jetting action under the caisson, washing out so much sand under one end that the caisson settled, became "hog-backed," and, under the pounding of the waves, was soon a total wreck. The same action would have taken place if the caisson had been set in its final project location. Inadequate attention had been given to the problem of foundations and the method of placing the caissons.

These instances constitute no criticism of the use of concrete caissons in breakwater construction. Too many breakwaters of this type have been successfully built and maintained to gainsay their serviceability and practicability. Their chief usefulness, like rock-filled cribs, is where rock of suitable character and size is not available or is too expensive. Where durable rock can be obtained in large pieces and at a reasonable price, a rubble-mound breakwater has the distinct advantage of simplicity of design and ease of repair.

It is simple enough to design a safe structure, if no regard is given to cost. To attain an economic engineering solution, one with an adequate but not excessive factor of safety, is not so simple. The lay mind (and many engineering minds) would dictate a structure such that no storm damage could ever occur. One may well question the economy of such procedure, however. Therein lies the advantage of a breakwater or sea-wall design such that it will resist the stress of usual storms, but which is susceptible of sustaining slight damage in an abornmal storm without complete destruction. When reduced to an annual basis the cost of such infrequent repairs is likely to be much less than the interest on the capital investment in a heavier breakwater, adequate to resist all storms.

The theoretical value for the wave pressure produced by a wave striking a breakwater obliquely is given in Equation (19). The actual pressure is likely to exceed this value, as the friction of the wave against the breakwater tends to drag back that flank of the wave front, causing a pressure more nearly normal to the breakwater face than the general line of wave front would indicate, and also producing the heaping up of the wave so familiar to engineers who have observed such occurrences.

The paper is devoted chiefly to vertical-faced breakwaters, although some consideration is given to the form of superstructure. A vertical face naturally will involve the maximum wave stress on the structure. The author points out that it is always preferable to render the wave pressure on the structure less severe by adopting a superstructure design that does not obstruct the wave in its entirety on a vertical surface. The same reasoning holds true, where feasible, of the entire breakwater cross-section, as thereby the horizontal component of the wave force will be reduced and, also, the component normal to the surface will then have less overturning or sliding effect.

This result may be accomplished, as stated, by a sloping surface, but in a concrete structure a curved profile that turns the wave back upon itself may be preferable. Still better is a step-faced type which may be said to resist the wave shock "on the installment plan." There are numerous examples of the successful use of this principle in the construction of sea-walls.

Where rock is not too expensive to prevent its use in a rubble mound, this is ordinarily the preferable type, not merely from the standpoint of economy, and because of ease of repair, but particularly from the standpoint of safety. In this case, the waves are broken and dispersed and the wave force is dissipated in a multitude of directions. If, perchance, an unusually severe storm beats down a part of such a breakwater, it makes a broader and more stable foundation, on which the rebuilt structure will be safer than before.

The author does well to call attention to the hydrostatic pressure which wave impact produces inside a breakwater structure and which is transmitted to all parts of it. This pressure, and confined air pressure, account for many of the apparently paradoxical effects produced by storms.

The three points most commonly lost sight of by the novice are: (1) Loss of weight by submergence; (2) transmission of hydraulic pressure equally in all directions; and (3) difference in pressure, dependent on whether or not the wave is breaking.

Additional emphasis and discussion might well have been devoted to the effect of the part of a wave that is not stopped by a breakwater. Not only should consideration be given to the question of whether or not ships are to be moored close to the harbor side of the breakwater, but the stability of the of the breakwater structure itself is vitally involved. It is not sufficient that "the wall structure must be capable of resisting this static equivalent [of the energy of the part of the wave stopped by the wall] without exceeding certain requirements of structural stability and safety." Many of the recorded breakwater failures have been caused primarily by the part of the wave that overtops the structure. The falling water may produce such an impact on parts of the superstructure as to shatter it, or the substructure or foundation on the lee side may be so weakened as to result in overturning.

In conclusion, the writer wishes to commend Mr. Molitor for the service he has done the profession by this contribution to the clarification and simplification of an involved subject.

A. A. EREMIN,[16] ASSOC. M. AM. SOC. C. E. (by letter).—An original method of analyzing wave pressures on sea-walls is presented in this paper. There always has been an urgent demand for a reliable mathematical expression for wave pressures in the design of sea-walls. In 1926, the International Congress for Navigation appointed a special committee of five engineers to study the factors that influence the stability of sea-walls. Professor J. Lira, a member of this Committee, made an extensive study of the causes of failures of walls at various seaports.[17] He found that the wave pressures computed by the method introduced in 1929 by G. Sainflou[18] are in agreement with the actual forces that caused the failures of the walls.

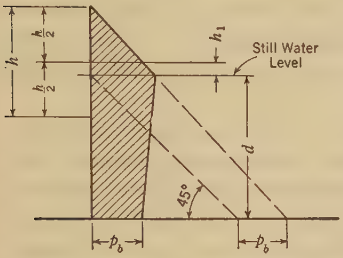


FIG. 8.—WAVE PRESSURE ON VERTICAL WALL ASSUMED BY SAINFLOU.

A coefficient of safety slightly greater than unity has been found sufficient in designing new sections of walls to replace sections that have failed. It is instructive to compare the pressures computed by Mr. Molitor in Fig. 7, with those computed by the Sainflou method. Fig. 8 defines wave pressures according to Sainflou's method, the notation being same as that used by the author. The point of maximum pressure, h_1, above still-water level, is given by the expression:

$$h_1 = \frac{\pi h}{4 L} \coth \frac{2 \pi d}{L} \quad \ldots\ldots\ldots\ldots\ldots\ldots (22)$$

and, the wave pressure at the bottom of the wall is,

$$p_b = \frac{h}{2 \cosh \frac{2 \pi d}{L}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (23)$$

The overturning moment at the bottom of the wall in terms of volume of water is:

$$M = \frac{\left(d + h_1 + \frac{h}{2}\right)^2 \times (d + p_b)}{6} - \frac{d^3}{6} \quad \ldots\ldots\ldots\ldots (24)$$

The dimensions given by Mr. Molitor for Magann's Pier (Fig. 7) are as follows: The depth of still water is $d = 9.0$ ft; and the height and length of the reduced wave are 6.9 ft and 106 ft, respectively. Therefore, from Equations (22) and (23) the distance of the point of maximum pressure above still-water level and the pressure at the bottom of the wall are, respectively, $h_1 = 0.73$ ft and $p_b = 3.02$ ft.

The maximum overturning moment at the bottom of the wall in the direction of wave action, from Equation (24), is $M = 14\,150$ ft-lb. The overturning moment normal to the face of the wall is $M = 14\,150 \times 0.846 = 11\,971$ ft-lb. The overturning moment due to the pressure, P_o, computed by the author, is $M = 1\,580 \times 10 = 15\,800$ ft-lb, or, about 30% on the safe side as compared with that computed by Sainflou's method.

---

[16] Associate Bridge Designing Engr., Div. of Highways, State Dept. of Public Works, Sacramento, Calif.

[17] *Le Genie Civil*, April 6, 1933.

[18] *Loc. cit.*, January 26, 1929.

It is interesting to note also that Sainflou assumed the wave pressure to extend to the bottom of the wall (see Fig. 8).

T. L. CONDRON[19] AND CHESTER L. POST[20], MEMBERS, AM. SOC. C. E. (by letter).—A helpful service to those members of the profession called upon to design or investigate marine structures has been rendered by Mr. Molitor in his paper. So much has been gathered from widely scattered and often inaccessible sources and has been presented so clearly and concisely, that problems now coming under this title may be more readily investigated. Particularly is this true, since Captain Gaillard's paper[2] is out of print and hence no longer easily obtainable.

The first part of the paper presents the several steps to be taken in determining the wave-pressure curve based on a known fetch and a definite wind velocity. From this wave-pressure curve the theoretical effect of the wave on the structure is determined. The author gives the complete solution of such problems and then applies it to two actual structures, one at Harbor Beach, Mich., the other at Magann's Pier, Toronto, Ont., Canada.

In order to aid in establishing the dependence that may be placed on the author's recommended method, the writers have applied this method of calculating the wave force to two breakwater structures, recently investigated, and reported upon by them. The first is a rock-filled timber crib breakwater, built about 1905, and the second, a proposed rock-filled, steel, sheet-pile cellular type of breakwater, the actual construction of which has since been completed for a length of about 1 500 ft. The latter structure, a continuation of the former, is to be 5 025 ft long and, being in the same line, is exposed to the same intensity of wave action. These two structures are at Calumet Harbor, Ill., near the south end of Lake Michigan.

Through the courtesy of Lt. Col. Donald H. Connolly, Corps of Engineers, U. S. A., M. Am. Soc. C. E., District Engineer of Chicago, Ill., the writers are able to offer the results of their investigation of the designs of these two structures. If the theoretical analysis, outlined by the author, leads to findings in conformity with the actual behavior of the timber structure and indicates a greater factor of safety for the steel structure than for the timber structure, then as he states (see heading "Stability of Breakwater Cribs Subjected to Wave Force"), "the speculative features involved in the method will have been reduced to a negligible quantity."

Following the author's method of investigating the breakwater at Harbor Beach, illustrated by Fig. 6, the writers have determined the various factors for both the Calumet Harbor structures, most of which factors are shown in Figs. 9 and 10. Those not shown there, and the results of the calculations made, are given in Table 6.

---

[19] Cons. Engr. (Condron & Post), Chicago, Ill.

[20] Cons. and Designing Engr. (Condron & Post), Chicago, Ill.

In order to make a comparison with the foregoing results, the writers have also determined a wave-pressure curve by plotting on these same diagrams (Figs. 9 and 10), three points, the ordinates from the faces of the walls representing the maximum pressures observed by Captain Gaillard for 18-ft waves, as given by the author in Table 4, these dynamometer readings being plotted at the corresponding heights above still water. The maximum wave height recorded by Captain Gaillard is 18 ft, while the calculated maximum wave height, determined according to the author's formulas, is 21.83 ft, which may or may not be greater than might reasonably be expected at Calumet Harbor.

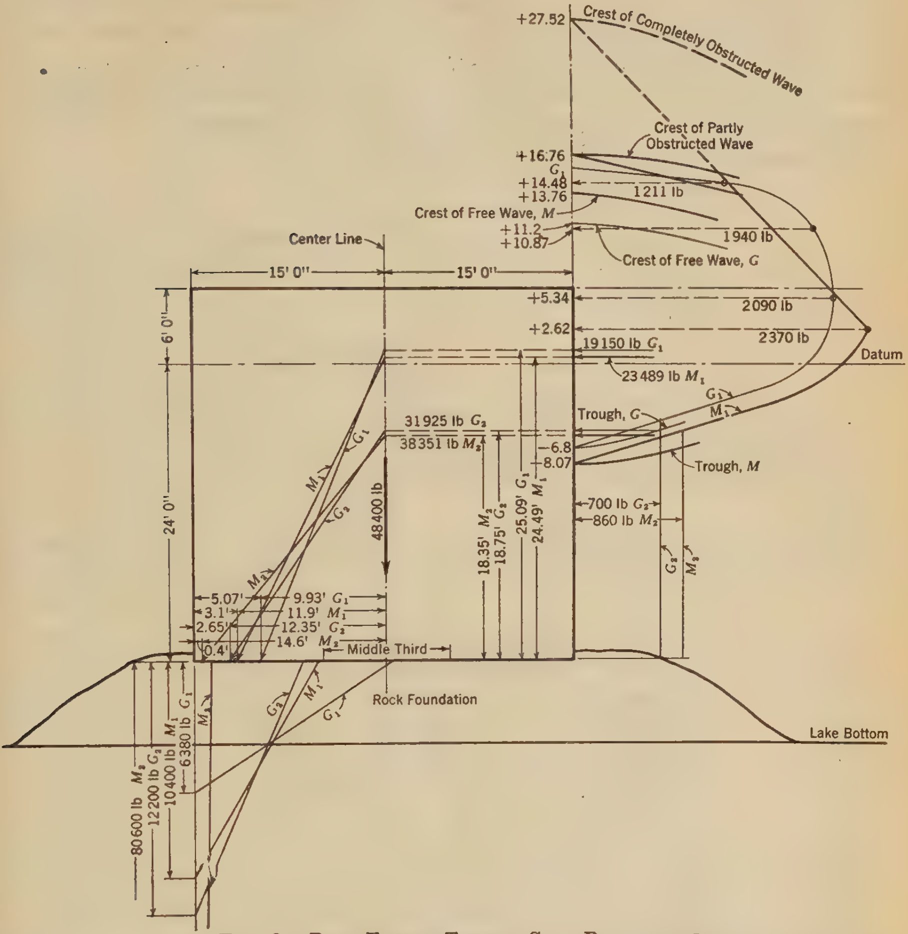

+27.52
Crest of Completely Obstructed Wave
Crest of Partly Obstructed Wave
+16.76
G_1
+14.48
+13.76
1211 lb
Crest of Free Wave, M
+11.2
+10.87
1940 lb
Crest of Free Wave, G
Center Line
15' 0"
15' 0"
6' 0"
+5.34
2090 lb
+2.62
2370 lb
19150 lb G_1
Datum
23489 lb M_1
G_1
M_1
Trough, G
31925 lb G_2
38351 lb M_2
−6.8
−8.07
Trough, M
G_1
M_1
G_2
M_2
24' 0"
48400 lb
18.35' M_2
18.75' G_2
25.09' G_1
24.49' M_1
700 lb G_2
860 lb M_2
G_2
M_2
5.07'
3.1'
2.65'
0.4'
9.93' G_1
11.9' M_1
12.35' G_2
14.6' M_2
Middle Third
Rock Foundation
G_2
M_1
G_1
M_2
Lake Bottom
80600 lb M_2
12200 lb G_2
10400 lb M_1
6380 lb G_1

FIG. 9.—ROCK-FILLED TIMBER CRIB BREAKWATER.

The distances from the toe of the intersections with the base of the resultants of the vertical and horizontal forces acting on both the timber and the steel structures, as well as the toe pressures, are shown on Figs. 9 and 10 and in Table 6.

Apparently, neither the author, nor the earlier investigators quoted by him, have taken into account the additional unbalanced pressure on the breakwater structure, due solely to the momentary difference in the depths of the water on the windward and leeward sides. The writers have considered this static pressure in addition to the dynamic wave pressure and have determined the centers of action of the combined pressures, the distances from the toes of the walls of the intersections of the resultants of the vertical and these increased horizontal forces, and the corresponding toe pressures, all of which are also shown on Figs. 9 and 10 and in Table 6.

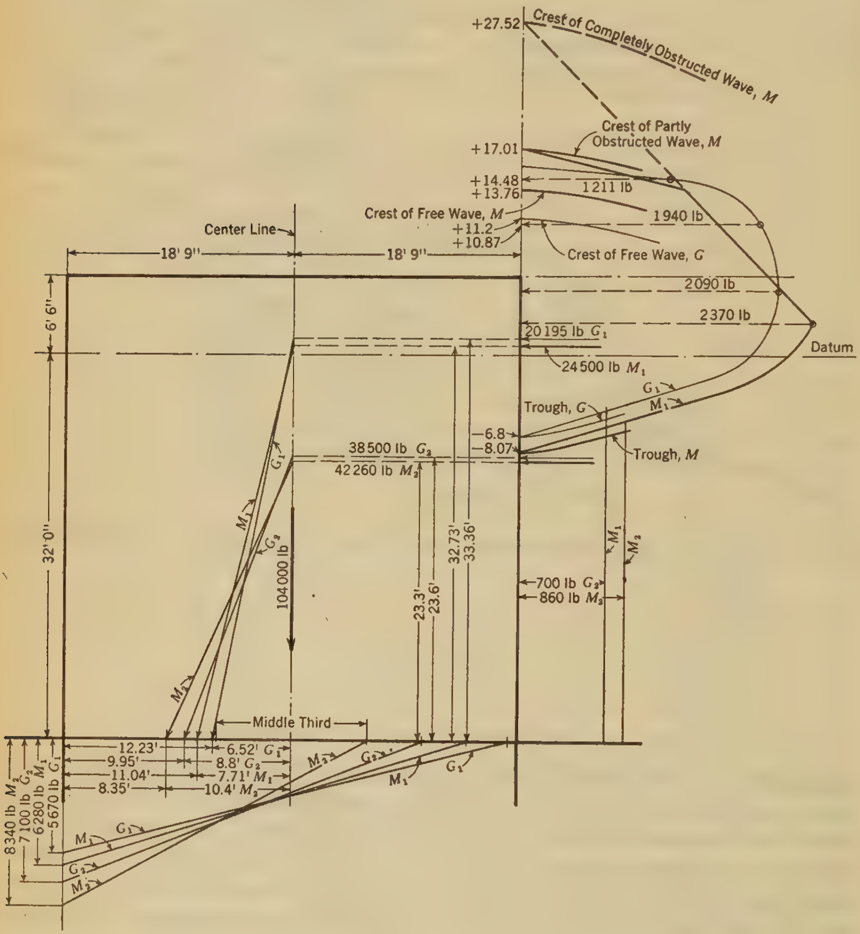

+27.52
Crest of Completely Obstructed Wave, M
Crest of Partly Obstructed Wave, M
+17.01
+14.48
+13.76
1211 lb
1940 lb
Crest of Free Wave, M
+11.2
+10.87
Crest of Free Wave, G
Center Line
18' 9"
18' 9"
2090 lb
2370 lb
Datum
6' 6"
20 195 lb G₁
24 500 lb M₁
G₁
M₁
Trough, G
−6.8
−8.07
Trough, M
38 500 lb G₂
42 260 lb M₂
G₁
M₁
G₂
104 000 lb
32'0"
23.3'
23.6'
32.73'
33.36'
M₁
M₂
700 lb G₂
860 lb M₂
M₂
Middle Third
12.23'
9.95'
11.04'
8.35'
6.52' G₁
8.8' G₂
7.71' M₁
10.4' M₂
M₂
G₂
M₁
G₁
8340 lb M₂
7100 lb G₂
6280 lb M₁
5670 lb G₁
G₁
M₁
G₂
M₂

FIG. 10.—ROCK-FILLED STEEL-PILE CELLULAR BREAKWATER.

Considering first, the comparison between the author's theoretical method and the empirical method of applying to the structures the wave forces reported by Captain Gaillard, it will be seen that the total horizontal pressures

TABLE 6.—COMPARISON OF FORCES ON A BREAKWATER

(D = 300 miles, northeast; angle, 60°; V = 55 miles per hr, northeast; and L = 334 ft)

Description	(a) WAVE FORCE ONLY BASED ON:					(b) WAVE FORCE PLUS HYDROSTATIC PRESSURE BASED ON:				
	Notation	Author's Formulas for Structure of:		Gaillard's Recorded Pressures for Structure of:		Notation	Author's Formulas for Structure of:		Gaillard's Recorded Pressures for Structure of:	
	(1)	Timber (2)	Steel (3)	Timber (4)	Steel (5)	(6)	Timber (7)	Steel (8)	Timber (9)	Steel (10)
Width of base, in feet.....	B	30	37.5	30	37.5	....	......	......	......	......
Weight, in pounds per linear foot.............	W	48 400	104 000	48 400	104 000	....	......	......	......	......
Horizontal pressure, in pounds per linear foot*..	P	23 489	24 500	19 150	20 195	P'	38 351	46 260	31 925	38 500
Height of pressure, P, above base, in feet......	X	24.49	32.73	25.09	33.36	X'	18.35	23.30	18.75	23.60
Distance from toe of structure to intersections of the resultant of W and P with the base, in feet....	Y	3.1	11.4	5.07	12.23	Y'	0.40	8.35	2.65	9.95
Ratio of Y to B expressed as percentages..........	R	10.3	29.4	16.9	32.6	R'	1.33	22.3	8.83	26.6
Toe pressure, in pounds per square foot............	T	10 400	6 230	6 380	5 670	T'	80 600	8 340	12 200	7 100

* The wave force, P, is taken as the force normal to the face of the breakwater considering the direction of the wave as 70° to the face of the breakwater.

are about proportional to the relative depth of the waves (that is, as 21.83 is to 18.0) and that the elevations above the base of the centers of actions of these forces are practically identical.

The greater weight and width of the steel structure gives it twice as much resistance to overturning as the timber structure, notwithstanding the fact that the base of the steel structure is 8 ft deeper in the water than the timber crib. In this entire study the effect of rip-rap protection (which is provided for both structures), and the passive resistance of the water, have been disregarded.

According to the author's method, the Calumet Harbor timber structure would be subject to a wave force that would cause the resultant of the horizontal and vertical forces to intersect the base 3.1 ft from the toe instead of 10 ft (one-third of the base), and would cause a toe pressure of 10 400 lb per sq ft. on the rock-fill, which, in turn, is supported by the clay of the lake bottom. According to the Gaillard observations the resultant from an 18-ft wave would intersect the base at 5.07 ft from the toe and result in a toe pressure of 6 380 lb per sq ft.

This timber crib structure has withstood successfully the storms of the past thirty years. In the great storm of 1929, for example, the lake level was abnormally high and a northeast wind blew almost continuously for two days, attaining velocities of 50 to 55 miles per hr for several hours duration. It is improbable that the waves of that storm produced as great wave pressure as that determined for this structure by Mr. Molitor's method.

When the author's method is applied to the steel cellular structure at Calumet Harbor, the resultant of the horizontal and vertical forces intersects the base 11.4 ft from the toe, or 0.6 ft outside the middle third, and results in a toe pressure of 6 230 lb per sq ft on the clay bottom of the lake. Applying the wave force based on Gaillard's observations, the resultant intersects the base 0.23 ft inside the middle third and the toe pressure is 5 670 lb per sq ft.

Certainly the author's method appears from this study to be conservative and to give results that may be safely depended upon for a structure designed to be built for the harbors of Lake Michigan and the other Great Lakes.

The addition of the effect of difference in hydrostatic pressure used by the writers in this study would appear to be an unnecessary addition to the formulas of the paper, but certainly it introduces a factor that must be present during wave action, and if the formulas for wave action truly evaluated that force, the factor of the additional hydrostatic pressure would have to be taken into account. It is only because of the probability that the author's formulas err on the side of safety that it may be unnecessary to add the hydrostatic factor.

David A. Molitor,[21] M. Am. Soc. C. E. (by letter).—In offering this paper the writer attempted to outline a method of evaluating the total wave force which may be expected in any locality where a sea-wall or a breakwater is proposed, so that a safe and relatively economic design could be evolved.

The formulas for height of a wave in terms of wind velocity and fetch are admittedly very approximate. When direct observations are available, as in the cases cited by Mr. Fowler, recourse to speculative estimates becomes unnecessary.

In the absence of reliable measurements of wave dimensions for any given locality, the data here presented give very acceptable answers for most inland bodies of water, more particularly for the Great Lakes, for any known values of wind velocity and fetch; actually, the time element or the duration of the storm also enters as an important factor.

The reason for revising the Stevenson formulas, based on a 78-mile wind, should be apparent, as in many cases the maximum wind velocity reported may be as low as 40 miles per hr, whereas in some localities a 100-mile wind may be encountered for the particular fetch and exposure.

Major Leeds has added considerable material relative to the validity of the Stevenson formulas which will prove instructive. He has supplied what the writer left unsaid for the sake of brevity, and, finally, comes to the same conclusions presented in the paper, namely, that observational data when available are preferable to results from grossly approximate formulas. The introduction of wind velocity into the Stevenson formulas renders them applicable to varying conditions. Naturally, a 40-mile wind will produce a lesser wave height than a 100-mile wind, whereas the Stevenson formulas apply only to a 78-mile wind. There are many cases on the Great Lakes in which only a 40-mile wind has occurred in the special direction affecting a given breakwater.

---

[21] Structural Engr., Procurement Div., Public Works Branch, Treasury Dept., Washington, D. C.

Reference is made to the value of k in Equation (12) which it is stated may have a possible maximum value of 2.00. This value might be utilized for wind velocities of about 100 miles per hr, and for storms of long duration. Smaller values of 1.3 to 1.7 are perfectly safe for waves produced by wind velocities from 30 to 70 miles, respectively, lasting not more than 24 hr.

The writer agrees with the preference expressed by Major Leeds for rubble-mound breakwater construction whenever such type is suitable. A factor of safety of, say, two against sliding and overturning, for crib designs, is likewise considered adequate for maximum storm conditions of record. The cost of infrequent repairs resulting from damage due to abnormal storms is also regarded as more economical over long periods of time.

The use of Equation (19) should be restricted to waves striking in rather deep water. Whenever deep-water waves approach a beach, the shoaling causes the waves to veer materially. Finally, they break on the beach along lines nearly parallel to the shore, in which case the formula would give values that are too small.

Mr. Eremin presents a curious method attributed to Sainflou, which he applies to Magann's Pier and finds a rather close agreement with the writer's analysis. However, this must be regarded as quite accidental and should not be accepted as a criterion for other cases.

The discussion contributed by Messrs. Condron and Post brings out some interesting points which deserve special attention. Their analysis of the Calumet Harbor Breakwater, serves as a splendid illustration of the tendency to over-estimate rather than under-estimate the wave force.

Although there is no question regarding the stability of this structure as designed, the writer wishes to record the results of an analysis such as he considers in accordance with the method advocated in the paper.

The 55-mile wind from the northeast agrees with the records of the U. S. Weather Bureau as given in Fig. 5. However, a fetch of 300 miles, which is the entire length of Lake Michigan, presupposes a storm of wider extent and longer duration in this direction than has ever occurred. It is writer's judgment that a fetch of 150 miles from a northeast direction would better fit this case, with the wave approaching normally to the breakwater, instead of at an angle. Furthermore, as the depth of water in front of the structure is 32 ft, the deep-water wave will not undergo any shoaling effect before striking.

It is quite inadmissible to apply wave pressures observed at Duluth Harbor directly to the Calumet Harbor Breakwater without due allowance for the different exposures. Each case must be treated in strict accordance with the special conditions governing the same. Finally, it must be clear that, in order to build up full wave pressure, the resisting surface must be of sufficient height (about 17.5 ft at Calumet Harbor) while the top of the pier is only 6.5 ft above still-water level. Therefore, this wave encounters only a relatively small obstruction, and does not build up pressures as great as those estimated by Messrs. Condron and Post.

The analysis herein presented is believed to approximate the true conditions more accurately, and will illustrate the correct application of the writer's method to the steel pile structure at Calumet Harbor as shown in Figs. 10 and 11.

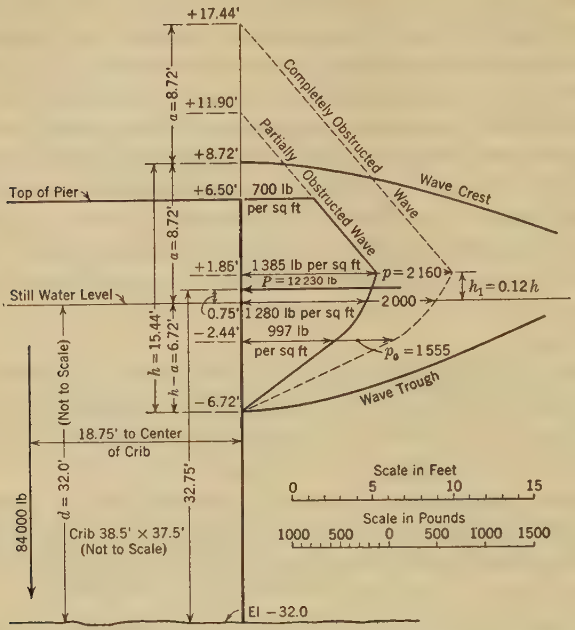

+17.44'
a = 8.72'
+11.90'
+8.72'
Completely Obstructed Wave
Partially Obstructed Wave
Wave Crest
Top of Pier
+6.50'
700 lb per sq ft
a = 8.72'
+1.86'
1 385 lb per sq ft
P = 12 230 lb
p = 2 160
h₁ = 0.12 h
Still Water Level
2 000
0.75'
1 280 lb per sq ft
−2.44'
997 lb per sq ft
p₀ = 1 555
h = 15.44'
h − a = 6.72'
(Not to Scale)
−6.72'
Wave Trough
18.75' to Center of Crib
Scale in Feet
0
5
10
15
Scale in Pounds
1000
500
0
500
1000
1500
84 000 lb
d = 32.0'
32.75'
Crib 38.5' × 37.5' (Not to Scale)
El −32.0

FIG. 11.—CALUMET HARBOR BREAKWATER, LAKE MICHIGAN.

Hence, for a maximum wind velocity of 55 miles per hr and a normal fetch of 150 miles, the following results are obtained: By Equation (1), $h = 0.17\sqrt{55 \times 150} = 15.44$ ft; by Equation (3), with $\frac{L}{h} = 15.3$ for $V = 55$, gives $L = 15.3\,h = 236$ ft; by Equation (4*a*) for $d > 1.84\,h$, $a = \frac{h}{2} + \frac{h^2}{L} = 8.72$ ft and $h - a = 6.72$ ft. By Equation (7) for $\frac{d_0}{L} = \frac{d}{L} + 2\left(\frac{h}{L}\right)^2 = 0.144$ (for which Table 3 gives $c = 0.858$, $\beta = 1.358$, and $\mu = 1.165$), find $v = 2.26\,c\sqrt{L} = 29.67$ ft per sec.

Furthermore, Equation (14) gives $v_0 = 7.11\,\mu\frac{h}{\sqrt{L}} = 8.3$ ft per sec. Equation (15) then gives $p = 1.5\,(v + v_0)^2 = 2\,160$ lb, and $p_0 = 0.72\,p = 1\,555$ lb per sq ft; also, $h_1 = 0.12\,h = 1.85$ ft.

The foregoing calculations make it possible to plot the pressure curve for the completely obstructed wave as shown in Fig. 11, from which the pressure curve for the partly obstructed wave is deduced.

The total effective pressure, $P = 12\,230$ lb per lin ft of structure, is obtained by fixing the height of the pressure area at Elevation 11.9, which is a mean level between the top of the pier and the full wave height at Elevation 17.44. This gives a maximum pressure of 1 385 lb per sq ft, as compared with the 2 370 lb used by Messrs. Condron and Post.

Assuming the weight of the submerged part of the crib at 50 lb per cu ft and the part above water at 100 lb, the crib would weigh only 84 400 lb per lin ft. Hence, the factor of safety would be as follows: Against overturning: $\frac{84\,400 \times 18.75}{12\,228 \times 32.75} = 4.17$; and, against sliding: $\frac{84\,400 \times 0.5}{12\,228} = 3.45$. The writer considers that a factor of safety of two is sufficient.

The introduction of an hydrostatic head due to a momentary difference in water level between the front and the back of the breakwater is not considered a proper addition to the wave force.

Although not mentioned in the paper, this matter was given careful consideration. No evidence of such action was actually observable as dynamometers placed below the level of the wave trough have not indicated any pressure, and none was expected as the wave effect is entirely dynamic and confined to surface action over a height equal to the height of the wave and due to the moving mass of water.

In concluding this discussion, it may be of interest to recall briefly the circumstances which led the writer to undertake an exhaustive study of this subject which many years later furnished the material for his paper.

When the safety of the design of the breakwater at Toronto Harbor was challenged by the contractor, before its actual construction was undertaken in 1915, it became necessary to establish proof of adequate design. In the absence of any authenticated method of procedure, it was necessary to devise a method of appraising approximately, the probable wave force for the particular exposure and depth of water, and then to show by comparison that the same method of analysis, when applied to numerous existing structures, gave results which agreed well with their actual behavior after they had weathered many severe storms. It was thus a case of first inventing a method of analysis and then of proving the method to be reliable.

After the claim of inadequate design was refuted by the writer's method of approach, a thorough investigation was made by a Board of Consulting Engineers, with the result that the structures were built without increased dimensions. They have now (1935) weathered all storms for the past nineteen years without showing any signs of weakness.

The writer wishes to express his thanks and appreciation to those who have contributed to this discussion. The points brought out will serve to emphasize certain matters on which divergence of opinions may be expected.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 1914

FLOW OF WATER AROUND BENDS IN PIPES[1]

BY DAVID L. YARNELL[2], AND FLOYD A. NAGLER[3],
MEMBERS, AM. SOC. C. E.

WITH DISCUSSION BY MESSRS. D. BENJAMEN GUMENSKY, WALLACE M. LANSFORD, F. T. MAVIS, AND DAVID L. YARNELL.

SYNOPSIS

In condensed form, this paper presents the more outstanding results of a series of experiments on the flow of water around bends of various shapes and various degrees of curvature in 6-in. pipes. Only the most salient points are discussed. These experiments are believed to be unique in that, with the same quantity of flow, the effect on the loss of head resulting from unequal velocity distribution in the pipe approaching the bend was fully investigated.

The experiments show: (1) That it is possible to have conditions such that the resistance to flow may be very small or unusually large in the same pipe bend carrying identical quantities of water; (2) that in a standard 90°, 6-in. pipe bend, for the same quantity of flow, with high velocity on the inside, and low velocity on the outside, of the approach pipe, the loss of head may be four times as much as would be measured in the same bend when high velocity exists on the outside, and low velocity on the inside, of the tangent leading to the bend; (3) that present formulas for computing loss of head due to bends appear to apply only to cases in which approximately uniform velocity distribution exists in the approach pipe; (4) that the losses of head in the bends experimented upon appear to vary as the square of the velocity, and not as the 2.25 power as suggested by some writers; (5) that a pipe bend may be as useful as any other device for the measurement of discharge; (6) that the

---

[1] Presented at the Joint Meeting of the Power Division of the Society and the Hydraulics Division of the American Society of Mechanical Engineers at the Annual Convention, Chicago, Ill., June 29, 1933, and published in August, 1934, *Proceedings*.

[2] Senior Drainage Engr., U. S. Dept. of Agriculture, Hydr. Laboratory, Iowa City, Iowa.

[3] Prof. of Hydr. Eng., Univ. of Iowa, Iowa City, Iowa. Professor Nagler died on November 10, 1933.

direction of flow of the secondary currents in pipe bends depends entirely upon the velocity distribution in the approach pipe; and (7) that the same fundamental laws of flow through bends apply to both closed conduits and open channels.

INTRODUCTION

The constant efforts of designers of pumping plants and hydro-electric plants to increase the efficiencies of their operation requires continuous research in hydraulics. Any change in design that may reduce the loss of head in the pipes and bends means increased efficiency. A bend acts as an obstruction to flow, causing additional loss of head in the case of both open and closed conduits. Certain types of bends cause internal changes in velocity to such an extent that the efficiency of the bends and down-stream channels as hydraulic conduits is considerably reduced. As a rule, disturbed flow means inefficient flow, and the maximum usefulness of the entire cross-section of the channel is not realized.

The studies have developed new data on velocity and pressure changes in bends and in the tangents adjacent. The data will be useful to designers of new plants and to engineers engaged in making plant-efficiency tests, because they show the effects that may be expected when piezometric connections are made at different points on a conduit. The data also demonstrate the necessity of taking into consideration the disturbing effects of conduit bends upon the performance of the down-stream tangents. Such conditions are often encountered in penstocks and spiral casings approaching water turbines as well as in draft-tube conduits of the elbow type.

Available manuscripts show that hydraulic research on bends has engaged the attention of engineers for more than 150 years. Apparently, the first tests were made by du Buat[4] about 1786. These tests were on pipes 1 in. and 2 in. in diameter. Numerous other investigators conducted experiments on pipe bends during the succeeding years. One of the first discussions on flow of water in open-channel bends was written by James Thomson, in England, in 1876.[5] The first field experiments on open-channel bends appear to have been made by M. Leliavski[6] on the Dnieper River, at Kiev, Russia, in 1892.

The investigations described herein were undertaken for the purpose of determining the laws governing the changes in pressure and velocity in different parts of the flowing stream, as the moving water undergoes the transition from motion along a straight path to motion around a curve, and, again, as it undergoes the opposite transition back to final straight-line motion. This condition of transitional flow also may be considered as representative of what always occurs whenever water flows in a bend or a crooked channel, or whenever it meets a bridge pier or other form of obstruction. A knowledge of these laws is fundamental to an understanding of the effects, harmful or otherwise, of crooked and obstructed channels on the flow of water, and to a determination of the best means for alleviating or relieving any objectionable or troublesome effects.

---

[4] "Principles Hydraulique," par E. du Buat, Paris, 1786, Pt. 1, pp. 141–151.

[5] *Proceedings*, Royal Soc. of London, May 4, 1876.

[6] "Currents in Streams and the Formation of Stream Beds," by M. Leliavski, Sixth International Congress of Internal Navigation, The Hague, The Netherlands, 1894.

A novel feature of the investigation was the use of transparent material for the walls of the testing channel. This type of construction enabled the observation of the direction of secondary currents by means of threads, color, and floats that could readily be seen and photographed through the exterior walls of the conduit.

Laboratory Investigations

The first tests were made on a 180° bend of square cross-section, 10 by 10 in., with a 5-in. inner radius. This shape was tried first because it seemed easier to build of transparent material, and because of the convenience of dividing the channel into elementary strips for studying flow conditions. The approach tangent was 25 ft long and the discharge tangent, 28 ft. The bend and 8 ft of the two tangents adjacent to it were made of transparent celluloid. Velocity traverses were made at eight different sections on the bend and at eleven sections on the two tangents. In each test velocities were taken at more than 800 different points for each quantity of flow. Pressures were measured at 296 different points. About 48 man-hr were required to get all the measurements for a single test.

Tests were next made on 180° bends of rectangular cross-section, 5 in. wide by 10 in. deep. One bend had a 5-in. and the other, a 10-in., inner radius. Tests were made on the bends flowing full and partly full, and with both uniform and non-uniform velocity distribution in the channel approaching the bend.

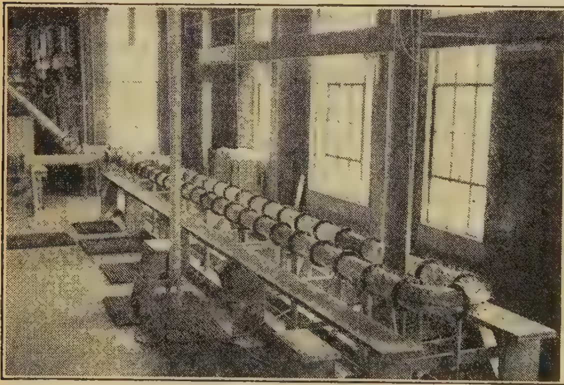


Fig. 1.—Installation of 6-Inch Celluloid Pipe for Test of 180° Bend.

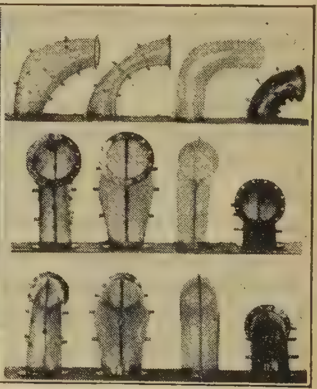


Fig. 2.—Types of 90° Celluloid Bends Tested.

The next set of experiments was made on bends of circular cross-section, 6 in. in diameter. The bend (standard elbows with a $5\frac{1}{4}$-in. inner radius), approach, and discharge tangents were made of transparent celluloid. Tests were made on a 180° bend continuous in one direction (Fig. 1), a standard 90° ell, an abrupt 90° elbow, a 180° reverse curve bend, and a 270° bend made up of three 90° bends all in a horizontal plane, but twice reversing the direction of curvature. The approach and discharge tangents were each 25 ft long.

Bends of irregular cross-section were also included in the experiments. These bends included the hyperbolic and elliptical shapes as well as a circular bend with varying radius of curvature. Fig. 2 shows the front, side, and rear views of four specimens. The one at the right is a standard elbow and the other three are special designs made by Charles A. Mockmore, M. Am. Soc. C. E., for studies on draft-tubes.

Velocity measurements were taken at seven different points on each of four diameters at each section. There were nine velocity sections on each of the approach and discharge tangents, in addition to velocity traverses at intervals of 22°.5 on the bend. For each quantity of flow, velocities were taken at 200 different points, and pressure readings were taken at about 230 different points, in the bend and on the tangents.

Velocity and pressure measurements were recorded for various flows in the pipe with a normal uniform distribution of velocity in the approach tangent. A series of tests having non-uniform velocities approaching the bend was also made. These tests consisted of a high velocity at one side and a low velocity at the opposite side of the pipe. A test was then made with the high velocity at the top of the pipe. Next, this high velocity was created at the inside of the pipe, then at the bottom of the pipe, and, finally, outside the pipe.

It is impractical to present, in a limited space, all the data obtained in the investigation.

Velocity Conditions Within the Bend

When water flows in a straight conduit or channel, the transverse water-surface profile is presumably a straight level line. On a curve or bend, due to centrifugal force, the transverse water-surface profile cannot be level. Water naturally moves in a straight line unless deflected by the action of some unbalanced force. When water moves around a bend, an unbalanced force directed toward the center of curvature acts against the water.

As the water approaches the bend, the filaments of flow along the inside of the bend speed up because the grade is steeper, the distance around the bend being less along its inner radius than along its outer radius. Thus, the filaments next to the inside of the bend have a greater velocity than those along the outside. Neglecting friction, the total energy in any filament is constant; therefore, the greater the velocity along the inner radius, the lower the pressure will be. As the water moves around the bend, the centrifugal force on the filaments of water will be balanced by the radial difference in pressure.

That this condition exists may be seen in Fig. 3, which shows the velocity and pressure conditions in a 180° bend. At Section 0°, the beginning of the bend, the filaments of flow at the inside, have already acquired a velocity considerably greater than that along the outside, of the bend. The greatest velocity occurs at a point next to the inside wall, approximately midway around the bend. The thread of maximum velocity in the channel gradually moves to the outside of the bend as the water travels around, as may be seen at Section + 0.43 ft in Fig. 3. For a given discharge, the sharper the radius of curvature the greater the velocity along the inner radius.

The velocity distribution in circular conduit bends is similar to that in bends with channels of either square or rectangular cross-section. The velocity in a 6-in., 90° bend is shown in Fig. 4. Attention is called to the shifting of the high velocity from the inside to the outside at the end of the bend. The typical section, Fig. 4, shows the location of piezometer connections and Pitot tube readings.

Pressure Changes Within the Bend

When a stream of water meets an obstruction that diverts the various particles of water from motion along a straight line to motion in a curved path, the speed of the particles of water must be changed by unequal amounts; that is, the velocity of some parts of the water must be increased and that of other parts must be decreased. Accompanying these changes of velocity are related changes in pressure.

The pressures against the top and bottom of the rectangular channels flowing full are shown in Fig. 3 as circles connected by solid lines. Since flow around bends follows the law of free vortex motion, the profile of transverse pressure is convex upward, and has the greatest slope on the convex or inner bank and the flattest slope on the concave or outer bank. It will be noted that as the water moves around the bend the pressure is greatest near the outer side of the bend and is much lower along the inner side. This condition was true even when the channels were flowing only partly full.

The peripheral pressures at various sections around bends of square and rectangular cross-sections are very enlightening. Peripheral pressures were measured at Sections − 1.0 ft, 0°, 45°, 90°, 135°, 180°, and + 1.0 ft on the 10 by 10-in. 180° bend. Pressures were measured at twenty-two different points on the periphery for various quantities of flow, seven on each of the outside and inside walls, three on the bottom, and five on the top.

It will be noted that the pressure, measured as above the channel bottom, is uniform on the four walls of the conduit in the approach channel to the bend (Section − 1.0 ft (Fig. 5)). As the water moves around the bend the pressures on the inside wall of the bend decrease, as shown in Sections 45°, 90°, 135°, and 180° (Fig. 5). The pressure against the outside wall of the bend at any one section is practically uniform.

When non-uniform velocities prevail in the channel approaching the bend, the periphery pressures within the bend become much more irregular, as may be seen in Fig. 6. In this case the bend was 5 in. wide by 10 in. deep, with a 5-in. inner radius. Attention is called to the large differences in periphery pressure at the 45° section, as shown by comparing Figs. 5 and 6.

The pressure distribution in circular conduit bends is similar to that in bends on either square or rectangular cross-section (see Figs. 3 and 4).

The pressure differences between the inner and outer sides of a 90° bend in a 6-in. pipe, on horizontal diameters, are shown in Fig. 7. Pressure differ-

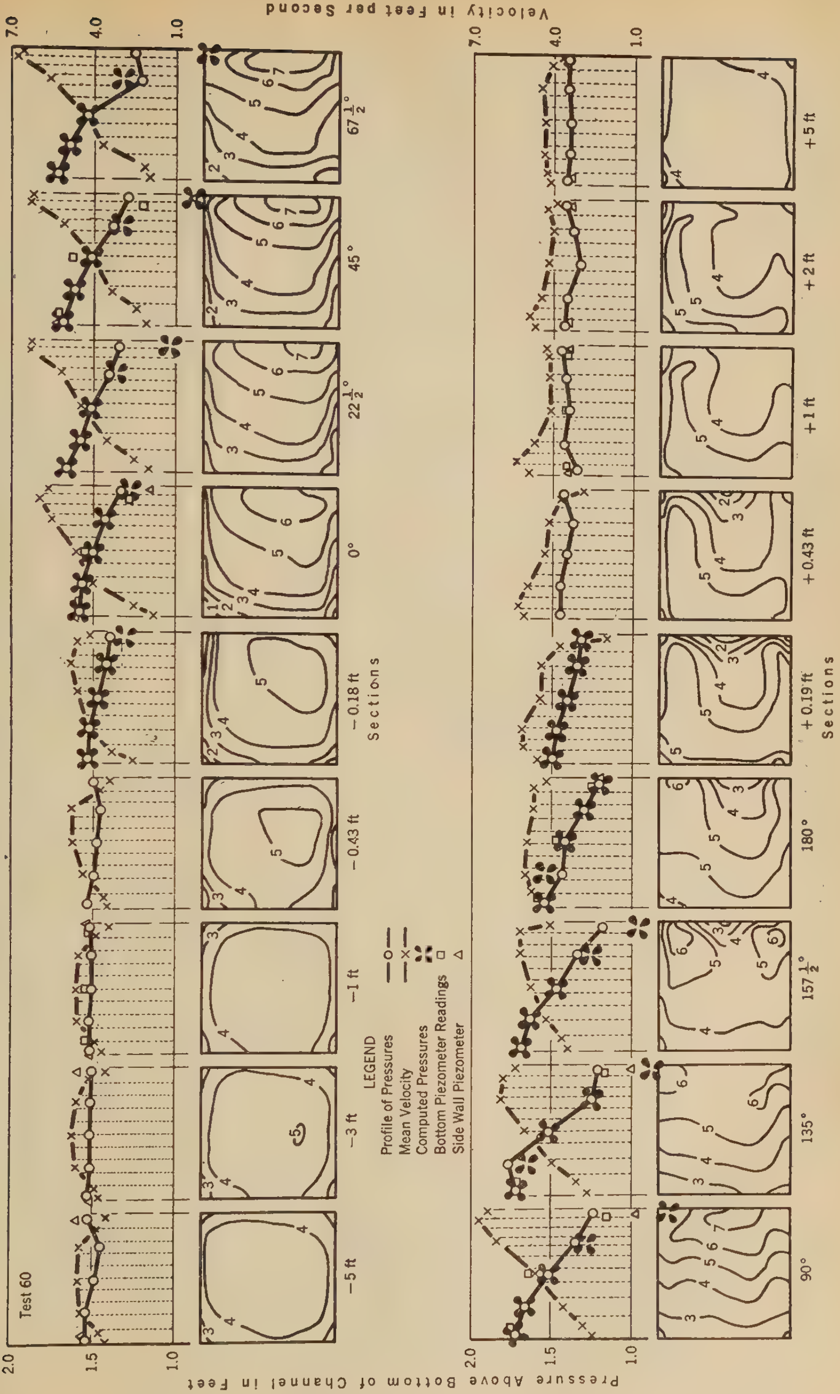


FIG. 3.—VELOCITY AND PRESSURE CONDITIONS IN 180° BEND.

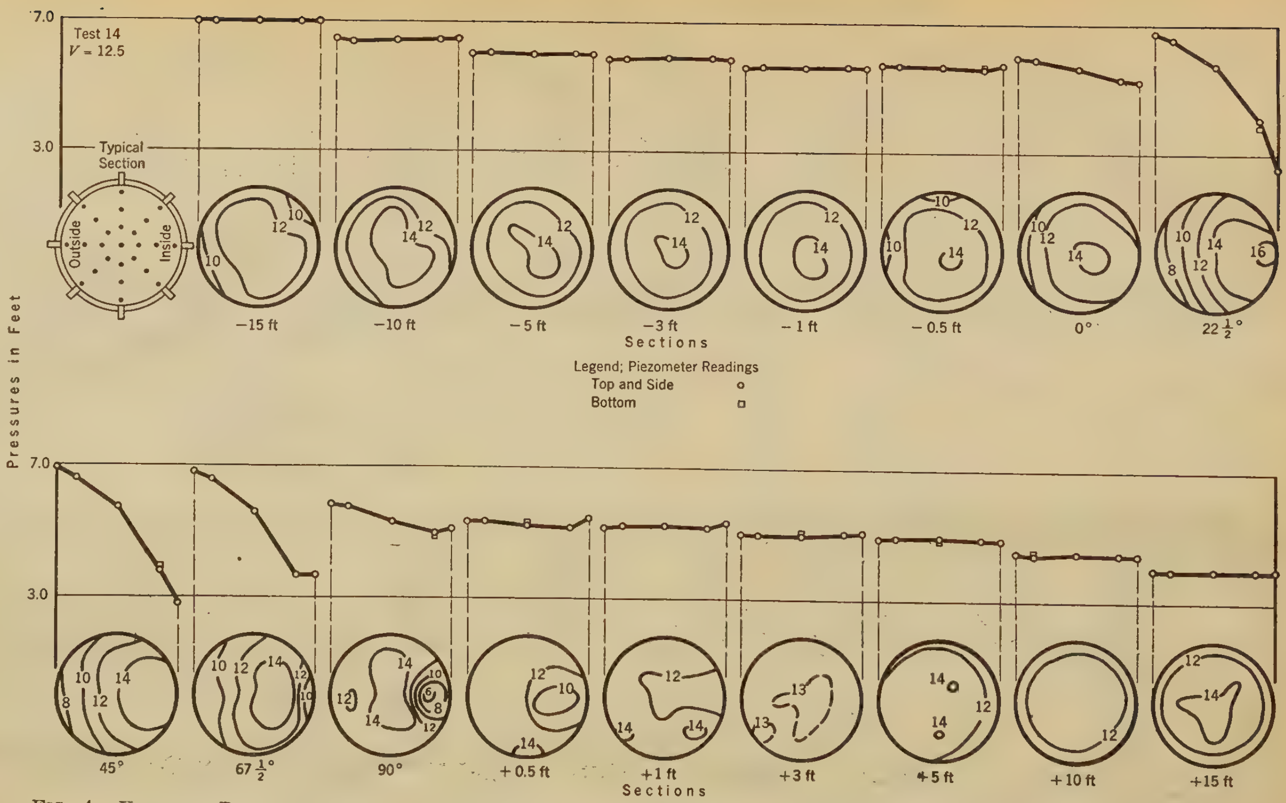


FIG. 4.—VELOCITY DISTRIBUTION AT VARIOUS SECTIONS IN 90° BEND IN 6-INCH PIPE (RADIUS OF CENTER LINE 8¼ INCHES; MEAN VELOCITY, 12.5 FEET PER SECOND; APPROXIMATELY UNIFORM DISTRIBUTION IN APPROACH TANGENT; TRANSVERSE PROFILE OF PRESSURE SHOWN ABOVE EACH SECTION).

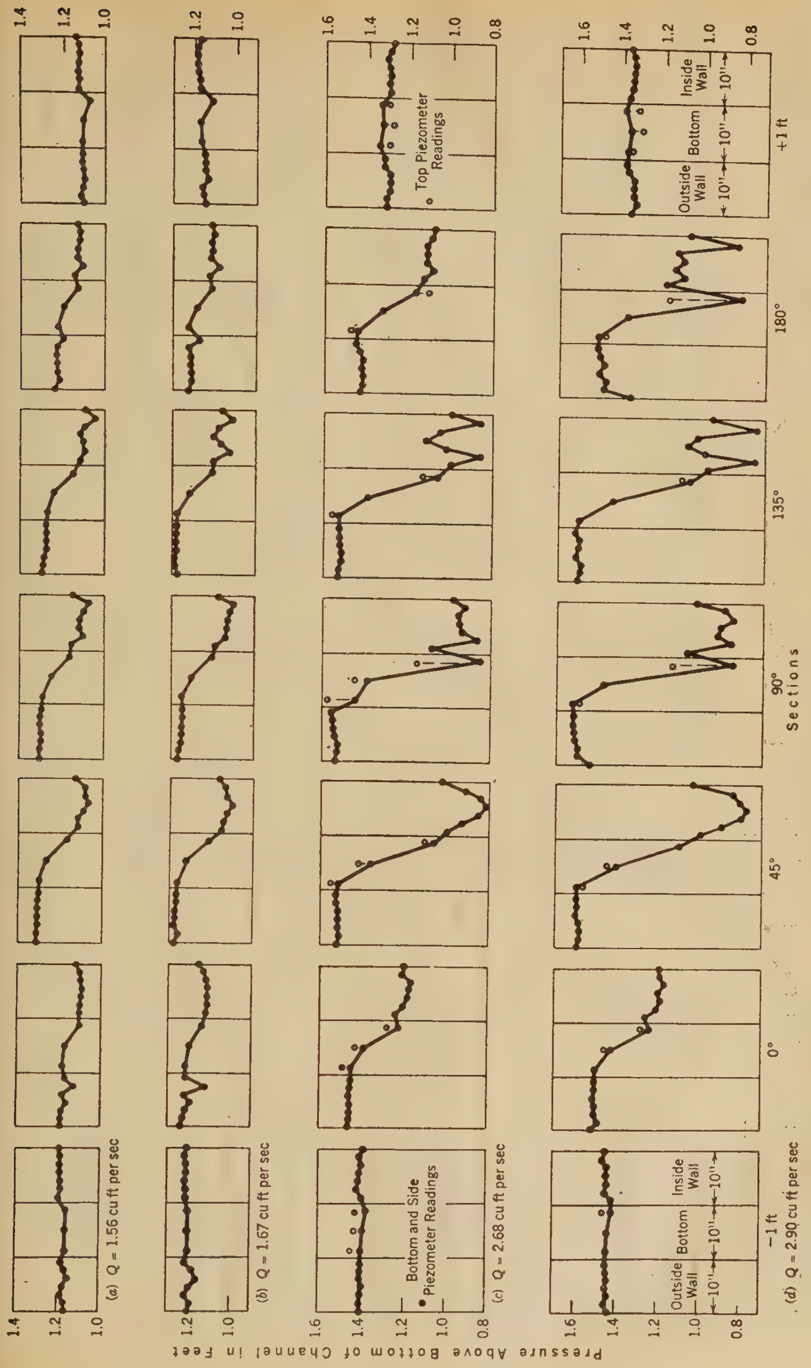


FIG. 5.—PRESSURES ON PERIPHERY OF 10-INCH SQUARE CHANNEL ON 180° BEND OF 5-INCH INNER RADIUS, WITH UNIFORM DISTRIBUTION OF VELOCITY IN APPROACH CHANNEL (FLOW, 1.56 TO 2.90 CUBIC FEET PER SECOND).

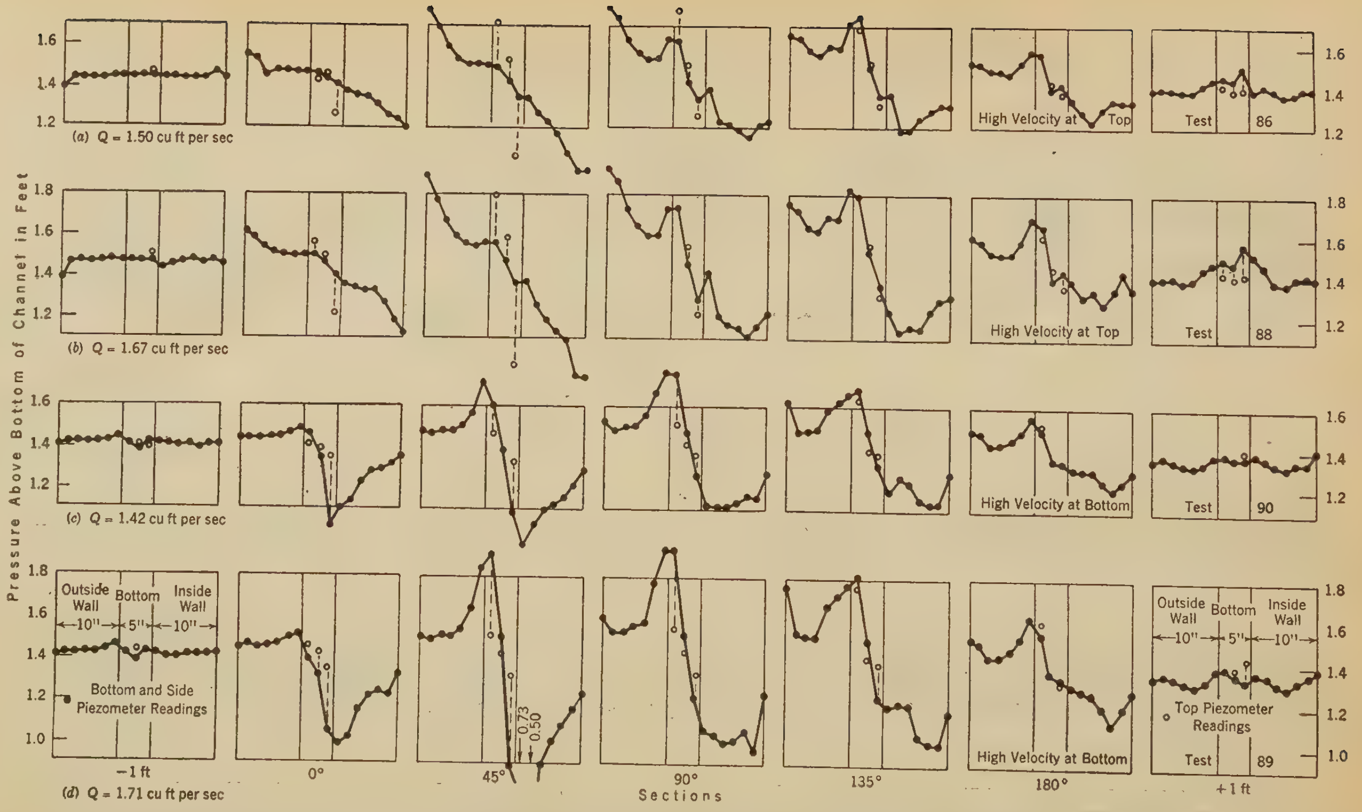


FIG. 6.—PRESSURES ON PERIPHERY OF 5 BY 10-INCH CHANNEL ON 180° BEND OF 5-INCH RADIUS WITH NON-UNIFORM DISTRIBUTION OF VELOCITY IN APPROACH CHANNEL AS SHOWN (FLOW, 1.50 TO 1.71 CUBIC FEET PER SECOND).

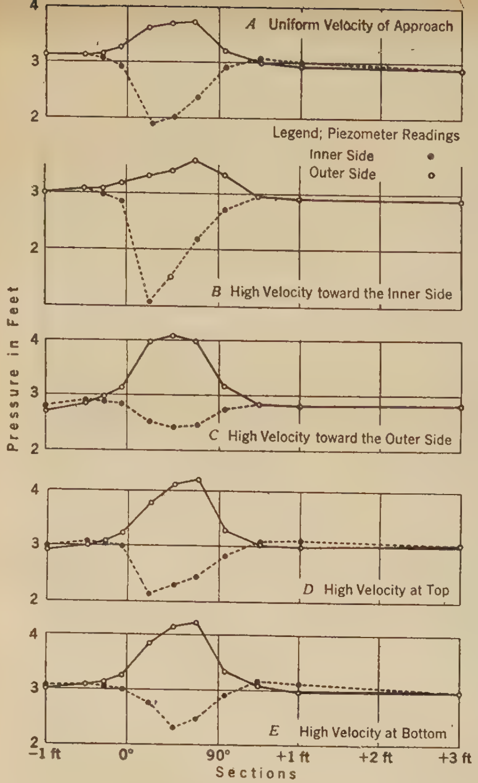


FIG. 7.—PRESSURES ALONG INNER AND OUTER SIDES OF 90° BEND IN 6-INCH PIPE ON HORIZONTAL DIAMETER, WITH VARIOUS VELOCITY DISTRIBUTIONS IN APPROACH CHANNELS. (RADIUS OF CURVATURE OF CENTER LINES 8½ INCHES; FLOW FROM LEFT TO RIGHT; DISTANCES MEASURED ON CENTER LINE.)

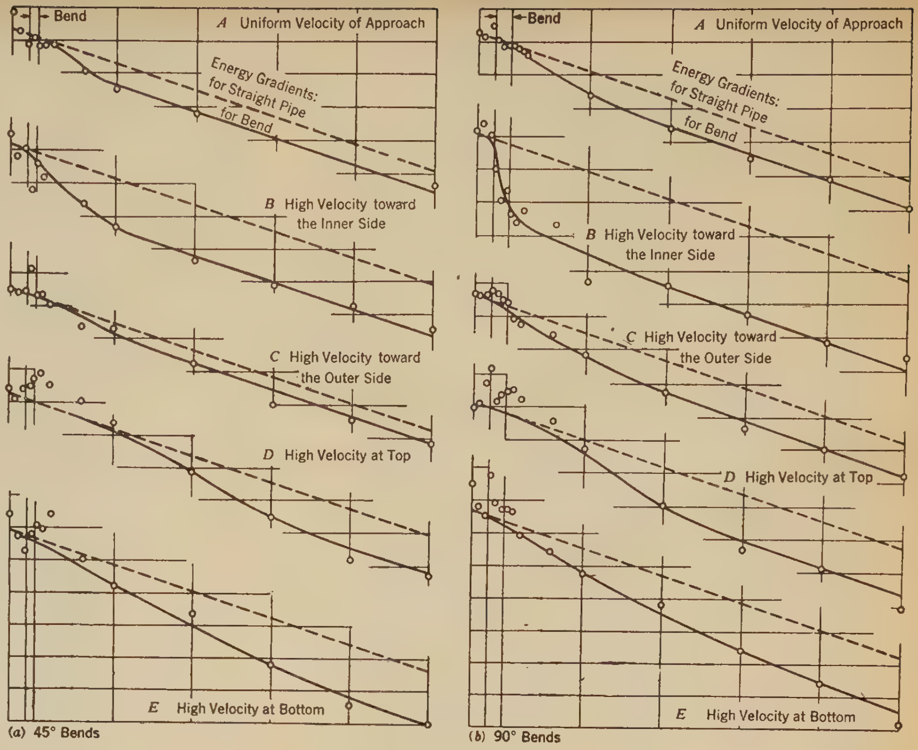


FIG. 8.—SECTIONS SHOWING LOSSES OF HEAD IN 45° AND 90° BENDS IN 6-INCH PIPE FOR DIFFERENT VELOCITY DISTRIBUTIONS. (HORIZONTAL INTERVAL, 5 FEET; VERTICAL INTERVAL, 0.2 FOOT.)

ences are shown for five different conditions of velocity distribution in the approach pipe to the bend, for a mean velocity of 8.3 ft per sec. Tests show that, for a given velocity distribution approaching the bend, there is a marked similarity of pressure differences near the beginning of the bend, regardless of the degree of curvature. In bends of square and rectangular cross-section, the longitudinal pressure curves are similar to those obtained with circular pipe bends.

Differences in pressure between the inside and outside of pipe bends may exceed several feet of water column even with ordinary velocities. With a mean velocity of 10 ft per sec in a 6-in. pipe, a 90° elbow caused a difference of about 6 ft of water column, but the addition of another 90° elbow did not increase this difference. A 6-in., abrupt 90° ell gave a difference of more than 10 ft, with a mean velocity of 10 ft per sec.

Knowledge of these differences of pressure in bends is of importance to engineers engaged in pump efficiency tests, particularly in plants in which gauge readings in the suction and discharge pipes are used to determine the total head pumped against. If a gauge must be attached to a bend, it is desirable to have four piezometric connections made at right angles to each other and carried to a common manifold to which the gauge is attached.

Loss of Head in Bends

The loss of head caused by a pipe bend depends on several factors, a very important one of which is the kind of velocity distribution that occurs in the pipe approaching the bend. It is possible to have such conditions of flow that the loss of head may be very little or unusually large in the same bend for identical discharges. Fig. 8 (*a*) shows the losses in 45°, and Fig. 8 (*b*), the losses in 90°, bends in 6-in. pipe for different velocity distributions in the pipe approaching the bend, with a quantity of flow giving a mean velocity of 8.3 ft per sec. This unequal velocity distribution was created by a special screen placed 5 ft up stream from the beginning of the bend. The obstruction gave a low velocity of about 4 ft per sec, and a high velocity of 12 ft per sec, at the beginning of the bend.

With the velocity in the approach pipe high toward the inner side, and low toward the outer side, of the bend, the loss of head may be two to four times as much as would be measured in the same bend with uniform velocity distribution existing in the approach pipe. On the other hand, with the approach velocity high toward the outer, and low toward the inner, side of the bend, the loss of head may be less than that caused by the same bend when uniform velocity distribution prevails in the approach tangent. With high velocity either at the bottom or at the top in the approach, the loss of head caused by a bend is considerably more than would occur if uniform velocity distribution prevailed in the approach tangent.

The data obtained show that the present formulas for computing loss of head in bends apply when uniform velocity distribution exists in the pipe approaching the bend.

The loss in a 45° bend (see Fig 8 (*a*)) is about three-fourths that in a 90° bend (Fig. 8 (*b*)) of the same radius. The loss in a 90° abrupt elbow is about 1.2 times the velocity head, or about 7.5 times that in a standard 90° elbow.

SECONDARY CURRENTS IN BENDS

Secondary currents within the bend are caused by the difference in the centrifugal forces of the filaments of varying velocity; that is, transverse flow occurs in a pipe or channel bend in addition to the forward motion of the fluid. These induced, or secondary, currents usually originate in that section of the pipe next to the top and bottom and gradually increase in strength and intensity as the water moves around the bend.

The types of secondary currents that prevail in a bend depend on the velocity distribution in the approach tangent. The unequal pressures against the inside wall of the 180° bend of square section (Sections 135° and 180° (Fig. 5)), are caused by the secondary currents set up within the bend. If a high velocity occurs at the top, and a low velocity at the bottom, of the approach channel, the secondary current will have a counterclockwise motion, as viewed looking down stream, in a conduit that bends to the right; if the high velocity is at the bottom in the approach channel, the direction of the secondary current will be clockwise. If uniform velocity distribution exists in the approach pipe, two secondary currents will be set up in the bend, one clockwise in the upper portion and the other clockwise in the lower portion, of the cross-section. In Fig. 9, the yarns next to the outside wall diverge, while those next to the inside wall converge. Secondary currents exist also in open-channel bends, but usually there is only one, even with uniform velocity distribution in the channel approaching the bend.

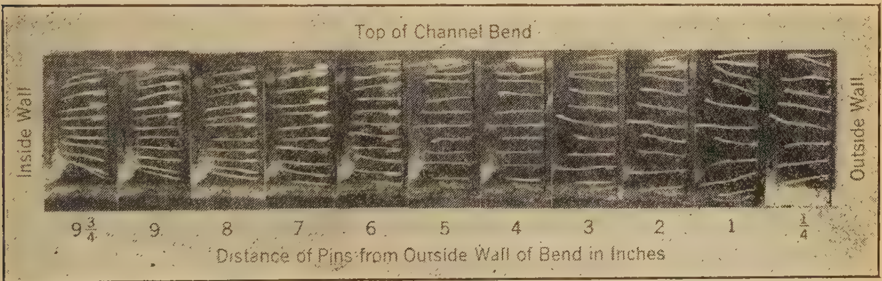


FIG. 9.

The loss of head in bends is due primarily to two causes: (1) Skin friction of the forward flowing water against the walls of the pipe; and (2), internal friction of the induced or secondary currents within and following the bend. The strength of the secondary currents in bends is greater in wide channels than in narrow channels. Hence, the practice of using blade turns in 90° elbows and guide-vanes in quarter-turn draft-tubes greatly increases the efficiency of these bends, due to reduction in the magnitude of the secondary currents.

In general, the greater the velocity, the more pronounced will be the secondary currents in bends, and any means devised to reduce these currents and to create a greater "effective area" of forward flow will reduce the energy loss and hence improve the efficiency.

Pipe Bends as Flow Meters

It would appear that advantage might be taken of the differences in pressure between the outside and the inside of a pipe bend, and that the bend could be used as a flow meter. The difference in pressure between the inside and outside of a bend may be fully as large as, or even larger than, that produced in a Venturi meter or Pitot tube and, as in those other measuring devices, it varies with the second power of the velocity. The experiments indicate that the errors likely to be involved are no greater than those experienced in these other devices. Each bend, however, should receive individual calibration where great accuracy is desired.

It can readily be proved that the difference in elevation of the water surface between the two sides of the bend is equal to:

$$\frac{V^2 B}{g R} \qquad \text{.............................(1)}$$

in which, V is the mean velocity at the section; B, the width of that section; g, the acceleration due to gravity; and R, the mean radius.

The formula, $H = \frac{V^2 B}{g R}$, can be written for all pipe bends, as follows:

$$V = c \sqrt{H} \sqrt{\frac{g R}{B}} \qquad \text{......................(2)}$$

in which, c is a coefficient depending on the kind of pipe. The tests with 6-in. celluloid pipe gave a value of c of about 0.91 for the 90° bend and a few tests with 6-in. cast-iron pipe also gave a value of c of about 0.91 for the standard 90° elbow. The pressure difference in the celluloid pipe was taken at 22°.5, and in the cast-iron pipe, 21°.5, from the beginning of the bend. It would appear that Equation (2) would give very close results in determining the quantity of flow. Further tests will be made on bends to ascertain the variation in the coefficient with the variation in position of measuring the pressure difference.

Equation (2), simplified for a single pipe bend, may be written as:

$$Q = c' \sqrt{H} \qquad \text{............................(3)}$$

in which, H is the pressure difference in the bend, and c', a constant. For the celluloid 90° bend, taking the difference in pressure at a section 22°.5 from the beginning of the bend, c' was found to be 1.14. For the 6-in., 90°, cast-iron bend, c' was 1.17, with the difference in pressure measured (of necessity) at 21°.5 from the beginning of the bend.

When unequal velocity distribution prevails in the approach channel to the bend, the best results probably would be obtained by measuring the pressure differences at the 45° point from the beginning of the bend, although further tests are needed to verify this point. For this condition the variation in c may be somewhat greater, particularly when high velocities prevail on the inside of the approach pipe to the bend unless the particular bend in question is calibrated.

New Types of Pipe Bends

Studies of the energy losses in pipe bends have resulted in the development of several new types of bends. In pipe bends carrying water, comparatively little has been done in suggesting changes of shape to reduce the loss of head. Lack of effort in this direction has been due mainly to the fact that in such pipes the velocity of flow is not always high. In pumping plants and similar units, long-radius elbows of uniform section or tapered reducing sections frequently are used. It would appear that some effort might profitably be expended in investigating and experimenting on various shapes of elbows.

Many shapes of bends are suggested, but only tests can show their good or their bad qualities. Some have suggested bends of the hyperbolic type, while still others have devised bends of elliptical shape so that the difference in velocities between the inside and the outside of the bend will be comparatively small. The difference in velocity helps to start secondary currents and the resultant inefficient flow. A new type of radius for a bend also has been suggested (see Fig. 3), in which the length of the radius at the point of tangency is infinite and decreases until, midway around the bend, it becomes a minimum, equal to the radius of the regular bend. The shape of the last half of the bend is similar to that of the first half.

The special bends shown in Fig. 3 are little better than the standard 90° bend tested, in so far as loss of head is concerned. Due to lack of funds, it was impossible to test other special shapes.

Conclusions

The following conclusions have been drawn, based on the data thus far collected in the investigations:

(1) All bends act as obstructions to flow, resulting in additional loss of head.

(2) With uniform velocity distribution in the approach channel to the bend, the velocities of the filaments along the inside wall of the bend are increased while those along the outside wall are reduced.

(3) For a given pipe bend and quantity of flow, the head lost in the bend depends upon the velocity distribution in the approach tangent.

(4) In a standard 90°, 6-in. pipe bend, for the same quantity of flow, with high velocity on the inside and low velocity on the outside of the approach pipe to the bend, the loss of head may be four times as much as would occur in the same bend when high velocity occurs on the outside and low velocity on the inside of the tangent leading to the bend.

(5) Knowing the maximum difference in pressure between the inside and the outside of the bend, and the radius and size of the bend, it is possible to compute the mean velocity in the pipe, and, therefore, the discharge.

(6) After a pipe bend has been calibrated, it may be used as a flow meter and discharges may be determined by measuring the difference in pressure between the inside and outside of the bend.

(7) The losses in the pipe bends experimented upon appear to vary as the square of the velocity and not as the 2.25 power as suggested by some writers.

(8) The fundamental laws that apply to closed conduit bends, also apply to open channel bends.

ACKNOWLEDGMENTS

The investigations described herein were made under the direction of S. H. McCrory, and L. A. Jones, Members, Am. Soc. C. E. Sherman M. Woodward, M. Am. Soc. C. E., acted in an advisory capacity. Frank W. Edwards, H. P. Evans, Jr., G. A. Marston, Roland A. Kampmeier, Ross N. Brudenell, Juniors, Am. Soc. C. E., and Mr. Cecil H. Morris assisted in the investigations, which were conducted by the Bureau of Agricultural Engineering, U. S. Department of Agriculture, at the Hydraulic Laboratory of the University of Iowa, Iowa City, Iowa.

DISCUSSION

D. Benjamen Gumensky,[7] Assoc. M. Am. Soc. C. E. (by letter).—The explanation of the phenomena of flow around bends, contained in this paper, is very elucidating. Particularly clear and useful are the sections, "Velocity Conditions Within the Bend" and "Pressure Changes Within the Bend." The section on "Loss of Head in Bends" is of extreme interest to the designer of large pipes where the cost of head is high. In such cases the loss of head due to bends may have considerable influence on the selection of the most economical size for the pipe.

The loss of head due to bends was a subject of study in connection with the design of large siphons in the Colorado River Aqueduct. This aqueduct, now under construction (1934) by The Metropolitan Water District of Southern California, has in its main part 150 siphons, aggregating 27 miles in length. In these siphons, head losses due to bends are computed by the empirical formula:

$$h_b = K\left(\frac{V^2}{2g}\right) \qquad \text{(4)}$$

in which, h_b is the loss in one bend; V, the mean velocity in the pipe; and g, the acceleration due to gravity; and,

$$K = C\sqrt{\frac{\Delta}{90}} \qquad \text{(5)}$$

in which, C is a coefficient depending on the ratio of pipe diameter to radius of bend; and Δ is the deflection angle, in degrees.

Equation (4) was devised several years ago by Julian Hinds, M. Am. Soc. C. E., in an arbitrary manner, after a study of the extremely discordant literature on the subject available at that time.

Although the total head allowed in siphon bends on the aqueduct is appreciable, the effect of bends is small in comparison with other factors, and it was considered safe to proceed with Equation (4) without attempting to substantiate it experimentally. However, the writer and others were interested in the problem and attempted to set up a small-scale experiment on their own initiative. R. R. Proctor, Assoc. M. Am. Soc. C. E., and members of his staff, made water and facilities available and assisted with the experiment. It is described herein in order to supply additional experimental data, and in the hope that the several points of discussion will merit Mr. Yarnell's attention in the closing discussion.

The experiment consisted in measuring the bend losses in a 2 by 2-in. square pipe for different angles of bend and different radii of curvature. Measurements were made on bends of 180°, 135°, 90°, 45°, and 20° for radii of curvature of 20, 8, and 4 in. Velocities varied from 5 to 16 ft per sec.

The experimental set-up is shown in Figs. 10, 11, and 12. The water was pumped from a well through a calibrated meter and a reducer into an 8-ft tangent of 2 by 2-in. square pipe; then it was passed through a bend, an 11-ft tangent, and discharged into the air. The outlet end of the straight tangent had to be constricted in order to build up back pressure. The same two tangents were used for all set-ups; and each bend was carefully fitted and aligned between the tangents. The connections were made by soldering the seams from the outside and being careful that no solder would project into the section of the pipe and form an obstruction.

---

[7] Asst. Engr., The Metropolitan Water Dist. of Southern California, Los Angeles, Calif.

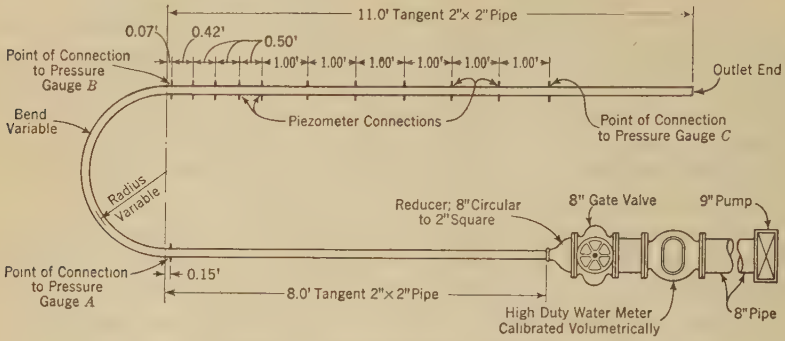

11.0' Tangent 2"× 2" Pipe
0.07'
0.42'
0.50'
1.00'
1.00'
1.00'
1.00'
1.00'
1.00'
Point of Connection to Pressure Gauge B
Outlet End
Bend Variable
Piezometer Connections
Point of Connection to Pressure Gauge C
Radius Variable
Reducer; 8" Circular to 2" Square
8" Gate Valve
9" Pump
Point of Connection to Pressure Gauge A
0.15'
8.0' Tangent 2"×2" Pipe
High Duty Water Meter Calibrated Volumetrically
8" Pipe

FIG. 10.—EXPERIMENTAL SET-UP

With a differential pressure gauge the loss of head was measured from a point 2 in. "up stream" from Bend (*A*) to a point 1 in. "down stream" from the end of Bend (*B*), and from the latter point to a point 8 ft farther along the tangent to Point (*C*). Then, as a check on the two measurements, an over-all loss was measured from the point above the bend to the point near the outlet end, Gauge (*A*) to Gauge (*C*). This measured total loss of head was compared with the experimentally determined loss of head in the identical lengths of straight pipe for the same velocity. The difference was considered to be the loss due to the bend.

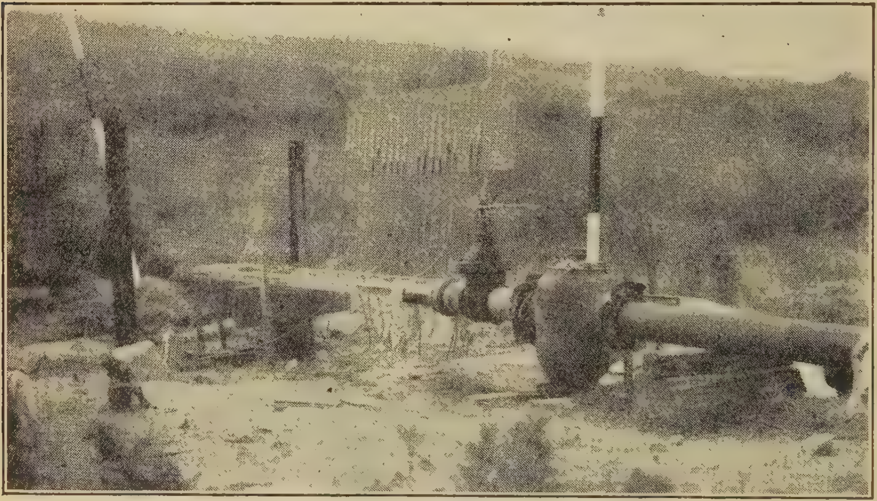


FIG. 11.—SET-UP FOR 180 DEGREE BEND

The results reduced to coefficients of velocity head in Equation (4) are shown in Fig. 13. Although the work was done with rather crude equipment the results are reasonably consistent. The general trend of variation and the actual values of losses seem to coincide quite definitely with those reported by the authors. The solid, horizontal lines denoting values of $C\sqrt{\frac{\Delta}{90}}$ were defined by the results of the test. The curve for K by Equation (5) (with $C = 0.25$) is that used by The Metropolitan Water District of Southern California.

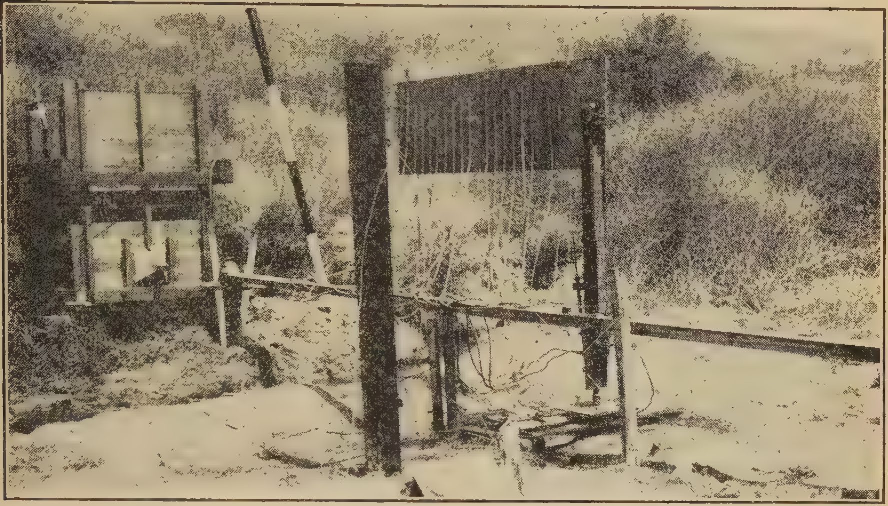


Fig. 12.—Set-Up for 10 Degree Bend

All the observed losses are smaller than those computed from Equation (4). Ignoring the observations for curves of 180° deflection, the average of all readings falls almost exactly on the line:

$$K = 0.16\left(\frac{\Delta}{90}\right)^{0.845} \qquad \text{(6)}$$

The coefficients of bend loss for 180° fall out of the line with others and proportionately are much smaller. No satisfactory explanation of this deviation can be offered. No regular variation of the value of K with radius was discovered. If the observations are segregated as to velocity, it is found that the value of K increases with increasing velocity. This would indicate that the bend loss is proportional to a power of V greater than two, which is contrary to Conclusion (7) of the paper. The character of the experiment and the results obtained do not warrant an attempt to determine the exact value of this power.

Actual observations have shown that the pressure on both sides of the straight pipe below the bend varied, changing from the position above the average value to that below it, as if the water were bouncing from one side of the pipe to the other. This was disclosed by observation on piezometers connected to both sides of the tangent below the curve, which can be seen on Figs. 11 and 12. Fig. 14 shows the typical measurements and the losses. The curves shown are not smoothed out, but are drawn through points of individual measurements.

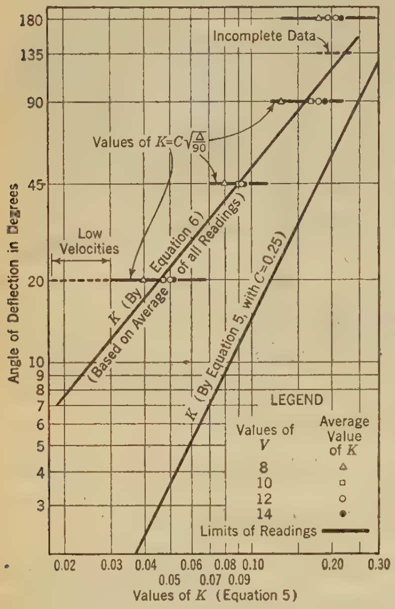


FIG. 13

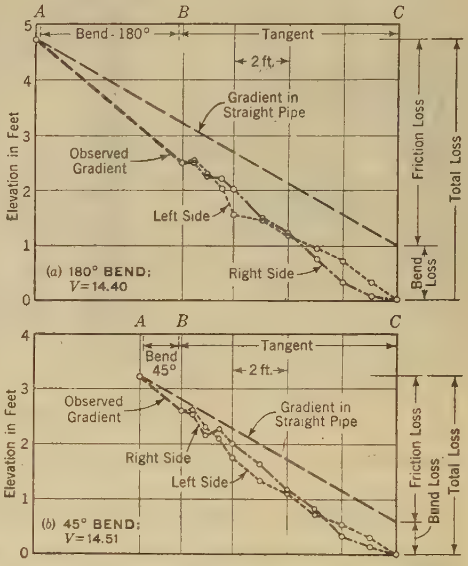


FIG. 14.—TYPICAL BEND LOSS MEASUREMENTS

The loss of head due to bend was measured not entirely in the bend, but also in the straight pipe following. This is well shown on Fig. 14. Similar results were found by Mr. A. W. Brightmore[8] in his experiments on elbows and bends in 3-in. and 4-in. pipe. Examining the authors' Fig. 8, it is readily seen that conditions were similar to those described herein. The energy gradient in a tangent following the bend is steeper than that for the straight pipe and does not become parallel to it for a distance varying between 5 and 15 ft from the end of the bend.

It is a regrettable fact that all the observations made thus far have been on pipes of small diameters. Application of principles of similitude to the existing experimental data in order to determine the probable losses in large pipes would be uncertain and speculative, although in actual practice engineers are forced to assign definite values to bend losses in large pipes.

WALLACE M. LANSFORD,[9] ASSOC. M. AM. SOC. C. E. (by letter).—The suggestion of the authors to use pipe bends as flow meters is a valuable one, although they are not the first[10] to propose such a possibility. The writer first heard of this method of measuring the flow of water in the discussion of this paper by the late Professor Nagler following its oral presentation.[1] In the summer of 1933 the writer made a series of tests which seem to verify two conclusions reached by the authors, first, that an elbow in a pipe line could be used as a flow meter; and, second, that where great accuracy is required, each elbow should be calibrated.

---

[8] *Minutes of Proceedings*, Inst. C. E., Vol. 169, pp. 315 *et seq.*

[9] Assoc., Theoretical and Applied Mechanics, Unv. of Illinois, Urbana, Ill.

Since relatively few experimental data are given in the paper, the writer's experiments will be stated in some detail. Tests were made on three cast-iron, flanged elbows located in a pipe line in the Hydraulics Laboratory of the University of Illinois, under the direction of M. L. Enger, M. Am. Soc. C. E. The radius of the inside bend of the elbows and the nominal diameter of both pipe and elbows were 4 in. The length of straight pipe preceding each elbow was 150 ft 2 in.; 8 ft 6 in.; and 128 ft, respectively. The pressure connections, $\frac{1}{4}$ in. in diameter, were made on both the inside and outside bends of each elbow at points approximately 45° from each flange in the plane of symmetry of the bend. An individual differential gauge was used for each elbow. The fluid (carbon tetrachloride and gasoline dyed red) used in all three gauges was taken from one mixture, thereby assuring identical coefficients. The velocity in the pipe was controlled by a valve at the outlet end. The discharge was measured by letting the water flow into a calibrated pit tank which had a cross-sectional area of 28.20 sq ft, and noting the rise of the water and the elapsed time of each run. The water could be turned quickly into the pit tank or diverted from it by means of a swinging pipe.

During any one run all three gauges were read when the water was flowing into the pit tank, thus assuring the same discharge through each elbow for any one set of readings. Time was measured by a stop-watch. Water was taken from a stand-pipe, 6 ft in diameter and 65 ft high, near the top of which was an overflow weir approximately 15 ft long. Water flowed over this weir continuously, thereby maintaining constant pressure throughout all the tests. Data were taken in several runs with velocities varying from 0.77 ft per sec to 5.72 ft per sec.

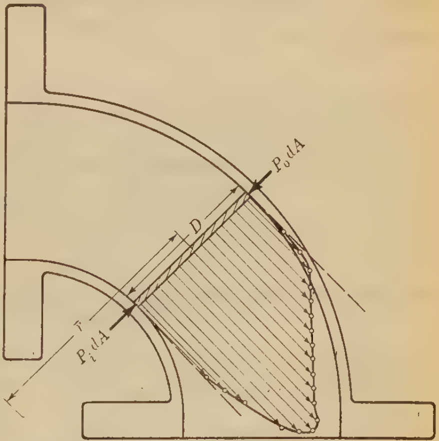


FIG. 15.—SECTION THROUGH CENTER OF ELBOW SHOWING VELOCITY DISTRIBUTION

---

[10] "New Method of Water Measurement by Use of Elbows in a Pipe Line," Gaskell S. Jacobs and Francis A. Sooy, *Journal of Electricity, Power, and Gas*, July 22, 1911, Vol. 27; "Die Turbinen für Wasserkraftbetrieb," von A. Pfaar, 1912. (He gives the following references: Isaachsen in *Zivilingenieur*, 1886, p. 338, and 1896, p. 352; Kankelwitz (Stuttgart), about 1870); "A Flow Metering Apparatus," by A. M. Levin, *Journal*, A. S. M. E., September, 1914; "The Hyperbo-Electric Flow Meter," *Power*, June 26, 1923; and "The Flow in Curved Pipes and Its Stability," by Otogoro Miyagi, Vol. XI, No. 1, Technology Repts., Tohoku Imperial Univ.

Using a small prism of water of length, *L*, and of cross-sectional area, *dA*, as shown in Fig. 15:

$$(p_o - p_i)\; dA = \left(D\,\frac{V^2}{\bar{r}}\right)\frac{w}{g}\; dA \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

and,

$$h = \left(\frac{p_o}{w} - \frac{p_i}{w}\right) = \left(\frac{2\,D}{\bar{r}}\right)\frac{V^2}{2g} = k\;\frac{V^2}{2g} \;\ldots\ldots\ldots\ldots\ldots (8)$$

in which, V = mean velocity of water in pipe, in feet per second; p_i = unit intensity of pressure at inside of bend, in pounds per square foot; p_o = unit intensity of pressure at outside of bend, in pounds per square foot; $\bar{r}$ = radius of the mass center of the prism, in feet; and D = diameter of elbow, in feet.

Equation (8) shows that the difference in pressure head between the outside and inside of an elbow through which water is flowing, is a constant times the velocity head (based on the average velocity), provided the value of $\bar{r}$ does not change for different rates of flow. Ordinarily, the radius, $\bar{r}$, is less than the radius of the center line of the elbow, because the velocity of flow is highest near the inside of the bend. However, with abnormal distribution of velocity at the entrance of the elbow, the radius, r, might be increased or decreased.

The curves in Fig. 16 show the relation between c_k and the average velocity in the pipe. It can be seen from these curves that c_k is a constant for each elbow when the velocity of the water in the pipe is approximately 1.5 ft per sec, or greater. This agrees with the conclusion of the authors that the elbow is valuable as a meter, and that each elbow should be calibrated for accurate use. The more constant value of c_k for each bend was, as follows (see Fig. 16):

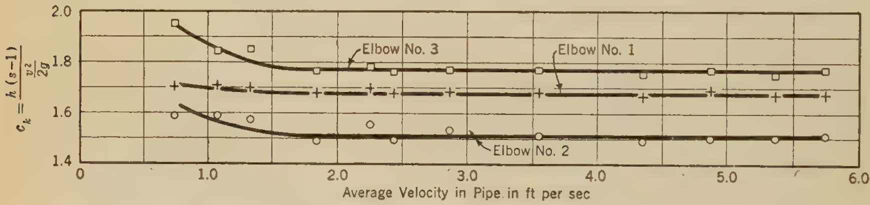


FIG. 16.—RELATION BETWEEN c_k AND VELOCITY IN PIPE FOR THREE 4-INCH ELBOWS (RADIUS OF INSIDE BEND, 4 INCHES)

Elbow No.	Value of c_k
1	1.68
2	1.51
3	1.77

Later, all elbows were interchanged in all positions in the pipe line, but the direction of flow through them was not changed. Values of c_k for the elbows, when in the latter positions, deviated about 3% from those shown in Fig. 16. This difference could be due to the effect of changing gaskets, or to the eccentricity of the elbows caused by a large clearance of the bolts in the holes through the flanges. The length of straight pipe preceding each position of the elbows seemed to be sufficient not to affect the value of c_k.

From Equation (8) the theoretical value of c_k for the elbows tested would be $c_k = \frac{2D}{\bar{r}} = 1.33$, if $\bar{r}$ is taken as the radius to the center of the elbow. In order to investigate the discrepancy between the actual and theoretical values of c_k, a Pitot tube traverse was made of Elbow No. 2 when in the southeast position where it was preceded by 150 ft 2 in. of straight pipe. The measured velocity curve is shown in Fig. 15. This curve is quite different from that assumed in the theoretical analysis. A rough calculation of $\bar{r}$ for this velocity distribution, assuming the water to flow in stream lines, shows it to be about 5.72 in. instead of 6 in. Therefore, c_k, would be $\frac{2D}{5.72} = 1.40$. It will be noted that a small variation in $\bar{r}$ changes the value of c_k materially; thus, when $\bar{r}$ is 5.33 in., $c_k = 1.50$. This would probably account for some of the difference between the actual and theoretical values of c_k.

On the basis of the information obtained from the tests herein reported, a useful device known as a flow indicator has been built and is being used successfully in the Hydraulics Laboratory of the University of Illinois. The centrifugal pumps that supply the water are on the basement floor, approximately 5 ft above the level of the water in the supply channel. The control switches for the motors that drive the pumps are on the floor above. When a pump is started it is convenient and important to know whether or not it is actually delivering water. The device is shown in Figs. 17 and 18. It consists essentially of two closed steel cylinders approximately $1\frac{3}{4}$ in. in inside diameter and 3 in. long, mounted end-wise on a steel bar and balanced on the bolt at mid-span as a fulcrum. The cylinders, about one-half full of mercury, are directly connected beneath by $\frac{1}{8}$-in. pipe. The $\frac{3}{16}$-in. copper tube that connects

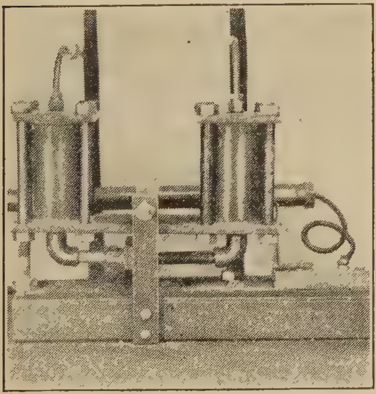


FIG. 17.—THE FLOW INDICATOR

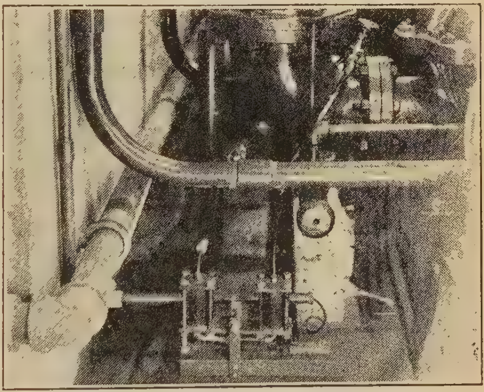


FIG. 18.—THE FLOW INDICATOR INSTALLED

the left cylinder to the outside bend of the elbow and the right cylinder to the inside is shown in Fig. 18. Water flowing through the elbow produces a high pressure at the outside of the bend and a low pressure at the inside, which causes the mercury to flow from the left cylinder to the right cylinder. The unbalanced weight causes the right end of the indicator to deflect and to make contact with an electric terminal. This closes an electric circuit which, in turn, lights a small electric light above the control switch on the first floor. When the pump is stopped the mercury flows back into the left cylinder, thus equalizing the weight and breaking the electric circuit. Two of these indicators are used on two pumps having a discharge capacity of 1 000 gal per min, each, and two on two pumps having a discharge capacity of 2 000 gal per min, each. All the indicators were installed in January, 1934, and have given good service.

F. T. MAVIS,[11] ASSOC. M. AM. SOC. C. E. (by letter).—A valuable summary of a long series of painstaking tests is presented in this paper. The authors deserve much credit for reviving the use of a bend as a flow meter and for their leadership in this particular research, which has also been included in the programs of other laboratories in the United States.

Attention should be called to the work of other investigators who have made significant contributions to the investigations of the hydraulics of bends in pipes. The pioneer work of Weisbach[12] in Germany about the middle of the Nineteenth Century is often referred to in current hydraulic literature. Twenty-five years or more ago, Alexander[13] and Brightmore[14] were conducting tests in England, and Williams, Hubbell, and Fenkell,[15] and Schoder[16] were making tests in this country. Among the current investigators might be mentioned the names of Hoffmann[17] and Schubart[18] in Munich, Nippert,[19] in Danzig, and Giesecke,[20] and the Staff of the National Hydraulics Laboratory in the United States. Most of these experiments were conducted on pipes of circular cross-section, although some tests were made on pipe of square or rectangular section. Doubtless, there are many other published and unpublished investigations which could be brought together, re-appraised, and correlated to make an exceedingly valuable reference work on the subject of flow of water around bends.

---

[11] Associate Prof., Dept. of Mechanics and Hydraulics, and Associate Director in Chg. of Laboratory, Iowa Inst. of Hydraulics Research, Univ. of Iowa, Iowa City, Iowa.

[12] "Lehrbuch der Ingenieur und Maschinen Mechanik," von J. Weisbach, p. 439, Leipzig, 1845.

[13] "The Resistance Offered to the Flow of Water in Pipes by Bends and Elbows," by C. W. L. Alexander, *Minutes of Proceedings*, Inst. C. E., Vol. 159, 1904–05, Pt. I, pp. 341–364.

[14] "Loss of Pressure in Water Flowing Through Straight and Curved Pipes," by A. W. Brightmore, *Minutes of Proceedings*, Inst. C. E., Vol. 169, 1907, p. 315.

[15] "Experiments at Detroit, Michigan, on the Effect of Curvature on the Flow of Water in Pipes," by the late Gardner S. Williams, M. Am. Soc. C. E., and Clarence W. Hubbell and George H. Fenkell, Members, Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. XLVII (1902), pp. 1 to 369.

[16] "Curve Resistance in Water Pipes," by E. W. Schoder, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. LXII (1909), pp. 67 to 112.

[17] "Neue Untersuchungen über den Druckverlust in Rohrkrümmern," von A. Hoffmann, *Mitteilungen* des Hydr. Inst. der Techn. Hochschule, München, Heft 2, pp. 70–71, 1928; also, Heft 3, pp. 45 ff, 1929.

[18] "Der Energieverlust in Kniestüchen bei glatter und rauher Wandung," von W. Schubart, *Mitteilungen* des Hydr. Inst. der Techn. Hochschule, München, Heft 3, pp. 121–144, 1929. ("Energy Loss in Smoooth and Rough Bends and Curves in Pipe Lines," tr. by F. T. Mavis, Assoc. M. Am. Soc. C. E.)

[19] "Uber den Strömungsverlust in gekrümmten Kanälen," von H. Nippert, *Forschungsarbeiten*, Heft 32, 1929.

[20] "Friction of Water in Elbows," by F. E. Giesecke, M. Am. Soc C. E., *Transactions* Am. Soc. of Heating and Ventilating Engrs., Vol. 32, 1926, pp. 303–314.

Many tests have been made at the University of Illinois by M. L. Enger, M. Am. Soc. C. E., and W. M. Lansford, Assoc. M. Am. Soc. C. E., to study the properties of bends as flow meters. In 1916, the idea of tapping the inside and outside of a bend and using the pressure difference in these two points as a means of determining the flow of water through the bend, was credited to Mr. Buckner Speed by the late J. C. Trautwine, Jr.,[21] Affiliate, Am. Soc. C. E. Mr. Trautwine also referred to tests reported by Messrs. G. S. Jacobs and F. A. Soog,[22] which led to an equation expressing the relation between the velocity in the pipe, V, the pressure difference, h, in feet of water, the radius of the bend, r, and the diameter of the pipe, D, as follows:

$$V = c \sqrt[u]{h \frac{r}{D}} \qquad \text{.........................(9)}$$

in which, $c = 5.6$ and $u = 1.9$. Other data and calibration tests of a similar flow-metering apparatus were reported in 1914, by Mr. A. M. Levin.[23]

The problem of the hydraulics of bends has been investigated in some detail from an analytical basis. zur Nedden has presented certain aspects of the problem which merit the mention of his paper[24] in particular. He calls attention to the fact that a bend may perform quite differently, depending on whether the flow of the fluid through it is laminar or turbulent. He cites specifically the case of a bend of rectangular cross-section the inner and outer curves of which were hyperbolic, as causing a lower head loss than a circular bend as long as the flow remained laminar; as soon as the flow became turbulent, the hyperbolic bend caused a greater head loss than the circular bend. This behavior would be apparent, of course, if the coefficients of velocity head, representing the excess head loss in the bend, were plotted as a function of Reynolds' number.

To-day, it is generally recognized that in flowing around a bend under turbulent flow conditions, a liquid moves in a so-called "spiral" or "helical" path, and that the induced currents and attendant impact are largely responsible for excess losses of head in bends. In 1902, the late H. W. Brinckerhoff, M. Am. Soc. C. E., discussing the paper by Messrs. Williams, Hubbell, and Fenkell,[25] explained the fact that a bend offers greater resistance than an equal length of straight pipe on an hypothesis of spiral flow which he described in its essentials precisely as other writers have done. In their closing discussion, Messrs. Williams, Hubbell, and Fenkell replied to Mr. Brinckerhoff's comments as follows:

> "At one time, one of them [the writers] was disposed to explain certain phenomena on the theory of a spiral motion, but in spite of attempts to locate such motion, no direct experimental evidence of its existence has yet been

---

[21] *Transactions*, Am. Soc. C. E., Vol. LXXX (1916), p. 911.

[22] *Journal of Electricity, Power, and Gas*, San Francisco, Calif., July 22, 1911.

[23] "Flow Metering Apparatus," by A. M. Levin, *Transactions*, A. S. M. E., Vol. 36, 1914, pp. 239–254.

[24] "Induced Currents in Fluids," by F. zur Nedden, *Transactions*, Am. Soc. C. E., Vol. LXXX (1916), pp. 844–913.

obtained, and, though the writers consider that Mr. Brinckerhoff's theory is ingenious and possibly correct, they are not prepared to endorse it."

Transparent model bends, such as those used by the authors and C. A. Mockmore,[25] M. Am. Soc. C. E., permit the observer to see this spiral or helical flow. These transparent models have done much to create a better understanding and a more intimate knowledge of the phenomena of hydraulics. To-day, this spiral flow in bends can be observed; three decades ago, it was an hypothesis which eminent hydraulic engineers were then "not prepared to endorse."

David L. Yarnell,[26] M. Am. Soc. C. E. (by letter).—The writer is indeed gratified at the additional data brought out by Messrs. Gumensky, Lansford, and Mavis in their discussions of this paper. Since it was necessary to limit the length of the paper, only the more important findings of the investigation could be presented. The fact that the tests were made on smooth pyralin pipe instead of commercial pipe should not be overlooked. It is quite probable that losses in commercial pipe of the same diameter and radius of curvature would be greater than those obtained in the smooth pyralin pipe.

It was unfortunate that tests could not have been made on pipe of varying radii, but lack of funds prevented such a program. It is a source of satisfaction that the tests reported by Mr. Gumensky as conducted under the direction of R. R. Proctor, Assoc. M. Am. Soc. C. E., have been published in connection with this paper.

The following formulas for loss of head in 6-in. pyralin pipe bends were obtained on the basis of the writer's tests:

For a 45° bend,

$$H = 0.11 \frac{V^2}{2g} \qquad \text{(10)}$$

for a 90° bend,

$$H = 0.15 \frac{V^2}{2g} \qquad \text{(11)}$$

and, for a 180° bend,

$$H = 0.19 \frac{V^2}{2g} \qquad \text{(12)}$$

It will be noted that the paper reports, for a 90° bend, a lower value of K (namely, 0.15), than that used by The Metropolitan Water District of Southern California. On commercial pipe (particularly the larger sizes) the value of K would be considerably greater than 0.15.

When unequal velocity distribution occurs at the beginning of the bend and excessive turbulent flow prevails within, and down stream from it, the loss of head due to the bend and the turbulent flow will occur many feet down stream.

---

[25] "Flow in Bends of Quarter-Turn Draft-Tubes," by C. A. Mockmore (Abstract), *Civil Engineering*, September, 1934, p. 460.

[26] Senior Drainage Engr., U. S. Dept. of Agriculture, Hydr. Laboratory, Iowa City, Iowa.

Although it is realized that the writer and the late Professor Nagler did not originate the idea of using a bend as a flow meter, it seemed that this fact had been overlooked by many engineers. Therefore, it was re-emphasized to show that bends in pipes, objectionable as they are, had certain good features. Thus, it is encouraging to study the additional tests by Mr. Lansford on bends as flow meters. Mr. Lansford states that the coefficient is a constant for velocities of 1.5 ft per sec, and greater. This might not be true when unequal velocity distribution prevails at the beginning of the bend. Mr. Lansford's use of a bend as a flow meter in connection with pumps in the Hydraulics Laboratory of the University of Illinois is very unique.

The references listed by Professor Mavis are valuable additions to the paper, As he states, transparent models have done much to throw light on conditions of flow within a pipe or a bend.

The writer wishes to acknowledge the assistance of Mr. Edward Soucek on the computations connected with these tests. In closing, he wants to emphasize what he considers probably the most important finding in the investigation, namely, that it is possible to have such conditions of flow that the loss of head may be very little, or unusually large, in the same bend for identical discharges.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 1915

STREET THOROUGHFARES
A SYMPOSIUM

Foreword. PAGE
By George H. Herrold, M. Am. Soc. C. E. 1046

Definitions, General Principles, and Recommendations: Report of the Committee of the City Planning Division on Street Thoroughfares Manual.
W. W. Crosby, M. Am. Soc. C. E., *Chairman*. 1048

Relation of Street Thoroughfares to Other Arteries of Transportation.
By Jacob L. Crane, Jr., M. Am. Soc. C. E. 1056

Design of Boulevards and Parkways.
By Jay Downer, M. Am. Soc. C. E. 1060

Super-Highways.
By J. P. Hallihan, M. Am. Soc. C. E. 1062

Thoroughfare Layouts in Relation to Drainage Systems.
By W. W. Horner, M. Am. Soc. C. E. 1064

Express Highways and By-Pass Routes.
By Daniel L. Turner, M. Am. Soc. C. E. 1066

Paying for Street Thoroughfares.
By George H. Herrold, M. Am. Soc. C. E. 1068

Bibliography on Street Thoroughfares.
By the Committee of the City Planning Division on Street Thoroughfares Manual 1076

With Discussion by Messrs. H. Shifrin, Donald M. Baker, Frederic Bass, Hugh E. Young, and Committee of City Planning Division on Street Thoroughfares Manual.

Note.—Published in August, 1934, *Proceedings*.

FOREWORD

BY GEORGE H. HERROLD,[1] M. AM. SOC. C. E.

In July, 1930, a Division Committee on Manuals was re-organized and the general subject of a City Planning Manual was discussed at a Division meeting in Cleveland, Ohio. It was decided that there should be a series of committees to study and report on the various subjects involved in such a manual and it was agreed that the first report to be completed should deal with Street Thoroughfares.

A Committee on Street Thoroughfares Manual was created by the Executive Committee of the City Planning Division on January 22, 1931. The Committee began active work in the spring of that year and developed a program to produce definite results within a reasonable time. A progress report of the Committee was presented at the meeting in Tacoma, Wash., in July, 1931, and included a bibliography and definitions. At this same meeting papers were also presented on allied subjects; that is, "Population Density Related to Roadway Area," "By-Passing Traffic Around Small Towns," and "Increasing Highway Efficiency."

Since this date the Committee has been actively engaged in the preparation of this Symposium. Its purpose is to suggest methods of identifying street thoroughfares according to their use and capacity and of outlining their design according to the best usage and well-defined trends of the present day.

The word, "thoroughfare," may be given the broadest possible interpretation of all words meaning a place for the movement of commodities and people.

In 1386, Chaucer wrote: "The world is nothing but a thoroughfare", and in Shakespeare's "Henry the VIII" (1540), is found "Chauncerie Lane and Pewter Lane being thoroughfares and passages from Fleet St. into Holborne." Dalton's "Country Justices" (1630) states: "In towns which are not thoroughfares the justices shall be sparing of allowing any ale-houses," and in Cooke's "Voyages in the South Seas" (1712): "There is a thoroughfare in the midst of it where we rode with our ships." In Emerson's "English Traits" (1856), is the expression: "They have made the island a thoroughfare and London a shop inviting to strangers" and, quoting from the *Westminster Gazette* of June, 1908: "How seldom must these ancient walled villages communicate with the thoroughfare valleys."

A street thoroughfare, therefore, is a way through a street open at both ends, through which there is much passing. The place of the street thoroughfare in the city plan structure is the most important. It affords accessibility to land and makes it usable. It is a definite part of the traffic network of State, region, or city. It drains the down-town district of vehicles quickly at the close of the business day or after the theatre. It is the avenue connecting the home districts with the business district. It is the driveway through parks or park reservations—a thoroughfare for recreation and pleasure. It is the street over which commodities and people are moved. It makes living agreeable and business possible.

---

[1] Member, State Planning Board; Managing Director and Engr., The City Planning Board, St. Paul, Minn.

The street thoroughfare is created by the necessity of travel between terminals. It determines the uses of land along its course. It forms the pattern that moulds the city. It lies between the origin and destination of traffic. It is a right of way for tree planting—a thoroughfare glorified by landscaping. It is a right of way for subterranean or underground utilities. Street thoroughfares dominate the city plan by determining the controlling major or minor traffic movements. The conflicting demands of "access to property" and "continuous traffic movements" are supplied by it. It is the *ne plus ultra* for ground movements. It connects terminals and, at the same time, serves abutting property. It creates the mosaic of city, region, or State, and becomes the radial, the boundary, or the axis of planned and zoned areas.

The many miles of frontages along street thoroughfares both in urban and suburban territory present peculiar and difficult problems in planning their use. The passing traffic is increasingly of a character that requires no service from abutting property between terminals, leaving much land valueless for speculative ventures or business serving a traveling public. In suburban areas the land retaining its agricultural use is enhanced in value only by the shortening in time to a market, and reduced in value by excessive traffic and traffic hazards.

As the principal use of land in urban areas is for streets, parks, and building sites, the location of street thoroughfares in their proper relation to each other and to building sites and open spaces is the first element in city planning. The street thoroughfare includes all trafficways, until further defined and restricted—the lane, the avenue, the mercantile street, the boulevard, the expressway, etc.

It is hoped that this Symposium will assist in clarifying some loose thought and discourage careless writing through the specifications set up by it, creating a terminology that may be used by all city planners.

DEFINITIONS, GENERAL PRINCIPLES, AND RECOMMENDATIONS

REPORT OF THE COMMITTEE OF THE CITY PLANNING DIVISION ON STREET THOROUGHFARES MANUAL

W. W. CROSBY,[2] M. AM. SOC. C. E., *Chairman*

The Committee on Street Thoroughfares Manual, appointed on January 22, 1931, herewith submits the following report in which it has quite thoroughly considered a relatively new subject. After the results have become more generally known and discussed, and after some future developments have occurred in street-thoroughfare practices, a further reconsideration of the subject may be productive of more valuable and specific conclusions.

The work of the present Committee—widely scattered in its membership—has had to be conducted entirely by correspondence. However, by sending, contemporaneously, copies of all texts to each member, all decisions have been made by an equally advised membership, and finally have been unanimous. Except for the individual papers, the report text is that of the Committee as a whole.

To illustrate the operations of the Committee, the following outline may be given: After acceptances of membership on the Committee had been secured, suggestions as to its objectives were obtained from Messrs. Harold M. Lewis, Secretary of the City Planning Division, and George H. Herrold, Division Sponsor for the Committee. Pursuant thereto, Jacob L. Crane, Jr., Jay Downer, W. W. Horner, and Daniel L. Turner, Members, Am. Soc. C. E., were requested to prepare their respective papers and to define certain terms. The Chairman, on receipt of these responses, collated the definitions and resubmitted them with some others for discussion. John P. Hallihan, M. Am. Soc. C. E., was added to the Committee membership in August, 1931, and his paper was received soon after the request then made for it. He also contributed valuable suggestions for definitions.

The discussion concerning the definitions was prolonged and has now finally resulted in the expressions given.

Progress reports were submitted at the meetings at Tacoma, Wash., in July, 1931, and at St. Louis, Mo., in October, 1931. During 1932, the correspondence concerning definitions, papers, and other Committee affairs was continued.

In January, 1933, the Chairman submitted to the members of the Committee (with copies to Chairman Noyes, Mr. Herrold, and Secretary Lewis) a tentative draft of the Summary and Conclusions, abstracted mainly from the papers by the members and the correspondence of the Committee. Discussion again followed until full accord was reached, as submitted.

---

[2] Cons. Engr., Coronado, Calif.

The Committee proposes the following definitions of terms used in its report in order that its meanings may be clear. The general adoption of these terms and definitions is recommended in the interests of clarity and brevity.

DEFINITIONS

Alley.—A street less than three traffic lanes in width, usually in the rear of building lots, and designed for local service only.

Avenue.—See *Road* or *Street.*

Boulevard.—An important street of extraordinary width (not less than 120 ft wide), with ample provisions for shade trees or ornamental planting. A boulevard is not intended for trucking but is readily available and attractive to pleasure traffic, and adjacent to property that may be peculiarly adapted to furnishing imposing settings for public buildings or for the highest type of private residences. In its design ample or generous provisions are made for pedestrians, for restful recreation, or contemplation, and for ornamentation.

By-Pass Road or Route.—A road or route providing for the passage of through traffic around, instead of through, a locality.

Freeway.—A highway to which there is no vehicular access from abutting properties.

Highway.—The entire right of way devoted to public travel, including the sidewalks and other public spaces.[8]

Arterial Highway: A highway for vehicular traffic that is a main channel, with many tributaries, and the roadway of which is required, by traffic frequency, to be not less than four traffic lanes wide.

Express Highway: A highway having an express roadway.

Express Roadway: A roadway for fast minimum-stop traffic, with separated opposite-direction free-traffic lanes, and without grade crossings.

Express Traffic: Non-stop, fast traffic with distantly separated origin and destination.

Expressway: A highway for express traffic exclusively.

Major Highway: A highway forming an essential part of a highway system for a region, such as a State, and the right of way of which is not less than 100 ft wide.

Secondary Highway: (1) A highway with right of way less than 100 ft wide, tributary to, or parallel with, and of less importance than, a major highway; (2) a highway with right of way less than 100 ft wide.

Super-Highway: A highway to accommodate mass passenger transportation on rails, in addition to all other highway functions, including local service to abutting properties.

Lane.—A narrow secondary highway. A strip of roadway passed over by successive vehicles traveling in the same direction, including in its width an excess on both sides for the clearance of passed vehicles.

Loading Lane: The strip of roadway reserved, or customarily used, for the loading of vehicles.

---

[8] *Transactions*, Am. Soc. C. E., Vol. LXXXII (1918), p. 1425.

Parking Lane: The strip of roadway reserved, or customarily used, for the parking of vehicles.

Standing Lane: The strip of roadway reserved, or customarily used, for standing vehicles.

Free Traffic Lane: A lane of traffic not used for loading, parking, or standing, nor containing street railway tracks, along which traffic may proceed in one direction freely and without interruption, except by traffic signals and the usual requirements of safety.

Parking.—The standing of vehicles, attended or not, for any purpose except the observation of traffic regulation, congestion, or signals.

Parkway.—A thoroughfare, with a right of way normally not less than 300 ft wide, planted with trees or other decorative growth, where vehicular access to abutting properties is precluded except by supplementary roadways connecting with that of the parkway at widely separated intervals; and where grade crossings with main intersecting highways are eliminated.

Plaisance.—A short parkway, extraordinarily ornamented by plantations, and where parking spaces for the enjoyment of the views are provided for pleasure vehicles. A narrow park on which houses face, the street of vehicular approach to the house being in the rear.

Planting Space.—The strip or area of the highway set aside for grass, trees, shrubs, flowers, statuary, and other ornamentation. (Sometimes, regrettably and confusingly, the planting space is still referred to as "Parking.")

Road.—A highway outside an urban district.[4] This term is sometimes used to designate a meandering street discordant with the general street plan.

Roadway.—That part of the highway particularly devoted to the use of vehicles.[4]

Street.—A highway in a urban district.[5]

Heavy Traffic Street: A street carrying more than the average traffic per square yard of roadway.

Major Residential Street: A street through a purely residential district, so located as to be a satisfactory distributor to other residential streets of the district; it serves the district as a traffic-way in contrast to serving merely the block unit; and usually requires a roadway width of not less than three traffic lanes.

Mercantile Street: A street devoted to local commerce.

Street Thoroughfare.—A street carrying through traffic from and to other than the abutting properties.

Traffic.—The movement of goods and persons on the highways.

Fast Traffic: Motor traffic proceeding continuously at speeds greater than 40 miles per hr.

Heavy Traffic: Traffic of more than the average per square yard of roadway, with consideration of both the number and the weights of the traffic unit.

---

[4] *Transactions*, Am. Soc. C. E., Vol. LXXXII (1918), p. 1426.

[5] *Loc. cit.*, p. 1428.

General Summary

There is a general recognition of the fundamental necessity for street thoroughfares. A fairly widespread agreement also is apparent that the ordinary development of such arteries of traffic has been, and is, insufficient even when no regard for the morrow is taken into consideration. It is also evident that, unless more frequent advance planning for street thoroughfares is established, the accidental development of ordinary streets into street thoroughfares will continue to the disadvantage of traffic and with the deterioration of the adjacent property.

Furthermore, it seems that most city planners and those others more or less actually informed on such matters quite well agree that the methods of providing street thoroughfares by simply widening the existing streets or by laying out new streets of extraordinary width, sometimes do not provide the desired results except temporarily. Even then such results as are thus had may be: (1) Extravagant, in that they require tremendous expenditures proportionately; (2) insufficient, to the extent that more width and additional traffic lanes do not properly move traffic; or (3) often destructive of values in the abutting property in that, if such property is residential, it may become less desirable, and if mercantile, it may be hampered by the increased width of the thoroughfare.

In some cases, mercantile property may even be cut off from direct access to the thoroughfare. The results of widening existing streets may be abandoned in despair later, when officials in charge recognize the impossibility of further development and that express highways on other locations will have to be provided to meet the needs for the actual movement of traffic.

While each case calls for its individual solution, certain principles seem to underlie all the problems. The effort of the Committee has been to discover these principles, to suggest them in this report, and, at least, to make a beginning toward the desirable end of a general recognition and the ultimate development of these fundamentals.

Underlying Principles

Some of the "Principles" unanimously agreed on by the Committee may be stated, as follows:

Necessity for Street Thoroughfares.—

1.—Street thoroughfares are needed for: (*a*) Mass movements of general traffic, with a minimum of congestion and a maximum of speed and safety for the entire mass, as exemplified by major highways in and adjacent to a business district; (*b*) certain extra speedy traffic, with a minimum of danger; for this type, express highways are indicated; (*c*) certain leisurely traffic, with a maximum of safety and of pleasure to that traffic as accommodated on boulevards and parkways; and (*d*) all the traffic likely to use the highway, including the local traffic as well as the through traffic, with a minimum of danger or congestion and a maximum of safety and convenience to all, as in the case of major highways, or super-highways.

Segregation of Opposing Traffic and Traffic Lanes.—

2.—To accomplish the ends mentioned under Principle 1, it may often be necessary to segregate the classes of traffic and to provide separate kinds of thoroughfares under different circumstances.

3.—The segregation of traffic may even be extended to the physical separation of vehicular traffic lanes (by a planting space) on a two-way thoroughfare; or by the establishment of one-way thoroughfares, and the abolition of grade crossings where cross-traffic cannot be cared for satisfactorily by rotary or similar intersections.

4.—Traffic-separating strips on expressways may effectually be as narrow as 4 ft in width in order that sufficient of the total width of the right of way may be devoted to the protection of the highway on both sides of the double roadway. Where, however, a sufficiently wide right of way will permit, a wider separating strip may be desirable; and, where vehicular passage from one roadway to the other is to be permitted, the separating strip should preferably be as much as 40 ft wide, in order that the turning radius for the vehicle may be practicable and that the left-turning vehicle may come to a stop out of the traffic before entering the opposite main traffic lane.

5.—For express highways carrying traffic in both directions, the opposing traffic should be physically separated (by planting or otherwise) and each roadway should be of not less than two, preferably three, and not more than four, traffic lanes.

6.—A maximum of three lanes for moving vehicles is usually best for a one-way traffic roadway.

7.—Such segregation and separate provision, if not a matter of present need, should be contemplated as a possibility in the future. If possible, this should be provided for through the early acquisition of the width of right of way ultimately desirable, and then by the progressive gradual changes that may prove feasible as they become necessary.

8.—The complete segregation of pedestrian traffic—and even of street railway traffic also—from the vehicular traffic of street thoroughfares may be warranted in some cases.

Parking and Side Roads.—

9.—No standing of vehicles should be contemplated or allowed contiguous to the lanes for moving vehicles along a street thoroughfare. Any parking, even momentary, for service to adjacent property or for rest or recreation, should be provided for in a service roadway or in parking spaces off the thoroughfare roadway itself.

10.—Ingress and egress for such service roadways or parking spaces should be provided so as to interfere in the minimum degree with traffic along the thoroughfare.

Location of Street Thoroughfares.—

11.—In considering the selection or location of street thoroughfares, recognition should be accorded the possibility of satisfactory low-grade routes parallel with stream channels. Such thoroughfares could be developed in connection with the preservation and landscaping of the drainage ways, and would assume parkway characteristics to some extent. Furthermore, the topography that suggests a location for a canal or flume between two points will indicate even more strongly a route for a heavy traffic highway.

There are many other advantages in utilizing for thoroughfares, land along a watercourse, and proper regional planning should provide a system of thoroughfares bordering on reserved natural drainage ways.

12.—The highest function of a street thoroughfare is to supplement other traffic routes and thus to provide for satisfactory transportation not otherwise obtainable. This function may not be the only one demanded of a street thoroughfare, but the optimum results will be had when that function is not subordinated to, or seriously interfered with, by others.

13.—All highway users enjoy pleasing surroundings and, other things being equal, will tend to congregate on the more pleasant thoroughfares.

14.—Inadequate traffic solutions result only in impasses such as are everywhere to be found in cities that now realize they have more to do than they can pay for to relieve their traffic situations. Careful consideration of the relationship between practicable street thoroughfares and other arteries of transportation, with the highest kind of trained pre-vision in such matters, is necessary to prevent a constant repetition of such inadequate solutions.

Cross-Traffic and By-Passes.—

15.—Where traffic across a freeway must be provided for at grade, a possible solution is that of "staggering" the inlets and exits of the cross-ways so that, in effect, the conflicting traffic streams may be merged between the entrance and departure of these cross-streams, on the principles of a rotary or "traffic circle" intersection.

16.—By-pass highways should retain at least the traffic capacities of the main routes of which they form a part.

Costs.—

17.—The division of right-of-way costs for street thoroughfares among abutting properties, local and foreign traffic interests, the community, and other beneficiaries is so intricate a matter and especially one so affected by the peculiarities of each case, that only one conclusion concerning this division can be attempted herein, namely, that this right-of-way cost should be considered separately from the construction costs in assessing benefits and damages, or in otherwise providing funds to meet it.

18.—The cost of constructing street thoroughfares should seldom if ever be met by the abutting properties alone, but should largely, if not entirely, be provided directly or indirectly from the other-than-local traffic interests.

The minimum fraction of this cost should be borne by the abutting property when the access of this property to the thoroughfare is most restricted. The maximum fraction to be assumed by the abutting property will be when side or "service" roadways supplement the freeway of the thoroughfare so as to enable the abutting property to share fully the benefits of the latter. In this case that maximum fraction will include the costs of these service roadways to the extent that the portion borne by others will cover the extra cost of the thoroughfare as a whole over and above that of a normal highway not a street thoroughfare.

Co-Ordination of Street Thoroughfares in the Traffic Plan.—

19.—Selection of the types of thoroughfares best fitted to serve the purposes of any particular community or area is largely affected by the topography, the character of the existing development, the existing street and road system within and contiguous to the developed area, the necessity of providing for future economies in construction of public transit facilities, and the protection of industrial as well as residential territory.

The proper inter-relation of all these factors pre-supposes a general plan covering the future metropolitan area, with legislation and resources adequate to insure its execution under continued administrative authority. Acquisition of space for future thoroughfares in advance of need, when space is attainable economically, is a primary necessity, achievable only under the authority of an official plan.

20.—Ordinarily, streets are for the use of all persons. Property on the street has the right of access; vehicles operating in the street have the right to come to the curb to pick up or to discharge passengers or goods. Vehicles using the street for purposes of public transportation, operating under a franchise, have the right to the unimpeded progress of their patrons from the sidewalk to their vehicles at points where stops are established. The pedestrian who may at one moment be crossing the street merely to get to the other side, and at another moment be boarding or leaving a public transit vehicle, has the right to complete protection in such movements. All these rights may be exercised freely, but only to the point where they interfere with the rights or safety of others. To insure that none of these rights may be exercised unduly and that the optimum welfare of all may be secured, the city may impose controls through its police power, or otherwise. It may regulate the size, weight, character, and speed of vehicular traffic; as well as the stopping of vehicles in the roadway. It may limit the movement of pedestrians or animals; and even, in the case of thoroughfares, for example, it may deny to the abutting properties access, except by special provisions of construction outside the thoroughfare roadways.

These facts have an important bearing on the design of street thoroughfares and on the distribution and co-ordination of such thoroughfares in a general city or regional plan.

Recommendation

As suggested by the foregoing principles, the Committee believes that a new type of street for automobiles is now needed in some cases, and will soon be needed generally. That type may be illustrated by the railway with its right of way closed to access from the sides, and with the important crossings, at least, not at grade.

Such a type of street would offer a solution of many of the problems now due to or connected with modern traffic, which are not solved satisfactorily by all other means yet tried. In many cases, existing laws or customs seem to prevent the development of an ordinary highway right of way into a freeway, and that development might be impossible in any case in which the power of eminent domain of the State was not involved.

Therefore, this Committee recommends that in a city, county, or other subdivision of a State, this matter of legalizing the creation and maintenance of freeways in addition to highways, be given serious consideration, and that a study be made to the end that city and regional planners and authorities may make proper provision, when desirable, for such thoroughfares.

By the Committee on Street Thoroughfares Manual,

W. W. CROSBY, *Chairman,*
JACOB L. CRANE, JR.,
JAY DOWNER,
J. P. HALLIHAN,
W. W. HORNER,
DANIEL L. TURNER,
GEORGE H. HERROLD, *Sponsor.*

RELATION OF STREET THOROUGHFARES TO OTHER ARTERIES OF TRANSPORTATION

BY JACOB L. CRANE, JR.,[6] M. AM. SOC. C. E.

The term, "artery of transportation," is graphic, but the analogy to the arteries in the human body is not quite accurate. Physiologically, an artery has the specific function of carrying the flow of blood and only that; it serves that function and only that function from the first day to the last; it does not compete with other elements of communication; and its use is automatically co-ordinated with all other organs, and the flow through it is automatically and consistently controlled. An artery of transportation, on the contrary, has none of these qualities, although it should have them, and if and when city planning finally succeeds, it will have them.

As it stands now, any one of several types of arteries of transportation may be called upon to carry the flow of any one of several types of movement; the facility which it provides is in demand by several competing elements of traffic; it is seldom satisfactorily controlled in its use and almost never properly co-ordinated with other phases of circulation and communication. As long as cities continue to grow and change in a way which it is impossible to forecast accurately in detail, the thoroughfares cannot be brought into completely satisfactory use and control and co-ordination. Meanwhile, a city's requirements must be anticipated as nearly as possible to develop the principles applicable generally and, in each case, specifically, and to improve, at least temporarily, the manner in which the traffic arteries are used, co-ordinated, and controlled. The writer is concerned with setting forth a few of the basic considerations which have to do with the inter-relation between street thoroughfares (as defined in the report of the Committee) and other principal arteries of transportation within and adjacent to urban areas. The other more important arteries of transportation may be listed as follows: (1) The roadways of secondary streets, not classed as thoroughfares; (2) the highways just outside the city; (3) footways; (4) railways: steam lines (main lines, suburban lines); electric lines (main lines, interurban, suburban, street, elevated, subways, freight, and passenger); (5) waterways: rivers, canals, ports; and (6) airways.

The mere statement of these various arteries of transportation implies a definitive classification of function. The roadways of secondary streets have, as their special purpose, the carrying of wheeled vehicles for relatively short distances between points of desired access to property alongside. The outer highways are intended to collect and carry outward, and to bring in and distribute, traffic to and from the city. Footways accommodate pedestrians, bicycle riders, and roller skaters for comparatively short distances in each case. Main steam railways are designed primarily to transport passengers and goods between cities; suburban lines to carry them from down-town to outer residential areas; elevated lines and subways serve primarily for the movement of persons radially and transversely through a town. Waterways may have one or several functions ranging from bulk cargo barges and passenger liners to the outboard motor-boat. Thus far, airways are used almost entirely for long-distance inter-city passenger, mail, and express carriage. Some day, perhaps soon, a city plan will classify the types of movement and the arteries to carry them in even greater detail, and city planners will undertake to segregate and maintain each type of artery and each single artery for one, or at most, a few purposes.

---

[6] Cons. Engr., Chicago, Ill.

Now, the inter-relation between street thoroughfares and these other arteries of transportation is extremely intricate. The footway, while customarily in the same strip of land with a roadway, may have a less direct relationship to that roadway than any of the other elements of transportation. As long as the footways lead to points at which the pedestrian is picked up by automobile, street car, train, or otherwise, there is no good reason why the footway needs to follow the street; in fact, there are several good reasons, safety and agreeableness being chief among them, why footways should be located quite apart from the streets.

Conversely, the roadways of secondary streets and of the outer highways must be intimately interconnected with the street thoroughfares so that traffic can flow from one to the other safely and conveniently.

While a steam railroad right of way sometimes offers a convenient location for a parallel street thoroughfare alongside, the real relationship arises out of the requirement that the thoroughfares make it possible for wheeled traffic to reach the loading points and particularly the main terminals of the railroads. The same may be said of the relationship between transit lines and street thoroughfares. On this point, however, it becomes more and more evident that no street thoroughfare (and preferably no street) will have street-car tracks, or elevated railways, or even subways, within the same right-of-way lines with the roadway. The best arrangement, and the one which may eventually be set up as a definite objective, is that in which the transit lines have rights of way altogether separate from the streets. In most cases, this would make possible the open-cut depressed transit track, bridged at street crossings and accessible from the streets at all loading points. Plainly, the practice of combining into one right of way, a street roadway, sidewalks, a surface-car line, an elevated line, and a subway, enormously decreases the efficiency of each one—least seriously, perhaps, with the subway, and most seriously with the conflict between street cars and wheeled vehicles on the pavement. The logical inter-relationship is one of providing convenience of interchange at loading points, and only that.

In relation to waterways and airways, the street thoroughfares should give direct, convenient, and fast lines of flow to the points of interchange—the passenger docks, the commercial wharves, the yachting centers, and the airports. How far they fall short of this ideal!

It is seen, then, that there are, in the main, only two desirable relationships between street thoroughfares and other arteries of transportation, namely (1) that of the flow between street thoroughfares and other streets in such a way as to cause as little interruption and hazard as possible; and (2) the relationship at points of transfer from wheeled vehicles using street thoroughfares to the car, plane, or ship using other arteries of transportation. This is simple to state, but extremely complicated in application.

Furthermore, even when a planning genius succeeds in determining a sound and appropriate pattern for all the arteries of transportation in proper relation to each other, he would still have equally difficult problems to deal with: (*a*) In the relationship between these arteries and the changing and increasingly intensive use of private property; (*b*) in the matter of creating designs satisfactory from the architectural and landscape standpoint; and (*c*) in the baffling matter of public financing.

In the planning of street thoroughfares, as in every other phase of city planning, both the big task and the big opportunity lie in devising means to bring good city building within range of the capacity to pay—within the range of financing limitations. The impassé in which the American city now finds itself (the impassé of "more to do than it can pay for"), is the result not so much of poor administration and graft, but rather of poor planning, with the necessary sequence of tremendously high costs for tearing down, rebuilding, opening up and extending the city's physical services. It seems clear enough that the "checkmated" American city can work its way out only by the most skilful, long-term planning and "programming" of its construction and reconstruction. In this project the underlying and, perhaps, the most important element is that dealing with the classification, inter-relation, and co-ordination of the arteries of transportation.

The basic principle of design for arteries of transportation is implicit in the preceding statements. Each such artery should be designed to carry specific classes of traffic to and from specific points or areas, the critical limitation on the design being the matter of cost. The several classes of traffic are passenger cars, trucks, drays, street cars, elevated cars, sub-surface cars, and pedestrians. In China, the rikshaw is an extremely important factor in street design; and, in Russia, the main boulevards must make provision for the "grand procession."

The problem of design breaks up into several elements—location, terminals, width, cross-section, grade, intersection, type and weight of surfacing, drainage, lighting, access to abutting property, and underground services. The cross-sectional design requires that the total width be built up from the items required, and there is an almost infinite variety of designs to meet special conditions. One good type is: Four 10-ft middle lanes for through traffic, two 10-ft lanes for local traffic, two 8-ft lanes against the curb for loading and unloading, and, say, a 16-ft space on each side for sidewalks, or for sidewalks and planting strip—a total of 108 ft for right-of-way width. Such a street might be classed as a street thoroughfare, but if the local traffic is heavy, it would not operate well for through traffic. Then it would require re-study for location, or for greater width, or perhaps to separate the through lanes from the local lanes by islands, elevations, or other means.

One important consideration in design emerges from the experience of recent years, namely, that the points at which local traffic enters the thoroughfare must be infrequent, even as far apart as $\frac{1}{2}$ mile.

The principles are becoming more clear; the physical and fiscal planning is still mainly in the future.

DESIGN OF BOULEVARDS AND PARKWAYS

By Jay Downer,[7] M. Am. Soc. C. E.

The design of future boulevards and parkways in the light of modern traffic conditions and the accumulated experience that is a matter of common knowledge to the Engineering Profession would appear to require added consideration to the following elements:

1.—Relationship of privately owned property to the vehicular roadway or roadways of the boulevard or parkway, with reference to the extent and proximity of linear contact.

2.—Frequency of, and the arrangement of connections between, the boulevard and parkways and the secondary or minor thoroughfares.

Boulevards long antedate parkways in origin. Behind the boulevard lies some centuries of what might be termed an evolutionary process of adaptation, modification, and improvement. Its primitive beginnings were the utilization as a public thoroughfare, of the sites of bulwarks, fortifications, or walls surrounding a town. Incidentally, the encircling boulevard in many cases happened to be a scenic highway.

By extension and analogy the boulevard became a broad, tree-planted internal thoroughfare, either as the main axis or as radials of a city plan. It provides main traffic channels and imposing settings for public buildings, grand avenues for residential uses, or for commercial and amusement enterprises, as well as its characteristically European use for the leisurely enjoyment of life in the open air during a considerable part of the year. All these purposes will be served by the boulevards of future unbuilt cities or expansions of existing cities. In width, the boulevard may range from 100 to 200 ft of public right of way utilized in varying combinations of multiple roadways separated or flanked with grass and tree-planted strips. Many examples and typical arrangements are available for study as to detailed planning.

Further evolution or modifications in the design of future boulevards would seem to indicate a need for a greater degree of segregation of the use contacts of adjacent property from the principal traffic stream and a reduction in the number of connections with lesser thoroughfares.

Parkways are of much more recent origin and were evolved from a different concept. The name itself is suggestive of what was probably the earliest conception of a parkway as a park-like thoroughfare connecting an outlying park with the down-town commercial, industrial, or built-up center of a city. Expanding combinations of this idea may cover a connecting way having park attributes and joining a park with the congested city center, or two parks with each other. More recently, it has been applied to joining many more park areas or reservations into a unified system. At the same time such an arrangement provides a traffic circulation system regardless of the primary purpose of connecting parks, and has brought parkways into the category of greatly extended arterial thoroughfares.

---

[7] Chf. Engr., Westchester County Park Comm., White Plains, N. Y.

The modern parkway reservation or right of way may be, and is—in the environs of New York City, Boston, Mass., and Philadelphia, Pa.—any width from 200 to 1 200 ft. On the basis of a 40-ft paved roadway with flanking strips for seeding, planting, landscape treatment, or simply the preservation of original native beauty, the desirable minimum width is 300 ft. Intermediate widths from 400 to 800 ft may be dictated by varying topography and especially the inclusion of a stream and its immediate marginal lands. Widths up to 1 200 ft for a parkway are possible through sections of low-priced lands, such as swamps or rocky ridges.

Parkways are planned and designed on the principle of absolute exclusion of privately owned property from direct contact with the paved roadway, complete separation at all main intersecting thoroughfares of the roadway levels and their main traffic streams, and the minimum number of entrance and exit connections consistent with serving the requirements of the locality traversed. A desirable minimum spacing might be placed at ½ mile and range up to 2 miles.

Although originally they were created largely from esthetic motives, parkways have emerged as the most effective solution of the modern arterial road problem on the following purely economic grounds:

(*a*) They are efficient for passenger traffic movement because (1) they minimize interference with the traffic stream by eliminating roadside parking; (2) entrance and exit connections occur infrequently; and (3) grade crossings are generally eliminated.

(*b*) This effect on adjacent land is invariably to enhance values in contrast with a depression of values which may occur where arterial road frontage in direct contact with the pavement cannot be absorbed by business or industrial uses.

As to terminology, the Committee on the Regional Plan of New York and Its Environs, after considerable study of both legal and physical aspects, adopted the term, "boulevard," as denoting a highway that is furnished with trees, grass, or other park features, and the term, "parkway," to denote a way through, and within, a park.

These definitions are derived from the premise that, in law, there are only two classifications, namely, highways and parks. These terms have the advantage of brevity and of meeting rigid legal requirements. "Park," in a legal sense, is inclusive not only of the common conception of a park, but of a linear extension or ribbon into a parkway, even if it provides no other form of recreation than pleasure driving and enjoyment of landscape.

Although it might seem desirable to amplify the foregoing terminology as to "boulevard," the difficulty is that descriptive amplification quickly exceeds the limits of terse definition.

SUPER-HIGHWAYS

BY J. P. HALLIHAN,[8] M. AM. SOC. C. E.

As far as is known from the records, the term, "super-highway," was coined by Daniel L. Turner, M. Am. Soc. C. E., to describe a single-level, all-purpose highway designed to provide an exclusive right of way for rail transit in combination with superior facilities for movement of heavy-volume motor traffic created in metropolitan areas.

The design was applied by Mr. Turner as the keynote of his plan for a co-ordinated thoroughfare system within the developed section of Detroit, Mich., and its expansion into a regional plan for the Metropolitan Area, later adopted as the Master Plan for Detroit and the three counties affected.

The purpose of the design may be best described by a brief condensation from Mr. Turner's report as Consulting Engineer to the Rapid Transit Commission, dated April 10, 1924:

> The specifications provide for: (1) A central reservation of 84 ft for the exclusive use of rapid transit lines on rails and the tree and shrub planting to screen it; (2) a 20-ft roadway for express motor traffic on each side of the central space; (3) a 5-ft planting or safety zone strip, on the outside of the express roadway, separating express from local roadways, but with openings at intervals for transfer from local to express where the roadways are at the same level between stations; (4) local roadways, 20 ft wide; and (5) sidewalks, 15 ft wide, making a total of 204 ft.
>
> After acquisition of the right of way, the execution of the design is intended to be coincident with the local requirements of the area and, therefore, will proceed gradually as building development progresses and traffic needs increase. At first, all roadways will be at the same level and cross-street traffic will cross the rail reservation at all street intersections. The express roadways will be provided when the central reservation is occupied by rapid transit rails and the right of way is made exclusive by separation of grades at half-mile intervals at stations, the rails and express roadways going over the half depressed cross-street and the local roadways meeting the cross-street at grade. Cross-traffic coming in between stations may then turn into the local roadway, but may cross the express roadway and rail reservation only at the under-pass. Traffic from local roadways may enter the express roadway at the barrier opening provided where local and express roadways are at the same level. Grade separations may be built for express roadways in advance of rail occupancy of the central reservation if traffic needs require, under the same limitations as to crossing.

From a traffic standpoint, the super-highway design is seen to be a provision for insurance of ample roadway space for motor traffic under conditions permitting high speed with safety. It separates directional traffic, as well as through and local traffic, and permits greater freedom of movement by reducing traffic interferences. From a transit standpoint it is seen to provide for future economies in rail transportation by making it possible to construct the permanent way on the surface at one-fifth the cost of an underground structure and at one-half the cost of a wholly elevated structure. The noise of passing trains is reduced to the minimum by the width of street and the tree screens.

---

[8] Chf. Engr., Rapid Transit Comm., Detroit, Mich.

A utility highway of this nature is peculiarly adapted to the metropolitan areas surrounding great cities, where traffic volume assumes great proportions and where mass transportation facilities must eventually provide for a density beyond the capacity of a street car and bus system. Beyond those areas the safety provisions of separation of directional traffic and permissible elimination of left turns at intersections may be maintained by a boulevard design with a minimum of 30 ft for the central parkway, and with side parkways separating distance traffic from local traffic.

There is no economic reason why the all-purpose roads of the future should not provide for esthetic treatment by landscaping as well as for the utility needs, provided an adequate right of way is secured upon initial location, before it acquires a prohibitive value created by the facilities furnished.

THOROUGHFARE LAYOUTS IN RELATION TO DRAINAGE SYSTEMS

By W. W. Horner,[9] M. Am. Soc. C. E.

For the purpose of this paper, the "thoroughfare" is the general designation of a traffic-way of importance and will include streets or roads generally called "boulevards," "arterial highways," "major or secondary highways," and "parkways." The "drainage system" of the region comprises all facilities for the removal of storm water, ground-water, spring flow, and other excess liquids, exclusive of sewage and contaminated waste waters. These facilities may be the natural watercourses, such as creeks, brooks, dry runs, or ravines, or may be artificial drains, either open or covered.

The construction of covered masonry storm drains of large size, as is now common in large cities, is:

(1) The result of failure to develop and organize a proper system for the improvement and protection of natural watercourses. This failure is largely due to improper platting, which either disregards watercourses or leaves the valleys as waste land, or places the watercourses at the rear of lots, where they readily become rubbish dumps.

(2) Placing a high charge on platted property for which there is no adequate return and which is accordingly detrimental to simple and uniform development and tends to produce high density of population.

(3) Destructive of the greatest asset for a residential setting, an attractive natural scenery of a semi-pastoral character.

The alternative drainage policy involves the preservation of watercourses enlarged or supplemented to become adequate for the increased run-off from urban areas and the protection and preservation of the natural conditions. This involves;

(*a*) The setting aside of considerable areas of lowlands for permanent use as drainage ways, but with possibilities of considerable value also for recreational purposes;

(*b*) The subdivision of land so that drainage ways will be conspicuously evident, in order that their scenic value will be best appreciated and that they may not be readily restricted by filling or surreptitiously used for the disposal of rubbish; and

(*c*) A consequent requirement for lot frontage along the drainage ways which would then involve the platting of marginal streets along such ways. The valleys being generally easy gradient routes, such streets would naturally be thoroughfares in the direction of the valley trend.

GENERAL

Proper original planning should provide a system of thoroughfares bordering reserved natural drainage ways to which would be joined a complementary system crossing the valleys at the most advantageous points. This system has been followed for residential areas and has resulted in such major residential streets as those of the Turtle Creek Valley, in Dallas, Tex.; such boulevards as those of Kansas City, Mo.; and such parkway developments as in Westchester County, New York State. Common habit to the contrary, the system can be used also in commercial areas if people are willing to organize adequate protection and policing. The result would be to the ultimate benefit of the community from every angle. There is a popular reaction against the present ugly, uniform type of urban development which would welcome a change in this direction. Obviously, the scheme is not generally applicable to thoroughfare planning of a corrective character, in old settled communities, and it has no economic advantage where money has already been spent for large storm sewers.

---

[9] Cons. Engr., St. Louis, Mo.

EXPRESS HIGHWAYS AND BY-PASS ROUTES

By Daniel L. Turner,[10] M. Am. Soc. C. E.

Express highways and by-pass routes are recent evolutions of the motor-vehicle age. There was no necessity for them in a man-and-horse civilization. By-passes have been in use for a considerable time, but express highways have developed very slowly. The latter came into actual use only a short time ago. This, too, despite the fact that H. G. Wells anticipated such special types of roads for motor-vehicle use as early as 1901, and at the very beginning of the motor era, when he described the probable "Locomotion in the Twentieth Century," as follows:

> "The * * * conceivable ideal of locomotive convenience, * * *, is surely a highly mobile conveyance capable of travelling easily and swiftly to any desired point, traversing, at a reasonable controlled pace, the ordinary roads and streets, and having access for higher rates of speed and long-distance travelling to specialized ways restricted to swift traffic * * *. They (the specialized ways) will have to be very wide,— * * * just as wide as the courage of their promoters goes—and if the first are too narrow, there will be no question of gauge to limit the later ones. Their traffic in opposite directions will probably be strictly separated * * * and streams of traffic will not cross at a level * * *."

At another point, describing the use of the privately owned motor carriage on such special ways, he stated:

> "This, for all except the longest journeys, will add a fine sense of personal independence. * * * It will be capable of a day's journey of three hundred miles or more, * * *. One will be free to dine where one chooses, hurry when one chooses, travel asleep or awake, stop and pick flowers, * * * be free to travel up to the limit of their very highest possible speed."

He thought such segregation of motor traffic might possibly begin even in the decade from 1900 to 1910. People are just now (1934) beginning to think about it seriously. The foregoing is a remarkable vision of the conditions of to-day at a time when the tremendous growth of automobile use scarcely seemed conceivable.

The express feature of a highway applies only to the roadway as it is defined in the definitions by the Committee on Street Thoroughfares. Therefore, an express highway may be defined as a highway having an express roadway. On the same basis, there may be express boulevards, express parkways, express streets, express roads, etc. In fact, every type of highway would become express if provided with an express roadway.

An express roadway may be defined as a roadway constructed with separated opposite-direction lanes and without crossings at grade, so as to permit fast minimum-stop traffic.

The ideal express roadway should not be directly accessible to abutting property; and access to it from the local lanes of the highway of which it may be a part should be infrequent. Similarly, it should be accessible only infrequently to other highways. There should be lanes for trucks and buses as well as passenger cars. Grades should be low. Curvature should be reduced to the minimum. There should be at least two traffic lanes in each direction and, preferably, three.

---

[10] Cons. Engr., New York, N. Y.

Practical considerations will make necessary many departures from the ideal. Other types of highways have been, and will be, proposed to secure to as great a degree as possible the advantages of the ideal express roadway—without resort to the high construction costs and to such complete differentiation as the ideal requires.

Instead of including the express roadway as a feature of the highway it may be constructed as an entirely separate highway without any local traffic lanes, but designed for express traffic exclusively. In order to distinguish it from an express road or express street which provided for both local and express traffic, such a highway might be called an expressway; that is, defined as a highway for express traffic exclusively. This would be the ideal method for express service.

Along such expressways, at signal-controlled ingress and egress points, all the necessary service facilities should be segregated, restaurants, inns, or hotels, comfort and rest houses, oil and gas, repairs, emergency trucks, etc. These service stations should be as far apart as practicable, consistent with the development of the country side. Along the expressway, however, emergency pull-out spaces, accessible only from the roadway, should be provided at convenient distances apart and these should be equipped with telephone to the service stations. Here travelers could change tires and make minor repairs without resort to the station service, and without obstructing the traffic lanes.

Expressways will not be as practicable of construction in urban districts as in the open country. An elevated or depressed roadway in a city makes an ordinary street into an express street, it is not an expressway in the sense intended herein.

The most important city streets should be those planned for the accommodation of mass passenger transportation facilities on rails, as well as on rubber tires because, after all, probably three-fourths of the people travel along such thoroughfares. In fact, such streets, which may be termed super-highways, should be the base plan for any city street system. After they are planned to circulate and distribute the population in the best possible manner, all other streets, which are secondary, should fill in and complete the circulating system. Failure to recognize the necessity of providing ways for these mass travelers is one of the greatest oversights of which city planners have been guilty in the past. The present congested condition of cities is largely due to this oversight.

A super-highway may be defined as a highway designed to accommodate mass passenger transportation facilities on rails and rubber tires, in addition to all other street uses.

Since a by-pass route is only a connecting link around a locality between highways into, and out of, the locality, it should conform in all particulars to the type of the highways it is to connect permanently; that is, the by-pass should be of the expressway type if eventually it is to form a part of the expressway or, if it is to shunt an ordinary road around congestion, it should be an ordinary road.

PAYING FOR STREET THOROUGHFARES

BY GEORGE H. HERROLD,[11] M. AM. SOC. C. E.

HISTORICAL

In 1785 the Continental Congress established the township system originated in New England, and made it applicable to all Government-owned land. The State then adopted or created the section-line highway of 2, 3, or 4 rods in width, to make the lands accessible and to permit their development. This was practically the highway system of the United States—a rectangular system. The easement to the road being by prescription, the abutting owners, in turn, had access to the road at all points of contact for trade and barter of the products of the land. Originally, the township paid for the construction of the roads on the section and quarter-section lines; then, some one in New Jersey applied the "scientific method." By observation on a certain stretch of road, he found not only local township wagons, but wagons from adjoining townships and from townships far beyond. He reached the conclusion and was able to convince the Legislature that the State should contribute. The first legislative act was to require that the abutting property owner pay one-tenth of the cost, the State one-third, and the county the remainder. Thus began, in principle, the taxing of urban units for the development of country roads.

Toll Roads.—The first attempt at creating thoroughfares of communication was the establishment of toll roads in the Eighteenth Century. The first cost and maintenance was borne by chartered turnpike corporations, and the users paid a toll to reimburse the owners. These companies had the right of eminent domain and could appropriate existing highways when necessary. The turnpikes were public highways, in every respect, and were free to all except that the legal toll must be paid as a condition of use.

One of the great toll roads of history was constructed by the United States Government. The Cumberland Road, or National Turnpike, from Fort Cumberland (Baltimore, Md.), across Pennsylvania, Ohio, and Indiana, to Vandalia, Ill., about 800 miles, was a Government project. It was begun in 1806 and finished in 1844 under a Congressional Act of 1796. It was financed by the United States Treasury which was supposed to be reimbursed from a road fund. This road fund was created by setting aside one-tenth of the proceeds of the sale of Government lands in Ohio and possibly other States. The cost of the pike was $7 000 000, and it carried an annual traffic of 3 000 wagons.

Government aid for road building was stopped about 1840. It was then advocated that the States should build their own roads and the Government confine its financial aid to territorial and military roads.

Steam Roads.—Coincident with the development of the toll road, but beginning later, was the wonderful and rapid promotion of the steam railroad. By 1840, it had demonstrated its ability to compete with toll roads. It operated over a private right of way and its destination was a succession of terminals. Owners of abutting property had no right of access to its right of way except at terminals. Encroachment on the right of way or trespass was punishable by law. Land was often given for terminals by abutting owners who recouped by enhancement in value of their remaining land; or the railroad companies bought the land at a proposed terminal and reaped the benefit of this increase in value—a benefit brought to it by the railroad which made it accessible to other terminals.

---

[11] Member, State Planning Board; Managing Director and Engr., The City Planning Board, St. Paul, Minn.

Railroads were financed by private capital, land grants, gifts, and Government loans. Shippers paid the entire cost of moving their products or commodities over the railroads.

Trunk Highways.—The present-day road-building activities began with Federal aid. For twelve years, appropriation bills were written and killed prior to 1916, when the Act was successful. It provided that the Federal Government should extend aid to the States for the construction of rural post roads and for other purposes. These enormous expenditures ($125 000 000 per yr in 1931, 1932 and 1933, of Government aid matched by an equal amount from the States) made possible the great network of interconnecting trunk highways over the United States.

State funds are produced from gasoline tax, motor-vehicle license tax, wheelage tax, mileage tax, and bonds.

Urban Streets.—In urban centers the principal use of land is for streets, parks, and building sites. The street system is usually created by dedication in a town-site plat. Many cities were started during a great speculative era when more thought was given to land as a commodity and the profits to be made from it than the development of a street system for a city.

Widths of streets were determined by the vision, generosity, or cupidity of the owner or developer. In fact, so little was known about street widths that habit probably determined many of them, and the 60-ft street became fairly common. It was considered a suitable boundary for lots.

In some notable instances, thoroughfares of ample width were laid out to serve the street system of the business center, but more often no thoroughfares were contemplated. For speculative reasons certain uses were established on these streets. Some of these uses could have been better served on wider streets while others could be served on narrower ones.

From 1870 to 1920 city populations 'worried along" with this inadequate and poorly designed street system, and improper location of structures and uses.

The introduction of the motor vehicle has compelled a larger floor area for transportation uses. Thoroughfares have had to be created to serve centers and minor street systems. There has been much widening of mercantile streets and connections and construction of parkways for passenger cars, arterial streets, express highways or super-highways—all-purpose thoroughfares and cul-de-sac street systems in residential districts. Where not created in new land these have been obtained at an enormous cost.

Financing Public Improvements

The procedure for financing public improvements varies from place to place and from State to State; but, speaking generally, when a group of citizens want their street graded, widened, or paved, or when they want a sewer put in or a sidewalk built, they enter into a co-operative agreement with the municipality, under which the latter does the work and those who benefit by the construction pay for it. The State gives the right to a governmental unit to do this under quite elaborate legal protective restrictions.

The necessity of a municipality handling these co-operative individual investments in local improvements can be explained by one simple example. If a group of citizens wishes to construct a sidewalk the following are some of the questions that would arise. To what grade should the walk be laid? (Each owner will want the walk as near to the level of his property as possible.) Shall the walk be 4 ft or 6 ft, or some other width? Shall it be built of concrete, brick, or cement tile? Shall it be a cinder or board walk? Shall it be laid along the property line or in the middle of the reservation, or along the curb? Is a walk necessary at all? Who would underwrite the construction cost in order that a contract might be let? How would collection of each one's portion of cost be made?

It is not possible for a group of people to arrive at an agreement on these things, due to difference in culture, experience, knowledge, notions, and financial ability to pay, and if each laid his own walk a construction Babel would be the result. The municipality, therefore, carries out the work according to its engineering standards; pays the contractor from moneys which it has arranged beforehand to have on hand through a tax levy or bond issue or certificates of indebtedness, and collects back from the citizens the cost of the work, justly apportioned to each according to benefits, in installments with interest. If these costs are not paid they become a lien on the property and are added to the taxes, and when the taxes are paid the assessment must be paid at the same time, or the property is sold.

In many States a municipality also has the power to order such improvements as a public convenience and necessity and to collect the cost from owners benefited.

The three great powers of Government are: The power of eminent domain; the power of levying taxes; and the power of collecting the construction cost for local improvement work from property owners as a benefit assessment. No greater power exists. Ownership of land infers simply the right of possession contingent on payment of taxes and assessments; otherwise, possession passes to others.

In street thoroughfare widenings, the municipality takes the property required under its powers of eminent domain, orders buildings moved or cut off, resets the curb, widens the pavement, constructs sidewalks, changes grades, moves light standards, fixtures, etc., all under its power of investing the citizen's money in street improvements to enchance his business and social opportunities. It pays the owner for the land and building taken, the cost of new front remodeling, consequential damages due to the construction work, etc., and it pays the contractor. It then proceeds to collect the cost from citizens that are benefited. This may be done by a special assessment or through a tax levy, or both.

If it can be shown that certain property receives a benefit or peculiar advantage by construction work authorized by the municipality, that property may be assessed for the cost. It is the property owner's investment in an improvement. If the benefit or advantage is to the entire city, every piece of property in the city may be assessed. This is probably never done, as it is simpler to spread the cost on the tax roll and collect it with the real estate tax.

If the benefit or advantage is greatest to abutting property on a street, but also extends to a district or to the entire city, but in a lesser degree, a graduated assessment may be spread over the street or district, and the remainder may be collected as a tax on real estate, remembering that the ones assessed must also pay the tax, because a real estate tax covers every parcel. If the benefit or advantage is entirely to the street the abutting property on that street may be assessed for all the cost.

A benefit assessment is a bill for construction work done by the municipality for a group of property owners to enhance the social, economic, and physical usefulness of their property. To insure uniformity of construction, compliance with engineering specifications, and equitable distribution of the cost, the municipality makes the investment for the property owners and collects the bill.

To determine the value of thoroughfare improvement is at best a matter of forming an approximate estimate.

In Providence, R. I., all paving is paid for out of a street-paving fund set up in the budget each year on the theory that all citizens were benefited by paving. Cities and towns in Virginia are constitutionally prevented from levying special assessments for local improvements, except that cities of more than 500 inhabitants per sq mile may assess abutting property owners for sidewalks in existing streets, for improving and paving existing alleys, for constructing sewers, or for the use of sewers.

In St. Paul, Minn., it is assumed that 24 ft of paved roadway is sufficient to serve abutting property and that if a greater width is necessary it is for the benefit of the city at large. It is a charter provision that 24 ft, or less, of pavement may be the investment of the abutting property and that such an investment cannot be required of the owner oftener than once in fifteen years. The remainder of the cost is the city's investment and amounts to from 44 to 58% of the total cost. The amount to be assessed for a sewer is decided for each particular case; there is no limitation, the amount assessed depending on the value of the property and the benefits. On the other hand, water mains are assessed at $1.00 per front ft, payable in ten yearly installments.

In Minneapolis, Minn., one-third the cost of paving is paid for by abutting property, one-third from the current paving fund, and one-third from bonds running for thirty years. Abutting property owners may be assessed $2.50, or less, per front ft for a sewer, the remainder being paid from a bond issue or by general taxation.

These are arbitrary rules laid down for convenience and to prevent difficulties that would result from more refined methods. Sometimes they are inconsistent, but they work. The methods used in each city are usually considered, if not absolutely equitable, at least the best they can "get away with."

In street widening, or openings, the apportionment of cost is not so simple—land must be acquired and owners want money for land. No simple rules have yet been devised such as are used for other public improvements. The owner demands all he can get in damages and escapes all he can in the award of benefits. Land values are determined by appraisers; the building damages are determined by engineers, architects, or contractors.

The building damages can be determined accurately and beyond serious dispute, but the determination of land values is difficult. Expert testimony may develop differences that cannot be reconciled. Such expert testimony reflects the psychological attitude of the individual. It may be based on facts, or it may reflect an intoxication of optimism and imagination. The municipality must accept a reconciliation of these mental attitudes as representing the actual value and may proceed to pay the cost and to apportion the benefits to the property which, in its judgment, has received an advantage or benefit.

The apportionment of the amounts of assessment in each case is the most intricate problem confronting a city.

In 1913, the City of St. Paul widened Robert Street, a mercantile street. After considering various ways of financing the project, the majority of abutting owners on the street agreed with the City that they would pay the cost. The City instituted condemnation proceedings acquiring 20 ft on the west side from Second Street to Central Avenue, twelve blocks, of which six blocks were in the heart of the retail district. The street was widened from 55 to 75 ft. Theoretically, the owners on the east side of the street who were undisturbed paid one-half the value of the land and buildings taken on the west side. The owners of the property on the west side, which was cut off from the widening, accepted one-half the value of what they gave up. The entire street was repaved, and both sides paid. The cost was $1 000 000, or at the rate of $250 360 per mile.

In 1923, the widening of Robert Street was extended north 1 500 ft., connecting with University Avenue by cutting through a hill and separating the grade with one of the cross-streets (Cedar Street). This made a continuous thoroughfare from the University of Minnesota, in Minneapolis, to the heart of the business district of St. Paul, a distance of six miles. This extension cost $260 000, and it was assessed largely over that part of Robert Street originally assessed. It was regarded as a completion of that project.

In 1927, the new Robert Street Bridge with a 56-ft roadway was built over the Mississippi River, at a cost of $1 800 000, and through a period of three years other widenings were completed, making Robert Street a continuous thoroughfare to South St. Paul, three miles, and there connecting with a State trunk highway.

The value of property on Robert Street was never enhanced to equal the assessment of $1 260 000, although it was approaching it in 1928 when the value curve turned downward. The street was widened to preserve its business character, to make the business houses more accessible, and to prevent mutation. It was the desire of the owners in 1913 to make it an arterial street and this was accomplished.

On Robert Street, at Seventh, are two large department stores, each of which do an annual dollar volume business exceeding that of any other retail store in the Northwest. There are also two large clothing houses, banks, hotels, office building, brokerage houses, restaurant, and shops. The widening of the street started values upward. The increase of through motor travel pulled values downward.

Seventh Street, in St. Paul, runs at right angles to Robert Street. It is the leading retail street and on it may be found the highest land values in the retail district. It is a 60-ft street with a 40-ft roadway, and 10-ft sidewalks; and it accommodates a double-track car line. The Planning Board recommended that the street be arcaded for five blocks, placing the sidewalks within the building and widening the roadway to 56 ft. During the discussion, property owners asked for an estimate of cost for a physical widening, making an 86-ft street with a 56-ft roadway, and two 15-ft sidewalks. The estimated cost was approximately $6 500 000, but an analysis of these figures disclosed that if the property owners would give the land and each pay his actual reconstruction cost; that is, tearing down 13 ft of his building, constructing a new front, re-arranging the interior, paving an additional 8 ft, and building a 15-ft sidewalk, the actual cash outlay on the part of the property owners would have totaled approximately $1 000 000. This difference is the so-called value of the land and buildings taken. Now, if the City had proceeded with condemnation proceedings and allowed damages amounting to $6 500 000, and then declared benefits of $5 500 000, the same results would have been obtained, but the property owners and every one else knows that such an award of benefits would be attacked, although it is equitable and just, and the property owners would stand a good chance of winning their case in Court, because of the attitude of the Courts toward property values.

Where street widenings are necessary, it is because the original dedication of width was not sufficient. The original dedication for a street was made from the land, the owner absorbing his loss of area and benefiting by accessibility to all parts of the remaining portion. Where a street is widened by the power of eminent domain, it is to correct the error of the original dedication.

The L'Enfant Plan of Washington, D. C., provided 90, 110, and 160-ft streets. These wide streets were no burden to the city or to the property owners, as only that part was improved that was needed for traffic—the abutting owners using the excess width of street as yards. When it is necessary to widen a street in Washington, no condemnation proceedings are necessary, the roadway is widened by setting back curbs and sidewalks.

Users Should Pay

The widening of a street can only be supported by arguments based on surveys and a careful analysis of the survey data. Physical units are not the only ones to be counted. There is a social aspect, an economic condition, and a psychological viewpoint. Furthermore, if there is a chance of decreasing the cost of distribution, the advantage accrues to the owner, to the traffic, and to the customer.

In a city the character of use determines the width of the street and the width of the roadway, the latter being based on the almost uniform laws establishing vehicle widths. The following road width (in feet), will be considered standard:

For residence districts, minor streets	27
For residence districts, major streets	36
For approaches to residence districts, or arterial streets	36
For mercantile businesses, or for multiple-family homes	56

Arguments for these roadways may be found in all literature on the subject. They are the required roadway widths for such uses of property. The width of the street is the width of the roadway required, plus the width of the sidewalk required, plus planting and embellishment strips desired. The owners of abutting property should pay the entire cost of the right of way or street width in these cases.

The paving or surfacing of these roadways and the question of who shall pay the cost must be determined locally, depending on values, on use, and on general custom. There is always some through traffic on all streets in a varying volume. If owners of motor vehicles pay a personal property tax, a gas tax, and a wheelage tax (or some combination of the three), these moneys should be set aside for paving or surfacing purposes.

For arterial streets, express highways, super-highways, and trucking ways, consideration must be given to acquiring the land for a right of way or for a widened right of way through general taxation of, bonds of, and special assessments of, districts or terminals. It would be unusual for the owners of abutting lands to profit; the property may even be depreciated by these thoroughfares unless a buffer right of way is provided, broad enough to protect the abutting owner from the noise, heat, and fumes of traffic.

The right of way for a parkway should be wide not only for landscape purposes, but (more important) for protecting the near-by property owner from the undersirable features of traffic; and, where there is no access of abutting property, there should be no assessment except where the property may be a part of the assessment district. If service drives for abutting property are constructed on the parkway lands, the value of this part of the land and the construction cost should be assessed to the abutting property, providing the development has not taken away an existing frontage right, in which case the law would require that the service roadway be provided at the cost of the municipality building the parkway. In general, there is a right of access to all property which should be paid for by the owners of that property, but the right of access may not be by way of a parkway, artery, or super-highways, except as use is made of the same right of way.

In the case of highways between terminals the same principles apply. The right of way must be acquired and paid for by assessments of the district, or preferably the terminals, and the grading, paving, and structures by the users —motor-operated vehicles through license fees, gas tax, and wheelage tax.

It would seem to be a basic principle that land for streets or for street widenings, of the standard widths required by the use, should be provided by abutting property without cost; that is, in condemnation proceedings, benefits and damages should be made equal for land acquirements. Why should the owner be paid for the increment in value that has been created by the people of the city? The only difference in the value of a lot in the Sahara Desert and a lot on Fifth Avenue, in New York City, is the spirit of business that surrounds that lot, and that spirit of business was created by the people of New York. The owner receives his remuneration in the increased value of his remaining property and in obtaining the accessibility which his business required.

BIBLIOGRAPHY ON STREET THOROUGHFARES

BY THE COMMITTEE OF THE CITY PLANNING DIVISION ON THOROUGHFARES MANUAL

Autostrada: Papers and Reports of the International Road Congresses at Seville, Spain (1923), and Milan, Italy (1926).

Braided Crossing for Express Highway, by A. O. Child, *The Highway Magazine,* February, 1933, p. 40.

A City Planning Primer; Pamphlet of United States Department of Commerce, 1928.

Control of Roadside Development Through State and County Highway Planning, by Robert Whitten, *City Planning,* July, 1932, p. 157.

The Design of Street and Highway Intersections, by L. S. Tuttle and E. H. Holmes, Jun. Am. Soc. C. E., *Public Roads,* United States Department of Agriculture, July, 1932, p. 73.

Distribution of Costs of Assessable Improvements, *Civil Engineering,* April, 1933, p. 222.

Effect of Bridges on Transit Conditions (Paper No. 1554), by John A. Miller, M. Am. Soc. C. E., *Transactions,* Am. Soc. C. E. Vol. 88 (1925), p. 127.

Elements Governing the Development of Highway Traffic (Paper No. 1576), by A. N. Johnson, M. Am. Soc. C. E., *Transactions,* Am. Soc. C. E., Vol. 89 (1926), p. 259.

Elevated Highways for Chicago, Compared to Street Widenings, *Engineering News-Record,* April 27, 1933.

Equitable Distribution of Benefit Assessments, by Hyman Shifrin, M. Am. Soc. C. E., *Civil Engineering,* March, 1933, p. 158.

Excess Condemnation in City Planning: A Symposium (Paper No. 1585), *Transactions,* Am. Soc. C. E., Vol. 89 (1926), p. 791.

Express Highways in France (Editorial), *New York Times,* November 4, 1930.

The Expressway in the Region, by Robert Whitten, *City Planning,* June, 1932.

Financing Street and Highway Improvements (Paper No. 1828), by R. W. Crum, M. Am. Soc. C. E., *Transactions,* Am. Soc. C. E., Vol. 98 (1933), p. 74.

Graphic Regional Plan, Regional Plan of New York and Its Environs, Vol. 1 (1929).

Highway Location, Notes on, by W. W. Crosby and G. E. Goodwin, Members, Am. Soc. C. E., Gillette Pub. Co., 1928.

Highway Traffic, Regional Survey of New York and Its Environs, Vol. III (1927).

Highway Traffic Capacity, by A. N. Johnson, M. Am. Soc. C. E., *Public Roads,* United States Department of Agriculture, May, 1932.

Highways, by Gorden Whitnall, *Roads and Streets,* August, 1932, p. 335.

Increasing the Capacity of American Highways, by S. D. Waldon, *Traffic Regulation,* December, 1930, p. 51.

Increasing the Capacity of Existing Streets (Paper No. 1556), by Arthur S. Tuttle, M. Am. Soc. C. E., *Transactions,* Am. Soc. C. E., Vol. 88 (1925), p. 210.

Legal Problems Involved in Establishing Setback Lines (Paper No. 1808), by Clifton Williams, *Transactions,* Am. Soc. C. E., Vol. 96 (1932), p. 887.

Machine Made Leisure, by Paul T. Franke, *The Parkway,* p. 179, Harper and Brothers, 1932.

Misleading Propaganda Directed Against Highway Users (Staff Article), *Roads and Streets,* September, 1932, p. 375.

Municipal Engineering: Address of President George S. Webster, at the Annual Convention, at Houston, Tex. (Paper No. 1474), *Transactions,* Am. Soc. C. E., Vol. LXXXIV (1921), p. 516.

A New Type of Thoroughfare: The "Freeway", by L. C. Squire and H. M. Bassett, *The American City,* November, 1932, p. 64.

Ocean Beach Esplanade, San Francisco, California (Paper No. 1539), by M. M. O'Shaughnessy, M. Am. Soc. C. E., *Transactions,* Am. Soc. C. E., Vol. LXXXVII (1924), p. 492.

The Parkway as a Traffic Artery, by H. S. Swan, *The American City,* October and November, 1931.

Planning and Research at Civil Engineers' Meeting—Elevated Highways, *Engineering News-Record,* January 25, 1934.

Planning for Residential Districts: Report of the President's Conference on Home Building and Home Ownership, Washington, D. C., 1932.

Planning Highways and Street Intersections for Modern Traffic Needs, by Burton W. Marsh, M. Am. Soc. C. E., *The Municipal Index,* Edition of 1931, p. 123.

Planning of Street Systems, by H. M. Lewis, M. Am. Soc. C. E., *Highway Engineer and Contractor,* July, 1931, p. 33.

The Preservation of the Countryside: Lecture by the Earl of Cranford and Balcarres, with Discussion by Cass Gilbert, *Journal,* Royal Society of Arts, June, 1932, p. 662.

The Prevention of Blight in Residential Areas Adjacent to Expressways, by Robert Whitten, *Bulletin,* National Conference on City Planning, June, 1931.

Preventive and Palliative Measures for Street Traffic Relief, by Miller McClintock, *City Planning Quarterly,* April, 1930, p. 199.

Regional Plan of the Philadelphia Tri-State District: Findings and Proposals, 1932.

Regional Planning (Paper No. 1526), by Nelson P. Lewis, M. Am. Soc. C. E., *Transactions,* Am. Soc. C. E., Vol. LXXXVI (1923), p. 1326.

Regional Planning Theory, by A. C. Comey, M. Am. Soc. C. E., 1932 (Pamphlet).

Report (Final) of Special Committee on Materials for Road Construction, and Standards for Their Test and Use (Paper No. 1421), *Transactions,* Am. Soc. C. E., Vol. LXXXII (1918), p. 1384.

Report on "Manual of City Planning, Scope and Procedure", by Hugh E. Young, M. Am. Soc. C. E., before City Planning Div., Am. Soc. C. E., Cleveland, Ohio, July 10, 1930.

Report on "Manual on Subdivision of Urban Land", by Rolland S. Wallis, M. Am. Soc. C. E., before City Planning Div., Am. Soc. C. E., Cleveland, Ohio. July 10, 1930.

Russian Planning Norms, by J. L. Crane, Jr., M. Am. Soc. C. E., *City Planning,* July, 1932, p. 168.

Seattle, Plan of: Report of Municipal Plans Commission, Submitting Report of Virgil Bogue, M. Am. Soc. C. E., 1911.

Special Assessment Procedure: Report of Investment Bankers Assoc. of America, November, 1931.

Speed on Thoroughfares, by Maxwell Halsey, *Traffic Regulation,* May, 1931.

Traffic Survey Plan for Cleveland Metropolitan Area, by R. C. Chaney, *Engineering News-Record,* September 15, 1932.

Thoroughfare Plan for Boston, 1930, by Robert Whitten, *Transactions,* Am. Soc. C. E., Vol. 99 (1934), p. 789.

Through Streets, National Safety Council, Chicago, Ill., 1930 (Pamphlet).

Traffic Circles and Rotary Traffic, by Herbert S. Swan, *Civil Engineering,* July, 1932, p. 425.

A Traffic Planner Imagines a City, by Fritz Malcher, *The American City,* March, 1931, p. 134.

Traffic Separation Plan for Two Intersecting Roadways, by C. D. Feight, *Civil Engineering,* September, 1932, p. 588.

Widths (Preliminary Investigations), American Highway Engineers Handbook, Wiley and Sons, New York, 1919.

Zoning and Freeways, by E. M. Bassett, *City Planning,* July, 1933, p. 138.

DISCUSSION

H. SHIFRIN,[12] M. AM. SOC. C. E. (by letter).—This Symposium, and especially the Report of the Committee of the City Planning Division on Street Thoroughfares Manual on Definitions, General Principles, and Recommendations, should prove valuable to engineers interested in city planning or planning for metropolitan areas. Most of the principles can be applied profitably to those new communities that are now (1934) being set up by the Federal Government, such as subsistence homesteads, and those being constructed to house the personnel connected with the immense power projects under way. This use, however, even if basically sound, can not be applied so readily when considering relief measures necessary in connection with old established metropolitan areas and large cities.

In the definitions furnished by the Committee, it would seem that the term, "Freeway" is synonymous to "Expressway". An expressway providing a highway for its express traffic exclusively, in order to be efficient, could not have access to it from abutting property. With no such access, the expressway really becomes a private right of way for through uninterrupted traffic, with practically no intersections at grade, and with no access to it from the abutting property. Such a definition would make the expressway synonymous to a freeway. One of these terms should be sufficient, as use of more than one for the same purpose merely adds to confusion.

The writer agrees thoroughly with the statements of the Committee (see "General Summary") that simply to widen existing streets to extraordinary widths does not provide the desired results. Experiences of engineers in cities in which extensive widenings have occurred during the 10-yr period from 1920 to 1930, have clearly proved that the cost involved in widening streets to extraordinary widths without providing for uninterrupted traffic at street intersections can not be justified on the basis of the benefits received by the terminals served through the widened streets, by the abutting property, or by the users of the thoroughfares. Experiences gained as a result of these widenings, have also proved that the methods used for determining benefit assessments to defray the expenses involved are often unjust and inequitable.

In most old cities of the United States, property values in the downtown or loop district, which is the main terminal toward which traffic is directed, are, in general, too high to permit the construction of satisfactory express or super-highways which would contain lanes for the uninterrupted movement of express and mass traffic through such terminal areas. As a matter of fact, the more feasible plan for serving these areas is to provide thoroughfares along the extremities or limits of such areas, and to utilize the existing facilities through the interior of the loop district. With sufficient thoroughfares of the expressway type connecting the valuable center or the down-town area on all sides with the highways or arterial streets existing in the metropolitan area, the stabilization of property values (which is the crying need of most cities) could be accomplished.

---

[12] Cons. Engr., St. Louis, Mo.

The policy of the highway departments in most States in the past has excluded the construction of highways or thoroughfares, or the use of highway funds for work within large cities, notwithstanding the fact that a substantial percentage of the funds made available is received from the citizens residing in such cities. That this fact is being realized is evidenced by the number of highway departments which are setting aside a certain portion of their funds to provide extensions of highway facilities, not only within the metropolitan areas, but within the corporate limits of the municipalities. This attitude has been further encouraged through the regulation of the U. S. Bureau of Public Roads in the expenditure of the Public Works funds set aside for highways, which regulations require that at least 25% of such funds be spent in cities. Use of such funds should properly be made only for the construction of expressways or freeways connecting the arterial or super-highways with the valuable centers or down-town districts of large cities, and no part of such funds should be used on ordinary city streets.

In addition to expressways or freeways, it would seem necessary that within the city proper, definite provisions should be made for mass transportation. Unless super-highways are made available, where a portion of the highway is physically segregated for the use of mass transportation only, it would seem desirable to designate certain existing thoroughfares of ordinary width for use of mass-transportation vehicles primarily. The satisfactory solution of providing mass-transportation facilities is urgent in practically all the cities of this country, and has been further aggravated by the recent widening of streets where no segregation has occurred to separate fast-moving traffic from mass transportation.

Donald M. Baker,[13] M. Am. Soc. C. E. (by letter).—Just as many thousand miles of steam railroads were built before the economic principles of railroad construction were evolved and put in practice, so thousands of miles of highways were constructed and improved before such principles underlying their development were formulated and used. The various papers in this Symposium well set forth the principles underlying the use, design, and location of street thoroughfares. The definitions contained therein will be of material assistance to those engaged in solving traffic and vehicular transportation problems.

Of these principles, now fairly well recognized, least is known concerning those relating to highway financing, particularly in urban areas. Current practice is, through gasoline and motor-vehicle taxes, to place the major portion of the cost of constructing and maintaining highways located outside urban areas upon the users of such highways, while in urban areas such costs are borne to a large extent by property served, or assumed to be served.

---

[13] Cons. Engr., Los Angeles, Calif.

The reasons leading to the development of such practices are of more practical than scientific soundness. It was soon found that values of suburban and rural property were insufficient to support the costs involved, and that the benefits accruing to such property were small in proportion to those accruing to the users of such highways. Therefore, if such highways were to be constructed and maintained to proper standards and in sufficient number to satisfy the growing demand, their costs would have to be borne principally by those using them.

In the case of highways in urban areas, a different situation existed. Urban property values were many times higher than those in outside areas, and such properties at the outset were considered well able to bear construction and maintenance costs. During the decade, 1920–1930, which saw rising traffic congestion becoming a serious menace and, during which, many street thoroughfares were developed, rising prices and the general real estate "boom" throughout the country caused, at the outset, an increase in property values in areas served by these new thoroughfares, out of proportion to the general rise in property values. A change in the traveling habits of the public, brought about by the increasing use of the motor vehicle, likewise contributed to this rise.

Sales of property were made at large profit, more thoroughfares were developed, and more profitable sales were made. No objection was raised by property owners to this method of financing thoroughfares because of these increased values or because such costs were largely paid by subsequent purchasers of their property. Gradually, however, the operation of economic laws came into play. The margin of profit between benefits, measured in terms of enhanced property values, and costs charged against the property which paid the costs were reduced and in some cases disappeared. This condition was greatly aggravated by the deflation in property values which took place in the period following 1929.

The need for additional street space persisted. Since local tax revenues, reduced through deflated property values and burdened by unemployment relief measures, were not sufficient to carry the cost of providing this additional street space, a demand arose for a greater allocation of gasoline and motor-vehicle tax revenues to urban highway construction and maintenance.

The problem of highway financing has four phases: (1) The determination of the different interests benefited by the construction and continued operation of the highway; (2) the evaluation of the benefits accruing to each of the interests thus benefited; (3) the allocation of construction and maintenance costs among the benefited interests in an equitable manner; and (4) the development of a method of collecting such costs from such interests.

The following principles should be observed as controlling in any policy of highway financing, whether it apply to urban or outside highways: (*a*) The benefits received may be direct or indirect; (*b*) the benefits must ultimately be expressed in terms of dollars and cents; (*c*) the costs should be allocated in proportion to, but must not exceed, the benefits derived; (*d*) the costs should be within the ability of the benefited interests to pay; and, (*e*) the method of collecting such costs must be simple and practical, yet sufficiently equitable as to meet with popular approval and support.

The problem does not admit of an exact solution, but an approximate one may be developed. The interests benefited fall into the following classifications: (1) Users of the highways, principally passenger and commercial motor vehicles; (2) property served by the highways, including (*a*) property abutting the highway; (*b*) property in an area adjacent to the highway, but not abutting thereon; and (*c*) a larger area within the community which is indirectly served and, therefore, benefited.

The gasoline and motor-vehicle tax appears to be generally accepted as a satisfactory method of financing highways outside urban areas on account of its approximate equity and its ease of collection. Unless large inroads are made upon these revenues, for urban highways and for other purposes, this method will probably continue to prove satisfactory for future needs. Because of changed conditions, a new and more equitable method of financing urban highways, particularly street thoroughfares, appears to be necessary. Determination of the interests benefited, and the evaluation of benefits accruing must be made in a more scientific manner. The development and application of sound principles for financing these thoroughfares is the crux of the entire problem. The writer agrees in the recommendation of the Committee that "a new type of street for automobiles is now needed in some cases, and will soon be needed generally", but does not believe that such new type of street can ever be achieved under present or near future economic conditions, until different methods of financing have been developed.

The writer does not agree with the principle presented that right-of-way and construction costs should be considered separately in assessing benefits and damages. This principle appears in the report to be based upon the idea that, since inadequate rights of way were dedicated by the original owners of property, their successors in interest should bear the burden of rectifying this mistake. Such inadequate rights of way were dedicated because no one asked for adequate rights of way at the time of dedication, in the case of the older streets, and probably public authorities at the time had no conception of present traffic congestion, or could have asked or intelligently requested, such right of way had they so desired.

During the years, 1927 to 1930, the Los Angeles (Calif.) Board of Planning Commissioners reviewed approximately a thousand subdivisions. Practically every subdivider was quite willing to dedicate street space of ample width and in proper location when asked to do so. To place the burden of rectifying past mistakes of previous owners upon existing owners, when such past mistakes were unconsciously made, appears to be an unfair principle. If a fair and equitable method of ascertaining benefited interests and of evaluating benefits accruing is developed, there will be no need of considering, separately, right-of-way and construction costs.

Frederic Bass,[14] M. Am. Soc. C. E. (by letter).—The conclusion is advanced in this Symposium that, in the division of right-of-way costs for street thoroughfares among various beneficiaries, the land values and building costs should be determined separately. This principle is supported by Mr. Herrold on the ground that building costs may be estimated accurately, whereas estimation of land values is likely to be affected by irrelevant influences, and opinions of experts are likely to be supported by insufficient data. Mr. Herrold further states, as a basic principle, that land for streets or for street widenings should be provided by abutting property without cost. In other words, he claims that, in condemnation proceedings, benefits and damages "should be made equal for land acquirements," since the owner receives his remuneration in the increased value of his remaining property and in obtaining the accessibility which his business requires.

It is a fact that when real property is completely transferred, the purchaser assumes all liabilities and encumbrances attached to the property. It is also a matter of common experience that payment for mistakes, whether consciously or unconsciously made, must be equally met. Where it cannot be shown that increased values of remaining property result and accessibility is improved for any particular property, the owner should no doubt be compensated for land taken. Ordinarily, however, the property does increase in value, and accessibility is improved by the street widening, and, to that extent, the owner should contribute. Mr. Herrold's principle appears to facilitate that contribution.

Hugh E. Young,[15] M. Am. Soc. C. E. (by letter).—A statement of the principle that land for streets or for street widenings of the standard widths required by the use should be provided by abutting property owners without cost, is an important feature of the paper by Mr. Herrold. His observations and deductions merit the most careful consideration at this time when the development of comprehensive plans for much needed improvements in street transportation systems seems quite remote because of the financial situation arising from dwindling tax resources.

This basic principle is well fortified by fact and logic. It is true, as Mr. Herrold states, that street widths were determined by vision, generosity, cupidity, and probably habit; that the introduction of the motor vehicle has compelled a tremendous increase in the floor area that transportation uses; and that groups of citizens want their streets improved to preserve their business character, to make business houses more accessible, and to prevent mutation.

It is also a fact (see the heading "Users Should Pay") that "in a city the character of use determines the width of the street and the width of the roadway, the latter being based on the almost uniform laws establishing vehicle widths." The roadway widths recommended by Mr. Herrold are standard. As a matter of basic principle it is equitable that the owner of abutting property should pay the major part of the entire cost of the right-of-way or street width required for standard uses of property—the pavement to be paid for as determined by use. This judgment is based, not on the theory that the original dedication was insufficient, because it probably was sufficient at the time it was made, but rather on the theory that added accessibility accruing to the property and the greater value thereby produced is at least an offset to the area of land given up by the owner.

---

[14] Cons. Engr.; Prof. of Municipal and San. Eng., Univ. of Minnesota, Minneapolis. Minn.

[15] Chf. Engr., Chicago Plan Comm., Chicago, Ill.

However, as with all principles the mere formula cannot be applied with equal justice in every instance as, for example, where an owner has a piece of property 50 ft deep on an existing street developed with a building adequate for his purposes, and from it 25 ft are taken to widen the street. No theory of inadequate dedication or added value would stand, if the remainder of his land is no longer usable. There is nothing to do in a case like this other than to compensate him for the land taken and damaged.

The basic principle applies to street requirements in a city where the normal use determines the width of the street and the width of the roadway. For arterial streets, express highways, super-highways, and trucking ways, as Mr. Herrold states, the widened right of way must be acquired through special assessment districts or terminals, or other usual proceedings.

Since there are exceptions, it would seem that taking into consideration all the factors (and especially the basic principle that in condemnation proceedings benefits and damages should be made substantially equal for land acquirements), the most practicable plan would be to include under one proceeding land condemnation, damages, construction cost, including sidewalks and lighting system—in fact, everything necessary for the completed improvement. A benefit should be assessed against the abutting property in a sum not less than the property damage minus reconstruction costs, and damage claim should be allowed to the remainder, if any. All other costs should be assessed against the municipality or other public body as a public benefit to be paid from gas tax, vehicle tax, or other tax.

For arterial highways, express highways, super-highways, trucking ways, etc., the same principle should be applied except that where the width of the improvement is greater than that required for standard uses of property, the cost of the excess land and building taken should be paid for by the user, or by the public in general, for the obvious reason that the greater width of thoroughfare is for the express benefit of traffic. In other words, all cost in excess of that which ordinarily would be assessed against the abutting property for the standard widening, as outlined by Mr. Herrold, would be assessed against the municipality as a public benefit.

Committee of the City Planning Division on Street Thoroughfares Manual (by letter).—The Committee wishes first to express its appreciation of the thoughtful comments by Messrs. Baker, Shifrin, Young, and Professor Bass.

The discussion has been directed mainly to the subject of "costs." Some agreement and some disagreement with the Committee's conclusions appear.

With conditions existing as to the participation of the Federal Government in highway work, and the present uncertainties in this connection, the Committee feels that it is useless now (1935) to prolong these arguments or to make the effort for a revision of its earlier statements.

The Committee has nothing further to submit, beyond stating that the importance of the statements originally made in its report under "Segregation of Opposing Traffic" seems to be gaining general recognition. This is demonstrated by actual construction in many localities. Moreover, even where the extraordinary width required for an "expressway" or "parkway" may seem impracticable of acquirement in a few built-up sections, some ingenuity may enable the ultimate effects to be largely achieved even there, and opportunities should be sought and quickly grasped, not only for establishing expressways, and parkways through unbuilt areas, but also into the built-up areas of cities.

For the Committee,

W. W. Crosby, *Chairman.*

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 1916

ECCENTRIC RIVETED CONNECTIONS

By Eugene A. Dubin,[1] Esq.

With Discussion by Messrs. Carlton T. Bishop, James R. Bole, Albert Wertheimer, Kenneth L. DeBlois, Jonathan Jones, Armin Rozman, A E. R. de Jonge, Fang-Yin Tsai, Odd Albert, and Eugene A. Dubin.

Synopsis

Formulas and corresponding curves for designing riveted connections, subjected to eccentric loads, are developed in this paper. The general equations express, directly, the relationship between the load, the eccentricity of the load, the stress in the extreme rivet, the spacing of the rivets, and the number of rivets in the group.

Although riveted connections subjected to eccentric loads are frequently encountered in practice, the method of designing them is essentially a matter of "cut and try"; that is, the rivet group is assumed and the stress in the extreme rivet is checked. Hitherto, "families" of curves have been made for use in designing this type of connection, but it has been difficult to interpolate for values between such curves, and beyond the range of the curves the "cut-and-try" method must still be used. Furthermore, the probability of reading values, incorrectly, is great, due to the complicated nature of such curves. Tables are also available, but they, too, offer no accurate means of interpolation for values other than those listed.

The formulas and charts presented with this paper are suggested to replace those now in current use. Accurate and quick results are possible with these curves due to the inherent advantages of an alignment chart over a "family" of curves. The general formulas herein developed, express directly the relationship between the load, the eccentricity of the load, the stress in the extreme rivet, the spacing of the rivets, and the number of rivets in the group; but the resulting function of *n* (the number of rivets), is difficult of solution algebraically. However, a graphical solution of this function is possible by means of an alignment chart. The problem has been restricted to a con-

---

Note.—Published in August, 1934, *Proceedings*.

[1] Washington, D. C.

sideration of the case in which the action line of the load is parallel to the rows of rivets in the group. Figs. 1 to 4 are offered for the solution of cases that occur most commonly. The charts are for groups of rivets containing from one to four rows, and for the one case of 3-in. spacing of rivets. Fig. 2 is made to cover all cases in which the spacing between rows of rivets varies from 3 to 10 in. For connections of three and four rows of rivets only one case is presented—that of 3-in. spacing between rows. For cases beyond the range of the charts, in which the "cut-and-try" method is used, the formulas for the polar moment of inertia of the rivet group, and the distance from the center of gravity of the group to the extreme rivet will aid materially in the solution.

For checking the strength of connections not covered by the charts the formulas developed are quick and easy of solution. Fig. 1 is useful for designing plate-girder web splices when the common assumption is made that the stress in each rivet, due to bending, is in direct proportion to its distance from the neutral axis of the group. To design web splices, then, it is only necessary to divide the shear and moment of the web uniformly between the assumed number of rows of rivets on each side of the splice, determine the eccentricity of the shear (the moment divided by the shear), and proceed to use the charts as for any eccentric load. In the same manner, Fig. 1 may be used for a preliminary design for connections of more than one row of rivets that lie beyond the range of the other charts.

FORMULAS AND DERIVATIONS

To use the charts, lay a straight-edge across the three scales of the chart, intersecting two of them at known values. The point at which the straight-edge intersects the third scale is the desired value. The following notation is presented for the convenience of discussers:

a = a number; thus: $a\ (a + 1)$, $(2a + 1)$, etc.;
d = diameter of rivet;
e = eccentricity of load with respect to the center of gravity of the rivet group;
n = total number of rivets in a group;
p = pitch;
r = distance from the center of gravity of the group of rivets; r_0, to the extreme rivet; r_1, r_2, etc., to Rivets 1, 2, etc., of a group; and $\Sigma\, r^2$ = polar moment of a group;
s = total stress in extreme rivet, or allowable total stress;
w = width of a strip, or distance between rows of rivets parallel to the axis of the member;
x = an unknown variable;
$A = \dfrac{P}{s}$
P = load, with line of action parallel to the rivet rows;
S = section modulus of a group of rivets;
θ = an angle between vectors (Fig. 2(d));
Σ = summation; $\Sigma\, r^2$ = polar moment of inertia of a group of rivets; and,
ϕ = an angle measured at the center of gravity of a rivet group (see Fig. 2(d)).

Example 1.—One Row of Rivets.—The stress in the extreme rivet (Fig. 1(*a*)) is the resultant of the shearing stress and the stress due to bending, which, for one row of rivets, is at right angles. Therefore, $s^2 = \left(\frac{P}{n}\right)^2 + \left(\frac{P\,e\,r_0}{\Sigma\, r^2}\right)^2$; or,

$$\left(\frac{s}{P}\right)^2 = \left(\frac{1}{n}\right)^2 + \left(\frac{e\,r_0}{\Sigma\, r^2}\right)^2 \quad \text{.......................(1)}$$

in which,

$$r_0 = p\,\frac{(n-1)}{2} \quad \text{.........................(2)}$$

and $\Sigma\, r^2 = r_1^2 + r_2^2 + \;.\;.\;.\;.\; + r_0^2$. With an odd number of rivets in the row: $r_1 = p$, $r_2 = 2p$, etc.; and,

$$\Sigma\, r^2 = 2 \sum_{1}^{0.5\,(n-1)} (p\,x)^2 = 2\,p^2 \sum_{1}^{0.5\,(n-1)} x^2$$

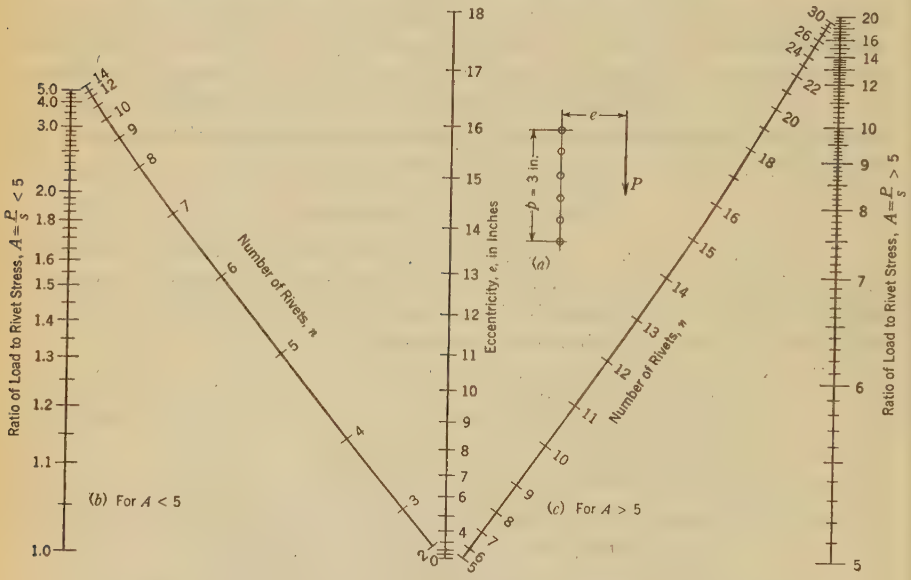


FIG. 1.—ONE ROW OF RIVETS WITH A 3-INCH PITCH.

The sum of a series in x^2 between the limits, 1 and *a*, is given by the expression, $\sum_{1}^{a} x^2 = \frac{a\,(a+1)\,(2\,a+1)}{6}$. Therefore,

$$\Sigma\, r^2 = \frac{2\,p^2}{6}\left[\left(\frac{n-1}{2}\right)\left(\frac{n-1}{2}+1\right)(n-1+1)\right]$$

$$= \frac{p^2\,n}{12}\,(n-1)\,(n+1) \quad \text{.....................(3)}$$

Although the expression for the polar moment of the rivet group (Equation (3)) was derived for a row with an odd number of rivets, it can be proved true for a row with an even number of rivets.

Equations (2) and (3) may be substituted in Equation (1) and the result written as,

$$\left(\frac{1}{A}\right)^2 - e^2\left[\frac{6}{pn\,(n+1)}\right]^2 = \frac{1}{n^2} \quad \text{.................(4)}$$

in which, $A = \frac{P}{s}$. For constant values of p this equation takes the form,

$$f_1(A) + f_2(e) \times f_3(n) = f_4(n) \quad \text{.................(5)}$$

a standard type of equation soluble by an alignment chart.

The section modulus, S, of any group of rivets is:

$$S = \frac{\Sigma\, r^2}{r_o} = \frac{p\,n}{6}\,(n+1) \quad \text{.....................(6)}$$

and, therefore, Equation (4) may also be written:

$$\frac{1}{A^2} = \frac{1}{n^2} + \frac{e^2\,r^2_o}{(\Sigma\, r^2)^2} = \frac{1}{n^2} + \frac{e^2}{S^2} \quad \text{.................(7)}$$

The alignment chart, Fig. 1, is a device for solving the foregoing equations. With any two of three values known (required value of A, number of rivets, or eccentricity), the missing characteristic of an eccentric riveted connection can be determined.

Let $A = \frac{P}{s}$ $(< 5) = 1.23$, and $e = 10$ in. The broken line from $A = 1.23$ (Fig. 1) to $e = 10$, intersects the diagonal line to indicate that such a joint requires a single row of five rivets.

Example 2.—Two Rows of Rivets.—The stress in the extreme rivet (Fig. 2(*a*)) is the resultant of the shearing stress and the stress due to bending, as shown in Fig. 2(*d*). Then,

$$s^2 = \left(\frac{P}{n}\right)^2 + \left(\frac{P\,e\,r_o}{\Sigma\, r^2}\right)^2 - 2\left(\frac{P}{n}\right)\left(\frac{P\,e\,r_o}{\Sigma\, r^2}\right)\cos\,\theta$$

and, $\cos\,\theta = -\cos\,\phi = -\frac{w}{2\,r_o}$. Therefore,

$$s^2 = \left(\frac{P}{n}\right)^2 + \left(\frac{P\,e\,r_o}{\Sigma\, r^2}\right)^2 + \frac{P^2\,e\,w}{n\,\Sigma\, r^2}$$

or,

$$\left(\frac{s}{P}\right)^2 = \left(\frac{1}{n}\right)^2 + \left(\frac{e\,r_o}{\Sigma\, r^2}\right)^2 + \frac{e\;w}{n\,\Sigma\, r^2} \quad \text{................(8)}$$

The vertical component of r_o may be obtained from Equation (2), and the horizontal component added to it vectorially; thus

$$r_o = \left[p^2\left(\frac{\frac{n}{2} - 1}{2}\right)^2 + \frac{w^2}{4}\right]^{\frac{1}{2}} = \left[\frac{p^2}{16}\,(n-2)^2 + \frac{w^2}{4}\right]^{\frac{1}{2}} \quad \text{........(9)}$$

The vertical components of Σr^2 for one row of rivets are obtained from Equation (3), with n replaced by $0.5\,n$, and the result is multiplied by 2 for the two rows of rivets. The horizontal components of Σr^2 are $n\left(\frac{w}{2}\right)^2$; therefore,

$$\Sigma r^2 = \frac{2\,p^2}{12}\left(\frac{n}{2}\right)\left(\frac{n}{2}-1\right)\left(\frac{n}{2}+1\right)+\frac{n\,w^2}{4}$$

$$= \frac{p^2\,n}{48}\,(n-2)\,(n+2)+\frac{n\,w^2}{4} \quad \text{.................} \quad (10)$$

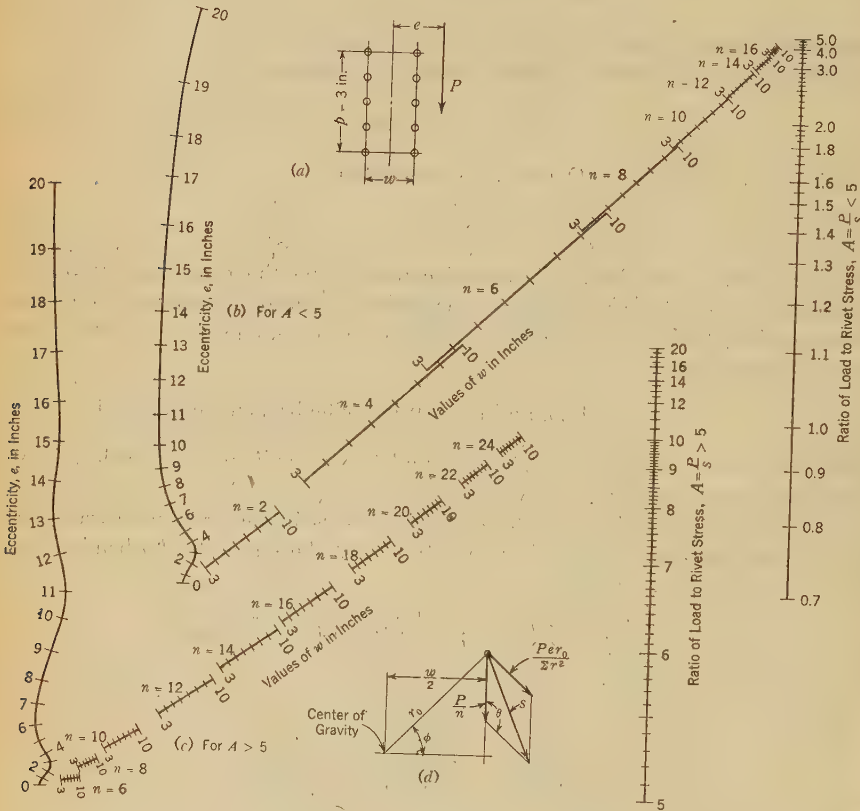


FIG. 2.—TWO ROWS OF RIVETS WITH A 3-INCH PITCH.

Equations (9) and (10) may be substituted in Equation (8), but the resulting function cannot be transformed to fit any standard form for which an alignment chart may be plotted. Nevertheless, a graphical solution is possible by a method developed by Mr. A. Wertheimer, of the Bureau of Ordnance, United States Navy Department. The result may be brought to any desired degree of accuracy by successive approximations. The charts for two, three, and four rows of rivets were constructed by this method. In no case will the error in the charts exceed 5 per cent.

Referring to Fig. 2, a formula similar to Equation (7) may be written:

$$\frac{1}{A^2} = \frac{1}{n^2} + \frac{e^2\, r^2_0}{(\Sigma r^2)^2} + \frac{e\, w}{n\, \Sigma\, r^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (11)$$

For this case,

$$r_0 = \left[\frac{p^2}{16}\,(n-2)^2 + \frac{w^2}{4}\right]^{\frac{1}{2}} \ldots\ldots\ldots\ldots\ldots\ldots (12)$$

and,

$$\Sigma\, r^2 = \frac{p^2\, n}{48}\,(n-2)\,(n+2) + \frac{n\, w^2}{4} \quad \ldots\ldots\ldots\ldots (13)$$

The n-curves of Fig. 2 are a series of curves for constant values of n and varying values of w, in which, w varies from 3 in. to 10 in. The n-curves are graduated in units of $w = 1$ in.

Fig. 2 is used precisely the same as Fig. 1. With the numerical data in Example 1 enter Fig. 2(b) as shown by the broken line, which indicates a joint of four rivets, about $8\frac{1}{4}$ in. apart along the axis, and a 3-in. pitch. If A is more than 5, the problem is solved in Fig. 2(c).

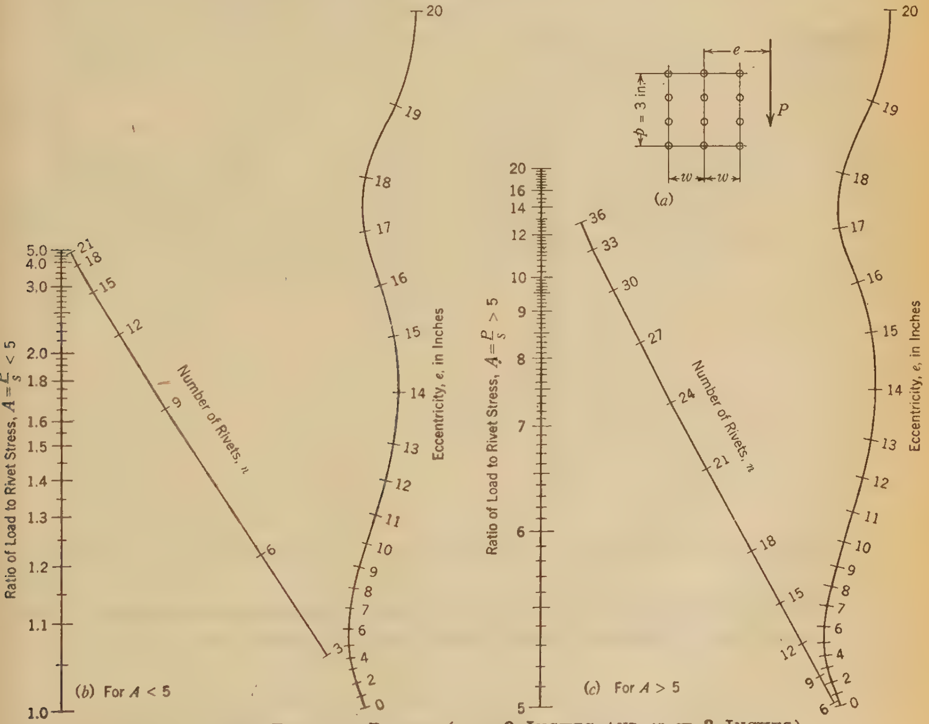


Fig. 3.—Three Rows of Rivets ($p = 3$ Inches and $w = 3$ Inches).

Example 3.—Three Rows of Rivets.—Similar to Equation (11), for the case shown in Fig. 3(a):

$$\frac{1}{A^2} = \frac{1}{n^2} + \frac{e^2\, r^2_0}{(\Sigma\, r^2)^2} + \frac{2\, e\, w}{n\, \Sigma\, r^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (14)$$

in which,

$$r_0 = \left\{ \left[\frac{p}{6}\,(n-3) \right]^2 + w^2 \right\}^{\frac{1}{2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (15)$$

and,

$$\Sigma\, r^2 = \frac{p^2\, n}{108}\,(n-3)\,(n+3) + \frac{2\, n\, w^2}{3} \quad \ldots\ldots\ldots\ldots\ldots (16)$$

As before, entering Fig. 3(*b*) with $A = 1.23$ and $e = 10$, the broken line intersects the diagonal at slightly more than $n = 6$, say, $n = 9$.

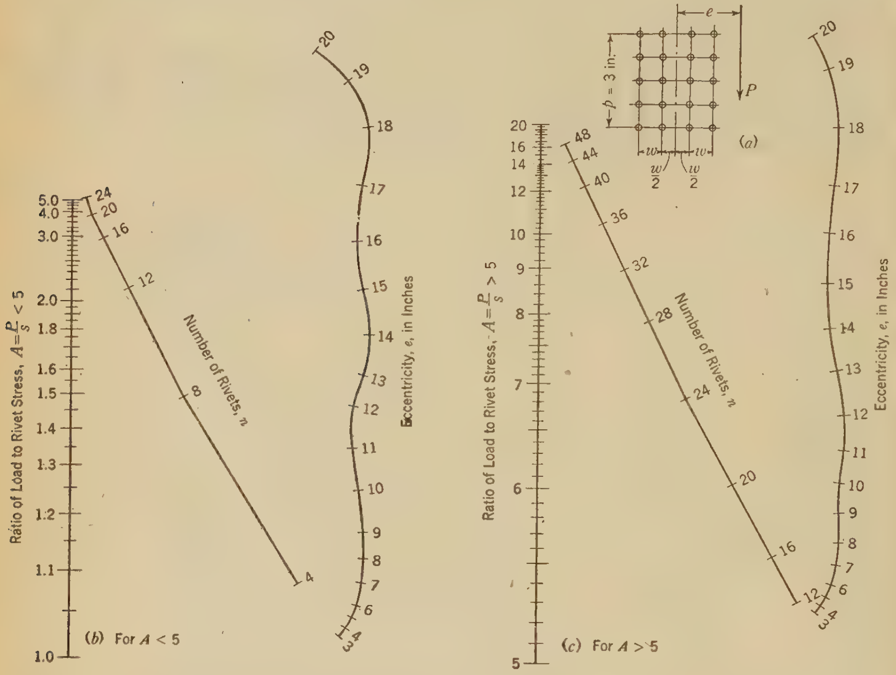


FIG. 4.—FOUR ROWS OF RIVETS (p = 3 INCHES; r_g = 1½ INCHES; AND w = 3 INCHES).

Example 4.—Four Rows of Rivets.—If w_1 = distance between inner rows of rivets, and w_2 = distance between inner row and outer row:

$$\frac{1}{A^2} = \frac{1}{n^2} + \frac{e^2\, r_0^2}{(\Sigma\, r^2)^2} + \frac{2\, e \left(\dfrac{w_1}{2} + w_2 \right)}{n\, \Sigma\, r^2} \quad \ldots\ldots\ldots\ldots (17)$$

in which,

$$r_0 = \left\{ \left[\frac{p}{8}\,(n-4) \right]^2 + \left(\frac{w_1}{2} + w_2 \right)^2 \right\}^{\frac{1}{2}} \quad \ldots\ldots\ldots\ldots (18)$$

and,

$$\Sigma r^2 = \frac{p^2 n}{192} (n - 4)(n + 4) + \frac{n}{2}\left[\frac{w^2{}_1}{4} + \left(\frac{w_1}{2} + w_2\right)^2\right] \ldots\ldots (19)$$

For the case, $w_1 = w_2 = w$:

$$\frac{1}{A^2} = \frac{1}{n^2} + \frac{e^2 r^2{}_0}{(\Sigma r^2)^2} + \frac{3 e w}{n \Sigma r^2} \ldots\ldots (20)$$

in which,

$$r_0 = \left\{\left[\frac{p}{8}(n - 4)\right]^2 + \left(\frac{3 w}{2}\right)^2\right\}^{\frac{1}{2}} \ldots\ldots (21)$$

and,

$$\Sigma r^2 = \frac{p^2 n}{192} (n - 4)(n + 4) + \frac{5 n w^2}{4} \ldots\ldots (22)$$

Entering Fig. 4(*b*) with $A = 1.23$ and $e = 10$, the broken line intersects the diagonal at about $n = 5$, say, $n = 8$.

The formulas for Examples 3 and 4 are developed in a manner similar to those in Example 2.

CONCLUSION

The problem of eccentric riveted connections can be simplified by the use of an equation expressing the number and spacing of the rivets required, as a function of the rivet stress, and the load and its eccentricity.

The method and charts presented in this paper can be extended to cover cases other than those illustrated.

DISCUSSION

Carlton T. Bishop,[2] M. Am. Soc. C. E. (by letter).—The alignment charts proposed by Mr. Dubin satisfy a long-felt need, because they provide a simple and direct method of determining the required number of rivets in an eccentric connection without assumption. His formulas are based on the usual method of calculation, and his derivations are sound.

The writer has checked the problems in his files by means of the charts with complete satisfaction. Problems with one or two rows of rivets seemed in perfect agreement, while the few tests made for three and four rows were well within the 5% which the author mentions. In all cases, the results were sufficiently accurate for practical use. Should greater accuracy be desired, larger charts may be constructed. The results obtained from the given charts are as accurate as is consistent with the assumption that the vertical components due to shear are distributed equally among the rivets, whereas the vertical components due to moment are not. This assumption is consistent with the usual theory that a concentric load is distributed equally among a group of rivets, notwithstanding the fact that absurd results may be obtained when it is applied to extreme and impractical problems.

The use of these charts will create a demand for additional similar charts, and individuals will doubtless make them to meet their own requirements. For example, a horizontal distance, $w = 5\frac{1}{2}$, for two rows of rivets, and $w_1 = 5\frac{1}{2}$ for the inner distance for four rows, would conform to the usual rivet spacing in columns and probably would meet with greater favor than the 3-in. distance. Similarly, it may be desirable to use a vertical distance other than 3 in.; it is often better to spread the rivets to $3\frac{1}{2}$ or 4 in. than to add more rivets.

In this connection attention should be called to similar charts by Odd Albert, Assoc. M. Am. Soc. C. E.,[3] which may be used for different spacing, both horizontal and vertical. For rivets in a single row the latter chart may be used to determine the number of rivets directly, but the chart for two or more rows can be used only for an assumed number of rivets. Even so, the correct number can be found more readily than by calculation.

James R. Bole,[4] Jun. Am. Soc. C. E. (by letter).—An interesting and useful solution of the problem of determining stresses in riveted connections, subjected to eccentric loads, is presented in this paper. The writer does not believe, however, that alignment charts are best suited for a graphical solution to the cases of three and four rows of rivets. Fig. 5, for example, gives a solution, by means of a family of curves, for a case of four rows of rivets, in which $w_1 = 5\frac{1}{2}$ and $w_2 = 3$. This would be the standard spacing of rivet lines in the flange of a heavy column.

---

[2] Associate Prof., Structural Eng., Yale Univ., New Haven, Conn.

[3] *Civil Engineering*, February, 1931, p. 413.

[4] Long Beach, Calif.

By substituting Equations (18) and (19) in Equation (17), the following result is obtained,

$$A = \frac{n\,M}{\sqrt{[e\,(n-4)\,p]^2 + [4\,e\,(w_1 + 2\,w_2) + M]^2}} \quad \ldots\ldots\ldots \quad (23)$$

in which, $M = (w_1)^2 + (w_1 + 2w_2)^2 + \frac{1}{24}\,(n^2 - 16)\,p^2$.

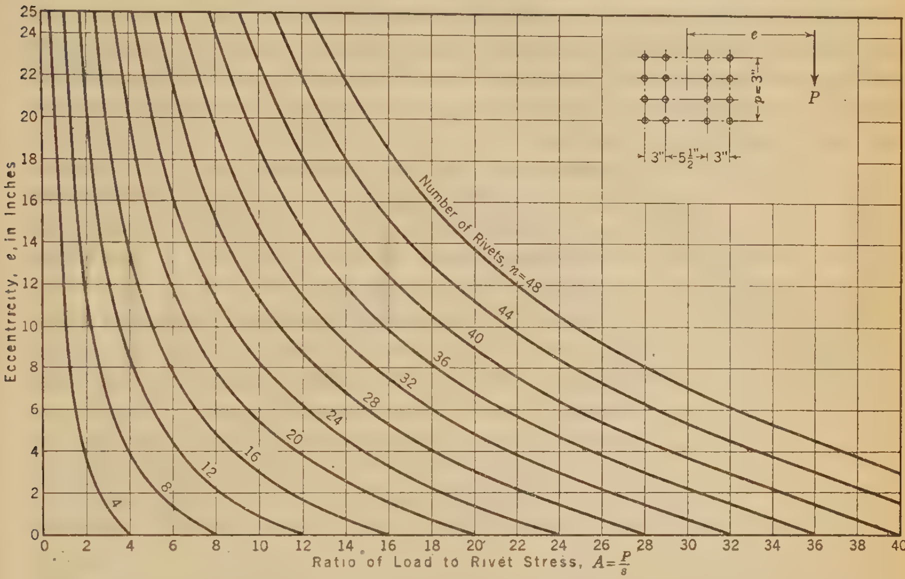


FIG. 5.

Fig. 5 was constructed by solving Equation (23) for various values of *e* and *n*. Charts of this type have several advantages over those presented by Mr. Dubin. Their construction does not require knowledge of alignment charts, or of the special method mentioned in the paper; they are more accurate and more compact.

ALBERT WERTHEIMER,[5] ESQ. (by letter).—The manner in which the complicated equations of this paper are finally presented in the form of alignment charts is of special interest. The method is quite general, in that it can be applied to any set of data in three variables, given either in tabular form, or as a family of curves. The general theory and application of this method have been discussed by the writer elsewhere.[6]

The chart obtained from any given set of data is not unique; as a rule, several types of charts can be obtained to represent the same data even with the same degree of accuracy. The choice of any particular type depends upon other factors, such as the desirability of having one of the scales a straight line, etc.

---

[5] Care, Bureau of Ordnance, U. S. Navy Dept., Washington, D. C.

[6] "The Graphical Transformation of a Family of Curves into Straight Lines and the Construction of Alignment Charts", by Albert Wertheimer, *Journal*, Franklin Inst., Vol. 219, No. 3, March, 1935, pp. 343–363.

The excessive waviness of one of the scales in each chart of this paper could probably have been eliminated, if the other scales had been allowed to become curved instead of straight lines. However, this is only a matter of appearance, as the charts in their present form are sufficiently accurate for all practical purposes.

Kenneth L. DeBlois,[7] Assoc. M. Am. Soc. C. E. (by letter).—A method of solving certain eccentric riveted joints directly instead of by the usual "cut and try" method, is presented in this paper. The author is to be commended for his clear and concise presentation of the analytical and nomographical solutions. As he states, the problem has been restricted to a consideration of the case in which the action line of the load is parallel to the rows of rivets in the group. The four examples given will fit numerous cases of web splices, crane brackets, cantilever beam connections, and bracing angles, in which the riveted connection is often eccentric to the center of gravity of the member.

In bridge truss design the author's examples might be of use in the top chord joints where the line of action of the load is usually eccentric to the rivet group. Practical considerations make the direct solution of heavy members more difficult, due to the different spacing of rivet lines and the number of rivets per row, to staggering, and to the unsymmetrical arrangement of rivets. The case may occur in which some rivets are in single shear, and the others in bearing, in the same joint. Each joint of a heavy truss is a special problem and the "cut and try" method is probably the most convenient. When the author's charts do not apply and when it is desired to find the polar moment of inertia, Σr^2, of a symmetrical joint, Equations (3), (10), (16), and (22) afford a quick method.

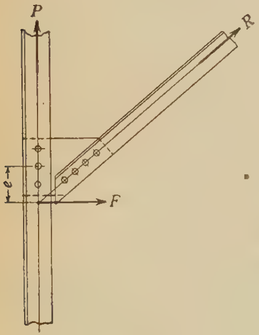


Fig. 6.

Example 1 can be extended to cover the case as shown in Fig. 6. The author's nomenclature is used with the addition of the force, F, normal to the line of action of the rivet row. In Equation (24) F and P are components of the load, R:

$$s^2 = \left(\frac{P}{n}\right)^2 + \left(\frac{F\,e\,r_0}{\Sigma\,r^2} + \frac{F}{n}\right)^2 \quad \text{..................(24)}$$

Substituting values of r_0, Σr^2, and S, as given in Equations (2), (3), and (6), respectively, Equation (24) is convertible to the form,

$$\frac{1}{A^2} = \frac{1}{n^2} + \frac{F^2}{P^2}\left(\frac{e}{S} + \frac{1}{n}\right)^2 \quad \text{..................(25)}$$

corresponding to Equation (7). In a similar manner, Example 1 can be extended to two, three, and four rows of rivets.

---

[7] Assoc. Highway Bridge Engr., U. S. Bureau of Public Roads, San Francisco, Calif.

JONATHAN JONES,[8] M. AM. SOC. C. E. (by letter).—The topic of this paper has been treated in the Handbook[9] of the American Institute of Steel Construction, the solutions in both sources being restricted to certain frequently occurring special cases, for each of which a general formula is given.

The principal difference is that the author presents alignment charts for ready solution of the several formulas, whereas, the Handbook presents tables, which, for values of eccentricity other than the exact values tabulated, require interpolation. It must be granted that interpolation is sufficiently accurate for the purpose; the choice becomes, then, one of individual preference, and the writer feels that most draftsmen will prefer the tables, which are an extension of similar tables long since published in handbooks on steel construction.

There is one distinct advantage, however, in the Handbook table over the author's chart for the case of four rows of rivets, when the most usual application is considered; that is, in eccentric connections to building columns. The author uses 3 in. for the distance between gauge lines, which distance is not practical for column flange detailing. In the Handbook tables, the distances between gauge lines are those which have been standardized by the fabricating industry for column detailing. It is believed, therefore, that the presentation in the Handbook, is a superior one for the usual structural drafting-room.

ARMIN ROZMAN,[10] ESQ. (by letter).—The method of developing alignment charts and formulas for eccentric riveted connections presented by the author is a valuable contribution to the field of designing details. The construction of this type of chart becomes comparatively simple and rapid. In his paper, Mr. Dubin presents the most commonly used charts which only need to be copied for application. Other eccentric problems can be easily solved by his method.

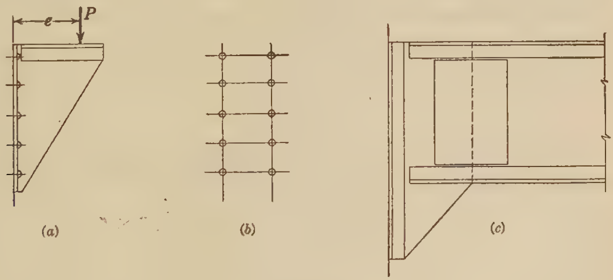


FIG. 7.

For example, consider the detail of bracket connections, such as those shown in Fig. 7. The stress in the extreme rivet is the resultant of the vertical shearing stress and the moment-produced horizontal stress, or tensional stress. Theoretically, the computation of this horizontal stress is quite complicated. The moment is resisted by couples composed of concentrated tensional stresses on the rivets and uniform compression stresses, depending on the strength of the bracket itself. The problem is somewhat similar to that of reinforced concrete beams. It involves too many unknowns to be solved by a method practical for ordinary usage and for designing charts. In practice, however, it is assumed that the moment is resisted by couples acting around the gravity center line of the rivets, the couples being composed of compressional as well as tensional concentrated stresses on the rivets. As a result, the connection is stronger than necessary, and the "short-cut" is on the safe side.

---

[8] Chf. Engr., McClintic-Marshall Corporation, Bethlehem, Pa.

[9] "Steel Construction," 1934 Edition, pp. 222-223.

[10] Asst. Structural Engr., U. S. Treasury Dept., Washington, D. C.

The formulas will be similar to Equations (1) to (4) in Example 1: Equation (1) remains unchanged; Equation (2) becomes:

$$r_0 = p \, \frac{\left(\frac{n}{2} - 1\right)}{2} = \frac{p\,(n - 2)}{4} \quad \dots\dots\dots\dots (26)$$

Equation (3) becomes:

$$\Sigma r^2 = \frac{p^2\, n}{48}\,(n - 2)\,(n + 2) \quad \dots\dots\dots\dots (27)$$

and Equation (4) becomes:

$$\left(\frac{1}{A}\right)^2 - e^2 \left[\frac{12}{p\, n\,(n + 2)}\right]^2 = \frac{1}{n^2} \quad \dots\dots\dots\dots (28)$$

With a standard value of p, Equation (27) can be solved by an alignment chart.

In the design of plate-girder web splices, Fig. 1 cannot have as general use as would appear on first thought. For illustration examine first a simple girder with one concentrated load (see Fig. 8(*a*)). At any section the moment is equal to the shear (in this case, R_1) times the arm, x, which varies from 0 to L_1. Next, examine a simple girder with uniform load (see Fig. 8(*b*)). At any section the moment is equal to the shear times the arm. The shear is equal to the resultant of all the forces to the left of the section, or $R_1 - u\,x$, and the point of action of the resultant force is outside the span. For a section taken at the center of the span the point of action of the resultant force is at infinity and the force is equal to 0.

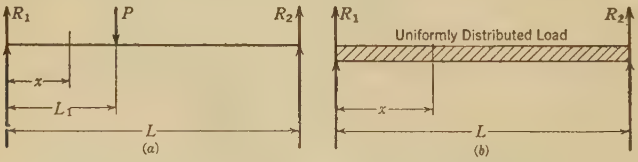


FIG. 8.

Similar reasoning can be offered for other types of loading, and, therefore, the arm can never be less than the distance between the splice and the support (as in the case of one concentrated load), and its maximum is infinity (as in the case of uniform loading).

The moment is maximum where the shear is zero, and it follows that at any point of maximum moment the arm is infinitely long; that is,

$$\frac{\text{Moment}}{\text{Shear}} = \text{arm}; \frac{\text{Maximum moment}}{0} = \infty$$

A plate girder web is usually spliced if the girder is too long to be fabricated or to be shipped in one piece, and then it is spliced somewhere between the third point and the center. A girder less than 40 to 50 ft long is seldom spliced, and, therefore, a chart to meet the foregoing requirements must have as its limits of eccentricity about 15 ft to infinity, which does not seem practical. A chart for web splices, however, can be utilized profitably in designing wind-bracing brackets of plate girders, in which case it is commonly assumed that the moment is equal to a vertical concentrated load times one-half the span. The eccentricity is always one-half the span. Moreover, since generally the vertical shear in the girder due to the wind forces is neglected, Equation (4) and, consequently, Fig. 1 can be simplified. Naturally, the splice must be investigated independently for the shear due to the gravity load.

Care should be exercised, however, in computing the area, A, when using the chart, because s is not the allowable rivet value as in the case of simple eccentric connections, but depends on the location of the extreme rivet as compared with that of the extreme fiber.

A. E. R. de Jonge,[11] M. Am. Soc. C. E. (by letter).—The subject of eccentric riveted connections has already been dealt with many times by numerous investigators.[12] Thus, the subject is not a novel one.

As the author states at the beginning, he has set himself the task of eliminating the "cut-and-try" method necessitated by the "families of curves" used by his predecessors. For these families of curves he substitutes an alignment chart.

In itself, there can be no objection to the use of an alignment chart if it is easily constructed and allows of sufficient accuracy in obtaining the results. In order to construct an alignment chart, the author develops formulas which express the relationship between load, eccentricity, "stress" in the extreme rivet, spacing of rivets, and number of rivets in the group. At this place the critical reader misses the diameter of the rivet which, of course, is another important variable in this problem.

Furthermore, the author restricts himself to the case of a force, the line of action of which is parallel to the rows of the rivet groups, and to a rivet spacing (rivet pitch) of 3 in. In the case of two rivet rows, he considers a spacing of the rows from 3 to 10 in., while in the case of three and four rows he deals with a 3-in. spacing between rows only. Thus, it follows from the beginning that the author has covered only a few of the possible cases, so that his alignment charts require considerable extension before they can cover all the cases that may be solved by the families of curves used by his predecessors. In fact, he too would require families of curves to cover all cases possible, and, in the alignment charts, these families of curves would be far more complicated (note the wavy curves and the overlapping w-curves for various values of n in the examples treated) than those of his predecessors. Besides they would make the reading off of the values much more troublesome, quite apart from the far greater amount of labor required in constructing these charts.

---

[11] With Babcock & Wilcox Co., New York, N. Y.

[12] See "Rivet Joints; An Historical Survey of Their Development, with a Bibliography and Abstracts from the Literature", by A. E. R. de Jonge, Research Publications, A. S. M. E. (1934).

Moreover, alignment charts are simple and accurate only when they consist of straight lines. The mere fact that the curves must actually be drawn involves errors, which it is impossible to predict and to guard against, and considerably complicates the task of laying out these charts. Undoubtedly, the severity of this task would have become more evident had the author described how he actually arrived at the curves and constructed the charts, and since he has covered only a small fraction of the possible cases, this description of the process of construction is the very subject that is of a far greater interest to the average engineer than the few ready charts included in the paper.

Another fact that should not be overlooked is the accuracy of the charts for certain conditions. For example, in Fig. 1, let the ratio of the load to the rivet stress be 4.5, while the eccentricity be 2 in.; then, the intersection between the n-curve and the straight line joining the two points on the respective guide scales is so flat that the slightest deviation from the true location, due to errors committed in laying out or drawing the curves, changes the result considerably. In this case, the uncertainty of the result lies between five and seven rivets. The same holds for the other charts in similar cases, as, for example, in Fig. 2 for an eccentricity of 2 in. and a ratio of load to rivet stress of 3, in which case the uncertainty of the spacing between rivet rows is between $w = 6$ and $w = 9$ in. for $n = 4$ rivets. Similar conditions prevail in the remaining charts of Figs. 3 and 4. Thus, it cannot be said that the accuracy in the cases cited is great, an obvious drawback to the use of the alignment charts. Further disadvantages are the uneven increments of the various scales (since they likewise affect the accuracy of the alignment charts) and the necessity of keeping a ruler handy when using the charts.

Furthermore, the author's use of the term, "total stress", in the extreme rivet is unfortunate because stress is now understood to denote what was formerly called "unit stress". In fact, the author obviously means "load on the extreme rivet." This ambiguity could have been avoided easily if, in the list of notations, the dimensions of the various quantities had been stated definitely. On the other hand, the introduction of the "load" on the extreme rivet is the reason why the rivet diameter, this rather important variable, does not appear in the author's formulas and charts. The very objection to the older methods which the author intends to eliminate, namely the "cut and try" method, is thus re-introduced by him in that an assumption must be made with regard to the diameter which, if it should prove unsuitable, would have to be amended by a different choice. This, however, is nothing but a "cut and try" process.

There remains then the setting up of the alignment charts which, for Example 1, can be accomplished readily by the perusal of any treatise on graphic representations and alignment charts, although it would be necessary for the reader to obtain such a treatise before he was able to construct the chart, unless he is already conversant with the methods used.

In the case of Examples 2 to 4, the author admits that even such treatises would not help the reader, as there does not as yet exist a standard method that would readily allow of the construction of the alignment charts which represent the respective formulas. In fact, the author refers to a method by A. Wertheimer, of the Bureau of Ordnance, United States Navy Department, which, by a process of successive approximations, allows of setting up such an alignment chart. Unfortunately, the author has given no reference as to where the description of this method may be found; nor has he stated whether it has been published at all. Under these circumstances it is neither possible to check the charts for Examples 2 to 4, nor to make use of alignment charts for cases other than those covered by the charts given in the paper. Thus, it would be of value to the profession if these omissions were rectified by the author.

After having discussed the shortcomings of the paper with regard to detail it seems appropriate to review it also from a more general point of view.

What has the author really accomplished? What new and permanent good has his paper brought the profession?

The answer to these questions can only be that he has introduced a new method of plotting results that are already known. As far as advancing the theory of structures is concerned, the author does not bring forward any new points since his assumptions are the same as those used by all his predecessors, namely: (1) A perfectly rigid plate; (2) an even distribution of the direct load over all the rivets of the joints; (3) direct contact between rivet shank and plate; (4) all the rivets act in shear; and, (5) deformation of the rivet shanks in proportion to their distance from the center of gravity of the rivet group.

It is well known that none of these assumptions holds true in practice. The plate is not perfectly rigid; and the rivets of the joint do not carry an equal share of the direct load, but the outermost rivets of the group (in the direction of the line of action of the force) carry by far the greater proportion of the load as was clearly shown by A. Hrennikoff in his paper[13] and in the discussion which ensued, particularly by the literature cited at that time by the present writer. Furthermore, the rivet shanks do not bear against the plate at all, at least not if the connection has not been over-strained previously so that permanent slip and deformation have occurred. This is due to the fact that, at working loads, the riveted joint carries the load by the frictional resistance developed between the plates. Finally, the bending moment distorts the outer parts of the bracket or connection plate more than the inner parts, due to the greater stresses which act in the outer parts. Thus, the assumption that the rivets, in supporting the bending moment, carry a load in proportion to their distance from the center of gravity of the group is likewise incorrect, the outer rivets supporting, in this case, too, the greater part of the moment.

---

[13] "Work of Rivets in Riveted Joints", by A. Hrennikoff, *Transactions*, Am. Soc. C. E., Vol. 99 (1934), pp. 437–489.

It is perfectly clear, therefore, that the outer rivets carry by far the greater part of the eccentric load, contrary to the results shown by the older methods of calculation, including that by the author's alignment charts. Thus, these charts do not give the correct loading that comes on the rivets, and the paper can only be of academic interest as establishing a new method of presenting, in a new mathematical form, data already known to be incorrect.

In this connection, a few words on the excessive use of mathematics in cases where it is not essential, are in order. It is a great pity that, at present, many investigators take up only the mathematical side of engineering problems without duly considering the physical (and practical) foundations underlying them. Many highly complicated treatises are written nowadays based on greatly simplified assumptions, thus being of doubtful value in practice. This is particularly the case in the theory of structures, and a simple reflection often shows that the results arrived at must be fallacious on that very account, as they do not take into consideration the underlying physical conditions which, generally, are of a very much more complex nature than the simplified assumptions made.

To give an example: What could be simpler than an ordinary pin such as is often used in bridge or truss construction. It is generally assumed that it is subjected to a bending moment and shear, the former giving a stress generally far in excess of the shear stress. If, however, instead of the Navier theory, which makes simplifying assumptions, the more correct mathematical theory of stress is used, as was done by Friedrick Bleich,[14] it is found that the stresses near the ends of the pin are far in excess of both those due to bending and those due to shear. Another example is the distribution of stress at points of fixation, a problem which only in quite recent times is beginning to be studied.[15] Furthermore, in all problems it is assumed that the material is perfectly homogeneous and isotropic, a requirement which no engineering material fulfills. A little blow-hole or inclusion may alter the calculated stresses far beyond the yield point. These are all facts that are often completely overlooked; and yet, it is the correct balance between the physical foundations of a problem, the simplifying assumptions, the mathematical treatment (graphical or analytical treatment), and the accuracy in calculation (which often is driven unnecessarily far beyond requirements), that constitute a practically useful solution of a problem.

---

[14] "Der gerade Stab mit Rechteckquerschnitt als ebenes Problem", von Friedrich Bleich, *Der Bauingenieur,* Vol. 4, pp. 255–259, May 15; pp. 304–307, May 31; pp. 327–331, June, 1923. See, also, his "Theorie und Berechnung eiserner Brücken", Chapter 4, Paragraph 15, Julius Springer, Berlin, 1924.

[15] For example, "Der Einfluss von Einspann-und Kraftangriffsstellen auf die Dauerhaltbarkeit der Konstrukionen", von A. Thum and F. Wunderlich, *Zeitschrift* für Verein deutscher Ingenieure, Vol. 77, No. 31, pp. 851–853, August 5, 1933.

A further example of the trend of the times is the following: When the writer recently was asked for advice by a University Research Fellow regarding a certain stress problem, and when he suggested that a graphical solution might yield results of sufficient accuracy for practical purposes where the analytic methods would give either no results, due to the complexity of the problem, or results which (due to the simplifying assumptions necessary to solve the differential equations) would not be in conformity with the actual stresses obtaining in the problem, the gentleman in question concurred readily that a graphic treatment might yield a result, but stated that such a solution would not be as highly scientific (meaning mathematical). This answer is typical of the attitude often found.

Lord Kelvin is credited with having stated that "a problem is never solved until it has been reduced to its simplest terms". To this must be added that its foundation must be sound and should be investigated first.

It is often more necessary for engineers to display sound common sense (as witness the early pioneers who, with their very incomplete knowledge of the theory of stresses, erected buildings that have stood the test of time), than to rely on calculations of doubtful value. In this respect, the writer recalls Sir Frederic Bramwell's definition of "engineering" as: "The art of drawing sufficient conclusions from insufficient premises". In fact it is often better to rely on such conclusion drawn by common sense than on mathematical derivations which are based on insufficient premises. If this would be constantly kept in mind, the rapidly growing technical literature would benefit by the absence of many papers and articles which only tend to confuse the mind of the engineer earnestly searching for solutions of problems that conform as closely to the actual conditions as is possible with the means presented by the respective state of development of science.

Thus, if, for example, the author had investigated eccentric loaded connections by the photo-elastic method, he would have come nearer the truth than by the alignment charts presented as the result of his endeavors.

FANG-YIN TSAI,[16] ASSOC. M. AM. SOC. C. E. (by letter).—The formulas and alignment charts presented in this paper are certainly great improvements over similar devices for the solution of the problem, and the author is to be commended for his service to the Engineering Profession. Four alignment charts are presented for the four common cases (group of rivets consisting of one, two, three, and four vertical lines), in which the vertical pitch, *p*, is fixed at 3 in. uniformly. Although a uniform 3-in. pitch is usually to be preferred in structural detailing, this is not always possible, and, in the worst case, a varying pitch may have to be used to fit some unusual condition. If a uniform pitch, p, is also taken as a variable within a certain range (say, from $2\frac{1}{4}$ in. to 6 in., with a variation of $\frac{1}{4}$ in.) in constructing those charts, they would certainly have a much wider range of application.

---

[16] Prof., Dept. of Civ. Eng., National Tsing Hua Univ., Peiping, China.

Four sets of formulas are also presented for the four common cases. These formulas are not only too cumbersome, but too limited in their application, because no equation is given for any rivet group consisting of more than four vertical lines. Elsewhere, the writer has derived[17] a general formula that may be applied to a rivet group consisting of any number of vertical lines, with the limitations that all vertical lines have the same number of rivets and the same uniform pitch, and that the line of action of the eccentric load is parallel to the vertical line of rivets.

Although the formulas therein presented are somewhat lengthy they are general and comprehensive and are preferable to shorter formulas applicable to particular cases only. Alignment charts constructed on the basis of the writer's more general formulas would undoubtedly have a much wider application than those presented by the author.

Odd Albert,[18] Assoc. M. Am. Soc. C. E. (by letter).—Credit is due Mr. Dubin for the novel manner in which he presents his problems. Unfortunately, the method advanced in the paper will not allow the use of more complicated formulas, because as the author modestly admits in connection with Equation (10) "the resulting function cannot be transformed to fit any standard form for which an alignment chart may be plotted."

By using the so-called "successive charts" it is possible, however, to express graphically almost any kind of formulas; but it is most important to have these formulas first transformed into a certain type that can be used in plotting a chart. Very often this is a problem in itself. As a case in point, consider the following formula:[19]

$$s = \frac{P}{n}\sqrt{C^2 B + C + 1} = \frac{P}{n}E \ldots\ldots\ldots\ldots\ldots (29)$$

in which, in addition to the notation of the paper,

$$C = \frac{e n w (N-1)}{\Sigma r^2} = \frac{12\, e\, w\, (N-1)}{p^2\,(n_0^2 - 1) + w^2\,(N^2 - 1)} \ldots\ldots\ldots (30)$$

and,

$$B = \frac{r_0^2}{w^2\,(N-1)^2} = \frac{p^2\,(n_0 - 1)^2 + w^2\,(N-1)^2}{4\,w^2\,(N-1)^2} \ldots\ldots\ldots (31)$$

In Equations (30) and (31) n_0 = number of rivets in each row, and N = number of vertical rivet rows. Equation (29) corresponds with Equation (8) of the paper, except that it is more general and applies to any number of rivet rows.

---

[17] *Civil Engineering*, March, 1935, p. 178.

[18] Faculty Lecturer, Advanced Design of Structures, New York Univ., New York, N. Y.

[19] *Civil Engineering*, February, 1935, p. 414, Equation [2].

If applied to two rows of rivets, Equations (30) and (31) take the form:

$$C = \frac{12\, e\, w}{p^2\,(n^2_o - 1) + 3\, w^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (32)$$

and,

$$B = \frac{p^2\,(n_o - 1)^2 + w^2}{4\, w^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (33)$$

The adaptation of Equation (29) to graphical solution may be demonstrated by reference to Fig. 9, to determine the maximum rivet stress for the extreme rivet in an eccentrically loaded rivet group under the condition that the vertical pitch, p, and the horizontal spacing, w, are equal. For this case,

$$C = \frac{12\, e}{w\,(n^2_o + 2)} \quad \ldots\ldots\ldots\ldots\ldots\ldots (34)$$

and,

$$B = \frac{(n_o - 1)^2 + 1}{4} \quad \ldots\ldots\ldots\ldots\ldots\ldots (35)$$

Consider a connection to be designed with ten rivets in two vertical rows. The load, P, = 28 000 lb (28 kips), applied with an eccentricity of 10 in. Let $w = p = 3$ in. In Fig. 9, beginning at $e = 10$ in., intersect $p = w = 3$, horizontally. From this intersection produce a vertical line to an intersection with $n = 10$. A horizontal line from this latter intersection will cut $P = 28$ such that the final vertical line yields $s = 9\,650$ lb.

For the case treated in Example 2 of the paper, Equation (29) may be used again. From Equations (30) and (31):

$$C = \frac{12\, e\, w}{9\,(n^2_o - 1) + 3\, w^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (36)$$

and,

$$B = \frac{9\,(n_o - 1)^2 + w^2}{4\, w^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (37)$$

In this connection it may be mentioned that the polar moment of inertia, $\Sigma\, r^2$ can be expressed as follows:

$$\Sigma\, r^2 = \frac{[(n^2_o - 1)\, p^2 + 3\, w^2]\, n}{12} \quad \ldots\ldots\ldots\ldots\ldots\ldots (38)$$

which is a simpler expression than Equation (10). For a rivet group of N rows of rivets the following expression may be derived:

$$\Sigma\, r^2 = \frac{[(n^2_o - 1)\, p^2 + (N^2 - 1)\, w^2]\, n}{12} = \frac{e\, n\, w\,(N - 1)}{C} \quad \ldots\ldots (39)$$

Equation (39) is easily converted into the forms of Equations (16) and (22) by making $N = 3$ and 4, respectively, and $n_o = \frac{n}{3}$ and $\frac{n}{4}$, respectively.

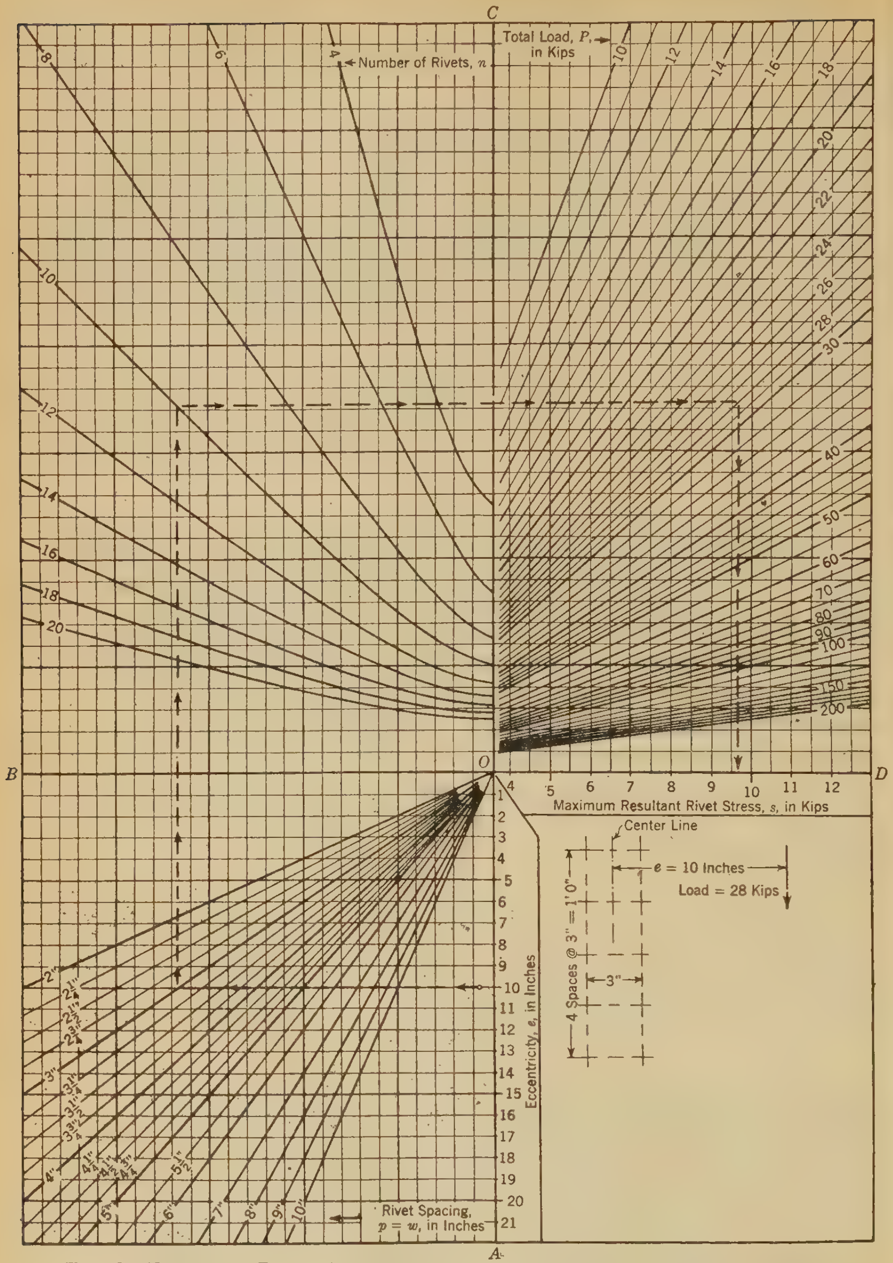


Fig. 9—Eccentric Rivet Connections with Two Vertical Rows of Rivets.

For the distance, r_0:

$$r_0 = w\,(N-1)\,\sqrt{B} \quad \text{.......................} (40)$$

and, for two vertical rivet rows, for example:

$$r_0 = w\,\sqrt{B} = \tfrac{1}{2}\,\sqrt{p^2\,(n_0 - 1)^2 + w^2} \quad \text{...........} (41)$$

It will be seen that Equation (40) is very simple as compared to Equations (9) and (18), and has a further advantage in being applicable to any number of vertical rivet rows, as has been shown by Equation (41).

If a rivet group is subject to bending only, the maximum rivet stress will be,

$$s = \frac{P}{n}\,C\,\sqrt{B} \quad \text{.......................} (42)$$

in which the values of C and B refer to Equations (30) and (31).

Thus far, the methods presented and charts described are mathematically correct, with no approximations. Mr. Dubin states that the results determined by using his charts may be in error as much as 5 per cent. If accuracy may be sacrificed, somewhat, in the interest of speed, it should be possible to save considerable time by assuming that: The rivets are in one, two, or more, vertical rows, symmetrically arranged in respect to a horizontal axis; in every row, pairs of rivets at the same distance from and on opposite sides of the neutral axis form couples and take moments in proportion to the distance between them; and the spacing between them in both directions equals three to five times the rivet diameter. Then, for one vertical row of rivets, the stress, s_B, on the extreme rivet due to the bending alone is,

$$s_B = \frac{6\,P\,e}{p\,n\,(n+1)} = \frac{P\,e}{p\,K} \quad \text{...................} (43)$$

in which the values of K for given numbers of rivets in one vertical row are as follows:

Number of rivets in each row	Corresponding value of K	Number of rivets in each row	Corresponding value of K
4	$3\frac{1}{3}$	10	$18\frac{1}{3}$
5	5	11	22
6	7	12	26
7	$9\frac{1}{3}$	13	$30\frac{1}{3}$
8	12	14	35
9	15	15	40

If there are several rows of rivets, the sum of the moments that each row is capable of withstanding, is assumed to be equal to the total outside moment. Thus, for instance, in the case of three rows of rivets the moment equals:

For the first row,

$$M_1 = p\,s_{B1}\,K_1$$

for the second row,

$$M_2 = p\,s_{B2}\,K_2$$

and, for the third row,

$$M_3 = p\ s_{B3}\ K_3$$

If the extreme rivets are at the same distance from the neutral axis, the s-stresses are alike. Furthermore, the outside moment equals $M_1 + M_2 + M_3$; and, consequently,

$$M = p\ s_B\ (K_1 + K_2 + K_3) \dots\dots\dots\dots\dots\dots (44)$$

and the maximum rivet stress caused by bending alone will be,

$$s_B = \frac{P\ e}{p\ \Sigma K} \dots\dots\dots\dots\dots\dots\dots\dots (45)$$

in which ΣK equals the sum of as many terms as there are vertical rivet rows. The number of rivets in each row only determines the value of K for this row.

The Resultant Stress.—As an eccentric load always produces a shear stress, s_A (due to the load divided by the number of rivets), in addition to the s_B-stress, the resultant stress will be,

$$s = \sqrt{s^2_A + s^2_B} \dots\dots\dots\dots\dots\dots\dots\dots (46)$$

which corresponds to Equations (1), (8), and (11). The s_A-stress is always vertical, and the s_B - stress is assumed to be horizontal. As soon as there are two or more rows, this is not true. As the last terms shown in Equations (8), (11), and (14) are omitted, the error caused by this omission is partly offset by the assumption that the s_B-stress is horizontal.

As an example, consider a rivet group, consisting of four vertical rows with thirteen rivets in each row. A vertical load of 100 000 lb is applied with an eccentricity of 32.7 in. If the vertical and the horizontal spacing is $4\frac{7}{16}$ in., the maximum rivet stress is determined as follows: The stress due to shear, $s_A = \frac{100\ 000}{52} = 1\ 922$ lb, and the value of K (from the list of K-values) $= 30\frac{1}{3}$; therefore, for four rows, $\Sigma K = 4 \times 30\frac{1}{3} = 121.32$; and $s_B = \frac{100\ 000 \times 32.7}{4.48 \times 121.32}$ $= 6\ 020$ lb. Therefore, the resultant rivet stress will be, $s = \sqrt{1\ 922^2 + 6\ 020^2}$ $= 6\ 310$ lb. The maximum stress should have been 6 490 lb, indicating an error of only 2.85 per cent.

The foregoing approximation is correct for one row of rivets and is accurate well within 5% for two or more rows. Allowing the same tolerance, it is quicker than that proposed by Mr. Dubin.

Eugene A. Dubin,[20] Esq. (by letter).—It is almost axiomatic that the best structural member is one that has the least number of component parts. The most simple and direct design is always to be preferred. Nevertheless, certain structural details of a complicated nature are necessary. A riveted connection is one such necessary detail.

---

[20] Washington, D. C.

Mr. de Jonge has indicated the nature of the problem by enumerating the assumptions upon which the paper is based. It is correct that these assumptions do not all hold true in practice; yet the nature of the action of a riveted connection is such that the simplifying assumptions used seem justified.

In any riveted connection, whether the load is eccentric or concentric, the direct stress is not distributed uniformly between the rivets. The distribution of the load among the rivets of a connection of many rows of rivets is quite indeterminate, since the load on one row of rivets depends on the yielding of the rivets in the rows nearer the load, and on the yielding of the plates. In a connection of many rows of rivets, the outermost rivets initially carry more of the direct load than those farther from the load. Over-stress in these rivets causes local yielding of the plates and the rivets, and tends to distribute the load more uniformly among the rivets. In connections of a ductile material, this is not serious. It must be borne in mind that abrupt changes in section of any member, introducing holes in a member (rivet holes), etc., cause localized stresses in the member at the hole, change in section, etc., that may reach considerable magnitude. The material around rivet holes is seriously damaged by punching. Rivet holes do not match perfectly. Rivets cannot be driven uniformly, especially in the field. Field rivets may not be heated uniformly. These factors all influence the actual distribution of the load.

Mr. de Jonge states,

> "* * * the rivet shanks do not bear against the plate at all, at least not if the connection has not been over-strained previously * * *. This is due to the fact that, at working loads, the riveted joint carries the load by the frictional resistance developed between the plates."

A well-driven rivet is assumed to fill the hole. The contraction of the rivet during cooling does introduce clamping action on the plates which develops frictional resistance. Tests have shown,[21] however, that slip may occur at ordinary working stresses.

It is well also to consider the nature of the stress in the rivets. The contraction of the rivet while cooling introduces an indeterminate degree of tension; the rivet is subject to bending (which may have an important influence on long rivets); and it is subjected to a shearing force. A determination of the actual resultant stress in the rivet is impossible. Obviously, neither the shearing stress nor the bearing stress (which is not uniform over the length of the rivet in bearing), is actually the governing stress. However, by tests, empirical values for the allowable shear and bearing have been determined, which offer criteria that make it possible to design riveted connections safely, considering only these stresses. The method presented by the writer offers a solution based upon these criteria. The nature of the problem warrants Professor Bishop's warning, "that absurd results may be obtained when it is applied to extreme and impractical problems."

---

[21] "Tests of Nickel-Steel Riveted Joints," by Arthur N. Talbot, Past-President and Hon. M., Am. Soc. C. E., and Herbert F. Moore, *Bulletin No. 49*, Eng. Experiment Station, Univ. of Illinois, Urbana, Ill.

Mr. de Jonge states that "alignment charts are simple and accurate only when they consist of straight lines." The writer wishes to point out that an alignment chart does not necessarily consist of straight lines. In fact, an exact, mathematically constructed chart for some equations may give one scale that is a curve resembling the letter, *S*, etc. The method of constructing alignment charts is well presented in standard works on the subject.[6]

The charts originally presented were restricted to groups of rivets with a 3-in. pitch, and, in Examples 3 and 4, to a spacing between rows of $w = 3$ in. Professor Bishop, Professor Tsai, and Mr. de Jonge have suggested that the charts would be more useful if they were extended to cases in which the pitch differed from 3 in. Mr. John Krippner suggested to the writer that the ratio, $\frac{e}{p}$, instead of e be plotted on one scale. By means of this device the charts can be used for any equal vertical spacing of rivets.

Consider Example 1 of the paper, in which $\frac{e}{p} = m$. For this case, Equation (4) becomes,

$$\left(\frac{1}{A}\right)^2 - m^2 \left[\frac{6}{n\,(n+1)}\right]^2 = \left(\frac{1}{n}\right)^2 \quad \ldots\ldots\ldots\ldots\ldots (47)$$

Since $p = 3$ in. was used to plot Fig. 1, this chart may be revised by dividing the numerical values of the "eccentricity" scale by 3, and marking it "Values of m" instead of "Eccentricity, e, in inches."

Mr. Rozman states that Fig. 1 cannot have any use in designing plate-girder web splices, since a chart for such purposes requires that the range of the eccentricity be "about 15 ft to infinity." Fig. 1 does cover only a limited range, but Mr. Rozman's statement is in error. The moment in the web is not the total moment in the girder. It is common to assume that one-eighth the area of the web acts as flange material. The moment in the web is, $M_w = \frac{\frac{1}{8} A_w}{A_f + \frac{1}{8} A_w} \times M_G$, in which M_w = moment in the web; A_w = area of the web; A_f = area of the flange; and, M_G = total moment in the girder. The moment in the web approximates 5% to 15% of M_G.

Assuming that $M_w = 0.1\, M_G$, consider Mr. Rozman's examples. In Fig. 8(*a*), assume P at the mid-span, L, varying from 30 ft to 100 ft, and web splices at $\frac{L}{3}$. At the web splice, the moment in the web due to P is $\frac{PL}{4} \times \frac{2}{3} \times \frac{1}{10} = \frac{PL}{60}$, and the shear due to P is $\frac{P}{2}$. The eccentricity then is $\frac{L}{30}$, or 1 ft to 3.33 ft for the range in spans considered. In Fig. 8(*b*), with a uniform load per foot, W, for the same range in spans and splices at the same points,

$$e = \frac{WL^2}{9} \times \frac{1}{10} \div \frac{WL}{6} = \frac{L}{15}$$

or 2 ft to 6.66 ft for the range of spans considered. Fig. 1 of the paper did not cover this range; therefore, Fig. 10, similar to Fig. 1, is extended to $m = 14$, which is equivalent to $e = 42$ in. when $p = 3$ in.

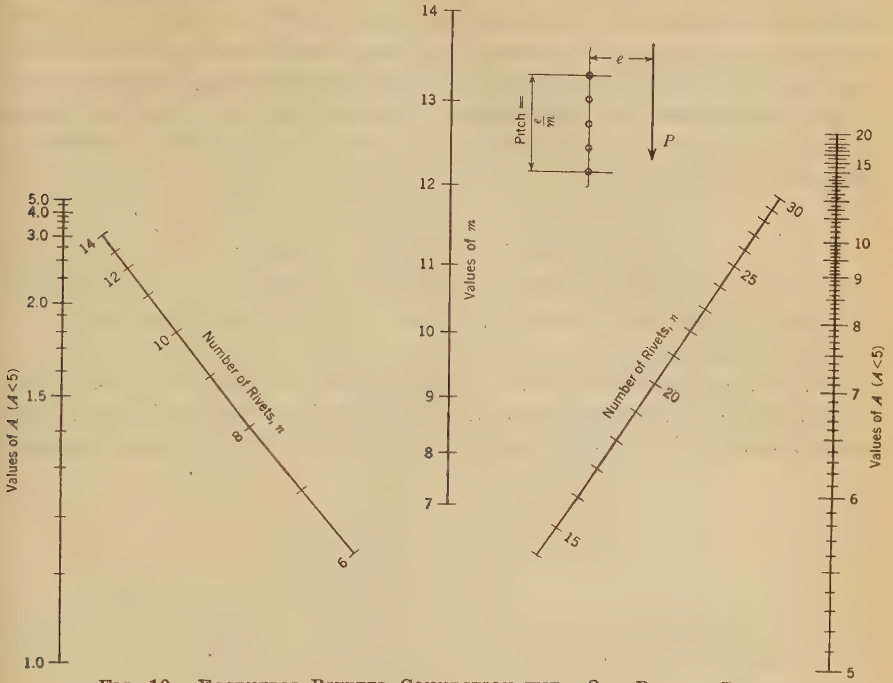


FIG. 10.—ECCENTRIC RIVETED CONNECTION WITH ONE ROW OF RIVETS; $m = 7$ TO 14.

The charts for Examples 2, 3, and 4 of the paper may be revised to provide for any equal vertical rivet spacing and any distance between rivet rows in the manner used for Example 1. For example, consider Example 2; substituting the values of r_0 and Σr^2 in Equation (11):

$$\left(\frac{1}{A}\right)^2 = \frac{1}{n^2}\left\{1 + \frac{4e^2\left[\frac{p^2}{4}(n-2)^2 + w^2\right]}{\left[\frac{p^2}{12}(n-2)(n+2) + w^2\right]^2} + \frac{4\,ew}{\left[\frac{p^2}{12}(n-2)(n+2) + w^2\right]}\right\} \quad \text{. . (48)}$$

or, if $e = m\,p$; and $w = m'p$:

$$\left(\frac{1}{A}\right)^2 = \frac{1}{n^2}\left\{1 + \frac{4\,m^2\left[\frac{1}{4}(n-2)^2 + (m')^2\right]}{\left[\frac{1}{12}(n-2)(n+2) + (m')^2\right]^2} + \frac{4\,m'\,m}{\left[\frac{1}{12}(n-2)(n+2) + (m')^2\right]}\right\} \quad \text{. . . . . (49)}$$

Equation (49) may be plotted with an m-scale and an m'-scale substituted for the e and w scales, respectively, of Example 2. Then, the numerical values of e and w, in Fig. 2, divided by 3, are the numerical values of m and m', respectively. The charts for Examples 3 and 4 may be extended to cover more than the one case originally presented, and constructed in a manner similar to Fig. 2, with m and m' substituted for e and w, respectively.

Mr. DeBlois has extended Example 1 to cover the case of a rivet group of one row subjected to an eccentric load with its line of action inclined to the axis of the rivet group. A more convenient form of expressing this might be,

$$\left(\frac{1}{A}\right)^2 = \left(\frac{1}{n}\right)^2 + \frac{e}{S}\left(\frac{e}{S} + \frac{2 \sin \alpha}{n}\right) \quad \ldots\ldots\ldots\ldots\ldots (50)$$

in which α is the angle between the axis of the rivet group and the action line of the load (see Fig. 11).

Mr. Rozman has developed a formula for the solution of bracketed connections. His assumption that the neutral axis of the tension and compression areas is the center of gravity of the rivet group, although commonly made, is not correct, theoretically. As he states, however, the assumption is on the side of safety. It must be borne in mind, nevertheless, that the tension in the rivets is normal to the plane upon which the shear occurs, and, therefore, Equation (28) does not hold. A bracketed connection must satisfy three requirements: (1) The bearing of the rivets must not exceed the allowable value of the bearing for the direct load; (2) the maximum shear on the rivet shank due to the direct load and the tension must not exceed the allowable shear value of the rivet; and (3) the maximum tension in the rivet due to the shear and the tension caused by the moment, must not exceed the allowable value of the rivet in tension. The design or investigation of a bracket for bearing on the rivets due to the direct load needs no discussion.

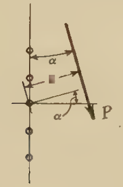


FIG. 11.

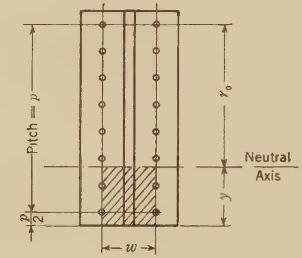


FIG. 12.

The maximum unit shearing stress on a member subjected to normal tension and direct shear is,

$$s'_s = \tfrac{1}{2} \sqrt{s_t^2 + 4\, s_s^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (51)$$

in which s_t = the tensile unit stress; and, s_s = the shearing unit stress. The maximum unit tensile stress is,

$$s'_t = \tfrac{1}{2}\, s_t + \tfrac{1}{2} \sqrt{s^2_t + 4\, s^2_s} \quad\ldots\ldots\ldots\ldots\ldots\ldots (52)$$

Since, for steel rivets the allowable tensile unit stress is equal to the allowable shearing unit stress, only the maximum tension need be considered when the rivets are of steel. In the bracketed connection shown in Fig. (7*a*),

$$s_t = \frac{P\, e\, r_0}{I\, a_r} \quad\ldots\ldots\ldots\ldots\ldots\ldots (53)$$

in which r_0 = the distance from the neutral axis of the rivets in tension and the area in bearing, to the topmost rivet; a_r = the area of a rivet; and I = the moment of inertia of the rivets in tension and the area in bearing about the neutral axis. Similarly,

$$s_s = \frac{P}{a_r\, n} \quad\ldots\ldots\ldots\ldots\ldots\ldots (54)$$

and, $s'_t\, a_r = \dfrac{1}{2} \times \dfrac{P\, e\, r_0}{I} + \dfrac{1}{2} \sqrt{\left(\dfrac{P\, e\, r_0}{I}\right)^2 + 4 \left(\dfrac{P}{n}\right)^2}$; from which,

$$\frac{2}{A} = \frac{e}{S} + \sqrt{\left(\frac{e}{S}\right)^2 + 4 \left(\frac{1}{n}\right)^2} \quad\ldots\ldots\ldots\ldots\ldots\ldots (55)$$

It is possible to determine the location of the neutral axis with a reasonable degree of accuracy. Assume that the width of angles between the rivets is in bearing, and that the distance from the bottom of the angles to the bottom horizontal row of rivets is $\dfrac{p}{2}$. Assume, also, that the neutral axis lies midway between two horizontal rows of rivets (see Fig. 12). The statical moments of the tension and compression areas about the neutral axis are equal, and,

$$\frac{w\, y^2}{2} = 2\, a_r\, \Sigma\, r \quad\ldots\ldots\ldots\ldots\ldots\ldots (56)$$

in which y (the distance from the neutral axis to the bottom of the angles) $= n_2\, p$; $\Sigma\, r = \dfrac{p\, n^2_1}{2}$; n_2 = the number of rivets below the neutral axis in one row; and n_1 = the number of rivets above the neutral axis in one row. Therefore,

$$n_2 = n_1 \sqrt{\frac{2\, a_r}{w\, p}} \quad\ldots\ldots\ldots\ldots\ldots\ldots (57)$$

or, $n_2 = \tfrac{1}{2}\, n - n_1$; and,

$$n_1 = \frac{n}{2\left(1 + \sqrt{\dfrac{2\, a_r}{w\, p}}\right)} \quad\ldots\ldots\ldots\ldots\ldots\ldots (58)$$

Neglecting the moment of inertia of the rivet areas about their neutral axis, $I = 2\,a_r\,\Sigma\,r^2 + \dfrac{w y^3}{12} + w\,y\left(\dfrac{y}{2}\right)^2$; or:

$$I = \frac{a_r\,p^2\,n_1}{6}\left[(2\,n_1 + 1)\,(2\,n_1 - 1) + 4\,n^2_1\sqrt{\frac{2\,a_r}{w p}}\right] \quad \text{........(59)}$$

Furthermore, $r_0 = \dfrac{p}{2}\,(2\,n_1 - 1)$; and:

$$S = \frac{I}{r_0} = \frac{a_r\,p\,n_1}{3\,(2\,n_1 - 1)}\left[(2\,n_1 + 1)\,(2\,n_1 - 1) + 4\,n^2_1\sqrt{\frac{2\,a_r}{w\,p}}\right] \quad \text{...(60)}$$

Equations (58) and (60) may be substituted in Equation (55), and for constant values of a_r, p, and w, an alignment chart may readily be constructed. Thus, the problem does not involve too many unknowns for a practical solution which observes theoretical considerations.

Mr. Bole prefers a family of curves to an alignment chart and Mr. Jones prefers tables. In these matters the individual habits of the engineer govern. Contrary to the opinion of Mr. Bole, however, alignment charts are accurate, and are not difficult to construct.

As Mr. Wertheimer points out, any set of data in three variables may be solved by the use of his method. The method involves a series of successive approximations whereby the results obtained may be brought to any desired degree of accuracy. The charts for Examples 2 to 4, inclusive, are accurate to within 5%, as shown, but they could be brought to a greater degree of accuracy. In view of the nature of the problem, this does not seem justified.

Mr. Albert has extended the problem to a solution similar to that of the writer by the use of a "successive chart." The chart presented in Fig. 9 offers a solution similar to the charts presented in Fig. 2. Fig. 9 is more limited in its scope, however, more cumbersome to use, and no more accurate in results. It is restricted to equal horizontal and vertical spacing of rivets, whereas Fig. 2 may be used for any equal vertical spacing between $m = 0$ and $m = 6\tfrac{2}{3}$, and any horizontal spacing between $m' = 1$ and $m' = 3\tfrac{1}{3}$. To solve the problem presented by Mr. Albert, lay a straight-edge across the three scales of Fig. 2(*b*) intersecting e at 10 in. (or $\dfrac{e}{p} = m$ at $3\tfrac{1}{3}$) and $n = 10$ at $w = 3$ in. (or $\dfrac{w}{p} = m' = 1$). The intersection on the A-scale gives $A = 2.9$; then $s = \dfrac{P}{A} = \dfrac{28\,000}{2.9} = 9\,650$ lb. The result agrees with that obtained by Fig. 9, and the solution was simpler.

Both Professor Tsai[22] and Mr. Albert have extended the problem to cover any number of vertical rows of rivets. The solution of this general case by either an alignment chart or a "successive chart" is not possible without reducing the number of variables involved by considering a specific number of vertical rows of rivets.

---

[22] *Civil Engineering*, Vol. 5, No. 3, March, 1935, pp. 178–179.

To facilitate the solution of an eccentric connection of more than one row of rivets, Mr. Albert proposes the assumption that "pairs of rivets at the same distance from and on opposite sides of the neutral axis form couples and take moments in proportion to the distance between them." He has presented a simplified formula and a table of K-values to aid in solving the formula. The method is the usual "cut-and-try" procedure of assuming the number of rivets first and then checking the connection to see that it is satisfactory. The assumption is justified, however, only when there are many rivets in each vertical row, and the distance between rows is not great.

Mr. Albert's proposal is essentially that the stress in each rivet due to bending shall be in proportion to the distance of the rivet from the neutral axis. What Mr. Albert does in his approximate solution is to divide the load uniformly between the number of rows of rivets in the connection and treat it as a connection of one row of rivets. In the introduction to the paper, the writer suggested that, with the foregoing conditions, "Fig. 1 may be used for a preliminary design for connections of more than one row of rivets that lie beyond the range of the other charts." The design is preliminary in character when the number of rivets in each vertical row is not large, or when the distance between vertical rows is great.

Fig. 1 (or Fig. 10, which is Fig. 1 extended) entirely eliminates the necessity for Mr. Albert's formulas and tables. Consider Mr. Albert's problem involving four vertical rows of thirteen rivets, under the assumption proposed herein: $P = 100\,000$ lb; $e = 32.7$ in.; and $p = 4\frac{7}{16}$ in. To find the resultant load on the extreme rivet, lay a straight-edge across the scales of Fig. 10 intersecting m at $\frac{e}{p} = \frac{32.7}{4\frac{7}{16}} = 7.39$ and $n = 13$. Thus, $\frac{P}{s} = 3.75$ and $s = \frac{25\,000}{3.75}$, or 6 660 lb. Since the theoretical value of $s = 6\,490$ lb, the error is 2.62 per cent. The error is acceptable. The solution is more readily accomplished than by use of Mr. Albert's tables and formulas.

Mr. Albert states that "the approximation is accurate well within 5% for two or more rows." This is not true, except as outlined previously. Consider the problem of four vertical rows of rivets with two rivets in each row: $p = w = 3$ in; $P = 10\,000$ lb; and $e = 15$ in. From Fig. 4 (*b*), $\frac{P}{s} = 1.3$, or $s = 7\,700$ lb. By Mr. Albert's method: $K = \frac{n\,(n + 1)}{6} = 1$; and, from Equation (45), $S_B = \frac{10\,000 \times 15}{3 \times 4 \times 1} = 12\,500$ lb and $S_A = \frac{10\,000}{8} = 1\,250$ lb. By solving Equation (46), $S = 12\,580$ lb. Consequently, the error is between 60 and 65 per cent.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

TRANSACTIONS

Paper No. 1917

DETERMINATION OF TRAPEZOIDAL PROFILES FOR RETAINING WALLS

By A. J. Sutton Pippard,[1] M. Am. Soc. C. E.

With Discussion by Messrs. E. S. Lindley, Kenneth L. DéBlois, M. A. Drucker, and A. J. Sutton Pippard.

Synopsis

In spite of recent advances in the theory of earth pressures, that due to Rankine is still widely used by engineers in designing profiles for retaining walls. The design is usually a matter of trial and error, a suitable section being guessed and then verified to see that it satisfies the conditions of stress required. A method of direct design is given herein which enables a profile to be determined rapidly. Cases dealt with are those of walls with vertical and sloping backs and with the earth level with the top of the wall, or surcharged. The base width of a suitable wall can be determined quickly for any value of height and top width by the use of diagrams and simple mathematical formulas.

(1) Whatever may be thought of the merits of the Rankine theory of earth pressure in the light of modern research it is still used widely as a basis for design. The object of this paper is to present a convenient and rapid method of determining the profile of a retaining wall, which will just satisfy the usual criteria for strength when it is subjected to an earth pressure calculated by this theory.

The walls considered are trapezoidal in section. The earth surface is either inclined at an angle or level with the top of the wall; that is, the wall may or may not be surcharged.

This problem generally involves either a somewhat laborious computation or a process of trial and error in combination with a graphical analysis. By the help of the diagrams given, the work is reduced considerably.

Note.—Published in August, 1934, *Proceedings*.

[1] Prof. of Civ. Eng., Univ. of London, Imperial Coll., City and Guilds Coll. (Eng.). London, England.

(2) The late Sir Benjamin Baker, Hon. M. Am. Soc. C. E., has stated[2] that,

> "Experiment has shown the actual lateral thrust of good filling to be equivalent to that of a fluid weighing about 10 lbs per cubic foot, and allowing for variations in the ground, vibration, and contingencies, a factor of safety of 2, the wall should be able to sustain at least 20 lbs fluid pressure, which will be the case if $\frac{1}{4}$ of the height in thickness.
>
> "It has been similarly proved by experience that under no ordinary conditions of surcharge or heavy backing is it necessary to make a retaining wall on a solid foundation more than double the above, or $\frac{1}{2}$ of the height in thickness. Within these limits the engineer must vary the strength in accordance with the conditions affecting the particular case."

The assumption of a fluid pressure of 20 lb per cu ft only gives the same result as a design based on the Rankine hypotheses for certain specific combinations of earth density and angle of repose. Thus, if ρ_e is the unit weight of earth fill, ϕ, the angle of repose, and H, the height of the wall, the lateral pressure on the vertical face of a non-surcharged wall is, by the Rankine theory,

$$P = \frac{\rho_e \, H^2}{2} \left(\frac{1 - \sin \phi}{1 + \sin \phi} \right) \ldots\ldots\ldots\ldots (1)$$

whereas, that on a wall retaining a fluid of density, ρ_L, is,

$$P = \frac{\rho_L \, H^2}{2} \ldots\ldots\ldots\ldots (2)$$

For these to be equal it is necessary that,

$$\rho_L = \rho_e \left(\frac{1 - \sin \phi}{1 + \sin \phi} \right) \ldots\ldots\ldots\ldots (3)$$

This is a general relationship, but if ρ_L is made equal to 20 lb per cu ft, Equation (3) becomes,

$$\frac{1 - \sin \phi}{1 + \sin \phi} = \frac{20}{\rho_e} \ldots\ldots\ldots\ldots (4)$$

and for any value of ρ_e there will be a corresponding value of ϕ, which will give the same pressure as that calculated from Equation (1).

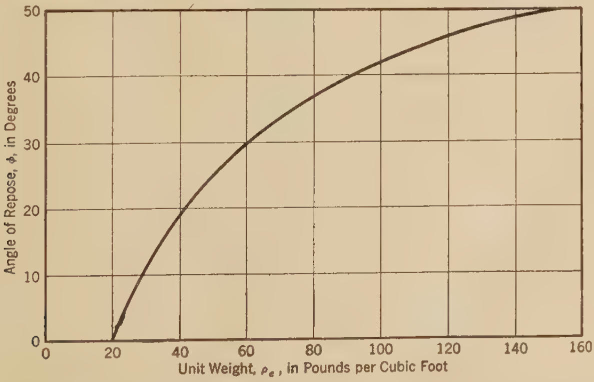


FIG. 1.

---

[2] "The Actual Lateral Pressure of Earthwork," by the late Sir Benjamin Baker, Hon. M. Am. Soc. C. E., *Minutes of Proceedings*, Inst. C. E., Vol. LXV, p. 183.

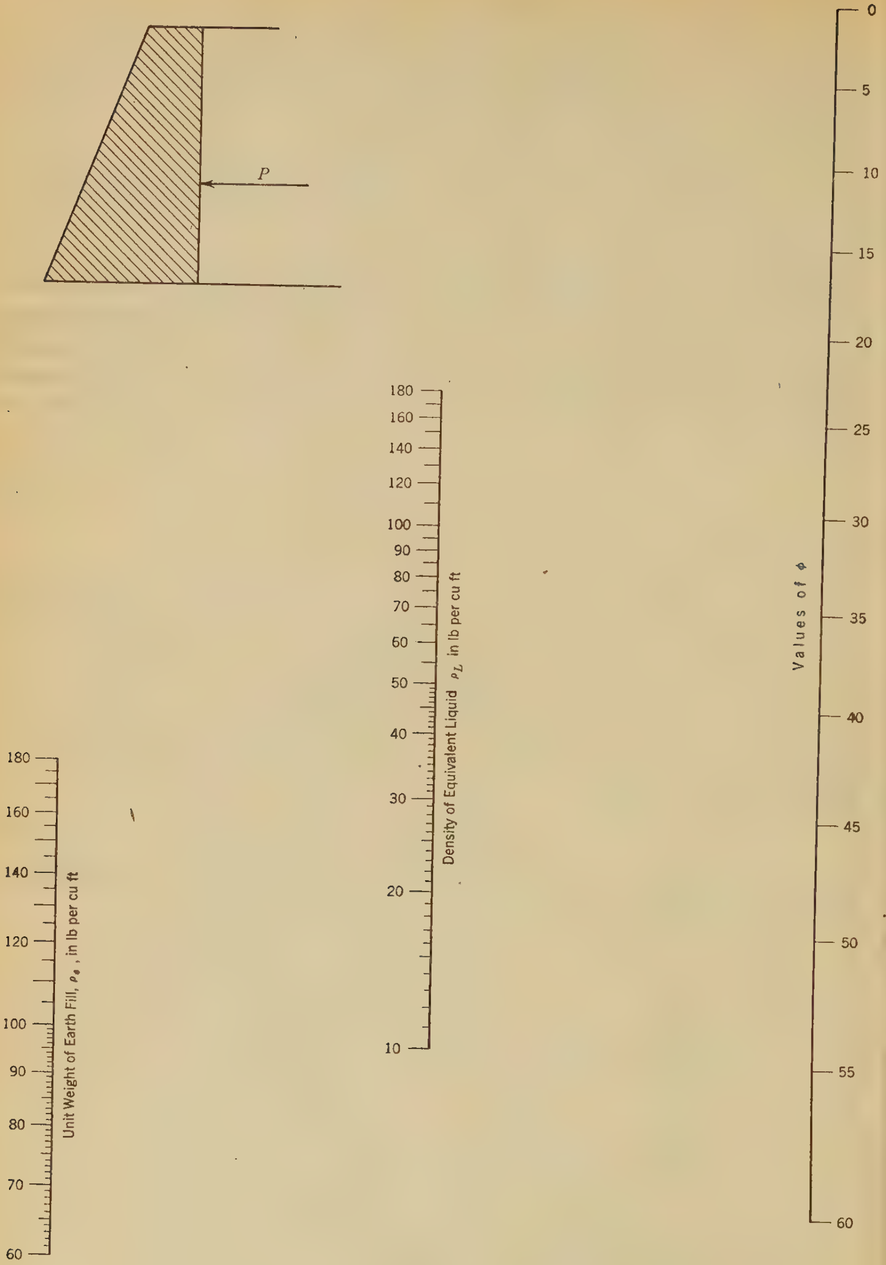

P
Unit Weight of Earth Fill, ρe, in lb per cu ft
Density of Equivalent Liquid ρL in lb per cu ft
Values of φ

FIG. 2.

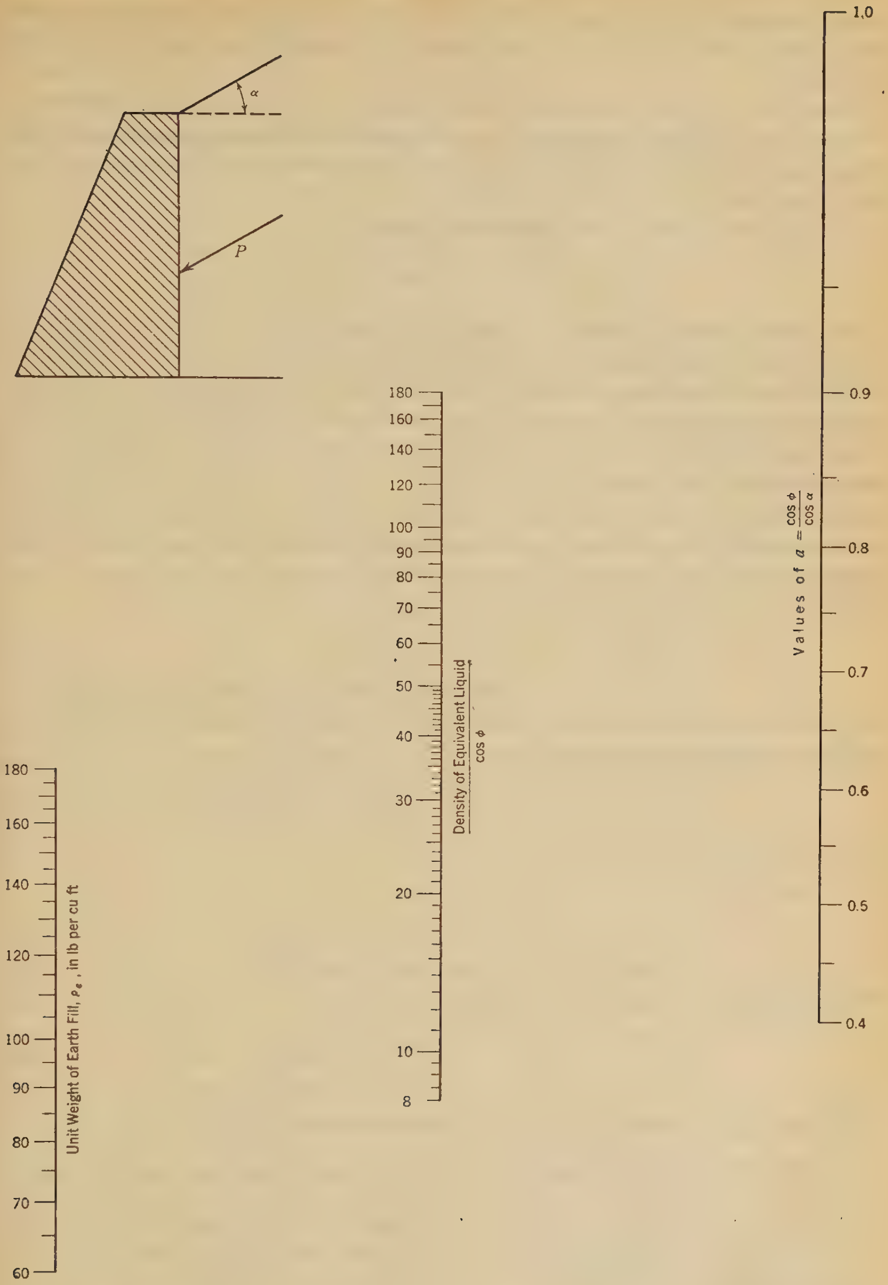

α
P
Unit Weight of Earth Fill, ρ_e, in lb per cu ft
180
160
140
120
100
90
80
70
60
Density of Equivalent Liquid / cos φ
180
160
140
120
100
90
80
70
60
50
40
30
20
10
8
Values of $a = \frac{\cos \phi}{\cos \alpha}$
1.0
0.9
0.8
0.7
0.6
0.5
0.4

FIG. 3

Fig. 1 shows the range of combinations of ρ_e and ϕ that will satisfy Equation (4), and it is evident that only a small region on this curve gives simultaneous values of ρ_e and ϕ which are likely to be associated in practical material.

(3) This method of viewing the problem can be used more generally, however, since for any values of ρ_e and ϕ, a value of ρ_L can be found from Equation (3), which will give exactly the same "Rankine" pressure as the earth considered.

This equation can be written in the form,

$$\log \rho_L = \log \rho_e + \log \left(\frac{1 - \sin \phi}{1 + \sin \phi}\right) \dots\dots\dots\dots\dots (5)$$

which lends itself to simple nomographic representation as shown in Fig. 2. If a line is drawn on this diagram joining the known values of ρ_e and ϕ on the two outsides lines, its intersection with the middle line gives ρ_L at once. As will be shown later, however, there is no need to calculate the value of P from Equation (2) because a suitable profile can be obtained directly from the value of ρ_L thus found.

(4) The case of the surcharged wall is not quite as simple as that just dealt with but, nevertheless, can be exhibited in a similar form. If α is the angle of surcharge, the earth pressure acting at one-third the height from the base is, according to the Rankine theory,

$$P = \frac{\rho_e H^2}{2} \cos \alpha \frac{\cos \alpha - \sqrt{\cos^2 \alpha - \cos^2 \phi}}{\cos \alpha + \sqrt{\cos^2 \alpha - \cos^2 \phi}} \dots\dots\dots\dots (6)$$

and this pressure is assumed to act parallel to the earth surface. The unit weight of the equivalent liquid is now,

$$\rho_L = \rho_e \cos \alpha \frac{\cos \alpha - \sqrt{\cos^2 \alpha - \cos^2 \phi}}{\cos \alpha + \sqrt{\cos^2 \alpha - \cos^2 \phi}} \dots\dots\dots\dots (7)$$

and, if $\frac{\cos \phi}{\cos \alpha} = a$, this becomes,

$$\frac{\rho_L}{\cos \phi} = \frac{\rho_e}{a} \left(\frac{1 - \sqrt{1 - a^2}}{1 + \sqrt{1 - a^2}}\right) \dots\dots\dots\dots\dots\dots (8)$$

Writing Equation (8) in the logarithmic form:

$$\log \frac{\rho_L}{\cos \phi} = \log \rho_e + \log \frac{1 - \sqrt{1 - a^2}}{a\,(1 + \sqrt{1 - a^2})} \dots\dots\dots\dots (9)$$

which can also be expressed as a nomogram as shown in Fig. 3.

To use this diagram the value of $\frac{\cos \phi}{\cos \alpha}$ is first calculated, and the point on the right-hand line corresponding to it is joined by a straight line to the value of ρ_e on the left-hand line. The intersection of this connecting line with the middle line of the diagram gives $\frac{\rho_L}{\cos \phi}$, and this needs only to be multiplied by the value of $\cos \phi$ already found, to give ρ_L.

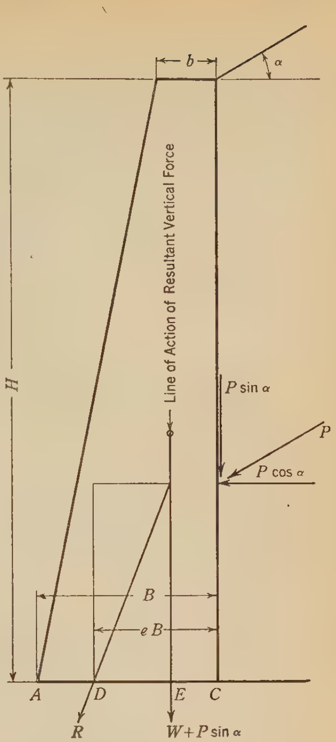


FIG. 4.

(5) The value of ρ_L having thus been found either from Fig. 2 or Fig. 3 (as appropriate), the profile of the wall must be determined. Fig. 4 shows the section of a trapezoidal wall of height, H. The known top width is b and the base width, to be determined, is B.

The pressure, P, acts at one-third the height from the base and is parallel to the earth surface. The resultant of the inclined pressure, P, and the weight of the wall, acting through the center of gravity of the profile cuts the base joint at Point D, a distance, $e\,B$, from Point C, the heel of the wall. It is convenient to resolve the inclined pressure, P, into a horizontal force, $P \cos \alpha$, and a vertical force, $P \sin \alpha$.

By taking moments about C, the distance of the center of gravity of the vertical forces from C is found to be:

$$\overline{CE} = \frac{B^2 + B\,b + b^2}{3\,(B + b) + \dfrac{6\,P \sin \alpha}{\rho_m\,H}}$$

in which, ρ_m is the unit weight of the masonry of which the wall is built. Then, $\overline{DE} = e\,B - \overline{CE}$; and, $\dfrac{\text{total vertical force}}{P \cos \alpha} = \dfrac{H}{3\,D\,E}$. When P is made equal to $\dfrac{\rho_L\,H^2}{2}$ this leads to the result:

$$B^2\,(3\,e - 1) + B\,b\,(3\,e - 1) - b^2 + 3\,e\,\sigma\,B\,H \sin \alpha = \sigma\,H^2 \cos \alpha \;\ldots(10)$$

in which, σ is the ratio, $\dfrac{\rho_L}{\rho_m}$.

(6) If the usual condition is adopted, that there shall be no tensile stress at the back of the wall, e must be two-thirds, and Equation (10) then becomes:

$$B^2 + B\,(b + 2\,\sigma\,H \sin \alpha) - (\sigma\,H^2 \cos \alpha + b^2) = 0$$

and the solution of this equation is:

$$2\,B = -\,(b + 2\,\sigma\,H \sin \alpha) + \sqrt{5\,b^2 + 4\,\sigma\,H\,(H \cos \alpha + \sigma\,H \sin^2 \alpha + b \sin \alpha)}\,.(11)$$

If b is made zero, the profile of the wall is a triangle and,

$$\frac{B}{H} = \sqrt{\sigma^2 \sin^2 \alpha + \sigma \cos \alpha} - \sigma \sin \alpha \;\ldots\ldots\ldots\ldots(12)$$

With $\alpha = 0$, Equation (12) reduces to the well-known result for a non-surcharged wall:

$$B = H\sqrt{\sigma} \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(13)$$

The family of curves plotted in Fig. 5 enables $\frac{B}{H}$ to be found for any values of σ and α, and the profile of a triangular wall, to satisfy the Rankine theory, is thus determined directly from the value of ρ_L found from Fig. 3 without the necessity of calculating the value of P.

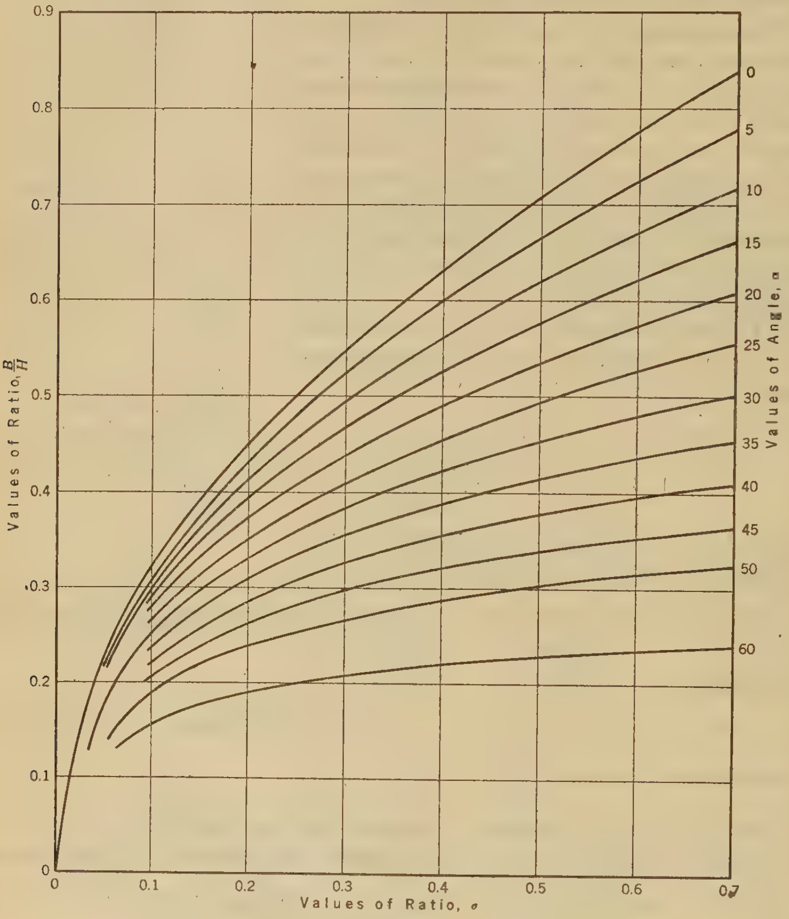


FIG. 5.

(7) To determine the trapezoidal profile of top width, b, proceed as follows: Let B_b be the necessary width of the base of the trapezoidal section given by Equation (11), and B, the base width of the triangular wall, which would satisfy the conditions and which can be found as explained previously. Then, the reduction in the base that can be made, due to the provision of a top width, is, $B - B_b = B'$. Substituting from Equations (11) and (12):

$$\frac{B'}{H} = \frac{n}{2} - \frac{1}{2} \sqrt{4 \sigma (\cos \alpha + \sigma \sin^2 \alpha)} \left\{ \sqrt{\frac{5 n^2 + 4 \sigma n \sin \alpha}{4 \sigma (\cos \alpha + \sigma \sin^2 \alpha)}} + 1 - 1 \right\}$$

in which, n is written for $\frac{b}{H}$. Expanding the root in the bracket and retaining only the first two terms as giving a sufficiently accurate result:

$$\frac{B'}{H} = \frac{n}{2} - \frac{1}{8}\left(\frac{5\,n^2 + 4\,\sigma\,n \sin\alpha}{\sqrt{\sigma^2 \sin^2\alpha + \sigma \cos\alpha}}\right)$$

Substituting for $\sqrt{\sigma^2 \sin^2\alpha + \sigma\cos\alpha}$ from Equation (12) this reduces to,

$$B' = \frac{b}{2}\left(\frac{\frac{B}{H} - \frac{5\,b}{4\,H}}{\frac{B}{H} + \sigma \sin\alpha}\right) \quad\text{..................(14)}$$

In this connection, $\frac{B}{H}$ is taken from the curves of Fig. 5. Owing to the omission of terms in the expansion of the root of the equation for $\frac{B'}{H}$, this correction is slightly smaller than an exact result would give, but the small error introduced is negligible in view of the nature of the problem and, in any case, is on the right side.

(8) Rankine suggested that the resultant action in the case of earth retaining walls might be allowed to cut the base within the middle three-eighths instead of the middle third. This makes $e = \frac{11}{16}$ in Equation (10), and it is found that the resulting base width of the wall is 3% less than that determined by the curves in this paper.

(9) As an example of the method suppose a profile is required for a wall 20 ft high, retaining earth weighing 120 lb per cu ft and having an angle of repose of 45 degrees. The surcharge angle is 30°, the weight of the masonry, 140 lb per cu ft, and the top width, 2 ft.

Since $\cos\alpha = 0.866$ and $\cos\phi = 0.707$, $\frac{\cos\phi}{\cos\alpha} = 0.816$. Then, from Fig. 3, $\frac{\rho_L}{\cos\phi} = 39.5$, and $\rho_L = 39.5 \times 0.707 = 27.9$ lb per cu ft. Then,

$$\sigma = \frac{\rho_L}{\rho_m} = \frac{27.9}{140} = 0.199$$

Thus, from Fig. 5, the value of $\frac{B}{H}$ for a triangular wall is 0.328; that is, $B = 0.328 \times 20 = 6.56$ ft. Since the top width is 2 ft, a correction may be applied as found by Equation (14), or,

$$B' = \frac{0.328 - 0.125}{0.328 + 0.098} = 0.48$$

and the base width is $6.56 - 0.48 = 6.0$ ft, say. It will be noticed that this thickness is well within the limits suggested by the late Sir Benjamin Baker quoted in Section (2).

This profile is shown in Fig. 6 and a graphical analysis has been made to check it; thus, the weight of the wall $= 4.0 \times 20 \times 140 = 11\,200$ lb per lin. ft. The earth pressure, by Equation (6), is equal to,

$$\frac{60 \times 400 \times 0.866 \times 0.366}{1.366} = 5\,560 \text{ lb per ft}$$

The resultant of these forces is shown in Fig. 6 and cuts the base just inside the middle third, thus checking the result obtained by the use of the curves.

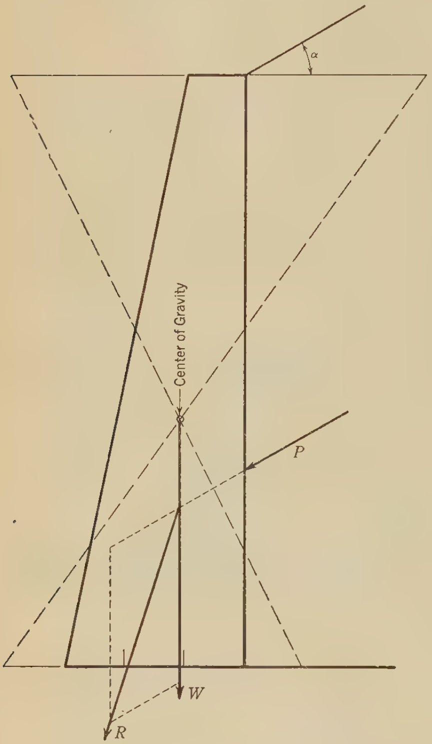


FIG. 6.

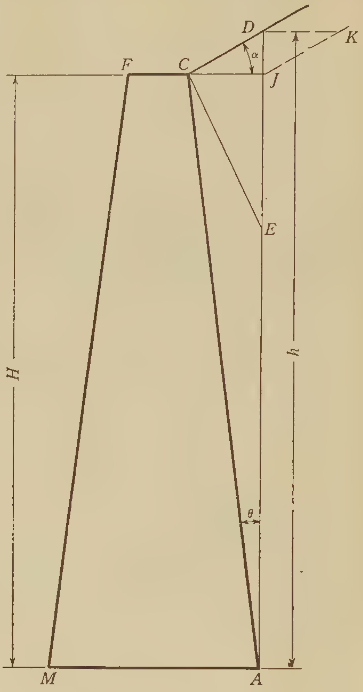


FIG. 7.

(10) When the back of the wall is battered, the problem is more complicated, but the treatment to be given has the advantage of being a direct design method which is easily applied. In Fig. 7, let $A\,C$ be the back of the wall battered at an angle, θ, to the vertical. Then, the resultant action on the back, $A\,C$, is compounded of the weight of earth represented by $C\,D\,A$ and the earth pressure on $A\,D$, of height: $h = H\,(1 + \tan\alpha \tan\theta) = m\,H$.

If $A\,E$ is made equal to $r\,h$, in which, r is the ratio, $\frac{\rho_e}{\rho_m}$, the masonry triangle, $C\,E\,A$, is equivalent to the earth triangle, $C\,D\,A$, as regards both weight and moment about A. Hence, the section, $F\,C\,E\,A\,M$, is the equivalent wall to resist the earth pressure on the face, $D\,A$.

The pressure on DA is proportional to $\rho_e\, m^2\, H^2$, and is equal in magnitude to the pressure that would be exerted on JA if the filling had a density, $\rho_e' = \rho_e\, m^2$, and was surcharged to the angle, α. The earth surface for this equivalent filling is JK in Fig. 7.

The procedure adopted is to design a wall of trapezoidal profile, $FJAM$ (Fig. 7), by the methods previously explained, and to correct the base width thus found to allow for the inaccuracies involved. These inaccuracies are as follows: (*a*) The absence of the masonry triangle, CJE, from the section reduces the weight of the wall from that designed, and also moves the center of gravity away from the back; and, (*b*) the center of pressure of the force acting on DA is at a height of $\dfrac{h}{3}$ from A instead of $\dfrac{H}{3}$, as assumed in the design.

The effects are small and can be dealt with as follows: In Fig. 8, let l be the height of the center of pressure above Point A; k, the distance of the center of gravity of vertical forces from Line AJ; and, V, the total vertical force acting through the center of gravity $= \rho_m \left(\dfrac{B+b}{2}\right) H + P \sin \alpha$.

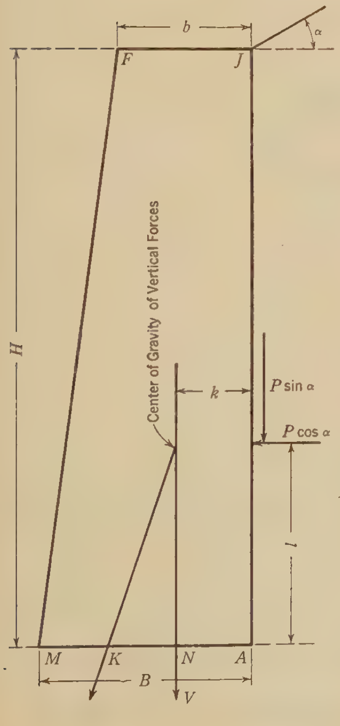

b
α
F
J
H
Center of Gravity of Vertical Forces
k
P sin α
P cos α
l
M
K
N
A
B
V

Fig. 8.

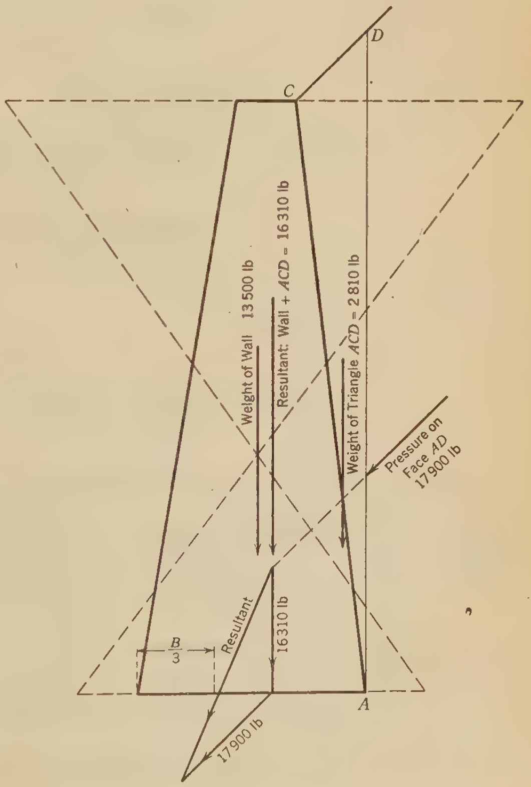

D
C
Weight of Wall 13 500 lb
Resultant: Wall + ACD = 16 310 lb
Weight of Triangle ACD = 2 810 lb
Pressure on Face AD 17 900 lb
B/3
Resultant
16 310 lb
17 900 lb
A

Fig. 9.

Taking moments about Point K:

$$l\ P \cos \alpha = V \left(\frac{2\ B}{3} - k \right)$$

since $\overline{MK} = \frac{B}{3}$. The effect of small changes in V, l, and k, on B is given by:

$$\delta\ B = \frac{\partial\ B}{\partial\ V}\ \delta\ V + \frac{\partial\ B}{\partial\ l}\ \delta\ l + \frac{\partial\ B}{\partial\ k}\ \delta\ k$$

in which, $\frac{\partial\ B}{\partial\ V}$, $\frac{\partial\ B}{\partial\ l}$, and $\frac{\partial\ B}{\partial\ k}$ are the partial differential coefficients of B with respect to V, l, and k, and $\delta\ V$, $\delta\ l$, and $\delta\ k$ are the small increments of these quantities. It follows that $\frac{\partial\ B}{\partial\ V} = \frac{3\ k - 2\ B}{2\ V}$; $\frac{\partial\ B}{\partial\ l} = \frac{3\ P \cos \alpha}{2\ V}$; and, $\frac{\partial\ B}{\partial\ k} = \frac{3}{2}$. In the present case:

$$\delta\ V = - \frac{\rho_m}{2}\ (1 - m\,r)\ H^2 \tan \theta$$

$$\delta\ l = \frac{H}{3}\ (m - 1)$$

$$\delta\ k = \frac{\rho_m\ (1 - m\,r)\ H^3 \tan^2 \theta}{6\ V}$$

$$k = \frac{\rho_m\ H\ (B^2 + B\ b + b^2)}{6\ V}$$

$$l = \frac{H}{3}$$

and,

$$\frac{P \cos \alpha}{V} = \frac{\overline{KN}}{\overline{GN}} = \frac{2\ B - 3\ k}{H}$$

Substituting these values and reducing;

$$\frac{\delta\ B}{H} = \frac{\tan \theta}{2} \left[\frac{(1 - m\,r) \left(\tan \theta + \frac{2\ B - 3\ k}{H} \right)}{\sigma' \sin \alpha + \frac{B}{H} + \frac{b}{H}} + \frac{2\ B - 3\ k}{H} \tan \alpha \right] \quad \text{..(15)}$$

in which, σ' is the value of σ for the equivalent filling; that is, $\frac{\rho_L}{\rho_m}\ (1 + \tan \theta \tan \alpha)^2$. The terms in this expression are readily found from the designed profile, $F\ J\ A\ M$ (Fig. 8), and the increase in base width thus determined.

(11) As an example of the method, consider a wall to the following requirements: Height, H, = 20 ft; top width, b, = 2 ft; batter of back = 1 in 8 (that is, $\theta = 7°.1$); weight of masonry, ρ_m, = 140 lb per cu ft; weight of earth fill, $\rho_e = 100$ lb per cu ft; angle of repose, ϕ, = 45°; and angle of surcharge, α, = 45 degrees.

The necessary width of base, B, is to be determined. The equivalent unit weight of earth is $\rho_e' = 100\ (1 + \frac{1}{8})^2 = 126.5$ lb per cu ft. In the first place, the curves are used as in the example of Section (9) to find the necessary width of a wall with a vertical back and a top width of $(2 + H \tan \theta) = 4.5$ ft. From Fig. 3, $\rho_L = 126.5 \cos \theta = 89.5$ lb per cu ft, and $\sigma' = \frac{89.5}{140} = 0.64$. From Fig. 5, $\frac{B}{H}$ for a triangular wall $= 0.36$, or $B = 7.2$ ft. The correction to be applied for the width is, from Equation (14): $B' = \frac{4.5}{2} \left(\frac{0.36 - 0.281}{0.36 + 0.452} \right) = 0.22$ ft. The base width of the vertical back wall is thus: $7.2 - 0.22 = 7.0$ ft, say. The pressure of earth acting at 45° on the vertical back is, $P = \frac{\rho_e' H^2}{2} \cos \alpha = 17\,900$ lb per ft of wall. The vertical component of this is, $17\,900 \times 0.707 = 12\,650$ lb per ft. The weight of the wall is $140 \times 11.5 \times 10 = 16\,100$ lb per ft. Therefore, $V = 28\,750$ lb and,

$$k = \frac{\rho_m H (B^2 + B b + b^2)}{6 V} = \frac{140 \times 20 \times 100.75}{6 \times 28\,750} = 1.64 \text{ ft}$$

which, if desired, could have been found graphically.

Let $\tan \theta = \frac{1}{8}$; $(1 - m r) = 1 - \frac{5}{7} (1 + \tan \alpha \tan \theta) = 1 - \frac{5}{7} (1.125) = 0.196$; $\frac{B}{H} = 0.35$; $\frac{b}{H} = 0.225$; and $k = 1.64$. Substituting these values in Equation (15): $\frac{\delta B}{H} = 0.034$; or, $\delta B = 0.68$ ft. The necessary base width is, thus, $7.0 + 0.68 = 7.68$ ft.

As a check, this profile (shown in Fig. 9) will be examined graphically. The weight of the wall per foot of run is $(2 + 7.65)\ 10 \times 140$ lb $= 13\,500$ lb. The weight of the wedge of earth resting on the battered back is $(2.5 \times 22.50)\ 50 = 2\,810$ lb. The line of action of the resultant of these two vertical forces is found graphically. The force acting on the vertical earth back is, $P = \frac{\rho_e h^2}{2} \cos \alpha = 50 \times 22.5^2 \times 0.707 = 17\,900$ lb, and is inclined at 45° to the vertical. Combining these forces the resultant is found to cut the base closely to $\frac{B}{3}$ from the toe of the wall as required.

DISCUSSION

E. S. Lindley,[3] M. Am. Soc. C. E. (by letter).—In connection with this paper it may be of interest to call attention to another nomographic solution of Rankine's formulas[4] which seems to be simpler than that presented by Professor Pippard. To allow for surcharge, the wall is calculated for an increased height. The factor by which the height is to be multiplied is a function of the angle of surcharge, and is given in plain scales.

For a wall with a vertical face and back, a single isopleth gives the solution. The fixed points for this isopleth are, on one scale, the angle of repose, and on another, the ratio of density of masonry to that of soil. The solution is to determine the ratio of thickness to height. There is no need to find cosines from tables, as in Professor Pippard's method, and the ratios to be computed are simpler.

For a wall with a vertical face and battered back the solution is in three steps; in addition to the two given factors stated in the preceding paragraph, there is the ratio of top thickness to height. The solution is to determine the ratio of bottom thickness to height.

Kenneth L. DeBlois,[5] Assoc. M. Am. Soc. C. E. (by letter).—The direct solution of gravity retaining walls algebraically and by nomographic charts, as presented by the author, will help to eliminate the "cut-and-try" method. Most retaining wall stems are designed as reinforced concrete cantilever beams. Except in high walls the vertical component of the earth pressure and the weight of the stem are neglected as affecting the bending stresses but little. Many cases, however, require a gravity section for which the author's formulas and charts will apply. Railroad companies frequently use the gravity type as being more suited to supporting fills subject to heavy locomotive loads and impacts.

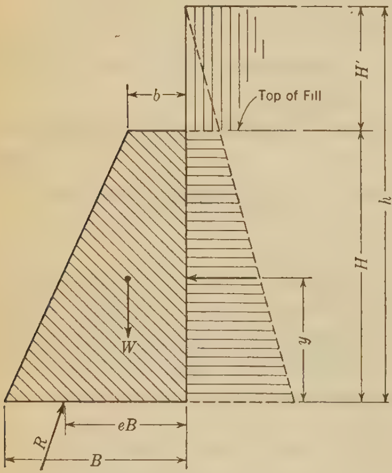


Fig. 10.

The equations in this paper can be extended to cover the case of a wall subject to railroad or highway live load surcharge as shown in Fig. 10. In addition to the notation of the paper, let H' = the equivalent surcharge due to live load. Then,

$$B^2 (3e - 1) + Bb (3e - 1) - b^2 = \sigma H \ (H + 3H') \quad \ldots\ldots(16)$$

---

[3] Superintending Engr., Indian Public Works Dept., Panjab Irrig. Branch, Wotton-under-Edge, Gloucestershire, England.

[4] "Panjab Irrigation Nomograms," by E. S. Lindley, M. Am. Soc. C. E., Panjab Irrig. Branch, Public Works Dept., Panjab, India.

[5] Assoc. Highway Bridge Engr., U. S. Bureau of Public Roads, San Francisco, Calif.

which corresponds to Equation (10). When $e = \frac{2}{3}$:

$$2B = -b + \sqrt{5b^2 + 4\sigma H(H + 3H')} \quad \ldots\ldots\ldots(17)$$

which corresponds to Equation (11).

Formulas can be written for a similar direct solution when the back of the wall is battered. For this case, however, the writer is of the opinion that it is easier to solve each individual problem, using the numerical values.

M. A. DRUCKER,[6] ESQ. (by letter).—The analysis of the design of trapezoidal retaining walls, including diagrams, presented by the author, should greatly facilitate the design of such walls. However, the illustrative example solved in Section (9), if followed by others, would lead to irrational designs. In this example the necessary width of base is determined for a wall 20 ft high to support a fill having a sloping surcharge of 30°, whereas the angle of repose is assumed to be 45 degrees. With an assumed top width of 2 ft, the width of base is found to be 6 ft, in order that the resultant shall intersect the base at the outer third point.

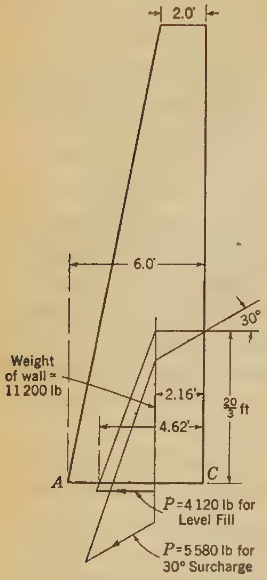
It would seem reasonable to assume that a wall designed to resist the pressure of a fill having a 30° surcharge should be capable of withstanding the pressure due to a level fill of the same soil; the factor of safety for the latter case being at least as large as for the former. However, assuming the same wall section, with a 6-ft base and 2-ft top width, and all other conditions being the same as given by the author, the results, as indicated on Fig. 11, were obtained for a level fill. The resultant was found to intersect the base at a distance of 4.62 ft from the back of the wall, which is considerably outside the middle third of the base. As the center of gravity of the wall is 2.16 from the back of the wall, the factor of safety against rotation for this condition would be equal only to $\frac{6.0 - 2.16}{4.62 - 2.16} = \frac{3.84}{2.46} = 1.56$. Fig. 11 also indicates the pressure and resultant for the author's design for the condition of a 30° surcharge.

It is not intended to imply that the author's analysis or diagrams are in error; the same results would be obtained by any method using the Rankine theory. Such irrational results are due to, and are inherent in, the Rankine assumption that the direction of the resultant earth pressure is parallel to the ground surface. In accordance with this method, while the pressure increases with the angle of surcharge, as given in Column (2) of Table 1 (notation in Fig. 12), the tendency to overturn does not vary directly with the slope of the ground surface. This is illustrated by Fig. 13(*a*) which shows the points of application, at the base of the wall, of the resultant due to the earth pressure and weight of wall for several angles of surcharge. The wall, 20 ft high, was designed to support a level fill with the resultant for this condition intersecting the base at the outer third point. the necessary width of base being 7.0 ft.

---

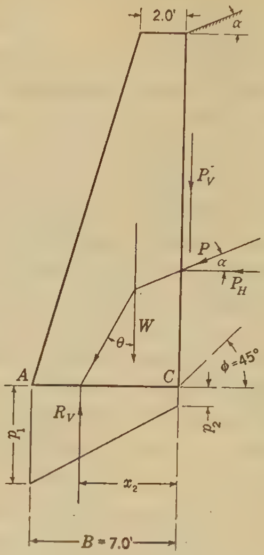
[6] Designer in Chg., Board of Transportation, New York, N. Y.

It may be seen from Fig. 13 that for all other cases, governed by the author's conditions as to weights and angle of repose, including that for the maximum surcharge of 45°, the point of application of the resultant is within the middle-third area of the base of the wall designed for a level fill. This denotes that, as regards rotation, the wall would be in a safer condition for sloping fills than for a level fill. It may be advisable, therefore, when designing a wall for a sloping surcharge, to make sure that the wall would be equally safe in case it is decided later not to place the fill on the



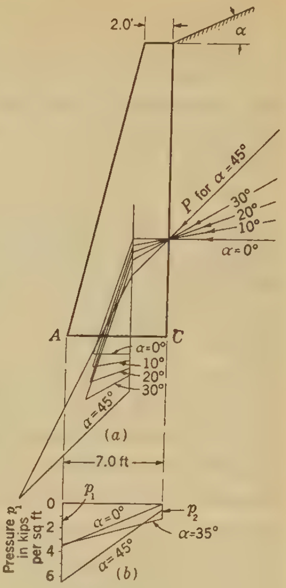
2.0'
6.0'
30°
Weight of wall = 11 200 lb
2.16'
$\frac{20}{3}$ ft
4.62'
A
C
P = 4 120 lb for Level Fill
P = 5 580 lb for 30° Surcharge

FIG. 11.



2.0'
α
P'_V
P
α
P_H
W
θ
A
C
φ = 45°
R_V
p_1
p_2
x_2
B = 7.0'

FIG. 12.



2.0'
α
P for α = 45°
30°
20°
10°
α = 0°
A
C
α = 0°
10°
20°
30°
α = 45°
(a)
7.0 ft
p_1
p_2
α = 0°
α = 35°
α = 45°
(b)
Pressure p in kips per sq ft
0
2
4
6

FIG. 13.

TABLE 1.—POINT OF APPLICATION OF RESULTANT, SLIDING TENDENCY, AND PRESSURES, AT BASE OF WALL

Angle of surcharge, α, in degrees	APPLIED PRESSURE, IN POUNDS			Moment, M_C,* in foot-pounds, of P and W about Point C, Fig. 12	Vertical component of resultant, R_V (= W + P_V), in pounds	Distance, X_1 (in feet) $= \frac{M_C}{R_V}$	INTENSITY OF PRESSURE, IN POUNDS PER SQUARE FOOT		Value of tan θ $\left(= \frac{P_H}{R_V}\right)$
	P	P_V	P_H				At toe, P_1	At head, P_2	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
0......	4 120	0	4 120	58 800	12 600	4.67	3 600	0	0.328
5......	4 130	360	4 120	58 800	12 960	4.53	3 490	215	0.318
10......	4 260	740	4 190	59 300	13 340	4.45	3 450	350	0.315
15......	4 370	1 130	4 220	59 400	13 730	4.32	3 340	580	0.308
20......	4 610	1 570	4 330	60 200	14 170	4.25	3 330	720	0.306
25......	5 020	2 120	4 550	61 700	14 720	4.19	3 340	860	0.309
30......	5 580	2 790	4 840	63 600	15 390	4.14	3 410	990	0.317
35......	6 480	3 720	5 300	66 700	16 320	4.09	3 510	1 150	0.325
40......	8 170	5 250	6 260	73 100	17 850	4.10	3 860	1 240	0.350
45......	17 000	12 000	12 000	111 300	24 600	4.59	6 630	405	0.488

* M_C = moment of P and weight of wall about Point C.

slope assumed. Otherwise, one might find himself in the embarrassing position of having to insist either that the fill must be placed at the angle assumed in the design or that the wall should be made wider.

It may also be pointed out that the author's analysis and diagrams aim at obtaining a base of such width as to have the resultant pressure intersect it at the outer third point. While such a condition provides for bearing on the entire base, with a maximum pressure at the toe and zero pressure at the heel, it does not give an exact indication of the factor of safety of the wall against rotation because, with such a limitation, this factor would be 2 for a triangular wall and 3 for a wall rectangular in shape. Consequently, for any particular case, the factor of safety against overturning would vary between these limits.

In the design of retaining walls, of course, other conditions are to be considered, in addition to that of rotation treated by the author. One other phase of design that should be considered is that of sliding on the base. The tendency to slide may be measured by the tangent of the angle between the resultant pressure line and the vertical. Fig. 13(*a*) shows that this angle would be larger for the case of a level fill than for a sloping surcharge up to about 35 degrees. This is demonstrated more clearly in Column (10), Table 1.

Another condition that should be investigated is that of maximum pressure on the soil at the toe of the wall. From Fig. 13(*b*), which shows the variation in pressures under the wall, as obtained for a level fill as well as for several angles of surcharge, it is clear that a level fill will cause greater pressures than sloping surcharges up to about 35 degrees. Column (8), Table 1, shows the variation in pressures for various angles of surcharge. It may also be noted that while, as regards overturning, the wall is safer to resist the pressure due to the 45° surcharge than that due to the level fill, the pressure at the toe of the wall is nearly twice as large for the former condition as for the latter. Therefore, a design based only on the stability to resist overturning may not reveal other conditions which should be taken into consideration.

From the preceding it may be seen that, if the magnitude and direction of pressures be obtained by following the Rankine theory, the pressures of sloping surcharges up to an angle of about 75% of the assumed angle of repose, need not be considered if they act against a wall having a vertical back.

A. J. SUTTON PIPPARD,[7] M. AM. SOC. C. E. (by letter).—The three discussions of this paper were helpful contributions. Mr. Lindley refers to earlier nomograms devised for the same purpose as those given; it is evident that the results of formulas of the Rankine type can be expressed in a variety of ways and it is not surprising that the nomographic representation has been adopted previously. The publication to which Mr. Lindley refers[4] is, however, not readily accessible.

---

[7] Prof. of Civ. Eng., Univ. of London, Imperial Coll., City and Guilds Coll. (Eng.), London, England.

The extensions of Equations (10) and (11) given by Mr. DeBlois for the case in which a live load is carried on the fill, are simple and useful.

Mr. Drucker points out the apparent anomaly that a wall designed on the Rankine hypotheses is safer when carrying a surcharge that when subjected to the pressure from a level fill. As he mentions, this arises from the fact that the resultant earth thrust is assumed to act parallel to the slope of the fill and is inherent in Rankine's treatment. In the simplest form of the wedge theory the pressure is assumed to act normally to the back of the wall and walls designed on this basis are too heavy. By introducing consideration of the frictional resistance of the wall the direction of thrust is modified and a nearer approach to the Rankine result is obtained, but the value to be used for the coefficient of friction between earth and wall is a debatable point. Until a complete theory of the behavior of earth is obtained, the design of retaining walls must be largely a matter of judgment; fortunately, workers in many countries are helping toward a much more complete understanding of the intricate problems involved.

The writer realized that other considerations, mentioned by Mr. Drucker, must be taken into account in the design of a wall, but once a profile has been obtained which eliminates the possibility of tension on mortar joints it is a simple matter to check the remaining possibilities of failure. In many cases these will be found to be amply safeguarded.

In conclusion, the writer would like especially to thank Mr. Drucker for the trouble he has taken in illustrating the points mentioned in his discussion.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 1918

A GENERALIZED DEFLECTION THEORY FOR SUSPENSION BRIDGES

By D. B. Steinman,[1] M. Am. Soc. C. E.

With Discussion by Messrs. E. L. Pavlo, Jonathan Jones, A. Müllenhoff, H. Cecil Booth, Jacob Feld, Glenn B. Woodruff, Howard C. Wood, and Ralph A. Tudor, L. J. Mensch, A. A. Eremin, Hans H. Bleich, F. H. Frankland, Gustav Lindenthal, Julian W. Shields, A. W. Fischer, J. M. Frankland, Fredrik Vogt, Leon S. Moisseiff, A. Mitchell and G. T. Parkin, Sterling Johnston, Harold E. Wessman, C. H. Gronquist, W. R. Frederick, Jr., William H. Yates, and D. B. Steinman.

Synopsis

An improved and extended Deflection Theory for suspension bridges is presented in this paper. The theory is generalized to include structures of any number of spans, continuous or non-continuous, symmetrical or unsymmetrical, and with or without tie cables.

The more general adoption of the continuous type of suspension bridge, offering advantages of economy and rigidity, has been retarded by the lack of an accurate theory for its analysis. The Deflection Theory for simple-span suspension bridges has been available to the Engineering Profession for more than forty-five years; but, thus far, the corresponding theory for the suspension bridge with continuous stiffening truss has been lacking.

In order to supply this deficiency, the writer has undertaken to develop a Generalized Deflection Theory, applicable to both continuous and non-continuous, three-span, and multiple-span structures. By simply dropping the recognizable terms due to continuity, the formulas are reduced to those for the simpler case of stiffening trusses hinged at the towers.

In the development of the analysis herein presented, maximum simplicity of formulas and ease of practical application have been governing considera-

---

Note.—Published in March, 1934, *Proceedings*.

[1] Cons. Engr. (Robinson & Steinman), New York, N. Y.

tions. Incidentally, new simplifications are here developed and introduced in the working formulas hitherto published for the two-hinged type.

To show the practical workability of the Generalized Theory, the paper includes a numerical example of the application of the formulas to the calculation of the stresses and deflections in a continuous suspension bridge of 800-ft main span.

1.—INTRODUCTION

The common or approximate theory for the stress analysis of stiffened suspension bridges is known as the Elastic Theory. The values of the bending moments and shears yielded by this method are too high; they satisfy safety but not economy. The error increases with the flexibility of the structure, the span length, and the ratio of dead load to live load.

A more exact method of analysis, which takes into account the deformed configuration of the structure, is known as the Deflection Theory. It yields lower stresses and a consequent saving (ranging normally from 20 to 65%) of metal in the stiffening truss.

The Deflection Theory, or "More Exact Theory," as applied to non-continuous suspension bridges, was originated by J. Melan and was first published by him in 1888 in his classic work "Theorie der eisernen Bogenbrücken und der Hängebrücken." (It was republished in 1906 in the Third Edition, which was translated by the writer[2] in 1909. In the Fourth Edition (1925), the Deflection Theory appears again, in amplified form.) The working formulas were amplified by L. S. Moisseiff, M. Am. Soc. C. E. (and independently by the writer), in 1909, to cover the case of suspended side spans, and the theory has since been published in its extended form by F. E. Turneaure, Hon. M. Am. Soc. C. E.,[3] and by the writer.[4]

The Deflection Theory, as hitherto developed, is not directly applicable to suspension bridges with continuous stiffening trusses. A number of such structures have been built, including the Rondout Bridge, at Kingston, N. Y., with a 705-ft main span (1922), the General U. S. Grant Bridge over the Ohio River, at Portsmouth, Ohio, with a 700-ft main span (1927), the Sixth, Seventh, and Ninth Street Bridges over the Allegheny River, at Pittsburgh, Pa., and several bridges in Europe; but the designers have been handicapped by the lack of an accurate method of analysis. Without correct proportioning, the full economy of this type of suspension bridge can not be secured.

Continuous stiffening trusses offer greater efficiency than the conventional two-hinged type. This increased efficiency may be in the form of superior economy, or superior rigidity, or both, depending on the proportions adopted. The hingeless type offers incidental advantages in greater efficiency of the continuous lateral truss, in improved and simplified supporting details at the towers, and in reduced variation between minimum and maximum sections. For suspension bridges of less than 1 000-ft main span, the continuous stiffening truss may well be considered the superior type.

---

[2] "Theory of Arches and Suspension Bridges," by D. B. Steinman, M. Am. Soc. C. E., M. C. Clark Pub. Co., Chic., 1913.

[3] "Modern Framed Structures," by J. B. Johnson, C. W. Bryan, and F. E. Turneaure, Ninth Edition, Pt. 2 N. Y., John Wiley & Sons, 1911, and succeeding editions.

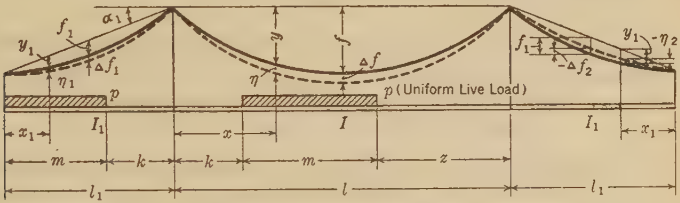
[4] "A Practical Treatise on Suspension Bridges," by D. B. Steinman, M. Am. Soc. C. E., Second Edition, N. Y., John Wiley & Sons, 1929.

In past designs of continuous suspension bridges, the deflection corrections have been either neglected or conservatively approximated. Had the theory and formulas presented in this paper been then available, the stresses calculated could have been reduced approximately 30% for the Rondout Bridge, and 18% for the Portsmouth Bridge. For the more modern stiffening truss design of 800-ft main span, used as a practical example in this paper, the theory here developed yields approximately 45% reduction in stresses and sections. With the Deflection Theory for such continuous structures available, more scientifically proportioned designs can be made, and the economic utilization of this bridge type will be facilitated.

Multiple-span suspension bridges also offer possibilities that have not been fully explored. The generalization of the Deflection Theory to permit the accurate analysis of multiple-span structures should facilitate their further investigation..

2.—NOTATION

The general notation to be used is shown in Fig. 1. The subscripts (as in I_1, l_1, f_1, x_1, y_1, η_1) are added to distinguish corresponding side-span magnitudes. The initial loading, w, for which the stiffening truss is unstressed and undeflected, will be called the "dead load." The load, p, applied subsequently, which produces stress and deflection, will be termed the "live load."



Cable in Full Line = Dead Load Position
Cable in Dotted Line = Deflected Position

FIG. 1.—NOTATION DIAGRAM

The notation used in the formulas (with the numbers of pertinent equations, for convenience of reference) is as follows:

H_w = horizontal tension in cable due to dead load $(w + g)$.
H = horizontal tension in cable due to live load, p, and temperature, t. (Equations (24), (27), (37), and (47).)
M_0 = simple beam bending moment due to live load, p.
M = total resultant bending moment at any section, x, of stiffening truss. (Equations (4), (6), (9), and (29).)
T = bending moment at any section, x, due to continuity. (Equations (3).)
$T_1, T_2, \ldots$ = bending moments in truss at towers. (Equations (19) and Article 11.)
V = total shear at any section, x, of stiffening truss. (Equation (10).)

η, η_1 = deflection of truss at any section, x, x_1. (Equation (7).)
I, I_1 = moments of inertia of stiffening truss.
C_1, C_2 = pair of integration constants. (Equations (12), (13), and (30).)
E = elastic modulus of truss; E_c = elastic modulus of cable.
L_s, L_t = cable-length functions, defined by Equations (21).
p_x = intensity of live load at any section, x.
p_s = live load carried by the cable. (Equation (11).)
p_t = live load carried by the truss. (Equation (11).)
t = temperature change.
ω = coefficient of expansion.
A_c = cable section at any point; A_0 = cable section at mid-span.
α = slope of cable chord in any span. (Equations (21).)
s = length of cable arc. (Equations (21).)
l, l_1 = span lengths.
f, f_1 = cable sags.
$n = \dfrac{f}{l}$.
$n_1 = \dfrac{f_1}{l_1}$.
$\Delta f, \Delta f_1$ = mid-span deflections. (Equations (33).)
$r = \dfrac{l^2}{8f}$; $r_1 = \dfrac{l^2_1}{8f_1}$. (Equations (8).)
$K_1 = \dfrac{r}{r_1}$; $(K = 1)$. (Equation (23).)
x, x_1 = abscissas. (x_1 measured from free end of each side span.)
y, y_1 = cable ordinates.
$c^2 = \dfrac{H_w + H}{EI}$; $c^2_1 = \dfrac{H_w + H}{EI_1}$. (Equations (5).)
$d = \dfrac{e^{cl} - 1}{e^{cl} + 1}$; $d_1 = \dfrac{e^{c_1 l_1} - 1}{e^{c_1 l_1} + 1}$. (Equations (14) and (15).)
$h = 1 - \sqrt{1 - d^2}$; $h_1 = 1 - \sqrt{1 - d^2_1}$. (Equations (31).)
$a = cd + \dfrac{c}{d} - \dfrac{2}{l}$; $a_1 = c_1 d_1 + \dfrac{c_1}{d_1} - \dfrac{2}{l_1}$. (Equation (41*a*).)
$b = -cd + \dfrac{c}{d} - \dfrac{2}{l}$. (Equation (41*b*).)
$L_c = l - \dfrac{2d}{c}$; $L_{c1} = l_1 - \dfrac{2d_1}{c_1}$. (Equations (26).)
N = denominator of T-formula. (Equation (19*a*).)
F = term in T-formula (Equation (19*a*)) $= \Sigma' (KL_c)$. (Equation (19*c*).)
A = load function used in H-formulas; tabulated in Article 9.
B_1, B_2 = load functions in T-formulas; tabulated in Article 9.
G_1, G_2, G_3 = load functions in formulas for C_1 and C_2; tabulated in Article 9.
W_1, W_2 = work quantities. (Article 7.)
Σ = summation of similar terms for all spans.
Σ' = weighted summation for all spans with one-half weight for end spans.
S, S_1, S_2 = load functions, defined in Equations (28).

k, m, z = span segments. (Article 9.)
R_L, R_R = simple-beam end reactions at $x = 0$ and $x = l$. (Equations (17), (20*a*), and (38).)
$U_1, U_2 \ldots$ = slope-change functions at towers (Equations (18) and (38).)
Y_L, Y_R = functions of R and G. (Equations (20).)
Y'_L, Y'_R. = functions of R, G, and H. (Equations (42).)

3.—FUNDAMENTAL ASSUMPTIONS

The Deflection Theory for the analysis of continuous and multiple-span suspension bridges is based on the same assumptions as the corresponding theory for two-hinged suspension bridges, namely: (1) The initial curve of the cable is a parabola (in practice, the greatest ordinate deviation from a true parabola is seldom as great as $\frac{1}{2}\%$); and, (2) the initial dead load, w, is carried by the cable (producing the initial horizontal tension, H_w), without causing stress in the stiffening truss.

Unlike the Elastic Theory, the Deflection Theory does not assume that the ordinates, y, of the cable curve remain unaltered on application of the loading. In other words, the alteration of the lever arms of the cable forces is taken into account. This change in cable ordinates or lever arms makes the initial cable tension, H_w, significant.

The theory that follows is applicable to either continuous or non-continuous suspension bridges, with or without suspenders in the side spans. For simplicity of notation, a symmetrical three-span suspension bridge is first assumed. All the formulas, however, except where noted, are also applicable to other cases. The few necessary modifications for complete generality, to cover unsymmetrical and multiple-span structures, are presented in Article 11.

4.—FUNDAMENTAL EQUATIONS

If the deflections of the span are neglected, the bending moment at any point of a two-hinged suspension span is given by the basic formula of the Elastic Theory:

$$M = M_0 - H\,y \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

in which, M_0 denotes the simple-span bending moment and $H\,y$ represents the relieving moment due to live-load cable tension, H, acting through the suspender forces.

In consequence of the deflection, η, the bending moments are relieved by an additional amount, $(H_w + H)\eta$, and the expression for M becomes:

$$M = (M_0 - H\,y) - (H_w + H)\eta \ldots\ldots\ldots\ldots\ldots (2)$$

If the spans are continuous at the towers, let T_1 and T_2 denote the total resultant bending moments (including correction for deflections) at the left and right towers, respectively. Then the contribution of continuity to the bending moment at any section will be:

For the Main Span:

$$T = \frac{l - x}{l}\,T_1 + \frac{x}{l}\,T_2 \ldots\ldots\ldots\ldots\ldots\ldots (3a)$$

For the Side Spans:

$$T = \frac{x_1}{l_1}\, T_{1,2} \qquad \text{(3b)}$$

With this contribution, T, added to the expression of Equation (2), the total resultant bending moment at any section of a continuous suspension bridge will be:

$$M = (M_0 - H\,y) - (H_w + H)\,\eta + T \qquad \text{(4)}$$

This is the basic equation of the Deflection Theory for continuous suspension bridges. It is also the basic equation of the Generalized Deflection Theory, applicable to simple, continuous, and multiple-span suspension bridges.

By introducing the symbol, c^2,

For the Main Span:

$$c^2 = \frac{H_w + H}{E\,I} \qquad \text{(5a)}$$

For the Side Spans:

$$c^2_1 = \frac{H_w + H}{E\,I_1} \qquad \text{(5b)}$$

Equation (4) becomes:

$$M = (M_0 - H\,y) - (E\,I\,c^2)\eta + T \qquad \text{(6)}$$

Neglecting the elongation of the suspenders, the truss at any point will have the same deflection, η, as the cable at that point.

By the common theory of flexure applied to the truss, $\frac{d^2\eta}{dx^2} = -\frac{M}{E\,I}$. Substituting Equation (6),

$$\frac{d^2\eta}{dx^2} = c^2\eta - \frac{1}{E\,I}\,(M_0 - H\,y + T)$$

The solution of this differential equation yields the general formula for deflections, or the equation of the deflection curve:

$$\eta = \frac{1}{E\,I\,c^2}\left[(C_1\,e^{cx} + C_2\,e^{-cx}) - \frac{1}{c^2}\left(p_x - \frac{H}{r}\right) + (M_0 + T - H\,y)\right]. \qquad \text{(7)}$$

in which, p_x is the live load per unit length at the section, x, and r is the parameter of the cable parabola, given by:

$$r = \frac{l^2}{8\,f} \qquad \text{(8a)}$$

and,

$$r_1 = \frac{l^2_1}{8\,f_1} \qquad \text{(8b)}$$

Substituting Equation (7) in Equation (6), the general formula for M, or the equation of the M-curve, is:

$$M = -\left[(C_1 e^{cx} + C_2 e^{-cx}) - \frac{1}{c^2}\left(p_x - \frac{H}{r}\right)\right] \ldots\ldots\ldots\ldots (9)$$

From Equation (9) it is observed that the bending moment, M, is not simply proportional to the load, p, that produces it. Equation (4) also shows that the value of M is affected by the dead load stress, H_w, in the cable before the application of the live load. In the Deflection Theory, influence lines (in the ordinary sense) cannot be used. Stresses producible by combinations of loadings cannot be found by adding algebraically the respective stresses producible by the component loadings.

The general formula for shears, V, or the equation of the V-curve, is obtained by differentiating Equation (9), which gives:

$$V = \frac{dM}{dx} = -c\left[C_1 e^{cx} - C_2 e^{-cx}\right] \ldots\ldots\ldots\ldots (10)$$

Differentiating Equation (10), an expression is obtained for the live load per unit length that is actually carried by the stiffening truss at any point, x, as follows:

$$p_t = (p_x - p_s) = -\frac{d^2 M}{dx^2} = -\frac{dV}{dx} = c^2\left[C_1 e^{cx} + C_2 e^{-cx}\right] \ldots (11)$$

Equation (11) shows that the suspender loading, p_s, is no longer constant, as in the Elastic Theory, but becomes a variable in the Deflection Theory.

Equations (4), (6), (7), (9), (10), and (11) are applicable to points in the side spans as well as to those in the main span; all that is necessary is to write the subscript symbols, x_1, y_1, l_1, f_1, r_1, I_1, c_1, and η_1, instead of the corresponding main-span quantities. For non-continuous suspension bridges, simply write zero for T wherever it occurs in this Article. The formulas of this Article are directly applicable, without change, to multiple-span suspension bridges as well as to unsymmetrical structures.

5.—Evaluation of the Integration Constants

The constants of integration, C_1 and C_2, appearing in Equations (7), (9), (10), and (11), as well as later in the basic equations for T_1, T_2, and H, must be determined for each different condition of loading. For each span segment having a constant value of p and of I, there is a pair of values for C_1 and C_2.

In the treatment that follows, it will be assumed (as is usually done for the sake of simplicity), that the moment of inertia, I (or I_1), is constant throughout the length of any span under consideration, although it may have different respective values for the three different spans. The error of ignoring the variation of I within a span is found to be practically negligible and on the side of safety.

The procedure for writing working formulas for C_1 and C_2 for the various conditions of loading, is as described in the following paragraphs.

One Loading Segment.—For the case of the main span fully loaded with a uniform applied load, p, and assuming constant moment of inertia, I, the quantities, C_1 and C_2, are obtained from the two known conditions that, for $x = 0$ and $x = l$, $\eta = 0$ in Equation (7), or $M = T_1$ and T_2, respectively, in Equation (9). Substitute these values and solve the resulting two independent equations for C_1 and C_2.

Two Loading Segments.—For the case of a partial loading of the main span with a uniform load, p, per unit length extending a distance from either end of the span, the constants, C_1 and C_2, for the left segment, and the constants, C'_1 and C'_2, for the right segment, are obtained from the four known conditions that moment and shear at the right end of the left segment must be equal, respectively, to those at the left end of the right segment, and that $\eta = 0$ (or $M = T_{1,2}$) at each end of the span. Substitute these four relations in Equations (7), (9), and (10), and solve the resulting four equations for the two pairs of integration constants.

Three Loading Segments.—If the main span is divided into three segments, $k + m + z = l$, having different uniform loads, or alternately loaded and unloaded, the three corresponding pairs of integration constants are obtained from the six known conditions that M and V at the right end of the first segment must be equal, respectively, to M and V at the left end of the second segment; that the same two equalities hold at the junction of the second and third segments; and that $\eta = 0$ (or $M = T_{1,2}$) at each end of the span. Substitute these six relations in Equations (7), (9), and (10), and solve the resulting six independent equations for the three pairs of integration constants.

By the foregoing procedure, the following working formulas are obtained for the pair of integration constants in any loaded or unloaded segment of any span:

In the Main Span:

$$C_1 = -\frac{(1-d)}{4}\left[(T_1 + T_2) - \frac{1}{d}(T_1 - T_2) + \frac{2}{c^2}\left(\frac{H}{r} - G_1\right)\right] \ldots (12a)$$

$$C_2 = -\frac{(1+d)}{4}\left[(T_1 + T_2) + \frac{1}{d}(T_1 - T_2) + \frac{2}{c^2}\left(\frac{H}{r} - G_2\right)\right] \ldots (12b)$$

and,

$$C_1 + C_2 = -T_1 - \frac{1}{c^2}\left(\frac{H}{r} - G_3\right) \ldots\ldots\ldots\ldots\ldots (12c)$$

In the Side Spans:

$$C_1 = -\frac{(1-d_1)}{4}\left[\frac{(1+d_1)}{d_1} T_{1,2} + \frac{2}{c^2_1}\left(\frac{H}{r_1} - G_1\right)\right] \ldots\ldots (13a)$$

$$C_2 = +\frac{(1+d_1)}{4}\left[\frac{(1-d_1)}{d_1} T_{1,2} - \frac{2}{c^2_1}\left(\frac{H}{r_1} - G_2\right)\right] \ldots\ldots (13b)$$

and

$$C_1 + C_2 = -\frac{1}{c^2_1}\left(\frac{H}{r_1} - G_3\right) \ldots\ldots\ldots\ldots\ldots (13c)$$

Equations (13) for the side spans may be written from the corresponding Equations (12) for the main span by simply substituting the end moments, 0 and $T_{1,2}$, of the side spans for the end moments, T_1 and T_2, respectively, of the main span, and writing c_1, d_1, and r_1 instead of c, d, and r.

In Equations (12) and (13), the quantities, G_1, G_2, and G_3, are functions of the loading. The expressions for G_1 and G_3 are tabulated for the different loading cases in Article 9. The expressions for G_2 are not tabulated, because they may easily be written, if desired, from the parallel expressions for G_1 by simply substituting $-d$ for d throughout (in the same manner as the C_2 formulas may be written from the parallel formulas for C_1). For any span fully unloaded, G_1, G_2, and G_3 are zero. For any span fully loaded, $G_1 = G_2 = G_3 = p$.

A general formula for G_1, G_2, G_3, in any segment, m, of any span having any number of segments with any uniform load (or zero loading) in any segment, may be written easily, but is here omitted.

A new abbreviation is introduced in Equations (12) and (13):

$$d = \frac{(e^{cl} - 1)}{(e^{cl} + 1)} \qquad \text{(14a)}$$

and,

$$d_1 = \frac{(e^{c_1 l_1} - 1)}{(e^{c_1 l_1} + 1)} \qquad \text{(14b)}$$

This substitution eliminates, from the working formulas of this theory, the various exponential functions involving e^{cl} or $\epsilon^{c_1 l_1}$, thereby facilitating analytical operations and numerical application. The convenience of this abbreviation in simplifying the analysis is illustrated by the following typical resultant substitutions for exponential functions that, otherwise, would occur, singly or in combination, in the formulas of this paper:

$$\frac{(e^{cl} + e^{-cl} - 2)}{(e^{cl} - e^{-cl})} = d \qquad \frac{(e^{cl} + e^{-cl} + 2)}{(e^{cl} - e^{-cl})} = \frac{1}{d}$$

$$\frac{1}{(e^{cl} + 1)} = \frac{1}{2}\left(1 - d\right) \qquad \frac{1}{(e^{cl} - 1)} = \frac{1}{2}\left(\frac{1}{d} - 1\right)$$

$$\frac{e^{cl}}{(e^{cl} + 1)} = \frac{1}{2}\left(1 + d\right) \qquad \frac{e^{cl}}{(e^{cl} - 1)} = \frac{1}{2}\left(\frac{1}{d} + 1\right)$$

$$\frac{(e^{cl} + e^{-cl})}{(e^{cl} - e^{-cl})} = \frac{1}{2}\left(d + \frac{1}{d}\right) \qquad \frac{2}{(e^{cl} - e^{-cl})} = \frac{1}{2}\left(\frac{1}{d} - d\right)$$

A table (Table 1) or graph giving the values of d (or d_1) for different values of $c\,l$ (or $c_1\,l_1$) may be computed by Equations (14) or, with the aid of a table of natural hyperbolic functions, using the relation,

$$d = \frac{\sinh c\,l}{\cosh c\,l + 1} = \frac{\cosh c\,l - 1}{\sinh c\,l} = \tanh \frac{c\,l}{2} \qquad \text{(15)}$$

The direct use of Table 1 in the numerical application of the theory takes the place of tables of exponential or hyperbolic functions, or the equivalent logarithmic operations, and eliminates the need of working with the various expressions involving the exponentials, e^{cl}, e^{-cl}, $e^{0.5cl}$, $e^{-0.5cl}$, which heretofore have been conspicuous in Deflection Theory formulas. This substitution of *d*-terms for exponential expressions may easily be extended, if so desired, to the few remaining exponentials in the working formulas of this theory, thereby further condensing the formulas and completely eliminating exponential functions from the numerical solution.

TABLE 1.—VALUES OF *d*

cl	*d*	*cl*	*d*	*cl*	*d*	*cl*	*d*	*cl*	*d*	*cl*	*d*
2.5	0.84828	3.4	0.93541	4.3	0.97323	5.2	0.98903	6.1	0.99552	7.0	0.99818
2.6	0.86172	3.5	0.94138	4.4	0.97574	5.3	0.99007	6.2	0.99595	7.1	0.99835
2.7	0.87405	3.6	0.94681	4.5	0.97803	5.4	0.99101	6.3	0.99633	7.2	0.99851
2.8	0.88535	3.7	0.95175	4.6	0.98010	5.5	0.99186	6.4	0.99668	7.4	0.99878
2.9	0.89569	3.8	0.95624	4.7	0.98197	5.6	0.99263	6.5	0.99700	7.6	0.99900
3.0	0.90515	3.9	0.96032	4.8	0.98367	5.7	0.99333	6.6	0.99728	7.8	0.99918
3.1	0.91379	4.0	0.96403	4.9	0.98522	5.8	0.99396	6.7	0.99754	8.0	0.99933
3.2	0.92167	4.1	0.96740	5.0	0.98661	5.9	0.99454	6.8	0.99777	8.2	0.99945
3.3	0.92886	4.2	0.97045	5.1	0.98788	6.0	0.99505	6.9	0.99799	8.4	0.99955

In actual design it will generally be found that the values of d are between 0.96 and 0.9999, and that the values of d_1 are between 0.85 and 0.96.

It should be noted that unsymmetrically loaded spans are not reversible left to right without altering the values of the integration constants given by Equations (12) and (13), unless the origin of x is also reversed. Equations (13) for the integration constants in the side spans are correct only if x_1 is measured from the free end of the span. It should also be noted that the expressions for the integration constants, C, for any loading condition in a span, are unaffected by the loading conditions in the other spans.

Upon substituting zero for T_1 and T_2 (representing continuity), the foregoing formulas for C_1 and C_2 will reduce to the corresponding formulas for non-continuous suspension bridges. The functions, G_1, G_2, and G_3, are identical for continuous and non-continuous stiffening trusses.

The formulas of this Article for any span are not affected by the number or the proportions of the other spans. Hence, the foregoing formulas (including the G-functions) are directly applicable, without change, to multiple-span suspension bridges and to unsymmetrical structures. For the end spans of multiple-span bridges, use the side-span formulas; for any intermediate span, use the main-span formulas.

6.—EVALUATION OF T_1 AND T_2

The continuity moments, T_1 and T_2, are to be evaluated by equating deflection slopes at the towers.

Differentiating Equation (7), the general equation for the slope of the elastic curve of the stiffening truss at any point, is as follows:

$$\frac{d\eta}{dx} = \frac{1}{E I c^2}\left[c\,(C_1\,e^{cx} - C_2\,e^{-cx}) + \left(\frac{T_2 - T_1}{l} + \frac{dM_0}{dx}\right) - H\,\frac{4\,f}{l^2}\,(l - 2\,x)\right] \quad (16)$$

At the tower ends of the spans, Equation (16) yields the following values of the slope:

Main span, at $x = 0$:

$$\frac{d\eta}{dx} = \frac{1}{EI\,c^2}\left[c\,(C_1 - C_2)_{2L} + \left(\frac{T_2 - T_1}{l} + R_{2L}\right) - 4\,H\,n\right] \ldots (17a)$$

Main Span, at $x = l$:

$$\frac{d\eta}{dx} = \frac{1}{EI\,c^2}\left[c\,(C_1\,e^{cl} - C_2\,e^{-cl})_{2R} + \left(\frac{T_2 - T_1}{l} - R_{2R}\right) + 4\,H\,n\right] . (17b)$$

Left Side Span, at $x_1 = l_1$:

$$\frac{d\eta}{dx} = \frac{1}{EI\,c^2}\left[c_1\,(C_1\,e^{c_1 l_1} - C_2\,e^{-c_1 l_1})_{1R} + \left(\frac{T_1}{l_1} - R_{1R}\right) + 4\,H\,n_1\right] \ldots (17c)$$

Right Side Span, at $x_1 = l_1$:

$$-\frac{d\eta}{dx} = -\frac{1}{EI\,c^2}\left[c_1\,(C_1\,e^{c_1 l_1} - C_2\,e^{-c_1 l_1})_{3R} + \left(\frac{T_2}{l_1} - R_{3R}\right) + 4\,H\,n_1\right] \ldots (17d)$$

in which, R (with designating subscripts) denotes the simple-beam live-load reaction at the end of a span, and C_1 and C_2, with the same designating subscripts, are the pair of integration constants in the span segment at the same end of the same span.

The foregoing subscript notation will be found to be convenient later. The numeral subscript after a symbol or expression designates the number of the span, counting from left to right. The letter subscript designates the left or right end of the span (Subscript L at $x = 0$, and Subscript R at $x = l$).

Equating the two slopes at the left tower (Equations (17a) and (17c)), and the two slopes at the right tower (Equations (17b) and (17d)), and solving the resulting two simultaneous equations for T_1 and T_2:

$$(T_1 + T_2) = l_1\,(U_1 + U_2) \ldots\ldots\ldots\ldots\ldots\ldots (18a)$$

and

$$(T_1 - T_2) = \frac{l_1}{1 + 2\,\dfrac{l_1}{l}}\,(U_1 - U_2) \ldots\ldots\ldots\ldots\ldots (18b)$$

in which,

$$U_1 = (R_{1R} + R_{2L}) - c_1\,(C_1\,e^{c_1 l_1} - C_2\,e^{-c_1 l_1})_{1R} + c\,(C_1 - C_2)_{2L} - 4\,H\,(n + n_1) \ldots\ldots\ldots\ldots\ldots\ldots (18c)$$

and,

$$U_2 = (R_{2R} + R_{3R}) - c\,(C_1\,e^{cl} - C_2\,e^{-cl})_{2R} - c_1\,(C_1\,e^{c_1 l_1} - C_2\,e^{-c_1 l_1})_{3R} - 4\,H\,(n + n_1) \ldots\ldots\ldots\ldots\ldots\ldots (18d)$$

Substituting, in Equations (18), the general expressions for the integration constants given by Equations (12) and (13), and then solving for

$(T_1 + T_2)$ and $(T_1 - T_2)$, the following working formulas are obtained for evaluating T_1 and T_2:

$$(T_1 + T_2) = \frac{\dfrac{H}{r} F - \Sigma B_1}{cd + \dfrac{c_1}{2}\left(d_1 + \dfrac{1}{d_1}\right) - \dfrac{1}{l_1}} = \frac{\dfrac{H}{r} F - \Sigma B_1}{N} \quad \ldots (19a)$$

and,

$$(T_1 - T_2) = \frac{- \Sigma B_2}{\dfrac{c}{d} + \dfrac{c_1}{2}\left(d_1 + \dfrac{1}{d_1}\right) - \left(\dfrac{1}{l_1} + \dfrac{2}{l}\right)} = \frac{- \Sigma B_2}{(N + b)} \quad \ldots (19b)$$

in which,

$$F = 2\, r\left[4\,(n + n_1) - \left(\frac{d}{r\,c} + \frac{d_1}{r_1\, c_1}\right)\right] = \Sigma'\,(K\, L_c) \ldots\ldots (19c)$$

and in which, ΣB_1 and ΣB_2 are functions of the loading, p. These functions are related to previously derived functions, as follows: Let,

$$Y_{L,\,R} = R_{L,\,R} - \left[\frac{d}{2\,c}\,(G_1 + G_2) \mp \frac{1}{2\,c}\,(G_1 - G_2)\right]_{L,R} \quad \ldots\ldots (20a)$$

Then the values of B_1 and B_2 for the respective spans, are:

For the Main Span:

$$B_1 = (Y_L + Y_R);\; B_2 = (Y_L - Y_R) \ldots\ldots\ldots\ldots (20b)$$

For the Left Side Span:

$$B_1 = B_2 = Y_R \ldots\ldots\ldots\ldots (20c)$$

For the Right Side Span:

$$B_1 = - B_2 = Y_R \ldots\ldots\ldots\ldots (20d)$$

These contributions, B_1 and B_2, from the individual span loads are tabulated for the various loading cases in Article 9. The contributions from all three spans must be included in ΣB_1 and ΣB_2, respectively, for substitution in Equations (19). For any span fully unloaded (stressed only by the H from temperature and from any loading in the other spans), the contributions, B_1 and B_2, are zero. For any loading in the left side span, $B_2 = B_1$; for any loading in the right side span, $B_2 = - B_1$. For any loading symmetrical about the center line of the bridge, $\Sigma B_2 = 0$ (making $T_1 = T_2$). For any span fully loaded, $G_1 = G_2$ $(= p)$; hence, $Y_L = Y_R$.

For unloaded back-stays, write $n_1 = 0$ and $\dfrac{1}{r_1} = 0$ (or $K_1 = 0$) in the foregoing formulas. Equations (18) and (19) (the T-formulas) are written for symmetrical three-span continuous suspension bridges. For multiple-span continuous suspension bridges, and for unsymmetrical structures, new formulas for T_1 and T_2 must be written. Such formulas are derived in Article 11.

For non-continuous suspension bridges, of any number and proportions of spans, $T_1 = T_2 = 0$, and the formulas of this Article become unnecessary.

7.—Evaluation of H

The horizontal cable tension, H, due to any live load, p (including any supplemental dead load), and temperature change, t, all following the condition represented by the initial tension, H_w, may be evaluated as follows:

The total work, W_1, done in the vertical displacements of the suspender loads, $w + p_s$, and the cable weight, g, must equal the total work, W_2, done by the cable tension, $H_w + H$, in stretching the cable. These work quantities, W_1 and W_2, are expressed as the integrated products of the mean forces and their respective displacements, as follows, using the symbol, Σ, to denote the summation of similar expressions for all the spans:

$$W_1 = \sum \int_0^l \left(g + w + \frac{p_s}{2}\right) \eta \, dx = \sum \frac{8f}{l^2}\left(H_w + \frac{H}{2}\right)\int_0^l \eta \, dx$$

and,

$$W_2 = \left(H_w + \frac{H}{2}\right)\left[\frac{H}{E_c A_0} L_s \pm \omega t L_t\right]$$

in which,

$$L_s = \sum \int_0^l \frac{A_0}{A_c} \frac{ds^3}{dx^2} \quad \text{.....................(21a)}$$

and,

$$L_t = \sum \int_0^l \frac{ds^2}{dx} \quad \text{.......................(21b)}$$

For a parabolic wire cable, having uniform A_c, it will be sufficiently accurate to write:

$$L_s = \Sigma\, l\,(\sec^3 \alpha + 8\, n^2 \sec \alpha) \quad \text{.................(21c)}$$

and,

$$L_t = \Sigma\, l \left(\sec^2 \alpha + \frac{16}{3}\, n^2\right) \quad \text{.................(21d)}$$

Similarly, for a parabolic eye-bar cable, assuming A_c varying with the slope secant, $\frac{ds}{dx}$, it will be sufficiently accurate to write Equation (21d) for both L_s and L_t.

Equating the foregoing expressions for W_1 and W_2, the following work equation is obtained:

$$\sum \frac{8f}{l^2} \int_0^l \eta \, dx = \frac{H}{E_c A_0} L_s \pm \omega t L_t \quad \text{..............(22)}$$

Substituting the expression for η from Equation (7), and solving for H, the basic H-equation is:

$$H = \frac{\sum K \int_0^l \left(M_0 - \frac{p_x}{c^2}\right) dx \mp r c^2 EI \omega t L_t + \frac{(T_1 + T_2)}{2} \Sigma' (K l)}{\sum K \left[\frac{2}{3} f l - \frac{l}{r c^2} - \frac{1}{H}\int_0^l (C_1 e^{cx} + C_2 e^{-cx})\, dx\right] + r c^2 \frac{E}{E_c} \frac{I L_s}{A_0}} \quad \text{...(23)}$$

in which, E is the modulus of elasticity of the truss material and E_c that of the cable.

The summations, Σ, in Equation (23) embrace the corresponding expressions for all the spans; the symbol, Σ', likewise denotes a summation of corresponding expressions for all the spans, except that the side-span contributions are to be multiplied by $\frac{1}{2}$ before adding them.

The coefficient, K, occurring in the summations, denotes the ratio of $\frac{f}{l^2}$ for any span to $\frac{f}{l^2}$ of the main span; hence, $K = 1$ for the main span and $K_1 = \frac{r}{r_1}$ for the side spans. (Generally the ratio, K_1, for suspended side spans is between 1.00 and 1.05, representing the ratio of side-span weight to main-span weight per unit length.) For "unloaded" or "straight" back-stays, $K_1 = 0$, and all side-span terms vanish from the summations in Equation (23).

It should be noted that some approximations are involved in the foregoing derivation of the H-equation (Equation (23)), but the effect on the resulting stresses is found to be practically negligible.

In Equation (23) only the last term in the numerator represents the effect of continuity. With this term omitted (since $T_1 = T_2 = 0$ for hinges at the towers), Equation (23) reduces to the basic H-equation for non-continuous suspension bridges.

For any given loading condition, substitute the appropriate values of M_0, p_x, C_1, and C_2 in the integration terms of Equation (23), carry the two integrations over all span segments, and then re-solve the equation for H. This procedure yields the following general formula for H for all loading cases:

$$H = \frac{\Sigma\,(KA) + \frac{(T_1 + T_2)}{2}\,F \mp r\,c^2\,E\,I\,\omega\,t\,L_t}{\frac{2}{3}\,\Sigma\,(K\,f\,l) - \sum\left(\frac{K\,L_c}{r\,c^2}\right) + r\,c^2\,\frac{E\,I}{E_c\,A_0}\,L_s} \quad \ldots\ldots\ldots(24)$$

in which, A for each span is a function of the loading in that span. A general expression for A for any loading in any span is:

$$A = \int_0^l M_0\,dx - \frac{1}{c^2}\,(Y_L + Y_R) \quad \ldots\ldots\ldots\ldots\ldots\ldots(25)$$

Values of A for different loading cases are tabulated in Article 9. After deriving the expression for A for any segmental loading (such as Case III, Article 9) by the foregoing procedure, the expressions for all other loading cases may be written by simple processes of substitution, addition, and subtraction. Use can be made of the fact that the expression for A or $\Sigma\,(KA)$ (although not its value) is algebraically additive for combinations of loading.

In Equation (24) there is introduced the abbreviation:

$$L_c = l - \frac{2}{c}\,\frac{(e^{cl}-1)}{(e^{cl}+1)} = l - \frac{2\,d}{c} = \text{(approx.)}\ l - \frac{2}{c} \quad \ldots\ldots(26a)$$

and,

$$L_{c_1} = l_1 - \frac{2}{c_1}\,\frac{(e^{c_1 l_1}-1)}{(e^{c_1 l_1}+1)} = l_1 - \frac{2\,d_1}{c_1} \quad \ldots\ldots\ldots\ldots(26b)$$

This abbreviation will be found convenient in condensing the formulas and applications of the Deflection Theory.

All the foregoing formulas in this Article are directly applicable to symmetrical three-span suspension bridges, continuous or non-continuous; they may also be applied to non-continuous multiple-span suspension bridges, symmetrical or unsymmetrical. Applicability also to continuous multiple-span suspension bridges, symmetrical or unsymmetrical, is obtained with only a slight change in the form of the continuity term in the numerators of Equations (23) and (24) for H. This slight modification for complete generality of the H-formula is recorded in Article 11.

Substituting for $(T_1 + T_2)$ in Equation (24), the expression given by Equation (19a), and then re-solving for H, the working formula for H for symmetrical three-span suspension bridges, either continuous or non-continuous, is obtained as follows:

$$H = \frac{\Sigma\,(KA) - \dfrac{F}{2\,N}\,\Sigma\,B_1 \mp r\,c^2\,E\,I\,\omega\,t\,L_t}{D = \dfrac{2}{3}\,\Sigma\,(K\,f\,l) - \sum\left(\dfrac{K\,L_c}{r\,c^2}\right) + r\,c^2\,\dfrac{E\,I}{E_c\,A_0}\,L_s - \dfrac{F^2}{2\,r\,N}} \quad \ldots.(27)$$

in which, N is the denominator of Equation (19a).

In the working formula for H, Equation (27), the first term, $\Sigma\,(KA)$, in the numerator is the load term with continuity disregarded. The second term in the numerator, containing ΣB_1, is the load term due to continuity. The remaining term in the numerator, containing t, represents the effect of temperature change; it has the minus sign for a rise in temperature above normal, and the plus sign for a drop in temperature below normal.

In the denominator of Equation (27), the fourth term represents the contribution of the geometrical constants of the structure to the correction for continuity. In both numerator and denominator, the terms due to continuity are identified by the fact that they contain F $(= \Sigma'\,K\,L_c)$. For application to non-continuous suspension bridges, these terms vanish.

It is of interest to note that unsymmetrically loaded spans are reversible (left to right) without affecting the value of $\Sigma\,(KA)$. Furthermore, an unsymmetrically loaded main span is reversible (left to right) and unequally loaded side spans are interchangeable without affecting the value of H, since the shifting of any load to a symmetrical position about the center line of the entire structure does not alter the value of $(T_1 + T_2)$ which is represented by ΣB_1; but an unsymmetrically loaded side span cannot be reversed about its own center line without altering the value of B_1 and, therefore, of H. (This last distinction does not apply to non-continuous suspension bridges.)

The denominator of the H-formula (Equation (27)) will be denoted by the symbol, D. Although the expression for D remains unchanged for all loading conditions, it contains the variable, c, which depends upon H, and, therefore, the numerical value of D varies with the loading. The calculations for a given structure are facilitated by computing in advance the values of D for varying values of H, and tabulating or plotting the results for reference in the subsequent computations. (For an illustration, see Table 2, in Article 13.)

8.—Loading Conditions for Maximum Moments and Shears

In the continuous suspension bridge, loading conditions for maximum stress are not as uniform and definite as in the non-continuous structure; nor are they as easily predictable from a study of the basic equations. A study of the loading conditions that produced maximum calculated moments and shears for continuous suspension bridges that have been designed yields the guiding indications for loading placement that are tabulated in Fig. 2.

LOADINGS FOR MAXIMUM MOMENTS AND SHEARS

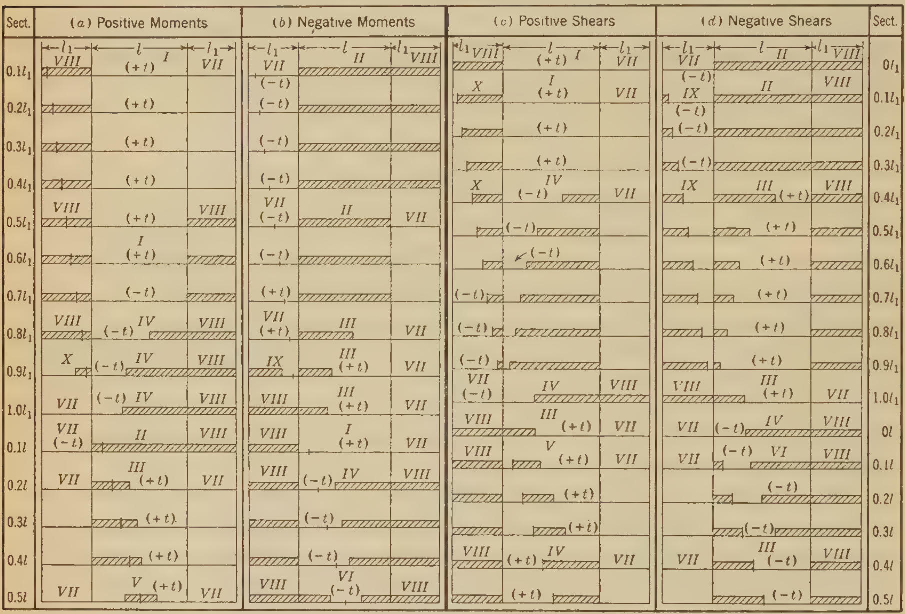


Fig. 2

The load lengths for maximum positive and negative moments and for shears generally have to be found by trial in the Deflection Theory. Generally, three trial values of the load length suffice to determine the maximum value of the function sought. In Fig. 2 the Roman numerals over the loading diagrams in the individual spans refer to the numbered designations of the respective basic loading cases tabulated in Article 9. Thus, for instance, the load condition shown in Fig. 2(a) for maximum positive moment at 0.1 l is resolved into the elementary Loading Cases VII, II, and VIII, the formulas for which are listed in Article 9.

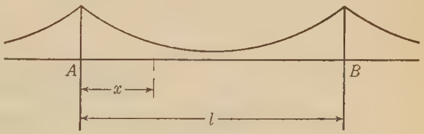
The symbols, $+t$ and $-t$, on the loading diagrams indicate whether a rise or fall in temperature should be assumed in conjunction with the indicated loading to yield the maximum value of the stress sought. A convenient rule, in practical application, is generally to use the highest temperature (in conjunction with the appropriate live-load placement) to obtain the absolute maximum M (disregarding sign) at any point of any span, and the lowest temperature (with the complementary live-load placement) to obtain the maximum of opposite sign. It may also be shown that the division points at which the temperature moment changes sign are, approximately, at $x = 0.15\,l$ and $0.85\,l$ in the main span and at $x_1 = 0.65\,l_1$ in the side spans. For sections near the towers, between the adjacent division points, use the lowest temperatures for maximum positive moment; for all other sections of the main and side spans, use the highest temperature for maximum positive moment.

In the case of structures of unusual geometric proportions, the loading diagrams in Fig. 2 may be subject to some modification. A preliminary analysis by the Elastic Theory will generally indicate the proper loading placements, as well as the trial values for the load lengths. The ten component loading cases given in Article 9 are fundamental, and are expected to cover all conditions arising in practical application.

9.—Working Formulas for Various Loadings

By considering each span separately, the various loading conditions that are useful in design (as illustrated in Article 8) may be resolved into ten primary loading cases: Six for main-span conditions, and four for side-span conditions. These ten cases, in combination, cover all loading conditions of practical importance; and they are presented in complementary pairs as follows:

Case I.—No Load in Main Span.—



Case *I*

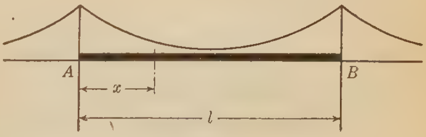
For calculating H, T_1, and T_2:

$$A = B_1 = B_2 = 0$$

For calculating C_1 and C_2 in Segment $A\,B$:

$$G_1 = G_2 = G_3 = 0$$

Case II.—Main Span Fully Loaded.—



Case *II*

For calculating H, T_1, and T_2:

$$A = \frac{p\,l^2}{12} - \frac{1}{c^2}\,B_1$$

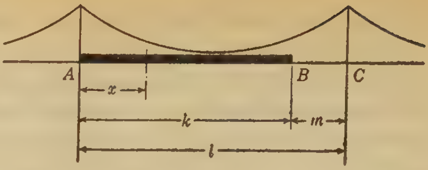
$$B_1 = p\left(l - \frac{2\,d}{c}\right) = p\,L_c$$

and,

$$B_2 = 0$$

For calculating C_1 and C_2 in Segment $A\,B$:

$$G_1 = G_2 = G_3 = p$$



Case III

Case III.—Main Span Loaded from Left End.—

For calculating *H*, T_1, and T_2:

$$A = p\,\frac{k^2}{12}\,(3\,l - 2\,k) - \frac{1}{c^2}\,B_1$$

$$B_1 = p\left[k - \frac{d}{c} - \frac{(1-d)}{2\,c}\,(e^{ck} - e^{cm})\right]$$

and,

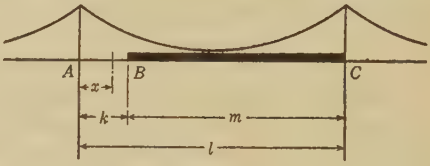
$$B_2 = p\left[\frac{k\,m}{l} - \frac{1}{c\,d} + \frac{(1-d)}{2\,c\,d}\,(e^{ck} + e^{cm})\right]$$

For calculating C_1 and C_2 in Segment *A B*:

$$G_1 = \frac{p}{4\,d}\left[(1+d)\,(e^{cm} + e^{-cm}) - 2\,(1-d)\right]$$

and,

$$G_2 = p$$



Case IV

Case IV.—Main Span Loaded from Right End.—

For calculating *H*, T_1, and T_2:

$$A = p\,\frac{m^2}{12}\,(3\,l - 2\,m) - \frac{1}{c^2}\,B_1$$

$$B_1 = p\left[m - \frac{d}{c} + \frac{(1-d)}{2\,c}\,(e^{ck} - e^{cm})\right]$$

and,

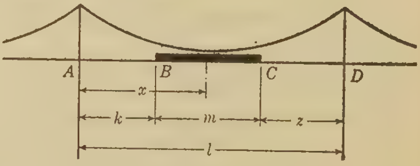
$$B_2 = -\,p\left[\frac{k\,m}{l} - \frac{1}{c\,d} + \frac{(1-d)}{2\,c\,d}\,(e^{ck} + e^{cm})\right]$$

For calculating C_1 and C_2 in Segment *A B*:

$$G_1 = -\,p\,\frac{(1+d)}{4\,d}\,(e^{cm} + e^{-cm} - 2)$$

and,

$$G_2 = 0$$



Case V

Case V.—Main Span Loaded in Middle Segment.—

For calculating *H*, T_1, and T_2:

$$A = p\,\frac{m}{4}\left[k\,(l-k) + z\,(l-z) + \frac{m^2}{3}\right] - \frac{1}{c^2}\,B_1$$

$$B_1 = p\left[m - \frac{(1-d)}{2\,c}\,(e^{cm} - 1)\,(e^{ck} + e^{cz})\right]$$

and,

$$B_2 = -\,p\left[m\,\frac{k-z}{l} - \frac{(1-d)}{2\,c\,d}\,(e^{cm} - 1)\,(e^{ck} - e^{cz})\right]$$

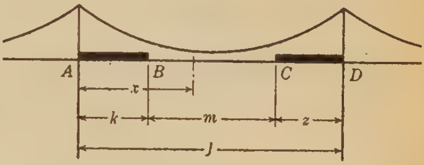
For calculating C_1 and C_2 in Segment $B\,C$:

$$G_1 = -\,\frac{p}{4\,d}\left[(1-d)\,(e^{ck} + e^{-ck}) - (1+d)\,(e^{cz} + e^{-cz})\right]$$

and,

$$G_3 = \frac{p}{2}\,(e^{ck} + e^{-ck})$$

Case VI.—Main Span Loaded in End Segments.—



Case VI

For calculating H, T_1, and T_2:

$$A = p\,\frac{k^3}{12}\,(3\,l - 2\,k) + p\,\frac{z^3}{12}\,(3\,l - 2\,z) - \frac{1}{c^2}\,B_1$$

$$B_1 = p\left[(k+z) - \frac{2\,d}{c} + \frac{(1-d)}{2\,c}\,(e^{cm} - 1)\,(e^{ck} + e^{cz})\right]$$

and,

$$B_2 = p\left[m\,\frac{k-z}{l} - \frac{(1-d)}{2\,c\,d}\,(e^{cm} - 1)\,(e^{ck} - e^{cz})\right]$$

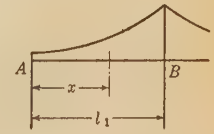
For calculating C_1 and C_2 in Segment $B\,C$:

$$G_1 = \frac{p}{4\,d}\left[(1-d)\,(e^{ck} + e^{-ck} - 2) - (1+d)\,(e^{cz} + e^{-cz} - 2)\right]$$

and,

$$G_3 = -\,\frac{p}{2}\,(e^{ck} + e^{-ck} - 2)$$

Case VII.—No Load in Side Span.—



Case VII

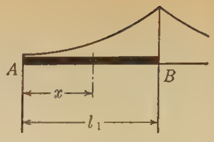
For calculating H, T_1, and T_2:

$$A = B_1 = B_2 = 0$$

For calculating C_1 and C_2 in Segment $A\,B$:

$$G_1 = G_2 = G_3 = 0$$

Case VIII.—Side Span Fully Loaded.—



Case *VIII*

For calculating H, T_1, and T_2:

$$A = p\,\frac{l^3_1}{12} - \frac{2}{c^2_1}\,B_1$$

$$B_1 = \frac{p}{2}\left(l_1 - \frac{2\,d_1}{c_1}\right) = \frac{p}{2}\,L_{c1}$$

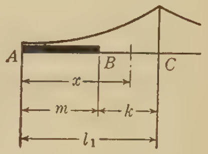
and,

$$B_2 = \pm\,B_1$$

For calculating C_1 and C_2 in Segment $A\,B$:

$$G_1 = G_2 = G_3 = p$$

Case IX.—Side Span Loaded from Free End.—



Case *IX*

For calculating H, T_1, and T_2:

$$A = p\,\frac{m^2}{12}\,(3\,l_1 - 2\,m) - \frac{p}{c^2_1}\left[m - \frac{d_1}{c_1} - \frac{(1-d_1)}{2\,c_1}\,(e^{c_1 m} - e^{c_1 k})\right]$$

$$B_1 = \frac{p}{2}\left[\frac{m^2}{l_1} - \frac{(1-d^2_1)}{2\,c_1\,d_1}\,(e^{c_1 m} + e^{-c_1 m} - 2)\right]$$

and,

$$B_2 = \pm\,B_1$$

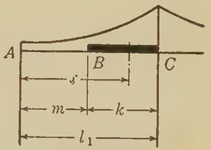
For calculating C_1 and C_2 in Segment $B\,C$:

$$G_1 = \frac{p}{4\,d_1}\,(1-d_1)\,(e^{c_1 m} + e^{-c_1 m} - 2)$$

and,

$$G_3 = -\,\frac{p}{2}\,(e^{c_1 m} + e^{-c_1 m} - 2)$$

Case X.—Side Span Loaded from Tower End.—



Case *X*

For calculating H, T_1, and T_2:

$$A = p\,\frac{k^2}{12}\,(3\,l_1 - 2\,k) - \frac{p}{c^2_1}\left[k - \frac{d_1}{c_1} - \frac{(1-d_1)}{2\,c_1}\,(e^{c_1 k} - e^{c_1 m})\right]$$

$$B_1 = \frac{p}{2}\left[\frac{l^2_1 - m^2}{l_1} - \frac{2d_1}{c_1} + \frac{(1-d^2_1)}{2\,c_1\,d_1}\,(e^{c_1 m} + e^{-c_1 m} - 2)\right]$$

and,

$$B_2 = \pm\,B_1$$

For calculating C_1 and C_2 in Segment $B\,C$:

$$G_1 = -\frac{p}{4\,d_1}\left[(1 - d_1)\,(e^{c_1 m} + e^{-c_1 m}) - 2\,(1 + d_1)\right]$$

and,

$$G_3 = \frac{p}{2}\,(e^{c_1 m} + e^{-c_1 m})$$

For each primary loading case, the pertinent working formulas are given in Cases *I* to *X* for the following load functions:

A = a load term in the numerator of the H-formula (Equation (24), or Equation (27)). The values of A contributed by the three spans are to be combined as $\Sigma\,(KA)$ for substitution in the H-formula. (The expressions for A are independent of continuity, and are the same for two-hinged suspension bridges.)

B_1 = a load term occurring as $\Sigma\,B_1$ in the continuity term of the numerator of the H-formula (Equation (27)), and in the numerator of the formula for $T_1 + T_2$ (Equation (19*a*)). The values of B_1 contributed by the three spans are combined as $\Sigma\,B_1$. (B_1 also occurs as a convenient abbreviation in all but the last two formulas for A.)

B_2 = a load term used in the numerator, ΣB_2, of the formula for $T_1 - T_2$ (Equation (19*b*)). The values of B_2 contributed by the three spans are to be combined as $\Sigma\,B_2$. (B_1 and B_2 are also useful in the T-formulas of Article 11 for multiple-span and unsymmetrical structures.)

G_1 = a load term used in the general working formula (Equation (12*a*) or Equation (13*a*)) for the integration constant, C_1. (Formulas for G_2, not tabulated, may be written if desired by simply changing the sign of d wherever it occurs in the formulas for G_1.)

G_3 = a load term used in the formula for $C_1 + C_2$ (Equation (12*c*), or Equation (13*c*)).

10.—Simplified Formulas for Spans Fully Loaded or Unloaded

Let p, p_1, and p_2 be the intensities of loading covering the main span, the left side span, and the right side span, respectively. Then, by simply writing zero for one or more of these loading intensities, the fully unloaded condition of the corresponding span or spans is represented.

Introduce the following abbreviations for the three respective spans:

$$S = p - \frac{H}{r} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(28a)$$

$$S_1 = p_1 - \frac{H}{r_1} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(28b)$$

and,

$$S_2 = p_2 - \frac{H}{r_2} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(28c)$$

As thus defined, S in any span represents, approximately, the uniform load actually carried by the stiffening truss.

For any condition of loading, the general formula for moments (Equation (9)) may be written in the form:

$$M = \frac{1}{c^2} S - (C_1 e^{cx} + C_2 e^{-cx}) \ldots\ldots\ldots\ldots\ldots\ldots (29)$$

provided p_x is used in the formula for S.

For the main span fully loaded or unloaded (Case I or Case II), the constants of integration become:

$$C_1 = \frac{1-d}{2}\left(\frac{S}{c^2} - T_1\right) + \frac{1-d^2}{4d}(T_1 - T_2) \ldots\ldots\ldots (30a)$$

and,

$$C_2 = -C_1 + \left(\frac{S}{c^2} - T_1\right) \ldots\ldots\ldots\ldots\ldots\ldots (30b)$$

and those for the left side or right side span fully loaded or unloaded (Case VII, or Case VIII), become:

$$C_1 = \frac{1-d_1}{2}\,\frac{S_{1,2}}{c^2_1} - \frac{1-d^2_1}{4d_1}\,T_{1,2}, \ldots\ldots\ldots\ldots\ldots\ldots (30c)$$

and,

$$C_2 = -C_1 + \frac{S_{1,2}}{c^2_1} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (30d)$$

Introduce the abbreviations:

$$h = 1 - \sqrt{1-d^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (31a)$$

and,

$$h_1 = 1 - \sqrt{1-d^2_1} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (31b)$$

Substitution of Equations (30) in the general formula for moments (Equation (29)), yields the following simplified formulas:

At $x = 0.5\,l$,

$$M = \frac{h}{c^2} S + (1-h)\frac{T_1 + T_2}{2} = (\text{approx.})\ \frac{h}{c^2} S \ldots\ldots\ldots (32a)$$

At $x_1 = 0.5\,l_1$,

$$M = \frac{h_1}{c^2_1} S_{1,2} + (1-h_1)\frac{T_{1,2}}{2} = (\text{approx.})\ \frac{h_1}{c^2_1} S_{1,2} \ldots\ldots\ldots (32b)$$

The approximate value for M in the main span is correct within 2%, on the safe side. Hence, $\frac{h}{c^2} S$ represents, very closely, the total bending moment at the middle of the main span. The approximate value for M in the side span is less permissible, although it is also on the safe side. For a non-continuous structure, both approximate values become exact.

Substituting Equations (30) in the general formula for deflections (Equation (7)), or substituting Equations (32) in Equation (6), the following simplified formulas are obtained for mid-span deflections, Δf and $\Delta f_{1,2}$:

$$EIc^2 \, \Delta f = \left(\frac{l^2}{8} - \frac{h}{c^2} \right) S + \frac{h}{2} (T_1 + T_2) \, \ldots\ldots\ldots\ldots (33a)$$

and,

$$EIc^2 \, \Delta f_{1,2} = \left(\frac{l^2_1}{8} - \frac{h_1}{c^2_1} \right) S_{1,2} + \frac{h_1}{2} T_{1,2} \, \ldots\ldots\ldots\ldots (33b)$$

These are convenient working formulas for computing maximum deflections in any span of any suspension bridge. (For non-continuous structures, the T-terms vanish and only the term containing S remains in each expression.)

Correspondingly simple formulas for shears may be written by substituting Equations (30) in Equation (10). Only the following approximate values are here presented (correct within a fraction of 1%):

At $x = 0$, or l,

$$V = \text{(approx.)} \pm \left(\frac{S}{c} - c \, T_{1,2} \right) \ldots\ldots\ldots\ldots (34a)$$

At $x_1 = l_1$,

$$V = \text{(approx.)} - \left(\frac{d_1}{c_1} S_{1,2} - c_1 \, T_{1,2} \right) \ldots\ldots\ldots\ldots (34b)$$

Equations (28) to (34), inclusive, are correct in any span fully loaded or unloaded, no matter what the loading may be (full, zero, or partial) in the other spans. This fact increases the usefulness of the foregoing formulas in practical application. These equations are directly applicable to three-span and multiple-span suspension bridges, whether continuous or non- continuous, symmetrical or unsymmetrical. The formulas that follow are written only for symmetrical, three-span, continuous structures. For the loading under consideration (each span either fully loaded or unloaded), Equations (19) yield the following formulas for evaluating T_1 and T_2:

$$(T_1 + T_2) = \frac{\frac{H}{r} F - \Sigma' (p \, L_c)}{N} = -\frac{1}{N} \Sigma' (S \, L_{c_1}) \ldots\ldots\ldots (35a)$$

$$(T_1 - T_2) = -\frac{\frac{1}{2} (p_1 - p_2) L_c}{N + b} \ldots\ldots\ldots\ldots (35b)$$

or, approximately,

$$T_{1,2} = -\frac{1}{2 \, N} \left[S \, L_c + S_{1,2} \, L_{c_1} \right] \ldots\ldots\ldots\ldots (36)$$

(This approximate value is exact for symmetrical loading.) Substituting in Equation (27) the values of $\Sigma (KA)$ and ΣB_1 for spans fully loaded or unloaded:

$$H = \frac{\sum K \left(\frac{p \, l^3}{12} - \frac{1}{c^2} p \, L_c \right) - \frac{F}{2 \, N} \Sigma' (p \, L_c) \mp r \, c^2 \, E \, I \, \omega \, t \, L_t}{D} \ldots (37)$$

For the effect of temperature variation with no load on the spans (Cases I and VII), $p = 0$, $p_1 = 0$, $p_2 = 0$, $A = 0$, $B_1 = 0$, $B_2 = 0$, $G_1 = 0$, $G_2 = 0$, and $G_3 = 0$, and Equations (28) to (37) are further simplified.

11.—APPLICATION TO MULTIPLE-SPAN SUSPENSION BRIDGES AND UNSYMMETRICAL STRUCTURES

For non-continuous three-span and multiple-span suspension bridges, whether symmetrical or unsymmetrical, Equations (1) to (37), inclusive, are completely valid and sufficient, without any modification. (For any non-continuous structure, simply omit the recognizable terms representing continuity.)

For continuous multiple-span and unsymmetrical suspension bridges, all these formulas are directly applicable except those for T_1 and T_2 and the continuity terms in the H-formulas.

All formulas in Article 4 ("Fundamental Equations") and in Article 5 ("Integration Constants") remain valid without modification for unsymmetrical or multiple spans. The formulas established for the "side spans" of the three-span bridge should be applied to the "end spans" of the multiple-span structure, and those for the "main span" should be applied to each "intermediate span" of the multiple-span suspension bridge.

In Article 7 ("Evaluation of H") all formulas remain unmodified except the continuity terms in the numerator and denominator of the H-equations. In Equation (23), instead of $\frac{(T_1 + T_2)}{2} \sum{}' (K\,l)$ write $\sum \left(\frac{T_L + T_R}{2} K\,l \right)$. In Equation (24), instead of $\frac{(T_1 + T_2)}{2} F = \frac{(T_1 + T_2)}{2} \sum{}' (K\,L_c)$ write $\sum \left(\frac{T_L + T_R}{2} K\,L_c \right)$. With these slight modifications, the continuity terms are generalized to cover multiple-span as well as unsymmetrical three-span structures. For evaluating H in multiple-span and unsymmetrical suspension bridges, Equation (24), as thus modified for complete generality, should be used instead of Equation (27) and its derivative, Equation (37), since the continuity terms in the latter formulas are valid only for symmetrical three-span structures and would be replaced by more complicated expressions for the general case.

All the working formulas tabulated in Article 9 ("Loading Cases") remain valid without modification for unsymmetrical or multiple spans as well as all the formulas through Equations (34) in Article 10 ("Simplified Formulas").

The formulas for T_1 and T_2 presented in Article 6 (Equations (18) and (19)), and the derived simplified T-formulas in Article 10 (Equations (35) and (36)) are valid only for symmetrical three-span structures. For the more general case of multiple-span suspension bridges, of any number and proportions of spans, new T-formulas are needed and are presented hereinafter. (Note that Equations (20) are general and remain valid.)

Following the procedure of Article 6, let the break in slope at the first, second, third, etc., towers (before neutralization by the tower moments. T. and omitting the uniform factor, $EI\,c^2$, be denoted by U_1, U_2, U_3, etc. Then (compare Equation (18*c*)) the expression for U at any tower, say, at the second tower, is of the general form:

$$U_2 = (R_{2R} + R_{3L}) - c_2\,(C_1 e^{c_2 l_2} - C_2 e^{-c_2 l_2})_{2R} + c_3\,(C_1 - C_2)_{3L} - 4\,H\,(n_2 + n_3) \quad \text{.......................(38)}$$

In Equation (38) the numeral and letter subscripts have the significance explained in Article 6. Advance the numeral subscripts in writing the similar expressions for U at the successive towers. (The expression for U at the last tower would be similarly written, if x in the last span were measured from left to right; otherwise, the third term has the reversal modification shown in Equation (18*d*)).

The generalized basic formulas for T_1, T_2, etc., at the n successive towers are then as follows:

$$\left.\begin{array}{l}
\dfrac{T_1 - 0}{l_1} - \dfrac{T_2 - T_1}{l_2} = U_1 \\[2ex]
\dfrac{T_2 - T_1}{l_2} - \dfrac{T_3 - T_2}{l_3} = U_2 \\[2ex]
\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\[2ex]
\dfrac{T_n - T_{n-1}}{l_n} - \dfrac{0 - T_n}{l_{n+1}} = U_n
\end{array}\right\} \quad \text{.....................(39)}$$

Since there are as many of these equations as there are towers, Equations (39) can be solved for the n-values of T. (It is interesting to note that the functions, U and T, appear in Equations (39) in exactly the same relationship as concentrated loads and bending moments, respectively, on a simple beam of span, $\Sigma\, l$. Consequently, a solution of Equations (39) for the value of T at any tower may be written directly as the bending moment at that point producible by the functions, U, applied as concentrated loads at the respective tower points on the span, $\Sigma\, l$.)

In Equations (39), substitute the respective values of U as defined by Equation (38), utilizing the relations given by Equations (12), (13), and (20*a*). In this way the following generalized working formulas for T, for any number of spans of any proportions, are obtained:

$$\left.\begin{array}{rl}
0 - (a_1 + a_2)\;\; T_1 \;\; + b_2\, T_2 & = 2\,(Y'_{1R} + Y'_{2L}) \\
b_2\, T_1 - (a_2 + a_3)\;\; T_2 \;\; + b_3\, T_3 & = 2\,(Y'_{2R} + Y'_{3L}) \\
b_3\, T_2 - (a_3 + a_4)\;\; T_3 \;\; + b_4\, T_4 & = 2\,(Y'_{3R} + Y'_{4L}) \\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots & \cdots\cdots\cdots\cdots \\
b_n\, T_{n-1} - (a_n + a_{n+1})\; T_n + \;\; 0 & = 2\,(Y'_{nR} + Y'_{(n+1)R})
\end{array}\right\} \quad \text{.....(40)}$$

Equations (40) are applicable, with complete generality, to all types of continuous suspension bridges considered in this paper, whether symmetrical or unsymmetrical, three-span or multiple-span, with tie cables or without tie cables, and with or without tower resistance to be considered.

In writing Equations (40), the following abbreviations are used: In any span,

$$a = cd + \frac{c}{d} - \frac{2}{l} \qquad \text{.....................(41a)}$$

and,

$$b = -\,cd + \frac{c}{d} - \frac{2}{l} \qquad \text{...................(41b)}$$

At either end of any span,

$$Y'_{L,R} = Y_{L,R} - \frac{H\,L_c}{2\,r} \qquad \text{......................(42)}$$

(Note that the subscripts, L and R, refer to the left and right ends, respectively, of any span; except that, in the last span, where the direction of measuring x is reversed, Y'_R and Y_R refer to the left or tower end.) By Equations (20), in any intermediate spans,

$$Y'_{L,R} = \frac{1}{2}\,(B_1 \pm B_2) - \frac{H\,L_c}{2\,r} \qquad \text{..................(42a)}$$

and in either end span,

$$Y'_R = B_1 - \frac{H\,L_c}{2\,r} \qquad \text{..........................(42b)}$$

Hence, in any span fully loaded,

$$Y'_L = Y'_R = \frac{1}{2}\,L_c\;S \qquad \text{.....................(42c)}$$

and in any unloaded span,

$$Y'_L = Y'_R = -\,\frac{H\,L_c}{2\,r} \qquad \text{.....................(42d)}$$

For a symmetrical three-span continuous suspension bridge, Equations (39) reduce to Equations (18), and Equations (40) reduce to Equations (19). For an unsymmetrical three-span continuous suspension bridge (as well as for any three-span continuous suspension bridge with tie cables), Equations (40) reduce to:

$$\left.\begin{aligned} T_1 &= -\,2\,\frac{(a + a_3)\,(Y'_{1R} + Y'_{2L}) + b\,(Y'_{2R} + Y'_{3R})}{(a + a_1)\,(a + a_3) - b^2} \\ &\text{and,} \\ T_2 &= -\,2\,\frac{b\,(Y'_{1R} + Y'_{2L}) + (a + a_1)\,(Y'_{2R} + Y'_{3R})}{(a + a_1)\,(a + a_3) - b^2} \end{aligned}\right\} \text{..... (43)}$$

Simply write $a_1 = a_3$ to obtain Equations (19) for the symmetrical case. For a symmetrical continuous four-span suspension bridge, Equations (40) reduce to the following working formulas for T:

$$\left.\begin{aligned}
T_2 &= -\frac{b\,(Y'_{1R} + Y_2{}'_L + Y'_{3R} + Y'_{4R}) + (a + a_1)\,(Y'_{2R} + Y'_{3L})}{a\,(a + a_1) - b^2} \\
\frac{1}{2}\,(T_1 + T_3) &= -\frac{a\,(Y'_{1R} + Y'_{2L} + Y'_{3R} + Y'_{4R}) + b\,(Y'_{2R} + Y'_{3L})}{a\,(a + a_1) - b^2} \\
\frac{1}{2}\,(T_1 - T_3) &= -\frac{(Y'_{1R} - Y'_{4R}) + (Y'_{2L} - Y'_{3R})}{(a + a_1)}
\end{aligned}\right\} \quad \ldots.(44)$$

With the aid of Equations (42*a*) and (42*b*), these working formulas may be re-written in terms of B_1 and B_2 which are tabulated in Article 9. If the successive spans are fully loaded with uniform loads of p_1, p_2, p_3, and p_4, respectively (where any or all of these loads may be zero, as in Article 10), Equations (43) and (44) may be re-written in simple form in terms of S_1, S_2, S_3, and S_4, upon substituting Equation (42*c*).

Always starting from the completely general Equations (40) (in the same manner as Equations (43) and (44) were written for the three-span and four-span cases, respectively) similar working formulas may be written for other cases of continuous multiple-span or unsymmetrical suspension bridges, of any number of spans.

The formulas in the preceding Articles included non-continuous suspension bridges in general, and symmetrical three-span continuous structures. With the T-formulas (and modifications of the H-formulas) presented in this Article, the Deflection Theory is further generalized and extended to cover also multiple-span and unsymmetrical continuous suspension bridges.

12.—Application to Multiple-Span Suspension Bridges with Tie Cables

Multiple-span suspension bridges may be increased in efficiency by the provision of tie cables connecting the tower tops. The function of the tie cables is to reduce relative tower-top movements or span elongations, $\Delta\, l$. The idea was invented about 75 years ago, and, more recently, several structures of this type have been built in France and projected in the United States; but an accurate analysis for this type of bridge has been lacking thus far. This practical problem affords an opportunity to show the comprehensiveness and convenience of the "Generalized Deflection Theory" as a foundation for further extensions of suspension-bridge theory. Solution of the problem of analysis is, of course, a prerequisite to correct economic evaluation and application of a structural type.

For multiple-span suspension bridges with tie cables, all the formulas in the preceding Articles remain valid except the formulas for H and some of the formulas for T (namely, the T-formulas written for symmetrical structures). These formulas require some modification when tie cables are used, because H then varies from span to span. All other formulas of this paper remain directly applicable without change, including specifically all the formulas in Article 4 ("Fundamental Equations"), Article 5 ("Integration Constants"), and Article 9 ("Working Formulas"), as well as Equations (28) to (34), inclusive, of Article 10 ("Simplified Formulas"), and Equations (39) to (43), inclusive, of Article 11 ("Multiple Spans and Unsymmetrical Structures").

The modified procedure for determining H in each span is, as follows: Let H_0 be the initial horizontal tension in the tie cable, corresponding to H_w in the main cable. Then $(H_0 + H_w)$ is the same in all spans (except for a minor correction, usually negligible, for any initial stress in the towers in the direction of cable tension). Let H_T be the additional horizontal tension induced in the tie cable (by span elongation and temperature change), corresponding to H (due to live load, span elongation, and temperature change) in the main cable. Then $(H + H_T)$ has the same value in all spans, except for a minor correction due to the deflection resistance of the towers. If j is the resistance to unit deflection of the tower top,

$$(H + H_T)_1 = (H + H_T)_2 - j_1\,\Delta l_1$$
$$= (H + H_T)_3 - j_1\,\Delta l_1 - j_2\,(\Delta l_1 + \Delta l_2) = \ldots \text{ etc.} \ldots\ldots\ldots (45)$$

The subscripts designate the numbers of the successive spans and of the successive towers, counting from one end of the bridge.

From the geometry of the tie cable before and after deflection, one may write with sufficient accuracy:

$$H_T = P\,\Delta l + X \ldots\ldots\ldots\ldots\ldots\ldots (46)$$

in which,

$$P = \frac{E_T\,A_T}{(L_s)_T} \ldots\ldots\ldots\ldots\ldots\ldots (46a)$$

and,

$$X = E_T\,A_T\left[\frac{8}{3}\,n^2{}_T\,(R^2{}_0 - 1) - \omega t\right] \ldots\ldots\ldots\ldots (46b)$$

In Equations (46*a*) and (46*b*), the subscripts, T, designate tie-cable quantities, and,

$$R_0 = \frac{H_0}{H_0 + H_T} \ldots\ldots\ldots\ldots\ldots\ldots (46c)$$

To write the corresponding H-formula for the main cable, follow the procedure of Article 7, equating external and internal work, but limit consideration to a single span. In this case, the expression for external work, W_1, contains the additional term, $\left(H_w + \frac{H}{2}\right)\Delta l$, representing the work done against cable tension by the span elongation, Δl. With this modification, Equation (24) takes the form:

$$H = Q\,\Delta l + Z \ldots\ldots\ldots\ldots\ldots\ldots (47)$$

in which,

$$Q = \frac{1}{D'}\,(r\,c^2\,E\,I) \ldots\ldots\ldots\ldots\ldots\ldots (47a$$

and,

$$Z = \frac{1}{D'}\left[A + \frac{L_c}{2}(T_L + T_R) - r\,c^2\,E\,I\,\omega\,t\,L_t\right] \ldots\ldots\ldots (47b)$$

In Equations (47*a*) and (47*b*), D' is the denominator of the H-formula for a single span:

$$D' = \frac{2}{3}\,f\,l - \frac{L_c}{r\,c^2} + r\,c^2\,\frac{E\,I}{E_c\,A_0}\,L_s \ldots\ldots\ldots\ldots\ldots\ldots (47c)$$

In Equations (46*a*) and (47*b*), L_s and L_t are the values for a single span. From Equations (46) and (47) combined, the elongation of any span is given by,

$$\Delta\,l = \frac{(H + H_T) - (X + Z)}{(P + Q)} \ldots\ldots\ldots\ldots\ldots\ldots (48)$$

The values of Δl for all the spans must always satisfy the primary governing conditions:

$$\Sigma\,(\Delta l) = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (48a)$$

Upon substituting Equations (45) and (48), Equation (48*a*) yields:

$$(H + H_T)_1 = \frac{\sum\left(\frac{X + Z}{P + Q}\right) - \sum\left(\frac{J\,\Delta l}{P + Q}\right)}{\sum\left(\frac{1}{P + Q}\right)} \ldots\ldots\ldots (49)$$

which is the key formula for the solution of multiple-span suspension bridges with tie cables. In Equation (49),

$$\sum\left(\frac{J\,\Delta l}{P + Q}\right) = \Delta l_1\left[\frac{j_1}{(P + Q)_2} + \frac{(j_1 + j_2)}{(P + Q)_3} + \frac{(j_1 + j_2 + j_3)}{(P + Q)_4} + \ldots\right]$$
$$+ \;\Delta l_2\left[\frac{j_2}{(P + Q)_3} + \frac{(j_2 + j_3)}{(P + Q)_4} + \ldots\right] + \Delta l_3\left[\frac{j_3}{(P + Q)_4} + \ldots\right] + \ldots \;.(49a)$$

Equation (49*a*) is a minor term in Equation (49) and is usually negligible, The other quantities in Equation (49) are the simple summations of the respective indicated expressions for all the spans.

If the elastic resistances or deflections of the towers are negligible, or if rocker towers are used, one may write $j = 0$ and Equations (45) and (49) reduce to:

$$(H + H_T)_1 = (H + H_T)_2 = (H + H_T)_3 = \ldots = \frac{\sum\left(\frac{X + Z}{P + Q}\right)}{\sum\left(\frac{1}{P + Q}\right)} \ldots . (50)$$

In any case, Equation (50) may be used as a first approximation. It will generally be sufficiently accurate except in cases of rigid towers or extreme distortion of the structure.

After $(H + H_T)$ is determined for all spans by Equation (50), or for the individual spans by Equations (49) and (45), the value of Δl for any span is given by Equation (48), and the value of H for the span is then given by Equation (47). The values of M, V, η, etc., are then given by the formulas of Articles 4, 5, and 9.

The solution of the problem to this point suffices if the stiffening truss is non-continuous at the towers. In such case, simply write $T_L = T_R = 0$ in Equation (47*b*).

If the multiple-span suspension bridge with tie cables has the stiffening truss continuous at the towers, the correct expressions for T are given for any number of spans of any proportions by the general T-formulas, Equations (39) and (40) of Article 11. Equations (18) and (43) also remain valid. In the expressions for U, typified by Equation (38), a slight modification is required to provide for the varying H: Instead of $4\ H\ (n_2 + n_3)$, write $4\ (H_2\ n_2 + H_3\ n_3)$. The forms in which the general T-formulas (Equations (40)) are written, as well as any T-formulas for unsymmetrical structures, automatically take care of this correction.

The formulas of this Article also cover suspension bridges without tie cables, but with deflection resistances of towers to be considered. In such cases, simply write zero for H_0, H_T, P, and X, in the foregoing formulas. If, in addition, $j = 0$, the formulas of this Article reduce to those of Article 7.

13.—Practical Application to Continuous Spans

In order to test the practical applicability of the Generalized Deflection Theory and, at the same time, to establish data for comparisons of types of structure and theories of analysis, the theory and formulas developed in this paper have been applied to the analysis of a three-span continuous suspension bridge.

Design I

The structure selected for the numerical application of the theory has an 800-ft main span and two 400-ft side spans, and had been previously designed as a two-hinged suspension bridge. The trusses have a constant depth of 12 ft throughout and are spaced 45 ft, center to center. For the first comparative design (herein referred to as Design I), the same moments of inertia, ($I = 1\,960$ in.^{2} ft^2, and $I_1 = 2\,420$ in.^{2} ft^2), were assumed as in the two-hinged design, in order to ascertain comparative rigidity under conditions of equal economy.

Comparison with Elastic Theory.—The stresses in the continuous spans were first computed by the Elastic Theory. This preliminary analysis incidentally yielded the approximate loading conditions to be used as a guide for assuming trial load lengths in the more exact analysis. The stresses were then computed more accurately by the Generalized Deflection Theory, using the formulas and procedure developed in this paper. The maximum bending moments yielded for Design I by the two respective theories are plotted in

Fig. 3. The percentage reductions obtained by the application of the Deflection Theory are also plotted. These reductions in maximum bending moments range from 11% at the tower to 50% in each span. Except for a comparatively short length close to the tower (including about one-eighth of the side span and one-twentieth of the main span) where the average reduction effected by the Deflection Theory is only about 13%, the reductions are approximately 44% in the main span and 48% in the side span. A comparison of the total areas under the bending-moment graphs for the two respective theories, as plotted in Fig. 3, shows that the reduction or saving yielded by applying the Deflection Theory to a continuous stiffening truss is 44.5% as an average for the entire length of the structure.

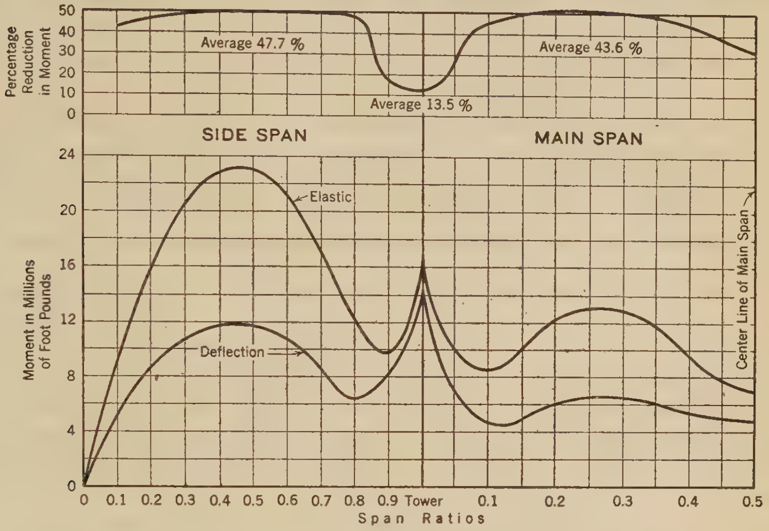


FIG. 3.—COMPARISON OF MAXIMUM MOMENTS IN STIFFENING TRUSSES BY ELASTIC AND DEFLECTION THEORIES, WITH PERCENTAGES OF REDUCTION IN MOMENT.

The percentages of reduction from the Elastic Theory in the case of continuous spans are closely comparable to the reduction percentages previously established for two-hinged suspension bridges. The direct application of any approximate factor of reduction, however, is modified in the continuous structure by the variation of reduction ratio near the towers.

Comparison of Deflections.—The maximum deflections in the two-hinged design, computed by the Deflection Theory (using Equations (33), with $T_1 = T_2 = 0$) were 5.16 ft in the main span and 3.30 ft in the side spans. In the continuous structure of Design I (having the same assumed moments of inertia to represent equal economy), the maximum deflections were also computed by Equations (33) and were found to be 5.28 ft in the main span and only 2.81 ft in the side spans. The comparison of maximum deflections shows a reduction of 14.9% in the side spans of the continuous design and an increase of 2.3% in the center span, or an average reduction of 6.3% for the entire structure. Hence, for designs of equal economy, the continuous structure of 800-ft main span is, on the whole, about 6% more rigid than the two-hinged type.

Design II

In order to ascertain comparative economy for equal rigidity, the assumed moments of inertia for the continuous structure were modified for a second analysis. Following the indications of the comparison of deflections yielded by Design I, the new assumptions for Design II were $I = I_1 = 1\,960$ in.^{2} ft^2 (as compared with the values of $I = 1\,960$ and $I_1 = 2\,420$ in the two-hinged design). Typical computations for Design II, illustrating the application of the Generalized Deflection Theory to a three-span continuous suspension bridge, are herewith presented.

General Data.—Calculation of Constants.—The following are the dimensional constants: Main span: $l = 64$ panels $= 800$ ft; $f = 84$ ft; and $n = 0.105$; side spans: $l_1 = 32$ panels $= 400$ ft; $f_1 = 21$ ft; and $n_1 = 0.0525$; $\sec \alpha = 1.03484$; $r = r_1 = 952.381$; $K_1 = 1.00$; and $A = 87.8$ in.^{2}

The following are the loading constants (all values per cable): Dead load: $w = 3\,850$ lb per ft; live load: $p = 1\,300$ lb per ft; $H_w = \dfrac{w\,l^2}{8\,f} = 3\,667$ kips; temperature: $t = \pm 60°$ F; $E = 29\,000\,000$; $\omega = 0.0000065$; and, $E\,\omega\,t = 11\,310$ lb per sq in.

By Equations (21): $L_s = 2\,075$, and $L_t = 1\,998$. $E_c = 25\,000\,000$. The truss constants are as follows: Main span: $I = 1\,960$ in.^{2} ft^2, and $E\,I = 56\,840\,000$ ft^2 kips; side span: $I_1 = 1\,960$ in.^{2} ft^2, and $E\,I_1 = 56\,840\,000$ ft^2 kips.

Calculation of Values of D.—Preparatory to obtaining values of H for various conditions of loading, the values of D (the denominator of the formula for H) were calculated by Equation (27) for different values of H from -100 to $+1\,200$ kips. The values of d and d_1 for the various values of $c\,l$ and $c_1\,l_1$ were written directly from Table 1 (Article 5). Computations for the values of D are made by a systematic tabular method which embraces the step-by-step numerical operations. The principal constants and the values of D for the different values of H are given in Table 2. The values of D used in computing H for various loadings may be obtained for any trial H by interpolation in this tabulation.

TABLE 2.—COMPUTATION OF *D*.

H, in kips	$c = c_1$	$cl = 2\,c_1l_1$	d	d_1	D
—100	0.007921	6.3368	0.99647	0.91927	24 226
0	0.008032	6.4256	0.99677	0.92262	24 600
+100	0.008141	6.5128	0.99704	0.92579	24 955
200	0.008248	6.5984	0.99728	0.92880	25 325
300	0.008354	6.6832	0.99750	0.93165	25 671
400	0.008458	6.7664	0.99770	0.93436	26 020
600	0.008664	6.9312	0.99805	0.93940	26 685
800	0.008865	7.0920	0.99834	0.94393	27 342
1 000	0.009061	7.2488	0.99858	0.94806	27 972
1 200	0.009253	7.4024	0.99878	0.95181	28 584

Values of H for Advancing Uniform Load in Main Span.—The values of H, T_1, and T_2 for varying lengths of a continuous advancing uniform live load in the main span, at highest temperature and with no load in the side spans, are necessary in the computation of the maximum positive moments at several points in the main span. By the use of Equations (27) and (19) with the formulas of Cases III and VII, the values of H, T_1, and T_2 are calculated for the aforementioned loading for successive lengths varying by 0.1 of the span. Agreement with the trial value of H is usually obtained on the second trial, and the values of T_1 and T_2 are then computed. The results are plotted in Fig. 4.

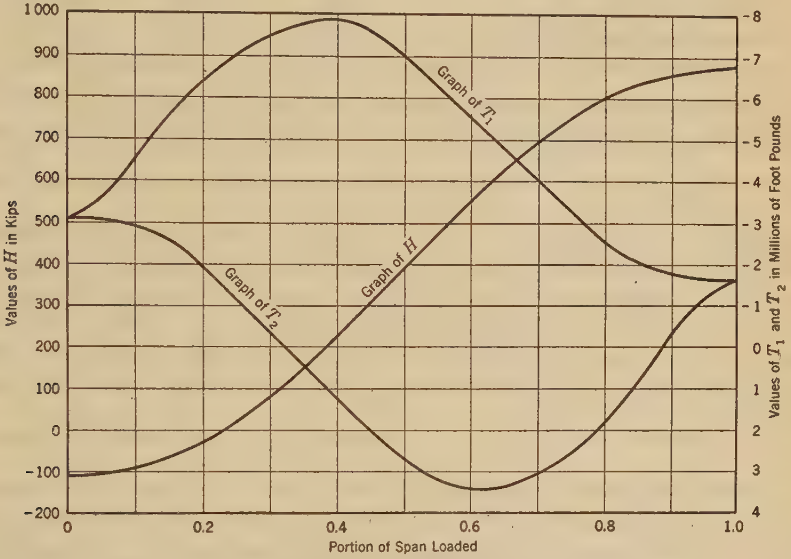


FIG. 4.—GRAPHS OF H, T_1, AND T_2 FOR ADVANCING UNIFORM LOAD IN MAIN SPAN AT HIGHEST TEMPERATURE

Maximum Positive Moments in Main Span.—Maximum moments are positive for Points $\frac{x}{l} = 0.2$ to 0.5, and negative for Points $\frac{x}{l} = 0$ and 0.1. Maximum positive moments for Points $\frac{x}{l} = 0.2$, 0.3, and 0.4 are calculated by Equations (9) for the load condition of Case III (as indicated in Fig. 2(a)). For each section, x, the value of M is calculated for three or four different trial load lengths, k, until the maximum value of M for the section is determined. The values of H, T_1, and T_2 for each load length are taken from the H and T-graphs, Fig. 4. The governing quantities, in abbreviated form, are recorded in Table 3.

Maximum Positive Moment at Mid-Span.—Maximum positive moment at the center of the main span, as indicated in Fig. 2(a), is produced when the central part of the span is symmetrically loaded with a uniform load for a length, m, at highest temperature and with no load on the side spans. In order to find the value of m for which the moment at this point is a maximum, it is necessary to assume different trial values of m and calculate the corresponding values of M by use of the formulas of Case V. Before thus computing each M, however, it is also necessary to obtain, by trial, the value of H corresponding to the assumed value of m. The final quantities, in condensed form, are: $\frac{m}{l}$ (assumed) $= 0.33$; H (assumed trial value) $= 420$ kips; c (by Equations (5)) $= 0.00848$; $e^{cm} = 9.37$; $e^{ck} = 9.69$; D (from Table 2) $= 26\,085$; H (by Equation (27)) $= 419$ kips; T_1 (by Equations (19) and formulas of Case V) $= +\ 450$ kips; C_1 (by Equation (12a) and formulas of Case V) $= 92.6$; C_2 (by Equation (12b) and formulas of Case V) $= 81\,560$; $e^{cx} = e^{0.5cl} = 29.71$; and, the maximum moment, M (by Equation (9)) $= +\ 6\,440$ ft-kips.

TABLE 3.

Section: $\frac{x}{l}$	0.2	0.3	0.4
Load: k (assumed)	$0.41l$	$0.475l$	$0.53l$
H, (from Fig. 4)	242	348	440
c (by Equation (5))	0.00829	0.00841	0.00850
T_1 (from Fig. 4)	−7 840	−7 330	−6 515
T_2 (from Fig. 4)	+1 430	+2 415	+3 115
C_1 (by Equation (12a))	604	365.5	234.6
C_2 (by Equation (12b))	22 435	20 160	17 890
ecx	3.764	7.53	15.18
Maximum moment, (by Equation (9))	+6 970	+7 770	+6 870

Maximum Negative Moment in Main Span Near Tower.—The maximum negative moment at the Point $\frac{x}{l} = 0.1$ occurs when the adjacent side span is fully loaded at highest temperature and the other two spans are unloaded. For this condition of loading, H is determined by trial, using the formulas of Case VIII. The value of M is then calculated by the formulas of Cases I and VIII, and Equation (9), or Equation (29). A summary of the principal resulting values is as follows: H (final value by Equation (27), or Equation (37)) $= -\ 87.4$ kips; c (by Equations (5)) $= 0.00793$; d (from Table 1) $= 0.99649$; T_1 (by Equations (19) and formulas of Case VIII) $= -\ 11\,550$ ft-kips; T_2 (by Equations (19) and formulas of Case VIII) $= -\ 1\,550$ ft-kips; C_1 (by Equation (30a)) $= 4.97$; C_2 (by Equation (30b)) $= 13\,000$; $e^{cx} = 1.886$; and, M (by Equation (29)) $= 5\,450$ ft-kips.

Maximum Negative Moment at Tower.—To obtain the maximum negative moment at the tower, it is necessary to load the adjacent side span fully and a part of the main span next to the tower, at highest temperature. The main span load length for which the tower moment is a maximum must be determined, by the formulas of Cases III and VIII, as previously outlined for the computation of maximum moment at mid-span. This computation is briefly summarized as follows: H (trial value) $= 150$ kips; c (by Equations (5)) $= 0.00819$; d (from Table 1) $= 0.99715$; d_1 (from Table 1) $= 0.92725$; $\frac{k}{l} = 0.34$;

$e^{ck} = 9.29$; $e^{cm} = 75.69$; D (from Table 2) $= 25\,140$; H (by Equation (27) and formulas of Cases III and VIII) $= 158.2$ kips; and T_1 (by Equations (19) and formulas of Cases III and VIII) $= 16\,300$ ft-kips.

Maximum Positive Moments Near Free End of Side Span.—Maximum moments are positive for all sections in the side span, except Points $\frac{x_1}{l_1} = 0.8$ and 0.9 near the tower. The loading condition producing maximum positive moments at Sections $\frac{x_1}{l_1} = 0.1$ to 0.4, is shown in Fig. 2(*a*) to be the same as that for negative moment at Point $\frac{x}{l} = 0.1$ in the main span, the side span being fully loaded at highest temperature, with no load on the other two spans. The computations for H, T_1, and T_2 have already been indicated for this loading condition; the remaining calculations for the determination of the moment at Point $\frac{x_1}{l_1} = 0.4$ are: C_1 (by Equation (30*c*)) $= 1\,374$; C_2 (by Equation (30*d*)) $= 20\,745$; $e^{cx} = 3.56$; and, maximum moment (by Equation (29)) $= +\,11\,410$ ft-kips.

Maximum Positive Moments Near Center of Side Span.—The maximum moment at Point $\frac{x_1}{l_1} = 0.7$ is positive and occurs when both side spans are fully loaded at lowest temperature. The maximum positive moment at Points $\frac{x_1}{l_1} = 0.5$ and 0.6 is produced by fully loading both side spans at highest temperature. The computations for the moment at mid-span are: H (trial value) $= -\,65$ kips; $c_1 = c$ (by Equations (5)) $= 0.00796$; $D = 24\,357$; H (by Equation (37)) $= -\,64.9$ kips; $T_1 = T_2$ (by Equation (36)) $= -\,9\,980$ ft-kips; C_1 (by Equation (30*c*)) $= 1\,273.6$; C_2 by Equation (30*d*)) $= 20\,315$; $e^{0.5 c_1 l_1} = 4.91$; and M (by Equation (32*b*)) $= +\,11\,205$ ft-kips.

Maximum Negative Moment in Side Span Near Tower.—The loading condition for maximum negative moment at Point $\frac{x_1}{l_1} = 0.8$ is shown in Fig. 2(*b*) to be the same as for positive moments in the main span. This moment is computed in the same manner as for maximum positive moments at Sections $\frac{x}{l} = 0.2$, 0.3, and 0.4 in the main span, except for the use in this computation of the integration constants of Case VII.

The maximum negative moment at Point $\frac{x_1}{l_1} = 0.9$ occurs under a partial loading of the side span and the main span, at highest temperature, as shown in Fig. 2. In obtaining the maximum value of this moment, it must be computed by use of the formulas of Cases III and IX for several trial load lengths in both spans.

Minimum Moments.—The loading conditions under which minimum moments are produced in the trusses are also shown in Fig. 2. The values of these moments are obtained in the same general manner, as outlined for the calculations of maximum moments, and usually occur at lowest temperature in combination with the indicated live loadings.

Maximum Shears.—The maximum positive and negative shears are calculated for the loading conditions indicated in Fig. 2(c) and Fig. 2(d). The method of procedure is the same as that for the calculation of maximum moments, involving the assumption of trial values of H and trial load lengths to secure the values for which the shears are a maximum or minimum. The shear is computed from the values of c, C_1, and C_2 by Equations (10) and (34b).

Maximum Deflection in Main Span.—Maximum deflection in the main span is produced by loading that span fully, at highest temperature. The value of H is obtained by trial from Equation (37), with the formulas of Case II, the result having been plotted in the H-curve for advancing load in the main span (Fig. 4). The calculation of the deflection at the center of the span in condensed form is, as follows (the calculation of H, T_1, and T_2 was indicated previously): H (by Equation (37)) = 873 kips; $T_1 = T_2$ (by Equation (36)) = − 1 575 ft-kips; S (by Equation (28a)) = 0.3845 kips per ft; h (by Equation (31a)) = 0.94415; and Δf (by Equation (33a)) = 5.44 ft.

Maximum Deflection in Side Span.—Maximum downward side-span deflection occurs when the two side spans are both fully loaded at highest temperature. The computation of the necessary constants for this loading condition has already been indicated in the calculation of positive moment at the center of the side span. The deflection at the center of the side span may be computed from Equation (7), or from Equation (33b), and is found to be 3.10 ft.

It will be noted that the side-span deflection is 6% less than in the two-hinged design, while the main-span deflection is 5.4% greater. The average rigidity over the entire length differs from that of the two-hinged design by only 0.3 per cent.

Comparison of Maximum Moments for Continuous and Two-Hinged Designs.—The maximum bending moments produced by live load and temperature in the continuous spans of Design II, computed as herein outlined, are plotted in Fig. 5. For comparison, Fig. 5 also includes a curve of the maximum bending moments produced by live load and temperature in the corresponding two-hinged design. The saving in chord material by the adoption of the continuous type, as indicated by the percentage of difference between the moment areas under these two respective graphs, is 5%, and this value is substantiated by the actual design of the truss members.

Advantages of the Continuous Type.—The foregoing comparative design studies indicate that, for a suspension bridge of 800-ft main span, the continuous type is approximately 6% more rigid than a two-hinged design of the same economy, and about 5% more economical than a two-hinged design of the same rigidity. These percentage differences in favor of the continuous type will be greater in shorter spans or with deeper trusses.

As the length of span or its flexibility is increased, the effect of continuity at the towers is lost on the spans at points proportionately nearer and nearer the towers. It is for this reason that the advantage of the continuous type is greater with shorter spans and deeper stiffening trusses. In general, the advantage of the continuous type over the two-hinged design will be governed by the stiffness factor, $S' = \frac{1}{l}\sqrt{\frac{E\,I}{H_w}} = \frac{1}{l^2}\sqrt{\frac{8\,f\,E\,I}{w}}$, which also governs the percentage correction between the results of the Elastic and the Deflection Theories.

The continuous type has an advantage in respect to behavior under lateral forces. The lateral rigidity is greater than in the two-hinged design, and there is a better distribution and absorption of stresses from lateral loading. In fact, for spans of 800 ft, or less, for which the ratio of width to span is greater than 1:20, the chord sections in a continuous design are not affected by wind stresses, since the better distribution of these stresses brings them under the 25% increase in allowable stress permitted under this loading by the usual design specifications.

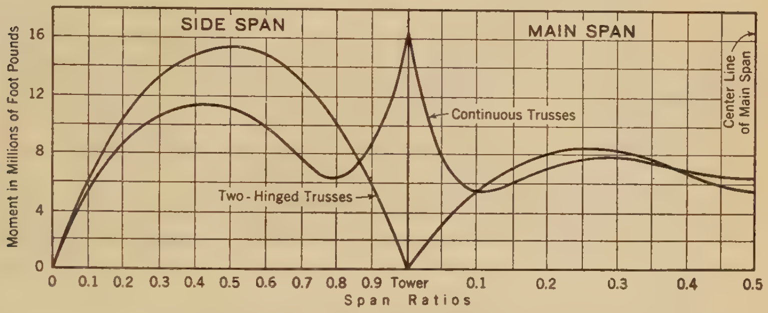


FIG. 5.—COMPARISON OF MAXIMUM MOMENTS FOR LIVE LOAD AND TEMPERATURE IN CONTINUOUS AND TWO-HINGED STIFFENING TRUSSES.

As a general conclusion, it may be stated that the continuous type of suspension bridge offers advantages over the two-hinged type for spans under 1 000 ft, designed for highway loading, and for longer spans when designed for railroad loading.

ACKNOWLEDGMENT

This paper, in different form, was originally presented by the writer at the Joint Session of the Structural Division of the Society and the Applied Mechanics Division of the American Society of Mechanical Engineers, at the Annual Convention, at Chicago, Ill., June 30, 1933. In the original paper, the continuity moments at the towers were evaluated by applying the "theorem of three moments" to the usual assumption of uniform suspender loading in each span. In his discussion, Dr. S. Timoshenko suggested the evaluation of these continuity moments by equating slopes at the towers. This sug-

gestion has proved valuable, yielding a simpler and more accurate solution and prompting a revision of the paper to the present condensed and improved form. The writer is gratefully indebted to Dr. Timoshenko for this helpful suggestion. Acknowledgment is also due C. H. Gronquist, Assoc. M. Am. Soc. C. E., for valuable assistance in the preparation of the paper.

DISCUSSION

E. L. Pavlo,[5] Esq. (by letter).—The Deflection Theory for suspension bridges with stiffening trusses hinged at the towers became generally known to the profession through its practical development and application to the design of the Manhattan Bridge by L. S. Moisseiff, M. Am. Soc. C. E., in 1904. Since that time several bridges have been built with the stiffening trusses continuous over the towers because of their estimated superior rigidity and economy over the hinged type. Because there was no exact theory for the analysis of stresses in continuous trusses, the design of these bridges has necessarily been approximate and the advantages of the continuous type could not have been definitely ascertained.

Mr. Steinman eliminates this deficiency and also extends the use of the Deflection Theory to the cases of multiple suspension bridges with or without tie cables. His paper is a notable scientific contribution to the theory of suspension bridges. The analysis of the stresses in the continuous stiffening trusses is a difficult and involved mathematical problem. Aside from the theoretical complexity it presents an algebraic difficulty in transforming and condensing the extremely complex formulas into workable expressions. The paper demonstrates the remarkable ingenuity of the author in overcoming these difficulties. Only those who have attempted the same task can fully appreciate the time and energy required.

In 1932 the writer undertook to verify, by an independent method, the conclusions of Mr. Steinman's original paper, and to obtain some additional data for the economic comparison of the hinged and continuous types of stiffening trusses. For this purpose the writer developed an analysis quite similar to that of the present paper, which is likewise based on the equality of the slopes to the elastic curve at the towers. Being primarily interested in the practical applications of the theory, rather than in developing a finished treatise on the subject, the writer strove to correlate his derivation with the already existing theory for the hinged trusses. In doing so he devised an easy means of obtaining equations for continuous trusses from the existing equations[6] derived for the hinged condition, simply by adding certain constant quantities, say, Δ_1, Δ_2, and β to the existing equations for C_1, C_2, and H, respectively. This procedure will be particularly useful to those already familiar with the Deflection Theory as applied to the hinged stiffening trusses.

In evaluating the constants of integration, C_1 and C_2, the only difference between the derivations for the hinged and continuous conditions is that in the case of hinged trusses M is equated to zero for $x = 0$ and $x = l$; whereas, in the case of continuous trusses M is equated to T_1 and T_2. This change in the boundary conditions creates additional constant terms, representing the effect of continuity, in the expressions of C_1 and C_2, for various loading conditions. These terms are:

---

[5] White Plains, N. Y.

In the main span,

$$\Delta_1 = \frac{T_1\, e^{-cl} - T_2}{H\,(e^{cl} - e^{-cl})} \qquad (51)$$

and,

$$\Delta_2 = -\frac{T_1}{H} \qquad (52)$$

In the side spans,

$$\Delta_1 = -\frac{T_{1,2}}{H\,(e^{c_1 l_1} - e^{-c_1 l_1})} \qquad (53)$$

and,

$$\Delta_2 = 0 \qquad (54)$$

All the formulas for the integration constants may thus be easily written from the formulas of the hinged case, given in textbooks, by simply adding Equation (51) or Equation (53) to C_1 and Equation (52) or Equation (54) to C_2.

For symmetrical loading cases $T_1 = T_2$, and Equation (51) becomes,

$$\Delta_1 = -\frac{T_1}{H\,(e^{cl} + 1)} \qquad (55)$$

To obtain the formulas of H for various loading conditions it is sufficient to add the term,

$$\beta = (T_1 + T_2)\,\frac{F}{2} \qquad (56)$$

to the numerators of the known formulas for the hinged trusses, the denominator being left unchanged. The use of H-formulas containing temperature terms both in the numerator and denominator is not permissible for the present purpose and will introduce an error. The general form of the H-formula is given by Equation (24). The writer finds it easier to use this type of equation than Equation (27) in which the term $(T_1 + T_2)$ is partly eliminated at the expense of complicating the denominator, for the reason that $(T_1 + T_2)$ must be found in any case for use in the equations of C_1 and C_2. In case of computations for trial values of H the writer also finds it more advantageous to use Equation (24).

The continuity at the towers introduces the additional two unknowns, T_1 and T_2, which are treated as the known quantities although they vary with each loading condition. At the same time the continuity of the elastic curve at the towers provides the additional two equations for the two unknowns it created. These equations are obtained by equating the slopes to the elastic curves on each side of the towers.

In the case of a bridge fully loaded with the load, p, in the center span and the load, p_1, in the side spans the application of the slope equations yields,

$$T_1 = T_2 = H\,\frac{\left[\dfrac{d}{c}\left(\dfrac{p}{H} - \dfrac{1}{r}\right) + \dfrac{d_1}{c_1}\left(\dfrac{p_1}{H} - \dfrac{1}{r_1}\right) + 4\,(n + n_1) - \dfrac{(p\,l + p_1\,l_1)}{2\,H}\right]}{\left[cd + \dfrac{c_1}{2}\left(d_1 + \dfrac{1}{d_1}\right) - \dfrac{1}{l_1}\right]} \qquad (57)$$

---

[6] "A Practical Treatise on Suspension Bridges," by D. B. Steinman, Second Edition John Wiley & Son, 1929.

In the case of the center span fully loaded and no load on the side spans, place $p_1 = 0$ in Equation (57). In the case of side spans fully loaded with the load, p_1, and no load on the center span, place $p = 0$ in Equation (57).

In the case of partial or full unsymmetrical loadings, the general algebraic equations for T_1 and T_2 become too complicated for practical use and it is more advantageous in these cases to write out the expressions for $(T_1 + T_2)$ and $(T_1 - T_2)$ by substituting the appropriate contributions, B_1 and B_2 (tabulated in Article 9 of the paper) into Equations (19*a*), (19*b*), (35*a*), and (35*b*), and to solve numerically the resulting two equations for T_1 and T_2.

In the practical application of the Deflection Theory for continuous trusses, the problem of finding the moments of inertia, I and I_1, requires further elucidation. The writer quite agrees that the effect of the variation of I within a span may be ignored; the difficulty, however, lies in the fact that it is impossible to find correct single values of the moments of inertia to apply for all loading conditions as in the case of hinged stiffening trusses. The deflection theory is based upon the deflections of the stiffening member as expressed by the beam flexure equation, $\frac{d^2 n}{dx^2} = - \frac{M}{EI}$.

Inasmuch as in most cases the stiffening member is a truss and not a beam, it is necessary, for the application of the theory, to find an equivalent beam such that its deflection characteristics would approximate those of a given truss. For the two-hinged trusses this equivalent beam is found by equating computed deflections of the truss under certain loadings to the corresponding beam deflection formulas and solving for I. The average of the values of I for various loading conditions represents the so-called "equivalent moment of inertia" of the truss. It is found that for the two-hinged trusses the individual values of I for various loading conditions differ but little from each other; and, therefore, it is assumed that their average value can apply for all the loading cases.

In case of a three-span continuous truss of symmetrical design, the determination of equivalent values of I for the center span and I_1 for the side spans requires finding deflections in a statically indeterminate structure. After finding these deflections in one side span and in one center span under a certain loading, they are equated to the corresponding equations of the three-span beam deflections which contain both I and I_1. In this way two equations are obtained which are solved simultaneously for I and I_1. In this manner the writer analyzed a normal design of a three-span continuous truss for various full-span loadings and found that the individual values of equivalent moments of inertia differed considerably for some loadings.

Since equivalent moments of inertia that can apply for all loading cases can not be found, it is necessary, in the practical application of the deflection theory, to prepare a set of moments of inertia for the several representative loading conditions. The problem is further complicated by the fact that the representative loading conditions can only be known tentatively since the actual loading on the truss is variable on account of the variable suspender pull.

The method of finding equivalent moments of inertia by multiplying the average chord area by the square of one-half the height of the truss should not be used in the case of continuous stiffening trusses. In the case of two-hinged trusses this method is sometimes used since it gives values close enough to those obtained by equating the deflections. In the case of continuous trusses it has no such justification, and the deflection method of finding one equivalent beam is the only rational method to be used.

The example of finding the stresses and deflections in a three-span continuous suspension bridge is very instructive. The economic comparison of hinged and continuous trusses is also of great interest to the designing engineer. The writer is of the opinion that specifications for suspension bridges should contain rigidity requirements in terms of maximum permissible change in grade instead of deflection ratios. On that basis the comparison of the hinged and continuous trusses would be more advantageous for the latter. The writer agrees with Mr. Steinman that a 1 000-ft span marks the upper limit of the economic advantages of continuous suspension bridges in so far as the saving of material in trusses is concerned. For long spans the continuity is comparatively unimportant since it affects relatively small lengths of span immediately adjacent to the towers. In long-span suspension bridges the rigidity is derived through the preponderance of the dead load over the live load. If the ratio of dead to live load is sufficiently high the stiffening trusses may be dispensed with entirely as was done in the design of the George Washington Bridge. In shorter spans, the continuity assumes a far greater importance.

There is a misconception on the part of many engineers that suspension bridges can apply economically only for long-span crossings. By building the short-span suspension bridges with trusses continuous over the towers the requisite rigidity can be attained and a saving in material effected. Suspension bridges can thus penetrate economically into the domain hitherto believed to be reserved for other types of bridge construction. The multiple, short-span suspension bridges also present an excellent economic solution for relatively shallow crossings. The present paper affords a means of designing such suspension bridges rationally, and therein lies its great importance.

JONATHAN JONES,[7] M. AM. SOC. C. E. (by letter).—Mathematical tools are provided, in this valuable paper, for more accurate study of suspension bridge economy in two categories—the span of less than 1 000 ft, and the continuous span with tie cables—both of which, from present indications of the trend, should engage the attention of many highway bridge engineers in the United States. The author has been able to present an orderly process for making the computations, as well as a development of the theory, in a compact and usable form.

Two of the practical conclusions in the paper seem to warrant comment:

(1) In the "Introduction," the statement is made that the continuous truss offers advantages "in improved and simplified supporting details at the towers."

---

[7] Chf. Engr., McClintic-Marshall Corporation, Bethlehem, Pa.

This is true when the tower legs are battered so that the trusses pass between the legs and rest on a transverse girder. In that case one bearing per truss, instead of two, will be more simply accommodated. For towers with vertical legs in the truss planes, the opposite is true; it is not so simple to pass a truss through a tower leg, at a point of portal bending stress, as to pin it off front and back. However, as the author indicates that the probable field for the continuous truss is in shorter spans, to which past practice also indicates that the battered tower is appropriate, the statement here criticized will hold in general.

(2) In the "Comparison of Deflections," there occurs an averaging of main-span and side-span deflections, which seems to fit no conception that the writer can appreciate. The standard conception is that a deflection, occurring over a certain length of span, represents a change in rate of grade which may or may not be unsatisfactory for the conditions. Now it may happen that the main-span deflection in a given bridge is the criterion of unsatisfactoriness or, in some other bridge, the side-span deflection. An unsatisfactory main-span deflection, however, is not made satisfactory by the circumstance that the side-span deflection is moderate, nor *vice versa*. Therefore, it appears to the writer that the author's examples have shown the 800-ft continuous truss to be slightly more flexible in the main span and considerably less flexible in the side spans than the discontinuous truss of the same economy, and that there the comparison rests. Similarly, in the second study, the continuous truss which costs 5% less than the discontinuous, is a truss 5.4% more flexible in the main span and 6% less flexible in the side spans, whichever of those two percentages may be significant.

Economic comparisons of suspension bridge types, to be valid, must be based on equality of maximum deflection in that span (main span or side span) where deflection is significant. This is true because two suspension bridges may be the same as to loads and unit stresses (that is, as to safety), and yet be as far apart as the poles in flexibility. Unless they both fall within whatever criterion of flexibility is applicable, while one may be an acceptable bridge, certainly both are not.

The design of other types of bridges is based upon their carrying certain loads at certain unit stresses; but a suspension bridge design cannot even be begun until there are stipulated, in addition to the loads and unit stresses, the criteria for maximum admissible flexibility. To a certain extent these criteria must be psychological, possibly as much so as mechanical. Whether such criteria of flexibility as have been and are being chosen are valid, and to what extent deviations therefrom would be regretted, is the most important remaining question in the field of suspension bridge design.

A. MÜLLENHOFF,[8] ESQ. (by letter).—The theory presented in this paper is as complete and simple as is possible in dealing with such an intricate subject as the exact theory of suspension bridges. Under "Introduction," the author gives a short history of the exact theory and states that it was originated by J. Melan. From the bibliography in Melan's work cited by Mr. Steinman, credit in turn may be traced to Müller-Breslau. In his paper[9] for the first time Müller-Breslau presents the fundamental differential equation:

---

[8] Prof., Technische Hochschule, Aachen, Germany.

$$\frac{d^2\,\eta}{dx^2} - c^2\,\eta = c^2\,f(x) \quad \text{.......................(58)}$$

and its solution,

$$\eta = C_1\,e^{cx} + C_2\,e^{-cx} - f(x) - \frac{f''(x)}{c^2} \quad \text{...............(59)}$$

in which the notation is that of Mr. Steinman's paper. At that time, Müller-Breslau had not developed formulas for use in computations by that method and, evidently, he did not then recognize the fundamental difference and the superior value of his solution over the ordinary method of computation. Nevertheless, this paragraph in his paper seems to be the first reference to the deflection or exact theory and, therefore, Müller-Breslau should not be omitted from any historical reference to this subject.

The introduction of the function, *d,* in the paper seems to be a great improvement (see Equations (14) and (15)). The profession should be grateful for it, and this discussion is confined to a few suggestions as to what else could be done along this line.

Cantilever stiffening trusses which have been advocated in several instances (as, for example, in Schachenmeyer's design of the Hudson River Bridge),[10] are not covered by the theory of this paper, of course. However, it is improbable that bridges of this type will ever be built.

The author leaves only two questions to be answered: (1) The variation of the coefficients of reduction which need to be applied to the values for H and the moments computed by the elastic theory in order to obtain preliminary values of H and c with varying values of the moment of inertia; and (2) the exact theory applying to the influence of wind loads.

In designing a structure by the exact theory it is advisable first to prepare a preliminary design by the ordinary elastic theory and to reduce the values of H and I (which depend on the moments) by certain percentages, in order to obtain approximate values for the final design. These percentages depend mostly on the rigidity of the stiffening truss, or on its moment of inertia, which can vary between very wide limits from zero to almost any value. The author gives the percentages of reduction for a certain case (see Fig. 3), but these do not apply for a design with widely different proportions. It seems desirable to arrange these reduction factors graphically or by formula in such a way that they can be selected and applied to every case.

Perhaps it would be possible to determine this relation either in a general way or by solving a number of cases. The results could be plotted in a graph using as the argument, $\rho = \frac{M_0}{M}$, in which, M is the moment that the stiffening truss is capable of resisting, and M_0, the moment producible by the live load in a beam of the same span length, without cable suspension. Under "General Data—Calculation of Constants," the author uses the following data: $l = 800$ ft; $l_1 = 400$ ft; $p = 1\,300$ lb per ft; and $I = 1\,960$ in.^{2}–ft.^{2} The depth of the truss, h', is given as 14 ft. Then, if the allowable stress is, say, 20 000 lb per sq in., $M = \frac{2 \times 1\,960 \times 20\,000}{14} = 5.6 \times 10^6$ ft–lb.

---

[9] "Theorie der durch einen Balken versteiften Kette," by Müller-Breslau, *Zeitschrift* d. Architektur und Ingenieur-Vereins zu Hannover, 1881, p. 76.

[10] *Transactions,* Am. Soc. C. E., Vol. 97 (1933), p. 36, Fig. 19(*b*).

A few trials will be sufficient to decide whether M_0 may be taken as the moment of a simple beam the length of the main span (that is, $\frac{1\,300 \times 800^2}{8} = 104 \times 10^6$ ft–lb), or whether it is necessary to use the maximum or the minimum moment in a continuous beam of the same dimensions.

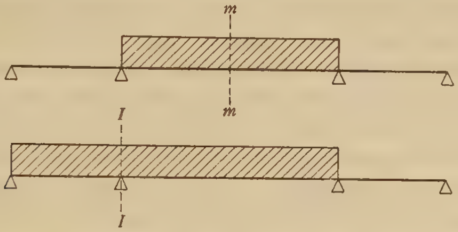


FIG. 6.

In Fig. 6, for example, the maximum moment, M_m, equals 52×10^6 ft-lb, and the minimum moment, M_I, equals -62×10^6 ft–lb. For the first case, therefore, $\rho = \frac{104}{5.6} = 18.6$, and for the second case, $\rho = \frac{62}{5.6} = 11$; or, $\rho = \frac{52}{5.6} = 9.3$. The advantage of this basis of comparison would be its independence from the values of l, I, and h', since ρ is a dimensionless number.

H. CECIL BOOTH,[11] ESQ. (by letter).—With the present paper by Mr. Steinman and the work of the late Max Am Ende,[12] the theory of suspension bridges seems to have been so thoroughly investigated as to leave little more to be added.

The entire question of the design of a suspension bridge for a particular case depends primarily upon a practical determination of the deck deflection that is to be permitted. Economy in design is dependent upon the extent of this deflection.

No writer on this subject, including the author, seems to have given an opinion as to the maximum allowable deck deflection (say, in terms of deck deflection to span) for suspension bridges of varying spans and for different classes of traffic; and it would be very useful if the author would establish his recommendations.

---

[11] Cons. Engr., Westminster, London, S. W. 1, England.

[12] "Suspension Bridges with Stiffening Girders," by Max Am Ende, *Minutes of Proceedings*, Inst. C. E., Vol. CXXXVII (1898–99).

JACOB FELD,[13] M. AM. SOC. C. E. (by letter).—The assumptions from which the basic formulas of this paper are derived, are clearly stated and the limitations of each resulting formula are carefully explained. Although there is a great deal of summarized mathematics, the logical steps required for the derivation of each formula can easily be filled in by any one so inclined.

The determination of stresses in structures that are classified as rigid, in which changes of shape are caused entirely by the change in lengths of members due to elastic deformation, for the determination of principal stresses, requires the rules of statics only. However, when structures include cables for total or partial support, the geometrical shape of the cable is quite important in determining the stress, not only in the cable but in all the rigid members. The inter-relationship of the stresses in the cable and those in the stiffening truss members are clearly stated by the author. It is also pointed out that the geometrical shape of the cable is closely related to that of the stiffening truss. However, to avoid unwieldy complications for the formulas, it is assumed that the main cable remains a parabola under all conditions of dead and live load. A weightless cable assumes the shape of a parabola when it is acted upon by a series of equal loads, equally spaced. In a suspension bridge these loads, of course, are the suspender loads. As indicated by Equation (11), the deflection theory shows that the suspender loadings are not constant, but depend on deflections in the stiffening truss. This is especially true in short spans where, in addition, the ratio of live load to dead load is much greater than in long spans. The inequality of suspender loads will necessarily change the shape of curve in the cable. Such change involves a revision of the cable tension as well as a change in all stresses that depend on the cable tension for their valuation.

In addition, especially for short spans, the condition of unbalanced live load, such as that shown in Cases III and IV can only result in a curve that is not a symmetrical parabola as originally assumed, but that can be taken approximately as two half parabolas, of different characteristics. The lowest point of the curve will not be at the center line of the span.

For long-span suspension bridges, the aforementioned refinements are negligible. However, for short-span bridges, if the ratio of live load to dead load approximates unity, a noticeable change in shape of curve can be determined. The amount of change will increase with an increase in ratio of sag to span, and for the shorter spans the sag ratio is usually greater than for the long spans. Such increase of sag ratio tends to produce a better looking bridge for short spans.

The profession is greatly indebted to the author for such a complete derivation of formulas for all possible cases of suspension bridges in accordance with the deflection theory. With this study available, there is probably no further reason for using the more approximate elastic theory and, in that replacement, this paper serves as a milestone in the advancement of engineering as a science. It could well serve as a model for future reports on the theoretical developments in any branch of engineering.

---

[13] Cons. Engr., New York, N. Y.

Glenn B. Woodruff,[14] M. Am. Soc. C. E., **Howard C. Wood,**[15] Assoc. M. Am. Soc. C. E., and **Ralph A. Tudor,**[16] Assoc. M. Am. Soc. C. E. (by letter).—A commendable contribution to the theory of suspension-bridge analysis has been made by Mr. Steinman, and his paper merits a most careful study. In particular, his extension of the "Deflection Theory" to include (1) the effects of stiffening trusses continuous through the towers; and (2) multiple-span bridges with and without auxiliary tie cables, gives the designing engineer two new tools for which there may well be a growing need in the future.

As pointed out in the paper, continuous trusses have already been used on several structures. The author indicates the advantages that this feature possesses, but it is believed that he has given too little emphasis to the added rigidity contributed.

In the usual layout the maximum roadway grade is to be found on the side spans. Live loads produce a change in the grade which is often of serious proportions. A continuous truss will reduce this change materially and particularly so on the side spans. Furthermore, continuity of the truss will forestall any abrupt break in the roadway grade.

Probably the most important advantage to be derived from this feature is the reduction of the maximum wind stresses in the truss. This fact is brought out by Mr. Steinman, but is of such importance as to bear repetition.

Together with the advantages to be derived from continuity, there are certain practical considerations that must not be overlooked. A suspension bridge is somewhat peculiar in that it suffers relatively large distortions. Because of this, the expansion joints at the truss ends must be built to accommodate movements much greater than for temperature alone. Hence, it is not unlikely that for the longer suspension bridges a continuous truss introduces expansion difficulties that cannot be met reasonably. This problem becomes increasingly serious for multiple spans. A combination of a truss continuous through all towers and multiple-span construction is not probable.

Multiple Spans.—The use of a multiple-span suspension bridge has been generally condemned by engineers on the grounds that such a structure would be too flexible. However, there are several possible means of increasing the rigidity, and some are sufficiently meritorious to bear serious consideration:

(*a*) The dead load of the structure may be increased to such an extent that the ratio of dead load to live load becomes larger and the effect of live load becomes small. This would be an uneconomical means of stiffening the bridge and is not worthy of further thought.

(*b*) The moment of inertia (vertical) of the stiffening truss may be increased. Since the stiffness of the truss has such a relatively small effect on the deflections of a suspension bridge, it would be necessary to increase the moment of inertia by such an amount as to make the economics of this method extremely doubtful.

---

[14] Engr. of Design, San Francisco-Oakland Bay Bridge Comm., San Francisco, Calif.

[15] Senior Bridge Designing Engr., San Francisco-Oakland Bay Bridge, San Francisco, Calif.

[16] Senior Bridge Designing Engr., Div. of Highways, State Dept. of Public Works, San Francisco-Oakland Bay Bridge, San Francisco, Calif.

(*c*) The sag ratio of the main supporting cable may be reduced. The stiffness is increased by this means approximately in proportion to the decrease in sag, while the size of the cable must be increased at about the same rate. Thus, if it is desired to reduce the deflections by one-half, it becomes necessary to double approximately the size of the cable. This is a method that may prove the most satisfactory for a given set of conditions when economic, construction and all other phases of the problem are considered.

(*d*) Some system of guys may be used to reduce the movement of the tower tops. An auxiliary "tie cable" running between the tops of successive towers and extending from anchorage to anchorage is such a system. By varying the size and sag ratio of this tie cable the deflections of a multiple-span bridge can be reduced effectively. Economically, this will compare favorably with any of the other methods.

(*e*) Due to very heavy live loads or to other elements contributing to the flexibility of the structure, it may be advisable to introduce cable anchorage in an intermediate position. This essentially produces independent suspension bridges placed end to end. The west channel structure of the San Francisco-Oakland Bay Bridge has been so designed. The 900-ft unloaded back-stay, beyond the west side span, together with a relatively heavy live load, made this type of construction advisable.

Incident to the investigation of various multiple-span suspension bridge combinations for the west channel crossing of the San Francisco-Oakland Bay Bridge, means were developed for the necessary calculations. These investigations resolved themselves into two quite distinct phases, the first of which was the preliminary work involving the determination of general dimensions, cable sizes, and distortions. This was followed by an exact analysis of the combination selected to determine moments and shears in the stiffening truss, and other items not obtainable during the first phase.

General Dimensions, Cable Sizes, and Distortions.—For this first phase of the work a modification of the Dana method[17] was developed. This device is only applicable for loadings that are entirely continuous over one or more spans. Since maximum tower and truss deflections and maximum cable stresses are produced by such loadings, these items can be readily obtained. The advantages of the method lie in its relative simplicity, its flexibility, and its accuracy. It involves no calculus. It can be readily made to accommodate any changes in dimensions, loads, or other span constants, and it is particularly well adapted to investigations of multiple spans either with or without tie cables. Finally, it has been found by comparison with model tests to give more accurate values for deflections than those obtained by the "deflection theory." The error in the latter theory is doubtless due largely to the neglect of the change in span lengths.

Final Moments and Shears.—After the establishment of dimensions of the bridge, an exact analysis of the structure (the second phase) to determine the remaining items, was pursued. This investigation was based on equations similar in nature to those presented by Mr. Steinman, the only essential difference being in the treatment of the tie-cable factor. Equation (46) is based on the geometry of the cable. The writers equated the internal work,

---

[17] "Tests on Structural Models of Proposed San Francisco-Oakland Suspension Bridge," Appendix *B*, Univ. of California *Publications in Engineering*, Vol. 8, No. 2.

$$\left(\frac{H_T}{2} + H_0\right)\left(\frac{H_T L_{sT}}{A_T E_T} \pm w\, t\, L_{TT}\right),$$

to the external work,

$$\left(\frac{H_T}{2} + H_0\right) \Delta\, l + w_T \int_0^l \eta\, dx$$

The second term of this expression represents the work accomplished in moving the dead load of the tie cable (w_T) up or down to its new location. The value of η at the center of the span is equal to the change in tie-cable sag, which is expressed as $\Delta\, f_T$. From the equation of the parabola,

$$\Delta\, f_T = \frac{w_T\, l^2}{8}\left(\frac{1}{H_0 + H_T} - \frac{1}{H_0}\right) = - \frac{w_T\, l^2\, H_T}{8\, H_0\, (H_0 + H_T)} \quad\ldots\ldots (60)$$

For any other point in the span, η may be expressed in terms of $\Delta\, f_T$, thus:

$$\eta = \frac{4\, \Delta\, f_T\, x\, (l - x)}{l^2} = \frac{w_T\, H_T\, x\, (x - l)}{2\, H_0\, (H_0 + H_T)} \quad\ldots\ldots\ldots\ldots (61)$$

The expression for the external work then becomes,

$$\left(\frac{H_T}{2} + H_0\right) \Delta\, l - \frac{w^2{}_T\, l^3\, H_T}{12\, H_0\, (H_0 + H_T)}$$
$$= \left(\frac{H_T}{2} + H_0\right) \Delta\, l - \frac{16\, f^2{}_T\, H_0\, H_T}{3\, l\, (H_0 + H_T)} \quad\ldots\ldots\ldots\ldots (62)$$

and,

$$\Delta\, l = \frac{H_T\, L_{sT}}{A_T\ E_T} \pm w\, t\, L_{TT} + \frac{16\, f^2{}_T\, H_0\, H_T}{3\, l\, (H_0 + H_T)\left(H_0 + \frac{H_T}{2}\right)} \quad\ldots\ldots (63)$$

An equation for the main cable was then established in the same manner. The internal work, $\left(\frac{H_1}{2} + H_w\right)\left(\frac{H\, L_s}{A\, E} \pm w\, t\, L_T\right)$, being equated to the external work, $\left(\frac{H}{2} + H_w\right) \Delta\, l + \left(\frac{H}{2} + H_w\right) \frac{8\, f}{l^2} \int_0^l \eta\, dx$. This gave,

$$\Delta\, l = \frac{H\, L_s}{A\, E} \pm w\, t\, L_T - \frac{8\, f}{l^2} \int_0^l \eta\, dx \quad\ldots\ldots\ldots\ldots\ldots (64)$$

which is the same as Equation (47) with the terms transposed and expressed in the form, $\Delta\, l = \frac{H - Z}{Q}$.

The writers anticipated that it would be extremely difficult to solve a single equation for the various simultaneous values of H and H_T. Such a solution would require that these values be first assumed and then the assumptions tested by the equation. In view of this fact, a semi-graphical method for the determination of the simultaneous values of H and H_T was developed and used in lieu of an equation in the form given by Mr. Steinman (Equations (49) and (50)).

This method may be explained best by considering a single span only without reference to the remaining spans of the bridge. In Equation (63) there are only two variables for a given temperature condition. Hence, it is a simple matter to draw a curve expressing the relation between Δl and H_T. In Equation (64) the relation between Δl and H can be established for any given condition of live load and temperature, and a curve drawn. It can be seen that a different curve must be drawn for the main cable for each loading condition.

All curves for one span were drawn on the same sheet. Values of Δl were expressed as abscissa and those of H and H_T as ordinates. Positive values of H were measured up from the origin and of H_T down. In this manner, a vertical scale could be placed on a given value of Δl and the total $H + H_T$ for the span determined by reading the intercept between the proper curves. If, on the other hand, a value of $H + H_T$ was assumed, the corresponding value of Δl could be determined by moving a vertical scale to a position such that it measures an intercept equal to this assumed value of $H + H_T$. The horizontal distance of the scale from the origin measured the value of Δl.

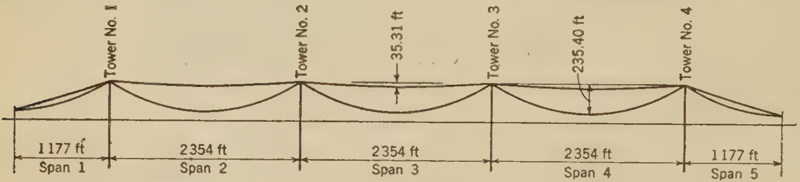


FIG. 7.—GENERAL DIMENSIONS, MULTIPLE-SPAN SUSPENSION BRIDGE.

An outline of the structure, with basic dimensions, is shown in Fig. 7. Other design data are:

Cross-Section Areas, in Square Inches:	
Main cable	494
Tie cable	150
Moment of Inertia, in Inches² Feet²:	
Truss	57 500
Loads, in Pounds per Foot:	
Dead load (excluding tie cable)	8 900
Dead load of tie cable	530
Live load	2 625

In Fig. 8 a set of curves has been drawn for the main spans. These curves include those for the tie cable, and the main cable with no live load on the structure, with live load covering 0.4 of the span, and with live load covering the entire span. It should be realized that for a complete analysis several other curves for other conditions of live load and temperature would be needed. It would also be necessary to extend some of the curves beyond the limits now shown. In Fig. 9, curves for the side span cables are presented, the main cable curve being for an unloaded condition.

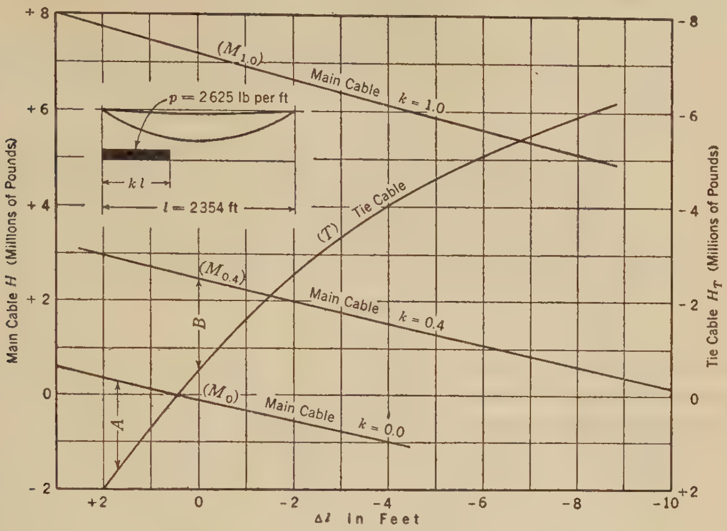


FIG. 8.—*H*-CURVES FOR SPAN 2, 3, OR 4, FIG. 7 (TEMPERATURE, 30 DEGREES FAHRENHEIT ABOVE NORMAL).

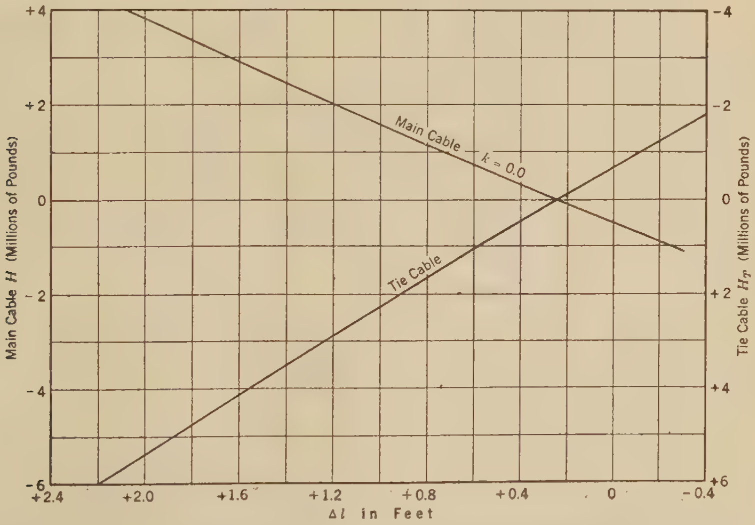


FIG. 9.—*H*-CURVES FOR SPAN 1 OR SPAN 5 (TEMPERATURE, 30 DEGREES FAHRENHEIT ABOVE NORMAL).

As an illustration of the use of the curves, assume that there is no live load on a main span and that a value of $H + H_T$ is assumed as + 2 000 000 lb. With a scale, an intercept is measured between the tie-cable curve and the main cable curve for no live load (Fig. 8) equal to this value of $H + H_T$ and is found at A (Fig. 8), with the value of Δl of + 1.75 ft. The respective values of H and H_T are found by measuring the ordinate to each of the curves at this point. In this case, H is found to be + 300 000 lb and H_T, + 1 700 000 lb.

If a live load covering 0.4 of the span is present, the intercept is measured between the tie-cable curve and the main cable curve for this loading. A value of $H + H_T$ of + 2 000 000 lb is located at B (Fig. 8), and Δl is found to be + 0.03 ft. H is then + 2 460 000 lb and H_T, − 460 000 lb.

A set of curves was drawn for each different span. For the usual conditions with a symmetrical layout, this requires only one set for main spans and another for side spans.

The determination of the several values of H and H_T in the various spans was a matter of "cut and try." A value of $H + H_T$ was assumed and the corresponding values of Δl for each span were determined from the curves, as explained. The value of $\Sigma \Delta l$ was then determined for this value of $H + H_T$, and if it was not equal to zero another value of $H + H_T$ was tried. It was found expedient to solve $\Sigma \Delta l$ for about four assumed values of $H + H_T$ and by means of the plotted relation between the two to determine the satisfying value for $\Sigma \Delta l = 0$. With the proper value of $H + H_T$ determined, the value of H and H_T for each span was readily ascertained from the curves.

It is not difficult to introduce the difference in the total horizontal cable pull between spans due to the resistance of the towers to bending, but normally this can be neglected. Certainly this item will have only a minor effect on truss moments, since the latter are not appreciably affected by variations in H of as much as 10 per cent.

Strictly speaking, this method is not applicable in the case of a bridge with a continuous stiffening truss. However, it appears that, except in the case of short spans with a relatively stiff truss, the effect of continuity may be neglected without introducing serious errors when calculating H-values. The latter could then be used in Equations (9) and (10) to determine moments and shears in the truss. Values of the integration constants must include the continuity factors.

It is significant that a set of curves such as Figs. 8 and 9 can also be used to analyze a bridge without tie cables and of any number of spans. The method could be used readily in conjunction with a structure of the usual type consisting of a single main span.

Practical Considerations.—There are certain difficulties presented by multiple-span suspension bridges with tie cables that should be emphasized. Of first importance is the necessity for providing adequate anchorage between the main and the auxiliary cables at all tower tops. This provision is imperative, since there is a transfer of load from one to the other which runs into millions of pounds on a major structure. The solution of this problem is not impossible, but it is difficult. If the cables are continuous over the towers, some highly effective clamping action must be developed. Discontinuous cables of socketed bridge strands may offer a solution.

It is not difficult to proportion this type of structure so that the vertical deflections of the trusses are relatively small; but it is somewhat of a problem to keep the bending of the towers toward the center of the crossing within reasonable limits. These towers must be strengthened adequately to perform this additional duty. The possibility of hinged towers presents itself, and, under favorable conditions, may be the answer.

Several new construction problems would certainly present themselves, but doubtless the ingenuity of American erectors would be fully equal to the occasion.

L. J. MENSCH,[18] M. AM. SOC. C. E. (by letter).—While it is true that Professor Melan used a certain tool of the elastic theory, he knew all the limitations of his assumptions and called his method the "approximate theory" rather than the "elastic theory." He also knew that his formulas for the secondary stresses caused by the deflection of the cables form only an approximate theory and, therefore, called it a more precise theory, and did not use the expression, "exact theory," so liberally found in European technical literature.

The differential equations occurring in this masterly paper by Mr. Steinman are used in many problems which the modern engineer must solve. In the design of thin reservoir and ships plates, Dr. P. Forchheimer[19] shows that one cannot neglect the change of length of the neutral axis of the continuous thin plates, and he treats the resulting suspension action in the same manner as the author, a problem which has been previously discussed by F. Grashof.[20] Dr. Forchheimer simplified the design greatly without the use of any differential equation, and the writer is convinced that Mr. Steinman will be able to simplify his formulas by following Dr. Forchheimer's lead.

The same differential equations occur in the problems of computing the secondary stresses in cylinders held at the ends, such as tanks and boilers, as well as in rectangular tanks to find the stresses in the vertical corners. They occur also in the design of road slabs in finding the stresses due to concentrated loads at the middle or at the edges. All these problems have been simplified so that differential equations are not required for their solution.

A. A. EREMIN,[21] ASSOC. M. AM. SOC. C. E. (by letter).—Simple equations are developed in this paper, for an accurate calculation of the stresses in a continuous suspension bridge. Furthermore, the author has provided instructive diagrams for the stresses in a suspension bridge continuous over three spans.

---

[18] Civ. Engr. and Constructor, Chicago, Ill.

[19] "Die Berechnung ebener und gekrümter Behälterböden", von P. Forchheimer. Second Edition, Berlin, 1909.

[20] "Theorie der Elasticität und Festigkeit", von F. Grashof, Berlin, 1878, p. 152.

[21] Assoc. Bridge Designing Engr., Div. of Highways, State Dept. of Public Works, Sacramento, Calif.

Fig. 10 may be found convenient in computing the loading terms. For example, the loading terms in Case V, Article 9, may be expressed as follows, to compute values of H, T_1, and T_2:

$$A = pu\left[x\,(l - x) - \frac{u^2}{3}\right] - \frac{1}{c^2}\,B_1 \;\ldots\ldots\ldots\ldots\;(65a)$$

$$B_1 = p\left[2\,u - \frac{(1 - d)}{2\,c}\,\frac{(e^{2cu} - 1)\,(e^{2cx} + e^{cl})}{e^{c(x+u)}}\right] \;\ldots\ldots\ldots\;(65b)$$

and,

$$B_2 = -\,p\left[\frac{2\,u\,(2\,x - l)}{l} - \frac{(1 - d)}{2\,c\,d}\,\frac{(e^{2cu} - 1)\,(e^{2cx} - e^{cl})}{e^{c(x+u)}}\right] \ldots (65c)$$

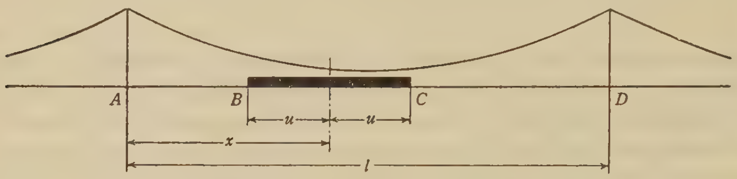


FIG. 10.—MAIN SPAN PARTLY LOADED NEAR CENTER.

For calculating the constants, C_1 and C_2, in Segment BC:

$$G_1 = -\,\frac{p}{4\,d}\left[\frac{(1 - d)\,(e^{2cx} + e^{2cu})}{e^{c(x+u)}} - \frac{(1 + d)\,(e^{2cl} - e^{2c(x+u)})}{e^{c(l+x+u)}}\right] \ldots (66a)$$

and,

$$G_3 = \frac{p}{2}\,\frac{(e^{2cx} + e^{2cu})}{e^{c(x+u)}} \;\ldots\ldots\ldots\ldots\ldots\ldots\;(66b)$$

By varying the dimensions, u and x, Equations (65) and (66) may be used for the loading uniformly distributed over any length and at any part in the span. The loading terms for Case VI, Article 9, may be obtained by calculating the terms for the case in which the span is fully loaded and thereafter subtracting the terms calculated with Equations (65) and (66) for loading over the middle part.

Equations (65) and (66) are convenient for computing the loading terms because they contain like factors for all cases of distribution. Furthermore, these equations may easily be simplified for the loading symmetrical about mid-span.

The author has developed equations for the stresses and deflections in a suspension bridge with the assumption that the stiffening truss and the suspension cable deflect at any section for the same amount. However, considering the elongation of the hangers, $\Delta\,h$, the bending moment in a stiffening truss will be,

$$M = M_0 - Hy + T - (H + H_w)\,(\eta - \Delta\,h) \ldots\ldots\ldots\ldots (67)$$

nd the differential equation for the deflection of the stiffening truss may e written,

$$\frac{d^2\eta}{dx^2} = c^2\eta - c^2 \,\Delta\, h - \frac{1}{EI}\,(M_0 + T - Hy) \text{ ....(68)}$$

An elongation in the hanger sustaining the loading from cable and emperature stresses may be expressed as,

$$\Delta\, h = \left(\frac{H}{r\,A_h\,E_h} + \omega_h\, t\right)(F_c + f - y) \text{ ....(69)}$$

n which, A_h = section area of hanger per unit length along the truss; E_h = elastic modulus of hanger; F_c = hanger length at mid-span; and ω_h = coefficient of expansion of hanger.

Substituting Equation (69) in Equation (68) and solving for the deflection of the truss,

$$\eta = \frac{1}{E\,I\,c^2}\Bigg\{C_1\,e^{cx} + C_2\,e^{-cx} + M_0 - H\,y + T + E\,I\,c^2\,\Delta\,h$$

$$- \frac{1}{c^2}\left[p_x - \frac{H}{r}\left(1 + \left[\frac{H}{r\,A_h\,E_h} + \omega_h\,t\right]\frac{E\,I\,c^2}{H}\right)\right]\Bigg\} \text{ ....(70)}$$

and the bending moment in the stiffening truss will be:

$$M = -\Bigg\{(C_1\,e^{cx} + C_2\,e^{-cx})$$

$$- \frac{1}{c^2}\left[p_x - \frac{H}{r}\left(1 + \left[\frac{H}{r\,A_h\,E_h} + \omega_h\,t\right]\frac{E\,I\,c^2}{H}\right)\right]\Bigg\} \text{ ....(71)}$$

The constants, C_1 *and* C_2, in Equation (71) are the same as those in Equation (9). Therefore, the moment in Equation (71) differs from the moment in Equation (9) in the term, $\left[\frac{H}{r\,A_h\,E_h} + \omega_h\,t\right]\frac{E\,I\,c^2}{H}$. This term has been shown to be small.[22] Therefore, without material error, the effect of the extension of hangers may be neglected as has been stated correctly by the author.

Mr. Steinman has contributed a most outstanding paper to the Engineering Profession. The range of application of suspension bridges is increasing. Therefore, the design method of continuous suspension bridges presented in the paper is timely.

Dr. Ing. Hans H. Bleich,[23] (by letter).—Following the same idea as that of the author, namely, that suspension bridges with continuous stiffening trusses must have certain advantages over the usual simple-span suspension bridges, the writer has likewise developed a generalized analysis applicable to continuous as well as to non-continuous suspension spans.

---

[22] "Modern Framed Structures" by Johnson, Bryan, and Turneaure, 1917 Edition, Pt. 2, p. 300.

[23] Vienna, Austria.

The writer's procedure is based on the known relationship between the deflection curves of the stiffening truss due to any live loading and the deformation of the stiffening truss due to buckling as a long column under a longitudinal load. This analysis, differing in method of attack, yields final working formulas and numerical results practically equivalent to those obtained by Mr. Steinman, thus affording a substantial check.

One advantage of the writer's method of analysis is that it facilitates consideration of the variable moment of inertia of the stiffening truss. The influence of a varying moment of inertia is particularly important in suspension bridges with continuous stiffening trusses.

The author's claims of superior efficiency for the continuous type of suspension bridge are conservative. He has limited his comparisons to mid-span deflections. By considering also quarter-point deflections, more marked advantages are found even in longer spans.

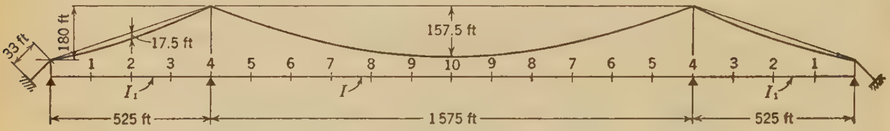


FIG. 11.

The following example of a three-span structure with 1 575-ft main span will show advantages of continuity beyond those pointed out by the author. Referring to Fig. 11, the following general data apply to the problem (using the author's notation): $l = 1\,575$ ft; $l_1 = 525$ ft; $f = 157.5$ ft; $f_1 = 17.5$ ft; $I = I_1 = 33\,400$ in.^{2} ft^2; $A_c = 620$ in.^{2}; $E_c = 23\,500\,000$ lb per sq in.; $w = 13\,440$ lb per ft; $p = 4\,030$ lb per ft; $t = \pm 35°$ C; and $\omega = 0.0000125$.

The maximum bending moment curves for live load and temperature for a continuous truss, and for a two-hinged stiffening truss, both proportioned for the same uniform moment of inertia, are recorded in Fig. 12 (compare with Fig. 5). A comparison of the areas of Designs I and II, Fig. 12, shows that, for the same depth, the continuous truss would require a 6% larger chord section than the two-hinged stiffening truss.

The maximum deflections of the two systems of bridges show that the continuous design (without the 6% addition of chord material) is marked by a notable increase of rigidity over the other system. Table 4 contains the maximum deflections at the center and at the quarter-point of the main span and at the center of the side span. The greatest and most important deflection occurs at the quarter-point of the main span. For this critical deflection, the continuous suspension bridge of 1 575-ft main span, is 17% stiffer than the two-hinged type (although both are proportioned for the same moment of inertia). The corresponding increase in the side-span rigidity is 20 per cent.

The reductions in bending moments from the elastic theory to the deflection theory are found, for this example (see Design I, Fig. 12), to be of similar magnitude to those found by Mr. Steinman for his design with 800-ft main span. These reductions for the design with 1 575-ft main span, amount to about 19% at the tower and about 45% in the main span (compare Fig. 3).

A true knowledge of the economic advantage of the continuous type is obtained by comparing the bending moments in two designs of equal rigidity.

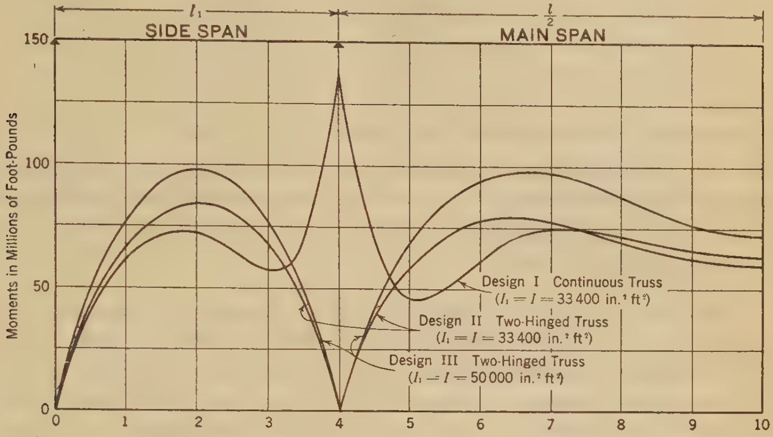


FIG. 12.

The moments of inertia of the two designs should be chosen to yield approximately the same average deflections.

Table 5 shows that a two-hinged suspension bridge with $I = I_1 = 50\,000$ in.^{2} ft^2 (Design III), has about the same rigidity as the continuous bridge with $I = I_1 = 33\,400$ in.^{2} ft^2 (Design I). The maximum bending moments from live load and temperature for this bridge (Design III) are also recorded in Fig. 12. The area of this maximum moment graph is found to be 13% larger than for the continuous bridge (Design I). Hence, when proportioned for equal rigidity, the continuous design with 1 575-ft main span is 13% more economical than the two-hinged design. (A comparison of the respective assumed moments of inertia, for equal rigidity, shows an economy of 33 per cent.)

TABLE 4.—COMPARISON OF MAXIMUM DEFLECTIONS, IN FEET (SEE FIG. 12).

Design	SIDE SPAN	MAIN SPAN	
	Center	Quarter-point	Center
I.	$1.72 = \frac{l_1}{334}$	$4.11 = \frac{l}{383}$	$3.91 = \frac{l}{403}$
II.	$2.15 = \frac{l_1}{244}$	$4.96 = \frac{l}{317}$	$4.03 = \frac{l}{391}$
Percentage difference	20	17	3

The writer has thus confirmed Mr. Steinman's findings of the superior efficiency of the continuous type and has extended these findings to greater span lengths. The same numerical values and comparisons are yielded by

TABLE 5.—Comparison of Maximum Deflections, in Feet, Designs of Equal Rigidity (See Fig. 12).

Design	Side Span	Main Span	
	Center	Quarter-point	Center
I	1.72	4.11	3.91
III	1.68	4.35	3.78
Percentage difference	—2.5	+6.0	—3.0

either form of the generalized deflection theory. The author would have found the same magnitude and range of superior economy and rigidity if he had considered quarter-point deflections instead of limiting his comparisons to mid-span deflections.

F. H. Frankland,[24] M. Am. Soc. C. E. (by letter).—The generalized deflection theory of design for suspension bridges presented in this paper, by which the economy realized by the use of the exact or deflection theory, instead of the elastic theory, in the design of bridges with simple-span stiffening girders or trusses, is now available for continuously stiffened, and for multiple-span, suspension bridges. This contribution is undoubtedly one of the outstanding recent advances in bridge design.

The design of suspension bridges has advanced greatly during the past few years, and this development has been much more marked than with other types of bridges. Twenty years ago (1914) it was said with truth that little was known regarding the science of design as relating to this type of structure.

For long crossings there will be many instances where multiple-span suspension bridges will offer the most economic and efficient solution over a considerable range of conditions. These conditions vary from case to case, often in great degree, but in many instances, because of the fact that such structures for highway traffic carry relatively small live loads with fairly large permissible deflections, it is possible to achieve economy, efficiency, practicability, and good appearance in a greater degree with a multiple-span suspension bridge of truly modern design than with any other type. The incorporation of steel floors as a unit with the stiffening girders and wind girder, has permitted hitherto unrealized economies in some recently constructed bridges. Furthermore, in some cases, it is possible to reduce the metal in stringers materially by constructing them continuous over the floor-beams, a feature of special value for self-anchored suspension bridges. When this is done, the stringers become a part of the stiffening system, with corresponding savings in the main stiffening girders.

Gustav Lindenthal,[25] Hon. M. Am. Soc. C. E. (by letter).—The generalized deflection theory offered in this paper presents an ingeniously deduced basis for computing stresses in stiffened, suspension bridges whether of one span with end spans, or of multiple spans, assuming continuous parallel-chord trusses through the towers as a recognized feature of economy. It offers a welcome check on other methods of calculation and a refinement toward greater economy, provided the economically most suitable relations of cables and stiffening trusses are first chosen by trial for the given case.

---

[24] Director, Eng. Service, Am. Inst. of Steel Constr., Inc., New York, N. Y.

[25] Pres. and Chf. Engr., North River Bridge Co.; Cons. Engr., Jersey City, N. J. Mr. Lindenthal died on July 31, 1935.

It is a singular fact that, whereas the ascertainment of stresses in a suspension bridge from uniform loads, as for a suspended aqueduct, is a simple matter, that of the stresses from live loads or from moving loads presents certain complexities resulting from the different behaviors of cables and suspended trusses under such loads, further influenced by temperature effects and elastic restraints not wholly statically determinate or subject to elimination by the deflection theory, as, for instance, the elastic elongations of suspenders which for simplicity the author assumes to be negligible.

Therefore, the computed results for any given case, even with the perfected theories now extant, are more of the nature of close approximations than of absolute accuracy, similar to all structures not wholly statically determinate. They are sufficiently accurate, however, for safety and close economy in suspension bridges, which are different from other truss forms in that their carrying capacity is not affected by their deflection under moving loads; only their rigidity is affected, and this can be kept within predetermined limits.

For long spans and concentrated loads it is advisable to make comparisons and checks with models. Suspension structures considered as suspended arches are always in stable equilibrium for any loading. They are unlike the unstable equilibrium of the erect arch in which the center of gravity is far above the points of support, and in which the breaking of a stiffening member may produce collapse; in the suspended form nothing more serious would result than local deformation.

Far in advance for long spans of either the arch or girder types, this anciently known fact early led to the construction of important suspension bridges (600-ft spans over the Menai Strait between the Island of Anglesey and Northwest Wales, in 1832, and over the Danube at Budapest, Hungary, in 1843) stiffened only nominally with hand-rails. Nothing serious can happen to a suspension bridge as long as the anchorage, towers, and cables are sound. In the long-span Brooklyn Bridge, for instance, fractures in the chords of the stiffening trusses, or in the suspenders, have occurred without seriously endangering the structure as a whole.

The first theory for the stiffening of a suspended span by the English engineer and mathematician, Rankine (1858), was not used in the United States as far as known, because it gave heavier and more expensive trusses than the empirical Roebling System (low stiffening trusses with stays up to the towers) successfully used for road bridges in this country. The deflection and stresses in the Brooklyn Bridge were studied with the aid of a model which indicated for the long span a sufficiently stiff structure under the exigencies of highway traffic; but a highway and railroad bridge of this system with an 800-ft span, over the Niagara River, did not prove a success. It was a "wobbly" structure, which was later replaced by a rigid steel arch bridge.

The first analysis in the United States of stresses in suspension bridges stiffened with trusses was made by the late Augustus Jay Dubois, M. Am. Soc. C. E.[26] In his treatment of the subject Professor Dubois maintained that the curve of the cable does not remain parabolic as the generally received theory assumed, but that it takes the curve of equilibrium due to loading. He criticized the Roebling System as unscientific and wasteful, and no more bridges of this type were built thereafter.

When Professor Josef Melan, of Prague, Czecho-Slovakia, published, in 1888, his theory of stiffening suspension bridges, he was not aware of the equilibrium theory of Professor Dubois. Professor Melan assumed his stiffening girders hinged at the ends and at the center in order to avoid temperature stresses, but this assumption was corrected in the later editions of his book when the writer had pointed out[27] that the middle hinge does not eliminate temperature stresses but merely re-distributes them.

From the first, Professor Dubois assumed continuous girders fastened down at the ends to resist end bending moments. Both writers assumed only single spans with parallel chord trusses of uniform moments of inertia. Professor Melan's theory, enlarged and supplemented in the later editions of his book, became the accepted theory abroad and in the United States. Other contributors followed with specialized investigations, among whom were the German authors, C. M. Bohny, J. Fritsche, K. Girkman, and F. Glaser. A quite meticulous mathematical treatment of the subject was presented in a new deflection theory (published in English in 1930) by the late Hans Henrik Rode, M. Am. Soc. C. E. Professor Rode's program to extend his method of investigation to suspended and erect braced arches remained incomplete because he died suddenly on July 18, 1930.

Special cases of suspension structures may arise requiring independent investigations. This may happen, for instance, in the possible strengthening of the Brooklyn Bridge for heavier live loads. When this question came up before the writer in 1902, he made a study of different methods of strengthening with the purpose of decreasing the load on top of the stone towers and of transferring part of the load of the superstructure directly to the tower foundations. The stone masonry in the towers is sound, but it is exposed to bending stresses which increase the pressure on the stone to more than 1 000 lb per sq in., and this pressure should be relieved. Additional cables resting on the tops of the towers must be avoided. The study assumed stiffening trusses of greater height and greater moment of inertia at the towers than at the center, or at the anchorages, to take up, through cantilever action, a large proportion of the suspended dead load on both sides of the towers and transfer it directly to the foundations. The arrangement would require a special analysis checked by tests on a model.

---

[26] *Journal*, Franklin Inst., 1882.

[27] Rept. of U. S. Board of Engr. Officers, on Maximum Span Practicable for Suspension Bridges, 1894, Appendix D, p. 68, "Temperature Stresses in Three-Hinged Arches", by Gustav Lindenthal, Hon. M. Am. Soc. C. E.

In Section 12 the author discusses the application of his method to multiple suspension bridges with tie-cables connecting the tower tops. Such an arrangement would mar the architectural appearance of a suspension bridge. It would look more like a derrick structure than a bridge, and seems to be avoidable without much loss of economy, at least in highway bridges in which the suspended load is several times heavier than the live load.

The writer bases his belief on his observation of the behavior of a suspension bridge with ten successive spans of 180 ft each under exceptionally heavy street traffic. This was the old Monongahela Bridge, at Pittsburgh, Pa., before it was rebuilt in 1882. Originally, a timber bridge had been built in 1810. When it burned down in 1843, it was rebuilt on the same piers by the late John Augustus Roebling, M. Am. Soc. C. E., as a wire cable suspension bridge of ten spans. Each span was carried by two cradled cables, made on shore, of parallel wrapped wires.

The cables had loops around an iron shoe at each end suspended from short hangers in the top of the cast-iron towers, about 20 ft high and having bases about 8 ft square on the piers. The sag of the cable was one-thirteenth of the span, and on each side was a frail wooden hand-rail of no value as a stiffening element. Heavy horse teams and crowded horse-cars were passing continually in opposite directions. The proportion of live load to dead load was about 1 to 2, and yet the deflections of the roadway under heavy loads were relatively small. The rod iron links from which the cables were suspended in the towers were in constant pendulum-like motion. The pins on which they were suspended were badly worn when the bridge was dismantled.

It was notable that the entire suspended structure of ten spans (total length, 1 800 ft), with all its faults was remarkably steady, and that the time element for the propagation of wave motion in the suspended structure had a deterrent effect on the deflections. If this arrangement was comparatively steady for small spans, it could be made so for long highway spans on which the dead load, including a concrete floor, is several times greater than the live load, and on which the deflection wave must travel longer distances. The better architectural appearance of the bridge may justify the combination of flat catenaries with stiffening trusses continuous through the towers to avoid sagging ties between the tower tops. The widening of tower bases need only be sufficient to include the resultant from the tower top from unbalanced horizontal tension in the cables, which, in a highway bridge, can be kept between narrow limits. Of course, such an arrangement would not do for a railroad bridge, which must be designed to carry concentrated loads at high speeds.

One encounters here, a considerable difference in weight of metal and cost between suspension bridges for more or less distributed highway loads on a flexible structure and that of a structure for the concentrated loads of railroad trains at fast speeds, requiring a rigid track.

In order that a suspension railroad bridge shall be equivalent to a bridge of rigid type (truss, cantilever, or arch type), its local deflections under passing trains should not materially exceed those in these rigid types for equal carrying capacity; but in the suspension bridge stiffened with trusses the weight of steel required for such a purpose may exceed very considerably the weight of the cables which carry the entire dead load and live load, because the size and weight of the stiffening trusses increase approximately in inverse ratio to the predetermined deflection under passing loads, other things being equal.

If this condition would be adhered to strictly, the suspension bridge with stiffening trusses, for the same span, would become nearly as costly for railroad traffic as a bridge of the rigid truss type. The increased cost would not be for increased carrying capacity, but for greater stiffness.

In this respect the study[28] of the late George Shattuck Morison, Past-President, Am. Soc. C. E., for a long-span suspension railroad bridge remains instructive regarding the guiding elements in such a structure stiffened by stiffening trusses. His method of calculation furnished sufficiently close results that would not be affected materially by the author's deflection theory, making due allowance for the higher unit stresses with higher steels that now would be assumed for such a study and that would substantially reduce the weights of metal.

The discussion of Mr. Morison's paper was illuminating in that all the contributors endorsed the decided economy and advantage of stiffening trusses continuous through the tower (the feature adopted also by the author). The discussers also recognized the greater economy of a suspension bridge in the form of suspended braced arches, in which the stiffening web, taken as it were from the trusses, is placed between the parallel cables which act as chords. A criterion of this economy is thus obtained by comparing the weight of steel in each system required merely for stiffening with the weight of the cables.

In his study for a 3 200-ft span, Mr. Morison assumed a total dead and live load of 50 000 lb per lin ft of span. He found the weight of cables was 10 900 lb. The weight of stiffening trusses for a live load of 12 000 lb was 15 780 lb (truss chords, 11 600 lb; web, 3 240 lb; and cross-bracing between top chords, 1 440 lb). In other words, each 100 lb of live load required 90 lb of wire cable and 131 lb of steel purely for stiffening trusses in order to prevent a deflection of the floor in excess of 3½ ft at the quarter-points and to resist the bending and shearing stresses from live load for that deflection. By this comparison the stiffening trusses are, therefore, 31% heavier than the live load and 45% heavier than the wire cables, which carry the entire dead and live load. The weight of the stiffening trusses could be reduced only at the expense of rigidity. Trains would then be compelled to slow up over the bridge, which means it would lose its merit and value as a railroad structure.

On a highway bridge of the same magnitude the live load would be more or less distributed, which would permit a large reduction in the weight of stiffening trusses, eventually less than 25% of the cable weight, without impairment of the capacity and usefulness of the bridge. Traffic regulations can prevent local concentration and congestion of highway traffic, as is done, for instance, on the Brooklyn Bridge. In this way deflections can be kept within moderate limits; but in a railroad suspension structure in which the deflection wave from passing loads must be kept at a minimum, the weight of the stiffening trusses cannot be reduced arbitrarily without impairing the usefulness of the bridge.

---

[28] *Transactions*, Am. Soc. C. E., Vol. XXXVI (1896), p. 359.

Next, assume a braced suspension arch bridge of the same span in which the web taken from the stiffening truss is inserted between the cables (60 ft apart). Then, suspended braced arches are obtained with a saving assumed roughly at 12 500 lb of steel per lin ft of span over the stiffening truss bridge. Since the suspended dead load in Mr. Morison's study is found to be 39 000 lb, the saving of steel in a suspension braced arch over the stiffening truss type would be 35% on this one item. All values are approximate and are used merely to illustrate the economy of suspended braced arches for the same live loads.

The theoretical treatment of a suspended framed arch bridge is outside the theories for a suspension bridge stiffened with trusses. For the suspended framed arch type, with parallel or nearly parallel chords or cables, the analysis must deal with a structure computable on the theory of elasticity, which for the case of a middle span and two side spans is statically indeterminate in the fifth degree, but with certain permissible assumptions may be reduced to the third degree. When the results are plotted they show that the stresses in the cables from bending moments in the arches can be kept between close limits and require no additional material in the cables. For a completely satisfactory solution checking of the theoretical results on a model is advisable, as is already the practice for large suspension bridges. Such tests will permit verification of computed results and may suggest their further refinement, through application of that part of the author's deflection theory which relates to changes in the curve of the cables from moving loads.

The only suspension bridge type which is statically determinate for moving loads is that with intersecting chords (eye-bar chains), as in the side spans of the Tower Bridge over the Thames, at London, England, or as was proposed in the writer's competing design[29] for the railroad suspension bridge with three spans (659 ft 3 in., 1 758 ft, and 659 ft 3 in.) over the St. Lawrence River, at Quebec, Que., Canada.

The deflections from full live load and temperature (but not from local live loads) are somewhat greater than in equivalent cantilever structures, but are still comparatively small and unobjectionable and even so can occur only rarely. The suspension structure is about 16% cheaper than the cantilever structure, and that its architectural merits are superior to the latter is too obvious for comment.

Julian W. Shields,[30] Esq. (by letter).—With the publication of this paper, a more general adoption of the suspension type of bridge for relatively short highway spans should follow. The writer has long been interested in the suspension bridge for short spans, recognizing its superior æsthetic properties over the conventional types now in use. More important, an accurately designed suspension structure will prove itself economically justifiable. Actual bidding, in the past, has confirmed the belief that the suspension type can compete successfully with other types in the short-span field.

---

[29] *Engineering News*, November 23, 1911.

[30] Instr. in Civ. Eng., Rensselaer Polytechnic Inst., Troy, N. Y.

The major problem of short-span suspension bridge design is to secure sufficient, yet economical, stiffening of the floor system. The hingeless stiffening truss seems best suited to meet the situation. It is admittedly more rigid and, if accurately analyzed, more economical than the two-hinged stiffening truss. Hence, with maximum economy available in the form of the more efficient hingeless stiffening truss, accurately analyzed, the suspension bridge should be able to compete more advantageously than ever before in the short-span field.

A 400-ft span suspension bridge, with 200-ft suspended side spans, was selected in order to study the relative economy of the hingeless and two-hinged stiffening trusses. Following the method outlined in Mr. Steinman's paper, the two truss types were compared, using equal center-span deflections. It is realized that perhaps the truss types should be compared for conditions of equal maximum change in slope. Nevertheless, the results are most gratifying. The hingeless type showed a superior economy of approximately 12% over the the two-hinged truss. This percentage of economy in favor of the hingeless stiffening truss might be increased somewhat if the two truss types were compared under conditions of equal maximum change in grade.

The loadings for maximum moments were found to agree closely with the chart of loadings (Fig. 2) published in the paper. However, there were several exceptions in the example presented by the writer. These exceptions were probably due to the fact that the span lengths adopted were much shorter than those of the example given by Mr. Steinman. This might be expected, as the author indicates. The maximum moments at the sections, $\frac{x_1}{l_1} = 0.6$ and $\frac{x_1}{l_1} = 0.7$, were negative and occurred for an advancing live load in the main span at the highest temperature. The maximum moment at the middle of the main span was positive, but occurred for an advancing live load in that span at highest temperature. The chart calls for a part of the main span loaded symmetrically about the center line at highest temperature. It must be admitted that the difference between the maximum functions at the center of the main span, obtained from the two different conditions of loading, was small.

As stated in the paper, the effect of continuity at the towers is lost in the spans at points proportionately nearer and nearer the towers as the span length increases. Conversely, the opposite is true for decreasing span lengths. In the example, solved by the writer, this is strikingly brought out by the fact that the effect of continuity at the towers is felt in the side spans for a distance of two-fifths of the span, whereas, in Mr. Steinman's example, with the span lengths just double, this effect is felt in the side span for a distance of only one-fifth of the span. Perhaps, this accounts for the disagreement with the loading chart at the sections, $\frac{x_1}{l_1} = 0.6$ and $\frac{x_1}{l_1} = 0.7$.

Some may question the advisability of using the deflection theory for the analysis of the stiffening trusses for such short span lengths, feeling that the approximate theory will give results sufficiently accurate for practical purposes. Using a 3 to 1 ratio of dead load to live load, the writer found that the average deflection correction for live load alone amounted to 16%, which seems to justify the use of the deflection theory formulas even for comparatively short spans.

While, unquestionably, the great worth of this paper lies in its presentation of a workable theory for hingeless and multiple-span suspension bridges, the ease with which the deflection theory may be applied in the solution of two-hinged stiffening trusses, is worthy of note. This feature alone should commend it to the profession. The writer finds that by using the simplified equations of the paper, it is possible to obtain a solution for a two-hinged stiffening truss by the deflection theory almost as rapidly as by the approximate theory. The paper demonstrates a skill in condensing and simplifying difficult algebraic equations, and describes a complicated theory in a form both easy to understand and to apply.

A. W. Fischer,[31] Esq. (by letter).—Remarkable ingenuity is demonstrated by the derivation of the general formulas in this paper. These formulas can be used for continuous stiffening trusses—and for non-continuous suspension bridges—by simply writing zero for T wherever it occurs. If hyperbolic functions are used, formulas can be derived so that the moments and shears can be solved in less time than it requires to solve the formulas as given by the author.

Using the same notation and the differential equation given by the author:

$$\frac{d^2\eta}{dx^2} - c^2\eta = -\frac{1}{E\,I}(M_0 - H\,y + T) \ldots\ldots\ldots\ldots (72)$$

Equation (72) is a linear differential equation with constant coefficients, the second member of which is a function of x. The complete integral of this formula consists of two parts, a complementary function and a particular integral, the complementary function being the complete solution of the equation formed by making the first member of Equation (72) equal to zero.[32]

The solution of the particular integral is not difficult, but is somewhat lengthy.[33] The complete solution of Equation (72), is:

$$\eta = C_1 \sinh cx + C_2 \cosh cx - \frac{1}{E\,I\,c^2}\left[\left(\frac{p_x}{c^2} - \frac{H}{r\,c^2}\right) - M_0 + H\,y - T\right] . (73)$$

---

[31] Philadelphia, Pa.

[32] "Differential Equations", by Abraham Cohen, First Edition, p. 96.

[33] "Differential Equations", by Daniel A. Murray, Eighth Impression, p. 72.

which is similar to Equation (7) of the paper, except that hyperbolic functions enter into it.

The second derivative of Equation (73) gives the value of moment,

$$M = -(H_w + H)(C_1 \sinh cx + C_2 \cosh cx) - \frac{H}{r\,c^2} + \frac{p_x}{c^2} \ldots (74)$$

and the third derivative gives the value of shear,

$$V = -(c)(H_w + H)(C_1 \cosh cx + C_2 \sinh cx) \ldots\ldots\ldots (75)$$

For a uniform load of p_x per unit of length extending a distance, k, from the left end of the span (which is Case III of the paper), the constants of integration for the loaded section, AB, and the unloaded section, BC, may be determined at the same time. The constants for the loaded sections will be denoted by C_1 and C_2; those for the unloaded part by C_3 and C_4. The additional formulas now required for the four constants are obtained from the condition that the moments and shears at the right end of Section AB are equal to those at the left end of Section BC.

From Equations (74) and (75), the moments and shear for the various sections are:

$$M_{AB} = -(H_w + H)(C_1 \sinh cx + C_2 \cosh cx) - \frac{H}{r\,c^2} + \frac{p_x}{c^2} \ldots (76a)$$

and,

$$M_{BC} = -(H_w + H)(C_3 \sinh cx + C_4 \cosh cx) - \frac{H}{r\,c^2} + 0 \ldots (76b)$$

$$V_{AB} = -c\,(H_w + H)(C_1 \cosh cx + C_2 \sinh cx) \ldots\ldots (77a)$$

and,

$$V_{BC} = -c\,(H_w + H)(C_3 \cosh cx + C_4 \sinh cx) \ldots\ldots (77b)$$

From Equations (76) and (77), and the conditions that when $x = 0$, $M = T_1$ in Equation (76a) and for $x = l$, $M = T_2$ in Equation (76b) and from the conditions stated previously—the values of the constants can be determined. Solving for C_1:

$$C_1 = \frac{1}{(H_w + H)\sinh c\,l}\left[\frac{H}{r\,c^2}(\cosh c\,l - 1) - \frac{p_x}{c^2}(\cosh c\,l - \cosh c\,k \cosh c\,l + \sinh c\,l \sinh c\,k) + T_1 \cosh c\,l - T_2\right] \ldots\ldots\ldots\ldots (78)$$

and,

$$C_2 = \frac{1}{H_w + H}\left[\frac{p_x}{c^2} - \frac{H}{r\,c^2} - T_1\right] \ldots\ldots\ldots\ldots\ldots (79)$$

Substituting the values of C_1 and C_2 in Equation (76*a*) and reducing, an expression is derived for the moment between the points, *A* and *B*, as follows:

$$M_{AB} = \left[\begin{array}{l} + \dfrac{p_x}{c^2}\left\{1 - \dfrac{\sinh c\,x \cosh c\,(l-k) + \sinh c\,(l-x)}{\sinh c\,l}\right\} \\[2ex] - \dfrac{H}{r\,c^2}\left\{1 - \dfrac{\sinh c\,x + \sinh c\,(l-x)}{\sinh c\,l}\right\} \\[2ex] + \dfrac{T_1 \sinh c\,(l-x)}{\sinh c\,l} + \dfrac{T_2 \sinh c\,x}{\sinh c\,l} \end{array}\right] \quad \ldots . (80)$$

Differentiating Equation (80), an expression is derived for shear in the main span, which gives:

$$V_{AB} = \left[\begin{array}{l} - \dfrac{p_x}{c}\left\{\dfrac{\cosh c\,x \cosh c\,(l-k) - \cosh c\,(l-x)}{\sinh c\,l}\right\} \\[2ex] + \dfrac{H}{r\,c}\left\{\dfrac{\cosh c\,x - \cosh c\,(l-x)}{\sinh c\,l}\right\} \\[2ex] - \dfrac{c\,T_1 \cosh c\,(l-x)}{\sinh c\,l} + \dfrac{c\,T_2 \cosh c\,x}{\sinh c\,l} \end{array}\right] \quad \ldots . . (81)$$

The expression for moments and shears seems lengthy, but if a table of hyperbolic functions and a 20-in. slide-rule are used, the values can be found in about one-half the time it requires to compute the moments and shears by the formulas as given by the author. On close examination it can be seen that the moment using hyperbolic functions can be solved in less time than is required for the constant, C_1, as given by the author. A 20-in. slide-rule does not give reliable results using the author's formulas for Case III loading.

For a given load on the main span as shown for Case III loading the location of the greatest bending moment can be determined by the method of finding the maximum of any continuous function. At the point of greatest bending moment, the differential of the moment, or the shear, must be equal to zero. This results in,

$$\text{inh}\; c x = \frac{1}{\left\{\left[\dfrac{\left(\dfrac{H}{r} - p_x + c^2\,T_1\right)\sinh c\,l}{p_x \cosh c(l-k) - \dfrac{H}{r} - \left(p_x - \dfrac{H}{r}\right)\cosh c\,l + c^2\,T_1 \cosh c\,l - c^2\,T_2}\right]^2 - 1\right\}^{\frac{1}{2}}} \quad . (82)$$

as the formula of the condition for greatest moment, for a certain value of *k*. From Equation (82) *x* can be solved readily. Furthermore, by means of Equation (82), the point of maximum moment for an approaching load on the main span can be determined and, in this way, maximum positive moments can be calculated for the main span from the end to near the center. The approaching load can be increased for various values of k until the maximum positive moment is found.

Example 1.—As an example take the author's values from Table 3 when $k = 0.475\ l$, to solve the value of x where the moment is greatest for a load on the main span extending from A to B (Case III loading); thus: $\sinh c\,l = \sinh 6.728 = 417.7$; $\cosh c\,(l - k) = \cosh 3.531 = 17.09$; and $\cosh c\,l = \cosh 6.728 = 417.7$.

The values of the hyperbolic functions are taken from a table in the writer's possession,[34] but for work of this nature a more complete table should be used.[35]

Substituting the foregoing values in Equation (82) and reducing:

$$\sinh c\,x = \frac{1}{\left\{\left[\dfrac{-\,606.0}{-\,585.0261}\right]^2 - 1\right\}^{\frac{1}{2}}} = \frac{1}{\{1.073 - 1\}^{\frac{1}{2}}} = \frac{1}{0.2701} = 3.705$$

Therefore, $c x = 2.02$, and $x = \dfrac{2.02}{0.00841} = 240.4$ ft. The author uses a value of 240 ft.

Equation (82) seems long, but it is easier to calculate $c x$ from this formula than to use the "cut-and-try" system of locating the point of maximum moment for a given load on the main span from AB.

Example 2.—Using the same values as in Example 1, with $x = 0.3\ l$, to solve for the moment, M, in Case III when the load extends from A to B: $\sinh c x = \sinh 2.019 = 3.699$; $\cosh c\,(l - k) = \cosh 3.531 = 17.09$; $\sinh c\,(l - x) = \sinh 4.709 = 55.47$; and $\sinh c\,l = \sinh 6.728 = 417.7$.

Substituting in Equation (80):

$$M_{AB} = \frac{1.300}{0.00007064}\,(1 - 0.2843) - 5\,165\,(1 - 0.1416) - 953$$
$$= 13\,170 - 4\,435 - 953 = +\,7\,782 \text{ ft-kips}$$

The author gives + 7 770 ft-kips. For a two-hinged stiffening truss with the same loading condition, the moment would be 13 170 − 4 435 = + 8 735 ft-kips. Equations (78) to (82), inclusive, are true for a two-hinged stiffening truss, if the terms containing T_1 and T_2 are zero. The solutions of T_1 and T_2 as given by the author are quite long and hyperbolic functions can be used to advantage.

In view of the simple formulas that can be developed using hyperbolic functions for the solution of moments and shears the writer believes that after formulas are written for the various loading conditions, they will be used in nearly all cases for the solution of moments and shears for continuous or non-continuous suspension bridge stiffening trusses.

As suspension bridges will be used more and more for either long or short spans, the simplest, shortest, and yet most exact analysis should be used.

---

[34] Mechanical Engineers' Handbook, by Lionel S. Marks.

[35] See Smithsonian Tables of Hyperbolic Functions.

J. M. FRANKLAND,[36] ESQ. (by letter).—Ingenuity was shown in arranging the complicated working formulas of this paper so that they assume the most convenient forms for computation. The use of a single function, *d*, from which all the hyperbolic-function terms may be derived, and the publication of a table of its values (Table 1), are of much practical advantage.

It should be noted, however, that, to maintain accuracy in the use of *d* in its various combinations, the values of *d* itself should be known to a higher degree of accuracy, to five figures at least. Linear interpolation for intermediate values of *c l* in the published table is not sufficiently accurate at low values. Therefore, the following interpolation formula is suggested:

$$\Delta d = \frac{(1 - d_0)\,(1 + d_0)}{2 + d_0\,\Delta(c\,l)}\,\Delta(c\,l) \qquad (83)$$

in which, Δd is the increase in d due to an increase in $c\,l$ equal to $\Delta\,(c\,l)$, and d_0 is the value of d from which the increase is reckoned. The values obtained are accurate throughout Table 1 to the nearest figure in the fifth decimal place.

Numerous objections have been raised from time to time with regard to the conventional deflection-theory derivation (that is, as given in Article 7) of the equation for the component of cable tension due to live load. The author[37] himself has written:

> "It should be noticed that two approximations are involved in the foregoing [conventional] derivation of the H-equation * * * it is assumed that the suspender and cable loading is uniformly distributed over the span; this is contrary to the actual condition * * *. The second approximation consists in writing the original cable sag (f) instead of the augmented cable sag $(f + \eta)$ in the expression for W_1 * * *".

A further objection can be made to the calculation of the work of deformation in terms of the mean forces acting during the displacement, since in a suspension bridge the forces are not linear functions of the displacement.

A kinematical analysis shows that Equation (22), the usual formula for *H*, is more accurate than the assumptions involved in its derivation and that, in fact, the foregoing objections do not apply to the final equation when the towers are hinged at the base. This analysis may be briefly sketched as follows: The cable under dead load and normal temperature is assumed to have the form of a parabola. Let ϕ be the angle to the original cable curve and the x-axis (assumed horizontal). When the temperature alters and live load is applied, the point, (x, y), of the cable curve is displaced to (x', y'), and ϕ changes to ϕ'. Let,

$$\eta = y' - y \qquad (84)$$

and,

$$\Delta\,\phi = \phi' - \phi \qquad (85)$$

---

[36] Cons. Physicist, New York, N. Y.

[37] "Modern Suspension Bridges", by D. B. Steinman, M. Am. Soc. C. E., Second Edition, p. 252, New York, John Wiley and Sons, 1929.

If one neglects the change in vertical dimensions of towers and suspenders due to alteration of temperature and stress, as is customary, the stiffening truss has the same deflection, η, as the cable.

If ds is the element of arc in the original cable curve, and ds', the corresponding element of arc in the strained cable,

$$ds' = (1 + \varepsilon)\, ds \ldots\ldots\ldots\ldots (86)$$

in which,

$$\varepsilon = \frac{H \sec \phi'}{E_c\, A_c} \pm \omega\, t \ldots\ldots\ldots\ldots (87)$$

The horizontal projection of the element of strained cable (correct to quantities of the second order) is:

$$\begin{aligned} ds' \cos \phi' &= (1 + \varepsilon)\, ds \cos(\phi + \Delta\phi) \\ &= [(1 + \varepsilon) \cos \phi - \Delta\phi \sin \phi]\, ds \ldots\ldots\ldots\ldots (88) \end{aligned}$$

If the cable span, l, increases by Δl,

$$\begin{aligned} l + \Delta l &= \int_0^l [(1 + \epsilon) \cos \phi - \Delta\phi \sin \phi] \frac{ds}{dx}\, dx \\ &= \int_0^l (1 + \epsilon - \Delta\phi \tan \phi)\, dx \ldots\ldots\ldots\ldots (89) \end{aligned}$$

and,

$$\Delta l = \int_0^l (\varepsilon - \Delta\phi \tan \phi)\, dx \ldots\ldots\ldots\ldots (90)$$

Next, a value of $\Delta\phi$ is wanted in terms of known quantities:

$$\frac{dy'}{ds'} = \sin \phi' = \sin \phi + \Delta\phi \cos \phi \ldots\ldots\ldots\ldots (91)$$

Also,

$$\frac{dy'}{ds'} = \frac{1}{1 + \epsilon} \frac{dy'}{ds} = \frac{1}{1 + \epsilon}\left(\frac{dy}{ds} + \frac{d\eta}{dx}\frac{dx}{ds}\right) = \frac{1}{1 + \epsilon}\left(\sin \phi + \frac{d\eta}{dx} \cos \phi\right) \ldots (92)$$

Equating the two values of $\dfrac{dy'}{ds'}$ and solving,

$$\Delta\phi = \frac{d\eta}{dx} - \varepsilon \tan \phi \ldots\ldots\ldots\ldots (93)$$

Hence, $\varepsilon - \Delta\phi \tan \phi = \varepsilon \sec^2 \phi - \dfrac{d\eta}{dx}\dfrac{dy}{dx}$; and,

$$\Delta l = \int_0^l \epsilon \sec^2 \phi\, dx - \int_0^l \frac{d\eta}{dx}\frac{dy}{dx}\, dx \ldots\ldots\ldots\ldots (94)$$

It is sufficiently accurate to replace ϕ' by ϕ in the expression for ε. The first integral then becomes:

$$\int_0^l \varepsilon \sec^2 \phi\, dx = \frac{H}{E_c\, A_0} l_s \pm \omega\, t\, l_t \ldots\ldots\ldots\ldots (95)$$

in which,

$$l_s = \int_0^l \frac{A_0}{A_c} \left(\frac{ds}{dx} \right)^3 dx \ldots\ldots\ldots\ldots\ldots\ldots (96)$$

and,

$$l_t = \int_0^l \left(\frac{ds}{dx} \right)^2 dx \ldots\ldots\ldots\ldots\ldots\ldots (97)$$

On integrating by parts, the second integral gives:

$$\int_0^l \frac{d\eta}{dx} \frac{dy}{dx} dx = \frac{8f}{l^2} \int_0^l \eta \, dx \ldots\ldots\ldots\ldots\ldots\ldots (98)$$

Thus, the increase in cable span is given by:

$$\Delta l = \frac{H}{E_c A_0} l_s \pm \omega t l_t - \frac{8f}{l^2} \int_0^l \eta \, dx \ldots\ldots\ldots\ldots (99)$$

Summing from anchorage to anchorage, $\Sigma \Delta l = 0$; $\Sigma l_s = L_s$; and, $\Sigma l_t = L_t$.

In the case of hinged towers with the same value of H in all spans,

$$\sum \frac{8f}{l^2} \int_0^l \eta \, dx = \frac{H}{E_c A_0} L_s \pm \omega L_t \ldots\ldots\ldots\ldots (100)$$

which is identical with Equation (22). When the towers are fixed, it will be satisfactory in ordinary cases to use Equation (100) and interpret H as the mean value for the bridge, giving the side spans the weight one-half.

The derivation is not particularly complicated by dispensing with the assumption that vertical dimensions are unchanged. The effect of temperature is greater than that of suspender elongation under stress combined with the shortening in the towers. Considering only the temperature effect on vertical dimensions and taking the same coefficient of thermal expansion for both towers and cables, Equations (99) and (100) become:

$$\Delta l = \frac{H}{E_c A_0} l_s \pm \omega t l - \frac{8f}{l^2} \int_0^l \eta \, dx \ldots\ldots\ldots\ldots (101)$$

and,

$$\sum \frac{8f}{l^2} \int_0^l \eta \, dx = \frac{H}{E_c A_0} L_s \pm \omega t L \ldots\ldots\ldots\ldots (102)$$

in which, L is the distance between anchorages. There is little difference between the values of H given by Equations (100) and (102), but the latter is a little simpler, since there is no need to calculate the integral, L_t. To take into consideration the effect of elongation of suspenders under stress gives one a negligible gain in accuracy at the expense of complication of the formulas and is not to be recommended.

After obtaining the foregoing derivation independently, the writer found that Professor G. G. Krivoshein[38] had previously made an equivalent analysis to apply to the case of the Delaware River Bridge.

---

[38] "Simplified Calculation of Statically Indeterminate Structures: Appendix", by G. G. Krivoshein, Prague, the Author, 1930.

The most satisfactory method of analyzing suspension bridges with tie cables, or with fixed towers, is by the use of Equations (101) and (102). The free-hanging tie cable is completely determined when the change in span is known.

It may not be amiss to point out a corollary to be drawn from the H-equation. The loss of gravitational potential of the dead load is:

$$\sum \int_0^l w\, \eta\, dx = H_w \sum \frac{8f}{l^2} \int_0^l \eta\, dx \quad \ldots\ldots\ldots\ldots\ldots\ldots (103)$$

At normal temperature Equation (103) is equal (by Equation (102)) to $\frac{H\ H_w}{E_c\ A_0} L_s$, which is always a positive quantity at normal temperature. This shows that the center of gravity of the dead load drops in every case when live load is applied. Thus, the dead load, as such, does not resist the live load deformation. Of course, the initial tension, H_w, in the cable resists the live load deformation, but this must be viewed as a secondary (not immediate) effect of dead load. Since it is conceivable that other means than dead load alone may be used to obtain H_w (such as connecting suspenders to the stiffening truss in such a manner as to give zero bending moment at an elevated temperature), the writer prefers to call H_w the initial tension rather than the dead load tension.

The paper represents a distinct advance in the design theory of suspension bridges, supplying the engineer for the first time with an accurate means of computing the behavior of definite structures of those types the peculiar advantages of which have become increasingly apparent as experience in this field has developed. The formulas of the deflection theory, however, do not exhibit clearly the relations among the essential variables which the designer has at his control. For this reason, its use is restricted almost entirely to computing the behavior of designs obtained primarily by other methods. This unfortunate condition has hindered the economic development of suspension bridge design. The inadequacy of the elastic theory has been demonstrated clearly; apart from its quantitative inaccuracy, the qualitative picture it furnishes is far from true. It is earnestly to be hoped that, even at the expense of quantitative accuracy, simpler formulas will be developed on the basis of the deflection theory that will demonstrate directly in their form the true functioning of suspension bridges. The problem is not an easy one because there are many unexpected traps for the unwary.

FREDRIK VOGT,[39] ASSOC. M. AM. SOC. C. E. (by letter).—A reference to the studies[40] of the late Hans H. Rode, M. Am. Soc. C. E., is pertinent to this paper.

Following Equation (9), the author states that "in the Deflection Theory, influence lines (in the ordinary sense) cannot be used". Professor Rode proved that such lines can be used for computation of the additional cable stress due to live load and the other effects of live loads, such as bending moments, etc. No doubt, the use of influence lines is limited to the case of proportionality between load and effect, and no exact proportionality exists for suspension bridges. However, on long suspension bridges, the dead load is large as compared with the live loads, and within the range from zero to full live load the proportionality is rather close, whereby the use of influence lines is made possible. Strictly speaking, one influence line should be computed for each particular value of the horizontal cable stress, *H*, between maximum and minimum, but Professor Rode found that sufficient accuracy was obtained by the use of two influence lines, one for a maximum, and one for a minimum, value of *H* (Tension *H* depending on temperature and load), between which a linear interpolation can be applied.

---

[39] Prof. of Mechanics, Norges Tekniske Hoiskole, Trondhjem, Norway.

[40] "New Deflection Theory", by the late Hans H. Rode, M. Am. Soc. C. E., *Transactions,* Det Kgl. Norske Videnskabers Selskab (1930), No. 3. (In English.)

The importance of using influence lines, of course, is that the load lengths for maximum positive and negative moments, etc., can be determined directly instead of by trial as used by the author. Furthermore, it should be mentioned, that the method developed by Professor Rode can be applied for systems of any shape, statically indeterminate in any degree, and the stiffness may vary arbitrarily from panel to panel. Professor Rode takes into account the effect of introducing the deflection theory for the elastic theory by means of a "replacement system", which is identical with the original, except for the addition of certain elastic restraints, some or all of which may be statically indeterminate. This replacement system can be analyzed by the ordinary methods; that is, by the elastic theory. Since the replacement system is statically indeterminate to a high degree and the analysis, consequently, is rather complicated, Professor Rode also developed[40] methods for numerical computations.

LEON S. MOISSEIFF,[41] M. AM. SOC. C. E. (by letter).—The Deflection Theory has been known to American bridge engineers for some time and has formed the basis of the design of almost all important suspension bridges in this country for the last twenty-five years. It will be of interest to relate herein some of its early history in the United States.

In the summer of 1901, due to a combination of causes, a number of suspension rods in the main span of the Brooklyn Bridge failed. At that time this was the only crossing over the East River and carried most of the passenger traffic from Brooklyn, N. Y., to New York City. The Williamsburg Bridge was then in the process of construction. The uninterrupted travel over the Brooklyn Bridge was then of vital importance to the inhabitants of New York. Great public interest was aroused by the incident and an engineering investigation and report was made at the direction of the District Attorney by the late Edwin Duryea, Jr., and Joseph Mayer, Members, Am. Soc. C. E.[42] During the controversy on the behavior and safety of the Brooklyn Bridge which followed the publication of the engineering report, the writer worked on an analysis of the stresses and deformations of that bridge. He found that while there were no official measurements of the deflection of the bridge under load, such deflections as could be observed daily were of much less magnitude than would result from computations based on the Elastic Theory. Further consideration showed that, neglecting altogether the effect of the stiffening trusses on the deflection of the bridge and going back to the simple computations of the equilibrium polygons under dead load and under dead and live load, much closer agreement with the actual behavior of the bridge would result. It became apparent that the tacit neglect by the elastic theory of the effect of the dead load on the deformation of the bridge under live load was a serious error for large bridges. This is explained by the fact that the Rankine-Ritter theory was based on a rigid stiffening truss and thus logically introduced the error. It was later taken over without further scrutiny by the "fathers" of the elastic theory. Due to this, the latter theory is grossly in error for suspension bridges.

---

[41] Cons. Engr., New York, N. Y.

[42] *Engineering News*, October 10, 1901.

While working on the development of a more correct analysis, the writer directed his attention to the short treatment given by Melan to what he called "The More Exact Theory."[43] Strangely, Melan did not put much emphasis on the practical results of the theory. The writer found that Melan's analysis of a one-span suspension bridge, taking into account the effect of the dead load on the deformation of the bridge under live load, gave solutions which closely approached actual conditions.

The Manhattan Bridge, as built, has a center span and two symmetrical side spans. To make a practical application of the theory to the design, it became the task of the writer to develop it for a three-span bridge and to establish the equations for the various loadings and temperature variations causing greatest stress. Bearing in mind the importance of the bridge which was designed for a congested live load of 16 000 lb per lin ft, the analysis was made with much hesitation and caution, and various methods were devised to furnish verifications of the computed results. Realizing that there can be no theory which, applied to a practical structure, will be absolutely exact, the writer subsequently denoted the developed theory by the name of "The Deflection Theory," having in mind that it is based on the deflections of the cable and of the truss. Thus, the Manhattan Bridge was designed in 1904 by the deflection theory, and may be considered the first modern suspension bridge.

Previous to that date, the writer in a discussion of the paper by the late Joseph Mayer, M. Am. Soc. C. E., entitled "The Stiffening System of Long-Span Suspension Bridges for Railway Trains," presented an application of the deflection theory to determine the temperature stresses in a three-hinged stiffening truss.[44]

In the beginning of 1909, when the Manhattan Bridge was approaching completion, the Commissioner of Bridges appointed Ralph Modjeski, M. Am. Soc. C. E., to report on the design and construction of the bridge. Mr. Modjeski engaged F. E. Turneaure, Hon. M. Am. Soc. C. E., for the analytical work. The writer submitted to the latter the fully developed theory and formulas used in the design of the main structure.[45]

---

[43] "Eiserne Bogenbrücken und Hängebrücken, von J. Melan, 1888, Wilh. Engelmann, Leipzig, Germany.

[44] *Transactions*, Am. Soc. C. E., Vol. XLVIII (1902), pp. 422 *et seq.*

In the Final Report of the Board of Engineers on the Bridge over the Delaware River in 1927 (see Appendix D), the writer presented the deflection theory as applied to that bridge.

As developed and published the deflection theory was adapted for single-span, and three-span, suspension bridges with stiffening trusses discontinuous over their supports, and all long-span suspension bridges built during the last twenty-five years have been so constructed. The engineers who designed the several bridges with continuous trusses, mentioned by the author, evidently did not think it worth while to utilize the deflection theory.

Attention is called to the fact that Professor Turneaure, whose work is cited in the paper, states:[46] "For this case [suspended side spans] the constants of integration are determined by equating the moments at the towers for main span with those for side spans, and the deflections at the towers equal to zero." This is the same method used in the paper.

The author shows that some saving in materials and cost, or increased stiffness, could be realized by making the trusses continuous over the tower supports. He, therefore, has extended the formulas of the deflection theory to include continuous stiffening trusses and multiple tied spans. The work principle which is utilized in the derivation of the equations of the deflection theory lends itself well to expansion over these types. The author has succeeded in preparing a presentation of the theory which elegantly embraces the common types of suspension bridge as well as the one with continuous trusses and multiple tied spans.

In generalizing a theory which in practice has been found workable and sufficiently accurate for the required purposes, it is important to scrutinize the original assumptions on which the theory has been based. Usually, some of the assumptions are apparent and have been explicitly stated and some are implied and must be brought out by a trenchant analysis. It is in the character of science to guard its steps. The verity of deduced applications which are extended beyond common practice must be checked by quantitative studies and by physical observations. The application of the deflection theory to continuous stiffening trusses may serve to illustrate the limitations of general theory.

The deflection theory assumes that the effect of the elongation of the suspenders on the total work of the structure is negligible and that its effect on the value of the cable pull is insignificant. This assumption has been justified by computations in several instances as well as by observations on models. The effect of suspender elongation cannot, however, be neglected locally. Near the towers, where the suspenders are longest, the effect of their lengthening under live load on the stiffening truss cannot always be neglected and in long-span bridges it may seriously affect the stresses in the panels nearer the towers. Careful and extensive studies of the effect of suspender elongation on long spans have shown that the moments and shears in the trusses will increase considerably toward the tower ends and, in some panels, will require an increase of as much as 25% in sectional area of the chord members and diagonals. In shorter spans, this effect, of course, will be less, but it will not decrease proportionally. This has a pertinent bearing on the design of continuous trusses.

---

[45] In Mr. Modjeski's report, the theory is given in Appendix B. In the Ninth Edition, 1911, of "Modern Framed Structures," by J. B. Johnson, C. B. Bryan, and F. E. Turneaure, it can be found in the Chapter on "Suspension Bridges" under "Exact Methods of Calculation."

[46] "Modern Framed Structures," by J. B. Johnson, C. W. Bryan, and F. E. Turneaure, Ninth Edition, Pt. 2, John Wiley & Sons, 1911, p. 283.

In the case of trusses hinged at the towers, the chord sections in the panels nearer the ends commonly over-run and the excess of section is in most instances sufficient to take care of the hanger effect. The writer has followed the practice of designing the rockers and diagonals near the ends for an excess stress of one-third to one-half. In the case of continuous trusses the heaviest chord stresses are at the towers, and an excess of section, due to over-run, will scarcely be available. The effect of suspender elongation, because of the stiffness of the truss due to continuity and heavy chord sections, will here be much more conspicuous and will require still larger sections. Much of the theoretical saving may be nullified by it. It will be found desirable to study the effect of suspender elongation and tower shortening before reaching conclusions concerning continuous stiffening trusses.

In considering the behavior of continuous trusses, it should not be forgotten that the trusses are suspended from the cables for their dead load and that they must follow the deflections of the cable. The cable curve does not reverse its direction near the towers; the continuous truss, on the contrary, tends to do so. Such behavior must result in straining the truss unduly.

Another instance in which the assumptions of the deflection theory for the case of hinged trusses are sufficiently close is the use of a constant equivalent moment of inertia for each individual span. For discontinuous trusses, simple methods, based on various loadings, are used to compute the equivalent moment of inertia. These methods give close results. For continuous trusses the accurate determination of an equivalent moment of inertia is much more difficult and involved.

The writer does not wish to imply that practical problems will not be encountered in which continuity of the stiffening truss will be desirable. Small and light spans which commonly have an over-run of materials because of practical considerations may be given increased stiffness by continuity, and special conditions may make it advisable to resort to continuity in other instances.

To avoid misunderstandings, the writer calls attention to the fact that in the original publication of the deflection theory the values of the constants, C, represent lines or distances, while in the paper they represent moments, distance times H. The writer finds that the original notation is more helpful for the physical interpretation of the various terms.

For purposes of comparison the moment curves, as computed by the elastic and the deflection theories shown in Fig. 3 are instructive. They prove again how much in error the elastic theory is for suspension bridges. It is timely to point out that the elastic theory may well be abandoned for suspension bridges and that attempts to tie it to the deflection theory, or any other advanced analysis, are of an atavistic nature.

In conclusion, the writer wishes to express his appreciation of the fine work done by Mr. Steinman in extending the deflection theory for continuous stiffening trusses and multiple tied spans.

A. MITCHELL,[47] AND G. T. PARKIN,[48] JUNIORS, AM. SOC. C. E. (by letter).—A major contribution to the mathematical treatment of suspension bridges is contained in this paper. New developments in construction, together with the practical form of the author's method of designing, should bring about an increasing use of short-span highway suspension bridges of the continuous type.

Since "stresses producible by combinations of loadings cannot be found by adding algebraically the respective stresses producible by the component loadings" (see following Equation (9)), it is desirable to have general formulas for the load functions, *A*, *B*, and *G*. Equations of the suggested form are readily reducible to the special cases treated by the author by noting that e^{-cl} $(1 + d) = 1 - d$ and e^{cl} $(1 - d) = 1 + d$. Thus, for *G*:

$$G_{1z} = \frac{1 - d}{4\,d}\left\{p_k\ (e^{-ck} + e^{ck} - 2) - p_m\ (e^{-ck} + e^{ck} - e^{-c(l-z)} - e^{c(l-z)})\right.$$
$$\left. - p_z\ (e^{-c(l-z)} + e^{c(l-z)} - 2\ e^{cl})\right\} \ldots\ldots\ldots\ldots\ldots (104)$$

$$G_{2z} = \frac{1 + d}{4\,d}\left\{- p_k\ (e^{-ck} + e^{ck} - 2) + p_m\ (e^{-ck} + e^{ck} - e^{-c(l-z)} - e^{c(l-z)})\right.$$
$$\left. + p_z\ (e^{-c(l-z)} + e^{c(l-z)} - 2\ e^{-cl})\right\} \ldots\ldots\ldots\ldots\ldots (105)$$

$$G_{3z} = \frac{1}{2}\left\{- p_k\ (e^{-ck} + e^{ck} - 2) + p_m\ (e^{-ck} + e^{ck} - e^{-c(l-z)} - e^{c(l-z)})\right.$$
$$\left. + p_z\ (e^{-c(l-z)} + e^{c(l-z)})\right\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (106)$$

$$G_{1m} = \frac{1 + d}{4\,d}\left\{p_k\ (e^{-c(l+k)} + e^{-c(l-k)} - 2\ e^{-cl}) - p_m\ (e^{-c(l+k)} + e^{-c(l-k)}\right.$$
$$\left. - e^{-cz} - e^{cz}) - p_z\ (e^{-cz} + e^{cz} - 2)\right\} \ldots\ldots\ldots\ldots\ldots (107)$$

$$G_{2m} = \frac{1 - d}{4\,d}\left\{- p_k\ (e^{c(l-k)} + e^{c(l+k)} - 2\ e^{cl}) + p_m\ (e^{c(l-k)} + e^{c(l+k)}\right.$$
$$\left. - e^{cz} - e^{-cz}) + p_z\ (e^{-cz} + e^{cz} - 2)\right\} \ldots\ldots\ldots\ldots\ldots (108)$$

$$G_{3m} = \frac{1}{2}\left\{- p_k\ (e^{-ck} + e^{ck} - 2) + p_m\ (e^{-ck} + e^{ck})\right\} \ldots\ldots\ldots\ldots\ldots\ldots (109)$$

---

[47] Structural Designer, State Highway Dept., Santa Fé. N. Mex.

[48] Designer, State Highway Comm., Bridge Dept., Raleigh, N. C.

$$G_{1k} = \frac{1 + d}{4\,d} \left\{ p_k \left(e^{c(l-k)} + e^{-c(l-k)} - 2\,e^{-cl} \right) - p_m \left(e^{c(l-k)} + e^{-c(l-k)} - e^{cz} - e^{-cz} \right) - p_z \left(e^{cz} + e^{-cz} - 2 \right) \right\} \qquad (110)$$

$$G_{2k} = \frac{1 - d}{4\,d} \left\{ - p_k \left(e^{c(l-k)} + e^{-c(l-k)} - 2\,e^{cl} \right) + p_m \left(e^{c(l-k)} + e^{-c(l-k)} - e^{cz} - e^{-cz} \right) + p_z \left(e^{cz} + e^{-cz} - 2 \right) \right\} \qquad (111)$$

and,

$$G_{3k} = p_k \qquad (112)$$

in which, in addition to the notation of the paper, the subscripts, *z*, *m*, or *k*, are added to *p* and *G* to indicate the portion of the span to which they apply (see Fig. 1).

STERLING JOHNSTON,[49] M. AM. SOC. C. E. (by letter).—Although this paper, as well as other highly commendable papers on suspension bridges, bears witness to the present high standards of development, there still remains a question of fundamental importance that is rarely, and then only sketchily, treated in print, namely, that of establishing an acceptable, upper limit of flexibility to which a suspension bridge may be designed. The desire of the writer, therefore, is to call attention to the need of a more uniform standard of measurement of the elastic behavior of a suspension bridge; point out the bearing that this question may have on the author's analysis of "Designs I and II"; and lay proper emphasis on its economic importance.

In Mr. Steinman's analysis of the comparative rigidity and economy of the continuous and two-hinged types of stiffening trusses, the degree of rigidity is based on the maximum deflections at the centers of the main and side spans. While this method is commonly used for making similar comparisons, it may well be questioned as to whether such a gauge is comparable for a case of this kind, in which two types of stiffening trusses of widely different deflection characteristics are being compared. Although the maximum center deflections are necessary for establishing navigable clearances, they do not, in the case of the main span, correspond with the deflections for maximum bending moment or for maximum change of grade.

Another unit of measurement, commonly used for such comparisons, is the percentage of change of grade under the live loads placed in the position or positions, for maximum change; which simply means the change, in degrees, in the slope of the roadway due to the influence of the live load. This is the more preferable gauge which should be used in the comparisons made by the author and, in fact, should be more generally adopted for all such comparisons. If it is used as a basis of comparison for "Designs I and II," the degree of rigidity of each design should be based on the relative changes in the slopes of the main and side-span stiffening trusses at, or near, the towers rather than on the maximum deflections at the centers of the main and side spans.

---

[49] Engr., McClintic-Marshall Corporation, Bethlehem, Pa.

The determination of the limiting unit of flexibility to which a suspension bridge may be designed is fully as important, both economically and practically, as the determination of the live loads to be carried, or the unit stresses to be used. What then is the upper limit of flexibility to which a suspension bridge may be designed, and how is it to be measured?

In considering this "unit of flexibility," care should be used to distinguish between the unit itself, and the functions of the unit. To illustrate: The unit working stress for a given steel is a function of its physical properties, factor of safety, shape, etc. Similarly, the unit that measures the degree of flexibility of a suspension bridge, is a function of the sag ratio of the cable; the depth ratio of the stiffening truss; the ratio of live to dead load; and possibly temperature changes. (The writer is inclined to believe that "temperature changes" should be considered a function of the stresses only.) While all these "functions" are necessary factors in the analysis of the deflection problem, no single one of them can be used as a gauge for a well-balanced design. They are simply a means to an end, and while the problem of how to keep the deflections within the prescribed limits is an all-important one, its consideration is beyond the scope of this discussion. The writer is primarily concerned with the unit itself, and more especially with the magnitude of the unit. In other words, the writer's query is not a question of how to stiffen a suspension bridge, but how much to stiffen it because, if "percentage of change of grade" is accepted as the basic unit with which to measure the degree of flexibility of a suspension bridge, the question naturally arises as to what this "percentage" should be.

While these two questions of how to stiffen a suspension bridge, and how much to stiffen it, are closely related, they are, nevertheless, independent questions, each of which must be approached from an entirely different angle. The question of "how" involves a highly technical analysis of the component parts of the structure, whereas the question of "how much" depends almost entirely on the nature and demands of the traffic for which the bridge is to be designed. The connecting link between "how" and "how much" is found in the effect of the dead weight of the suspended structure on its own stiffness. The technical analysis, for example, is based largely on the dead load as a controlling factor. Similarly, the degree of flexibility is a function of the dead load, the stiffness of the stiffening truss, and the sag ratio of the cable.

The practice of utilizing the dead load as a stiffening factor is of comparatively recent origin, and its economic importance is probably not yet fully appreciated. For spans of moderate length, the degree of stiffness is largely under the control of the stiffening trusses. For spans of great length, however, the degree of stiffness, incidental to the dead load alone of a given type of construction, may be such as to more than satisfy the criteria set up as the permissible flexibility. In such a case, a lighter, cheaper, and more flexible type of construction may be justified; provided, of course, that the deflections are kept within the prescribed limits.

If, on the other hand, the degree of stiffness, incidental to the dead load alone, is inadequate; then special provision must be made for the additional stiffness required. Any such provision necessarily adds to the cost. In fact, the stiffness and cost of a suspension bridge are very closely related; other factors being comparable, the more rigid the bridge is, the more costly it will be. It follows, therefore, that both the degree of stiffness and its unit of measurement are factors of fundamental economic importance.

The writer has reached the conclusion that the required degree of stiffness for any bridge depends almost entirely upon the nature and demands of the traffic for which the bridge is to be designed. A suspension foot-bridge, for example, may be so flexible as to cause alarm to some of its passengers, and yet be perfectly safe and serviceable. On the other hand, the deflections of a modern railway bridge must be held within such narrow limits as generally to exclude the suspension type from serious consideration when placed in competition with the arch or cantilever types.

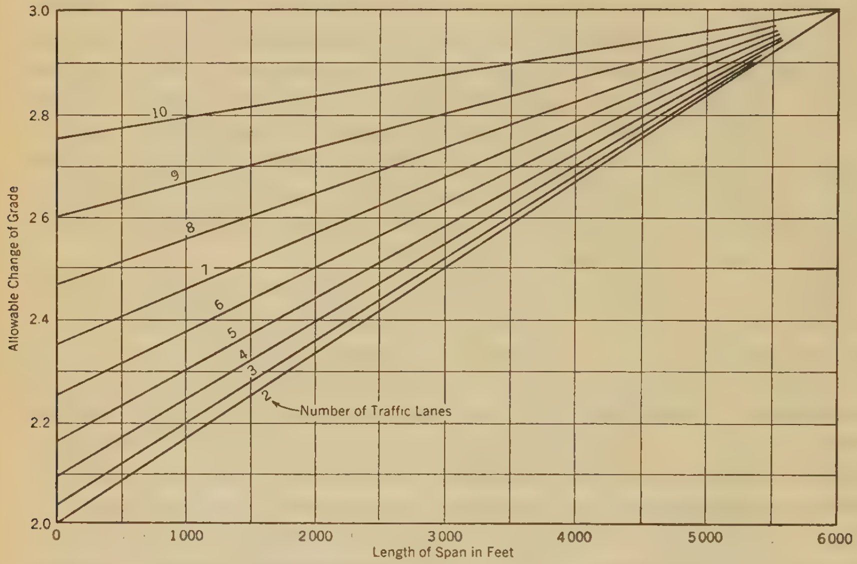


FIG. 13.—ALLOWABLE CHANGE OF GRADE FOR HIGHWAY SUSPENSION BRIDGES (FOR LIVE LOAD ONLY)

The degree of stiffness required of a highway bridge, like that of the railway type, must be based on the nature and demands of the traffic for which the bridge is to be designed. The nature of the traffic carried by these two types of bridges, of course, is radically different. In the case of a single-track railway suspension bridge, for example, the entire moving load for which the bridge is designed comes on to the span at one time, at high speed, and with heavy impact. In the case of a large highway suspension bridge, on the other hand, the total live load for which the bridge is designed is seldom applied and even if it is, the application, as a whole, is so gradual as to cause practically no impact. Moreover, the positions of the congested live loads which produce the maximum changes of grade are such as may never occur during the life of the bridge. Therefore, while the principles of design are the same for both railway and highway types, the difference in the nature of the traffic permits of a much wider range in the computed changes of grade for the highway type.

Fig. 13 is submitted as a possible answer to the question that has been raised, as to what constitutes a permissible upper limit of flexibility for highway suspension bridges. It is based on the generally accepted thesis that an increase in the length of span, or an increase in the number of traffic lanes, permits a corresponding increase in the computed change of grade. The chart includes permissible changes of grades for live load only. It does not include temperature changes. The total grade, including temperature changes and normal grade, however, should not exceed 5 per cent.

The author is to be congratulated for this contribution to the literature on suspension bridge design. His generalized deflection theory furnishes a much needed, rational analysis of the continuous stiffening truss that has been the riddle of this type of bridge for so many years.

Harold E. Wessman,[50] Assoc. M. Am. Soc. C. E. (by letter).—It is quite evident to any one who has studied this field that a vast amount of painstaking labor lies behind this paper. Mr. Steinman is to be highly congratulated on his new contribution to the theory of analysis of suspension structures.

After reading the literature of suspension bridges, one might conclude at first that the subject has been completely dissected and analyzed, and that nothing is left for future investigators; but when one recalls the notable contributions in recent years, it is apparent that the entire field is experiencing a critical review; that much thinking has been done recently; and, what is most important, much thinking and study still remain to be done.

Earlier concepts of the action of suspension bridges placed too much emphasis on a mistaken interpretation of the function of the stiffening truss. There was, and for that matter still is, a tendency to consider the stiffening truss as the major structural element, with the cable as a minor element added to assist the truss. This assignment of relative importance of the units was analogous to "placing the cart before the horse."

The major structural element in a suspension bridge is the cable. The stiffening truss is added to help the cable. If the cable does not need help, the stiffening truss may well be omitted, as was done on the George Washington Bridge across the Hudson River in New York City. Does the deflection theory apply in a case such as this? Perhaps. Even if the machinery of analysis may be the same, regardless of how one classifies cable and truss, it is important that engineers approach this problem with a rational perspective, a correct philosophy. It is well to keep in mind the valuable discussion[51] by O. H. Ammann, M. Am. Soc. C. E., of the paper entitled, "Costs of Suspension Bridges," by J. A. L. Waddell, M. Am. Soc. C. E.

---

[50] Assoc. Prof. of Mechanics and Structural Eng., Univ. of Iowa, Iowa City, Iowa.

[51] *Transactions*, Am. Soc. C. E., Vol. 91 (1927), p. 932.

With reference to the author's theory, one minor point is worth noting. His basic differential equation for the deflection, $\frac{d^2\eta}{dx^2} = -\frac{M}{EI}$, is based upon beam deflections due to flexure only. Shear deflection is ignored. The basic differential equation for beam deflection is sometimes given in the following form,

$$\frac{\partial^2\eta}{\partial x^2} = -\frac{\partial\epsilon_x}{\partial y} + \frac{\partial\gamma_{xy}}{\partial x} \dots\dots\dots\dots\dots\dots(113)$$

in which, the second term represents shear effects. The first term leads to $\frac{M}{EI}$ when Poisson's ratio is ignored. Ordinarily, shear deflection in a beam is so small, relative to that caused by moment, that it is of no consequence. In a truss, however, the deformation of the web system, which is a measure of shear deflection, may be considerable for part span loadings.

Even if the shear term is omitted in the differential equation, however, the method of selecting an equivalent moment of inertia for the stiffening truss will offset error due to its omission. If the deformation of the web members is taken into account in determining truss deflection, and then an equivalent beam of uniform moment of inertia is selected that will give this same deflection, shear effect is accounted for to a certain extent. The author's viewpoint in this matter would be of some interest and value.

C. H. Gronquist,[52] Assoc. M. Am. Soc. C. E. (by letter).—The deflection theory presented in this paper, for distributed loading, can be modified to include the effect of any number of concentrated loads in any span simply by adding to the general load functions, A, B_1, and B_2, the new terms for these concentrated loads. The expressions for M_o and $R_{L,R}$ will also naturally be augmented by terms contributed by the concentrated loads. Since $\frac{d^2M_o}{dx^2} = 0$ for concentrated loads, there will be no contributions to the G-functions. Thus, there will be found only one set of integration constants, C_1, C_2, for each span subjected only to concentrated loads.

The contributions of the concentrated loads to B_1 in each span will be the sum of the simple beam reactions produced by these loads. The contributions to B_2 in each span will be the difference of the reactions. In the side spans only the reaction at the tower is considered. The contributions in the several spans are:

In the Main Span:

$$B_1 = R_L + R_R = \Sigma\, P \dots\dots\dots\dots\dots\dots(114)$$

and,

$$B_2 = R_L - R_R = \Sigma\, P\,\frac{(l-2\,k)}{l} \dots\dots\dots\dots\dots(115)$$

---

[52] With Robinson & Steinman, Cons. Engrs., New York, N. Y.

In the Left Side Span:

$$B_1 = B_2 = R_B = \Sigma\, P_1 \frac{k_1}{l_1} \ldots\ldots\ldots\ldots\ldots\ldots (116)$$

and, in the Right Side Span:

$$B_1 = -B_2 = R_B = \Sigma\, P_2 \frac{k_2}{l_2} \ldots\ldots\ldots\ldots\ldots\ldots (117)$$

In Equations (114) to (117), k, k_1, and k_2, are the distances to the concentrated loads, P, P_1, and P_2, from the left end of the main span and the free end of the side spans.

The contributions of the concentrations to A in each span are:

$$A = \int_0^l M_0\, dx = \Sigma\, P \frac{k}{2} (l - k) \ldots\ldots\ldots\ldots (118)$$

The contributions to B_1 do not enter into the expression for the contribution to A for concentrated loads.

The formulas for the load functions given in Equations (114) to (118) may be applied without modification to unsymmetrical three-span and multiple-span bridges as well as to symmetrical three-span bridges with or without continuity. Several formulas of the paper for the case of spans fully loaded or unloaded would be modified, however, in accordance with the contributions due to concentrated loading.

To obtain the effect of concentrated loads without any distributed loading, it is necessary merely to make p in the formulas of the paper equal to zero. The total values of the load functions, A, B_1, and B_2, are then those of the foregoing contributions of the concentrated loads.

W. R. Frederick, Jr.,[53] Jun. Am. Soc. C. E. (by letter).—The simplified working formulas presented in this paper are a great improvement over those heretofore found in the literature on the subject and, with the generalization of the theory, provide a useful tool for the engineer.

It is claimed in the paper that a saving of 5% in truss chord material can be effected by using a continuous truss (Design II) rather than a hinged truss. This may be true when the designs are developed in full detail and the truss depth is adjusted. However, Design I affords greater economy in truss chord material than Design II. This phase will be discussed subsequently. It is interesting to note that the 5% saving claimed would amount to about 0.25% of the total cost of constructing a suspension bridge of the assumed dimensions. The value of this paper lies not in the saving of truss chord material, but rather in the dependability of the results obtained by its use and in the relatively simple form in which it is presented.

This discussion deals directly with the three-span, symmetrical suspension bridge, but much of its content can be extended to include any type of suspension bridge.

The Measure of Rigidity.—The stiffening trusses are used for the purpose of reducing deflections, thereby providing a facility for, and inspiring a sense of security in, the user and insuring for him the least of maximum roadway grades that is practical. Determination of the desired stiffness of the trusses and the measure of this stiffness should be based on this fact. The writer will attempt to show that, in a structure of the approximate dimensions assumed for the numerical example in the paper, the vertical deflections are practically meaningless to an individual using or viewing the bridge and that only the angular deflections are appreciable to his senses and thus affect his comfort and feeling of security.

---

[53] Park Engr., Long Island State Park Comm., Forest Hills, N. Y.

Vertical Deflection.—The maximum vertical live load deflections are caused by full span loads, and the length of time required for them to occur under most favorable conditions is between 10 and 20 sec. An individual standing in the middle of a span would be dropped a distance of about 3 ft in this length of time. Consideration of elevator speeds and the nature of the acceleration will show this movement to be insensible. A car moving across the bridge with a large group of other vehicles placed so as to occasion maximum deflection at the center of a span, would traverse a continuous curve varying slightly from the roadway grade of the unloaded bridge. The variation of the vertical deflection, or of the curve from the grade, cannot be detected by any normal human sense.

The contribution of temperature to the vertical deflection need not be considered. It occurs slowly and depends less on the moment of inertia of the stiffening truss than the live load deflection. The vertical deflection does affect the structure in that, in order to maintain the minimum vertical clearance when fully deflected, it may be necessary to raise the theoretical profile. The expense involved is slight.

Angular Deflection.—Angular deflections, especially breaks in continuity, are visible and, if not unsightly, at least, may indicate weakness to the beholder. Furthermore, the angular deflection increases the grade for some vehicles using the structure. For others, the grade will be relieved. The conclusion is that vertical deflections are not satisfactory for measuring the rigidity and that angular deflections supply the most useful yardstick for this purpose. Designers may prefer to follow an intermediate course and use both vertical and angular deflections in measuring rigidity.

Deflections of Continuous and Hinged Stiffening Trusses.—Suspension bridge trusses should be designed of such stiffness that the angular deflection that increases the grade near the tower is under a specified maximum allowable deflection (or what amounts to the same thing), that the maximum grade along the deflected structure does not exceed a definite amount. This general rule should be modified by judgment and experience since extremely flexible trusses may result from assuming a high value for the allowable grade increase (Fig. 14). The maximum angular deflection at the approach end of the side span does not, for the usual structure, increase the roadway grade. However, the abrupt change of grade due to this deflection should be considered.

The points at which the grade increase results in the maximum grade for a vehicle to climb, are near the towers. The angular changes in the trusses may be greater at other points, but the resulting grades are not likely to be maximum since in most suspension bridges there is a camber of several feet in the main span and the side spans are on a grade up to the center span.

If the principle of limiting the maximum grade of the deflected structure is applied to the hinged truss, the controlling angular deflection is likely to be at the tower end of the side span. If the side spans are shorter than one-half the main span this may not be true. Since the deflection curve for a hinged truss is broken at the towers while the curve for a hingeless truss is continuous, the hingeless truss has an immediate advantage in appearance if the angular change allowed is the same for both types of trusses.

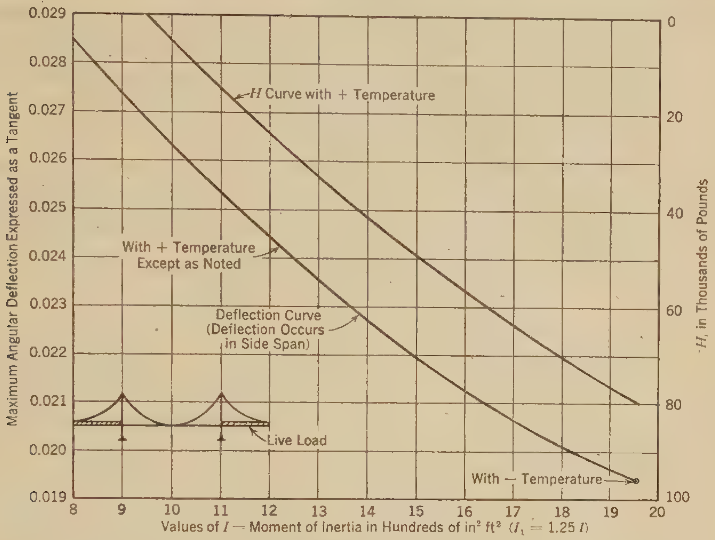


FIG. 14.

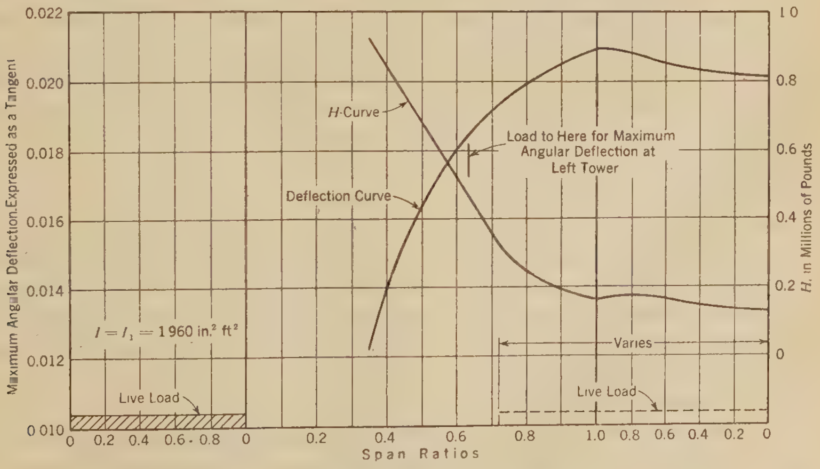


FIG. 15.

In attempting to reduce angular deflections to a practical minimum it will be well to keep in mind that the reduction of side span relative to center span will accomplish much by shifting the point where the maximum grade occurs away from the side span toward, and even on to, the main span. Fig. 15 showing the angular deflection plotted against the load position will demonstrate the argument. The point noted on the curve as the slope change at the left tower is much less than the maximum slope change which occurs in the side span, 52.5 ft away from the tower under a different loading. Shorter side spans also straighten the side cables and thus have the secondary effect of stiffening the structure.

Table 6 shows the maximum deflections for four structures. The maximum vertical deflection at the center of the side and main spans of Design III has been computed and is shown with the same deflections for the hinged design and Designs I and II as given by the author.

TABLE 6.—MAXIMUM DEFLECTIONS

Design	ANGULAR DEFLECTION, EXPRESSED AS A TANGENT		VERTICAL DEFLECTION AT CENTER LINE, IN FEET	
	Location	Amount	Side span	Center span
Hinged....	Side span at tower	0.0268	3.30	5.16
I........	Side span, 22.3 ft from tower	0.0194	2.81	5.28
II........	Side span, 52.5 ft from tower	0.0211	3.10	5.44
III........	Side span, 48 ft from tower	0.0227	3.31	5.66

The dimensional and loading constants for Design III are identical with those for Design I and Design II, except that $I = 1\,400$ in.^{2} ft^2 and $I_1 = 1\,750$ in.^{2} ft^2, and the depth of the truss is undetermined. The truss is continuous at the towers. It was not considered practical for Design III to use a truss that would allow the same angular deflection as the hinged truss (Table 6). The resulting moments of inertia would have been $I = 950$ in.^{2} ft^2 and $I_1 = 1\,190$ in.^{2} ft^2 (Fig. 14).

The maximum angular deflection (Table 6) occurs for the hinged truss with one side span fully loaded, with positive temperature, and with no load on the other two spans, and for Design II and Design III with both side spans fully loaded, with positive temperature, and with no load in the main span. For Design I it occurs under the same live loading as Design II and with negative temperature.

The maximum vertical deflection at the center of the side span (Table 6) for the hinged truss occurs with the side span fully loaded, with positive temperature, and with no load on any other span, and for Designs I, II, and III under the same loading and temperature as obtains for the maximum angular deflection of Designs II and III. The maximum vertical deflection in the center span occurs for all designs with that span fully loaded and with positive temperature.

The combination of heavy live loading, the exact placement of the live loading, and the temperature for which the deflections in Table 6 are computed, is not likely to occur. The probable actual deflections of the truss are much smaller than those shown in Table 6.

In arriving at the values for I and I_1 of Design III, I_1 was assumed to be about 25% greater than I. This is roughly in agreement with the ratio of average moments in the side spans to those in the main span as taken from Figs. 3 and 5 of the paper, and it enables the economical use of the same depth of truss for the side and for the center spans. In Design II the author has chosen $I = I_1$, and of such a value that the percentage deflection decrease at the center of the side span and the percentage deflection increase at the center of the main span relative to the same deflections of the hinged design are nearly equal. Granting that vertical deflections measure the rigidity, the result obtained in Design II is pointless unless the depth of truss in the main span is increased over that in the side span. A design of sections to obtain the condition, $I = I_1$, to maintain a constant truss depth throughout, and to carry the moments at stresses equal to, or less than, the allowable unit stress, would involve a considerable waste of metal in the chords of the main span truss.

Fig. 15 shows the variation with respect to load of the maximum numerical values of the negative angular deflection, expressed as a tangent, near the left tower of the continuous truss of Design II. The changing load consists of live loading extending from the right end of the structure. The left end of this load retreats from a position at 0.35 l of the main span until no load remains on the right. The full load on the left span does not vary. The temperature used in computing values along the curve is negative. The H-curve for the same loading conditions is shown. The position of the load for maximum angular change at the left tower is marked and noted. When the load extends beyond this point to the left the maximum angular change is in the main span and when all the load lies to the right of this point, except the left span load, the maximum angular change occurs in the side span. The greatest angular deflection occurs, however, when the temperature is positive and the value shown in Table 6 is this maximum.

Comparison of Designs.—Table 7 shows the relation between the moments of inertia, moments, truss depths, and chord material in the side and main spans of four designs. The data presented are intended to serve as a guide in determining relative economy. The moments shown are the averages of the maximum moments in each span for each design and the moments of inertia are those used in computing the moments. The theoretical truss sections determined are used to compute new moments of inertia which are shown in Column (7). The theoretical weight of chord material for a complete structure is shown in Column (9).

If carbon and high-strength alloy steels are used in a truss of any single span, the preparation of data such as those presented in Table 7 is considerably complicated, since a number of sections will have to be designed to determine the depth that will carry the imposed moments and supply the average moment of inertia used in computing the maximum moments.

A choice between the designs based on vertical deflection is difficult. The angular deflections (Table 6) show Design I to be definitely superior to the other designs. This is in agreement with the relative economy as far as this is affected by the chord area (Table 7). Design I requires 21% less weight in the chord sections than its nearest competitor, Design III.

To clarify the relation of the various factors involved in the design of stiffening trusses a formula is developed, as follows: Assume a hypothetical truss having a moment of inertia, I; depth, q; chord areas, A_t, each, and carrying a moment, M, at a chord stress of σ; all in appropriate units. Then, $I = \frac{1}{2} A_t q^2$ and $A_t = \frac{M}{\sigma q}$. If all factors are varied as indicated by subscripts, except σ, the following equation can be written showing the relation between the old and new chord areas:

TABLE 7.—Relation Between Moments of Inertia, Moments, Truss Depths, and Chord Material

(Unit stresses, in pounds per square inch; tension, 27 000 on net section; and compression, $27\,000 - 100 \frac{l}{r}$, with a maximum of 23 000.)

Design (1)	Span (2)	Moment of inertia I, in inches² feet² (3)	Average of maximum moments, in foot-kips (4)	Depth, d, in feet (5)	Total theoretical area, top and bottom chords, in square inches (6)	Moment of inertia, I, for section designed, in inches² feet² (7)	Chord weight per foot of one truss, in pounds (8)	Total weight of chords of two trusses, three spans, in pounds (9)
Hinged	Side	2 420	10 475	10.4	89.5	2 420	304	702 000
	Main	1 960	6 375	14.0	39.8	1 944	135	
I	Side	2 420	8 850	12.7	60.1	2 420	204	523 000
	Main	1 960	6 083	14.7	36.1	1 945	122.5	
II	Side	1 960	8 763	10.1	76.7	1 957	261	692 000
	Main	1 960	7 315	12.5	50.3	1 962	171	
III	Side	1 750	8 203	9.7	74.8	1 759	254	661 000
	Main	1 400	5 942	10.9	46.9	1 387	159.5	

$$A_{t1} = A_t \frac{I}{I_1} \left(\frac{M_1}{M} \right)^2 \qquad (119)$$

The conditions assumed for developing Equation (119) do not exactly fit those encountered in practice, but they do lead to a clear presentation of the effect of the moment and moment of inertia on the chord areas of a truss. The economy in truss web members, structural and design details, and handling is not represented in Equation (119). The chord areas of all the trusses listed in Table 7 can be checked one against another with this formula. The results agree closely.

If the depth of the hypothetical truss and the unit stress in the chords are kept constant, it can be demonstrated that the moments of inertia vary directly with the moments. Thus, in a constant depth stiffening (suspension bridge) truss, the average moment and the average moment of inertia occur at the same point, in theory at least.

Fig. 16 shows the curves of maximum moments for Design III and the hinged design. The loading positions for maximum moments as given by the author proved to be correct for Design III, which design effects a 16% saving in total moment area when compared to the hinged design. The saving in truss chord material cannot be predicted from the saving in moment area. The average of the side-span moments is 39% greater than the average of the main span moments. The assumed relation, $I_1 = 1.25\ I$, is checked fairly well by the computed depths of the main and side span trusses (Table 7).

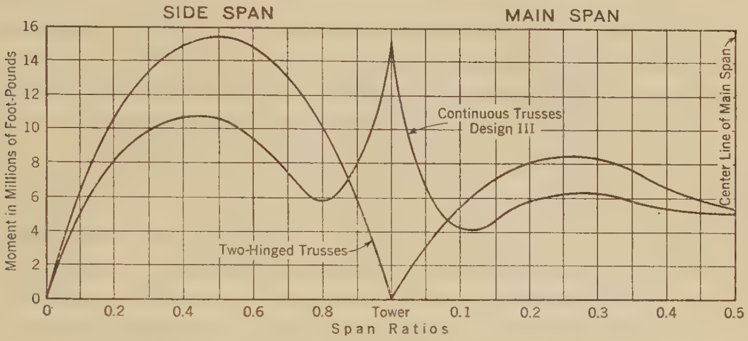


FIG. 16.—COMPARISON OF MAXIMUM MOMENTS FOR LIVE LOAD AND TEMPERATURE IN CONTINUOUS AND TWO-HINGED STIFFENING TRUSSES.

The average of maximum moments can be determined by computing the maximum moments at the points along the truss shown in Table 8. These points are practically identical for the three continuous designs.

TABLE 8.—SHOWING LOCATIONS ALONG TRUSS WHERE THE AVERAGE OF MAXIMUM MOMENTS (SHOWN IN TABLE 7) OCCUR. POINTS ALONG THE MAIN SPAN ARE SYMMETRICAL ABOUT THE CENTER LINE AND ONLY ONE-HALF ARE SHOWN.

Design	Span	Distance from left end
Hinged	Left side Main	$0.203l_1$ and $0.790l_1$ $0.124l$ and $0.416l$
I	Left side Main	$0.205l_1$, $0.695l_1$, and $0.915l_1$ $0.060l$, $0.205l$, and $0.3475l$
II	Left side Main	$0.215l_1$, $0.655l_1$, and $0.900l_1$ $0.0575l$, $0.210l$, and $0.3775l$
III	Left side Main	$0.205l_1$, $0.665l_1$, and $0.905l_1$ $0.0575l$, $0.2175l$, and $0.340l$

Column (6), in Table 7, is determined by assuming sections having the approximate areas required, calculating the allowable stress in the compression chord, determining the ratio of gross to net area for the tension chord, and using these values for determining the gross chord area shown. This is called a theoretical area because the detail of the section, when determined, may provide excess area. It may be possible to improve any of the designs by changing the moments of inertia so as to obtain more nearly equal depths in the side and main spans.

The author's comparison between hinged and continuous stiffening trusses is misleading since he takes no account of the effect on truss depth of the moment of inertia and the corresponding maximum moments. Economy in truss chord material depends on a number of factors and not solely or directly on the saving in moment area of one design over another. The data in Table 7 support this conclusion.

The truss properties for a structure afford maximum economy consistent with the desired rigidity and satisfactory details can be selected from a tabulation similar to Table 7 and a curve such as is shown in Fig. 14. For example, varying moments of inertia can be assumed and a number of designs similar to Design I and Design III prepared by computing the maximum monents at 0.21 l and 0.34 l of the main span and 0.205 l_1 and 0.67 l_1 of the side span and considering these to be the average moments in their respective spans (Table 8) for the purpose of determining relative economy.

William H. Yates,[54] M. Am. Soc. C. E. (by letter).—This paper is a welcome addition to a neglected subject—the design of suspension bridges with spans of less than 1 000 ft. The Rondout Bridge, at Kingston, N. Y., is cited by the author as a type of structure (with continuous stiffening trusses) for which, until now, the deflection theory was not applicable. When it was designed in 1920 a popular impression that suspension bridges deflected excessively, had to be overcome. Consequently, a design was proposed that would have only one-half the deflection customary at that time. This was done by making the stiffening trusses continuous between anchorages instead of designing the usual type with a hinge at each tower. In fact, the Kingston Bridge is an unusually rigid structure, due not only to the continuous stiffening trusses, but also to the heavy reinforced concrete deck which made the dead load stresses about double the live load stresses. With a lighter deck the reverse would be true. The width of this deck and its location about midway between the upper and lower chords of the stiffening truss permitted an independent horizontal wind truss about 35 ft in width or depth. This is about one-twentieth of the length of the channel span, which is 705 ft long.

This span is generally regarded as too short for the economical use of the suspension type. Since its completion in 1922 several other bridges of that type have been built with spans of about the same length. The Kingston Bridge, therefore, was a pioneer among short-span suspension bridges designed for modern heavy highway loading.

Many variations of live load were assumed in the design of the chords and web members of the continuous stiffening trusses to determine the maximum moments and shears. The formulas made available in this paper would have reduced the number of computations greatly.

D. B. Steinman,[55] M. Am. Soc. C. E. (by letter).—For the many stimulating and informative discussions that have been contributed, the writer is sincerely grateful. He is glad that his paper has been the means of evoking so valuable a symposium, enriching the science and art of suspension bridge design. The individual discussions are here reviewed in the order of their publication.

---

[54] Civ. Engr., New York, N. Y.

[55] Cons. Engr. (Robinson & Steinman), New York, N. Y.

Mr. Pavlo presents in convenient form, separate expressions representing the contribution of the tower moments. The addition of these expressions to the respective formulas of the deflection theory for the two-hinged truss yields the more complete formulas for the continuous stiffening truss presented by the writer. This synthetic procedure is only the reverse of the writer's analytical presentation.

The writer's method of obtaining the equivalent or effective moment of inertia of the continuous truss in any span is that of simply multiplying the average chord area by one-half the square of the truss depth. Investigation for several typical designs has indicated, contrary to the opinion of Mr. Pavlo, that this procedure is the most practical method and is sufficiently accurate. Computation of equivalent moments of inertia by comparing unsuspended truss deflections, as recommended by Mr. Pavlo, yields widely discrepant results for some cases of loading; moreover, the results cannot be accepted as correct, since the method does not take into account the effects of the variable suspender loads and of the dead load on the truss deflections.

An accepted scientific measure of rigidity of suspension bridge spans is lacking. The writer agrees with Mr. Jones that change of gradient is a better criterion of comparative rigidity of two designs than mere comparison of maximum or average deflections. The latter procedure, used in the paper for obvious reasons of convenience, familiarity, and expedition, erred on the side of under-statement of the advantages of the continuous type.

The results of the preferred criterion, by comparison of maximum grade changes under live load and temperature, applied to the numerical design examples of this paper, are recorded in Table 9. For the two designs of equal economy, this table shows that the continuous truss is more rigid than the two-hinged type by 40% in the main span and by 10% in the side spans. If only increase in gradient is considered, the rigidity advantages are 6% in the main span and 36% in the side spans. The claim of 6% superior rigidity made in the original paper, therefore, was conservative.

For Design II, representing "equal rigidity" by the method of average deflections, the tabulated values of maximum grade changes show that the continuous truss is more rigid than the two-hinged type by 31% in the main span and by 0% in the side spans. If only increase in gradient is considered, the rigidity is found to be 1% less for the continuous type in the main and 24% greater in the side spans. Hence Design II, which was found to be 5% more economical than the two-hinged design, is also from − 1% to + 31% more rigid in the main span, and from 0% to 24% more rigid in the side spans. Calling it a design of "equal rigidity" erred on the conservative side.

If the average of maximum increase and maximum decrease of gradients for each span is used, the hingeless truss shows 23% superior rigidity in the main span and 22% superior rigidity in the side spans for Design I with equal economy; and Design II, with 5% superior economy, shows 16% superior rigidity in the main span and 11% superior rigidity in the side spans as compared with the two-hinged design. These findings of superior advantages of the continuous type may be summarized in an approximate empirical formula for suspension bridges of 800-ft main span:

$$\text{Gain in Rigidity} + 2\ (\text{Gain in Economy}) = 23\% \ldots\ldots (120)$$

The numerical percentage in this formula, representing combined advantages of the continuous type, will be less for longer spans and greater for bridges of shorter span.

From the viewpoint of ruling gradient, the most significant flexibility characteristic of a suspension bridge is increase of grade in the side spans. Table 9 shows that the superior resistance to increase of gradient in the side spans of the continuous truss, as compared with the two-hinged truss, is marked, amounting to 36% for Design I and to 24% for Design II. The corresponding deflections as computed by Mr. Frederick yield rigidity gains of 38% and 27%, respectively.

Professor Müllenhoff traces the origin of the deflection theory for suspension bridges to Müller-Breslau in a work published in 1881, seven years before its publication in more developed form by Melan. He inquires as to the possibility of establishing coefficients for reducing the values of *H* and *M* given by the elastic theory in order to obtain preliminary design values corresponding to the deflection theory. For *M* and *V* in two-hinged stiffening trusses, this has already been done.[56] Similarly, reduction coefficients might be evaluated empirically and recorded for continuous suspension bridges, if such procedure is deemed desirable.

TABLE 9.—COMPARISON OF DEFLECTION GRADIENTS

Description	TWO-HINGED TRUSSES	CONTINUOUS TRUSSES	
	Design I	Design I	Design II
Maximum Increase in Grade (Live Load Plus Temperature):			
Main span	2.00%	1.89%	2.02%
Side spans	2.70%	1.98%	2.17%
Maximum Decrease in Grade (Live Load Plus Temperature):			
Main span	2.80%	2.00%	2.13%
Side spans	2.70%	2.44%	2.69%
Grade Change from Temperature Alone:			
Main span	0.49%	0.18%	0.20%
Side spans	0.13%	0.10%	0.11%

To expedite the application of the deflection theory, tables giving stresses and deflections for spans of various lengths and constants have been developed, but such design tables have not yet been extended to cover continuous and multiple spans.

As a contribution toward facilitating the numerical application of the generalized deflection theory, particularly for continuous suspension bridges, the writer submits Table 10, which gives values of the constants, d, $\frac{L_c}{l}$, h, $\frac{8\,h}{c^2\,l^2}$, al, and $-bl$ for values of cl increasing from 0.1 to 10.0. Most of these constants (with or without the factor, l) occur repeatedly in the working formulas of the paper, and Table 10 expedites their evaluation. The numerical values in Table 10 will also facilitate the setting up of approximate empirical relations.

---

[56] "Suspension Bridges," by D. B. Steinman, M. Am. Soc. C. E., John Wiley & Sons, New York, 1929, pp. 276–282.

The function, $\frac{8\,h}{c^2\,l^2}$, in Column (5) Table 10, offers interesting possibilities as a reduction factor for obtaining deflection theory moments and deflections from values given by the elastic theory. Let M_E and M_D denote mid-span bending moments for full-span loading (in any span), computed by the elastic and deflection theories, respectively. Then, upon substitution of the familiar expressions for M_E, Equations (32) yield:

$$M_D = \frac{8\,h}{c^2\,l^2}\,M_E \qquad (121)$$

Similarly, Equations (33) yield the following convenient formulas for mid-span deflections under full-span loading (in any span):

$$\Delta f = \frac{M_E - M_D}{E\,I\,c^2} \qquad (122a)$$

and,

$$\Delta f = \frac{M_E}{E\,I\,c^2}\left(1 - \frac{8\,h}{c^2\,l^2}\right) \qquad (122b)$$

Equations (121) and (122) are exact for non-continuous spans and closely approximate for continuous spans. The reduction factor, $\frac{8\,h}{c^2\,l^2}$, is conveniently furnished, for varying values of cl, by Table 10. The following approximate empirical relations, deduced from Table 10, may also be convenient:

$$\frac{8\,h}{c^2\,l^2} = 1.1 - h = 0.9 - \frac{L_c}{l} \qquad (123)$$

Hence, Equations (121) and (122) may be written approximately:

$$M_D = (1.1 - h)\,M_E \qquad (124)$$

and,

$$\Delta f = \frac{M_E}{H_w + H}\,(h - 0.1) \qquad (125)$$

Mr. Booth properly emphasizes the need for a specification of maximum allowable deflections for suspension bridges of varying spans and for different classes of traffic. Mr. Johnston makes a valuable pioneer contribution of such a specification for span rigidity. For some time the writer has been engaged

TABLE 10.—VALUES OF CONSTANTS

cl (1)	d (2)	$\frac{L_e}{l}$ (3)	h (4)	$\frac{8h}{c^2l^2}$ (5)	al (6)	$-bl$ (7)	cl (1)	d (2)	$\frac{L_e}{l}$ (3)	h (4)	$\frac{8h}{c^2l^2}$ (5)	al (6)	$-bl$ (7)
0.1	0.04996	0.0008	0.00125	1.000	0.00660	0.00340	5.1	0.98788	0.61260	0.845	0.2598	8.20076	1.87562
0.2	0.09967	0.0033	0.00498	0.996	0.02655	0.01331	5.2	0.98903	0.61960	0.852	0.2522	8.40064	1.88528
0.3	0.14889	0.0074	0.01114	0.9911	0.05958	0.02976	5.3	0.99007	0.62639	0.859	0.2448	8.60053	1.89421
0.4	0.19738	0.0131	0.01967	0.9835	0.10550	0.05240	5.4	0.99101	0.63296	0.866	0.2376	8.80044	1.90246
0.5	0.24492	0.02032	0.03046	0.9747	0.16394	0.08098	5.5	0.99186	0.63932	0.873	0.2308	9.00037	1.91009
0.6	0.29131	0.02897	0.04337	0.9638	0.23445	0.11513	5.6	0.99263	0.64549	0.879	0.2242	9.20030	1.91716
0.7	0.33638	0.03891	0.05827	0.9513	0.31645	0.15449	5.7	0.99333	0.65146	0.885	0.2178	9.40025	1.92371
0.8	0.37995	0.05012	0.07499	0.9374	0.40950	0.19842	5.8	0.99396	0.65726	0.890	0.2117	9.60021	1.92973
0.9	0.42190	0.06244	0.09336	0.92207	0.51292	0.24650	5.9	0.99454	0.66287	0.896	0.2058	9.80018	1.93540
1.0	0.46212	0.07576	0.11318	0.90544	0.62606	0.29818	6.0	0.99505	0.66832	0.901	0.2001	10.00015	1.94045
1.1	0.50052	0.08996	0.13427	0.88774	0.74828	0.35286	6.1	0.99552	0.67360	0.905	0.1947	10.20012	1.94522
1.2	0.53705	0.10492	0.15645	0.86917	0.87889	0.41003	6.2	0.99595	0.67873	0.910	0.1894	10.40010	1.94968
1.3	0.57167	0.12051	0.17952	0.84980	1.01721	0.46913	6.3	0.99633	0.68370	0.914	0.1843	10.60009	1.95367
1.4	0.60437	0.13661	0.20330	0.82980	1.16258	0.52966	6.4	0.99668	0.68854	0.919	0.1794	10.80007	1.95743
1.5	0.63515	0.15313	0.22761	0.80928	1.31438	0.59108	6.5	0.99700	0.69323	0.923	0.1747	11.00006	1.96094
1.6	0.66404	0.16995	0.25230	0.78844	1.47195	0.65297	6.6	0.99728	0.69779	0.926	0.1701	11.20005	1.96405
1.7	0.69107	0.18698	0.27721	0.76736	1.63477	0.71487	6.7	0.99754	0.70223	0.930	0.1657	11.40004	1.96700
1.8	0.71630	0.20411	0.30221	0.74620	1.80225	0.77643	6.8	0.99777	0.70654	0.933	0.1615	11.60004	1.96964
1.9	0.73978	0.22128	0.32715	0.72499	1.97391	0.83725	6.9	0.99799	0.71073	0.937	0.1574	11.80003	1.97223
2.0	0.76159	0.23841	0.35194	0.70388	2.14926	0.89710	7.0	0.99818	0.71481	0.940	0.1534	12.00002	1.97450
2.1	0.78181	0.25542	0.37648	0.68296	2.32787	0.95573	7.1	0.99835	0.71877	0.943	0.1496	12.20002	1.97656
2.2	0.80050	0.27227	0.40067	0.66226	2.50938	1.01282	7.2	0.99851	0.72264	0.945	0.1459	12.40021	1.97853
2.3	0.81775	0.28891	0.42443	0.64186	2.69343	1.06823	7.3	0.99865	0.72640	0.948	0.1423	12.60002	1.98028
2.4	0.83365	0.30529	0.44771	0.62182	2.87967	1.12185	7.4	0.99878	0.73006	0.951	0.1389	12.80001	1.98193
2.5	0.84828	0.32138	0.47045	0.60218	3.06784	1.17356	7.5	0.99889	0.73363	0.953	0.1355	13.00001	1.98335
2.6	0.86172	0.33714	0.49262	0.58298	3.25769	1.22325	7.6	0.99900	0.73711	0.955	0.1323	13.20001	1.98479
2.7	0.87405	0.35256	0.51416	0.56424	3.44901	1.27087	7.7	0.99909	0.74050	0.957	0.1292	13.40000	1.98598
2.8	0.88535	0.36761	0.53507	0.54599	3.64157	1.31639	7.8	0.99918	0.74380	0.959	0.1262	13.60000	1.98720
2.9	0.89569	0.38228	0.55532	0.52825	3.83523	1.35977	7.9	0.99926	0.74702	0.961	0.1233	13.80000	1.98830
3.0	0.90515	0.39657	0.57491	0.51103	4.02982	1.40108	8.0	0.99933	0.75017	0.963	0.1204	14.00000	1.98928
3.1	0.91379	0.41046	0.59381	0.49433	4.22521	1.44029	8.1	0.99939	0.75324	0.965	0.1177	14.20000	1.99012
3.2	0.92167	0.42396	0.61203	0.47815	4.42130	1.47738	8.2	0.99945	0.75623	0.967	0.1150	14.40000	1.99098
3.3	0.92886	0.43705	0.62957	0.46249	4.61798	1.51250	8.3	0.99950	0.75916	0.968	0.1125	14.60000	1.99170
3.4	0.93541	0.44976	0.64644	0.44736	4.81516	1.54562	8.4	0.99955	0.76201	0.970	0.1100	14.80000	1.99244
3.5	0.94138	0.46207	0.66265	0.43275	5.01278	1.57688	8.5	0.99959	0.76480	0.971	0.1076	15.00000	1.99304
3.6	0.94681	0.47399	0.67821	0.41865	5.21076	1.60628	8.6	0.99963	0.76753	0.973	0.1052	15.20000	1.99364
3.7	0.95175	0.48554	0.69313	0.40504	5.40906	1.63390	8.7	0.99967	0.77019	0.974	0.1030	15.40000	1.99426
3.8	0.95624	0.49672	0.70742	0.39192	5.60761	1.65981	8.8	0.99970	0.77280	0.975	0.1008	15.60000	1.99472
3.9	0.96032	0.50753	0.72110	0.37928	5.80640	1.68410	8.9	0.99973	0.77534	0.977	0.0987	15.80000	1.99520
4.0	0.96403	0.51798	0.73421	0.36711	6.00537	1.70687	9.0	0.99975	0.77783	0.978	0.0966	16.00000	1.99550
4.1	0.96740	0.52810	0.7467	0.3554	6.20450	1.72818	9.1	0.99978	0.78027	0.979	0.0946	16.20000	1.99600
4.2	0.97045	0.53788	0.7587	0.3441	6.40378	1.74800	9.2	0.99980	0.78265	0.980	0.0926	16.40000	1.99632
4.3	0.97323	0.54733	0.7702	0.3332	6.60317	1.76661	9.3	0.99982	0.78492	0.981	0.0907	16.60000	1.99666
4.4	0.97574	0.55648	0.7811	0.3228	6.80266	1.78386	9.4	0.99983	0.78727	0.982	0.0889	16.80000	1.99680
4.5	0.97803	0.56532	0.7915	0.3127	7.00223	1.80005	9.5	0.99985	0.78951	0.983	0.0871	17.00000	1.99715
4.6	0.98010	0.57387	0.8015	0.3030	7.20186	1.81506	9.6	0.99986	0.79170	0.983	0.0854	17.20000	1.99732
4.7	0.98197	0.58214	0.8109	0.2937	7.40156	1.82896	9.7	0.99988	0.79384	0.984	0.0837	17.40000	1.99768
4.8	0.98367	0.59014	0.8200	0.2847	7.60131	1.84193	9.8	0.99989	0.79594	0.985	0.0821	17.60000	1.99784
4.9	0.98522	0.59787	0.8287	0.2761	7.80109	1.85407	9.9	0.99990	0.79800	0.986	0.0805	17.80000	1.99802
5.0	0.98661	0.60536	0.8369	0.2678	8.00091	1.86519	10.0	0.99991	0.80002	0.987	0.0789	18.00000	1.99820

on a study of this problem, involving not only the development of a specification based on the flexibility constants of existing satisfactory structures, but also embracing the translation of these rigidity requirements into convenient rules for the quick selection of the best truss depths or moments of inertia for suspension bridges of various types and spans.

Mr. Feld refers to the deviation of the deflected cable curve from the parabola in short spans and the possible effect of such deviation upon the accuracy of the formulas of the deflection theory. Deviations from a parabolic curve are relatively slight in any adequately stiffened suspension bridge, even of short span, and the resulting effect on theoretical stresses is found to be negligible.

Referring to the excellent discussion by Messrs. Woodruff, Wood and Tudor, the writer appreciates their clear and amplifying endorsement of the advantages claimed for the continuous type of suspension bridge as well as their presentation and analysis of the problem of multiple spans.

In their reference to the modified Dana method, which was used in the preliminary phase of the multiple-span studies for the San Francisco-Oakland Bay Bridge, they state that "it has been found by comparison with model tests to give more accurate values for deflections than those obtained by the 'deflection theory.' The error in the latter theory is doubtless due largely to the neglect of the change in span lengths." This defect may have been true of the deflection theory as previously available; but in the formulas of the generalized deflection theory as presented in the paper for multiple-span suspension bridges, full consideration is given to the changes in span lengths.

In the final design formulas presented by the discussers, Equation (64) for the main cable is, as they state, identical with the writer's Equation (47) with the terms transposed; and Equation (63) for the tie cable can be shown to be effectively identical with Equation (46), which the writer has reduced from the more exact mathematical relationship to more compact, workable form with certain simplifying approximations, after having verified their numerical permissibility.

After setting up these two essential formulas corresponding to Equations (46) and (47) of the paper, the discussers present an attractive semi-graphical method for recording and applying the resulting numerical values. In either the graphic or analytical procedures, a method of "cut and try" or successive approximation, must be used in satisfying the governing relationship of Equation (48*a*). The graphic method has the advantage of facilitating visual inspection and interpolation of results in the various intermediate steps. The writer presented the analysis in the form of compact summation equations for the sake of maximum directness and maximum generality of application, covering not only the simplest case, but also the conditions of tower resistance and truss continuity.

Mr. Mensch cites precedents in other fields of structural analysis where the problems have been simplified so that differential equations are not required for their solution, and he suggests a possibility of similar simplification of the suspension bridge theory. Opinions may differ, however, on the importance of this objective. In the derivations of the paper, the application of differential equations is negligible, and the resulting working formulas have been reduced to a form requiring not even a knowledge of the calculus for their application. In the solution of such problems of structural analysis, the first essential step is, with the use of suitable mathematical tools, to establish correct equations for the unknowns sought; then comes the simplification of these formulas to compact, workable form for practical application; and, finally, there follows the development of tables or charts computed from the formulas, for expediting design.

The types of structural problems cited by Mr. Mensch bear little or no similarity to the suspension bridge problem. Any opportunity for further material simplification of the derivation and working formulas of the deflection theory is not apparent.

Mr. Eremin contributes generalized formulas for the loading terms, A, B_1, B_2, G_1, and G_3, applicable to a variety of loading cases. These are of interest, although they would generally be less convenient in practical application than the reduced formulas given in the paper for the specific loading cases. The demonstration by Mr. Eremin that the effect of suspender elongation may be neglected, is a matter of useful record.

Dr. Bleich contributes an independent confirmation of the writer's results and conclusions, particularly in reference to the claims of superior efficiency for the continuous type of suspension bridge, and he extends the indication of superior advantage to a range of greater span lengths than was anticipated by the writer. Dr. Bleich correctly points out that the comparison of quarter-point deflections in the main span is a better criterion of comparative rigidity of two designs than a comparison of mid-span deflections. On this basis he finds, for a span-length of 1 575 ft, that the continuous design is 17% more rigid than the two-hinged design of the same economy; the comparison of side span deflections in the same study shows 20% advantage in rigidity for the continuous design. For two designs of equal depth and rigidity at this span length, Dr. Bleich finds for the continuous design an economic advantage of 13% by comparison of moment graphs and 33% by comparison of assumed moments of inertia. This apparent economic advantage, however, vanishes if the two designs under comparison are adjusted to different truss depths to balance the relative values of I and M. This explains the anomalous showing of greater economic advantage at a span length of 1 575 ft than was found by the writer at a span length of 800 ft. The writer does not advocate continuous construction for highway suspension bridges exceeding 1 000-ft main span, but a longer span may be the economic limit for continuity in suspension structures designed for railway use.

Various possibilities, hitherto unrealized, of advantage and economy for bridges of the suspension type are pointed out by Mr. F. H. Frankland. In the short-span and multiple-span field, suspension construction should find increased application. In this connection it may be noted that, as pointed out by Mr. Lindenthal, there are conditions under which multiple-span suspension bridges can be used economically and effectively without the provision of tie cables.

Mr. Lindenthal contributes an interesting and valued discussion relating to the development of suspension bridge theory and practice. He emphasizes the fact that the minor approximations made in the derivation of the formulas of the paper are completely justified, since the stiffening truss of a suspension bridge is only a secondary member, on which the safety of the structure is not dependent. He states: "Nothing serious can happen to a suspension bridge as long as the anchorage, towers, and cables are sound." This important and governing fact should be kept in mind in discussing any further refinements in stiffening truss analysis.

Confirming and extending the writer's conclusions, Mr. Shields presents the results of the application of the generalized deflection theory to a suspension design of 400-ft span. The superior economy of the continuous type over the two-hinged truss, indicated by the writer as 5% for an 800-ft span, is found by Mr. Shields to increase to 12% for the shorter span length of 400 ft. The finding of increased economy for shorter spans was predicted and, as mentioned also by Messrs. Pavlo and F. H. Frankland, demonstrates that the continuous type of stiffening truss with the improved analytical theory will increase the importance of the suspension bridge in the short-span field. If the comparisons were made on the basis of equal deflection gradients instead of merely equating mid-span deflections, the hingeless type would show still greater economic advantage. The discussion of Dr. Bleich covering the comparative results obtained for a suspension design of 1 575-ft main span, combined with the report of Mr. Shields on results for a 400-ft span, together serve to extend the range and generalize the practical application and conclusions of the theory as given by the writer.

As was anticipated, Mr. Shields finds that the shortness of span introduces some modifications of the general loading conditions outlined in the paper.

The use of hyperbolic functions instead of exponential functions in the formulas of the deflection theory, as advocated by Mr. Fischer, has previously appeared in the work of other writers. The basis of a choice between the two forms will generally be that of personal preference. The writer has found, by experience and comparison, that the exponential form with the introduction of the *d*-function (which may be considered either a hyperbolic or an exponential function) yields maximum compactness of formulas and facility of analysis. A table of exponential functions will facilitate substitution in those sections of the working formulas where the general *d*-function has not already simplified their form and application.

In an attempt to offer a simplified procedure, Mr. Fischer applies the familiar principle that the point of maximum bending moment for a fixed loading of length, k, is defined by writing the expression for shear equal to zero. In the notation of the paper, using Equation (10), this value of x for a given value of k is simply given by the formula,

$$e^{2cx} = \frac{C_2}{C_1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (126)$$

which should be compared for brevity with Mr. Fischer's Equation (82).

Unfortunately, however, this proposed short-cut, eliminating the use of trial load lengths, does not yield the results sought. The problem is not to find the maximum bending moment producible by a given loading, but to find the loading producing the maximum bending moment at a given point of the span and then to find the resulting value of this maximum bending moment.

In general, the maximum bending moment thus obtained for a given section will be greater than the more restricted value yielded by Mr. Fischer's proposed procedure; the two values will agree only at or near the points of absolute maximum bending moments in the span such as those accidentally selected by Mr. Fischer for his numerical illustration. If a bending moment curve is drawn for a given load length, k, some point on this moment curve other than the maximum may be a governing design value; the envelope of such moment curves for various values of k will be the curve of maximum bending moments used in design. Instead of this comprehensive and lengthy procedure, the use of a few trial load lengths for each section, as illustrated in the paper, is a convenient practical expedient, yielding the desired results.

The interpolation formula (Equation (83)) contributed by Mr. J. M. Frankland for use in connection with Table 1, or Table 10, for values of d should prove useful when an adequate table of hyperbolic functions is not available. Ordinarily, linear interpolation between the tabulated values of $c\,l$ should suffice, and closer interpolation should be necessary only for the lower values of d in Table 1, or in Table 10.

The independent derivation by kinematical analysis of Equation (100) identical with Equation (22) is a welcome additional check of this equation which underlies the H-formulas of the paper. It is re-assuring to know that this important equation is accurate despite the several assumptions involved in its simplified derivation as a work equation.

Mr. Frankland's suggested correction (Equation (102)) of the temperature term in Equation (22) is also welcome. This correction, writing L = total distance between anchorages, in place of L_t, in all formulas for suspension bridges, although negligibly affecting the results, should be accepted as an improvement. It has the advantage of greater simplicity, while correcting the H-formulas for the previously neglected effect of temperature change in the steel towers and in the suspenders. The correctness of this modification may be verified, easily and independently, by adding temperature terms for towers and suspenders in the formula for the internal work, W_2, preceding Equation (22); substitution and reduction then yields L instead of L_t in Equation (22) and all subsequent formulas.

A slight additional correction in the temperature term can be written to take into account differences of elevation of steel tower bases and anchorage ends of cables, but such further correction is negligible.

Professor Vogt refers to the influence line solution proposed by the late Hans H. Rode, M. Am. Soc. C. E. In this method of obtaining the load lengths for maximum moments or shears, the values of influence ordinates for M or V at a given point for unit concentrations moving along the span are computed for two assumed values of c^2, corresponding to maximum and minimum probable values of $(H_w + H)$, in order to account for the lack of direct proportion between load and stress.

On the assumption that there will not be an excessive spread between the influence lines thus plotted for the two values of c^2, the approximate load length for maximum moment or shear can be scaled, the value of H for this loading can be found by interpolation from the two influence lines for H, and the final load length and moment or shear can then likewise be found by interpolation between the two influence lines for M or V. If the final values of H and M or V are computed by the formulas for distributed loading, it should be necessary to obtain the influence line ordinates only near the critical points of zero moment or shear.

This method of Professor Rode is not, strictly speaking, an influence-line solution; but, if it is not too lengthy or inaccurate, it may have an advantage, in some instances, over the generally more practical solution by trial. At any rate, this suggested influence-line procedure holds some possibilities for further development. The values of H and M or V for the unit loads would be computed from the general formulas of the deflection theory for concentrated loads.

Mr. Moisseiff questions whether including the effect of suspender elongation and tower shortening will not serve to increase the tower moments of the continuous stiffening truss as analyzed by the formulas of the paper. As stated elsewhere, the effect upon H of temperature changes in the lengths of towers and suspenders is included in the analysis by merely substituting for L_t in the formulas the simpler quantity, L, which is the sum of the horizontal cable spans. Due to the great sectional area of the towers as compared with the cross-section of the cables, the shortening of the towers under live load is relatively negligible.

In the continuous type, at and near the towers, the effect of live load elongation of suspenders is of interest only for that case of loading which produces, simultaneously, maximum negative tower moment and maximum positive shear at the tower. For this loading condition, with the near side span fully loaded and the main span only partly loaded (for a minor fraction of its length out from the tower), the value of H in the cable is small; in the numerical example of the paper, it amounts to only about one-sixth of the maximum value of H. Consequently, the suspender forces, and therefore the changes in suspender lengths, will also be relatively small. The generalized deflection theory shows, moreover, that for this case of loading the continuous truss, under the action of live load and temperature change, relieves the cable slightly of dead load near the tower, so that several suspenders adjacent to the tower actually shorten rather than elongate. This would tend to decrease the negative tower moment. Actual computation for the effect of suspender elongation on the stiffening truss of Design II indicates a decrease in both the negative moment and the shear at the tower, which decrease amounts, however, to only a fraction of 1 per cent. It thus appears conclusive that the effect of change of length of suspenders on the analysis of the continuous stiffening truss may be neglected safely. This conclusion is confirmed independently by analysis in the discussion by Mr. Eremin.

Near the tower, the continuous truss does not follow the deflections of the cable; consequently, the truss is not "unduly" strained as surmised by Mr. Moisseiff. Near the tower, the continuous truss deflects less than the cable; hence the suspender shortening. The theory assumes that the truss follows the cable deflections and, therefore, yields near the tower more severe stresses than actually occur. The difference, however, is negligible.

In the remainder of the span, where positive bending moments govern, the suspenders elongate and the truss deflects slightly more than the cable. Suspender elongation tends to reduce positive bending moments, so that here, also, the theory yields more severe stresses than actually occur. Here, again, the difference is negligible (a small fraction of 1 per cent). In addition to being negligible in effect, the assumptions that have been challenged are on the safe side.

Messrs. Mitchell and Parkin present formulas for the G-functions which they have derived for the case of a span divided into three loading segments. Completely general formulas for the G-functions, as mentioned in the paper, may be written as follows for any segment, m, of any span having any number of loading segments with any uniform loads (or zero loading) in any segments:

Let p_1, p_2, p_3 . . . = the uniform load in the first, second, third, etc., segments from the left end of the span, up to p_m in Segment m (note $p_0 = 0$); k_1, k_2, k_3 . . . = the clear distance of the first, second, third, etc., segments from the left end of the span, up to k_m for Segment m (note $k_1 = 0$); q_1, q_2, q_3 . . . = the uniform load in the first, second, third, etc., segments from the right end of the span, up to $q_m = p_m$, in Segment m (note $q_0 = 0$); and, z_1, z_2, z_3 . . . = the clear distance of the first, second, third, etc., segments from the right end of the span, up to z_m for Segment m (note $z_1 = 0$, and $k_m + m + z_m = l$). Then, in any segment (m) of any span,

$$G_1 = -\frac{(1-d)}{4\,d}\,\Sigma_L\,[(p_n - p_{n-1})\,(e^{ck} + e^{-ck})]$$

$$+\frac{1+d}{4\,d}\,\Sigma_R\,[(q_n - q_{n-1})\,(e^{cz} + e^{-cz})]\ldots\ldots\ldots\ldots(127a)$$

$$G_2 = \frac{1+d}{4\,d}\,\Sigma_L\,[(p_n - p_{n-1})\,(e^{ck} + e^{-ck})]$$

$$-\frac{1-d}{4\,d}\,\Sigma_R\,[(q_n - q_{n-1})\,(e^{cz} + e^{-cz})]\ldots\ldots\ldots\ldots(127b)$$

and,

$$G_3 = \tfrac{1}{2}\,\Sigma_L\,[(p_n - p_{n-1})\,(e^{ck} + e^{-ck})]\ldots\ldots\ldots\ldots(127c)$$

in which Σ_L denotes the summation for all the segments from the left end of the span up to and including the segment, m, and Σ_R denotes the similar summation for all the segments from the right end of the span up to and including the segment, m.

Applying the general rule expressed by Equations (127), all the G-formulas for the different loading cases tabulated in Article 9 may be written by inspection. *G*-formulas for other loading cases or for other span segments may likewise be written directly from Equation (127).

Mr. Johnston makes a pioneer contribution of a specification for rigidity of suspension bridges. He presents a chart (Fig. 13) which gives his recommendations for allowable change of gradient for highway suspension bridges of varying lengths of spans and numbers of traffic lanes. For a 4-lane structure of 800-ft main span, the chart yields an allowable grade change of 2.22%, which should be compared with the computed values recorded in Table 9.

Change of gradient in a span is generally significant only when the change is such as to increase the normal (unloaded) grade of the roadway. If Fig. 13 were to be interpreted as for allowable increase of grade and the allowable base percentage of 2.00% were to be increased to 2.25 or 2.50%, the chart would better fit established and proved practice and would constitute a practical specification for suspension bridge rigidity. It should be noted, in this connection, that the distinction between increase and change of grade is ordinarily of consequence only in the continuous type.

A working specification of allowable flexibility of a highway suspension bridge should also be governed by the location of the bridge, since a grade change which would be considered excessive in a large city or for heavy trucking traffic might be altogether proper in a less heavily traveled site. For computing the maximum change of grade in any span, a particular design loading as to length and placement in each span must be assumed. If it appears that such a theoretical loading is seldom likely to occur, that fact should be borne in mind in choosing the value of the maximum permissible grade change.

The recommendation of Mr. Johnston that the total augmented grade of the roadway should never exceed 5% is likewise subject to qualification. Higher ruling gradients are demanded and approved in some localities, and appear to be justified by the improvements that have been made in automotive equipment. Urban traffic conditions which might set the maximum augmented grade of a span at 5% would not obtain in rural districts or in regions where steeper grades are the rule.

Professor Wessman calls attention to the omission of the shear effect in writing the differential equation for the truss deflection. It is the writer's opinion that the relative shallowness of the stiffening truss fully justifies the neglect of shear contribution to the deflection, even in the case of the continuous type. This view is substantiated by the results obtained in the computations for effective moments of inertia of the truss; in these computations, the contribution made by the diagonals to the total deflections is found to be negligibly small in comparison with that made by the chords.

The formulas for concentrated loads, contributed by Mr. Gronquist, will be found useful in computing deflections under heavy concentrations, and in applying the partial influence-line solution of Professor Rode as suggested by Professor Vogt.

In the thoughtful discussion contributed by Mr. Frederick, he supports the thesis that angular deflection or grade change is a better index of comparative rigidity than mere comparison of maximum deflections. In his studies of comparative economy, he uses the refinement of varying the truss depths, even as between main span and side spans of the same design. For the brief numerical comparisons at the end of the paper the writer hesitated to go to this extreme, although it would have facilitated the presentation of "balanced" designs.

Of course, investigators using different methods of evaluating relative economy and rigidity will obtain varying numerical results. Although quantitative findings vary, the qualitative conclusions are in accord: All agree on the important point that continuous suspension design for span lengths of the range under consideration are more rigid or more economical (possibly both) than the two-hinged type.

The maximum angular deflections reported by Mr. Frederick in Table 6 are in sufficiently close agreement with the side-span grade increases recorded by the writer in Table 9, but the economic comparisons in Table 7 do not appear entirely consistent. Nevertheless, the final results of Mr. Frederick's analysis serve only to support and strengthen the writer's claim of superior advantage for the continuous type. In comparison with the two-hinged design, Mr. Frederick's results show 25% gain in economy plus 38% gain in rigidity for Continuous Design I; more than 1% gain in economy plus 27% gain in rigidity for Continuous Design II; and 6% gain in economy plus 18% gain in rigidity for Continuous Design III. All these combinations exceed the measure of superior advantage for this span length recorded by the writer in summary form in Equation (120), and the conclusion recorded in Equation (120), in turn, exceeds the more conservative claims made in the original paper.

Mr. Yates properly refers to the Rondout Bridge, at Kingston, N. Y., as a pioneer among modern suspension structures, from the viewpoints of continuity, short span, concrete deck, and design for modern heavy highway loading. Had it been available in 1920, the generalized deflection theory would have facilitated and improved the stress analysis and proportioning of sections for that bridge, and would have yielded a more efficient design.

Conclusions.—In conclusion, it may be stated that the objectives of the paper have been confirmed and amplified by the discussions. The deflection theory for suspension bridge analysis has been generalized and extended to cover structures of any number of spans, continuous or non-continuous, symmetrical or unsymmetrical, and with or without tie cables. Theory and formulas have been reduced to a new degree of compactness and simplicity, and procedures have been developed to facilitate and expedite application. The advantages of continuous stiffening trusses, in economy and rigidity, have been confirmed. Correct analysis as a prerequisite to the development of possibilities in the multiple-span field of suspension bridge design has been presented. Increased possibilities for the suspension type in the short-span bridge field have been suggested. Criteria for comparing and specifying the rigidity of suspension structures have been introduced. The reliability, economy, and workability of the deflection theory have been confirmed, and its range of application extended, so as to relegate the comparatively wasteful and unscientific elastic theory to obsolescence.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 1919

SECURITY FROM UNDER-SEEPAGE MASONRY DAMS ON EARTH FOUNDATIONS

By E. W. Lane[1], M. Am. Soc. C. E.

With Discussion by Messrs. William P. Creager, L. F. Harza, Joel D. Justin, Louis E. Ayres, W. M. Griffith, E. Mckenzie Taylor, Donald J. Hebert, Arthur Casagrande, Calvin V. Davis, C. H. Eiffert, E. H. Burroughs, Alexander Potter, Robert E. Kennedy, A. N. Khosla, A. Streiff, and E. W. Lane.

Synopsis

This paper gives the results of an investigation of more than two hundred masonry dams with various kinds of earth foundation, to determine the length of the percolation path necessary to prevent failure from under-seepage or piping. Based on this study, a new method of analysis of such structures has been developed, which generally permits the use of smaller seepage distances than are ordinarily considered to be safe.

When a masonry dam is founded on earth, some of the water from the reservoir percolates beneath it and appears on the down-stream side. If the velocity of the flow where it emerges is sufficient, particles of the foundation material will be carried away by the water, and thus decrease the resistance to the percolation. The result is an increased velocity and greater erosion, ultimately causing the formation of a channel or "pipe" beneath the dam, which may enlarge rapidly and cause the failure of the structure. This process is known as piping. To prevent piping it is necessary to design the dam so that the velocity of the seepage water as it emerges on the down-stream side is insufficient to remove the foundation material. This is accomplished by making the route along which the water may percolate of such a length that the velocity of flow is reduced to a safe value. The investi-

---

Note.—Published in September, 1934, *Proceedings*.

[1] Prof. of Hydr. Eng., State Univ. of Iowa, Iowa City, Iowa; Formerly Research Engr., U. S. Bureau of Reclamation, Denver, Colo.

gation described herein was undertaken to determine from data on actual dams the length of the percolation path necessary to insure safety for dams founded on various kinds of material.

A more complete report, including drawings that show dimensions and typical sections of all the dams, as well as a comprehensive bibliography has been filed for research purposes in Engineering Societies Library, New York, N. Y.

THEORETICAL CONSIDERATIONS

The law of flow of water through earth was determined many years ago by H. Darcy[2]. Later work was done in the United States by the late Allen Hazen, M. Am. Soc. C. E.[3], and C. S. Slichter[4], and in India by Col. J. Clibborn[5] and Mr. J. S. Beresford. Their experiments showed that (neglecting temperature effects) the discharge through a column of soil of a given composition varied directly with the head, directly with the cross-sectional area, and inversely with the length. This relation, commonly known as Darcy's law, may be expressed as:

$$Q = c_1 \frac{H\,A}{L} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

in which, Q is the discharge, in cubic feet per second; H, the head, in feet; A, the cross-sectional area, in square feet; L, the length, in feet; and c_1, a coefficient which depends on the character of the material.

Substituting for Q the value, $A\,V$, Equation (1) becomes,

$$L = c_1 \frac{H}{V} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

For a given class of material there is a definite maximum velocity, V_m, at which the water can emerge below the dam without carrying away the foundation material and causing the failure of the structure. Combining this value of V_m with c_1, which also depends on the material, to form a new coefficient, $c_2 = \frac{c_1}{V_m}$, the expression,

$$L_n = c_2\,H \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

is obtained, in which, L_n is the minimum safe length of travel path and c_2 is a coefficient depending on the foundation material.

The foregoing theory seems to be generally accepted. In applying it, two difficulties arise: (1) How shall the length of the travel path, L_n, be measured? and (2) what value of c_2 can safely be used for various classes of material? Before going further into a discussion of these points it may be well to give the history of the development of this phase of dam design.

---

[2] Pub. in Paris, France, in 1858.

[3] Rept., Massachusetts State Board of Health, 1892.

[4] *Water Supply Papers Nos. 67 and 140*, U. S. Geological Survey.

[5] "Experiments on the Passage of Water Through Sand", Govt. of India, Central Printing Office, 1902.

Historical

The first rational basis for the design of masonry dams on earth foundations seems to have been developed in India, as a result of the investigations of Colonel Clibborn[5] and Mr. Beresford. Colonel Clibborn was at one time Executive Engineer in charge of the Rolhikhand Canal Division of the United Provinces, India, where there were a number of dams founded on light sand, which had often given trouble. After he became Principal of Thomason College, together with Mr. Beresford, he carried out a classical set of experiments on the law of flow through sand. From these experiments Mr. Beresford concluded that the Narora Weir on the Ganges River was unsafe, because of excessive upward pressure on the apron, and made a report to that effect. At the time no special trouble had been experienced with the dam, but as a result of this report pressure pipes were placed in the apron to indicate the upward pressure beneath it. The pressure indicated in these pipes confirmed Mr. Beresford's conventions and by coincidence, the next day after the readings were taken (March 30, 1898), the apron at another part of the dam was blown up, resulting in a breach of the weir. The failure of an important structure, following so promptly after the declaration of its instability, profoundly impressed the engineers of the United Provinces; and the hydraulic gradient theory of design became generally accepted there about 1898.

The first edition of "Practical Design of Irrigation Works", by W. G. Bligh, appeared in 1907, in which the theory was advanced that the stability of a weir on a porous foundation depended on the weight of the structure and not on the ratio of the percolation distance to the head. In his second edition, published in 1910, Bligh admitted the fallacy of his original contentions and explained his well-known theory that the safety of masonry dams on earth foundations depends on the length of the percolation path, which is along the line of contact of the structure and its foundation.

About the same time, this conclusion was also reached independently by Mr. W. M. Griffith whose paper[6] has not received the recognition in the United States which its value justifies. Before it was published, Bligh's second edition appeared, in which he also proposed that idea. The same conclusion seems to have been reached about that time by the late William Wolcott Tefft, M. Am. Soc. C. E., but did not find its way into print. The widespread use of Bligh's book, together with the publication of other articles and books[7] by him, has led engineers generally to give the credit for the idea to Bligh.

In 1911, a paper appeared, by Mr. Arnold C. Koenig,[8] giving rules for the design of masonry dams on earth foundations, which contains a number of valuable ideas.

---

[6] "The Stability of Weir Foundations on Sand and Soil Subject to Hydrostatic Pressure," *Minutes of Proceedings*, Inst. C. E., Vol. 197, Pt. III, 1913–14, p. 221.

[7] "Dams and Weirs"; "Dams, Barrages and Weirs on Porous Foundations", *Engineering News*, December 29, 1910, p. 708, and January 12, 1911, p. 52; "Weirs on Porous Foundations and with Pervious Floors", *Engineering News*, April 13, 1911, p. 444; "Lessons from the Failure of a Weir and Sluices on Porous Foundations", *Engineering News*, February 6, 1913, p. 266; and "Irrigation Headworks Repair and Dam Failure", *Engineering News*, June 8, 1916, p. 1070.

[8] *Transactions*, Am. Soc. C. E., Vol. LXXIII (1911) p. 175.

Little appeared in technical literature for many years after these publications to aid the engineer in the practical design of masonry dams on earth foundations, but a number of measurements of upward pressure on actual and model dams have been published from time to time. Recently, methods have been devised for determining the upward pressure and path of percolation for various conditions. One of these is that used by Charles Terzaghi, M. Am. Soc. C. E.[9] It seems to have been first used for dam foundations by Phillip Forchheimer[10], but Professor Terzaghi's paper appears to be the first explanation of this use in English.

By means of a flow net[11], the direction of the currents of the water and the pressure throughout the material beneath the dam can be computed for various assumed conditions. Another interesting method of obtaining the same information has been devised by Professor N. N. Pavlovsky[12], who makes use of the similarity of the laws of flow of the electric current through a conductor and the flow of water through soil. With the methods of either Professor Terzaghi or Professor Pavlovsky it is possible to obtain a rigid solution for the given assumptions, and they are very useful in securing a picture of the flow of the water beneath the dam. As will be discussed later, however, the results obtained, if the ordinary assumptions are used, must be applied to actual cases with caution.

Four interesting and instructive papers have recently appeared, which throw much light on the problem. Two of these are papers presented at the Punjab Engineering Congress of 1930, by A. N. Khosla,[13] M. Am. Soc. C. E. The third is a paper by Mr. S. Leliavsky.[14] The fourth is a paper recently presented by Mr. Khosla[15] before the Punjab Engineering Congress, giving the results of measurements of upward pressure on the Panjnad Weir.

COURSE OF SEEPAGE BENEATH A DAM

The path that the water takes in flowing beneath a dam might be determined with reasonable accuracy by either the flow net, or by the electrical, methods, if all the conditions governing the flow were known accurately. This is never the case, however, and for purposes of design certain assumptions must be made. In the second edition of his "Practical Design of Irrigation Works", Bligh proposes an analysis on the assumption that the water follows a path along the line of contact of the dam foundation (including the sheet-piling) with the foundation material. The same method was suggested by Mr. Griffith. This contact between the dam and the foundation material is sometimes called the line of creep, and the method may be called the line-of-creep method. Another method that has been advocated to some extent, may be called the short-path method, and is based on the assumption that the course taken by the percolating water is the shortest path through the pervious material between the head-water and the tail-water. Neither of these methods gives a true picture of the actual conditions of flow beneath dams, but they are useful methods of practical design. The line-of-creep method has gained a wide acceptance, and most masonry dams on earth foundations have been designed according to it. It has been used in the irrigation works of the United States Bureau of Reclamation for many years with satisfactory results. The short-path theory has been but little used.

---

[9] *Technical Publication No. 215*, Am. Inst. of Mining and Metallurgical Engrs., p. 31.

[10] "Hydraulics", by P. Forchheimer, 1924, p. 448.

[11] "Hydraulic Laboratory Practice", by the late John R. Freeman, Past-President and Hon. M., Am. Soc. C. E., Am. Soc. Mech. Engrs., 1929, p. 605.

[12] *Engineering News-Record*, Vol. 112, June 14, 1934, p. 765.

[13] "Hydraulic Gradients in Subsoil Water Flow in Relation to Stability of Structures Resting on Saturated Soils", *Paper No. 138*; and "Stability of Weirs and Canal Works: An Application of the New Theory of Hydraulic Gradient", *Paper No. 142*, Punjab Eng. Congress, 1930.

[14] "On Percolation under Aprons of Irrigation Works", by S. Leliavsky (published privately).

[15] *Paper No. 162*, Punjab Eng. Congress.

In designing a masonry dam on an earth foundation it is customary to assume that the path of the water flowing beneath it is always normal to the axis of the dam. Analyses are made, therefore, of cross-sections of the dam perpendicular to the axis. If the material on which the structure is founded varies along its length, it may be necessary to investigate more than one section. The assumption of normal flow is sufficiently accurate for most conditions. In unusual cases, however, a more accurate analysis may be necessary. In analyzing the security of the dam at the abutments, normal flow cannot be assumed.

Comparison of Design Methods

Although the line-of-creep method has been widely used in dam design, it has been subject to some criticism. Part of this criticism is believed to be due to an improper presentation of the case in its favor. Bligh states that the water follows the line of creep and not the path of least resistance. This statement is believed to be in error, because the fact that water would take the path of least resistance seems almost axiomatic. If water flows along the line of creep instead of the shorter path directly through the foundation material it is because the resistance to travel along the line of creep is less than along the shorter path. That resistance along the line of creep may be less than through the foundation material seems quite reasonable, on account of the difficulty of securing as intimate a contact between the more or less plain surfaces and the foundation material as between the individual particles of the foundation material.

It should be remembered that the line-of-creep method is intended to give a dam safety at all points. This requires that it apply to the worst condition that will happen with every reasonable care in construction. The seepage may not follow the creep line at many cross-sections of the dam, but the points where there is most danger of failure are likely to be those where contact between the dam and the foundation material is not so close, and, therefore, where the line-of-creep method best applies.

The principal weakness of the line-of-creep theory is that it assumes the resistance to flow along all parts of the contact between the dam and the foundation to be the same. In practice, the contact between vertical and steeply sloping surfaces is more likely to be intimate than that along horizontal or slightly sloping surfaces. In a masonry dam on earth foundations there is likely to be unequal settlement which will cause less pressure at some points than at others, in places, even a slight lifting up of the masonry from its contact with the earth beneath. The earth beneath a dam may not be compact, and may settle after the dam is built, leaving void spaces beneath the floor, especially where the dam is founded on piles. This action is sometimes called "roofing".

Another cause of roofing is given by J. C. Oakes[16], M. Am. Soc. C. E., as follows:

> "If the bed of the foundation [of a dam] is below the water level, and pumping is required, there will be a small space between the base of the masonry and the sand, caused by erosion by the flow of water from under one block of masonry while the next one is being placed. In other words the structure is ordinarily supported wholly by the piles, with almost a certainty that there will be a space between the sand and the masonry. Under such circumstances, any upward pressure that develops under the dam will be uniform from the sheet-piling to the toe, and will depend on the tightness of the sheeting and the ease of escape below the dam, * * *.
>
> "* * * the writer has watched the placing of concrete, and in no case has he found, on the works under discussion [Ohio River Dams, Nos. 43 and 48] that the concrete rested on the sand after sufficient time had been given it to set. The bed of the foundation of the various parts of these structures is about 10 ft below low water. Construction has been carried on within coffer-dams, and during stages of the river from low water to 14 ft above it. Owing to the permeable nature of the material, there has always been considerable percolation, which has required pumping to keep the pit sufficiently clear of water to enable construction to proceed. The water escaping from under the concrete already placed, carries away the fine material directly under the concrete, leaving a space between it and the sand through which the transmission of pressure will be direct, and, consequently, any pressure which may be developed will be uniformly exerted over the whole base of the structure."

A study of the results of upward pressure measurements on actual dams discloses much evidence to support Colonel Oakes' conclusions.

On vertical or steeply sloping faces, roofing will not occur because the void spaces, if they should be formed, would be filled again by the inability of the earth to maintain the steep slope; the earth from the steep bank would move down and fill the void. For this reason the contact of the foundation with such surfaces is close and offers more security against piping than contact beneath horizontal surfaces.

D. C. Henny, M. Am. Soc. C. E., has suggested that the vertical creep may be more effective than the horizontal creep because the stream-bed material is likely to be laid down in nearly horizontal layers or lenses of varying permeability. By building cut-offs through these layers, the flow through the relatively permeable ones is stopped, and the water is forced either to pass through the more impermeable layers or to seek a much longer route, with consequent reduction of velocity.

Piping failure, therefore, should be considered as possible from two largely independent causes: (1) Direct percolation through the foundation material itself; and (2) percolation along the contact of the dam and sheet-piling with the foundation material. Considerable light on the first of these causes has been obtained by experiment, but Cause (2) can only be evaluated by investigating a large number of structures. A weakness of the ordinary line-of-creep theory is that it considers only the second of these causes. Another weakness is that it is possible to drive lines of sheet-piling so close together that the short path may be so small that failure can occur by flow which will not follow the path of creep at all.

---

[16] *Transactions*, Am. Soc. C. E., Vol. LXXX (1916), pp. 469–470.

The weakness of the short-path theory is that it takes no account of the greater probability of percolation along the line of contact of the structure and its foundation. In cases where the lines of sheet-piling are very close together, however, it may be more reliable than the creep theory. The short-path principle would seem to be obviously inferior to the line-of-creep analysis as applied to clay or hardpan foundation, since the foundation material in this case would be so nearly impermeable that the seepage along the line of creep would be much greater. Another difficulty with the short-path analysis is that it gives no method of estimating the magnitude or distribution of upward pressure beneath the dam. As some estimate of this pressure is required in order to determine the required thickness of the apron, additional assumptions are necessary if the dam is to be designed by the short-path theory throughout.

The flow net and electric analogy methods are essentially the same. Both should give the same results for the same assumptions. Professor Terzaghi[9] does not take into consideration the greater probability of percolation along the line of contact. Whether or not Professor Pavlovsky[12] does, is not known. It would be possible to do it by either method, however, by assuming the relative permeability along this line as compared with that through the foundation material. The weakness of both methods is the necessity of a detailed knowledge of sub-surface conditions and the lack of data relating the results obtained, as shown by the flow net, to the safe limits for the various classes of material.

Nevertheless, the flow net and electric analogy methods may prove to be useful tools in analyzing unusual conditions, and in forming a mental picture of what takes place under certain conditions. It is hoped that further studies along this line will be made to clear up some of the problems of design. There is a great opportunity for useful experimentation in this field. The results that would be obtained by the flow net or by the electrical analysis would roughly correspond in a homogeneous medium to that obtained by the short-path method. Both are based on the consideration of flow directly through the foundation material. The short-path method may be considered, therefore, as a rough approximation of the flow net, or electrical analysis.

A weakness of all the methods, as ordinarily applied, is that the flow is considered as taking place only in a single plane. This is not necessarily the case however, because, as Mr. Griffith has pointed out,[17] when a pipe tends to form, it provides a line of lowered pressure, and water from both sides flows toward it, as well as that in the plane of the incipient "pipe". Analyzing the stability of a dam by means of its cross-section is a useful device, but one is likely to be blinded to the true conditions by thinking too rigidly in terms of cross-section only.

---

[17] *Minutes of Proceedings*, Inst. C. E., Vol. 197, 1913–14, Pt. III, p. 223.

REQUIRED LENGTH OF PERCOLATION PATH

Not only have there been differences of opinion regarding the path along which the percolation should be assumed to occur, but this is true also regarding the length of the path necessary to insure safety from piping failure.

As the result of his study of dams and dam failures, Bligh arrived at values of the creep-head ratio which he believed would make dams safe from piping failure. The description of the various classes of material and the values of the ratios varies somewhat in different publications by Bligh. The following list gives the classification in his latest publication:[18]

	Safe ratio
River beds of light silt or sand of which 60% passes the 100-mesh sieve, as those of the Nile or the Mississippi Rivers	18
Fine micaceous sand of which 80% passes a 75-mesh sieve, as in Himalayan rivers and in such rivers as the Colorado	15
Coarse-grained sands, as in Central and South India	12
Boulders or shingle and gravel and sand mixed	5 to 9

Just how extensive a study of dams was made by Bligh as a basis for his data is not known, but the amount of material published is quite meager. It includes only two dam failures, both on fine sand foundations; and in one or both of these cases, unsound conclusions were drawn. There were no dam failures on silt, coarse sand, gravel, or boulders. In a later publication, Bligh discusses the failure of a head-gate of the Southern Alberta Land and Irrigation Company. The published data on this structure are so conflicting as to throw serious doubt on the reliability of his conclusion.[19] These statements are not made to disparage Bligh's work, which has formed the basis of the design of scores of safe dams, but merely to indicate that, apparently, it was based on meager data and that with much more extensive data, and better construction materials, a reduction of Bligh's values of C might be made without conflicting with well-established facts.

In an abridged paper,[6] Mr. Griffith gave the following values of the ratio of creep distance to head which "had been found sufficient to ensure stability" in the United Provinces of India:

Material	Limiting safe value of C
Fine micaceous sand	14½ to 16
Fine quartz sand	12½ to 14
Coarse quartz sand	10 to 12
Shingle	8
Boulders	4

---

[18] "Dams and Weirs", 1916, p. 155.

[19] *Engineering Record*, Vol. 63, p. 589; Vol. 66, p. 376; and *Engineering News*, Vol. 69, p. 266; and Vol. 75, p. 1070.

Concerning shingles and boulders, Mr. Griffith states that "the question of loss by leakage may make higher values of $\frac{L}{H}$ advisable in these cases."

In an unpublished part of this paper he suggested "a 20% reduction in the values given where reliable vertical staunching of 10-foot depth was used."

In the third edition of Bligh's "Practical Design of Irrigation Works", which was revised and brought up to date by F. F. Woods, Chief Engineer of Irrigation Works, Punjab, India, Mr. Wood contends[20] that Bligh should have used a ratio of 11 for ordinary sand instead of 15. It will thus be seen that both Messrs. Griffith and Woods advocate somewhat lower values of C than Bligh. Independently, the writer arrived at the same conclusion from a study of the data on most of the dams included in the tables of this paper.

A New Method of Analysis

As has already been pointed out, the existing methods of analysis are open to serious objections. The commonly used method, as advocated by Bligh, does not consider the greater resistance to flow along vertical contacts as compared with horizontal ones. The short-path method and the more exact flow net and electrical methods do not consider the lesser resistance along the contact of the masonry and foundation material as compared with that directly through the foundation material. All these methods have elements of truth, but all have weaknesses. A method should be devised which will combine the virtues of both without including their faults.

In the present state of knowledge the only method of analyzing the probability of failure from flow along the creep line seems to be a study of the action of actual dams. No exact data have yet been presented to show the relative resistance to the flow along the contacts as compared with that through the foundation material. It will be very difficult to obtain these data because points where danger of flow along the contact is great, will only occur, occasionally, and would probably be discovered only by extensive observations. The flow through the foundation material can be obtained with more exactness by experimental methods; but even in that case the unknown conditions of the foundations and the difficulty in determining the safety of the structure (even if the velocity of flow is known) make this method difficult to apply. It seems obvious, therefore, that while research along this line should be given every encouragement, the main reliance in dam design, for a long time in the future, must be on a somewhat empirical basis.

As the studies made in connection with this paper indicated faults of the existing methods of analysis, as well as the insufficiency of the factors used in the ordinary methods, the writer has attempted to develop a more rational method of design, as well as to establish more accurate coefficients. In order to devise a method that could readily be used by the designing engineer in planning actual structures, in addition to the aforementioned reason, the method developed was necessarily a somewhat empirical one.

---

[20] See "Preface", p. VI.

For clarity in presenting the data on which writer's conclusions are based, it is necessary here to review, briefly, the basic facts of the new method developed.

From a study of all the available data it appeared that there were two distinct forms of piping, one in which the water passed along the line of contact of the structure and its foundation, as assumed in the Bligh theory, and the other in which it passed directly through the voids in the foundation material. In the former path the contact of the foundation on vertical or steeply inclined surfaces can be relied upon to offer more resistance to flow than along horizontal or slightly sloping contacts. In computing the safety of a structure, therefore, from this type of failure, the creep distances along horizontal or slightly sloping surfaces should be given less weight than those along vertical or steeply inclined surfaces. This method of estimating the stability of a structure may be called the weighted-creep method. As a result of these studies a weight of one-third is given to the horizontal or slightly inclined creep as compared with the other section of the path. For example, if a dam sustains a head of 10 ft and has a creep distance along horizontal or slightly inclined surfaces of 60 ft and along vertical and steeply inclined surfaces of 10 ft, the weighted creep distance would be $60 \times \frac{1}{3} + 10$, or 30 ft, and the weighted-creep-head ratio would be $30 \div 10 = 3.0$.

For brevity in the remainder of this paper, creep along vertical surfaces or surfaces sloping more than 45° with the horizontal will be called "vertical creep", and other surfaces, "horizontal creep." It should be noted that "vertical or steeply inclined" refers to the position of the surface against which creep takes place, and not to the direction of the creep, which sometimes is not in the same direction as the inclination of the surface. The weighted-creep distance is the vertical creep plus one-third the horizontal creep, which, therefore, is practically always less than the creep distance.

As the water follows the line of least resistance, if the resistance to flow along the creep line is much greater than directly through the voids in the foundation material, much more water may take the latter course and failure from piping from this cause may result. The short path is a rough measure of the resistance through the voids. Both the weighted-creep and the short-path ratios, therefore, must be greater than their respective critical values for the type of material on which the dam is founded.

METHOD OF ANALYZING DATA ON EXISTING STRUCTURES

In this study an intensive search was made in all available engineering literature and other sources of information. All masonry or concrete dams on earth foundations were analyzed when the data were sufficiently complete. In many cases there is considerable uncertainty. When more than one description was found of a single structure there was often a surprising lack of agreement between them.

Frequently, there is considerable room for the exercise of judgment as to just where the travel path should be assumed to end. The travel path of the water was ordinarily assumed to end at its junction with the rip-rap down stream from the dam or at a reverse filter. If wooden cribs or loose or articulated concrete blocks were used, it was assumed to end at the up-stream edge of the rip-rap or filter. Although it is true that rip-rap, cribbing, or blocks may assist in preventing piping or a blow-out, it is believed that less uncertainty is introduced by considering that they do not add to the travel path than to assume that they do.

Some discrepancies were found in checking over the data given by Bligh with those available from other sources. When the data on some dams indicated that a certain creep-head ratio had been used in design, it was usually not possible to check this exactly from the dimensions of the structure. In order that all the data might be on the same basis, the same method was used throughout, although this might not give the same ratio that the designer of the dam believed he was using. In all cases the writer's best judgment was used in determining the most probable values for use in this paper. At several points in the analysis of some of the dams it was necessary to make assumptions. One of these was in the case of rows of sheet-piling close together.

In computing the weighted-creep distance it was necessary to determine the division point between steeply sloping and slightly sloping contacts. This division should be at the steepest slope at which, unquestionably, a bank of earth under water would be unstable; that is, at which the bank undoubtedly would be certain to slip. This was assumed to be on a 1 on 1 slope. Any contact with a slope of 1 on 1, or steeper, was considered to be a "vertical" contact and, at a flatter slope, a horizontal contact. In a few cases a puddle or earth-fill against a dam rested on top of a part of the dam masonry. Although this was a horizontal contact, it was assumed to be a "vertical" contact, because the earth would press as closely on the masonry under these conditions as on a vertical surface. There is some evidence to indicate that the contact of puddle with earth is so close that it would be equivalent to vertical creep. In order to be conservative, however, this assumption was not made in computing the ratios used in this paper.

In several cases dams have been built with a filling of dry or broken stone beneath them. This would offer much less resistance to seepage than the contact between solid masonry and foundation material. Where the floor above this stone filling has not been vented, to allow the water to escape, the resistance of the creep line along this section has been assumed to be one-half as great as for solid masonry. This is a conservative estimate from the standpoint of this study, but would not be conservative if used in design. The use of such filling seems to be confined to old dams and is a bad practice. In one case it is believed to have contributed to the failure of a structure. In the case of a number of dams of the buttressed concrete type, no floor was provided between the buttresses, and the water passing beneath the up-stream cut-off could rise in this space and escape through vents provided in the down-stream face of the dam. If these vents were closed, the water would have to pass beneath another section of the dam to reach the tail-water. For dams of this type, the distances and ratios for both conditions have been computed although the shorter creep distance is probably the effective one.

Considerable uncertainty has been introduced in the determination of the creep distance by the presence of weep-holes or vents. In determining the distances used in this paper, weep-holes have not been considered as reducing the creep distance, but a separate analysis of them has been made.

There was sometimes uncertainty as to the head to use in estimating the ratios. In ordinary over-fall dams it was measured from the crest of the dam to tail-water elevation, or, if the latter was not given, to the stream bed below the dam. If crest gates or flash-boards were used, it was measured from their tops. The ordinary operating head was used, although, in some cases (especially in movable navigation dams), the structures, no doubt, were subjected occasionally to greater heads.

Several cases have been found in which dams were built on a layer of porous material, such as gravel or sand, which was underlaid by an impervious layer of clay or hardpan, into which the cut-off walls or sheet-piling extended. These cases are not subject to analysis in the ordinary way and, therefore, the data on them are given without analysis. Masonry dams on earth in which the cut-off was carried to solid rock were not analyzed.

Classification of Foundation Materials

There is a great need of a more accurate and scientific classification of foundation materials than the common terms, gravel, coarse sand, etc. Several classifications on the basis of grain size have been made, but none is entirely satisfactory. One developed by the United States Bureau of Soils[21] has been used extensively in the classification of soils and of materials for earth dams. For dam foundation materials, however, it is objectionable because in its classifications, fine sand, coarse sand, etc., are finer material than the engineer usually has in mind when using these terms. Mechanical analyses of foundation materials could be obtained for only a few dams.

Results of Analysis of Existing Structures

The results of the analysis of the weighted-creep relations for all structures for which sufficient data could be obtained, are given in Table 1. Some of the description in Column (9) of this table can be further elaborated, as follows:

Table 1(a).—Dam No. 12.—A firm, impermeable, red clay, with at least 50% boulders.

Table 1(a).—Dam No. 20.—A very compact and unyielding gravelly hardpan.

Table 1(a).—Dam No. 27.—A part of this dam was situated on clay overlaid by gravel and sand.

Table 1(a).—Dam No. 28.—A part of this dam was situated on clay overlaid by a thin layer of gravel.

Table 1(b).—Dam No. 3.—A layer of gravel and fine material, underlaid by an impervious layer which, in turn, was underlaid by pervious fine material and boulders.

---

[21] "Grouping of Soils on the Basis of Mechanical Analysis", by R. O. E. Davis and H. H. Bennett, *Circular No. 419*, U. S. Dept. of Agriculture.

Table 1(b).—Dam No. 44.—From sand with a small percentage of gravel at the surface to heavy boulders with some sand and gravel at the bottom of the sheet-piles.

Table 1(c).—Dam No. 17.—Graded from silt and fine sand at the surface to coarse sand at the bottom of the sheet-piling.

Table 1(d).—Dam No. 33.—Fine sand at the surface, grading to coarse sand below.

Table 1(d).—Dam No. 37.—Beneath most of this dam there was a layer of clay; but there was a running layer of sand and mud below part of it.

The source of this information is given in detail form in the original manuscript filed in Engineering Societies Library.

It is probable that the larger structures in the United States are still giving reasonably satisfactory results or a record of the failure would have been published. The smaller structures in this country and those in foreign countries are probably still in use, although there is a slight chance that they may have failed.

Most of the failures are dams of poor design, but they serve to show approximately the limits of good design. Well-designed dams rarely fail, but more lessons can be learned from failures than from successes. Some well-known failures are not included, because the data are insufficient or too conflicting to permit reliable conclusions. These include the Hauser Lake Dam, Missouri River, the Grand Barrage on the Nile, and the head-gate of the Southern Alberta Land and Irrigation Company.

Several cases were found in which the data were so complex that they were difficult to analyze. This was the case in the Alcona and Upper Alameda Creek Dams; the dam in the Scioto River, at Columbus, Ohio; that on the Guadalupe River, in Texas; and the Sacramento, Calif., Weir. Additional information as to the reliability of the data on the various structures, the length of the period of service, and other pertinent facts, is given in the original record of this research, previously mentioned.

Important Facts Shown by Experience with Existing Dams

A study of all the available records of percolation distances of existing masonry dams on earth foundations, and of those that have failed from piping, brings out three important facts: (1) Several dams have failed from piping with percolation distances which, judged by the ordinary standards. should be safe; (2) many dams have stood successfully with percolation distances much less than those previously recommended; and (3) the dams that failed had very little of their creep paths along vertical or steeply sloping surfaces, while those that stood, with much smaller distances, had a considerable proportion of such creep.

Failures have occurred with creep distances which ordinary standards would consider safe in the case of the Kulli Bye-Wash (Table 1(*d*), Dam No. 10), and the Deoha Barrage (Table 1(*d*), Dam No. 4). The first of these structures was founded on fine sand and failed with a creep-head ratio of between 14 and 16. For fine sand, Bligh recommends a ratio of

TABLE 1.—DATA FOR ANALYSIS OF WEIGHTED-CREEP RELATIONS

Dam No. (1)	Name (2)	Location (3)	Head, in feet (4)	CREEP DISTANCE, IN FEET		WEIGHTED CREEP, $\frac{H}{3}$		Foundation material (9)
				Vertical (5)	Horizontal (6)	Distance, in feet (7)	Ratio (8)	
			(a) DAMS ON CLAY AND HARDPAN					
	Failures:							
1	Woodward	Flanders Brook, Hill, N. H.	30	24	8	27	0.9	Hard material, probably hardpan
2	Dolgarrog	L. Eigian, N. Wales	10	3	15	8	0.8	Hard blue clay; boulders
3	Fergus Falls	Red River, Minn	23	17	28	26	1.1	Hardpan
	*Non-Failures:**							
10	French Landing Plant	Huron, Mich	32	50	54	68	2.1	Clay
			32	50	8	53	1.6	....
11	Dolgarrog	L. Eigian, N. Wales	10	13	15	18	1.8	Hard blue clay; boulders
12	Marysville	Washwaak River, N. B.	22	24	68	47	2.1	Red clay; 50% boulders
13	Fergus Falls	Red River, Minn	23	70	34	82	3.6	Hardpan
14	Kettle Creek	Kettle Creek, Ont	20.5	18	48	34	1.7	Hard blue clay
15	Shataipore Weir	Vellar River, India	9	27	44	42	4.6	Clay
16	Rupar Weir	Sutlej River, India	11.5	18	100	51	4.5	Clay with boulders
17	Cooke	Au Sable R., Mich	40	76	289	172	4.3	Clay
18	Sangum Anicut	Pennen, India	9.0	52	98	85.0	9.4	Sand on clay
19	Shawano	Wolf River, Wis	15	57	81	84	5.6	Stiff greasy clay
20	Whiting St	Holyoke, Mass	18	20	17	26	1.5	Gravelly hardpan
21	N. Diversion	Orland Project, Calif	4.7	......	5	2	0.4	Red clay; gravel
22	Big Coulee	Big Coulee R., Mont	6	13	15	18	3.0	Clay
23	Sidhnai	Ravi River, India	7.5	35	46	50	6.7	Clay
24	Dam No. 2	Ouashita River, La	14.4	63	31	73	5.1	Dense clay
25	Dam No. 5	Ouashita River, La	6.9	77	34	88	12.8	Hard clay
26	Dam No. 8	Ouashita River, Ark	10.5	41	22	48	4.6	Mostly clay; some sand
			13.5	97	34	108	8.0	
27	Dam No. 5	Mohawk River, N. Y	15	63	77	89	5.9	On clay
28	Dam No. 6	Mohawk River, N. Y	15	63	77	89	5.9	On clay
29	Junction	Manistee River, Mich	50	199	165	254	5.1	Hard clay
			50	205	225	280	5.6	....
30	Hodenphyl	Manistee River, Mich	65	144	381	271	4.2	Clay
31	L. Washington	Seattle, Wash	25	60	190	123	5.0	Hard blue clay
32	Acatlan	Mexico	17.7	51	90	81	4.6	Compact clay
33	Aganoa River	Aganoa, P. I	9.4	67	40	81.0	8.6	Blue clay and sand
34	Santa Barbara	Philippine Islands	9.9	54	48	70	7.0	Clay
35	Cascade Plant	Thornapple, Mich	28	72	93.0	103	3.7	Tough grey clay
36	Edenville Plant	Tittabawassee, Mich	45	47	123	88	2.0	Clay-gravel and hardpan
			45	62	124	103	2.3	Hardpan (very hard)
37	Riley Plant	St. Joseph, Mich	14	48.5	45.0	63.5	4.5	Clay and gravel
38	Sanford Plant	Tittabawassee, Mich	27.5	34.0	68	56	2.1	Clay-gravel and hardpan
			27.5	42.0	68	65	2.4	Clay, gravel, hardpan
39	Secords Plant	Tittabawassee, Mich	48	59	116	98	2.1	Clay-gravel and hardpan
40	Smallwood Plant	Tittabawassee, Mich	27.5	38	84	66	2.4	Clay-gravel and hardpan
			27.5	27.0	68	50	1.8	Clay, gravel, hardpan
41	Tobacco Plant	Tobacco, Mich	45	38.0	128	81	1.8	Clay-gravel and hardpan
42	Barton Dam	Huron, Mich	25	40	53	58	2.3	Gravel and hardpan
43	Black River	Black R., Mich	25	40	8	43	1.7	....
			15	58	52	75	5.0	Clay
44	Built by	Des Moines Elec. Co., Iowa	15	32	3	33	2.2	....
			8	18	4	19	2.4	Clay.
			8	22	33	33	4.1	Clay
45	For Sturgis	St. Joseph, Mich	25	12	8	15	0.6	Hardpan
46	Bassano	Alberta, Canada	48	34	165	89	1.8	Clay over sand

* No failures recorded.

TABLE 1.—(*Continued*)

Dam No. (1)	Name (2)	Location (3)	Head, in feet (4)	Creep Distance, in Feet: Vertical (5)	Creep Distance, in Feet: Horizontal (6)	Weighted Creep, $\frac{H}{3}$: Distance, in feet (7)	Weighted Creep, $\frac{H}{3}$: Ratio (8)	Foundation material (9)
	*Non-Failures:**							
47	Lake St. Francis	St. Francis R., Que	37	46	48	62	1.7	Stiff clay with small boulders and gravel
			37	53	68	76	2.1	
48	Vandalia	Milk River, Montana	26	113	125	155	6.0	Stiff clay and clay with fine gravel
49	Upper Grays	Black Creek, Utica, N. Y.	30	45	110	82	2.7	Good clay, underlaid by hardpan
50	Scranton	Scranton, Pa	15.5	31	12	35	2.3	Sand and yellow clay
51	Baltic Main	Baltic, Conn	28.7	28	40	41	1.4	Clay and gravel
52	Baltic Side	Baltic, Conn	13.7	16	20	23	1.7	Clay and gravel
53	Batavia	Towanda Cr., N. Y	10	16	35	28	2.8	Clay and gravel
54	Bethlehem	L. Ammanoosuc, N. H	23	100	56	119	5.2	Boulders, sand, clay
55	Burnham	Burnham, Me	32	68	105	103	3.2	Clay
56	Kearsley Cr	Flint, Mich	25	106	138	152	6.1	Clay
57	Logan	Logan River, Utah	30	24	56	43	1.4	Sand and gravel
58	Mayfield R.	Mayfield, N. Y	30	50	105	85	2.8	Hardpan
59	Massillon	Nimisilla Cr., Ohio	25.5	43	59	63	2.5	Clay and sand
60	New London	Conn	24	26	31.5	37	1.5	Clay and gravel
61	Newport	Sugar R., Nwpt., N. H	16.5	42	56	61	3.7	Clay with gravel
62	Patillas	Guayama, Porto Rico	5	16	25	24	4.9	Clay and marl
63	Prince Albert	Sask. R., Sask., Can	23	50	125	92	4.0	Gravel and clay
64	Somerville	New Jersey	7.5	27	13	31	4.2	Clay and coarse sand
65	Reservoir	St. Paul, Minn	25	10	25	18.3	0.7	Clay, 50% sand, gravel
66	Wakeman	Vermillion R., Ohio	15	30	42	44	2.9	Clay and shale
67	Wonder Lake	Nippersink Cr., Ill	20	25	68	48	2.4	Clay over sand; gravel
68	Hightstown	Rocky Brook, N. J	11	30	28	39	3.6	Clay, sand and silt
69	Pittsfield No. 2	Millbrook R., Mass	28.7	30	38	42.7	1.5	Clay and gravel
70	Stoney River	Dobbin, W. Va	51	85	60	105	2.1	Boulders, clay, gravel
71	Rodriguez	Tijuana, Mexico	180	300	256	385.3	2.1	Soft clay
72	Swift Current Dam	Sask., Canada	19	63	63	84	4.4	Clay
73	Gonzalez	Texas	18	52	53	70	3.9	Clay
74	Williamson	Sandy Cr., Cisco, Tex	63	72	45	87	1.4	Clay
75	Portland Dam	Portland, Mich	11	12	38	24.7	2.3	Clay
76	Foote Dam	Au Sable River Oscada, Mich.	31.5	100	118	139.3	4.4	Clay
77	Five Channels		26	50	150	100	2.8	Clay
78	Loud Dam		34	53	160	106.3	4.4	Clay
79	Stronach	Manistee River, Wellston, Mich	20	37	30	47.0	2.3	Clay
80	Springfield	Springfield, Vt	15	28	23	36	2.4	Clay
81	Quaro Dam	Texas	30	40	35	51.6	1.7	Clay
82	Redlands	Gunnison R., Colo	12	50	57	69	5.7	Clay and gravel
83	Coleman Br	Chester, N. J	9	26	26	34.7	4.0	Boulders and clay soil
84	Springbank	London, Ont	20	70	34	81	4.0	Dense clay
85	Delhi Weir	Sone River, India	8	26	112	64	8.0	Clay and sand
	(*b*) Dams on Gravel, Cobbles, and Boulders							
	Failures:							
1	Puentes	Gwadalantin R., Spain	143	44	157	96.3	0.67	Sand and gravel; classification uncertain
2	Port Angeles	Elwha River, Wash	79	72	97	104	1.3	Gravel, and coarse sand
3	Pittsfield	West Brook, Mass	33	29	47	45	1.4	Gravel, fines, boulders
4	Coon Rapids	Miss. R., Minn	48	35	127	46	0.95	Sand and boulders
5	Plattsburg	New York	34	44	45	59	1.75	....

---

* No failures recorded.

TABLE 1.—(*Continued*)

Dam No.	Name	Location	Head, in feet	Creep Distance, in Feet		Weighted Creep, $\frac{H}{3}$		Foundation material
				Vertical	Horizontal	Distance, in feet	Ratio	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	Failures:							
6	At Durance	Durance R., France	22	11.2	44.6	50.2	2.3	Founded on gravel
7	Stoney River	Stoney River, W. Va.	39	90	126	132	3.9	Boulders, clay, gravel
8	Retaining wall	Ansonia Canal, Conn.	18	24	36	36	2.0	Founded on gravel with horizontal creep in silt
	*Non-Failures:**							
11	Dam No. 5	Ohio River, Penn.	7.5	43	32	54	7.2	Gravel
12	Dam No. 10	Ohio R., Ohio-W. Va.	8.4	49	35	61	7.2	Gravel
13	Dam No. 6	Ouachita R., Ark.	6.2	47	33	58	9.4	Gravel with fine sand
14	Dam No. 45	Ohio R., Ind.-Ky.	9.0	87	35	99	11.0	Sand and small gravel
15	Dam No. 46	Ohio R., Ind.-Ky.	9.0	87	35	99	11.0	Sand and small gravel
16	Dam No. 51	Ohio R., Ill-Ky.	8.0	87	35	99	12.3	Fine sand and gravel
17	Dam No. 35	Ohio R., Ohio-Ky.	6.4	47	35	59	9.1	Sand and gravel
18	Dam No. 36	Ohio R., Ohio-Ky.	7.9	84	35	96	12.0	Sand and gravel
			7.9	89	80	116	14.6	Sand and gravel
			7.9	80	20	87	10.9	Sand and gravel
19	Dam No. 43	Ohio R., Ind-Ky.	9.0	86	40	99	11.0	Sand and gravel
20	Dam No. 52	Ohio R., Ill-Ky.	12.0	85	40	99	8.2	Sand, gravel, some clay
21	Dam No. 53	Ohio R., Ill-Ky.	13.4	87	35	99	7.4	Sand, gravel, some clay
22	Pinhook	Maquoketa R., Iowa	25	68	61.5	89	3.5	....
23	Richmond	Indiana	5	11	16	16.3	3.3	Gravel and sand
24	Power Intake	Salt River, Ariz.	6.5	43	18	49	7.5	Gravel and sand
25	Island Park	Miami River, Ohio	7.5	37	29	47	6.2	....
26	Puentes	Spain (see No. 1)	75	51	288	147	2.0	
27	Miller Cr.	Miller Creek, Ore.	5.0	14	14	19	3.8	Sand, gravel, and earth
28	Sardorab	Arax R., Russia	16.7	57	108	93	5.5	Coarse sand and gravel
29	Cherrecksky	Arpachay R., Armenia	13.3	35	59	55	4.1	Coarse sand and gravel
30	Limbourg	Meuse R., Holland	12	84	131	128	10.7	Coarse gravel and sand
31	Herr Island	Allegheny R., Penn.	7	45	48	61	8.7	Gravel
32	Mirowitz	Czecho-slovakia	12.8	60	33	71	5.5	Gravel
33	Hamilton	Miami, Ohio	7.5	31	31	41.3	5.5	Gravel
34	Fremont	Sacramento R., Calif.	3.5	21	35	33	9.4	Gravel
35	Dam No. 7	Mohawk R., N. Y.	12	32	63	53	4.4	Gravel
36	Dam No. 13	Mohawk R., N. Y.	8	24	52	41.3	5.2	Gravel
37	Horse Creek	Horse Creek, Wyo.	6.4	18	17	24	3.8	Coarse sand and gravel
38	Mesilla	Rio Grande, N. Mex.	8.7	24	40	37.3	4.3	Medium sand with fair sprinkling of gravel
39	Mandalay	Madaya R., Burma	11.3	10	78	36	3.2	Gravel and shingle
40	Gr. Valley	Colorado R., Colo.	18.2	52	85	80	4.4	Coarse gravel; cobbles
41	Boise	Boise R., Idaho	33	48	48	64	1.9	Compact gravel; cobbles
42	Sherman Island	Hudson River, N. Y.	53	182	161	236	4.4	Sand and boulders
43	Hyatt	Oswegatchie R., N. Y.	23	45	37	57	2.6	Sand; glacial boulders
44	Percha	Rio Grande, N. Mex.	11	59	73	83	7.6	Sand and gravel†
45	Harper	Malheur River, Ore.	12	29	57	48	4.0	Sand and gravel
46	Moore Weir	Cache Creek, Calif.	8	36	30	46	5.8	Sand and gravel
47	Western Jumna Weir	Jumna River, India	8	36	74	61	7.6	Cobbles and boulders

* No failures recorded.

† See further description in text.

TABLE 1.—(*Continued*)

Dam No.	Name	Location	Head, in feet	Creep Distance, in Feet		Weighted Creep, $\frac{H}{3}$		Foundation material
				Vertical	Horizontal	Distance, in feet	Ratio	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	*Non-Failures:**							
48	Dam No. 44	Ohio R., Ind.-Ky	9.0	87	35	99	11	Sand, gravel, boulders
49	Granite Reef	Salt River, Ariz	20	45	32	56	2.8	Gravel and boulders
50	Marseilles Weir	Durance R., France	4.3	51	33	62	14.3	Gravel and boulders
51	Weber-Provo	Weber River, Utah	8.5	28	36	40	4.7	Gravel and boulders
52	Strawberry	Span. Fork R., Utah	15	42	63	63	4.2	Gravel and boulders
53	Greeley Intake	Cache La Poudre, Colo	5	34	50	51	10.2	Sand, gravel, boulders
54	Avigononet	France	56	95	171	152	2.7	Gravel, from cobble size to very fine sand
55	Byrmus Paper Company	W. Dudley, Mass	7.6	64	19	70	9.3	Gravel with boulders
56	Radin	Germany	9.7	39	46	54	5.6	Firm gravel
57	Talavera R	Philippine Islands	8.2	18	57	37	4.5	Sand, gravel, boulders
58	Lujan Weir	Argentina	8.2	32	37	45	5.4	Gravel and sand
59	Puntilla Weir	Argentina	15.8	32	50	49	3.1	Gravel and sand
60	Tunuyan Weir	Argentina	8.5	19	31	29	3.5	Gravel and sand
61	Twin City (St. Paul)	Miss. R., Minn	28.5	66	101	99.6	3.5	Sand, gravel, and broken limestone slabs
62	St. Mary's	Sun R. Project, Mont	5.5	20	38	33	6.0	Sand, gravel, boulders
63	Scotland	Shetucket R., Conn	24.4	83	134	128	5.3	Gravel, small boulders
			24.4	82	102	116	4.8	
64	Camp Humphries	Springfield, Va	8	40	12	44	5.5	Coarse sand and gravel
66	Flint R	Flint River, Ga	24	30	105	65	2.7	Broken limestone
67	Oneonta	Susquehanna, N. Y	11	25	30	35	3.2	Sand and gravel
68	Piedmont	West Virginia	12	18	14	23	1.9	Boulders
69	Post Brook	Post Brook, N. J	7.5	50	21	57	7.6	Gravel and boulders
70	Rahway	Raritan R., N. J	7	45	19	51	7.3	Porous gravel
71	Rolling Fork River	Lebanon, Ky	8	30	15	35	4.4	Gravel and boulders
72	Sarda Barrage	Sarda River, India	14	36	138	82	5.9	Shingles and boulders
73	Stamford	Rippowan R., Conn	10	20	16	25	2.5	Gravel and sand
74	Walkill	Front Brook, N. Y	17	14	22	21	1.2	Boulders and gravel
75	Woodstock	Vermont	24	40	40	53	2.2	Loose gravel
76	Juniata River	Huntington, Pa	26	50	60	70	2.7	Cemented gravel overlying hardpan
77	Shoshone	Big Horn R., Wyo	49.5	67	83	94.6	1.9	Boulders
78	Cochiti	Rio Grande, N. Mex	8	17	80	44	5.5	Sand, gravel, cobbles
79	Sixth Lake	Inlet, N. Y	11	18	30.5	28.2	2.6	Gravel and boulders
80	Reservoir	Muskogee, Okla	30	6	105	41	1.4	Gravel and boulders
81	Newton Lower Falls	Boston, Mass	9	40	20	46.7	5.2	Boulders and gravel
82	Dunbarton	Greenwich, Conn	13	36	48	52	4.0	Gravel and boulders
83	Dellwood	Joliet, Ill	20	42	26	50.7	2.5	Sand and gravel
84	Graham Lake	Union River, Me	32	104	90	134	4.2	Gravel and boulders
85	Grants Pass	Rogue River, Ore	38	64	70	87.3	2.3	Gravel
86	Upper Creek	Alameda Cr., Calif	29	70	110	106.6	3.7	Gravel
87	Dam No. 1	Nueces River, Tex	50	120	70	143.3	2.9	Gravel
88	Dam No. 2	Nueces River, Tex	20	62	77	87.7	4.4	Gravel

* No failures recorded.

TABLE 1.—(*Continued*)

Dam No. (1)	Name (2)	Location (3)	Head, in feet (4)	CREEP DISTANCE, IN FEET		WEIGHTED CREEP, $\frac{H}{3}$		Foundation material (9)
				Vertical (5)	Horizontal (6)	Distance, in feet (7)	Ratio (8)	
			(*c*) DAMS ON COARSE SAND					
	Failures:							
1	Flederborn:							
	(Probable)......	Kuddow, Germany.......	23	49	89	79	3.4	Sharp sand underlaid by fine sand
	(Possible)......		23	76	89	106	4.6	
	*Non-Failures:**							
11	Upper Coloroon...	Coleroon R., India.........	5.3	24	44	39	7.4	Pure, coarse sand
12	Burra...........	Mahanuddee R., India.....	12	40	69	63	5.3	Coarse sand
13	Riverdale.........	Apple River, Wis..........	24	39	62	60	2.5	Torpedo sand
14	Sidhnai...........	Ravi R., Punjab, India....	7.5	49	53	67	8.9	Coarse sand
15	Godaveri R.......	Madras, India............	10.8	48	52	65	6.0	Coarse sand
16	Moss Bluff.......	Oklawaha River, Fla.......	11.5	159	40	172	15	Graded stratified sand
17	Florence..........	Gila River, Ariz..........	10.5	68	102	102	9.7	Silt and sand†
18	Dam No. 1.......	Big Sunflower R., Miss.....	16.9	105.4	34.5	116.9	6.92	Sand with a little clay, gravel, and clayey sand
20	Mezali...........	Mon Canal, Burma........	11	23	199	89	8.1	Pebbles and sand
21	Prairie du Sac.....	Wisconsin River, Wis.....						Pure, coarse sand
	Power-house....		32	160	141	207	6.5	
	Dam...........		32	130	68	153	4.8	
	Dam-vents open		29	100	24	108	3.7	
22	Columbia........	Paulinskill, N. J...........	26	66	31	76	2.9	Coarse sand
			26	64	35	76	2.9	
23	Cranford.........	Rahway River, N. J.......	6	16	20	23	3.8	Coarse sand
24	Gloucester........	Massachusetts............	23.5	42	35	54	2.3	Coarse sand
			19.5	42	68	65	3.3	
25	C. Humphries.....	Accotink Cr., Va..........	8	40	12	44	5.5	Coarse sand to gravel
26	Bound Brook.....	Raritan R. and Millstone R., N. J.......	5.7	6	24	14	2.4	Coarse sand
27	Upper Nile Canal..	Colorado.................	16	60	31	70.3	4.4	Coarse sand
28	Watertown.......	Wisconsin...............	15	48	20	54.7	3.65	Coarse sand
29	Leavittsburg......	Ohio....................	15	42	58	61.3	4.09	Coarse sand
30	Highland Pk......	Detroit, Mich.............	14.5	4	140	50.7	3.5	Coarse sand
31	Gillespie..........	Gila River, Ariz..........	20	60	68	82.7	4.1	Sand and gravel
32	Garfield..........	S. Pasadena, Calif.........	16	6	80	32.7	2.05	Sand and gravel
33	Wausau..........	Wisconsin R., Wis.........	32	230	98	262.6	8.20	Coarse sand
			(*d*) DAMS ON SAND, FINE SAND, AND SILT					
	Failures:							
1	Narora...........	Ganges, India............	13	45 44 20	50 99 50	62 77 37	4.7 5.9 2.8	Fine sand
2	Khanki (Lower Chenab)........	Punjab, India.............	12	15	104	50	4.2	Fine sand
3	Corpus Christi....	Nueces, Texas............	37	39	115	77	2.1	Sand
4	Deoha...........	Deoha, India.............	12	60	180	120	10.0	Fine sand
8	Kitcha Bye.......	Rohilkhand, India.........	8	22	36	34	4.3	Dry sand
9	Kulli Dam........	Rohilkhand, India.........	......	......	......	......	......	Fine sand
10	Kulli Bye........	Rohilkhand, India.........	......	......	......	......	......	Fine sand
11	Nadrai Escape Fall	Lower Ganges, India.......	20	33	76	58	2.9	Light sandy soil
12	Harmon Park.....	Lebanon, Ohio...........	6 6.5	6 8.5	9.6 11	9.0 12	1.5 1.9	Quicksand and gravel
	*Non-Failures:**							
18	Deoha...........	Deoha R., India..........	12	140	180	200	16.7	Sand
19	Kabo Weir.......	Mu River, Burma.........	10.8 10.0	42 58	177 129	101 101	9.4 9.4	Sand
20	Delhi............	Sone, India..............	8.0	26	112	63	7.9	Clay and sand
21	Dowlarshwaram...	Godaveri Delta, India......	10	26	164	81	8.1	Sand
22	Ferozepore Weir..	Sutlej, India..............	22	92	192	155	7.0	Fine sand at surface; then coarse sand to clay
23	Islam Weir.......	Sutlej, Punjab...........	18	79	175	137	7.6	Sand
24	Jamrao...........	Sind, India..............	8	42	111	79	9.9	Fine sand

* No failures recorded. † See further description in text.

TABLE 1.—(*Continued*)

Dam No.	Name	Location	Head, in feet	Creep Distance, in Feet		Weighted Creep, $\frac{H}{3}$		Foundation material
				Vertical	Horizontal	Distance, in feet	Ratio	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	*Non-Failures:**							
25	Khanki	Punjab, India	13	46	168	102	7.8	Fine sand
			13	75	135	120	9.2	Fine sand
26	Adimapalli	Penner, India	8.5	54	68	77	9.0	Sand and rock
27	Lloyd	India	18.5	114	190	177	9.6	Sand and silt
28	L. Jhelum	Jhelum River, India	10	57	195	122	12.2	Fine sand
			10	92	134	137	13.7	Fine sand
29	Merala	Chenab River, India	12.5	131	121	171	13.7	Fine sand
30	Narora	Ganges R., India	13	59	142	106	8.2	Fine sand
31	Nadrai Escape Fall	Lower Ganges, India	20	19	176	78	3.9	Light sandy soil
32	Nzeswaran	Godaveri Delta, India	10.7	46	130	89	8.3	Sand
33	Panjnad Weir	India	9	92	202	159	17.7	Sand †
	Panjnad Annex Weir	India	10	139	163	193	19.3	
34	Assuit	Nile River, Egypt	8.2	53	138	99	12	Water-laden fine sand
35	Isna (Esna)	Nile River, Egypt	8.2	52	139	98	12	Fine sand
36	Nag Hannadi	Nile River, Egypt	14.7	72	195	137	9.3	Fine sand; some coarse
37	Zifta	Nile River, Egypt	13	54	156	106	8.1	Clay, sand, and mud †
38	Hindia	Euphrates, Mesopotamia	16.4	69	292	166	10	Fairly hard silt
39	Sultan Bend	Murgab R., Turkestan	24	168	207	238	10	Probably loess
40	Su Chuang:							
	Regulator	Pei Ho, China	11.5	83	91	113	9.9	....
	Sluice		13.1	83	116	121	9.3	Loess and sand
41	Versen	Ems River, Germany	8.6	76	28	85	10	Sand
42	Ketchetovskaia	Don River, Russia	11.2	90	116	129	11.5	Sand
43	Angat River	Angat, Philippine Islands	9.8	83.4	53.2	101	10.2	Sand, gravel; some mud
44	Dam No. 48	Ohio R., Ind.-Ky	9	84	35	96	10.6	Very fine sand
45	Dam No. 51	Ohio R., Ill.-Ky	8	87	35	99	12.3	Fine sand; some gravel
46	Dam No. 47	Ohio R., Ind.-Ky	9	87	35	99	11.0	Fine to medium sand
47	Rockwood Heading	Colorado R., Calif	4.5	56	45	71	15.8	Very fine silt
48	Middle Loup	Mid. Loup R., Nebr	10	134	13	138	13.8	Sand, probably fine
49	Iron Mt	Menominee R., Mich	30	119	138	165	5.5	Fine sand
50	Mendota	San Joaquin R., Calif	10	56	50	72.6	7.3	Drifted sand
51	Cambian	Nepan River, N. S. W.	6	28	62	49	8.2	Sand
52	Sandy Lake	Minnesota	12	129	115	167	14.0	Sand
53	Bonnet Carre	Mississippi R., La	14.5	149	103	183	12.6	Loam, silt, and clay
54	BeardsleyC'n'lWeir	Kern River, Calif	7	32	84	60	8.6	Fine sand and silt
55	Leasburg	Rio Grande, N. M. and Tex.	7	50	30	60	8.6	Sand
56	Mottville Plant:							
	Sec. BB	St. Joseph, Mich	13	38	66	60	4.6	Quicksand
	Sec. CC		13	52	41	66	5.0	
	Sec. DD		13	48	40.3	61.4	4.7	
57	Built for	Lowell, Mich	17.5	70	28	79	4.5	Sand
			17.5	52	2	53	3.0	
58	Built for	Bellaire, Mich	15	54	44	69	4.6	Fine, clear sand
			15	36	30	76	3.1	
59	Akron	Cuyahoga, Ohio	35	115	100	148	4.2	Sand and silt
60	Anadarko	Washita, Okla	26.5	65	57	84	3.2	Sand
61	Barre	Massachusetts	18	40	30	50	2.8	Sand, fine sand, silt
			15	50	67	72	4.8	
62	Miami	Miami R., Ohio	10	28	39	41	4.1	Sandy silt
63	Oklahoma	Canadian R., Okla	8	40	30	50	6.3	Sand and shale
64	Wiggins	Nice Canal, Colo	10	48	41	62	6.2	Quicksand
65	Okla. City	N. Canadian R., Okla	12.0	38	34	49.3	4.1	Fine sand and silt
66	Cotulla Dam	Nueces River, Tex	22	60	75	85	3.9	Fine sand and silt
67	Rogers Dam	Muskegon R., Mich	35	90	89	119.6	3.4	Sand and silt
68	Oradell Dam	Hackensack R., N. J	16	102	72	126	7.9	Sand and silt
69	Milliken	Nueces River, Tex	22	52	82	79.3	3.6	Fine sand and silt
70	Croton Dam	Muskegon R., Mich	39	128	275	219.6	5.6	Sand and silt
71	Wichita River	Wichita Falls, Tex	9.5	32	180	92	9.7	Sand
72	Rice's Rip	Maine	28	96	66	118	4.2	Sand

* No failures recorded.

† See further description in text.

15 and Griffith, 14½ to 18 for fine micaceous sand, and 12½ to 14 for fine quartz sand. The Deoha Barrage on fine sand was threatened with a creep-head ratio of 17 and, finally, failed with a piping path with a length at least 26.6 times the head. None of these structures had vertical cut-offs.

The second fact is based on the data on dams given in Table 2, which show that much lower ratios than were recommended by Bligh or Griffith have been used successfully. As Bligh and Griffith do not give ratios for material of the exact description of that existing at some of these dams, the writer has used his best judgment in estimating from the classifications which they did give, what they would have probably recommended.

TABLE 2.—Comparison of Creep-Head Ratios

Dam	Material	Creep-head ratio	Recommended Ratios (Approximate)		Actual Ratio in Percentage of Recommended:	
			Bligh	Griffith *	Bligh	Griffith
Dam No. 48, Ohio River	Very fine sand	13.2	18	11.6–12.8	73	103–113
Dam No. 47, Ohio River	Sand, very fine to medium; a little fine gravel	13.5	18	11.6–12.8	75	105–116
Barre, Mass	Fine sand and silt	8.0	18	11.6–12.8	44	62– 69
Iron Mountain	Fine sand	8.6	15	10 –11.2	57	77– 86
Riverdale	Coarse sand	4.2	12	8 – 9.6	35	44– 52
Prairie du Sac	Coarse sand	4.3	12	8 – 9.6	36	45– 54
		6.2	12	8 – 9.6	52	65– 77
Columbia	Coarse sand	4.0	12	8 – 9.6	33	41– 50
Dam No. 1, Big Sunflower River	Sand with a little clay, gravel, and clayey sand	6.6	14	8	47	83
Tunuyan Weir	Sand and gravel	6.0	9	6.4	66	94
Granite Reef	Gravel and boulders	3.2	5–9	4.8	36–64	66
Walkill, N. Y	Gravel and boulders	2.6	5–9	4.8	30–52	54
Oswegatchie River	Sand and boulders	3.6	5–9	6.4	40–72	54
Piedmont	Boulders	2.7	4	3.2	68	85

* These values include the reduction for vertical staunching.

Considering the two failures previously mentioned, in which the creep distances were substantially equal to the recommended values in connection with the ratios in Table 2 (in which the ratios are much less than recommended), it is a significant fact that the group that failed had nearly all the creep distance horizontal and the other had substantial parts of its percolation distances along vertical or steeply sloping surfaces. In fact, not a single case was found in which piping unquestionably took place under a deep cut-off, and only one case, the Flederborn Dam (Table 1(*c*) Dam No. 1), in which it may have. The data on this failure are so incomplete that the piping may have been behind the abutments, rather than under the dam; but even if it was under the dam it probably took place at ratios much less than those recommended in this paper and, certainly, at ratios somewhat less. The failures of the Coon Rapids and Plattsburgh Dams (Table 1(*b*), Dams Nos. 4 and 5) were apparently due to defective construction, and that of the Pittsfield Dam (Table 1(*b*), Dam No. 3), to a blow-out directly through the foundation material from a porous layer beneath the impervious layer on which the dam was founded and not along the line of creep.

The logical conclusion from these facts is that in the design of masonry dams on earth foundations, greater weight should be given to the creep along vertical or steeply sloping surfaces than along horizontal or slightly sloping ones. This has led the writer to the development of the weighted-creep analysis of existing dams and dam failures.

ORIGIN OF THE WEIGHTED-CREEP ANALYSIS

The writer's attention was first directed to the greater value of vertical creep by the difficulty of explaining the stability of the Prairie du Sac Dam, in Wisconsin (Table 1(*c*), Dam No. 21), with its extremely low plain creep ratio of 4.3 on a coarse sand foundation. The evidence from one structure, however, was not sufficient to justify a general conclusion.

Frequently, when one forms a correct impression and thoroughly studies the literature of the subject he finds the same idea expressed by others. There are few absolutely unique ideas in engineering. The idea apparently was expressed first by Griffith, who suggested in his abridged paper[6] that a reduction could be made in his suggested creep-head values for reliable vertical staunching. He explains the greater effectiveness of vertical creep as follows:

> "First, the line of creep is subject to a greater pressure where carried to a greater depth; secondly, at the greater depth larger and heavier sand will probably be met; and, thirdly, sand carried out must be lifted up the curtain wall, presumably requiring greater velocity of percolation."

Mr. Griffith kindly supplied the data on the Kulli Bye-Wash and expressed the conviction that his creep-head ratios were probably too low unless some vertical staunching was used. The writer's explanation for the difference seems to have been independently reached by Mr. A. A. Musto[22].

The effect of sheet-piling in reducing upward pressure and cutting down the flow through the voids in the foundation material has been discussed at length by Khosla[15] and Leliavsky[14]. Both seem to have reached the conclusion that such piling reduces the pressure by a greater amount per unit of contact length than horizontal creep.

RELATIVE WEIGHTS OF VERTICAL AND HORIZONTAL CREEP

The proper relative weight to give to vertical and horizontal creep can be determined only by the analysis of actual dams, although some light may be thrown on it by an analysis of observation of upward pressure on actual dams. While the results of the analyses of all available data on percolation distances of dams do show conclusively that the horizontal creep distance is not as effective in resisting piping as vertical creep distance, they do not indicate exactly what their relative effectivenesses are. Although several dams without appreciable vertical creep have failed with distances that normally would be considered sound, no failures were found with considerable vertical creep, except at ratios much less than those ordinarily considered safe, and, therefore, the upper limit of weight must remain uncertain.

A careful study of the upward pressure measurements on several structures on which data were available shows that the drop in upward pressure along horizontal concrete surfaces is almost zero, and along a puddle surface somewhat more, indicating that for horizontal concrete surfaces the ratio might be as low as zero. It does not necessarily follow, however, that the safe ratio for horizontal creep in considering piping failure is the same as in considering upward pressure. More study along this line is very desirable.

---

[22] *Paper No. 142*, Punjab Eng. Congress, 1930, p. 212 *i*.

In arriving at the decision to use a weight of one-third for horizontal creep the writer listed all the structures having creep distances less than, or near, what might be considered the lowest safe limit, together with the material on which they were founded and the weighted creeps for each, with weights for horizontal creep of 0.5 and 0.33. At the same time, a table was made of the weighted-creep ratios which the results of analyses of actual dams indicated to be safe for the various creep ratios.

In view of the uncertainty as to how low a ratio to use for horizontal creep, it is advisable to be conservative. As the present use of unity for this weight has generally produced safe structures (although frequently unnecessarily conservative), it is not desirable to depart too far from the present standards until data definitely establishing the safety of the new ones are available. The writer, therefore, recommends a weight of one-third for the horizontal creep, although there are considerable data to indicate that a higher ratio might be better.

It should be recognized that the one-third weight may not be the best possible ratio in all cases. Further research may show that it should be varied, depending on a number of factors, and study to bring about such relations should be encouraged. The ratio recommended, however, seems to be the best that can be adopted with the present knowledge. To recommend a definite quantity involves the danger that it will be followed blindly by some, without consideration being given to the manner in which it was derived. There seems to be no way by which this danger can be avoided. Since the studies have shown that, for the present, the one-third ratio seems to be the best that can be reached, to fail to give it would require that each individual dam designer go through a long process of deriving a value which he considered satisfactory, and unless he had many new data, it is unlikely that he would arrive at one materially better than that suggested. It is doubtful whether many designers, particularly when working on small dams, would have the time available for such studies. The best method, therefore, seemed to be to give the value which from the available information, seemed to be most satisfactory, with the warning that it probably did not represent a final solution, and to present, as fully as possible, the data on which it was based, in order that those who desired might arrive at their own conclusions, and that when further data became available a new determination might be made. If no value of weighted creep was suggested, those who might adopt it blindly would equally blindly adopt one of the old Bligh values; and since this cannot be prevented, it would seem better to have them adopt a weight of one-third, since it would probably give them a better creep distance.

SUGGESTED WEIGHTED-CREEP RATIOS

In Table 3 are given the weighted-creep ratios which the analysis of all available data from existing structures (and particularly from those in Tables 1 and 2) indicate as necessary for safety against failure from piping along the contact of the structure and its foundation. In order to use these values with safety the cut-offs must be of solid masonry built in contact with the earth sides of the trench, or of interlocking steel or concrete piling driven so that the interlock is not broken, and so that it is satisfactorily embedded at the top in the masonry structure. They also assume competent supervision during construction and efficient maintenance afterward. These values are for major structures. Somewhat smaller values may be used for less important structures, ranging down to perhap 80% of those given for those of minor importance.

TABLE 3.—COMPARISON OF WEIGHTED-CREEP RATIOS
(Weight of Horizontal Creep, One-Third)

Material	Safe weighted-creep ratio	Bligh's value
Very fine sand or silt	8.5	18
Fine sand	7.0	15
Medium sand	6.0	...
Coarse sand	5.0	12
Fine gravel	4.0	...
Medium gravel	3.5	...
Gravel and sand	...	9
Coarse gravel, including cobbles	3.0	...
Boulders with some cobbles and gravel	2.5	...
Boulders, gravel, and sand	...	4 to 6
Soft clay	3.0	...
Medium clay	2.0	...
Hard clay	1.8	...
Very hard clay, or hardpan	1.6	...

There are so many types of foundation material that it is impossible to give values for all. Only the usual types are given, therefore, and the other conditions can be determined by comparison with these. The values for medium and soft clay are somewhat uncertain as no record was found of dams founded on these materials. If the requirements of bearing pressures can be met, the values given would seem to be sufficiently conservative.

RECOMMENDED VALUES ARE CONSERVATIVE

The recommended values have intentionally been made quite conservative. Not a single failure was found where the dam had creep-head ratios as large as those given. It is possible that future experience may show that lower values are unquestionably safe. Quite a number of dams with lower values have been built and have given satisfactory service over a period of years. Some experienced engineers use considerably lower values. In a number of cases, however, failures have occurred suddenly and without previous warning in structures that have given satisfactory service over a long period, ranging up to twenty years. If a set of values which were somewhat too low came into general use, it might be a long time before their insufficiency would be demonstrated and a large number of insecure structures would be built in the meantime. It is better, therefore, to take a conservative view and make a small step in the right direction permitting more experience to accumulate before taking another step, than to run the risk of overstepping and having to go back.

The present form of this paper has been a gradual development and is the third step in this process which the writer has made. In the first report which was based on data from approximately 100 dams, he recommended the plain creep theory with somewhat lower values than those recommended by Bligh. As data from the Deoha Barrage and Kulli Bye-Wash became available, the weakness of the plain creep analysis became evident and a second report was prepared, based on data from 120 dams recommending a set of values with a weight for horizontal creep of one-half. Later data, largely furnished by the firms of Ambursen Dam Company; Ayres, Lewis, Norris, and May; and Holland, Ackerman, and Holland which included many dams with a high proportion of vertical creep, gave adequate bases for using smaller relative weight to horizontal creep. The present paper was prepared, therefore, based on a study of more than 200 dams, using a weight for horizontal creep of one-third.

Low values of creep for dams founded on sand have been recommended by Koenig.[8] The shape of dam and sheet-piling advocated by him give weighted creeps varying from 8.5 for low dams to 3.6, or less, for high ones. It is doubtful, however, whether his recommendations for higher heads were based on actual experience.

An examination of minimum weighted-creep ratios suggested as a result of this study shows that they are somewhat less than one-half those recommended by Bligh. Table 3 offers a comparison.

This does not mean that Bligh's values are more than 100% too large, or that dams designed according to the weighted-creep ratios will have only one-half the percolation distance that dams have had in the past. The difference does not represent as radical a change from the present custom as a comparison of the ratios would suggest. The requirements of length to protect against scour, and to secure sufficient bearing pressure, will still necessitate considerable horizontal creep. For dams, on coarse gravel, including cobbles and boulders with some cobbles and gravel, the necessity of conserving all the water in some cases may require greater percolation distances, because the values given may permit too much seepage.

The effect of the adoption of the weighted-creep method will be to put the emphasis on deep cut-offs, and will lead to their use as far as other limitations will permit. It will tend to eliminate the wide masonry-floor type without substantial cut-offs. For example to meet the weighted-creep requirements for a weir on fine sand with no vertical creep, would require an ordinary creep distance of 3 × 7 = 21 times the head, while the ordinary creep distance according to Bligh should be 15 and according to Griffith, 12½ to 16.

Where it is possible to obtain a detailed knowledge of the extent and position of impervious or relatively impervious layers that may exist, the dam should be designed in the light of this knowledge in which case the recommended values would be merely guides. They are intended for use directly only where such information is not available, or where the borings indicate no persistent or well-defined impervious parts of the foundation material.

Because of the emphasis which the weighted-creep method puts on deep cut-offs if the greatest economy is to be secured, it will be necessary to investigate more fully than before the length of apron necessary to prevent scour, and the best shape of dam to accomplish this purpose. The pressure on the designer will be toward using as short an apron as possible and unless careful studies are made there will be a tendency to adopt one that is too short. It should be kept in mind that the weighted-creep method deals with piping only, and the requirements of scour may dictate much more expensive structures than safety from piping would necessitate. The recent advances in hydraulic model tests have clearly shown the reliability of this means of determining the conditions necessary to prevent scour. The design of important structures, therefore, should include a hydraulic model study.

Because of this emphasis on deep cut-offs there will be a tendency to use deep, single lines of sheet-piling. Evidence is accumulating, however, which shows that it is unwise to "put all one's eggs in one basket," and for important structures especially two lines of shorter piling would be much preferable to one line of long piling, as there would then be a "second line of defense". Failures of these structures have already been recorded due to defects in the sheet-piling. These sometimes occur even with reliable engineers and contractors, and the probability of serious results is much less with two lines of piling.

Grouting Beneath Horizontal Parts of Dams

Since the available evidence indicates that there is likely to be a poor contact between the foundation and horizontal parts of dam foundations, improving this contact by grouting is worthy of consideration. This could usually be done without great expense through holes drilled in the floor-slabs. High pressures could not be used because they would lift the floor. By starting at one end of the structure and working toward the other, it is believed that good contact could be secured. If possible, the grouting should be done before the head came on the structure, as otherwise much of the cement might be carried away in the flowing water.

Reverse Filters, Weep-Holes, and Drains

Another factor that should be considered in selecting the proper ratio to use in the design of a masonry dam on a porous foundation is the possibility of using reverse filters, weep-holes, or drains. The reverse filter was probably first developed by Mr. J. S. Beresford and seems to have been used extensively about 1914. It was used on the Zifta and Assuit Barrages, on the Nile River, and on the Hindia Barrage, in Mesopotamia (Table 1(*d*), Dams Nos. 34, 37, and 38). It has been used in this country on Dam No. 7, at Amsterdam, N. Y., and Dam No. 13, at Yost, N. Y., on the Mohawk River (Table 1(*b*), Dams Nos. 35 and 36). It consists of a filter placed at the lower edge of the down-stream apron, built up of a bottom layer of fine material surmounted by other layers of progressively coarser material. It is called a reverse filter because the position of the layers is the reverse of that in ordinary filters, but the water passes first through the fine and then through the coarse material as in other filters. Its only function is to filter the water seeping beneath the dam and to prevent the removal of any of the fine foundation material. Although usually covered with rip-rap, the reverse filter is likely to be damaged or destroyed by the water passing over the dam at high velocity.

Weep-holes through the apron of the dam or the wing-walls have been extensively used in America, but no examples have been found of their use in other countries. Most of the available literature on India and Egypt, however, was published many years ago, and it is possible that they have come into use there since then. Their function is usually to relieve upward pressure beneath the apron of the dam and thus to permit the use of a thinner apron without danger of its being lifted from under-pressure. In recent years, pipe drains have been utilized in some cases instead of weep-holes. The greatest advantage is secured by combining the functions of the reverse filter and weep-hole by constructing a filter behind the weep-holes or around the pipe drains. In this position the filter is protected from scour.

Since piping failure results from the removal of foundation material from beneath the dam, if a filter prevents the removal of this material, the dam will be safe, even if large quantities of water are passing beneath it. The quantity that passes through the filter depends on the head acting on it. In ordinary filtration practice, a filter is frequently subjected to a head of water several times its thickness. If an ordinary reverse filter were subjected to such a head it would be lifted bodily. Such filters, therefore, cannot be subject to much head. However, if the filter is weighted down by a sufficient thickness of concrete apron, it can be subjected to a much higher head than a reverse filter, and the dam will thus be made safe for the passage of a greater quantity of water beneath it. With such a filter near the end of the percolation path of the water flowing beneath a dam, a greater velocity of flow in this water is permissible and, therefore, a shorter percolation path can safely be used.

Weep-holes and drains have been used with very short percolation distances between them and the head-water. In one case, a flow through the weep-holes sufficient to cause geysers several inches high occurred without apparent ill effects.[23]

A striking example of the effectiveness of vents came to light at the Coon Rapids Dam, in Minnesota (Table 1(*b*), Dam No. 4). This is an ogee structure having a thin down-stream apron with steel sheet-piling cut-offs under the heel of the dam at the down-stream edge of the thin apron. Immediately behind the up-stream cut-off a drain was constructed, consisting of a reverse filter discharging into a small tunnel built in the dam. Due to an accident, one of the steel sheet-piles of the up-stream cut-off was pushed down far beyond its proper position, leaving an opening in the cut-off, and reducing the creep distance to the drain to 0.6 times the head. In spite of this the flow through the drain does not seem to have been sufficient to excite alarm and the defect in the cut-off was not discovered until a large hole washed below the dam was pumped out during repairs to the apron and a blow-out occurred. Had the drain not existed it seems quite probable that the thin concrete apron below the dam would have been blown up.

---

[23] *Transactions*, Am. Soc. C. E., Vol. 88 (1925), p. 319.

Table 4 gives data on dams using weep-holes, drains, or reverse filters, together with the creep head, weighted-creep head, and short-path ratios to the most up-stream vent, and other pertinent data. The low ratio resulting from the short distance from the head-water to the up-stream vent in many cases is striking.

The writer does not believe that it is safe to use low ratios for drains or weep-holes, except where they are combined with a reverse filter. It is probable that the use of a drain without a filter contributed to the failure of the abutment of the Corpus Christi Dam (Table 1(*d*), Dam No. 3). For many years, the U. S. Bureau of Reclamation has used weep-holes with filters of gravel beneath with good success. Experience at the Lost River Dam[24] shows that weep-holes without filters, or other coarse material, behind them are likely to clog. Although weep-holes and drains are valuable devices, the greatest care must be exercised in their construction. One that will permit the foundation material beneath the dam to pass through in effect, is a pipe already partly formed, and certainly no such weak spot in the protection of the dam should be constructed deliberately with the impression that it was adding to the safety of the structure.

Weep-holes or drains have been used on dams of practically all classes of material. The necessity of a well-constructed filter is especially evident in the case of weep-holes or drains with clay, silt, or fine sand foundations, as the fine material could readily pass out through them unless restrained by an effective filter. In gravel, a filter probably would form naturally by the washing out of the fine material near the weep-hole or drain, in the same manner as a filter tends to form around the screen of a driven well. However, in view of the small expense in installing a filter behind the weep-hole, it is believed that their construction is justified even in gravel. They are absolutely necessary in clay or silt, if weep-holes are depended upon to reduce the length of the percolation path required.

The location of the weep-holes and filters should be given considerable study in dam design. From the standpoint of the prevention of piping, the best location for the filter is probably beneath the apron at the down-stream end just up stream from the cut-off wall or row of sheet-piling which is usually placed at the down-stream edge of the structure, the weep-holes discharging through the wall or piling into the rip-rap or other scour protection material. The down-stream wall or row of piling in this case does not act so effectively as a cut-off, but is very desirable to prevent undermining. It need be no longer, therefore, than is necessary to accomplish

---

[24] *Engineering News*, Vol. 71, April 30, 1914, p. 968.

TABLE 4.—DATA ON WEEP-HOLES, DRAINS, AND REVERSE FILTERS (FOR DESCRIPTIVE NOTES, SEE TABLE 1)

Dam	Type	Head	PLAIN METHOD				WEIGHTED METHOD, $\frac{H}{3}$				SHORT-PATH METHOD				Remarks
			Total distance	Creep ratio	Total distance to weep-holes	Vent ratio	Total distance	Creep ratio	Total distance to weep-holes	Vent ratio	Total distance	Creep ratio	Total distance to weep-holes	Vent ratio	
(a) WEEP-HOLES															
Agauca R.; Irr. Sys.	Ogee, crest and apron	9.4	106.9	11.4	86.2	9.2	81	8.6	60	6.4	74	7.9	63.1	6.8	Weep-holes
Santa Barbara; Irr. Sys.	Ogee, crest and apron	9.9	101.5	10.3	94.7	9.6	70	7.0	70	7.0	70.9	7.2	67.9	6.9	Weep-holes
Bassano, Canada	Ambursen	46	199	4.3	36	0.8	89	1.5	27	0.6	177.5	3.9	29	0.6	....
Vandalia	Ambursen	27	304	11.3	151	5.6	177	6.5	99	3.7	....	....	....	....	....
Upper Grays	Ambursen	30	155	5.2	30	1.0	82	2.7	27	0.9	....	....	....	....	....
Batavia, N. Y.	Ambursen	10	51	5.1	14	1.4	28	2.8	13	1.3	41.6	4.2	12.3	1.2	....
Bethlehem, N. H.	Ambursen	23	156	6.8	66	2.9	119	5.2	62	2.7	....	....	66	2.9	....
Burnham, Me.	Ambursen	32	173	5.4	35	1.1	103	3.2	31.7	1.0	....	....	35	1.1	....
Kearsley Creek, Mich.	Ambursen	25	244	9.8	67	2.7	152	6.1	66.3	2.7	....	....	67	2.7	....
Mayfield, N. Y.	Ambursen	30	185	5.1	45	1.5	85	2.8	41.7	1.4	....	....	45	1.5	....
Massillon, Ohio	Ambursen	25.5	102	4.0	32	1.3	63	2.5	30	1.2	....	....	32	1.3	....
Newport, N. H.	Ambursen	16.5	98	5.9	35	2.1	61	3.7	34.3	2.1	....	....	35	2.1	....
Prince Albert, Can.	Ambursen	23	175	7.6	28	1.2	92	4.0	23	1.0	138.2	6.0	23	1.0	....
Wakeman, Ohio	Ambursen	15	72	5.1	18	1.2	44	2.9	16.7	1.1	57	3.8	16	1.1	....
Hightstown, N. J.	Ambursen	11	58	5.3	29	2.6	39	3.6	27	2.5	....	....	29	2.6	....
Wonder Lake, Ill.	Ambursen	20	93	4.7	21	1.1	48	2.4	20.3	1.0	....	....	21	1.1	....
Pittsfield	Ambursen	33	76	2.3	24	0.7	45	1.4	20	0.6	63	1.9	21.5	0.7	....
Miller	Concrete stop-logs	5	38	7.6	28	5.6	28	5.6	19	3.9	25	5.0	20	4.0	....
Fremont	Ogee, crest and apron	3.5	56	16.0	48	13.7	32	9.1	28	8.7	43	12.3	39	11.1	....
Horse Creek	....	6.4	36	5.6	29.7	4.6	26	4	21	3.3	29	4.5	25.7	4.2	....
Mesilla	Ogee, crest gates, apron	6.7	67	7.7	57	6.6	39	4.5	30	3.5	51	5.9	45	5.2	....
Grand Valley	Ogee, with roller crest	18.3	137	7.5	117	6.4	80	4.4	63	3.5	100	5.5	92	5.0	....
Oswegatchie	Ambursen	23	82	3.6	37	1.6	57	2.5	22	1.0	57	2.5	30	1.4	....
Hyatt, N. Y.	Ambursen	26	86	3.3	37	1.4	57	2.2	33.6	1.3	60	2.3	32.2	1.2	....
Percha	Ogee, with apron	11	132	12	100	9.1	83	7.6	66	6.0	86	7.8	64	5.8	....
Harper Diversion Dam	Gate with apron	12	86	7.2	78	6.5	48	4	40	3.3	69	5.8	63	5.3	....
Granite Reef	Ogee, with apron	20	77	3.8	58	2.9	56	2.8	41	2.0	58	2.9	47	2.4	....
Weber-Provo	Ogee, with apron	8.6	64	7.5	52	6.1	40	4.7	29	3.4	50	5.9	45.	5.3	....
Strawberry	Ogee, with apron	15	105	7.0	59	3.9	63	4.2	42	2.8	84	5.6	43	2.9	....
Burmis Paper Co. Dam	Ogee, with apron	7.6	83	10.9	74	9.7	70	9.2	62	8.1	66	8.7	59	7.8	....
Talavera River; Irr. Sys.	Ogee, crest and apron	8.2	75	9.2	71.0	8.7	37	4.5	33	4.0	68.5	8.4	66.5	8.1	....
		7.2	99	13.7	96.2	13.3	45.8	6.3	42.8	5.9	86.0	11.9	84.3	11.7	Section at east half of dam
Twin City Dam	Ambursen	30.5	197	6.5	134	4.4	104	3.4	80	2.6	165	5.4	107	3.5	Section at west half of dam
St. Mary's Intake Dam	Ambursen	5.5	57	10.4	48	8.6	33	6	83	4.2	43	7.8	39	7.1	Weep-holes
Uncas (Scotland Dam)	Ambursen	24.4	184	7.5	33	1.4	128	5.3	33	1.4	114	4.7	33	1.4	....
Rahway, N. J.	Ambursen	7.3	64	9.1	23.5	3.4	51	7.3	23	3.3	40.5	5.8	23	3.3	....

(a) Weep-Holes — (*Continued*)															
Wallkill	Ambursen	17	35.5	2.1	10.5	0.6	21	1.3	8.8	0.5	36	−2.1	9.0	0.5	....
Juniata	Ambursen	26	110	4.2	30	1.2	70	2.7	30	1.2	81.5	3.2	30	1.2	....
Sixth Lake Dam	Ambursen	11	48.5	14.4	18.5	1.7	28.2	2.5	15.5	1.4	40	3.6	18.5	1.7	....
Florence	Ogee, with apron	10.5	170.0	16.2	143	13.6	102	9.7	93	8.9	112	10.7	85	8.1	....
(b) Drains															
French Landing Power Plant	Multiple-arch hollow dam	32	104	3.2	58	1.8	68	2.1	53	1.7	100	3.3	50	1.6	Drains installed
Shawano	Ogee, crest gates, apron	15	138	9.2	76	5.1	84	5.6	43	2.9	....	....	....	....	Gravel drain
Barton Power Plant	Multiple-arch dam	25	93	3.7	48	1.9	58	2.3	43	1.6	85	3.4	41.5	1.7	Drains installed
Black River Storage Dam	Multiple-arch dam	15	110	7.4	35	2.3	75	5.0	33	2.2	97.5	6.5	34	2.3	Drains installed
Des Moines Elec. Co., Ia.	Hollow multiple-arch dam	8	55	6.8	22	2.8	33	4.1	19.3	2.4	58	7.3	22	2.8	Drains used
City of Sturgis, Michigan	Hollow multiple-arch dam	25	....	....	52	2.1	....	....	45	1.8	....	....	43.9	1.8	Drains installed
		25	....	....	20	0.8	....	....	15	0.6	....	....	20	0.8	....
Coon Rapids	Ogee, crest gates, apron	48	131	2.7	....	....	45.8	9.5	....	....	130	2.7	....	....	With filter
		20	....	....	12	0.6	....	....	9	0.5	....	....	12	0.6	With filter
Iron Mountain	Ogee, crest gates, apron	30	257	8.6	171	5.7	165	5.5	122	4.0	180	6.0	129	4.3	....
Village of Lowell, Mich.	Ambursen	17.5	100	5.6	54	3.1	81	4.6	53	3.0	86	4.9	50	2.9	Spillway section Drains installed
Village of Bellaire, Mich.	Ogee, crest gates, apron	15	98	6.6	30	2.0	69	4.6	31	2.1	83.4	5.6	30	2.0	Section through spillway
		15	66	4.4	32	2.1	46	3.1	31	2.1	64.4	4.3	32	2.1	Section through power house Drains installed
(c) Reverse Filters															
Sandorab	Ogee, with apron	16.7	165	9.9	....	....	93	5.6	....	....	113	6.8	....	....	Filter at end of dam
Dam No. 7, Mohawk (Amsterdam)	Bridge	12	95	7.9	....	....	53	4.4	....	....	67	5.6	....	....	Filter at end of dam
Dam No.31, Mohawk (Yost)	Bridge	8	76	9.5	....	....	42	5.3	....	....	59	7.4	....	....	Filter at end of dam
Hindia Barrage	Stoney gate	11.5	....	....	115	10.0	....	....	52.5	4.6	....	....	102	8.9	....
		16.4	361	22.0	....	....	164	10	....	....	309	18.9	....	....	....
Panjhad Annex Weir	Flat weir	10	363.0	36.3	305	30.5	235	23.5	19.6	19.6	222	22.2	199	19.9	....
Assuit Barrage	Stoney gate	8.2	191	23.3	....	....	98.5	12	....	....	148	18	....	....	Filter at end of dam
Zifta Barrage	Stoney gate	13	211	16	....	....	106	8.2	....	....	160	12.3	....	....	Filter at end of dam
Leasburg Dam	Ogee, crest gates, apron	7	80.7	11.4	70	10.0	60	8.6	50	7.1	58.5	8.4	156	8	Sheet-piling was not tight

this purpose. To reduce the upward pressure on the apron it may be necessary to place the weep-holes farther up stream. They should not be placed farther than necessary for this purpose (unless required by safety from sliding) because to do so shortens the percolation path with no advantage, but brings a greater pressure on the filter and, therefore, increases the danger of failure. For this reason the installation of a drain immediately down stream from the up-stream cut-off is not good practice. On sand or silt foundations where flexible block or book-slab pavements are used down stream from the solid apron of the dam, it may be desirable to install weep-holes and a filter at the down-stream edge of the apron. With the block protection, the water passing beneath the dam is prevented from escaping from a large part of the area just down stream from the apron, but is free to escape through the cracks between the blocks. This tends to produce concentrated flows through these cracks and causes the removal of fine material, making an incipient pipe.

A word of caution also may not be amiss regarding drains beneath the apron of dams where a hydraulic jump is formed. If the drain discharges down stream from the jump while its position under the apron is up stream from the jump, the pressure beneath the apron will be equal to, or greater than, that where the drain discharges, which, being below the jump, will be greater than the pressure acting downward on the apron, due to the thin sheet of water up stream from the jump. This unbalanced pressure may be enough to lift the apron. In the Iron Mountain Dam, in Michigan (Table 1(*d*), Dam No. 49), the drain discharges at the toe of the ogee section, up stream from any jump that may form. In this case the drain may be down stream from the jump and discharge up stream, which would cause an unbalanced pressure acting downward on the apron, increasing its stability. Weep-holes that discharge directly through the apron equalize the pressure on the two sides, but they should be constructed in such a manner that the high velocities can not suck foundation material through them.

If it can be done conveniently, it is advisable to have drains discharge where the flow from them can be observed, in order that any failure to function properly may become known. In Northern climates, reliance should not be placed in weep-holes or drains that may freeze up and thus become inoperative.

By the use of weep-holes or drains constructed with reverse filters that prevent the removal of the foundation material, a reduction of the safe weighted distances given in Table 3 may be permitted. This assumes that the vents are: (1) Of sufficient capacity; (2) free from freezing; and (3) located a sufficient distance up stream from the lower end of the percolation path so that practically all the under-seepage will reach the tail-water through them. No fixed rule can be given on these requirements, but the exercise of reasonable judgment should prevent serious mistakes. For the location just up stream from the down-stream cut-off a reduction of 10% might be permitted, and smaller reductions could be allowed for less favorable locations.

LIMITATIONS OF CREEP ANALYSIS

As has already been stated, piping may occur as a result of percolation along the lines of contact of the dam with its foundations, or by flow through the foundation material itself. In a given case failure would result by travel along the path which offered the least resistance.

In order to determine experimentally the length of percolation path necessary to prevent piping from flow through the material, Griffith performed a series of experiments with a masonry trough $1\frac{1}{2}$ by $1\frac{1}{2}$ ft in section, filled with fine micaceous sand, and covered with earth, to determine the head of water necessary[25] to cause a blow-out. The results showed that a blow-out did not occur until the ratio of length of creep to head reached values of 3.33 to 2.5 although experience had shown that creep-head ratios of about $14\frac{1}{2}$ were necessary to secure safety in dams built on this sand. Griffith found that the flow of water through the sand column increased directly with the head until failure occurred, which came suddenly, without the previous formation of springs to form a channel and carry away the material. He also found that loading the sand decreased the tendency to blow out, and that water brought up suddenly on dry sand increased that tendency.

Much the same results were obtained by Professor Terzaghi[26], who found that the water percolating through the sand increased with the head until a point was reached at which the permeability of the sand suddenly decreased, and the velocity through it consequently increased, and a blow-out took place.

This increase in permeability is the result of a change of position of the sand grains to a less compact arrangement, giving them an increased bulk and greater voids. The effect of the added weight in Griffith's experiments was probably to resist this change of volume and, therefore, to increase the head necessary to cause a blow-out. This phenomenon has also been studied by Osborne-Reynolds[27], and by Warren J. Mead[28], Affiliate, Am. Soc. C. E.

Leliavsky has discussed a number of experiments to establish the head ratio at which blow-out occurs. In his paper[14] he explains the difference in the results obtained by various experimenters. This is due largely to the slope of the surface of the material where the water emerges. If the surface is inclined, less pressure is necessary to move the particles than if it is level, since gravity assists in the movement and a blow-out will take place, therefore, with lower head.

The data on the head necessary to cause blow-outs by flow directly through the material from actual dams are meager. The only dam in which failure unquestionably took place by flow directly through the foundation material is that at Coon Rapids, in Minnesota (Table 1(*b*), Dam No. 4). The tail-water level at this dam was so far beneath the apron that percolation could not occur along the line of creep, at least for part of the distance. This dam was on gravel and failed with a short-path ratio of 2.7. It is probable, however, that the surface where the blow-out occurred was on a slope, and, therefore, a blow-out occurred at a less head than would have been the case if the surface was level.

---

[25] Data from an unpublished part of Griffith's paper previously mentioned.

[26] *Technical Publication No. 215*, Am. Inst. of Min. and Metallurgical Engrs., p. 36

[27] "Scientific Papers", by Osborne-Reynolds, Vol. II (1901), p. 217.

[28] "The Geologic Rôle of Dilatancy", *Journal of Geology*, Vol. XXXIII, No. October–November, 1925.

The failures at the dam, in Port Angeles, Wash., and the Puentes Dam, in Spain (Table 1(*b*), Dams Nos. 1 and 2), may have been directly through the foundation material. In the former this seems especially likely. The situation in this case, however, was complicated by the piling driven just below the dam, which probably concentrated the flow and caused the blow-out (which took place with a short-path ratio of 1.9) to occur at a less ratio than would otherwise have been the case. The Puentes Dam failed with a short-path ratio of 1.4 and probably would have held at one of 2.4, although it had not sustained this ratio long enough to be certain.

It would seem that considerable light on this subject would be shed by experience with coffer-dams, as a great many of these have been constructed. When coffer-dams sunk in a saturated material are pumped out, water flows up through the bottom, tending to raise the foundation material and make it flow into the coffer-dam. If the water level outside the coffer-dam is at the same level as the ground surface, and the excavation in the coffer-dam is at the level of the bottom of the sheet-piling when the water inside the coffer-dam is pumped out, the head acting is approximately equal to the length of percolation path, making the short-path (as well as the creep) ratio practically 1.0. The length of path might be somewhat greater than the head, but this would probably be more than offset by the fact that the downward flow of the water outside the coffer-dam would take place through a much larger area than the flow upward into the coffer-dam. The condition would be that of flow through a channel of gradually decreasing cross-section and the velocity at the smaller end (in the coffer-dam), therefore would be greater than for flow through a uniform prism of material with a length equal to the head.

Two examples that have come to the writer's attention may be mentioned. In the case of a number of bridge pier foundations in the gravel of the Arkansas River bed at Pueblo, Colo., there was no tendency of the gravel to flow up into the coffer-dams. In a number of coffer-dams in Florida, with sand of the fineness of table salt, there was a considerable flow. These cases would indicate that a short-path ratio of 1.0 would be sufficient for gravel and not sufficient for fine sand. Griffith has mentioned a case of a coffer-dam in fine quartz sand with small stones that failed with a short-path ratio of 1.95.

The data are not sufficient to define closely the safe limit of length from the standpoint of flow directly through the foundation material. They do indicate in a rough way, however, that the assumption is justified that the flow takes place directly through the material when the distance is less than one-half the weighted-creep distance. On this basis one might assume that the limit of safe short-path distance would one-half that of safe weighted creep. There is another factor that enters, however: Even with well designed and maintained dams, scour may occur occasionally at the down-stream edge. Under these conditions the material on the side of the hole is more easily moved by percolating water than if it were on a flat surface. Moreover, the percolating water reduces the effective weight of the particles at the down-stream edge of the dam and makes them more subject to scour. For this reason, it is believed advisable to limit the short-path ratios to eight-tenths of those given for weighted creep. This limitation will seldom control the dimensions of the dam.

As previously stated, there is a defect in the creep analysis when rows of sheet-piling are too close together. Bligh recognized this and stated that the percolation followed the line of creep as long as rows of sheet-piling were not closer together than twice their depth.[29] Some such limitation of sheet-piling spacing is necessary because it is obvious that two rows of long sheet-piling very close together would offer little more resistance to piping than one row. If the rows were a great distance apart, however, they would offer considerably more resistance than one row. Bligh did not suggest any reason for using the limit of twice the depth rather than some other value; nor did he discuss what would take place if piling rows were spaced closer than his limiting value.

As the flow follows the path of least resistance, Bligh's limit seems to be equivalent to stating that when the piles are spaced at twice their depth, the resistance along the creep line is equal to that directly through the material, and when closer than twice their depth the resistance directly through the material is less than along the line of creep. When the lines of piles are spaced twice their depth apart, the creep distance between their lower ends is twice the short-path distance; and since for this condition the resistance along these two paths is equal, the resistance per unit length along the short path is twice that per unit length along the creep line. Wherever there are two points on the creep path so close together that a path exists directly through the foundation material which is less than one-half the weighted-creep distance between the points, the flow would take place directly through the material with a resistance twice that along the creep path. Where several overlapping short paths are thus possible, the one that gives the smallest combined weighted-creep and short-path distance for the entire structure is that on which the travel would take place.

For example, two lines of steel sheet-piling were driven 31 ft apart for the Middle Loup River Dam, in Nebraska (Table 1(*d*), Dam No. 48). One line extends 29 ft, and the other, 39 ft, below the bottom of the dam. One possible short path that fulfills the foregoing conditions extends horizontally from the end of the 29-ft piles to the longer ones. Other possible paths extend from various points on the shorter row of piling to various points along the longer row. The path extending between the lower ends of the sheet-piling rows gives the lowest combined weighted-creep and short-path distance for the structure of all the possible paths, and, therefore, should be adopted as the path to be used. When the water follows the short path, the path connecting the ends of the two adjacent sheet-piling rows will usually give the lowest weighted creep for the structure.

---

[29] "The Practical Design of Irrigation Works", Second Edition, p. 168.

The foregoing reasoning applies also to the weighted creep. In this case, the horizontal creep is considered to have a weight of one-third, the vertical creep a weight of one, and the short-path creep a weight of two. In computing the plain and weighted creeps for existing dams, these rules have been used wherever applicable.

The rule that the flow follows the short path if its length is less than one-half the weighted-creep distance between any two points in the creep path permits somewhat smaller pile spacing than Bligh's rule, since the horizontal creep, which, in Bligh's rule, has a weight of one in computing the creep distance, has a weight of only one-third in computing the weighted-creep distance.

If boring records indicate a very pervious layer extending beneath the dam and coming near the surface at the down-stream toe, more conservatism in the short-path distance will be necessary because, as Professor Terzaghi[9] has pointed out, this condition is conducive to high velocities where the water emerges and, therefore, danger of piping.

UPWARD PRESSURE

In masonry dams, upward pressure is important because of its tendency to reduce the factor of safety against sliding. Sliding is not so important a factor in the design of masonry dams on earth foundations. The principal danger from upward pressure on a dam on earth foundations is that it will be sufficient to lift some part of the dam, breaking a new outlet opening for the percolation which will give such a short percolation path that the dam will fail. Upward pressure, therefore, is a factor in the failure of dams from piping.

As previously shown, dams may fail from piping by flow along the line of contact of the structure with its foundation, or by flow directly through the foundation material. In all dams some flow takes place directly through the foundation material, and no doubt the usual condition is that most of the flow occurs in this way. If a structure is designed so as to be safe against piping, the upward pressure under practically all of it will probably depend more on the conditions of flow through the foundation material than along the line of contact. Even if a pipe did form, it would cover only a small area and the foundation masonry would bridge over it. Therefore, although it is necessary to give creep along the contact the pre-eminent place in determining the safety against piping, it may have less bearing in regard to upward pressure.

Under conditions in which the foundation material and its distribution are accurately known, it seems probable that some light on the upward pressure to be provided for, could be obtained with considerable accuracy by the flow net or electric analogy, although these results would be modified somewhat by the case of creep along horizontal surfaces. The writer hopes, in the near future, to investigate this further by a comparison of observed upward pressures on actual structures and the results obtained by the electric analogy method. Where the foundation conditions are complicated or unknown, some approximate rule must be used. Measurement of upward pressure on actual dams and research by model experiment, flow net, and electro-hydraulic analogy methods seem to indicate that the drop in pressure caused by a cut-off is greater in proportion to the creep distance than for the horizontal contacts. Leliavsky[30], after a study of the works of Coleman, Forchheimer, Terzaghi, and Pavlovsky, states,

> "Thus, in calculating the hydraulic gradient the depth of the sheet-piling should be multiplied by a coefficient = 2, (sic) whereas there is already sufficient evidence to show that this coefficient should be three or four or more."

In other words, the horizontal creep should be weighted three-fourths, one-half, or less. Until more definite data are obtained, therefore, it is suggested that when it is necessary to use an empirical rule, the upward pressure be assumed to vary along the contact line in proportion to the weighted creep; (in other words, that the pressure drop along the vertical or steeply sloping contacts per unit of length be assumed to be three times that along horizontal contacts or slightly sloping contacts). This is believed to be a conservative rule as upward pressure experiments frequently indicate that a weight for horizontal creep of zero would give reasonably close results.

Where lines of sheet-piling are so close together that the flow may be considered to follow the short path between them rather than the line of creep, the upward pressure along the creep line between the ends of the short path may be considered to vary according to the weighted-creep distance between these points.

Special conditions sometimes cause unusual distribution of upward pressure. A dam founded on pervious material underlaid with impervious material and with a cut-off extending nearly to the impervious material, will have a lower uplift force down stream from the cut-off. A dam with such a foundation, and in which the cross-section of pervious material under the down-stream end of the apron is less than that at the up-stream edge, will have a greater upward pressure over the entire base than one founded on pervious material of great depth. This condition was noted in the upward pressure measurements on the Grand Valley Dam[31], in Colorado (Table 1(*b*), Dam No. 40). A contracting of the space through which the water flows beneath the dam acts as a partly closed valve; it raises the pressure up stream from it and lowers it down stream.

In computing upward pressure it should be remembered that the conservative assumptions may be different from those for safety from piping. In the latter case, the creep under flexible aprons and similar construction down stream from the solid part of the dam, should be neglected in estimating the security of the dam against piping, as their effect is uncertain and unreliable. In computing the upward pressure they should be given consideration, however, as they may cause greater upward pressure to occur at the down-stream edge of the solid part of the dam than the other assumption would indicate. For example, the upward pressure experiments on the Pinhook Dam[32], in Iowa (Table 1(*b*), Dam No. 22), showed a considerable upward pressure at the down-stream edge of the solid part of the dam, due no doubt to the flexible apron down stream, although if the ordinary assumption for the design of such structures for safety against piping were used in determining the position of the end of the creep path, there would be no upward pressure at that point.

---

[30] "On Percolation Under Aprons of Irrigation Works", p. 46.

[31] *Transactions*, Am. Soc. C. E., Vol. 93 (1929), p. 1532.

At this point it is worth while to suggest that the upward pressure be assumed to be applied at the bottom of the foundation; it really is so applied and has a magnitude equal to the difference in elevation between this point and the piezometric line. The total weight of the masonry (in air) and of any water which may be above it, can then be considered as resisting this pressure. This will eliminate any uncertainty as to whether or not the masonry ought to be considered as submerged (and, therefore, as having lost weight) and will prevent a mistake sometimes made, of measuring the upward pressure by taking the difference between the elevation of the top of the apron and the piezometric line, while still assuming the full weight of the masonry available to resist it. Experiments by H. de B. Parsons, M. Am. Soc. C. E.,[33] and others, have shown the necessity of assuming that the upward pressure acts over the entire area of the base of the dam.

Bligh advocated a factor of safety in determining the thickness of the apron required to resist the upward pressure, by making it heavy enough to resist four-thirds of the computed upward pressure. It would be desirable to analyze all existing upward pressure measurements to fix more accurately what factors of safety are required, since there is considerable evidence that the pressures will be less than computed, either by the Bligh, or by weighted-creep, rules. Until further information is available, however, it is believed that the factor of safety of four-thirds used by Bligh should also be used with the weighted-creep rule.

Conclusions

The ordinary method of analyzing a masonry dam on an earth foundation to secure safety against piping, which is usually ascribed to Bligh, is faulty in that it does not consider the greater probability of percolation along level or slightly sloping contacts between the dam and its foundation, than along vertical or steeply sloping contacts. The flow-net and electric-analogy methods are faulty in neglecting the lesser resistance along the line of contact of a dam with its foundation as compared with that directly through the foundation material.

Piping may occur in two distinct ways: (1) By flow along the line of contact of the structure and its foundation; or (2) by flow directly through the foundation material. Flow ordinarily occurs along both these paths in inverse proportion to their relative resistances. Considerable light on the probability of failure from the first of these causes can be obtained by experimental methods, but the second cause must be studied largely by an analysis of the action of actual dams.

---

[32] *Transactions*, Am. Soc. C. E., Vol. 93 (1929), p. 1551.

[33] *Loc. cit.*, p. 1317.

From the result of an analysis of the action of more than 200 dams, it has been found that creep along contact surfaces having slopes with the horizontal of less than 45° should be considered to offer only one-third the resistance to piping as those with slopes of 45°, or more.

Analysis on this basis may be called the weighted-creep analysis, the creep along surfaces that slope less than 45° being termed "horizontal creep" and that of 45°, or more, "vertical creep". It should be noted that these slopes are those of the surface of contact. The slope of the path taken by the water of a dam may be different. The weighted-creep distance of a cross-section of a dam is the sum of the vertical creep distances plus one-third the sum of the horizontal creep distances, and the weighted-creep-head ratio is the weighted creep divided by the effective head.

A schedule of safe weighted-creep-head ratios for use in the design of major structures is given in Table 3. These values can only be used for solid masonry cut-offs built directly against the earth, or for interlocking steel or concrete sheet-piling driven so that the interlock is not broken and with the top of the piling satisfactorily embedded in the masonry of the dam. Competent supervision during construction and efficient maintenance are assumed. If all these do not exist higher values must be used. For less important structures these ratios may be reduced somewhat, to perhaps 80% of the values given, for very minor structures.

Reverse filters, weep-holes, and drains are aids to security, and weighted-creep-head ratios may be reduced as much as 10% if they are used. The best form is a weep-hole with a reverse filter behind it. For best results vents should be far enough from the end of the travel path to insure that most of the flow passes through them. Except to reduce upward pressure, they should not be farther up stream than necessary to accomplish this purpose. Usually, the best location, from the standpoint of piping, is just above the down-stream cut-off, venting through the cut-off. The position of the hydraulic jump should be considered in locating vents. Great care must be exercised in constructing vents or drains.

In all cases care must be exercised to insure that cut-offs are properly tied in at the ends, so that the water will not outflank them, and that there is no short route behind or under the abutments through which a channel may be formed.

In order to prevent failure of dams by percolation directly through the foundation material the short-path-head ratios should not be less than eight-tenths of those recommended for the weighted creep.

If any two points on the creep line are so close together that the short path between them is less than one-half the weighted-creep distance between them, flow may be considered to take place directly through the material, the length of this travel being given a weight of two. Where more than one short path is possible between the same parts of the creep path, the one that gives the smallest total weighted creep for the structure should be used.

The upward pressure to be used in design may be estimated by assuming that the drop in pressure from head-water to tail-water along the contact line of dam and foundation is proportional to the weighted-creep distance. Between sheet-pile lines that are so close together that the short-path limitations apply, the total pressure drop may be computed as proportional to the short-path distance with its weight of two and may be distributed between the two ends of the short path in proportion to the weighted creep between these points.

ACKNOWLEDGMENTS

It would have been impossible to have collected the amount of data analyzed in this paper without the hearty co-operation of a large number of engineers. The sources of data on the individual dams is acknowledged in full in the original manuscript. Other acknowledgments are made in a number of places in the text. In addition, the writer wishes to express his indebtedness to the following:

To the Ambursen Dam Company for the data on a large number of dams of the Ambursen type, on earth foundations, with most of which that Company was connected as designer or contractor; to Messrs. Ayres, Lewis, Norris, and May, for data on dams designed by them, or by the late Gardner S. Williams, M. Am. Soc. C. E.; to Mr. W. M. Griffith for many valuable suggestions and ideas and for data on several dams; to Messrs. Holland, Ackerman, and Holland for data on a number of dams designed by them; to the Department of Public Works, Philippine Islands, for data on dams built by it; to the Verein Deutscher Ingenieure and the Deutsche Wasserwirtschafts and Wasserkraftverband for assistance in securing information on failures in Germany; to the U. S. War Department and its various District Offices for data on dams built by the U. S. Corps of Engineers; to D. W. Mead, Hon. M. Am. Soc. C. E.; A. L. Alin, H. E. Babbitt, J. L. Burkholder, E. L. Chandler, C. H. Eiffert, B. A. Etcheverry, D. C. Henny, L. C. Hill, Ivan E. Houk, R. D. Goodrich, John H. Gregory, Fort Kurtz, R. B. McWhorter, and the late William W. Tefft, Members, Am. Soc. C. E.; to George R. Clemens, Assoc. M. Am. Soc. C. E.; to Messrs. P. Bier, G. P. F. Boese, H. N. Eaton, W. H. Frazer, Charles G. Gablemen, C. C. Neher, Paul A. Nelson, A. D. Williams, and numerous others, for assisting in securing data and for valuable suggestions. The writer was assisted in preparing the data and drawings by Messrs. Walter H. Price, W. F. Bingham, and Richard J. Willson.

This paper represents the result of a study made for the U. S. Bureau of Reclamation to assist in the design of an intake dam on the Colorado River for the All-American Canal. All the work of this Bureau is under the direction of Elwood Mead, M. Am. Soc. C. E., Commissioner of Reclamation. All engineering work is in charge of R. F. Walter, M. Am. Soc. C. E., Chief Engineer. The study was carried out under the immediate supervision of J. L. Savage, M. Am. Soc. C. E., Chief Designing Engineer.

DISCUSSION

WILLIAM P. CREAGER,[34] M. AM. SOC. C. E. (by letter).—It has been suspected for some time that Bligh's coefficients were too conservative, and the author has been successful in proving it. However, it is the writer's opinion that even Mr. Lane's coefficients are too conservative for scientifically designed structures.

Two methods are available for the determination of the correct design to insure the safety of a dam on earth foundations as regards the possibility of piping. The first is to model the dam after those of similar kinds that have stood successfully. The second consists in the direct determination of the safe escape velocity and in the design of the dam and its appurtenances to keep this velocity within proper limits. The paper deals with the first method.

Unfortunately, the data collected by the author are extremely difficult to classify properly. Only by the remotest chance could one of the dams cited resemble another closely. Unadjustable differences consist of: (1) Depth of heel cut-off; (2) depth of toe cut-off; (3) relative length of cut-offs to length of base of dam; (4) relative resistance to flow for horizontal and vertical creep lines (quite arbitrarily, the author has adopted a ratio per foot of length of 1 to 3); and (5) the character of the foundation material. The material cannot be classified as "fine sand", "medium clay", etc., with any precision, as each class has great variations.

In Table 1(*d*) the author has tabulated seventy-five dams on silt, fine sand, and medium sand. These have been plotted in Fig. 1 in the order of magnitude of the ratio of weighted-creep distance to head. Failures are noted by circles, and non-failures by dots. The author's safe values for these materials from Table 3 have also been shown. It will be noted that most of the failures were for very short weighted-creep-distance ratios, showing that creep distance has perhaps the preponderating effect on safety. On the other hand the great divergence of the ratio for the dams that failed shows that the creep distance is by no means the only influence.

Fig. 1 indicates that, for the twenty-nine dams designed with ratios less than the author's minimum recommended value of 6, twenty, or 70% of them, are still standing. Furthermore, most small earthen dams, built of fine sand without core-walls, have paths of percolation ratios of less than 6. Therefore, it would seem perfectly feasible to design for ratios less than 6, provided it is done scientifically, with full knowledge of the action of water seeping under dams. Since piping begins below the dam it will not occur if it can be prevented at that point. Therefore, it is only the velocity of the vertical seepage below the dam in which the designer is interested.

The results of electro-hydro-dynamical experiments by Professor N. P. N. Puzyrevsky,[35] which the writer was privileged to see in Moscow, tests made by the writer on both small scale sand dams, and with electro-hydro-dynamical experiments, as well as tests made by L. F. Harza, M. Am. Soc. C. E., with the hydraulic-electric analogy tray,[36] all show a crowding of the flow and very high velocities around any projections in the base or foundations.

---

[34] Cons. Engr., Buffalo, N. Y.

[35] *Engineering News-Record*, June 14, 1934, p. 761.

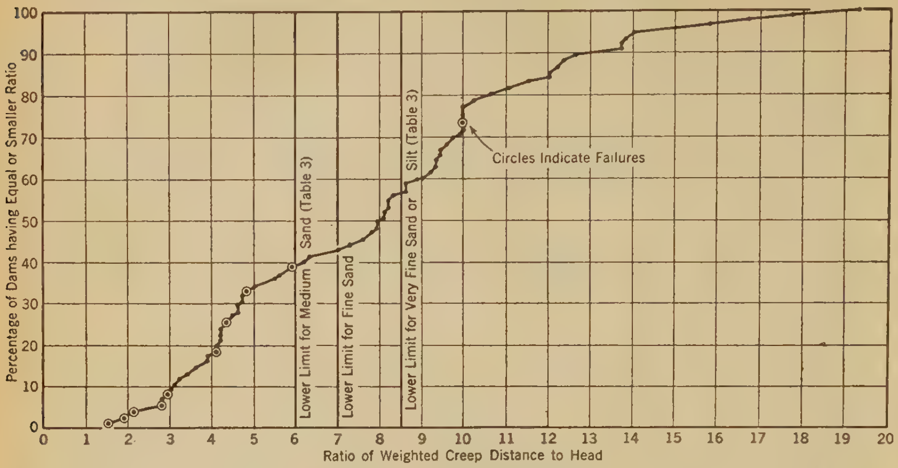


FIG. 1.—SEVENTY-FIVE DAMS ON SILT, FINE SAND, AND MEDIUM SAND, FROM TABLE 1(*d*)

Where no cut-offs are provided, this zone of high velocity is concentrated at the angles of the base of the dam at the up-stream and down-stream ends. The water enters the foundation close to the up-stream face and leaves it close to the down-stream face at many times the average velocity, or the velocity that would obtain without such angles. The high velocity at the up-stream edge is of no importance, of course, as the foundation material is confined, but at the down-stream edge it is the direct cause of piping. If a cut-off is placed at the up-stream end of the base, the zone of high velocity is transferred to the end of the cut-off, as shown by Mr. Harza.[37] A down-stream cut-off, which should always be used, would transfer the zone of high velocity away from the surface to the bottom of the cut-off and, if properly designed, would reduce the escape velocity to a safe amount. All this testimony indicates the importance of the details of design as well as the limitation of the line of creep.

The velocity of escape at which any class of material will flow to cause piping can readily be determined from experiments on samples from the foundations. The coefficient of percolation can also be determined experimentally. Then, the proper length of base and length of up-stream and down-stream cut-offs to provide a safe velocity of escape can be determined by the electro-hydro-dynamical method more readily than by any other for any degree of perviousness along the horizontal creep, that may be assumed.

By the utilization of different arrangements of cut-offs, it is possible to design two dams for the same foundation with exactly the same weighted-creep distance, but one will have a much greater velocity of escape than the other and, consequently, will be much weaker as regards piping.

---

[36] See p. 1353.

[37] See p. 1368, Fig. 10.

The writer does not contend that the author's compilation of creep distances is without value. On the contrary, it is of exceedingly great value for comparative purposes and for designs made without the aid of experimentation. However, the writer does take exception to Mr. Lane's statement (under the heading "A New Method of Analysis") that: "In the present state of knowledge the only method of analyzing the probability of failure from flow along the creep line [piping failure] seems to be a study of the actions of actual dams."

L. F. Harza,[38] M. Am. Soc. C. E. (by letter).—While it contains much other interesting material of practical value, this paper is, nevertheless, devoted mostly to the exposition of Mr. Lane's "weighted-creep" principle, in which horizontal creep distance is given a value of only one-third that of vertical creep. The author's approach is entirely empirical, which has led to the erroneous conclusion that the apparent lower resistance of horizontal seepage is necessarily the result of poor contact, or actual "roofing," due to a combination of stratification and a settlement of the foundation material away from the bottom of the structure.

Rational analysis by mathematics and its equivalent, the electric analogy,[39] proves conclusively that even in a homogenous foundation in perfect contact with the structure, apparent horizontal creep resistance along the base may have a value much less than one-third the average resistance to vertical seepage around the cut-offs, the actual value being related to the relative depth of cut-offs in proportion to their distances apart.

Thus, for example, in case of both heel and toe cut-offs of equal depth, d, as compared with the spacing, b, the hydraulic gradient or creep resistance per unit distance along the base, as compared with the creep resistance per unit distance along the vertical faces of cut-offs (equivalent to Mr. Lane's weighting of horizontal creep), should be, as follows:

Space, b, between heel and toe cut-offs.	Approximate weighted value of horizontal creep.
$5.0d$	1/3
$2.5d$	1/4.2
$2.0d$	1/7
$1.5d$	1/9

The weighting of horizontal creep resistance by any fixed relation to the vertical is thus obviously incorrect in basic principle.

Considering the meager extent of available quantitative data, the author has done well to discover and verify empirically the fact that horizontal creep offers less resistance than vertical creep; but to try to express this ratio quantitatively and at a fixed value of one-third is going far beyond the accuracy justified by the data. Unfortunately, the relation is far too complicated to be expressed by any such simple ratio.

The foregoing comments apply to the condition of perfect contact between the bottom of the structure and the foundation material; that is, without roofing. It is granted that roofing is a condition almost inevitable in the case of a dam with smooth under-surface, founded on piles. It is also quite possible if the dam is founded directly on the earth. If there is even a strong possibility of roofing, however, horizontal creep distance should be given no weight whatever, even one-third would be an unsafe assumption. The writer believes in the use of longitudinal scoring of the sand bed ahead of the placing of concrete to furnish what might be called a "corrugated foundation" on the under side of the concrete to insure against roofing.

---

[38] Cons. Engr. and Pres., Harza Eng. Co., Chicago, Ill.

[39] "Uplift and Seepage Under Dams on Sand", by L. F. Harza, M. Am. Soc. C. E., see p. 1353.

The effect of stratification, although not experimentally demonstrated, is believed to be opposite from that assumed by the author, unless in the case of the occurrence of an exceptionally porous stratum. It may be stated as a general principle that the rate at which seepage head dissipates will be rapid and, consequently, the hydraulic gradient will be steep in proportion to the tendency of the flow to concentrate. Stratification in general will tend to concentrate seepage flow rather than permit its diffusion.

The writer is not a believer in reliance alone upon abstract mathematics unsupported by experimental data for the solution of the problem of seepage under dams where non-homogeneity of material may so greatly affect the conclusions. He does believe, however, that mathematics and the electric analogy can be relied upon to establish fundamental principles, around which to build the experimental data, and that the problem should be approached in that manner.

Joel D. Justin,[40] M. Am. Soc. C. E. (by letter).—A thorough research is evidenced by this paper and to the data collected, the author has applied hard common sense and good judgment, demonstrating a thorough knowledge of sound design and construction.

That a vertical cut-off will be more effective in reducing the velocity of flow through foundation materials than an equivalent horizontal distance along the base, seems entirely logical. This is doubtless true in many cases, even if the contact of the base with the foundation material is insured by extensive and thorough grouting operations. One important reason is that, in actual cases, the foundation material is rarely, if ever, homogeneous, but occurs in strata and lenses of varying degrees of permeability. With a vertical cut-off, the various strata or lenses are penetrated, forcing the water to pass through the strata in a vertical direction (more or less), including some strata of more impervious material than that immediately below the base of the dam, thus decreasing the velocity of the seepage.

The fact that the author's weighted-creep criterion produces ratios lower than those of Bligh, may lead some to assume that it is less conservative than that of Bligh. Mr. Lane states that his weighted-creep ratios are somewhat less than one-half the ratios of Bligh. However, the application of the weighted-creep ratios given in the paper will frequently yield base widths greater than those obtained by using the Bligh coefficients. Actually, the author's method is more logical and gives results which are, in general, more conservative than would be obtained by the use of the Bligh coefficients. This is indicated by the data in Table 5.

---

[40] Cons. Engr., Philadelphia, Pa.

It will be noted from Table 5 that, for a dam with a 30-ft head, founded on coarse sand, with a 60-ft cut-off, the author would obtain a base width of only 90 ft as against 240 ft, which would be obtained by the use of Bligh's method. In this case Mr. Lane appears to be less conservative than Bligh, but in the case of an over-flow dam, it is probable that the base width required to secure a structure to smooth out the velocity of the water passing over the dam and to dissipate this velocity so that there will be no erosion, would become the controlling factor. Table 5 also shows the emphasis placed on vertical cut-offs by the author and indicates that it may frequently prove more economical to secure safety by deep cut-offs than by a great width of base.

TABLE 5.—COMPARISON OF REQUIRED BASE WIDTHS OF DAM BY LANE'S RATIO AND BLIGH'S RATIO; HEAD, 30 FEET.

Description (1)	(a) DAM ON FINE SAND, OR SILT: Base Width, in Feet: By Lane's coefficient (2)	(a) Base Width, in Feet: By Bligh's coefficient (3)	(a) Ratio: Column (2) / Column (3) (4)	(b) DAM ON COARSE SAND: Base Width, in Feet: By Lane's coefficient (5)	(b) Base Width, in Feet: By Bligh's coefficient (6)	(b) Ratio: Column (5) / Column (6) (7)
Creep ratios	8.5*	18	....	5*	12	....
Depths of Cut-Off, in Feet:						
0	765	540	1.42	450	360	1.25
15 (= 0.5 H)	675	510	1.32	360	330	1.09
30 (= 1.0 H)	585	480	1.22	270	300	0.90
60 (= 2.0 H)	405	420	0.96	90	240	0.38

* Weight of horizontal creep, one-third.

Cut-offs of steel sheet-piling do not always give the same assurance of safety as a concrete cut-off may provide. If the foundation contains boulders, these piles may curl and split without this fact being detected at the time, with the result that the cut-off is quite largely ineffective. However, with most sandy materials, the use of an adequate high-pressure jet will permit driving the sheet-piling even to great depths with some assurance that the diaphram thus formed is continuous.

As pointed out in the paper, the weighted-creep method does not provide a complete criterion for securing dams on sands and soils safe against piping. Data on the foundation material of existing dams are far from precise. The occurrence of ledge rock or other impervious strata near the base of the dam, but at depths not reached by the cut-off, may affect, materially, the velocity of the water that may issue from the stream bed just down stream from the toe. Local conditions at or near the toe, such as lenses of coarse material, may cause a concentration of flow and may increase the velocity of the water at the egress to a point where there is danger of piping.

The author shows that it is necessary, for safety against piping, that the velocity of the water at the point of egress be so low that there is no danger of its carrying away the finer particles of the foundation material. For expressing this requirement the writer has devised formulas[41] for the necessary length of the path of percolation, base width of dam, and necessary depth of cut-off, all based on Darcy's law. Thus, for the simple case of no cut-off, the formula is:

$$b = \frac{K H}{P V} \qquad \text{.........................(4)}$$

in which, b is the necessary width of base; H, the head from head-water to stream bed at the toe of the dam; P, the porosity of the foundation material; K, the transmission constant of the material intercepted, or the weighted average transmission constant of the materials intercepted by the line of flow; and V, the safe velocity for the water at the point of egress, such that it will not move any of the finest foundation material (with a factor of safety applied).

Of course, Equation (4) is merely a straight derivation from Darcy's law. The formula is simple and is based on an entirely logical concept. The difficulty in applying it is in the choice of V, the safe velocity of the water at the point of egress. When sufficient experimental data become available, this formula and others dealing with the necessary depth of cut-off on a similar basis, will prove quite useful for aiding in the more precise analysis of the design of masonry dams on soils and sands, where relatively complete sub-surface investigations are made.

The use of the author's ratios will prove desirable as a basis for preliminary design, just as ratios are used in the preliminary design of masonry dams on ledge-rock foundations. Then, a thorough investigation at the site, including borings, test pits, mechanical analysis, and percolation tests should be made. In the case of important structures, model tests should be included, by all means. Finally, all these data should be interpreted in the light of experience, and the final design prepared accordingly.

Louis E. Ayres,[42] M. Am. Soc. C. E. (by letter).—The author has made a valuable contribution to the subject with which he deals. He has collected and critically analyzed much pertinent practical experience, and his comments thereon indicate a clear grasp of the design elements involved in dams on earth foundations. It is to be regretted that the "drawings that show dimensions and typical sections" of several of the dams investigated—at least, in sufficient number for illustration—could not have accompanied the printed text.

This paper presents a new set of arbitrary creep ratios which, on their face, appear to be less than one-half those heretofore recommended by Bligh and others. This apparent reduction has been accomplished by giving a greater weight to vertical than to horizontal creep. Obviously, Mr. Lane might have presented equivalent creep restrictions, with values that would have appeared higher than those of Bligh, had he multiplied the vertical creep, instead of dividing the horizontal creep, by 3. This is merely stating that the new ratios are lower because of a change in definition and in emphasis. Actually, the new ratios permit of a reduction in the bottom width of a dam only if it is provided with a relatively deep up-stream cut-off; with a shallow cut-off, the bottom width required is greatly increased. The author's emphasis on the importance of adequate vertical creep is eminently proper. His recommended weighted ratios are an improvement on past usage and afford a conservative arbitrary standard, based on much experience.

---

[41] "Earth Dam Projects," by Joel D. Justin, M. Am. Soc. C. E., pp. 174–181, inclusive, John Wiley & Sons, Inc., 1932.

[42] Civ. and Elec. Engr. (Ayres, Lewis, Norris & May), Ann Arbor, Mich.

A further logical step in the direction taken by Mr. Lane would be a recommendation to eliminate horizontal creep entirely from one's computations where it is practicable to obtain an adequate vertical cut-off. Many dams in recent years have been designed on this principle and many others, on porous soils, actually depend for their security solely on their vertical creep distances. The author quotes J. C. Oakes, M. Am. Soc. C. E., to the effect that under dams on piles in sand, where pumping is required during construction, it is "almost a certainty that there will be a space between the sand and the masonry." Whether or not, however, clearly defined open spaces exist, an average sand and gravel foundation will not offer much resistance to ordinary seepage along a smooth horizontal masonry surface; and if the seepage escapes readily at the toe, the structure becomes fairly well underdrained throughout. Certainly a dam's water-tightness under such conditions, if not its security, depends mainly on the up-stream cut-off; and the assumed advantage of horizontal creep has largely disappeared.

The author's investigations disclosed "many dams with a high proportion of vertical creep." These structures, in the main, are hollow dams of the multiple-arch and flat-slab types in some of which the supporting abutments rest directly on the soil, or on piles; such types depend for a cut-off solely on a line of steel sheet-piling driven along the up-stream edge. Where there is no floor between the abutments of such dams there is no bottom along which water must creep, and the creep distance becomes, therefore, twice the length of the cut-off. Obviously, the security of such a structure depends on the adequacy of the cut-off, which must penetrate the foundation material to a depth sufficient to prevent the possibility of a blow-out.

Although one may admit, with the author, that such a design is putting "all one's eggs in one basket," the evidence of experience shows that "basket" to be quite a reliable one. The author states that "not a single case was found in which piping unquestionably took place under a deep cut-off." He reports one failure because "one of the steel sheet-piles of the up-stream cut-off was pushed down far beyond its proper position, leaving an opening in the cut-off"; a second failure was due to "defective construction"; and a third came from "a blow-out directly through the foundation material from a porous layer beneath the impervious layer on which the dam was founded"; and, in this instance, the vertical creep distance was only 29 ft with a head of 33 ft.

Where there is no provision for horizontal creep under a dam, either because a floor is lacking, or the floor is underdrained, smaller creep distances than those shown in Table 3 would seem to be warranted. A range in depth of cut-off from a minimum of one-half the head in firm, homogeneous clay to about twice the head in sand would appear to be sufficient on the basis of much experience. An important caution in this connection is that proper depths may not be determined arbitrarily, but only on the evidence of an adequate soil investigation carried well below the bottom of the cut-off.

In his computation of vertical creep distance Mr. Lane considered "any contact with a slope of 1 on 1, or steeper," to be "vertical" contact. The most valuable part, by far, of the creep distance is the vertical path downward, along the up-stream face of the cut-off. The vertical path, upward, is certainly less valuable. For this reason the back-filling of clay, or other impervious material, against the up-stream face of a dam contributes to security. It is doubtful whether relatively shallow longitudinal ribs on the bottoms of dams, particularly in porous soil, are as effective per foot of depth as an equivalent depth added to the up-stream cut-off.

The author deals with the advantages of drainage under dams and aprons and he points out, very properly, the need for caution in location and for care in design. Many engineers are still reluctant to utilize drainage under dams on earth foundations and only a few seem to have taken full advantage of the judicious use of drains under such dams. There is a feeling that it is unwise to approach too closely to the up-stream cut-off with a drainage system and that down-stream and apron drainage only is consistent with safe design. There is an assumption of security in a long horizontal seepage surface and a fear of defects in a vertical cut-off. Does experience justify either the assumption or the fear? It is important, obviously, that drains be well designed and carefully constructed; that they be not so placed as to afford a short-cut between head and tail-water; nor so built as to be crushed under load or settlement, or become plugged or otherwise ineffective. Drainage under dams, in embankments and in connection with many other structures subject to upward pressure, affords a valuable tool in experienced hands, which has been used too sparingly.

Except under certain special conditions such as: (*a*) In some very fine sands or river silts where a "floated" dam may be required; or (*b*) in soil containing numerous large boulders or which, for other reasons, is difficult to penetrate with steel sheet-piling, a dam provided with an adequate cut-off and a reliable under-drainage system is a more secure and less expensive structure than a dam without drainage, depending on its weight to hold down upward pressure and a relatively long horizontal seepage surface to overcome percolation.

W. M. GRIFFITH,[43] ESQ. (by letter).—In general, the writer agrees with the weighted-creep theory presented in this paper. As the extra stability of the material along the vertical creep must be due, to some extent, to the higher intensity of pressure within the material at the greater depth, the extra stability of the material in the vertical creep will depend, to some extent, on the depth of the vertical creep. This factor should also be considered in applying the theory to special cases. The relative value of the horizontal creep must also depend, to some extent, on the internal pressure in the material in immediate contact with the foundations, and this pressure will be that resulting from the downward pressure of the structure on the material. Its intensity will depend on the design of the structure. As a result of the application of the hydraulic gradient theory, such structures are now designed more lightly than in former years, and, in modern structures, the resultant pressure along the line of horizontal creep is less than in old designs. The relatively higher value of vertical creep, therefore, is intensified in modern structures.

---

[43] London, England.

The author has referred to a paper by the writer, on this subject,[6] published in 1913–14. At that time, interlocking steel piling was unknown on irrigation works in the United Provinces (India) and, where used, sheet-piles were of timber, driven by native labor and, consequently, were not especially reliable. Subsequent experience with interlocking steel piling has caused the writer to modify, to some extent, the views expressed in his paper.

In suggesting a reduction of 20% in the length of creep, where reliable vertical sheathing to a depth of 10 ft was used, the writer had in mind a single line of piling and dams of the class with which he was then (1913) dealing (which, in general, were founded on fine sand and supporting heads of about 10 ft). The value assumed for the safe creep ratio being about 15, such structures would require a 150-ft length of horizontal creep or, with 20% deduction, a 120-ft length of combined horizontal and vertical creep. The vertical creep for a 10-ft depth of piling being 20 ft this 120-ft length of combined creep would be divided into a 100-ft length of horizontal, and a 20-ft length of vertical, creep. The writer, therefore, was giving the 20-ft length of vertical creep the equivalent of a 50-ft length of horizontal creep—a ratio of $2\frac{1}{2}:1$.

The author's value of this ratio is 3 to 1, but it will be seen from his Table 3 that his safe values for simple horizontal creep (that is, three times his weighted creep) are somewhat higher than those the writer was proposing. As a result of the hydraulic gradient theory, several dams were designed and built in the Rohilkhand Canal Division (United Provinces, India) from about 1905 to 1908, that were founded on fine light sand, relying entirely on horizontal staunching aprons, with no vertical cut-offs. Apparently, these dams were satisfactory for some years but, later, exhibited signs of failure by settlement of some part of the up-stream apron near the crest line. These defects were repaired, but eventually the structures failed completely. Piping was the direct cause of failure, and experience showed that this action can be slow and accumulative. It was generally considered that such action could be detected in time, by observing that springs down stream carried away small particles of sand.

The sudden and complete failure of Kulli Dam by under-seepage, after standing satisfactorily for nine years, showed that this indication could not always be relied upon, as no such indication was observed in this case. In consequence of this experience, the writer would not recommend founding a dam on fine light sand, without one line of vertical piling, designed to carry the line of creep to a depth at which the internal pressure could be relied on to ensure good consolidation.

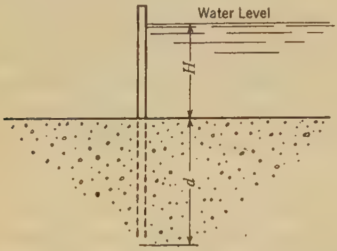


FIG. 2.

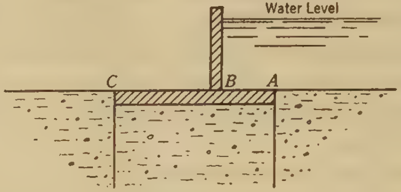


FIG. 3.

The same reasoning does not apply, however, to dams founded on coarse heavy sand, or shingle; the writer has known a number of such structures designed without vertical staunching and has never known a failure from under-seepage.

A good test of Mr. Lane's theory and of his values for the safe weighted-creep ratios, is to consider the extreme case of a dam having only vertical sheet-piling. The case of a coffer-dam, of interlocking steel piling, driven in the alluvial bed of a river to provide a dry spot for foundation work, affords a common example of this, as depicted in Fig. 2. The length of creep, L, is equal to $2\,d$ and the head is H. Then, the weighted-creep ratio, $\alpha, = \frac{H}{2\,d}$; or,

$$d = \frac{\alpha\,H}{2} \quad \text{..........................} (5)$$

The safe values of d required to resist a 10-ft head of water as calculated from the author's Table 3 for different materials, are given in Table 6. These values appear materially in excess of those usually adopted for temporary structures; and they seem to be safe values, and to support the author's theory.

TABLE 6.—WEIGHTED-CREEP RATIOS REQUIRED TO RESIST A 10-FOOT HEAD OF WATER

Class of material	Depth of driven piling necessary to hold up a 10-ft head of water from values in Table 3, in feet.	Class of material	Depth of driven piling necessary to hold up a 10-ft head of water from values in Table 3, in feet.
Fine sand	35	Coarse gravel, including cobbles	15
Medium sand	30	Soft clay	15
Coarse sand	25	Medium clay	10
Fine gravel	20	Hard clay	9
Medium gravel	17.5		

The writer does not agree with Mr. Lane's recommendation that two lines of sheet-piling are preferable to one deep line (see heading "Recommended Values Are Conservative"), believing that one deep line, if correctly placed, is more reliable. Interlocking steel piling forms a reliable vertical cut-off and one deep line placed under the crest or line of gates gives the maximum security for the material used.

To illustrate this point consider the case of a dam having a horizontal floor with a line of piles at the up-stream end and down-stream end, as sketched in Fig. 3. From Point *A* to Point *B* the downward pressure on the apron is greater than the uplift in the line of creep, and this difference is intensified by the piles at Point *A*. If this apron is a thin one (as is usual), and if piping or bad consolidation occurs beneath it, it may fail near Point *B* where the difference in pressure is greatest. Such failure at one point will at once shorten the line of creep by the length of the apron and the line of piles at Point *A*. This may reduce the creep distance sufficiently to cause a blow-out; in any case it would increase greatly the uplift beneath the floor, *BC*, and this floor would be likely to blow up under the increased upward pressure, in which case a blow-out would be certain. An initial failure in Apron *AB* might not be noticed at first because it is submerged. The failure of the Deoha Barrage was undoubtedly an example of this type.

With two lines of piles stability then depends not only on the reliability of both lines, but also on that of the aprons. If the two lines of piling are placed near together, one on each side of the crest line, *B*, the chance of failure through failure of the apron is reduced, but in that case the short path may prove the critical line of creep, and the value of the two lines is then greatly reduced.

Finally, one line of deep piles will carry the line of creep to an area at which the internal pressure in the material is twice as great as in the case of two lines carried to one-half the depth. Where two lines of piling are used, one being situated at the down-stream end of the apron, the writer would favor the construction of "weep-holes" with inverted filters below in the down-stream apron.

Mr. Lane has presented an interesting and valuable paper; the writer is greatly impressed by the complete data collected, and by the author's method of analysis.

Dr. E. McKenzie Taylor[44] (by letter).—In considering the percolation path, it is important to distinguish between weirs protected by lines of wells or masonry curtain-walls, upon which Bligh formulated his theory, and those protected by sheet-piles. It is impossible to obtain a perfect seal between the wells or masonry curtain-walls and the permeable material on which the dam is constructed. It follows that in all probability Bligh's theory of creep was correct for this form of construction. When sheet-piles are used for protection there is no disturbance of the foundation material and, hence, the flow postulated by Bligh does not occur. A large number of model experiments[45] have been made to study the lines of flow when a work is protected by sheet-piles, and in no case does the flow occur vertically up the downstream face of the pile line.

---

[44] Director, Irrig. Research, Punjab, Lahore, India.

Two instances of weirs confirm the results of the model experiments. Panjnad Weir (Punjab) has three lines of sheet-piles. It has been found possible to reproduce the actual uplift pressures on a model which shows that the pile lines are functioning as they do in the model. The second instance is that of Bay 4, Khanki Weir (Punjab). During reconstruction in 1934, lines of piles were driven. The effect of these piles was tested on a scale model and the results of the actual uplift pressures as observed on the floor of the weir are in close agreement with those obtained in the model. In the data presented in this paper no distinction is made between the two forms of construction. A re-examination of the data taking into consideration the differences in the percolation paths associated with the two methods of construction would be of the greatest value.

The author urges caution in applying the results of model tests, whether obtained by means of the flow net or by the hydraulic electric analogy, to constructional practice. Recent work in connection with a study of Panjnad Weir,[46] has established the following principles:

(*a*) The variation in uplift pressure recorded on the work can be reproduced on a model by varying the deposit of silt up stream.

(*b*) A work designed from the results of a model on a homogeneous permeable foundation will withstand the worst conditions as regards uplift pressure to which it will be subjected.

(*c*) The uplift pressures on a work on a permeable foundation are independent of the grade of the foundation material if it is uniform.

Even under much more complex underground conditions, it has now been shown that, if the conditions are known, they can be reproduced in a model. This was done in the case of Bay 4, of Khanki Weir, in which a clay stratum was found some distance below the sand surface. As a result of these investigations, the results of model experiments were applied to the design of other Bays of Khanki Weir. Since a design based on the results of model experiments deals with the worst conditions that are likely to be experienced, it appears that model work can be applied with confidence.

Under "Relative Weights of Vertical and Horizontal Creep," the author states that "the upward pressure measurements on several structures on which data were available shows that the drop in upward pressure along horizontal concrete surfaces is almost zero." This statement cannot apply to works in a good state of repair. Such a condition in the Punjab would indicate the immediate necessity for investigating the safety of the weir. Two instances of no drop in pressure along the floor have come to the writer's notice recently. In both cases an investigation of the dam showed that flow, as distinct from percolation, was occurring due to the unsound condition of the masonry. In cases where sheet-piling has been used in the Punjab a drop in pressure invariably occurs along the floor.

---

[45] *Proceedings*, Punjab Eng. Congress, Vol. XXII, 1934, pp. 51–78.
[46] *Loc. cit.*, Vol. XXIII, 1935.

It is indicated, therefore, that construction which involves the use of wells or masonry curtain-walls is unsound and should be abandoned. In the future, sheet-piling should be used for increasing the percolation path. If this is done, the deductions based on the analysis of data for works in which a line of wells or masonry curtain-walls exist, are likely to be inapplicable. The insertion of pressures pipes and their regular observation is now a feature on some weirs in the Punjab. The importance of inserting pressure pipes in a weir cannot be emphasized too much and the alteration in pressure is a valuable indication of conditions under the floor. Such observations have already enabled action to be taken before the situation became serious.

From his investigation the author concludes that greater weight should be given to the creep along the vertical, than along the horizontal, surfaces. While this may be the case in dams with masonry curtain-walls, the weighting suggested is not borne out either in model experiments or in observations on actual works in which sheet-piles are used. The main drop in pressure occurs around the foot of the pile and is due to the curvature imposed on the flow lines. This drop in pressure is usually four times that experienced on the up-stream face of the pile. The drop in pressure on the down-stream face of the pile amounts to about one-twelfth of that on the up-stream face. It is impossible at present to state any rule for determining the optimum length of pile with reference to the length of floor. The only satisfactory method at present available for determining the effect of sheet-piles on the uplift pressures, in a complicated structure, is the examination of these piles on a scale model.

A further point of considerable importance in connection with uplift pressures is the "unbalanced head." The normal practice in the Irrigation Research Institute, Punjab, is to examine a model of the work in the flume and plot the water profile for a variety of conditions that are likely to be experienced on the work. From the position of the standing wave for the worst condition and the uplift pressures determined by means of the flow net the unbalanced head can be plotted.

DONALD J. HEBERT,[47] JUN. AM. SOC. C. E. (by letter).—The recommendation in this paper of a design method for securing dams from under-seepage, based on an exhaustive and systematic compilation of data in existing structures, represents a valuable contribution. With the problem in its present state, the approach toward a solution that has been made by the author is the reasonable and practical one. Until the theoretical aspects of the problem are more clearly understood, an empirical solution is necessary, and the one advanced in this paper is the most satisfactory from a consideration of economy and safe design.

The writer performed some experiments for the U. S. Bureau of Reclamation[48] which he proposes to describe in this discussion, because they have a direct bearing on the subject of the paper by Mr. Lane. The results, which are set forth herein, demonstrate quite clearly the reliability of the present empirical and theoretical solution. Only a part of the results are included, the remainder being filed for reference in Engineering Societies Library, New York, N. Y.

---

[47] Jun. Engr., U. S. Bureau of Reclamation, Denver, Colo.

[48] "Hydrostatic Uplift Pressures Under Dams on Pervious Earth Foundations", by Donald J. Hebert, Jun. Am. Soc. C. E., *Memorandum No. 384*, Bureau of Reclamation, U. S. Dept. of the Interior, 1935.

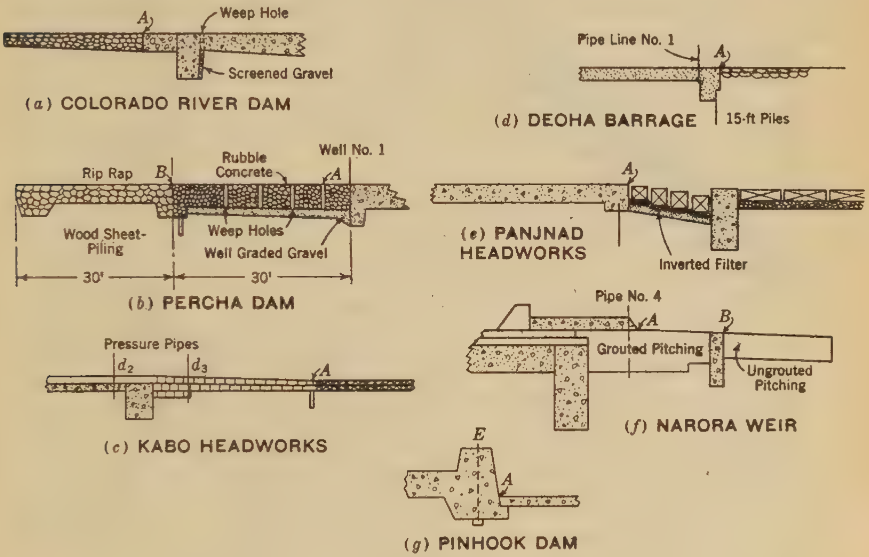
The purpose of the study for which these experiments were performed was to investigate the existing methods of estimating the magnitude of uplift pressures, due to a head of water, under masonry dams on porous foundations. A comparison of the various methods is made on the basis of a collection of observed pressures under a number of existing structures. Pressures at significant points under each structure were computed by each of the design methods, and these values were compared, by means of curves, with the actual pressures measured at the dam. The data as received consisted of readings of water levels in pressure pipes placed in various positions under the dams during construction. These readings were expressed in terms of head lost to the point under consideration. This loss is represented by the difference in elevation between the level in the reservoir and the water level in the piezometer pipe rising from the point in the subsoil. The loss of head was then expressed as a percentage of the total head on the dam. The pressure expressed in this way was assumed to be constant at any point.

From Darcy's law for flow through soil, $V = C_1 \frac{h}{L}$, or $h = \frac{VL}{C_1}$. Since $\frac{L}{C_1}$ is constant for any point in a homogeneous soil, h is directly proportional to the velocity, V, which, in turn, depends on the total head, H, that causes flow. When H increases, V also increases, and there is a greater loss due to friction. Thus, h increases as H increases, keeping the ratio between them theoretically constant. By expressing the loss in this way the effects of fluctuations in head-water and tail-water were eliminated.

Creep distance (which Mr. Lane defines as the distance of travel along the under surface of the dam) was divided into two classes for this study. The creep used in the Bligh method is called plain creep and the creep as introduced in the method advocated by Mr. Lane is designated as weighted creep. All the data were reduced to the same basis and expressed in the same terms. To make the comparison between theoretical and observed pressures a graph was plotted for each dam. The percentage-creep was plotted along the abscissa and the percentage-head loss was plotted along the ordinate. The values at the various observation pipes were connected by straight lines. In order to compute the percentage of plain creep for the horizontal control it was necessary to designate some point as the end of creep. The fact that tail-water potential must be assumed to exist at the end of creep complicated the determination of this point. Due to the presence of weep-holes, long down-stream aprons, grouted and ungrouted rubble masonry, the plane of the tail-water potential, or creep end, could not be located rigidly. An analysis of the conditions at the toe of each dam is included herein accompanied by a sketch of the dam (see Fig. 4). The point at which the end of creep was assumed to occur is indicated on each sketch.

Colorado River Dam.—In the case of the Colorado River Dam, the last pressure pipe was located 29 ft 3 in. up stream from Point *A* (Fig. 4(*a*)). Down stream from this pipe there was no method of determining the pressure. The location of the actual tail-water was not given in the data. Beyond Point *A* the water can escape freely to the tail-water so that a pressure corresponding to the tail-water elevation was assumed to exist down stream from Point *A*. The weep-hole was not taken as the end of creep because it was small and because it was in front of a concrete wall that would tend to create a pressure.

Percha Dam.—Two different locations of creep-end were used in drawing the curves for Percha Dam. In the first case the assumption was made that the rubble concrete, even with the weep-holes through it, offered enough resistance to retain some of the uplift pressure, and the end point was taken at Point *B* (Fig. 4(*b*)). In the second case the first weep-hole was taken as the end point and designated as Point *A* (Fig. 4(*b*)). This is based on the assumption that the weep-hole is large enough to permit the subsoil water to pass through freely.


A
Weep Hole
Screened Gravel
Pipe Line No. 1
A
15-ft Piles
Rip Rap
B
Rubble Concrete
Well No. 1
A
Wood Sheet-Piling
Weep Holes
Well Graded Gravel
30'
30'
A
Inverted Filter
Pressure Pipes
d_2
d_3
A
Pipe No. 4
A
B
Grouted Pitching
Ungrouted Pitching
E
A

(*a*) COLORADO RIVER DAM

(*d*) DEOHA BARRAGE

(*b*) PERCHA DAM

(*e*) PANJNAD HEADWORKS

(*c*) KABO HEADWORKS

(*f*) NARORA WEIR

(*g*) PINHOOK DAM

FIG. 4.

Kabo Head-Works.—The end of creep at Kabo Head-Works was taken at Point *A*, in Fig. 4(*c*). The original data did not indicate the location of the tail-water potential. The observations of water levels in Pipe d_3 (Fig. 4(*c*)) indicated a residual pressure, and, therefore, the end of the brick pitching was assumed to be the creep-end.

Deoha Barrage.—The end point in the case of Deoha Barrage is rather definitely fixed. The original data included computations which assumed Point *A* (Fig. 4(*d*)) to be at tail-water potential, and, consequently, this point was used.

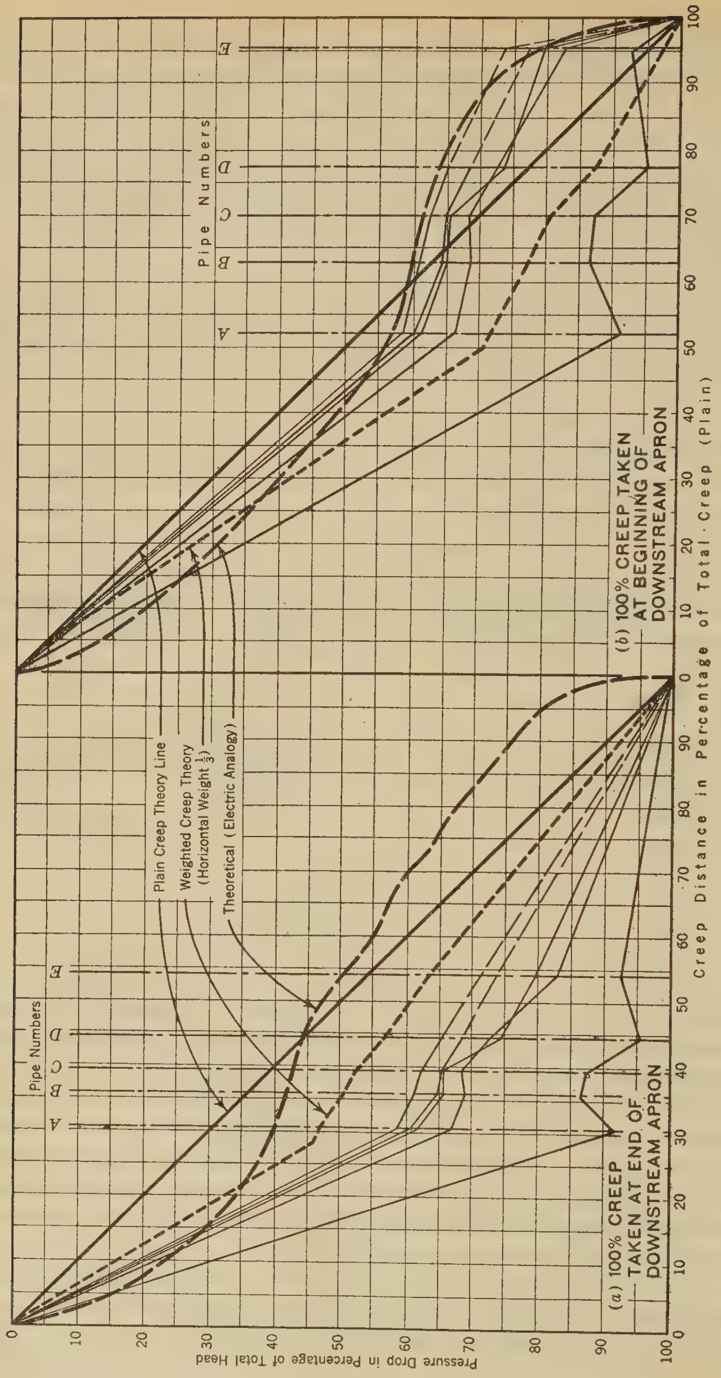

Pressure Drop in Percentage of Total Head
0
10
20
30
40
50
60
70
80
90
100
Creep Distance in Percentage of Total Creep (Plain)
Pipe Numbers
A
B
C
D
E
Plain Creep Theory Line
Weighted Creep Theory (Horizontal Weight 1/3)
Theoretical (Electric Analogy)
(a) 100% CREEP TAKEN AT END OF DOWNSTREAM APRON
(b) 100% CREEP TAKEN AT BEGINNING OF DOWNSTREAM APRON

FIG. 5.

Panjnad Head-Works.—The beginning of the filter at Point *A* at the Panjnad Head-Works (Fig. 4(*e*)) was taken as 100% creep. A residual head of about 9.4% occurs at this point. It decreased to 7.4% at the end of the filter, but increased again to 8.2% at the beginning of the down-stream rip-rap. This distribution of pressures can be attributed to a dividing of the flow after passing the last line of sheet-piling. The water probably escapes through the filter as well as the rip-rap. The magnitude of the residual pressure can be explained when the assumption is made that the layer of clay which is shown on a geological section of other parts of the subsoil extends into the section containing the pressure pipes. This layer of clay would create the pressures indicated.

Narora Weir.—In an analysis which accompanied the data for this weir the assumption was made that the creep at Narora Weir ended at Point *A* (Fig. 4(*f*)). This seems to conflict with the sketches that indicate grouted pitching beyond this point. The condition of this pitching might explain this contradiction, but no description of it was included. The question was decided by using two different points for the end of creep: First, Point *A* was used; then the set-up was changed, using Point *B* (Fig. 4(*f*)) for the end. The results are included for both cases to show how the plots change with the choice of the creep-end.

Pinhook Dam.—Because of the inadequacy of the data for Pinhook Dam it was again necessary to take two end points—one at the beginning and one (Fig. 4(*g*)) at the end of the down-stream apron. Fig. 5, which affords a comparison of the various design methods, is practically self-explanatory. It might be well, however, to point out a few of the features demonstrated.

Perhaps the most definite conclusion drawn from the pressure curve of this dam was that the Bligh method for predicting pressures fails to give anything like a true picture of the pressure distribution. The line computed by Mr. Lane's method follows the general trend of the measurements more closely than the Bligh line. It tends to correct the very definite error in the Bligh line, which leads to an over-estimate in the up-stream, and an under-estimate in the down-stream, parts of the foundation.

ARTHUR CASAGRANDE,[49] ESQ. (by letter).—In this study of the relationship of the stability of weirs and dams to the percolation of water beneath their foundations, the author has covered so large a number of structures all over the world that the paper represents the most comprehensive study of this kind that has ever been undertaken. For this reason the writer was greatly interested to study this paper in detail, including the unpublished supplement (filed in Engineering Societies Library) containing the cross-sections of most of the structures that were used by the author in his analysis.

Although the writer cannot agree with the general viewpoint expressed by the author, he has found the study of this paper very stimulating. It should certainly be helpful in inspiring engineers to pay more attention to the question of the effect of seepage on the stability of such structures.

---

[49] Asst. Prof. of Civ. Eng., Graduate School of Eng., Harvard Univ. Cambridge, Mass.

Mr. Lane suggests an improvement of Bligh's coefficient of percolation by introducing what he terms "weighted creep." The writer believes that the basic shortcoming of Bligh's method is not eliminated by the introduction of the weighted creep.

In a publication[50] by Charles Terzaghi, M. Am. Soc. C. E., which appeared in 1922, it was demonstrated convincingly by theory as well as by experiments that it is the magnitude and distribution of seepage forces near the toe of water-impounding structures (particularly the discharge gradient at the toe) which determines the degree of safety against piping or underground erosion.

In order to judge how misleading the use of Bligh's coefficient of percolation, or any modification of it, really is, compare the five solutions for a low dam on pervious ground shown in Fig. 6. The flow nets were constructed according to the method which Forchheimer developed in 1911.[51] In Table 7 are assembled quantities which were derived from the graphical solutions in Fig. 6 and which are helpful for a comparative study of the effects of seepage on the stability of the structure for the various designs shown.

TABLE 7.—Quantities Derived from the Graphical Solutions in Fig. 6

Case	Type of soil (1)	Seepage factor (2)	Discharge gradient at the toe (3)	Factor of safety against piping (4)	Effect of scour on factor of safety against piping (5)	Bligh's coefficient of percolation (6)	Lane's weighted creep ratio (7)	Average uplift, in tons per square foot, beneath downstream third of weir (8)	Remarks on Stability; Safety Against:	
									Piping (9)	Uplift (10)
A	Homogeneous.	0.33	0.22	4.6	Negligible	5.6	3.9	0.33	Safe	Unsafe
B	Homogeneous.	0.33	0.44	2.3	Very large	5.6	3.9	0.17	Decidedly unsafe	Safe
C	Homogeneous.	0.56	0.11	9.0	Negligible	3.8	1.9	None	Safe	Very safe
D	Homogeneous.	0.25	0.05	20.0	Negligible	9.8	3.9	None	Very safe	Very safe
E	Slighty stratified........	0.45	0.30	3.3	Negligible	5.6	3.9	0.46	Higher degree of safety recommended	Unsafe

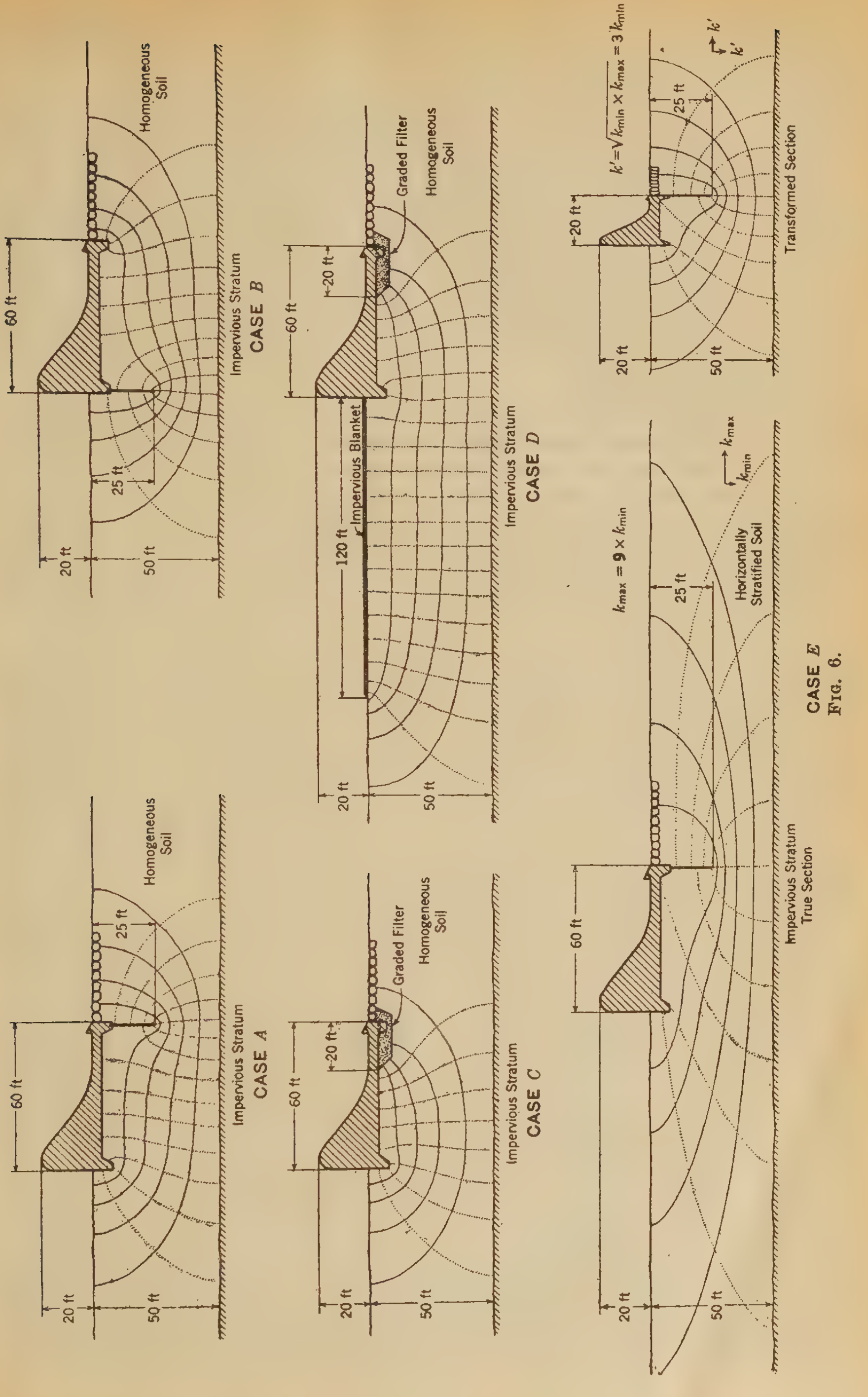
The seepage loss per unit of length of the dam is found (all quantities measured in a consistent system of units) by multiplying the coefficient of permeability with the difference between head and tail-water level and with the seepage factor, the latter being the ratio of the number of squares between any two neighboring lines of equal head (equi-potential lines) over the number of squares between any two neighboring flow lines. The seepage factor contained in Column (2) of Table 7 may be used, therefore, as a relative measure for the seepage losses of the various cases shown in Fig. 6.

Column (3), Table 7, contains the hydraulic gradient along the flow line at the point of discharge at the toe of the structure, for which the writer is using the term, "discharge gradient." According to Professor Terzaghi's

---

[50] "Der Grundbruch an Staumauern und seine Verhütung," von Charles Terzaghi, *Die Wasserkraft*, 1922.

[51] "Hydraulik", by Phillip Forchheimer, Berlin, 1930 Edition, p. 82.



CASE *E*
FIG. 6.

60 ft
25 ft
20 ft
50 ft
Homogeneous Soil
Impervious Stratum
CASE *A*
60 ft
25 ft
20 ft
50 ft
Homogeneous Soil
Impervious Stratum
CASE *B*
60 ft
20 ft
Graded Filter
Homogeneous Soil
20 ft
50 ft
Impervious Stratum
CASE *C*
60 ft
20 ft
Graded Filter
Homogeneous Soil
Impervious Blanket
120 ft
20 ft
50 ft
Impervious Stratum
CASE *D*
$k_{max} = 9 \times k_{min}$
k_{max}
k_{min}
25 ft
Horizontally Stratified Soil
60 ft
20 ft
50 ft
Impervious Stratum True Section
$k' = \sqrt{k_{min} \times k_{max}} = 3\,k_{min}$
k'
k'
25 ft
20 ft
20 ft
50 ft
Transformed Section

theoretical and experimental investigations,[50, 52] the soil at the toe of the structure will blow up as soon as the discharge gradient reaches a critical value which for average sands is equal to 1.

Column (4), Table 7, contains a factor of safety against piping based on a critical hydraulic gradient of 1. In Column (5) is stated the extent to which the discharge gradient is affected by scouring at the toe of the structure, a factor which should always be considered in the design of such structures. Column (6) contains Bligh's coefficient of percolation, and Column (7), Mr. Lane's weighted-creep ratio. The average hydrostatic uplift on the down-stream third portion of the structure, in tons per square foot, is assembled in Column (8). Columns (9) and (10) contain general remarks as to the stability of the structure as affected by seepage pressures and by hydrostatic uplift.

A comparison of the five cases in Table 7 indicates clearly that neither Bligh's nor Lane's coefficients reflect the degree of safety against piping. For example, Case *B* has only a small degree of safety against piping which would disappear entirely if scouring took place at the toe. Therefore, this design must be considered unsafe. On the other hand, Case *C* has a factor of safety of 9 and scouring has little effect on the degree of safety. However, according to Mr. Lane's weighted-creep ratio, Case *B* appears to be twice as safe as Case *C*.

The examples shown in Fig. 6 and the corresponding data in Table 7 also permit a conclusion regarding the effectiveness of a properly designed filter. It is apparent that such filters beneath the down-stream portion of the structure are more effective in reducing the discharge gradient than an increase in the width of the weir or a sheet-pile wall of average depth. It was Professor Terzaghi[50] who first pointed out the advantages of such filters that are loaded by the weight of the structure itself and are properly protected from scouring action.

From the foregoing, it will be understood why the writer cannot agree with Mr. Lane that a reverse filter is good only for a 10% increase in security.

Under the heading, "Reverse Filters, Weep-Holes, and Drains," the author makes a number of important remarks, conceding that designs of such structures have been used with very short percolation distances. The example of Coon Rapids Dam, in Minnesota, where the coefficient of percolation was reduced accidentally to 0.6 without ill effects, shows strikingly the effectiveness of loaded reverse filters as a method for preventing piping as well as unfavorable uplift pressures. The writer wishes to emphasize that there is no other way of knowing whether or not a filter and drains will be effective than by Professor Terzaghi's theoretical and experimental approach to these problems.

During the ten years, 1925–35, Professor Terzaghi and others have shown that the average permeability of a natural deposit of soil in a horizontal direction (k_{max}) is always larger than that in a vertical direction (k_{min}); and that $k_{max} = n k_{min}$, in which n varies between 2 and 10. For distinctly

---

[52] "Erdbaumechanik", von Charles Terzaghi, M. Am. Soc. C. E., Vienna, 1925, pp. 369–377.

stratified soils the value of n may become much larger than 10. Such soil conditions reduce the safety of a structure if compared with the safety of a similar structure on homogeneous soil with an average permeability, $k' = \sqrt{k_{\max} \times k_{\min}}$, which will yield the same amount of seepage. As an example, consider Case *A* in Fig. 6, and assume that the average coefficient of permeability of the soil in a horizontal direction, $k_{\max}$, is nine times greater than $k_{\min}$ for the vertical direction. As a result of the anisotropic character of the soil, the flow net will be distorted as shown in Case *E*, Fig. 6. Flow lines and lines of equal head do not intersect at right angles, except for certain positions, and the squares are elongated in the horizontal direction. For the construction of this flow net, the writer used a method which was originally developed and applied to seepage problems by A. F. Samsioe, of Stockholm, Sweden, in 1930. According to this method, one can find the actual flow net for an anisotropic soil by plotting an ordinary flow net in a transformed cross-section of the structure in which all horizontal dimensions are reduced by the factor, $\sqrt{\frac{k_{\min}}{k_{\max}}}$. In the present example, this factor is $\frac{1}{3}$. The transformed section and flow net are shown in Case *E*, Fig. 6. After reversing the transformation, the actual shape of the flow net is as shown in Case *E*, "True Section."[53]

A comparison of the structure on homogeneous soil (Fig. 6, Case *A*) with the same structure on slightly stratified soil (Case *E*), is afforded by the data contained in Table 7. The stratification has increased the seepage losses by about 30% and decreased the factor of safety by approximately the same amount. For distinctly stratified soils, the discharge gradient may easily reach the critical value. This example demonstrates the tremendous importance of stratification of the soil and of other "minor geologic details"[54] on the safety of such structures against piping.

Mr. Lane's statistics reflect the effect of such stratification and to a certain extent he succeeds in refining Bligh's method in the right direction. For example, in Case *A* (Fig. 6), if one wished to increase the safety by increasing the horizontal path or the vertical path of percolation, Bligh's coefficient would indicate no preference, while Mr. Lane's method gives preference to the vertical faces of a sheet-pile wall. However, the ratio for this preference is fixed instead of being dependent on actual soil conditions. In addition, the "weighted creep" as well as Bligh's coefficient disregard entirely the fundamental importance of the relative position of vertical faces to the toe of the structure, for which there is no other method of analysis available than that developed by Professor Terzaghi.

The writer believes that any statistical approach to this problem as attempted by Bligh and the author can be defended only as long as the mechanics of piping is not properly understood and as long as there are no scientific methods available to analyze seepage pressures and particularly their effect on the stability of the soil near the toe of such structures.

---

[53] A general derivation for this transformation formula is contained in a paper by Professor Schaffernak, entitled "Erforschung der physikalischen Gesetze, nach welchen die Durchsickerung des Wassers durch eine Talsperre oder durch den Untergrund stattfindet," *Die Wasserwirtschaft*, 1933, No. 30.

[54] "Effect of Minor Geologic Details on the Safety of Dams," by Charles Terzaghi, M. Am. Soc. C. E., Am. Inst. of Min. and Metallurgical Engrs., *Technical Publication No. 215*, February, 1929.

The safety of a structure designed on the basis of Bligh's or Lane's coefficients of percolation is subject to the law of probability on which these methods are based, thus meaning that in most cases the structure will be many times over-designed, whereas occasionally local soil conditions, perhaps combined with an unfavorable cross-section, will invite failure. However, Professor Terzaghi's analysis of the seepage forces, if based on extensive investigations of the soil conditions, does provide a satisfactory basis for choosing a safe and economical cross-section which will best fit the peculiarities of the foundation conditions in each case.

The general character of, and any uncertainties in, the soil conditions can easily be taken care of by choosing an appropriate factor of safety. The writer believes that the factor of safety against a blow-up at the toe should be at least 3 for homogeneous soil strata with a coefficient of permeability, k_{max}, of not more than two or three times k_{min}, and with a sufficiently high vertical face at the toe so that scouring will have only a small effect on the magnitude of the discharge gradient. However, if the soil conditions are not sufficiently explored to permit reliable conclusions regarding their uniformity, or if such exploration and soil tests indicate the presence of stratified or irregular deposits (particularly if consisting of uniform fine-grained sands) the factor of safety should be at least 10, and still larger for very uncertain foundation conditions. It was on such a basis that the writer arrived at the concluding remarks contained in Columns (9) and (10) in Table 7.

In conclusion, the writer wishes to state that the study of Mr. Lane's paper has fortified his belief that Professor Terzaghi's theory on piping and underground erosion and his methods for analyzing the stability of weirs and dams are in excellent agreement with practical experience and that they also represent the only existing approach to these problems which is satisfactory from a scientific standpoint. Their application is strongly recommended in the design of new structures, as well as in the analysis of the stability of many existing weirs and dams.

CALVIN V. DAVIS,[55] M. AM. SOC. C. E. (by letter).—An exhaustive research, which covers the characteristics of a considerable number of masonry dams on soft or porous foundation, forms the basis of this valuable and timely paper. The conclusions, therefore, have been crystallized from the practical experiences of the very limited group of engineers who have labored in this highly special field. As experience is by far the most important factor in the design of such structures, these conclusions are entitled to both respect and thoughtful consideration. No better method can be found as a starting point for the creation of a new method of analysis of creep-flow ratios than a thorough examination of the record of as many existing dams of reasonable height as may be available and the requirements of design that were followed.

---

[55] Chf. Designer, Ambursen Dam Co., Inc., New York, N. Y.

The results of the author's comprehensive survey have been expressed in terms of a modified form of Bligh's ratio in which the vertical surfaces of cut-off walls are assumed to have three times the resistance to water travel possessed by the under sides of horizontal floors and hearths. Although Mr. Lane's research confirms the fact that the resistance of vertical surfaces to flow is greater than that of many forms of horizontal surfaces, the results are not conclusive enough to give each a definite weight. Actually, the relation between the resistance to water travel of horizontal-creep and vertical-creep surfaces will vary with both the foundation material and the type of structure built on it.

It is certain that under conceivable conditions the resistance of a horizontal foundation slab might be considerably less than one-third that of the vertical surfaces of a good cut-off wall. If, however, a foundation were serrated as shown by Fig. 7, the under side of the floor should have a water-travel resistance equal to that of a vertical cut-off surface. The floor of the dam shown by Fig. 7 would be tightly pressed against the foundation material by the inclined components of buttress stress, which are considerably greater in magnitude than the vertical components acting on a horizontal floor. Sliding is also partly resisted by compressive and shearing forces rather than by friction, and this also would have a tendency to tighten the foundation material against the floor. It would not be necessary for the inclined surfaces of this floor to be as steep as 45° to be as effective as vertical surfaces, as suggested by Mr. Lane.

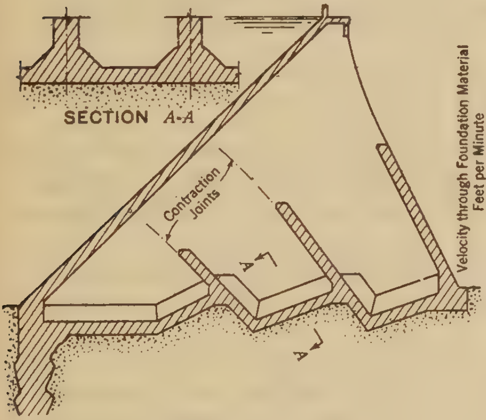


Fig. 7.

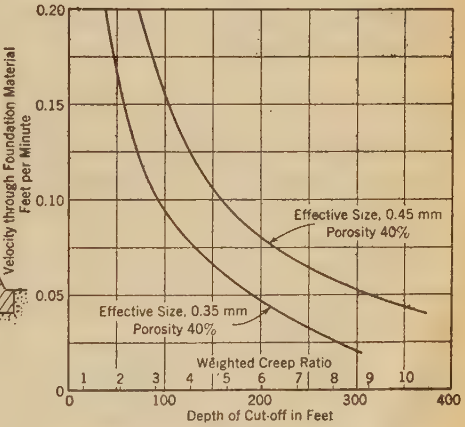


Fig. 8.—Height of Dam, 75 Feet; Effective Horizontal Creep, 45 Feet; Foundation Material, Medium Sand.

The writer has prepared various designs for this type of construction for specific projects, varying from 25 ft to 210 ft in height, and for different kinds of earth foundations. Although as yet none has reached the construction stage, a careful analysis of the designs indicated that this type possesses considerable merit, particularly for high dams in which the introduction of a contraction joint along the trajectory of first principal stress may be indicated. Its resistance to creep is obvious.

The uncertainties connected with the theoretical basis of the investigation as developed by the author from Darcy's law do not allow fixed weighted-creep ratios to be established with consistency. In applying Darcy's law as expressed by the author in Equations (1), (2), and (3), the required length of creep may be determined by the direct method of applying coefficients which experience has proved to be amply safe—or by assuming a velocity of flow through the foundation material that would appear to give a large margin of safety against piping failure—and then determining the depth of cut-off required to hold to this velocity. The latter method is commonly based on the formula derived by Professor C. S. Slichter which was also referred to by the author.

A comparison of these two different approaches to the same problem reveals a decided lack of agreement. Typical examples will illustrate this discrepancy.

Assume a spillway dam of the Ambursen type on continuous floor, founded on medium sand, having a height of 75 ft and a total horizontal creep of 135 ft. The depth of cut-off required for this dam by the author's method (adopting his recommended weighted-creep ratio of 6) would be:

$$d = \frac{1}{2}\left[(6 \times 75 \text{ ft}) - \frac{135}{3} \right] = 202.5 \text{ ft.}$$

In determining the cut-off depth by the second method, Darcy's law is modified to include Slichter's formula. The resulting formula for the depth of the cut-off wall is:

$$d = \frac{K\,H}{2\,P\,V} - \frac{b^2\,P\,V}{2\,K\,H} \quad \text{.......................} (6)$$

in which, d = the depth of cut-off; K = a transmission constant; H = the head, in feet; P = the porosity of the material expressed as a decimal; V = permissible velocity, in feet per minute; and, b = the effective base width of the dam.

Equation (6) is based on the short-path method which is correct if the safe velocity through the foundation material is to be used as a criterion of foundation safety.

An examination of Slichter's tables, however, shows that there is no fixed definition for a medium sand; a so-called medium sand has a range of effective sizes varying from 0.25 mm to 0.45 mm. The properties of sands having this variation in size are by no means the same, and yet by arbitrarily using the fixed-creep ratio of 6 recommended by Mr. Lane the same treatment would be given to all the foundation materials coming within this classification.

To illustrate, Fig. 8 shows the relation (based on Equation (6)) between depth of cut-off, velocity, and weighted-creep ratio for the dam in the foregoing problem if two sands each having a porosity of 40% and effective sizes of 0.35 mm and 0.45 mm, respectively, are assumed. It will be observed from

Fig. 8 that the weighted-creep ratio for the finer material need only be about 62% of that required for the coarser material in order to give the same velocity through the foundation materials or equal theoretical factors of safety against failure by piping.

A velocity of 0.2 ft per min is considered by Joel D. Justin,[56] M. Am. Soc. C. E., and other authorities to be ultra safe for a medium sand. A weighted-creep ratio of 6 gives a depth of cut-off 2.7 times that required to give the velocity through the coarser material and 5.0 times that required to give it through the finer material.

The depth as determined from Slichter's formula, however, is based on the short-path flow. The results obtained in the foregoing computations in one respect substantiate Mr. Lane's conclusion as to the weakness of the short-path theory. Actually, a much greater length of creep is necessary than that required to give merely a safe velocity through the underlying foundation material. Both experience and theory in this case also indicate that there is a greater probability of seepage following the plane of contact of the dam with the foundation material.

The writer has found, however, that the length of water travel need not be as great as required by the weighted-creep ratios recommended by the author if proper construction methods are followed. His experience with the design of permeable foundation dams has been that lower ratios may be used with safety if the foundation structures consist of continuous, heavily reinforced, concrete slabs flexibly connected to substantial vertical cut-off walls at their up-stream and down-stream ends. The under side of these slabs may contain intermediate shallow cut-off walls and should be in intimate contact with the foundation material.

Where a very great length of travel is indicated it would probably be broken into at least two vertical cut-off walls, and additional horizontal travel might be installed up stream of the dam in the form of an impervious fill or a concrete mat to reduce the depths of the cut-off wall. There is no question but that the use of an area of clay blanket or other impervious fill above the dam, well connected to the up-stream deck, is in many cases the cheapest way to secure a limited amount of additional creep. Although the writer has sometimes used a concrete slab for this purpose, he believes that where the material is available a well-placed clay blanket, because of its plastic nature, is a preferable form of construction. The ideal up-stream mat is a layer of clay, carrying upon it an articulated concrete slab, with a further clay fill upon the concrete.

Empirical designing rules and theoretical assumptions must always have their part in dam design, but lack of care or skill in actual construction may be more dangerous to the stability of a dam than errors in designing assumptions. The writer fully agrees with the soundness of Mr. Lane's judgment in preferring a concrete cut-off properly built in contact with the sides of the trench to a steel or timber sheet-piling. The writer would use the latter in no case, except where the depth to which it is to be driven is relatively small and the material one susceptible of easy penetration. Wandering, brooming, and splitting is the rule rather than the exception for Wakefield, splined, or tongue-and-groove sheeting.

---

[56] "Earth Dam Projects," by Joel D. Justin, John Wiley & Son, 1932.

In the case of steel sheet-piling, its advantages are fully realized, but its uncertainties in hard driving are also recognized as well. Furthermore, the writer has never been satisfied that the use of such piling in certain types of foundation materials may not reduce the resistance to water travel by inviting pipes along its smooth sides, particularly along the lines of the structural irregularities necessary to its interlock. The use of jets for sinking difficult cut-off piling may well impair its creep-resisting value in many materials through displacement of well-bedded particles and the disturbance of all the immediately adjacent material.

In the case of the conventional concrete cut-off wall, when constructed in almost any material other than hardpan, well-cemented gravel, or very stiff clay, it is more often than not subject to faulty construction and its creep value may even be less than good horizontal creep. These field defects are inherent in wet foundation construction where men are working at a disadvantage and where flowing water will carry with it the finer materials adjacent to temporary sheeting. Even if performed in the dry, if sheeting and bracing is necessary, the withdrawal of the sheeting invites displacement of foundation material and faulty contact between the concrete and the sides of the trench. The writer recalls a recent example of the human errors which may nullify the most careful design. Despite the specifications, a cut-off into fractured shale, instead of being poured solidly into the trench, was neatly formed. The remaining area was back-filled with broken shale, reducing the value against creep to the bare thickness of the wall at the bottom where it contacted the seamed and partly disintegrated shale. In another and older case, a cut-off wall was built in gravel, boulders, and sand, and the up-stream part of the trench was actually back-filled with construction débris as the sheeting was withdrawn. Early failure of this structure was readily predicted by the designer unless the trench were re-excavated and a puddle fill substituted; it occurred a month or two later while the city officials were still meditating on his recommendations.

Many attempts to grout porous foundation materials have convinced the writer that improvement in the value of horizontal creep cannot be assured by this method, as suggested by the author. Operative difficulties limit the value of grouting beneath a poorly constructed foundation slab. Perhaps some of the methods of chemical impregnation or permanent gelatinization may yet be developed to the point where suspected areas may be closed against percolating water, but the conventional cement grout has been tried many times and although found to be helpful in some cases, limited reliance on its efficacy is advised.

For horizontal creep, the writer has always advised against placing too much reliance upon long hearths below the dam, whether monolithic or articulated, but particularly the latter. These offer little, if any, resistance to the underflow of water and are entitled to slight consideration in determining the proper depth to which the vertical cut-off should be carried. An extensive program of experimental research is urgently needed to establish fundamental principles.

C. H. EIFFERT,[67] M. AM. SOC. C. E. (by letter).—The writer has always been of the opinion that, in sand and gravel foundations such as those for most low-head dams in the southwestern part of Ohio, the seepage or percolation is more likely to take the short path than to follow the line of creep. The writer agrees absolutely with Mr. Lane in the statement that for clay or hardpan foundations the short-path principle is inferior to the line-of-creep analysis. However, it seems quite possible that in a porous foundation of sand and gravel the line of least resistance would be the short path through this material and, as Mr. Lane states, it seems axiomatic that the line of least resistance will be taken by the seepage.

It seems reasonable to assume that as a rule there will be greater resistance to flow along vertical joints than along those which are horizontal. The weight of the material will cause it to remain in close contact with a vertical row of sheet-piling and, for the same reason, it may draw away from the horizontal surface of the bottom of the dam, at least in places (especially if there is pumping during construction which may draw out the finer material). However, one should not overlook the fact that, although the sheet-piling will have a smooth surface, the concrete bottom of the dam will not be smooth

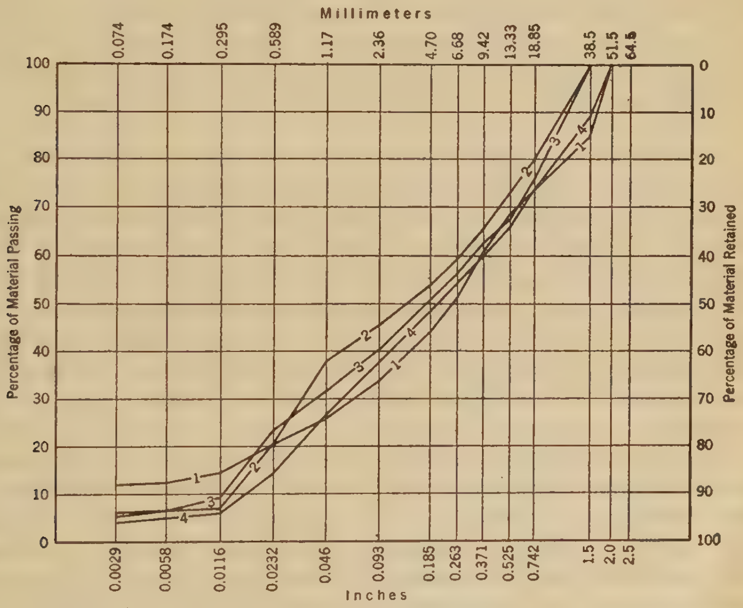


FIG 9.

---

[67] Chf. Engr. and Gen. Mgr., The Miami Conservancy Dist., Dayton, Ohio.

if poured on a gravel foundation; it will be forced down into the voids of the gravel so as to form a more or less interlocking bond which, if settlement does not take place, may offer greater resistance to flow than the contact with the smooth surface of steel sheet-piling or the short path through the material itself.

In Table 1 (*b*), the Hamilton, Ohio, Dam (No. 33), is listed as having a weighted-creep ratio of 5.5. The material under this dam is a dense, well graded gravel, varying in size from boulders to sand. A few years ago, in making repairs to the paving below the apron, a trench about 6 ft deep was dug the entire length of the apron along its down-stream edge. This was unwatered in sections and practically no seepage could be detected coming from beneath the dam. It is possible, therefore, that the length of travel for seepage under this dam is greater than necessary as the conditions in actual use will be much less severe than those imposed during the repair work. However, it is also quite probable that the bed of the stream above the dam has been sealed by silt deposits so that a greater ratio was necessary immediately after construction than is now required.

No mechanical analyses have been made of materials coming from the foundation of this dam, but several such analyses were made of materials taken within a few hundred feet of the dam and which appear to be the same type of gravel. These are presented in Fig. 9, and although it is not possible to determine the required ratio from them, they do establish the fact that, for this type of material, the existing ratio at this dam is sufficient.

E. H. Burroughs, Jr.,[58] M. Am. Soc. C. E. (by letter).—The increase in the sum of engineering knowledge as to adequate treatment for porous foundations during the past two or three decades is made apparent by a comparison of Mr. Lane's paper with that presented in 1911, by Mr. Arnold C. Koenig.[59] Reading the author's paper in the light of this earlier analysis of the state of the art demonstrates, strikingly, the advancement that has been made.

Mr. Lane's paper thus directs the attention of the Engineering Profession at large to the vast changes in the field of masonry dam design and construction during the past thirty years. Essential water storage and power developments once thought impossible because of foundation conditions formerly considered unconquerable have been successfully consummated through an arduous and painstaking solution of the many theoretical and practical factors controlling the safe utilization of the site. During the writer's span of professional practice he has seen the design and construction of masonry dams on permeable foundations become well-established engineering practice. Thirty years ago masonry dams on other than ledge foundations were purely speculative, except for insignificant heads. Of these small dams many had been built and an astonishingly large percentage had gone down the river, for reasons usually unknown. To-day, the building of safe low dams on earth foundations is relatively common, and in the field of higher heads the upward limit has not been reached.

---

[58] Vice-Pres., Ambursen Constr. Co., Inc., New York, N. Y.

[59] "Dams on Sand Foundations: Some Principles Involved in Their Design, and the Law Governing the Depth of Penetration Required for Sheet Piling," *Transactions*, Am. Soc. C. E., Vol. LXXIII (1911), p. 175.

The results presented in this paper should not be used arbitrarily as the necessary fundamentals for the design of dams on soft foundations, owing to the considerable height and importance of many of the dams accompanying modern hydraulic projects. The design of a high masonry dam on foundations other than sound ledge rock presents a problem that differs from the design of low weirs, such as comprise more than one-half the dams listed by Mr. Lane. It would have been more desirable to have segregated these dams according to height, but even this would not tell the complete story. Each high dam on permeable foundations (much more so than in the case of low-head dams), presents an individual problem that may bear little relation to the design of any other such dam. The inclusion of a multitude of small dams in the paper adds to its value as a compendium of experience, but whether the study of the design and performance of these low-head structures adds substantially to the value of a weighted-creep ratio theory, or is actually misleading, is a question.

The securing of a high factor of safety for a low dam usually requires small investment, particularly if the depth of vertical up-stream cut-off theoretically required may be small. This is generally the case, due to the relatively great length of base required for stream-bed protection below small spillway diversion weirs on soil foundation. A few additional feet in depth of sheet-piling does not proportionately increase the cost of the cut-off and is cheap insurance. Therefore, a greater study of the higher structures may well be justified, as the cost of providing an adequate factor of safety for a high dam for power or for storage on earth foundation becomes one which may actually determine the economic feasibility of the entire development. It is fallacious and misleading to apply to the higher range of dams on permeable foundations arbitrary coefficients derived from observation of a large number of unimportant low dams with heads of 15 ft, or less. A more valuable measure of proper requirements for future structures of considerable height is experience in the design and construction of similar structures and observation of their subsequent behavior over a long period of years.

The primary purpose of the design for a dam on porous foundations is to produce a structure with the largest factor of safety for the smallest money investment, within reasonable limits. Wasteful use of material on the one hand must not be encouraged; on the other hand, a design must be produced which meets all requirements for an individual site, which will be safe against unforeseen events, and which will require as little maintenance as possible.

The writer is reluctant to suggest that the author's ratios, already well below those of Bligh, should be reduced still further in general practice. However, it does appear that the rather meager data concerning failures, which have controlled his safe percolation distances are neither sufficiently authentic nor sufficiently comprehensive to justify the reliance that he has placed upon them. Undoubtedly, in the case of most of these failures, the true cause has never been fully established. Unknown defects in construction and changes in operating circumstances or natural conditions may have contributed even to those failures which are specifically classed as due to under-seepage.

Table 8 is offered to prove that the creep ratios for existing low-head dams influence more important structures unfavorably from the viewpoint of economical design, and, furthermore, to demonstrate that dams of higher heads may be built safely with lower creep ratios than those of lower heads. The tabulation includes 186 dams and power plants, on soft foundations, assembled from the writer's experience and records and from those of his associate in professional practice, A. Streiff, M. Am. Soc. C. E. Full acknowledgment is made to him for his aid in the laborious compilation of these new data, as well as to H. S. Hunt and W. G. Fargo, Members, Am. Soc. C. E., for the exact data on dams designed or constructed under their supervision. Many of these structures are quite old, and, therefore, caused the writer much difficulty in assembling correct foundation records and substructure data, but the basic information is derived from contact with each project. It is difficult to place many of these projects, accurately, in a somewhat arbitrary class of foundation material, but the grouping used in Table 8 is substantially along the same line as that of the author, and, in general, follows his classifications.

The large number of structures covered by Table 8 will justify a further examination of the safe weighted-creep ratios laid down by the author. For the purpose of making the data comparable with those of Mr. Lane, in preparing Table 8 the writer has shown, in one column, the author's recommendation for the weight of horizontal creep as one-third, although with reservations as discussed elsewhere. He also shows the actual (unweighted) creep ratio.

The writer wishes to make it very plain that in giving the mean creep ratio, whether weighted or actual, it should not be inferred that any such rule-of-thumb determination of a safe ratio is even implied for application to other projects on seemingly similar foundation materials. It is well to state at this point that the great variation in the creep ratios for individual structures on similar foundation material has been due to many factors, not the least of which has been the profitable knowledge of the methods of design and construction successfully followed in earlier structures. Furthermore, particularly in the case of the weighted-creep ratios, the shape of the structure, the details of the design, the necessary erosion protection, and numerous other circumstances have been material factors in these variations.

A few of the dams tabulated have not been included in the computations of the ratios; foundation data were not dependable, and, in some cases, the cut-off wall was carried into impervious material. As in Mr. Lane's lists, they are included without analysis. In a few of the structures the use of weep-holes in the floor tends to shorten the creep distance somewhat, but this has been ignored in the analysis, as was also done by Mr. Lane in his tables.

Table 8 shows in a striking manner that a creep ratio for all dams on any classification of foundation material, if based on a mean of the existing dams,

TABLE 8.—COMPARATIVE DATA, WEIGHTED-CREEP RELATIONS

Dam No.	Project	Stream	State	Head, in feet	Creep Distance, in Feet		Creep Ratios	
					Vertical	Horizontal	Actual	Weighted
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
(*a*) DAMS ON VERY FINE SAND OR SILT								
1	Oklahoma City By-Pass	North Canadian	Oklahoma	6	38	34	12.00	8.22
2	Oklahoma City Diversion	North Canadian	Oklahoma	10	40	30	7.00	5.00
3	Middletown	Miami	Ohio	10	28	39	6.70	4.10
4	San Benito Lock	Rio Grande	Texas	10	20	95.5	11.55	5.20
5	Wiggin	Bijou Creek	Colorado	11	48	41	8.09	5.55
6	Hightstown No. 1	Rock Brook	New Jersey	12	30	28	4.83	3.28
7	Otsego	Kalamazoo	Michigan	14	81	28.5	7.83	6.45
8	Barre	Prince	Massachusetts	15	50	67	7.80	4.82
	Mean for dams, 15 ft., or less			....	....	....	7.93	5.18
9	Sommerset	Apple	Wisconsin	20	40	45	4.25	2.75
10	Cotulla	Nueces	Texas	22	60	75	6.14	3.86
11	Millikin	Nueces	Texas	22	52	82	6.09	3.60
12	Anadarko	Washita	Oklahoma	28	65	57	4.36	3.00
13	Akron	Cuyahoga	Ohio	35	115	100	6.14	4.24
14	Rogers	Muskegon	Michigan	36	100	89	5.25	3.60
15	Croton	Muskegon	Michigan	39	180	275	11.67	6.95
16	Pittsfield No. 1	Ashley Brook	Massachusetts	41.5	....	....	....	....
	Mean for dams higher than 15 ft.			....	....	....	6.61	4.22
	Mean for Table 8 (*a*)			....	....	....	7.01	4.51
	Creep ratios recommended by Lane			....	....	....	....	8.50
(*b*) DAMS ON MEDIUM SAND								
17	Lake Erie	Lake Erie Inlet	Ohio	6	10	17	4.50	2.61
18	Oliver	Okanagan	British Columbia	8	15	30	5.62	3.13
19	Portland	Looking Glass	Michigan	10	36	38	7.40	4.87
20	Wichita	Wichita	Texas	12	32	180	17.67	7.67
21	Thompsonville	Bear Creek	Michigan	13	44	80	9.53	5.42
22	Tecumseh	Raisin	Michigan	15	53	56	7.25	4.75
	Mean for dams, 15 ft., or less			....	....	....	9.24	5.05
23	Pasadena	Reservoir	California	16	....	....	....	....
24	Grand Junction	Gunnison	Colorado	18	45	60	5.83	3.61
25	Berrien Springs	St. Joseph	Michigan	23	106	62	7.32	5.50
26	Rice's Rip	Messalonskee	Maine	28	96	66	5.79	4.22
27	Mio	Au Sable	Michigan	29	112	100	7.31	5.02
28	Traverse City	Boardman	Michigan	29	61	54	3.95	2.72
29	Bassano	Bow	Alberta	50	25	165	3.80	1.60
	Mean for dams higher than 15 ft.			....	....	....	5.38	3.47
	Mean for Table 8 (*b*)			....	....	....	6.40	3.89
	Creep ratios recommended by Lane			....	....	....	....	6.00
(*c*) DAMS ON COARSE SAND								
30	Bound Brook No. 1	Raritan	New Jersey	6	6	24	5.00	2.34
31	Cranford	Rahway	New Jersey	6	16	20	6.00	3.78
32	Bound Brook No. 2	Millstone	New Jersey	7	8	30	5.42	2.57
33	Upper Nile	Upper Nile Canal	Colorado	10	60	31	9.10	7.03
34	Watertown	Rock	Wisconsin	15	48	20	4.53	3.64
35	Highland Park	Reservoir	Michigan	15	4	140	9.60	3.34
36	Leavittsburg	Mahoning	Ohio	15	42	58	6.63	4.08
	Mean for dams, 15 ft., or less			....	....	....	6.85	3.94
37	Caro	Cass	Michigan	17	50	22	4.53	3.37
38	Columbia	Paulins Kill	New Jersey	20	64	2	3.30	3.27
39	Gloucester	Ipswich	Massachusetts	21	42	70	5.33	3.12
40	Wausau	Wisconsin	Wisconsin	32	100	98	6.19	4.14
41	Pitt	Pitt River	California	58	136	58	3.34	2.69
	Mean for dams higher than 15 ft.			....	....	....	4.34	3.22
	Mean for Table 8 (*c*)			....	....	....	5.17	3.45
	Creep ratios recommended by Lane			....	....	....	....	5.00

TABLE 8.—(*Continued*)

Dam No.	Project	Stream	State	Head, in feet	Creep Distance, in Feet		Creep Ratios	
					Vertical	Horizontal	Actual	Weighted
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
(*d*) Dams on Medium Gravel								
42	Charlotte	Battle Creek	Michigan	5	8	15	4.60	2.60
43	Dimondale	Grand	Michigan	8	16	32	6.00	3.32
44	Hayden	Raisin	Michigan	8	14	25	4.88	2.78
45	Centerville	St. Joseph	Michigan	8	12	36	6.00	3.00
46	Kalamazoo	Kalamazoo	Michigan	8	17	40	7.12	3.79
47	Smithville	Grand	Michigan	8	18	25	5.38	3.28
48	Pompton Lakes	Post Brook	New Jersey	9	48	21	7.67	6.12
49	Joliet No. 2	Desplaines	Illinois	12	40	30	5.82	4.17
50	Concord	Kalamazoo	Michigan	12	36	42	6.50	4.20
51	Woonsocket	Providence	Rhode Island	13	30	36	5.07	3.24
	Mean for dams, 15 ft., or less			....	....	....	5.95	3.73
52	Watertown	Rock	Wisconsin	17	40	48	5.18	3.41
53	Crystal City No. 2	Nueces	Texas	20	62	77	6.95	4.42
54	Cartersville	Etowah	Georgia	21	42	30	3.43	2.48
55	Woodstock	Ottauquechee	Vermont	24	40	40	3.33	2.23
56	Uncas	Shetucket	Connecticut	28	90	80	6.07	4.17
57	Williams	White	Indiana	29	....	....	....	....
58	Alameda	Alameda Creek	California	31	70	140	6.77	3.76
59	Plattsburg	West Brook	New York	35	....	....	....	....
60	Grants Pass	Rogue	Oregon	38	64	70	3.54	2.30
61	Crystal City No. 1	Nueces	Texas	50	70	120	3.80	2.20
	Mean for dams higher than 15 ft			....	....	....	4.73	2.97
	Mean for Table 8 (*d*)			....	....	....	5.07	3.18
	Creep ratio recommended by Lane			....	....	....	....	3.50
(*e*) Dams on Gravel and Sand								
62	Albion	Kalamazoo	Michigan	8	18	16	4.25	2.92
63	Fort Humphrey	Accotink Creek	Virginia	9	40	12	5.77	4.88
64	Oneonta	Susquehanna	New York	9	25	30	6.12	3.90
65	Greenville	Flat	Michigan	10	20	50	7.00	3.66
66	Balding	Flat	Michigan	10	53	39	9.20	6.60
67	Stamford	Mill Creek	Connecticut	10	20	3	2.30	2.10
68	Smyrna	Flat	Michigan	10	34	21	5.50	4.10
69	Jonesville	St. Joseph	Michigan	11	25	52	7.00	3.85
70	Flint	Flint	Michigan	11	44	40	7.63	5.20
71	Homer	Kalamazoo	Michigan	11	22	23	4.08	2.87
72	Saranac	Grand	Michigan	12	25	26	4.25	2.81
73	River Raisin	Raisin	Michigan	12	39	28	5.58	4.02
74	Allegan	Kalamazoo	Michigan	13	70	30	7.70	6.15
	Mean for dams, 15 ft., or less			....	....	....	5.92	4.11
75	Reed City	Hershey	Michigan	16	29	25	3.38	2.33
76	Ypsilanti	Huron	Michigan	17	58	28	5.07	3.97
77	Garfield	Garfield Creek	California	18	6	80	4.77	1.80
78	Joliet No. 1	Desplaines	Illinois	20	42	26	3.40	2.54
79	Gillespie	Gila	Arizona	22	60	68	5.82	3.76
80	Twin Branch	St. Joseph	Indiana	22	60	....	2.72	2.72
81	Jim Falls	Chippewa	Wisconsin	25	62	36	3.92	2.96
82	Hyatts	Oswegatchie	New York	28	40	64	2.96	2.19
	Mean for dams, higher than 15 ft			....	....	....	4.07	2.78
	Mean for Table 8 (*e*)			....	....	....	4.89	3.37
	Creep ratio recommended by Lane			....	....	....	....	....

TABLE 8.—(*Continued*)

Dam No. (1)	Project (2)	Stream (3)	State (4)	Head, in feet (5)	CREEP DISTANCE, IN FEET		CREEP RATIOS	
					Vertical (6)	Horizontal (7)	Actual (8)	Weighted (9)
	(*f*) DAMS ON BOULDERS WITH COBBLES AND GRAVEL							
83	Newton Lower Falls	Charles	Massachusetts	7	40	20	8.57	6.67
84	Rolling Fork	Rolling Fork	Kentucky	8	30	15	5.63	4.38
85	Picton	Raritan	New Jersey	8	43	19	7.75	6.17
86	Post Brook	Post Brook	New Jersey	9	50	21	7.89	6.33
87	Brome	Brome Lake Inlet	Quebec	10	38	17	5.50	4.37
88	Dunbarton	Battenkill	New York	11	36	48	7.63	4.73
89	Piedmont	Potomac	West Virginia	12	18	14	2.67	1.89
90	Sixth Lake	Sixth Lake Inlet	New York	12	18	31	4.08	2.36
	Mean for dams, 15 ft., or less	....	....	....	....	....	5.95	4.35
91	Walkill	Front Brook	New York	17	14	30	2.59	1.41
92	Flint River	Flint	Georgia	24	30	105	5.63	2.71
93	Muskogee	Reservoir	Oklahoma	30	6	105	3.70	1.37
94	Twin City	Mississippi	Minnesota	31	74	101	5.65	3.47
95	Jonquiere	Jonquiere	Quebec	35	38	66	2.97	1.72
96	Shoshone	Big Horn	Wyoming	58	67	83	2.59	1.63
	Mean for dams higher than 15 ft	....	....	....	....	....	3.69	2.01
	Mean for Table 8 (*f*)	....	....	....	....	....	4.32	2.67
	Creep ratio recommended by Lane	....	....	....	....	....	....	2.50
	(*g*) DAMS ON BOULDERS, GRAVEL, AND SAND							
97	St. Marys	Sun	Montana	7	20	38	8.29	4.66
98	Chester	Coleman Brook	New Jersey	10	30	28	5.80	3.93
99	Shiatown	Shiawassee	Michigan	10	46	38	8.40	5.87
100	Ionia	Grand	Michigan	10	68	45	11.30	8.30
101	Mishawaka	St. Joseph	Indiana	10	34	40	7.40	4.73
102	Corbett	Shoshone	Wyoming	12	....	....	....	....
103	Lowell	Grand	Michigan	12	14	30	3.67	2.00
104	Monarch	Kalamazoo	Michigan	13	51	58	8.38	5.40
105	Plainwell	Kalamazoo	Michigan	14	32	22	3.86	2.81
106	Buchanan	St. Joseph	Michigan	14	85	24	7.78	6.50
107	Ceresco	Kalamazoo	Michigan	15	39	33	4.80	3.34
	Mean for dams, 15 ft., or less	....	....	....	....	....	6.74	4.68
108	Carson City	Fish	Michigan	20	45	61	5.30	3.27
109	Bethlehem	Ammonoosuc	New Hampshire	23	100	56	6.78	5.15
110	Bakerton	West Brook	Pennsylvania	24	....	....	....	....
111	Decorah	Iowa	Iowa	24	72	38	4.58	3.53
112	Dobbin	Stony	West Virginia	36	90	126	6.00	3.67
113	Lansford	Nesquahoning Cr.	Pennsylvania	50	....	....	....	....
	Mean for dams higher than 15 ft	....	....	....	....	....	5.71	3.89
	Mean for Table 8 (*g*)	....	....	....	....	....	6.26	4.30
	Creep ratio recommended by Lane	....	....	....	....	....	....	....
	(*h*) DAMS ON MEDIUM CLAY							
114	Somerville	Raritan	New Jersey	8	27	13	5.00	3.92
115	Hightstown No. 2	Rock Brook	New Jersey	10	30	20	5.00	3.67
116	Cedar Rapids	Cedar	Iowa	10	36	67	10.30	5.83
117	South Bend	St. Joseph	Indiana	10	37	45	8.20	5.20
118	Elk Rapids	Elk Lake Outlet	Michigan	11	32	49	7.36	4.40
119	Springfield	Black	Vermont	12	28	23	4.25	2.97
120	Three Rivers Power House	St. Joseph	Michigan	12	53	40	7.75	5.52
121	Utah Avenue	Flint	Michigan	12	106	85	15.90	11.20
122	Redland	Gunnison	Colorado	13	50	57	8.23	5.31
	Mean for dams, 15 ft., or less	....	....	....	....	....	8.15	5.43

TABLE 8.—(*Continued*)

Dam No. (1)	Project (2)	Stream (3)	State (4)	Head, in feet (5)	Creep Distance, in Feet		Creep Ratios	
					Vertical (6)	Horizontal (7)	Actual (8)	Weighted (9)
	(*h*) Dams on Medium Clay — (*Continued*)							
123	Goshen	Elkhart	Indiana	16	40	69	6.82	3.94
124	Scranton	Roaring Brook	Pennsylvania	17	31	12	2.53	2.06
125	Elkhart	St. Joseph	Indiana	18	32	30	3.44	2.34
126	Gonzalez No. 1	Guadalupe	Texas	18	52	53	5.83	3.87
127	Swift Current	Swift Current	Saskatchewan	20	68	68	6.80	4.53
128	Trowbridge	Kalamazoo	Michigan	21	42	32	3.52	2.51
129	Stronach	Manistee	Michigan	23	37	30	2.91	2.04
130	St. Paul	Reservoir	Minnesota	25	10	120	5.20	2.00
131	Webber Spillway	Grand	Michigan	26	60	52	4.31	2.97
132	Webber Power House	Grand	Michigan	26	33	43	2.92	1.82
133	Texas Power No. 5, Spillway	Guadalupe	Texas	27	73	63	5.04	3.48
134	Texas Power No. 5, Power House	Guadalupe	Texas	27	49	49	3.63	2.42
135	Massillon	Nimisilla Creek	Ohio	28	43	59	3.64	2.24
136	Kearsley	Kearsley Creek	Michigan	28	138	106	8.72	6.18
137	Hunt Devel. No. 5, Power House	Guadalupe	Texas	28	49	57	3.79	2.43
138	Hunt Devel. No. 4, Spillway	Guadalupe	Texas	28	57	68	4.46	2.85
139	Hunt Devel. No. 4, Power House	Guadalupe	Texas	28	49	57	3.79	2.43
140	Hunt Devel. No. 5, Spillway	Guadalupe	Texas	28	57	68	4.46	2.85
141	Cleveland	Hinckley	Ohio	29	24	32	1.93	1.20
142	Texas Power No. 4, Spillway	Guadalupe	Texas	30	36	62	3.27	1.89
143	Texas Power No. 4, Power House	Guadalupe	Texas	30	62	61	4.10	2.74
144	Cuero	Guadalupe	Texas	30	40	35	2.50	1.72
145	McQueeney, Power House	Guadalupe	Texas	31	43	57	3.23	2.00
146	McQueeney, Spillway	Guadalupe	Texas	31	60	70	4.20	2.69
147	Prince Albert	N. Saskatchewan	Saskatchewan	31	50	125	5.65	2.96
148	Grays	Black Creek	New York	32	40	112	4.75	2.42
149	Loud	Au Sable	Michigan	32	50	250	9.37	4.17
150	Burnham	Sebasticook	Maine	34	68	105	5.09	3.03
151	Alcona	Au Sable	Michigan	35	180	200	10.86	7.05
152	Cooke	Au Sable	Michigan	40	76	289	9.12	4.30
153	Five Channels	Au Sable	Michigan	44	90	255	7.84	3.97
154	Williamson No. 1	Sandy Creek	Texas	45	72	45	2.60	1.93
155	Texas Power No. 1, Spillway	Guadalupe	Texas	46	55	73	2.78	1.73
156	Texas Power No. 1, Power House	Guadalupe	Texas	46	74	67	3.06	2.10
157	Foote	Au Sable	Michigan	48	150	200	7.29	4.51
158	Junction	Manistee	Michigan	50	84	48	2.64	2.00
159	Mathis Dike	Tallulah	Georgia	50	199	165	7.28	5.10
160	Williamson No. 2	Sandy Creek	Texas	64	50	84	2.10	1.22
161	Hodenpyl	Manistee	Michigan	65	144	381	8.08	4.20
162	Sand Springs	Shell Creek	Oklahoma	65	....	....	....	....
163	Rodriguez	Tijuana	Mexico	250	400	275	2.70	1.97
	Mean for dams higher than 15 ft.			....	....	....	4.58	2.83
	Mean for Table 8 (*h*)			....	....	....	4.80	2.98
	Creep ratio recommended by Lane			....	....	....	....	2.00
	(*i*) Dams on Very Hard Clay and Hardpan							
164	Patillas	Patillas	Puerto Rico	5	16	25	8.20	4.87
165	Key West	Ocean Inlet	Florida	8	6	10	2.00	1.17
166	Batavia	Towanda Creek	New York	11	16	35	4.64	2.52
167	Fullerton	Cedar	Nebraska	13	24	34	4.46	2.72
168	Baltic No. 1	Shetucket	Connecticut	15	16	20	2.40	1.51
169	Wakeman	Vermilion	Ohio	15	30	42	4.80	2.94
	Mean for dams, 15 ft., or less			....	....	....	4.10	2.44

TABLE 8.—(*Continued*)

Dam No. (1)	Project (2)	Stream (3)	State (4)	Head, in feet (5)	Creep Distance, in Feet		Creep Ratios	
					Vertical (6)	Horizontal (7)	Actual (8)	Weighted (9)
	(*i*) Dams on Very Hard Clay and Hardpan — (*Continued*)							
170	Newport	Sugar	New Hampshire	18	42	56	5.45	3.37
171	Wonder Lake	Nippersink Cr.	Illinois	21	25	68	4.43	2.27
172	New London	Bogue Brook	Connecticut	22	26	30	2.55	1.64
173	Grand Ledge	Lookingglass	Michigan	23	16	30	2.00	1.13
174	Amherst	Amherst Creek	Nova Scotia	25	40	50	3.60	2.26
175	Vandalia	Milk	Montana	27	113	191	11.26	6.65
176	Juniata	Juniata	Pennsylvania	28	....	....	....	....
177	Baltic No. 2	Shetucket	Connecticut	30	28	40	2.27	1.38
178	Pittsfield No. 2	Mill Brook	Massachusetts	30	30	38	2.27	1.42
179	Logan	Logan	Utah	31	26	56	2.65	1.44
180	Mayfield	Mayfield	New York	32	50	105	4.85	2.66
181	River Falls	Conecuh	Alabama	37	20	100	3.24	1.44
182	St. Francis Dam	St. Francis	Quebec	40	40	48	2.20	1.40
183	Prattville	Tallapoosa	Alabama	42	80	110	4.52	2.78
184	Girard	Meander Creek	Ohio	60	20	68	1.47	0.71
185	Rapidan	Blue Earth	Minnesota	64	....	....	....	....
186	La Prele	La Prele	Wyoming	135	....	....	....	....
	Mean for dams higher than 15 ft	....	....	....	....	....	3.53	2.02
	Mean for Table 8 (*i*)	....	....	....	....	....	3.60	2.08
	Creep ratio recommended by Lane	....	....	....	....	....	....	1.60

is greatly increased by the inclusion of the structures with heads of 15 ft, or less. Although the author did not determine his recommended new weighted-creep ratios from the mean creep ratios of his Table 1(*a*), 1(*b*), 1(*c*), and 1(*d*), (being governed instead by the ratio at which no dam failure has been recorded) the relatively few failures which have controlled his ratios, in most cases, cannot be conclusively shown to have occurred from piping alone. A creep ratio might well be determined from a consideration of the behavior of successful dams, if they are included in a large enough foundation group and are old enough to justify confidence.

For example, Table 1(*d*), from which data the author's creep ratios for the three classifications of "very fine sand or silt," "fine sand," and "medium sand," are derived, contains data on sixty-five dams; nine failed, most of them being low-head Indian weirs. Of the fifty-six remaining, thirty-nine, or 70%, have heads of 15 ft or less, leaving only seventeen relatively important non-failures in Mr. Lane's three highest-ratio foundation classifications. In Table 1(*c*) (coarse sand), 59% of the non-failures are 15 ft, or less, in height; in Table 1(*b*) (gravel, cobbles, and boulders), 69%; and, in Table 1(*a*) (clay and hardpan), 38 per cent. Of the four tables combined, the low dams account for 56% of the numerical total. Although insisting on conservatism in the design of a dam of any size on permeable foundations, the writer believes that creep ratios so heavily influenced by a high proportion of low-head structures may penalize, unnecessarily, the designers of higher-head developments.

In the mean ratios derived from Mr. Lane's tables this influence of the low-head dams is even more pronounced than in the case of Table 8, as shown by Table 9. For convenience of reference and for the purpose of comparison the writer's data have been grouped in the same four general foundation classifications as the author's tables. It should be noted in this connection that Table 8 includes data concerning approximately ninety dams which were originally furnished to Mr. Lane and included in the complete report on file at Engineering Societies Library. The remainder are new to the discussion as foundation details were unavailable in time to be included by the author. To prove the writer's view that, in accordance with his own experience, high dams may be designed properly with lower creep ratios, whether weighted or actual, than low dams, he has segregated into "Author's Group" (Table 9) the mean ratios for those dams in which data did not originate with the writer and Mr. Streiff, and has shown in "Writer's Group" the mean ratios of all those dams covered by Table 8. All ratios are for successful structures.

TABLE 9.—COMPARISON OF CREEP RATIOS

Table No.	Foundation classifications	MEAN CREEP RATIOS									PROPOSED RATIOS		
		Dams of 15 Feet or Less			Dams Higher Than 15 Feet			All Heads Combined					
		Number of dams	Weighted creep	Actual creep	Number of dams	Weighted creep	Actual creep	Number of dams	Weighted creep	Actual creep	Lane	Bligh	Griffith
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
8(*a*)	Very fine sand and silt	8	5.18	7.93	7	4.22	6.61	15	4.51	7.01	8.50	18	11.6 to 12.8
8(*b*)	Fine sand	0	....	....	0	....	....	0	....	....	7.00	15	10.0 to 11.2
	Medium sand	6	5.05	9.24	6	3.47	5.38	12	3.89	6.40	6.00	.....	.....
.....	Writer's group, Table 8(*a*) and 8(*b*)	14	5.15	....	13	3.88	....	27	4.22	....	....	.....	.....
.....	Author's group, Table 1(*d*)*	41	9.90	....	10	6.88	....	51	8.95	....	....	.....	.....
.....	Both groups combined	55	8.66	....	23	4.92	....	78	6.80	....	....	.....	.....
8(*c*)	Coarse sand	7	3.94	6 85	5	3.22	4.34	12	3.45	5.17	5.00	12	8 to 9.6
....	Author's group, Table 1(*c*)*	7	8.72	....	5	4 80	....	12	6.14	....	....	.....	.....
....	Both groups combined	14	6.20	....	10	4 00	....	24	4.72	....	....	.....	.....
....	Fine gravel	0	....	....	0	....	....	0	....	....	4.00	.....	.....
8(*d*)	Medium gravel	10	3.73	5.95	8	2.97	4.73	18	3.18	5.07	3.50	.....	.....
8(*e*)	Gravel and sand	13	4.11	5.92	8	2.78	4.07	21	3.37	4.89	....	9	6.4
....	Coarse gravel, including cobbles	0	....	....	0	....	....	0	....	....	3.00	.....	.....
8(*f*)	Boulders, cobbles, and gravel	8	4.35	5.95	6	2.01	3.69	14	2.67	4.32	2.50	.....	4.8
8(*g*)	Boulders, gravel, and sand	10	4.68	6.74	4	3.89	5.71	14	4.30	6.26	....	4 to 6	6.4
....	Writer's group, Table 8(*d*), 8(*e*), 8(*f*), and 8(*g*)	41	4.22	....	26	2.80	....	67	3.33	....	....	.....	.....
....	Author's group, Table 1(*b*)*	44	7.14	....	11	3.12	....	55	5.13	....	....	.....	.....
....	Both groups combined	85	5.62	....	37	2.92	....	122	4.05	....	....	.....	.....
....	Soft clay	0	....	....	0	....	....	0	....	....	3.00	.....	.....
8(*h*)	Medium clay	9	5.43	8.15	40	2.83	4.58	49	2.98	4.80	2.00	.....	.....
	Hard clay	0	....	....	0	....	....	0	....	....	1.80	.....	.....
8(*i*)	Very hard clay and hardpan	6	2.44	4.10	14	2.02	3.53	20	2.08	3.60	1.60	.....	.....
....	Writer's group, Table 8(*h*) and 8(*i*)	15	4.20	....	54	2.64	....	69	2.76	....	....	.....	.....
....	Author's group, Table 1(*a*)*	22	5.45	....	21	2.30	....	43	3.18	....	....	.....	.....
....	Both groups combined	37	4.95	....	75	2.57	....	112	2.88	....	....	.....	.....

* Omitting writer's data.

To obtain the greatest possible accuracy and scope for Table 9, of the 186 listed structures in Table 8, eleven have not been included because of uncertainty as to exact percolation factors; and six shown are power houses with creep ratios differing from those of the dams built in conjunction with them. In computing the means for the author's group (being the basic 140 dams remaining in his original Tables 1(*a*), 1(*b*), 1(*c*), and 1(*d*), after those on the writer's lists have been eliminated) twenty-one creep ratios have been added which occurred in different parts of certain individual dams. By the foregoing method 336 different creep ratios are analyzed in Table 9. Therefore, in support of the writer's conclusion that high dams can be constructed safely with lower creep ratios than low dams, he not only relies upon his own lists (Tables 8 and 9) (11 dams with indeterminate ratios have not been analyzed) showing 77 structures 15 ft or less in height and 98 (or 56%) with heads greater than 15 ft; but also upon the author's group of 161 creep ratios, of which 114 are from dams of 15 ft, or less, head, and 47 (or 29%) with heads greater than 15 ft.

The greater disparity between the mean creep ratios of the low-head dams and the dams of higher head in Mr. Lane's tables seems partly due to the inclusion in his lists of a large number of small Indian and Egyptian weirs and low U. S. Government dams, having quite high ratios. Many of these structures are on pile foundations or consist largely of considerable horizontal hearth or paving, having horizontal percolation distances that are probably of slight value; yet they heavily influence the mean ratios of his tables. Furthermore, the high proportion of 71% of low dams to 29% of high ones widens the gap. As hereinafter shown, a favorable influence for the higher-head structures is their increasing value of horizontal creep due to properly designed supporting floor construction.

From the multitude of low weirs that have failed in the past much valuable information might have been secured. However, their small size, remoteness, and relative unimportance permitted the event to pass usually with only local notice. In most cases the true cause of the failure was never determined and recorded. Valuable lessons may be drawn from a careful study of the failure of the most obscure dam on permeable foundations, and it is regrettable that no representative of the Engineering Profession at large has had the duty of analyzing such minor failures. For many years the writer has assembled data on reported dam failures which now cover about 300 instances. In less than 15% of the cases was the cause of failure ever determined and published in such a form as to be useful or even available to the Engineering Profession. Only twenty-one failures on porous foundations are listed by the author, and the writer ventures the statement that the direct cause of few of these failures can be attributed, exclusively and with certainty, to excessive under-seepage.

The difficulty that would be experienced in applying Mr. Lane's weighted creep ratios from Table 3 to actual foundations will be the same as the writer has had, for example, in classifying his sand structures according to the four sand classes cited by the author. This is not mentioned in a critical sense, as all engineers experienced with foundation materials, particularly complex river-bed deposits, realize the impossibility in most cases of placing any specific site entirely under any one of the fourteen classifications used by Mr. Lane. There seems to be no exact method of classifying different materials with accuracy or uniformity for the determination of creep resistance. Two sands in different localities, of the same fineness or effective size, may have entirely different degrees of percolation resistance due to the natural bedding and stratification, or to the different shapes and irregularities of the grains. Divergence of views as to the proper foundation classification into which to cast any single structure built on varying material is naturally to be expected. The writer doubtless has placed certain of his structures which also appear in the author's tables in a different classification of foundation material than did Mr. Lane for establishing the data in his Table 3, but the foundation classifications for the dams incorporated in the writer's tables were based upon an examination of the foundation material in the field in nearly all cases. However, this has little effect on the divergence of the mean ratios for low and high dams.

Except in certain localities, such as parts of Michigan, for example, and in some of the Southwestern States, river-bed deposits are generally too heterogeneous to permit the safe application of any arbitrary creep ratio unless the requisite creep distance for the most porous deposits to be expected is applied. Mr. Lane properly offers his weighted-creep values for use only where accurate foundation data as to the extent and position of underlying impervious strata are not available, or where the data do not indicate whether or not such strata exist. The extreme variation of foundation material that is so frequently encountered at any one site serves to emphasize the writer's assertion that each dam on difficult foundations presents an individual problem. No solution of any individual problem can be for general application. It is applicable only to identical foundation material and identical design of the dam itself, including height, length of base, spillway requirements, back-water conditions, etc. The writer doubts that identical conditions will ever be found for two structures.

The proper classification of foundation types has probably been one of Mr. Lane's most difficult tasks. To force a listed dam somewhat arbitrarily into any classification might well result in indicating a structure with an impaired factor of safety against percolation, or, conversely, as having been "saddled" with an excessive construction cost, as the case might be.

At this point the writer desires to emphasize that nothing can take the place of careful and complete soil investigations as a prerequisite for the design of any dam on porous or yielding foundations. Based upon the writer's examination of many dam foundations at sites that were developed eventually, he estimates that not more than 15% of these projects had a foundation material that, during the actual construction of the work, was found to be uniformly of the anticipated characteristics as indicated by previous exploration. Of course, where such unexpected diversity of material is found the worst condition must be met by subsequent revisions of the design. Because of the probability that even the most careful sub-surface investigation will fail to reveal lenticular deposits or strata of a more porous material than that indicated by the exploration, caution must be exercised in selecting a creep ratio. Greater conservatism must be used in the design of permeable foundation dams than in the case of almost any other engineering structure.

Mr. Lane understates the important effect which the design of the superstructure may have on the value of horizontal creep. Probably this is one reason why most of the dams in Table 8 show creep ratios so much lower in general than those recommended by the author, when his weighted value of one-third is applied to the horizontal percolation-resisting elements of these dams. Experience has convinced the writer that proper design and construction are much more important than theoretical considerations relating to creep ratios, although the latter should not be neglected as an important, although not necessarily the controlling, factor in the final design.

As stated by the author, the best shape of dam must first be determined to meet other conditions, before the depths of the vertical cut-offs are established. The escape velocity of water beneath the dam is controllable by the actual design of the superstructure as well as by the design and location of the cut-off walls. Moreover, the depth to which the up-stream cut-off wall must be carried (and the down-stream one also, if one is used), is controlled by the shape of the superstructure of the dam itself and its resulting length of base. This design, in turn, is governed by factors which will vary widely for different sites, thus nullifying any arbitrary ratio based on consideration of the foundation material alone. These factors include the quantity of flood water to be discharged and the resulting shape of the rollway; the details of stilling pools, or other energy-dissipating devices; back-water conditions; the geographical location of the dam as to ice-pressure conditions; the type of crest gates, piers, or bridge that are to be superimposed on the structure; and the bearing value of the foundation itself. It is also necessary to determine the permissible limits of leakage under the structure, which, depending somewhat upon the economic factors, may be extended very close to the safety point. These are among the elements which must first be determined before a solution of the cut-off problem is undertaken, and the results of these investigations will usually go far toward determining the proper value to be given to the resulting horizontal creep.

This point is made clear by an example, which is not the most extreme that could be selected. Consider the design of the Cooke Dam on medium clay (Dam No. 152, Table 8). The exigencies of the site required that the spillway be designed to care for a 12½-ft depth of flood discharge. Then, scour prevention forced such an added base length as to create a horizontal creep distance of 289 ft for a 40-ft head, which alone gives the structure a creep ratio of 7.25, actual, and 2.41, weighted. The addition of the necessary minimum of vertical cut-off walls permitted by good judgment increased the creep ratio to 9.12, actual, and 4.30, weighted. Contrast this with the weighted mean of 2.83 for forty high dams on similar material in Table 8, and with the author's recommendation of 2.00, and the basis becomes evident for the writer's contention that design controls the creep ratio to a far greater extent than creep ratios control the design.

A further example will suffice. On medium sand, the 25-yr old Bassano Dam (Dam No. 29, Table 8) has a weighted-creep ratio of 1.60, 42% of the actual creep ratio of 3.80. On the other hand, the Berrian Springs Dam (Dam No. 25) has a weighted-creep ratio of 5.50, 75% of the actual creep ratio of 7.32. Both are considered equally good designs. Each misses the mean weighted creep of 3.47 for their class by a wide margin, and neither reaches the author's recommendation of 6.00.

Thus, it becomes also readily apparent that empirical creep ratios in hands unskilled in the manifold requirements for correct designs for erodible and permeable foundations may become a real danger. It is the solving of the various problems of design for each individual site that causes the great variation in creep ratios within any one foundation class in Table 8. Theoretical considerations and laboratory conclusions are valueless under these practical circumstances.

Nearly all the writer's 186 listed dams have certain design characteristics in common. Substantial vertical cut-offs are located at the up-stream and down-stream ends of continuous, well reinforced, foundation slabs. These foundation slabs are supported directly by the foundation material and bear heavily upon it, and are often keyed to it by intermediate shallow walls. Although deep-river bed deposits are usually of a rather heterogeneous nature, the method of their deposition and the long period of time during which settlement by saturation has been occurring, have given to most sites a well equalized bearing value throughout their area despite the varying resistances to percolation. Although a value of one-third for percolation distance along the under side of a concrete hearth or apron may be justified, the writer is not willing to accept so low a value where this horizontal surface is the actual foundation slab of a dam of moderate height. To carry the author's theory of a value of one-third for horizontal creep a step further, the value of vertical creep should be increased according to the depth to which the cut-off wall is carried. It would be quite logical, and fundamentally consistent with Mr. Lane's theory, to give the percolation path at the bottom of the deepest cut-off wall a value of unity, and then to decrease this value progressively for the upper portions of the cut-off to its juncture with the dam, and continue some progressive reduction in the creep value to the zero point where the percolation emerges freely beyond the down-stream toe. The writer agrees with the author, however, that percolation resistance along the line of creep is probably less than through the foundation material itself, which path is in most cases that of least resistance. This condition is quite variable, however, depending almost entirely on the texture of the foundation material.

For a continuous-floor dam, properly designed and constructed, the horizontal creep value should be more than one-third compared with the vertical creep, whereas in the case of any type of dam having a base supported on piles horizontal creep has little or no value. This is true, of course, in the case of structures with supporting buttresses that are carried on piles, with no connecting slab, and in those rare cases where the buttresses rest upon spread footings. Where the writer refers to a floor, it is intended to mean only the characteristic continuous, monolithic, reinforced concrete foundation slab upon which the typical Ambursen type of dam for soft foundations is constructed. He does not refer to what the author terms "the wide masonry-floor type" which is understood to describe the so-called Indian weir structures with a long down-stream hearth or paving serving as a stilling pool. The writer, like Mr. Lane, places little reliance on the value of horizontal creep beneath such hearths.

The continuous floor of an Ambursen dam, whereby the weight of the structure and its water load is transmitted directly and entirely to the underlying foundation material without the use of bearing piles, has also a direct and favorable bearing on the usefulness of the vertical creep. In the case of many, if not all, permeable soil foundations there is an inevitable displacement of foundation materials or a transfer of foundation pressures caused by the application of the weight of the dam and its water load as a surcharge. This displacement, consolidation, or flow of foundation material tends to tighten the contact against both the up-stream and the down-stream faces of a (somewhat flexible) sheet-pile, up-stream, cut-off wall, and against at least the down-stream side of a masonry cut-off wall. This transmission of pressure through foundation material is so thoroughly established, it seems obvious that the application of the weight of the dam and its load to a soil foundation must inevitably retard the rate of seepage against all vertical faces of cut-off piling, whether at the heel or at the toe of the dam. Accepting this as a certainty, there must also be some consolidation at least within the upper portion of the foundation material itself to which the load of the dam is applied, with resulting decrease in voids and a tighter contact against the foundation slab, thus giving the horizontal creep a high value. The drop in upward pressure below some horizontal concrete surfaces, as stated by Mr. Lane, may be almost zero in certain cases, but where this concrete surface forms the properly constructed supporting base of the dam itself, and the structure is of substantial height, different results are obtained.

The author's quoted statement of Mr. Griffith, that "the line of creep is subject to a greater pressure where carried to a greater depth" comes from the same logical reasoning.

As shown in Fig. 10, by careful designing and the use of a continuous floor-slab to support the buttresses (a design which possesses many structural and construction advantages as well as usually making for economy as compared to a piled foundation), the writer has constructed dams successfully on saturated fine sand or silt, that have stood for more than twenty years.

The writer's preference for the use of a continuous foundation slab of reinforced concrete is due not only to his belief that this form of structure distributes the load better over the undisturbed foundation material than a piled foundation (thereby increasing its resistance to percolation instead of decreasing it), but that it further affords an admirable tie for the entire structure, acting somewhat as the bottom chord of a truss. Its value in the case of a blowout caused by piping was fully demonstrated in 1908 at Pittsfield, Mass., where the undamaged structure spanned a gap in the foundation material 50 ft in width, after the reservoir emptied safely. Repairs to the cut-off and replacement of supporting foundations were made, and this dam has been in successful use ever since.

Griffith's valuable experiments correctly pointed out that by loading the foundation material the tendency to blow out was decreased. The resistance to flow is increased materially, therefore, if the weight of the dam and the water load are transmitted directly through the floor-slab to the foundation. The use of piles is to be avoided owing to the inevitable tendency of this type of construction to cause roofing. If the foundation material is properly confined by a suitable arrangement of cut-off walls, bearing piles can serve no useful purpose. As Professor W. M. Patton pithily wrote, "Sand will hold your structure if you can hold the sand." Of the 186 earth-foundation dams listed herein only eight are on piles, and of late years the writer has abandoned their use entirely in designing the conventional Ambursen type of dam. Where used, the value of horizontal creep under the dam is negligible. While the use of bearing piles beneath a gravity dam is usually required because of the heavy and unequally distributed dead load of the structure itself, no such design (except of relatively low height) can be constructed safely on a soil foundation without excessive cost. Further, just as the driving of timber or of steel sheet-piling for a cut-off wall may impair its own value as a medium for increasing percolation distance by providing

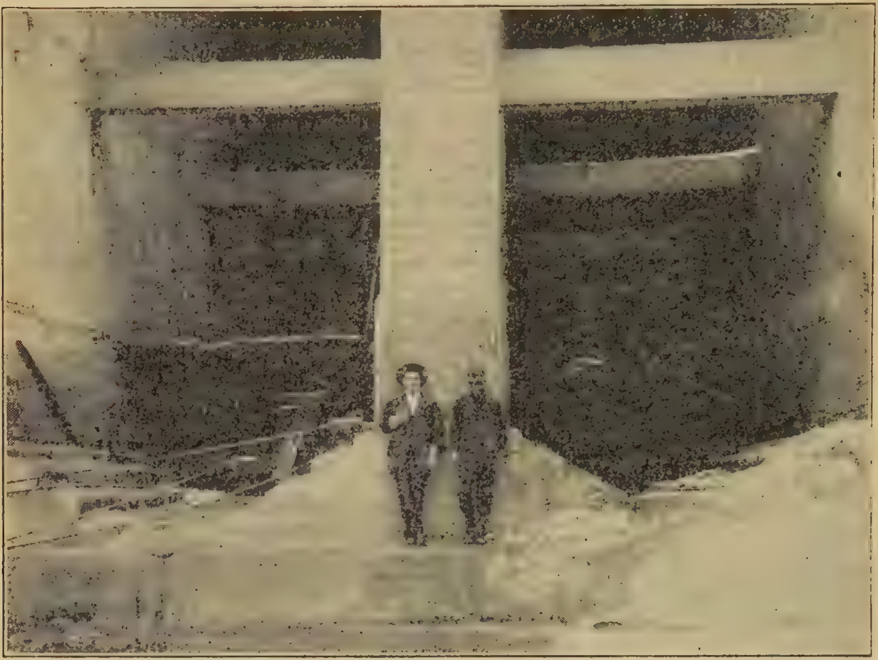


FIG. 10.—TYPICAL AMBURSEN FLOOR-SLAB UNDER DAM 135 FEET HIGH, LA PRELE, WYOMING; CLAY AND HARDPAN FOUNDATIONS; BUILT IN 1908.

a smooth surface along which water under pressure may rise from one stratum to another, so may the driving of bearing piles under a dam likewise afford a shorter upward path for percolation along their sides. These perforations through varying strata may serve as pipes to the roofed space under the piled dam.

As to the use of spread buttress footings beneath a dam for soil foundations, without piles, the writer has always avoided this dangerous form of construction. Little, if any, initial economy is obtained and, without "splitting hairs" as to the exact weight to be given to horizontal creep beneath a foundation floor, the latter has numerous advantages and no disadvantages, when compared to spread footings.

The writer agrees with Mr. Lane that a single deep up-stream cut-off wall has greater value for percolation resistance than the same total vertical depth divided into two or more walls, not only because it avoids the possibility of short-path travel, but because the resistance to percolation increases in a given material at a greater depth. The connection of the dam superstructure proper to the top of the cut-off wall should be made so as to permit slight movement without rupture of the joint and loss of water-tightness. There is nearly always some measurable although slight subsidence of the foundation material as the weight of the dam with its water load is applied. The author's statement that, "in order to use these values with safety the cut-offs must be of solid masonry built in contact with the sides of the trench, * * *," emphasizes the importance of proper design and construction methods for this critical part of the dam. Steel sheet-piling has almost entirely displaced Wakefield or other timber sheet-piling for cut-off walls, not only because of the better sections and interlocks that are marketed to-day, but because of the extremely bad record of timber sheet-piling when used in connection with hydraulic structures. Nevertheless, steel sheet-piling still leaves much to be desired as a cut-off material, even under the most favorable driving conditions.

The writer considers the up-stream cut-off the most valuable for restricting, to a permissible quantity, the seepage through a permeable foundation. Nevertheless, he has always considered it good practice in the case of a floor dam also to install substantial sheet-piling (or a masonry cut-off) either beneath the down-stream edge of the foundation slab itself, or at the down-stream edge of the stilling pool, or hearth, and in many cases at both points. This has usually been done not merely to increase the effective creep distance, but to protect the structure against scour. Mr. Lane's statement that his creep-ratio values are based on the assumption of "competent supervision during construction and efficient maintenance afterward," affords the writer the opportunity to state that, improbable as it might seem to the layman or to an engineer not experienced in this field, these two factors are not the easiest to obtain for dams on earth foundations. Obviously many, if not most, of the failures incorporated in Mr. Lane's tables, were due to lack of one or the other. Over a long period of time the writer has observed that adequate maintenance of dams on erodible foundation material is by no means always assured. This is true principally in the case of the smaller structures, although it is also found often on larger projects where there is a permanent back-water condition making close observation of scour inconvenient. As this erosion, of course, occurs even in rock foundations below important and adequately maintained structures (notable examples being the McCall Ferry Dam, in Pennsylvania, and the Keokuk Dam, in Iowa), a sturdy down-stream cut-off is always advisable for earth foundations.

The conclusion might be drawn from the cumulative weight of the behavior records of 186 dams on permeable foundations listed by the writer (many having been in use for more than 25 yr) that a more or less arbitrary creep ratio, weighted or otherwise, based upon an analysis of Tables 8 and 9 would be safe. However, the writer is not willing to advocate this practice yet for the reason, perhaps repetitiously mentioned, that every individual dam on soil foundations is a special and separate problem in itself that may have little or no relation to any other such structure. Therefore, until considerably more research has been done, and more is known as to the actual behavior of under-seepage beneath dams of substantial height under the many different conditions controlling its flow, the writer is not prepared to recommend any reduction for general use from the figures tentatively set up by Mr. Lane. Certainly, his new weighted-creep ratios for the more permeable foundations seem adequate, and for the higher dams his ratios might be substantially reduced in special cases if proper design, construction, and maintenance were assured. This is in disagreement with the author's views that his values are suitable for major structures, and that smaller values may be used for less important ones. The writer believes that higher factors of safety for smaller dams can usually be had so cheaply that they are justified, since any earth foundation development, in the last analysis, must be classed as hazardous.

Mr. Lane's statement that some light may be thrown on the relative value of vertical and horizontal creep through a study of upward pressure on actual dams is correct. The most authentic and reliable information that can be procured is through the medium of tests of pressure and flow under structures, such as the system installed in the Rodriguez Dam, in Mexico (250 ft high) to permit the observation and recording of upward pressure beneath the base of this dam, as well as at the bottom of its cut-off wall, 300 ft deep. Similar recording installations where possible should be provided in all future dams 15 ft, or more, in height on permeable foundations. Nevertheless, Mr. Lane properly states that "while research along this line should be given every encouragement, the main reliance in dam design, for a long time in the future, must be on a somewhat empirical basis." Small-scale model experiments, laboratory tests, and the flow-net or electric analogy methods have been in common use for many years. They still remain merely aids to good judgment and experience which latter must form the basis for successful design and construction of masonry dams on soil foundations. In inexperienced hands, the application of such theoretical factors may be more dangerous than beneficial. If sufficiently complete information were available as to the exact conditions of the foundation material, greater reliance might be placed on the electric analogy or flow-net methods, but in the presence of such detailed knowledge, adequate and economical design for percolation resistance could be made without resort to them.

The treatment of each class of foundation material must be fundamentally a matter of sound judgment based on experience. Economy in design and construction (which, in the last analysis, is the objective of most engineering work) should flow only from this source. The writer's experience with the design, construction, and maintenance of a large number of dams on difficult foundations, has led him to conclude that no method of theoretical or empirical analysis will ever be satisfactory and uniformly applicable to such structures. Much more study of actual rates and the behavior of under-flow and of upward pressures is needed before Mr. Lane's recommendations can be accepted entirely; but as more quantitative and qualitative data become available (particularly as to failures), the writer is convinced that further reductions will be made in the percolation ratios now considered necessary. The ratios of both Bligh and Griffith have the serious weakness of being based on observations merely of a relatively few low-head dams, and their available data were also restricted as to classes of foundation materials. Moreover, the number of observable structures was insufficient to justify any attitude other than the greatest conservatism.

In conclusion, where conditions vary so widely between different sites and structures there are naturally divergencies of opinion as to what constitutes the best, or at least the favored, practice from the viewpoint of different engineers trained in this field. All may be meritorious, but they must be so proved in practice and by the greatest test of all—time. The writer, because of his nearly thirty years experience with the Ambursen type of dam, has naturally favored the type of structure and details of design and construction that in his experience have proved satisfactory. He does so with the greatest respect for the individual preferences and opinions of other engineers who may be skilled and experienced in the same field.

Among the most important conclusions and advice to be drawn from Mr. Lane's invaluable paper are the following:

(1) Explore as fully as may be economically feasible, the sub-surface area beneath and adjacent to the dam;

(2) Use a continuous well-reinforced concrete floor beneath any buttress dam for soft foundations, regardless of differences of opinion as to the exact creep value of such construction;

(3) Avoid the use of pile foundations or spread footings for dams of any but the lowest heads;

(4) Avoid reliance on horizontal creep under long down-stream pavements or stilling basins, particularly where articulated;

(5) Take advantage of the economical up-stream impervious fill to increase seepage distance;

(6) Rely principally on a properly constructed up-stream masonry cut-off wall for control of under-seepage and piping;

(7) Install a good down-stream cut-off as insurance against removal of foundation material by scour, and as Mr. Lane's "second line of defense"; and,

(8) Where rock or other impervious material may be reached without prohibitive cost, carry a substantial concrete cut-off wall down to it, regardless of theoretical creep limitations.

These principles are among the fundamentals for soft foundation dam design and construction, and are believed worthy of emphasis. There is no structure justifying a higher factor of safety than a dam on soil foundations nor, as may be judged from the diversity of views shown, one less susceptible to determination based upon laboratory or theoretical considerations. Mr. Lane's paper, serving as it does to call attention to the great advances made during thirty years in the treatment of difficult dam foundations, is not merely a tribute to the pioneering work of those engineers who had the courage to break with precedent and devise new types and methods for solving the problems inherent to this field of engineering. It also serves as an incentive to others to aid in the expansion and distribution of this knowledge by research, experimentation, and observation. Experience breeds confidence, and confidence broadens the engineering horizon. The paper is a useful encyclopædia of experience gained in developing this difficult field, in addition to the unquestionable value of Mr. Lane's effort to fix a needed new standard for security from under-seepage, with resulting economies in construction cost. Further advancement must follow as a direct result. This painstaking and laborious paper coincides with a transition period in the art of all dam design, and is presented at a time when similar research is most needed. The profession is indebted to Mr. Lane for his efforts.

ALEXANDER POTTER,[60] M. AM. SOC. C. E. (by letter).—In 1911, in a discussion[61] of the paper by Mr. Arnold C. Koenig, the writer ventured the prediction that the increasing municipal and commercial needs of rapidly developing areas of the country would force the harnessing of rivers and streams with unstable foundations, correct solutions for which had not then been found. That this prediction was timely has been borne out by developments during nearly twenty-five years which have elapsed since that time. It is evidenced, furthermore, by the long list of successfully constructed dams shown by the author, which tabulation adds a double value to his paper.

In compiling the mass of data presented in this paper, occasional errors are unavoidable. For instance, it is doubtful whether the Muskogee Reservoir, constructed by the writer in 1912, should properly be included in the tables accompanying the paper. This reservoir consisted of thirty-six bays of the Ambursen type of dam, 30 ft. high, which formed substantially a circular reservoir, the bottom of which was fully paved with 6 in. of reinforced concrete, so that the element of creep on the stability of the structure, carries little significance, notwithstanding the fact that it was built almost entirely above the ground, the foundation consisting of sand, gravel, and cobbles. After more than twenty years' of use, the reservoir is still giving satisfactory service, and the leakage is scarcely measurable.

---

[60] Cons. Engr., New York. N. Y.

[61] *Transactions*, Am. Soc. C. E., Vol. LXXIII (1911), p. 191.

Mr. Lane's comment on the value of weep-holes or drains is useful and along the lines of the writer's experience. Mr. Koenig did not lay sufficient emphasis on the value of these drains for relieving upward pressure and reducing the danger against piping and this point was emphasized by the writer at the time.

The great strides which have been made in the field of soft foundation dams is evidenced by a comparison of Mr. Koenig's paper and the present work by Mr. Lane, who deserves great credit for the painstaking and thorough study he has made of existing dams. It was time that some courageous engineer should have attacked the ratios suggested by Bligh and Griffith which originated from an experience covering a limited variety of foundation materials. It has been almost an impossibility to proportion or amend these ratios intelligently for completely different types of materials. Furthermore, they were derived from contact with what to-day are rather small head developments. When such arbitrary ratios are applied to the modern soft-foundation dams of considerable height and great economic importance, they are likely to burden the development with extravagant cost.

The recommendation for a value of one-third for horizontal creep (even if it merely affords a justification to cut down previously recommended ratios to reasonable figures commensurate with modern practice), will be of service to the Engineering Profession. It points the way to reduced construction costs and will affect, favorably, the future storage and power developments that are close to the border line of economic feasibility.

Mr. Koenig's useful and studious paper and the discussions thereon, marked a summing up of the meager engineering experience with sand foundations that was available in 1911. It is well that Mr. Lane has had the time and patience to summarize the present state of the art. Mr. Koenig was one of the earliest engineers to direct attention to the great bearing value of a saturated sand, provided it could be confined adequately; he was also among the first to recommend the use of a stilling pool for absorbing the kinetic energy of spillway discharge. While his conception of adequate stream-bed protection below the stilling pool may have been defective, it was a useful attempt, nevertheless, to start the engineer's thoughts along the proper channel. Mr. Lane properly advises great care against erosion below the spillway with resulting sudden drop in percolation resistance, the cause of many failures.

It would have been desirable to incorporate in the paper more data on dam failures. The writer has frequently had occasion to refer to the records of failures and finds a surprising lack of uniformity of opinion as to their causes, particularly with respect to soil foundations. The application of a study of soil mechanics to a determination of the value of percolation paths, while desirable, would be difficult to develop owing to the non-homogeneity of the materials at most dam sites.

In the hands of experienced designers, the writer believes that Mr. Lane's ratios can be lowered even more, but agrees that in this respect it is better to advance slowly.

ROBERT E. KENNEDY,[62] M. AM. SOC. C. E. (by letter).—Many small earth dams have been built or proposed in the Western Dakota and Eastern Montana area for drought and unemployment relief, erosion control, stock water, and general water conservation.

Constructed largely with no special compacting equipment permitted and frequently under the supervision of young and inexperienced college graduates, otherwise unemployed, the chief protection against percolation through the dam is an impervious core-wall, usually of wood. Table 3 of the paper gives a ready means of determining the necessary penetration of this core-wall, if the computation is based upon the shortest possible path, which is always shorter than the probable path, as shown in Fig. 11.

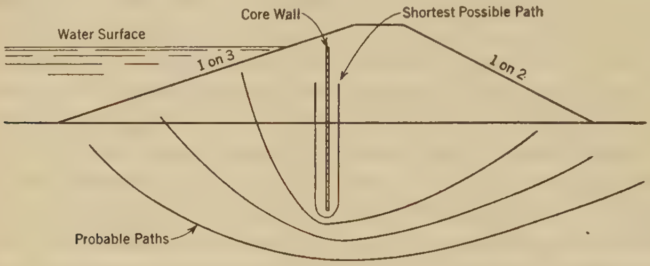


FIG. 11

For illustration assume that the water is to be 10 ft behind an earth dam placed on a coarse sandy foundation. The factor is 5. The minimum path should be 50 ft. If the earth is so loose as to have little value in retarding percolation the core-wall would extend 25 ft into the foundation, making a total height of 35 ft.

If the material in the dam is likely to be of the same general classification as that in the foundation, and if it is likely to receive a similar degree of compaction during construction, the penetration need be only 15 ft, making a total height of 25 ft.

For the ordinary soils with the usual methods of construction it may be considered safe to assume that the material in the dam will be about one-half as compact and dense as the material in the foundation. Then the necessary penetration in the illustration would be 20 ft.

For the benefit of those who would like to use a formula the following may serve, if used with discretion: $d = \frac{h}{2}(f - 1)$, in which d is the desired penetration; h is the depth of water behind the dam; and f is the suitable factor selected from Table 3.

A. N. KHOSLA,[63] M. AM. SOC. C. E. (by letter).—Quite properly Mr. Lane stresses the greater effectiveness of the vertical as compared with the horizontal contacts between a dam and its foundation. His explanation for this phenomenon, however, is open to question. The author believes that the horizontal contacts are less effective because of the subsidence of the foundation material. If arching does occur from this source the resistance along horizontal planes will be zero and a weight of one-third for this case would be decidedly unsafe.

---

[62] With Dept. of Investigations and Project Studies, U. S. Bureau of Reclamation, Denver, Colo.

[63] Executive Engr., Khanki Div. (Punjab Irrig.), Khanki, Punjab, India.

The method which Mr. Lane advances is certainly a considerable improvement over Bligh's method and in a number of cases it will yield results that are quite close to the actual condition. The paper has the merit of simplicity. Nevertheless, being based on arbitrary assumptions, it is empirical and, therefore, cannot be accepted as a basis for design. A reference to Fig. 12 and Table 10 will demonstrate how widely the relative effectiveness of various planes of contact differ from those given by the weighted-creep method advanced in this paper.

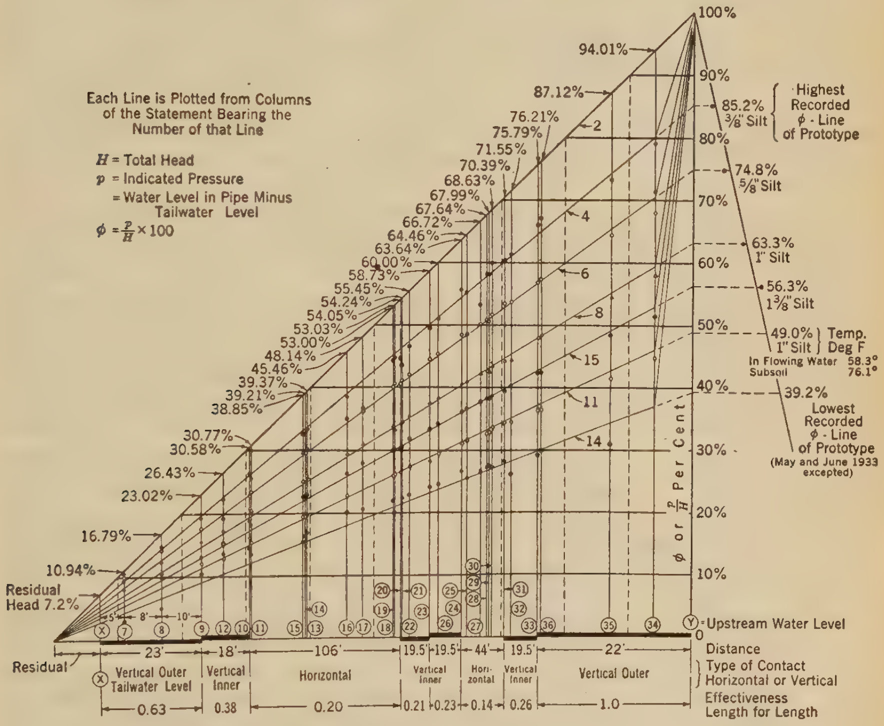


FIG. 12.—RELATIVE EFFECTIVENESS OF VERTICAL AND HORIZONTAL CONTACTS, PANJNAD WEIR (1:50) MODEL AND PROTOTYPE.

Not only does Mr. Lane assign equal weight to all vertical contact planes, but to all planes inclined at angles greater than 45 degrees. This is essentially erroneous as has been proved conclusively by pressure observations at the Panjnad Weir.[64] The outer faces of cut-off walls at the extreme up-stream and down-stream sides of a dam are many times more effective than the inner

---

[64] *Paper No. 162*, Punjab Eng. Congress, 1933.

TABLE 10.—COMPARISON OF UPLIFT PRESSURES ON PANJNAD WEIR WITH A 1:50 SCALE MODEL

Description	READING 1		READING 2		READING 3		READING 4			READING 5			READING 6	READING 7	
								Prototype			Prototype				
(1)	Scale model (2)	Electric analogy (3)	Scale model (4)	Prototype (5)	Scale model (6)	Prototype (7)	Scale model (8)	(*a*) (9)	(*b*) (10)	Scale model (11)	(*a*) (12)	(*b*) (13)	Prototype (14)	Scale model (15)	Prototype (16)
(*a*) GENERAL DATA															
Date of test	.....	.....	.....	9/17/33	.....	8/12/34	.....	9/5/9/34	1/24/34	.....	11/29/33	4/18/34	6/27/34	.....	4/11/34
Head, *H*, in feet	8.2	16.4	10.26	6.3	8.32	5.76	11.97	8.18	5.09	11.97	12.55	4.74	7.93	11.12	6.95
Depth of Silt, in Feet:															
Up stream	*	*	1.6	1.2	2.6	1.6	4.2	2.8	2.9	4.2	2.8	2.9	2.4	5.7	2.8
Down stream	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*
Temperature, in Degrees Fahrenheit:															
River	76†	.....	76†	89.5†	76†	87.5†	76.0†	85.5†	41.5†	58.3†	58.5†	76.0†	86.0†	76†	76.5†
Subsoil	76†	.....	76†	85.0†	76†	82.0†	76.0†	78.5†	73.0†	76.1†	78.0†	74.5†	76.0†	76†	74.5†
(*b*) VALUES OF ϕ, OR UPLIFT PRESSURES EXPRESSED AS PERCENTAGES OF HEAD, *H*															
Pipe No. 34	94.01	95.0	79.12	90.00	68.04	72.60	58.20	65.50	87.8	45.10	74.90	80.2	71.4	51.59	84.0
Pipe No. 35	87.12	86.5	73.21	71.90	64.55	62.91	54.73	53.00	55.45	41.90	42.60	41.00	31.00	48.26	48.1
Pipe No. 36	76.12	76.5	67.24	64.40	57.42	57.42	48.31	45.90	47.74	36.89	35.80	37.30	30.00	42.39	42.7
Pipe No. 33	75.79	75.00	66.04	64.50	56.93	56.87	48.09	47.50	48.50	36.84	37.10	34.90	29.30	42.25	41.8
Pipe No. 32	71.55	71.00	61.69	62.10	53.71	55.22	45.39	45.30	46.43	34.75	35.20	34.80	26.00	40.20	40.4
Pipe No. 31	70.39	70.50	60.44	60.80	53.61	53.30	45.24	45.00	45.49	34.60	35.30	32.90	28.00	39.59	40.0
Pipe No. 30	68.63	67.30	60.18	82.8	51.60	59.2	43.55	51.3	46.5	33.62	38.5	34.80	27.20	38.76	46.1
Pipe No. 29	67.99	66.30	58.36	60.00	50.56	52.75	42.70	43.80	44.74	32.99	34.00	32.90	27.50	38.03	39.0
Pipe No. 28	67.64	66.70	58.37	58.90	50.75	51.37	42.79	43.20	43.98	32.86	33.70	30.00	27.10	38.08	38.4
Pipe No. 27	66.72	66.25	53.45	53.30	50.21	52.3	42.29	43.10	45.2	32.21	32.90	32.50	26.50	37.81	38.0
Pipe No. 24	63.64	64.00	55.83	59.00	48.50	52.75	40.70	42.80	42.11	31.25	34.00	31.20	26.50	36.06	39.6
Pipe No. 25	64.46	62.50	55.40	76.8	48.53	59.7	40.95	60.5	62.6	31.64	42.50	49.8	25.50	36.51	36.3
Pipe No. 26	60.00	59.20	51.06	59.0	45.34	55.9	38.11	45.8	39.85	29.54	33.8	29.10	24.40	34.13	39.3
Pipe No. 23	58.73	58.30	49.76	52.30	44.53	46.43	37.38	38.50	38.91	28.73	30.20	29.10	24.00	33.25	34.0
Pipe No. 22	55.45	55.00	46.89	49.00	42.12	43.68	35.58	37.00	36.84	27.53	28.80	26.30	22.70	31.72	32.5
Pipe No. 21	54.24	53.80	43.74	47.60	41.12	42.58	34.47	35.70	34.96	26.82	28.20	26.40	22.30	30.57	31.1
Pipe No. 18	53.00	54.20	44.36	45.70	40.03	40.93	33.50	33.70	34.21	26.07	27.40	25.10	21.70	29.89	30.4
Pipe No. 19	53.03	53.10	43.95	45.40	40.49	39.6	33.99	35.20	36.9	26.31	27.20	23.20	20.00	30.18	30.8
Pipe No. 20	54.05	54.70	44.88	....	40.85	....	34.28	....	....	26.70	....	....	....	30.32	....
Pipe No. 17	48.14	47.80	36.53	44.30	36.57	39.01	31.01	33.10	33.08	23.94	26.2	24.20	20.80	27.75	29.2
Pipe No. 16	45.67	45.80	38.57	42.00	35.00	37.09	29.58	31.70	31.77	22.79	25.30	23.20	20.00	26.21	27.8
Pipe No. 13	39.37	40.70	33.51	35.10	30.20	31.87	25.83	25.80	28.20	19.87	21.50	20.10	17.00	22.98	23.7
Pipe No 14	39.21	40.50	32.88	36.00	29.78	32.69	25.58	32.4	28.38	19.78	22.10	20.30	16.40	22.90	24.8
Pipe No. 15	38.85	39.40	32.59	34.30	29.72	31.8	25.08	28.70	29.9	19.52	21.70	19.90	15.50	22.40	23.8
Pipe No. 10	30.58	31.00	26.29	30.30	24.30	25.00	21.06	22.90	22.93	16.14	18.00	16.40	14.50	18.89	21.3
Pipe No. 11	30.77	30.50	25.64	30.00	23.43	26.2	20.71	21.20	24.9	15.78	18.40	16.90	13.70	18.42	20.0
Pipe No. 12	26.43	27.50	22.20	38.1	19.25	29.7	17.26	26.8	32.6	13.12	22.1	21.8	13.50	15.16	26.3
Pipe No. 9	23.02	23.00	19.30	23.8	17.67	20.62	15.69	16.00	20.6	12.19	15.10	13.80	10.50	14.20	15.8
Pipe No. 8	16.79	14.50	14.89	14.80	14.22	11.81	12.71	8.00	12.9	10.16	10.10	8.80	5.00	11.48	9.5
Pipe No. 7	10.94	8.00	10.70	11.60	10.37	7.69	10.24	1.00	11.6	8.15	8.44	8.80	3.00	9.24	9.1

* Negligible † Approximately.

faces of those vertical cut-offs, or of intermediate cut-off walls, unless in the latter case the depth of penetration is greater than in the former. If intermediate cut-offs are driven lower than the outer ones, the extra depth of sheeting which faces the direction of flow will be nearly as effective as the outer faces of the end cut-offs.

Mr. Harbans Lal Uppal has conducted experiments on a 1:50 scale model of the Panjnad Weir, which support and confirm the foregoing statement. These experiments were carried out at the Irrigation Research Institute, at Lahore, Punjab, India. The unusually careful technique developed in the conduct of these experiments insures their verity and importance; for example, the water used for seepage purposes was freed of all silt and suspended matter, carefully and completely, by passing it through glass wool filters; the lines of vertical piling were made absolutely water-tight; and the readings were of a high order of accuracy.

The results obtained with no silt up stream or down stream from the dam, and with the seepage water and the foundation material at a uniform temperature, are given in Column (2), Table 10. These values represent normal conditions such as those that might be encountered in the prototype. In Column (3), Table 10, comparative results are recorded as scaled from a flow-net diagram constructed by Mr. D. J. Hebert.[65] The agreement between values in Columns (2) and (3), Table 10, is remarkably close, so close as to be a definite "landmark" in the development of a rational foundation design; but these conclusions in themselves would not assist a designer materially unless it were also possible to determine why the results obtained from the scale model or from the electric analogy model differ so much from those obtained in the prototype. Consequently, Mr. Uppal conducted further tests with a scale model by adding silt to the bed of the up-stream approach in varying depths. Typical results of this further study are given in Columns (4), (6), (8), and (11) of Table 10. In Columns (5), (7), (9), (10), (12), (13), and (14), the results from the prototype are recorded, and the agreement between the two sets of data is again remarkably close. The process was finally reversed by gradually reducing the depth of silt, and the results for corresponding depths of silt deposit were reproduced exactly as in the cycle of increasing depth of silt.

Still further tests were made with the scale model by lowering the temperature of the seepage water to 15° C, in the meantime maintaining the temperature of the foundation material at 24° C. The results with a 1-in. layer of silt and with river water at 24° C are given in Column (8), Table 10, and the results with seepage water at 15° C are given in Column (11). Corresponding results for the prototype are recorded in Column (12). The silt in the model (Column (11)) is 4.2 ft and that in the prototype (Column (12)), 2.8 ft. The corresponding temperature of the foundation material is 58.3° F and 58.5° F, and that of the river water is 76.1° F and 78.0° F, respectively. The controlling factors were thus reproduced in the model as nearly as possible, and the results of observations on the model and the prototype could not agree any better.

---

[65] *Technical Memorandum No. 484*, U. S. Bureau of Reclamation, Fig. 70.

In his conclusions, Mr. Lane states that "the flow-net and electric-analogy methods are faulty in neglecting the lesser resistance along the line of contact of a dam with its foundation as compared with that directly through the foundation material." The writer does not agree with this conclusion, since the electric analogy model and the scale model are equally effective in determining correctly the pressures involved.

Fig. 12 reveals the inconsistency in assigning equal weights to all vertical planes of contact; it demonstrates the underlying fallacy of the weighted-creep method. This diagram was plotted in a somewhat novel but effective manner with the percentages of pressure from 0 to 100 plotted as ordinates. These pressures, for zero depth of silt and for uniform temperature (see Column (2), Table 10), are projected to Diagonal Line No. 2 which locates the various pressure points from Nos. 34 to 7. The points at which these projections intersect the diagonal line are projected again to the horizontal base, by vertical lines, which latter are used subsequently in plotting the pressures at the respective points for conditions other than the normal; that is, for varying depths of silt deposits and for different temperatures. Fig. 12 illustrates the relative efficiency of various planes of contact.

The results are listed in Table 11, from which it will be noted that there is very little drop between pile lines. The outer faces of the end rows of sheet-piles are thus seen to be three to four times as effective as intermediate cut-offs, or as horizontal planes of contact. This is under normal conditions. When silt deposits are to be considered, or the effect of differences in temperature, the relative efficiency of the up-stream cut-off increases considerably. A comparison of Columns (4) and (5), Table 11 (see Items Nos. 2 to 7), will show that the weighted-creep method is fundamentally incorrect. (Note particularly the wide disparity in Column (6), Table 11, between Items Nos. 2, 4, 5, and 7).

TABLE 11.—Analysis of Fig. 10

Item No.	Description	Length, in feet (1)	Percentage drop (2)	Ratio Column (2) / Column (1) (3)	Relative Effectiveness: Actual (4)	Relative Effectiveness: Lane's method (5)	Departure (6)
1	Up-stream face of up-stream pile line (up-stream floor to Pipe No. 33)	22	24.2	11.0	1.0	1.0	0.0
2	Down-stream face of pile line (Pipes Nos. 33 to 31)	19.5	5.4	2.8	0.26	1.0	0.76
3	Horizontal line of creep (Pipes Nos. 31 to 24)	44	6.75	1.5	0.13	0.33	0.20
4	Up-stream face of middle pile line (Pipes Nos. 24 to 23)	19.5	4.91	2.5	0.23	1.0	0.78
5	Down-stream face of middle pile line (Pipes Nos. 23 to 21)	19.5	4.49	2.3	0.21	1.0	0.79
6	Horizontal line of creep (Pipes Nos. 21 to 10)	106	23.66	2.2	0.20	0.33	0.13
7	Up-stream face of down-stream pile line (Pipes Nos. 10–9)	18	7.56	4.2	0.38	1.0	0.62
8	Down-stream face of down-stream pile line (Pipe No. 9 to down-stream floor)	23	15.82	6.9	0.69	1.0	0.31
9	Residual head	......	7.2	....	......	......	......
			100.0				

The fact that all the points of any particular set of observations lie on a straight line shows that the relative effectiveness of all parts of the bed of the stream remain unaltered. A reduction of pressure due to an increased deposit of silt or to changes in temperature is equivalent to reducing the hydrostatic head at the point where seepage water enters the foundation. The pressures in all pipes will rise or fall in proportion to the pressures recorded in them. Thus, if ϕ_{34} represents the percentage of pressure in Pipe No. 34 for any given set of conditions and ϕ'_{34} represents the percentage pressure in the same pipe for another set of conditions, the pressures in all other pipes will vary in the ratio of $\frac{\phi_{34}}{\phi'_{34}}$ for that change of conditions. In other words, for Pipe No. 10, ϕ_{10} will become $\phi'_{10} = \phi_{10} \times \frac{\phi'_{34}}{\phi_{34}}$. The silt and temperature effects have been studied at some length by the Central Board of Irrigation.[66]

For the purpose of insuring a safe design a most important factor is the discharge gradient which is the same as the residual head. It is important in design to insure that this value is within safe limits. Extensive and painstaking research is required to determine such gradients for different classes of soils under different types of base profiles. If the distribution of pressures can be obtained reliably by models or by simple formulas derived from models, and if the safe discharge gradient or residual head can be obtained, the task of developing safe and economical designs should present no difficulty.

A. STREIFF,[67] M. AM. SOC. C. E. (by letter).—Regarding the design of dams on soft foundations, experience is the best teacher. It is of great value to practitioners, therefore, to peruse the carefully compiled tables in this paper and to study the collected data. Mr. Lane has rightly concluded from an empirical study of these data that, in most cases, the resistance to flow of vertical creep is much greater than that of horizontal creep. Additional theoretical and practical considerations, herein presented, substantiate this finding. As a matter of fact, it may be demonstrated that, in many cases of dams on difficult foundations, safety depends upon vertical cut-off walls alone.

Mr. Lane derives the theoretical basis for the adoption of the percolation coefficient by placing $L = C_1 \frac{H}{V_m}$, in which V_m is the definite safe maximum velocity at which water can emerge below the dam without carrying off material. It appears, however, that on this basis the coefficient, $C_2 = \frac{C_1}{V_m}$, proposed by either Bligh or Lane, gives seepage velocities which are only on the order of 0.001 of the actual entrainment, or so-called "competent" velocities. A factor of safety of 1 000 to 1 is quite unusual in engineering, and the reasons for such departure from common practice are herein further discussed.

---

[66] *Publication No. 8*, Central Board of Irrigation, Revised Edition.

[67] Vice-Pres., Ambursen Dam Co., Inc., San Antonio, Tex.

That the ratio of 1 000:1 really exists, and requires further explanation before Mr. Lane's Equation (2) can be accepted as a basis for the percolation coefficients proposed, may be shown as follows: The seepage velocities for the average percolation ratios are small values. For medium sand (porosity, 40%, and grain size, 0.45 mm), Slichter's table gives 0.00049 ft per sec for a percolation ratio of 6 as proposed by Mr. Lane.

The "competent" velocities of entrainment for various sizes can be compared. These velocities have been investigated many times, both recently and some time ago. Prominent investigators were Dubuat, Fargue,[68] Lechelas,[69] Leslie,[70] Deacon,[71] Gilbert,[72] Dubois,[73] Strickler,[74] and the German hydraulic laboratories.[75] Gilbert[72] gives the formula in terms of mean velocity,

$$\frac{0.0042}{G-1} v^{2.7}_m = D \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

in which D equals the diameter, in feet, of sand grains; G is the density; and v_m equals the competent mean velocity. For the grain sizes shown, Equation (7) yields the values listed in Table 12.

TABLE 12.—COMPETENT VELOCITIES OF ENTRAINMENT

Sieve measures per square inch	Diameter of sand grain, D, in millimeters	Competent mean velocity, v_m, in feet per second. (Very few grains moving.)
50 to 60	0.305	0.74
40 to 50	0.375	0.80
30 to 40	0.502	0.99

Another method by which to arrive at the competent velocity of flotation would be the formula of Stokes for small spheres dropped in a viscous fluid:

$$P = 6\,\pi\,\mu\,a\,v \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

in which P equals the weight of a particle; a, the radius of a spherical particle; μ, the viscosity coefficient; and v, the competent velocity of flotation. It is found that, for the foregoing example, a competent velocity of about 0.5 ft per sec is compared with a seepage velocity of 0.00049 ft per sec; or a ratio of 1:1 000.

It is necessary to explain the use of such an apparently extravagant factor of safety in Mr. Lane's proposed coefficients. One explanation has been mentioned by Mr. Lane, who quotes Professor Terzaghi in attributing failure to a greater than average gradient at the down-stream exit, and causing "flotation," decreased permeability, and blow-out. An examination, however, will show that these concepts are incorrect and that remedial measures at the downstream toe of a dam are inconsequential.

---

[68] *Annales des Ponts et Chaussées*, 1894.

[69] *Loc. cit.*, 1871.

[70] *Cambridge Philosophical Transactions*, 1844.

[71] *Minutes of Proceedings*, Inst. C. E., 1894.

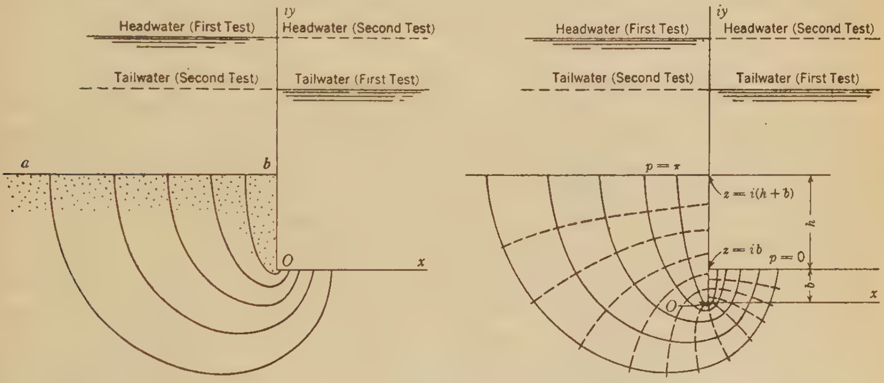
[72] *Professional Paper 86*, U. S. Geological Survey, 1914.

[73] "Le Rhône et les rivières a lit affouillable," *Annales des Ponts et Chaussées*, 1879, II.

[74] Mitteilungen des Amtes für Wasserwirtschaft, Berne, 1923.

[75] Publications by Schoklitsch, Kreuter, Krey, Schaffernak, Engels, and others.

Beginning about thirty-five years ago (1900) those excellent engineers practicing in India — Messrs. Beresford, Clibborn, Griffith, Parker, Bligh, and others—advanced such "fountaining" as a cause of failure. Twenty-two years later this "fountaining" was taken up anew by Terzaghi under the term, "flotation," and pronounced to be the exclusive cause of piping and under-seepage, based on escape gradients greater than average. Spurred by Beresford's tests[76] on flotation made in 1904, the writer and the late W. W. Tefft, M. Am. Soc. C. E., made the following tests to determine whether flotation by steep exit gradients was the exclusive cause of piping. These tests were made in 1914, and demonstrated the fallacy of flotation as the exclusive cause of piping, quoted by Mr. Lane in discussing the limitations of creep analysis. Consider a vessel with a diaphragm in the center, the lower edge of which is placed at a certain height above the bottom. Fill it with sand on one side to the lower edge, and on the other side somewhat higher. It will be found that if the head of water is on the high side, the blow-out value is greater than if the condition is reversed. Apparently, then, the value against blow-outs in an up-stream fill is greater than that of a weighted apron, and fountaining down stream can be prevented by an up-stream fill, regardless of a theoretically infinite escape gradient.



(*a*) WITHOUT CUT-OFF (*b*) WITH CUT-OFF

FIG. 13.—FLOW NET FOR FLOTATION EXPERIMENTS.

The flow net for this experiment is easily computed by conformal mapping with the Schwarz-Christoffel method. By double transformation the following relations between flow and pressure lines are obtained:

$$x = f - \sinh f \cos p \qquad (9)$$

and,

$$y = p - \cosh f \sin p \qquad (10)$$

which determines the flow net shown in Fig. 13(*a*), in which $f = f_1(x, y)$ and $p = f_2(x, y)$ are conjugate functions and denote the flow and pressure lines.

---

[76] *Proceedings*, Punjab Inst. of Civ. Engrs., Vol. 158, p. 77.

From these lines the gradient at any point is easily computed, and shows, of course, much steeper gradients along Line OX (approaching ∞ at 0) than along Line $a\,b$, which is apparent also without this computation.

Contrary to the flotation theory then, the blow-out value is greater with greater escape gradient, whereas, if the theory were correct, the reverse should obtain. In other words, as Mr. Lane points out, quoting Griffith, piping is dependent on other factors besides flotation, and the practice of designing dams on flotation considerations is not supported by the facts.

Consequently such flotation has been always ignored by the writer, and was never considered in the seventy structures with which he has been connected (see Table 8), and which are operating successfully under heads as high as 65 ft.

Further tests were made with a diaphragm extending into the sand, as in the case of sheet-piling with unequal fill on both sides. The flow net is given by the Schwarz-Christoffel method, transforming directly from the w-plane into the z-plane:

$$z = A\left(\frac{1+a}{2}\right)\cosh w - \frac{A}{2}\,(1-a)\;w \,\ldots\ldots\ldots\,(11)$$

in which, $z = x + i\,y$; $A = i\,b + \dfrac{h + 2\,b}{\pi}$; $w = f + i\,p$; and, $a = \dfrac{i\,b - \left(\dfrac{h+2\,b}{\pi}\right)}{i\,b + \left(\dfrac{h+2\,b}{\pi}\right)}$;

and is illustrated in Fig. 13(b). As before, $f = f_1(x, y)$ and $p = f_2(x, y)$.

Here, too, it was found that blow occurred sooner from the low to the high side than *vice versa*, showing that exit gradients were just of opposite effect to that shown by the "fountaining" theory of Beresford.

In applying flow nets for seepage flow it becomes necessary to take into account further limitations of the theory, in addition to Mr. Lane's criticism that the flow net does not take into account the lesser resistance along the line of creep. The theory breaks down at the so-called "suction points," that is, the external corners, such as the tips of sheet-piling or base-slab corners. The entire flow net is based on the law of Darcy, which no longer applies when gradients become very steep and the flow becomes turbulent;[77] hence, in those points, neither does the flow net. It is easy to see that according to Bernouilli the pressure would drop to $-\infty$ at a point where the gradient, and hence the flow, is ∞. This can not occur, and the infinite gradients are actually non-existent. The flow net is a mere geometrical creation based on the fact that the Laplace equation, $C_1 \nabla \nabla p = 0$, gives, as a general solution, a complex system of conjugate functions. Herein, the law of Bernouilli takes no part whatever, and later is introduced to give the pressure at every point. In seepage flow nets this is not done, and it is plain that the flow net only applies where velocities are very small; but, where the gradients are steep, the theory no longer applies, and the infinite gradients at the "suction points" are illusory.

---

[77] Chilton and Colburn, *Industrial Engineering Chemistry*, Vol. 23, 1931; Fair and Hatch, *Journal* Am. Water Works Assoc., Vol. 25, 1933; Fancher and Lewis, *Industrial Engineering Chemistry*, Vol. 25, 1933; Hulburt and Feben, *Journal*, Am. Water Works Assoc., Vol. 25, 1933; Lindquist, Premier Congrès des grands Barrages, Stockholm, 1933; and Waddell, *Journal*, Franklin Inst., Vol. 217, 1934.

Therefore, one can discard the contention that the Lane and Bligh coefficients with their seemingly extravagant factor of safety are based on the concentrated escape gradients as the cause for piping. Nevertheless, these coefficients can not be diminished materially without danger, and it becomes necessary to offer a reason for this fact.

Mr. Lane's discussion of roofing gives the key to the solution of the problem. There may be a danger that incipient pipes are present under the roof created by the base slabs, and it is this circumstance which prevents a radical reduction of the Lane or Bligh coefficients.

This is particularly true for low dams of small weight. Under certain conditions clay and rock strata may also serve as roofs and thus initiate piping. Only in this manner, and in the absence of proper cut-offs, can the seepage velocities through remaining "plugs" attain sufficiently high values to transport fines gradually and, finally, cause blow-outs. Experience demonstrates that blow-outs may occur years after a structure has been completed.

Hence, in order to prevent under-seepage, it is necessary to counteract the existence of incipient pipes or roofing under the base slab. In all the structures within the writer's experience, this under-seepage was prevented, with uniform success, by placing a zone of safe filtration (that is, a properly built cut-off wall) up stream from the zone of roofing. Mr. Lane's study of all the available records of percolation distances of existing masonry dams on earth foundations shows the importance of such cut-off walls. An examination of the records of certain canal and river structures that have been in existence for hundreds of years will serve to strengthen Mr. Lane's conclusions and also show that roofing may easily be corrected. Such examples may be found in the navigation locks and dry docks, many of which have been built on pile foundations, in the Netherlands, for many centuries. These structures are cited because their design introduces the presence of a more clearly established roof than those built in the United States. The latter are usually of concrete, and are sometimes cast around pile-butts, resting on the foundation material at intermediate points. In the Netherlands, however, the soil is often soft mud and silt, and for many centuries before the introduction of concrete, the practice of using brick required the construction of a timber floor on top of the piles.

The range of heads for which these locks were constructed is about equal to those found for the dams in India, quoted by Mr. Lane, heads of 10 ft seldom being exceeded. The horizontal length across the base slab of the lock heads, however, is very much shorter than the length of apron of the Indian dams; they are placed on a perfect roof (the timber floor being placed on piles) and their record of service covers centuries. Failures such as those that occurred to the Narora Weir, Chenab Dam, Deoha Barrage, Kulli Bye-Wash, etc., are extremely rare. The safety of these locks rests entirely on the application of vertical cut-offs consisting of carefully constructed tongue-and-grooved wood sheet-piling. This piling is carefully notched into the under-side of a plank floor, and the joint is tightened with oakum. Two double rows of sheet-piles are commonly used, one at each end of the lock head; often a fifth intermediate row is driven. Between the double rows of piles a so-called "clay chest" is carefully puddled. This clay chest is not of any considerable depth, however. The upward pressure is taken care of by placing heavy timbers, called "zwalpen," on top of the timber floor, reaching from one lock-wall to another. These timbers prevent accidents, such as the blowing up of the Narora apron, or the floor of the Pearl Harbor Dry Dock. In addition to the "zwalpen," the pile-butts are dovetailed and wedged into the cap timbers, in order to withstand the upward pull. Between these "zwalpen," the brick floor is placed in "tras" and lime mortar, tras being a puzzolana cement.

As these locks and dry docks have been in use for centuries, it follows that a perfect roof does not necessarily preclude safe construction. The same experience, although necessarily covering a shorter period (about 30 yr) prevails in the case of dams on pile foundations, of which many have been constructed operating under various heads as high as 40 ft. Of these dams, two examples are considered.

The Mio Dam was built on the Ausable River, in Michigan, in 1915. The foundation material was sand with an admixture of some gravel. The head is 29 ft. In addition to the Tainter gate spillway of the Ambursen type, the dam contains under-sluices through the power house. The entire structure was placed on a pile foundation, the concrete slab being cast on the pile-butts and resting on the sand between the piles. The adjoining embankments consist of fine sand.

It was decided to provide this dam with ten pipes for the measurement of upward pressure under the power house, arranged in two rows of five each, in the up-stream and down-stream directions, in order to establish the magnitude of the upward pressures. As before, stability was secured by a zone of filtration up stream of the zone of roofing, by means of an up-stream cut-off of 20-ft sheet-piling and a sand fill, 22 ft thick, on a 1 on 4 slope, resting against the up-stream inclined deck slab of the dam and power house.

Fig. 14 shows the resulting pressures under the base. The first set of readings was made from January 19 to February 17, 1917, and the second set during August, 1917. It may be seen that the up-stream cut-off reduced the head by about 50 per cent. The up-stream half of the base was apparently a perfect roof with a "plug" between Points 3, 8, and 2, 7, possibly as a result of an additional 5-ft wooden pile cut-off, causing a drop of approximately 7 ft between these points. In August, the pressure at Point 7 reached approximately the same value as in the up-stream points, indicating that a pipe had apparently reached this point, and that the space down stream from it had become plugged. Points 2, 2*a*, and 7*a* gave readings, approximately the same as Point 1, 1*a*, 6, and 6*a*, at the end of the floor. Here, the pressure was still about 6 ft above tail-water level, due to short sheet-piling and the concrete paving in the tail-race necessitated by the under-sluices. It is interesting to note that, although the upward pressures indicate roofing and do not diminish gradually, the loss of head through the up-stream filter is approximately as computed by Professor Weaver for a dam with sheet-piling at the heel, namely, about 50% for the prevailing proportions.

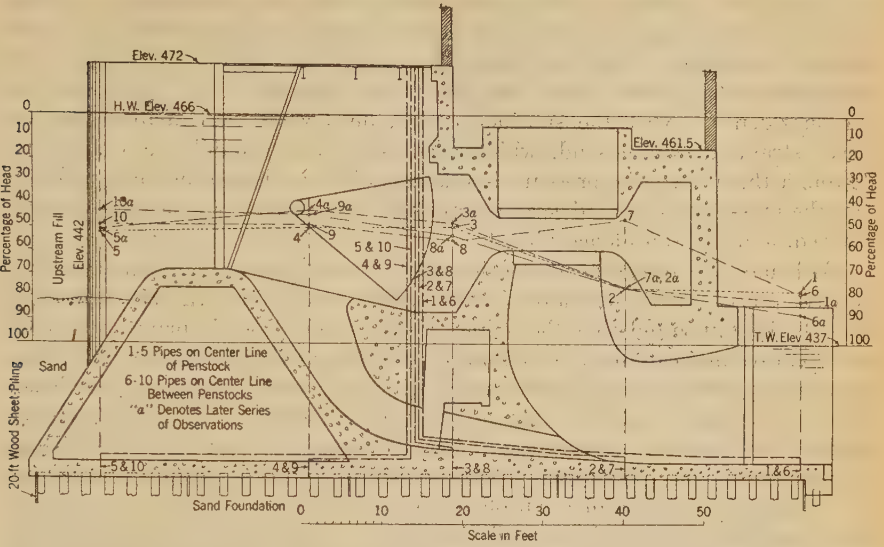

Elev. 472
H.W. Elev. 466
Percentage of Head
0
10
20
30
40
50
60
70
80
90
100
Upstream Fill
Elev. 442
10a
10
5a
5
4a
9a
4
9
5 & 10
4 & 9
3a
3
8a
8
3&8
2&7
1&6
7
Elev. 461.5
7a, 2a
2
1
6
1a
6a
T.W. Elev 437
Sand
20-ft Wood Sheet-Piling
1-5 Pipes on Center Line of Penstock
6-10 Pipes on Center Line Between Penstocks
"a" Denotes Later Series of Observations
5&10
4&9
3&8
2&7
1&6
Sand Foundation
0
10
20
30
40
50
Scale in Feet

FIG. 14.—PIPING FOR THE MEASUREMENT OF UPWARD PRESSURES UNDER FOUNDATION, MIO DEVELOPMENT, EASTERN MICHIGAN POWER COMPANY, JACKSON, MICH.

Alcona Dam is also on the Ausable River, in Michigan. It is of the conventional Ambursen type and is probably the most extreme example of the non-applicability of a percolation ratio on account of complex foundation conditions. The head is 40 ft, and the dam has been in operation for thirteen years. It consists of a two-unit power house with under-sluice spillway, and with adjoining sand embankments. The site consists of a 30-ft layer of top soil and clay, resting on a bed of sand more than 100 ft in thickness (principally quicksand), which gradually becomes less fine with increasing depth. It is subjected to artesian water pressure from a line of hills to the south. While the clay foundation pit was being excavated, escaping quicksand caused the remaining thickness of clay to shear off along the sides of the foundation pit, and to subside. Subsequently, long timber piles were jetted through the clay and quicksand into the coarser sand strata below, and a box of long steel sheet-piles was placed around the structure. The concrete substructure of the power house was then placed on the piles and the clay slab. In places, a flow of 150 gal per min emerged alongside a driven steel sheet-pile.

In this case, again, the pile supports make a roof over the horizontal creep. In addition to the long steel sheet-pile box, security against under-seepage is again obtained by a vertical fill up stream of the zone of roofing, reaching from the base slab to the sill of the intake of the turbines and under-sluices.

By these means the structure has been made perfectly stable for the thirteen years since 1922. Although the dam has safely carried a head of 40 ft without movement, the unstable conditions in the down-stream river channel have continued, owing to the escape of the quicksand through vents in the clay cover. At the down-stream end of the tail-race subsidence of the retaining walls has continued until one corner of the wall disappeared entirely below the water line. The bottom of the tail-race slopes downward in a down-stream direction. This condition, if continued, eventually would have endangered the power house, and, therefore, measures were taken to stop the subsidence. Near the subsidence area deep wells were drilled into the lower strata of coarse sand, which acts as a filter around the well-casing. Through these wells the pressure is released without entraining the quicksand and the pressure on the quicksand vents was lowered so that no more material escaped, and the subsidence ceased.

Although the foregoing discussion serves to emphasize the importance of vertical creep and to substantiate Mr. Lane's conclusions in this respect, it must be kept in mind that the value of such creep depends primarily on the construction methods used in obtaining it. Actually, there are several kinds of vertical creep each of which has a different value as far as percolation resistance is concerned. A line of sheet-piles driven in clay, for instance, might be of no value, opening up a line of cleavage rather than establishing a creep resistance. The best type of vertical creep is at the up-stream face of the dam in the form of a cut-off of sheet-piling or masonry, and inasmuch as an up-stream fill can be placed with every care required, such fill must be considered as one of the best forms of vertical creep obtainable, especially if its gradual sloughing into any roofed space under the dam is prevented by a proper up-stream cut-off wall or sheet-piling.

Minimum seepage is a necessity for a dam, and can best be controlled by the use of proper material in the up-stream fill. The more impervious the material, the steeper the gradient that can be applied; but it should not be overlooked that steep gradients and short distances require perfection of workmanship and as quality of workmanship cannot always be fully controlled in the field, it is better to avoid them.

As long as the up-stream zone of filtration is secure title drainage under the base slab undoubtedly reduces pressure better than weep-holes. The latter are of little use, as will be demonstrated later, and their existence does not justify the use of low creep ratios, as pointed out by Mr. Lane. The writer does not agree with Mr. Lane's statement, however, that a reverse filter will make such weep-holes more effective.

The ineffectiveness of weep-holes may be illustrated by Fig. 15, in which, OY represents the bottom of the base slab, with O the center of a weep-hole. Furthermore, let the pressure at the weep-hole be p_0, and under the base slab in the immediate vicinity of the weep-hole, the pressure, p_1, is uniform. Curve OA represents the pressure drop from p_1 to p_0. No roofing is considered. Curve OA be-

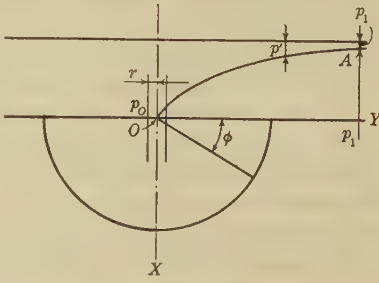


FIG. 15.

comes tangential to the horizontal. The hydrostatic potential in any point, $P_{R\phi}$, is constant. Hence, the gradients are only dependent on R and not on the angle, ϕ. For the discharge through the weephole:

$$Q = K \int_0^{\frac{\pi}{2}} 2 \pi R^2 \cos \phi \, d \phi \, \frac{dp'}{dR} = 2 \pi R^2 K \frac{dp'}{dR} \quad \text{........} (12)$$

Hence,

$$-\frac{1}{R} = \frac{2 \pi K}{Q} p' + C \quad \text{..................} (13)$$

In Equation (13), for $p_o = 0$, $C = -\frac{1}{r}$, and r = the radius of the weep-hole: which (for $p_o = 0$) gives:

$$Q = 2 \pi K r p_1 \quad \text{........................} (14)$$

As an example consider a 3-in. weep-hole and an under-pressure of 15 ft at the hole. The sand is of medium grade with $K = 0.01$ (Slichter); and, $P = 40$ per cent. Substituting these values in Equation (14) it is found that $Q = 0.118$ cu ft per min.

The velocity through the hole is 0.04 ft per sec which is much less than the competent velocity. The gradient at the weep-hole is 120 ft per ft; a gravel pocket under the hole will take care of that condition. The gravel need not be graded as the velocities are too small. For a 1-ft radius the gradient is only 1.9 ft per ft and the seepage velocity is only 0.00031 ft per sec. The pressure for a 1-ft radius is 13.15 ft; hence, the effect of the weep-hole in reducing pressure is practically nil 1 ft from the center of the drain.

Even if the weep-holes are spaced at only 2-ft centers, the effect on upward pressure would be little. In that case the pressures would be restored to the original a few feet under the base slab and the net effect would be some added weight to the structure against under-pressure.

The foregoing discussion not only shows the ineffectiveness of weep-holes in reducing under-pressure, but also shows that inverted filters would be of little aid at the weep-hole.

Mr. Lane has performed a great service to the Engineering Profession in covering so thoroughly the many elements which must be considered in preparing adequate designs of dams on permeable foundations. One of the most important results will be the revision of previous views as to the necessity for creep ratios as great as those recommended by Bligh after much early experience (although in a rather narrow field) many years ago. From a review of the author's tables it becomes apparent, however, that it will always be dangerous, if not impossible, to establish a more or less arbitrary creep ratio for application to all structures on any selected foundation material, as it has been proved that the dams with the highest heads have generally the smallest percolation ratios. Naturally, the requirements of design for special field and operating conditions will also continue greatly to affect any percolation ratio that may be developed as a general guide.

E. W. LANE,[78] M. AM. SOC. C. E. (by letter).—The extent of the discussion evoked by this paper indicates that the subject has been one of considerable interest. The discussions have added data, and valuable conclusions based on extensive experience. The presentation of the various viewpoints has been very beneficial since to arrive at the truth it is necessary that all possibilities be considered and weighed to determine their relative merits.

A general view of the discussions shows that there are two schools of thought regarding security from seepage under masonry dams on earth foundations, one which emphasizes the flow through the foundation material and the other which views failure as taking place because of imperfect contact of the dam and foundation. In many cases those who hold one or the other of these views give no consideration whatever to the possibilities of the action of the other type.

The paper was completed in 1932, and in the intervening three years a great mass of enlightening information has appeared, causing modifications of some of the original ideas. Nevertheless, the principal conclusions of the paper are still sound. The writer stated that the water flowing beneath a dam follows not only the path of least resistance, but takes all available paths in quantities varying inversely as the resistance of the several paths. Much of it flows through the void spaces between the grains of the foundation material and, in most cases, it is possible that practically all of it takes such a path. Thus, ordinarily, there is little danger of failure for dams of any reasonable cross-section; but there is a possibility of such failure. The greater danger in such dams arises from an imperfect contact of the dam with its foundation (or "roofing," as it is sometimes called), which permits a channel to form, through which the foundation material can be carried; this allows the channel to enlarge, ultimately resulting in failure. Such a condition is not general beneath a dam; it occurs only under unusual conditions in local areas and, therefore, may not be detected, even with rather extensive observations of upward pressure. Like a flaw in the rails of a railroad, it may occur only at one point in a great length; yet this one flaw is sufficient to cause failure. It may be concluded therefore, that failure may occur by either of two methods: (1) By flow along the imperfect contact of the dam and its foundation; and (2) by flow directly through the foundation material; Method (1) is the more common. A number of those who have objected to the writer's analysis have ignored his discussion of the second type of failure and his recommendations for avoiding it. The velocity gradient method of analysis described by Mr. Harza, Mr. Creager, and Professor Casagrande, is simply a more refined method of analyzing the flow through the foundation material and its effect on piping. The writer advocated limiting the short path to 80% of the weighted creep, as the best method then apparent of securing safety against flow through the foundation material, which should indicate that he does not belong strictly to either of the foregoing schools of thought. The emphasis in the paper was placed upon flow along the creep line because it was believed that this is by far the most common method of failure; but both methods of failure were considered.

---

[78] Prof. of Hydr. Eng., State Univ. of Iowa, Iowa City, Iowa; formerly Research Engr., U. S. Bureau of Reclamation, Denver, Colo.

Perhaps it would aid in understanding the situation to consider the analogous case of a simple beam. Such a beam can fail by being loaded until the fiber stress in tension (or compression) exceeds the ultimate value that the material will stand; or it may fail by causing the shearing stress to exceed its ultimate value. These are separate methods of failure, and the methods of analysis are entirely different; yet to be secure, a beam must be safe against failure by either of the two methods, and the consideration of only one of the possibilities is not enough to assure safety. Long beams are more likely to fail by exceeding the tensile (or compressive) stress and short beams by exceeding the shearing stress; but in either case the result is the same, the beam fails. If, in a long beam, failure due to exceeding the fiber stress is prevented by some method, it will fail by shear, ultimately, if the load is increased sufficiently.

Similarly, a masonry dam on an earth foundation may fail by piping along the line of contact with its foundation, or by flow directly through the foundation. In either case the result is a failure of the structure, yet the methods of analysis of the two types of failure are different, and the use of the method of analysis suitable for one method of failure cannot be justly criticized because it does not conform to the analysis of the other. If a dam is built by the usual present-day method it is believed (for reasons that will be given subsequently), that it is more likely to fail by piping along the line of contact with the foundation material. However, if this method of failure does not occur and the head is sufficiently increased, failure by flow directly through the foundation material will occur eventually.

Relative Probability of the Two Kinds of Failure.—The emphasis that should be given to failure by either one of the two processes previously described should depend, logically, upon the probability of failure by that method. If one type of failure is relatively frequent and the other is rare, a more complete treatment of the frequent type is justified. The type of failure that is most frequent in actual dams can be determined only by observations on actual cases. Model tests or analogies cannot be depended upon for the answer. A careful reading of all the available literature on failures of this type of dam has convinced the writer that failures by flow along the line of contact of the base of the dam and its foundation is relatively frequent and, therefore, is very important, whereas failure from flow through the base material is infrequent and occurs only under very unusual conditions.

As previously mentioned, the only case found in which failure appeared certainly to have been due to percolation through the material was at the Coon Rapids Dam, where a gap in the sheet-piling cut-off combined with the pumping out of a deep scour hole below the dam, caused a blow-out. Two other cases in which that type of failure may have occurred were the Puentes and Port Angeles Dams. Both these structures were of very unusual type; no one with knowledge of the elements of design of masonry dams on earth foundations would build dams like them. Not a single conclusive example was found of failure of a dam by flow through the foundation material where the structure had a form which would be proposed by one familiar with the principles of design of such structures. It is probably not without significance that the advocates of flotation as the method of failure have confined themselves to a discussion of flow nets, pressure gradients, and model tests, and have not cited a single case of actual dam failure due to flotation.

In addition to the evidence of imperfect contact or roofing quoted from J. C. Oakes,[16] M. Am. Soc. C. E., there are many other reports of dam failures and difficulties that demonstrate the occurrence of imperfect contact and the tendency to fail by flow along the contact of the dam and its foundation. The failure of the Deoha Barrage[79] is one of these. A channel was formed under the floor of the structure for a distance of more than 300 ft with a 12-ft head. This channel started at a gap in a line of sheet-piles and took a path much longer than had existed along practically the entire length of the structure when first completed. The lack of perfect contact below the Stony River Dam has been demonstrated by F. W. Scheidenhelm, M. Am. Soc. C. E.[80] Cavities have recently been found beneath the floor of the Khanki Weir, in India, which showed by the deposit of silt in them that they were not formed by water passing through the foundation material.[81]

The failure of the Corpus Christi Dam, occurred obviously along the line of contact of the dam and its foundation. The channel formed down the up-stream face of the dam at the contact of this face with the up-stream earth blanket; and it was present for some time before failure, as shown by the stains on the face of the dam. The remainder of the path (under the wing-wall, over the top of the sheet-piling, and back under the wing-wall to the apron that was blown up) although not proved by direct evidence, is so strongly indicated by the circumstances as to be very good evidence. Further confirming Colonel Oakes' observations,[16] it is reported that in the reconstruction of one of the Ohio River navigation dams farther up stream than those mentioned by him, the evidence of lack of perfect contact of the dam with the foundation, and the flow of water along this contact was unmistakable.

The pressure readings under the Mio Dam given by Mr. Streiff show the variability of the pressure beneath a dam and cannot be explained on the assumption that the water passed through the foundation material. No amount of evidence by models or electric analogy can prove that phenomena observed on actual structures did not happen. Models and the analogy can show how actual dams may fail, but they cannot prove that methods of failure actually observed are impossible.

As a result of consideration of the foregoing examples, it is concluded that piping failure may occur in dams of conservative appearing designs, but failure by flotation occurs only in unusual cases; and for dams designed with the factors given in the paper, flotation failure is very improbable. However, if further experience indicates that lower creep-head ratios than those suggested by the writer can be used safely, that horizontal creep might in some cases be given greater weight, or, if some method is found of securing perfect contact of the dam with its foundations, then the probability of failure by flotation will be increased. For this reason it should not be overlooked entirely.

---

[79] *Minutes of Proceedings*, Inst. C. E., Vol. 233, 1931–32, pp. 152, 181, and 184.

[80] *Transactions*, Am. Soc. C. E., Vol. LXXXI (1917), Fig. 23, p. 971.

[81] "An Investigation of the Flow of Water under Khanki Weir and the Pressure on the Floor," by Taylor and Uppal, *Bulletin*, Punjab Irrig. Research Inst., Vol. 2, No. 6, February, 1934.

Recommended Creep-Head Ratios.—There seems to be no opinion that the weighted-creep ratios suggested by the writer are too small. On the other hand, quite a few of the discussers considered them to be too conservative. The tables compiled by Messrs. Burroughs and Streiff give considerable support to this conclusion, especially for high dams. Mr. Ayres' suggestion of values for vertical creep only, ranging from 1.0 for firm, homogeneous clay to 4.0 for sand is also based on extensive experience. The ratios suggested by the writer were (as stated) recognized to be very conservative and the possibility of a later revision downward was mentioned. The fact that such ratios might be used generally (in many cases by persons with little knowledge of the principles involved or experience in hydraulic structures) and, no doubt, in some instances that they may be applied to cases in which they had little application, led toward the adoption of conservative values. Furthermore, the frequency with which failures occur without warning after many years of satisfactory service counsels caution in adopting values based upon the record of dams, many of which have only a short period of service. Similar thoughts seem to have led Mr. Burroughs to state in spite of the tables he presented, that he was "not prepared to recommend any reduction for general use from the figures tentatively set up by Mr. Lane," and also caused Mr Potter to remark that he agrees that in the matter of reducing ratios "it is better to advance slowly." At this time, therefore, the writer does not advocate any reduction, for general use, in the creep-head ratios previously suggested, but should a considerable period of time elapse without bringing about failure in the structures with low-creep values given in the tables prepared by Messrs. Burroughs and Streiff, it could be concluded, reasonably, that lower values were justified.

Relative Values of Horizontal and Vertical Creep.—The discussers seemed to be in general agreement that vertical creep was more effective than horizontal creep, or (which is not the same thing) that a vertical cut-off usually caused a greater drop in pressure than occurred along an equivalent creep length of the base of the dam. There was no general agreement, however, as to the relative value of horizontal to vertical creep. Messrs. Burroughs and Streiff contend that where the base of the dam rests directly on the earth, without being supported by piles, if the pressures are high (as is the case under dams holding back considerable heads of water) the contact between the bottom of the dam and the foundation material will be more perfect than if the pressures are low, as in the case of small dams. For this reason they contend that a greater value can be given to horizontal creep for high dams than for low ones. This contention is supported by an extensive collection of data. Mr. Griffith also appears to take this view.

Mention was made in the paper, of the possibility that a value of one-third for the horizontal creep might not be the best in all cases and that further study might show the value to depend upon other factors. There is strong evidence that Messrs. Burroughs, Streiff, and Griffith have pointed out one of these factors, and when this information has been considered carefully, in all its aspects, the relations between pressure and creep values may become more clearly defined.

Messrs Burroughs and Ayres contend that when dams are supported on piles, no allowance at all should be made for horizontal creep. There is much evidence to support this viewpoint.

There was some contention that deeper cut-offs could be given greater weight because of the greater pressures involved at the bottom of them. There would seem to be some justification for this if a perfect cut-off was assured. However, since practically all deep cut-offs are of sheet-piling, and the deeper the piling the greater the probability of imperfections, it would seem best to balance any advantage secured from greater pressure against the greater probability of imperfections and adopt a constant value for vertical cut-offs.

Non-Agreement of Pressures Computed by Weighted Creep and Electric-Analogy Analysis no Proof of the Fallacy of the Former.—A number of those who have discussed this paper have pointed out that the pressures beneath a dam, when computed according to the weighted-creep relations, do not give the same results as when the electric analogy is used. From this they have drawn the erroneous conclusion that the weighted-creep method of analysis is unsound. This error involves two fallacies: (1) That the drop in pressure shown by a model or the electric analogy is a measure of the resistance to piping; and (2), that the analogy or models will give exactly the conditions that would exist on the prototype.

As a proof of the fallacy of the weighted-creep analysis, it was cited that: (*a*) The pressure drop on the up-stream side of the up-stream row of sheet-piling (for example, as indicated by the electric analogy), is usually greater than that on the down-stream side; and (*b*) since the weighted-creep analysis is based on the assumption that the resistance to piping is the same on both sides, it must be unsound. The fallacy in this reasoning is that the pressure gradient at any point beneath a dam is not a measure of the resistance to piping at that point. Mr. Harza[82] and others have shown that the direction of the pressure is as important a factor as the pressure drop. The greater pressure drop on the up-stream side of the piling, combined with its downward direction may involve no greater tendency to piping than the lesser pressure drop on the down-stream side, combined with its upward direction. Still other factors may be involved, but in any event pressure gradient is no indication of piping resistance.

---

[82] See p. 1360.

If perfect contact is assumed, there is more basis for argument that the upward pressure beneath a dam could not be predicted by assuming that the pressure gradients along vertical creep surfaces are three times as great as those along horizontal creep surfaces. It should be noted however, that the writer recommended the weighted-creep method for upward pressure only where foundation conditions were complicated or unknown. He expected that where the foundation material was known accurately, closer predictions of upward pressure could be made with the electric analogy. It was somewhat of a surprise, therefore, to find that even for upward pressure the electric analogy did not seem to give better results than the weighted-creep distribution. One of the reasons for this phenomenon may be that the drop in pressure acting below a dam, due to a line of sheet-piling, is the sum of the drops on the up-stream and the down-stream sides of the piling, and the greater drop on one side is likely to be offset by the lesser drop on the other side.

Another error in the argument used to prove the weighted-creep analysis to be unsound is the assumption that the electric analogy or models will give accurately the conditions on the prototype. As stated by Mr. Hebert, the results of his comparisons between the electric-analogy analysis, the weighted-creep analysis, and the observed pressures beneath all dams for which data were available, do not indicate that any closer agreement to the observation on actual structures is obtained by the electric analogy method than by the weighted-creep method. Since the electric analogy should indicate the same pressures as a model if the distribution of permeability of the conductor corresponds to that of the soil, the foregoing results no doubt hold also for models. If the distribution in pressures beneath a dam, therefore, shows the probability of failure by piping, the electric analogy or models (at least when using homogeneous mediums), does not give a picture any more accurate than the weighted-creep analysis. Therefore, the argument that the weighted-creep analysis is fallacious because it does not agree with the analogy or model is invalid.

As a general proposition it is not possible to prove the weighted-creep method fallacious by using an argument based on an assumption of perfect contact, since the conditions leading to the conclusion are inherent in the assumptions. The validity of a conclusion based on an assumption of perfect contact, therefore, is only as good as the assumption. This leads back to the question, previously discussed, regarding the method that causes failure in actual dams.

As an example of how divergent are the two conceptions of the method by which dams actually fail, suppose observations on a dam at a given cross-section showed a negligible drop in pressure for a considerable distance across the base. From the viewpoint of a perfect contact this would indicate the entire absence of danger of piping; but from the viewpoint of imperfect contact it would indicate that a pipe had already formed and, therefore, that an undersirable condition already existed.

Uncertainties of the Pressure Gradient Method.—It has been stated that the pressure-gradient method is a more exact analysis than the short-path method, of the possibility of failure by flow through the foundation material. The pressure-gradient conception makes a striking impression, particularly when combined with the beautiful solutions that are possible of simple cases with the aid of the electric analogy and models, and it is, therefore, not surprising that it has had a great effect on the thought of many engineers. A deeper study of it, however, will show that it possesses serious limitations.

One of the greatest objections is that mentioned by Mr. Streiff, that the velocity gradients determined from the flow net do not hold when they are sufficiently large to produce turbulent flow. Another serious objection is that the velocity-gradient method does not analyze a condition that causes failure, but only a condition that may lead to failure. Assuming a perfect contact of the dam with its foundation, it is true, of course, that if the velocity gradient at the point of exit from beneath the dam is insufficient to move any of the material, a failure will not occur; but by no means does it follow that if movement does take place, the structure will fail. The experiments[83] by C. A. Mockmore, M. Am. Soc. C. E., and John W. Dougherty, Jun. Am. Soc. C. E., show that when the material is non-uniform, the fine material may be carried away, leaving the coarse material behind. This has been confirmed by other experiments. Such an action would tend to produce a reverse filter, through which the water could escape harmlessly. However, should all the material within the zone in which the pressure gradient exceeds the theoretical value necessary to cause motion be moved out, and a new boundary of the foundation material be thus formed (giving rise to a new shape of flow net which, in turn, might have areas in which the gradient was still sufficient to move out more material), it does not follow that this process would necessarily continue until failure resulted. Consider, for example, Case *B* mentioned by Professor Casagrande. Suppose piping had continued until a large quantity of material had been removed, as shown in Fig. 16. The conditions would then resemble that in his Case *A*, which he regards as safe. It is true that the conditions would be worse than in Case *A*, but the factor of safety is 4.6 and it is unlikely that this margin would be overcome. It is evident, therefore, that more study should be given to conditions that follow the initial movement, to determine under what conditions such movement would result in failure and what others would produce an eventual stabilization.

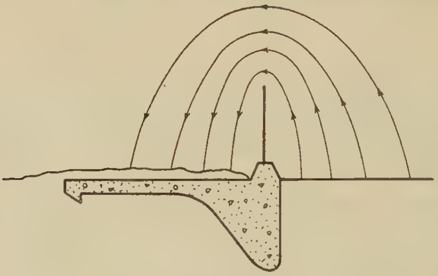


FIG. 16.

---

[83] See p. 1402.

Another of the uncertainties of the pressure-gradient analysis is that it has not been shown to act on actual dams, as the ordinary means of analysis indicate that it should act.

The study by Mr. Hebert covered all the data available at the time, on pressure under actual dams. On account of the uncertainty as to where the path of seepage should be considered to end, the results were determined for the various possible assumptions, even if they were improbable. Thus, in the right-hand diagram of Fig. 5, Mr. Hebert considered the path to end at the down-stream edge of the solid apron and neglected entirely the effect of 83 ft of paving in large solid slabs, with joints that allowed a certain amount of settlement. That these slabs influenced the flow of water is proved by the fact that they caused a tendency of the sand to be forced out through the cracks. It would be unreasonable to neglect these slabs in computing upward pressure, as a basis for design. The purpose of using the assumption in Mr. Hebert's work was to show that no assumption with even a remote justification would cause an agreement between observed and computed pressures.

The principal conclusion which may be drawn from Mr. Hebert's studies is that upward pressure is extremely variable, and apparently depends upon such a great number of factors that, with present knowledge, it cannot be predicted closely. None of the common methods agrees with actual results closely. The results, however, show very little to support the idea of a high-pressure gradient at the down-stream end of the dam, which the electric analogy indicates and which Mr. Harza proposed as the proper basis for design. In more than half the dams there was no evidence of this high gradient, and in half of those that did show a high gradient it could be ascribed just as reasonably to some other cause (such as greater impermeability of the surface material or the existence of a flexible apron) as to the cause described by Mr. Harza. It would seem, therefore, that there should be more proof of the existence of the high-pressure gradient at the down-stream edge of actual dams and that the factors affecting it should be more fully known before it can be used as a safe basis for design.

Another uncertainty of the pressure-gradient theory is the extreme variability of the premeability of earth materials and the difficulty in determining this variation in the foundation for a dam. The range in qualities of most materials that engineers encounter is not more than 1 000% from lowest to highest; but the range of permeability in earth material is more than 1 000 000 000 per cent. The following list of representative values of the permeability coefficient, was compiled mostly by S. P. Wing, Assoc. M. Am. C. E.:

Material	Coefficient[84]
River sand	0.041 to 0.266
Dune sand	0.0185
Beach sand	0.0089 to 0.0216
Undisturbed soil, fine sand to gravel	0.0163 to 0.316
Clay	0.000000023 to 0.000023

---

[84] In cubic centimeters per second per square centimeter for 1-cm thickness and 1-cm head.

This list shows the tremendous range of permeability of common foundation material. Moreover, it is not at all uncommon to find, side by side in a foundation, layers of river sand and clay, with permeabilities in the ratio of 1:100 000. According to Slichter, a decrease in grain size from 0.35 to 0.25 mm would increase the resistance to flow about 100% and a change of voids from 40 to 30% would increase the resistance 150 to 250 per cent.

In addition to the extremes of permeability in natural material, there is the difficulty of determining the value of the permeability of the material in place. This may be done by testing small samples removed by drill holes without disturbing them, or by driving pipes and pouring down, or pumping out, water. Mr. Burroughs rightly stresses the desirability of extensive borings as an aid in accurate design, but even the most extensive borings that could reasonably be expected would give only an imperfect picture of the permeability distribution beneath a dam; and, in most cases, the designer must be content with far less.

Even if the permeability distribution were perfectly known, there is no accurate method of determining the pressure gradient if the material is variable. There is no question that if one could duplicate perfectly, in the conducting medium of the electric analogy, the permeability conditions of the prototype the results would be practically perfect. The difficulty is the practical one of duplicating the conditions. An analysis in two dimensions could be made, with some difficulty, but the flow beneath a dam is in a three-dimensional medium. The results of observations on the Panjnad Weir, for exampde, shows pressure distributions that are highly improbable except in a three-dimensional medium. As far as known no practical method has been devised of simulating a highly variable material in three dimensions for use in the electric analogy, and it would be very difficult, if not practically impossible, to do so in models.

Another uncertainty in regard to the application of the pressure-gradient theory is the factors of safety that should be applied. The only one of those advocating the pressure-gradient analysis as the sole criterion for design, who suggested factors that could be used in design, was Professor Casagrande. He suggested a factor of safety of at least 3 for homogeneous soil under favorable conditions, ranging to at least 10 for insufficiently explored foundations or where stratified or irregular deposits are present, and still larger values for very uncertain foundation conditions. Since no information is given as to how these values were determined, it is concluded that they were not based on analysis of existing structures, or at least on no large number of them. In fact, at the present time, it would probably be impossible to determine such values from more than a few existing structures because the number of structures on which the foundation exploration was sufficiently comprehensive is probably very small.

Another reason for questioning the reliability of the pressure-gradient theory as the sole method of analysis for masonry dams on earth foundations is that it indicates no difference in results for various classes of material. The pressure gradient is the same for a dam of a given shape whether founded on the finest silt or on a coarse gravel, provided both are homogeneous, and that the flow through them is laminar. Experience has certainly shown, however, that it is more difficult to maintain a dam on a very fine, than on a coarse, material. Greater factors of safety could be used, no doubt, for fine material with the pressure-gradient theory as well as with the imperfect contact theory, but there is nothing in the pressure-gradient theory to indicate that they are necessary.

Still another reason for questioning pressure-gradient analysis as the sole solution is that dams with sand boils have stood without failure and, in some cases, quicksand conditions have been observed below structures which have stood many years. As the velocity-gradient method of analysis is presented by its proponents, this condition should bring about failure.

Reliability of Conclusions Based on Models, or Electric-Analogy Experiments.—There can be no question that models and the electric analogy have their place in many fields of hydraulic design. They are excellent tools for the situations in which they are applicable; but they have their limitations and these must be kept in mind when they are used. Hydraulic models, for example, can be relied upon to give very close results in certain fields; in other fields, they cannot be expected to give more than approximate answers; and, for still others, the best that can be said is that they give results closer than any other known method.

The "elegant" manner in which the electric analogy yields the solution for certain relatively complex problems has led some engineers to overlook the differences between the conditions that the analogy set-up represents and the actual conditions at a dam. It is not uncommon to try to make electric-analogy analyses for dams on variable material, of little known composition, using a homogeneous medium such as tinfoil or a salt solution. The extreme variability of foundation materials has been mentioned previously. One would not think of determining the strength of a beam composed of a mixture of unknown distribution of material, varying in strength from that of steel to that of cheese, by testing a beam of uniform material; but the permeability of a dam foundation may have as great a range as the strength of materials that are as dissimilar as steel and cheese. The analogy gives the answer for a homogeneous medium, but how close this approaches the results on an actual structure depends upon how closely the conditions in the analogy set-up reflect those in the actual structure. The writer believes in, and advocates the use of, the electric analogy and models; but he insists that they must be used with a proper respect for their limitations.

He has had considerable experience with experiments on the flow through granular material and beneath or through model structures, and he finds them to be difficult to manipulate so as to secure satisfactory results. By using a homogeneous material, placed with the greatest care to secure uniformity, with constant temperature to prevent variations of the viscosity of the water, and boiled water to prevent the giving up of entrained air within the material, it is possible to obtain results with a reasonable degree of consistency; but no where in Nature are these conditions duplicated.

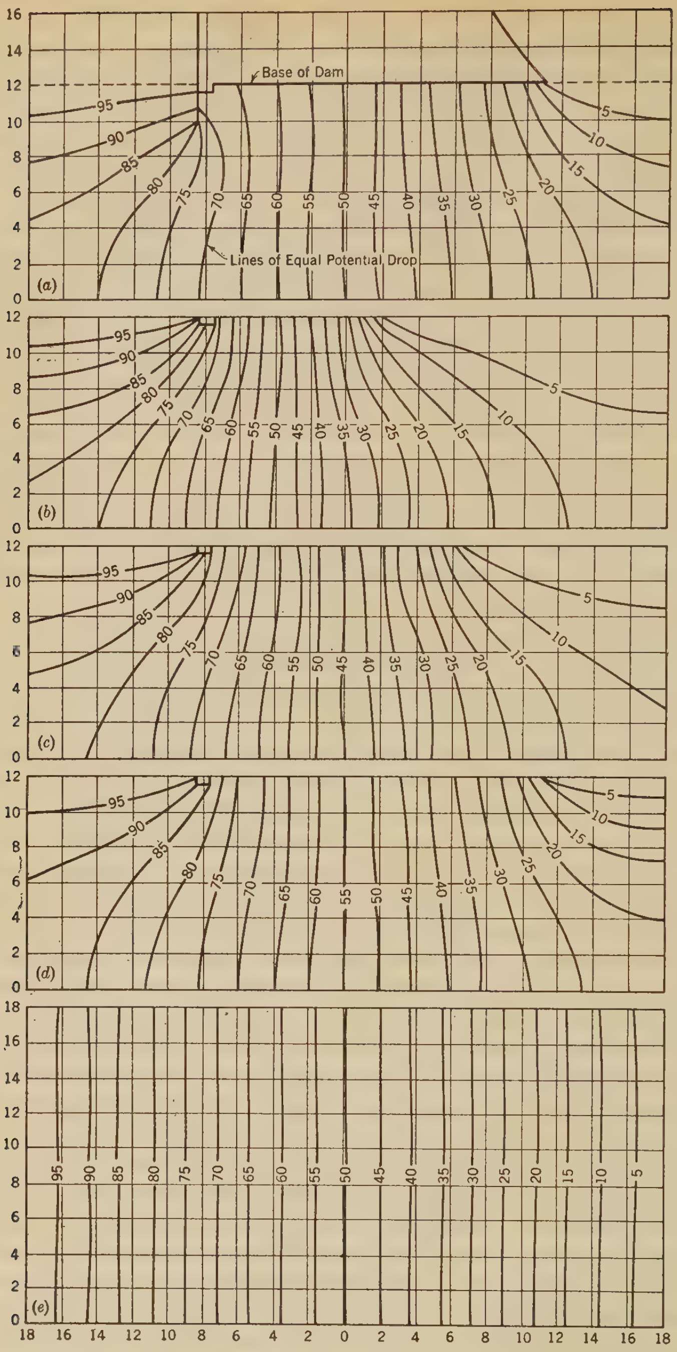


Fig. 17.—The Electric Analogy.

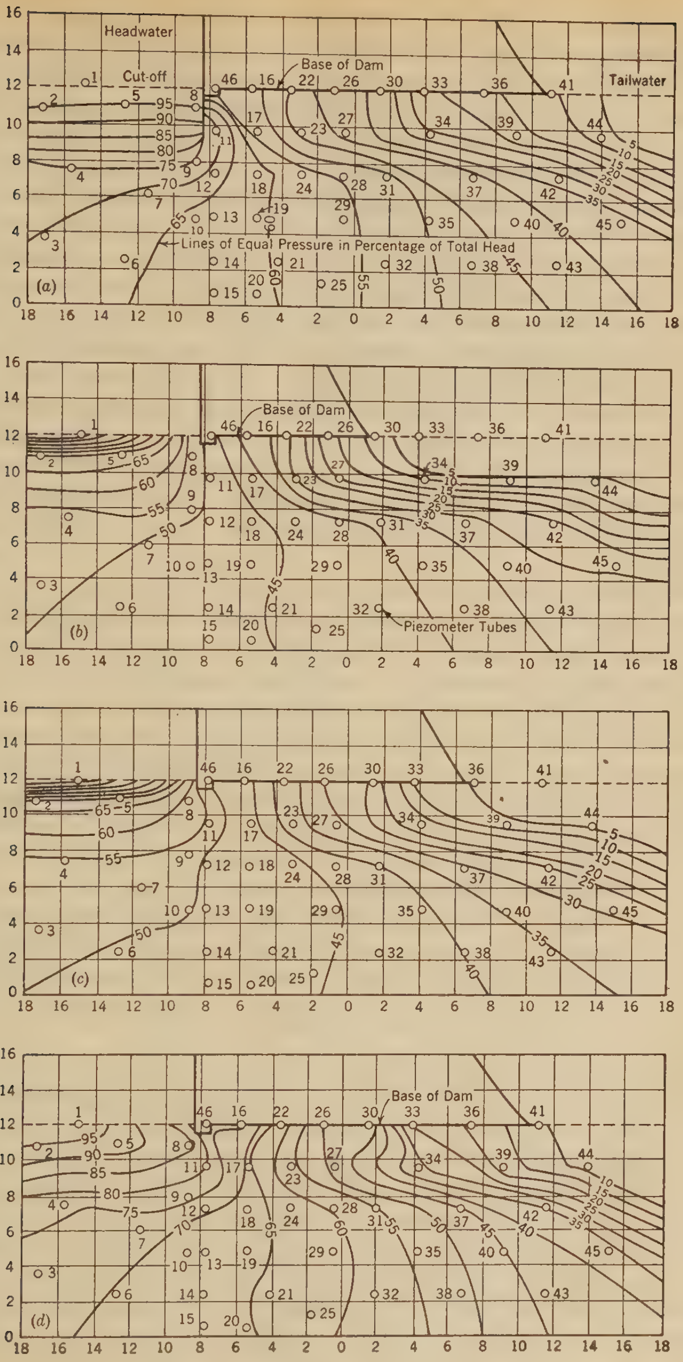


FIG. 18.—COLMAN'S MODEL DAM ON SAND.

An examination of the data submitted by Mr. Khosla does not indicate the close agreement that the statement, that the results on the prototype have been reproduced to within 4% on the scale model, would lead one to expect. The tables show several pipes in which the difference was much greater than 4%, but these are dismissed with the note that for these pipes the model and prototype differ widely throughout. It is true that the results on these pipes cannot be explained if two-dimensional flow is assumed, but they are entirely possible with three-dimensional flow, and an examination of the original records fails to show any reason to believe that the pressures indicated did not exist under the dam at those points. The agreement also is somewhat fortuitous, since the model and electric-analogy results were secured with a homogeneous medium and the dam rested on a foundation which increased materially in permeability with depth, as is proved by the nature of the flow net developed from the observed pressures, and indicated by borings made in the vicinity. The fact that the model and electric-analogy give results with such close agreement proves only that the high-class technique used had produced a uniformity of permeability within the model soil closely comparable to the uniformity of the electrical resistance which existed in the salt solution of the electric analogy.

To illustrate the wide difference that may be obtained, even under relatively favorable conditions, between the electric analogy and the flow of water through actual material, a comparison is afforded by Figs. 17 and 18 between the results obtained by Colman[85] and those obtained with the electric-analogy apparatus set up to represent what was believed to be similar conditions. Colman's results were obtained on a very large model using screened sand for the material, presumably placed with a reasonable amount of care, but not with great refinements. This was believed to represent a reasonably homogeneous medium, comparable to the homogeneity of the salt water used in the electric analogy. The comparison shows four different conditions of depth of sheet-piling and length of base. In Fig. 17(*e*) is also shown a check on the reliability of the electric analogy, obtained by placing terminals at both ends of the rectangular tray of the electric-analogy apparatus. A perfect check would have produced a series of straight, parallel, equally spaced lines. The results obtained approximate this ideal very closely.

The difference in the pattern of the pressure lines as given by the analogy and the model show the wide difference that can result even under the relatively favorable conditions of a laboratory, unless extreme precautions are used. If such a wide difference is secured under these conditions the dissimilarity that would probably result from the highly variable conditions which ordinarily occur in Nature may be readily visualized. It is interesting also to note in passing that although the velocity gradient at the downstream edge of this model was theoretically infinite, no tendency to blow out was recorded, even for cases in which the head acting on the model using a horizontal floor without sheet-piling exceeded the length of the floor.

---

[85] *Transactions*, Am. Soc. C. E., Vol. LXXX (1916), p. 421.

In using models or analogies, therefore, one should always keep in mind that their action in itself is of little importance; it only becomes important if it represents, accurately, the action of the prototype and hence their value depends entirely upon how faithfully they duplicate prototype conditions. They are of great value in showing what might happen in the prototype, but do not always show what does happen. To conclude from models or the analogy, for example, that there is no possibility of imperfect contact of a dam and its foundations because models or the analogy simulating a perfect contact do not show it, seems an unwarranted conclusion.

The results of Mr. Hebert's studies show that in the case of most of the dams on which upward pressure was observed, there was a considerable reduction of head near the upper end of the dam, which was probably due to the formation of a filter skin or layer of silt on the reservoir bottom up stream from the dam. This has been observed in many cases of dams and is a fruitful source of trouble in model experiments.[86] This formation is another source of difference between the prototype and model or analogy results. Fortunately, this results in a greater factor of safety than would be determined by model or analogy tests, but may lead to an unnecessarily large one.

Mr. Khosla has shown[87] that a variation in pressure should take place beneath dams due to a change of temperature of the water in the stream, and this has been observed on actual dams.[88] Beneath actual dams, however, there are surprising changes of pressure, which can scarcely be caused by temperature or silt. The change of pressure beneath the Mio Dam mentioned by Mr. Streiff is an example. The extent of the variation of these pressures under the Deoha Barrage is shown in Fig. 19. As the lines in this diagram represent the pressure loss to the point of observation in percentage of the total drop they should be straight lines if the ordinary assumptions of the velocity-gradient analysis hold.

Single Versus Double Rows of Sheet-Piling.—Messrs. Ayres and Griffith contend that ample safety is given by a single row of sheet-piling. Although no doubt there are cases in which it is impracticable to use more than one row of sheet-piling, the writer believes that failure from defects in a single row of piling have been frequent enough to justify the use of two rows wherever practicable. The probability of defects occurring in both rows of piling close enough together to cause failure is very remote. Of course, in the case of a defect in one line of piling, the other would be subjected to conditions much more severe than contemplated in the design, but this would almost certainly be taken care of by the factor of safety. The second row should be located far enough up stream so that the downward pressure of the dam would exceed the upward pressure and, therefore, uplift would not be important. It should not be close enough to the up-stream piling, however, to make short-path conditions control. Down-stream sheet-piling should rarely, if ever, be omitted from over-fall dams, as it is very useful

---

[86] *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), pp. 86 and 90.

[87] "Observation and Record of Pressures below Works on Permeable Foundations," Central Board of Irrig. (India), *Publication No. 8*.

[88] *Transactions*, Am. Soc. C. E., Vol. 88 (1925), p. 1288.

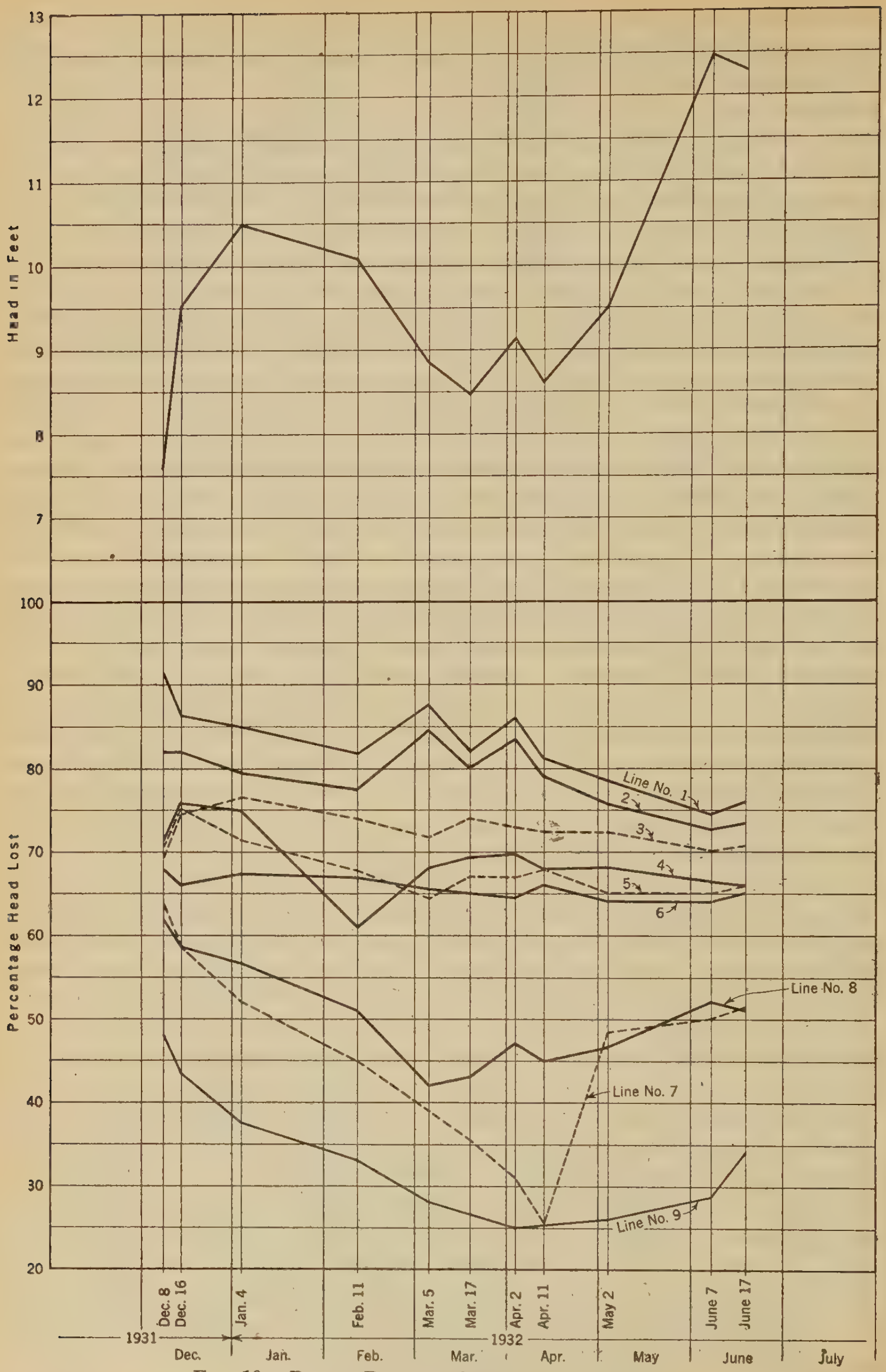


FIG. 19.—DEOHA BARRAGE OBSERVATION (MEANS).

as protection against under-cutting of the apron. It should be thoroughly vented through the piling at frequent intervals, however (preferably by a filter running just beneath the dam inside the piling), to reduce the upward pressure beneath the apron.

Blanket Up Stream From a Dam.—Messrs. Streiff, Burroughs, Ayres, and Davis mention the value of an up-stream impervious earth blanket in reducing the probability of piping beneath a dam. In many cases the use of this device is the most economical manner of securing protection. The writer, however, would not rely on such a blanket to the extent of dispensing entirely with cut-off walls. One danger with an impervious up-stream earth blanket is the difficulty of making an absolutely reliable contact between it and the dam. If the blanket material is merely piled against the up-stream face of the dam, the contact of the soil blanket with the face of the dam constitutes the line of creep. That failure may take place by percolation along such contact is shown in the case of the Corpus Christi Dam and at least one other similar case.

The natural silting above the dam greatly decreases the percolation, as shown by Mr. Eiffert, and thus increases its stability. Since the silting is not effective when the dam is first put into service it cannot be counted upon unless the filling goes on slowly enough to permit silting to occur before the full head is applied. In certain cases artificial silting might be used as the reservoir is filling. Silting often results in a material reduction of under-pressure on a dam. After some time the upward pressure beneath the Hamilton Dam, mentioned by Mr. Eiffert, was practically eliminated by silting.[89] At the Island Park Dam some distance up stream in the same river, the upward pressure from the river was entirely cut off, and the remaining pressure was due to ground-water, as was proved by the much lower temperature of the water.

Corrugations.—Mr. Harza advocates scoring the foundation prior to placing the concrete to produce a "corrugated foundation" on the under side of the concrete floor as a protection against roofing. Mr. Davis recommends a serrated foundation, in which the serrations are larger than those suggested by Mr. Harza, but the principle is the same. In both cases the dam, pressing down on the earth of the foundation would be more sure of a close contact. It is believed that both these suggestions have merit. The adoption of the 45° dividing line between horizontal and vertical creep was, of necessity, somewhat arbitrary. Lesser slopes would probably be better than horizontal surfaces but not so good as vertical contacts.

Weep-Holes and Drains.—Mr. Streiff's determination of the effect of weep-holes in reducing upward pressure is interesting. For flow only through the foundation material which his solution assumes, it demonstrates that little reliance can be placed on reduction of uplift from such weep-holes. The reduction might be increased by placing an inverted filter or pocket of gravel of considerable radius beneath the weep-hole, but even with these the results would probably be disappointing. The pressure reduction which would occur in the case of imperfect contact of the dam with its foundation would not be so easily analyzed, but would probably not be less than that given by Mr. Streiff's solution. It may be concluded, therefore, that careful study should be made before a reduction of under-pressure by means of weep-holes could be counted on. It does not follow from the foregoing, however, that weep-holes backed by gravel or a reverse filter would not be valuable in permitting the escape of the water flowing beneath a dam without removing the foundation material, and thus adding to the safety of the dam.

---

[89] *Transactions*, Am. Soc. C. E., Vol. 93 (1929), p. 1577.

Miscellaneous Remarks.—Mr. Davis has objected to the use of a value of 6 for medium sand, for example (suggested by the writer), on the grounds that there is a wide variation of properties within the range of that grade of sand. The effect would not be as great as Mr. Davis' values indicate, since he has not taken into consideration the fact that the 0.45-mm sand would have a higher competent velocity than the 0.25-mm size. Some thought was given by the writer as to the advisability of specifying the weighted-creep ratio as a function of grain-size dimension, but it is doubtful whether the present knowledge of safe values is close enough to justify such a refinement.

The practice of predicting the downward pressure on an apron by observing the water-surface profile on a model for a variety of conditions was mentioned by Dr. Taylor. A much more reliable procedure is to insert piezometers in the floor of the model and observe the actual pressure. It will be found that the pressure on the apron is less than that indicated by the surface of the jump, due to the inclusion of air in the water of the jump.

Mr. Potter has mentioned the desirability of more descriptions of actual dam failures. The complete paper, a copy of which is filed in the Engineering Societies Library, gives descriptions of many failures of dams of this type, as well as descriptions of dams with low-creep ratios which did not fail. Space limitations prevented their inclusion in the paper.

Summary.—The writer views this paper as being in the nature of a progress report on methods of obtaining the security of masonry dams on earth foundations against failure from under-seepage or piping. The position of a research engineer for an organization engaged in actual construction differs from that of one engaged in pure research, in that the former, if his work is to be justified on the project on which he is engaged, must reach results that are an improvement over preceding methods and can be applied to the project under consideration. He cannot, like the pure research engineer, be content as long as he makes progress in a forward direction without having the results immediately applicable to actual construction. No claim is made, therefore, that the views and results given represent the last word in this field. Undoubtedly, further progress will be made, and the writer hopes to be able to have a part in it. It would be unfortunate if the next decade produces as little progress as that immediately following the work of Bligh. Nevertheless, is is believed that this paper does form a step forward. The results in the paper and those presented in the discussions will be of considerable help in designing such structures in the immediate future, and will form a step in the progress toward the final solution of this problem.

The building of dams of the type under consideration will not be stopped by lack of a complete knowledge of the science of them. They will continue to be built with the best knowledge available. Except for the special case of no horizontal creep and the factors of safety given by Professor Casagrande (which are probably not based upon any analysis of actual structures) the discussions brought out no definite recommendations for proportioning structures which the designing engineer can use in his work. Therefore, the writer's recommendations will find considerable application until better ones are presented. Practically the same objections that were made to the writer's paper were made in a cruder form to the work of Bligh,[90] and an analysis of flow through the material beneath a dam was presented by Parker[91] prior to 1916; but these developments did not prevent the extensive use of Bligh's recommendations.

This paper demonstrates that masonry dams on earth foundations can fail either by seepage through the foundation material, or along the line of contact of the dam with its foundation, but that an analysis of actual dam failures proves that for dams of any reasonable section the latter is much the more likely form of failure. It is regretted that in maintaining this position the writer appears, at first, to be advocating a rule-of-thumb method against the more scientific appearing flow-net analysis, but experience with actual structures justifies it. However, dogmatism should be avoided in order that future progress may not be impeded, because it is possible that: (*a*) Should further experience show that lower weighted-creep values were unquestionably safe; (*b*) should greater weight be given to horizontal creep; or (*c*) should some method be found of insuring perfect contact of a dam with its foundation, the flow through the foundation would become more vital.

In connection with the design of the foundations of the Imperial Dam, on the Colorado River, by the U. S. Bureau of Reclamation, all methods of approach were used. Not only the weighted-creep theory, but also models and the electric analogy were used. The writer is in hearty accord with this use of all possible avenues of enlightenment, in order that all the facts bearing on the subject may come to light.

On one point it is believed that there will be unanimous agreement, that is, the desirability of more measurements of upward pressure on actual dams. Although actual failures are the most instructive, they are rare and frequently cannot be analyzed with certainty. Upward pressure pipes, however, can be placed with little expense in almost every dam. Future progress will probably be dependent largely upon the data obtained from this source and, therefore, it is strongly urged that all those who have charge of the design of such dams provide them with adequate devices for measuring upward pressure. In addition to furnishing scientific data, they will give valuable indications of the condition of the structure.

In closing, the writer wishes to express his appreciation to the authorities of the U. S. Bureau of Reclamation for their permission to publish the results of these studies.

---

[90] *Engineering News*, Vol. 65, January 26, 1911, p. 109.

[91] "Control of Water," by Philip A. Morley Parker, M. Am. Soc. C. E., 1916 Edition, p. 292.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

TRANSACTIONS

Paper No. 1920

UPLIFT AND SEEPAGE UNDER DAMS ON SAND

BY L. F. HARZA[1], M. AM. SOC. C. E.

WITH DISCUSSION BY MESSRS. EDWARD GODFREY, HIBBERT M. HILL, JOEL D. JUSTIN, CHARLES TERZAGHI, HARRY H. HATCH, DONALD J. HEBERT, C. A. MOCKMORE AND JOHN W. DOUGHERTY, AND L. F. HARZA.

SYNOPSIS

The basic principles of flow under dams on sand are presented briefly in this paper in the hope of stimulating scientific study of this much neglected subject. Analytical methods applicable to some familiar types of foundations, and graphical and electric analogy methods of general application are described, by which can be determined the theoretical laws governing: (1) The hydrostatic pressure along the foundation contact; (2) the hydraulic gradient with which the water escapes upward at the toe; and (3) the approximate leakage under the structure. These theoretical laws are to be used as scientific nuclei around which to build future experimental data and field observations to the end that the order of magnitude of field variations from scientific principles and the safe coefficients or factors of safety for practical applications may be determined.

INTRODUCTION

The problems encountered in the design of a masonry dam on a sand foundation are:

(1) To provide safe conduct for seepage water under and around it, so that neither foundation nor bank material will be removed.

(2) To reduce seepage losses under and around it to a reasonable and economic minimum, consistent with construction economy and the value of wasted water, safe conduct being assumed under Problem (1).

NOTE.—Published in September, 1934, *Proceedings*.

[1] Cons. Engr., and Pres., Harza Eng. Co., Chicago, Ill.

(3) To obtain stability of structures under all conditions of loading, uplift, overflow, buoyancy, tail-water fluctuation, etc.

(4) To design the structure so as to distribute the load and to minimize settlement, and to permit it to accommodate itself to unavoidable settlement so as to prevent internal stresses and cracking.

(5) To destroy the energy of waste water at the toe of the spillway in such a manner as to avoid injury to the apron and to minimize the erosion of stream bed and banks.

(6) To anticipate, in the design of all structures, the possible limit of degradation of river channel below the dam and consequent subsidence of tail-water level as affecting (*a*) undermining and stability of structures; (*b*) functioning of spillway; and (*c*) loss of draft head (if hydro-electric).

Problems (5) and (6) are related only to spillway structures and will not be considered herein because of prescribed space limitations of this paper Problems (1) to (4) require a knowledge of: (1) Uplift pressure; (2) upward escape gradient at the toe; and (3) seepage volume. This paper will be confined to a treatment of the fundamental principles of determining these conditions, indicating methods of investigation and study, but for the most part omitting illustrative examples of their application.

Bligh's Empirical Coefficient.—Modern practice in the design of dams on sand was developed to a great extent from experience in India, as elucidated by Mr. W. G. Bligh[2]. His empirical coefficient, C, now generally called the "percolation factor" or "coefficient", is the ratio of the percolation distance of the water passing under the dam to the applied head. It is thus the reciprocal of the average "hydraulic gradient". For four classes of material, Bligh recommends the percolation factors given in Table 1. As the percolation distance, Bligh uses what is sometimes called the "line of creep", defined as the total length of contact between structure and sand, from head-water to tail-water. This assumes that seepage will follow along the bottom of a structure until sheeting or some other cut-off wall is encountered; thence downward along and under this cut-off; thence upward on the down-stream side of the cut-off and again along the bottom of the structure. In lieu of the "line of creep", the "short path" distance of flow is sometimes used, which defines itself.

TABLE 1.—RECOMMENDED PERCOLATION FACTORS

Class	Description of material	Coefficient, C
1.......	River beds of light silt and mud, such as that of the Nile..................	18
2.......	Fine micaceous sand, as in the Himalayan rivers and in such rivers as the Colorado in the United States..	15
3	Coarse-grained sands, as in Central and South India (this is the most common type)	12
4.......	Boulders or shingle and gravel and sand mixed.........................	9 to 5*

*Varies

The Bligh coefficients were based upon meager data as to failures, and upon only superficial descriptions of the foundation materials. They were derived from observations of the behavior of low spillway weirs consisting frequently of loose rock-fill between longitudinal dividing walls. This type of weir was scarcely more than a well-paved, sloping stream bed where the erosion of the dam and the stream bed was perhaps the primary consideration. These coefficients have since been broadly applied to non-overflow and massive types for which they were not intended. The reliance which has thus been placed upon them is an indication of the primitive state of the art and of the tendency of the profession to lean upon precedent and rules after they have once been codified, as by Bligh. The writer feels that this, perhaps, too definite statement of coefficients has long served as a deterrent to further progress in the study of this problem. The man who does not use what others have used before him must accept more responsibility as the price of his initiative.

---

[2] "The Practical Design of Irrigation Works", by W. G. Bligh, D. Van Nostrand Co.

The writer hopes to reveal some basic principles which may help to establish a more scientific approach to the problem. He is fully aware that, if such approach is based upon the assumption of homogeneous foundation material, the conclusions may not be closely paralleled in practice. If based upon the best available data as to variations in material, they will at best be only rough approximations because of the difficulty and cost (if not impossibility) of obtaining accurate knowledge by borings, wells, pumping tests, and other available methods, of complex natural deposits. Nevertheless, a knowledge of the laws of underflow in uniform material is valuable in revealing basic principles. Such knowledge will form a basis for intelligent modification by the choice of a factor of safety dependent upon the degree of non-uniformity, and upon the extent of uncertainty as to existing conditions. Before attempting, however, to develop a rational rather than a purely empirical basis for the design of dams on sand, it is necessary to inquire rather minutely into the principles of the hydrodynamics of soil, and the forces exerted by seeping water.

Darcy's Law.—The principle stated by Darcy[8] that the velocity of seepage is proportional to the hydraulic gradient has been long the fundamental basis of attack, many times verified, for all types of granular material. Thus, with a quantity, q, flowing through a gross sectional area, a, with net effective velocity, v (applied to the gross section), through a length of path, l, under a head, h:

$$v = K \frac{h}{l} \qquad \text{(1)}$$

and,

$$q = av = aK \frac{h}{l} \qquad \text{(2)}$$

In Equation (2), K is the net effective velocity of seepage (as through the gross section rather than the pore section) through a volume of the material of unit length subjected to unit head; or it may be expressed as the velocity under a hydraulic gradient of unity, and is known as the "transmission constant". The constant, K, is dependent upon temperature, effective size and shape of grains, density, type of packing, and porosity. It has been investigated by the late Allen Hazen,[4] M. Am. Soc. C. E., and by Professor C. S. Slichter.[5]

---

[8] "Les Fontaines Publiques de la Ville de Dijon", par H. Darcy, 1856, p. 590.

General Principle of Pressure Due to Flow

Flow through a granular material can only occur as a result of, and in the direction of, a decrease of hydrostatic head as between two points in the material; and such decrease in head along the path of flow exerts a force upon this material, in the direction of flow, in the form of a higher pressure on the approaching side than on the receding side of each grain of material. Obviously, if the hydraulic gradient is unity the differential pressure on each particle will be equal to its displacement. For greater or smaller gradients the pressure exerted on each particle is proportionately greater or less than the displacement. The action is also complicated and increased by hydraulic friction and is of sufficient importance to analyze in detail. Consider some simple examples to illustrate the principle.

Pressure on Bottom of a Cylindrical Vessel.—Let s = specific gravity of sand grains = about 2.65 average value; w = unit weight of water; P = percentage voids in the material; $\Delta = w\ (1 - P)$ = displacement of one cubic unit of material; $w\ s$ = weight of one cubic unit of solid material; $w_s = w\ (s - 1)$ = submerged or buoyed unit weight of solid material; and $w_l = w\ (s - 1\ )\ (1 - P)$ = buoyed unit weight of loose material.

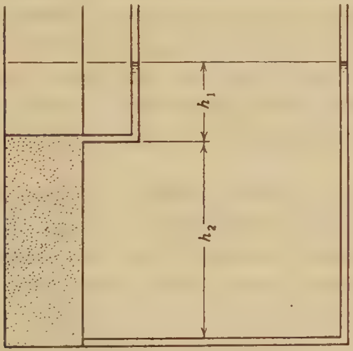


FIG. 1.

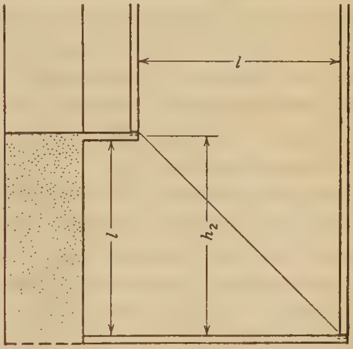


FIG. 2.

If sand and water are placed in a cylindrical vessel with a solid bottom, the pressure, p, on the bottom will be: (1) The independent weight of the contents; or it may be conceived as produced by, and equivalent to, (2) that of hydrostatic pressure transmitted undiminished through the sand and acting over the entire bottom area, plus the submerged or buoyed weight of sand. These conceptions can be expressed by the following algebraically equivalent equations, respectively (see Fig. 1):

$$p = wah_1 + waPh_2 + wah_2s\ (1 - P) \quad \text{(3)}$$

and,

$$p = wa\ (h_1 + h_2) + wah_2\ (s - 1)\ (1 - P) \quad \text{(4)}$$

---

[4] Rept., Mass. State Board of Health, 1892.

[5] *Water Supply and Irrigation Papers Nos. 67* and *140*, U. S. Geological Survey.

Cylinder with Screened Bottom.—If the bottom of the cylinder is screened instead of solid, one assumes water to be supplied at the top of a cylinder at the same rate as it escapes by seepage from the bottom, thus maintaining constant conditions. It is obvious that the load on the screened bottom must remain equal to the contents of the cylinder as on the solid bottom; and yet no part of this load is transmitted to the bottom by static hydraulic pressure as before. It is exerted entirely through the solid material, the water acting only to increase the effective weight of the particles and thus to increase the pressure on the screen of the bottom-most particles in contact with the screen.

Thus, Fig. 1 represents the solid bottom with piezometers both standing level, each sand grain being acted upon by buoyancy. Fig. 2 shows a screened bottom with the entire head, h_2, lost in friction through the sand, and with $h_1 = 0$ (except for a slight film of water to prevent capillary action). Then, the loss of head is 1 ft per ft of sand column, or just enough to overcome buoyancy; that is, the hydraulic gradient is 1.0. There is no static water pressure anywhere in the column, and, therefore, the pressure on the top and bottom of each sand grain is zero, indicating that there is neither buoyancy nor downward static pressure acting on the sand grains. The sand load, therefore, is its own dry weight plus only the hydraulic friction of the seepage flow. This friction, which results from a gradient of unity, must, therefore, equal the weight of pore water, $= w\, a\, h_2\, P$, if the total pressure on the screen is to remain equal to the pressure on the solid bottom. This force seems to be partly of the same nature as the forward thrust of flowing water caused by friction with the wall of a pipe; or, it may be contributed partly by the excess-dynamic force on the approaching side necessary to deflect the flow around each particle, and the corresponding deficient pressure on the down-stream side.

It is now evident that the forces exerted by water flowing through sand are composed of three elements: (1) A static differential pressure on the approaching and receding faces of each grain, resulting from the reduction in pressure along the route of flow and acting in the direction of flow; (2) a frictional or dynamic force acting in the same direction; and (3) buoyancy, acting upward and unrelated to the flow. For a gradient of unity, the differential static heads on the approaching and receding faces of each particle differ by the mean thickness of the particle, and, therefore, the force exerted is equal in amount to the displacement of the particle. Furthermore, the frictional-dynamic force is equal to the weight of the pore water, as previously proved, and the total force, therefore, is equal to the weight of water necessary to fill the total volume of the cylinder. With the hydraulic gradient greater or less than unity, obviously, the differential static pressures on each grain will increase or decrease in like proportion, and to maintain the accuracy of Darcy's law the frictional-dynamic forces must also vary in like ratio, the two combining to form the "gradient pressure".

The conclusion is justified, therefore, that under all conditions, the increment force, in addition to buoyancy (or "gradient pressure"), exerted by seeping water against a volume of granular material will be the difference in hydraulic head at the approaching and receding faces of the volume applied to the entire cross-section, as on a solid, instead of granular, piston, although the pressure is actually applied gradually through the material instead of against an impervious face. The aforementioned principle can be demonstrated mathematically in a much briefer manner, but the foregoing dissected method is chosen to give a clear conception of what transpires in the pores of the material. A few illustrations will further clarify this point.

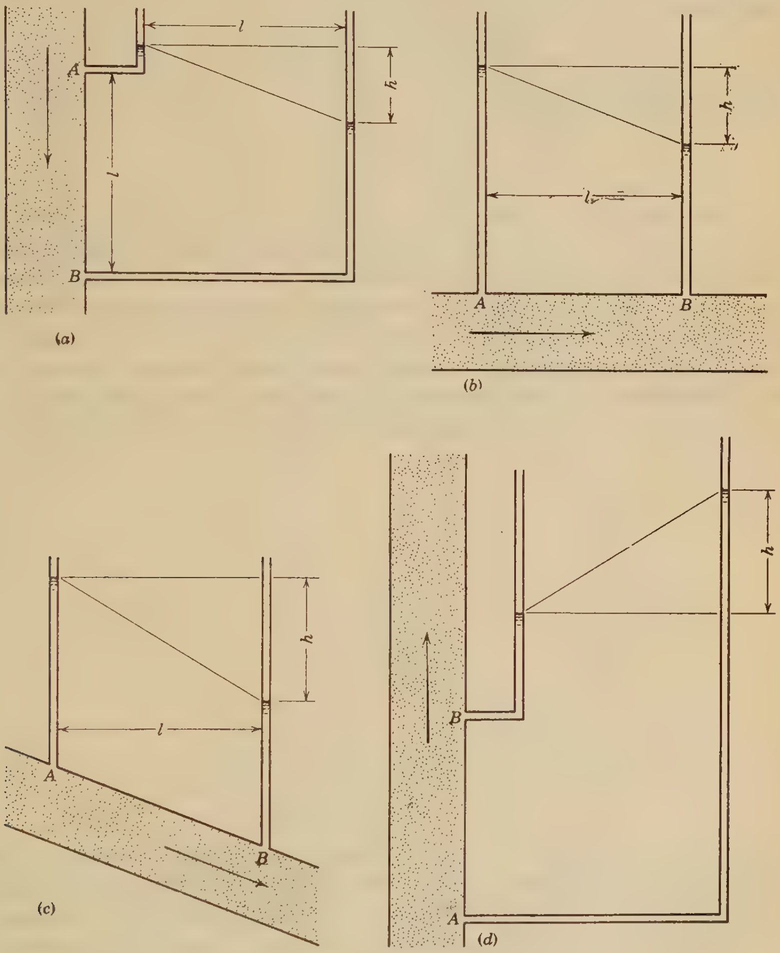


FIG. 3.—THE GENERAL CASE OF SEEPAGE THROUGH A SAND COLUMN.

In the general case (Fig. 3) of seepage through a sand column of Cross-Section *a,* the total differential force between Points *A* and *B* resulting from movement of the water will be *wah.* This must be added vectorially to the buoyant weight of sand, *wal* $(s - 1)$ $(1 - P)$, to obtain the total resultant force. In Fig. 3(*a*) the seepage force and buoyancy act opposite; in Fig. 3(*b*), normal to each other; in Fig. 3(*c*), at a variable angle; and in Fig. 3(*d*), in the same direction.

Quicksand, Sand Boils, Piping, "Flotation."—The most important case in its bearing upon the safety of a dam founded on sand is that of Fig. 3(*d*), in which water is flowing upward through the material. Here, the force of flow or "gradient pressure" combines with that of buoyancy to reduce the effective weight of the material and will actually float the material if the internal hydrostatic pressure of the seeping water becomes equal to, or greater than, the load of superimposed material. This critical condition occurs when:

$$w a h = w a l\,(1 - P)\,(s - 1) \qquad \text{(5)}$$

and the "'"flotation gradient" is:

$$F_c = \frac{h}{l} = (1 - P)\,(s - 1) \qquad \text{(6)}$$

Charles Terzaghi,[6] M. Am. Soc. C. E., has studied this phenomenon by platting a curve between the quantity of percolation upward through a cylinder of sand and the hydraulic gradient or "escape gradient" required to produce it. He finds a sudden increase in percolation when the critical or "flotation gradient" is reached (see Fig. 4) and explains it by calling attention to a sudden swelling of the sand into a more porous or "quick" condition. "Quicksand" is thus primarily a condition characterized by the flow of seepage water upward through the sand, rather than a characteristic of the material itself; and, yet, it is customary to associate the term, "quicksand", only with fine sand. This is the case because fine sand when once expanded with an excess of pore water and consequent voids, retains this condition for some time even with the removal of pressure because of the greater resistance to, and time required for, escape of the contained water through the small pores. Any granular material, such as coarse sand or gravel, may become "quick" with a sufficient supply of water, and under sufficient pressure, to satisfy the critical equation. The jetting of piles is merely the artificial creation of a "quick" condition to make the sand more fluid.

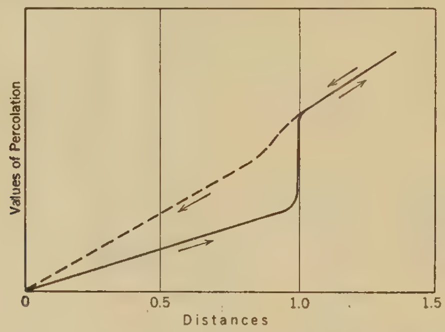


FIG. 4.—UPWARD GRADIENT.

---

[6] *Technologic Publication No. 215,* Am. Inst. of Min. and Metal. Engrs., p. 36.

Flotation at Toe of Dam.—Fig. 5 is a summary of the forces acting on a unit volume of sand along the route of seepage under a dam, exclusive of the load of the structure. In this diagram, G = the gradient or hydraulic thrust $= \frac{h}{l}$; W_s = the submerged weight of sand; and R = the resultant of G and W. It will be noted that the water entering the foundation sand on the up-stream side of the dam adds to the effective weight of the sand, and tends to compact it. This increases its resistance to seepage at entrance which is also increased by the formation of a filter skin. Under the middle of the dam the seepage causes a horizontal load on the sand while the buoyed weight is downward, and the resultant inclined. On the down-stream side of the dam the water rises to the surface and its gradient pressure tends to combine with the buoyancy to reduce the effective weight of the sand. From Fig. 5 it is evident that the sand is in a stable condition throughout the route of seepage, except as it rises to escape at the toe. Under critical conditions, therefore, there is no similarity to imminent pipe flow and the word, "piping", is not descriptive, but is actually misleading. The word, "flotation", is descriptive of this critical condition at the toe, and will be used herein. The "escape gradient" is vertical at the surface and tends always to reduce the effective weight of the material. If it reaches the critical value, or "flotation gradient", actual flotation results. The sand swells, causing an increase in porosity, and perhaps flows away with escap-

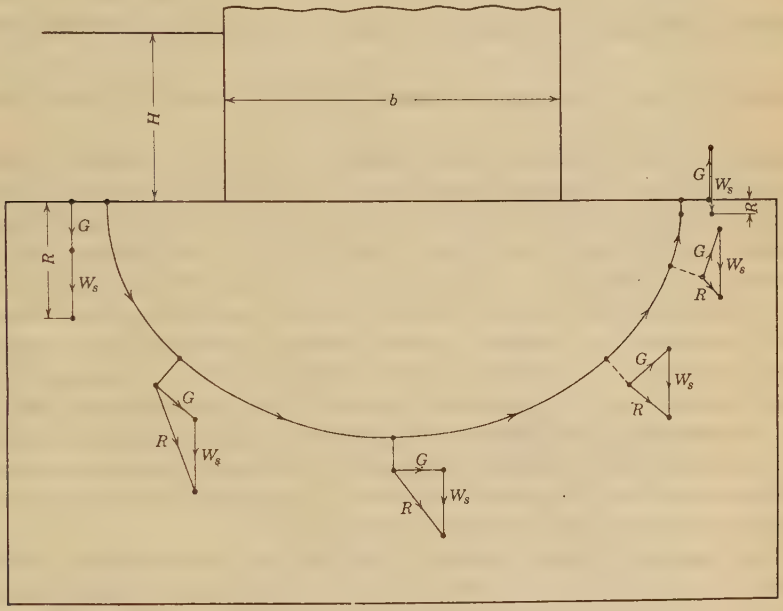


FIG. 5.—FORCES ACTING ON A UNIT VOLUME OF SAND ALONG THE ROUTE OF SEEPAGE UNDER A DAM.

ing water. Thereafter, the resistance to escape is further decreased which may further increase the flow and escape of sand. Thus, the sand may crumble back from the point of escape until a channel is progressively opened from tail-water back to head water under the dam. This result is evidently responsible for the word, "piping", and for the misconception of the nature of the phenomenon which the word implies. "Flotation" is the cause and "piping", the effect. If "flotation" is prevented, piping cannot occur. Therefore, a designer is concerned as to safety only with the conditions obtaining at the point of escape of the seepage water.

Uncertainties enter into the computation of the critical conditions of flotation at the toe because cross-flow may occur by convergence toward the easiest point of escape where the material is non-homogeneous, and the hydraulic gradient, $\frac{h}{l}$, at the point of escape is seldom equal to the average gradient, $\frac{H}{L}$, even in homogeneous material. Furthermore, if streaks, lenses, or strata of porous material are interbedded with finer material, the relative permeability may encourage concentration of flow from the finer into the coarser material near the outlet, giving the latter more than its proportionate quantity and a higher rate of loss of head at the point of escape. It is the function of the factor of safety, if not otherwise discovered and provided for, to allow for such conditions. This has been discussed by Professor Terzaghi under the subject of "Minor Geologic Details"[7], although it is often of major importance to the success of the structure. The existence of such "minor details" does not alter the origin of the failure as that of toe flotation. It does add to the difficulties of predicting the local flotation gradient. In fact, as stated by Professor Terzaghi[7], often such "features can be predicted neither from the results of careful investigations of a dam site nor by means of a reasonable amount of test borings".

Considered as a purely laboratory problem, the determination of critical or flotation gradient for a given material is reasonably definite once the characteristics of the material are known. In Professor Terzaghi's experiments on varied materials[8] the measured internal pressure to cause flotation always equalled or exceeded the theoretical, by ratios varying from 1.00 to 1.24. Thus, in general, the computed critical pressure apparently is on the safe side, and no factor of uncertainty need be introduced except to the extent that the relation of the escape gradient, $\frac{h}{l}$, to the known dimension of the structure is unknown and in so far as the characteristics of the sand and its stratification are unknown or incapable of interpretation in terms of the gradient at the point of escape of the water.

---

[7] *Technical Publication No. 213*, Am. Inst. of Min. and Metal. Engrs., p. 31.

[8] "Erdbaumechanik", p. 132.

CRITERIA OF DESIGN

The three conditions requiring determination in order to design or test a dam on sand are: (1) The uplift pressure under the base of the dam, as related to its weight; (2) the vertical escape gradient at the toe as related to the physical characteristics of the material; and (3) seepage volume, although not mathematically related to safety.

In all mathematical or experimental investigations of base uplift pressure or flotation gradient at the toe, in homogeneous material, the transmission constant, K (Equation (1)), cancels out. Neither Criterion (1) nor Criterion (2) is dependent upon quantity or velocity. Regardless of the size and porosity of the material, there is the same total head to dissipate in the same distance and, therefore, it will be dissipated at the same rate in any homogeneous material; the quantity of water that escapes will be directly proportional to the transmission constant; the water will escape with the same toe gradient and will cause the same uplift pressure under the base. Thus, in homogeneous material, the amount of leakage is largely an economic problem. Even a foundation of large boulders with no finer material might be safer with a large leakage than finer material with negligible leakage, or *vice versa* A method of estimating leakage will be introduced later.

Graphical Analysis.—Forchheimer has developed a very ingenious graphical method[9] of platting the lines of flow and equal pressure lines conveniently called a "flow net", illustrated in Fig. 6, and based upon flow in a plane normal to the axis of the dam. The same quantity of water is assumed to flow between each pair of consecutive flow lines. The velocity, therefore, must be inversely proportional to the spacing of these lines at each point. Since the drop in pressure is directly proportional to the velocity, the spacing of the equal pressure lines must everywhere bear a constant ratio to the spacing of the flow lines at that point. It is convenient to make the spacing of the two systems of lines equal and each figure an approximate square with the same average breadth and length.

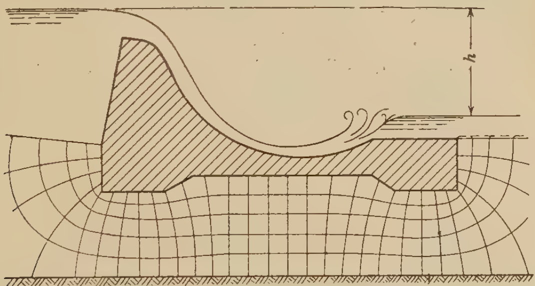


FIG. 6.—TYPICAL "FLOW NET" OR LINES OF FLOW AND EQUAL PRESSURE.

Since direction of flow is in the direction of drop of pressure, the lines must everywhere intersect at right angles. With these principles in mind the problem becomes purely geometrical, a process of successive approximations, or "trial and error". Obviously, the first flow line must coincide with the contact line of the structure with the sand. The last line must coincide, or must be parallel with, the rock line. On attempting to plat such a system of lines it will soon be discovered that they must be spaced closely at the protruding corners of the structure, and at the tip of the sheet-piling; otherwise, obvious distortion and reverse curvature and non-parallelism with the rock will result.

---

[9] "Hydraulik", von Ph. Forchheimer, 1930, p. 82.

Some practice will be required to make reasonably close assumptions as to the spacing of the first strip of squares adjacent to the structure. As the network is then built downward and outward, if inconsistencies develop, it is necessary to re-assume the spacing of the first row and proceed again until consistent curves are obtained. The process is a tedious one, but is mathematically sound within the limits of accuracy of the approximation of the small distorted figures to true squares. Strata of assumed greater or less permeability can be introduced by this method, involving complications which would defy mathematical analysis[10].

The Hydraulic Electric Analogy.—The fact that Ohm's law for flow of electricity, $\left(C = \frac{E}{R} = K' \frac{E}{l} a\right)$, is identical in form with Darcy's law of flow of water in granular material (see Equation (2)) permits the solution of underflow problems, by electric analogy utilizing the principle of a Wheatstone bridge. The writer's acquaintance with this principle was first obtained from E. W. Lane, M. Am. Soc. C. E.[11] Other investigators have platted complete "flow nets"[10] which are very fascinating and instructive, but the writer has chosen to determine only the three important criteria in order to economize time and cover the greatest number of conditions in a first survey of the field.

The writer's "tray" is connected somewhat differently from that of previous investigators, as shown in Fig. 7. It is 24 by 46 in. in size, utilizing a salt solution about 1 in. deep. Copper terminals, X and Y, permit simulating the base of a dam, 20 in. or less in width, A B, of any desired sectional design. With Switch E closed to F, the earphones, P, permit the determination of the proportionate voltage (or hydrostatic uplift head) drop at C along the base by selecting Point D to produce silence. Likewise, to determine the resistance of the solution between the copper strips, X and Y, Switch E is closed to F' through the known resistance, R, with earphones connected to C'.

To obtain unit resistance of the solution for parallel flow the two main terminals are changed from the copper strips, X and Y, to X' and Y', in a bakelite frame temporarily immersed; or the copper strips, X and Y, are replaced by strips along the entire ends of the tray while the model of the dam base is removed. Knowledge of the exact depth of the solution is not necessary and would be difficult to determine in such a shallow tray and likewise would be subject to frequent change by evaporation and replacement. It is only necessary to know the resistance between the terminals, X and Y, compared with the resistance producing parallel flow, as between X' and Y', in an equal depth of solution.

---

[10] *Technical Publication No. 215*, Am. Inst. of Min. and Metal. Engrs., p. 39.

[11] *Civil Engineering*, October, 1934.

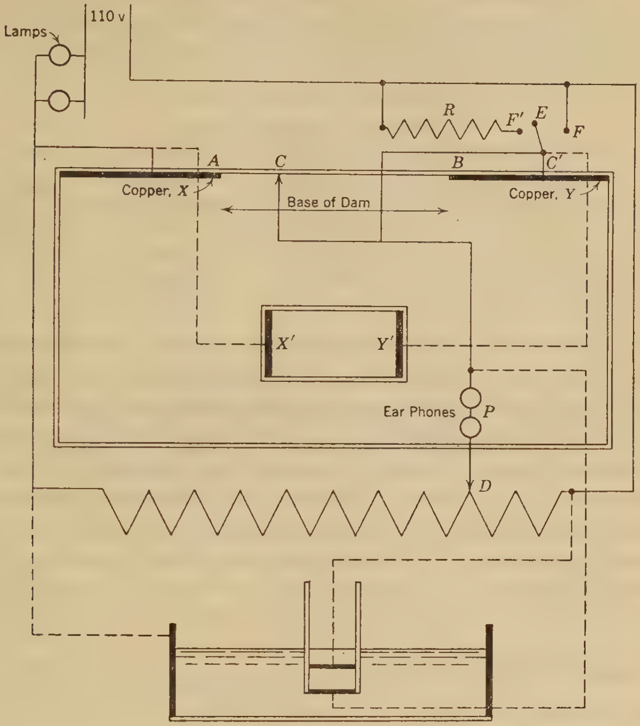


FIG. 7.

Quantity of Seepage.—In order to estimate the quantity of seepage to be expected under a given type of foundation, it becomes necessary to correlate the tray analogy with the physical characteristics of the material in the field. There is no present method known to the writer for determining the transmission constant, K, for sand in place except by Slichter's method of measuring velocity of underflow combined with test pipes for measuring slope. The pumping of a test well or test pit does not now give K because the information thus gained is not interpretable into K by any known law, due to the complex lines of flow converging through the material in three dimensions toward the outlet. The hydraulic electric analogy offers a rough basis for this solution. The side wall of a tub (Fig. 7), 18 in. in diameter was lined with copper and the tub was partly filled with salt solution. A piece of electrical fiber conduit was partly submerged in the center. This conduit was fitted with one circular metal disk that nearly closed the bottom and one close-fitting disk, adjustable in height above the bottom, both disks being connected to exterior binding posts by insulated conductors.

Without further detail it is evident that the Wheatstone bridge in Fig. 7 permitted the measurement of the relative resistance of the three-dimensional converging paths of flow toward the lower disk as compared with the resistance between disks within the tube representing parallel flow through the same solution. It was determined that the path between the two disks within the conduit, in order to offer the same total resistance as the converging flow from tub wall to lower disk, needed the disk spacing listed in Table 2, Column (3).

TABLE 2.—Disk Spacing

Diameter of duct, in inches (1)	Height of bottom of duct above bottom of tub, in inches (2)	Distance between disks, in inches (3)	Ratio of spacing to tube diameter (Column (3) divided by Column (1)) (4)
2	2	0.6	0.3
2	4	0.56	0.28
4	4	1.32	0.33

Nearly all the converging resistance is encountered close to the entrance where current density becomes high, and, therefore, the diameter of tub and the depth of water can be shown to have little effect. It may be stated, therefore, that within practical limits of accuracy of application, the draw-down head required for a given water yield of a test pit would discharge the same quantity of water longitudinally through a volume of the same material having the same cross-section as the test pit and a length equal to three-tenths of the test-pit diameter. Thus, one test pit, 6 ft in diameter, yielded 900 gal per min, or 2 cu ft per sec for a draw-down of 4 ft. For parallel flow, then, by Equation (2): $q = 2 = K \frac{4}{0.3 \times 6} 28$, or $K = 0.0322$ ft per sec, or 1.932 ft per min.

It is equally possible to mount metal tubing on bakelite rods with a varying ratio of length of tube to its diameter and then to submerge them in the same salt solution to obtain resistance of convergency to such a surface. This simulates a screened driven well, or well-point, as compared with the resistance to parallel flow, thus permitting determination of K by a similar analogy. The writer's data thus determined are too meager to report because this principle does not admit of as simple a statement as the test pit. The pumping of test pits and of driven well-points, or screened wells, thus furnishes, with the electric analogy, a basis for estimating the total seepage under any type of foundation design.

Thus, the resistance from X to Y (Fig. 7) under the model of a certain proposed design of power-house foundation was 7.6 ohms, while the resistance from end to end of the 24 by 46-in. tray was 9 ohms. The scale of the model was 25 ft to the inch. The parallel flow through a prototype section of the tested material, 1 ft thick, under a 65-ft head, would be, by Equation (2), $q = 0.0322 \frac{65}{46 \times 25} 24 \times 25 = 1.09$ cu ft per sec, in which it will be noted that the scale of the model is canceled. The flow under the proposed power house would then be $\frac{9.0}{7.6} \times 1.09 = 1.29$ cu ft per sec per foot length of the structure.

This computation ignores depth to rock or to impervious material. If known, the tray can be partitioned horizontally to simulate this depth; if not, it is believed the error will not be great and that usually it will be within the limits of accuracy of the method, as most of the seepage flow hugs close to the foundation profile of the structure.

Analyses of Special Conditions

Dam with Base Flush with Stream Bed, No Sheeting.—Weaver[12] has analyzed, mathematically, the problem of uplift pressure on the base for the case of a dam on homogeneous sand of indefinite depth, with the base of the dam resting on, and level with, the sand surface, with and without a single line of sheet-piling. He does not treat the equally important problem of escape gradient of the water at the toe. Weaver finds, under these simplified assumptions, with no sheet-piling, that the paths of flow are the lower halves of a family of confocal ellipses with foci at the heel and toe, and the lines of equal pressure are the conjugate family of confocal hyperbolas. In Fig. 8 the paths of flow are shown as full lines and the lines of equal pressure as dotted lines. Weaver has shown that the time-honored assumption of straight-line uplift gradient from heel to toe is fundamentally incorrect in principle, although not seriously so, and on the safe side. He finds that the uplift is given by the formula[13]:

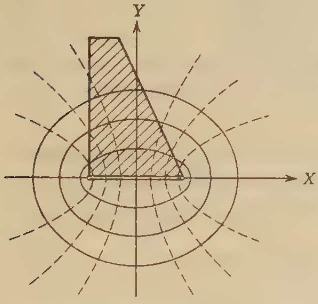


FIG. 8.—CONJUGATE ELLIPSES AND HYPERBOLAS.

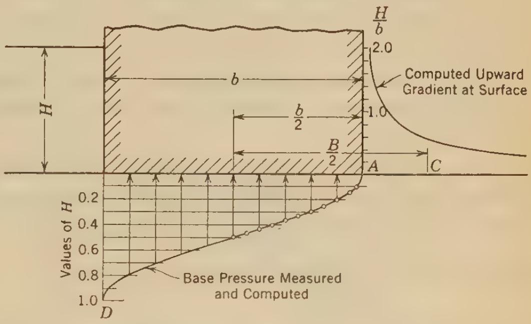


FIG. 9.—DAM ON SAND—NO CUT-OFFS.

$$\text{Uplift head} = \frac{H}{\pi} \cos^{-1} \frac{2\,x - b}{b} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

in which, the origin is at the heel; x is the abscissa; and b is the base width. The curve of Equation (7) is shown in *DA*, Fig. 9, indicating a steep hydraulic gradient near heel and toe (in fact, tangent to the vertical and, therefore, infinite immediately at the heel and toe), and a corresponding gentle intermediate gradient, dropping at the middle of the base to a value of $\frac{2}{\pi}$, or about two-thirds of the average gradient.

---

[12] *Journal of Mathematics and Physics*, Vol. XI, No. 2, June, 1932, Mass. Inst. Tech., Cambridge, Mass.

[13] *Loc. cit.*, Equation (12).

This curve is of basic importance not only with reference to dams on sand, but apparently in the case of dams on rock foundations. Its general shape is corroborated by field observations of existing dams reviewed by Houk[14]. It should be evident that the curve would take this general shape instead of being straight because of the tendency of the water to follow the shortest path and thus crowd close to both heel and toe, increasing the velocity and, therefore, the rate of loss of head at these places. This curve will appear so frequently that the term, *S*-curve (Equation (7)), will be used for future reference.

It should be noted that Weaver's analysis is based on sand of indefinite depth, whereas in all practical cases the depth would be finite. To test the magnitude of the error thus introduced, the writer first attempted to verify Equation (7) by his analogy tray, using 20 in. for the base of the dam. The resulting experimental points, shown in Fig. 9, coincided as accurately with the *S*-curve as they could be platted. This proved both the amazing accuracy and consistency that can be realized by this experiment, and also that the limited size of the analogy tray would not vitiate the accuracy of other deductions for which no mathematical check is available. Most of the flow evidently follows closely adjacent to the structure and, within measurable accuracy, ignores the size of tray for a structure not more than 20 in. in base width.

The writer has extended Weaver's analysis mathematically to determine the upward escape gradient in the stream bed below the dam, which is a matter equally as important as base pressure. If the origin is transferred to the center of the base (see Fig. 9), and x is made equal to $\frac{B}{2}$, which is greater than $\frac{b}{2}$, and y remains a variable, Weaver's v being the internal pressure head in the material, then $\frac{dv}{dy}$ is the vertical component of the gradient, or is $\frac{h}{l}$ along a vertical line at the point where $x = \frac{B}{2}$. This gradient is found to increase toward the surface especially close to the dam, and at the surface, it becomes:

$$\frac{h}{l} = \frac{2\,H}{\pi\,b\sqrt{\frac{B^2}{b^2} - 1}} \qquad \text{(8)}$$

in which, $B = 2\,x$; $\frac{h}{l}$ = the vertical escape gradient at the surface of the sand; H = head on the structure; and b = base width of dam.

---

[14] *Civil Engineering*, Vol. 2, No. 9, September, 1932, p. 578.

In Fig. 9 a curve of escape gradient is shown at the sand surface computed from Equation (8) for the condition of base width of twice the head, or $b = 2H$. The vertical scale of the escape gradient curve is in terms of $\frac{H}{b}$, or the mean hydraulic gradient along the base, the reciprocal of Bligh's percolation coefficient. The escape gradient at the immediate toe is infinite, indicating inevitable instability of sand at this point. The surface gradient decreases rapidly from infinite to a value of $\frac{H}{6} = 1$ at a distance down stream from the toe of about $0.1b$. Because of the excessive escape gradient at the immediate toe as well as for obvious practical reasons, such as rain erosion, the base of a dam on soft material should never be flush with the foundation surface at the toe. A depressed toe or toe cut-off is required for all dams on sand.

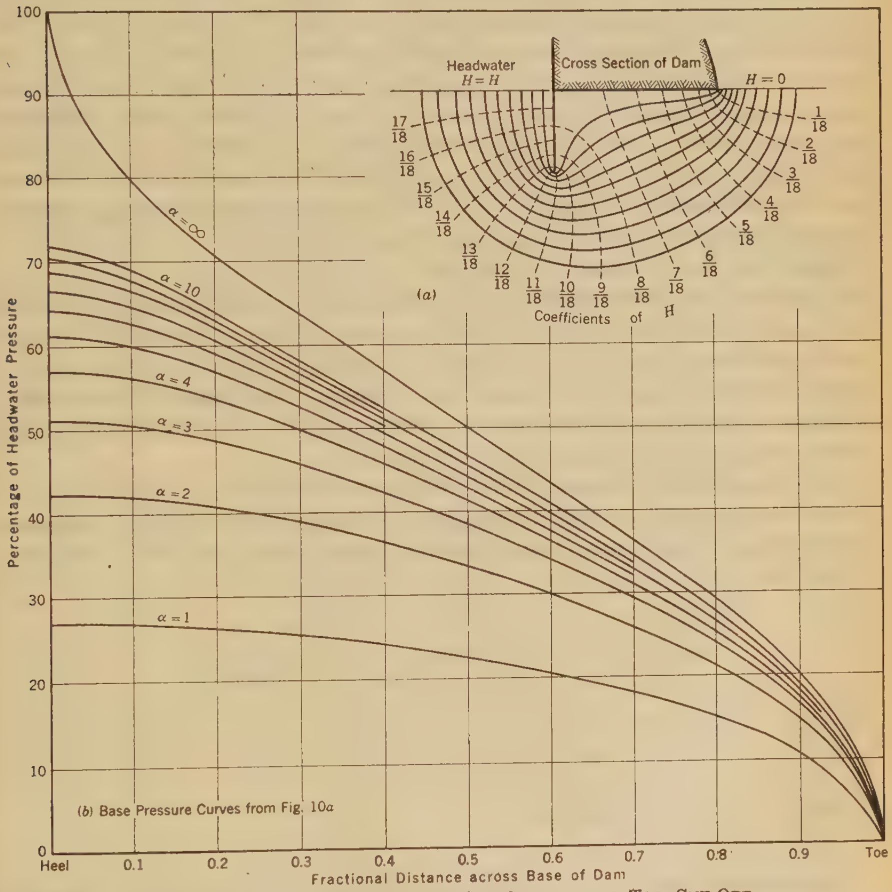


FIG. 10.—FLOW NET FOR HEEL CUT-OFF BUT NO TOE CUT-OFF.

Dam with Heel Sheeting but No Toe Sheeting.—For the case of heel sheeting, Weaver has determined mathematically, the curve of uplift pressure for various ratios of base width of dam to depth of sheeting. The lines of flow and equal pressure lines (flow net) as determined mathematically by him are illustrated in Fig. 10(*a*)[15], and agree with the flow net determined by H. M. Hill, Assoc. M. Am. Soc. C. E., between two closely spaced glass plates[16].

It will be noted that the pressure drop is nearly uniform along the up-stream face of the sheeting; and that it is very rapid due to crowding of flow lines around the tip of the sheeting, and again at the toe of the dam. The distributions of uplift pressure across the base of a dam are plotted in Fig. 10(*b*)[17] in percentages of head-water pressure, for various ratios, α, of width of base of dam to depth of sheet-piling at the heel. All these pressure curves become tangent to the vertical at the toe, which again indicates an infinite escape gradient and unstable material in close proximity to the toe as in the case of a dam without sheeting. This again should rule out this type of structure in which the base is flush with the foundation surface at the toe thus necessitating a depressed toe or toe cut-off.

Dam with Toe Sheeting but with Heel Apron in Lieu of Heel Sheeting.—This problem is mathematically and graphically the same as that of the last case except with the reversal of the direction of flow, Fig. 10(*a*). There would then be a rapid loss of pressure at the heel where the water enters the sand (which could do no harm), and a gradual and almost uniform rate of pressure drop as the water rises to escape on the down-stream side of the toe cut-off as shown by Fig. 10(*a*), if reversed.

The equation for pressure down the up-stream face of heel sheeting as derived by Weaver[18], by proper changes of sign for reversal of flow, becomes the equation for pressure at the depth, y, below the surface and along the lower face of the toe sheeting of the depth, d, as follows:

$$h = \frac{H}{\pi} \cos^{-1} \left[\frac{\lambda - 1 + \sqrt{1 - \frac{y^2}{d^2}}}{\lambda} \right] \quad \ldots\ldots\ldots\ldots\ldots (9)$$

in which, $\lambda = \dfrac{1 + \sqrt{1 + \frac{b^2}{d^2}}}{2}$.

Regardless of its apparent complexity, Equation (9) is essentially straight when platted, except the bottom quarter near the tip of the sheeting.

---

[15] *Journal of Mathematics and Physics*, Vol. XI, No. 2, June, 1932, Fig. 8.

[16] "Seepage Through Foundations and Embankments Studied by Glass Models", by H. M. Hill, *Civil Engineering*, January, 1934, p. 32.

[17] *Journal of Mathematics and Physics*, Vol. XI, No. 2, June, 1932, Fig. 5.

[18] *Loc. cit.*, Equation (27), p. 132.

In Fig. 11 is given a set of diagrams indicating the base pressure in percentage of head from Fig. 10(*b*), and the writer's computation, from Equation (9), of pressure along the down-stream side of the toe sheeting both in percentage of head at indicated percentages of the depth of the sheeting. To facilitate the platting and comparison of curves the length of sheeting and base curves are platted as constant which is equivalent to foreshortening the horizontal scale of the base of the dam. The small inset diagrams indicate the true proportions. Only one of these diagrams was checked in the analogy tray, that of $b = 5d$. The platted points indicate an accurate agreement with the curves computed by formulas. In Table 3, the escape gradient is expressed as a function of $\frac{H}{d}$, $\frac{H}{b}$, $\frac{H}{L_p}$, and $\frac{H}{L_c}$, in which, L_p and L_c denote "short path" and "creep distance", respectively.

TABLE 3.—COMPARISON OF ESCAPE GRADIENT, G

Ratio of base width to depth of sheet-piles, $\frac{b}{d} = a$	ESCAPE GRADIENTS IN TERMS OF THE RATIOS OF:			
	Head to depth of sheeting	Head to base width of dam	Head to creep distance	Head to short path of seepage
(*a*) DAM WITH TOE SHEETING AND HEEL APRON (SEE FIG. 11)				
1	$0.32 \frac{H}{d}$	$0.32 \frac{H}{b}$	$0.96 \frac{H}{L_c}$	$0.773 \frac{H}{L_p}$
2	$0.28 \frac{H}{d}$	$0.56 \frac{H}{b}$	$1.12 \frac{H}{L_c}$	$0.908 \frac{H}{L_p}$
3	$0.24 \frac{H}{d}$	$0.72 \frac{H}{b}$	$1.2 \frac{H}{L_c}$	$1.00 \frac{H}{L_p}$
4	$0.21 \frac{H}{d}$	$0.84 \frac{H}{b}$	$1.26 \frac{H}{L_c}$	$1.076 \frac{H}{L_p}$
5	$0.19 \frac{H}{d}$	$0.95 \frac{H}{b}$	$1.33 \frac{H}{L_c}$	$1.16 \frac{H}{L_p}$
6	$0.17 \frac{H}{d}$	$1.02 \frac{H}{b}$	$1.36 \frac{H}{L_c}$	$1.20 \frac{H}{L_p}$
8	$0.14 \frac{H}{d}$	$1.12 \frac{H}{b}$	$1.4 \frac{H}{L_c}$	$1.27 \frac{H}{L_p}$
10	$0.12 \frac{H}{d}$	$1.20 \frac{H}{b}$	$1.44 \frac{H}{L_c}$	$1.326 \frac{H}{L_p}$
(*b*) DAM WITH BOTH HEEL AND TOE SHEETING (SEE FIG. 12)				
1.5	$0.26 \frac{H}{d}$	$0.39 \frac{H}{b}$	$0.91 \frac{H}{L_c}$	$1.43 \frac{H}{L_p}$
2	$0.25 \frac{H}{d}$	$0.50 \frac{H}{b}$	$1.00 \frac{H}{L_c}$	$1.50 \frac{H}{L_p}$
2.5	$0.24 \frac{H}{d}$	$0.60 \frac{H}{b}$	$1.08 \frac{H}{L_c}$	$1.56 \frac{H}{L_p}$
5	$0.185 \frac{H}{d}$	$0.925 \frac{H}{b}$	$1.30 \frac{H}{L_c}$	$1.66 \frac{H}{L_p}$
(*c*) DAM WITH BASE SET 4½ INCHES INTO SAND (SEE FIG. 13)				
4.7	$0.16 \frac{H}{d}$	$0.75 \frac{H}{b}$	$1.05 \frac{H}{L_c}$	$1.05 \frac{H}{L_p}$

Dam with Both Heel and Toe Sheeting.—This case has not been analyzed mathematically by Weaver, and the writer has been unable to do so.

The close agreement of the electric analogy with the mathematical equations in the previous cases supports the belief that reliance can be placed on the former method. The writer has investigated four cases in his analogy tray the results of which are platted in Fig. 12. The first one used a 4-in. depth of sheeting on a 20-in. base width, all the others an 8-in. sheeting on base widths of 20, 16, and 12 in.

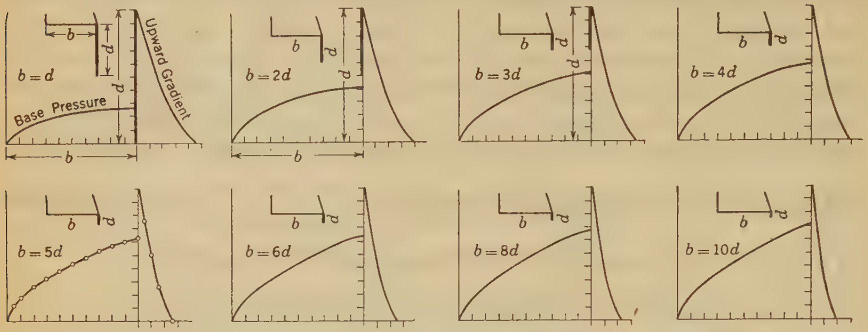


FIG. 11.—BASE PRESSURES, DAM WITH TOE SHEETING, AND HEEL APRON (SEE TABLE 3(*a*)).

Attention is called to the almost negligible difference in escape gradient for the three dams using 8-in. sheeting even with the large difference in base width (a = 1.5, 2, and 2.5 in Table 3(*b*)). This results, of course, from the small pressure drop under the base of the dam from the heel to the toe sheeting. The drop in pressure is much greater with the 4-in. sheeting as well as in the case of no heel sheeting. To assist in arriving at an explanation of this fact, the gap across the tips of the 4-in. sheeting was closed by a $\frac{1}{4}$-in. strip, thus simulating the condition of a flat base set a distance of $4\frac{1}{4}$ in. into the sand without heel or toe sheeting. The result-

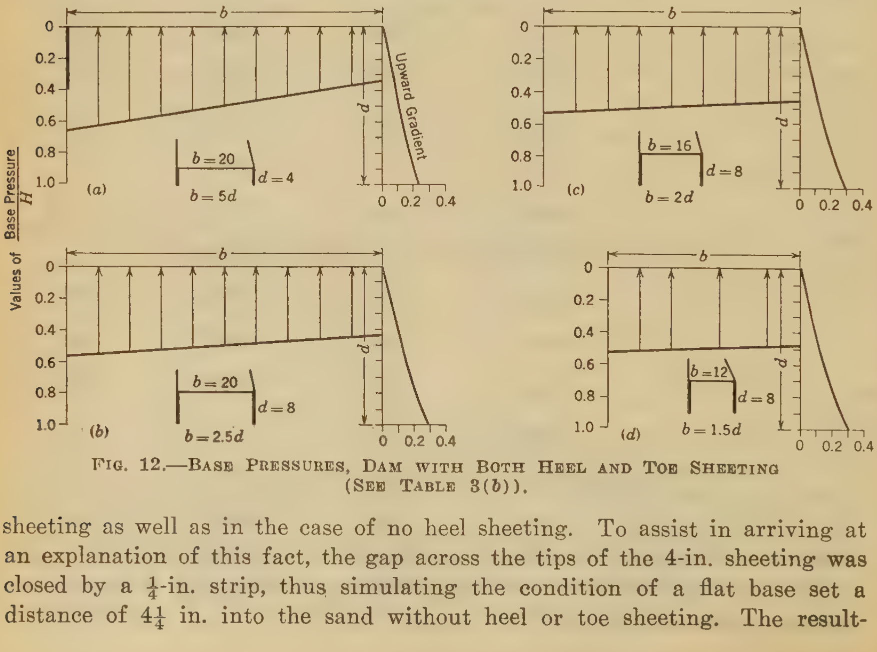


FIG. 12.—BASE PRESSURES, DAM WITH BOTH HEEL AND TOE SHEETING (SEE TABLE 3(*b*)).

ing loss of head along the base was much greater than with sheeting only (see Fig. 13) which served to reduce toe escape gradient from $0.925 \frac{H}{b}$ to $0.75 \frac{H}{b}$. These gradients reduce, respectively, to $1.66 \frac{H}{L_c}$ and $1.05 \frac{H}{L_c}$ (see Table 3(c)). Thus, for equal flotation safety, a Bligh percolation coefficient 1.58 times greater must be used for sheeting at the heel and the toe than for a base depressed equally into the sand. The explanation is apparently to be found in the fact that the water flowing under the dam tends to concentrate as much as possible along the shortest path. With the depressed base, this flow is restricted by the structure from diffusing upward and, hence, follows largely a narrow strip along the base. With only sheeting to this same depth, the flow lines can spread both upward and downward between the lines of sheeting, with resulting loss of head slightly more than one-half as great. With the 8-in. sheeting the spread of the flow lines upward and downward is still more nearly equalized. On this theory it would seem that the minimum gradient between sheeting lines would be realized with sheeting extending nearly half-way to rock bottom, giving equal opportunity for diffusion upward and downward. Likewise, it would be expected (although not confirmed) that stratification would create more rapid loss of head by encouraging concentration rather than diffusion of flow.

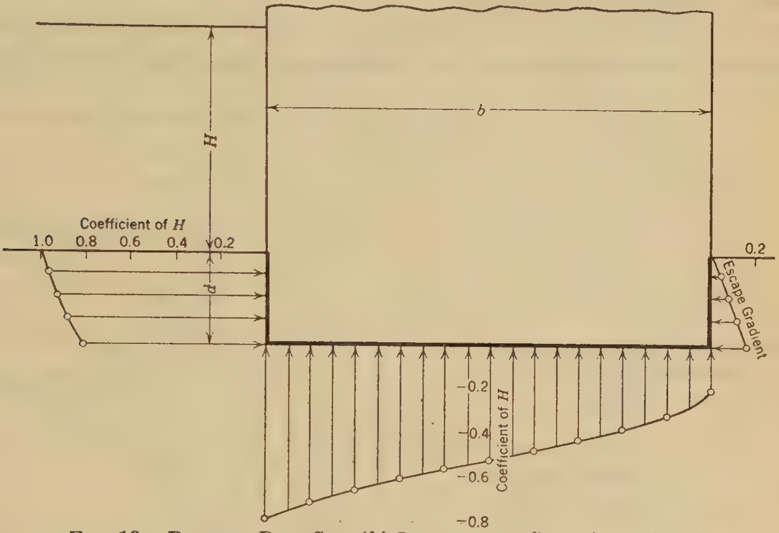


Fig. 13.—Base of Dam Set 4¼ Inches into Sand ($b = 4.7\ d$).

Single Diaphragm of Sheet-Piling.—Especially for purposes of construction, but occasionally in permanent structures, a single diaphragm of sheet-piling serves as a dam unaided by additional base width. This case yields readily to mathematical analysis by making the base width, $b = 0$, in Equation (9), thus resulting in:

$$h = \frac{H}{\pi} \sin^{-1} \frac{y}{d} \quad \text{.........................} \quad (10)$$

or,

$$\frac{dh}{dy} = \frac{H}{\pi\, d \sqrt{1 - \frac{y^2}{d^2}}} \quad \text{.....................} \quad (11)$$

in which, h is the static head in the sand along the face of the sheeting of penetration, d, at a depth, y, below the sand surface, the sand being at the same elevation on both sides.

Fig. 14 shows the curve of Equation (10) platted to scale, and the values observed in the writer's analogy tray for sheeting 4 in., 8 in., and 12 in. deep.

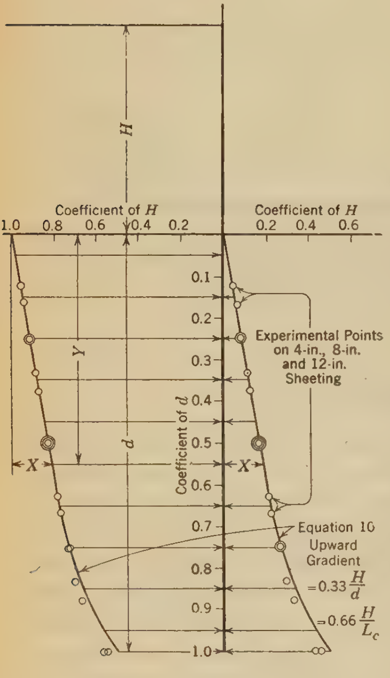

H
Coefficient of H
1.0
0.8
0.6
0.4
0.2
Coefficient of H
0.2
0.4
0.6
y
d
X
X
Coefficient of d
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
Experimental Points on 4-in., 8-in. and 12-in. Sheeting
Equation 10
Upward Gradient
$=0.33\frac{H}{d}$
$=0.66\frac{H}{L_c}$

FIG. 14.—DAM WITH SINGLE ROW OF SHEET-PILING

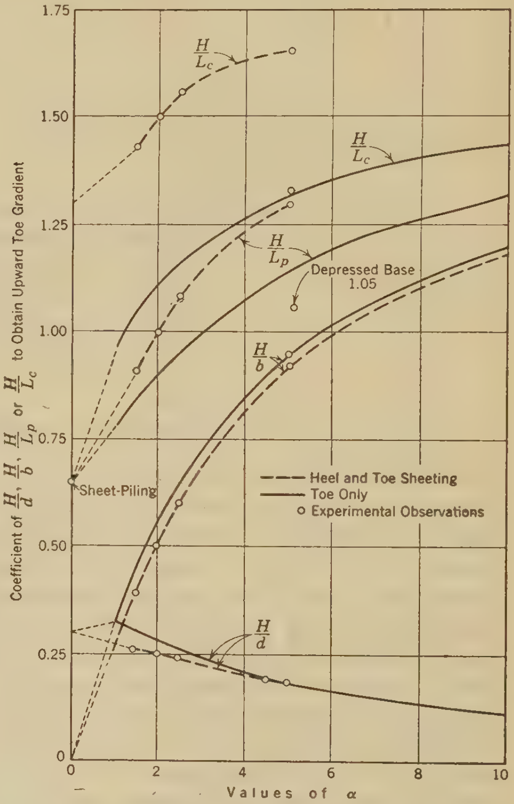


FIG. 15.—MATHEMATICAL AND EXPERIMENTAL DETERMINATION OF ESCAPE GRADIENTS.

The verification is close within the possible limits of accuracy in platting, except near the bottoms of the curves. This is explainable by the fact that the sheeting used in the analogy tray was $\frac{1}{4}$ in. thick and the bottom observations were at the lower corners, slightly more than $\frac{1}{4}$ in. apart, instead of on the bottom of the sheeting strip at the center line. The loss of head around the point of sheeting is rapid. The escape gradient up the upper half of the face of the sheeting is:

$$G = 0.66 \frac{H}{2\,d} \quad \text{.........................} (12)$$

or two-thirds of that which would be expected from uniform loss of head along the creep distance. The case of unequal sand levels on opposite sides of the sheeting has not as yet been investigated.

SIGNIFICANCE OF "LINE-OF-CREEP" AND "SHORT-PATH" THEORIES

The logic of the application of Bligh's coefficients to the long path or line of creep, or to the short-path distance can now be investigated. The results of mathematical and experimental determination of escape gradients from Figs. 11 and 12 for the cases of toe sheeting only, and for both toe and heel sheeting of equal length, are assembled in curve form in Fig. 15 which is platted against a horizontal scale of $\alpha = \frac{b}{d}$ for comparison. In each curve the flotation gradient at the toe is obtained by application of the coefficient at the left to $\frac{H}{d}$, $\frac{H}{b}$, $\frac{H}{L_p}$, or $\frac{H}{L_c}$, all arriving at the same result. These curves are dotted to the points where they should intersect the line, $\alpha = 0$, equivalent to a single row of sheeting at 0.33 for $\frac{H}{d}$; 0 for $\frac{H}{b}$; 0.66 for $\frac{H}{L_p}$ and $\frac{H}{L_c}$, for one cut-off, and 1.32 for $\frac{H}{L_c}$ for a double cut-off. The fact that these curves do not tend exactly toward the foregoing values for $\frac{b}{d} = 0$, is believed to result from the thickness of the model sheeting, the same cause explained under "Single Diaphragm of Sheet-Piling".

If the factor of safety against percolation could be represented correctly by applying a coefficient to any of the foregoing ratios, the curve of such expression in Fig. 15 would be a horizontal line. This is far from the case except for sheet-piling, which is a single point and for which the line-of-creep theory is, therefore, correct in principle. The other curves vary through a wide range of coefficients indicating that none of the heretofore accepted bases applies correctly to a solution of the problem. The writer has been unable to devise any method of weighting the several factors involved which would yield a nearly horizontal line in Fig. 15. This is not surprising when the complexity of the problem is observed from Equation (9).

The required increase of coefficients with increasing ratios of $\frac{b}{d}$ (Fig. 15), indicates that a constant coefficient as by Bligh would provide a smaller factor of safety for increasing values of $\frac{b}{d}$. Bligh's factors thus tend to yield a higher factor of safety for modern types of dams, both because of their smaller values of $\frac{b}{d}$ and because of the monolithic types of construction as compared with the older types, which often consisted of dry rock or rubble fills between concrete walls, and shallow cut-offs, if any. In fact, Bligh's coefficients were developed for some dams in which seepage could escape progressively along the down-stream slope instead of being confined until reaching the toe.

Consistency of Bligh Coefficients.—It is desirable to inquire into the consistency with which these coefficients can be applied to different types of foundations. Thus, assume $\frac{b}{d} = 5$, with an escape gradient of 0.25, and determine what value of the Bligh coefficient, C, will be required to assure this gradient.

For a single line of sheeting, (Fig. 15), $0.25 = 0.66 \frac{H}{L_c}$; or,

$$C = \frac{L_c}{H} \doteq \frac{0.66}{0.25} \ldots\ldots\ldots\ldots\ldots\ldots = 2.64$$

For other types the values of C are in the same ratios as the ordinates of the $\frac{H}{L_c}$ curves in Fig. 15 for $\frac{b}{d} = 5$; or,

$$C, \text{ for depressed base (Fig. 13) } = \frac{1.05}{0.66} \times 2.64 \ldots\ldots = 4.17$$

$$C, \text{ for heel apron and toe sheeting } = \frac{1.33}{0.66} \times 2.64 \ldots\ldots = 5.3$$

$$C, \text{ for heel and toe sheeting } = \frac{1.66}{0.66} \times 2.64 \ldots\ldots\ldots\ldots = 6.6$$

Thus, for dams of equal safety from toe flotation, C varies from 2.64 to 6.6. This fact indicates that these coefficients, based upon creep distance, are not representative of true resistance; they are even less representative from a practical rather than a theoretical standpoint when the effect of stratification and possible roofing is considered. It would seem self-evident that the art of design against percolation admits of improvement.

The high resistance furnished by sheeting will also serve to explain the previously surprising performance record of the Prairie du Sac Dam, in Wisconsin, which was dependent for several years on 50-ft. heel sheeting alone for a head of about 34 ft on medium fine sand. This represents a percolation coefficient of about 3, which is generally considered too small for safety; and yet, in reality, it represents in this case a factor of safety of at least 4, perhaps 5, against flotation (knowledge of porosity of the material is not available).

Observation of Fig. 16 for heel and toe sheeting reveals that there is little dissipation of head on the down-stream side of the heel sheeting, on the up-stream side of the toe sheeting, and in the horizontal run between the sheeting. The result is that combined heel and toe sheeting acts much like a single line with little gain in effectiveness because the dissipation of head largely occurs on the upper side of the heel sheeting, on the lower side of the toe sheeting, and around the tips of the sheeting.

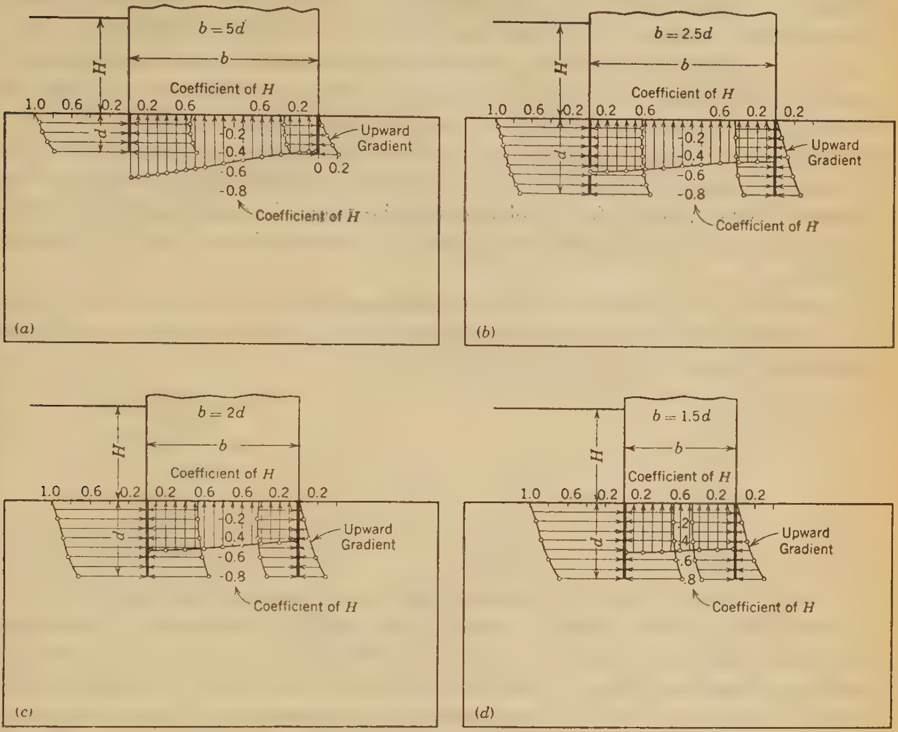

b = 5d
b
H
d
Coefficient of H
1.0 0.6 0.2 0.2 0.6 0.6 0.2
0.2
0.4
-0.6
-0.8
0 0.2
Upward Gradient
Coefficient of H
(a)
b = 2.5d
b
H
d
Coefficient of H
1.0 0.6 0.2 0.2 0.6 0.6 0.2 0.2
0.2
0.4
-0.6
-0.8
Upward Gradient
Coefficient of H
(b)
b = 2d
b
H
d
Coefficient of H
1.0 0.6 0.2 0.2 0.6 0.6 0.2 0.2
0.2
0.4
-0.6
-0.8
Upward Gradient
Coefficient of H
(c)
b = 1.5d
b
H
d
Coefficient of H
1.0 0.6 0.2 0.2 0.6 0.2 0.2
.2
.4
.6
.8
Upward Gradient
Coefficient of H
(d)

FIG. 16.—HEEL AND TOE CUT-OFFS.

PROPOSED RATIONAL APPROACH

The writer proposes, therefore, the gradual abandonment of Bligh's coefficients as fast as data regarding existing dams can be converted into the form of a safety factor or coefficient, f, to be applied to the critical flotation gradient, Equation (6), to arrive at a safe escape gradient, G, or,

$$G = \frac{F_c}{f} = \frac{1}{f} (1 - P)(s - 1) \quad \ldots\ldots\ldots\ldots\ldots\ldots (13)$$

The value of f is to be chosen with reference to the geological details, uniformity of the sand, stratified condition, and extent of knowledge of the sub-surface conditions. This factor, f, will then be dependent entirely upon judgment, empirical data, and field knowledge of foundation conditions constituting a true coefficient, whereas Bligh's coefficients involve also a part of the mathematical principles of the problem, which should be segregated. Once the basic equations and curves are available, it can be made as easy to use correct mathematical principles with modifying coefficients or factors of safety (which are truly dependent only on the non-mathematical and undiscernible elements of the problem), as to use Bligh's coefficients. Considerable research by the analogy method will be necessary to produce designing curves for all types of foundations, such as the double line of sheeting of unequal length, three or more lines, of sheeting, sloping surfaces, etc. In general, such research must be left to be conducted for the specific problem at hand. However, enough has been done herein to furnish a basis for the design of three types: (*a*) Heel and toe sheeting of equal length; (*b*) heel apron combined with toe sheeting; and (*c*) a single line of sheeting acting by itself.

Application of Escape Gradient to Design Problems.—The critical flotation gradient for average quartz sand with $s = 2.65$ for sands of various porosity as computed from Equation (6), are:

Porosity	Critical Flotation Gradient, F_c
0.30	1.15
0.35	1.07
0.40	0.99
0.45	0.91
0.50	0.825

It will be noted that the foregoing critical or flotation gradients, F_c, are approximately unity. Their reciprocals or Bligh's coefficients, therefore, would also approximate unity without a factor of safety if the average gradient along the line of creep were equal to the upward component at the point of escape. Fig. 15 furnishes a practical solution of design problems for foundation types to which it applies. Thus, if G is the escape gradient; m, the coefficient of $\frac{H}{b}$; and n, the coefficient of $\frac{H}{d}$, as ordinates at the left of Fig. 15, then, for any common value of $\frac{b}{d}$, $G = \frac{m\,H}{b} = \frac{n\,H}{d}$; or,

$$\frac{H}{G} = \frac{b}{m} = \frac{d}{n} \quad \text{.........................} \quad (14)$$

The head, H, is always known; the allowable value of G is determined from the physical properties of the sand modified by a factor of safety, Equation (13), and either b or d must be assumed. Thus, assume a double-sheeted dam with a head, H, of 30 ft and a quartz sand of about 40% voids, with a critical flotation gradient of 1.0 and a factor of safety of 4 yielding a working escape gradient of 0.25, then, $\frac{H}{G} = \frac{30}{0.25} = 120 = \frac{b}{m} = \frac{d}{n}$. If $b = 90$, then $m = 0.75$, $\frac{b}{d}$ from Fig. 15 $= 3.5$, and, hence, $d = \frac{90}{3.5} = 26$ ft.

DRAINAGE

The principle of drainage can be applied both to reduce upward pressure and to increase the factor of safety against flotation at the toe. Drainage is dangerous in the manner in which it is often carelessly installed (such as fragile tile in ungraded gravel trenches), and may permit the escape of material from beneath the structure. To be safe, drainage must follow the principle of an inverted sanitary water filter. There must be at least two

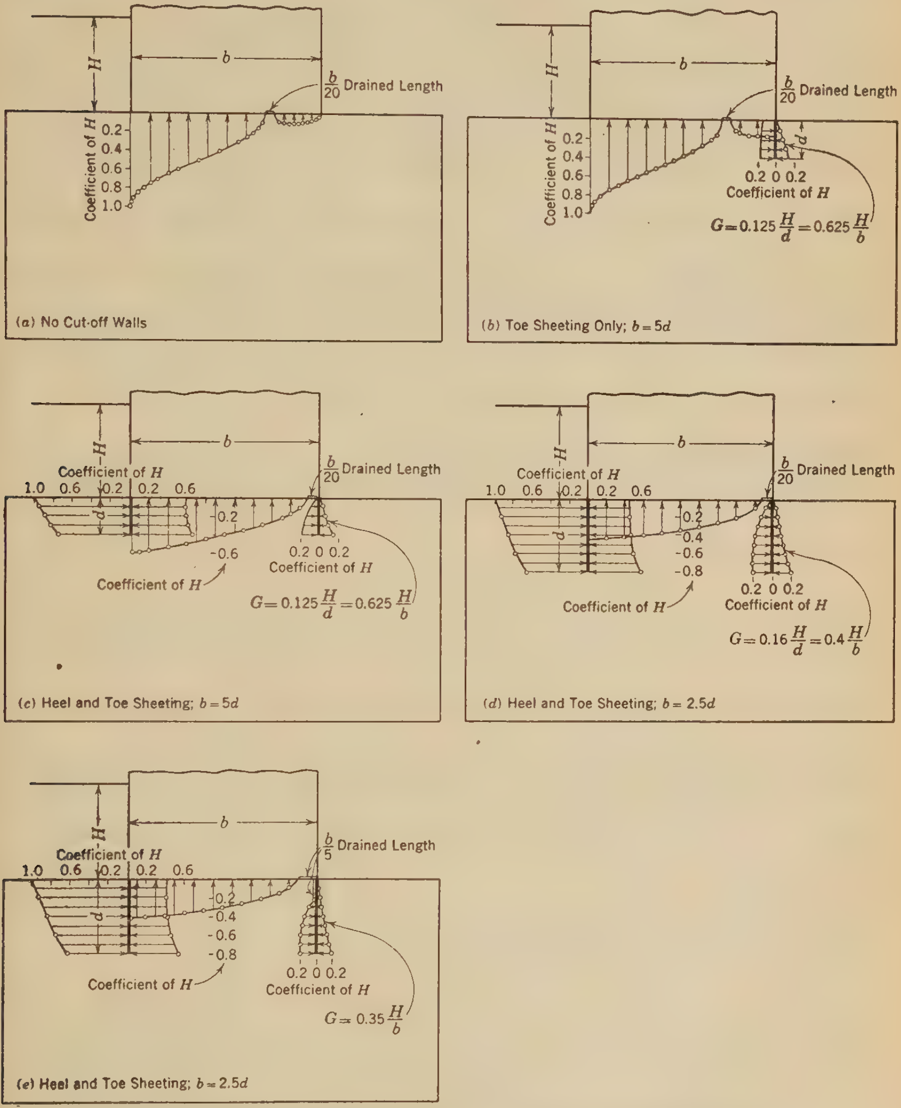

H
b
$\frac{b}{20}$ Drained Length
Coefficient of H
0.2
0.4
0.6
0.8
1.0
(a) No Cut-off Walls
H
b
$\frac{b}{20}$ Drained Length
Coefficient of H
0.2
0.4
0.6
0.8
1.0
d
0.2 0 0.2
Coefficient of H
$G = 0.125\frac{H}{d} = 0.625\frac{H}{b}$
(b) Toe Sheeting Only; b = 5d
H
b
$\frac{b}{20}$ Drained Length
Coefficient of H
1.0 0.6 0.2 0.2 0.6
d
0.2
-0.6
Coefficient of H
0.2 0 0.2
Coefficient of H
$G = 0.125\frac{H}{d} = 0.625\frac{H}{b}$
(c) Heel and Toe Sheeting; b = 5d
H
b
$\frac{b}{20}$ Drained Length
Coefficient of H
1.0 0.6 0.2 0.2 0.6
d
0.2
-0.4
-0.6
-0.8
Coefficient of H
0.2 0 0.2
Coefficient of H
$G = 0.16\frac{H}{d} = 0.4\frac{H}{b}$
(d) Heel and Toe Sheeting; b = 2.5d
H
b
$\frac{b}{5}$ Drained Length
Coefficient of H
1.0 0.6 0.2 0.2 0.6
d
0.2
-0.4
-0.6
-0.8
Coefficient of H
0.2 0 0.2
Coefficient of H
$G = 0.35\frac{H}{b}$
(e) Heel and Toe Sheeting; b = 2.5d

FIG. 17.—EFFECT OF DRAINAGE ON A DAM WITH FLUSH FOUNDATION.

layers of sand and one of gravel screened to such sizes that the material of one layer cannot penetrate the pores of the next layer above. Perforated plates or similar venting means must permit free escape of water through the structure.

The effect of drainage has been investigated by the writer in the analogy tray. Thus, in Fig. 17(*a*), is shown the result of draining the 15th-in. space of a 20-in. foundation without heel or toe sheeting. The uplift pressure curve to the point of drainage is the basic *S*-curve, Fig. 9, with the base reduced to three-fourths length. Beyond the point of drainage a small uplift pressure is resumed with a curve tangent to the vertical at the toe. This results in an infinite upward gradient at the immediate toe as in the basic *S*-curve. In Fig. 17(*b*) the same conditions are repeated except with a length of toe sheeting equal to $\frac{b}{5}$. The sheeting has no effect up stream from the drainage and serves principally to change the shape of the pressure curve at the toe to give a finite escape gradient. Comparing this with the same design ($b = 5\,d$ in Fig. 11), except without drainage, the reduction in escape gradient is found to be from $0.19\,\frac{H}{d}$ to $0.125\,\frac{H}{d}$, or a decrease of 34%, which is not as great as might be expected.

In Fig. 17(*c*) is shown heel and toe sheeting ($b = 5d$), with drainage 1 in. long immediately ahead of the toe sheeting; the resulting upward toe gradient, $0.125\,\frac{H}{d}$, agrees with that of the last case (see Table 3(*e*) and Table 3(*f*)). Another test was made of a dam with an 8-in. heel and toe sheeting, in which $b = 2.5\,d$, with a 1-in. drainage strip as in Fig. 17(*c*). The upward toe gradient is $0.4\,\frac{H}{b}$ as compared with $0.6\,\frac{H}{b}$ without drainage (compare Table 3(*g*) and Table 3(*b*)). This test was repeated with a 2-in. drainage strip immediately above the sheeting which resulted in a gradient of $0.35\,\frac{H}{b}$. This was again repeated with drainage along the upper 2 in. of the toe sheeting as well as 2 in. of the adjacent base width and with the upward toe gradient of $0.35\,\frac{H}{b}$ unchanged from the last one.

As in the case of earth dams there are two schools of thought with reference to drainage, those generally for, and those generally against, it. The writer believes that it provides a useful factor of safety especially in uncertain ground. He believes in the principle of imposing effective resistance to seepage through such a distance as is required until the quantity of leakage is sufficiently reduced, and then providing ample artificial, permanent, and well-constructed drainage works which will relieve as much as possible through a controlled route where its escape will be easier than through the material and where it can do no harm. Only the escape of seepage through the material is hazardous and it is better to provide a route through which it can escape more freely that through the material itself. It must be kept in mind that seepage will follow all routes of escape in inverse proportion to their resistances; and drainage, therefore, becomes a safety factor, not an elimination of toe-flotation possibilities.

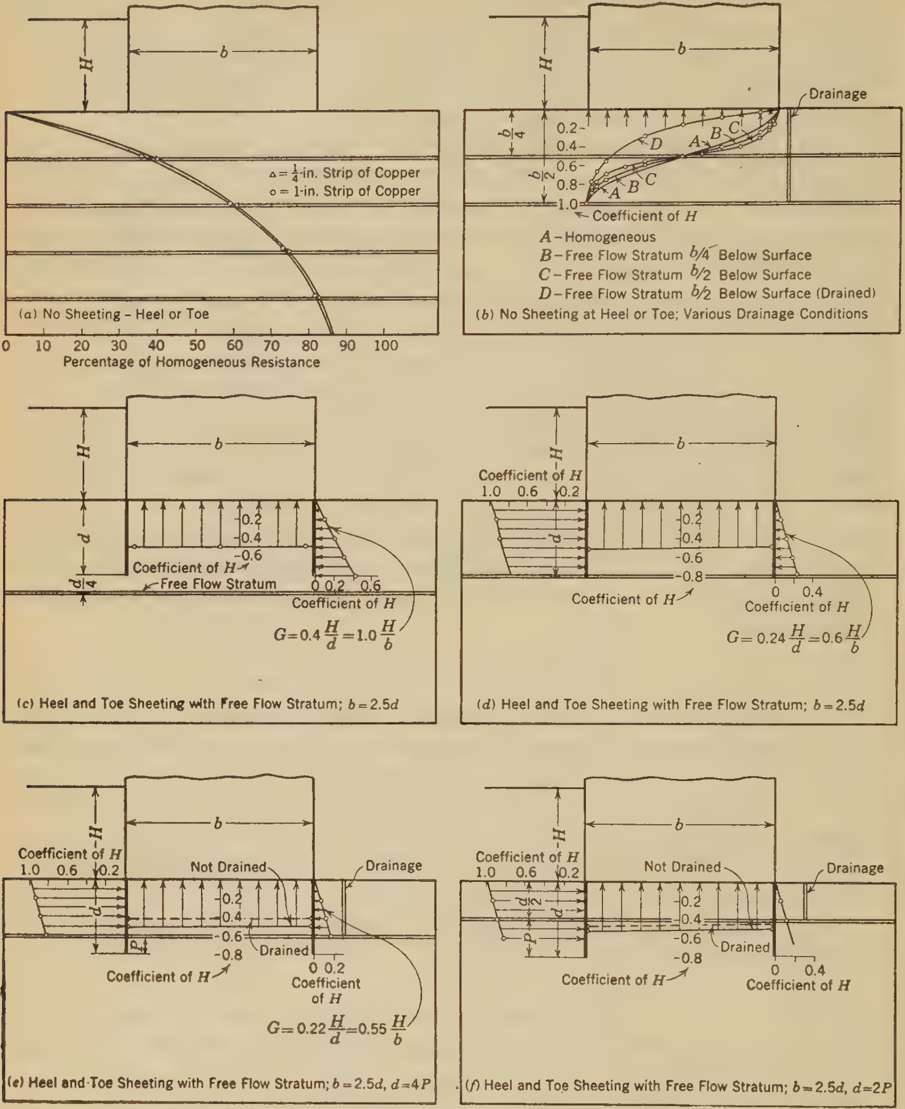
Thus, any pervious stratum or lense that may conduct excessive pressure beneath a dam, possibly to burst upward into tail-water, should be intercepted or blanketed near the heel where possible; and then near the toe it should be drained as freely as possible. Water will not burst upward through the sand from internal pressure if there is an easier route of escape than by lifting the sand. This line of escape may be created by well-points, screened driven wells, or intercepting trench if necessary to gain capacity, or (if local) by a dug well. Drainage wells can be connected by a header or filter gallery under or in the apron or toe of the dam and carried to a pump sump if it is desirable to hold the drained level below tail-water. This need might readily arise to protect an apron against uplift ahead of the hydraulic jump. Water would then drain toward the filter gallery from both head-water and tail-water. A complete line of drainage along the down-stream face of the toe sheeting could be used to eliminate most of the upflow through the material itself in an unstable situation requiring such precaution. Drainage is a useful tool for the designer and is capable of a great variety of useful applications.

HORIZONTAL POROUS STRATIFICATION

No Cut-Offs.—It was next attempted to gain some idea of the influence of horizontal pervious strata. To do so a 22-gauge, a ¼-in. strip and, alternately, a 1-in. strip of copper were submerged in the tray, successively, at the four positions shown in Fig. 18(*a*). The conductivity of the copper so greatly exceeded the salt solution as to be equivalent to a free-flow, no-resistance, stratum. Practically no difference was found in the ohmic resistance between forebay and tail-race as between the 1-in. and ¼-in. strip. Therefore, the latter was used exclusively afterward. Fig. 18(*a*) shows the variation in resistance of the bath with the varying positions of the copper strip.

In Fig. 18(*b*) the uplift pressure curves under the base are platted for a dam with no cut-offs: (*A*) On homogeneous material, and with no copper strip (the typical *S*-curve of Fig. 9); (*B*) with the copper strip, $\frac{b}{2}$, or 10 in. below the base; and (*C*) with the strip, $\frac{b}{4}$, or 5 in. below the surface. Curve (*D*) applies to the last case (*C*), but with a wire jumper from the copper strip to the terminal at the toe to simulate the result of freely draining the pervious stratum to the tail-race; the form of curve is entirely changed and an infinite toe gradient is replaced with zero gradient.

With Cut-Offs.—The first experiment, Fig. 18(*c*), finds the copper strip 2 in. below the tips of the 8-in. sheeting and not intercepted by it, which results in an escape gradient, $0.4\ \dfrac{H}{d}$, or $1.0\ \dfrac{H}{b}$. The gradient is higher than the gradient of a single row of sheeting $\left(0.33\ \dfrac{H}{d}\right)$ because there is no resistance between sheeting and less than otherwise past the tips.


H
b
△ = ¼-in. Strip of Copper
○ = 1-in. Strip of Copper
0 10 20 30 40 50 60 70 80 90 100
Percentage of Homogeneous Resistance
Drainage
b/4
b/2
0.2
0.4
0.6
0.8
1.0
A B C D
Coefficient of H
A – Homogeneous
B – Free Flow Stratum b/4 Below Surface
C – Free Flow Stratum b/2 Below Surface
D – Free Flow Stratum b/2 Below Surface (Drained)
d
d/4
Free Flow Stratum
0 0.2 0.6
$G = 0.4\frac{H}{d} = 1.0\frac{H}{b}$
1.0 0.6 0.2
0 0.4
$G = 0.24\frac{H}{d} = 0.6\frac{H}{b}$
Not Drained
Drained
P
d/2
0 0.2
$G = 0.22\frac{H}{d} = 0.55\frac{H}{b}$

(*a*) No Sheeting – Heel or Toe

(*b*) No Sheeting at Heel or Toe; Various Drainage Conditions

(*c*) Heel and Toe Sheeting with Free Flow Stratum; $b = 2.5d$

(*d*) Heel and Toe Sheeting with Free Flow Stratum; $b = 2.5d$

(*e*) Heel and Toe Sheeting with Free Flow Stratum; $b = 2.5d$, $d = 4P$

(*f*) Heel and Toe Sheeting with Free Flow Stratum; $b = 2.5d$, $d = 2P$

FIG. 18.—EFFECT OF SUB-SURFACE FREE-FLOW STRATUM ON RESISTANCE TO FLOW.

To simulate an intercepted stratum the $\frac{1}{4}$-in. copper strip was next cut in sections to fit between the sheeting and the tray walls. The case of open stratum barely penetrated is shown in Fig. 18(*d*). The gradient is $0.24\,\frac{H}{d}$, or $0.6\,\frac{H}{b}$, the same as in the case of no pervious stratum. An open stratum penetrated a distance, $\frac{d}{4}$ (Fig. 18(*e*)), gives a gradient of $0.22\,\frac{H}{d} = 0.55\,\frac{H}{b}$, which is even less than with no pervious stratum. With penetration of 4 in. (Fig. 18(*f*)), the gradient was the same as in the case of Fig. 18(*e*). This apparent anomaly can only be explained by the tendency of the free-flow stratum to carry the flow entirely to the cut-off without much diffusion, from whence it is concentrated at a high-current density around the tip of the cut-off with consequently rapid dissipation of pressure. In fact, two distant lines of independent sheeting with no resistance between them would have an upward toe gradient each of one-half $0.33\,\frac{H}{d} = 0.165\,\frac{H}{d}$. Apparently, this limit is being approached as the copper strip is raised toward the base of the dam, but will not be reached without wider spacing of sheeting. This apparent anomaly is significant in connection with roofing, and indicates that an open space under the foundation tends toward greater safety from toe flotation, assuming, of course, in all cases herein the integrity of the cut-offs. In the last two cases a further experiment was made by draining the open stratum below the toe with the result that no pressure existed on the down-stream side of the toe sheeting and that the uplift pressures under the base were reduced as shown.

OTHER COMMENTS

Uplift Under the Base.—Not much has been said about the important subject of uplift pressures because the curves are obvious. It is to be noted that in all symmetrical foundations the uplift at the center line is 0.50 H. With a double row of sheeting the variation of this pressure from heel to toe becomes less as $\frac{b}{d}$ decreases.

The uplift pressures obtained from the analogy are static pressures. They take no account of the force in the direction of flow exerted by the movement of the water through the material. Thus, in the drained bases, a considerable lifting effort must necessarily be exerted as the water rises to the drains, but will not increase the base pressure unless the upward gradient exceeds the critical value (see Equation (7)).

Sheeting Cut-Offs.—Experience seems almost universally to confirm some degree of uncertainty of effective sheet-piling cut-offs, except in favorable material, due to the possible opening of interlocking joints in driving. Unfortunately, the interlocking sections with the greatest jaw strength are difficult to drive in sand and necessitate compacting the sand intensely to force it out of the interlock ahead of the sheeting being driven. Boulders are a source of trouble, sometimes causing splitting of piles or separation of interlock without the knowledge of the operator. Much depends on the skill and experience of the operator. The most experienced can perhaps detect when all is not well with the sheeting, but there always remains a lurking uncertainty unless, as was done in a recent project, each sheet-pile was driven, pulled to determine its condition, and driven again. This, of course, is not practicable. The driving of sheet-piles in sand and gravel is a tedious slow operation and delays the work on the superstructure until this is complete. There are often boulder beds through which the sheeting cannot be driven successfully.

Trenched Concrete Cut-Offs.—The provision of a concrete cut-off in an excavated trench is sometimes resorted to in order to avoid the uncertainty of sheeting, and this is probably to be preferred. However, it is usually more expensive, causes still greater delay in initiating work on the superstructure, and has some (although less) element of uncertainty due to possible cracking apart from the main structure.

Abutments.—Too many dams are designed from a cross-section which ignores abutment conditions (in La Fruta Dam, Corpus Christi, Tex., for example). The abutment seepage problems are similar to those of the dam tipped on end, but are often more serious and difficult than those encountered under the dam itself. In the latter, for example, the seepage under a 50-ft length of dam has a 50-ft. length of toe under which to escape; but the seepage around the abutment of a dam, 50 ft high, all tends to concentrate at or near the tail-water line. This requires special attention to provide both ample lateral cut-offs and drainage.

Research by Model Testing

The studies made by the writer and reported herein have scarcely "scratched the surface" of needed and possible research as to the almost infinite variations of types of foundations used for dams on sand. Only a few of the elementary types justify general research to establish general principles. It is recommended, however, that for specific design problems, the alternate plans under consideration, and their fundamental hydraulic differences, be investigated by the analogy method as a useful guide to judgment. It is very accurate in principle and application, thus excluding the necessity of mathematics, which becomes too involved except for the simplest cases.

It is also believed that the analogy principle can be applied to three-dimensional models, thus permitting investigation of such problems as the influence of diagonal flow and relative concentration, if any, in the thread of the underflow stream (deepest channel), abutment conditions, etc. For this purpose a model of the entire structure would be made as a water-tight boat of insulating material, such as bakelite, with the proposed abutments correctly reproduced. Metallic screws would penetrate the boat shell from the inside at frequent intervals, upon which voltage readings could be taken.

Sheet-metal surfaces would simulate the sand surface in the river bed up stream and down stream of the dam; and the voltage would be applied between them.

Existing Dams.—It is suggested that some properly equipped research organization should undertake the determination of fundamental hydraulics by the analogy method on base-profile models of existing dams of importance. The aim should be to determine the theoretical pressure along the entire line of creep; the dissipation of head by various details of base profile, etc.; and the comparison of this with field data wherever records are available of actual pressures along the line of creep. The accumulation of such comparisons would be of inestimable value to designers, in arriving at the magnitude of departures to be expected from theoretical conditions, and the factors of safety to be used in design, effect of filter skin, etc.

SUMMARY AND CONCLUSIONS

1.—The problem of the safe conduct of seepage under a dam is analogous to that of the safe conduct of water over a spillway, in the sense that it seeks means to dissipate the head without disturbing or carrying away the foundation or stream-bed material.

2.—The dissipation of head is gradual through the foundation material from head-water to tail-water. The difference in head, as between two points, exerts a force against the intervening material and also causes a flow of pore water to occur. The reduction in pressure is the "cause"; force and seepage are sister "effects", material as well as water tending to be moved by the reduction in internal hydrostatic pressure.

3.—Seepage and force both occur or act in the direction of maximum reduction of pressure; that is, in a direction normal to surfaces or lines of equal pressure. The force exerted on a volume of material is, therefore, in the direction of seepage and is equal to the difference in internal pressure or head acting on the approaching and receding faces of the volume as against a solid rather than a porous surface.

4.—Foundation material cannot move when it is confined, unless the entire foundation as a whole is unstable. If not collectively unstable, then no material can be carried away by seepage unless and until the material ahead of it, in the direction of seepage, is first removed, as otherwise it would have no place to go.

5.—Analysis indicates that, in general, material under a dam is in stable condition with forces having a safe component downward until the tail-water is approached, at which point the drop of internal pressure has a component upward, becoming vertical at the surface. The more rapid the upward reduction in pressure (that is, the higher the upward hydraulic gradient), the less becomes the effective weight of the material until at a critical value, when the escape gradient equals $\frac{h}{l} = (s - 1)\ (1 - P)$ (see Equation (6)), the material actually floats and may be carried away with resulting rapid crumbling from the toe backward under the dam, causing failure. The tendency of this phenomenon to localize at the point in the tail-water of easiest escape may cause local sand "boils" and may give the impression that the sand is moving simultaneously throughout the route from head-water, thus producing an effect known as "piping". Actually, it is not, as is evident from Fig. 5; the instability is a local toe phenomenon, best termed, toe "flotation"; the problem is to prevent flotation, and piping cannot occur. Whether or not the critical value is approached, there will always be some upflow at the toe and this will reduce the effective weight of the sand and make it more easily eroded by waves, current, or rain.

6.—There seems to be no logical basis for expressing safety against toe flotation in terms of "creep distance", "short path" of seepage, or any functions thereof.

7.—Safety from toe flotation is best promoted:

(*a*) By choosing a design with a depressed toe or toe cut-off and one which dissipates head rapidly along the early part of its route, leaving as little remaining head as possible to be lost during the upward flow into tail-water;
(*b*) By supplying a special fill down stream from the toe, an inverted filter in principle;
(*c*) By providing an inverted filter drain under the toe of the structure itself, ahead of the toe cut-off; and
(*d*) By means of wells, drainage galleries, etc., at the toe.

8.—Dissipation of head is proportionate to velocity, which, in turn, is dependent on the tendency of the flow to concentrate. Such concentration occurs, especially, around protruding corners of the structure and about the tips of cut-offs, as well as along the up-stream face of heel cut-offs and the down-stream face of toe cut-offs. A single line of sheeting is the most effective structure investigated per unit of creep distance for the dissipation of head.

9.—A depressed toe or toe cut-off is theoretically essential for any dam, in order to avoid infinite upward gradient at the immediate toe as well as to prevent undercutting by erosion of rain, current, and wind. A heel cut-off in homogeneous material of relatively great depth, is apparently little more effective as to toe escape gradient than a heel apron, and is more expensive.

10.—The subject of drainage permits of much greater elaboration and more varied use than it has ever received, and much can be accomplished by drainage to improve the present state of the art.

11.—Hereafter all dams on sand should be equipped with sounding wells, pressure-recording devices, and other means of observing constantly the pressure conditions along the entire line of creep.

12.—Mathematics and research by means of hydraulic electric analogy offer valuable information as to fundamental principles of dissipating head in homogeneous material and relative ability of different foundation types to promote the minimum upward hydraulic gradient at the toe. Although the heterogeneity of natural deposits and the difficulty of interpretation by test borings will always require the use of a factor of safety above theoretical requirements, the writer maintains that a study of the problem should start from a nucleus based upon the law of flow through homogeneous material, with modifications introduced in the analogy tray in each individual case based upon such local conditions as can be revealed. Field tests and experience should be grouped around such ideal assumption instead of about purely empirical and unscientific coefficients, such as those of Bligh, to the end of eventually determining the order of magnitude of variations from the ideal by coefficients, or by factors of safety, much as in the case of weirs, etc.

13.—It is submitted that the proper ultimate method for designing dams on sand is to compare various proposed alternate types of foundation cross-section and abutment designs by the analogy method (simulating impervious and pervious strata as nearly as known) for uplift and upward toe gradient, amount of seepage, cost, etc.; and then to modify these comparisons based on field conditions at the site and empirical data which will have been accumulated in the meantime to aid in interpreting such conditions into their bearing upon the criteria of safety.

14.—The extent that "roofing", stratification, and other effects of non-homogeneity may modify the theoretical pressure profile in practice, should be observed in the analogy tray, and in actual structures and should be compared with the pressure profile that would exist in homogeneous material. It is proposed, therefore, that analogy models be made of all existing dam foundations for which foundation pressure records exist, in order to segregate the mathematical law from the effect of non-homogeneity of material, filter skin, roofing, etc.

DISCUSSION

Edward Godfrey,[19] M. Am. Soc. C. E. (by letter).—As an example of the trend of thought in all recent papers on the subject of dams (particularly as to under-pressure), this paper is significant and timely. Prior to about 1910 books on dams, at least in the English language, contained no mention of under-pressure as a factor in design. Papers on the subject that referred to under-pressure were almost nil. Nearly to the present day, the reports of the many failures that have occurred, in no case attribute the failure to under-pressure. Many such reports, however, have placed the blame for the failure on the foundation, no analysis being made of the manner of occurrence.

By its very title, Mr. Harza's paper presupposes that dams may be built on the most unstable of material, where flowing water is a possibility, namely, sand.

A significant statement is that of Conclusion 4. If under-pressure lifts a dam, it is to be expected that the water will flow through and wash out the foundation. This is what has led engineers to conclude that the foundation was at fault in the failure. Masonry and concrete dams that have been lifted and floated down stream, or have slid on their foundations, or have overturned because of insufficient base width to resist under-pressure, have not failed because of foundation weakness, but because of insufficient weight of material.

On the other hand, blow-outs below a dam follow exactly the course described by Mr. Harza in Conclusion 4: Foundation material flows out at the blow-out and gives place to other material. Even this type of failure is due to under-pressure of the water-soaked earth or sand pressing upward on the earth crust below the dam and removing it, which allows the sub-surface material to flow.

Hibbert M. Hill,[20] Assoc. M. Am. Soc. C. E. (by letter).—A rational treatment of the subject of seepage under dams on permeable foundations will no doubt be greatly advanced by this paper. Without question, Mr. Harza's treatment is fundamentally correct.

It is worth noting that, for a homogeneous foundation, determination of the flow and pressure conditions within the foundation is a geometrical problem. The distribution of flow and pressure may be depicted by an orthogonal family of curves forming the flow net. In a homogeneous foundation, this net is independent of the permeability and of the head and tail-water pressures. Its form is dependent only upon that of the base of the dam and of the ground under it. There is, therefore, a simple linear relation between the flow nets of model and prototype.

For such homogeneous foundations it may be shown[21] that if the flow net is so drawn as to divide the space within the foundation into approximate squares, and that if there are N vertical squares (flow lines) and M horizontal squares (equi-pressure lines) then the flow is equal, approximately, to:

---

[19] Structural Engr., Pittsburgh, Pa.

[20] Senior Engr., U. S. Engr. Office, St. Paul, Minn.

[21] "Hydraulik", von Ph. Forchheimer, p. 81, Teubner, Berlin, 1930.

$$Q = k \frac{N}{M} h \qquad \text{(15)}$$

in which, N and M are determined from the model, and k and h from the prototype. As a practical matter, only the near vicinity of the structure need be considered, as the flow diminishes rapidly with distance from the structure.

Mr. Harza's values of pressure are given in decimal fractions of head. The constant value of the tail-water pressure must be added in all cases to obtain the total uplift.

Flow nets for his various experiments have been determined by the writer from observations on glass-plate models.[22] The profile of a dam is cut from a thin impervious material and placed between two plates of glass. Flow beneath an actual dam is simulated by allowing water to pass between the plates. When a dye is introduced at points along the up-stream edge of the model, the lines of flow become visible. As bearing on the geometrical properties of the flow net, it was interesting to observe that when the head in the glass-plate model was reversed, the dye retraced its former path exactly.

Mr. Harza's treatment of the escape gradient and flotation is especially worthy of study. This problem is of especial interest in the design of sand dams, in which stability of the toe must be obtained. The failure of such sloping faces (as may well be the case, also, for foundation failures), is the result of a combination of flotation and erosion by water issuing farther up the slope. The author's critical "flotation gradient" appears in such cases to be an upper limit. The writer's observations on sand models lead him to believe that there are various degrees of flotation—a progressive "quickening" of the sand, as the escape gradient increases. This quickening renders the sand more susceptible to movement by forces other than flotation, as by erosive forces, or gravity on a slope, so that, for particular conditions, failure may occur for values less than the author's flotation gradient. Under such circumstances, surprising results may be obtained with a filter of coarse material over the affected area. For example, a sand model, 40 cm in height, with a top width of 25 cm, and 1 on 3 slopes, began to fail progressively with a head-water of about 33 cm and tail-water of about 15 cm. A layer of $\frac{1}{4}$-in. coarse sand, placed on the down-stream slope, made the model stable even with no tail-water.

The general subject of seepage flow offers a fascinating and almost unexplored field for research, both mathematical and experimental. Various investigators have made a beginning on the subject. As Mr. Harza suggests in Conclusion 12, some organization could render valuable service by digesting and correlating the available data, and by encouraging further research.

---

[22] *Civil Engineering*, January, 1934, p. 32.

JOEL D. JUSTIN,[23] M. AM. SOC. C. E. (by letter).—Darcy's law, as expressed in Equation (1) of this paper, may also be written $\frac{v}{K} = \frac{h}{l}$. Referring, furthermore, to Equation (6), F_c is shown to be approximately equal to 1 for the usual values of P and s. It should be borne in mind that F_c is the critical gradient at the point of egress and may be very different from the average hydraulic gradient. Hence, in addition to the manner in which the author has stated the equation, the critical condition for flotation at the toe may be stated as occurring when $\frac{v}{K} = F_c = 1$; or, it may be stated in words, as follows: The critical condition of flotation at the toe occurs when the velocity at the point of egress is approximately equal to the transmission constant for the given material.

The fact that $\frac{v}{K}$ is equal to $\frac{h}{l}$ enables the author to state that in "all mathematical or experimental investigations of base uplift pressure or flotation gradient at the toe, in homogeneous material, the transmission constant, K (Equation (1)), cancels out." As shown in this discussion, however, it would be equally feasible to eliminate $\frac{h}{l}$ from consideration and use the velocity and the transmission constant as the criterion.

On the basis of the author's equations, it might be assumed by the casual reader that all earth dams or all masonry dams on sand, of equal head and equal base width, on whatever kind of sand they are founded (as long as it is homogeneous) would be equally safe against the action of water passing under the dam, ignoring, for the sake of simplicity, any possible cut-offs. Such a conclusion, however, would not be justified because the "escape gradient", or the hydraulic gradient at the point of egress may bear practically no relation to the average hydraulic gradient. In actual cases, this "escape gradient" may be, and usually is, determined by local conditions at and near the toe.

By the method advanced in this paper, the quantity of leakage and its velocity would become largely an economic matter. With the statement as circumscribed by the author, the writer is in entire agreement. However, in any actual case, there are other factors which may make the quantity of discharge and its velocity through the foundation material, a matter of the most vital importance to the safety of the structure. Some of these factors have not been sufficiently emphasized.

Thus, for illustrative purposes, two extreme cases will be considered. In Fig. 19, the dam is assumed to be built of entirely impervious material on a foundation of very pervious material. The foundation material is entirely homogeneous and of great depth and the ground-water, before building the dam, is at a considerable depth and on a relatively steep slope; that is, due to the topography and geology of the site, the foundation is free draining.

---

[23] Cons. Engr., Philadelphia, Pa.

After the construction of the dam there is a large amount of seepage under it, due to the pervious nature of the material, but, because the foundation has sufficient carrying capacity, this seepage is conducted away from the site without any of it appearing at the surface of the ground near the toe.

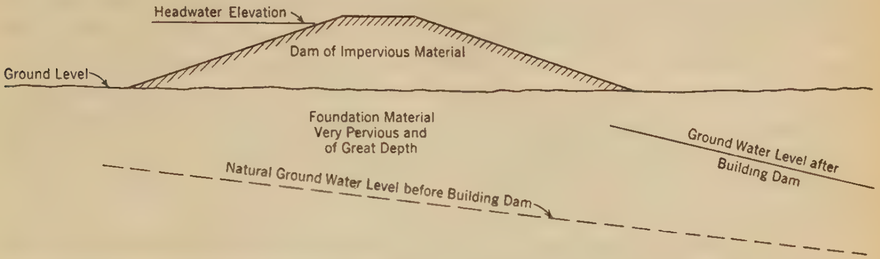

Headwater Elevation
Dam of Impervious Material
Ground Level
Foundation Material
Very Pervious and
of Great Depth
Ground Water Level after
Building Dam
Natural Ground Water Level before Building Dam

FIG. 19.—DAM OF IMPERVIOUS MATERIAL ON FOUNDATION OF VERY PERVIOUS MATERIAL OF GREAT DEPTH WITH LOW NATURAL GROUND-WATER LEVEL ON STEEP SLOPE.

In Fig. 20, the same conditions occur as in Fig. 19, except that ledge rock or other impervious strata occur relatively near the ground surface. The natural ground-water, before the building of the dam, is also near the surface and is on a relatively flat slope. When the dam in Fig. 20 is built and the head-water is raised, water issues from the ground surface near the toe of the dam. While Figs. 19 and 20 are entirely hypothetical, the conditions outlined measurably approach conditions at actual dams within the writer's experience.

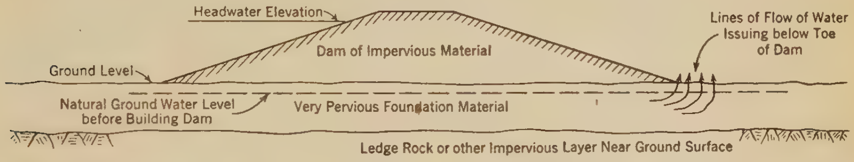

Headwater Elevation
Lines of Flow of Water
Issuing below Toe
of Dam
Dam of Impervious Material
Ground Level
Natural Ground Water Level
before Building Dam
Very Pervious Foundation Material
Ledge Rock or other Impervious Layer Near Ground Surface

FIG. 20.—DAM OF IMPERVIOUS MATERIAL ON FOUNDATION OF VERY PERVIOUS MATERIAL AS IN FIG. 19, BUT WITH LEDGE ROCK NEAR SURFACE AND NATURAL GROUND-WATER SURFACE NEAR GROUND LEVEL AND ON FLAT SLOPE.

There is no question as to the safety of the dam in Fig. 19 against any damage by the seepage of water under it, although the quantity of seepage carried away through the foundation may be considerable. On the other hand, the safety of the dam in Fig. 20 may be open to serious question. The "escape gradient" in Fig. 20 is evidently extremely steep, and its prediction would be difficult.

Attention is called again to the fact that this "escape gradient" may have practically no relationship to the average hydraulic gradient. In any actual case, conditions at and near the toe would determine very largely the actual "escape gradient". If the foundation material is extremely pervious (thus having a high transmission constant), the quantity of water carried would be large. Due to variations in the material, the flow may become concentrated and may issue from the ground below the toe with sufficient velocity to carry away the finer particles, thus starting the phenomenon commonly known as piping. In Fig. 20 conditions are favorable for the existence of a steep "escape gradient" and the possibility of "piping" is a real hazard, whereas in Fig. 19 there is no possibility of "piping".

As the "escape gradient" has almost no necessary relation to the average hydraulic gradient in most practical cases, it does not appear that safety against "flotation" or "piping" can be fully secured by a requirement that the base of the dam or that the length of the path of percolation shall be some given number of times the head, regardless of foundation conditions.

At any dam site where the foundation is sufficiently pervious so that large discharges through the foundation and high velocities may be anticipated, the possibility of "piping" requires careful consideration, and the same applies to earth dams built of very pervious material.

Thus, in the case of one dam built entirely of sand without a cut-off, it was considered that the anticipated seepage would not be of sufficient economic importance to justify an effective cut-off or diaphram. The anticipated discharge distributed throughout the cross-section gave a velocity low enough so that it did not appear that there was danger of moving any of the material. When the dam was first placed in service the actual seepage was about 20 cu ft per sec, which was a little less than that computed in advance of construction. However, the actual seepage was concentrated to a large extent which resulted in such high velocities at the egress that it carried with it the finer material from the surface. For a time it looked as if the dam might fail due to "piping" through the foundation at one of the abutments. Fortunately, the discharge finally worked back into gravel, too coarse to be moved. In the course of time, the seepage decreased considerably due, most likely, to the formation of a filter scum.

The possible effect of non-homogeneity in the material of the foundation and abutments, and the effect of ledge rock close to the surface causing a steep "escape gradient", had not (the event proved) been given sufficient consideration. In the case of a similar sand dam, built on a deep sand foundation, there was no trouble, and practically no seepage at the toe.

When considering the possible effect of seepage under dams on pervious foundations, it is of prime importance to the safety of the dam to consider the quantity of such seepage, its velocity through the material, and its possible velocity on leaving the material. In this connection the efficacy of cut-offs and of up-stream blankets merits consideration. Sometimes the natural blanket formed by a relatively thin layer of top soil will have great effect in reducing velocity and the amount of seepage. In at least one case, near disaster followed the removal of a part of such a natural blanket.

In cases where the escape of a large amount of seepage at the toes is anticipated, some kind of drainage system is indicated with the drains partly clogged by gravel and stone so as to form inverted filters, thus preventing any of the foundation material from being carried through. Heavy rock fills in the down-stream part of an earth dam with the foundation protected by an inverted filter may perform this function.

CHARLES TERZAGHI,[24] M. AM. SOC. C. E. (by letter).—In reading Mr. Harza's stimulating paper the writer notes with satisfaction that the escape gradient is beginning to receive the attention which it has deserved for a long time. Since the writer has used what Mr. Harza calls the "rational approach" to the problem for more than ten years, he wishes to call attention to some of the difficulties connected with a rational treatment of the problem and to the methods for handling them.

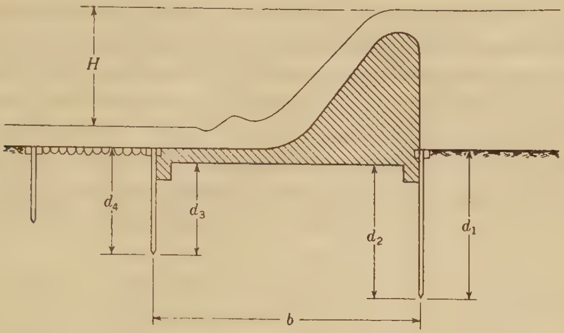


FIG. 21.

In 1922, when the writer derived[25] the equation for the flotation gradient (Equation (6) of the paper), he attempted at once to determine by experiment to what extent this factor can be used as a criterion for the safety of dams against piping. The results of experiments with perfectly homogeneous sands showed that the escape gradient at which piping occurred is practically equal to the flotation gradient. Thus, if the base of the dam rests on the horizontal surface of the sand without any sheet-piles intercepting the flow of seepage, as in the case shown in Fig. 5 of the paper, the escape gradient is equal to infinity irrespective of the creep-head ratio, $\frac{L}{H}$. Hence, if the escape and the flotation gradient are really identical, piping should occur at any hydraulic head. This actually took place, and, as a consequence, the writer was compelled to establish the base of his flat-bottom model dams at a certain depth below the surface, in order to be able to produce any hydraulic head at all.

For a single row of sheet-piles with a depth, d, the escape gradient is approximately $\frac{2}{3}\frac{H}{L} = \frac{2}{3}\frac{H}{2d}$. Piping occurred in the writer's tests at an escape gradient which was a trifle less than the flotation gradient. The results of some of these tests are presented elsewhere.[26] The fact that the critical escape gradient for single rows of sheet-piles is slightly below the flotation gradient has also been explained by the writer.[27] However, for practical purposes, the difference can be neglected.

---

[24] Dr. Ing.; Prof., Technische Hochschule, Vienna, Austria.

[25] "Der Grundbruch an Staumauern und seine Verhütung," von Charles Terzaghi, M. Am. Soc. C. E., *Die Wasserkraft*, December 15, 1922.

[26] "Modern Conceptions Concerning Foundation Engineering," by Charles Terzaghi, M. Am. Soc. C. E., *Journal*, Boston Soc. of Civ. Engrs., December, 1925, Fig. 17.

[27] "Erdbaumechanik," von Charles Terzaghi, M. Am. Soc. C. E., Vienna, 1925.

In order to demonstrate the practical consequences of these facts, consider the factor of safety against piping for the structure shown in Fig. 21. If a dam with such a cross-section is built on a perfectly homogeneous bed of sand, with no open space between the base and the sand, the escape gradient can easily be determined from the flow-net and expressed by a fraction, thus:

$$G = \frac{H}{\sigma\,(d_1 + d_2 + b + d_3 + d_4)} \quad \ldots\ldots\ldots\ldots\ldots\ldots (16)$$

in which, σ is a "shape factor" the value of which depends on the details of the section as shown by the formula. The shape factor represents the ratio between the average hydraulic gradient along the line of creep and the escape gradient. If the width, b, is not excessive compared with the depth of the sheet-piles, the shape factor is not far from unity. However, for unfavorable cross-sections, it can be very much smaller, and in the extreme case represented by Fig. 5 in Mr. Harza's paper it becomes equal to zero. The flotation gradient is $(1 - P)\ (s - 1)$. Hence, the theoretical factor of safety against piping is:

$$f = (1 - P)\ (s - 1)\ \sigma\ \frac{d_1 + d_2 + b + d_3 + d_4}{H} \quad \ldots\ldots\ldots\ldots (17)$$

The product, $(1 - P)\ (s - 1)$, has an average value of unity. The last factor represents the "percolation coefficient" of Bligh. Thus, the following conclusion can be drawn from Equation (17): For a given percolation coefficient, the factor of safety of a dam supported by a perfectly uniform sand depends on the value of the shape-factor, σ, which varies between 0 and more than 1.5.

In contrast to the assumptions on which Equation (17) is based, natural soil deposits are always more or less stratified and their average coefficient of permeability, k_v, in a vertical direction is always much smaller than the coefficient, k_h, in a horizontal direction. From a great number of permeability tests made in connection with the design of dams the writer has reached the conclusion that the value of the ratio, $\beta = \frac{k_h}{k_v}$, can range between any value from about 2 to more than 10. This important and decidedly disagreeable fact prevented the writer from attempting to establish any general rules concerning safety against piping. If the water percolates through the non-homogeneous deposit in a vertical direction, the hydraulic gradient required for maintaining a definite speed of flow is β times greater than that for the flow with the same speed in a horizontal direction. From the theory of seepage it is known that an increased permeability of the soil in an horizontal direction reduces the "weight" of the creep in that direction $\frac{1}{\sqrt{\beta}}$ times. Taking this fact into consideration, the factor of safety against piping, in a non-homogeneous soil, is obtained from Equation (17), in the following form:

$$f = (1 - P)\ (s - 1)\ \sigma\ \frac{d_1 + d_2 + \frac{1}{\sqrt{\beta}}\,b + d_3 + d_4}{H} \quad \ldots\ldots\ldots (18)$$

The last factor in Equation (18) represents the "weighted creep" factor which was proposed in the paper[28] entitled "Security from Under-Seepage—Masonry Dams on Earth Foundations," by E. W. Lane, M. Am. Soc. C. E. Mr. Lane assumes that $\sqrt{\beta}$ is equal to 3. Equation (18) shows that the shape factor, varying between 0 and more than 1.5, may have an enormous influence on the factor of safety of the dam. This formula also shows that, for the same cross-section of the foundation of the dam, the weighted creep depends on the value of $\sqrt{\beta}$, which may vary between 1.4 and 3.2. For this reason the writer welcomes Mr. Harza's suggestion that the analysis of existing dams be extended to include the influence of the cross-section of the foundation on the escape gradient and other vital factors.

If the soil profile is relatively simple, the ratio, β, can be estimated from the results of permeability tests. In this case the escape gradient can be determined from the known laws of the flow of water through non-homogeneous soils. According to methods developed for this purpose, the non-homogeneous material is replaced by a homogeneous material with a coefficient of permeability equal to $\sqrt{k_v \, k_h}$; and the horizontal scale of the section is reduced by $\sqrt{\frac{k_v}{k_h}}$. Although the writer always makes related investigations graphically, they may also be made by means of the electric analogy method proposed by Mr. Harza.

In most cases the failure of dams begins with the formation of springs which discharge a mixture of soil and water. For this reason, as soon as the writer recognized the importance of the flotation gradient, he concentrated his attention on the practical possibilities connected with reversed filters.[25] Since the effect of a filter on the factor of safety of a dam cannot be estimated by means of the formula for the flotation gradient, he was obliged to develop a theory for computing the forces which are exerted by the seepage water upon the soil beneath the filter. Let i = the hydraulic gradient in the direction of the lines of flow; s_1 = the specific gravity of water; and p_w = the force exerted by the flowing water in the direction of the lines of flow. Between these quantities the following relation was found:[25]

$$p_w = i \, s_1 \dots\dots\dots\dots\dots\dots (19)$$

Mr. Harza's Conclusion 3 expresses Equation (19) in words.

One of the essential requirements for making the filter effective is to prevent the flotation of the covered soil, by means of superimposed loads or by rigid obstacles. The results of tests related to this problem were presented in the paper cited previously.[26] Various possibilities for fulfilling this requirement are indicated in Fig. 22, which is reproduced from a priority claim filed by the writer at the Austrian Patent Office in 1922. Figs. 23 and 24 illustrate the practical application of the principles embodied in Fig. 22. Fig. 23 represents a section through a bear-trap weir.[29] The surcharge consists of the weight of the water enclosed within the movable part of the construction. Observations in the inspection wells showed that the quantity of seepage discharged through the vents decreased rapidly owing to the silting of the bottom of the storage reservoir. The dam shown in Fig. 23 is an example of Type *d* in Fig. 22. The design of this important structure was made by Vattenbyggnadsbyran (V. B. B.), Consulting Engineers, of Stockholm, Sweden. The great length of the apron was required, in order to insure the structure against sliding on its base which consisted exclusively of stratified clay to a great depth. The surcharge is produced by the weight of the water above the rear apron and by that of the dam itself. The efficiency of the filter was increased by several rows of filter wells F_1 to F_3 (Fig. 24).

---

[28] See p. 1235.

[29] "Die Halleiner Wasserkraftstufe im Zuge du Salzachregulierung," von J. Pfletschinger, *Die Wasserwirtschaft*, June 5, 1929.

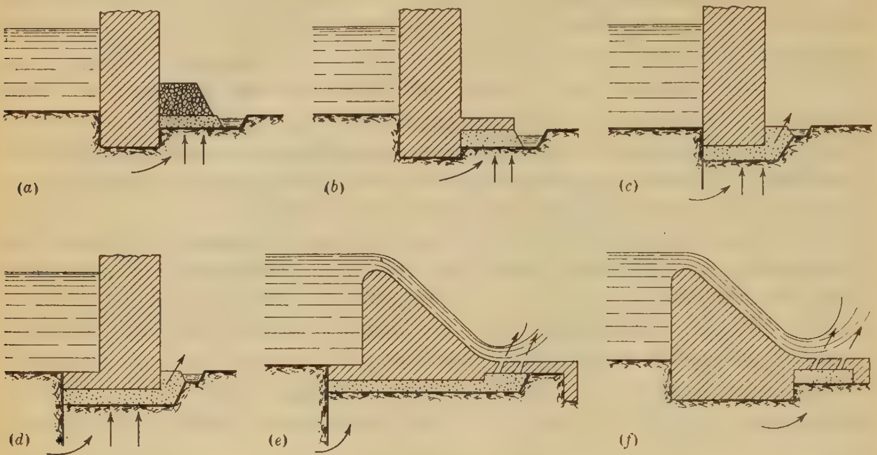


FIG. 22.

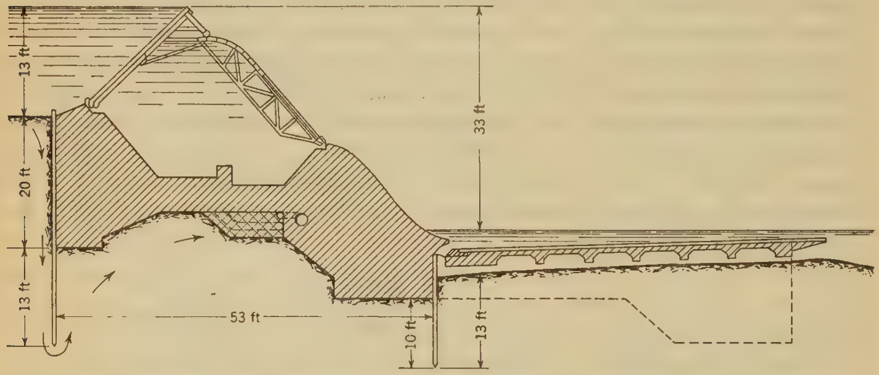


FIG. 23.

In North Africa, a loaded reverse filter covering an area of about 400 000 sq ft has been designed for the purpose of preventing piping from a storage reservoir with a depth of 160 ft.

The second requirement to be satisfied by a reverse filter is that the voids of the filter must be small enough to prevent a loss of soil without interfering appreciably with the discharge of the water. In his earlier designs, the writer specified that the average diameter of the grains of the lowest layer of the filter should be approximately five times as large as that of the largest grains of the covered soil. However, as his experience increased, he found it advisable to determine, experimentally, in every case the most satisfactory grain size for the bottom layer of the filter. Each one of the succeeding layers is about five times coarser than the preceding one.

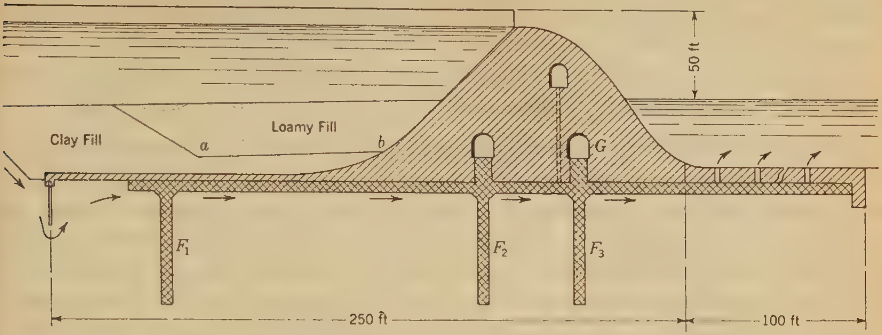


FIG. 24.

Conclusion 11 of Mr. Harza's paper certainly should be emphasized. Owing to the great number of purely empirical factors involved, any theory of dam design urgently requires supplementing by the results of observations on existing structures.

HARRY H. HATCH,[30] M. AM. SOC. C. E. (by letter).—Undoubtedly there is still considerable to be done in laboratory and field research, especially in obtaining dependable data from actual structures. The author should be commended for calling attention to a new field of investigation for the safety of masonry dams built on sand foundations.

One more factor may be added to those cited by the author effecting the value of the coefficient of permeability, K, in Equation (1), namely, the shape or the slope of the gradation curve below the effective size limit. It is known that the flatter the slope of this section of the gradation curve the smaller will be the value of K. Sample materials of the same physical characteristics with the same effective size and with almost identical particle sizes and gradation curves above the effective size limit, but with different slope gradation curves below the effective size, showed a variation in the values of K of as much as 300 per cent.

Confining his study to sand, Mr. Harza narrows the field of application of this important investigation. What is his definition of sand? In other words, what is the lower limit of the particle sizes of sand considered by the author? Why not extend the analysis so that the study may embody soils in general?

---

[30] Engr. in Chg., Cobble Mountain Reservoir, Springfield Water-Works, Springfield, Mass.

Soil is being studied with special interest from almost every angle; and appreciable progress has been made and considerable information is available on the subject. Had the author proposed his study for soils in general, it is believed that the principles advocated would have had a broader field of application, and that the new wealth of knowledge pertaining to the science of soils could be used to advantage. The principle introduced in the paper has a broader application and usefulness than if it were confined to sand alone.

Donald J. Hebert,[31] Jun. Am. Soc. C. E. (by letter).—The theory upon which the electric analogy method is based, has been adequately discussed in this paper. Before any attempt is made to adopt it as a basis for design, as Mr. Harza has attempted to do, an examination of the many curves should be made. The writer has found[32] that the line representing the pressures as predicted by an electric analogy analysis is for the most part no closer to the observed values than the line obtained by weighted creep computations. The discrepancy between actual and predicted pressures in the case of the analogy is particularly evident in the down-stream part of the under profile. Some of this discrepancy may be due to the uncertainty with which the end of creep was located. In this region the use of the Bligh method is questionable because it under-estimated the actual pressures. The electric analogy, on the other hand, might lead to an unnecessarily heavy and uneconomical design.

Before the electric analogy method can be used as a basis for rational design, it needs to be extended considerably. The technique of the method must permit at least an approximation of the various geological features of any particular subsoil. As a prelude to such an extension of the analogy technique, there is need for considerable progress in the methods for exploring the subsoil and for locating the various features which affect the flow of water through it. Charles Terzaghi, M. Am. Soc. C. E., has emphasized[33] this need by pointing out the possibility that certain geological features, known as "minor details," are often factors which determine that stability of a structure when built on a porous foundation.

Until the point is reached at which it is possible to investigate all the geological features of a subsoil and to reproduce them in an electrical set-up, the analogy method must be used with caution.

C. A. Mockmore,[34] M. Am. Soc. C. E., and John W. Dougherty,[35] Jun. Am. Soc. C. E. (by letter).—Since this paper appeared the writers have been interested in the verification of the formula (Equation (6)) given by Mr. Harza for the "flotation gradient." In keeping with the suggestion offered in the "Synopsis" that "these theoretical laws are to be used as scientific nuclei around which to build future experimental data," the writers have made some experiments at the Oregon State College with water flowing vertically through beds of different kinds of granular materials, and, in general, have succeeded in finding this formula valid.

---

[31] Denver, Colo.

[32] See Fig. 5, p. 1288.

[33] *Technical Publication No. 215*, Am. Inst. of Min. and Metallurgical Engrs.

[34] Head of Civ. Eng. Dept., Oregon State Coll., Corvallis, Ore.

[35] Portland, Ore.

Mr. Harza cites Professor Terzaghi's experiments in which the measured internal pressure to cause flotation always equalled or exceeded the theoretical by ratios varying from 1.0 to 1.24. This, the writers have been unable to verify, except as will be shown subsequently in the case where the granular material was lead shot.

A diagrammatical sketch of the apparatus used by the writers to simulate conditions described in Mr. Harza's Fig. 3 (*d*) is presented in Fig. 25. The same apparatus is shown in operation in Fig. 26. The sides of the box containing the sand were made of glass to assist in the study of the sand movement. In Fig. 25, the water entered at Point *A* and spread into a series of pipes, *B*. These pipes were formed in a series of three rows across the length of the tank and were perforated with small holes to permit even distribution of the water over the bottom of the tank in the space under the sand. The water could then flow vertically upward through the bed of sand

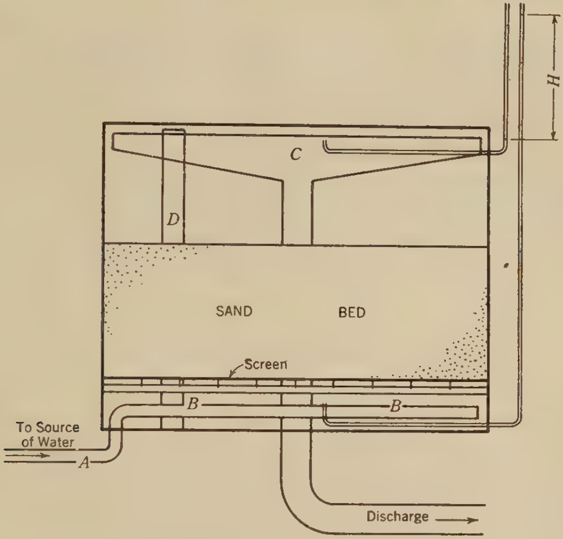


FIG. 25.—APPARATUS FOR DETERMINATION OF LOST HEAD IN FLOW OF WATER THROUGH SAND.

TABLE 4.—CHARACTERISTICS OF TEST SAMPLES

Definition (1)	Sample No. 1, white filter sand (2)	Sample No. 2, Columbia River sand (3)	Sample No. 3, Willamette River sand (4)	Sample No. 4, lead shot (5)	Definition (1)	Sample No. 1, white filter sand (2)	Sample No. 2, Columbia River sand (3)	Sample No. 3, Willamette River sand (4)	Sample No. 4, lead shot (5)
Average specific gravity........	2.59	2.69	2.59	11.36	Area of bed, in square feet....	1.97	1.97	1.97	0.196
Porosity (a percentage).......	38.9	37.6	32.8	43.2	Water temperature, in degrees Fahrenheit...	50	52	52	....
Depth of bed, in inches.........	6.7	6.4	7.8	3.5					

and proceed to the drains, *C* and *D*. The overflow, *C*, had a weir crest length of approximately 4 ft, and was arranged so that the water could be discharged through a vertical pipe which led to a weighing scales. Drain *D* was merely an overflow pipe to care for excess flow beyond the capacity of Drain *C*.

The characteristics of the samples and the test set-up are listed in Table 4. The sample to be tested was supported by a 50-mesh screen which rested in turn on a 4-mesh screen. This 4-mesh screen rested upon heavy tin strips, ½ in. high and extending across the box. The piezometers shown in Fig. 25 afforded an easy means of measuring the loss of head in flow through the sand. This set-up differed from Mr. Harza's Fig. 3 (*d*) in that his piezometer connection, *B*, was not above the upper surface of the sand bed.

With the sample in place the water was released slowly into the box, and at various readings of the lost head, *h*, the time necessary for 100 lb or 200 lb of water to flow through the material was recorded. The quantity of water flowing was increased after each measurement of weight and time, until the sand expanded and "boiling" or "piping" occurred. The lost head at this instant was difficult to measure because of its tendency to lower just as piping occurred (see Table 5).

TABLE 5.—LOSS OF HEAD IN FLOW OF WATER THROUGH SAND BEDS

Head loss, in inches (1)	Flow of water, in cubic feet per second (2)	Head loss, in inches (1)	Flow of water, in cubic feet per second (2)	Head loss, in inches (1)	Flow of water, in cubic feet per second (2)	Head loss, in inches (1)	Flow of water, in cubic feet per second (2)
(*a*) SAMPLE NO. 1		(*a*) SAMPLE NO. 1 (*Continued*)		(*c*) SAMPLE NO. 3		(*d*) SAMPLE NO. 4	
1.2	0.0018	5.35	0.0195¶	2.60	0.0069	1.92	0.0182
1.8	0.0031	5.45	0.0216	3.45	0.0096‖	3.42	0.0256
2.9	0.0052	5.50	0.0258	4.15	0.0119	4.77	0.0317
3.5	0.0064			4.85	0.0145	6.72	0.0386
4.0	0.0073	(*b*) SAMPLE NO. 2		5.30	0.0170¶	9.54	0.0475
4.5	0.0083			5.80	0.0200	12.11	0.0534
4.9	0.0092	1.8	0.0340	6.30	0.0219	13.92	0.0582
5.25	0.0098	3.3	0.0561	6.50	......*	17.16	0.0641
5.5	......*	4.2	0.0653	6.85	0.0276	18.11	0.0682
5.2†	0.0093	4.7	0.0711	6.85‡	0.0333	21.00	0.0697*
5.45	0.0118	4.9	0.0745	6.85§	0.0390	21.60	0.0763
5.45	0.0149	5.2	......*	6.85	0.0470	21.50	0.0942
5.45	0.0172	....	......	6.85	0.0695	23.05	0.1335

* Boiling began. † Dropped to 5.2 with continued boiling. ‡ Dropped to 6.85 from 7.2. § Dropped to 6.85 from 7.4. ‖ Small movement of fines. ¶ Good separation completed.

Mr. Harza mentions the possibility of conclusions, reached by testing homogeneous materials, which do not apply to the conditions in practice where foundation materials are not homogeneous. In fact, under laboratory control the conditions are none too good even when the sand sample is fairly well graded, and has had a chance to be separated by the flowing water.

The writers thought it best to make sieve analyses of the several granular materials after they had been tested in the apparatus. This would wash out the dust and fine clay that might be there, but it would also tend to cause separation of the fines from the coarser materials and make sampling for sieve analysis more difficult. However, with as much caution as possible after

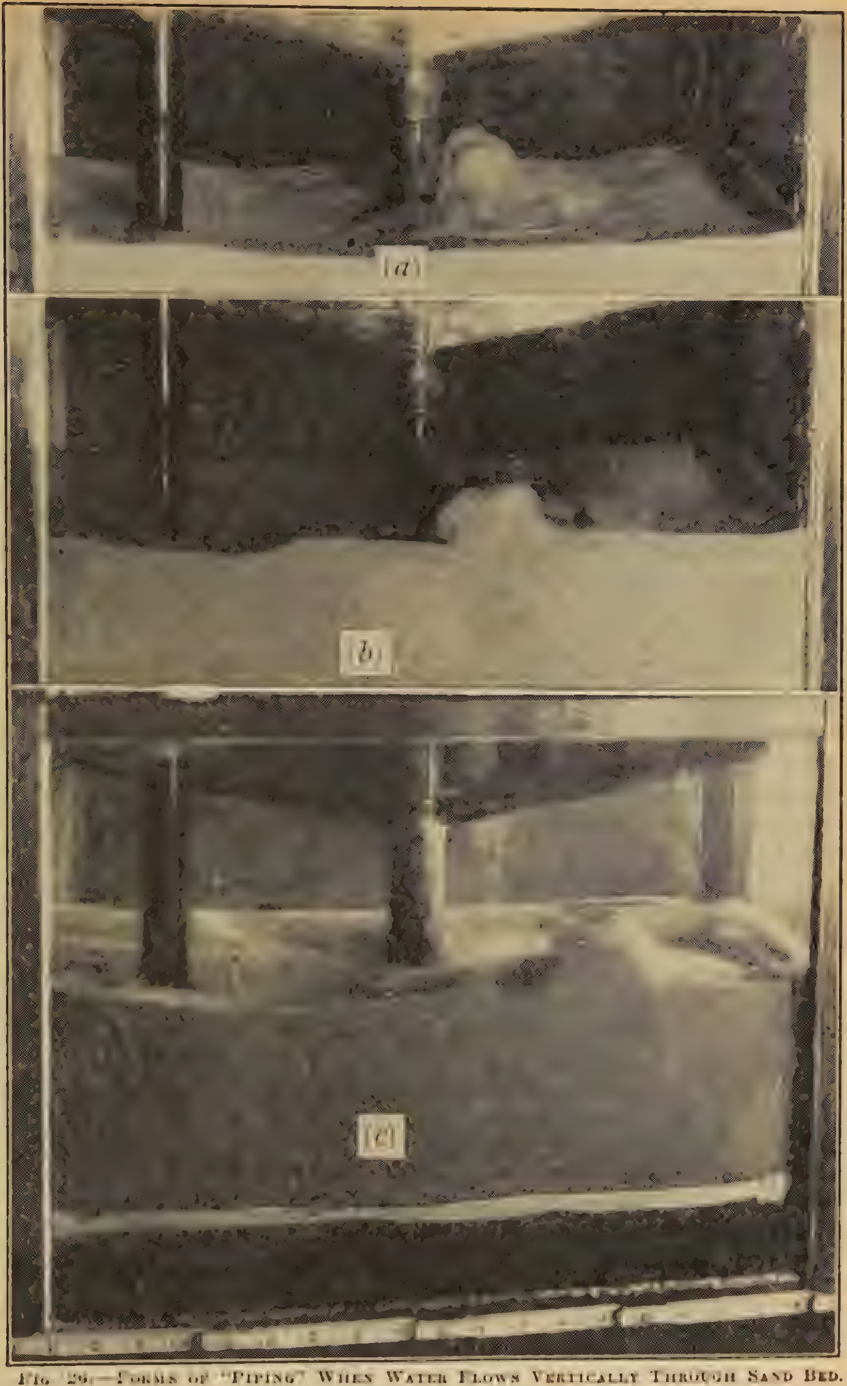


FIG. 29.—FORMS OF "PIPING" WHEN WATER FLOWS VERTICALLY THROUGH SAND BED.

each test for loss of head, a 2-kg or 3-kg sample of the material was taken and dried. This dried material was used in the American Society for Testing Materials standard tests for the determination of the apparent specific gravity and the approximate percentage of voids in inundated fine aggregate. The same tests were used in the case of the lead shot, although the tests are not actually standard when such a heavy material is used.

As shown in Table 4, Sample No 1 consisted of a fairly uniform white filter sand commonly used in sand beds for water filters. Sample No. 2 consisted of a fairly uniform Columbia River sand which was furnished by the U. S. Engineer Office, at Portland, Ore. Sample No. 3 consisted of "river run" Willamette River sand from which nearly all material except the sizes between the 6 and 28-mesh sieves was eliminated. Sample No. 4 was a nominal "BB" lead shot having an average diameter of 0.1733 in.

The views in Fig. 26 were taken when the test box was loaded with Sample No. 1. After the "piping" or the "boiling" appeared, it would move from place to place in a most fortuitous manner. At one instant it would be in the center of the box, as at Point *A*, in Fig. 26; in another instant, it would migrate to some such place as Point *B*, or Point *C*. When Fig. 26(*c*) was taken, the boil was moving toward the right. The operator's hand could penetrate the sand easily, as if it were in suspension, while the boiling would continue.

Point is made of this phenomenon in connection with Mr. Harza's statement that "the conclusion is justified that under all conditions" the increment force exerted by seeping water against a granular material acts "as on a solid piston, although the pressure is actually applied through the material instead of against an impervious face." This is evidently true as long as the rate of flow is not high enough to induce piping. After "piping" begins, the lost head, h, decreases. This indicates that the "piston" effect is limited to the area of the sand bed which is not yet piping or boiling (see Fig. 26). The points at which boiling began in each case, are indicated by footnote references in Table 5.

The apparatus for testing the lead shot differed from that shown in Fig. 26, only in the fact that a 6-in. transparent pyralin pipe was used instead of the glass-sided box. This made the cross-section of the bed of shot smaller than that of the sand beds.

Mr. Harza cites Professor Terzaghi's experiments on varied materials in which the measured internal pressure to cause flotation "always equalled or

TABLE 6.—RATIOS OF MEASURED INTERNAL PRESSURE TO THEORETICAL PRESSURES

Material	$\frac{h}{l}$ (Actual)	$(1 - P)\ (s - 1)$ (Theoretical)	Ratio of actual to theoretical
White filter sand	0.814	0.97	0.840
Columbia River sand	0.812	1.05	0.774
Willamette River sand	0.878	1.07	0.822
Lead shot	6.000	5.88	1.020

exceeded the theoretical, by ratios varying from 1.0 to 1.24." The writers have not been able to secure values as high as 1.0, except in the case of the lead shot. Assuming the actual gradient as h, and the theoretical gradient as $(1 - P)\ (s - 1)$, the ratios should be about as shown in Table 6. Sometimes in the case of the lead shot, the boiling seemed to begin when the lost head was somewhat less than 21.0 in., although an exact value was difficult to determine. If the lost head were actually 20.5 in., instead of 21.0 in., the ratio, $\frac{h}{l}$, would become 5.80, which checks very closely with the value for $(1 - P)\ (s - 1)$.

The data given in Table 5 may be shown graphically, if ratios of the actual to the theoretical flotation gradients are used as ordinates, and the rate of flow per square foot of sand bed is used as abscissas. When this was done (curves not published) the point at which "flotation" changed to "piping" was found to be rather indefinite. For Sample No. 2 the head, h, rose to 7.2 in. before "piping" began, and then it dropped to 6.85 in. This same phenomenon did not appear when the lead shot was used. These experiments indicate that the ratio of the measured to the theoretical head to cause flotation would be less than 1.0, rather than the higher value cited by Mr. Harza.

While testing Sample No. 3, which was not a very uniform material, it was noticed that at the incipient stage of flotation the finer portions of the sand would be raised to the top of the bed; then "boiling" or "piping" would take place in this fine aggregate before it would in the coarser material. Vigorous boiling would keep the sand bed quite well mixed, but the incipient stage of flotation would permit the smaller grains of sand to be raised above the larger. If this is compared with the critical conditions that Mr. Harza describes in Fig. 5 as cross-flow toward the easiest point of escape at the toe of a dam, it seems reasonable to conclude that this fine material might be easily washed away.

In making the tests described in this discussion, the writers have endeavored experimentally to verify the validity of Mr. Harza's "flotation" formula, Equation (6). In general, it is believed that the formula is correct, but that considerable care should be exercised in applying it to practical problems.

L. F. HARZA,[36] M. AM. SOC. C. E. (by letter).—It is quite disappointing that no discussion whatever was drawn out on what is believed to be an entirely novel method of estimating the "transmission constant," K, in Darcy's formula from results of test-pit or test-well pumping in undisturbed material, interpreted by electric analogy as shown at the bottom of Fig. 7 and discussed in the paragraph, "Quantity of Seepage." The advantages of determining this constant in place, instead of from laboratory samples in a percolation tank, is obvious. The latter test is vitiated by disturbing the material in digging it out and re-packing it into a tank. Applications of the new method at the Bonneville Dam, in Oregon, appear to corroborate the principle roughly. For its application, the draw-down of the test pit should be only enough for accurate measurement in order to avoid robbing the fines from the surrounding material and thus increasing the yield and apparent transmission constant. It is to be hoped that field applications of this principle will be extensive enough to ascertain and verify its practical value.

---

[36] Cons. Engr., and Pres., Harza Eng. Co., Chicago, Ill.

As to quantity of seepage under a dam, Mr. Hill has stated a rough approximation in his Equation (15). It can be stated further that the total seepage, per unit length of structure and per unit head, is theoretically independent of the size of the dam and dependent only upon its shape. Thus, in Fig. 7, if the rectangle, $X' Y'$, in the salt bath is doubled in both dimensions, retaining the same thickness or depth of fluid, the resistance will be unchanged. This is true of all geometrically similar figures, such as Fig. 6, regardless of size.

The resistance of a square per unit thickness is fixed regardless of dimensions. Therefore, it would be possible to determine and tabulate the conductivity (or its converse resistance) per unit thickness of each geometrical shape that might occur under a dam as bounded by the river bed both up stream and down stream, the base of the structure, and by bed-rock or other impervious bottom. This resistance should be expressed as a ratio to the resistance of a square of equal thickness of the same material. For example, the writer has found from the analogy tray that the ratio of total resistance beneath a flat base section, flush with the sand surface as in Fig. 5, to the resistance of a square is closely expressed by the formula:

$$\frac{R}{r} = 1 + 0.5 \frac{b}{D} \ldots\ldots\ldots\ldots\ldots\ldots (20)$$

in which R is the total resistance; r is the resistance of a square; b is the base width; and, D is the depth to rock. Similar treatment could be extended to apply to bases of any shape.

Mr. Hill correctly observes that material is more easily moved by erosive forces even before the uplift force reaches the critical value. This was less clearly stated by the writer in Conclusion 5, and is attributed to reduction in effective weight of the material which results from any upward flow, whether or not it is enough to overcome the weight of the material entirely.

Mr. Justin proposes a revision of theory based upon the assumption that the critical gradient, F_c, in Equation (6), is always approximately unity. The facts do not justify this assumption. The specific gravity, S, may vary in Nature roughly between about 1.8 and 2.8 and the voids between about 0.2 and 0.4 in the range from light, diffused river-bed silt to heavy, well-graded material, representing weights per cubic foot from 68 to 140 lb. These values would result in a range of critical gradient, F_c, between 1.44 and 0.48, or 3 to 1. The range is too great to justify the assumption of unity.

In his second paragraph Mr. Justin uses the fact that $\frac{v}{K}$ is equal to $\frac{h}{l}$ to argue that the former ratio can be used as well as the latter as the criterion for the critical condition and that v and K, therefore, do not cancel out in the stability equation. This reasoning seems incorrect; K itself is the velocity under a hydraulic gradient of unity. The expression, $\frac{v}{K}$, therefore, is the ratio of two velocities and is an abstract number. The number is, in fact, the ratio of actual velocity to that which would occur with a gradient of unity and is in proportion to the gradients, or to $\frac{h}{l} \div \frac{l}{l}$; or, $\frac{h}{l}$, leaving the principle unchanged.

In his third paragraph Mr. Justin misinterprets the writer's paper to mean, "that all earth dams or masonry dams on sand, of equal head and base width, on whatever kind of material founded (as long as it is homogeneous), would be equally safe against the action of water passing under the dam." This interpretation overlooks the principle point in the paper, which only indicates that the escape gradient would be equal in each case of homogenous foundation regardless of material. This is quite different from stating that they would be equally safe regardless of material, because as previously shown, there is a range of about 3 to 1 in the gradient which will cause flotation in different classes of material and, therefore, an equal range in permissible escape gradients. If one dam were otherwise comparable with another, but on a kind of foundation material unable to resist as large an escape gradient, something would need to be done to reduce the escape gradient to safe limits, such as the construction of a longer base, or cut-offs, or alternately the material at the point of escape could be removed and replaced with a material that would resist a higher gradient—an inverted filter, possibly.

Furthermore, Mr. Justin implies in the same paragraph, and again in his ninth and tenth paragraphs (although perhaps not so intended) that the writer claimed the hydraulic gradient at the point of egress to be equal to, or to have a definite relation to, the average hydraulic gradient. On the contrary, a large proportion of the paper is devoted to an effort to break down this fallacy which necessarily forms the basis of Bligh's method, and to the determination of the actual escape gradient for the actual profile of the base of structure in its relation to bed-rock profile. To do so the actual profile and stratification of materials are reproduced in the analogy tray as closely as known. With such treatment, the condition illustrated in Mr. Justin's Fig. 20, would be revealed and could be remedied, as previously suggested, within the limits of accuracy in which the conditions are known. No remedy could apply more accurately. A factor of safety would be needed to allow for inaccuracies in knowledge of conditions both general and at the point of escape.

There seems to be a common conception among several discussers that movement of material at the toe is the result of velocity and that the expected velocity must first be computed to compare with the velocity that the toe material will safely withstand. This seems to be a case of "putting the cart before the horse," although admittedly the question has some resemblance to the question of prior existence of the hen or the egg. The hydraulic gradient fundamentally arises from the gradual loss or dissipation through the route of flow of the head applied to the dam. This loss of head creates both the flow and the differential pressure on the sand grains, as analyzed in the paper, and it is this differential pressure or the rate of loss of head (or, in other words, the steepness of the hydraulic gradient) that determines whether the material will be floated.

However, if one wants to conceive of the velocity as the origin of pressure, he will arrive at a certain computed loss of head per unit distance for each velocity in a given material, and the safe velocity will be that which will not produce a sufficiently rapid loss of head or internal pressure to move the material. Then the problem gets back to the same basis as before; that is, the hydraulic gradient. The writer takes the position that coarse, heavy material will pass much more water than a fine, light material without causing flotation because the friction is less and a higher velocity is required to cause enough loss of head to produce the critical gradient, and that these differences in velocity are secondary and disappear from any rational analysis, being useful only in determining the gradient which can best be determined directly.

Professor Terzaghi draws the conclusion from Equation (17) that the factor of safety is proportional to the shape factor, σ. Although it is true that $(1 - P)$ $(s - 1)$ has an average value of unity, nevertheless it has a range of about 3 to 1 which the writer believes is too great to ignore. This would indicate that his rule should be stated thus: "For a given percolation coefficient, the factor of safety of a dam, supported by a perfectly uniform sand, depends upon the value of the product of the shape factor and the flotation gradient of the material."

If β is the ratio, $\frac{k_h}{k_v}$, as used by Professor Terzaghi, the hydraulic gradient required for maintaining a definite vertical speed of flow is β times greater than that required for maintaining the same horizontal speed of flow as stated by Professor Terzaghi. However, the speed of horizontal flow is governed as much by the available cross-section of flow channel as by k_h. After water passes around the bottom of the up-stream cut-off its path of flow will tend to diffuse widely in homogeneous material. It is this diffusion, resulting in low velocity, which causes the low rate of dissipation of head or low hydraulic gradient in horizontal flow. The writer would expect stratification in general to discourage such a wide diffusion and tend to concentrate the horizontal flow in some relatively porous stratum near the bottom of the cut-off. This would tend toward a higher horizontal velocity and hence a greater rate of head loss or steeper hydraulic gradient for horizontal flow in stratified than in homogeneous material. How closely this hydraulic grade line along the line of creep in stratified material corresponds with that for homogeneous material depends upon whether the proportionate increase is as great for horizontal as for vertical flow caused by stratification. In either case horizontal "creep resistance" would still seem to be less than the vertical by some variable ratio. The net result of stratification would then be merely to reduce the total seepage loss as compared with that of homogeneous material.

Professor Terzaghi's extensive use of the filter principle in Figs. 22, 23, and 24, emphasizes what the writer has long contended, namely, that free use of properly constructed drains under a dam can eliminate any relation between Bligh's "percolation factor" and safety, relegating the former to the status only of an indicator of amount of seepage loss, which is an economic question.

Mr. Hatch has brought out the fact that the writer used the too restricted term of "sand" foundation in his paper. Unfortunately, the English language supplies no inclusive word for granular material, and the writer used sand in such a generic manner. The language is in need of an inclusive word. In excavation language, the words "earth," "dirt," or "common" excavation are often used to mean all material that is not rock. The writer would like to suggest the word, "granulum."

The experiments reported by Messrs. Mockmore and Dougherty are very interesting and valuable. If the critical or flotation gradient in material of mixed sizes is not correctly expressed by $(1 - P)\ (s - 1)$ the fact should be known. Inasmuch as the experimenters admit that "the lost head at this instant was difficult to measure because of its tendency to lower just as piping occurred," it is possible that some of the discrepancy is explained by errors of observation. Any failure to record the head at exactly the right instant would tend to reduce the ratio in the last column of Table 6 below unity. It is suggested that any future experiments of this kind be provided with some type of indicating or recording "maximum" gauge so that the record of maximum pressure will be preserved without the necessity of recording it at a definite instant.

The fact that the lead shot gives the truest confirmation of theoretical principle (Table 6) might be accounted for by the larger head loss and the consequently higher degree of accuracy with which it could be measured. On the other hand, it was the only material of uniform grain size, which might imply that a graded material follows a different law. This should be verified by further research.

Quite recently the writer has been in correspondence with Mr. F. F. Haigh who reports experiments made in India.[37] His experiments consisted of testing to failure a series of models simulating the simpler forms of construction; namely, a single line of 2-in. piles, a 4-in. flush floor with a 2½-in. and a 1-in. cut-off, and combinations of these forms. As a result of his work Mr. Haigh offers the following coefficients to be introduced in the writer's formulas for the theoretical failure head to give the head determined experimentally:

Model	Coefficient
Line of 2-in piles........................	1.19
4-in. floor with a 2½-in. pile cut-off..........	1.04
4-in. floor with a 1-in. pile cut-off..........	1.20

These coefficients would seem to contradict the results obtained by Messrs. Mockmore and Dougherty to the extent that they can be compared.

Mr. Hebert is correct in warning that "the analogy method must be used with caution." The same is true of the "weighted-creep" method. All the unknown "minor details" of geological conditions of the foundation material disturb the accuracy of both methods. The former method requires an assumed factor of safety to compensate for conditions that cannot be analyzed mathematically and cannot be revealed by explorations and simulated in the analogy tray. The latter method involves an equivalent factor of safety, in the assumed ratio of horizontal to vertical creep resistance (which cannot have a fixed value as shown by rational analysis) and in the assumed percolation coefficient which demands a greater exertion of judgment than the first method because this coefficient must embody not only the fortuitous features, but the basic physical principles as well. As a general principle it may be stated that of several methods, that which leaves the least to assumptions based upon individual judgment, is the best.

---

[37] *Paper No. 182*, Punjab Irrig. Congress.

Apparently Mr. Hebert refers to Fig. 5[38] of his discussion of the paper[39] by E. W. Lane, M. Am. Soc. C. E., as disqualifying the electric analogy. On the contrary, these curves are closely corroborative and very re-assuring to the analogy method as far as they go (with the exception of one line) in case the right diagram labeled: "(*b*) 100% Creep Taken at Beginning of Downstream Apron," is chosen. Reference to Fig. 17 of the writer's paper will demonstrate clearly that 100% creep should be so taken if any such assumption is to be made which is quite unnecessary. If the analogy model were to reproduce the true foundation profile with its individual apron construction and drainage features, there would be no need and no possibility of two such different assumptions as in the two sets of curves, Fig. 5(*a*) and Fig. 5(*b*).[38]

Thus, Fig. 17 shows that the full typical shape of the pressure curve is completed at the first point of drainage as if it were the end of the structure, which indicates that the right diagram, Fig. 5(*b*)[38] is applicable approximately. Unfortunately, the five curves of Fig. 5(*b*),[38] applying to the field-recorded water pressure, are not labeled so that explanation of the one which departs widely from the electric analogy line is not possible. Conceivably, it represents a late curve after the establishment of a filter skin in the reservoir ahead of the dam.

In closing, it should be emphasized again that progress in hydraulics is only made by using what is known to be correct in basic principle and depending upon empirical coefficients only for the necessarily unknown and fortuitous features of a problem, such as non-homogeneity, effect of filter skin, rain and wave erosion, etc.

---

[38] See p. 1288.

[39] "Security from Under-Seepage—Masonry Dams on Earth Foundations," see p. 1235.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 1921

THE DEVELOPMENT OF THE AMERICAN SOCIETY OF CIVIL ENGINEERS

ADDRESS AT THE ANNUAL CONVENTION, LOS ANGELES, CALIFORNIA, JULY 3, 1935

BY ARTHUR S. TUTTLE,[1] PRESIDENT, AM. SOC. C. E.

The most significant impression one gets from a review of the history of the American Society of Civil Engineers is that of the wisdom and vision of its founders. A comparison of the definition of the objects to be attained by the Society, as written into the Constitution of 1852, with the definition in its present form will reveal their sound judgment and their keen appreciation of the responsibilities which the organization should assume. Such a comparison will disclose the remarkable fact that no change whatever has been made other than a re-arrangement of wording.

Though the Constitution has been amended as to eligibility for admission yet basically, in this particular, it remains the same as that drafted 83 years ago. Except that geologists are no longer recognized as belonging properly within our ranks unless their work is coupled with engineering experience, the admission requirements are unchanged. The succeeding years have merely brought recognition of the growth of specialization by setting forth the right of electrical, military, and naval engineers and of marine architects to membership. None of these changes is basically inconsistent with the original declaration, since from that first Constitution it is clear that our founders appreciated the desirability of bringing all the branches of the profession into a single organization for their common good, in order to make it as inclusive as would be consistent with the nature of the profession and our standards of achievement.

During all these intervening years, the Society has continuously observed a conservative, and at the same time, a far-sighted policy. It is to this policy—which, to reiterate, was that of our founders—that we owe our present strong position.

Meanwhile, certain changes of a specific nature have inevitably occurred with growth. A general review will show the important events, milestones of progress, which have been responsible for the remarkable increase in membership, in professional strength, in influence.

---

[1] Cons. Engr.; New York State Engr. and Acting State Director, Federal Emergency Administration of Public Works, New York, N. Y.

The first important event came nearly thirty years after the Society was founded; we achieved our first home, a modest building on East 23d Street. The effect of that step cannot be over-estimated, for it gave opportunity of increasing usefulness and it assured permanency. (See Fig. 1.)

Next followed two important constitutional amendments. In 1891, the Constitution was amended in such a way as to raise the requirements for full membership, with the effect of making the grade of Member a recognition of professional standing. In 1894, it was further amended by giving the Board of Direction the power to elect the Secretary in order to insure the selection of an officer with suitable qualifications.

In 1897, the Society, having outgrown its original quarters, moved to a new house in West 57th Street, one especially designed for its activities. (See Fig. 1.) Yet by 1905—after only eight years—this building, too, had been outgrown and an addition to it was erected to care for the rapidly increasing membership. In that same year (1905), moreover, came another evidence of growth: The first recognition of a Local Section was given by the Board of Direction.

But the Society was concerning itself not only with expansion. Following the ideals of the founders, it was interested also in maintenance of professional standards. Accordingly, in 1914, a Code of Ethics was adopted which has stood the test of time without change other than the addition of one paragraph.

In 1916 an agreement was entered into with the American Institute of Mining and Metallurgical Engineers, the American Society of Mechanical Engineers, and the American Institute of Electrical Engineers, as Founder Societies, under which our Society in the following year moved its Headquarters to the Engineering Societies Building in West 39th Street (see Fig. 1) and became one of the beneficiaries of the gift of $1 050 000 that had been made by Andrew Carnegie toward the cost of a building originally intended to provide a home for the four National Engineering Societies, which building had been completed and occupied by three of them since early in 1907. This change carried with it a merger of our library and participation in The Engineering Foundation.

More recent administrative changes have been for the purpose of making concrete those ideals which our Society has held from its inception: Expansion, usefulness, maintenance of high professional standards.

In 1920 the Board of Direction gave recognition to the first Student Chapter, and the next year the Constitution and By-Laws were revised, with important changes, including definite provision for Local Sections and Student Chapters, and for making the Secretary the Executive Officer of the Society.

In 1922 the By-Laws were amended by the creation of Technical Divisions.

In 1929 the Society became a member of American Engineering Council, pursuant to a majority vote of the membership in the preceding year.

In 1930 the Board of Direction authorized the publication of *Civil Engineering* at monthly intervals, with the effect of creating a humanizing

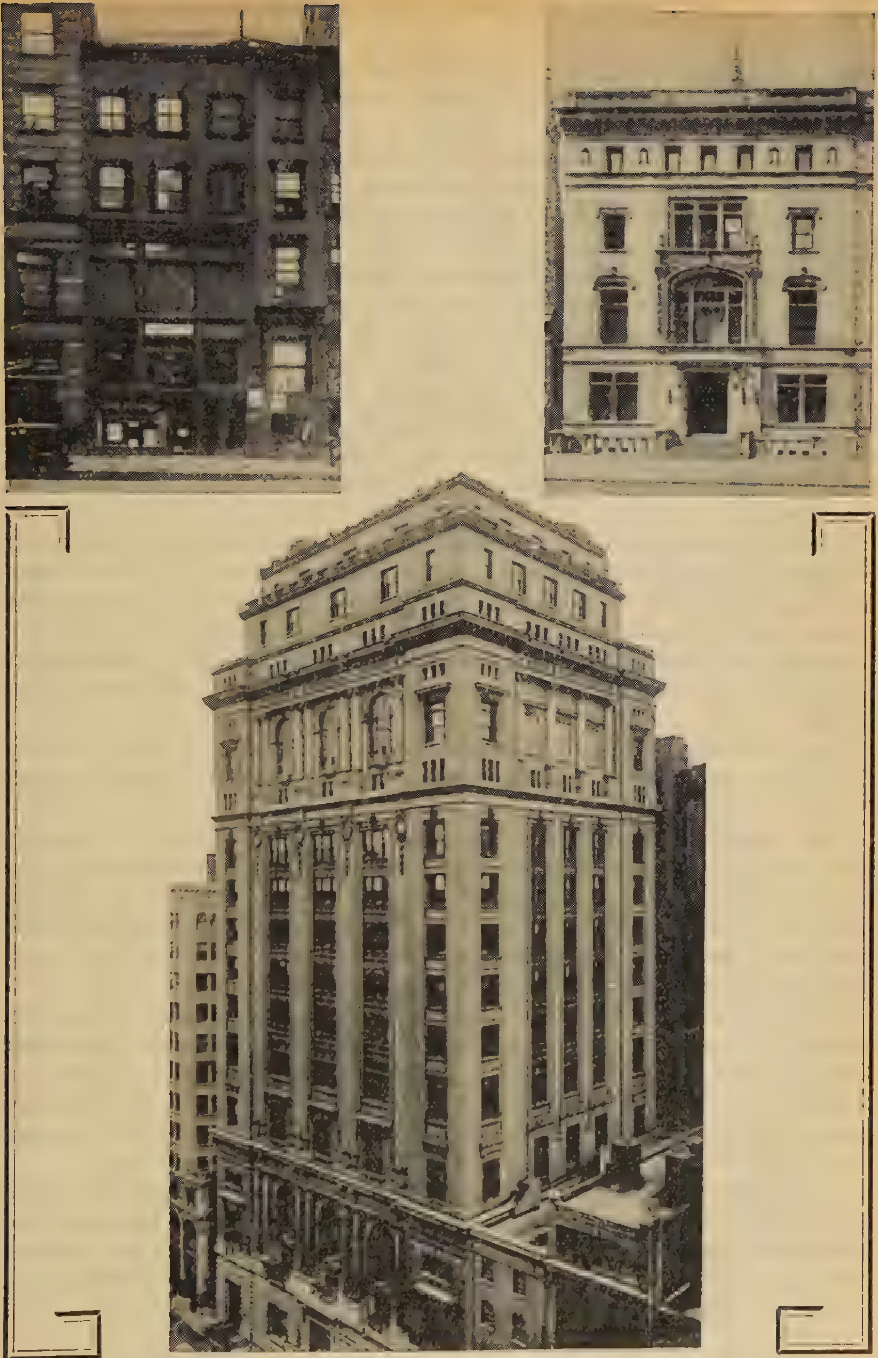


Fig. 1.—Views of Society Houses:
Upper Left-Hand Corner, East 23d Street Building as It Appears in 1935
Upper Right-Hand Corner, West 57th Street Building in 1897
Center, West 39th Street Building, Present House

agency of enormous value. This publication succeeded a modest four-page pamphlet containing current news items, issuance of which had been begun four years earlier.

In 1932 the Board of Direction concerned itself with the National Recovery movement, and promulgated a platform setting forth principles to be observed.

On April 4, 1935, the Board of Direction placed itself on record as favoring an amendment to the Constitution under which it is proposed to discontinue the Associate Member grade and to create a new grade, to be designated as "Fellow." Associate Members are to be transferred to the grade of Member, and men with the higher qualifications in the present grade of Member will be placed in the grade of "Fellow." It is also proposed to set up the grade of "Student Member." These changes are designed to harmonize with the recommendation of the Engineer Council for Professional Development with respect to uniformity in terminology of grades in all the Founder Societies, and will come before the business meeting of this Convention for consideration.

Most of these steps in development were made the subject of prolonged consideration before adoption, and those involving the move to the 39th Street Building suffered defeat once, while the joining with American Engineering Council was defeated twice, before final approval was given. This conservatism was also illustrated by the attitude of the Board toward the registration of engineers, approval of which was deferred until 1931 after consideration for 21 years.

The accompanying chart, Fig. 2, shows our membership from 1870 to the close of 1934, and it also discloses a similar record for the other three Founder Societies.

The continuous upward trend in our own growth, without interruption up to the close of 1931, is clearly evidenced, as is also the retention of our membership during the depression period. It is a record in which we can take great pride and particularly as we note that the upward acceleration in the membership scale can be coupled with the taking up of new activities which have invariably proved to be well worth-while and enduring.

It is quite possible that the holding up of our membership during the stress of the last two or three years, as compared with the records of the other three Founder Societies, may in a measure be due to greater leniency on the part of our Board of Direction in dealing with delinquents, but there seems to be an abundance of reason for believing that such leniency as may have been extended will have a substantial value in the days to come.

There are ample evidences that membership in our Society is prized among other things as of outstanding value since it carries a recognition of professional standing in degree second to no other agency, and that parting with it in time of financial stress means a loss to the engineer which he can ill afford to bear.

Activities

As the work of the Society has gradually expanded there has been a corresponding increase in demand for active service on the part of our membership to the end of putting new activities into successful operation.

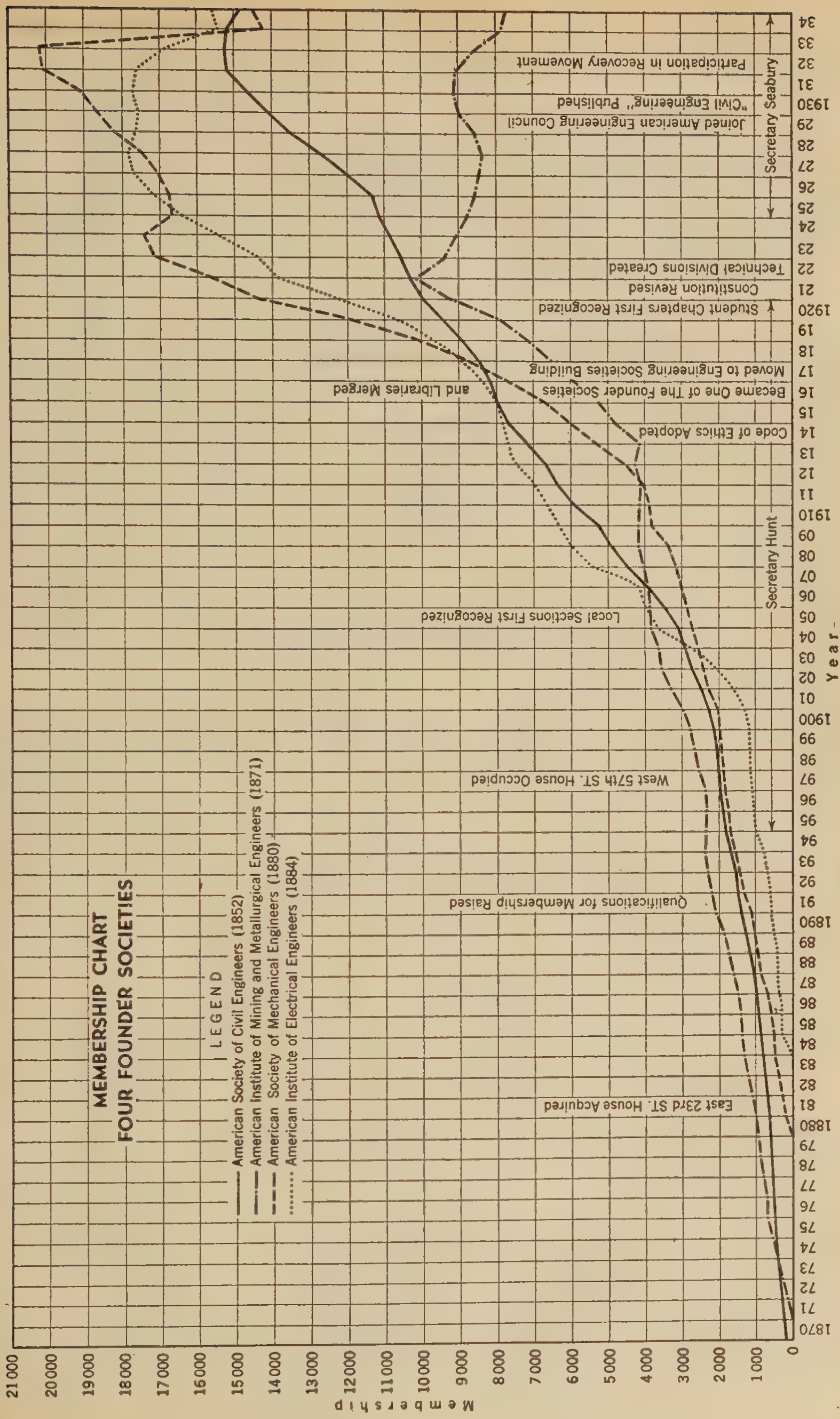


FIG. 2.—MEMBERSHIP CHART, FOUR FOUNDER SOCIETIES.

From the 1935 Year Book the following statistics as to Committee membership have been developed:

	No. of Committees	No. of Members
Board of Direction (Standing Committees)	6	31
Advisory, Administrative, Professional and Research	24	144
Technical Divisions	58	453
Joint Boards — Administrative	11	41
Joint Boards — Technical	11	56
Total	110	725

If to this list of active workers there be added the membership of special committees, the total number of assignments to active service in and for the Society, according to Secretary Seabury's estimate for 1935, is over 3 000, approximately 20% of the total. Responsibility of this character must incidentally have a strong influence in developing a spirit of loyalty to the Society on the part of those enlisted in its service, as well as in promoting our professional interests.

The budget for the year 1935 is set up to provide for a total expenditure of $373 000, of which only $213 000 is anticipated to be defrayed from dues—a ratio of $1.75 gross expenditure to each $1.00 collected as dues. Probably few organizations can show anything like such a large return to its members in proportion to their payments.

Additional income is derived from the 57th Street property and other investments, advertising, entrance fees, sale of publications, and returns for special services. These and a budget draft of $27 000 from surplus make possible the relatively large annual expenditures.

Some items of expenditure in addition to those providing for each member the very excellent technical publications, *Civil Engineering, Proceedings,* and *Transactions,* are as follows:

Employment Service	$6 700	
Library	8 726	
American Standards Association	550	
American Engineering Council	10 000	
Construction League of the United States	750	
Contributions to Joint Agencies		$26 726
Local Sections		$10 800
Technical Divisions		4 800
Technical, Administrative and Professional Committees		13 560
Meetings		6 925
Year Book		6 200
Total (without administration expense)		$69 011

The broadening out in Society activities has probably come about so gradually as to escape general notice and comment, and a review of them, appearing in the June, 1935, issue of *Civil Engineering*, and a report presented by our Committee on "Aims and Activities" and published in the February, 1935, issue of *Civil Engineering*, have doubtless brought surprise to those of our members who have not had a direct part in the expansion. A perusal of this report is commended to any member who has not read it. Since its submission the Board of Direction has definitely taken upon itself the responsibility for defending members of the Society who are unjustly accused of dereliction in duty or made the subject of oppression without being given an opportunity for defense; has adopted a scale of fees for engineering service; has placed the service of the Society at the disposal of the President of the United States for aid and advice in the setting up of the new Work Relief Program designed to bring about National Recovery; has gone on record as favoring the recognition of the value of our Junior Members by expressing the principle that so far as practicable they be given a place on all committees of the Society as well as on those of the Local Sections; has revived the Fall Meeting; has authorized the employment of a Field Secretary; has taken other steps to insure closer relations and better understanding with Local Sections and Student Chapters; and has committed itself to a program designed to insure proper publicity concerning its affairs.

As long ago as 1910 the subject of registration of engineers was under discussion. In 1925 the Board approved a "uniform" law drafted by a Society committee especially appointed for that purpose, and in 1931 it approved of the principle of registration.

In 1928 the Functional Expansion Program was adopted whereby nine new committees were instituted. Three were administrative committees designed to assist the Local Sections, the Juniors in the Society, and the Student Chapters. Six were professional committees designed to study matters with relation to the salaries of civil engineers, engineering fees, engineering education, the education of the public, legislation as it may affect civil engineers, and matters relating to the registration of engineers. Surveys and reports on the prevailing salaries of civil engineers, surveys on the degree of unemployment of civil engineers, recommendations with respect to fees, relations between the civil enginer and his client and with the contractor, suggested forms of contracts and other matters affecting the civil engineer in his personal experiences followed and were made available to authorities and to every member of the Society for local or personal application.

The function of a national organization like the Society is to gather information, digest it, devise policies, and to spread that information abroad for whom it may concern, in order that as individuals or as citizens they may avail themselves of the opportunities thus presented. This function is performed by committees, financed by the Society, whose observations are transmitted as authoritative information by publication, by correspondence, or by word of mouth. It is the function of the individual member or of the Local Section to translate that information or those policies into individual or local effect. In the past few years especially have these surveys, studies, and reports been undertaken in varying fields and in increasing quantity.

Local Sections

The recognition of local associations became a subject of discussion in 1872. The suggestion was rejected in 1882, but was finally approved by the Convention of 1905, although the canvass of leading centers made at that time did not develop much enthusiasm and in some cities the proposal met actual opposition. But the idea was evidently deemed of sufficient merit to proceed with the plan, and it was taken advantage of by San Francisco members in that year.

The next Association was organized in Colorado four years later, and in the first ten years 15 Associations were formed, of which 10 were located in territory west of the Mississippi, or in the area most remote from Society Headquarters. In the following six years, 20 new Sections were organized, of which 13 were east of the Mississippi. As a result of this spread of interest, direct recognition of Local Sections was given in our Constitution as revised in 1921.

At the beginning of the present year we had 57 Local Sections with a total membership of 6 254, averaging 110 per Section and ranging from 18 in the Oklahoma Section to 887 in the Metropolitan Section. These Sections are located in 34 States and in the District of Columbia, Puerto Rico, and Panama. New York and Ohio each has five Sections; California has four; Pennsylvania and Washington have three each; and Florida, Illinois, Minnesota, Missouri, and Tennessee have two each. The remaining twenty-four States in the list have one Section each.

But there is room for further activity in this field. The Society has no Local Sections in Delaware, Idaho, Kentucky, Maine, Mississippi, Montana, Nevada, New Hampshire, New Jersey, North Dakota, South Dakota, Vermont, West Virginia, Wyoming, Hawaii and The Philippines. These 14 States and 2 Possessions represent a total membership in the Society of 1 240, or an average of 78 each, ranging from 16 in Vermont to 570 in New Jersey. Three of the States (North Dakota, South Dakota, and Vermont) have memberships of less than the 25 required under the By-Laws for the formation of a Section, but eligibility exists in all other cases. At present Maine, Vermont, and New Hampshire members are identified with the Northeastern Section; New Jersey members are included in part in the membership of the Metropolitan Section and in part in that of the Philadelphia Section; and the Mid-South Section includes members in Arkansas and in portions of Illinois, Mississippi and Tennessee.

Under the present rules of the Board of Direction, as adopted in January, 1933, the Society contributes to the expense of maintaining a Local Section on the following basis, but subject to a maximum allotment at the rate of $3.00 per member paying Local Section dues during the preceding calendar year:

1.—For other than the Metropolitan Section, the Section allotment shall not exceed the amount paid for local dues during the preceding calendar year.

2.—For the Metropolitan Section, which has taken over some of the functions previously exercised by the Society and pays Society dues exceeding those of other districts by $5.00 and Local Section dues of $2.00, the allotment shall not exceed the amount paid for local dues during the preceding calendar year by more than 50 per cent.

3.—No allotment shall be made to a Local Section which fails to submit to the Secretary of the American Society of Civil Engineers, on or before October 1 of the year preceding the one for which the allotment is to be made, a statement as to the probable amount of the dues which will be collected during the calendar year, this statement being required for budget estimating purposes, nor shall the allotment be paid until there has been presented, after the close of the calendar year, a statement by the Section Treasurer and certified by the Section Secretary, showing the membership, the dues fixed, and the receipts and disbursements from all sources during the previous calendar year in such form as may be prescribed by the Board of Direction.

4.—The allotment shall not exceed $1.00 per member for Sections whose activity is confined wholly or largely to a center of population which holds less than five technical meetings a year.

5.—The allotment shall not exceed $1.00 per member for Sections whose activity is wholly or largely of a State or regional character and which hold less than two technical meetings a year.

6.—A technical meeting shall be deemed to be one at which some engineering work is discussed, notice of which has previously been sent to each member of the Section, to the Chairman of the Committee on Local Sections, and to the Secretary of the American Society of Civil Engineers. In case such meeting is held jointly with another society, advice as to the participation of the Local Section must appear in the notice.

7.—In case it becomes necessary at any time to curtail the allotment for Local Sections, the same shall be made proportionately for all Sections.

In a review of Section activities made by the Special Committee of the Board for 1933, it was brought out that while some of our Local Sections were exceedingly efficient, others were either to a considerable extent inactive or had become submerged in a local society. For these reasons the rules made at that time were intended to develop, where practicable, a procedure similar to that observed in the more active Sections but conducted on a scale commensurate with the size and opportunities of each individual Section.

Prior to the revision of the By-Laws in 1928, meetings of the Society were held once a month at Society Headquarters in New York, except during the summer season. The strictly technical papers which had appeared in *Proceedings* were presented at these meetings, being read either in whole or in part, or by title, with floor and written discussion. With the By-law change making provision for only four meetings during the year, the Metropolitan Section took over the responsibility for maintaining monthly meetings at Headquarters, and has since continued this plan, but the topics presented for discussion have been those of more popular interest, either to the profession at large or to engineers in the Metropolitan Area.

As is the case with papers presented at other Local Section meetings, many of these have been deemed of sufficient value to warrant reproduction in *Civil Engineering*, and those of the more technical character are eligible for acceptance for *Proceedings*, inclusion in *Transactions*, and for the Society prizes.

During the years succeeding the adoption of the allotment rules for Local Sections the financial condition of the Society has not permitted of paying the maximum amount set up under them; the payment for 1934 was $9 772.15, while the budgeted amount for 1935 is $10 800, an average of $1.73 per Section member. Nevertheless, it is hoped that the rules will stimulate interest in forming new Sections and in maintaining a high professional standing in those now existing, and that in time the Society will be able to assume a more generous share of the burden.

That the possible usefulness and value of the Local Section to the Society as a whole and to its scattered membership is being recognized by the Board of Direction is evidenced by the fact that in January last it determined to establish closer relations between and with the Local Sections through the appointment of a Field Secretary and by imposing upon the District Directors and the Vice-Presidents of the Zones the duty of visiting them to learn of local difficulties, and promote an *ésprit de corps*. Prior to this time contacts had been made only through general conferences as opportunity offered, through visits from the Secretary, whose time was necessarily burdened with other duties, and through infrequent visits by the President, the Vice-Presidents, and Directors.

The value of the Local Section in nationalizing the Society is clearly indicated on the membership diagram (Fig. 2), which shows a marked increase in growth beginning with the date of their first recognition. During the last three years there has been a falling off in membership due to causes readily understandable, but this will unquestionably be corrected with the return of better times.

In order to complete the nationalizing movement it would seem timely and desirable to amend our Constitution and By-Laws so as: First, to provide for definitely assigning each member of our Society residing in the United States or in any of its major possessions, or in Canada, to a Local Section, with provision for at least one such Section in each State or major possession; and second, to secure unity of action on the part of Local Sections in matters of State concern in case there is more than one Section in a State.

Incidentally, this step should be taken in such a way as to make a Local Section a real instrumentality of the Society and, at the same time, give it the maximum of freedom for vigorous participation in local affairs. Under the present Constitution a Section is restricted from speaking for the Society unless authorized by the Board of Direction. This condition, however, should not be construed as otherwise limiting its freedom, provided, of course, that it does not attempt to set up or endorse any policy conflicting with one adopted by the Society or by its Board of Direction.

Inasmuch as the success of such a plan would, at least to a considerable extent, be dependent upon the efficiency of a Section Secretary, it should be coupled with provision for the selection of the Section Secretary by the Governing Boards of the Sections as a paid officer, the selection to be subject to the approval of the Board of Direction of the Society. The salary allowed should be regulated with due regard to the membership in a Section, and it should have the psychological effect of imposing a recognized responsibility upon the holder of the office.

Functioning in this way, there would be brought to every member what Past-President Eddy has happily termed "Society consciousness", and irrespective of location the Sections should develop in such a way as to stimulate the feeling that their members are an essential agency of this Society and of the profession as a whole.

Through Section efforts and the proposed State combinations, and guided by our Board of Direction, the Local Sections should become agencies for increased usefulness and value both to individual engineers and to the public, and the prestige of our profession would inevitably be enhanced. There should be no doubt but that this increased expense would result in a widened interest in the affairs of the Society, and it is more than likely that it would soon attract enough additional members to fully justify the cost.

STUDENT CHAPTERS

In 1920 the Board authorized the formation of a Student Chapter at Stanford University, an action which was followed by the setting up of eight more Chapters in the same year and by the amendment of our By-Laws in 1921 to provide for their recognition and government. By the close of 1923 there were 63 such Chapters in existence, or more than one-half the present total of 111. Two of the 111 Chapters were formed only this year. The rapidity with which advantage has been taken of this opportunity to participate in our activities is amazing and is also abundant evidence of the existence of a real field for development and service.

The size of these Student Chapters, too, is significant. At the present time, eight have a membership of over 100, the largest being the one at the University of Illinois, with 194 members. The total membership is now 4 301, or an average of 39 per Chapter.

With such an army of fledglings under our guidance, it is important that those who direct them should clearly realize the aims and fully appreciate the possibilities of these college groups.

Primarily, a Student Chapter should function in such a way as to bring first-hand knowledge to the student body concerning activities in the engineering world and an opportunity to meet in person leaders of the profession. Indirectly and through the inspiration which should come from these sources, there should be developed in the student mind an incentive to join the National Society in order that he may take his place in the development of the profession and benefit himself through discussion with fellow practitioners of problems of mutual interest. He should also be made to appreciate the importance of organized effort for obtaining public recognition, to be mindful of the responsibility resting upon him individually to contribute of his time and brains to the advancement of the interests of the profession of which he is to be a member, and to recognize the obligation which he will be under for conscientious service to society.

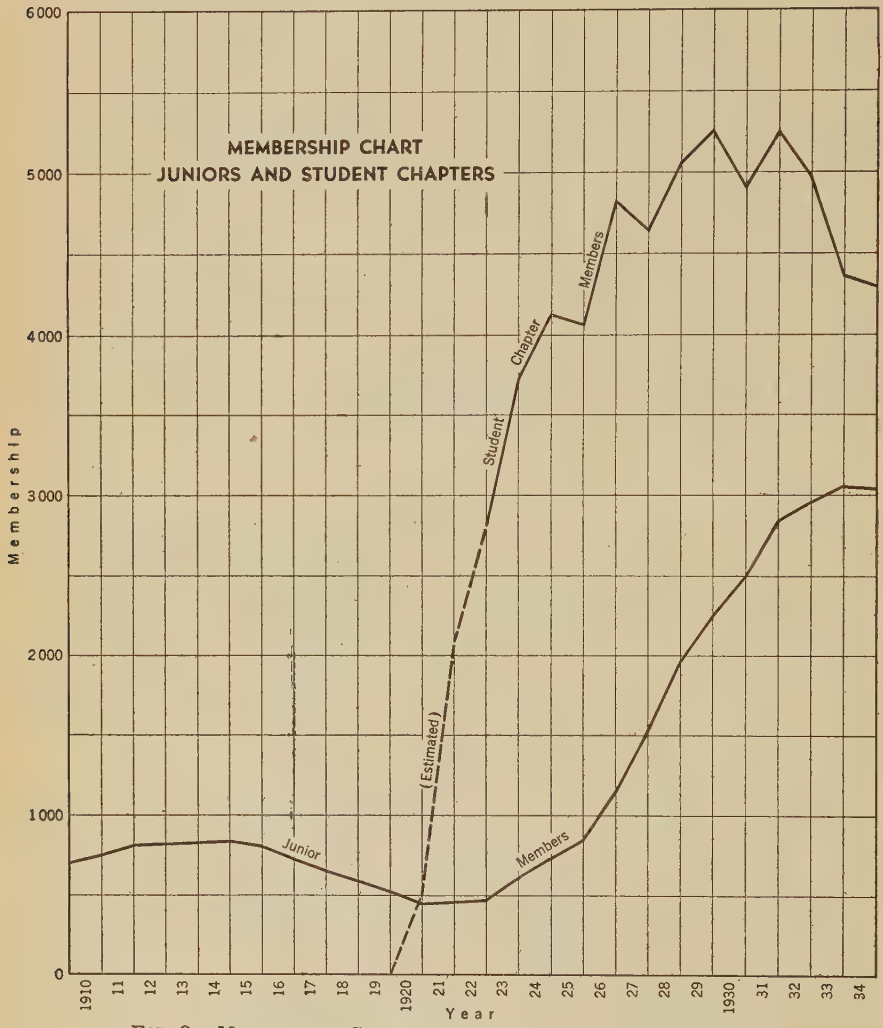


FIG. 3.—MEMBERSHIP CHART: JUNIORS AND STUDENT CHAPTERS.

This student body, now over 4 300 in size, should serve as a recruiting source for our Society to make up necessary replacements and maintain our growth. That it has functioned in this way is evident from an inspection of the accompanying chart (Fig. 3) showing the Junior membership in the Society since 1910, and also the Student Chapter membership since 1920, when Chapters were first organized. Such falling off in Student Chapter membership as there has been during the last three years, is clearly due to the depression.

In 1914 our Junior membership reached a peak of 840, and thereafter declined progressively until 1921, when there were only 448 registered, the loss in this period averaging 65 per annum. At the end of 1925, when definite steps were taken by the Board of Direction to encourage graduates to apply at once for membership in the Society, the effect of having this reservoir to draw upon became pronounced, and in the period which has since ensued the annual growth in Junior membership has averaged nearly 250, the membership now reaching a total of over 3 000. During the same period from 1926 to the close of 1934, the Society membership has increased from 11 281 to 14 910, averaging about 400 per annum. It therefore appears that over 60% of our membership gain since 1926 is made up of men in the Junior grade.

The membership chart also shows that the increase in the Junior grade brought about by the recognition of Student Chapters has been much more steady than that of the Society as a whole, and that the effect of the depression is less pronounced. It is also to be noted that our Juniors now constitute more than one-fifth of the entire membership, and we can take pride in having this substantial base for an enduring growth.

In view of the recognition so markedly extended by our Engineering Schools as to the value of the Student Chapter to them, it would seem desirable to place a larger and more direct measure of responsibility upon an accredited teaching institution for maintaining and governing the Chapter, rather than upon the individual members, and to require membership in it on the part of all registered students.

Technical Divisions

The directory of "Organizations of Engineers" recently compiled under the joint auspices of American Engineering Council, Engineers Council for Professional Development, and The Engineering Foundation, shows that there are in existence 63 engineering organizations of a national character, with a total membership of approximately 175 000, and 87 local or State societies with a total membership of over 37 000, making a total of 150 organizations with a combined membership of about 212 000. Included in this total are nearly 54 000 members of the American Society of Civil Engineers, the American Institute of Mining and Metallurgical Engineers, the American Society of Mechanical Engineers, and the American Institute of Electrical Engineers, these in the aggregate representing only about one-fourth the total.

It is now apparent that insufficient encouragement was accorded by our Society in the early years to those not practicing more particularly in the general field of Civil Engineering, with the result that new organizations were brought into existence to serve other branches of the profession. Several of these represent highly specialized groups, while others are open to men without technical training, the existence of either of which conditions would seem to preclude possibility of merger with any of the Founder Societies. The latest highly technical national engineering society is the American Institute of Chemical Engineers, which was organized in 1908. This Society has shown a remarkably steady growth, with a membership of 1 157 at the beginning of the present year. Each of these organizations has doubtless been created as a result of a well-developed need, to meet which there was no agency in existence, and it is more than likely that the leaders in many of them are members of one or more of the Founder Societies; their amazing growth is doubtless responsible for the increasing effort which this Society has been making in more recent years to extend its usefulness to the specialties and to bring its activities to its members wherever located through the activities of the Technical Divisions as created under an amendment in the By-Laws adopted in 1922.

Since their authorization the Technical Divisions, now 10 in number, have shown a steady growth. They comprise City Planning, Construction, Engineering-Economics and Finance, Highways, Irrigation, Power, Sanitary Engineering, Structural, Surveying and Mapping, and Waterways. At the 1935 Annual Meeting, seven of these Divisions held meetings, mostly with two sessions, and considered 33 papers. The programs set up for the Technical Divisions frequently make such heavy demands on available time that there is no opportunity for other than a business meeting of the Society. This condition seems unfortunate—even menacing. There should be an abundance of professional topics of common interest to warrant the bringing together of our entire membership at least once at each of the four general meetings for discussion of these professional topics, and it would seem wise to arrange our programs in such a way as to provide for this need.

Under the power given by the By-Laws, the Technical Divisions permit the enrollment of Division Affiliates who are not members of the Society, the clear intent being to secure the co-operation of specialists who could not qualify for corporate membership. It is evident that more definite allegiance to the Society would be insured if all such members were required to first become Affiliates in the Society, and it would seem reasonable to impose this obligation before the problem becomes a serious one.

With continued expansion of the Technical Division activities it should be possible to serve in one organization the technical needs of all Civil Engineering specialists, and thus check the movement toward the creation of new and independent agencies in these and other engineering fields, to the end that through combined effort a maximum of efficiency may be developed.

Relations with Other Engineering Societies

Up to 1916 the Society had generally held itself aloof from direct affiliation with any other engineering society. In 1903 it adhered to tradition when the project to join our three sister societies in the erection of the Engineering Societies Building was submitted to the membership, and was decisively rejected, the vote being 1 139 against acceptance of the offer to 662 in favor of it. But desire for isolation gradually abated. With the affiliation finally brought about in 1916, the libraries of the four Founder Societies were merged, and our Society became a co-trustee for the management of the funds of The Engineering Foundation.

Through its membership in the present United Engineering Trustees, then designated as the United Engineering Society, it also became one of the Trustees of the Engineering Societies Building and properties. Under the Constitution of the Trustees the objects are stated to be the advancement of "the engineering arts and sciences in all their branches". Accordingly, in the belief that this obligation required it to consider all matters of interest to the profession, the Trustees set up "Engineering Council" as a Committee to act for and advise them in this general capacity. This agency functioned until 1920, when it was replaced by American Engineering Council, which was designed to be a more democratic body with representation not only from the four Founder Societies but also from all other Engineering Societies, including State and local bodies. The membership of the American Society of Civil Engineers, however, took the position that leadership in the setting up and maintaining of policies should be confined to those National Societies maintaining high technical requirements as to membership, and that all local expression should be presented through the national body, in which it was presumed the local element was properly and fully represented; as a result the Society twice voted (1920 and 1923) against becoming affiliated with the new body. The question was again raised in 1928, when the membership reversed its original vote, and since April 3, 1929, we have been one of the constituent bodies of American Engineering Council, which is described in its Constitution as a central agency "to further the public welfare wherever technical and engineering knowledge and experience are involved, and to consider and act upon matters of common concern to the engineering and allied technical professions".

In 1923 the Institute of Mining and Metallurgical Engineers, which was one of the original members of American Engineering Council, decided to withdraw, so the advent of our Society still left Council with incomplete representation from the four Founder Societies. As a result of the depression, the large contributions from the Founder Societies were considerably decreased and Council was unable to carry on its activities with the same force and vigor as at the outset. Partly for financial reasons and partly for other reasons, the representation of the Founder Societies on Council was diminished with relation to the other member bodies. During the same period the idea was advanced that the existence of too many independent agencies was a serious hindrance against selling the professional needs to the public. Engineering Foundation was in need of financial strengthening so that it could undertake various investigations which had been proposed, but over which there was no clear-cut jurisdiction or well-defined responsibility.

In the hope that there might be developed a "better definition and co-ordination of joint organizations of Engineering Societies", the President of the United Engineering Trustees was authorized on February 15, 1932, to communicate with the four Founder Societies, requesting the appointment of "a small committee of the wisest men" who could be persuaded "to give the time and energy requisite for an extensive and thorough study". As a result of this request, a committee of three from each of the Founder Societies, together with three representatives of the United Engineering Trustees, was brought together on June 10, 1932, as a Coordinating Committee. The recorder for this meeting appended to the minutes a memorandum reading as follows:

> "Members of the societies, and even their boards have not absorbed information on the joint activities and organizations and therefore do not understand them. United Engineering Trustees recently has attempted arrangements for at least one Trustee from each Founder Society to attend meetings of his Society's board and discuss United Engineering Trustees actions; also to have an item for United Engineering Trustees matters on the agenda of each Founder Society board meeting. This Committee suggested similar arrangements for American Engineering Council. Too commonly members of joint bodies having performed their special duties, forget their obligation as a representative to keep their respective societies informed and to learn the desires of the societies. Interlocking of memberships of the societies boards and the joint organizations was recommended to the extent practicable."

Thereafter a large number of meetings of this Committee was held as well as of Sub-Committees appointed to deal with various phases of the problem.

The carrying forward of joint activities was also made a subject of conference at a joint meeting of officers of the four Founder Societies and the Engineering Trustees at Chicago, Ill., on June 27, 1933, at which it was brought out that there had been no similar general conference for more than thirteen years.

In the meantime, the whole subject had been given consideration by the Board of Direction of the American Society of Civil Engineers, with the result that at its meeting of September 26, 1933, and on the recommendation of a Special Committee, it adopted a plan for setting up a "Central Administrative agency for all joint activities capable of being administered by such an agency", these activities as then existing comprising United Engineering Trustees, The Engineering Foundation, Engineering Societies Library, Engineering Societies Employment Service, American Engineering Council, and Engineers Council for Professional Development, and recommended consideration of this proposition by the Coordinating Committee of Engineering Societies and by their governing boards.

In the final report of the Coordinating Committee, dated June 4, 1934, it was stated that "radically different views as to which agency might be expanded make it impossible for the Committee to agree on one", and it was recommended that an additional agency be set up to be called "The National Advisory Board of Engineers" designed to initiate or to consider projects and to allocate projects to existing agencies.

The minutes of the United Engineering Trustees of December 20, 1934, carries the following record:

> "Reports from the Founder Societies Secretaries regarding action on the report of the Coordination Committee, submitted to the Societies June 4,

1934, showed as follows: Civil Engineers, laid on table July 9, 1934; Mining Engineers, 'sees no need to warrant the establishment of additional joint board or other agency', June 21, 1934; Mechanical Engineers, approved July 22, 1934; Electrical Engineers, report expected January 1935."

It being evident that no unanimous agreement could be reached upon a plan, the Coordinating Committee was thereupon discharged.

It is clear from this review that during this period of more than two years, a great deal of conscientious work was done by outstanding men in the hope of providing a solution of the difficulties, and although the work appears to have been in vain in so far as direct accomplishment is concerned, there is not the slightest doubt but that it has brought about a better understanding among the existing agencies as to their mutual problems. It has also resulted in the submission of reports at regular intervals to the various Boards of Direction as to the work done by some of the joint agencies in which they participate and concerning which they should have knowledge, and it is doubtless responsible for the interesting joint meeting of all of the Governing Boards on May 20, 1935, under the auspices of the United Engineering Trustees, at which each of the activities of common interest was made the subject of formal discussion.

At some future time the question at issue will again be raised, and the study which has now been given to it should be helpful in ultimately bringing about an outcome satisfactory to all the Founder Societies.

FINANCIAL

The report of the Board of Direction for the year 1934 shows net worth as of December 31 greater than at any time in its history, and as follows:

Cash and other liquid assets	$26 858.32	
Marketable securities at market	25 881.67	
Accounts receivable, less doubtful accounts	14 096.03	
Publications and prepaid insurance	14 937.76	
Furniture	6 122.06	
Real estate, West 57th Street	597 638.84	
Share in joint holdings—West 39th Street Property	493 352.60	
Library	94 433.05	
Special funds (Book values)	145 731.33	
Net worth		$1 419 051.66

The special funds are all conditioned in such a way as to be used for purposes consistent with the activities of the Society.

The assets of the Society in 1892, or five years before the 57th Street house was occupied, were estimated at $60 000. The increase in value during the following 25 years was $700 000, making a total of $760 000 in 1917, when we removed our quarters to West 39th Street. This remarkable increase was partly due to contributions to the West 57th Street Building Fund, partly to sharing in the gift of Andrew Carnegie to the Founder Societies, and partly to excellent management. The further increase of nearly $660 000 during the period between 1917 and the beginning of this year, is chargeable partly to the estimated increase in the value of the West 57th Street property, which is probably still appraised at very much less than its selling value in a normal real estate market, partly to income from the property, partly to gifts, and partly to the policy which was long maintained of balancing the budget with a safe margin.

With the coming of the depression and the recognition in 1931 that members in many instances would be unable to pay their dues, the budgets for that year and for the succeeding years, 1932, 1933, 1934, and 1935, have all been set up with an anticipated draft upon surplus, in order that the essential activities of the Society might be maintained, and in order that there might be no substantial curtailment in usefulness during these serious times. Budgeted drafts upon surplus have been as high as $40 000, but due to various economies effected, the actual drafts in these years have been of lesser amount. The increase in income from dues in the year 1934 and thus far in the year 1935, and the increase in applications for membership, indicate the response made by the members and the profession to the work done in their behalf.

It is also particularly gratifying to note that during 1934 the final payment was made on the mortgage on the West 57th Street property, leaving the Society entirely free of debt and fixed charges, and that for the first time since the depression set in, more applications for admission were presented in 1934 than in the year preceding—a condition which the record for 1935 shows to be continuing.

For the year 1935, to permit of further expansion in efforts to serve members even more extensively than ever before, new activities have been undertaken which will again unbalance the budget to an estimated amount of over $25 000. But if history repeats itself, this money will flow back into our treasury in the course of time through revenues from increased membership, payment of back dues, and reinstatements. In any event, the Board of Direction has deemed it wise to adhere to the policy of increasing activities in every direction during these troublous times when substantial assistance could properly be given to the betterment of the present status of our members.

Looking back over our record, we can detect the outstanding events that have shaped a progress which has continuously been one of conservatism tempered with common sense, and can see that the more sweeping changes have each marked a period of increasing strength. Among these stand out the progressive betterments in quarters and in library facilities, the recognition of Local Sections, the setting up of Student Chapters, the creation of Technical Divisions, the publication of *Civil Engineering,* and the expansion of interest in human problems of direct concern to the individual.

At no time has the Society gone out to seek for members, its recognized value being so great to the practitioner as to bring applicants for admission on their own volition to our doors.

The recent valuable and forward-looking study made by our Special Committee on Aims and Activities and the splendid results which have grown out of its report give abundant evidence of the desirability of perpetuating its work, to the end that there may be in existence a unit charged definitely with the responsibility for reviewing, co-ordinating, and energizing all our technical and welfare activities, and bringing its conclusions before the Board of Direction from time to time. Such a Committee might be designated as a "Committee on Development" and be composed partly of Board members and partly of others of broad vision who have been active in Society affairs.

This activity might properly be considered as the subject for an amendment to our By-Laws and be given consideration in connection with the suggestions herein made with respect to the expansion of the Local Section influence, the strengthening of the Student Chapters, and the provision of complete identification with the Society on the part of members of Technical Divisions.

It has been said that the success of an organization is largely dependent upon the efficiency of its Secretary. Our Society has been particularly fortunate in having had the benefit of the brains and ability of the late Charles Warren Hunt, M. Am. Soc. C. E., who served in this capacity from 1895 to 1920, and in having retained in this office from 1925 to now, Secretary Seabury, likewise a man possessed of skill, energy, and loyalty. To the vision of these two men and their close and intelligent study of our problems, aided by an unusually capable staff, is to be given great credit for the remarkable progress which we have made and for the enviable place we hold in the list of professional societies. Recognition should also be given to devoted service on the part of our past officers, the list including the leaders of our profession, who have given freely of their time, more than frequently at great personal inconvenience, for the upbuilding of our Society.

Our position is a strong one, and we can face the future with confidence in continued progress toward the achievement of all the Society stands for—the advancement of our profession and of the interests of its members, and usefulness to our fellow man.

Note.—The writer desires to acknowledge the valuable assistance he has received in preparing this address from the paper by our late Secretary, Charles Warren Hunt, on "Twenty-five Years of Society Activities", which appeared in Volume LXXXII (1918) of our *Transactions;* from the article by Secretary George T. Seabury on "Eighty Years of Growth and Progress", published in *Civil Engineering,* November, 1932; from other publications of the Society, minutes of the Board of Direction, material furnished by the staff, and records of various committees with which he has been connected.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 1922

CONCRETE AND REINFORCED CONCRETE ARCHES

FINAL REPORT OF THE SPECIAL COMMITTEE

COMMITTEE

Clyde T. Morris, *Chairman*,
George E. Beggs, A. C. Janni,
E. H. Harder, Wilbur M. Wilson

November 22, 1934

Note.—Presented at the Annual Meeting, New York, N. Y., January 16, 1935.

CONTENTS

Page

Chapter I. — Summary and Conclusions........................ 1429

Chapter II. — Tests of Single-Span Arch Ribs.................. 1435

Chapter III. — Tests of Single-Span Arches with Decks.......... 1446

Chapter IV. — Tests of Three-Span Arch Ribs on Slender Piers....1468

Chapter V. — Tests of Three-Span Arch Bridges, with Decks on Slender Piers.................................... 1485

Chapter VI. — The Effects of Climatic Changes................. 1505

Chapter VII. — Movements of Foundations During Construction.... 1525

Chapter VIII. — The Effect of Time Yield in Concrete Upon the Deformation Stresses in a Reinforced Concrete Arch Bridge 1534

Chapter IX. — Reaction Components for Skew Barrel Arches...... 1551

Chapter X. — Combined Algebraic-and-Experimental Method of Analyzing Multiple-Span Arch Series............ 1573

CONCRETE AND REINFORCED CONCRETE ARCHES

FINAL REPORT OF THE SPECIAL COMMITTEE

I.—SUMMARY AND CONCLUSIONS

Article 1.—Work of the Committee.—The Special Committee on Concrete and Reinforced Concrete Arches was appointed by the President of the Society, on authorization of the Board of Direction, in May, 1923. The members invited to serve on the Committee were Clyde T. Morris, *Chairman*, George E. Beggs, John R. Chamberlin, E. H. Harder, A. C. Janni, and W. M. Wilson. Mr. Chamberlin died on December 15, 1925. In the later years of its work, the Committee secured the active co-operation of the United States Bureau of Public Roads, and E. F. Kelley, M. Am. Soc. C. E., Chief of the Division of Tests, with Mr. A. L. Gemeny, sat with the Committee in its deliberations.

The Committee has studied the design and construction of reinforced concrete arches since the time of its appointment, but many problems still remain, which it has not attempted to solve. Its work has been confined mainly to arch ribs and to arches with open spandrels. No work has been done on spandrel-filled arches with the exception of determining the reaction components for skewed barrel arches, and a large field of investigation is still open regarding the distribution of stresses and the effect of spandrel walls on this type of arch.

The Committee feels that its studies and investigations warrant certain conclusions, which are presented herein for the benefit of the profession.

A list of the papers and reports referred to herein, is appended in Article 4. These are numbered, and the numbers in parentheses following the paragraphs of the report indicate the references.

Among the more important investigations sponsored by the Committee are:

1.—An investigation of pier movements of multiple-span arch bridges during construction, in co-operation with the Ohio State Highway Department. (Bridges at Conneaut, Ashtabula, and Piqua, Ohio.)

2.—An investigation similar to Investigation No. 1, in co-operation with the Engineering Experiment Station of the University of Illinois. (Bridges at Yorkville and Rockton, Ill. and at Oliver Avenue and Kentucky Avenue, Indianapolis, Ind.) (1).

3.—An investigation of the effect of climatic changes upon a multiple-span arch bridge, in co-operation with the Engineering Experiment Station of the University of Illinois. (The Gilbert Street Bridge, at Danville, Ill.) (2).

4.—Laboratory tests of arch ribs, unsupported laterally, made in co-operation with the Engineering Experiment Station of the University of Illinois (3).

5.—A laboratory investigation by means of three dimensional models of the reaction components for skew barrel arches, by Professor Beggs, at Princeton University (11).

6.—A theoretical investigation of the effect of the elasticity of piers on the stress distribution in a multiple-span arch bridge, by Mr. Janni (4).

7.—A laboratory investigation of "time yield" or "flow" of concrete under load, in co-operation with the University of California, by Raymond E. Davis, M. Am. Soc. C. E. (5).

8.—A laboratory investigation of the stress distribution in single-span arches with and without decks, in co-operation with the Engineering Experiment Station of the University of Illinois and the U. S. Bureau of Public Roads, by Professor Wilson (3) (6) (8).

9.—A laboratory investigation of the stress distribution in multiple-span arch bridges with and without decks, in co-operation with the University of Illinois Engineering Experiment Station and the U. S. Bureau of Public Roads, by Professor Wilson (7) (8) (9).

10.—Model investigations of the stresses in single and multiple-span arch bridges, in co-operation with the Ohio State University Engineering Experiment Station, Princeton University, and the University of Illinois Engineering Experiment Station (6) (9).

11.—Laboratory investigation of the effect of time yield in concrete, upon temperature and shrinkage stresses in a reinforced concrete arch bridge, in co-operation with the U. S. Bureau of Public Roads and the University of Illinois, Engineering Experiment Station (12).

12.—Methods of algebraic analysis of multiple-span arches, in co-operation with the University of Illinois Engineering Experiment Station (19).

Article 2.—Acknowledgments.—The direct expenses of the investigations sponsored by the Committee were paid from funds contributed largely by the Society; the U. S. Bureau of Public Roads (Thomas H. MacDonald, Chief); and Engineering Foundation (Alfred D. Flinn, M. Am. Soc. C. E., Director). Acknowledgments are also hereby made to the following organizations and persons for funds and materials contributed and valuable assistance rendered:

Engineering Experiment Station of the University of Illinois (the late Milo S. Ketchum, Hon. M. Am. Soc. C. E., Dr. A. C. Willard, and M. L. Enger, M. Am. Soc. C. E., successively, Directors); the Testing Laboratories of the University of California (Charles Derleth, Jr., M. Am. Soc. C. E., Director, and Raymond E. Davis, M. Am. Soc. C. E., Professor of Civil Engineering); Princeton University (Arthur E. Greene, Dean of the School of Engineering); Engineering Experiment Station of the Ohio State University (Dean E. A. Hitchcock, Director); the Ohio Department of Highways (J. R. Burkey, Chief Engineer of Bridges); the Neal Gravel Company; the Portland Cement Association; the Universal Atlas Cement Company; the Illinois Steel Company; the American Bridge Company; Jones and Laughlin Steel Corporation; Interstates Sand and Gravel Company; Lincoln Sand and Gravel Company; the Fairbanks Morse Company; the United States Rubber Company; and the Ames Dial Company.

Article 3.—Conclusions.—The following conclusions have been reached by the Committee and are presented, with references to the supporting data:

1.—The elastic theory based upon the usual assumptions is adequate for determining the thrust and moment in reinforced concrete arch systems without continuous superstructure, with sufficient accuracy for purposes of design. The usual assumptions referred to are:

(*a*) A plane section before flexure remains a plane after bending.
(*b*) The moment of inertia for purposes of determining thrust and moment may be calculated upon the assumption that concrete takes tension.
(*c*) The modulus of elasticity of concrete may be assumed to have the same value at all stresses and sections (3) (6) (7) (8) (9) (15).

2.—A considerable accidental variation in the moment of inertia or in the modulus of elasticity will not materially affect the position or magnitude of the thrust due to loads in a single-span arch. This variation may be caused by non-homogeneity of the material or by cracks on the tension side of the rib (3) (6) (7) (8) (9) (10) (15).

3.—Failure of a reinforced concrete arch rib is very improbable except as it may be subjected to a large moment. In such a contingency the failure is primarily a flexural failure (3) (7) (8).

4.—The concrete in an arch rib will develop approximately the same ultimate strength as the same concrete in 6 by 12-in. cylinders (3) (6) (7) (8) (9) (12).

5.—If an arch rib is not subjected to lateral forces its strength is not affected by its slenderness, so long as the ratio of the unsupported length of the arch axis to width of the rib does not exceed 25 to 30 (3).

6.—In multiple-span arch systems the dead load unit stresses are sometimes greater than the corresponding unit stresses in a similar span having fixed ends, due to lack of homogeneity of the material and accidental variations in dimensions. This suggests that the design of multiple-span arch systems on slender piers should be more conservative than that of single-span arches (7) (8) (9).

7.—The following method of analyzing the stresses in a single-span arch bridge with open spandrel and deck is suggested in the design of structures for which deck participation may be important:

(*a*) Determine the dead load stresses analytically on the basis that the dead load is carried by the rib, unrestrained by the deck.
(*b*) Determine the live load stresses by an experimental method using elastic models, considering the structure as a whole and not the rib alone. This is important because of the effect of the deck participation upon the stresses in the deck and columns rather than because of its effect upon the stresses in the rib.
(*c*) Determine the shrinkage and temperature stresses by an experimental method using elastic models of the structure as a whole.

The theoretical analysis of an arch with open spandrel and deck is so complicated that few engineers have acquired the facility that justifies them in using it with confidence. Furthermore, the time required for such a theoretical analysis is excessive (16).

8.—The following method of analyzing a multiple-span arch bridge with open spandrel and deck is suggested, in the design of structures in which deck participation may be important:

(*a*) Determine the dead load stresses analytically on the basis that the dead load is carried by the rib unrestrained by the deck.
(*b*) Determine the live-load reaction components at the springings by an algebraic analysis based upon experimentally determined elastic constants, or by an all-experimental method, considering the structure as a whole and not the ribs alone.
(*c*) Determine the shrinkage and temperature reaction components at the springings from experimentally determined elastic constants, or by an all-experimental method.

The elastic constants may be determined from tests of an elastic model of a single span (7) (9) (16) (19).

9.—A deck without intermediate expansion joints, constructed so as to act with the rib in carrying loads, reduces the magnitude of the live load moment at the springing where the deck does not increase the strengh of the structure; and it increases the moment over the central portion of the span where the strengthening effect of the deck is greatest. Intermediate expansion joints in the deck reduce these effects (6) (7) (9).

10.—The moment at the springing due to shrinkage, time yield, temperature changes, and other causes equivalent to a change in span is greater for a structure having a continuous deck than for the same arch rib without a deck. Intermediate expansion joints in the deck may increase or may reduce this difference, depending upon the rise ratio of the rib and the proportions of the deck (6) (7) (9).

For an arch having a deck, the moment in the rib at a section near the crown and in a panel below an intermediate expansion joint in the deck, due to any causes equivalent to a change in span, will be greater than for a structure without expansion joints, and may be greater than for a rib without deck. If intermediate expansion joints are placed in the panel containing the intersection of the temperature thrust line and the axis of the arch, the temperature moment in the rib in the panel will be small (2) (6) (7) (9).

The general use of intermediate expansion joints in the deck is not recommended, except where an analysis shows that they are needed and are not harmful.

11.—The height of the deck above the rib of an arch bridge does not greatly influence the stresses in the rib due to loads; but increasing the height of the deck above the rib by reducing the rise of the arch greatly increases the stresses due to any cause equivalent to a change in span. From this it would follow that the rise of the rib should be made as great as the grade of the roadway will allow (6) (7) (9).

12.—The elastic deformation of the piers in multiple-span structures increases the value of the live-load positive moment over the central portion of the span. For arches with a continuous deck, this increase in moment occurs over that portion of the structure which is strengthened by the deck.

The elastic deformation of the piers decreases the live-load negative moment and increases the live-load positive moment at the springing of the center span, and increases the negative moment at the end abutment. The deck does not assist the rib in resisting moment at these points (4) (6) (7) (9).

13.—The bases of the piers and abutments of arch bridges should be designed so that the time yield of the soil due to dead load pressure will not cause the piers to tip. The uniformity of pressure distribution that is necessary depends upon the character of the soil but, with materials like hardpan and shale, which are particularly susceptible to time yield, an uneven distribution may cause considerable rotation even if the maximum pressure is not greater than would be permitted with uniform distribution (2).

14.—The piers and abutments of arch bridges should be kept under observation with precision instruments during the construction of the superstructure, in order that any excessive motion may be detected (1).

15.—For skew barrel arches tests of three-dimensional models confirm the theory for computation of reaction components presented by J. Charles Rathbun, M. Am. Soc. C. E. (11) (18).

16.—The length of the rib of a reinforced concrete arch shortens gradually after the concrete sets due to many causes, including initial drop in temperature, shrinkage, and time yield. The plastic flow of the concrete greatly reduces the flexural stress that would otherwise accompany this change in length. Plastic flow also reduces the stress that would otherwise accompany seasonal changes in temperature during the first year but possibly does not appreciably reduce temperature stresses later. It would appear, therefore, that if temperature stresses corresponding to the seasonal temperature range were taken into account in the design of an arch, the initial drop in temperature, shrinkage, and time yield might be neglected in determining stresses in the concrete. The initial drop in temperature, shrinkage, and time yield do, however, cause an appreciable permanent vertical movement of the rib at the crown, especially for flat arches, which should be considered in the design and construction of the superstructure. They also produce a high compressive stress in the steel, but this stress will not affect the load-carrying capacity of the arch if the steel is restrained against buckling.

Because, at stresses near the ultimate, a large increase in the strain of concrete is accompanied by only a very small increase in stress, it would appear that a considerable gradual movement of the abutments has very little effect upon the load-carrying capacity of an arch (5) (12) (14).

Article 4.—References to Supporting Data.—

(1) "Movements of Piers During the Construction of Multiple-Span Reinforced Concrete Arch Bridges", by W. M. Wilson, *Bulletin No. 234*, Univ. of Illinois Eng. Experiment Station.

(2) "Effect of Climatic Changes Upon a Multiple-Span Reinforced Concrete Arch Bridge", by W. M. Wilson, *Bulletin No. 174*, Univ. of Illinois Eng. Experiment Station.

(3) "Laboratory Tests of Reinforced Concrete Arch Ribs", by W. M. Wilson, *Bulletin No. 202*, Univ. of Illinois Eng. Experiment Station.

(4) "The Design of a Multiple-Span Arch System and Permissible Simplifications", by A. C. Janni, *Transactions,* Am. Soc. C. E., Vol. 88 (1925), p. 1142; also *Transactions,* Am. Soc. C. E., Vol. 91 (1927), p. 459.

(5) "Flow of Concrete Under Sustained Compressive Stress", by Raymond E. Davis, *Proceedings,* Am. Concrete Inst., Vol. 24 (1928), p. 303.

"Flow of Concrete Under Sustained Compressive Stress", by Raymond E. Davis and Harmer E. Davis, Jun. Am. Soc. C. E., *Proceedings,* A.S.T.M., Vol. 30 (1930), p. 707.

"Flow of Concrete Under Action of Sustained Loads", by Raymond E. Davis and Harmer E. Davis, *Proceedings,* Am. Concrete Inst., Vol. 27 (1931), p. 837.

(6) "Laboratory Tests of Reinforced Concrete Arches with Decks", by W. M. Wilson, *Bulletin No. 226,* Univ. of Illinois Eng. Experiment Station.

(7) "Laboratory Tests of Multiple-Span Reinforced Concrete Arch Bridges", by W. M. Wilson, *Transactions,* Am. Soc. C. E., Vol. 100 (1935), p. 424.

(8) "Laboratory Tests of Three Span Reinforced Concrete Arch Ribs on Slender Piers", by W. M. Wilson, and Ralph W. Kluge, Jun. Am. Soc. C. E., *Bulletin No. 269,* Univ. of Illinois Eng. Experiment Station.

(9) "Laboratory Tests of Three Span Reinforced Concrete Arch Bridges with Decks on Slender Piers", by W. M. Wilson and Ralph W. Kluge, *Bulletin No. 270,* Univ. of Illinois Eng. Experiment Station.

(10) "Dependability of the Theory of Concrete Arches", by Hardy Cross, M. Am. Soc. C. E., *Bulletin No. 203,* Univ. of Illinois Eng. Experiment Station.

(11) "Reaction Components for Skew Barrel Arches", by George E. Beggs, Chapter IX of this Report.

(12) "Effects of Time Yield on the Stresses in Arches", by W. M. Wilson, Chapter VIII of this Report.

(13) "Plastic Flow in Concrete Arches", by Lorenz G. Straub, Assoc. M. Am. Soc. C. E., *Transactions,* Am. Soc. C. E., Vol. 95 (1931), p. 613.

(14) "Plain and Reinforced Concrete Arches", by Charles S. Whitney, M. Am. Soc. C. E., *Proceedings,* Am. Concrete Inst., Vol. 28 (1932), p. 479.

(15) "Loading Tests on a Reinforced Concrete Arch", by Albin L. Gemeny and W. F. Hunter, *Public Roads,* Vol. 9, No. 10, December, 1928.

(16) "Model Analysis of a Reinforced Concrete Arch", by J. T. Thompson, M. Am. Soc. C. E., *Public Roads,* Vol. 9, No. 11, January, 1929. (Complete information regarding the tests of a model of the Yadkin River Bridge with continuous superstructure.)

"Deck Participation in Concrete Arch Bridges", by A. H. Finlay, Assoc. M. Am. Soc. C. E., *Civil Engineering,* Vol. 2 (1932), p. 685. (A method of obtaining influence lines and temperature reactions experimentally.)

"Examples of Deck Participation", by George E. Beggs, *Civil Engineering,* Vol. 3 (1933), p. 35. (A comparison of field and model tests.)

A general treatment of experimental methods of stress analysis will be found in the following references: "Elastic Arch Bridges", by McCullough and Thayer, Chapter VII; *Proceedings,* American Concrete Institute, Vol. 17

(1922), p. 58; *Journal*, Franklin Institute, March, 1927; and *Transactions*, Am. Soc. C. E., Vol. 88 (1925), p. 1208.

(17) "Continuous Frames of Reinforced Concrete", by Hardy Cross and N. D. Morgan, M. Am. Soc. C. E.

(18) "Analysis of the Stresses in the Ring of a Concrete Skew Arch", by J. Charles Rathbun, *Transactions*, Am. Soc. C. E., Vol. 87 (1924), p. 611.

(19) "Analysis of Multiple-Span Arches", by W. M. Wilson, Chapter X of this Report.

Article 5.—Laboratory Tests.—The Committee had not been studying its problems long before it was realized that much information of value could be gained by laboratory tests of arch bridges where the conditions could be controlled and the reactions and loads accurately weighed.

At the solicitation of the Committee the University of Illinois Engineering Experiment Station agreed to allow Professor Wilson to devote the major portion of his time to this investigation, and to provide space for the tests in its Materials Testing Laboratory.

The investigations undertaken were:

I.—Tests of single-span arch ribs.
II.—Tests of single-span arches with spandrel columns and decks.
III.—Tests of three-span arch ribs on slender piers.
IV.—Tests of three-span arch bridges with spandrel columns and decks, on slender piers.
V.—Tests to determine the effect of time yield in concrete upon the deformation stresses in a reinforced concrete arch bridge.

The detailed results of these tests are given in Publications Nos. (3), (6), (7), (8), (9), and (12), but a brief résumé of the tests and the results obtained are appended hereto.

Respectfully submitted,

Special Committee on Concrete and Reinforced Concrete Arches,

CLYDE T. MORRIS, *Chairman*,
GEORGE E. BEGGS,
E. H. HARDER,
A. C. JANNI,
WILBUR M. WILSON.

November 22, 1934.

II.—TESTS OF SINGLE-SPAN ARCH RIBS

Article 1.—Objects.—The objects of the investigation were:

1.—To compare measured stresses with those computed by the elastic theory, both when the stresses are produced by loads and when they are produced by abutment movements. The theory itself, being a proposition in geometry, needs no experimental verification, but it can only be applied to an arch by making certain assumptions relative to the behavior of the material. The validity of using the elastic theory as a means of analyzing the stresses in an arch depends upon the magnitude of the errors introduced in the analysis because of the errors in the assumptions.

2.—To compare the ultimate strength of the concrete as developed in the arch rib with the strength of the same concrete as developed in 6 by 12-in. control cylinders. The concrete in the arch rib is subjected to direct compression, bending, and shear, whereas that in the cylinders is subjected to axial compression only.

3.—To determine the effect upon the strength of an arch rib of its slenderness ratio. Arch ribs are long and slender, the ratio of unsupported length to width being sometimes greater than the permissible value of the same ratio for columns.

In this investigation thirteen reinforced concrete arch ribs were tested, having a span of 17 ft 6 in. and a rise of 4 ft 0$\frac{1}{16}$ in., measured to points on the arch axis. The intrados and extrados were circular arcs centered to give a depth at the crown of 5 in. and at the springing of about 10.8 in. The arch rib is shown diagrammatically in the testing machine in Fig. 1. The equipment was of such character that the abutments could either be held in position or moved at will, and means were provided for measuring the magnitude of the horizontal thrust, and for determining the position as well as the magnitude of the vertical reaction.

The four arches used in the study to determine the effect of the slenderness ratio of an arch rib upon its strength, had widths of 8 in., 6$\frac{1}{8}$ in., 4$\frac{1}{2}$ in., and 3$\frac{1}{8}$ in., respectively. Each arch contained the same amount of steel from end to end and was such that it produced 1.96% at the crown. Each of the other arches was 6$\frac{1}{8}$ in. wide.

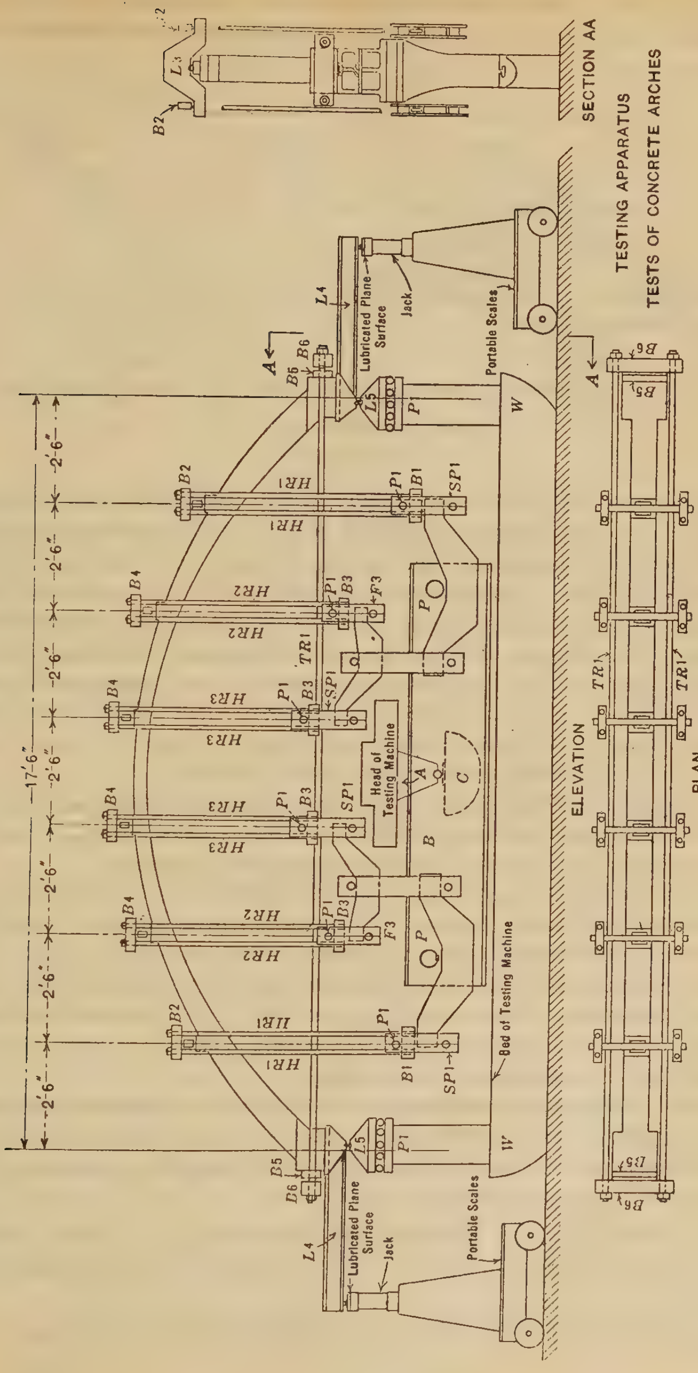
Three 6 by 12-in. control cylinders were poured for each arch. The arches and cylinders were stripped when two days old and wrapped in burlap, which was thoroughly soaked with water every other day for a period of two weeks and then removed. They were then stored in the laboratory until tested at an age of approximately 30 days for all except two, which were approximately 60 days old when tested.

The hangers through which the load was applied to the arch were arranged so that they exerted no horizontal or lateral force upon the arch. This feature is of special importance in the tests to determine the effect of the slenderness of the rib upon its strength.

Some of the arches were subjected to symmetrical and others to unsymmetrical loads. The symmetrical load was such that it gave a thrust line lying within the kern of the rib over its entire length. For the unsymmetrical loading the arrangement was such that the load on one-half the arch was 50% greater than on the other half, the purpose being to simulate the condition of dead load over the entire span and live load over one-half the span, the rate of loading being twice as great for dead load as for live load.

Article 2.—Verification of Elastic Theory.—Fig. 2 shows the relation between the theoretical and measured horizontal reaction for symmetrically and unsymmetrically loaded arches.

Similar diagrams were drawn for seven arches subjected to symmetrical, and for two arches subjected to unsymmetrical, loads. Although all the arches did not behave exactly alike, the discrepancy between the measured and the computed values did not exceed the tolerance of the apparatus.


SECTION AA
B2
L3
A
B5
B6
L4
Lubricated Plane Surface
Jack
Portable Scales
L5
P1
W
2'-6"
17'-6"
B2
B4
HR1
HR2
HR3
P1
B1
B3
SP1
F3
TR1
P
Head of Testing Machine
A
C
B
Bed of Testing Machine
ELEVATION
PLAN
TR1
B5
B6

TESTING APPARATUS
TESTS OF CONCRETE ARCHES

FIG. 1.

In Figs. 3 and 4 are shown the theoretical and measured positions of the thrust line at various loads for the same symmetrically and unsymmetrically loaded arches.

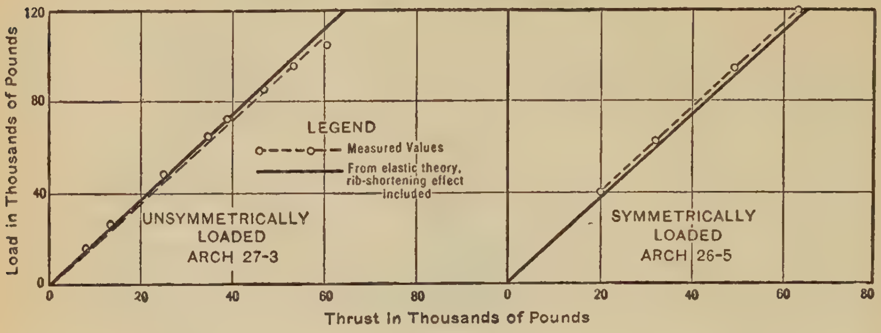


FIG. 2.

The theoretical and measured values of the unit stress in the concrete for the same arch are compared in Figs. 5 and 6. The theoretical values were corrected for rib-shortening and the measured values were obtained from the strain-gauge readings. The stress-strain relation used in converting strains into stresses was obtained for each section at which strains were measured, by computing the tangential thrust from the abutment reactions, deducting the measured stress in the longitudinal reinforcing steel, and considering the remainder of the tangential thrust to be taken by the concrete. The strain was measured at the top and bottom of each section so that the resulting diagram was really the average-stress, mean-strain diagram.

The theoretical and measured values are in excellent agreement at those sections for which the moment is small; but the measured stress is less than the theoretical at those sections having a large flexural stress.

Diagrams similar to Figs. 5 and 6 were drawn for three symmetrically and two unsymmetrically loaded arches. They are not exactly alike; yet all show about the same discrepancy between theoretical and measured values.

The relation between the ratio of flexural to axial stress and the ratio of theoretical to measured stress may be seen in Table 1 for symmetrically loaded arches and in Table 2 for those that are unsymmetrically loaded, the ratios being given at seven sections of each arch.

The effect of abutment movements on reactions was determined by measuring the changes in the reactions due to moving the abutments definite predetermined amounts, the total load on the arch remaining constant. There were four steps in the operation:

1.—The arch was loaded, the abutments being restrained, and all reactions, horizontal thrust, vertical reactions, and moments at the springing, were recorded.

2.—The abutments were spread, abutment rotation and change in load being prevented.

3.—The north abutment was rotated, the span, load, and angular position of the south abutment being kept constant.

4.—The south abutment was rotated, the span, load, and angular position of the north abutment being kept constant.

TABLE 1.—RELATION BETWEEN RATIO OF FLEXURAL TO AXIAL STRESS ON A SECTION AND RATIO OF THEORETICAL TO MEASURED STRESS ON SAME SECTION. SYMMETRICALLY LOADED ARCHES.

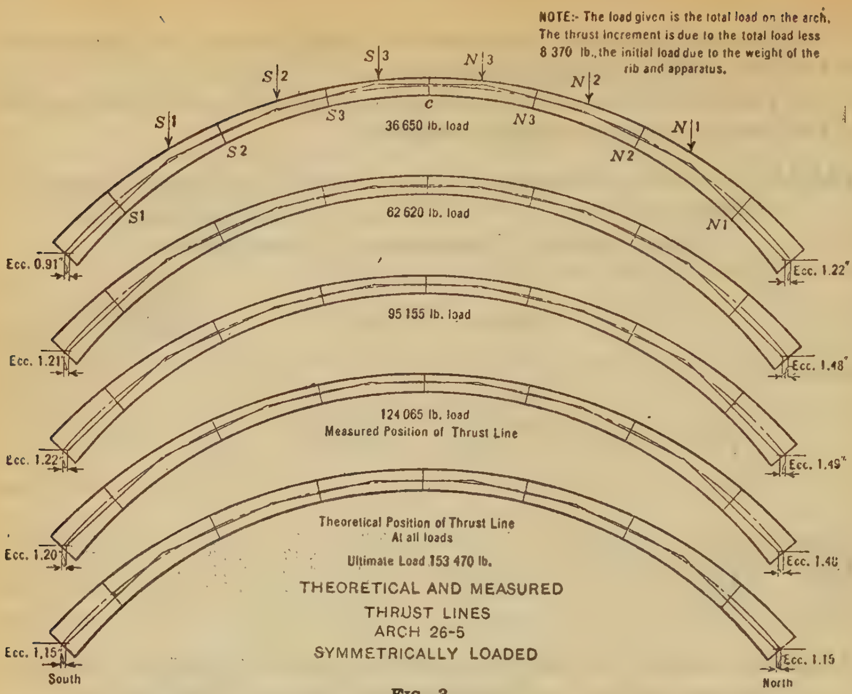
Section No.	RATIO OF THEORETICAL TO MEASURED STRESS.								THEORETICAL STRESS DUE TO 1 000-LB. LOAD.		Ratio, flexural to axial stress.
	Arch No. 26-1.	Arch No. 26-2.	Arch No. 26-3.	Arch No. 26-4.	Arch No. 26-5.	Arch No. 26-6.	Arch No. 26-10.	Average.	Top.	Bottom.	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
*N*1 and *S*1.	1.36	1.37	1.35	1.35	1.14	1.32	1.47	1.337	— 1.5	—21.5	0.87
*N*2 and *S*2.	1.34	1.09	1.21	0.98	1.09	1.11	1.22	1.149	— 9.6	—14.8	0.21
*N*3 and *S*3.	1.22	1.07	1.20	1.10	1.12	1.14	1.18	1.147	—12.4	—14.2	0.07
Center.....	1.00	1.06	1.60	1.10	1.11	1.27	1.04	1.168	—12.0	—15.6	0.15

TABLE 2.—RELATION BETWEEN RATIO OF FLEXURAL TO AXIAL UNIT STRESS ON A SECTION AND RATIO OF THEORETICAL TO MEASURED STRESS ON THE SAME SECTION. UNSYMMETRICALLY LOADED ARCHES.

Section No.	RATIO OF THEORETICAL TO MEASURED STRESS.			THEORETICAL STRESS DUE TO 1 000-LB. LOAD.		Ratio, flexural to axial stress.
	Arch No. 27-3.	Arch No. 27-4.	Average.	Top.	Bottom.	
(1)	(2)	(3)	(4)	(5)	(6)	(7)
N 1..............	1.53	1.43	1.48	+ 7.9	—32.2	1.64
N 2..............	0.99	0.97	0.98	—14.5	—10.8	0.17
N 3..............	1.38	1.47	1.43	—25.7	— 0.9	0.93
Center	1.06	0.94	1.00	—12.2	—15.4	0.12
S 3..............	1.42	1.38	1.40	+ 0.4	—27.0	1.03
S 2..............	1.30	1.13	1.22	— 5.0	—19.0	0.58
S 1..............	1.02	0.94	0.98	—11.0	—10.6	0.02

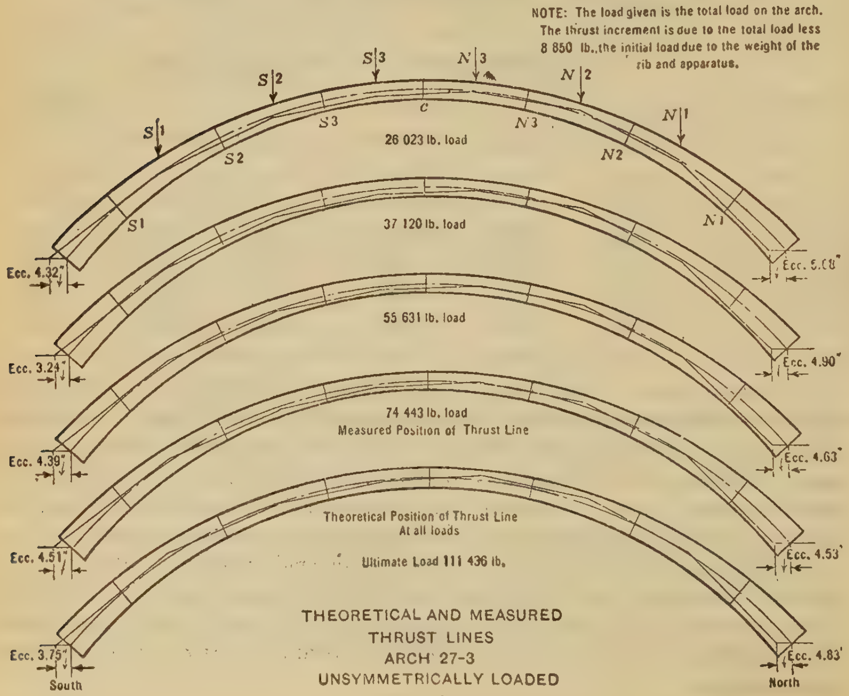
The observations made after each of these movements gave data from which the following quantities could be computed: (*a*) Thrust and moment at the springing due to unit change in span; (*b*) thrust and moment at the nearer springing, and moment at farther springing, due to unit rotation of north abutment; and (*c*) thrust and moment at the nearer springing, and moment at farther springing due to unit rotation of south abutment.

The theoretical value of these quantities depends upon the value of *E* for concrete. The tests have been interpreted by computing a composite *E* (Tables 3 and 4) which, if it were constant over the entire arch, would make equal the theoretical and measured reactions resulting from the abutment displacements.


NOTE:- The load given is the total load on the arch. The thrust increment is due to the total load less 8 370 lb., the initial load due to the weight of the rib and apparatus.

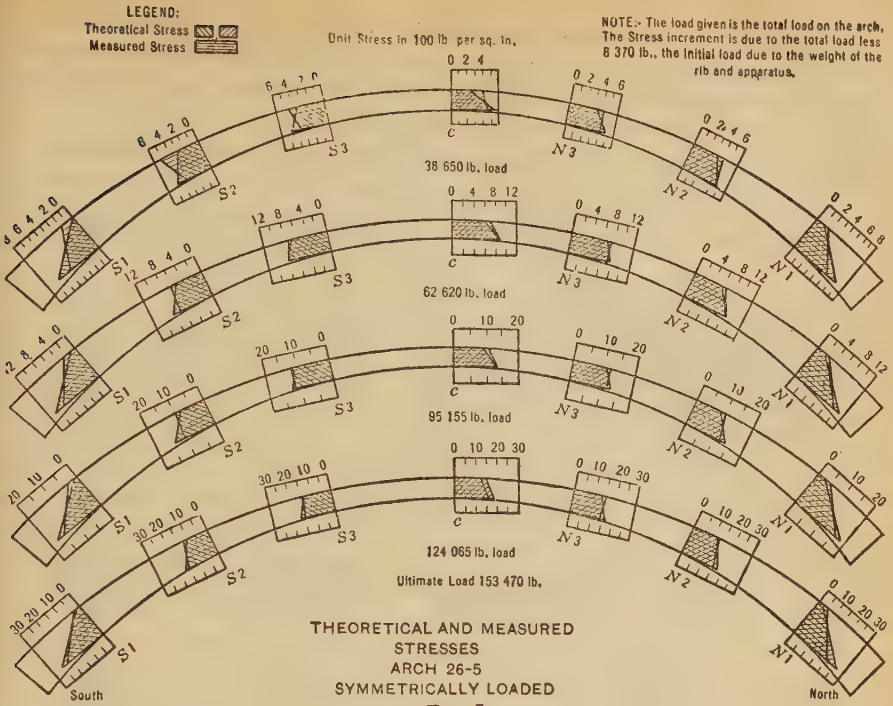
S|3
N|3
S|2
N|2
S|1
N|1
c
S3
N3
S2
N2
S1
N1
36 650 lb. load
62 620 lb. load
95 155 lb. load
124 065 lb. load
Measured Position of Thrust Line
Theoretical Position of Thrust Line
At all loads
Ultimate Load 153 470 lb.
THEORETICAL AND MEASURED
THRUST LINES
ARCH 26-5
SYMMETRICALLY LOADED
Ecc. 0.91"
Ecc. 1.22"
Ecc. 1.21"
Ecc. 1.48"
Ecc. 1.22"
Ecc. 1.49"
Ecc. 1.20"
Ecc. 1.48
Ecc. 1.15"
Ecc. 1.15
South
North

FIG. 3.


NOTE: The load given is the total load on the arch. The thrust increment is due to the total load less 8 850 lb., the initial load due to the weight of the rib and apparatus.

S|3
N|3
S|2
N|2
S|1
N|1
c
S3
N3
S2
N2
S1
N1
26 023 lb. load
37 120 lb. load
55 631 lb. load
74 443 lb. load
Measured Position of Thrust Line
Theoretical Position of Thrust Line
At all loads
Ultimate Load 111 436 lb.
THEORETICAL AND MEASURED
THRUST LINES
ARCH 27-3
UNSYMMETRICALLY LOADED
Ecc. 4.32"
Ecc. 5.08"
Ecc. 3.24"
Ecc. 4.90"
Ecc. 4.39"
Ecc. 4.63"
Ecc. 4.51"
Ecc. 4.53"
Ecc. 3.75"
Ecc. 4.83"
South
North

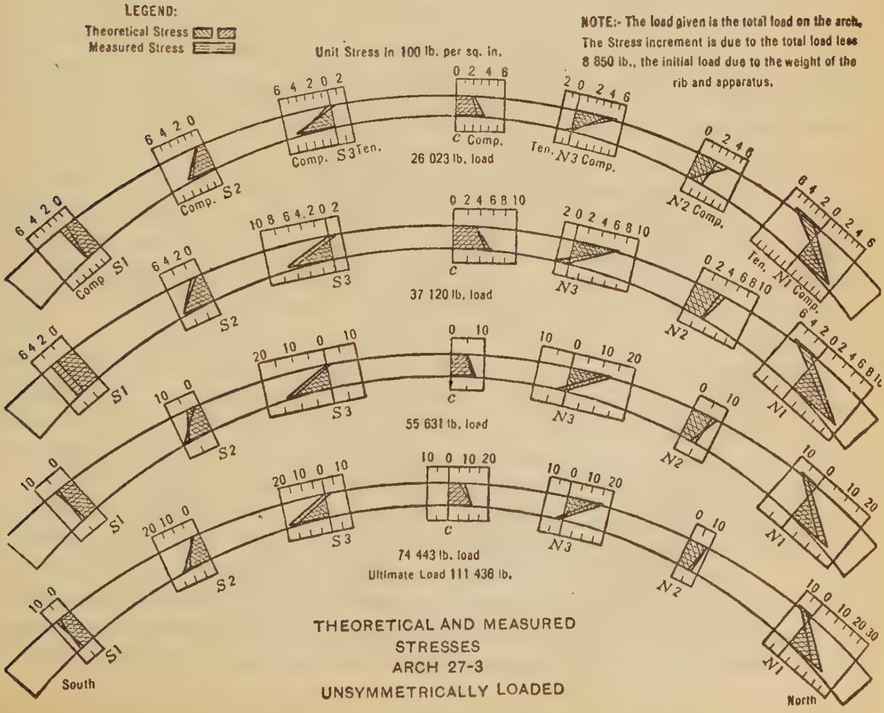
FIG. 4.


LEGEND:
Theoretical Stress ▨ ▨
Measured Stress ▤
Unit Stress in 100 lb. per sq. in.
South
North
S1
S2
S3
C
N3
N2
N1
38 650 lb. load
62 620 lb. load
95 155 lb. load
124 065 lb. load
Ultimate Load 153 470 lb.
0 2 4
0 4 8 12
0 10 20
0 10 20 30

NOTE:- The load given is the total load on the arch. The Stress increment is due to the total load less 8 370 lb., the initial load due to the weight of the rib and apparatus.

THEORETICAL AND MEASURED STRESSES
ARCH 26-5
SYMMETRICALLY LOADED

FIG. 5.


LEGEND:
Theoretical Stress ▨ ▨
Measured Stress ▤
Unit Stress in 100 lb. per sq. in.
South
North
S1
S2
S3
C
N3
N2
N1
Comp.
Ten.
26 023 lb. load
37 120 lb. load
55 631 lb. load
74 443 lb. load
Ultimate Load 111 438 lb.
0 2 4 6
0 2 4 6 8 10
0 10
10 0 10 20

NOTE:- The load given is the total load on the arch. The Stress increment is due to the total load less 8 850 lb., the initial load due to the weight of the rib and apparatus.

THEORETICAL AND MEASURED STRESSES
ARCH 27-3
UNSYMMETRICALLY LOADED

FIG. 6.

The computations for determining the effect of abutment movements on the reactions were based on the assumptions that E for the concrete has the same value at all sections and at all loads and, in determining the moment of inertia of a section, that the concrete resists tension.

TABLE 3.—VALUES OF COMPOSITE E OBTAINED FROM VARIOUS RELATIONS.

Relation from which E was determined. (1)	Maximum E. (2)	Minimum E. (3)	Average of five tests, E. (4)
(1) Moment at springing due to change in span	3 124 000	2 153 000	2 546 000
(2) Horizontal thrust due to change in span	3 575 000	2 827 000	3 169 000
(3) Horizontal thrust due to rotation of an abutment	2 809 000	1 863 000	2 564 000
(4) Moment at a springing due to the rotation of the adjacent abutment	2 772 000	2 125 000	2 390 000
(5) Moment at a springing due to the rotation of the opposite abutment	3 599 000	2 937 000	3 346 000

TABLE 4.—AVERAGE VALUES OF COMPOSITE E FOR VARIOUS ARCHES.

Arch No.	Load, in pounds.	Approximate average unit stress, in pounds per square inch.	Composite E, average of values obtained from five relations.	Strength of concrete as determined from tests of cylinders.
26-6	18 920	240	2 758 000*	2 918
26-6	22 440	280	2 605 000*	2 918
26-7	21 045	267	2 958 000	3 112
26-7	41 085	516	3 080 000	3 112
26-8	39 660	500	2 614 000	2 452

* Compare with value of 2 540 000 from Table 7.

The close agreement between theoretical and measured values, when the former were computed on this basis, would seem to justify the following conclusion:

In analyzing an arch to determine the thrust and moment, E may be assumed to have the same value at all sections and all loads, and the moment of inertia of a section may be based on the assumption that the concrete takes tension. It does not follow, however, that the moment of inertia used in computing the unit stress on a section due to a given moment and thrust should be based on the same assumption.

Article 3.—Comparison of Strengths of Concrete in Arch Ribs and Cylinders.—The relation between the strength of concrete as developed in an arch and that of the same concrete as developed in 6 by 12-in. control cylinders is of vital importance because of its influence on the unit stress to be used in the design. An accurate comparison of these strengths, however, is difficult because of the many factors affecting them. The large variations in the modulus of elasticity at the different sections of the same arch indicate that the concrete was not uniform in quality even if all parts of the arch were poured from the same mix and on the same day. For this reason the modulus of elasticity as well as the ultimate unit stress was used in studying the properties developed by the material in the arch and in the cylinders.

The theoretical unit stress corresponding to the ultimate load, at each of seven sections, is given in Tables 5 and 6. Table 7 shows the modulus of elasticity of the concrete at the same sections. Not a single arch broke at a section of maximum stress, but seven of the nine arches failed at a section for which the modulus of elasticity was a minimum, and for the two arches that did not fail at the section of minimum *E*, the minimum modulus of elasticity occurred at a section where the unit stress due to a given load was correspondingly lower than the modulus of elasticity.

The great variation in the modulus of elasticity of the concrete in an arch and the fact that the arches in general broke at sections of small modulus of elasticity rather than at those of large unit stress emphasizes the necessity of placing concrete in the same manner over the entire arch. The experimental arches were poured in a vertical position. The sides and bottom of the forms were in place when pouring began. Pouring started at the two ends simultaneously, and the top of the form was added as pouring progressed. It was difficult to place the concrete near the ends, but easier toward the center. The concrete which was most difficult to place received the most attention, and the consolidation was greater at the ends than at any other point, decreasing toward the crown. This is apparent from the values of *E* given in Table 7.

The unit stress for the arch (Column (2) Table 8) is the theoretical stress at the section where failure occurred, corresponding to the ultimate load; the unit stress for the cylinder (Column (3)) is the average value obtained from the tests of three cylinders for each arch, poured from batches that went into the structure. The moduli of elasticity of the concrete in the arch and in the cylinders are also given in Table 8. Only arches 6 in. or more in width are included in this comparison. If the modulus of elasticity is accepted as an indication of the strength of the concrete, the failure of all low-strength arches can be attributed to weak concrete at the point of failure.

The results presented in Tables 5, 6, 7, and 8 seem to justify the belief that concrete will develop approximately the same strength in an arch and in a control cylinder providing it is consolidated to the same degree in both. Furthermore, the strength of an arch is determined by the strength of its weakest section, and failure to consolidate the concrete at a single section may be as serious as a partial omission of cement, the use of poor aggregate, or the use of too much water.

The arches in these tests were poured more carefully than many concrete structures, yet the variations in *E*, and presumably in the strength, were very great.

Article 4.—Effect of Slenderness Ratio of Arch Rib on Its Strength.—Arches 26–1, 26–2, 26–3, and 26–4 were used in the study to determine the effect of the slenderness of the rib on its strength. They were identical in profile, all had the same percentage of reinforcement, and all were subjected to the same kind of loading; but they differed in width from 3½ in. to 8 in. As already noted, the loading hangers offered no resistance to horizontal motion of the arch.

TABLE 5.—ULTIMATE LOAD AND MAXIMUM THEORETICAL UNIT STRESS FOR SYMMETRICALLY LOADED ARCHES.

Arch No. (1)	Width of rib, in inches. (2)	Point of failure. (3)	Ultimate load, in pounds. (4)	Section No. (5)	Unit stress at ultimate load by elastic theory, in pounds per square inch. (6)
26-1	8	Center	193 820	*N* 1 and *S* 1	3 396
				N 2 and *S* 2	2 351
				N 3 and *S* 3	2 254
				Center	2 479*
26-2	6⅛	Load Point *S* 2	153 110	*N* 1 and *S* 1	3 494
				N 2 and *S* 2	2 428*
				N 3 and *S* 3	2 318
				Center	2 557
26-3	4⅛	Center	84 800	*N* 1 and *S* 1	2 636
				N 2 and *S* 2	1 834
				N 3 and *S* 3	1 744
				Center	1 942*
26-4	3⅛	Center	65 700	*N* 1 and *S* 1	2 936
				N 2 and *S* 2	2 032
				N 3 and *S* 3	1 952
				Center	2 169*
26-5	6⅛	Center	153 470	*N* 1 and *S* 1	3 300
				N 2 and *S* 2	2 271
				N 3 and *S* 3	2 179
				Center	2 394*
26-6	6⅛	Center	159 450	*N* 1 and *S* 1	3 428
				N 2 and *S* 2	2 360
				N 3 and *S* 3	2 264
				Center	2 487*
26-10	6⅛	Load Point *S* 4	147 860	*N* 1 and *S* 1	3 179
				N 2 and *S* 2	2 188
				N 3 and *S* 3	2 100
				Center	2 307*

* Sections where failure occurred.

TABLE 6.—ULTIMATE LOAD AND MAXIMUM THEORETICAL UNIT STRESS FOR UNSYMMETRICALLY LOADED ARCHES.

Arch No. (1)	Width of rib, in inches. (2)	Point of failure. (3)	Ultimate load, in pounds. (4)	Section No. (5)	Unit stress at ultimate load by elastic theory, in pounds per square inch. (6)
27-3	6⅛	Load Point *N* 4	111 436	*N* 1	3 600
				N 2	1 616
				N 3	2 864
				Center	1 716*
				S 3	3 009
				S 2	2 117
				S 1	1 226
27-4	6⅛	Section *S* 3	135 917	*N* 1	4 390
				N 2	1 971
				N 3	3 493
				Center	2 093
				S 3	3 670*
				S 2	2 582
				S 1	1 495

* Sections where failure occurred.

TABLE 7.—SECANT MODULUS OF CONCRETE IN ARCH AT STRESS OF 1 000 POUNDS PER SQUARE INCH, IN 1 000 000 POUNDS PER SQUARE INCH.

Arch No.	Section.							
	N 1.	*N* 2.	*N* 3.	Center.	*S* 3.	*S* 2.	*S* 1.	Average.
SYMMETRICALLY LOADED ARCHES								
26-1	3.12	2.73	2.41	2.25*	2.43	2.43	4.31	2.81
26-2	3.34	3.00	2.53	2.63	3.26	2.50*	3.25	2.93
26-3	3.45	2.28	2.21	1.18*	1.50	1.54	3.25	2.20
26-4	2.33	2.18	1.90	1.91*	2.16	2.10	2.55	2.16
26-5	3.92	2.56	2.46	2.06*	2.30	2.34	2.79	2.63
26-6	3.28	2.80	2.12	1.74*	2.50	2.36	2.99	2.54
26-10	2.62	1.70	1.88	1.48*	2.11	2.21	2.75	2.11
Average..........	....	....	....	....	....	....	...	2.48
UNSYMMETRICALLY LOADED ARCHES								
27-3	4.32	2.06	2.08	1.38*	1.62	1.93	1.74	2.16
27-4	5.41	2.42	2.25	1.82	2.15*	2.78	2.63	2.78
Average..........	....	....	....	...	....	...	...	2.47

* Sections at which failure occurred.

TABLE 8.—COMPARISON OF UNIT STRESSES DEVELOPED IN ARCHES AND IN CONTROL CYLINDERS.

Arch No.	UNIT STRESS, IN POUNDS PER SQUARE INCH.		SECANT MODULUS AT 1 000 POUNDS PER SQUARE INCH.‡			Section having maximum *E*.†
	Arch.	Cylinder.	Arch at section of failure.	Cylinder.	At section where it is a maximum.	
(1)	(2)	(3)	(4)	(5)	(6)	(7)
26-1	2 479	2 563	2.25	3.51	4.31	*S* 1
26-2	2 428	2 590	2.50	4.08	3.34	*N* 1
26-5	2 394	2 470	2.06	2.76	3.92	*N* 1
26-6	2 487	2 990	1.74	2.94	3.28	*N* 1
26-10	2 307	1 396*	1.48	1.41	2.75	*S* 1
27-3	1 716	3 380	1.38	2.95	4.32	*S* 1
27-4	3 670	3 575	2.15	4.00	5.41	*S* 1

* This is the value reported, but it is apparently in error.

† The maximum unit stress occurs at Sections *N*1 and *S*1 for all arches of the 26-series and at Sections *N*1 for those of the 27-series.

‡ Secant modulus is in 1 000 000 lb. per sq. in.

TABLE 9.—EFFECT OF SLENDERNESS RATIO ON STRENGTH OF AN ARCH RIB.

Arch No.	Width, in inches.	Slenderness ratio.	Ultimate load per inch width, in pounds.	Secant modulus at 1 000 lb. per sq. in. stress at section of failure.
26-1	8	30	24 230	2 250 000
26-2	6⅛	39	25 000	2 500 000
26-3	4½	53	18 840	1 118 000
26-4	3⅛	76	21 000*	1 910 000

* Buckling began at 18 700 lb. per in. width of rib and, if this load had been retained, failure would probably have eventually resulted without further increase in the load.

The relation between the load per inch width of arch and the lateral deflection at the crown is given in Fig. 7. The lateral deflection increased with the load up to the ultimate for all arches, but for only the one $3\frac{1}{8}$ in. wide did the failure partake of the character of buckling. For this arch the deflection was large and the failure was a typical buckling failure. The data pertaining to these arches are given in Table 9.

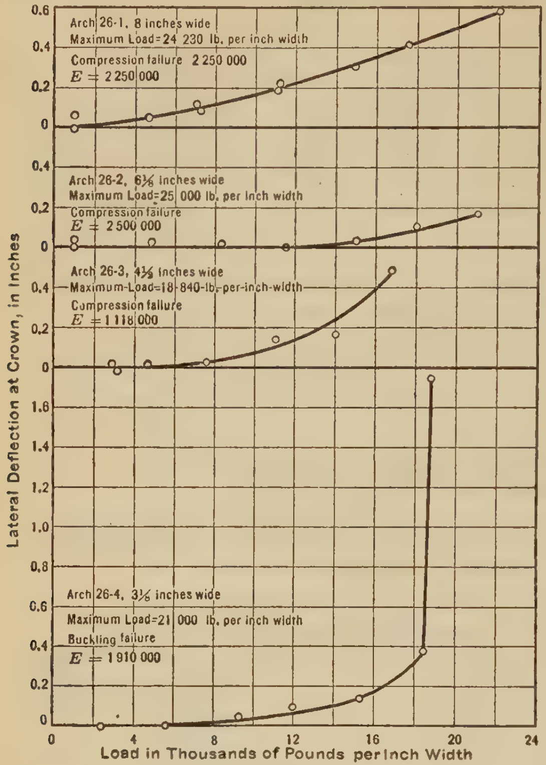


FIG. 7.

The results of these tests would seem to justify the conclusion that ribs having a ratio of unsupported length of arch axis to width, of 30, are not weakened by a tendency to buckle as long as they are not subjected to the action of lateral forces. Practical considerations should prevent the use of ribs of a greater slenderness ratio.

Acknowledgments.—The work described in this Chapter was done under the supervision of W. M. Wilson, M. Am. Soc. C. E., at the Engineering Experiment Station, University of Illinois, Urbana, Ill. (3).

III.—Tests of Single-Span Arches with Decks

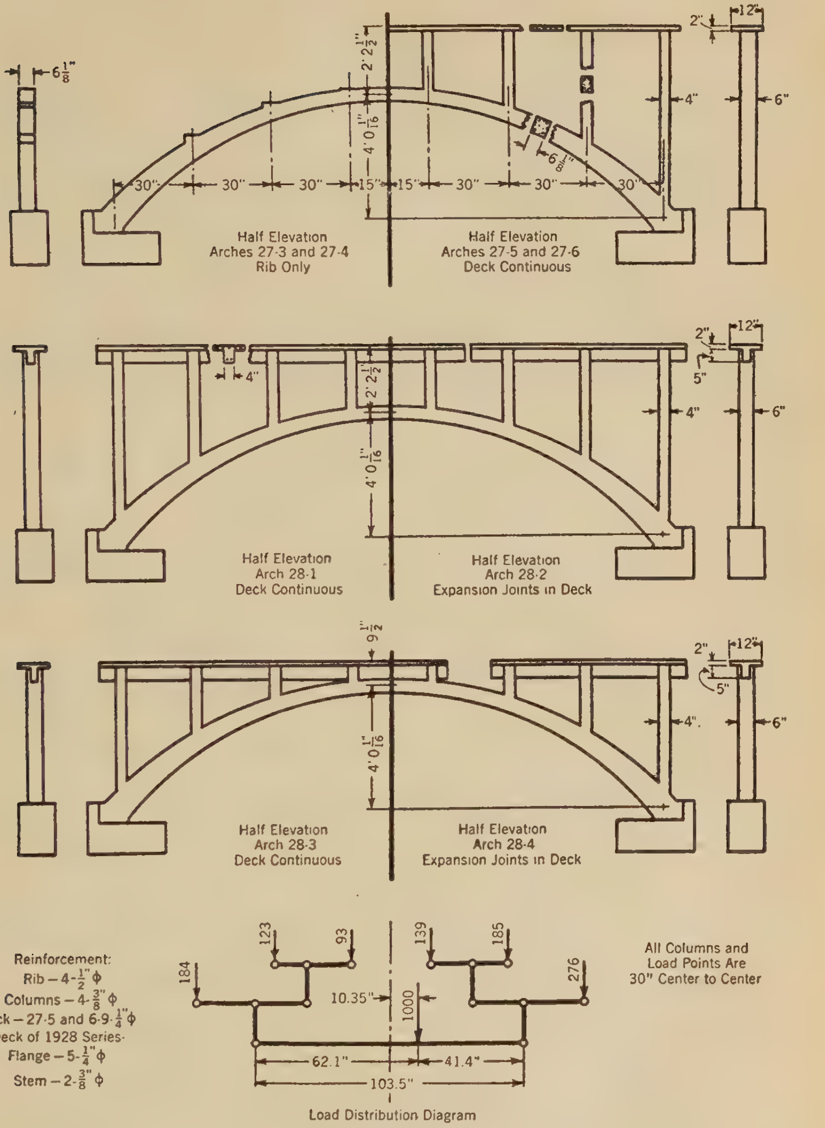
Article 1.—Objects.—The objects of the investigation were:

1.—To compare the behavior of a reinforced concrete monolithic structure consisting of an arch rib with spandrel columns and deck, with the behavior of a similar rib without deck;

2.—To determine the effect of varying the height of the deck above the crown of the arch rib; and,

3.—To determine the effect of, and the necessity for, intermediate expansion joints in the deck.

The tests were made upon six arches having ribs that were identical but decks that were different. The ribs were $6\frac{1}{8}$ in. wide and were of the same dimensions as those of the previous investigation of ribs without deck (see Fig. 1). The decks of two arches were low, being integral with the rib at


$6\frac{1}{8}$"
$2'\ 2\frac{1}{2}$"
$4'\ 0\frac{1}{16}$"
2"
12"
4"
6"
$6\frac{1}{8}$"
30"
30"
30"
15"
15"
30"
30"
30"
Half Elevation
Arches 27-3 and 27-4
Rib Only
Half Elevation
Arches 27-5 and 27-6
Deck Continuous
4"
$2'\ 2\frac{1}{2}$"
$4'\ 0\frac{1}{16}$"
2"
12"
5"
4"
6"
Half Elevation
Arch 28-1
Deck Continuous
Half Elevation
Arch 28-2
Expansion Joints in Deck
$9\frac{1}{2}$"
$4'\ 0\frac{1}{16}$"
2"
12"
5"
4"
6"
Half Elevation
Arch 28-3
Deck Continuous
Half Elevation
Arch 28-4
Expansion Joints in Deck
123
93
139
185
184
276
10.35"
1000
62.1"
41.4"
103.5"
Load Distribution Diagram

Reinforcement:
Rib – 4-$\frac{1}{2}$" ϕ
Columns – 4-$\frac{3}{8}$" ϕ
Deck – 27-5 and 6-9-$\frac{1}{4}$" ϕ
Deck of 1928 Series-
Flange – 5-$\frac{1}{4}$" ϕ
Stem – 2-$\frac{3}{8}$" ϕ

All Columns and
Load Points Are
30" Center to Center

FIG. 8. DETAILS OF DECKS

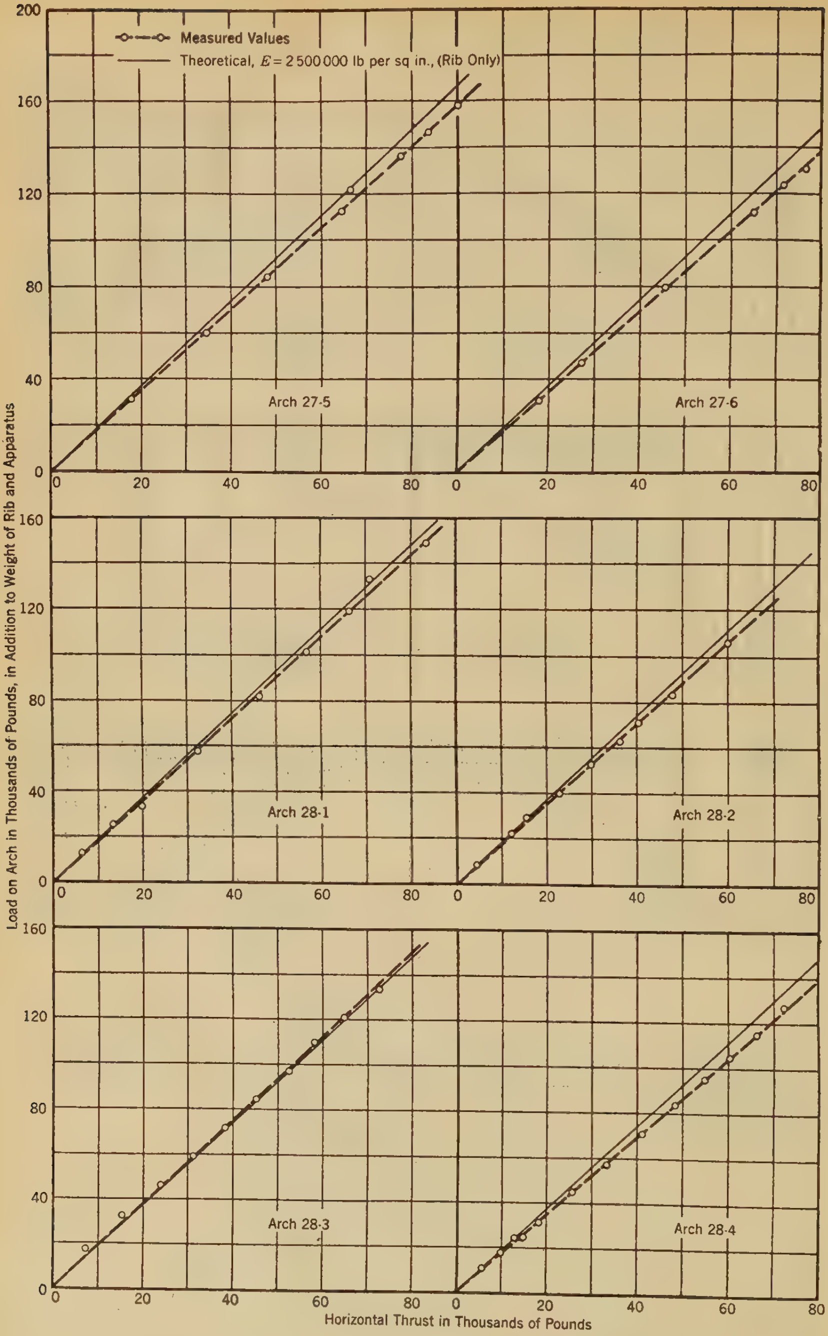


FIG. 9. RELATION BETWEEN LOAD AND HORIZONTAL THRUST, ARCH WITH DECK

the crown; the decks of four arches were high, being 2 ft above the rib at the crown. One arch with a high deck and one with a low deck had expansion joints in the deck near the third points. The other arches had decks without expansion joints. Fig. 8 gives the general dimensions of all the arches. The method of pouring and curing was similar to that already described for the arch ribs without decks (Chapter II).

The arches were subjected to an unsymmetrical load, the rate of loading being 50% greater for the north, than for the south, half of the arch. This loading was intended to simulate dead load over the entire span and live load over the north half, the rate of loading being one-half as great for live load as for dead load. The age of the arches when tested varied from 50 to 57 days.

Article 2.—Effect of Decks Upon Strength of Arches.—The relation between the load and the horizontal thrust is shown in Fig. 9. This may be compared with Fig. 2 which gives the same relation for a rib without deck.

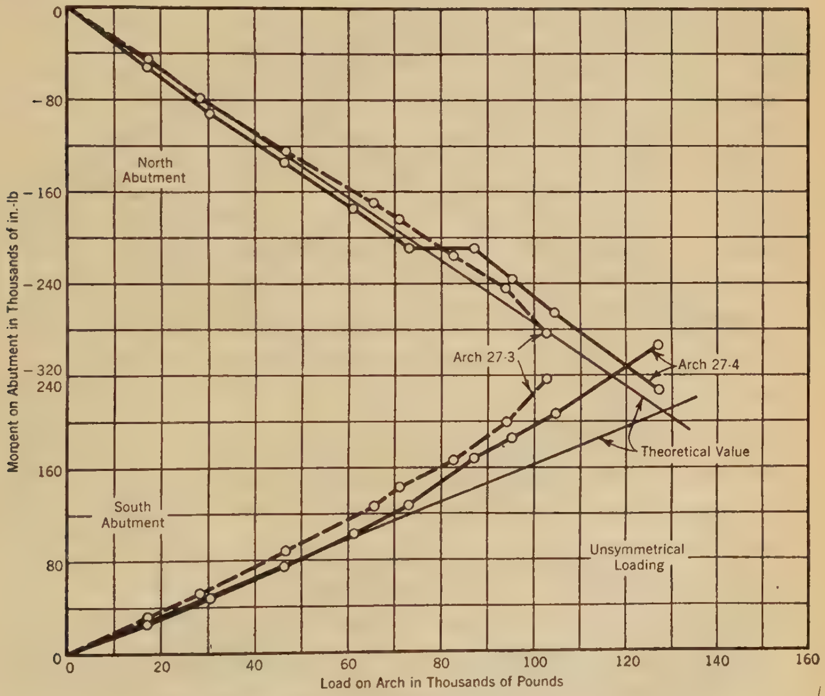


Fig. 10. Relation between Load and Moment on Abutments, Rib without Deck

Fig. 10 shows the relation between load and moment at the springing for the rib without deck and Fig. 11 gives the same relationship for the arches with decks.

The theoretical stresses in a rib without deck can be computed by the elastic theory. The stresses can also be computed for an arch consisting of a

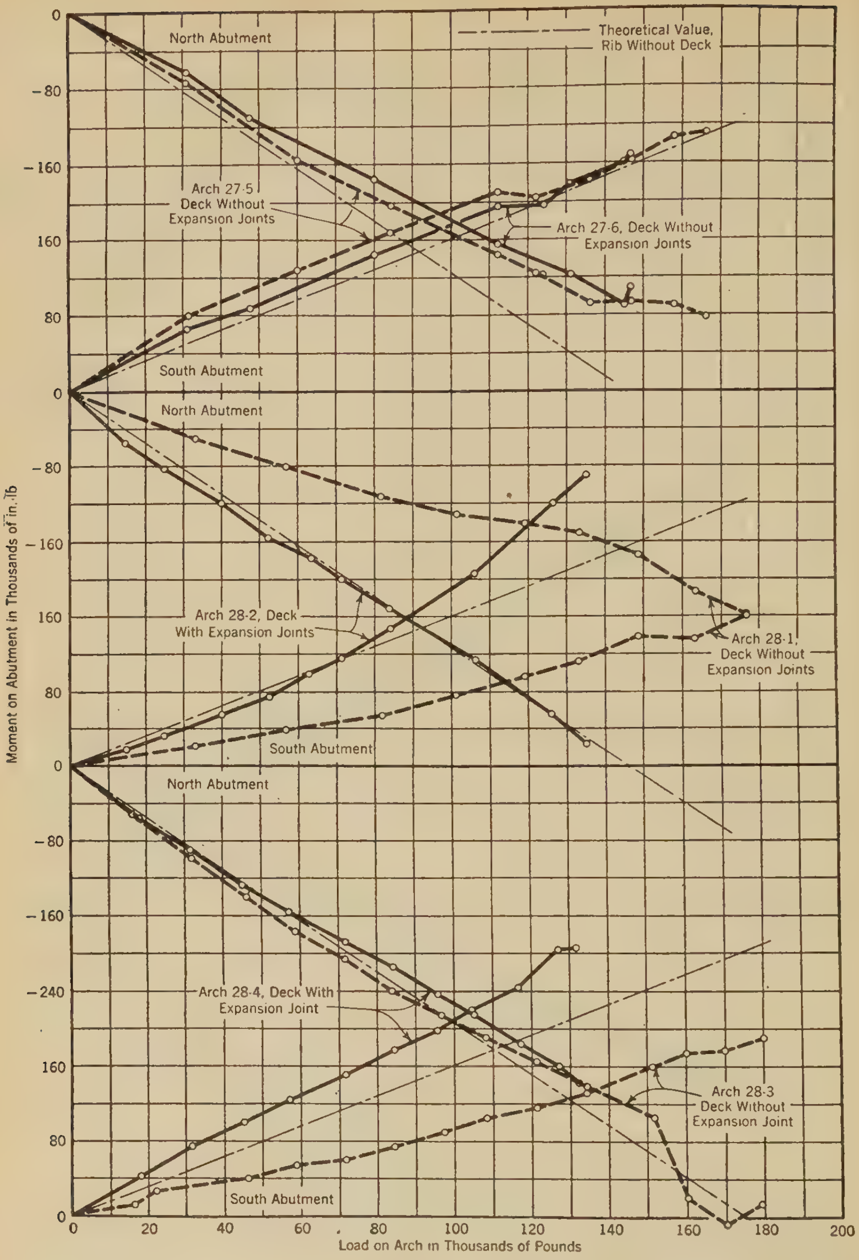


FIG. 11. RELATION BETWEEN LOAD AND MOMENT ON ABUTMENTS, ARCH WITH DECK

rib, spandrel columns, and deck, but the computations are so involved that they are seldom undertaken. They have not been made for the experimental arches.

The strains were measured at seven sections of the arch rib by means of a strain-gauge. The location of the sections at which strains were measured is shown in Fig. 12. Stress can be determined from strain providing the stress-strain relation for the material is known. The modulus of elasticity of steel is quite constant; for these tests it was taken as 30 000 000 lb per sq in. The stress-strain relation for concrete is a very uncertain quantity, however, and its value is influenced so seriously by the manner in which the specimen is poured that strain-gauge readings on an arch cannot be interpreted from the stress-strain relation of a control cylinder. Fortunately, however, the tangential thrust can be determined at every section of an arch from the measured loads and reactions. This thrust, less the stress taken by the steel (determined from strain-gauge readings on the latter), is the axial load on the concrete. This axial load, divided by the area of the concrete, gives the average unit stress on the concrete.

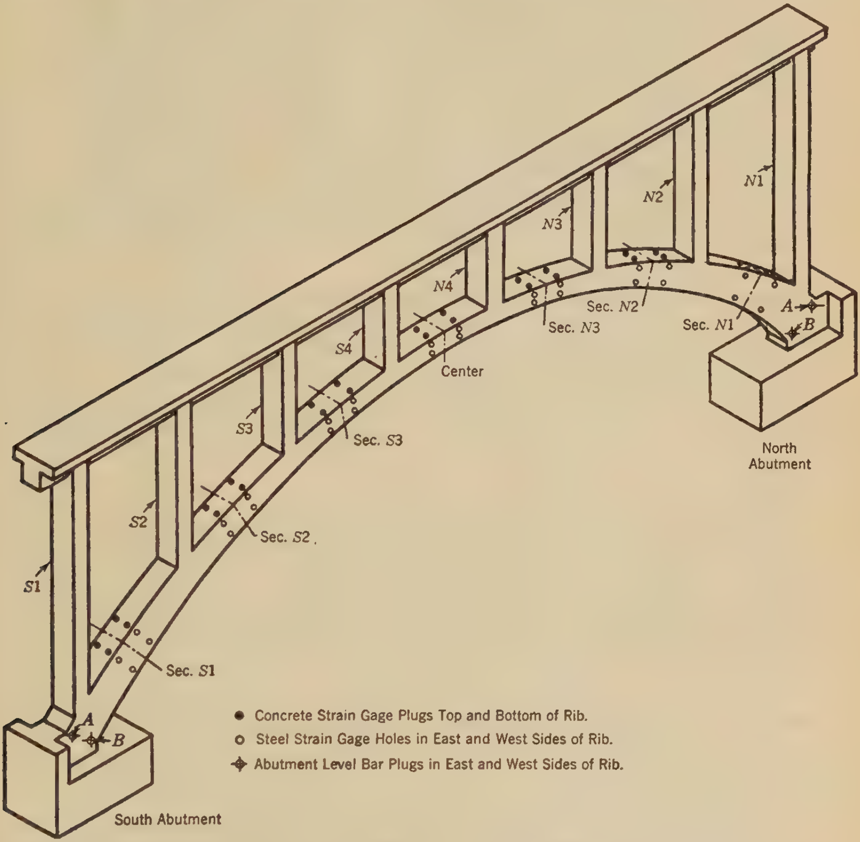

N1
N2
N3
N4
S4
S3
S2
S1
Sec. N2
Sec. N3
Sec. N1
A
B
Center
Sec. S3
Sec. S2
Sec. S1
North Abutment
A
B
South Abutment
● Concrete Strain Gage Plugs Top and Bottom of Rib.
○ Steel Strain Gage Holes in East and West Sides of Rib.
⌖ Abutment Level Bar Plugs in East and West Sides of Rib.

FIG. 12. SECTIONS AT WHICH STRAINS WERE MEASURED

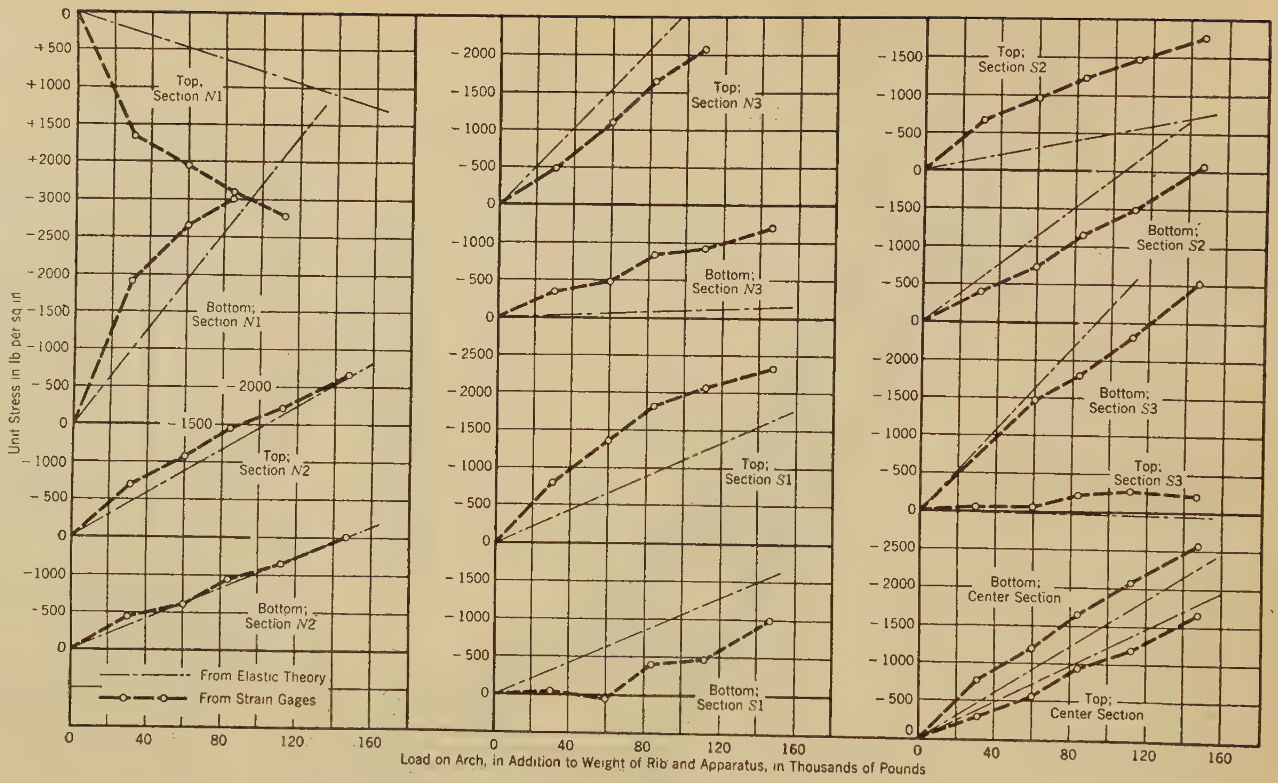


FIG. 13. RELATION BETWEEN LOAD AND UNIT STRESS, ARCH 27-5

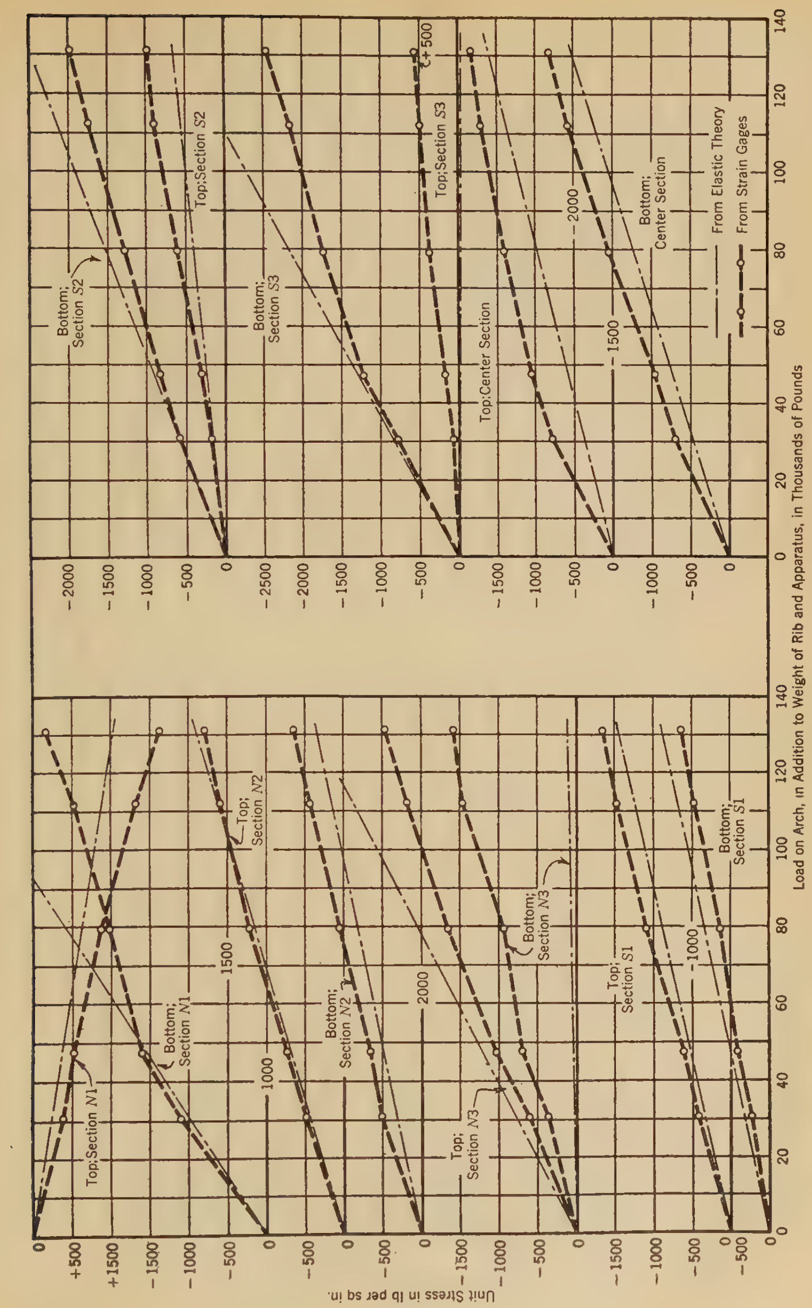


FIG. 14. RELATION BETWEEN LOAD AND UNIT STRESS, ARCH 27-6

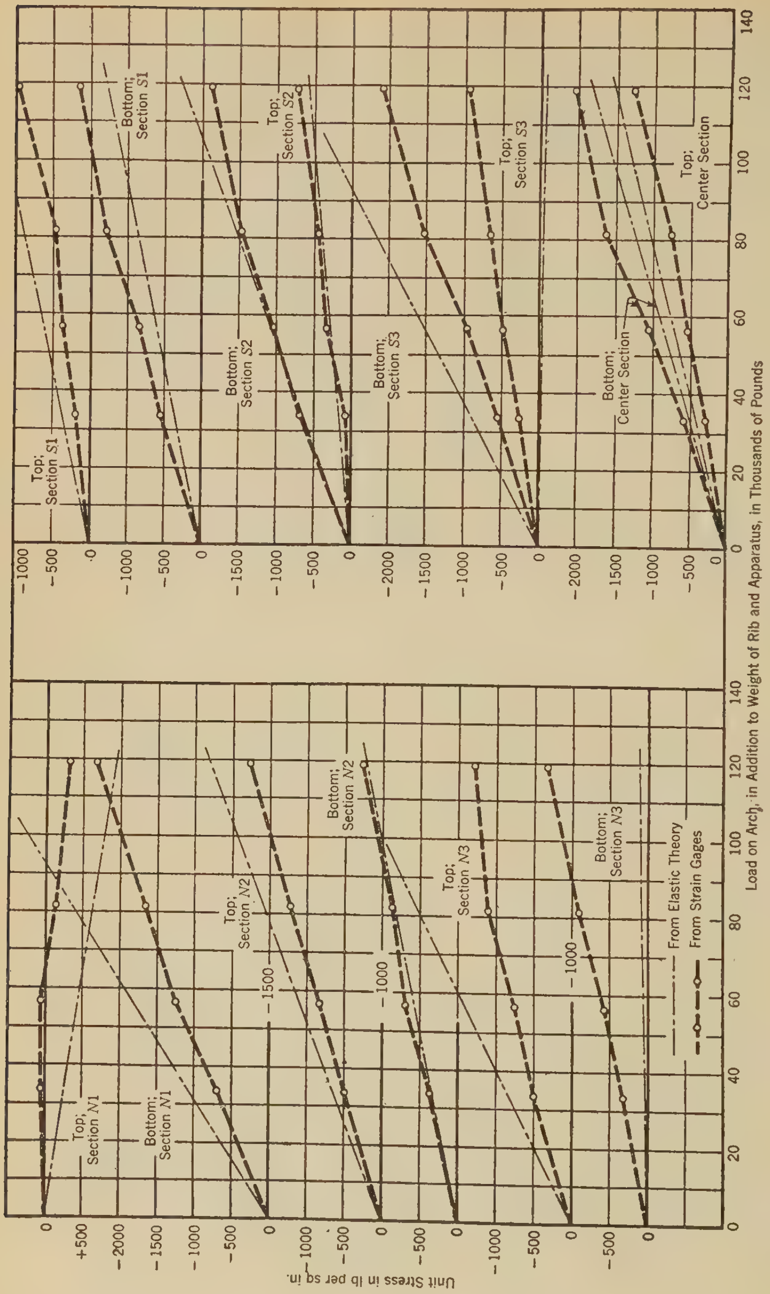


FIG. 15. RELATION BETWEEN LOAD AND UNIT STRESS, ARCH 28-1

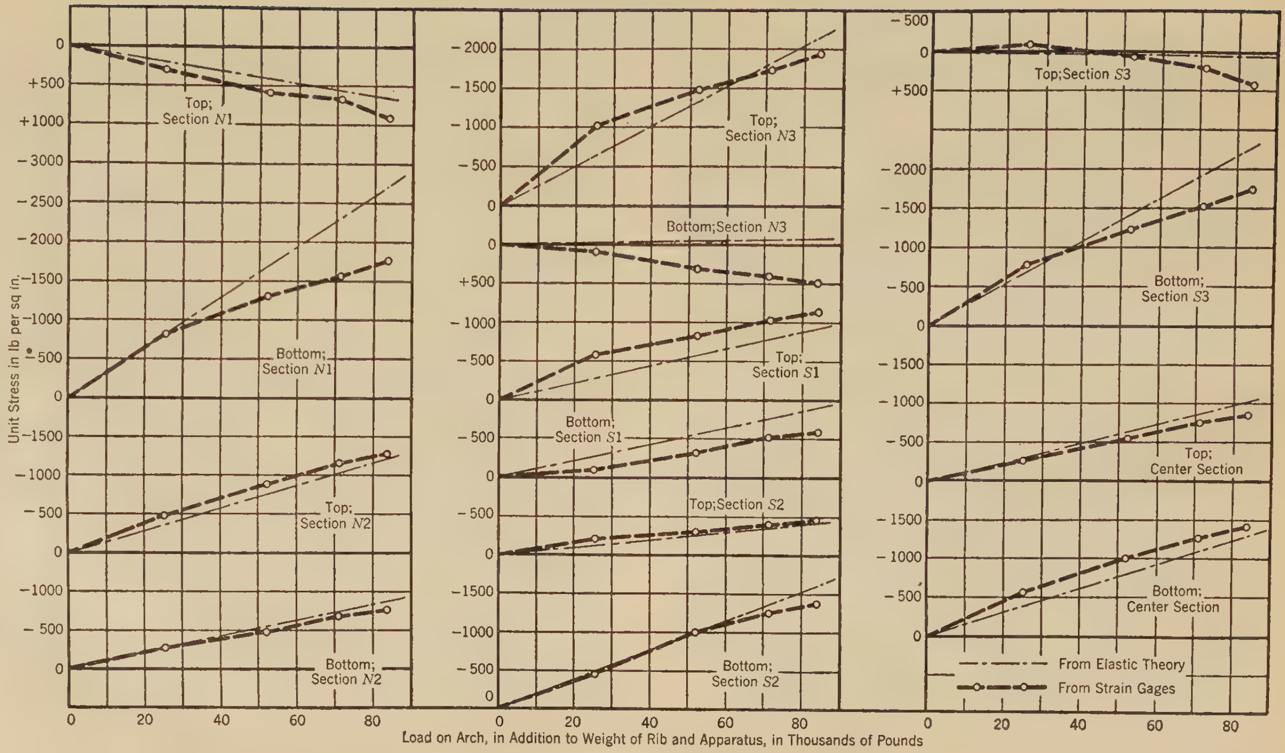


FIG. 16. RELATION BETWEEN LOAD AND UNIT STRESS, ARCH 28-2

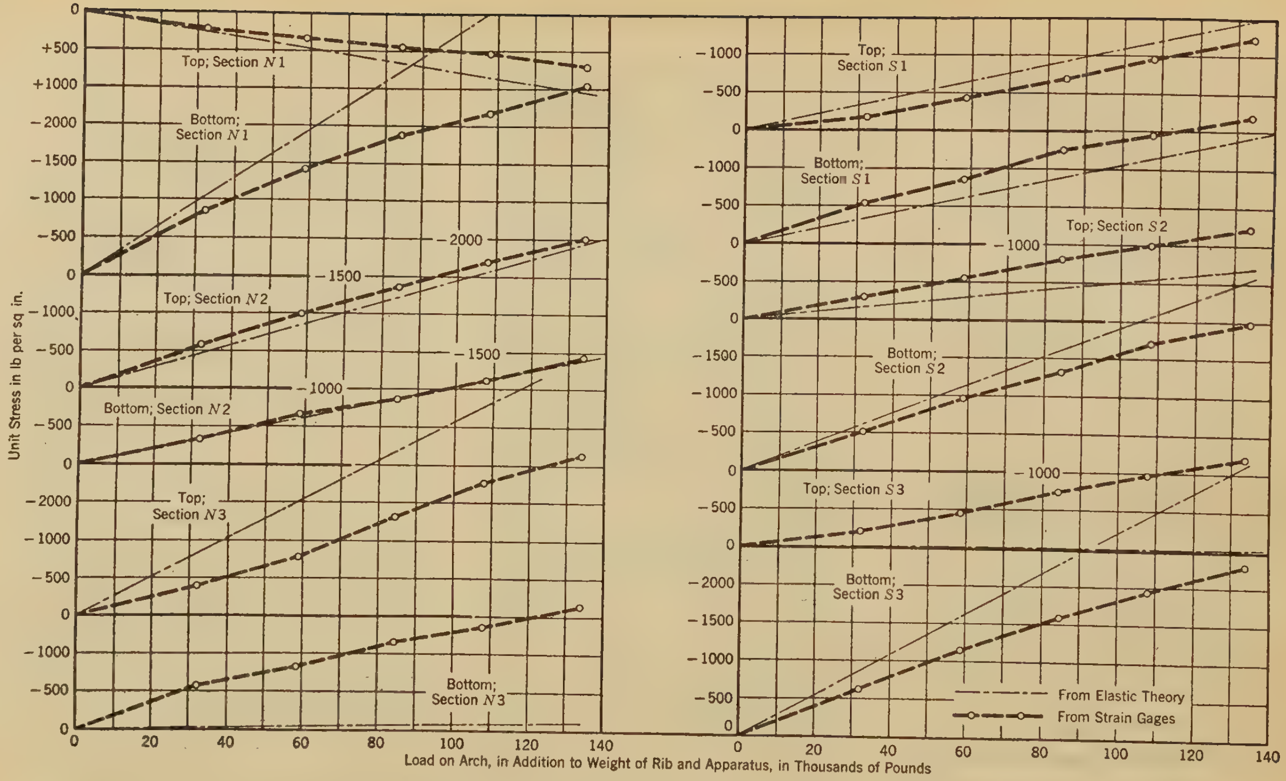


FIG. 17. RELATION BETWEEN LOAD AND UNIT STRESS, ARCH 28-3

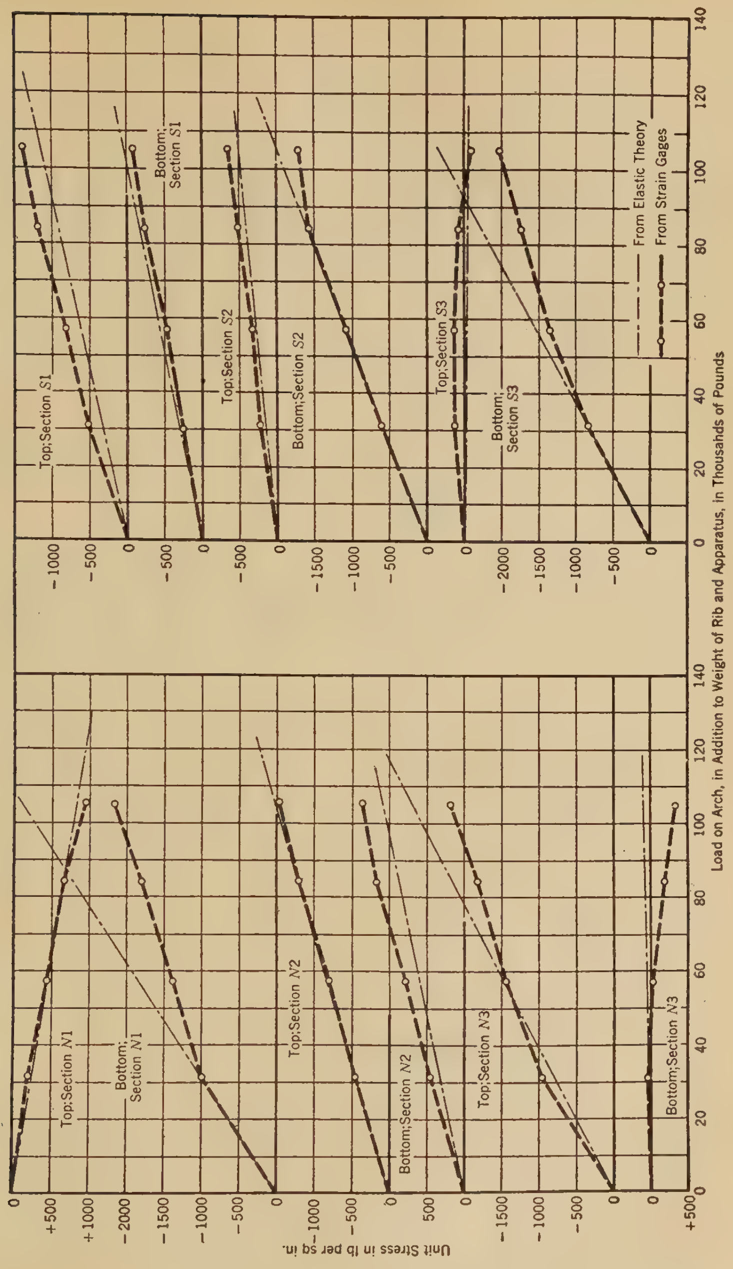


FIG. 18. RELATION BETWEEN LOAD AND UNIT STRESS, ARCH 28-4

For arches with decks, there is a slight error in the tangential thrust thus obtained, due to the shear in the spandrel columns. Although this shear is large enough to produce a large moment in the spandrel columns, its effect upon the tangential thrust in the rib is relatively small.

Figs. 13 to 18, inclusive, give the relation between load and unit stress for the arches with decks. In these diagrams, the light lines represent the theoretical values for a rib without deck.

Celluloid models were made of Arches 28–1, 28–2, 28–3, and 28–4, and the stresses in these models due to loads distributed in the same manner as the loads on the concrete arches, were determined by use of the Beggs deformeter gauge, under the supervision of Professor Beggs. The stresses thus determined agree closely with those obtained from the measured strains in the concrete arches.

The diagrams of Figs. 19 to 22, inclusive, show the relation between theoretical and measured stresses in the rib at the various sections. The theoretical stresses are for a rib without deck.

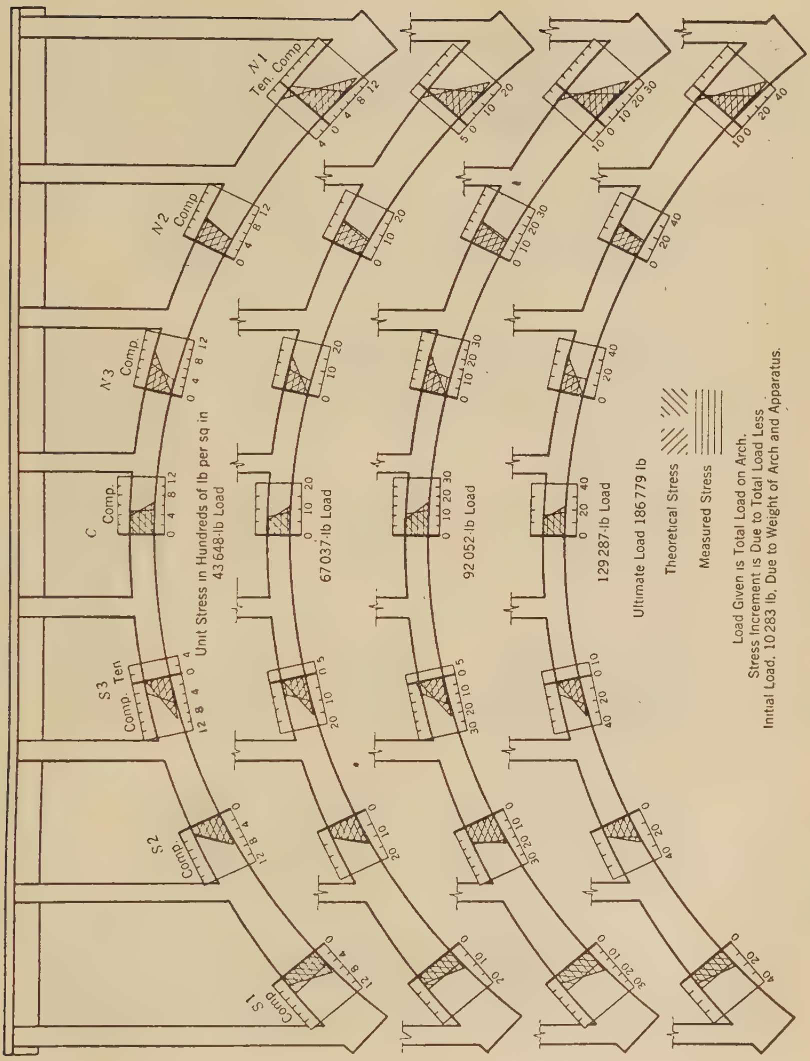
TABLE 10.—Influence of a Deck Upon Strength of Reinforced Concrete Arches

Arch No.	Kind of Deck	Ultimate Strength lb.	Age When Tested days	Strength of Control Cylinder lb. per sq. in.	Secant Modulus at 1000 lb. per sq. in. at Section in 1000 lb. per sq. in.			Section Where Failure Occurred
					N1	N3	S3	
27-3...	None	111 436	53	3382	2540	1540	1600	Just above N3
27-4...	None	135 917	52	3580	3180	1670	2130	S3
27-5...	High Deck, Rect. Section, No Exp. Joints	176 000	57	4006	5500	4170	3420	N1
27-6...	do.	156 300	50	3351	3330	2740 (2)	2840	N1
28-1...	High Deck, Tee Section, No Exp. Joints	186 779	57	3842	2920	3450 (1)	2780	Between N2 and N3
28-2...	High Deck, Tee Section, 2 Exp. Joints	145 465*	55	3985	2900	2860 (1)	2380	S3
28-3...	Low Deck, Tee Section, No Exp. Joints	189 731	56	3943	3850	5000	4390	N1
28-4...	Low Deck, Tee Section, 2 Exp. Joints	131 823	55	3485	3030	2880	1910	N3

*Probably would have failed at 136 650 pounds if north expansion joint had been wider.

(1) Average of N2 and center. Complete strain measurements were not taken at N3.
(2) At center. Complete strain measurements were not taken at N2 or N3.

The ultimate loads for the various arches are given in Table 10. The averages of the capacities of the two ribs without decks, of the four arches having decks without expansion joints, and of the two arches having decks with expansion joints are 123 676, 177 202, and 138 644 lb, respectively. If these values are altered in accordance with the variations in the strength of the concrete, as determined by the control cylinders, and a comparison is made on the basis of 3 500-lb concrete, these figures become 124 000, 164 000, and 130 000, respectively.


N1 Ten. Comp.
N2 Comp.
N3 Comp.
C Comp.
S3 Comp. Ten
S2 Comp.
S1 Comp.
Unit Stress in Hundreds of lb per sq in
43 648-lb Load
67 037-lb Load
92 052-lb Load
129 287-lb Load
Ultimate Load 186 779 lb
Theoretical Stress ▨
Measured Stress ▥

Load Given is Total Load on Arch.
Stress Increment is Due to Total Load Less
Initial Load, 10 283 lb, Due to Weight of Arch and Apparatus.

Fig. 19. Relation between Theoretical and Measured Stress, Arch 28-1

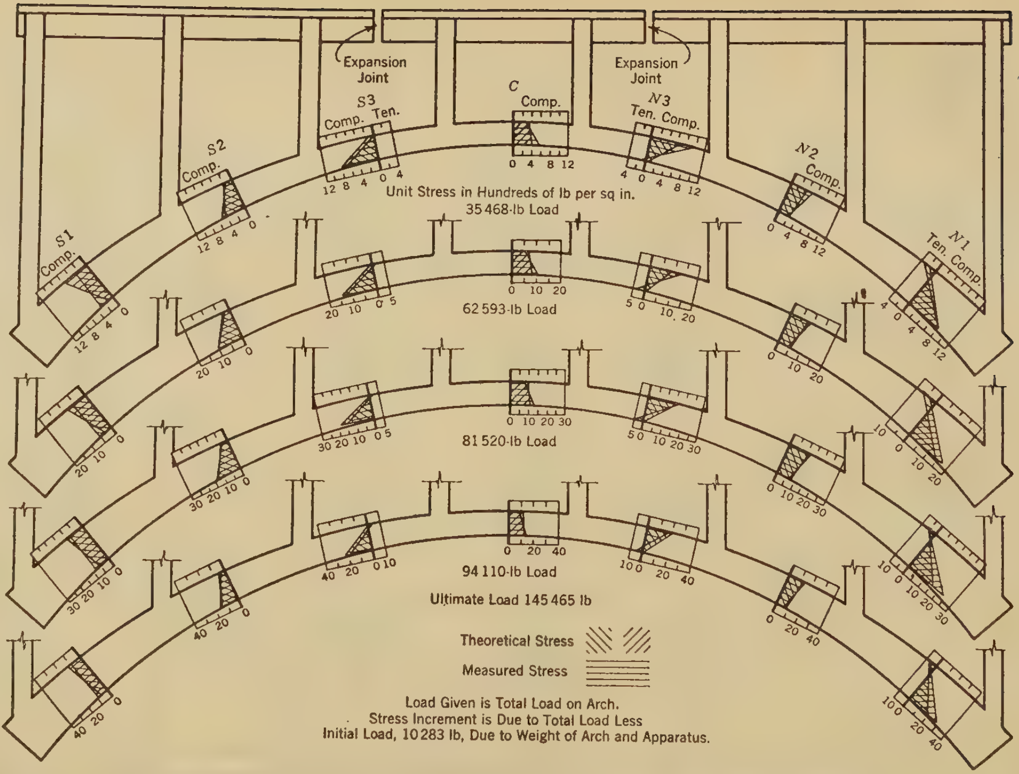

Expansion Joint
Expansion Joint
S1
Comp.
S2
Comp.
S3
Comp. Ten.
C
Comp.
N3
Ten. Comp.
N2
Comp.
N1
Ten. Comp.
Unit Stress in Hundreds of lb per sq in.
35 468-lb Load
62 593-lb Load
81 520-lb Load
94 110-lb Load
Ultimate Load 145 465 lb
Theoretical Stress
Measured Stress
Load Given is Total Load on Arch.
Stress Increment is Due to Total Load Less
Initial Load, 10 283 lb, Due to Weight of Arch and Apparatus.

FIG. 20. RELATION BETWEEN THEORETICAL AND MEASURED STRESS, ARCH 28-2

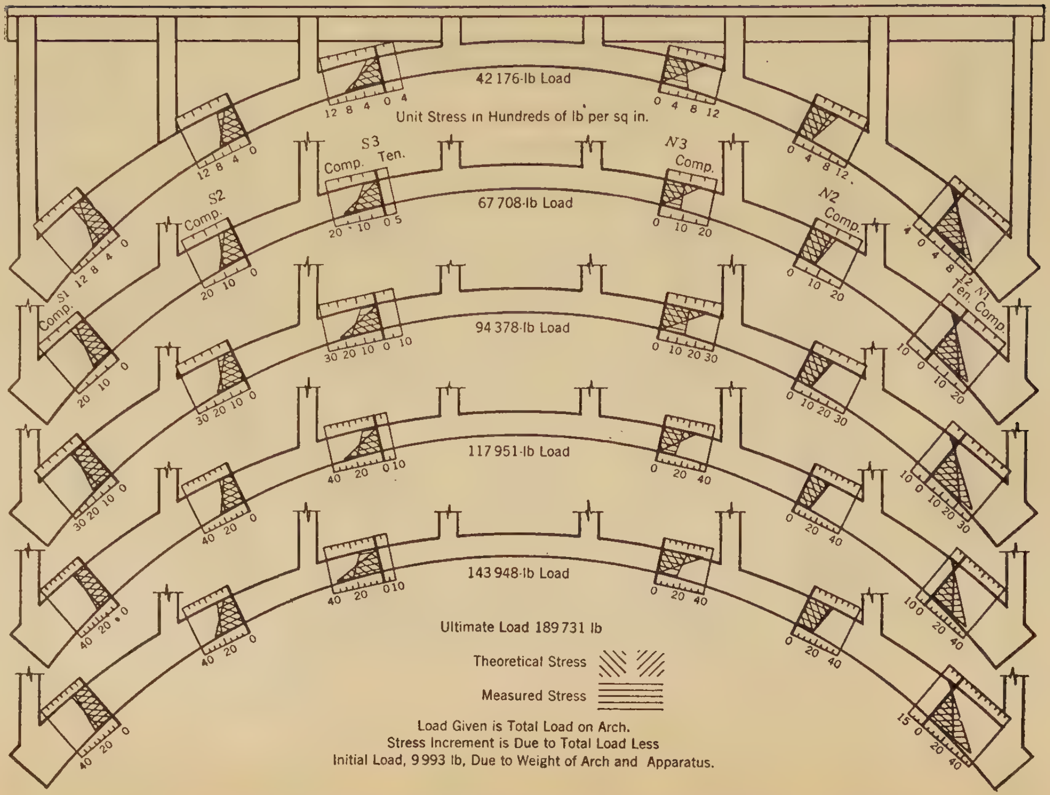
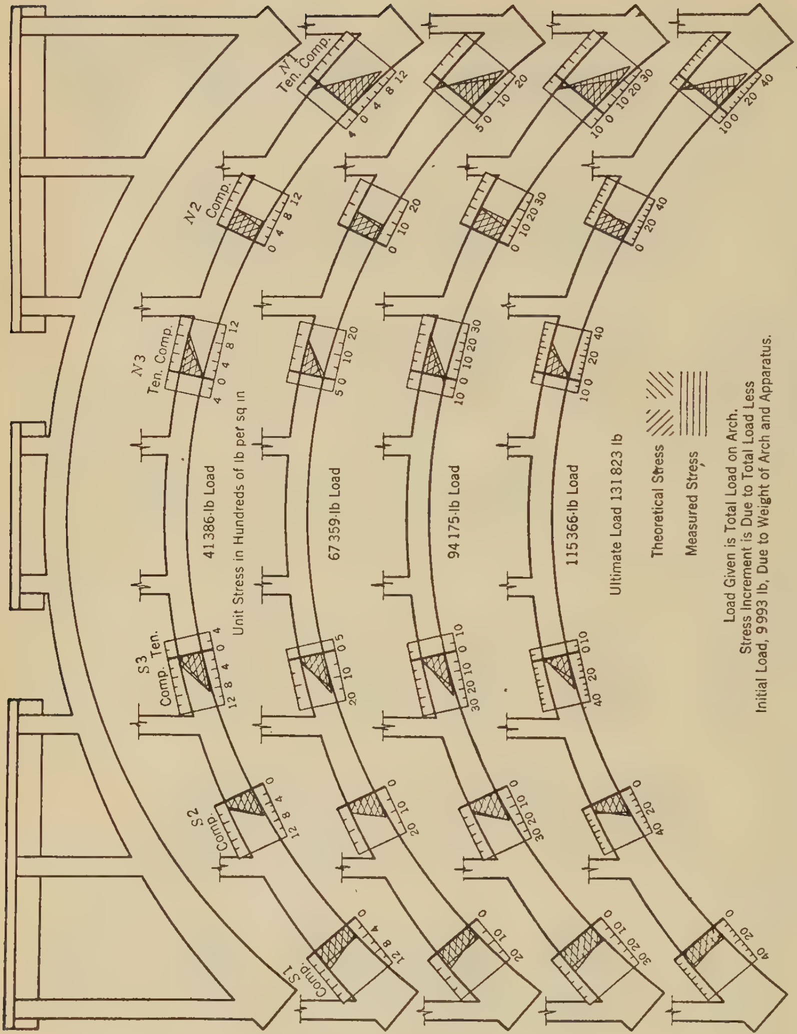

42 176-lb Load
Unit Stress in Hundreds of lb per sq in.
12 8 4 0 4
0 4 8 12
12 8 4 0
0 4 8 12
S1
Comp.
12 8 4 0
S2
Comp.
20 10 0
S3
Comp.
Ten.
20 10 0 5
67 708-lb Load
N3
Comp.
0 10 20
N2
Comp.
0 10 20
N1
Ten.
Comp.
4 0 4 8 12
20 10 0
94 378-lb Load
30 20 10 0
30 20 10 0 10
0 10 20 30
0 10 20 30
10 0 10 20
30 20 10 0
117 951-lb Load
40 20 0
40 20 0 10
0 20 40
0 20 40
10 0 10 20 30
40 20 0
143 948-lb Load
40 20 0
40 20 0 10
0 20 40
0 20 40
100 20 40
40 20 0
15 0 20 40
Ultimate Load 189 731 lb
Theoretical Stress ▧
Measured Stress ☰
Load Given is Total Load on Arch.
Stress Increment is Due to Total Load Less
Initial Load, 9 993 lb, Due to Weight of Arch and Apparatus.

FIG. 21. RELATION BETWEEN THEORETICAL AND MEASURED STRESS, ARCH 28-3


N1
Ten. Comp.
N2
Comp.
N3
Ten. Comp.
S3
Comp. Ten.
S2
Comp.
S1
Comp.
41 386-lb Load
Unit Stress in Hundreds of lb per sq in
67 359-lb Load
94 175-lb Load
115 366-lb Load
Ultimate Load 131 823 lb
Theoretical Stress
Measured Stress

Load Given is Total Load on Arch.
Stress Increment is Due to Total Load Less
Initial Load, 9993 lb, Due to Weight of Arch and Apparatus.

FIG. 22 RELATION BETWEEN THEORETICAL AND MEASURED STRESS, ARCH 28-4

From a study of the results of these tests the following conclusions seem justified:

(1) A deck without intermediate expansion joints and constructed so as to act with the rib in carrying loads, reduces the magnitude of the live-load moment at the springing where the deck does not increase the strength of the structure, and does increase the moment over the central part of the span where the strengthening effect of the deck is greatest. Intermediate expansion joints in the deck reduce these effects.

(2) The height of the deck above the rib does not greatly influence the stresses in the rib due to loads.

Article 3.—Temperature and Shrinkage Stresses.—For arches with deck, observations were made to determine the relation between changes in span and and in the horizontal thrust on the abutments, the object being to determine the effect of the deck upon the temperature thrust. The method of testing was to subject the arch to a small load, one-fourth to one-sixth its ultimate capacity, and then decrease the span, keeping the total load and angular position of the abutments as nearly constant as possible. Measurements were taken to determine the change in span. The load was kept nearly constant and the rotation of the abutments was very small. Corrections were made in the thrust for these small changes.

The relation between change in span and the horizontal thrust is shown in Fig. 23. The full lines represent the theoretical thrust for a rib without deck.

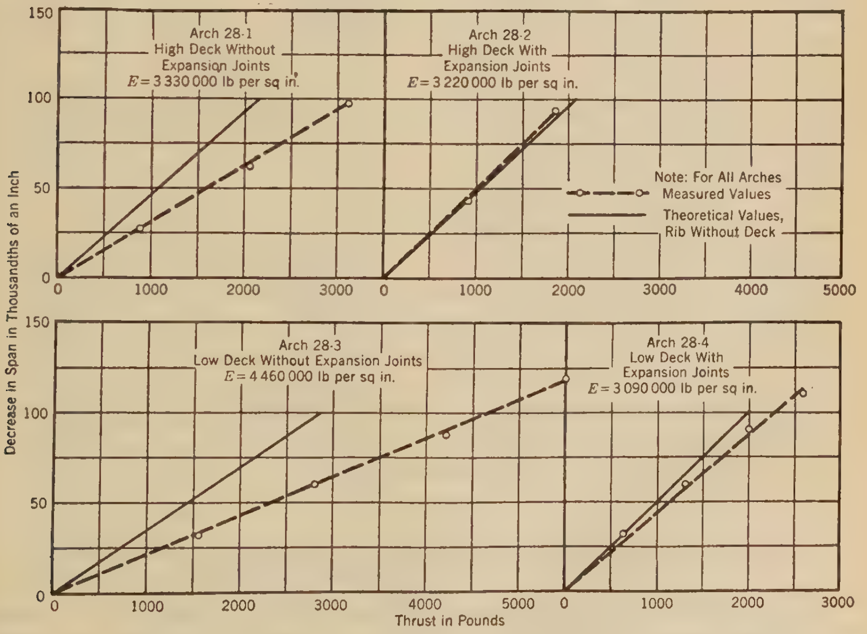


FIG. 23. RELATION BETWEEN CHANGE IN SPAN AND HORIZONTAL THRUST

This theoretical value depends upon the modulus of elasticity. In the computations, E was assumed constant for each arch, the value used being the average of those obtained from the measured strains at a stress of 500 lb per sq in. at the seven sections where strains were measured.

The value of E for each arch is given on the diagram. The ratios of the measured thrust to the theoretical thrust for a rib without deck are: 1.48 for Arch 28–1; 1.00 for Arch 28–2; 1.62 for Arch 28–3; and, 1.16 for Arch 28–4. Celluloid models of these arches were cut and the same ratios determined. The results were: 2.48 for Arch 28–1; 1.32 for Arch 28–2; 2.26 for Arch 28–3; and, 1.62 for Arch 28–4. If the concrete contained cracks the thrust due to a change in span was reduced.

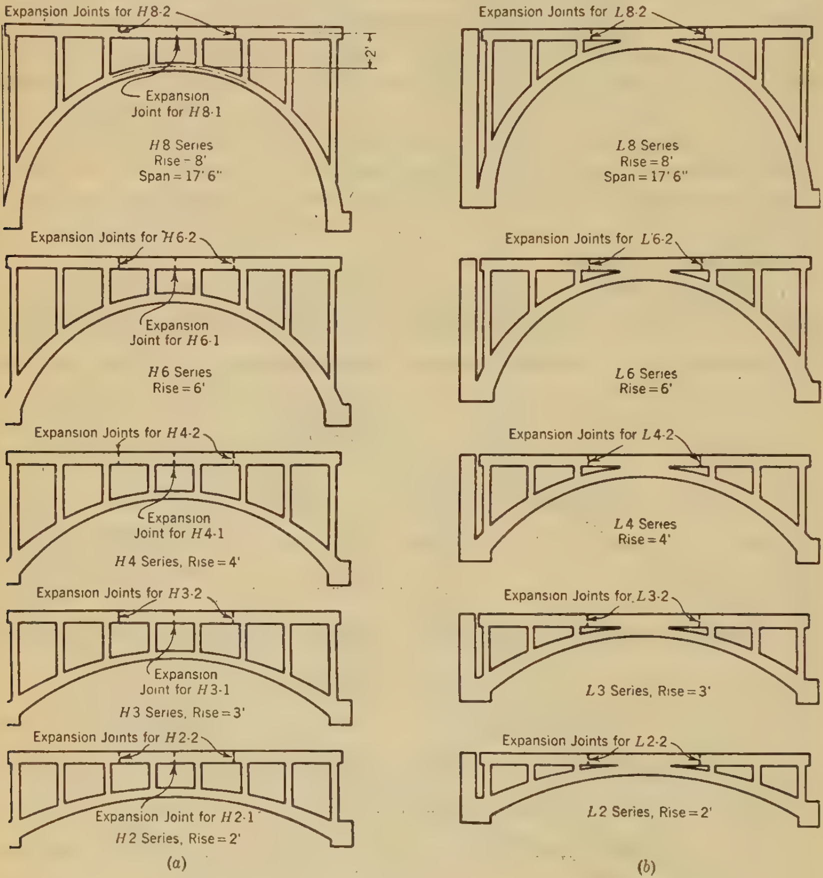

Expansion Joints for H8-2
2'
Expansion Joint for H8-1
H8 Series
Rise - 8'
Span = 17' 6"
Expansion Joints for H6-2
Expansion Joint for H6-1
H6 Series
Rise = 6'
Expansion Joints for H4-2
Expansion Joint for H4-1
H4 Series, Rise = 4'
Expansion Joints for H3-2
Expansion Joint for H3-1
H3 Series, Rise = 3'
Expansion Joints for H2-2
Expansion Joint for H2-1
H2 Series, Rise = 2'
(a)
Expansion Joints for L8-2
L8 Series
Rise = 8'
Span = 17' 6"
Expansion Joints for L6-2
L6 Series
Rise = 6'
Expansion Joints for L4-2
L4 Series
Rise = 4'
Expansion Joints for L3-2
L3 Series, Rise = 3'
Expansion Joints for L2-2
L2 Series, Rise = 2'
(b)

FIG. 24. CELLULOID MODELS FOR DETERMINING TEMPERATURE THRUST IN ARCHES HAVING VARIOUS GEOMETRICAL PROPERTIES

The study of temperature stresses was extended by tests of celluloid models to a wide variety of arches with decks. The models studied are shown in Fig. 24.

All these arches had the same span and the same depth of rib at the crown and springing as Arches 27–3 and 27–4. The models were made to a scale of 1 in. to 1 ft. Tests were made upon models with a deck having expansion joints, with a deck having two expansion joints near the third points, and in the arches with high deck having a single expansion joint at the center. The results of these tests are given in Tables 11 and 12.

Studies by means of celluloid models were also made to determine the effect of varying the length of the saddle which is integral with the rib at the crown. The variations tested are shown in Fig. 25 and the results of the tests are given in Table 13.

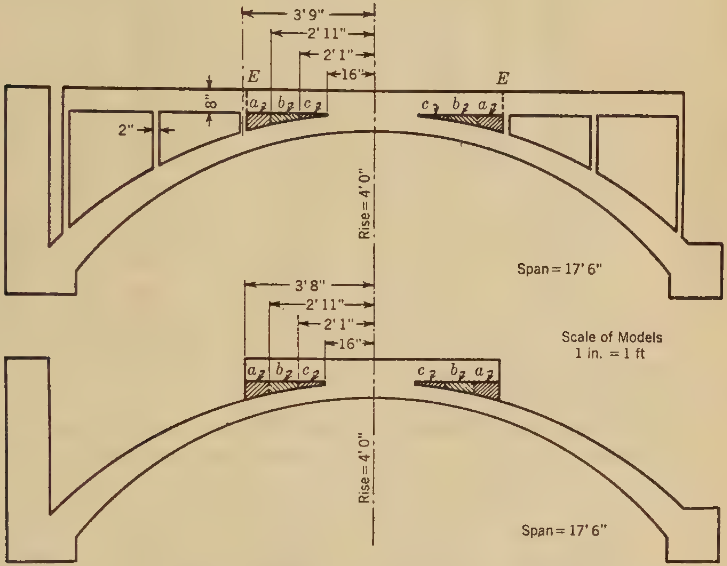


FIG. 25. CELLULOID MODELS FOR DETERMINING EFFECT OF LENGTH OF A SADDLE UPON TEMPERATURE MOMENTS

Tests of the celluloid models were made to determine the change in opening at the expansion joints due to a known change in span and also due to a known change in temperature. The results of these tests are given in Table 14.

In naming the arches the letter, *L,* indicates low deck and *H,* high deck. The number in the middle indicates the rise of the arch, in feet, and the last figure denotes the number of expansion joints in the deck.

The relation between the movement at the expansion joint and the rise of the arch is shown in Fig. 26.

These tests of reinforced concrete arches and the studies with celluloid models apparently justify the following conclusions:

(1) The moment at the springing due to shrinkage, time yield, temperature changes, and other causes equivalent to a change in span, is greater for structures having decks than for a rib without deck;

TABLE 11.—TEMPERATURE MOMENTS AS GIVEN BY TESTS OF CELLULOID MODELS, ARCHES HAVING DECK 2 FEET ABOVE RIB AT CROWN

Arch No.*	Temperature Thrust Relative to Same Quantity for Rib Without Deck	Distance From Springing to Temperature Thrust Line, Inches	Moment of Temp. Thrust About Point in Arch Axis, Relative to Same Quantity for Rib Without Deck, at		Temp. Moment in Rib at Exp. Joint, Relative to Temp. Moment at Crown of Rib Without Deck
			Springing	Exp. Joint	
H2R	1.00	19.9	1.00	1.00	....
H2F	2.08	13.5	1.41	....	....
H21	1.43	21.2	1.52	0.98	0.98
H22	1.36	20.6	1.41	1.10	0.96
H3R	1.00	29.6	1.00	1.00	....
H3F	2.36	20.6	1.64	....	....
H31	1.50	31.4	1.59	1.07	1.07
H32	1.65	31.1	1.73	1.19	1.00
H4R	1.00	39.7	1.00	1.00	....
H4F	2.48	27.7	1.73	....	....
H41	1.47	41.2	1.53	1.20	1.20
H42	1.32	40.9	1.36	1.10	0.94
H6R	1.00	59.2	1.00	1.00	...
H6F	2.52	40.5	1.72	....	....
H61	1.28	59.6	1.29	1.24	1.24
H62	1.41	62.0	1.48	1.06	0.94
H8R	1.00	78.0	1.00	1.00	....
H8F	2.68	53.0	1.82	....	....
H81	1.31	77.3	1.30	1.36	1.36
H82	1.33	80.3	1.37	1.14	1.05

*Middle symbol of arch number indicates rise of rib in feet; last symbol indicates type of deck, as follows: R = rib without deck, F = deck without expansion joint, 1 = one expansion joint at crown, and 2 = two expansion joints near one-third points.

TABLE 12.—TEMPERATURE MOMENTS AS GIVEN BY TESTS OF CELLULOID MODELS, ARCHES HAVING DECK INTEGRAL WITH RIB AT CROWN

Arch No.*	Temperature Thrust Relative to Same Quantity for Rib Without Deck	Distance From Springing to Temperature Thrust Line, Inches	Moment of Temp. Thrust About Point in Arch Axis, Relative to Same Quantity for Rib Without Deck, at		Temp. Moment in Rib at Exp. Joint, Relative to Temp. Moment at Crown of Rib Without Deck
			Springing	Exp. Joint	
L2R	1.00	19.9	1.00	1.00	...
L2F	1.84	17.6	1.63	....	....
L22	1.59	20.8	1.66	1.19	1.05
L2S	1.10	18.7	1.03	1.47	1.29
L3R	1.00	29.3	1.00	1.00	....
L3F	2.18	24.0	1.79	....	....
L32	1.70	30.7	1.78	1.28	1.09
L3S	1.20	28.0	1.15	1.47	1.25
L4R	1.00	39.0	1.00	1.00	....
L4F	2.26	30.3	1.76	....	....
L42	1.62	40.5	1.68	1.31	1.13
L4S	1.21	36.7	1.14	1.57	1.36
L6R	1.00	60.0	1.00	1.00	....
L6F	2.30	44.3	1.70	....	....
L62	1.41	61.1	1.44	1.26	1.10
L6S	1.13	57.0	1.07	1.45	1.27
L8R	1.00	79.2	1.00	1.00	....
L8F	2.18	57.0	1.57	....	....
L82	1.37	81.0	1.40	1.21	1.10
L8S	1.15	77.3	1.12	1.29	1.18

*Middle symbol of arch number indicates rise of rib in feet; last symbol indicates type of deck, as follows: R = rib without deck, F = deck without expansion joints, 2 = two expansion joints near one-third points, S = saddle but no other deck.

TABLE 13.—EFFECT OF SADDLE UPON TEMPERATURE MOMENTS

	Description of Structure	Temp. Thrust Relative to Same Quantity for Rib Without Deck	Distance From Springing to Temp. Thrust Line, Inches	Moment of Temp. Thrust About Point in Arch Axis, Relative to Same Quantity for Rib Without Deck, at		Temp. Moment in Rib at End of Saddle, Relative to Temp. Moment at Crown of Rib Without Deck
				Springing	End of Saddle	
Deck Full Length of Arch	1. Saddle Three Panels Long. No Exp. Jts.	2.43	29.6	1.81	20.10	
	2. Saddle Cut Away at *a*. Exp. Jt. at *E*...	1.80	36.4	1.64	3.33	1.62
	3. Saddle Cut Away at *a* and *b*. Exp. Jt. at *E*............	1.57	38.2	1.51	1.96	1.53
	4. Saddle Cut Away at *a*, *b*, and *c*. Exp. Jt. at *E*..........	1.45	40.0	1.46	1.41	1.29
	5. Saddle Cut Away at *a*, *b*, and *c*. Deck Cut Away Outside of Exp. Joints....	1.16	38.0	1.11	1.44	1.32
	6. Rib Without Saddle or Deck........	1.00	39.8	1.00	1.00	1.00
Rib With Saddle but no Deck	7. Saddle Three Panels Long............	1.80	32.6	1.47	12.91	1.93
	8. Saddle Cut Away at *a*	1.48	35.2	1.30	3.35	1.59
	9. Saddle Cut Away at *a* and *b*...........	1.30	36.4	1.18	2.05	1.59
	10. Saddle Cut Away at *a*, *b*, and *c*.......	1.15	38.2	1.10	1.43	1.31
	11. Rib Without Saddle or Deck...	1.00	40.0	1.00	1.00	1.00

TABLE 14.—MOVEMENT AT EXPANSION JOINTS FROM CELLULOID MODELS. SPAN FOR ALL MODELS 17.5 INCHES

Arch No.	Ratio Movement at Expansion Joint to Change in Span		Ratio $\frac{\text{From Change in Temp.}}{\text{From Change in Span}}$
	From Change in Span	From Change in Temp.	
L22..................	0.132	0.109	0.83
H21..................	0.895	0.980	1.10
H22..................	0.472	0.450	0.95
L32..................	0.141	0.130	0.92
H31..................	0.710	0.690	0.97
H32..................	0.378	0.370	0.98
L42..................	0.122	0.089	0.73
H41..................	0.575	0.580	1.01
H42..................	0.327	0.280	0.86
L62..................	0.100	0.090	0.90
H61..................	0.435	0.440	1.01
H62..................	0.257	0.400	1.56
L82..................	0.087	0.084	0.97
H81..................	0.317	0.480	1.51
H82..................	0.207	0.180	0.87
Span 2*..............	0.155	0.141	0.91

*Gilbert Street Bridge, Danville, Ill See Univ. of Ill. Eng. Exp. Sta. Bul. 174

(2) For structures having decks the moment in the rib at a section in a panel below an expansion joint in a deck, due to any cause equivalent to

a change in span, will be greater than for structures without expansion joints, and may be greater than for a rib without deck;

(3) Increasing the height of the deck above the rib, thereby decreasing the rise of the rib, greatly increases the stresses in the rib due to any cause equivalent to a change in span; and,

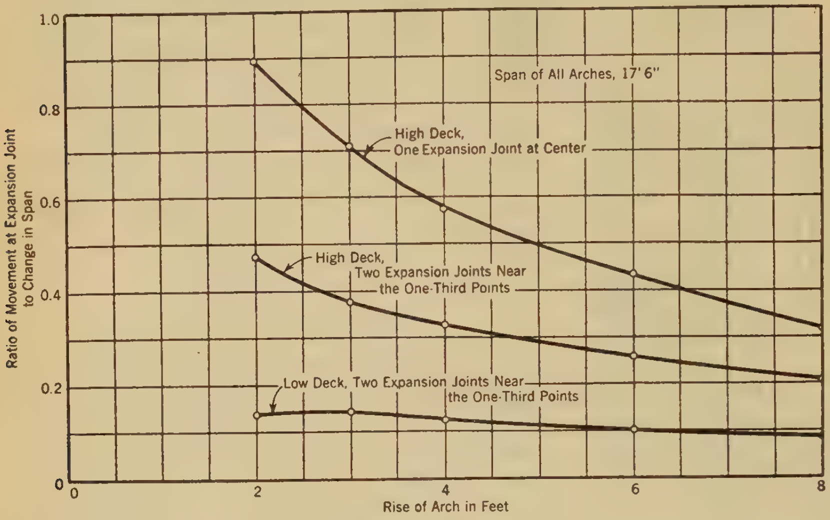


FIG. 26. RELATION BETWEEN MOVEMENT AT EXPANSION JOINT AND RISE OF RIB

(4) For important arches the analysis for live load and temperature stresses should be for the structure as a whole and not for the rib alone.

Acknowledgments.—The work described in this Chapter was done under the supervision of Professor Wilson (6).

IV.—TESTS OF THREE-SPAN ARCH RIBS ON SLENDER PIERS

Article 1.—Objects.—The objects of this investigation were to determine the effect, upon the stresses involved, of errors in the assumptions on which the elastic theory is based and to determine the effect of the movements of the pier tops upon the stresses in the arch ribs.

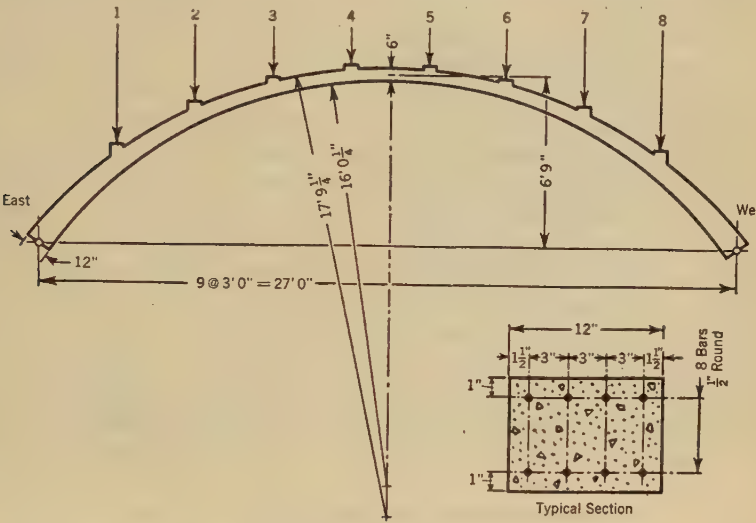
The tests of the single-span structures described in Chapters II and III were performed in a 300 000-lb beam, testing machine having wings with an over-all length of about 21 ft, thus making possible the testing of single arches of 17.5-ft span. Obviously, tests of the three-span structures could not be made in any testing machine, and the construction of special testing apparatus was necessary.

The three-span structures were made as large as could be housed conveniently in the Materials Testing Laboratory of the University of Illinois. This made possible the testing of three spans, each 27 ft center to center on piers 20 ft high.

Preliminary to the testing of the three-span structures tests were made on a single arch of 27-ft span. The purpose of this test was to determine the elastic constants of the single span and to test the weighing mechanisms and measuring apparatus necessary for the tests of the three-span structures.

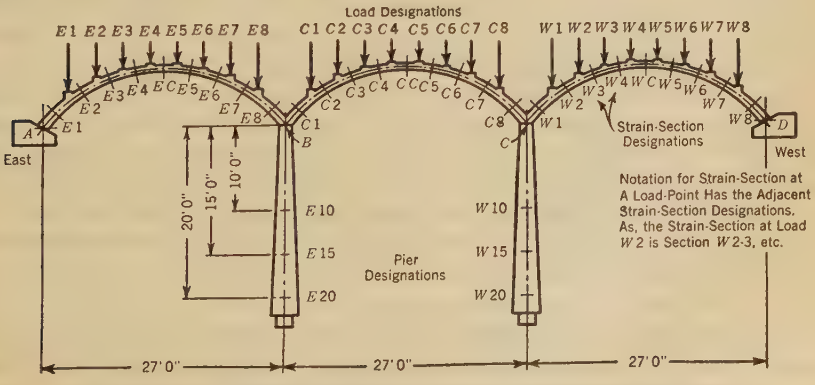
The results of the test of the single-span arch agree closely with computations by the elastic theory and confirm the conclusions from the tests of single-span arch ribs described in Chapter II.

Article 2.—Description of Specimens and Apparatus.—The dimensions of the arch ribs and arrangement of the reinforcement are shown in Fig. 27, and the general dimensions and notation for the three-span structure are shown in Fig. 28. The concrete was designed to have a strength of 2 200 lb per sq in. at 28 days. While the forms were on, the concrete was completely



1
2
3
4
5
6
7
8
6"
17'9¼"
16'0¼"
6'9"
East
West
12"
9@3'0" = 27'0"
12"
1½" 3" 3" 3" 1½"
1"
1"
8 Bars ½" Round
Typical Section

FIG. 27. DIMENSIONS OF ARCH RIB



Load Designations
E1 E2 E3 E4 E5 E6 E7 E8
C1 C2 C3 C4 C5 C6 C7 C8
W1 W2 W3 W4 W5 W6 W7 W8
E1 E2 E3 E4 EC E5 E6 E7 E8
C1 C2 C3 C4 CC C5 C6 C7 C8
W1 W2 W3 W4 WC W5 W6 W7 W8
A
B
C
D
East
West
Strain-Section Designations
20'0"
15'0"
10'0"
E10
E15
E20
W10
W15
W20
Pier Designations
Notation for Strain-Section at A Load-Point Has the Adjacent Strain-Section Designations. As, the Strain-Section at Load W2 is Section W2-3, etc.
27'0"
27'0"
27'0"

FIG. 28. THREE-SPAN STRUCTURE HAVING RIB WITHOUT DECK

enclosed, and the surfaces of the ribs and piers were moist when the forms were removed after about 10 days. Then the concrete was cured in the laboratory where the air is dry and the temperature is approximately 80° F.

Twenty 6 by 12-in. control cylinders were made, four from the batches that went into each pier, and four from the batches that went into each rib.

The multiple-span arch bridge differs from a single-span arch in that, for the latter, the ends are fixed, whereas for the former, the fixed points of the structure are the outer ends of the two end spans and the bases of the intermediate piers. As a live load crosses the structure the unequal loading of the two spans adjacent to a pier produces a horizontal thrust and moment at the top of the pier which move the top, even if the pier bases and abutments are fixed. This movement of the pier tops produces reactions at the end of the span differing materially from those that would be produced by fixed abutments supporting a similar single-span arch carrying the same load.

One of the principal objects of the investigation was to determine the effect of the elastic deformation of the piers upon the stresses in the arches. Inasmuch as the magnitude of this effect depends upon the slenderness of the piers, tests were made upon the structure when the piers had various heights. The effective height of a pier is the vertical distance from the springing of the arches down to the point in the pier that is fixed. By attaching instruments that indicate the position of the axis of the pier at distances below the springing of 20 ft, 15 ft, and 10 ft, the one structure was used in making tests of a three-span arch series that had these various pier heights.

The apparatus was designed and built especially for these tests. The load upon the structure was produced by suspending concrete blocks of known weight at the load points of the arch. For each load point there was one large concrete block suspended by four steel rods, which served as a loading platform on which to place smaller blocks that, in combination with the loading platform, constitute the desired load at the point. The arrangement is shown in Fig. 29. When the arch was unloaded the loading platforms rested upon supports provided for the purpose. To apply the load the turnbuckles in the suspension rods were tightened, shortening the rods until the loading beam came into contact with the loading shelf. The turnbuckles were turned, successively, by small amounts, back and forth from one end of the structure to the other, thereby transferring the load to the arches gradually.

Each abutment was supported on two vertical scales, the load being transmitted from the abutment to the scales by means of two jacks on each scale platform. The abutment could be raised or lowered without rotation by extending or depressing all jacks by the same amount, or an abutment could be rotated by extending both the east jacks and depressing the west jacks, or the reverse. The vertical scales were mounted on carefully machined steel rollers, 10 in. in diameter, which ran on a machined track so that the vertical scales offered practically no resistance to horizontal motion.

The horizontal reactions were converted into vertical loads by means of two bell-crank levers, one on the north side and one on the south side. These levers received the horizontal thrust through links and delivered it to a platform scale as a vertical force. An abutment could be moved horizontally by turning the link, since it was provided with right and left threads.

The vertical reactions at the pier bases were measured with apparatus similar to that used for measuring the abutment reactions. The horizontal reaction at each pier base was measured with two horizontal scales similar to those used on the abutments. These scales were capable of measuring reactions in one direction only. To overcome this limitation, a bell-crank supported on knife-edges and carrying a concrete block on the outer end of its horizontal arm, was provided for each pier base. This apparatus produced an initial horizontal thrust of about 1 000 lb acting against each horizontal scale. By this device the load on the horizontal scale was never reversed.

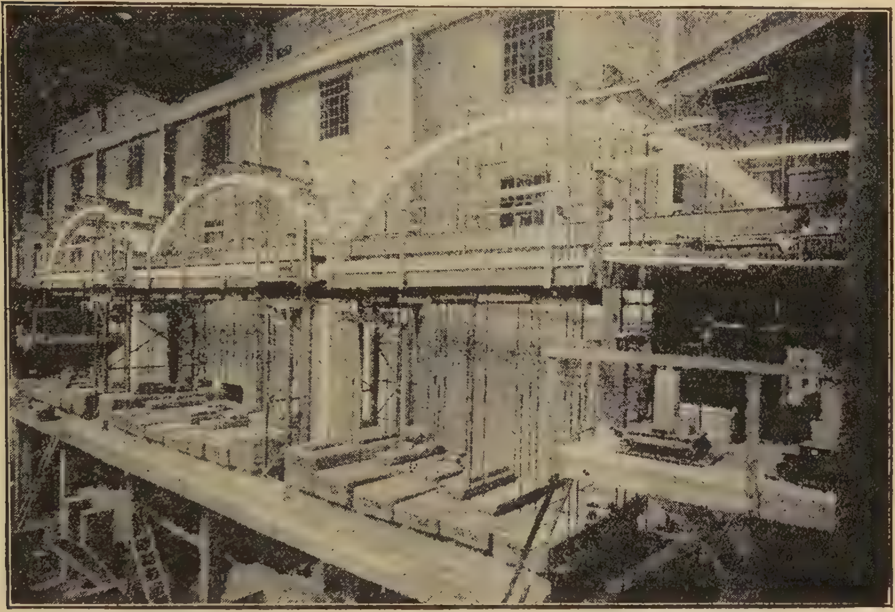


FIG. 29—THREE-SPAN STRUCTURE CONSISTING OF A RIB WITHOUT DECK WITH APPARATUS ATTACHED READY FOR TESTING.

Strains were measured with an 8-in. Berry strain-gauge and the angular position of the points where rotation was to be controlled or measured was determined with calibrated level-bubbles attached to the structure. Changes in span were measured by means of Ames dials attached to I-beams, one on each side of the structure. These beams were attached to the east springing of the east rib and were supported on rollers at the two piers and the west abutment.

The vertical movements of load points on the rib were measured relative to these I-beams by means of Ames dials; the vertical movement of the abutments and pier tops was measured by means of a hydrostatic gauge.

TABLE 15.—INFLUENCE ORDINATES FOR REACTIONS AT SPRINGING, CALCULATED BY ELASTIC THEORY (PIERS 20 FEET HIGH)

Load of 1 lb. at	Moment in. lb.			Horizontal Thrust lb.		Vertical Shear lb.		
	at *A* of *AB*	at *B* of *AB*	at *B* of *BC*	Span *AB*	Span *BC*	at *A* of *AB*	at *B* of *AB*	at *B* of *BC*
*E*1	−25.5097	+ 2.8676	+ 1.8334	+0.0900	+0.0219	+0.9765	+0.0235	−0.0022
*E*2	−30.9380	+ 8.8149	+ 6.3169	+0.3229	+0.0755	+0.9005	+0.0995	−0.0074
*E*3	−19.7244	+12.4852	+11.1693	+0.6089	+0.1342	+0.7661	+0.2339	−0.0129
*E*4	− 1.1952	+ 8.8567	+13.5495	+0.8156	+0.1641	+0.5866	+0.4134	−0.0152
*E*5	+14.0210	− 2.8940	+11.8108	+0.8407	+0.1452	+0.3922	+0.6078	−0.0126
*E*6	+19.2311	−17.5982	+ 6.7182	+0.6733	+0.0859	+0.2197	+0.7803	−0.0061
*E*7	+14.7950	−26.5022	+ 1.0914	+0.3985	+0.0188	+0.0948	+0.9052	+0.0006
*E*8	+ 6.2683	−21.6729	− 1.7976	+0.1425	−0.0176	+0.0249	+0.9751	+0.0034
*C*1	− 1.5672	− 2.7073	−23.1256	−0.0306	+0.1150	−0.0035	+0.0035	+0.9726
*C*2	+ 0.3369	− 0.5211	−30.0003	0.0000	+0.3302	−0.0027	+0.0027	+0.8983
*C*3	+ 4.4234	+ 4.8624	−22.8310	+0.0698	+0.5694	+0.0014	−0.0014	+0.7693
*C*4	+ 8.3858	+10.3127	− 8.5769	+0.1390	+0.7272	+0.0059	−0.0059	+0.5957
*C*5	+10.1127	+12.9018	+ 4.4140	+0.1704	+0.7272	+0.0086	−0.0086	+0.4043
*C*6	+ 8.8444	+11.4909	+10.4273	+0.1502	+0.5694	+0.0082	−0.0082	+0.2307
*C*7	+ 5.5271	+ 7.2606	+ 9.0443	+0.0944	+0.3302	+0.0054	−0.0054	+0.1017
*C*8	+ 2.0392	+ 2.6999	+ 4.0049	+0.0349	+0.1150	+0.0020	−0.0020	+0.0274
*W*1	− 0.3232	− 0.4301	− 0.7008	−0.0056	−0.0176	−0.0003	+0.0003	−0.0034
*W*2	+ 0.4166	+ 0.5661	+ 1.2806	+0.0072	+0.0188	+0.0005	−0.0005	−0.0006
*W*3	+ 1.7558	+ 2.3657	+ 4.7449	+0.0303	+0.0859	+0.0019	−0.0019	+0.0061
*W*4	+ 2.9310	+ 3.9436	+ 7.7375	+0.0506	+0.1452	+0.0031	−0.0031	+0.0126
*W*5	+ 3.2948	+ 4.4305	+ 8.6139	+0.0569	+0.1641	+0.0035	−0.0035	+0.0152
*W*6	+ 2.6870	+ 3.6122	+ 6.9886	+0.0464	+0.1342	+0.0029	−0.0029	+0.0129
*W*7	+ 1.5099	+ 2.0294	+ 3.9146	+0.0261	+0.0755	+0.0016	−0.0016	+0.0074
*W*8	+ 0.4365	+ 0.5866	+ 1.1295	+0.0075	+0.0219	+0.0005	−0.0005	+0.0022

Based on $n = 9$.
Plus (+) moment produces tension at intrados.

TABLE 16.—INFLUENCE ORDINATES FOR REACTIONS AT SPRINGING, CALCULATED BY ELASTIC THEORY (PIER 20 FEET HIGH; $E = 2.0$ IN 10^6 POUNDS PER SQUARE INCH; POSITIVE MOMENTS PRODUCE TENSION AT BOTTOM OF RIB)

Load of 1 lb. at	Moment in. lb.			Horizontal Thrust lb.		Vertical Shear lb.		
	at *A* of *AB*	at *B* of *AB*	at *B* of *BC*	Span *AB*	Span *BC*	at *A* of *AB*	at *B* of *AB*	at *B* of *BC*
*E*1	−25.4671	+ 2.7927	+ 1.8972	+0.0896	+0.0226	+0.9761	+0.0238	−0.0022
*E*2	−30.9122	+ 8.4870	+ 6.5013	+0.3202	+0.0776	+0.8993	+0.1006	−0.0077
*E*3	−19.9478	+11.8662	+11.4596	+0.6018	+0.1376	+0.7648	+0.2351	−0.0133
*E*4	− 1.7337	+ 8.0963	+13.8922	+0.8056	+0.1682	+0.5858	+0.4141	−0.0157
*E*5	+13.3835	− 3.4911	+12.1287	+0.8313	+0.1491	+0.3923	+0.6076	−0.0130
*E*6	+18.7601	−17.8037	+ 6.9439	+0.6674	+0.0889	+0.2204	+0.7795	−0.0063
*E*7	+14.5979	−26.3968	+ 1.1921	+0.3974	+0.0203	+0.0956	+0.9043	+0.0005
*E*8	+ 6.2735	−21.5367	− 1.8056	+0.1434	−0.0174	+0.0252	+0.9747	+0.0035
*C*1	− 1.5836	− 2.7704	−23.0293	−0.0313	+0.1151	−0.0036	+0.0036	+0.9721
*C*2	+ 0:3904	− 0.5153	−29.9289	+0.0004	+0.3279	−0.0027	+0.0027	+0.8971
*C*3	+ 4.5657	+ 4.9777	−23.0310	+0.0720	+0.5625	+0.0012	−0.0012	+0.7681
*C*4	+ 8.6051	+10.5352	− 9.1289	+0.1426	+0.7174	+0.0059	−0.0059	+0.5951
*C*5	+10.3713	+13.1928	+ 3.7134	+0.1748	+0.7174	+0.0087	−0.0087	+0.4048
*C*6	+ 9.0881	+11.7826	+ 9.8516	+0.1545	+0.5625	+0.0083	−0.0083	+0.2318
*C*7	+ 5.7075	+ 7.4854	+ 8.7323	+0.1975	+0.3279	+0.0054	−0.0054	+0.1028
*C*8	+ 2.1248	+ 2.8097	+ 3.9347	+0.0364	+0.1151	+0.0021	−0.0021	+0.0278
*W*1	− 0.3326	− 0.4417	− 0.6712	−0.0057	−0.0174	−0.0003	+0.0003	−0.0035
*W*2	+ 0.4680	+ 0.6357	+ 1.3745	+0.0081	+0.0203	+0.0005	−0.0005	−0.0005
*W*3	+ 1.8944	+ 2.5511	+ 4.8887	+0.0327	+0.0889	+0.0020	−0.0020	+0.0063
*W*4	+ 3.1400	+ 4.2221	+ 7.9056	+0.0543	+0.1491	+0.0033	−0.0033	+0.0130
*W*5	+ 3.5226	+ 4.7335	+ 8.7778	+0.0608	+0.1682	+0.0037	−0.0037	+0.0157
*W*6	+ 2.8741	+ 3.8607	+ 7.1218	+0.0496	+0.1376	+0.0030	−0.0030	+0.0133
*W*7	+ 1.6198	+ 2.1754	+ 4.0001	+0.0280	+0.0776	+0.0017	−0.0017	+0.0077
*W*8	+ 0.4708	+ 0.6322	+ 1.1600	+0.0082	+0.0226	+0.0004	−0.0004	+0.0022

Article 3.—Theoretical Analysis of Stresses.—The three-span structure without deck was analyzed separately by two men, one using an assumed value of $E = 2\,000\,000$ lb per sq in., and the other using a value of $E = 3\,330\,000$ lb per sq in. The latter value is consistent with the experimentally determined values.

The analyses were based upon the usual assumptions that plane sections remain plane, concrete takes tension, and that E has the same value at all sections and all stresses, both tension and compression. Table 15 gives the values of the reaction components as calculated for $n = 15$ and Table 16 gives the same reaction components for $n = 9$. Both these tables are for structures having 20-ft piers. The small change in the reactions due to a large change in the value of the modulus of elasticity of the concrete, so long as E remains constant for the structure, is of interest. The notation used in these tables is given in Fig. 28.

Article 4.—Loads.—The specimen is not a model to a definite scale of a larger structure, but the proportions were based on the design of a bridge of three spans, each 135 ft long, or five times the size of the laboratory specimen. Likewise, the selection of the live load to be used in testing the arch was based on the live load that would be used in designing a highway bridge with 135-ft arch spans, with two ribs and two lanes of traffic. The weight of the specimen was not great enough to produce a dead-load stress commensurate with the dead-load stresses encountered in the design of a full-sized bridge. A load, designated as the "design dead load," was selected, somewhat arbitrarily, such that the thrust line would remain within the kern of the

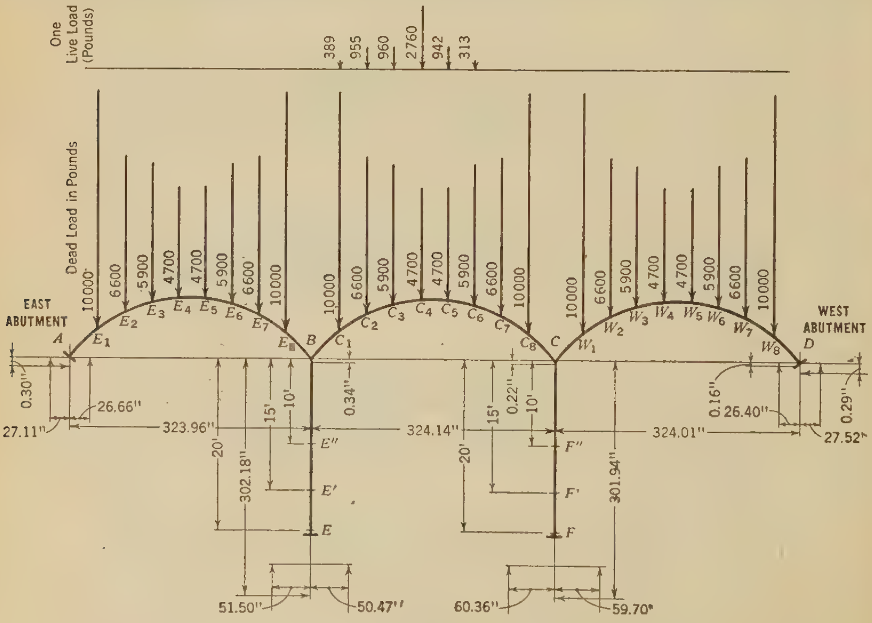

One Live Load (Pounds)
389
955
960
2760
942
313
Dead Load in Pounds
10000
6600
5900
4700
4700
5900
6600
10000
10000
6600
5900
4700
4700
5900
6600
10000
10000
6600
5900
4700
4700
5900
6600
10000
EAST ABUTMENT
A
E_1
E_2
E_3
E_4
E_5
E_6
E_7
E_8
B
C_1
C_2
C_3
C_4
C_5
C_6
C_7
C_8
C
W_1
W_2
W_3
W_4
W_5
W_6
W_7
W_8
WEST ABUTMENT
D
0.30"
26.66"
27.11"
323.96"
15'
10'
20'
302.18"
E''
E'
E
0.34"
324.14"
15'
0.22"
10'
20'
F''
F'
F
301.94"
324.01"
0.16"
26.40"
0.29"
27.52"
51.50"
50.47"
60.36"
59.70"

FIG. 30—POSITION OF MEASURED REACTIONS, THREE-SPAN RIB WITHOUT DECK.

rib, and of such magnitude that the maximum dead-load stress would be of the order of dead-load stresses existing in the prototype. The design live load consisted of a distributed load of 320 lb per lin ft, with a single concentration of 1 800 lb superimposed upon the distributed load. The live load, as usual, was distribtued for maximum stress at the section being studied in each instance. The design dead load and live load for maximum stress at Section C_4 are shown in Fig. 30.

Article 5.—Influence Lines.—The influence ordinates for the reactions at the springings of all three spans were determined experimentally by measuring the changes that occurred when a unit load was applied and removed, successively, at the various load points. The unit load was 2 000 lb, and the tests were made with the design dead load on the structure. The reactions were determined in two ways: (1) By weighing the reactions before and after the application and removal of the unit load; and (2) by measuring the displacement of the pier tops, due to application and removal of the load, and computing the reaction components corresponding to these displacements. The agreement between the results by the two methods was remarkably close. The influence ordinates were determined for all hights of piers, but only the results for the 20-ft piers are given in this summary. Figs. 31, 32, and 33 show the influence lines for the 20-ft pier height compared with those calculated by the elastic theory; and on Fig. 33 is shown the influence lines for a single span with fixed ends.

If an arch series is made of a homogeneous elastic material, a diagram showing the vertical movement of the load points due to a movement of a terminal is an influence line for the reaction at the terminal parallel to the given movement. Tests were made to determine the vertical deflection of all load points due to giving, successively, each of the four terminals (the two abutments and two pier bases) the following movements: (1) Rotation without horizontal or vertical translation; (2) horizontal translation without vertical movement or rotation; and, (3) vertical movement without rotation or horizontal translation. All these tests were made at pier heights of 10 ft, 15 ft, and 20 ft, making a total of thirty-six tests.

The arch carried the design dead load, and the movements were planned so as to keep the thrust line within the kern at all sections. The results of the tests are shown in Figs. 34 to 39, inclusive.

All influence lines for the vertical reactions (Figs. 34 and 35) are so nearly alike that the various lines can barely be distinguished. The influence lines for horizontal reaction at the abutment (Fig. 36) are all in fair agreement, but the influence lines for horizontal reaction at the pier bases (Fig. 37) differ materially. The statement just made relative to influence lines for horizontal reaction at the pier bases is equally applicable to the influence lines for moment at both the pier bases and the abutments.

Although the deflection diagrams differ materially from the influence lines, they have the same general characteristics and, if used as influence lines, would indicate approximately the same distribution of live load for maximum stress. The magnitude of live-load stress determined from deflec-

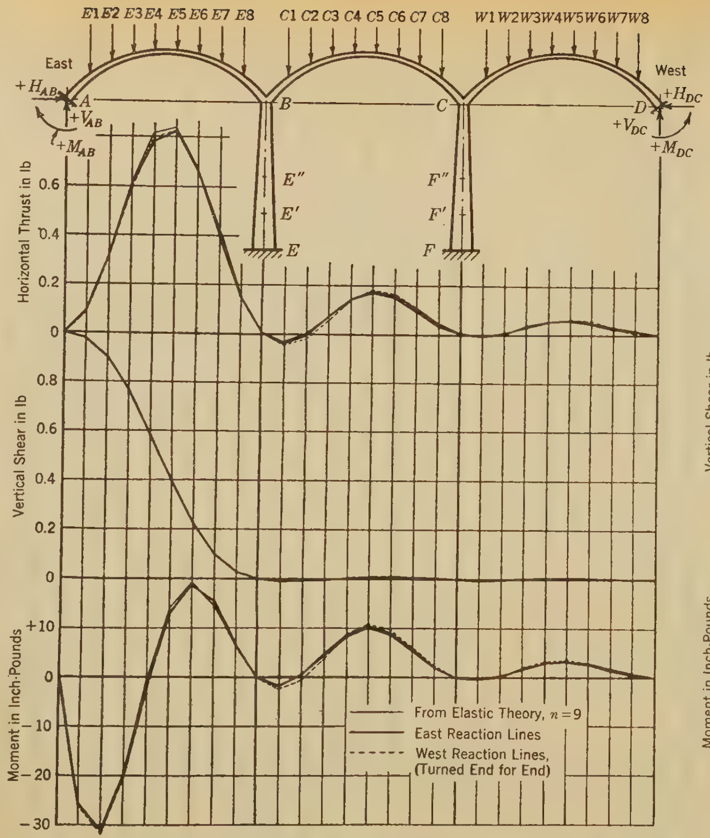


FIG. 31. INFLUENCE LINES BY UNIT LOADS FROM MOVEMENT OF PIER TOPS. END-SPAN ABUTMENT REACTIONS. 20-FOOT PIERS

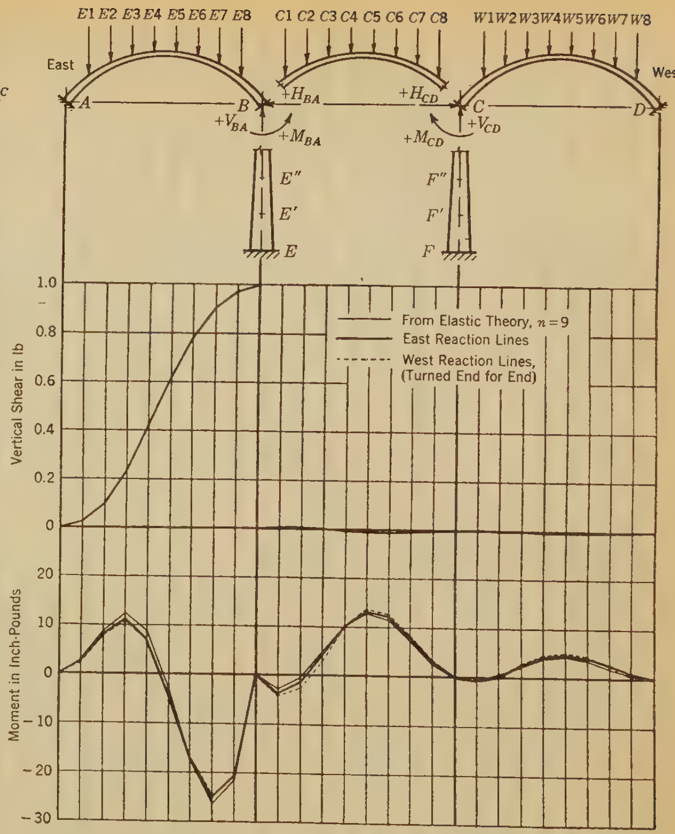


FIG. 32. INFLUENCE LINES BY UNIT LOADS FROM MOVEMENT OF PIER TOPS. END-SPAN PIER TOP REACTIONS. 20-FOOT PIERS

tion line and influence line would agree fairly well, but the values of the dead-load stress might differ greatly.

It is not surprising that the deflection diagrams for the concrete structure differ somewhat from the corresponding influence lines, since the theorem of reciprocal deflections is based upon assumptions as to the properties of the material, which it is known that concrete does not possess. The influence lines obtained by the use of unit loads agree very well with those obtained by the elastic theory, a fact that would seem to indicate that errors in the assumptions upon which the elastic theory is based do not produce appreciable errors in the results.

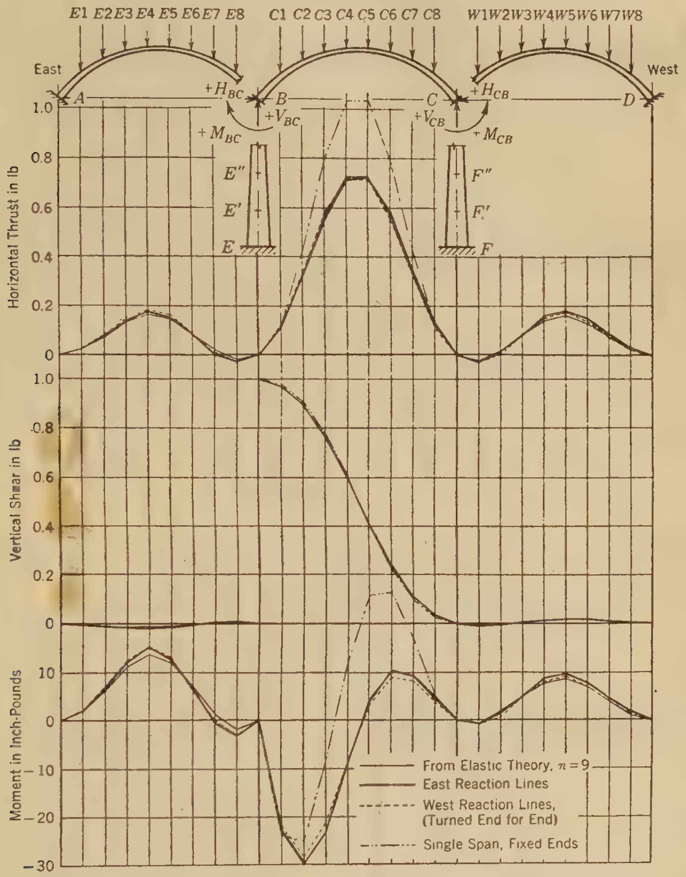


FIG. 33. INFLUENCE LINES BY UNIT LOADS FROM MOVEMENT OF PIER TOPS. CENTER-SPAN PIER TOP REACTIONS. 20-FOOT PIERS

Article 6.—Loading Tests.—Tests were made to determine the reactions and the location of the thrust line for the design dead load and one live load,

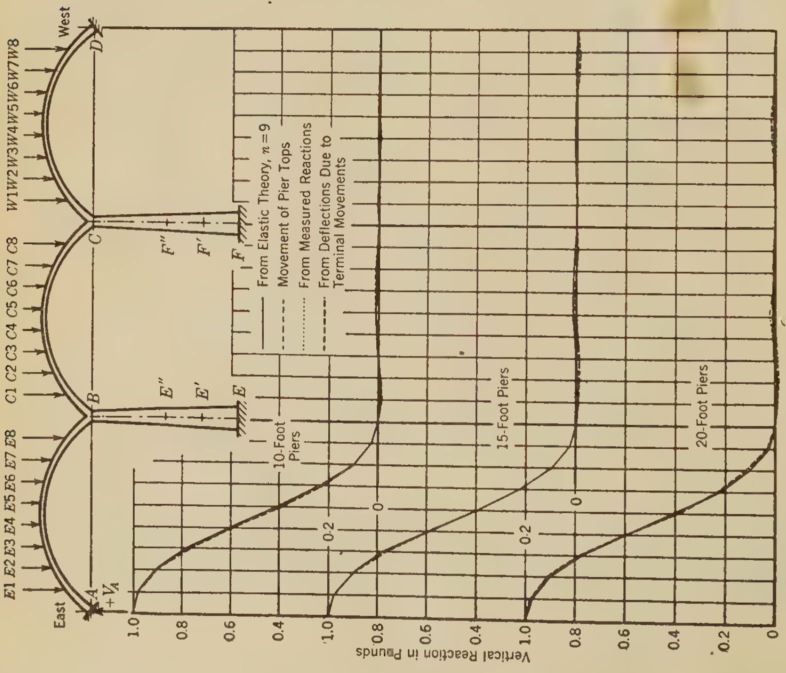


FIG. 34. INFLUENCE LINES FOR TERMINAL REACTIONS. VERTICAL REACTION AT ABUTMENT

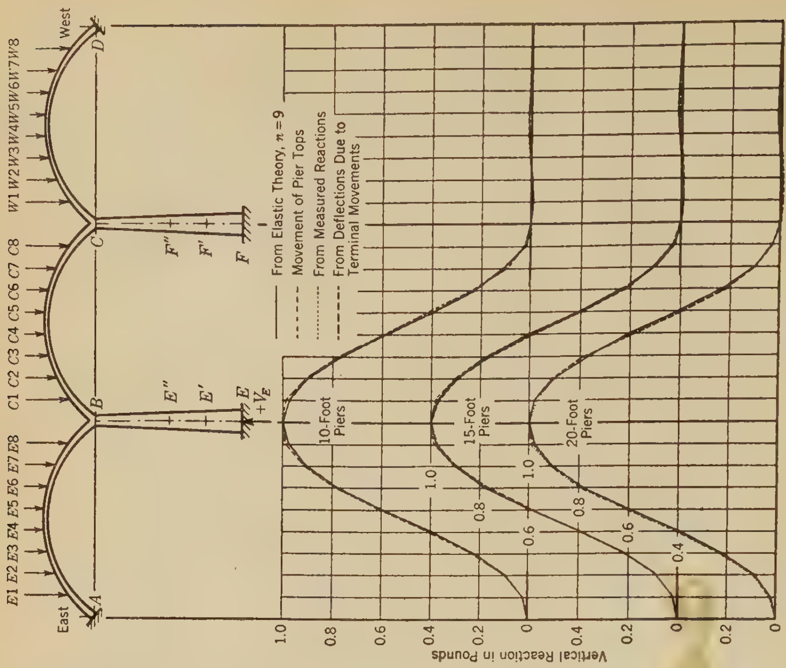


FIG. 35. INFLUENCE LINES FOR TERMINAL REACTIONS. VERTICAL REACTION AT PIER BASE

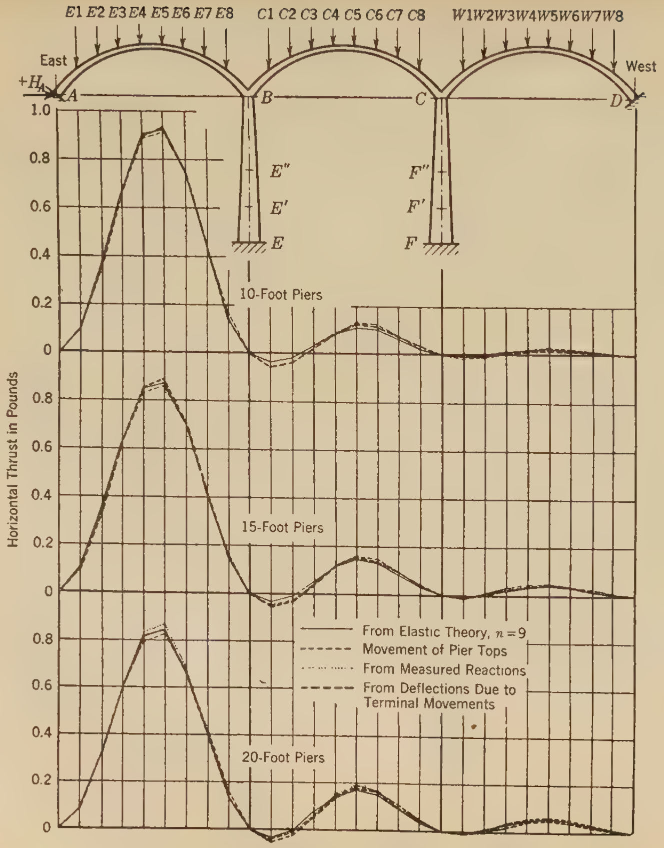


FIG. 36. INFLUENCE LINES FOR TERMINAL REACTIONS. HORIZONTAL REACTION AT ABUTMENT

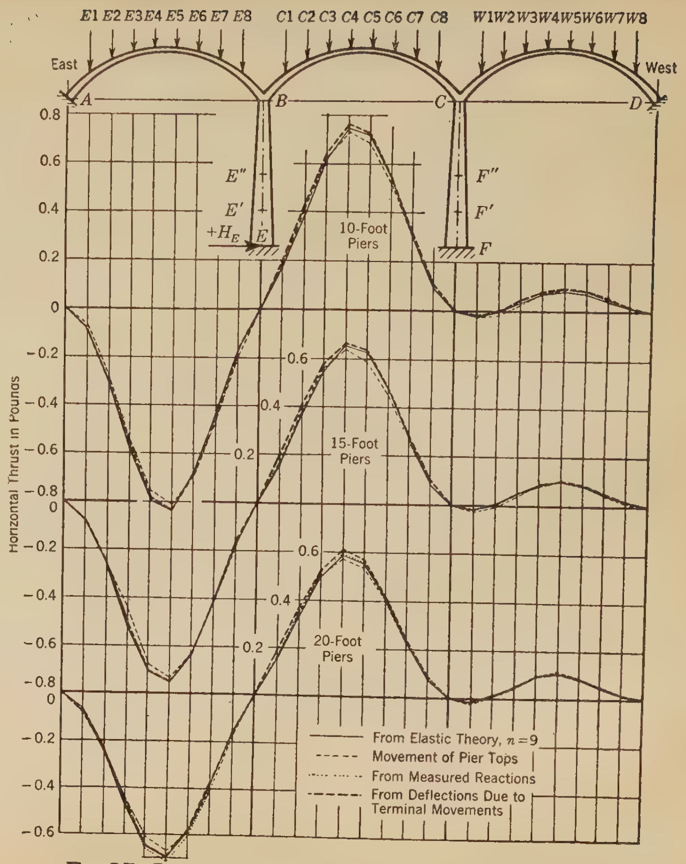


FIG. 37. INFLUENCE LINES FOR TERMINAL REACTIONS. HORIZONTAL REACTION AT PIER BASE

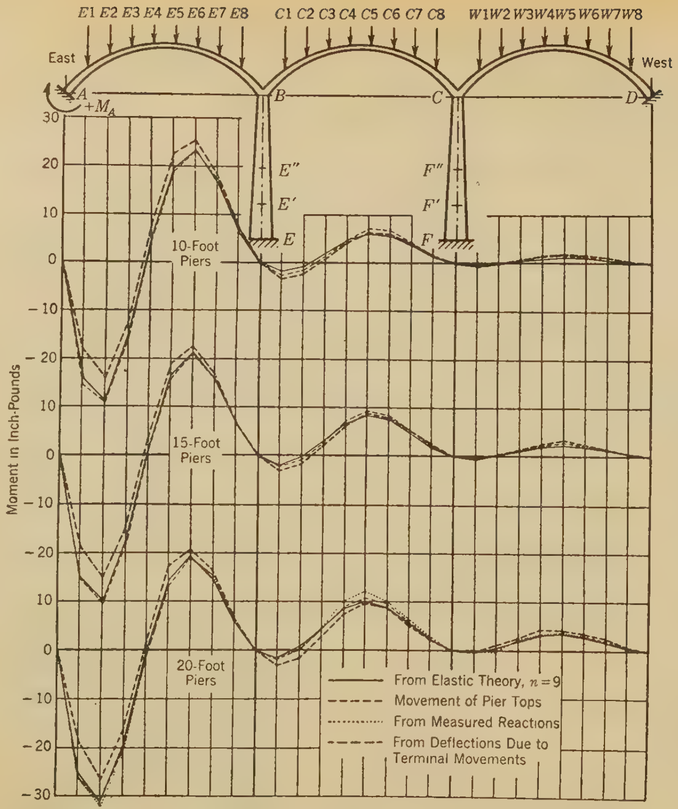


FIG. 38. INFLUENCE LINES FOR TERMINAL REACTIONS. MOMENT AT ABUTMENT

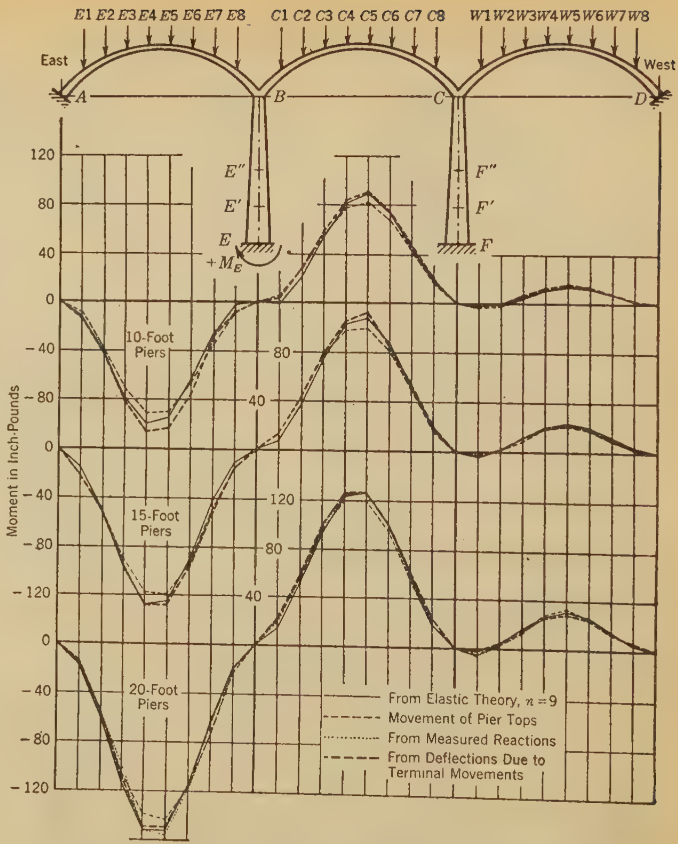


FIG. 39. INFLUENCE LINES FOR TERMINAL REACTIONS. MOMENT AT PIER BASE

as shown in Fig. 30. Complete sets of readings were taken to determine both the effect of applying and of removing the load. These tests were made on the structure for each of the three pier heights.

The strain in the concrete in the ribs was measured at the intrados and extrados on a section midway between each pair of adjacent load points. Since the reactions at the terminals were measured, the tangential thrust in the arch at each section could be determined. The portion of the tangential thrust taken by the steel was computed on the basis that the average strain in the steel was the same as the average strain in the adjacent concrete. The portion of the tangential thrust not taken by the steel was taken by the concrete and the average unit stress in the concrete could be computed for each section. The modulus of elasticity of the concrete for each section was then determined from the average stress and average strain. The modulus was determined for all sections of all spans for dead load alone and for dead load plus one live load. These data are given in Table 17. The values in this table indicate that the concrete in the east span had a much higher modulus than that in either the center span or the west span. The consistently low value of the modulus at the crown for all spans is of interest.

The load-carrying capacity of the structure was determined for a pier height of 20 ft. A complete set of readings was taken when the structure carried each of the following loads: (1) Its own weight, the zero or basic reading; (2) design dead load; (3) design dead load plus one live load; (4) design dead load plus two live loads; (5) design dead load plus three live loads; and (6) design dead load plus four live loads.

The structure was subjected to the design dead load plus four and one-half live loads, which it was able to carry, but no readings were taken although the terminals were returned to their normal position. When the load was increased to the dead load plus five live loads, the arch failed at Load Point C_4. It would seem, therefore, that the load-carrying capacity of the structure was between four and one-half and five times the design live load in addition to the design dead load. The rib was cracked in many places in all spans; but the cracks, although more than hair-cracks, were small at all sections except C_4, and there was no indication of impending failure at any other section. There were also small cracks in the piers near the bases.

The failure was a typical flexural failure for a concrete beam; a large crack opened on one side and the concrete spalled on the opposite side.

The positions of the thrust lines for the various loads are shown in Fig. 40. The small circles representing the centers of pressure as determined from the strain readings are given only at sections where the tension was not great enough to crack the concrete.

The fact that, even for loads near the ultimate, the position of the thrust line determined from measured reactions and by the elastic theory agrees so well, and the fact that the centers of pressure as determined from the measured strains fall upon these thrust lines, would appear to indicate that the position of the thrust line is given accurately by the elastic theory even if considerable portions of the rib contain many tension cracks.

TABLE 17.—MODULUS OF ELASTICITY OF CONCRETE IN ARCH RIBS.

Span	Load	Modulus of Elasticity, in 10^6 lb. per sq. in., at Section									
		1	2	3	4	C	5	6	7	8	Average
East	Dead	5.76	3.99	3.85	4.64	2.25	3.87	2.99	4.17	4.06	3.95
	Dead + live	5.58	3.60	3.68	4.45	2.33	3.95	2.84	4.10	3.80	3.81
	Average	5.67	3.80	3.76	4.54	2.29	3.91	2.91	4.13	3.93	3.88
Center	Dead	3.20	3.46	1.58	3.00	1.79	1.73	2.16	3.56	3.62	2.68
	Dead + live	2.87	2.72	1.69	2.89	1.43	1.86	3.59	4.67	3.44	2.80
	Average	3.04	3.09	1.63	2.95	1.61	1.79	2.88	4.11	3.53	2.74
West	Dead	2.78	3.33	3.33	2.16	1.86	2.77	3.13	3.51	4.10	3.00
	Dead + live	2.31	2.51	2.60	2.33	1.74	2.53	2.96	2.94	3.53	2.60
	Average	2.54	2.92	2.97	2.25	1.80	2.65	3.04	3.23	3.81	2.80
Average from Dead Load		....	...	....	....	....	....	....	....	....	3.21
Average from Dead + Live Load		....	...	....	....	....	....	....	....	....	3.07
Average of all		....	....	....	....	....	....	....	....	....	3.14

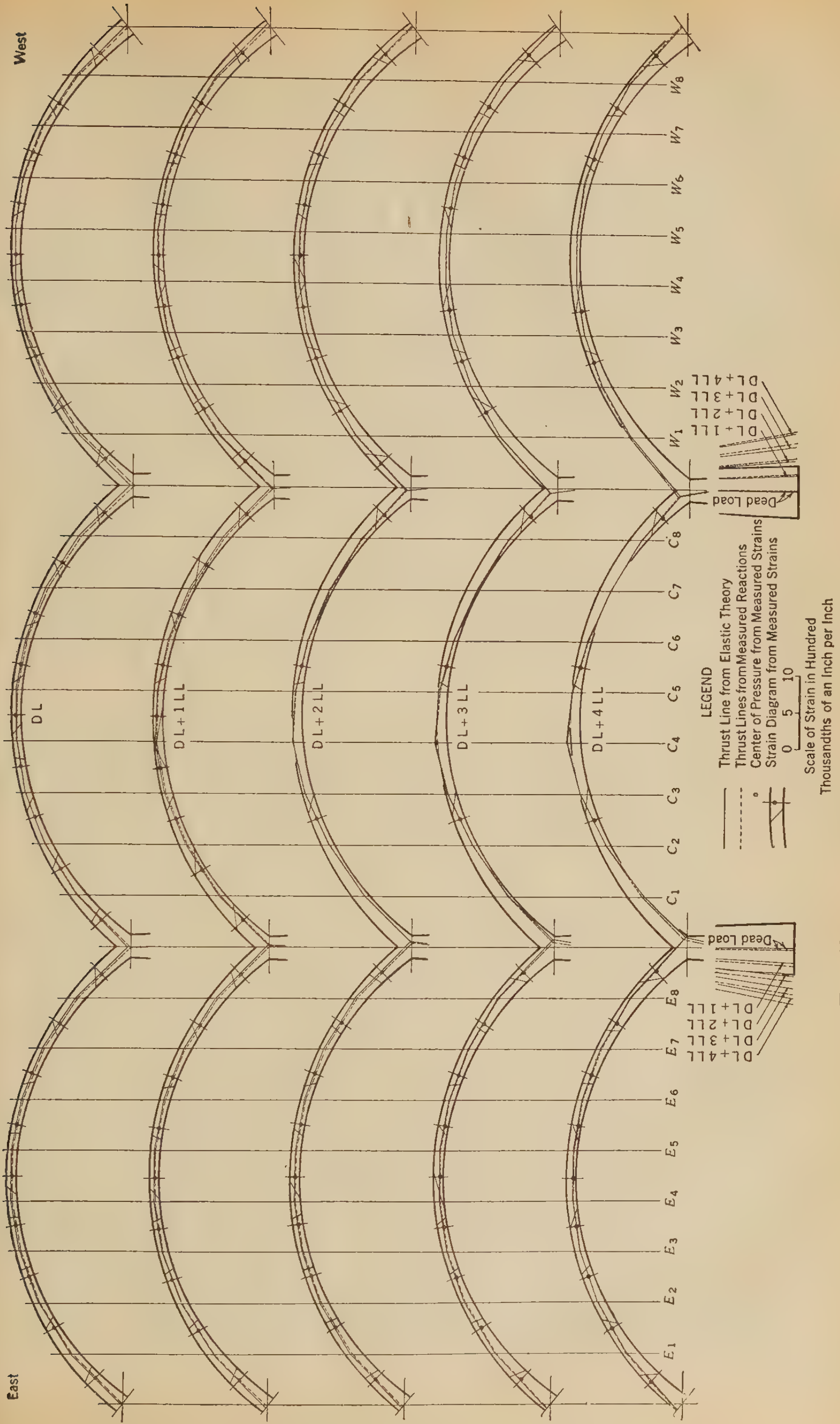

West
East
DL
DL+1LL
DL+2LL
DL+3LL
DL+4LL
W1
W2
W3
W4
W5
W6
W7
W8
C1
C2
C3
C4
C5
C6
C7
C8
E1
E2
E3
E4
E5
E6
E7
E8
Dead Load
DL+1LL
DL+2LL
DL+3LL
DL+4LL
Dead Load
DL+1LL
DL+2LL
DL+3LL
DL+4LL
LEGEND
Thrust Line from Elastic Theory
Thrust Lines from Measured Reactions
Center of Pressure from Measured Strains
Strain Diagram from Measured Strains
0 5 10
Scale of Strain in Hundred
Thousandths of an Inch per Inch

Fig. 40—Thrust Lines; Test to Failure (20-Foot Piers).

The structure carried the design dead load plus four and one-half live loads. The maximum unit stress at C_4 for this load computed by the elastic theory is 3 812 lb per sq in. The strength of the concrete as determined from the control cylinders was 3 310 lb per sq in.

For the structure tested, the modulus of elasticity of the concrete was much less at the crown than near the ends for all spans. For the west span, the modulus for the west half was much less than that for the east half. Moreover, the average modulus for the east span was about 40% greater than that for the center and for the west spans. These variations in the modulus do not appear to have influenced appreciably the stresses due to live load. The effect of the variations upon the dead-load stresses is not apparent.

The diagrams of Fig. 33 indicate that the influence ordinates for each of the components of the reaction at the springing of the center span are greatly affected by the elastic deformation of the piers. The stress at a given section, however, is a function of all three components of the reaction and, except for sections of the rib near the springing, is not necessarily so greatly affected as the individual components. The maximum unit stress in the three-span structure due to the design live load is 736 lb per sq in. in the extrados on the section through C_4, and the corresponding stress in the single span with fixed ends is 543 lb per sq in. at the extrados on the section through C_3; that is, the stress near the crown for design live load is increased about 32% by the flexure of the 20-ft piers. For a structure having 10-ft piers this increase is only about 20 per cent.

The maximum unit stress at the springing of the center span with 20-ft piers occurs at the intrados and is 555 lb per sq in. The maximum stress at the springing for the span with fixed ends occurs on the extrados and is 743 lb per sq in. Although the flexure of the piers decreases the negative moment at the springing of the center span, it increases the negative moment at the abutment of the end span.

Article 7.—Conclusions.—These tests of three-span arch ribs on slender piers seem to justify the following conclusions:

(1) The elastic theory based upon the usual assumptions gives values for the moment, thrust, and shear at various sections, that agree with the measured values within the tolerance of the tests, and with sufficient accuracy for purposes of design.

(2) In analyzing a multiple-span arch series, the moment of inertia of a section may be computed on the basis that the concrete takes tension.

(3) The value of E used in the analysis may vary considerably from the true value without seriously affecting the reactions due to loads.

(4) For the structure tested, the flexure of the piers increased the maximum live-load stress in the rib near the crown of the center span and near the abutment of the end span, but decreased the corresponding function for the springing of the center span.

(5) Considerable cracking of the arch ribs did not greatly alter the position or magnitude of the thrust.

(6) The concrete in the arch developed about the same maximum unit stress as the same concrete in the 6 by 12-in. control cylinders.

Acknowledgments.—The work described in this Chapter was done under the supervision of Professor Wilson (8).

V.—Tests of Three-Span Arch Bridges, with Decks on Slender Piers

Article 1.—Objects.—Arches consisting of a rib, spandrel columns, and a deck, are not readily susceptible of algebraic analysis. An experimental study of these structures is especially desirable as many of this type have been built. In making this investigation the following information was sought: (1) The elastic constants for a single span; (2) the influence ordinates for fixed-end reactions for a single span: (3) the influence ordinates for reactions at the springings of all spans of the three-span structures; (4) the magnitude and position of the thrust due to the design load; (5) the effect of intermediate expansion joints in the deck upon the load-carrying capacity of the three-span structures; and (6) the maximum load-carrying capacity of the three-span structures.

Article 2.—Description of Specimens and Apparatus.—Each specimen consisted of a three-span structure on high piers. The ribs and piers had the same dimensions and properties as the corresponding members of the three-span structure without deck described in Chapter IV and shown in Figs. 27 and 28. Two three-span structures were built and tested, one having a deck a considerable distance above the arch rib at the crown, and one having a deck so low as to be integral with the rib at the crown. The general dimensions of the two structures are given in Fig. 41. Each structure, as originally built, had no expansion joints in the deck except over the piers. After all tests had been made on this structure, except that for ultimate load-carrying capacity, intermediate expansion joints were cut in the deck as shown in Fig. 41. The structure with expansion joints was then tested and, finally, the load was carried to failure. This use of one structure for the two types was entirely feasible as the tests of the original structure did not crack the concrete.

Fig. 42 shows the notation used in connection with the tests, and Fig. 43 shows the specimen ready for testing.

The proportions of the concrete used in the structures with decks were as nearly like those used in the structure without deck as is practicable to obtain by the use of laboratory control methods. The equality of the concrete, however, varied greatly in different parts of the specimens due to the necessity of rodding the concrete more in some places than in others. This same variation has also been noted in all the arch structures previously tested. Control cylinders were poured from the batches that went into the arches and piers of both structures.

The method of curing and testing was similar to that already described for the three-span structure without deck, in Chapter IV. The design dead load is the same as that used in the previous tests with the following exception: The structures used in this investigation have two spandrel columns

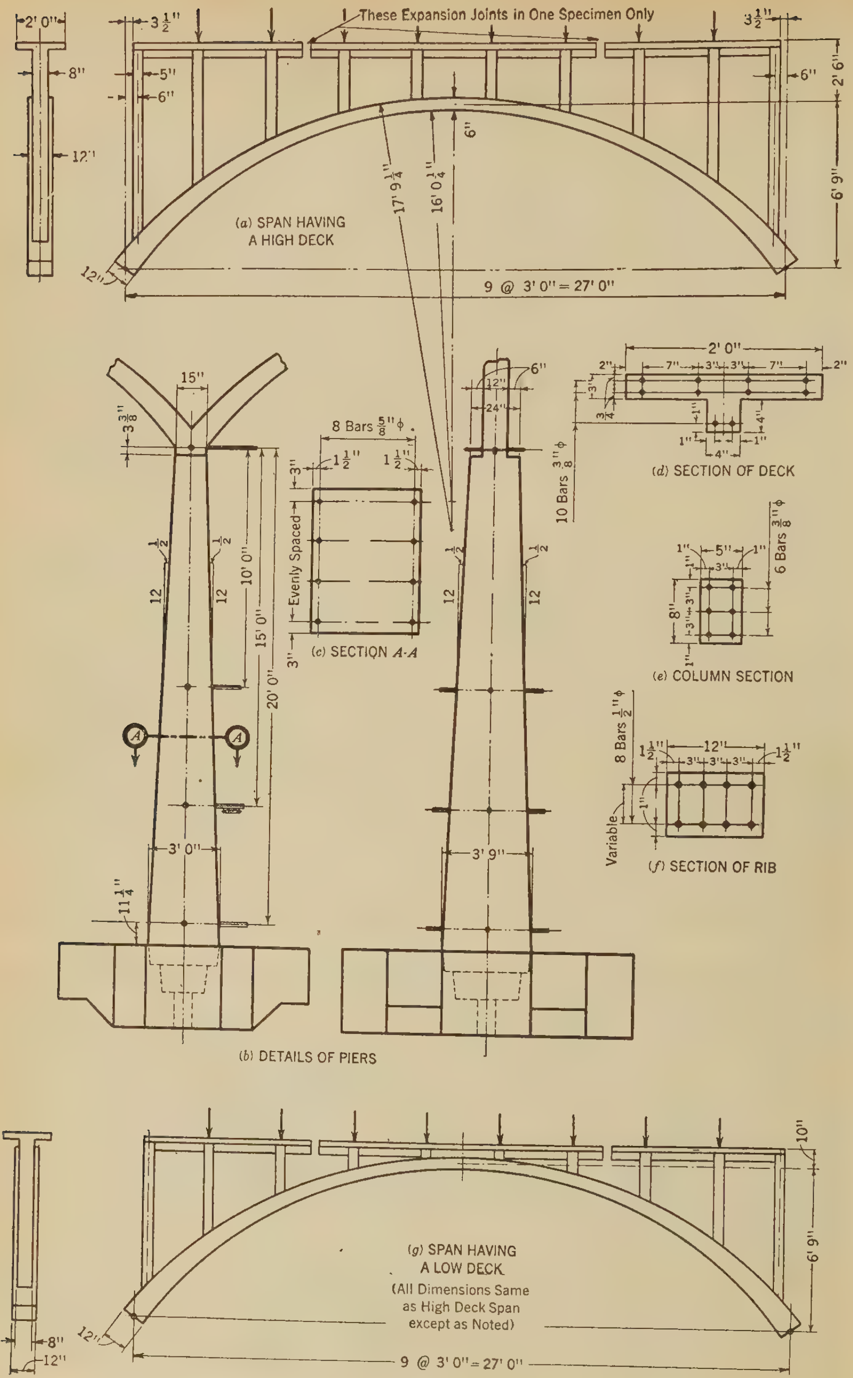
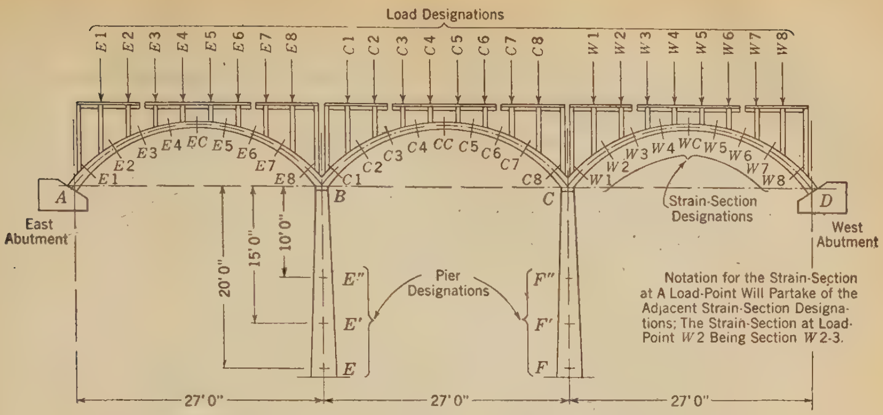

These Expansion Joints in One Specimen Only
2' 0"
8"
12"
3½"
5"
6"
6"
2' 6"
6'9"
17' 9¼"
16' 0¼"
12"
(a) SPAN HAVING A HIGH DECK
9 @ 3' 0" = 27' 0"
15"
3⅜"
½
12
10' 0"
15' 0"
20' 0"
A
3' 0"
11¼"
8 Bars ⅝"ϕ
1½"
3"
Evenly Spaced
(c) SECTION A-A
12"
24"
3' 9"
10 Bars ⅜"ϕ
2' 0"
2"
7"
3"
3¾
4"
1"
(d) SECTION OF DECK
6 Bars ⅜"ϕ
5"
(e) COLUMN SECTION
8 Bars ½"ϕ
Variable
(f) SECTION OF RIB
(b) DETAILS OF PIERS
(g) SPAN HAVING A LOW DECK
(All Dimensions Same as High Deck Span except as Noted)
10"

FIG. 41—DETAILS OF PIERS, RIBS, SPANDREL COLUMNS, AND DECK.


Load Designations
E1
E2
E3
E4
E5
E6
E7
E8
C1
C2
C3
C4
C5
C6
C7
C8
W1
W2
W3
W4
W5
W6
W7
W8
EC
CC
WC
A
B
C
D
East Abutment
West Abutment
20'0"
15'0"
10'0"
E"
E'
E
F"
F'
F
Pier Designations
Strain-Section Designations
27'0"
27'0"
27'0"

Notation for the Strain-Section at A Load-Point Will Partake of the Adjacent Strain-Section Designations; The Strain-Section at Load-Point $W2$ Being Section $W2$-3.

Fig. 42—Notation for Three-Span Arch Rib with High Deck.

over each pier which are close together. Any load coming upon these columns produces little stress in the rib, but increases the resistance to flexure of the columns. Inasmuch as the corresponding columns in an actual bridge carry considerable dead load, it seemed desirable to subject these columns of the experimental structure to an axial compression; therefore, the end columns of each span were subjected to an axial compression of 7 000 lb produced by means of springs. Springs were used instead of weights because the latter would interfere with the apparatus.

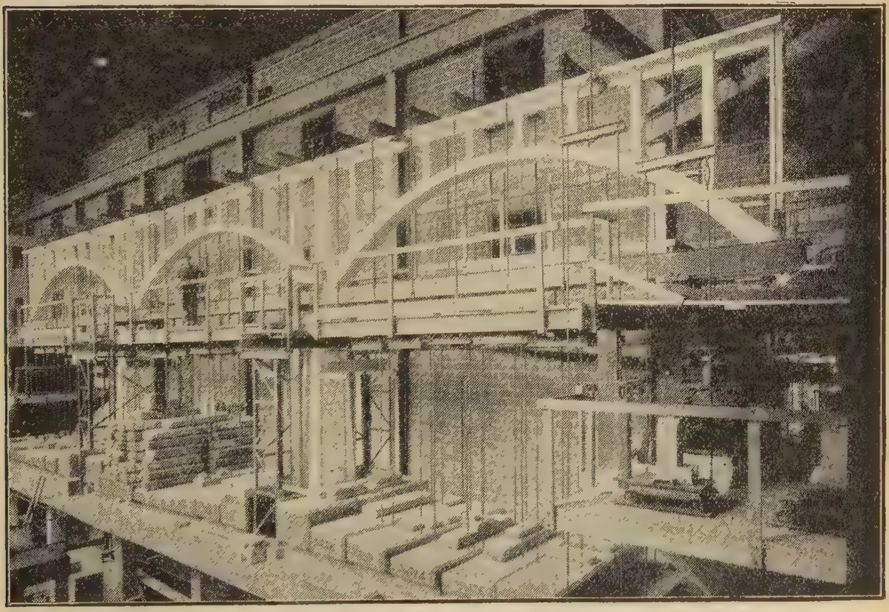


Fig. 43—The Three-Span Structure, with a High Deck.

Article 3.—Elastic Constants.—The influence ordinates for the fixed-end reactions of the east and west spans were determined by weighing, as a 2 000-lb load was applied and removed, successively, at the various load points. Before and after the application and removal of each load the abutments and pier tops, *A*, *B*, *C*, and *D*, Fig. 42, were brought to their normal positions, thus rendering each span a single arch with fixed ends. The design dead load was on the structure during the test.

These influence lines for fixed-end reactions are shown in Fig. 44, and the results confirm the conclusions given in Chapter III for the single span arches with decks.

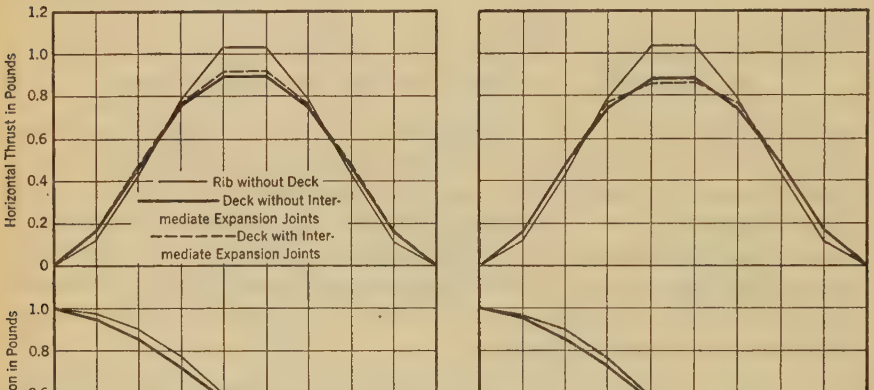


FIG. 44—INFLUENCE LINES FOR FIXED-END REACTIONS; VARIOUS TYPES OF STRUCTURES.

Tests were also run to determine the reactions at one springing of each span caused by a known movement of the other springing, horizontal and vertical; and tests were run on the two end spans to determine the effect of a known rotation of one end upon the reactions at both ends.

The elastic constants were determined from the movements of the springings of the arches with a view to using them to determine the reactions at the springings from the measured movements of the pier tops that accompanied the application and removal of a unit load of 2 000 lb at the various load points. All elastic constants were adjusted so that they were mutually statically consistent. The adjusted values of the elastic constants that have been used are given in Tables 18 and 19 for the structures with high deck and in Tables 20 and 21 for the structures with low deck.

TABLE 18.

INFLUENCE ORDINATES FOR HORIZONTAL THRUST ALGEBRAICALLY DETERMINED FROM EXPERIMENTAL ELASTIC CONSTANTS.

Structure with High Deck without Intermediate Expansion Joints. 20-ft. Piers.

Load of 1 lb. at	Horizontal Thrust in lb.				
	East Span	Center Span	West Span	East Pier	West Pier
*E*1	+0.11701	+0.03322	+0.01397	−0.08379	+0.01925
*E*2	+0.32884	+0.09427	+0.03965	−0.23457	+0.05462
*E*3	+0.55042	+0.15058	+0.06340	−0.39984	+0.08718
*E*4	+0.65766	+0.16854	+0.07110	−0.48912	+0.09744
*E*5	+0.67364	+0.15713	+0.06646	−0.51651	+0.09067
*E*6	+0.59414	+0.11934	+0.05072	−0.47480	+0.06862
*E*7	+0.38860	+0.05158	+0.02232	−0.33702	+0.02926
*E*8	+0.16314	+0.00027	+0.00059	−0.16287	−0.00032
*C*1	−0.01299	+0.13064	+0.04652	+0.14363	+0.08412
*C*2	+0.03425	+0.31665	+0.11134	+0.28240	+0.20531
*C*3	+0.10646	+0.49408	+0.16286	+0.38762	+0.33122
*C*4	+0.15214	+0.57121	+0.17276	+0.41907	+0.39845
*C*5	+0.17276	+0.57121	+0.15214	+0.39845	+0.41907
*C*6	+0.16286	+0.49408	+0.10646	+0.33122	+0.38762
*C*7	+0.11134	+0.31655	+0.03425	+0.20531	+0.28240
*C*8	+0.04652	+0.13064	−0.01299	+0.08412	+0.14363
*W*1	+0.00059	+0.00027	+0.16314	−0.00032	−0.16287
*W*2	+0.02232	+0.05158	+0.38860	+0.02926	−0.33702
*W*3	+0.05072	+0.11934	+0.59414	+0.06862	−0.47480
*W*4	+0.06646	+0.15713	+0.67364	+0.09067	−0.51651
*W*5	+0.07110	+0.16854	+0.65766	+0.09744	−0.48912
*W*6	+0.06340	+0.15058	+0.55042	+0.08718	−0.39984
*W*7	+0.03965	+0.09427	+0.32884	+0.05462	−0.23457
*W*8	+0.01397	+0.03322	+0.11701	+0.01925	−0.08379

Article 4.—Influence Lines.—Influence ordinates for the reactions at the springing of all three spans were determined experimentally by measuring the changes that occurred when a unit load was applied and removed, successively, at the various load points. The unit load was 2 000 lb, and the tests were made with the design dead load on the structure. The reactions were determined in two ways:

(1) By weighing the reactions at the abutments and pier bases before and after the application and removal of the unit load, and computing the reaction components at the springings of the arches by statics; and

(2) By measuring the displacements of the tops of the piers due to the application and removal of the unit load, and computing the reaction components at the springings of the arches corresponding to these displacements, using the experimentally determined elastic constants given in Tables 18 to 21, inclusive, and the fixed-end reactions shown in Fig. 44.

TABLE 19.

INFLUENCE ORDINATES FOR VERTICAL SHEAR ALGEBRAICALLY DETERMINED FROM EXPERIMENTAL ELASTIC CONSTANTS.

Structure with High Deck without Intermediate Expansion Joints. 20-ft. Piers.

Load of 1 lb. at	Vertical Shear in lb. at							
	A of *AB*	*B* of *AB*	*B* of *BC*	*C* of *BC*	*C* of *CD*	*D* of *CD*	East Pier	West Pier
*E*1	+0.95102	+0.04898	−0.00403	+0.00403	−0.00102	+0.00102	+0.04495	+0.00301
*E*2	+0.85356	+0.14644	−0.01150	+0.01150	−0.00290	+0.00290	+0.13494	+0.00860
*E*3	+0.71522	+0.28478	−0.01782	+0.01782	−0.00462	+0.00462	+0.26696	+0.01320
*E*4	+0.56416	+0.43584	−0.01906	+0.01906	−0.00519	+0.00519	+0.41678	+0.01357
*E*5	+0.41027	+0.58973	−0.01648	+0.01648	−0.00486	+0.00486	+0.57325	+0.01162
*E*6	+0.26456	+0.73544	−0.01075	+0.01075	−0.00372	+0.00372	+0.72469	+0.00703
*E*7	+0.13766	+0.86234	−0.00183	+0.00183	−0.00166	+0.00166	+0.86051	+0.00017
*E*8	+0.04948	+0.95052	+0.00343	−0.00343	−0.00007	+0.00007	+0.95395	−0.00350
*C*1	−0.00452	+0.00452	+0.94657	+0.05343	−0.00294	+0.00294	+0.95109	+0.05049
*C*2	−0.00272	+0.00272	+0.85250	+0.14750	−0.00695	+0.00695	+0.85522	+0.14055
*C*3	+0.00240	−0.00240	+0.72134	+0.27866	−0.00947	+0.00947	+0.71894	+0.26919
*C*4	+0.00642	−0.00642	+0.57554	+0.42446	−0.00901	+0.00901	+0.56912	+0.41545
*C*5	+0.00901	−0.00901	+0.42446	+0.57554	−0.00642	+0.00642	+0.41545	+0.56912
*C*6	+0.00947	−0.00947	+0.27866	+0.72134	−0.00240	+0.00240	+0.26919	+0.71894
*C*7	+0.00695	−0.00695	+0.14750	+0.85250	+0.00272	−0.00272	+0.14055	+0.85522
*C*8	+0.00294	−0.00294	+0.05343	+0.94657	+0.00452	−0.00452	+0.05049	+0.95109
*W*1	+0.00007	−0.00007	−0.00343	+0.00343	+0.95052	+0.04948	−0.00350	+0.95395
*W*2	+0.00166	−0.00166	+0.00183	−0.00183	+0.86234	+0.13766	+0.00017	+0.86051
*W*3	+0.00372	−0.00372	+0.01075	−0.01075	+0.73544	+0.26456	+0.00703	+0.72469
*W*4	+0.00486	−0.00486	+0.01648	−0.01648	+0.58973	+0.41027	+0.01162	+0.57325
*W*5	+0.00519	−0.00519	+0.01906	−0.01906	+0.43584	+0.56416	+0.01357	+0.41678
*W*6	+0.00462	−0.00462	+0.01782	−0.01782	+0.28478	+0.71522	+0.01320	+0.26696
*W*7	+0.00290	−0.00290	+0.01150	−0.01150	+0.14644	+0.85356	+0.00860	+0.13494
*W*8	+0.00102	−0.00102	+0.00403	−0.00403	+0.04898	+0.95102	+0.00301	+0.04495

TABLE 20.

INFLUENCE ORDINATES FOR MOMENT ALGEBRAICALLY DETERMINED FROM EXPERIMENTAL ELASTIC CONSTANTS.

Structure with High Deck without Intermediate Expansion Joints. 15-ft. Piers.

Load of 1 lb. at	Moment in in. lb. at							
	A of *AB*	*B* of *AB*	*B* of *BC*	*C* of *BC*	*C* of *CD*	*D* of *CD*	Base of East Pier	Base of West Pier
*E*1	−18.68411	+ 1.42290	+ 2.23191	+ 0.94806	+ 0.71604	+ 0.42921	−16.35399	+ 3.64302
*E*2	−20.13652	+ 4.35393	+ 6.35855	+ 2.69211	+ 2.03440	+ 1.21952	−46.06798	+10.35431
*E*3	−11.11861	+ 4.50374	+ 9.96662	+ 4.28808	+ 3.23154	+ 1.93664	−76.24632	+16.42854
*E*4	− 1.68338	+ 1.00239	+10.84423	+ 4.78048	+ 3.58789	+ 2 14938	−89.74136	+18.20979
*E*5	+ 6.26520	− 4.94775	+ 9.65570	+ 4.42857	+ 3.30223	+ 1.97705	−90.07195	+16.71254
*E*6	+10.63206	−11.77835	+ 6.71429	+ 3.32509	+ 2.44985	+ 1.46506	−77.14676	+12.33044
*E*7	+ 9.58992	−17.89865	+ 1.91362	+ 1.37601	+ 0.96607	+ 0.57502	−47.29893	+ 4.75334
*E*8	+ 4.26032	−15.75282	− 1.19892	− 0.06777	− 0.10855	− 0.06825	−17.30430	− 0.68102
*C*1	− 0.60553	− 1.98321	−16.76914	+ 1.91986	+ 2.76966	+ 1.73236	+13.66127	+17.03500
*C*2	+ 1.12257	+ 0.45425	−20.01574	+ 4.19743	+ 6.61200	+ 4.15151	+36.94461	+41.31463
*C*3	+ 3.83830	+ 4.94781	−14.60137	+ 3.11529	+ 9.45341	+ 6.05455	+60.71642	+63.91408
*C*4	+ 5.58142	+ 8.04486	− 7.75119	− 1.27681	+ 9.69138	+ 6.39133	+71.86395	+73.03061
*C*5	+ 6.39133	+ 9.69138	− 1.27681	− 7.75119	+ 8.04486	+ 5.58142	+73.03061	+71.86395
*C*6	+ 6.05455	+ 9.45341	+ 3.11529	−14.60137	+ 4.94781	+ 3.83830	+63.91408	+60.71642
*C*7	+ 4.15151	+ 6.61200	+ 4.19743	−20.01574	+ 0.45425	+ 1.12257	+41.31463	+36.94461
*C*8	+ 1.73236	+ 2.76966	+ 1.91986	−16.76914	− 1.98321	− 0.60553	+17.03500	+13.66127
*W*1	− 0.06825	− 0.10855	− 0.06777	− 1.19892	−15.75282	+ 4.26032	− 0.68102	−17.30430
*W*2	+ 0.57502	+ 0.96607	+ 1.37601	+ 1.91362	−17.89865	+ 9.58992	+ 4.75334	−47.29893
*W*3	+ 1.46506	+ 2.44985	+ 3.32509	+ 6.71429	−11.77835	+10.63206	+12.33044	−77.14676
*W*4	+ 1.97705	+ 3.30223	+ 4.42857	+ 9.65570	− 4.94775	+ 6.26520	+16.71254	−90.07195
*W*5	+ 2.14938	+ 3.58789	+ 4.78048	+10.84423	+ 1.00239	− 1.68338	+18.20979	−89.74136
*W*6	+ 1.93664	+ 3.23154	+ 4.28808	+ 9.96662	+ 4.50374	−11.11861	+16.42854	−76.24632
*W*7	+ 1.21952	+ 2.03440	+ 2.69211	+ 6.35855	+ 4.35393	−20.13652	+10.35431	−46.06798
*W*8	+ 0.42921	+ 0.71604	+ 0.94806	+ 2.23191	+ 1 42290	−18.68411	+ 3.64302	−16.35399

TABLE 21.

INFLUENCE ORDINATES FOR HORIZONTAL THRUST ALGEBRAICALLY DETERMINED FROM EXPERIMENTAL ELASTIC CONSTANTS.

Structure with High Deck without Intermediate Expansion Joints. 15-ft. Piers.

Load of 1 lb. at	Horizontal Thrust in lb.				
	East Span	Center Span	West Span	East Pier	West Pier
*E*1	+0.12459	+0.02924	+0.01029	−0.09535	+0.01895
*E*2	+0.35017	+0.08310	+0.02923	−0.26707	+0.05387
*E*3	+0.58576	+0.13182	+0.04642	−0.45394	+0.08540
*E*4	+0.69930	+0.14606	+0.05152	−0.55324	+0.09454
*E*5	+0.71551	+0.13398	+0.04739	−0.58153	+0.08659
*E*6	+0.63010	+0.09877	+0.03513	−0.53133	+0.06364
*E*7	+0.41077	+0.03793	+0.01380	−0.37284	+0.02413
*E*8	+0.17136	−0.00563	−0.00162	−0.17699	−0.00401
*C*1	−0.01753	+0.14051	+0.04115	+0.15804	+0.09936
*C*2	+0.02250	+0.34147	+0.09853	+0.31897	+0.24294
*C*3	+0.08748	+0.53340	+0.14311	+0.44592	+0.39029
*C*4	+0.12985	+0.61685	+0.15019	+0.48700	+0.46666
*C*5	+0.15019	+0.61685	+0.12985	+0.46666	+0.48700
*C*6	+0.14311	+0.53340	+0.08748	+0.39029	+0.44592
*C*7	+0.09853	+0.34147	+0.02250	+0.24294	+0.31897
*C*8	+0.04115	+0.14051	−0.01753	+0.09936	+0.15804
*W*1	−0.00162	−0.00563	+0.17136	−0.00401	−0.17699
*W*2	+0.01380	+0.03793	+0.41077	+0.02413	−0.37284
*W*3	+0.03513	+0.09877	+0.63010	+0.06364	−0.53133
*W*4	+0.04739	+0.13398	+0.71551	+0.08659	−0.58153
*W*5	+0.05152	+0.14606	+0.69930	+0.09454	−0.55324
*W*6	+0.04642	+0.13182	+0.58576	+0.08540	−0.45394
*W*7	+0.02923	+0.08310	+0.35017	+0.05387	−0.26707
*W*8	+0.01029	+0.02924	+0.12459	+0.01895	−0.09535

Of the structures with high deck, the one without intermediate expansion joints was tested with pier heights of 10, 15, and 20 ft, but the one with intermediate expansion joints was tested with pier heights of 10 and 20 ft only. Both structures with a low deck were tested with a pier height of 20 ft only.

The method of determining the reaction components at the springings from the movement of the pier tops is illustrated in Tables 22 and 23. The computations in these tables are for the center span. Each reaction component is made up of four parts: (1) The reaction due to spread; (2) the reaction due to rotation of the top of the east pier; (3) the reaction due to rotation of the top of the west pier; and, (4) the fixed-end reaction due to the unit load.

These components are listed separately in the various lines of Tables 22 and 23, and the resultant component is the algebraic sum of the various parts. The tables show the reaction components due to applying, and to removing, the unit load at the load points. The average of these two values is given for each reaction component, for each load point. Tables similar to Tables 22 and 23 were made up for each load point for all structures tested.

Influence lines for the reactions on the center span of four of the structures tested are given in Figs. 45 and 46. As in the case of the three-span structure without deck, tests were made to determine the accuracy of the use of deflection diagrams as influence lines. The vertical deflections of all load points were determined, due to giving, successively, each of the four terminals (the two abutments and the two pier bases), known movements of vertical translation, horizontal translation, and rotation. The arch carried the design dead load and the movements were planned to keep the thrust line within the kern at all sections.

All influence lines for vertical reactions were so nearly alike that the various lines could barely be distinguished. The influence lines for horizontal reaction at the abutment and at the pier bases were in fair agreement, but those for moment at the abutment and at the pier bases differed materially.

The maximum live load stress at any section of a three-span arch bridge is produced by placing loads on only those parts of the structure for which the influence ordinates for stress at the given section have the same sign. For this reason influence ordinates need not be determined with extreme accuracy if they are limited in their use to the determination of live load stress; but, because the influence line for stress lies partly above and partly below the base line with positive and negative areas nearly equal, the influence ordinates must be determined with extreme care if they are to be used to determine the dead load stress. The influence lines for stress at C_3 are shown in Fig. 47.

Article 5.—Algebraic Analysis Based on Elastic Constants.—The reaction components at the springings of the arches of a multiple-span arch bridge may be determined by the use of the elastic constants of the piers and the arch spans, together with the fixed-end influence ordinates for the arch span. These constants may be determined algebraically for an arch rib without deck, but the corresponding constants cannot be determined readily algebraically for an arch span with spandrel columns and deck. However, a method[1] was developed for analyzing these structures by using experimentally determined elastic constants for the arch spans in an otherwise algebraic method (19). The small circles in Figs. 45 and 46 represent the influence ordinates obtained by this combined experimental and algebraic method.

In order to determine the effect of variations in the elastic constants of adjacent arch spans upon the stresses, two structures were analyzed for which the properties of the material were assumed different for the several spans. Both structures had 20-ft piers. One had a high deck without expansion joints. The elastic constants for the center span were assumed to be as used previously and the constants for the end spans were assumed to be two-thirds as great. These data are shown in Figs. 48 and 49, and appear to indicate that a considerable variation in the elastic constants produces a relatively small variation in the influence ordinates for stress.

Article 6.—Loading Tests.—The design load test consisted of measuring the changes in the reactions at the terminals, the strain in the concrete of the rib, and the vertical movement of the load points due to applying first the dead load and then the live load and removing first the live load and then the dead load. The terminals were returned to their normal positions after each load change before readings were taken. The dead load was the

---

[1] "A Study of Multiple Span Arches," by Donald E. Larsen, Jun. Am. Soc. C. E. A thesis submitted to the University of Illinois in 1930, in partial fulfillment of the requirements for the degree of Master of Science.

TABLE 22.

INFLUENCE ORDINATES FOR VERTICAL SHEAR ALGEBRAICALLY DETERMINED FROM EXPERIMENTAL ELASTIC CONSTANTS.

Structure with High Deck without Intermediate Expansion Joints. 15-ft. Piers.

Load of 1 lb. at	Vertical Shear in lb. at							
	A of *AB*	*B* of *AB*	*B* of *BC*	*C* of *BC*	*C* of *CD*	*D* of *CD*	East Pier	West Pier
*E*1	+0.95095	+0.04905	−0.00396	+0.00396	−0.00089	+0.00089	+0.04509	+0.00307
*E*2	+0.85337	+0.14663	−0.01132	+0.01132	−0.00252	+0.00252	+0.13531	+0.00880
*E*3	+0.71488	+0.28512	−0.01753	+0.01753	−0.00400	+0.00400	+0.26759	+0.01353
*E*4	+0.56384	+0.43616	−0.01872	+0.01872	−0.00444	+0.00444	+0.41744	+0.01428
*E*5	+0.40984	+0.59016	−0.01613	+0.01613	−0.00409	+0.00409	+0.57403	+0.01204
*E*6	+0.26417	+0.73583	−0.01046	+0.01046	−0.00304	+0.00304	+0.72537	+0.00742
*E*7	+0.13738	+0.86262	−0.00166	+0.00166	−0.00121	+0.00121	+0.86096	+0.00045
*E*8	+0.04934	+0.95066	+0.00349	−0.00349	+0.00012	−0.00012	+0.95415	−0.00337
*C*1	−0.00425	+0.00425	+0.94657	+0.05343	−0.00320	+0.00320	+0.95082	+0.05023
*C*2	−0.00206	+0.00206	+0.85251	+0.14749	−0.00759	+0.00759	+0.85457	+0.13990
*C*3	+0.00342	−0.00342	+0.72135	+0.27865	−0.01049	+0.01049	+0.71793	+0.26816
*C*4	+0.00760	−0.00760	+0.57554	+0.42446	−0.01019	+0.01019	+0.56794	+0.41427
*C*5	+0.01019	−0.01019	+0.42446	+0.57554	−0.00760	+0.00760	+0.41427	+0.56794
*C*6	+0.01049	−0.01049	+0.27865	+0.72135	−0.00342	+0.00342	+0.26816	+0.71793
*C*7	+0.00759	−0.00759	+0.14749	+0.85251	+0.00206	−0.00206	+0.13990	+0.85457
*C*8	+0.00320	−0.00320	+0.05343	+0.94657	+0.00425	−0.00425	+0.05023	+0.95082
*W*1	−0.00012	+0.00012	−0.00349	+0.00349	+0.95066	+0.04934	−0.00337	+0.95415
*W*2	+0.00121	−0.00121	+0.00166	−0.00166	+0.86262	+0.13738	+0.00045	+0.86096
*W*3	+0.00304	−0.00304	+0.01046	−0.01046	+0.73583	+0.26417	+0.00742	+0.72537
*W*4	+0.00409	−0.00409	+0.01613	−0.01613	+0.59016	+0.40984	+0.01204	+0.57403
*W*5	+0.00444	−0.00444	+0.01872	−0.01872	+0.43616	+0.56384	+0.01428	+0.41744
*W*6	+0.00400	−0.00400	+0.01753	−0.01753	+0.28512	+0.71488	+0.01353	+0.26759
*W*7	+0.00252	−0.00252	+0.01132	−0.01132	+0.14663	+0.85337	+0.00880	+0.13531
*W*8	+0.00089	−0.00089	+0.00396	−0.00396	+0.04905	+0.95095	+0.00307	+0.04509

TABLE 23.

INFLUENCE ORDINATES FOR MOMENT ALGEBRAICALLY DETERMINED FROM EXPERIMENTAL ELASTIC CONSTANTS.

Structure with High Deck without Intermediate Expansion Joints. 10-ft. Piers.

Load of 1 lb. at	Moment in in. lb. at							
	A of *AB*	*B* of *AB*	*B* of *BC*	*C* of *BC*	*C* of *CD*	*D* of *CD*	Base of East Pier	Base of West Pier
*E*1	−18.21404	+ 1.93947	+ 1.90393	+ 0.74786	+ 0.44618	+ 0.24060	−13.45634	+ 2.37528
*E*2	−18.81030	+ 5.81178	+ 5.43485	+ 2.12936	+ 1.27094	+ 0.68536	−38.00413	+ 6.76722
*E*3	− 8.93994	+ 6.89570	+ 8.43637	+ 3.34722	+ 1.99371	+ 1.07494	−62.13373	+10.60671
*E*4	+ 0.85488	+ 3.78429	+ 9.04097	+ 3.65814	+ 2.17197	+ 1.17074	−71.91892	+11.53857
*E*5	+ 8.77656	− 2.20208	+ 7.84293	+ 3.28145	+ 1.93798	+ 1.04415	−70.49899	+10.27027
*E*6	+12.73715	− 9.48554	+ 5.15802	+ 2.31643	+ 1.35341	+ 0.72853	−58.24804	+ 7.13702
*E*7	+10.81556	−16.57629	+ 0.95436	+ 0.72059	+ 0.39585	+ 0.21193	−32.69175	+ 2.02634
*E*8	+ 4.65276	−15.34085	− 1.55435	− 0.33950	− 0.22926	− 0.12482	− 9.35670	− 1.28384
*C*1	− 0.80936	− 2.12534	−16.21351	+ 2.51957	+ 2.37379	+ 1.31886	+ 6.98143	+14.74018
*C*2	+ 0.43108	− 0.10466	−18.49371	+ 5.77657	+ 5.72436	+ 3.18837	+25.92455	+35.97781
*C*3	+ 2.59491	+ 3.89282	−12.10424	+ 5.65420	+ 8.15788	+ 4.61241	+47.64254	+54.99792
*C*4	+ 4.04870	+ 6.71510	− 4.81608	+ 1.67357	+ 8.27372	+ 4.78597	+58.96042	+61.64865
*C*5	+ 4.78597	+ 8.27372	+ 1.67357	− 4.81608	+ 6.71510	+ 4.04870	+61.64865	+58.96042
*C*6	+ 4.61241	+ 8.15788	+ 5.65420	−12.10424	+ 3.89282	+ 2.59491	+54.99792	+47.64254
*C*7	+ 3.18837	+ 5.72436	+ 5.77657	−18.49371	− 0.10466	+ 0.43108	+35.97781	+25.92455
*C*8	+ 1.31886	+ 2.37379	+ 2.51957	−16.21351	− 2.12534	− 0.80936	+14.74018	+ 6.98143
*W*1	− 0.12482	− 0.22926	− 0.33950	− 1.55435	−15.34085	+ 4.65276	− 1.28384	− 9.35670
*W*2	+ 0.21193	+ 0.39585	+ 0.72059	+ 0.95436	−16.57629	+10.81556	+ 2.02634	−32.69175
*W*3	+ 0.72853	+ 1.35341	+ 2.31643	+ 5.15802	− 9.48554	+12.73715	+ 7.13702	−58.24804
*W*4	+ 1.04415	+ 1.93798	+ 3.28145	+ 7.84293	− 2.20208	+ 8.77656	+10.27027	−70.49899
*W*5	+ 1.17074	+ 2.17197	+ 3.65814	+ 9.04097	+ 3.78429	+ 0.85488	+11.53857	−71.91892
*W*6	+ 1.07494	+ 1.99371	+ 3.34722	+ 8.43637	+ 6.89570	− 8.93994	+10.60671	−62.13373
*W*7	+ 0.68536	+ 1.27094	+ 2.12936	+ 5.43485	+ 5.81178	−18.81030	+ 6.76722	−38.00413
*W*8	+ 0.24060	+ 0.44618	+ 0.74786	+ 1.90393	+ 1.93947	−18.21404	+ 2.37528	−13.45634

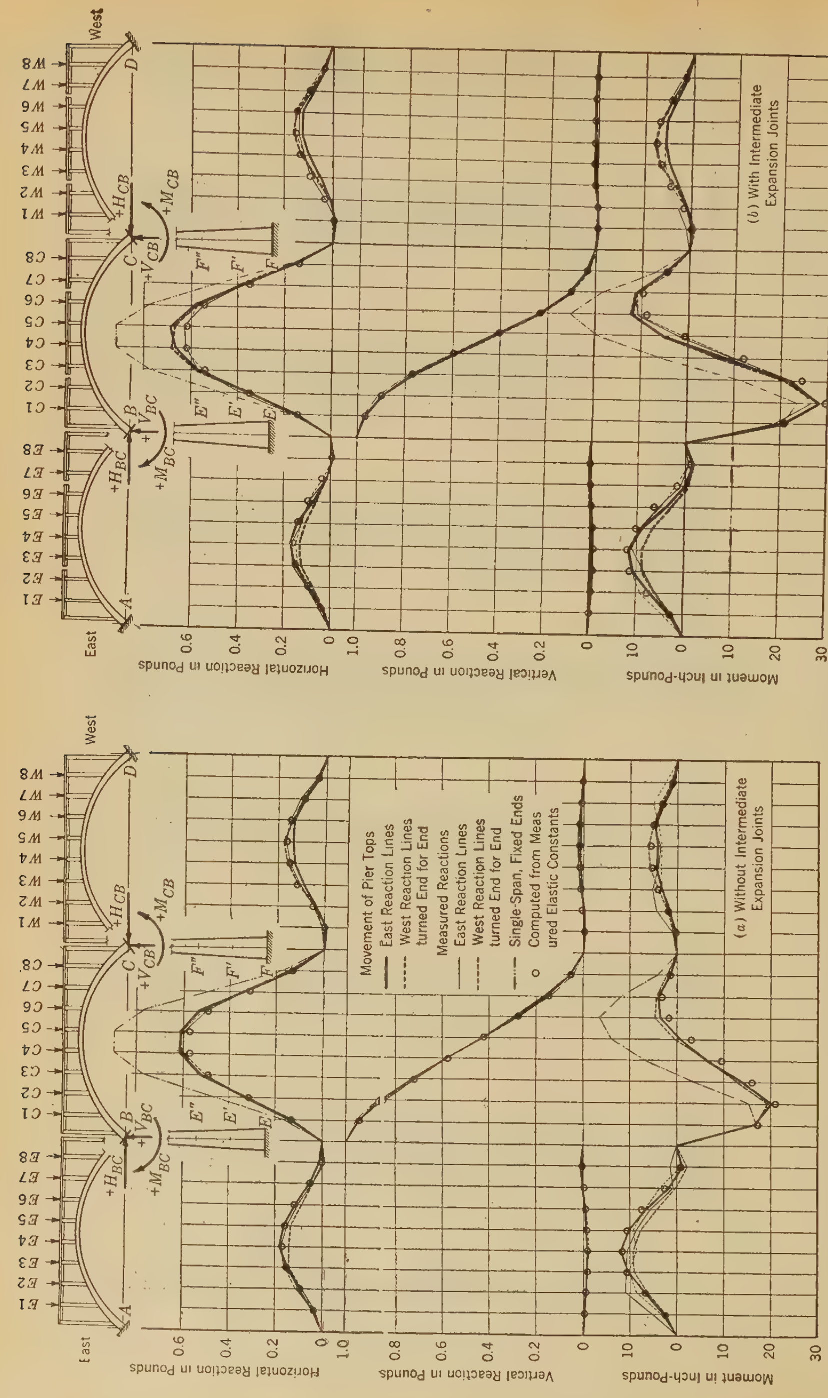


FIG. 45—INFLUENCE LINES, HIGH-DECK SPANS, BY UNIT LOADS; REACTIONS, TOPS OF 20-FOOT CENTER-SPAN PIERS.

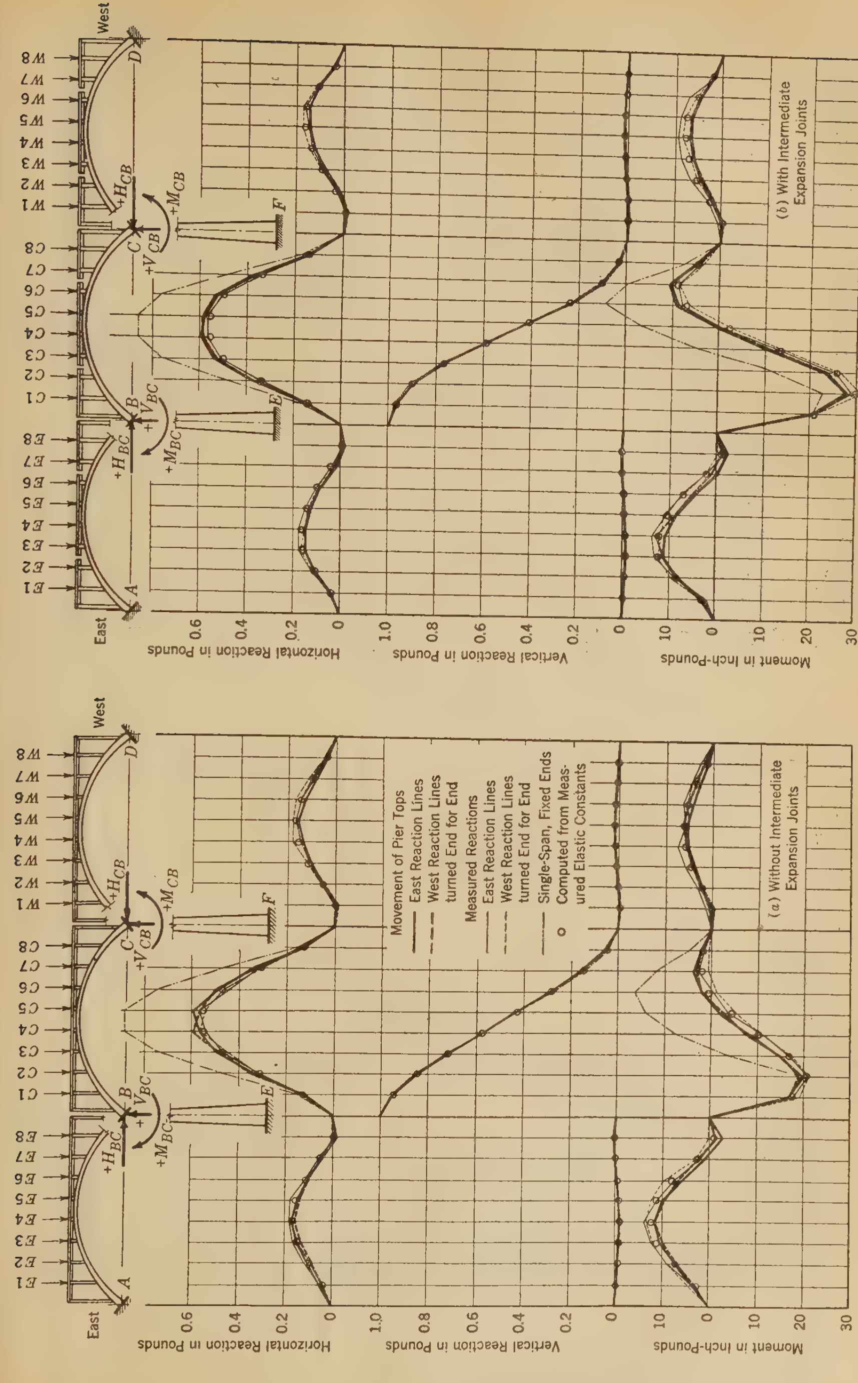


FIG. 46—INFLUENCE LINES, LOW-DECK SPANS, BY UNIT LOADS; REACTIONS, TOPS OF 20-FOOT CENTER-SPAN PIERS.

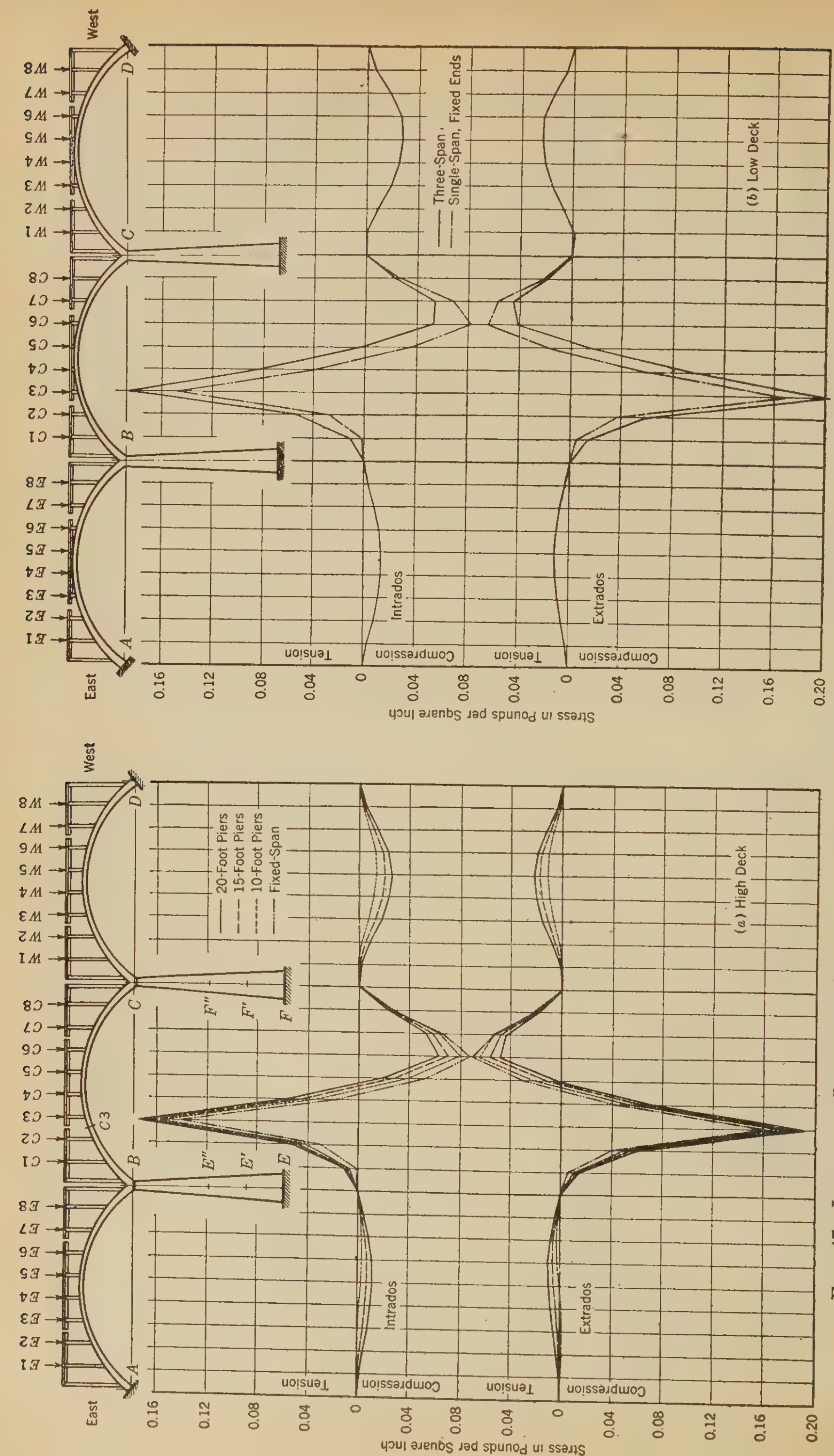


FIG. 47—INFLUENCE LINES FOR STRESS AT C_3 WITH INTERMEDIATE EXPANSION JOINTS (FROM ALGEBRAIC ANALYSIS).

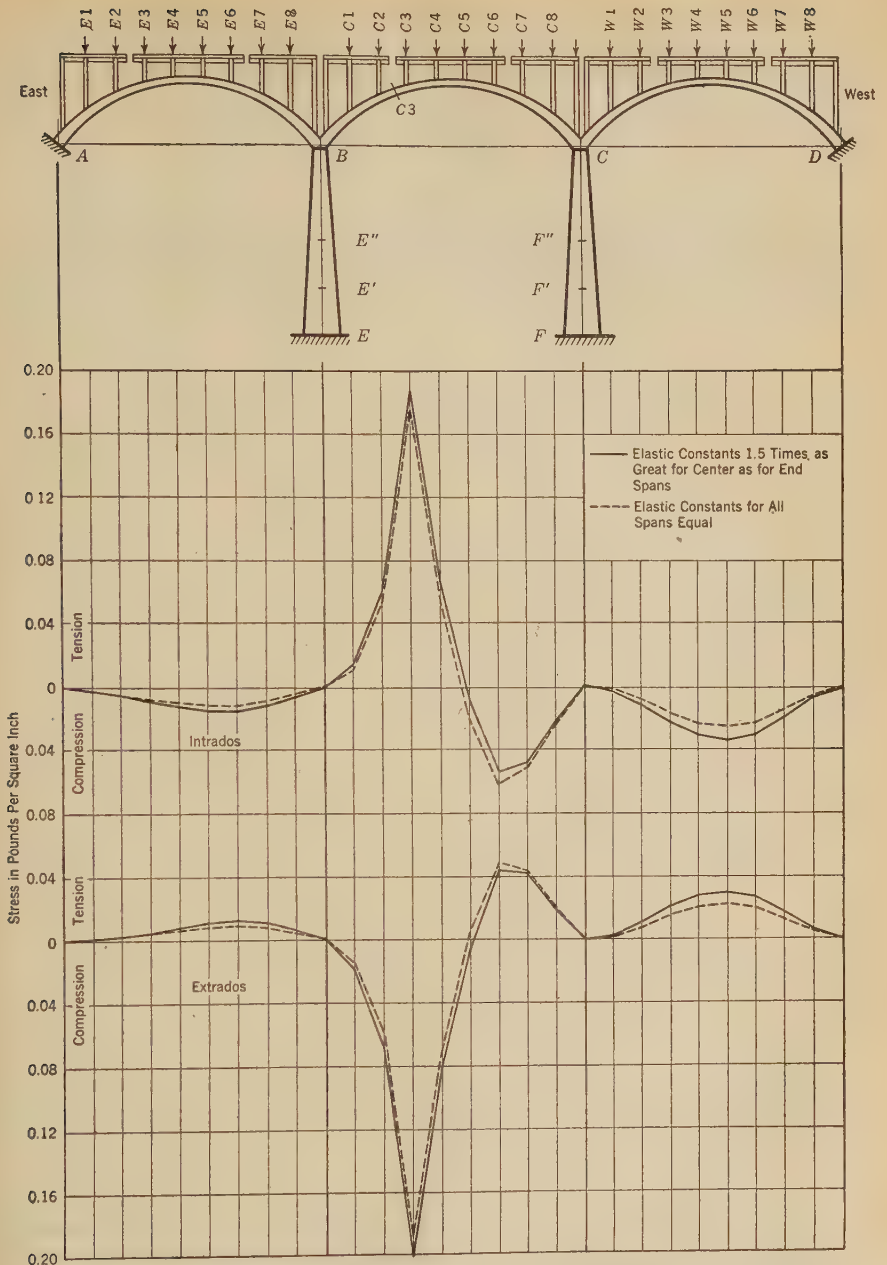


Fig. 48.—Influence Lines for Stress at *B* of *BC* from Algebraic Analysis Based on Assumed Elastic Constants. High Deck without Intermediate Expansion Joints. Expansion Joints, 20-Foot Piers.

same as that used for the tests of ribs without deck, described in Chapter IV. The live load was distributed so as to produce maximum stress at End *B* of Span *BC* for the structure without intermediate expansion joints and maximum stress at C_3 for the structure with intermediate expansion joints. Design load tests were made for the structure with high deck only.

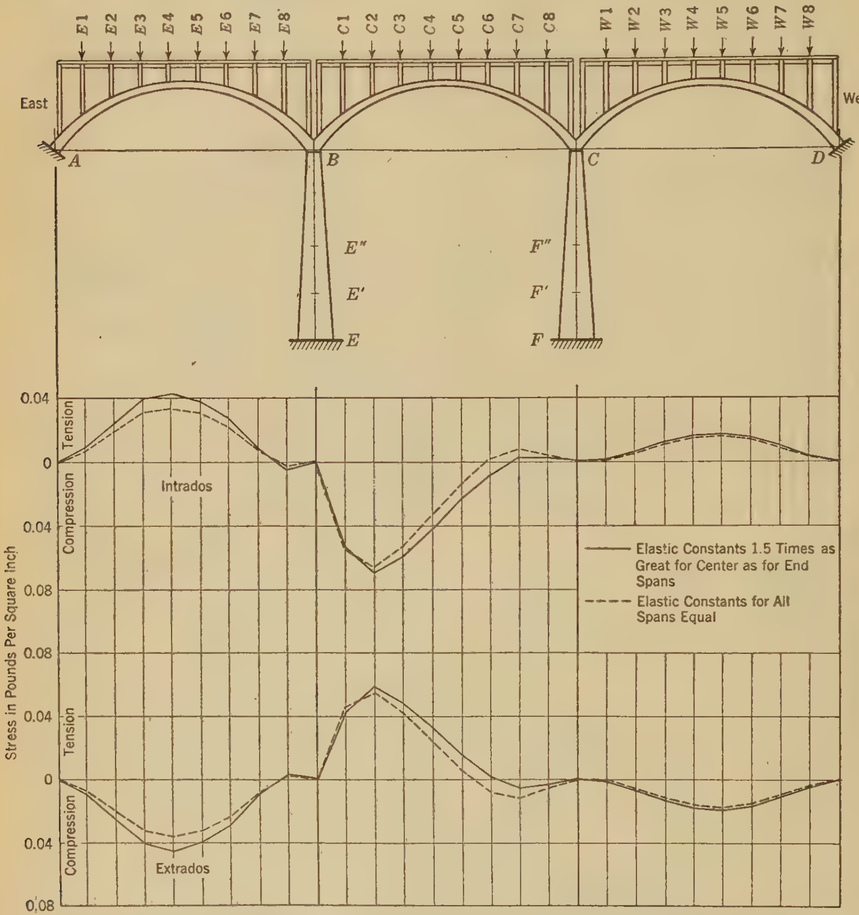


FIG. 49—INFLUENCE LINES FOR STRESS AT C_3 FROM ALGEBRAIC ANALYSIS BASED ON ASSUMED ELASTIC CONSTANTS. HIGH DECK WITH INTERMEDIATE EXPANSION JOINTS, 20-FOOT PIERS.

The capacity tests were made by applying the dead load and a live load (the latter in multiples of one live load) until the capacity of the structure had been reached. The readings taken were the same as those for the design load tests. The height of the piers for these tests was 20 ft.

Before the capacity tests were made the intermediate expansion joints in the deck were closed by connecting the reinforcing steel across the joint by means of sleeve-nuts and filling the joint with a rich sand-cement mortar.

None of the joints cracked during the tests, indicating that they were capable of resisting shear and tension as well as compression.

Two tests were made on each of the structures without intermediate expansion joints. For the first test the live load was distributed so as to give a maximum stress at End *B* of Span *BC*, and for the second it was distributed so as to give a maximum stress at C_4. Readings were taken when the structure carried no load, when it carried design dead load, dead load plus one live load, dead load plus three live loads, and dead load plus five live loads. Each of the tests was stopped when five live loads had been reached, as a number of small cracks had appeared in the spandrel columns, and it was desired to save the structure to be tested to failure as a structure with intermediate expansion joints.

After the tests just described had been completed the intermediate expansion joints in the deck were again opened, and the resulting structure was loaded to failure, the live load being distributed so as to produce failure by compression at the extrados at C_3,

For the structure with a high deck, a complete set of readings was taken when it carried no load, design load, dead load plus one live load, dead load plus two live loads, dead load plus four live loads, and dead load plus five live loads. Six live loads were added to the dead load and the terminals were returned to their normal positions. The rib was badly cracked at the intrados, and the extrados was spalled at the east edge of the spandrel column at C_3, but the structure carried the load for 15 or 20 min. It is possible that if this load had remained constant the structure would eventually have failed. Instead, additional loads were applied at C_3 and failure occurred after 700 lb. had been added.

For the structure having a low deck with intermediate expansion joints, a complete set of readings was taken when the structure carried no load, design dead load, dead load plus one live load, dead load plus three live loads, and dead load plus five live loads. The maximum load applied consisted of the dead load plus six live loads plus 960 lb at C_3.

The thrust lines and the trapezoids of strain for the structures with intermediate expansion joints are shown in Fig. 50, and those for the structure without intermediate expansion joints are shown in Fig. 51.

The results of the capacity tests are very re-assuring. Both structures without intermediate expansion joints carried five loads in addition to the dead load without signs of impending failure. The structure having ribs without deck carried four and one-half live loads, and both structures having a deck with intermediate expansion joints carried a little more than six live loads in addition to the dead load. This was true even when the computed stresses due to design load, given in Table 24 (supported by Table 24(*A*)), are higher than specifications usually allow.

The compressive stress at failure for the three-span structure without deck was 3 800 lb per sq in. which is comparable with the strength of the concrete in the control cylinders of 3 310 lb per sq in. The compressive stress at failure for the structure having a high deck with intermediate

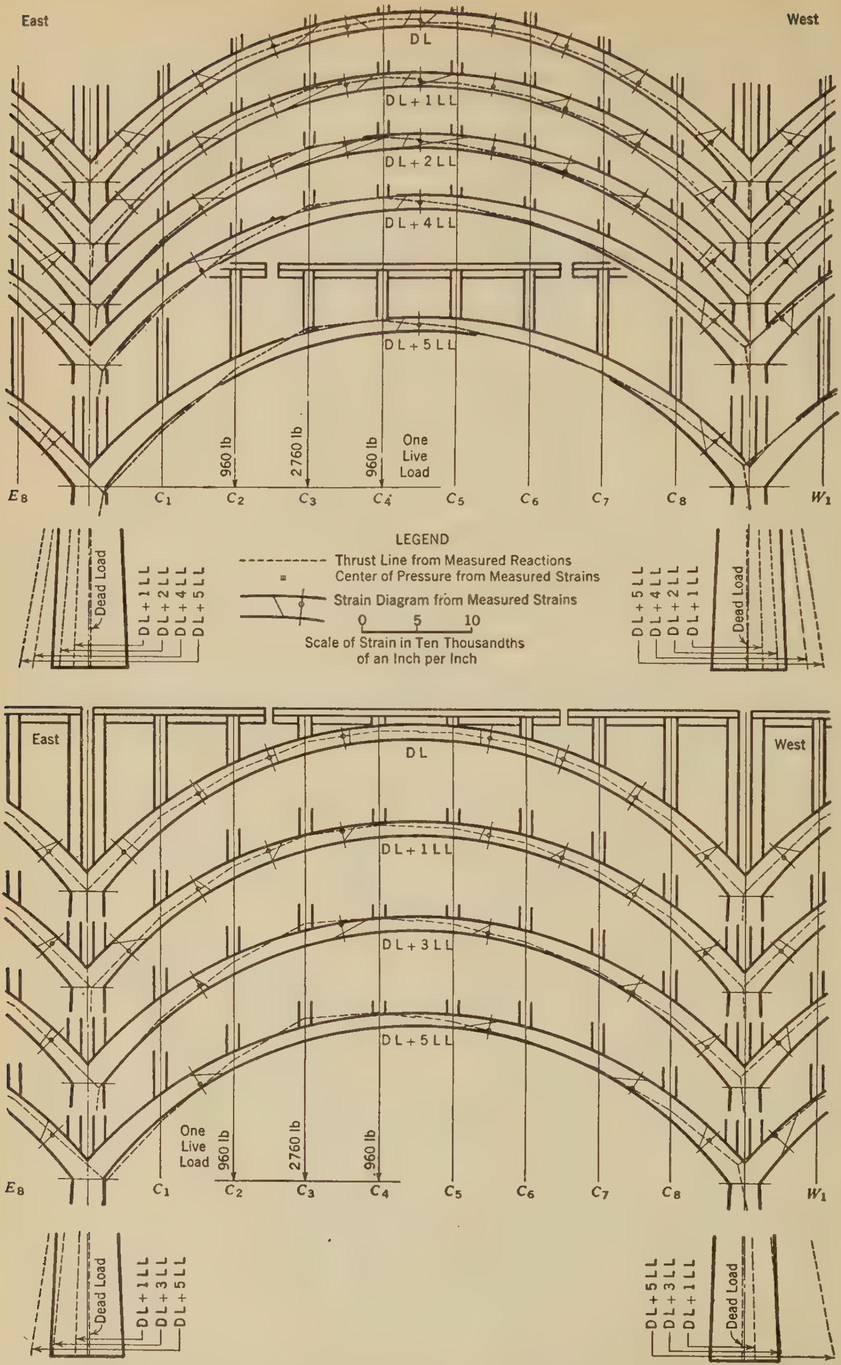

East
West
DL
DL + 1 LL
DL + 2 LL
DL + 4 LL
DL + 5 LL
960 lb
2760 lb
960 lb
One Live Load
E_8
C_1
C_2
C_3
C_4
C_5
C_6
C_7
C_8
W_1
Dead Load
LEGEND
Thrust Line from Measured Reactions
Center of Pressure from Measured Strains
Strain Diagram from Measured Strains
0 5 10
Scale of Strain in Ten Thousandths of an Inch per Inch
DL + 3 LL

Fig. 50—Thrust Lines for Capacity Load; Maximum Stress at C_3; with Intermediate Expansion Joints.

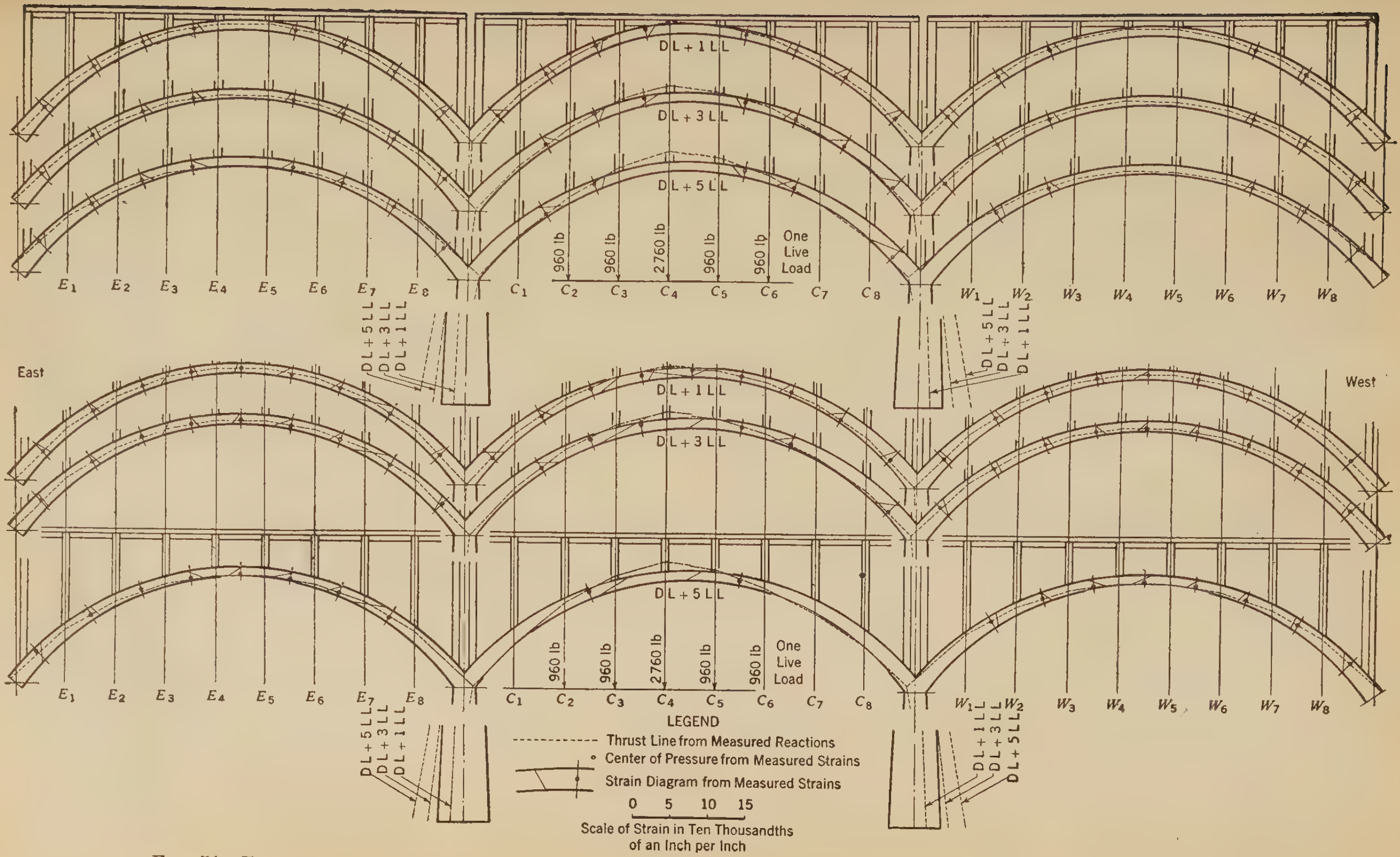

DL + 1 LL
DL + 3 LL
DL + 5 LL
960 lb
960 lb
2760 lb
960 lb
960 lb
One Live Load
E_1 E_2 E_3 E_4 E_5 E_6 E_7 E_8
C_1 C_2 C_3 C_4 C_5 C_6 C_7 C_8
W_1 W_2 W_3 W_4 W_5 W_6 W_7 W_8
East
West
LEGEND
Thrust Line from Measured Reactions
Center of Pressure from Measured Strains
Strain Diagram from Measured Strains
0 5 10 15
Scale of Strain in Ten Thousandths of an Inch per Inch

FIG. 51—THRUST LINES FOR CAPACITY LOAD; MAXIMUM STRESS AT C_4; WITHOUT INTERMEDIATE EXPANSION JOINTS.

expansion joints, based on measured reactions, was 3 200 lb per sq in., which is comparable with 3 421 lb per sq in. in the control cylinders. The compressive stress at failure for the structure having a low deck with intermediate expansion joints was 4 400 lb per sq in., which is comparable with 4 281 lb per sq in., as determined by the control cylinders.

TABLE 24.—MAXIMUM STRESSES FOR VARIOUS STRUCTURES DUE TO DESIGN LOAD

Structure (1)	Pier height, in feet (2)	Critical section (3)	Live* Load No. (4)	STRESS, IN POUNDS PER SQUARE INCH					
				Dead Load		Live Load		Dead Load + Live Load	
				Intrados (5)	Extrados (6)	Intrados (7)	Extrados (8)	Intrados (9)	Extrados (10)
Rib without deck	20	C_4	1	—142	—500	......	—736	......	—1 236
	15	C_4	2	—142	—500	......	—701	......	—1 201
	10	C_4	3	—142	—500	......	—665	......	—1 155
	Fixed end	3	4	— 92	—546	......	—543	......	—1 089
HIGH DECK									
Without expansion joints	20	*B* of Span *BC*	5	—287	—193	—334	......	—621	....
	15	*B* of Span *BC*	6	—251	—231	—310	......	—561	....
	10	*B* of Span *BC*	7	—192	—290	—276	......	—468	....
	Fixed end	East springing	8	—128	—342	......	—262	......	—604
With expansion joints	20	C_3	9	+152	—782	......	—665	......	—1 447
	10	C_3	10	+177	—809	......	—584	......	—1 393
	Fixed end	3	11	— 92	—546	......	—520	......	—1 066
LOW DECK									
Without expansion joints	20	*B* of Span *BC*	12	—137	—342	......	......	—480	....
	Fixed end	East springing	13	—128	—342	......	—264	......	—606
With expansion joints	20	C_3	14	+ 14	—610	......	—732	......	—1 342
	Fixed end	3	15	— 92	—546	......	—559	......	—1 105

* See Table 24(*A*) for description of these loads.

TABLE 24(*A*).—DISTRIBUTION OF LIVE LOAD FOR MAXIMUM STRESS

Live Load No.	SECTIONS							
	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
1	368	953	960	2 760	938	296	0	0
2	300	939	960	2 760	919	240	0	0
3	198	898	960	2 760	881	169	0	0
4	960	960	2 760	647	18	0	0	0
5	960	2 760	960	960	956	401	0	0
6	960	2 760	960	960	880	169	0	0
7	960	2 760	960	960	474	0	0	0
8	0	0	363	952	960	2 760	960	960
9	960	960	2 760	959	433	0	0	0
10	960	960	2 760	917	236	0	0	0
11	960	960	2 760	868	152	0	0	0
12	960	2 760	960	960	960	703	34	0
13	0	0	376	954	960	2 760	960	960
14	960	960	2 760	960	713	39	0	0
15	960	960	2 760	941	308	0	0	0

Article 7.—Conclusions.—Only a small part of the data pertaining to these tests have been given in this summary. However, all the data have been analyzed and the analysis appears to justify the following conclusions:

1.—The influence ordinates for fixed-end reactions are quite different for a structure having a deck than for one consisting of ribs without a deck. The effect of the deck is to reduce the moment at the springing, where it is all resisted by the rib, and to increase the moment over the middle of the span where the deck acts with the rib.

2.—The reactions at the springings of the arches can be determined by an algebraic method, using experimentally determined, fixed-end reactions and elastic constants for the arch spans. The constants may be obtained from tests of single-span concrete structures, or from elastic models. A considerable variation in the elastic constants will not seriously affect the resulting stresses, if the constants are statically consistent, but the fixed-end reactions must be determined accurately if the analysis is used to determine dead load stresses. Usually in such structures, the process of construction is such that all, or nearly all, the dead load is carried by the rib with little assistance from the spandrel columns and deck.

3.—A deck without intermediate expansion joints increases the stiffness and moment-carrying capacity of the central part of the span. Intermediate expansion joints reduce both these effects.

4.—There is considerable evidence indicating that the dead load stresses in a multiple-span structure may exceed the corresponding stresses in a similar single span having fixed ends.

5.—The concrete in the arches developed approximately the same ultimate stress as the same concrete in 6 by 12-in. control cylinders.

Acknowledgments.—The work described in this Chapter was done under the supervision of Professor Wilson (7) (9).

VI.—The Effects of Climatic Changes

Article 1.—Object of Investigation.—Variations in temperature and in the moisture content cause concrete to change in volume and to produce changes in the shape of the arch rib that are accompanied by direct and bending stresses in the rib, and by overturning thrusts upon the piers. The object of this investigation was to observe the changes that took place in a multiple-span arch bridge in order that their magnitude might be determined, that the measured and computed changes might be compared, and that the effects of these changes upon the stresses in the structure might be determined.

Article 2.—Description of the Structure.—The bridge on which the observations were made is a six-span, two-rib, highway bridge over the Vermillion River, at Gilbert Street, in Danville, Ill. The general dimensions are shown in Fig. 52. The bridge is of the open spandrel type except that for the middle three panels of each span, the rib is connected to the deck by a spandrel wall. As shown in Fig. 52, there are expansion joints in the deck over the piers and at the extremities of this saddle over the crown, of Spans Nos. 2, 3, 4, and 5, and over the piers only for Spans Nos. 1 and 6. The piers rest upon a good grade of shale.

Article 3.—Scope of Observations.—The observations included the rotation of the piers and the change in width of the expansion joints of all six spans. The west rib of Span No. 2 was selected for more extensive study, and on this rib observations were made for changes in temperature of the concrete,

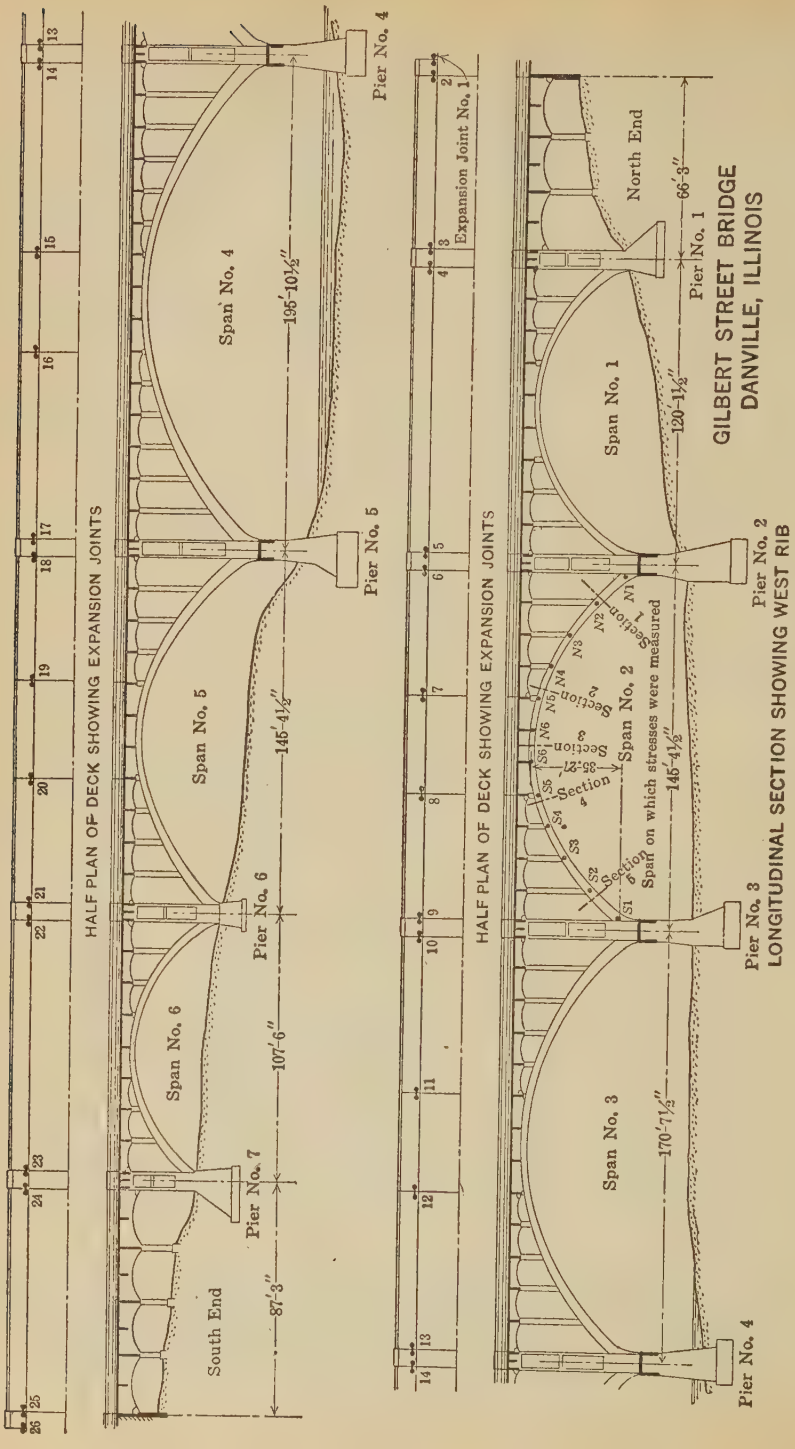

HALF PLAN OF DECK SHOWING EXPANSION JOINTS
13
14
15
16
17
18
19
20
21
22
23
24
25
26
Pier No. 4
Span No. 4
195′-10½″
Pier No. 5
Span No. 5
145′-4½″
Pier No. 6
Span No. 6
107′-6″
Pier No. 7
South End
87′-3″
HALF PLAN OF DECK SHOWING EXPANSION JOINTS
Expansion Joint No. 1
1
2
3
4
5
6
7
8
9
10
11
12
13
14
North End
66′-3″
Pier No. 1
Span No. 1
120′-1½″
Pier No. 2
N1
N2
N3
N4
N5
N6
S6
S5
S4
S3
S2
S1
Section 1
Section 2
Section 3
Section 4
Section 5
35.27
Span No. 2
Span on which stresses were measured
145′-4½″
Pier No. 3
Span No. 3
170′-7½″
Pier No. 4
LONGITUDINAL SECTION SHOWING WEST RIB
GILBERT STREET BRIDGE
DANVILLE, ILLINOIS

FIG. 52.

rise and fall of points on the arch rib, rotation of points on the arch rib, the transverse deformation of the concrete, and the longitudinal deformation of the steel and the concrete. Observations were made on eighteen different days distributed over a period of twenty months, including at least one extremely hot day each summer and one extremely cold day each winter. Each rib of Span No. 2 was 5 ft wide and varied in thickness from 27 in. at the crown to 48 in. at the springing.

Article 4.—Temperature Range.—The temperature of the concrete of the west rib of Span No. 2 was measured with mercury thermometers inserted in temperature wells. A well consisted of a hole in the concrete about 1 in. in diameter, in which two corks were inserted, one about 1 in. from the bottom and the other at the top of the hole. The part of the well below the lower cork was filled with cup grease. There was a hole at the center of each cork just large enough so that a thermometer forced into it would be held firmly in place. When a thermometer was in a well its bulb was in contact with the cup grease, a good conductor of heat that had already attained the temperature of the surrounding concrete. Cup grease was used in preference to a liquid because it will stay in place in a well on the under side of the rib. The dead stems of all the thermometers were so long that the thermometers could be read without being removed from the wells.

The temperature was observed at mid-depth of the rib and at the level of the reinforcing steel, a depth of approximately 3 in. The temperature was taken at mid-depth just below $N2$ (Fig. 52), where the thickness is about 40 in. and just below $N4$, where it is about 30 in.

The thermometer wells for obtaining the temperature adjacent to the reinforcing steel were placed at the middle of the strain-gauge lines for one of the inside and for each of the two outside bars at Sections 1, 2, 4, and 5, and for each of the two outside bars only at Section 3. These wells were located in both the top and the bottom of the rib. Wells for obtaining the temperature at a depth of 3 in. from the vertical surfaces were located on the east and west faces of the rib at Sections 1, 2, 4, and 5. The temperature at a given section was taken just before or just after the strain-gauge had been read at that section. The relation between the temperatures of the concrete and that of the air is given in Fig. 53. The temperature of the air is the mean of the maximum and minimum temperature for each day, as reported by the United States Weather Bureau Station, in Danville, situated about a mile from the bridge.

The record indicates that, except for a few points, the range in temperature in the 3-in. wells for any one day was quite small. The maximum range was 12.7° F. and usually it did not exceed 8 or 9 degrees. This range was due, not only to different degrees of shelter, but also to the fact that a period of from 4 to 6 hr would elapse between the first and the last readings.

The temperature of the concrete at the center of a section, following a sudden change in the temperature of the air, varied somewhat with the thickness of the rib. The greatest difference between the average temperature at

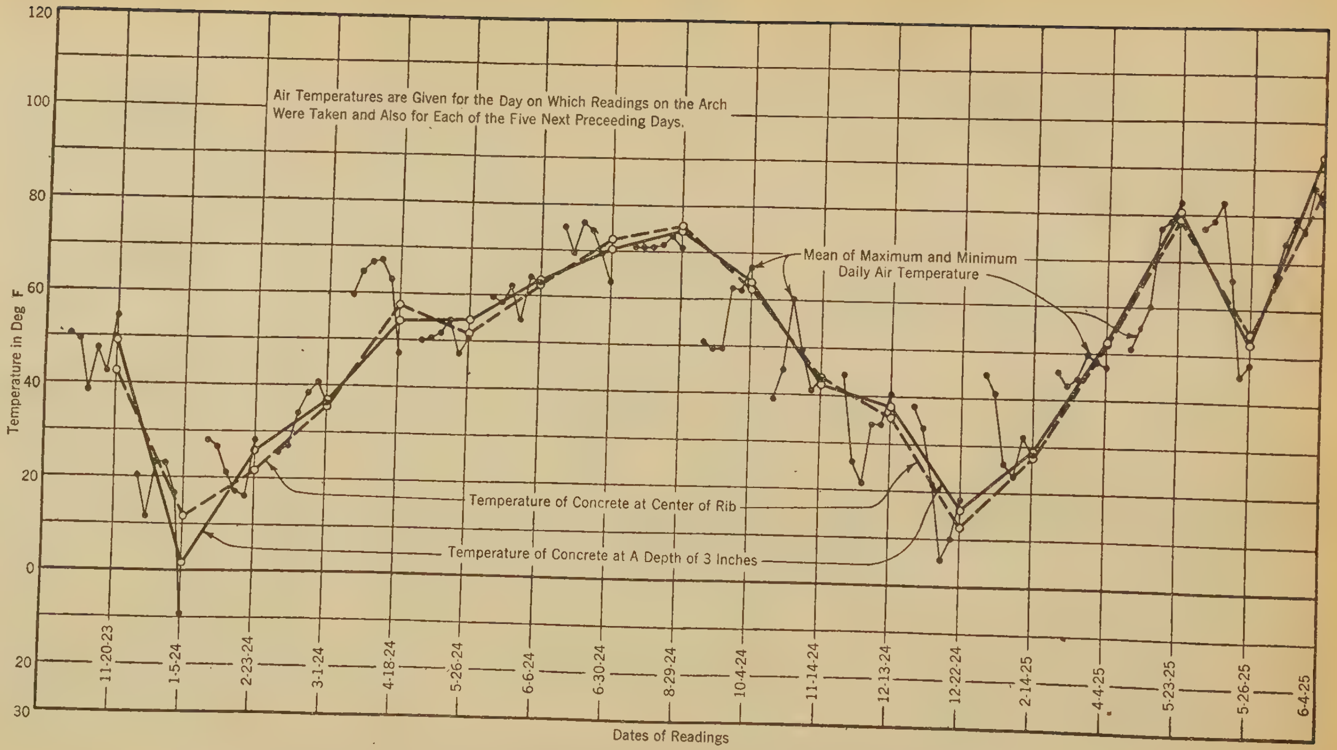


FIG. 53. RELATION OF TEMPERATURE OF AIR TO TEMPERATURE OF CONCRETE

the center of the rib and that 3 in. from the surface, occurred on January 5, 1924, just after a sudden drop in air temperature, the difference for that date being 10.2° F.

Using the mean between the average temperature at the center of the rib and at points 3 in. from the surface, respectively, as the mean temperature of the arch rib, the lowest temperature observed was 6° and the highest, 89° F., giving an extreme range of 83° F. A study of the range of air temperatures preceding the dates when readings were taken, indicates that the extreme range of concrete temperatures during the 20-month period was probably about 90 degrees.

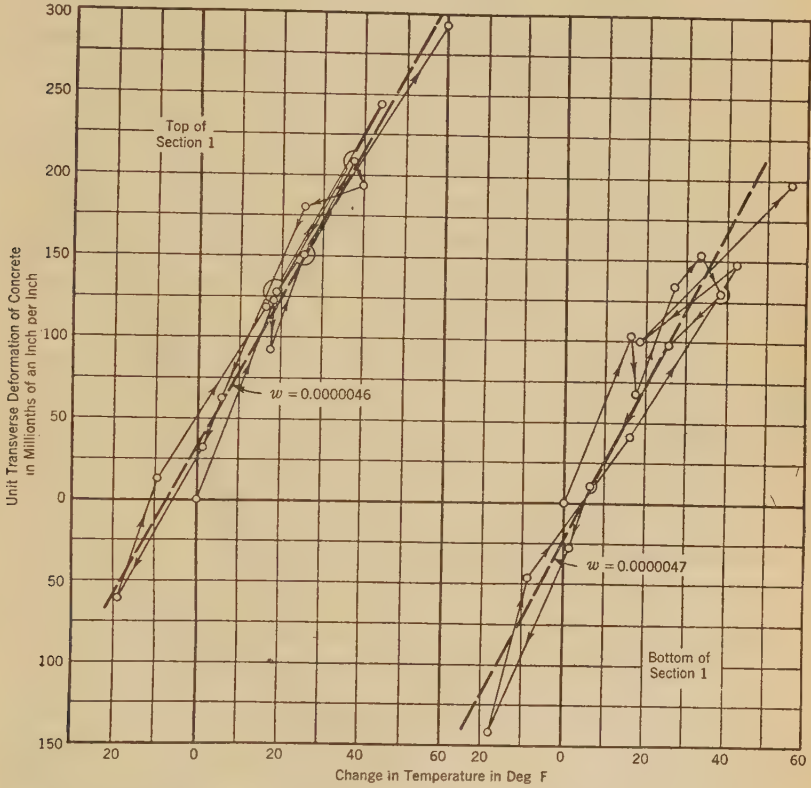


FIG. 54. RELATION BETWEEN VARIATION IN TEMPERATURE AND TRANSVERSE DEFORMATION OF CONCRETE, SECTION 1

Article 5.—Thermal Coefficient from Transverse Deformations.—The volume of concrete is affected by stress, by changes in temperature, and by changes in moisture content. Deformation readings on the concrete were taken transverse to the axis of the rib, a direction in which there is no primary stress. These readings should give the changes in dimension due to temperature and moisture changes. The curves of Figs. 54 to 58, inclusive, show the relation between the deformation and the change in temperature of the rib.

The observations on this bridge indicate that the transverse deformation of the concrete does not follow exactly the temperature changes, but that the average for all points does, indicating that the transverse deformation other than thermal is erratic. The average value of the thermal coefficient of this concrete, which was about three years old at the time the observations were made, is 0.0000049.

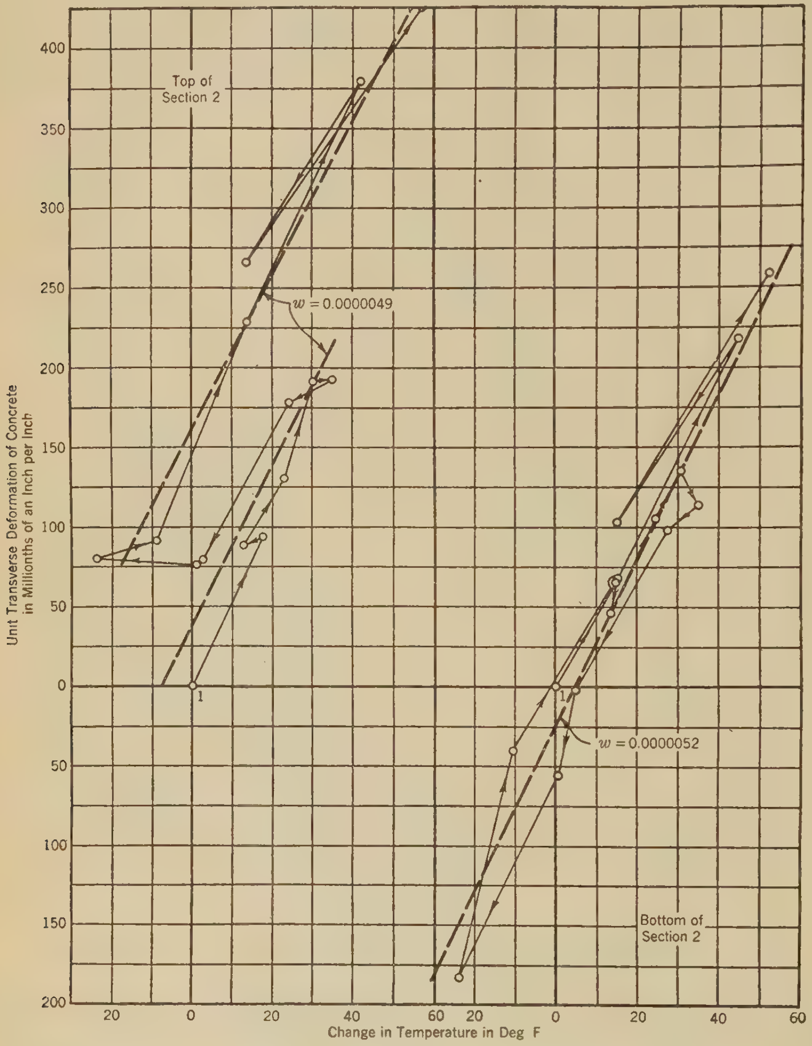


FIG. 55. RELATION BETWEEN VARIATION IN TEMPERATURE AND TRANSVERSE DEFORMATION OF CONCRETE, SECTION 2

Article 6.—Rise and Fall of the Rib.—The rise and fall of points on the west rib of Span No. 2 were measured at the twelve points, *N*1 to *S*1, indicated on Fig. 52. The relation between the rise and fall of the crown and the change in temperature of the concrete is shown in Fig. 59.

The relation between the temperature and the vertical movement of the crown was very constant, except for the period from November 14 to December 13, 1924. During this period, although the temperature of the arch fell 6.3°, the crown rose a little more than 0.04 in., and it retained this increment in its elevation throughout the remaining period of the observations.

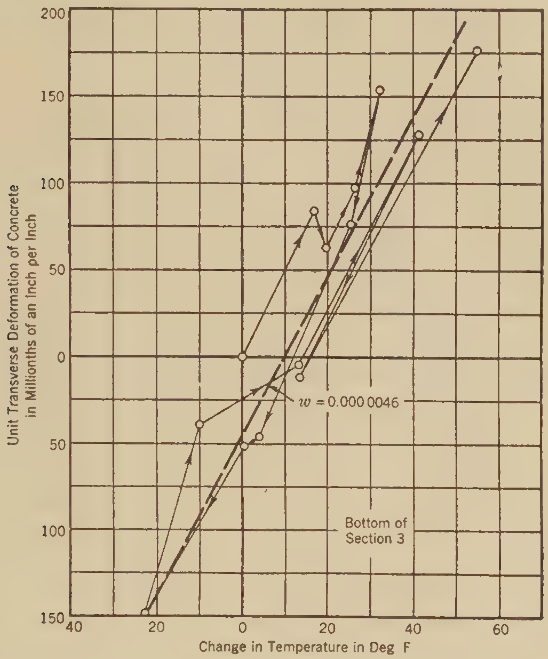


FIG. 56. RELATION BETWEEN VARIATION IN TEMPERATURE AND TRANSVERSE DEFORMATION OF CONCRETE, BOTTOM OF SECTION 3

The question naturally arises as to whether the "jump" shown in Fig. 59 was due to an actual change in the position of the crown or to an error in the observations. The latter could be occasioned in two ways: (1) By a change in the tape; or (2) by an error in individual readings. The only part of the tape that would affect the elevation of the crown relative to the springing is that which is used in taking readings at *N*6 and *S*6, and that which is not used at *N*1. This eliminates the end of the tape and the ring. The only alteration in the body of the tape that would affect the readings would be a break and an inaccurate splice. No breaks occurred, and as the same tape was used for all readings the possibility of alterations in the tape affecting the readings seems to be eliminated. Relative to errors in the individual readings, although mistakes do occur, the probability of the same error occurring for both the original and the check readings on eight successive days is extremely remote. Undoubtedly, Fig. 59 represents the actual movement of the crown.

Article 7.—Stresses Due to Thermal Changes.—Longitudinal deformations of the steel and of the concrete were measured at the sections (1 to 5) shown on Fig. 52. Measurements were taken on both the intrados and extrados of the rib. These deformations are in part due to temperature changes and in part to stresses in the rib induced by such changes. The part due to temperature change may be taken as equal to the transverse deformations discussed in Article 5. These are assumed to be free from stress influence, the Poisson ratio effect being disregarded. The difference between the longitudinal and transverse deformations in the concrete, then, may be taken as the measured deformation due to temperature stress.

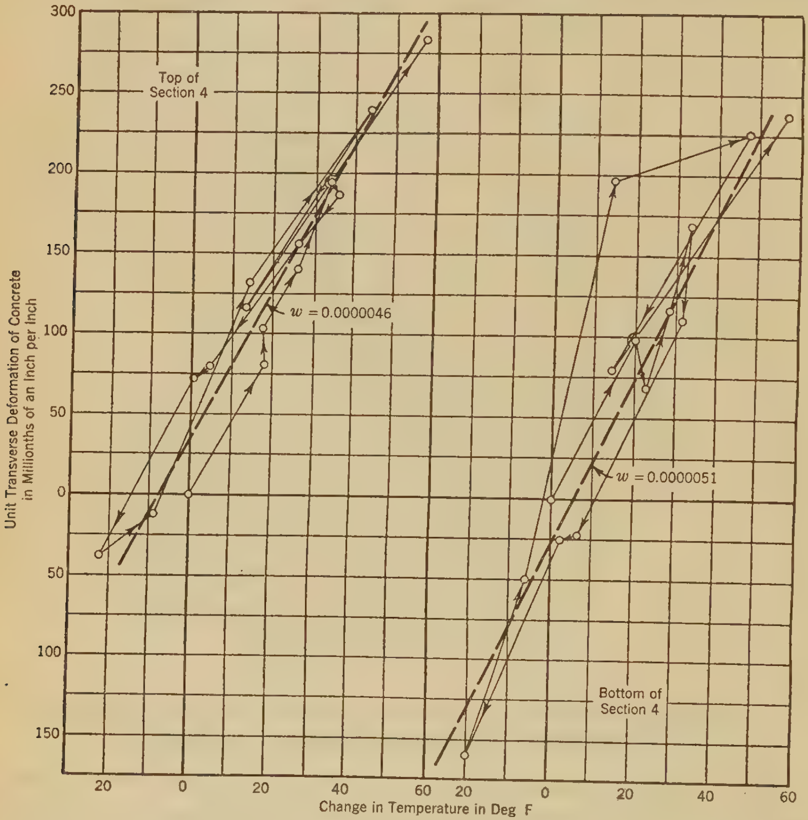


FIG. 57. RELATION BETWEEN VARIATION IN TEMPERATURE AND TRANSVERSE DEFORMATION OF CONCRETE, SECTION 4

These measured deformations compared with the computed values are shown in the curves of Figs. 60, 61, and 62. These curves show that the measured deformation of the concrete, due to stress, is greatest at Sections 2 and 4, in the panels below the expansion joints in the deck. Based on a value of $E = 2\,000\,000$ lb per sq in., the unit stress in the concrete would be not less than 340 lb per sq in. and might have reached 480 lb per sq in. at these sections. The maximum range in the temperature stress for the steel was about 6 000 lb per sq in. The measured stress deformation diagrams, for both the steel and the concrete, are very similar for corresponding sections on opposite sides of the center.

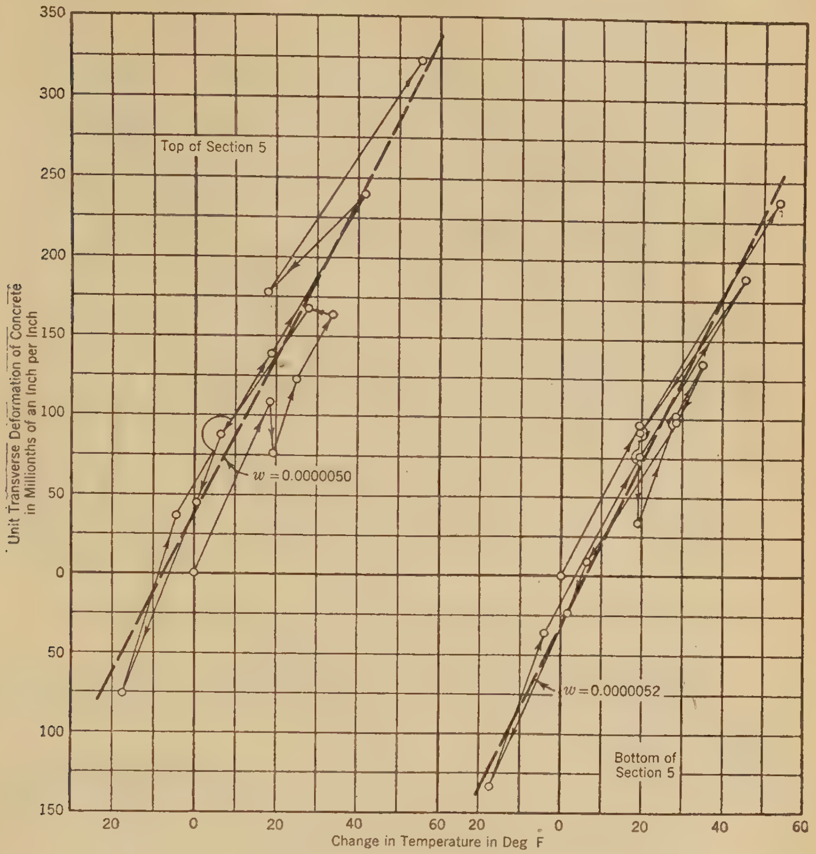


FIG. 58. RELATION BETWEEN VARIATION IN TEMPERATURE AND TRANSVERSE DEFORMATION OF CONCRETE, SECTION 5

The computed values of the stresses are based on the assumption that the rib is unrestrained by the superstructure. A study of the measured and computed values leads to the following conclusions:

(1) In general, the measured and the computed values of the deformation were consistent in sense, the two values for the same quantity increasing and decreasing together as the temperature changed.

(2) The measured value and the computed value of the stress in the steel agreed in magnitude remarkably well during the last six months of the test; the measured value exceeded the computed value during the first year of the test by as much as 100% during the summer of 1924. The greatest discrepancy occurred at the bottom of Sections 2 and 4, the sections that might be affected by the expansion joints.

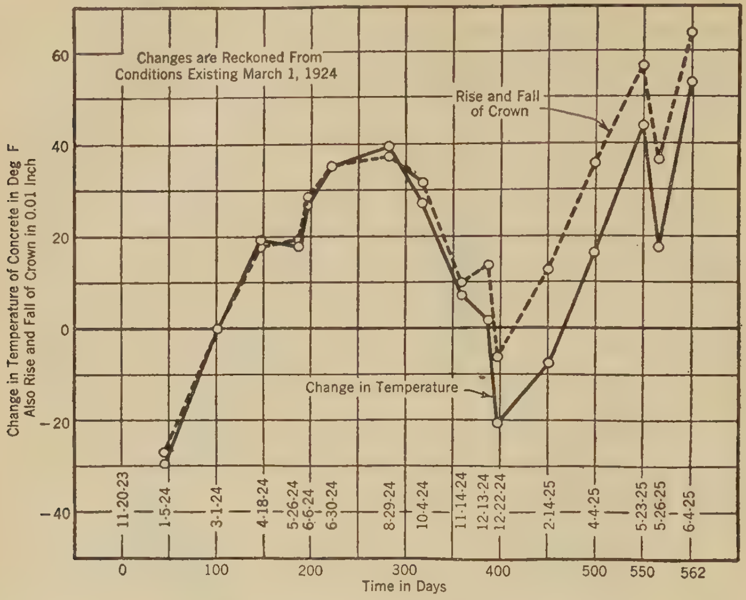


FIG. 59. RELATION BETWEEN RISE AND FALL OF CROWN AND CHANGE IN TEMPERATURE OF CONCRETE

(3) The measured and the computed values of the stress deformation in the concrete had approximately the same magnitude at the bottom of Sections 1 and 5, and at the top and bottom of Section 3; the measured value far exceeded the computed value at the top of Sections 1 and 5, and at the top and bottom of Sections 2 and 4.

(4) The temperature stress in the steel, due to the difference between the thermal coefficients of steel and concrete, exceeded that due to the flexural stress resulting from the temperature thrust.

(5) Some influence, presumably the expansion joints, accentuated the temperature stress in the rib adjacent to the saddle.

Article 8.—Expansion Joints.—The change in width of the opening at expansion joints was obtained by measuring the distance between center-punch marks on the ends of steel pins inserted, one on each side of a joint, in the vertical face of the west curb of the roadway. Duplicate readings were taken, and the two readings for a given point invariably checked within 0.01 in. The location of the expansion joints is given on Fig. 52, and their change in width is shown on Figs. 63 and 64.

The most interesting feature in connection with the movement at the expansion joints is the fact that the joints at the saddle did not change appreciably in width. An analysis of the motions which would accompany a rise in temperature shows that the increase in length of the deck would tend to close the joint, whereas the rotation of the bases of the columns which would be caused by the increase in rise of the arch rib would tend to open the joint. These two effects operated to cancel each other, resulting in no appreciable movement of these joints.

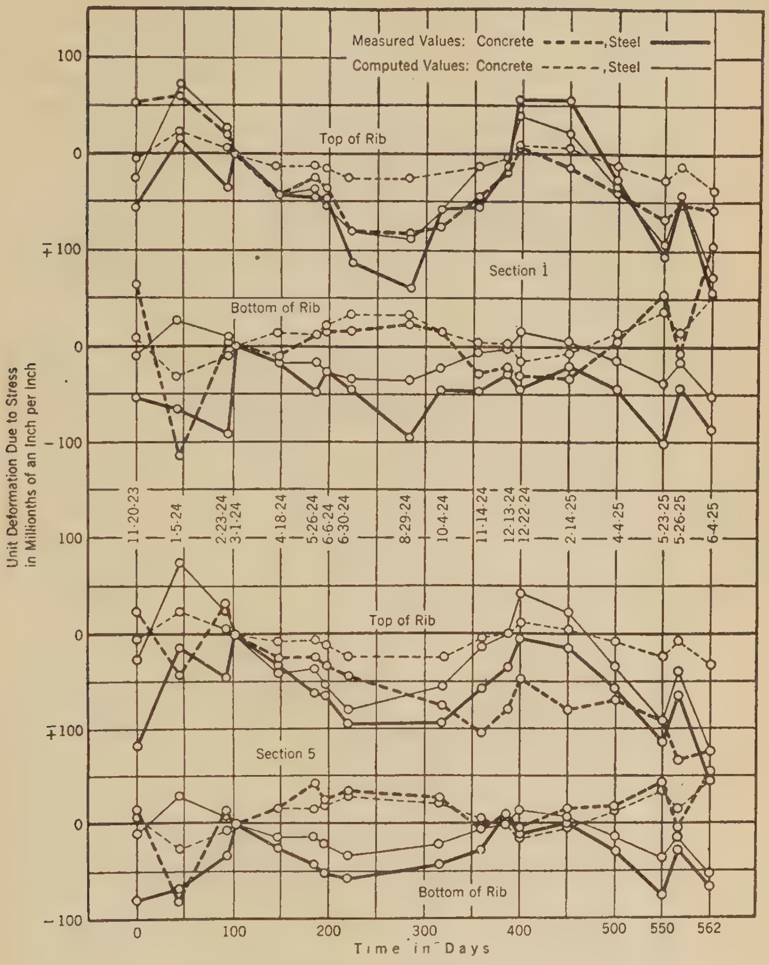


Fig. 60. Stress Deformation, Sections 1 and 5

The movements at all the joints from the city pavement on one end of the bridge to that on the other end were added and their sum was plotted against temperatures. The resulting diagram is shown in Fig. 65. The temperature of the rib at a depth of 3 in. was taken as the temperature of the deck, since no readings were taken of the latter. The maximum range in the sum of the movements is 5.37 in. This expansion and contraction occurred in a total length of bridge of 1 046 ft. This gives a thermal coefficient for the deck of 0.0000056 compared with 0.0000049 for the arch rib.

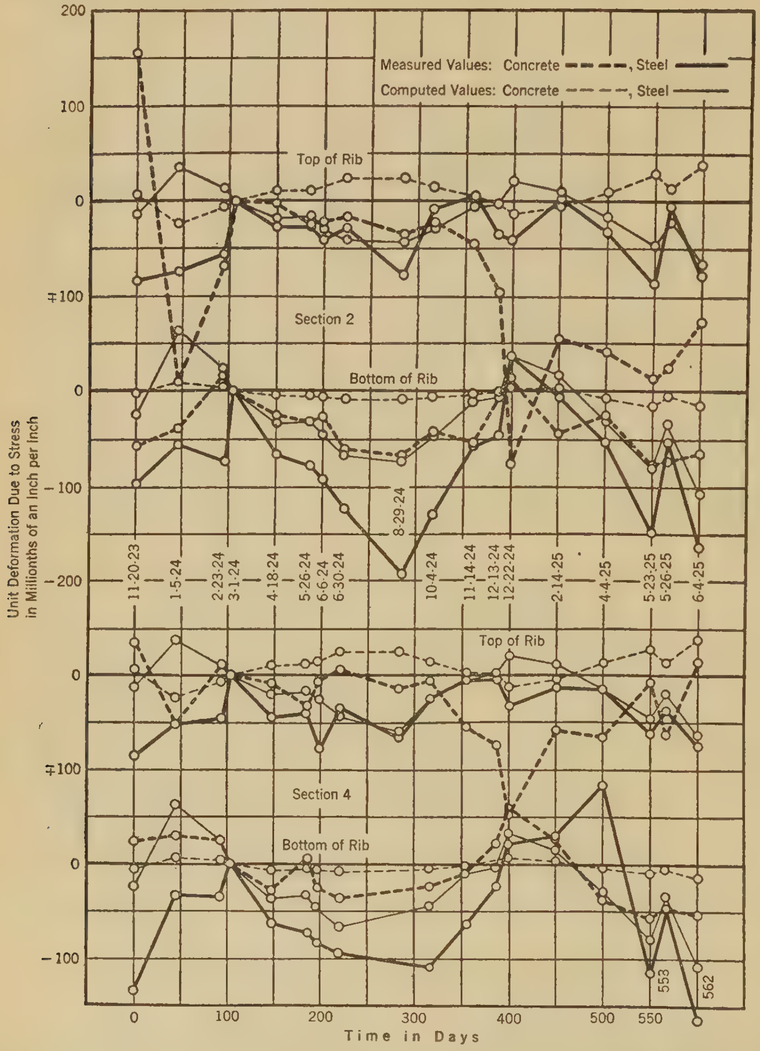


FIG. 61. STRESS DEFORMATION, SECTIONS 2 AND 4

Article 9.—Rotation of Piers.—The rotation of the piers was measured with a level-bar containing a 10″ bubble which, with careful manipulation, should show the angular position of a body to within 0.00001 radian, or about 2″ of arc.

The rotation of the piers was observed near the bottom, at the surface of the ground, and near the top, at the springings of the arches for Piers 2, 3, and 5, and at the springings only for Piers 1, 4, and 6. These observations are shown graphically in Fig. 66.

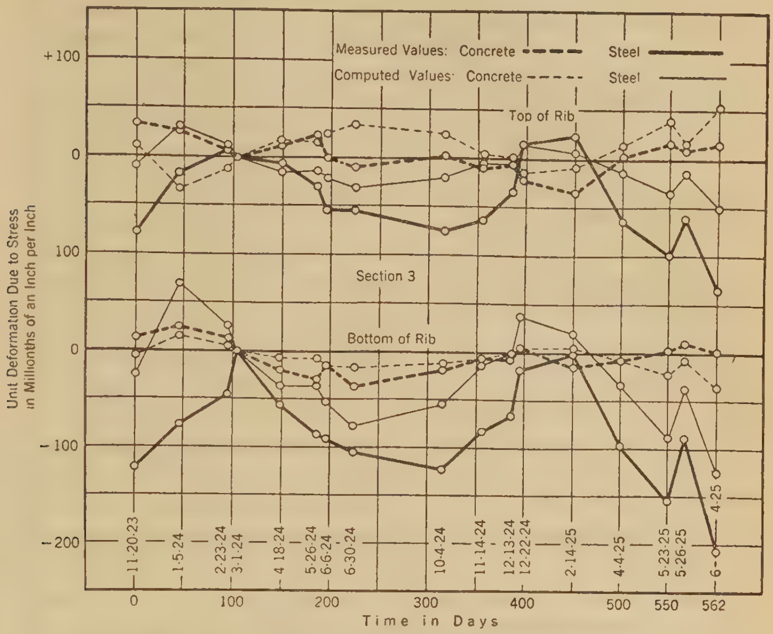


Fig. 62. Stress Deformation, Section 3

The most remarkable feature in connection with the rotation of the piers is the fact that, with a rise in temperature, the piers north of the longest span tipped south and those south of that span tipped north; that is, the piers tipped toward the long span with a rise in temperature.

The magnitude and position of the horizontal thrust due to a change in temperature of 1°, based upon the elastic theory, and neglecting the influence of the spandrel columns and deck, are given in Table 25. This shows that, as the temperature rises, the long span (No. 4), should tip Piers 4 and 5 outward, as its thrust is greater than that of the adjacent spans. Instead, Piers 2, 3, 4, and 5 have tipped in a direction opposite to that in which they should have tipped, if they were controlled by the thrusts of Table 25.

The condition of the expansion joints over the piers is given in Table 26. A study of Table 26 in connection with Figs. 63 and 64, shows that, in general, the expansion joints on the sides of the piers away from the long span were filled tightly, and that, consequently, their motion accompanying a rise in temperature was restricted. The tops of the piers at the roadway level were forced by the expansion of the deck away from the longer spans toward the shorter spans, and this motion agrees with the pier rotation shown in Fig. 66.

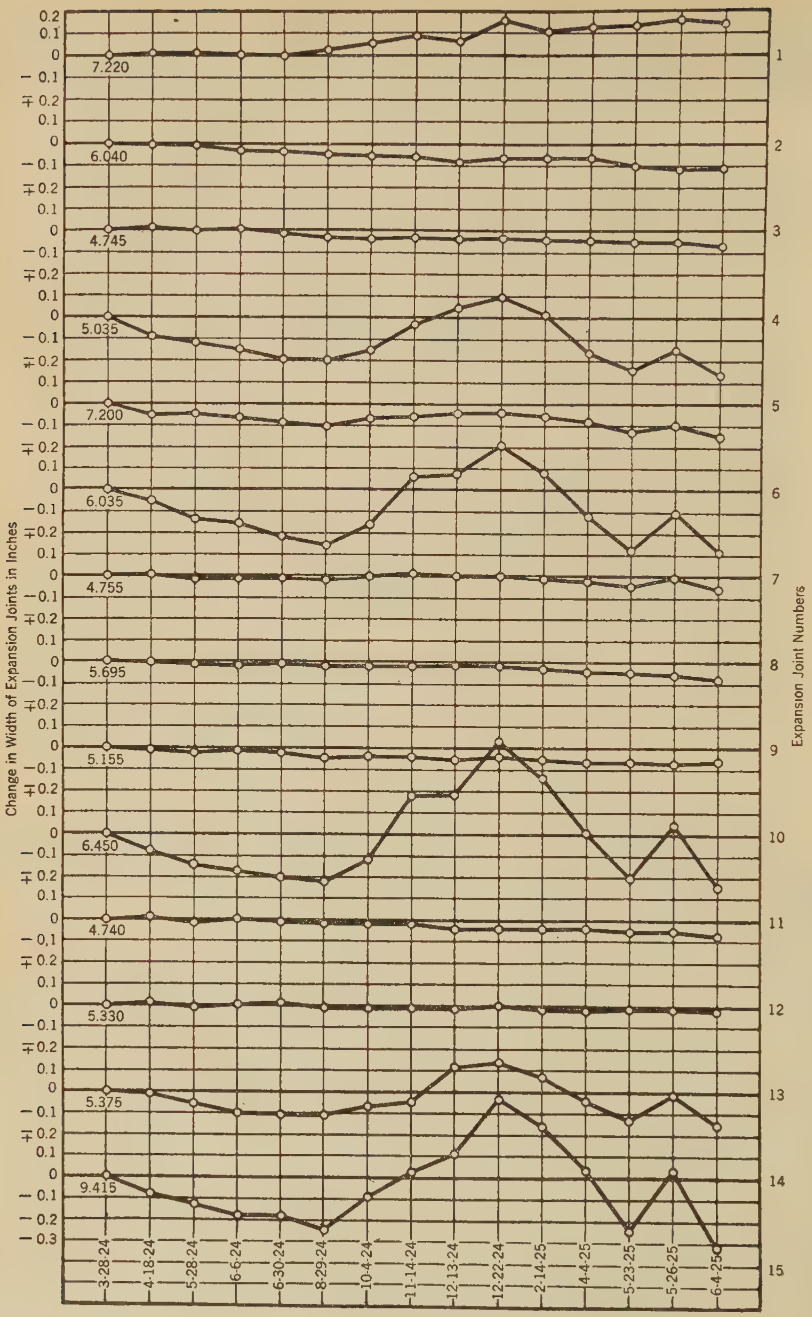


FIG. 63. CHANGE IN WIDTH OF EXPANSION JOINTS

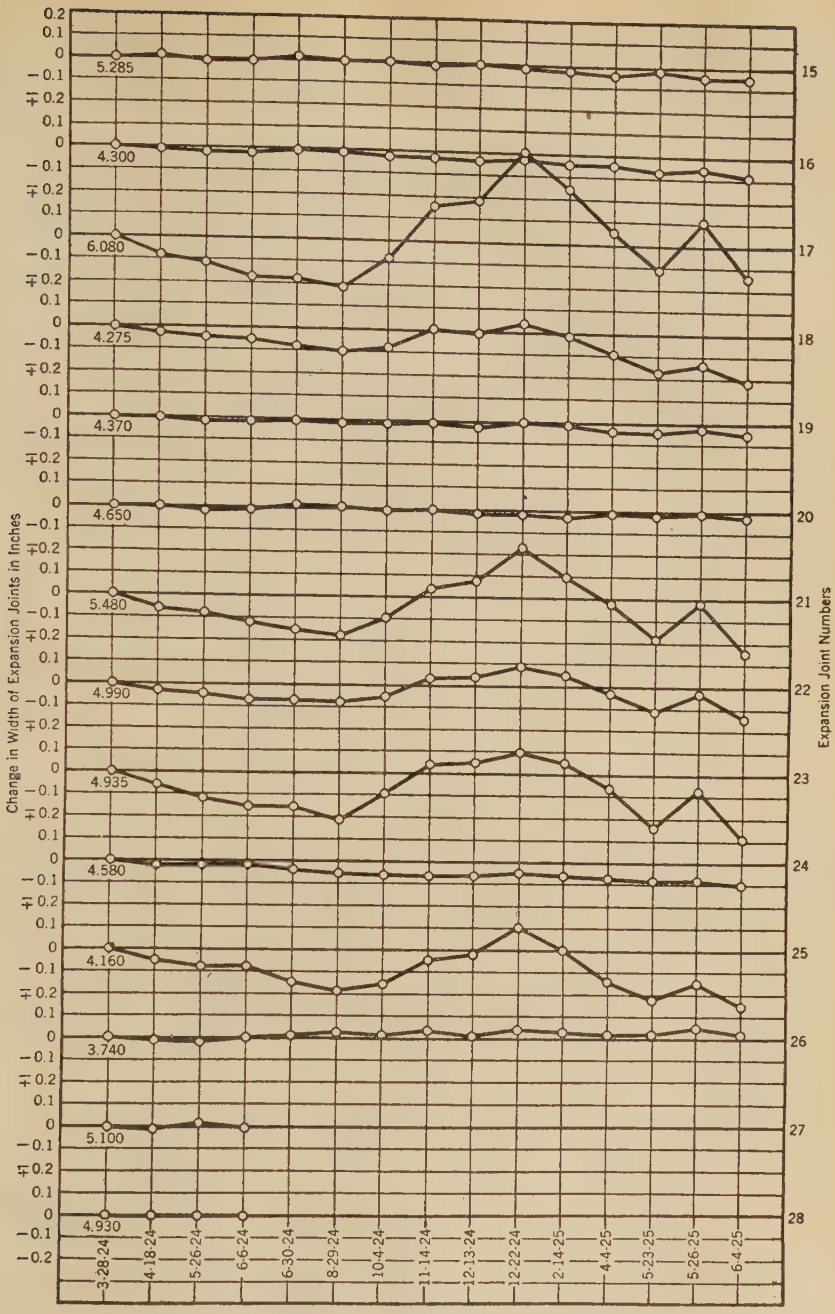


FIG. 64. CHANGE IN WIDTH OF EXPANSION JOINTS

Some explanation is necessary to account for the expansion joints on the shore side of the piers being closed while those on the channel side were open an undue amount. The plans of the bridge call for expansion joints equal in width on both sides of the piers, and no doubt the joints were of equal width when the bridge was built. Apparently, therefore, the deck had either moved inward relative to the piers or the piers had moved outward relative to the deck. Originally, the expansion joints were filled with asphalt 1 in. thick and where, at a given pier, the asphalt had been squeezed out of the joint on the shore side, the width of the channel side joint was invariably greater than the original, and the sum of the two joints was as great as ever. It is apparent, therefore, that the piers had tipped out rather than that the deck had crept in. The following explanation is offered to account for the movement of the piers.

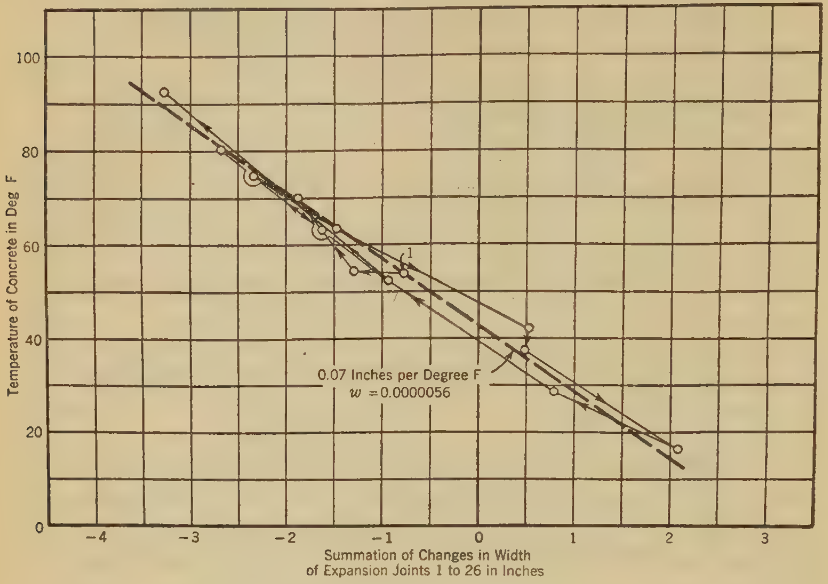


Fig. 65. Relation between Change in Temperature of Concrete and Movement at Expansion Joints

TABLE 25.—Thrust Due to One Degree Rise in Temperature

Span No.	Span length	Thrust, in pounds	Vertical distance, axis of rib at center line of span to thrust line, in feet	Angle of thrust line with the horizontal
1	114 ft 0 in.	111.35	7.22	2° 24′ 29″
2	139 ft 4 in.	145.66	8.16	0° 0′ 0″
3	164 ft 8 in.	227.63	8.49	0° 0′ 0″
4	190 ft 0 in.	329.16	9.31	0° 0′ 0″
5	139 ft 4 in.	185.88	7.58	3° 43′ 0″
6	101 ft 4 in.	138.32	5.46	3° 18′ 25″

The computed dead load pier reactions are given in Fig. 67, together with the distribution of pressure on the soil—hard shale in this case. The pressure in all cases is greater on the shore side of the pier than on the channel side, although the maximum pressure would not be considered excessive for the grade of material underlying these piers; but, with the variation in pressure shown, the progressive settlement due to time yield in the underlying shale would cause the piers to tip as is indicated by the changes in openings of the expansion joints shown in Table 26. The distance from the base of the pier

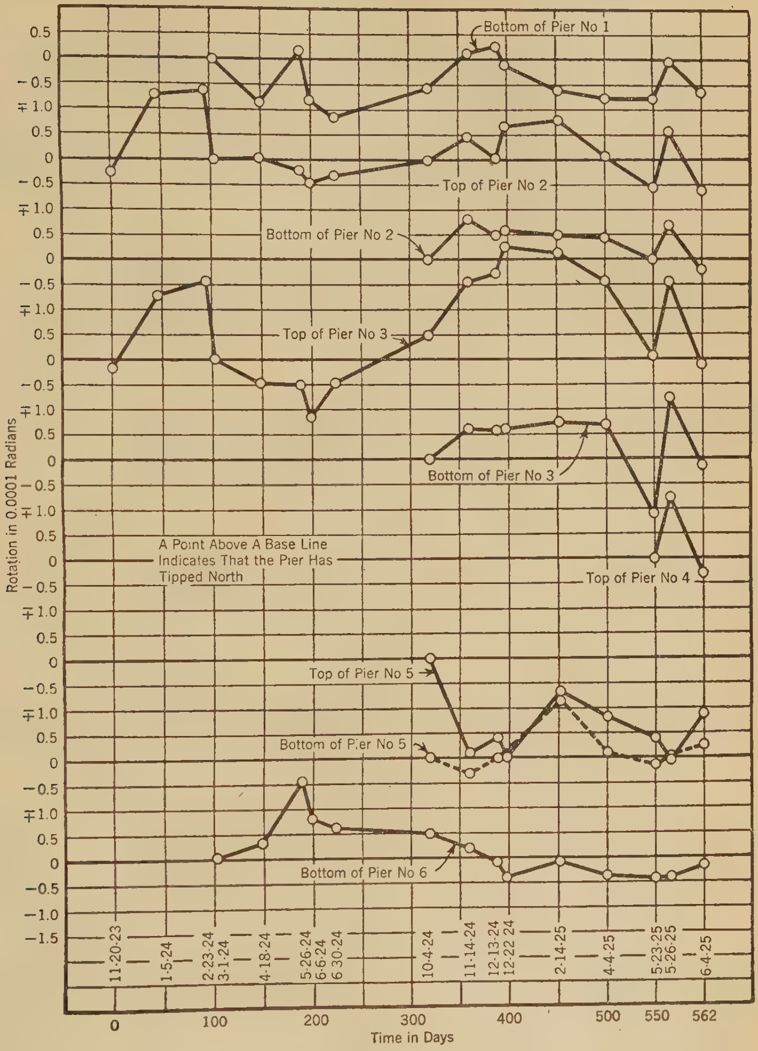


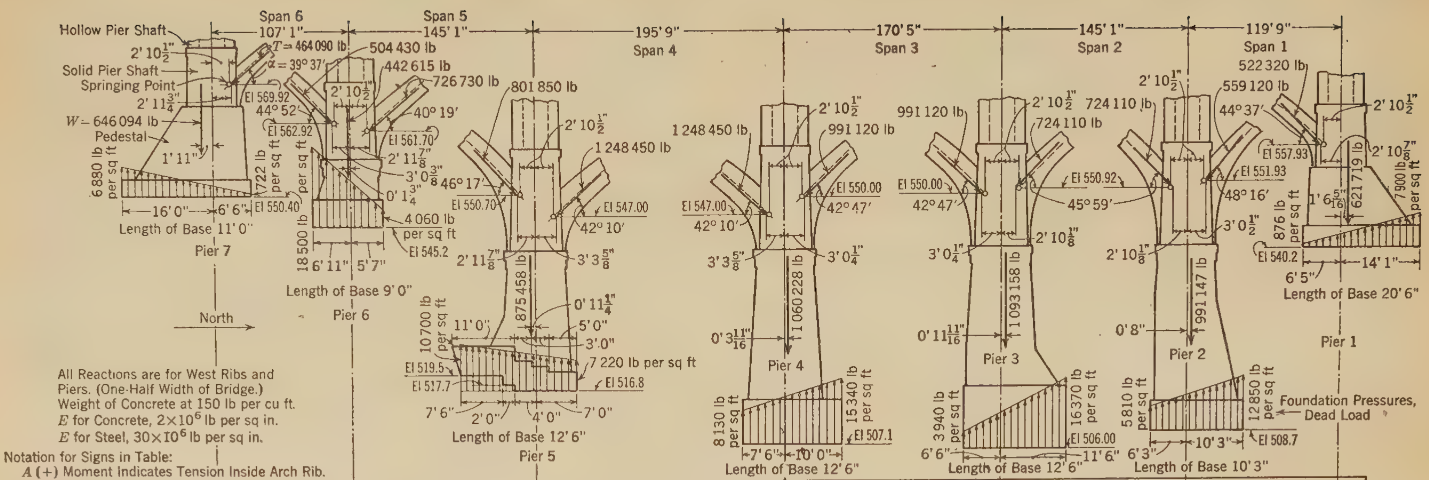
FIG. 66. ROTATION OF PIERS

to the deck is from four to five times the width of the base. The maximum movement of the top of the pier as indicated by the closing of the joint on the shore side probably did not exceed 1 in., a movement accounted for by an unequal settlement of 0.25 in. in a base width of 20 ft, an amount that could reasonably be expected with the pressure variation noted.

TABLE 26.—Notes on Expansion Joints

Joint on Side Toward Long Span			Joint on Side Away from Long Span		
Condition of Joint	Joint No.	Maximum Movement at Joint	Condition of Joint	Joint No.	Maximum Movement at Joint
Pier 1					
Width of opening in concrete, 0.38 in. Filler had fallen out except at curb.	4	0.36 in.	Width of opening in concrete, 0.31 in. Crack tightly filled with filler.	3	0.08 in.
Pier 2					
Width of opening in concrete, 0.75 in. Filler loose. Joint open at top of hand rail.	6	0.50 in.	Width of opening in concrete, 0.5 in. Nearly all of the filler had fallen out of joint in sidewalls. Joint tight at top of hand rail.	5	0.15 in.
Pier 3					
Width of opening in concrete, 1.75 in. Filler had all fallen out.	10	0.67 in.	Width of opening in concrete, 0.06 in. Crack tightly filled with filler.	9	0.08 in.
Pier 4					
Width of opening in concrete, 2 in. Filler had all fallen out.	14	0.68 in.	Width of opening in concrete, 0.56 in. Loose filler in opening.	13	0.29 in.
Pier 5					
Width of opening in concrete, 2 in. Filler had all fallen out.	17	0.62 in.	Width of opening in concrete, 0.19 in. Crack tightly filled with filler.	18	0.25 in.
Pier 6					
Width of opening in concrete, 0.75 in. A little loose filler in place.	21	0.47 in.	Width of opening in concrete, 0.5 in. Crack tightly filled with filler.	22	0.22 in.
Pier 7					
Width of opening in concrete, 0.75 in. A little loose filler in place.	23	0.39 in.	Width of opening in concrete, 0.12 in. Crack tightly filled with filler.	24	0.10 in.

The rotation of the piers due to unequal soil pressure is of interest because of the stresses it produces in the arch ribs. The largest stress would be pro-


Hollow Pier Shaft
2' 10½"
Solid Pier Shaft
Springing Point
2' 11¾"
W = 646 094 lb
Pedestal
1' 11"
6 880 lb per sq ft
722 lb per sq ft
16' 0"
6' 6"
El 550.40
Length of Base 11' 0"
Pier 7
Span 6
107' 1"
T = 464 090 lb
α = 39° 37'
El 569.92
44° 52'
El 562.92
504 430 lb
442 615 lb
2' 10½"
726 730 lb
40° 19'
El 561.70
2' 11⅞"
3' 0⅜"
0' 1¾"
46° 17'
4 060 lb per sq ft
El 545.2
18 500 lb per sq ft
6' 11"
5' 7"
Length of Base 9' 0"
Pier 6
North
Span 5
145' 1"
801 850 lb
2' 10½"
1 248 450 lb
El 550.70
El 547.00
42° 10'
2' 11⅞"
875 458 lb
3' 3⅝"
0' 11¼"
5' 0"
3' 0"
11' 0"
10 700 lb per sq ft
7 220 lb per sq ft
El 519.5
El 517.7
El 516.8
7' 6"
2' 0"
4' 0"
7' 0"
Length of Base 12' 6"
Pier 5
195' 9"
Span 4
1 248 450 lb
2' 10½"
991 120 lb
El 547.00
42° 10'
El 550.00
42° 47'
3' 3⅝"
1 060 228 lb
3' 0¼"
0' 3 11/16"
Pier 4
8 130 lb per sq ft
15 340 lb per sq ft
El 507.1
7' 6"
10' 0"
Length of Base 12' 6"
170' 5"
Span 3
991 120 lb
2' 10½"
724 110 lb
El 550.00
42° 47'
El 550.92
45° 59'
3' 0¼"
1 093 158 lb
2' 10⅛"
0' 11 11/16"
Pier 3
3 940 lb per sq ft
16 370 lb per sq ft
El 506.00
6' 6"
11' 6"
Length of Base 12' 6"
145' 1"
Span 2
2' 10½"
724 110 lb
2' 10⅛"
991 147 lb
0' 8"
Pier 2
5 810 lb per sq ft
12 850 lb per sq ft
6' 3"
10' 3"
El 508.7
Length of Base 10' 3"
119' 9"
Span 1
522 320 lb
559 120 lb
44° 37'
2' 10½"
El 557.93
El 551.93
48° 16'
3' 0½"
2' 10⅞"
1' 6 5/16"
621 719 lb
876 lb per sq ft
7 900 lb per sq ft
El 540.2
14' 1"
6' 5"
Length of Base 20' 6"
Pier 1
Foundation Pressures, Dead Load

All Reactions are for West Ribs and Piers. (One-Half Width of Bridge.)
Weight of Concrete at 150 lb per cu ft.
E for Concrete, 2×10^6 lb per sq in.
E for Steel, 30×10^6 lb per sq in.

Notation for Signs in Table:
A (+) Moment Indicates Tension Inside Arch Rib.

Dead Load Reactions at Springing, Corrected for Rib Shortening:												
Moment, M, in Inch-Pounds	−359 078	−672 017	−661 300	−763 233	−4 321 700	−4 321 700	−1 180 499	−1 180 499	+173 050	+173 050	−848 949	−149 393
Thrust:												
Total, T, in Pounds	464 086	504 432	726 733	801 855	1 248 450	1 248 450	991 121	991 121	724 109	724 109	559 117	522 322
Angle, α, With Horizontal	39° 37'	44° 52'	40° 19'	46° 17'	42° 10'	42° 10'	42° 47'	42° 47'	45° 59'	45° 59'	48° 16'	44° 37'
Horizontal Comp., H, in lb	357 457	357 457	554 139	554 139	924 650	924 650	727 324	727 324	508 210	508 210	371 769	371 769
Vertical Comp., V, in lb	295 973	355 916	470 182	579 569	838 840	838 840	673 271	673 271	515 806	515 806	417 612	366 889
Weight of Pier, W, in Pounds	646 094		442 615		875 458		1 060 228		1 093 158		991 147	621 719
Reactions at Springing Points Due to Abutment Displacement:												
1. If Right-Hand Abutment Rotates 0.001 Radians in a Clockwise Direction:												
Moment, M, in Inch-Pounds	−942 680	−2 037 701	−1 745 623	−3 832 685	−2 892 041	−6 476 201	−1 869 284	−4 212 820	−1 272 412	−2 875 860	−739 029	−1 783 182
Horizontal Comp., H, in lb	← 6 152	6 152 →	← 8 377	8 377 →	← 11 205	11 205 →	← 8 059	8 059 →	← 5 784	5 784 →	← 4 043	4 043 →
Vertical Comp., V, in lb	↓ 476	476 ↑	↓ 587	587 ↑	↓ 1 572	1 572 ↑	↓ 1 186	1 186 ↑	↓ 959	959 ↑	↓ 976	976 ↑
2. If the Length of the Span Increases 1 Inch Without the Piers Rotating:												
Moment, M, in Inch-Pounds	−4 958 221	−5 368 608	−6 709 370	−7 241 970	−11 204 190	−11 204 190	−8 058 885	−8 058 885	−5 784 190	−5 784 190	−4 790 493	−4 480 063
Horizontal Comp., H, in lb	← 23 177	23 177 →	← 22 641	22 641 →	← 29 463	29 463 →	← 23 509	23 509 →	← 17 780	17 780 →	← 16 597	16 597 →
Vertical Comp., V, in lb	↑ 1 263	1 263 ↓	↑ 1 388	1 388 ↓	0	0	0	0	0	0	↓ 647	647 ↑

FIG. 67.—PIER REACTIONS.

duced at the springing of Span No. 4, since for it, both piers tipped outward. Calculations based upon a value of E of 2 000 000 lb per sq in., result in a unit stress in the concrete due to this rotation of a little more than 300 lb per sq in.

Article 10.—Summary and Conclusions.—The conclusions resulting from the observations on the Gilbert Street Bridge at Danville, Ill., may be summarized, as follows:

(1) There was an observed range in the mean temperature of the arch rib of 83° F. The maximum range that occurred during the period of the observations was probably about 90 degrees.

(2) The most rapid change in the mean temperature of the rib was a drop of 26° in 3 days followed by a rise of 35.2° in the succeeding 9 days.

(3) The crown rose and fell with changes in the temperature, the relation between the movement and the temperature change being very constant and almost exactly (within 10%) equal to the theoretical relation. The maximum range in the vertical movement was 0.62 in.

(4) The thermal coefficient of the concrete was 0.0000049 for the rib and 0.0000056 for the deck.

(5) The change in the temperature caused the piers to rotate, but the movement was not of sufficient magnitude to produce appreciable stresses.

(6) The dead load thrusts produced an unequal distribution of the soil pressure on the footings of the piers and caused the piers to rotate until they came in contact with the deck on the side away from the long span.

(7) There was practically no movement at the expansion joints adjacent to the saddles. The sum of the movements of all the expansion joints was equal to the theoretical thermal expansion of a concrete slab extending from the pavement in the city streets at one end of the bridge to that at the other. The sum of the movements at all these joints had a maximum value of 5.37 in.

(8) The measured value of the temperature-stress deformation, in both the steel and the concrete, was greater than the computed value at all sections, but the difference was not great, except at Sections 2 and 4; at these sections the measured value was much greater than the computed value. The large temperature-stress deformation at these two sections was attributed to the abrupt change in the effective section at the end of the saddle. The rib, saddle, and deck apparently functioned as one solid piece, whereas the deck and rib outside the expansion joints adjacent to the saddle did not so function, there being an appreciable relative horizontal motion between the two.

(9) The expansion joints adjacent to the saddle are not needed to relieve the temperature stress in the deck, and they seem to weaken the structure by localizing the temperature deformation in the rib at the ends of the saddle. If these expansion joints had been omitted the bridge would have been cheaper, the roadway would have been smoother, and the structure would have been stronger.

(10) The uneven distribution of the dead load soil pressure on the footings of the piers caused the piers to tip, even though the maximum unit pressure

is no greater than would be permitted if the pressure were uniform. This action is attributed to the time yield in the underlying soil, which in this instance is shale.

Acknowledgments.—The work described in this Chapter was done under the supervision of Professor Wilson (2).

VII.—MOVEMENTS OF FOUNDATIONS DURING CONSTRUCTION

Part I

Article 1.—Objects.—The design of reinforced concrete arch bridges is based upon the assumptiin that the bases of the piers and the abutments are fixed in position. Knowing that in some cases piers of arch bridges have moved while the superstructure was being built, this investigation was undertaken in order to determine the magnitude and effect of these movements.

Article 2.—Scope of the Investigation.—Observations were made on five multiple-span arch bridges during the period of construction. The piers for two bridges rest upon rock; for one bridge, some piers rest upon rock and others upon piles driven into gravel; and for two bridges all piers are on piles driven into gravel. The first three bridges have twin-rib, open-spandrel, right arches, and the last two are spandrel-filled, skew, barrel arches. The angle of skew of the barrel arches is 6° 56′ and 46° 52′, respectively.

The observations on the twin rib arches included tipping of the piers, and, in one bridge, vertical movement of the piers and changes in the arch spans. The observations on the piers of the skew barrel arches included tipping, rotation about a vertical axis, and horizontal and vertical translation.

The tipping of a pier about a horizontal axis in the base produces stress in the rib or barrel due to the rotation at the springing and also due to a change in span from the tipping of the pier.

The ribs and barrels were analyzed by the elastic theory, neglecting the effect of the spandrel columns and deck; and the unit strain at the springing and crown were determined for a unit change in span and for a unit rotation of one abutment. The unit stress corresponding to unit strain can be determined only when the stress-strain relationship for the material is known, and, for concrete, this relationship varies with age, with the time interval over which the strain occurs, and with the nature of the aggregates and degree of consolidation when the concrete is placed. The unit strain, therefore, may be considered only as a rough indication of the value of the stress. The strain has been converted into stress on the basis that E for concrete is 1 000 000 lb per sq in., not because this is believed to be the true value, but because with the stress corresponding to this value known, the stress corresponding to any other value of E can be computed easily.

Article 3.—Yorkville Bridge.—Route 18 of the Illinois State Highway crosses the Fox River, at Yorkville, Ill., on two, three-span arch bridges separated by an island in the river. These bridges will be designated as the North Bridge and the South Bridge, respectively.

South Bridge.—The South Bridge has three spans, each 70 ft clear, with a rise of 10 ft. Each span is a twin-rib, open-spandrel arch supported on piers, 9 ft 6 in. high, resting on bed-rock. Observations were made on the two intermediate piers to determine the rotation which took place during construction. The rotation was measured with a 20-in. level-bar, on steel plugs set in the concrete near the springing of the arches.

Observations were started on July 15, 1927, before any of the arches were poured. The maximum rotation recorded is for Pier 1, on August 20, 1927, when the rotation was 0.000405 radian. With an assumed value of $E = 1\,000\,000$ lb per sq in., the theoretical unit stress at the springing produced by this rotation would be only 19 lb per sq in.

North Bridge.—The North Bridge has three spans, each 70 ft. clear, with a rise of 15 ft 6 in., and is supported on piers 18 ft 3 in. high which rest on rock.

Observations were started on April 30, 1927, before any of the arches were poured. The rotation of Pier 1, away from Span No. 1, increased from the time the first concrete was poured in the ribs of the first span until it reached a maximum when the superstructure of the first span was poured, the other spans having the ribs only in place. The maximum rotation was 0.001120 radian on July 8, 1927, which was reduced to a value of 0.000703 radian on August 20, 1927, when the bridge was completed. With an assumed value of $E = 1\,000\,000$ lb per sq in., the theoretical unit stress produced by a rotation of 0.001120 radian is 68 lb per sq in. If the pier rotated about a point in its base 200 in. below the springing, a rotation of 0.001120 radian would produce a horizontal motion of the springing of 0.224 in. This change in span would cause an additional stress at the springing of 70 lb per sq in. The change in span was not measured.

Article 4.—Washington Street Bridge at Rockton.—The highway bridge at Rockton, Ill., consists of eight, twin-ribbed, open-spandrel, reinforced concrete arches. The spans are each 65 ft clear, with a rise of 9 ft 9 in. The foundation conditions are unique in that the north abutment and Piers Nos. 1 to 4, inclusive, rest on piles driven into gravel whereas the other piers and the south abutment are founded on rock.

The rotation of the south abutment and of Piers Nos. 6 and 7, which were on rock, was very small, in all cases being less than 0.000160 radian. The rotation of the north abutment and of the other piers, although in no case excessive, is much greater. The maximum rotation recorded was 0.000597 radian for Pier No. 3. The maximum rotations for the other piers founded on piles, and for Pier No. 5 which was founded on the edge of the rock ledge, were nearly as great. With an assumed value of $E = 1\,000\,000$ lb per sq in., the theoretical unit stress produced by a rotation of a pier of 0.000600 radian is 42 lb per sq in. at the springing.

Readings were taken with a wye-level on two of the piers, which were founded on piles, to determine the settlement. A complete set of readings was not obtained, but the following fragmentary reports are of interest. The north abutment, which is supported on piles, settled 0.04 in. between

February 18 and February 28, 1926, the ribs of the adjacent spans being poured in the interim. Pier No. 1, also on piles, settled 0.0625 in. between December 19 and December 28, 1925, the rib of an adjacent span being poured in the interim. These settlements are insignificant.

Article 5.—Oliver Avenue Bridge, Indianapolis.—The Oliver Avenue Bridge over the White River in Indianapolis, Ind., consists of five, spandrel-filled, skew barrel arches. The angle of skew is 6° 56′ and the span and rise of the arches measured on a line parallel to the center line of the roadway, varied from 120 ft and 22 ft for the end spans, to 130 ft and 20 ft 7 in. for the middle span. The arches carry a 60-ft roadway and two 10-ft sidewalks, the barrels of the arches being approximately 60 ft long. All the piers and abutments are founded on wooden piles. The bottoms of the piers are 26 ft below the springings of the arches.

Observations were made during the construction of the arches, to determine the rotation, and the horizontal and vertical displacement of the piers. The maximum rotation occurred in the case of Pier No. 4 and was 0.000880 radian. This rotation, unaccompanied by a change in span or by a rotation of the other end of the span, and with an assumed value of $E = 1\,000\,000$ lb per sq in., produced a stress of 42 lb per sq in. at the springing.

The maximum horizontal movement occurred in Pier No. 2 and was 0.21 in. This change in span, unaccompanied by a rotation, would produce a unit stress of 45 lb per sq in. at the springing and 52 lb per sq in. at the crown.

Article 6.—Kentucky Avenue Bridge, Indianapolis.—The Kentucky Avenue Bridge over the White River in Indianapolis, Ind., consists of seven spandrel-filled, skew barrel arches. The span and rise vary from 122 ft and 22 ft 7½ in. for the end spans, to 138 ft and 22 ft for the middle span, measured on the center line of the roadway. The angle of skew is 46° 52′. The piers and abutments rest on wooden piles driven into gravel.

Observations were made to determine the rotation, and the vertical and horizontal displacement of the piers. The maximum rotation or tipping occurred for Pier No. 3 and was 0.001090 radian. This rotation, unaccompanied by a rotation of the other end of the span, or a change in span, will produce a theoretical unit stress at the springing of 57 lb per sq in., with an assumed value of $E = 1\,000\,000$ lb per sq in.

The maximum horizontal movement of a pier occurred at the north end of Pier No. 1, and was 0.41. A change in span of 0.4 in., unaccompanied by rotation, with $E = 1\,000\,000$ lb per sq in., would produce a unit stress of 79 lb per sq in. at the springing and 107 lb per sq in. at the crown.

Article 7.—Conclusions.—The movement of the piers, although small, produced an appreciable strain in the concrete. The strain often occurred when the concrete was green and did not occur instantaneously, but over a considerable period of time so that, although the modulus of elasticity is not known, it was small, and probably did not exceed the 1 000 000 lb per sq in. used herein in converting strain into stress. With a value of $E = 1\,000\,000$

lb per sq in., the stress corresponding to the unit strain[2] due to the pier movements although appreciable, is not large enough to be serious.

The movements reported were so small that they could not be detected without the use of precision instruments, yet the stress resulting from these movements is considerable. This is merely another way of stating that in arch construction caution should be exercised to prevent the piers from moving due to unbalanced thrusts during construction processes. It would not appear too great a precaution to require that the piers be kept under observation with precision instruments during construction, in order that any motion of the piers might be detected and steps taken to prevent its continuation.

On the whole, the results of the observations made on these bridges are re-assuring. They indicate that pier movements during construction will not be excessive if proper precautions are taken. The stability of piers on piles driven in gravel is of particular interest.

Article 8.—The Level-Bar.—The level-bar used in these observations of pier tipping was developed at the University of Illinois in connection with the tests on stresses in railway track which were under the supervision of Professor H. F. Moore. This instrument as used in the tests of arches has been described in *Bulletin No. 174* of the University of Illinois Engineering Experiment Station. The instrument is exceedingly simple and easy to manipulate and may be depended upon to give angular movements correctly within 2″ of arc. Its description is repeated here for the benefit of those who wish to keep records of pier movements during construction. The instrument is shown in Fig. 68.

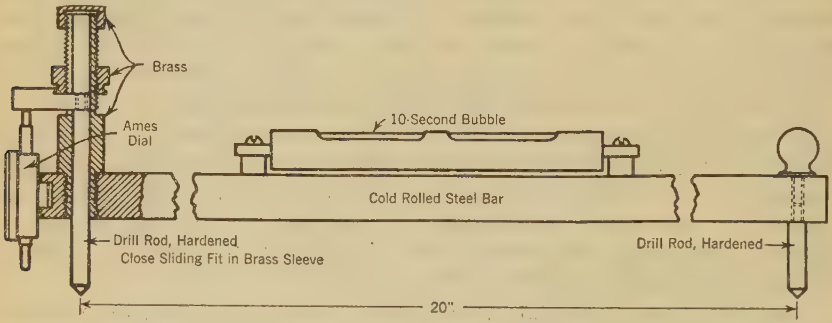


FIG. 68. LEVELBAR

In order to use the level-bar, two square pins are inserted in the body the rotation of which is to be measured. These pins are at the same level and are spaced 20 in. apart, with the top surfaces horizontal. One leg of the level-bar rests in a small round hole in one pin and the other leg rests in a slot in the second. In taking readings, the instrument is set on the pins, and the nut is turned until the bubble is in its mid-position and the reading of the dial is recorded. The instrument is then turned end for end and a second reading is taken. The difference between the two readings is twice the difference in level of the two pins. The change in the difference in level of the two pins divided by the distance between them, is the rotation of the body.

---

[2] The strains resulting from pier movements for the five bridges were computed and checked by Nathan M. Newmark, and Glenn Murphy, Juniors, Am. Soc. C. E., and Charles O. Harris, Research Graduate Assistants. The calculated value determined by the elastic theory applied to the rib alone, the effect of the deck and spandrel columns being neglected.

The Ames dial on the level-bar is graduated to 0.001 in. and fractions of a division are estimated with the eye, so that readings are recorded to 0.0001 in. Duplicate readings were taken in all cases and the two sets of readings usually agreed within 0.0004 in.

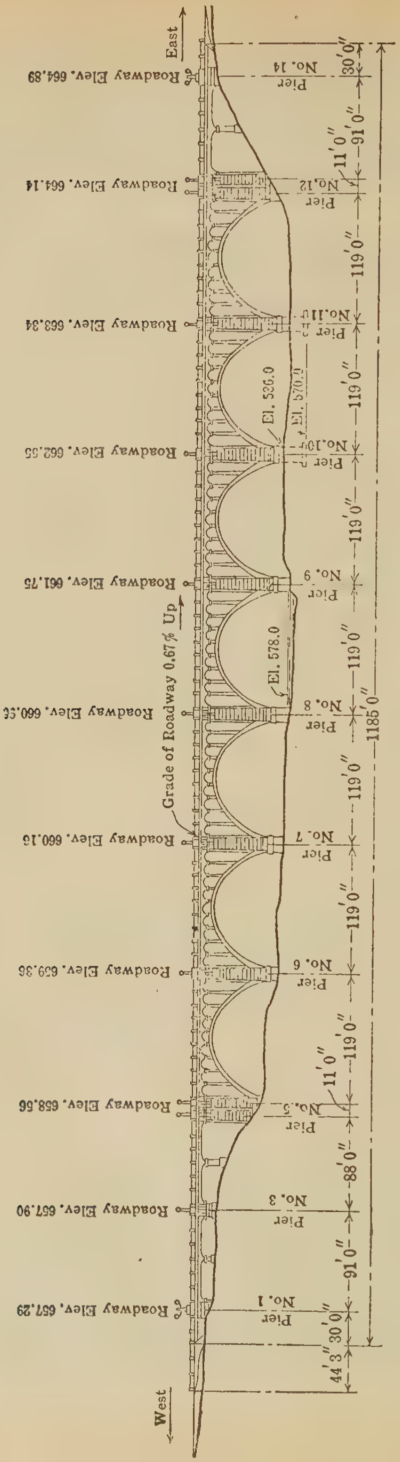
Part II

Article 9.—Scope of the Investigation.—Observations were made on three bridges in Ohio to determine the movement of piers during construction. The first of these was at Conneaut, where observations for rotation and settlement were made on only one pier and the two adjacent arch spans. The second bridge was at Ashtabula, where observations for rotation were made on all eight piers. The third bridge was at Piqua, where rotations were measured for the four piers and two abutments.

Article 10.—Conneaut Bridge.—The bridge at Conneaut consists of seven spans 119 ft each, with a rise of 32 ft, as shown in Fig. 69. All piers rest on a good shale foundation. Apparatus was installed for measuring the yielding of the foundations under both sides of Pier No. 11 and the movement of the top of the pier with respect to a point near the base was measured with a large theodolite. Holes for internal temperature readings were provided in the arch rings near the springing and at the crown. Observations as to the settlement of Pier No. 11 during construction were also made.

Article 11.—Temperature Variations at Conneaut.—When the concrete was placed, in two of the arch ribs, wells 1 in. in diameter were cored, extending 3 in. from the upper surface to the center, and to within 3 in. of the lower surface of the rib. One set of wells was placed at the springing and one set at the crown of each rib. The lower ends of these wells were filled with cup grease and the tops stopped with corks. Long-stemmed thermometers were inserted in the wells, with the bulbs buried in the cup grease when temperature readings were being taken.

The observations were carried on for a period of eleven months. The range of air temperatures was from 84° to −5° F. After the first two months, the concrete temperatures follow the mean air temperatures fairly well without approaching the extremes. The temperature record is shown in Fig. 70. The temperature of the concrete was higher than the air temperatures for the first two months. The ribs were 6 ft wide, 65 in. deep at the springing, and 36 in. at the crown. After the first two months the extreme temperature variation of the concrete was from 12° to 71° F, in a period of about five months.


East
West
Roadway Elev. 664.89
Pier No. 14
Roadway Elev. 664.14
Pier No. 12
Roadway Elev. 663.34
Pier No. 11
Roadway Elev. 662.55
Pier No. 10
El. 536.0
El. 570.0
Roadway Elev. 661.75
Pier No. 9
El. 578.0
Roadway Elev. 660.96
Pier No. 8
Grade of Roadway 0.67% Up
Roadway Elev. 660.16
Pier No. 7
Roadway Elev. 659.36
Pier No. 6
Roadway Elev. 658.56
Pier No. 5
Roadway Elev. 657.90
Pier No. 3
Roadway Elev. 657.29
Pier No. 1
30′0″
91′0″
11′0″
119′0″
119′0″
119′0″
119′0″
119′0″
119′0″
119′0″
11′0″
88′0″
91′0″
30′0″
44′3″
1185′0″

MULTIPLE ARCH BRIDGE
CLEVELAND-BUFFALO HIGHWAY
CONNEAUT, OHIO

FIG. 69.

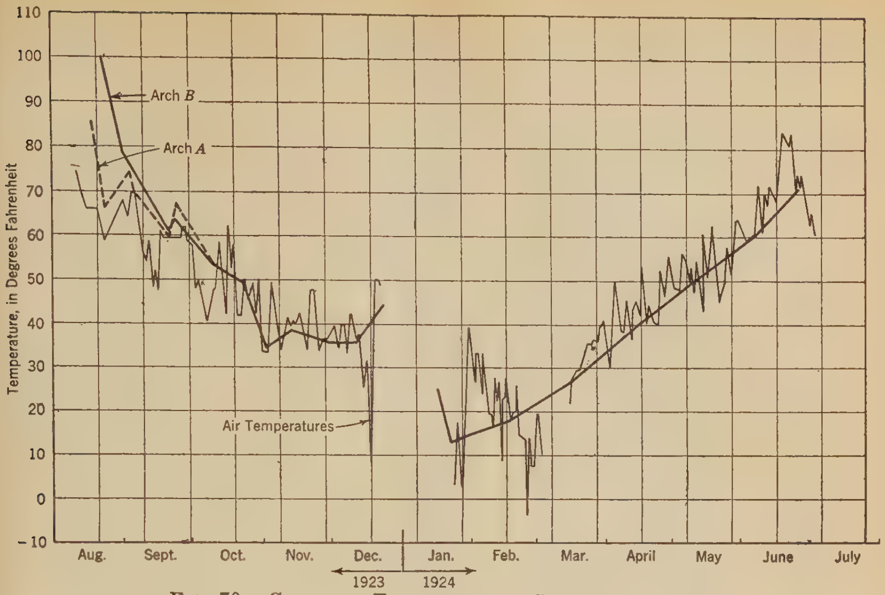


FIG. 70.—CONCRETE TEMPERATURES, CONNEAUT BRIDGE.

TABLE 27.—PIER TILTING, IN MILLIONTHS OF RADIANS

Date	Stage of construction	Determined by:			
		Theodolite		Rod Readings	
		West pier	East pier	West pier	East pier
1923:					
August 11	Falsework only	.....	.....	.....	67
August 12	Falsework only	.....	.....	.....	67
August 13	Sections 1 and 2 of West Arch ribs poured	125	.....	.....	50
August 14	West Arch ribs poured	.....	25	.....	17
August 15	West Arch ribs poured	.....	0	12	.....
August 23	Forms for East Arch in place	.....	0	37	.....
August 27	Both arch ribs poured	.....	.....	.....	88
August 28	Both arch ribs poured	.....	50	.....	.....
August 29	Both arch ribs poured	.....	251	.....	71
September 1	Centers lowered on west ribs	.....	.....	.....	46
September 2	Centers lowered on west ribs	.....	75	.....	.....
September 20	Centers lowered on west ribs	.....	176	.....	88
September 25	Centers lowered on both ribs	.....	176	.....	100
October 11	Centers lowered on both ribs	.....	.....	.....	33
October 22	Centers lowered on both ribs	.....	150	4	.....
November 6	Columns Nos. 2 to 6 on West Arch poured	.....	.....	129	.....
November 9	All columns on West Arch poured	.....	.....	137	.....
November 22	All columns on West Arch poured	.....	100	.....	.....
November 23	Columns Nos. 1 to 6 on East Arch poured	.....	.....	137	.....
November 24	Columns Nos. 1 to 7 on East Arch poured	.....	.....	154	.....
November 27	Columns Nos. 1 to 7 on East Arch poured	251	.....	.....	.....
November 28	6½ panels of deck on West Arch poured	176	.....	275	.....
November 30	West Arch deck complete	.....	.....	337	.....
December 3	West Arch deck complete	201	.....	.....	.....
December 12	West Arch deck complete	226	.....	.....	.....
December 21	5½ panels of deck on East Arch poured	76	.....	158	.....
December 22	East Arch deck complete	.....	25	.....	.....
1924:					
January 14	East Arch deck complete	50	.....	.....	.....
April 12	Deck on both spans complete	.....	.....	.....	71
May 7	Deck on both spans complete	0	.....	.....	.....
June 7	Deck on both spans complete	.....	.....	37	.....
June 24	Bridge complete	0	.....	46	.....

Article 12.—Pier Movements at Conneaut.—The movements of Pier No. 11 were determined by two methods, first by observing with a theodolite the relative horizontal movement of two points on the pier, one 33.2 ft above the other; and, second, by observing the change in elevation of two points on the footing, 20 ft apart, horizontally.

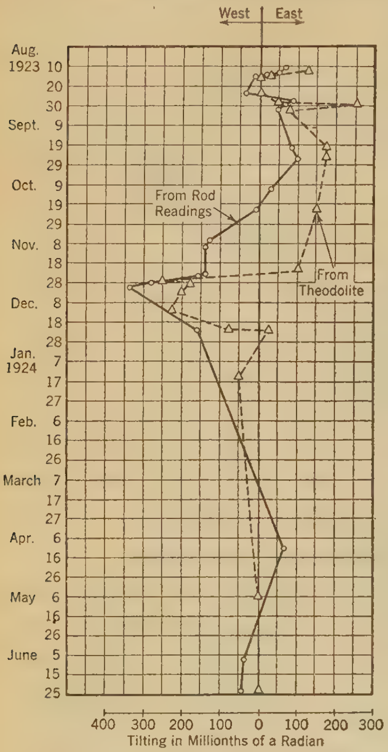


FIG. 71.—PIER TILTING, CONNEAUT BRIDGE.

The observations have been reduced to radians and are given in Table 27 and shown plotted in Fig. 71.

The maximum variation in angle as determined by the two methods was almost exactly the same. It was 0.000425 radian and occurred between September 25 and November 30. It is noteworthy that on the completion of the bridge the pier returned to its original alignment. The settlement of the pier during the construction of the bridge was about 0.13 in.

Article 13.—Ashtabula Bridge.—The bridge at Ashtabula, Ohio, contained seven spans, 135 ft, center to center, of piers and 121 ft clear, with a rise of 58.5 ft, as shown in Fig. 72. All piers rested on a good shale foundation.

The tipping of Piers Nos. 7 to 12, inclusive, was measured with a 20-in. level-bar on steel plugs set in the horizontal surface of the top of the pier just above the springing of the arches. Two sets of plugs were set in each pier, one on the center line of each arch rib, and are designated as *N* and *S*, respectively. The maximum movements are given in Table 28.

TABLE 28.—MAXIMUM MOVEMENTS, ASHTABULA BRIDGE

Pier No. (see Fig. 72)	Tilting, in radians	Pier No.	Tilting, in radians	Pier No.	Tilting, in radians
7*N*..........	0.000235	9*N*..........	0.000430	11*N*..........	0.000200
7*S*..........	0.000315	9S..........	0.000395	11*S*..........	0.000105
8*N*..........	0.000545	10*N*..........	0.000220	12*N*..........	0.000200
8*S*..........	0.000575	10*S*..........	0.000500	12*S*..........	0.000225

Article 14.—Piqua Bridge.—The bridge at Piqua, Ohio, consists of five half-through arch spans, 86 ft, center to center, of piers and 80-ft. clear span, measured parallel to the center line of the bridge. The bridge is on a 10° skew, as shown in Fig. 73. The foundations are supported on piles driven into a gravelly subsoil.

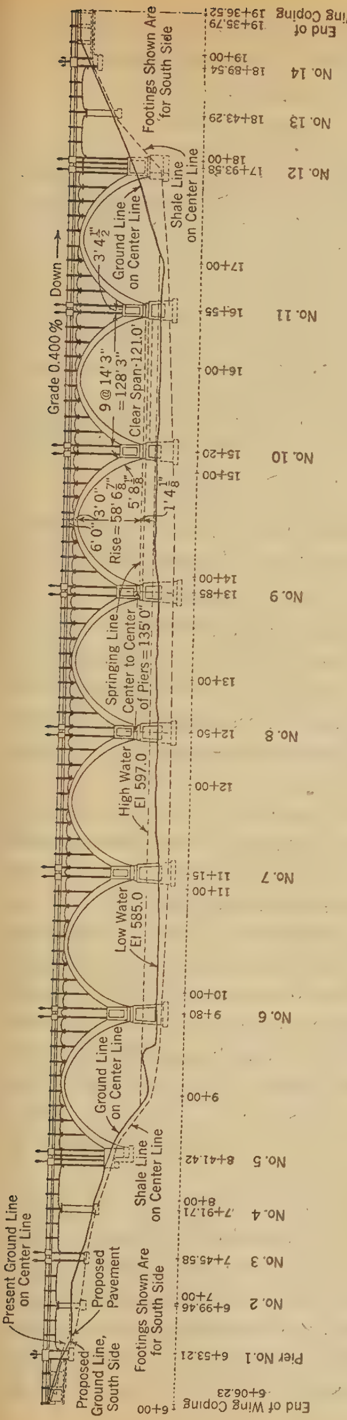

End of Wing Coping 19+36.52
End of 19+35.79
No. 14 18+89.54
19+00
No. 13 18+43.29
No. 12 17+93.58
18+00
17+00
No. 11 16+55
16+00
No. 10 15+20
15+00
No. 9 13+85
14+00
13+00
No. 8 12+50
12+00
No. 7 11+15
11+00
10+00
No. 6 9+80
9+00
No. 5 8+41.42
No. 4 7+91.71
8+00
No. 3 7+45.58
No. 2 6+99.46
7+00
Pier No. 1 6+53.21
End of Wing Coping 6+06.23
6+00
Footings Shown Are for South Side
Shale Line on Center Line
Ground Line on Center Line
3' 4½"
Grade 0.400 % Down
9 @ 14' 3" = 128' 3"
Clear Span 121.0'
6' 0"
3' 0"
Rise = 58' 6⅞"
5' 8⅛"
1' 4⅛"
Springing Line
Center to Center of Piers = 135' 0"
High Water El 597.0
Low Water El 585.0
Ground Line on Center Line
Shale Line on Center Line
Footings Shown Are for South Side
Present Ground Line on Center Line
Proposed Pavement
Proposed Ground Line, South Side

FIG. 72.

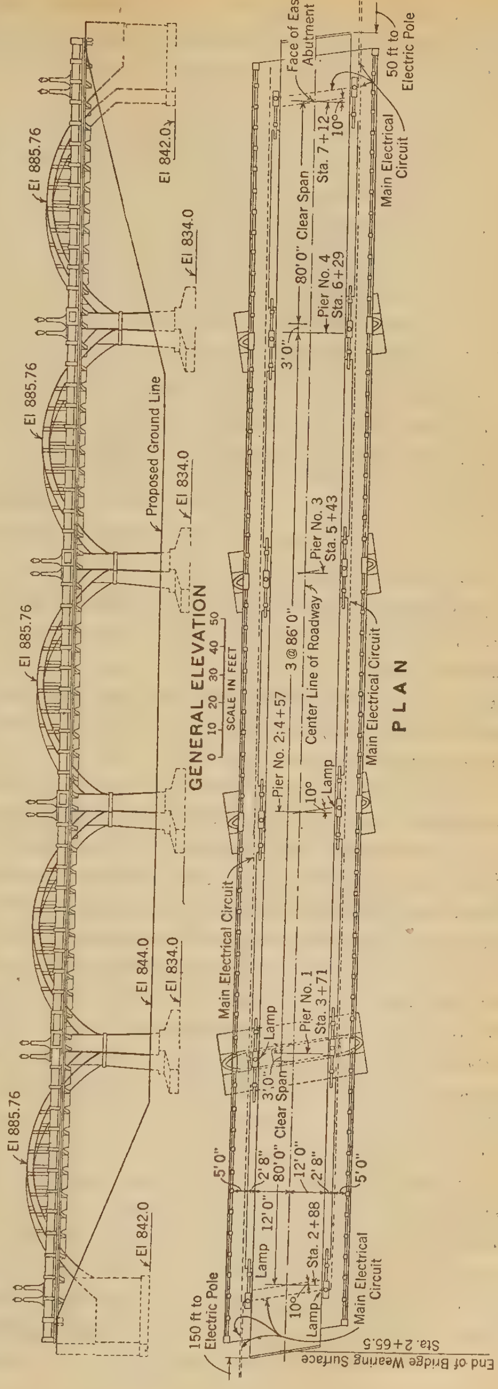

El 885.76
El 885.76
El 885.76
El 885.76
El 842.0
El 834.0
Proposed Ground Line
El 834.0
El 844.0
El 834.0
El 842.0
GENERAL ELEVATION
0 10 20 30 40 50
SCALE IN FEET
Face of East Abutment
50 ft to Electric Pole
Sta. 7+12
10°
80' 0" Clear Span
3' 0"
Pier No. 4 Sta. 6+29
Main Electrical Circuit
Pier No. 3 Sta. 5+43
3 @ 86' 0"
Center Line of Roadway
Pier No. 2; 4+57
10°
Lamp
Main Electrical Circuit
Main Electrical Circuit
PLAN
Lamp
Pier No. 1 Sta. 3+71
3' 0"
80' 0" Clear Span
5' 0"
2' 8"
12' 0"
2' 8"
5' 0"
Lamp
12' 0"
Sta. 2+88
10°
Lamp
Main Electrical Circuit
150 ft to Electric Pole
Sta. 2+65.5
End of Bridge Wearing Surface

FIG. 73.

The tipping of the piers and abutments was measured with a 20-in. level-bar on steel plugs set in the two ends of each pier at the springing of the arches. The maximum movements are given in Table 29.

TABLE 29.—MAXIMUM MOVEMENT, PIQUA BRIDGE

Pier Number	Maximum rotation, in radians
West Abutment	0.000099
Pier No. 1	0.000376
Pier No. 2	0.000279
Pier No. 3	0.000227
Pier No. 4	0.000165
East Abutment	0.000125

Acknowledgments.—The work described in Part I of this Chapter was done under the supervision of Professor Wilson (1); and that described in Part II was done under the supervision of Professor Morris.

VIII.—THE EFFECT OF TIME YIELD IN CONCRETE UPON THE DEFORMATION STRESSES IN A REINFORCED CONCRETE ARCH BRIDGE

Article 1.—Object and Scope of Investigation.—The stresses in a statically indeterminate structure are of two kinds: (1) Load stresses, which are due to the direct effect of the loads that the structure carries; and (2) deformation stresses, which are due to a movement of the supports. Stresses essentially the same as deformation stresses are also produced if the length of certain members change, due to a change in temperature or other causes, even though the supports remained fixed.

The length of a reinforced concrete arch rib changes: (*a*) Because of the immediate strain accompanying the application of a load, usually designated as rib-shortening; (*b*) because of the time yield in concrete; (*c*) because of temperature changes; and (*d*) because of the shrinkage in the concrete that takes place in drying. All changes in the length of the rib of an arch with fixed abutments change the shape of the rib, thereby producing flexural stresses similar to those produced by moving the abutments without changing the length of the rib. Since the stress is due to a change in shape and not to a load it is a deformation stress; and since the changes, except rib-shortening, take place slowly, it would appear possible that the time-yield property of the concrete might enable the rib to assume its new shape without incurring the stresses that would be produced if the same changes in length occurred quickly.

In view of this possibility, tests were made to determine the change in reactions at the springing of an arch due: (1) To the shrinkage of the concrete; (2) to the combined effect of shrinkage and time yield due to dead load; and (3) to changing the span by an amount equivalent to a change in temperature of 100° F. In order to simulate the gradual temperature trends throughout a season, the change in span was made in five equal increments, each of which was equivalent to a change of 20° F, and the changes were made at intervals of approximately 30 days. Readings were taken just before and just after each change in span to determine the immediate change in the reactions. Subsequent readings were taken at intervals of approximately one week to determine the change in reactions due to time yield.

A series of tests was also made to determine the effect of the gradual yielding of the abutments on the load-carrying capacity of the arch. The initial readings were taken when the design dead load was on the structure and when the abutments were in their normal position relative to each other. Subsequent readings were taken after each of a number of live loads had been added and after each of a number of movements of the abutments. In general, a period of one week was allowed to elapse between successive abutment displacements to permit time yield of the concrete to occur.

The temperature of the laboratory remained very constant, seldom varying more than 1° F during a single set of observations. The temperature of the air was recorded for all tests, and a standard temperature was designated at the beginning of each series. Subsequent tests were made on days when the temperature of the air in the laboratory was found to vary by not more than 1° or 2° from the standard temperature. Moreover, the ventilation of the building was controlled so that the temperature of the air did not vary by more than 1° or 2° for a period of 2 or 3 hr immediately preceding a test. It is believed that there are no appreciable errors in the results due to temperature changes.

Article 2.—Acknowledgments.—The tests described in this report are a part of the research work sponsored by the Committee. They also constitute a part of the investigation resulting from the co-operative agreement entered into by the Engineering Experiment Station of the University of Illinois, and the United States Bureau of Public Roads. The tests were planned in consultation with E. F. Kelley, Chief of the Division of Tests, and A. L. Gemeny, Senior Structural Engineer, both of the U. S. Bureau of Public Roads; and with Professors Morris and Beggs, and Messrs. Harder and Janni, of the Committee. The experimental work was done by Ralph W. Kluge and Nathan M. Newmark, Juniors, Am. Soc. C. E., Research Assistants in Civil Engineering at the University of Illinois, under the supervision of Professor Wilson.

Article 3.—Description of Specimen.—The specimen consisted of an open-spandrel reinforced concrete arch with a deck so low as to be integral with the rib at the crown. The dimensions of the arch were the same as those for one span of the three-span arch bridge with a deck, shown in Fig. 41 which was previously tested (9). The span was 27 ft and the rise 6 ft 9 in. The aggregate and the mix were as nearly as possible like the aggregate and mix used for the three-span structures. The mix was 1:3:3 by volume, with a 1.2 water-cement ratio. The quantities for a batch were determined by weight, correction being made for the moisture content of the aggregate. Each batch was mixed at least 4 min., and eight 6 by 12-in. control cylinders were made, one each from eight batches.

The arch was poured on June 24, and the forms were removed on July 5 and 6, 1933. The surfaces of the concrete were wet when the forms were removed, and the concrete probably had not shrunk an appreciable amount at that time.

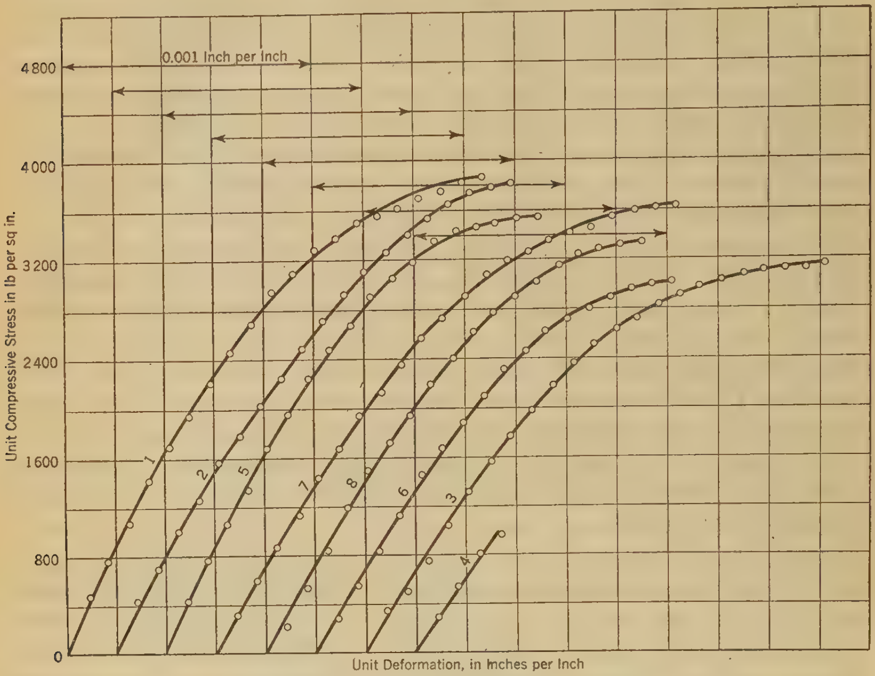


FIG. 74.—STRESS-STRAIN DIAGRAMS FOR CONTROL CYLINDERS.

The stress-strain diagrams for the control cylinders are given in Fig. 74 and the physical properties of the concrete, in Table 30. Fig. 75 shows the general appearance of the arch.

TABLE 30.—PHYSICAL PROPERTIES OF CONCRETE FROM CONTROL CYLINDERS

Cylinder No.	Ultimate stress, in pounds per square inch	Modulus of elasticity (10^6 lb per sq in.)	Cylinder No.	Ultimate stress, in pounds per square inch	Modulus of elasticity (10^6 lb per sq in.)
1........	3 860	4.3	5........	3 540	4.2
2........	3 810	4.0	6........	3 080	3.4
3........	3 180	3.3	7........	3 640	3.4
4........	2 780	3.1	8........	3 350	3.5
Average......	.....	...	....	3 400	3.7

Article 4.—Description of Control Specimens.—Four control specimens were made for use in studying the magnitude of shrinkage and time yield in a straight rectangular block of concrete. Each specimen was 4 ft long and had a 9 by 12-in., transverse section which is the average of the sections at

the crown and springing of the arch rib. Each control specimen was reinforced with eight $\frac{1}{2}$-in. plain round steel rods, the same as the reinforcement in the arch rib. The control specimens were poured the same day and of the same concrete as the arch, and were stored in the laboratory near the arch where the atmosphere was dry and warm. Two specimens were unloaded throughout the tests; the other two were subjected to an axial thrust of 240 lb per sq in., approximately the average dead load thrust on the arch rib. The change in length of these control specimens was measured on four reinforcing bars, two on the top and two on the bottom, with a 24-in. Berry strain-gauge.

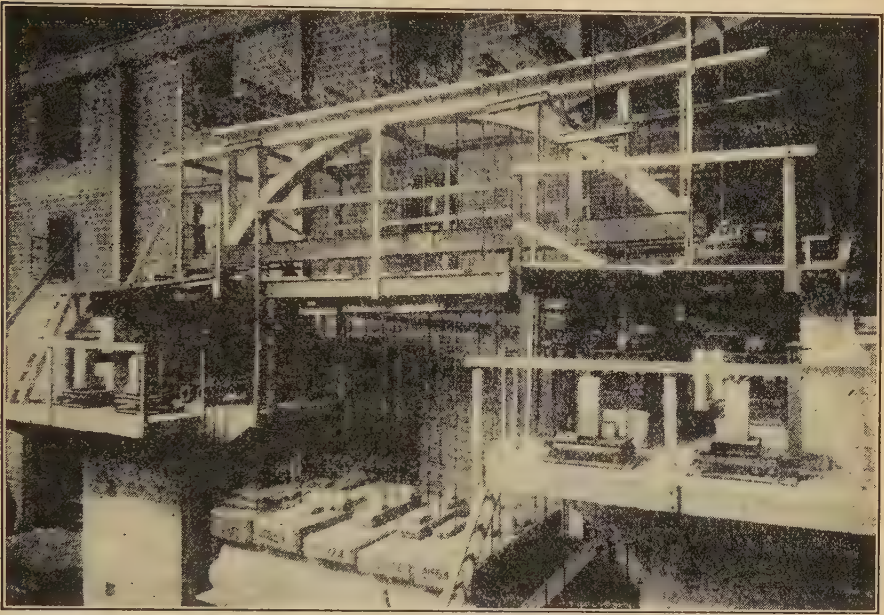


FIG. 75.—DETAILS OF ARCH.

Article 5.—Description of Apparatus.—Fig. 75 shows the general appearance of the specimen. The apparatus was designed and built for the three-span arch bridge that had previously been tested (8) (9).

The load on the structure was produced by suspending concrete blocks of known weight at the load points immediately over each spandrel column. For each load point there was one large concrete block suspended by four steel rods, which served as a loading platform on which to place smaller blocks that, in combination with the loading platform, constitute the total load to be applied at a particular point. When the arch was unloaded, the loading platform rested upon supports provided for the purpose, and the loading beam was carried on the steel suspension rods acting as struts.

To apply the load the turnbuckles in the suspension rods were turned, shortening the rods, until the loading beam came into contact with the loading shelf. The turnbuckles were then turned, successively, by small amounts in order, back and forth from one end of the structure to the other, thereby transferring the weight of the loading platforms gradually from the supports to the arch. Each load point on the arch was capped with a steel plate containing a $\frac{3}{4}$-in. steel ball in its top surface and at the point of application of the load. The loading beam had a small steel block attached at the center of its bottom flange. A small depression in the bottom of this block fitted over the top of the ball in the steel plate on the loading shelf, thus accurately locating the point of application of the load.

The load at a given load point for a particular test was obtained by placing concrete blocks of known weight upon the loading platform until the desired load had been obtained. Each abutment was supported on two vertical-load scales as shown in Fig. 76. The load was transmitted from the abutment to the scales by means of jacks, two for each scale, one on the north side and the other on the south side of the abutment. The contact between the abutments and the jacks was through knife-edges embedded in the abutment so that the line of action of the vertical forces weighed by each scale was accurately known. An abutment could be raised or lowered without rotation by extending or depressing all jacks by the same amount; or an abutment could be rotated about a horizontal north-and-south axis by extending both the east jacks and depressing both the west jacks, or the reverse. The vertical-load scales were mounted on carefully machined steel rollers, 10 in. in diameter, that ran upon a carefully machined track, and therefore offered practically no resistance to horizontal motion.

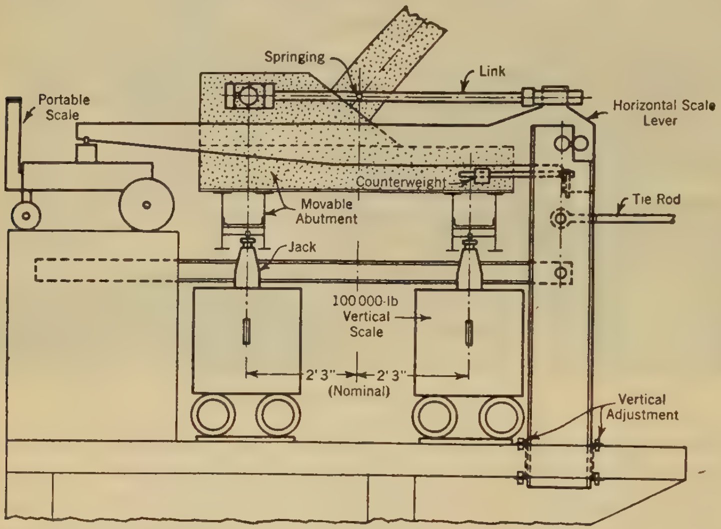


FIG. 76. WEIGHING APPARATUS AND SUPPORTS FOR ABUTMENTS

The horizontal reaction of each abutment was measured by means of a horizontal-load scale, as shown in Fig. 76. This scale consisted of two right-angle levers (bell-cranks), one on the north side and the other on the south side of the abutment, that received the horizontal thrust through links and converted it into a vertical force that was delivered to a platform scale. The bell-crank had a nominal multiplication ratio of 10 to 1, the actual multiplication ratio being determined for each. The link connecting an abutment and horizontal scale had knife-edge contacts at both ends. This link was maintained in a horizontal position and the line of action of the horizontal reaction was determined from the position of the knife-edge embedded in the abutment. An abutment could be moved horizontally by turning the link since it had a right-hand thread at one end and a left-hand thread at the other.

The strain in the concrete was measured with an 8-in. Berry strain-gauge. Readings were taken on two gauge lines on the intrados and two on the extrados at the section midway between each pair of adjacent load points.

The angular position of the abutments was determined by means of extremely sensitive level bubbles, one for each abutment. Each bubble was attached to a steel rod embedded in the concrete in the longitudinal central plane of the rib and normal to the axis at the springing.

The vertical movement of the abutments was measured with two hook-gauges, one at each abutment, connected to the same pipe line in such a manner that at any instant the water surface was at the same level for both gauges. The vertical movement of the points in the rib immediately below the spandrel columns was measured with reference to two steel **I**-beams, one on each side of the arch, that were supported on pins projecting from the sides of the rib at the springings. A $\frac{1}{2}$-in. vertical rod embedded in the concrete and projecting below the rib on its center line, directly below each spandrel column, had a small hole in the lower end. The tops of the **I**-beams were connected with steel battens directly beneath each spandrel column, and each batten contained a small hole at its center and directly below the rod projecting from the bottom of the rib. An instrument consisting of an Ames dial having a conical point on its plunger and mounted on a rod having a conical end, was used to measure the distance from the lower ends of the projecting rods to the top of the battens, thereby giving the movement of the points on the arch rib relative to the **I**-beams.

The **I**-beams from which the deflection was measured were also used to measure changes in the span of the arch. The beams were attached to the arch at the east springing, but they were supported on rollers at the west springing. An Ames dial was attached to the west end of each beam in such a manner that its plunger bore upon a steel pin projecting from the side of the arch at the west springing. This dial indicated changes in span of the arch.

The apparatus that has been described made it possible to give the abutments any movement that was desired; that is, each of the three components of movement (X, Y, or θ) could be given separately, to each abutment. Moreover, each component of movement could be measured. Furthermore, all three components of each abutment reaction (H, V, and M) could be determined in magnitude and position from the scale readings.

Article 6.—Tests to Determine Abutment Reactions Due to Shrinkage.—Previous to erecting the forms for the specimens, the scales supporting the abutments were read when they carried only the steel supporting the abutments. These readings were used as a base in computing the reactions at the springings of the arch from subsequent readings of the scales.

The arch was poured on June 24, and the forms were removed on July 5 and 6, 1933. After the forms had been removed and the apparatus attached to the arch, the abutments were brought to their normal position relative to each other. In making this adjustment, the span was changed until the horizontal thrust at the springing, as indicated by the horizontal scales, was equal to the value of the thrust as computed from the weight of the structure. This was approximately 3 075 lb. Likewise, in making the adjustment for the angular position of an abutment, the abutment was rotated until the moment at the springing, as indicated by the vertical scales, was equal to the value of the moment at the same point, as computed from the weight of the structure. This was approximately 16 000 in-lb. The bubble tubes attached to the arch at the springings were then adjusted so as to bring the bubbles to their mid-position, thus preserving the normal angular position of the abutments. The readings of the dials, indicating the normal span, and of the hook-gauges, indicating the normal relative elevation, were recorded and used, together with the bubbles, as a basis for maintaining the abutments in their normal position relative to each other during the subsequent tests.

These basic readings were taken on July 8, 1933, when the arch was fourteen days old and two days after the forms had been removed. Inasmuch as the surface of the concrete was wet when the forms were removed, the volume of the concrete had probably changed very little at the time the basic readings were taken.

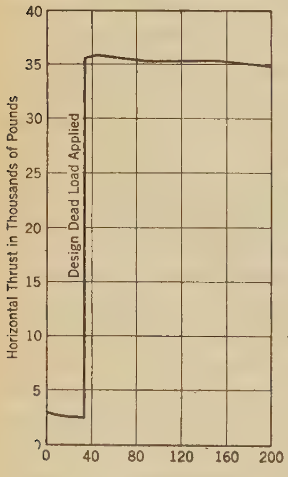


FIG. 77.—CHANGE IN THRUST DUE TO SHRINKAGE AND TIME YIELD.

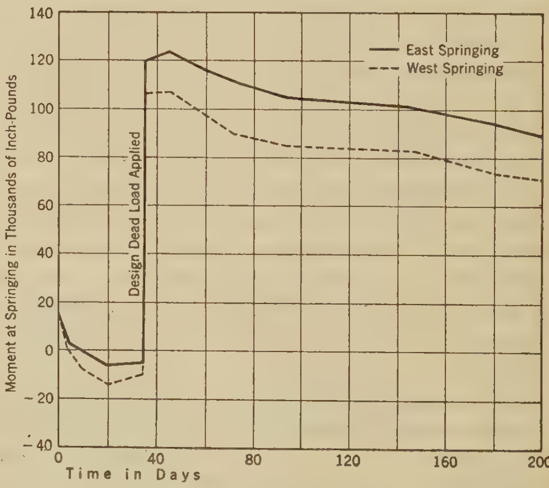


FIG. 78.—CHANGE IN MOMENT DUE TO SHRINKAGE AND TIME YIELD.

The arch carried no load except its own weight during the period from July 7 to August 11, 1933. The abutments were retained in their normal position, and the scale readings were recorded at frequent intervals to determine the change in the reactions at the springings due to the shrinkage of the concrete. The results of the observations are presented in the left-hand part of Figs. 77 and 78. These diagrams indicate that, for the period of 34 days beginning when the arch was 16 days old, the horizontal thrust fell off 540 lb, and the moment fell off 23 600 in-lb. at the springings.

During the same period the control specimens shrank, the average value being (see Fig. 79) 0.00010 in. per in. The shrinkage of the concrete in the arch rib, as determined from strain-gauge readings at sections midway between the spandrel columns, at the intrados and extrados (the average of the values for all sections), was 0.000092 in. per in. This corresponds to a change in the length of the 324-in. span of 0.0298 in. Using this value as the change in span that was prevented, the observed changes in reactions was equivalent to a horizontal thrust of 18 100 lb and a moment at the springing of 800 000 in-lb per in. of change in span. These observations cover a period from the time the arch was 16 days old until it was 50 days old.

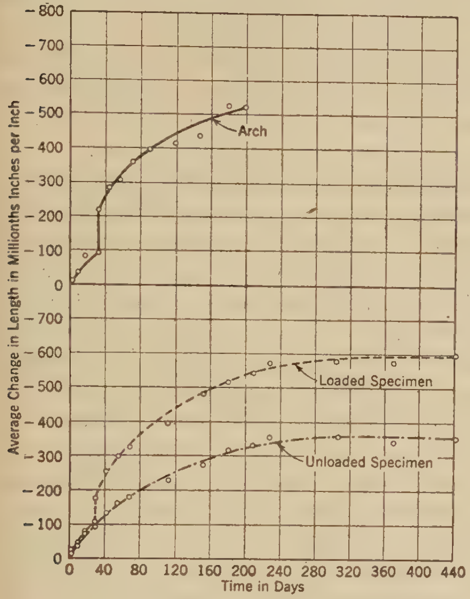


FIG. 79.—UNIT CHANGE IN LENGTH OF ARCH RIB AND CONTROL SPECIMENS DUE TO SHRINKAGE AND TIME YIELD.

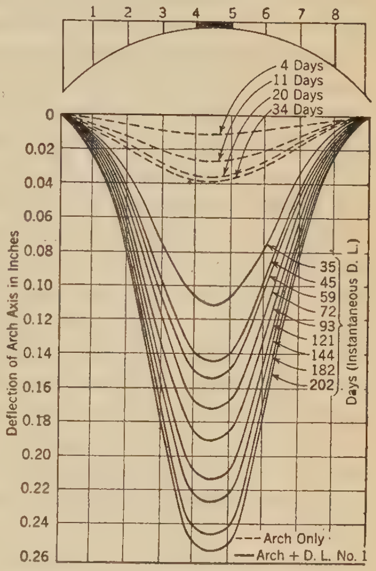


FIG. 80.—DEFLECTION OF ARCH AXIS DUE TO SHRINKAGE AND TIME YIELD.

The tests made to determine the immediate change in the reactions due to a unit change in span, described in Article 8, gave a change in thrust of 25 400 lb and a change in moment of 1 313 000 in-lb per in. of change in span. These observations covered a period from the time the arch was 216 days old until it was 415 days old. The ratio of the latter value of the thrust to the former is 1.40, and the ratio of the latter value of the moment to the former is 1.64. The fact that these ratios exceed unity may have been due in part to the low value of the modulus of elasticity of the concrete during these tests when the concrete was green, or it may have been due to the fact that the time yield in the concrete reduced the flexural stress due to shrinkage; but, whatever the cause, the unit stress due to shrinkage was much less than the stress due to sudden change in span equivalent to the unit shrinkage and based upon a value of E for the aged concrete.

The deflection of the arch rib due to shrinkage of the concrete is shown by the broken lines of Fig. 80. Inasmuch as a deflection diagram is an influence line for horizontal thrust, there should be a definite relation between the lines representing the deflection due to shrinkage, the influence line for horizontal thrust, and the unit shrinkage of the concrete in the rib. These relationships will be indicated for Load Points 4 and 5, the two adjacent to the crown. The experimentally determined influence ordinate for horizontal thrust for these points is 0.88.[3] This is shown graphically in Fig. 45.

The vertical movement due to shrinkage is made up of two parts. One part is due to the direct vertical shrinkage and is equal to the product of the unit shrinkage and the vertical projection of the part of the arch considered.[4] The other part is due to the flexural effect of changing the length of the arch axis without changing the span, and it equals the influence ordinate for the horizontal thrust at the point considered, 0.88 for Load Points 4 and 5, multiplied by the change in span that would have been produced by the shrinkage if the abutments had been free. The sum of these two parts should equal the vertical movement of Load Points 4 and 5. The average unit shrinkage at the end of the 34-day period, as determined by the strain-gauge, was 0.000092 in. per in., corresponding to a deflection in the 324-in. span of 0.0264 in. To this should be added the vertical movement due to direct vertical shortening which, for this case, is $0.000092 \times 80 = 0.0075$ in. The total computed value of the vertical movement is, therefore, $0.0264 + 0.0075 = 0.0339$ in.; the measured value, the average for Load Points 4 and 5, is 0.0380 in. The computed and measured values, respectively, of the vertical movement at Load Points 4 and 5 at the end of 2-day, 9-day, and 18-day periods are 0.003 in. and 0.01 in; 0.014 in. and 0.026 in.; and 0.032 in. and 0.035 in., respectively. The measured value exceeds the computed value in every instance; the difference is about 10% for the 18-day and 34-day periods, the only ones for which the values are great enough to be measured satisfactorily. This difference between measured and computed values is large, relatively, but the quantities themselves are so small that the absolute differences are not greater than the probable tolerance in the measurements and are not considered as having any significance.

Article 7.—Tests to Determine the Effect of Time Yield and Shrinkage of the Concrete Upon the Abutment Reactions.—Dead Load on the Structure.—

---

[3] See Table 9, *Bulletin No. 270*, Eng. Experiment Station, Univ. of Illinois, Urbana, Ill.

[4] "Temperature Deformations in Concrete Arches," by Hardy Cross, M. Am. Soc. C. E., *Engineering News-Record*, February 4, 1926, p. 190.

After the tests to determine the effect of shrinkage in concrete upon the reactions, described in Article 6, had been completed, the same structure was used in a series of tests to determine the effect of combined shrinkage and time yield of the concrete upon the reactions at the springings when the arch carried the dead load. The dead load used in this test (No. 1, Fig. 81) was purposely chosen so as to produce a large moment at the springing.

Preliminary to the tests, the abutments were brought to their normal position as indicated by the instruments provided for the purpose. The scales, the strain, and the vertical position of the points on the arch rib below each spandrel column were read when the structure carried no load except its own weight. The dead load was then applied to the arch, the abutments were returned to their normal position, and the readings were again recorded. The changes in the reactions at the springings due to the application of the dead load are shown in Figs. 77 and 78, and the vertical movement of points on the arch axis is shown in Fig. 80. The application of the dead load changed the moment on the east abutment from −5 000 in-lb to +120 000 in-lb, and on the west abutment from −10 000 in-lb to + 106 000 in-lb. (The plus sign indicates a moment that produces tension at the intrados.) The resulting change in moment at the springing, the average of the two ends, is 113 000 in-lb. The horizontal thrust at the springing increased from 2 500 lb to a little more than 35 500 lb. The stress across the section at the springing, computed from the weighed reactions on the basis that concrete takes tension, varied from 44 lb per sq in., in tension, at the intrados to 620 lb per sq in., in compression, at the extrados.

The dead load remained on the arch for a period of 167 days, and observations were made at frequent intervals during this time. For these subsequent readings, the abutments were returned to their normal position relative to each other so that, in so far as it was possible to bring it about, the only variations that entered to change the abutment reactions were the time yield and the shrinkage of the concrete. Both these actions shorten the axis of the rib. The shrinkage, by shortening the axis when the abutments are fixed, reduces the horizontal thrust and introduces a bending moment at all sections. The time yield of the concrete also shortens the arch axis. In addition to this direct effect of shortening of the axis, some engineers hold that time yield reduces the dead load flexural stress at sections where this portion of the stress is large. It had been hoped that the test would throw some light upon this question, but, if time yield did reduce dead load flexural stresses, the reduction was so small that it could not be detected.

The diagrams of Figs. 77 and 78 show that both the horizontal thrust and the moment at the springing decreased as the test continued. The question of interest is: Are the reductions in the reactions as great as they would have been if the shortening of the arch axis had occurred in a few hours instead of extending over a period of several months?

The change in length of the control specimens, described in Article 4, is given in Fig. 79. Two of the specimens, represented by the lowest diagram, remained unstressed, and the changes in length are attributed to the shrinkage of the concrete. Two other specimens, represented by the broken line, were loaded axially by means of calibrated springs to a unit stress equal, approximately, to the average unit dead load stress in the arch rib. The change in length of the specimens is due partly to shrinkage and partly to time yield. The upper line of Fig. 79 represents the change in length of the rib of the arch as determined from strain readings on four 8-in. gauge lines at sections midway between each pair of adjacent spandrel columns. The upper two lines indicate that the control specimens and the arch rib became shortened by nearly the same amount during the period of this test. The unit shortening of the arch axis was 0.00031 in. per in. This is equivalent to a shortening of the 324-in. span of 0.100 in.

The tests of Article 8 indicate that a quick change in span without any other movement of the abutments and without any change in load, produces a change in moment at the springing of 131 300 in-lb, due to a shortening of the span of 0.100 in. This is comparable to the reduction in the moment of 33 000 in-lb, which actually did take place as determined from the weighed reactions. The tests of Article 8 also indicate that a quick change in span of 0.10 in. produces a change in the horizontal thrust of 2 540 lb. This is comparable to the reduction in thrust of 640 lb which actually did take place; that is, the change in the length of the arch axis that occurred in a little less than six months, beginning when it was a little older than one month, caused a change in both the horizontal thrust and in the moment at the springing only one-fourth as great as would have been produced if the change in the length had taken place quickly.

As in the tests of Article 6, it is not clear whether the reduction in stress is due entirely to time yield or partly to time yield and partly to the fact that the value of E for the concrete was less at this time than later when the elastic constants were determined by a quick change in span; but the arch had been in a warm dry atmosphere for more than a month when the tests of this series were begun. Moreover, the tests from which the change in thrust and moment due to a quick change in span were determined, were made immediately after this series was completed. It would appear safe therefore, to assume that E did not vary greatly between the time of the tests of this series and the time when the elastic constants were determined by a quick change in span.

Article 8.—Tests to Determine the Effect of Time Yield Upon the Reactions, Accompanying a Change in Span.—After the tests described in Article 7 had been completed the load upon the structure was changed as indicated in Fig. 81 by removing 4 000 lb from each of Load Points 4 and 5, the resulting load being designated as Dead Load No. 2. This change was made so that the dead load moment at the springings would be small when the abutments were in their normal position, thereby making possible a considerable change in span without cracking the rib of the arch.

The abutments were brought to their normal position relative to each other, and a complete set of readings was recorded. The span of the arch was then reduced 0.036 in., which is equivalent to a rise in temperature of 20° F, and a second set of readings was recorded. Subsequent readings were taken at intervals of approximately a week for a period of about a month, the span, load, position of the abutments, and the temperature in the laboratory being the same for all readings. The span was then reduced another 0.036 in., and the observations were repeated. Five reductions in the span were made in this manner, each equivalent to a rise in temperature of 20 degrees. A complete set of readings was recorded immediately after each change and at intervals of approximately a week during the periods between changes, which were about a month apart.

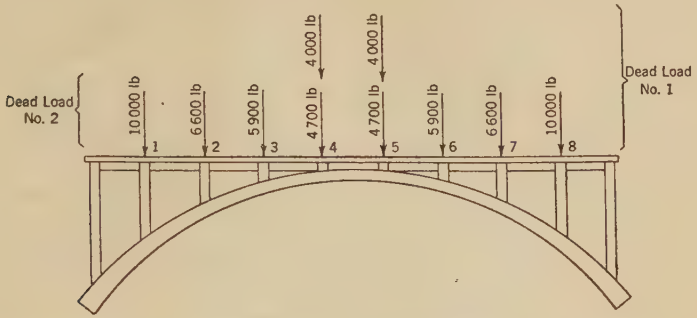


FIG. 81.—DEAD LOAD.

The changes in the reactions are shown in Figs. 82 and 83. The straight vertical parts of these diagrams represent changes that occurred immediately when the span was changed; the parts that slope downward to the right represent the changes that occurred during the period between the changes in span. The latter changes in the reactions are attributed to time yield and shrinkage in the concrete. The concrete was 206 days old when the series began. The unstressed control specimens contracted 0.000025 in. per in. during this period, and this shortening is attributed to shrinkage. The stressed control specimens contracted 0.000059 in. per in. The corresponding free contraction of the arch rib would shorten the span 0.02 in.

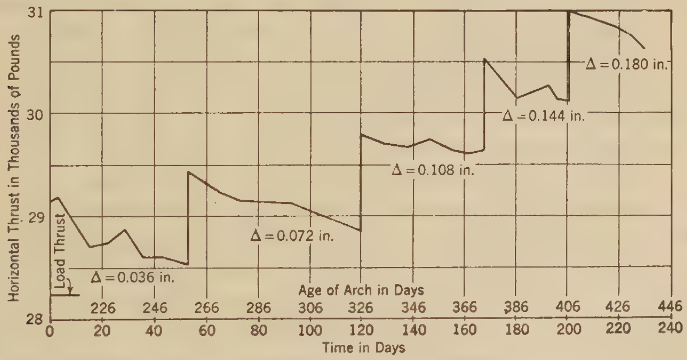


FIG. 82.—CHANGE IN THRUST DUE TO SHRINKAGE AND TIME YIELD FOR VARIOUS CHANGES IN SPAN LENGTH.

Figs. 82 and 83 indicate that a quick change in span of 1 in. is accompanied by a change in the horizontal thrust of 25 400 lb, and a change in the moment at the springing of 1 313 000 in-lb. The total shortening of the span in the period of the test was 0.18 in., of which 0.02 in. may be considered as being due to the free contraction of the rib. The remainder, or 0.16 in., would produce flexure. If this shortening had taken place quickly it would, according to the aforementioned values for unit change in span, have been accompanied by a change in horizontal thrust of 4 060 lb and in the moment at the springing of 210 000 in-lb. The actual change in the thrust was 2 350 lb and in the moment, 136 800 in-lb; that is, for this test, the effect of time yield in the concrete upon the horizontal reaction and moment at the springing, due to a change in span corresponding to a change in temperature of 100° F, in a period of 230 days, was as follows: The horizontal thrust decreased 42% and the moment at the springing decreased 35 per cent. It should be noted, however, that if time yield in concrete decreases with age and with repeated applications of loads (5), then the fact that it caused an appreciable reduction of the deformation stresses in this arch does not necessarily mean that it would have an equal effect in an arch several years old.

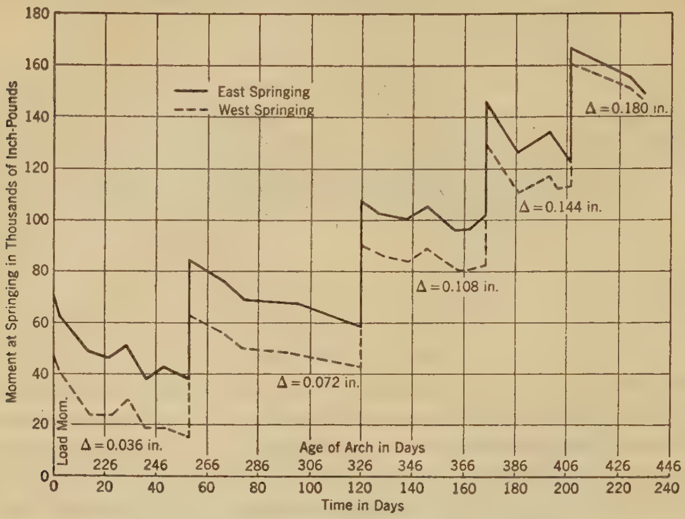


FIG. 83.—CHANGE IN MOMENT DUE TO SHRINKAGE AND TIME YIELD FOR VARIOUS CHANGES IN SPAN LENGTH.

Article 9.—Tests to Determine the Effect of Time Yield Upon Reactions Accompanying Combined Live Load and Abutment Movements.—After the tests described in Article 8 had been completed the abutments were returned to their normal positions preliminary to a series of tests planned to determine the combined effect of live load and abutment movements. The arch was about 480 days old when this series of tests began. Expansion joints were cut in the deck in the panels adjacent to and outside of Load Points 4 and 5.

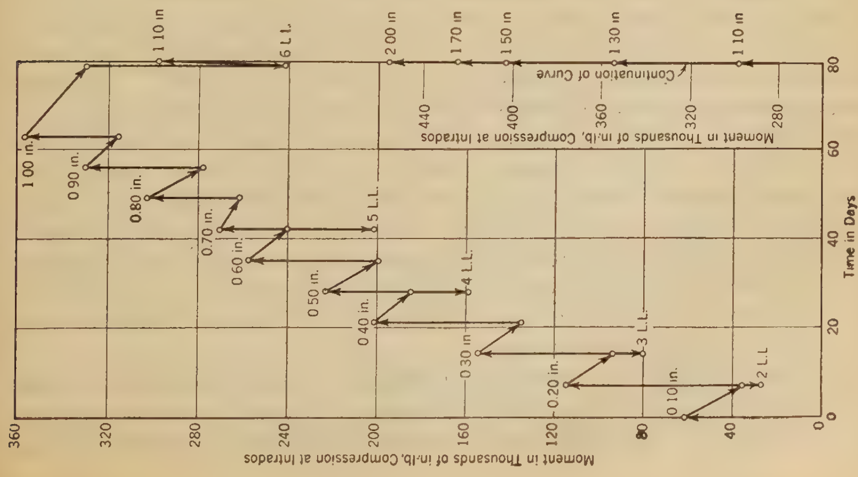


FIG. 84.—RELATION BETWEEN TIME AND MOMENT AT EAST SPRINGING.

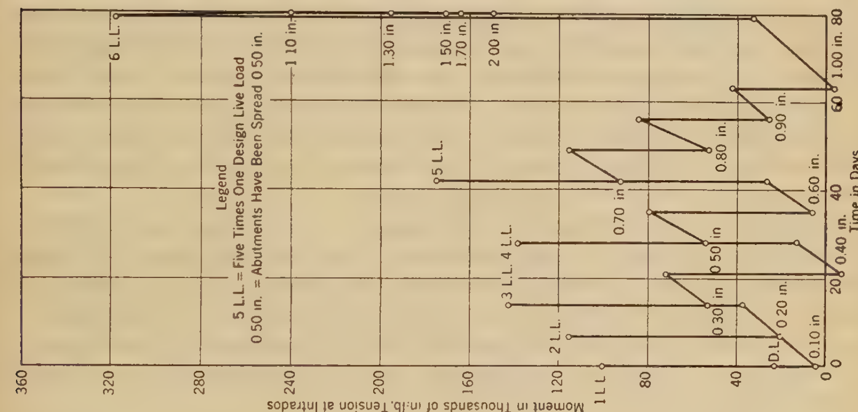


FIG. 85.—RELATION BETWEEN TIME AND MOMENT AT WEST SPRINGING.

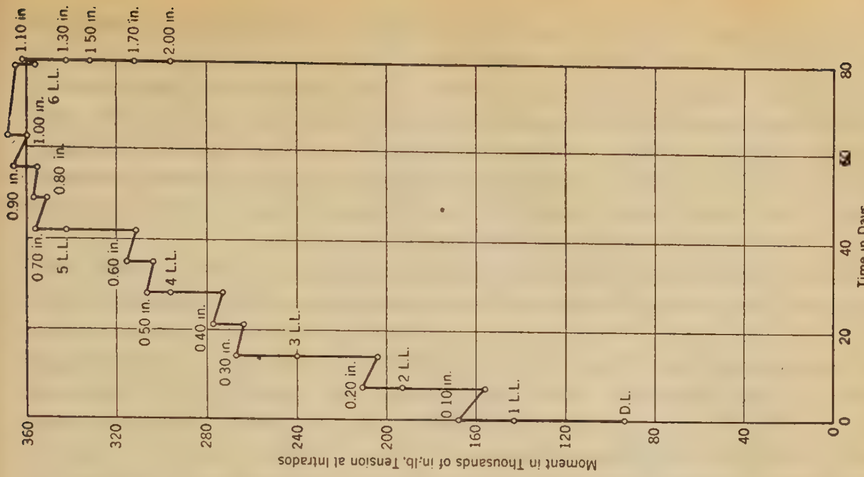


FIG. 86.—RELATION BETWEEN TIME AND MOMENT AT LOAD POINT 4.

This was necessary as the apparatus was not strong enough to break the structure if the deck remained continuous. After the expansion joints had been cut in the deck the weakest section of the rib was at Load Point 4, where the moment due to a change in span is large and its sense the same as the moment due to the live load when the latter is distributed for maximum stress at the section. The design live load used in the tests consisted of 960 lb at Load Point 3 and 2 760 lb at Load Point 4. The two loads together were designated as "one live load" (1 LL).

The initial readings were taken when Design Dead Load No. 2 (Fig. 81), was on the structure and when the abutments were in their normal position relative to each other. The second set of readings was taken after one live load had been applied to the arch and the abutments returned to their normal position. The abutments were then allowed to spread 0.10 in. without rotation or vertical movement, and without change in load. Readings were taken immediately after these changes had been made and again at the end of a week. Changes were made in this manner, increasing the number of live loads and the spread of the abutments until the arch carried six times the design live load, and the abutments had been spread 2.0 in. Readings were taken just before and just after each change and again in seven days, and, for some changes, a third set of readings was taken two weeks or more after the change in span or load. The reactions of the abutments were determined from the scale readings and include the effect of the weight of the structure as well as the superimposed load.

The relation between time and the moment at the springings is shown in Figs. 84 and 85; and the relation between time and the moment at Load Point 4, the point of maximum unit stress, is shown in Fig. 86. In these diagrams the legend, "0.30 in. and 0.40 in.," indicates that the abutments have been allowed to spread 0.30 in. and 0.40 in., respectively; the legend, "2 LL and 3 LL," indicates that the arch carried two times and three times the design live load, respectively.

The moment at the east abutment was negative (compression at the intrados) and that at the west abutment was positive except in two instances. The effect of the time yield was to reduce the negative moment at the east abutment and increase the positive moment at the west abutment. The rate of change was approximately 19 000 in-lb per week and had about the same value at the end, as at the beginning, of the 80-day period covered by this series. It should be noted, however, that the magnitude of the moment was much greater near the end of the period than at the beginning. The total change in the moment at the springing due to time yield was about 170 000 in-lb, and the absolute moment on the morning of the eightieth day, when the arch carried the design dead load and six times the live load, and the abutments were spread 1.00 in., was 240 000 in-lb for the east abutment and 320 000 in-lb for the west abutment.

The time yield in the concrete produced a smaller reduction in the moment at Load Point 4 than at the abutments. The total reduction during the 80-day period was 52 000 in-lb, whereas the absolute moment on the morning of the eightieth day was 356 000 in-lb. The increase in moment at Load Point 4, due to each live load, decreased as the load and the spread increased, the increase in moment for each additional design live load from 1 to 6 being, in 1 000 in-lb, 49, 37, 36, 23, 31, and −9, respectively. Likewise, the increase in moment due to each 0.10-in. increase in span decreased, somewhat irregularly, as the load and spread increased, the increase in moment for each additional increment in span from 0 to 1.1 in. being, in 1 000 in-lb; 25, 18, 27, 13, 11, 13, 14, 6, 11, 10, and 6. The moment at Load Point 4 actually decreased as the spread increased beyond 1.1 in., due to the fact that the concrete had spalled so badly that the section functioned as a hinge.

If each increment of live load had produced the same moment and thrust as the first increment, then the application of six times the live load in addition to the design dead load and the weight of the structure would have produced a unit stress equal to 4 535 lb per sq in. (The unit stress was determined from the thrust and moment on the basis that concrete takes tension. Although this method is not satisfactory, no other method is any more satisfactory for a badly cracked section.) If each 0.10 in. of spread had produced the same moment and thrust as the first increment, the spread of 2.0 in. would have produced a maximum unit stress at the section through Load Point 4 of 5 075 lb per sq in. This added to the hypothetical value of 4 535 lb per sq in., due to load, would make a total of 9 610 lb per sq in., a value greatly in excess of the strength of the concrete. Nevertheless, the arch carried six times the live load for several days after the abutments had been spread 2 in. It is true, however, that there were several large cracks in the rib, deck, and columns, and the concrete in the rib at Load Point 4 was badly spalled. The behavior of the arch was of interest under these extreme conditions, primarily, as demonstrating the "punishment" that a concrete arch can sustain and still carry a large load.

The condition of the structure when carrying four times the design live load with the abutments spread 0.60 in., is of interest. The stress due to load, based upon the assumption that each live load produced the same increment of stress as the first live load, is 3 640 lb per sq in.; and the stress due to the spread of the abutments, based upon a corresponding assumption, is 1 520 lb per sq in.; a total of 5 160 lb per sq in., which is considerably in excess of the strength of the control cylinders. Nevertheless, the structure was not seriously cracked.

The maximum stress as computed from the measured abutment reactions occurred when the abutments were spread 1 in. and the arch carried five times the live load, in addition to the design dead load and the weight of the structure. The maximum unit stress under these conditions, computed from the measured reactions on the basis that concrete takes tension, was 4 480 lb per sq in.

The vertical movements of the load points on the arch rib are shown in Fig. 87. The various deflections are due to changes in load and in span, as indicated. The deflection represented by the difference between a full line and the adjacent broken line is due to time yield.

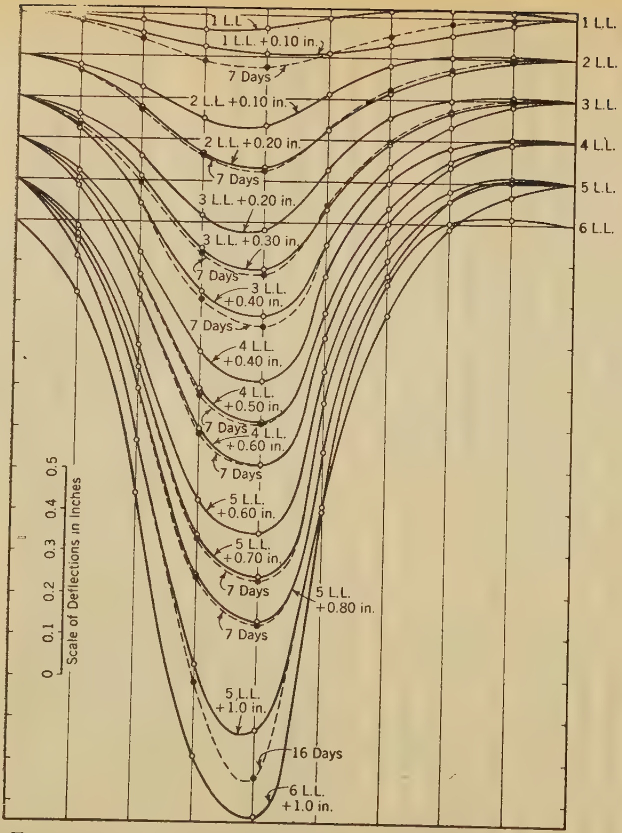


FIG. 87.—RELATION BETWEEN TIME AND DEFLECTION OF ARCH.

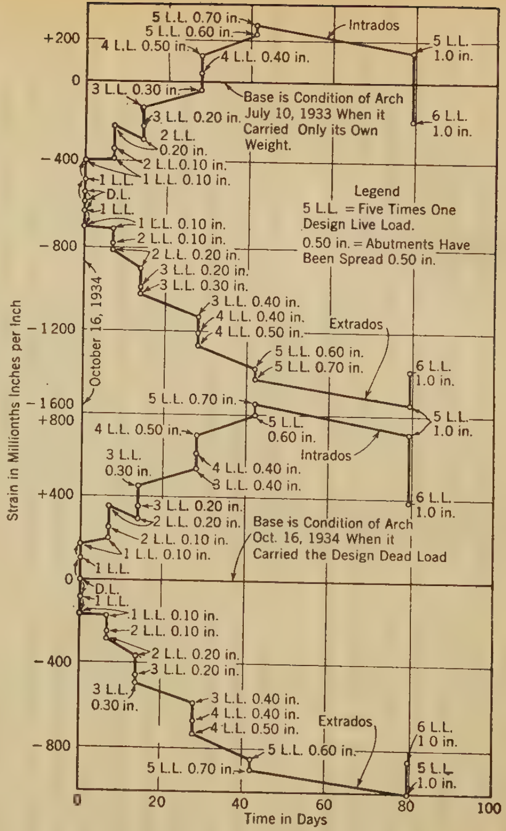


FIG. 88.—RELATION BETWEEN TIME AND STRAIN IN CONCRETE AT SECTION 4.

The strain in the concrete was measured on two gauge lines at the intrados and two at the extrados on a section midway between each pair of adjacent load points. For most sections there were cracks in the concrete that made the strain-gauge readings of little value, but there were no cracks across the gauge lines at Section 4 in the panel just east of Load Point 4 at any time during the test, and the strains on these lines are reported.

Fig. 88 shows the deformation in the concrete at Section 4 during the 80-day period covered by the capacity test. The upper part of the diagram shows the total deformation relative to conditions on July 10, 1933, and the lower part the deformation that occurred during the capacity tests, from October 16, 1934, to January 4, 1935. The deformation at the beginning of the period was −0.000536 in. per in. at the extrados and −0.000567 in. per in. at the intrados. This deformation is due to shrinkage, dead load, and time yield. It is interesting to note that on October 16, 1934, when the arch was approximately 16 months old and carried the design dead load in addition to its own weight, the average unit deformation in the concrete at Section 4 was 0.000552 and nearly uniform over the section. Assuming that the deformation in the steel was the same as in the adjacent concrete, the stress in the steel due to shrinkage, dead load, and time yield, was 16 500 lb per sq in.

Article 10.—Summary.—A single test has little statistical significance and has value primarily in that it may indicate the manner in which a structure functions and direct attention to phenomena that affect its behavior. The fact that time yield in concrete quickly eliminates a large part of the stress in the concrete of an arch that would otherwise accompany early volume changes (shrinkage, initial drop in temperature, and time yield), has been anticipated by rational methods (13) (14). This test supports the results of analysis, and it would appear safe to conclude that a considerable part of the early volume changes have little effect upon the stresses in the concrete of an arch rib. Although time yield of concrete reduces deformation stress in the concrete of an arch, it increases the stress in the steel, a phenomenon that is recognized in the design of reinforced concrete columns. It is believed that the high compression in the steel is not a serious source of weakness, providing the rods are supported against buckling. If time yield in concrete diminishes with age and the number of applications of a load (5), it is possible that temperature stresses in the concrete of arches a few years old may not be greatly relieved by time yield.

The fact that the large gradual change in the span of the arch did not greatly affect its load-carrying capacity has been attributed to the fact that near the ultimate a large gradual increase in strain is accompanied by a small increase in stress.

IX.—Reaction Components for Skew Barrel Arches

Article 1.—Introduction.—The skew barrel arch differs from the ordinary narrow arch rib with fixed ends as ordinarily designed, in that the former has in general six unknown reaction components at each abutment whereas the latter has only three. The skew barrel arch is generally a problem in three-dimensional stress analysis, whereas the narrow arch rib is a problem in co-planar stresses. It is recognized, however, that a right arch of the barrel type, loaded unsymmetrically transversely, is a limiting case of the skew barrel arch.

Article 2.—Object and Scope of Investigation.—Before the investigation was undertaken by the Committee, a paper was published by J. Charles Rathbun, M. Am. Soc. C. E.,[5] in which he developed a new theory of skew arch action by which the six components of the abutment reaction could be computed for any case of assumed loading. He had also made load tests on several plaster models in support of this new theory. Investigation of the nature of skew arch reactions had also been made, before the publication of Professor Rathbun's theory, by the U. S. Bureau of Public Roads.[6] The results of these tests could not be used conveniently as a check of Professor Rathbun's theory.

By reason of the importance of the problem, the Committee decided that it would be wise to build and test several models of skew arches, determine the six reaction components experimentally for various positions of loading, and compare the experimental results thus obtained with computations made independently by Professor Rathbun for the same skew arches. Accordingly, arrangements were made with Professor Rathbun to direct the computation of four 30° skew arches, and with Professor Beggs, to construct and test hard rubber models of the same arches. The computations were made independently at Rapid City, S. Dak., under the direction of Professor Rathbun,

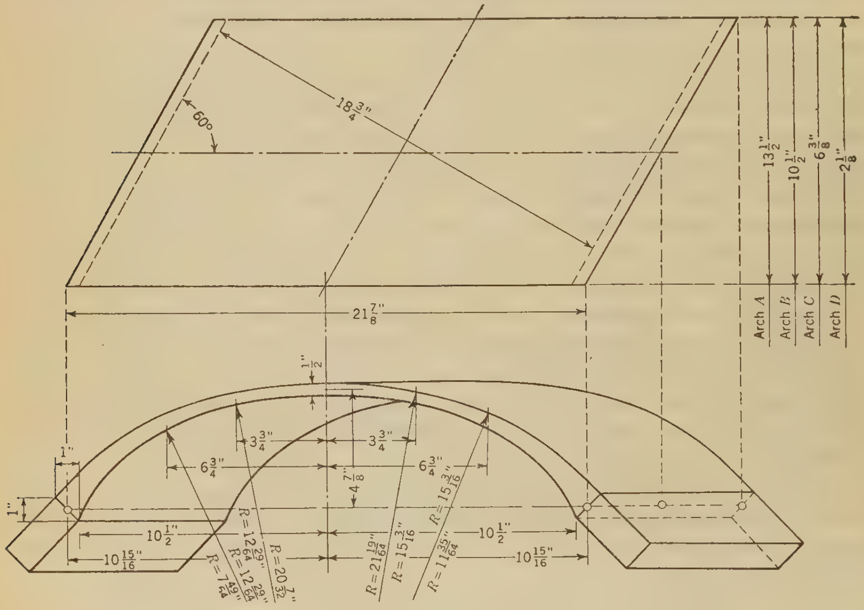


FIG. 89.—DIMENSIONS OF MODEL SKEW ARCHES, *A*, *B*, *C*, AND *D*.

---

[5] *Transactions*, Am. Soc. C. E., Vol 94, (1930) p. 135 and Vol. 98 (1933), p. 1.

[6] *Public Roads*, Vol. 6, No. 9, November, 1925.

while the experimental work was done under the direction of Professor Beggs by Messrs. Oscar L. Updike and John H. Thatcher, Graduate Students at Princeton University, in 1926. On the completion of the theoretical and experimental work, the results were compared for the first time at a meeting of the Committee held at Princeton, N. J., on June 6, 1926, attended also by Professor Rathbun and by Mr. George W. Davis, of the U. S. Bureau of Public Roads, who had been in charge of tests of concrete skew arches for that Bureau. At that time, the comparison of results was made only for the widest one of the four arches investigated. Computations and tests of the other three arches were completed at a later date.

Article 3.—Description of Specimens.—The skew arch models tested had the dimensions shown in Fig. 89. Arch *A* of the series corresponds at one-quarter scale to the concrete skew arch previously tested by the U. S. Bureau of Public Roads. Arches *B*, *C*, and *D* were the same as Arch *A*, except that the transverse width was made less for each successive arch. The arches were made of hard rubber of about the consistency of ordinary combs. One of these arches is shown attached to the testing apparatus in Fig. 90. Considerable investigation was necessary, with the help of the Research Department of the United States Rubber Company, to obtain a model of satisfactory quality and uniformity. The models were shaped in cast-iron moulds with plastic rubber, after which they were vulcanized. It is believed that the models were elastic within the small range of stress to which they were subjected when tested.

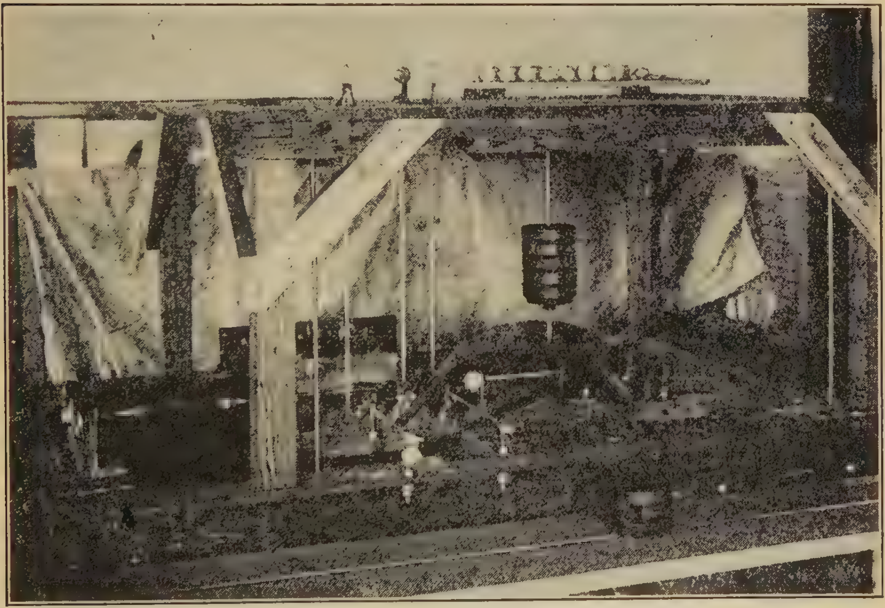


FIG. 90.—MODEL OF ARCH *A* SUBJECTED TO A 50-POUND CONCENTRATED LOAD.

Article 4.—Description of Apparatus.—Views of the testing apparatus are given by the photographs, Figs. 90, 91, and 92. Fig. 90 illustrates how the model was loaded when reaction components were measured by weighing. Figs. 91 and 92 show the arrangement of the apparatus when the reactions were determined by the deformeter method.[7] Practically all the test results were obtained by the deformeter method, which was more convenient and more rapid than the weighing method. However, a reasonable number of results was obtained by the weighing method, in order to provide an independent check of the reliability of the deformeter method.

The main base of the testing machine (Fig. 90) was made of deep **I**-beams riveted together so that the framework was extremely stiff in comparison with the hard rubber model. On the heavy base were mounted pairs of angle clamps, so placed as to receive the abutment ends of the model arch; one of these abutments was attached to a plate fixed to the base, the other to a plate subject to movement with respect to the base and restrained from such motion only by six turnbuckle struts, three of which may be seen in the several photographs while three more are below the movable left-hand abutment plate. These six struts were placed so as to resist the end thrust, side thrust, and vertical load of the abutment, and the three corresponding rotational tendencies of the same abutment, with reference to the chosen X, Y, and Z-axes defined by Figs. 93 to 108, inclusive.

By screwing in or out on one or two of the jack struts, properly selected, the abutment could be shifted a slight amount in any one of the X, Y, or Z directions, independently. Six Ames dials (0.0001-in.) were connected from the fixed frame to the movable abutment so that any motion of translation or rotation of this abutment with reference to the X, Y, or Z-axes might be recorded. When making the deformation test of the model, the vertical displacement of the assumed load points of the skew arch was communicated by wooden rods from the top of the skew arch to the Ames dials. These dial gauges are clearly seen in Fig. 91. When making a load test of a model, the deflection rods connecting to the surface of the arch were removed, and a 50-lb concentrated load was applied in the manner indicated by Fig. 90. In this case, delicate weighing scales (not shown), were placed below the testing frame in such manner that the vertical reaction and two of the abutment moments could be determined by direct weighing. No provision was made for weighing the other three reaction components resisted by the three turnbuckles visible in Fig. 90.

Article 5.—Procedure in Testing.—The results indicated by the influence contours in Figs. 93, 95, 97, 99, 101, 103, 105, and 107, for the model were obtained entirely by the deformeter method of operation. According to this method, the movable abutment was given a small motion (either vertical, outward, lateral, or rotational); then, the gauge dials, having been read before the movement of the abutment, were again read after the movement. The ratio of the arch dial reading to the abutment movement, made independently in the direction of the desired reaction component, multiplied by the load, gave

---

[7] *Transactions*, Am. Soc. C. E., Vol. 88 (1925), p. 1208.

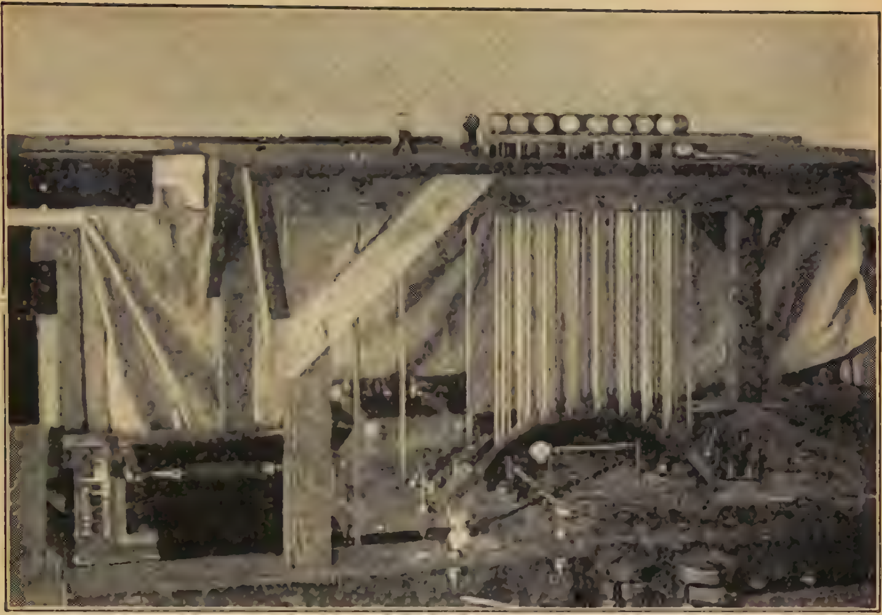


FIG. 91.—TESTING MACHINE FOR DETERMINING INFLUENCE CONTOURS FOR SIX REACTION COMPONENTS BY THE DEFORMETER METHOD.

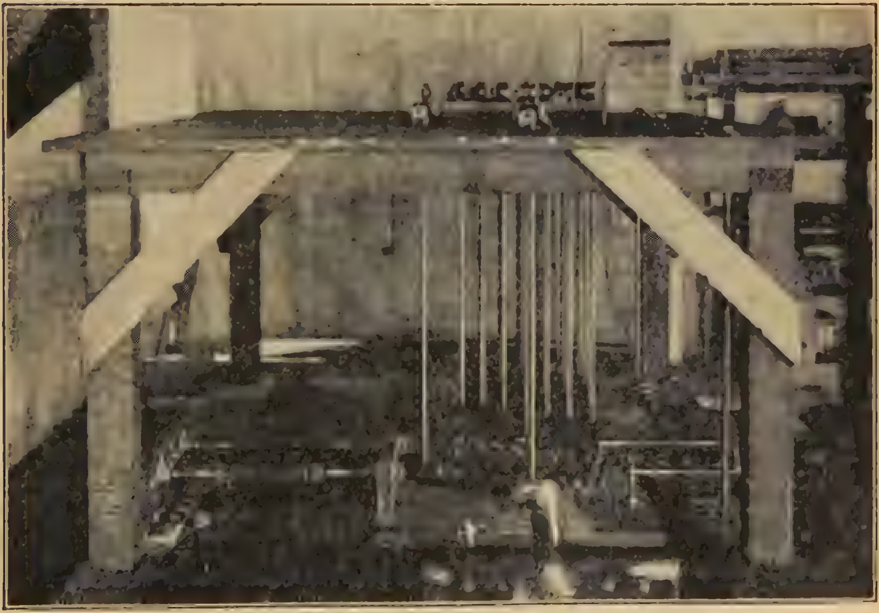
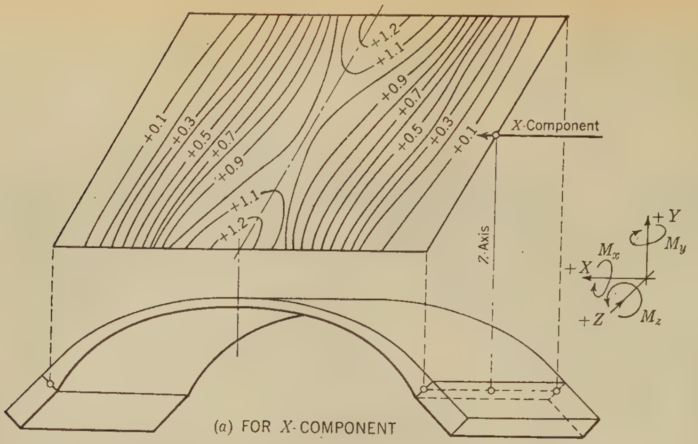
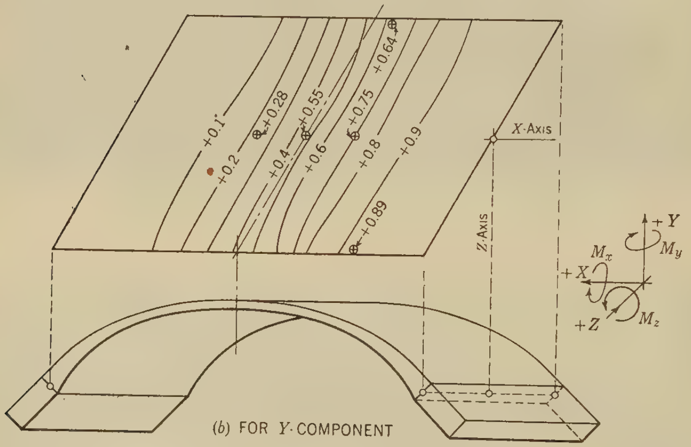


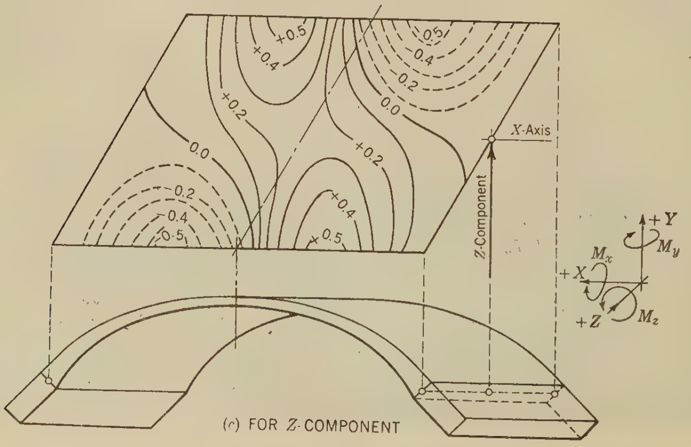
FIG. 92.—END VIEW OF TESTING MACHINE FOR DETERMINING INFLUENCE CONTOURS FOR REACTION COMPONENTS BY THE DEFORMETER METHOD.


+1.2
+1.1
+0.9
+0.7
+0.5
+0.3
+0.1
X-Component
+0.1
+0.3
+0.5
+0.7
+0.9
+1.1
+1.2
Z-Axis
+Y
M_y
M_x
+X
+Z
M_z

(a) FOR X-COMPONENT

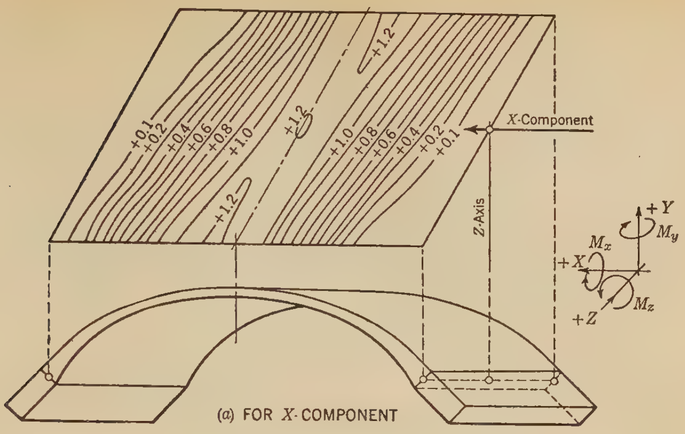

+0.64
+0.1
+0.28
+0.55
+0.75
+0.2
+0.4
+0.6
+0.8
+0.9
+0.89
X-Axis
Z-Axis
+Y
M_y
M_x
+X
+Z
M_z

(b) FOR Y-COMPONENT

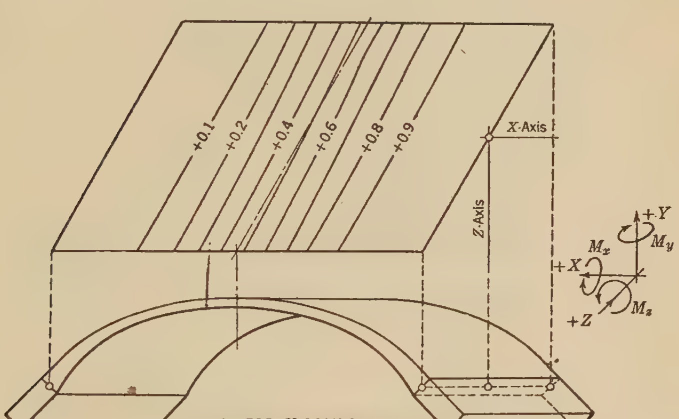

+0.5
+0.4
+0.2
0.5
0.4
0.2
0.0
X-Axis
0.0
+0.2
-0.2
-0.4
-0.5
+0.4
+0.5
Z-Component
+Y
M_y
M_x
+X
+Z
M_z

(c) FOR Z-COMPONENT

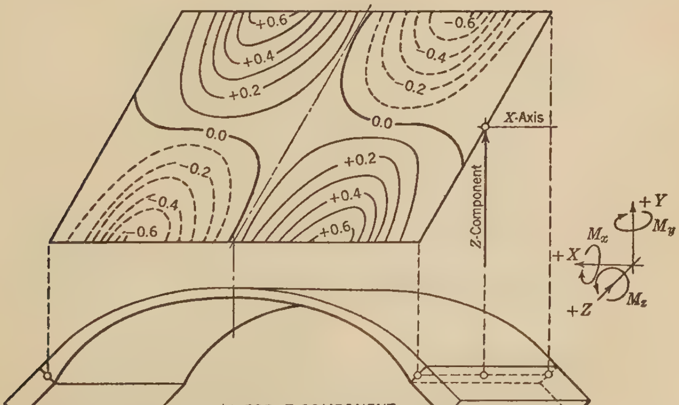
FIG. 93.—ARCH A, INFLUENCE CONTOURS FOR COMPONENTS OF REACTION AS DETERMINED BY MODEL.


+0.1
+0.2
+0.4
+0.6
+0.8
+1.0
+1.2
+1.2
+1.2
+1.0
+0.8
+0.6
+0.4
+0.2
+0.1
X-Component
Z-Axis
+Y
M_y
M_x
+X
M_z
+Z

(a) FOR X-COMPONENT

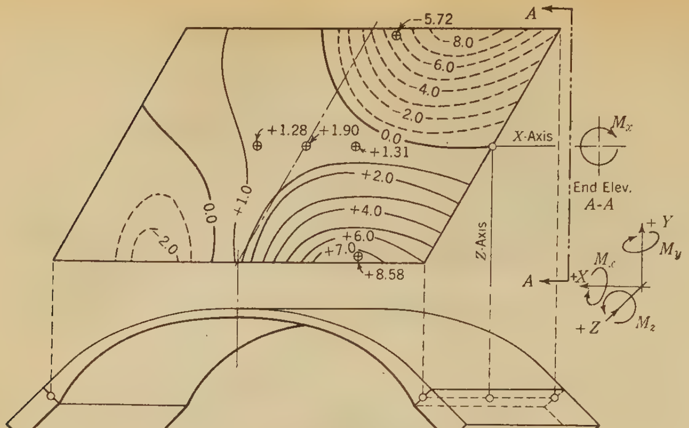

+0.1
+0.2
+0.4
+0.6
+0.8
+0.9
X-Axis
Z-Axis
+Y
M_y
M_x
+X
M_z
+Z

(b) FOR Y-COMPONENT

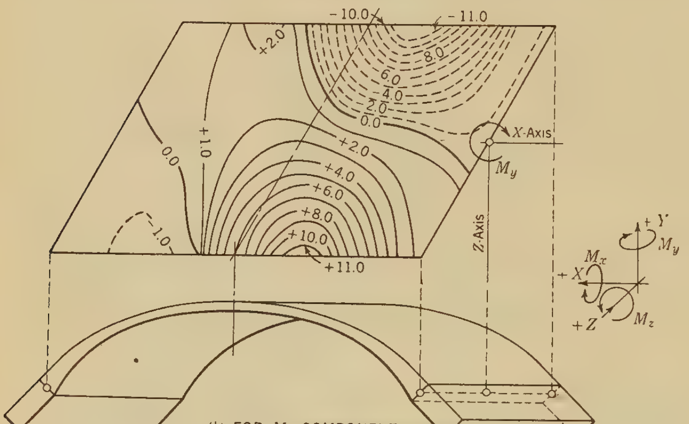

+0.6
+0.4
+0.2
0.0
-0.6
-0.4
-0.2
0.0
-0.2
-0.4
-0.6
+0.2
+0.4
+0.6
X-Axis
Z-Component
+Y
M_y
M_x
+X
M_z
+Z

(c) FOR Z-COMPONENT

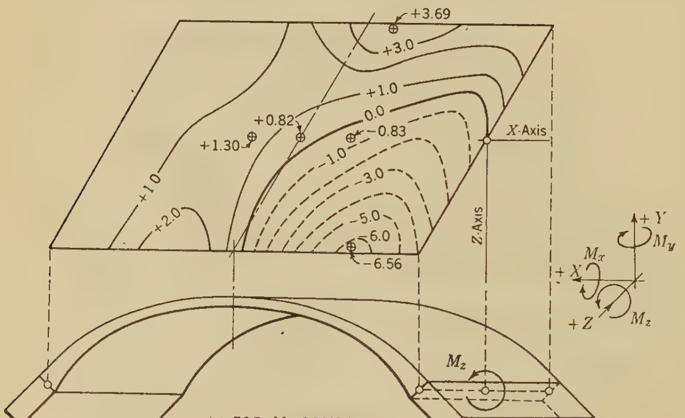
FIG. 94.—ARCH *A*: INFLUENCE CONTOURS FOR COMPONENTS OF REACTION AS DETERMINED BY THEORY.


-5.72
A
-8.0
-6.0
-4.0
-2.0
+1.28
+1.90
0.0
+1.31
X-Axis
M_x
End Elev.
A-A
+2.0
0.0
+1.0
+4.0
-2.0
+6.0
+7.0
+8.58
Z-Axis
+ Y
M_y
M_x
A
+X
+ Z
M_z

(a) FOR M_x-COMPONENT

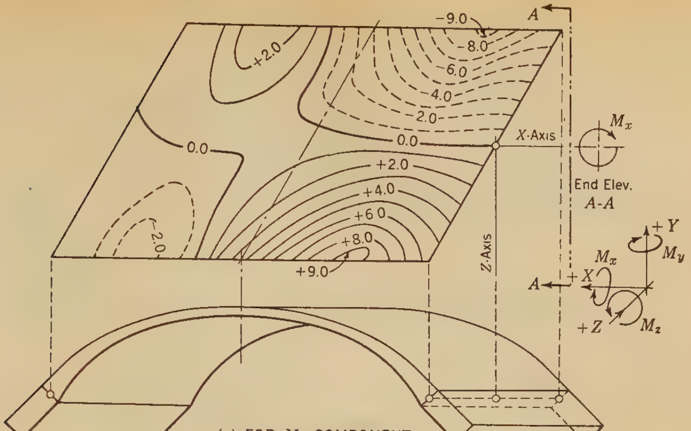

-10.0
-11.0
+2.0
8.0
6.0
-4.0
-2.0
0.0
X-Axis
M_y
0.0
+1.0
+2.0
+4.0
+6.0
+8.0
+10.0
+11.0
-1.0
Z-Axis
+ Y
M_y
M_x
+ X
+ Z
M_z

(b) FOR M_y-COMPONENT

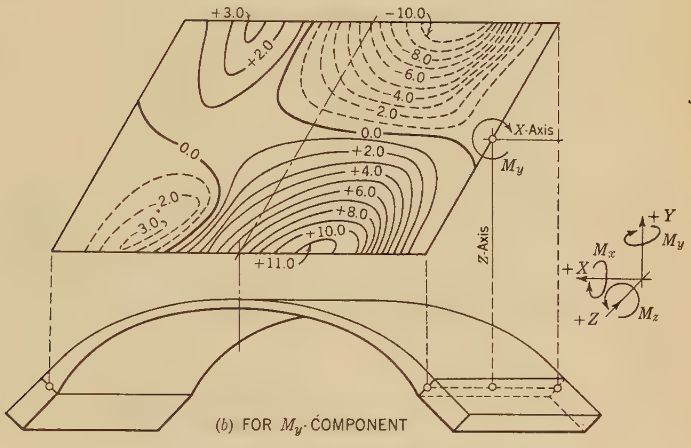

+3.69
+3.0
+1.0
0.0
+0.82
-0.83
+1.30
X-Axis
-1.0
+1.0
-3.0
+2.0
-5.0
-6.0
-6.56
Z-Axis
+ Y
M_y
M_x
+ X
+ Z
M_z
M_z

(c) FOR M_z-COMPONENT

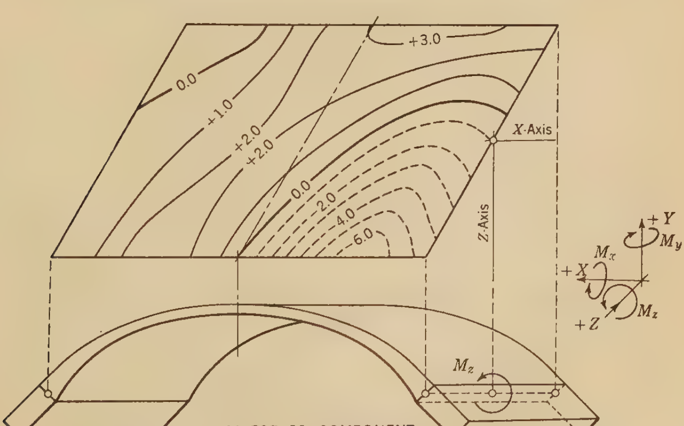
FIG. 95.—ARCH A: INFLUENCE CONTOURS FOR COMPONENTS OF REACTION AS DETERMINED BY MODEL.


−9.0
−8.0
−6.0
−4.0
−2.0
0.0
+2.0
0.0
+2.0
+4.0
+6.0
+8.0
+9.0
−2.0
A
X-Axis
M_x
End Elev.
A-A
Z-Axis
+Y
M_y
M_x
A
+X
+Z
M_z

(a) FOR M_x-COMPONENT

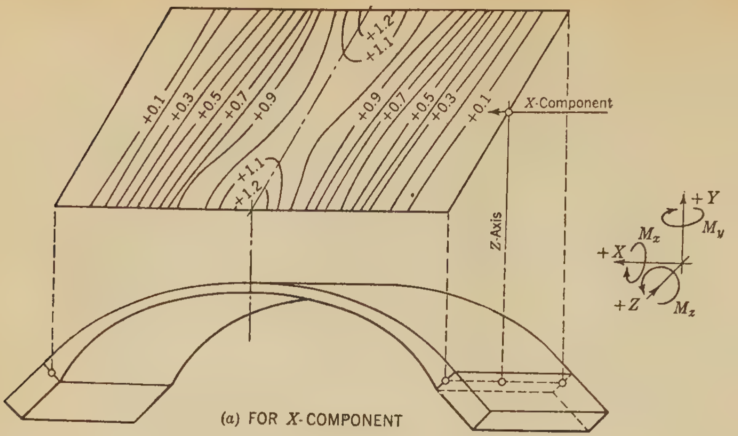

+3.0
+2.0
−10.0
−8.0
−6.0
−4.0
−2.0
0.0
0.0
+2.0
+4.0
+6.0
+8.0
+10.0
+11.0
−2.0
−3.0
X-Axis
M_y
Z-Axis
+Y
M_y
M_x
+X
+Z
M_z

(b) FOR M_y-COMPONENT

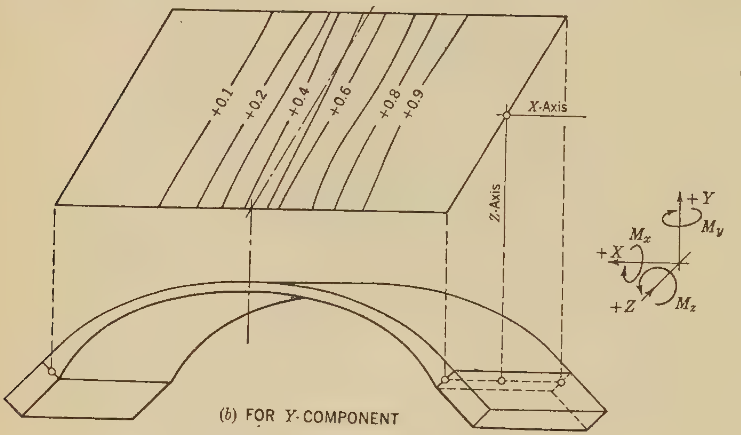

+3.0
0.0
+1.0
+2.0
+2.0
0.0
2.0
4.0
6.0
X-Axis
Z-Axis
+Y
M_y
M_x
+X
+Z
M_z
M_z

(c) FOR M_z-COMPONENT

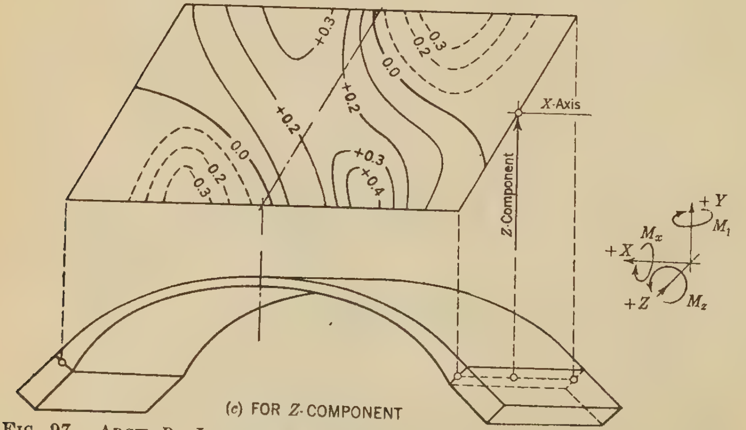
Fig. 96.—Arch A: Influence Contours for Components of Reaction as Determined by Theory.


+1.2
+1.1
+0.1
+0.3
+0.5
+0.7
+0.9
+1.1
+1.2
+0.9
+0.7
+0.5
+0.3
+0.1
X-Component
Z-Axis
+Y
M_y
M_x
+X
+Z
M_z

(*a*) FOR *X*-COMPONENT

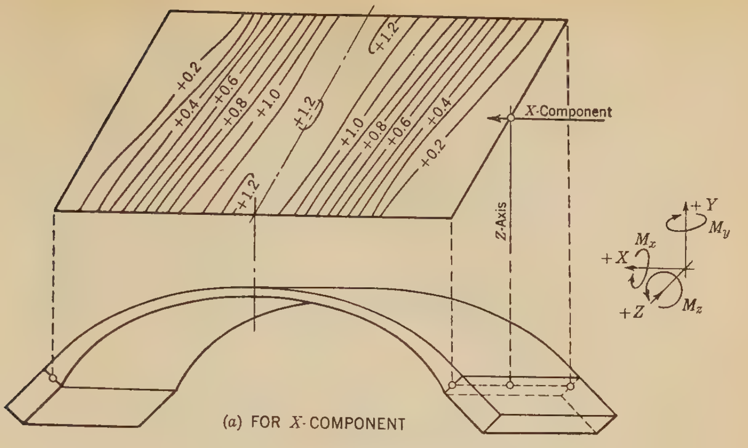

+0.1
+0.2
+0.4
+0.6
+0.8
+0.9
X-Axis
Z-Axis
+Y
M_y
M_x
+X
+Z
M_z

(*b*) FOR *Y*-COMPONENT

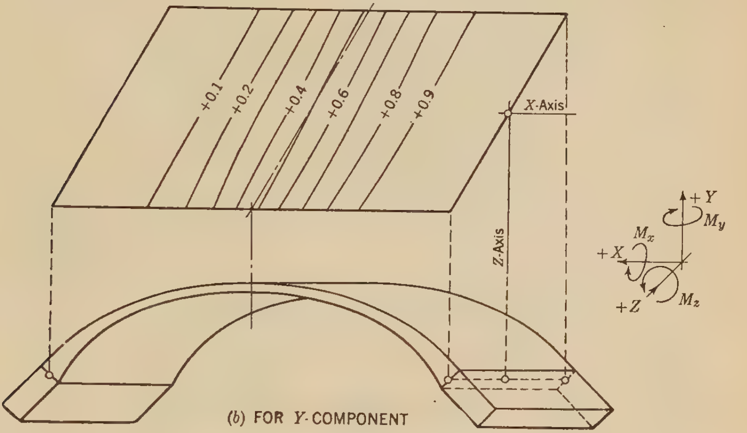

+0.3
-0.3
-0.2
0.0
+0.2
+0.2
0.0
-0.2
-0.3
+0.3
+0.4
X-Axis
Z-Component
+Y
M_1
M_x
+X
+Z
M_z

(*c*) FOR *Z*-COMPONENT

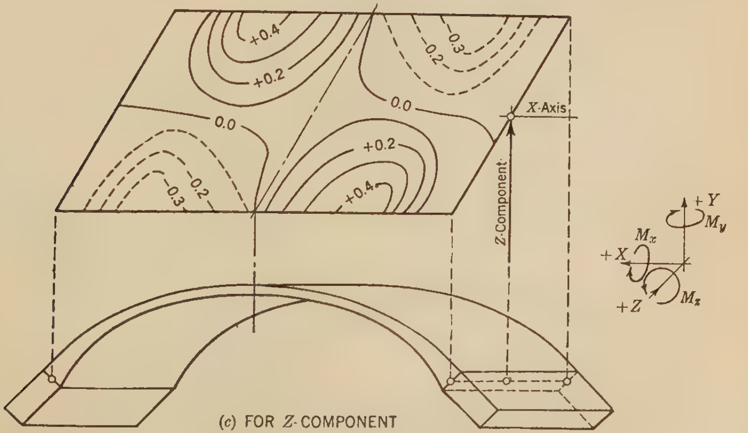
FIG. 97.—ARCH *B*: INFLUENCE CONTOURS FOR COMPONENTS OF REACTION AS DETERMINED BY MODEL.


+0.2
+0.4
+0.6
+0.8
+1.0
+1.2
+1.2
+1.2
+1.0
+0.8
+0.6
+0.4
+0.2
X-Component
Z-Axis
+Y
M_y
M_x
+X
+Z
M_z

(*a*) FOR *X*-COMPONENT

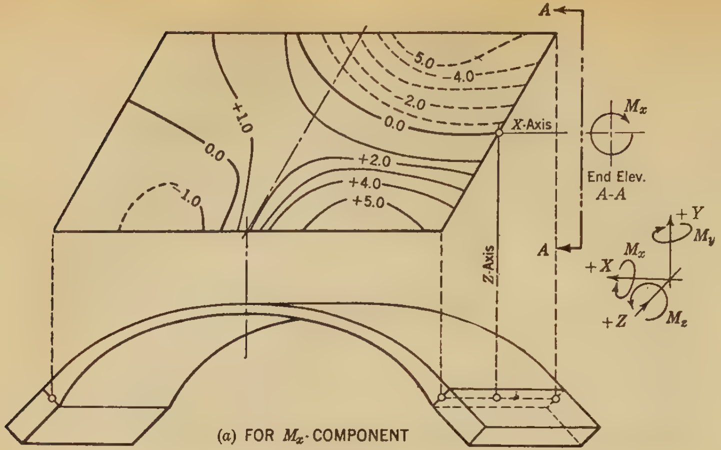

+0.1
+0.2
+0.4
+0.6
+0.8
+0.9
X-Axis
Z-Axis
+Y
M_y
M_x
+X
+Z
M_z

(*b*) FOR *Y*-COMPONENT

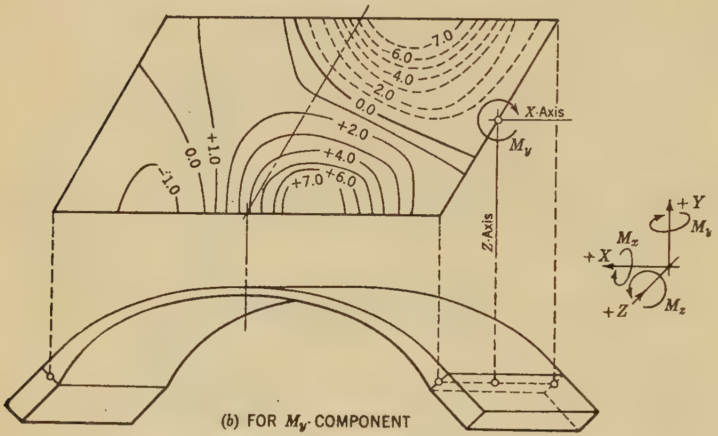

+0.4
+0.2
-0.3
-0.2
0.0
0.0
X-Axis
+0.2
+0.4
-0.2
-0.3
Z-Component
+Y
M_y
M_x
+X
+Z
M_z

(*c*) FOR *Z*-COMPONENT

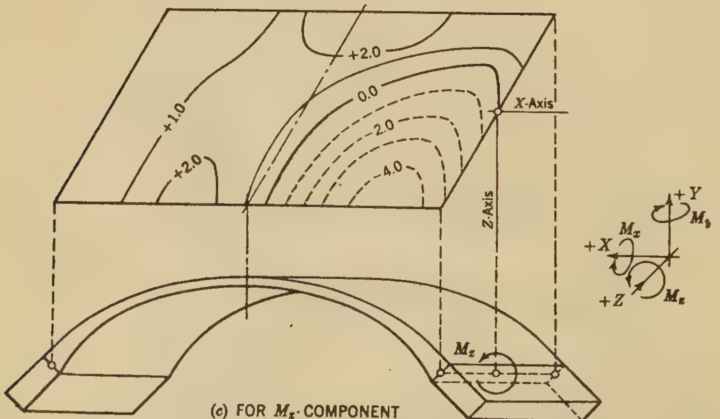
FIG. 98.—ARCH *B*: INFLUENCE CONTOURS FOR COMPONENTS OF REACTION AS DETERMINED BY THEORY.


A
5.0
4.0
2.0
0.0
+1.0
0.0
-1.0
+2.0
+4.0
+5.0
X-Axis
M_x
End Elev.
A-A
A
Z-Axis
+Y
M_y
M_x
+X
+Z
M_z

(a) FOR M_x-COMPONENT

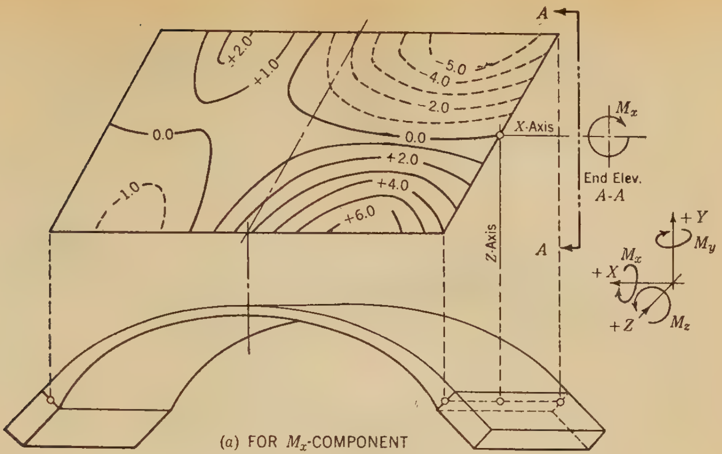

7.0
6.0
4.0
2.0
0.0
+2.0
+4.0
+6.0
+7.0
+1.0
0.0
-1.0
X-Axis
M_y
Z-Axis
+Y
M_y
M_x
+X
+Z
M_z

(b) FOR M_y-COMPONENT

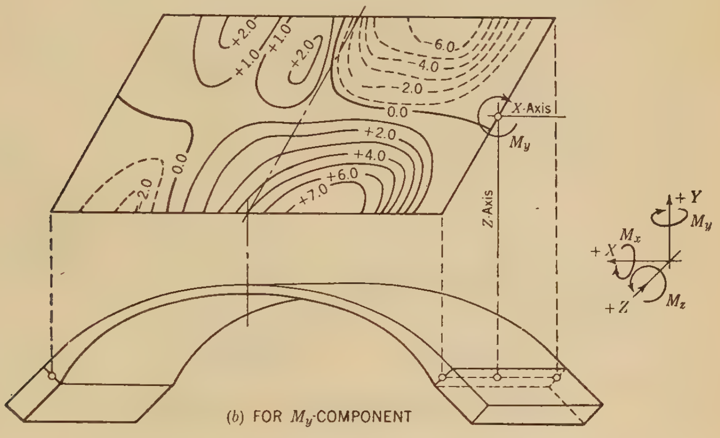

+2.0
0.0
-2.0
-4.0
+1.0
+2.0
X-Axis
Z-Axis
M_z
+Y
M_y
M_x
+X
+Z
M_z

(c) FOR M_z-COMPONENT

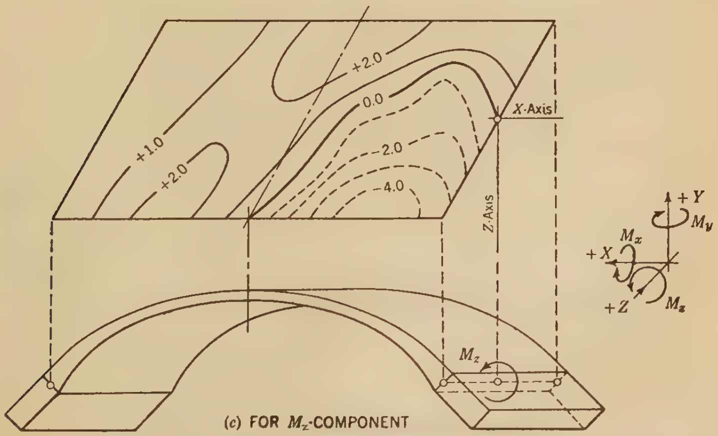
FIG. 99.—ARCH *B*: INFLUENCE CONTOURS FOR COMPONENTS OF REACTION AS DETERMINED BY MODEL.


A
A
+2.0
+1.0
-5.0
-4.0
-2.0
0.0
0.0
+2.0
+4.0
+6.0
-1.0
X-Axis
Z-Axis
M_x
End Elev.
A-A
+Y
M_y
M_x
+X
+Z
M_z

(a) FOR M_x-COMPONENT

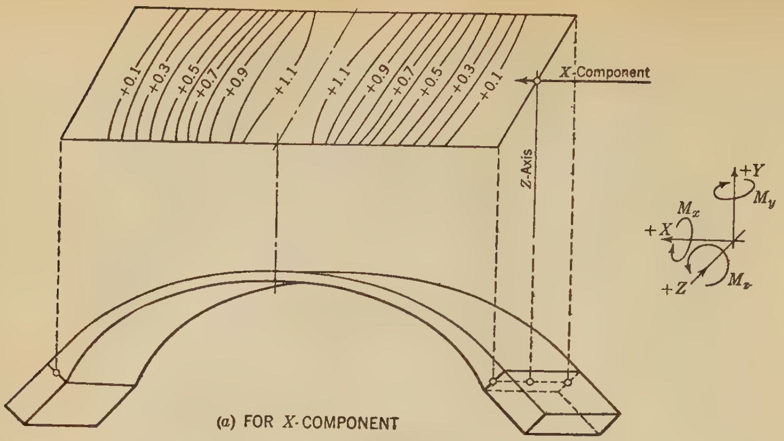

+2.0
+1.0
+1.0
+2.0
-6.0
-4.0
-2.0
0.0
+2.0
+4.0
+6.0
+7.0
0.0
-2.0
X-Axis
M_y
Z-Axis
+Y
M_y
M_x
+X
+Z
M_z

(b) FOR M_y-COMPONENT

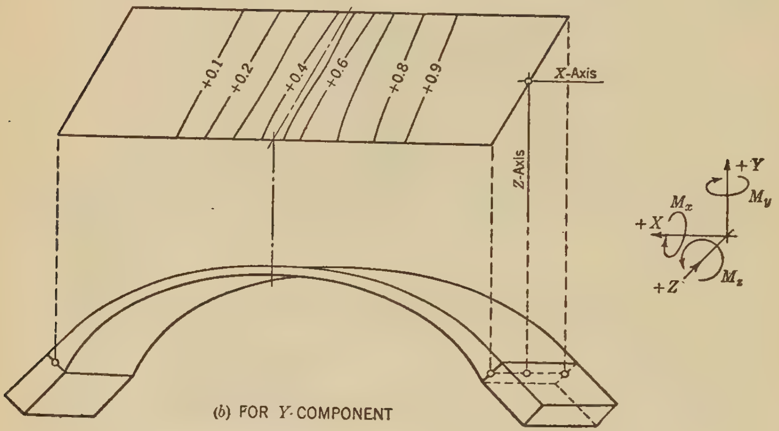

+2.0
0.0
+1.0
+2.0
-2.0
-4.0
X-Axis
Z-Axis
M_z
+Y
M_y
M_x
+X
+Z
M_z

(c) FOR M_z-COMPONENT

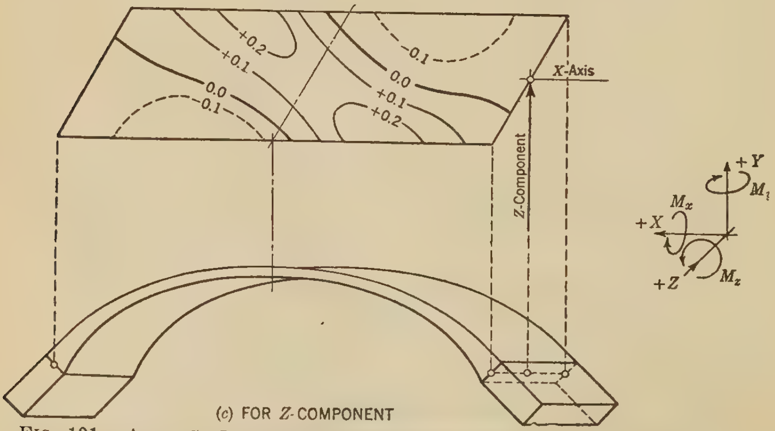
FIG. 100.—ARCH *B*: INFLUENCE CONTOURS FOR COMPONENTS OF REACTION AS DETERMINED BY THEORY.


+0.1
+0.3
+0.5
+0.7
+0.9
+1.1
+1.1
+0.9
+0.7
+0.5
+0.3
+0.1
X-Component
Z-Axis
+Y
M_y
M_x
+X
+Z
M_z

(a) FOR X-COMPONENT

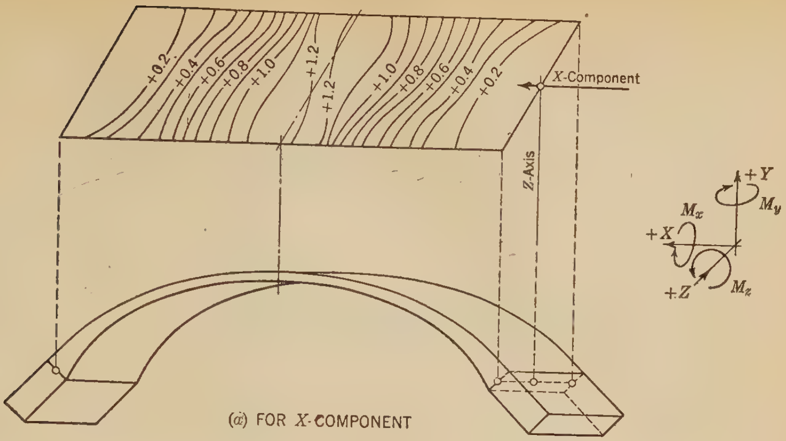

+0.1
+0.2
+0.4
+0.6
+0.8
+0.9
X-Axis
Z-Axis
+Y
M_y
M_x
+X
+Z
M_z

(b) FOR Y-COMPONENT

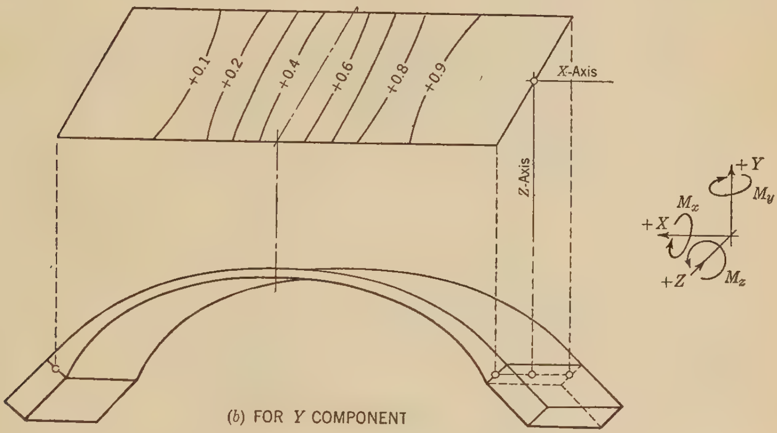

+0.2
+0.1
0.0
-0.1
-0.1
0.0
+0.1
+0.2
X-Axis
Z-Component
+Y
M_y
M_x
+X
+Z
M_z

(c) FOR Z-COMPONENT

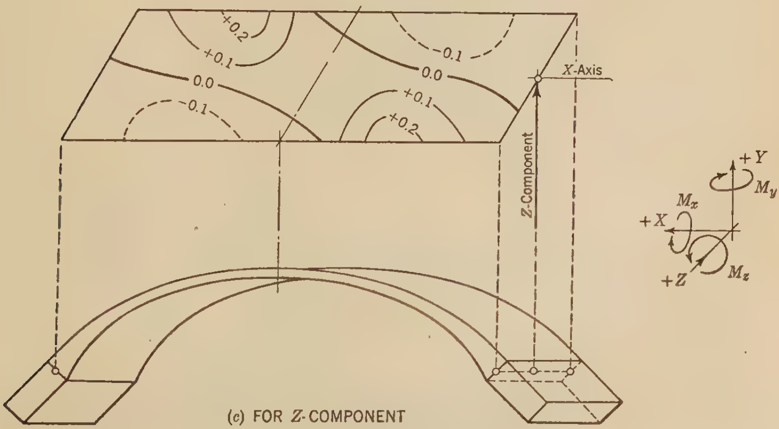
FIG. 101.—ARCH *C*: INFLUENCE CONTOURS FOR COMPONENTS OF REACTION AS DETERMINED BY MODEL.


+0.2
+0.4
+0.6
+0.8
+1.0
+1.2
+1.2
+1.0
+0.8
+0.6
+0.4
+0.2
X-Component
Z-Axis
+Y
M_y
M_x
+X
+Z
M_z

(a) FOR X-COMPONENT

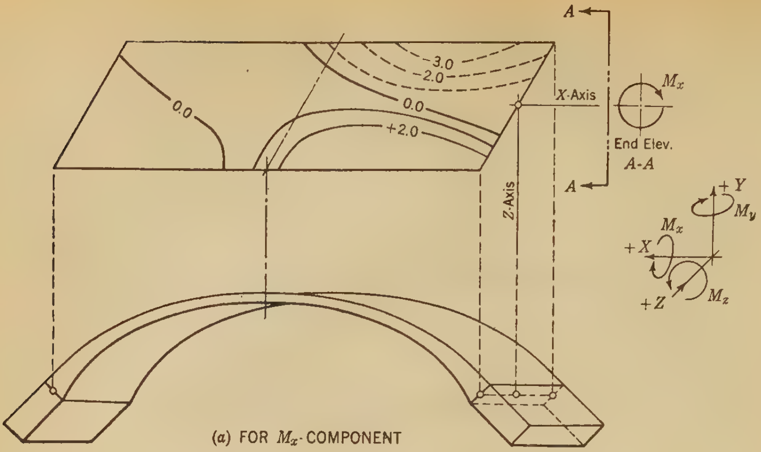

+0.1
+0.2
+0.4
+0.6
+0.8
+0.9
X-Axis
Z-Axis
+Y
M_y
M_x
+X
+Z
M_z

(b) FOR Y COMPONENT

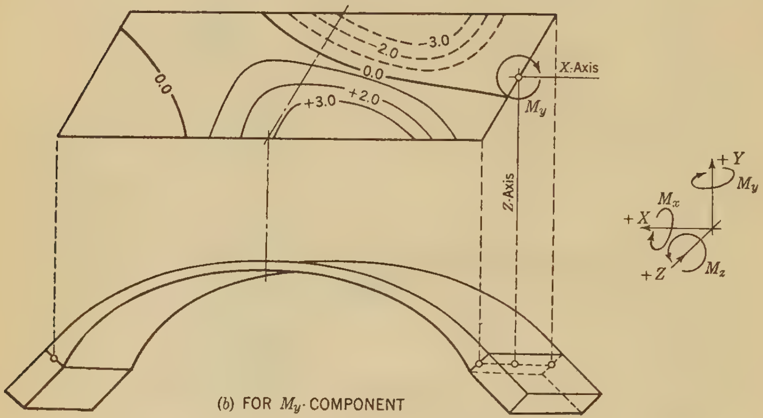

+0.2
+0.1
0.0
-0.1
-0.1
0.0
+0.1
+0.2
X-Axis
Z-Component
+Y
M_y
M_x
+X
+Z
M_z

(c) FOR Z-COMPONENT

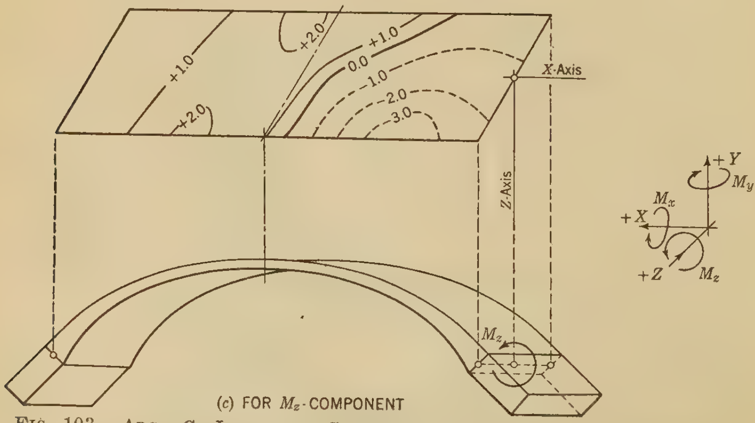
FIG. 102.—ARCH C: INFLUENCE CONTOURS FOR COMPONENTS OF REACTION AS DETERMINED BY THEORY.


A
-3.0
-2.0
0.0
0.0
+2.0
X-Axis
M_x
End Elev.
A-A
A
Z-Axis
+Y
M_y
M_x
+X
+Z
M_z

(a) FOR M_x-COMPONENT

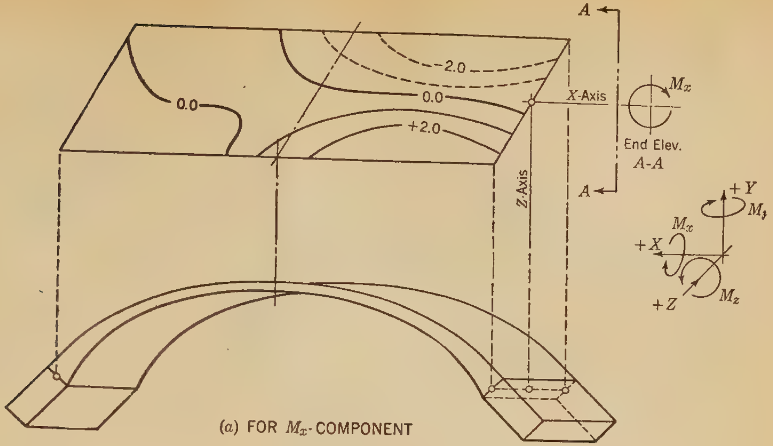

-3.0
-2.0
0.0
0.0
+2.0
+3.0
X-Axis
M_y
Z-Axis
+Y
M_y
M_x
+X
+Z
M_z

(b) FOR M_y-COMPONENT

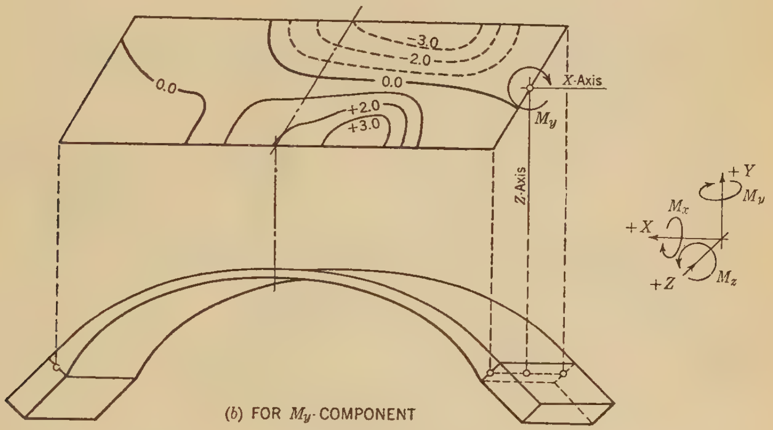

+1.0
+2.0
+2.0
+1.0
0.0
-1.0
-2.0
-3.0
X-Axis
Z-Axis
M_z
+Y
M_y
M_x
+X
+Z
M_z

(c) FOR M_z-COMPONENT

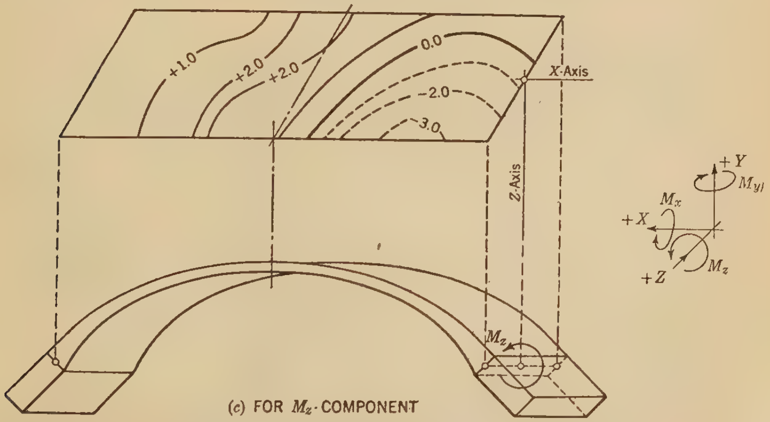
FIG. 103.—ARCH C: INFLUENCE CONTOURS FOR COMPONENTS OF REACTION AS DETERMINED BY MODEL.


A
0.0
−2.0
0.0
+2.0
X-Axis
M_x
End Elev.
A-A
Z-Axis
A
+Y
M_y
M_x
+X
+Z
M_z

(a) FOR M_x-COMPONENT

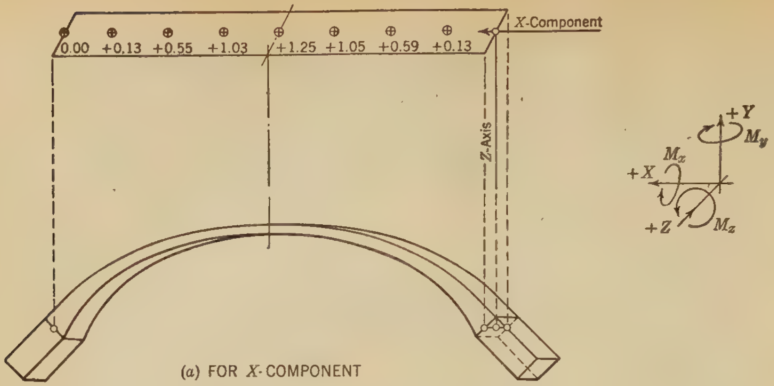

−3.0
−2.0
0.0
0.0
X-Axis
+2.0
+3.0
M_y
Z-Axis
+Y
M_y
M_x
+X
+Z
M_z

(b) FOR M_y-COMPONENT

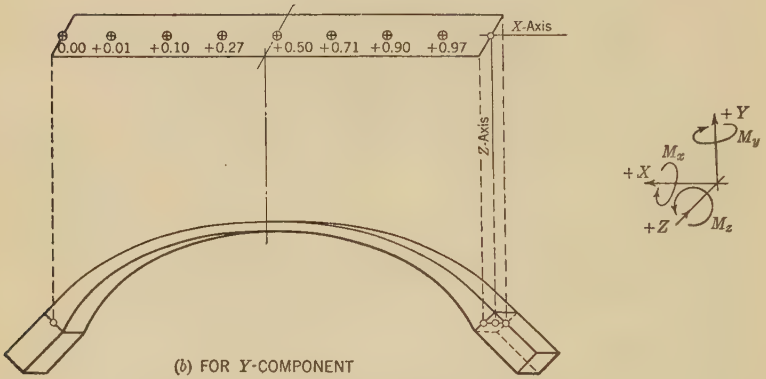

+1.0
+2.0
+2.0
0.0
X-Axis
−2.0
3.0
Z-Axis
+Y
M_y
M_x
+X
+Z
M_z
M_z

(c) FOR M_z-COMPONENT

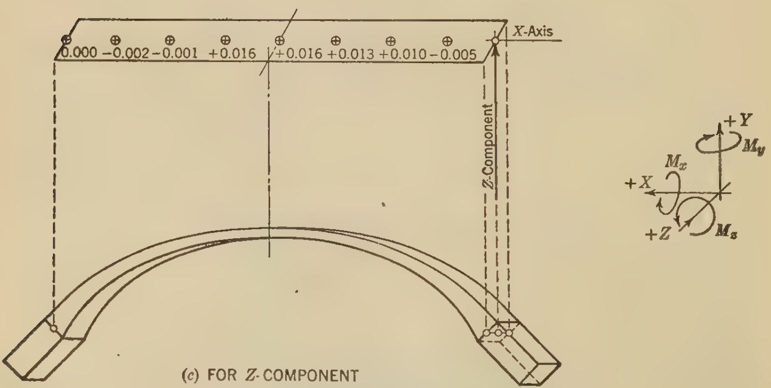
FIG. 104.—ARCH C: INFLUENCE CONTOURS FOR COMPONENTS OF REACTION AS DETERMINED BY THEORY.


0.00
+0.13
+0.55
+1.03
+1.25
+1.05
+0.59
+0.13
X-Component
Z-Axis
+Y
M_y
M_x
+X
+Z
M_z

(a) FOR X-COMPONENT

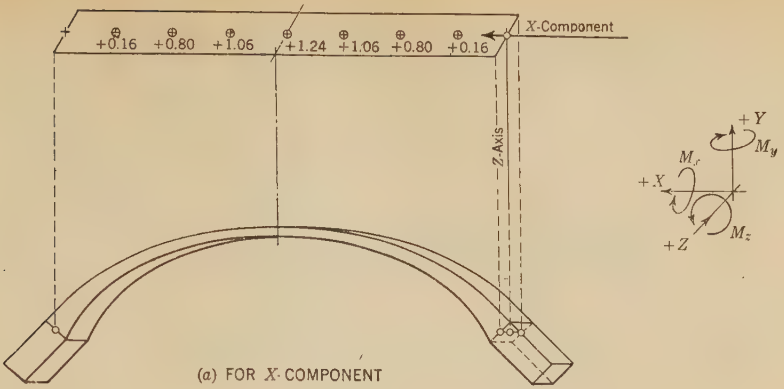

0.00
+0.01
+0.10
+0.27
+0.50
+0.71
+0.90
+0.97
X-Axis
Z-Axis
+Y
M_y
M_x
+X
+Z
M_z

(b) FOR Y-COMPONENT

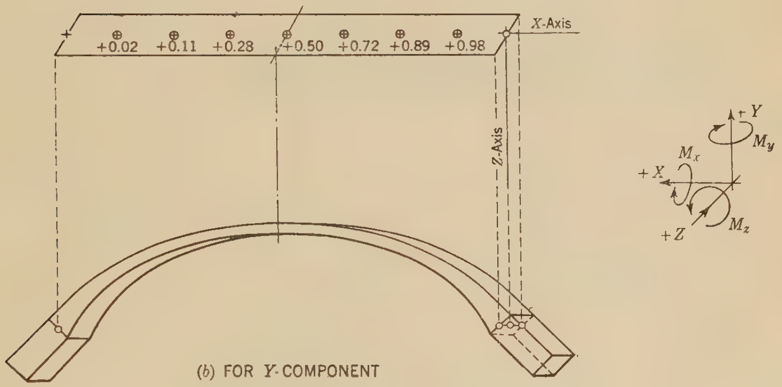

0.000
−0.002
−0.001
+0.016
+0.016
+0.013
+0.010
−0.005
X-Axis
Z-Component
+Y
M_y
M_x
+X
+Z
M_z

(c) FOR Z-COMPONENT

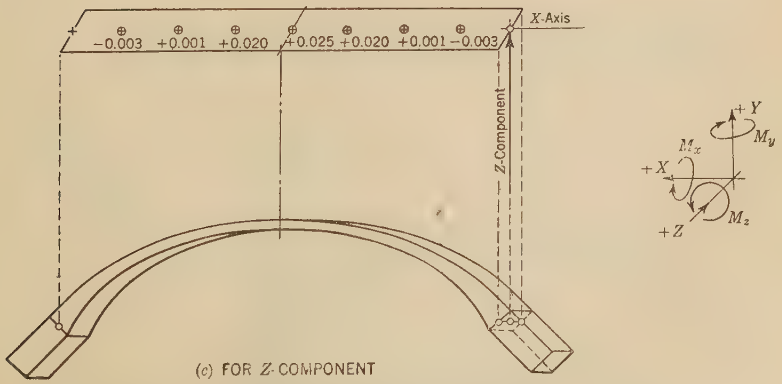
FIG. 105.—ARCH *D*: INFLUENCE CONTOURS FOR COMPONENTS OF REACTION AS DETERMINED BY MODEL.


+
+0.16
+0.80
+1.06
+1.24
+1.06
+0.80
+0.16
X-Component
Z-Axis
+Y
M_y
M_x
+X
+Z
M_z

(a) FOR X-COMPONENT

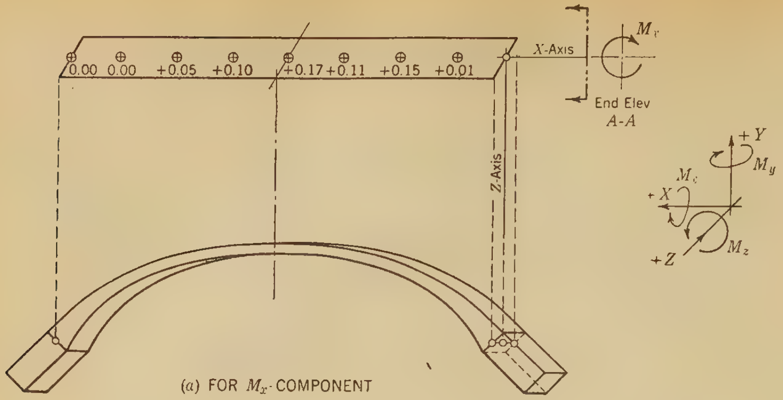

+
+0.02
+0.11
+0.28
+0.50
+0.72
+0.89
+0.98
X-Axis
Z-Axis
+Y
M_y
M_x
+X
+Z
M_z

(b) FOR Y-COMPONENT

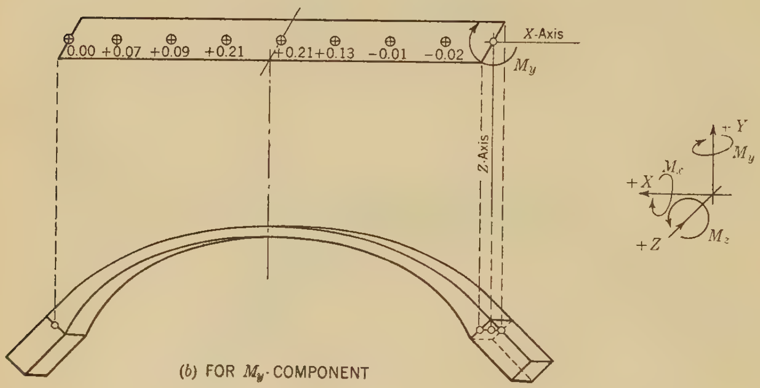

+
−0.003
+0.001
+0.020
+0.025
+0.020
+0.001
−0.003
X-Axis
Z-Component
+Y
M_y
M_x
+X
+Z
M_z

(c) FOR Z-COMPONENT

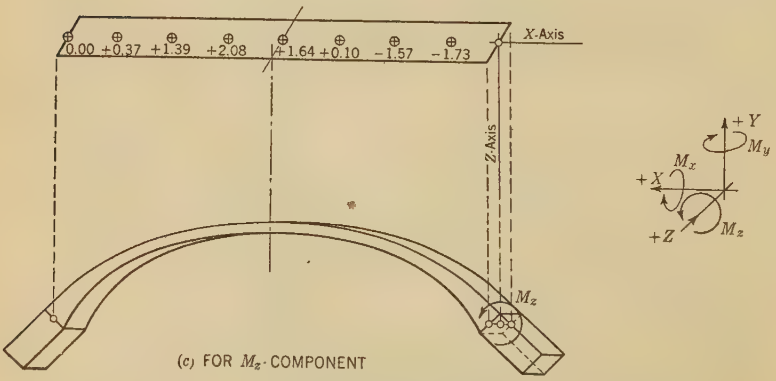
FIG. 106.—ARCH *D*: INFLUENCE VALUES FOR COMPONENTS OF REACTION AS DETERMINED BY THEORY.


0.00
0.00
+0.05
+0.10
+0.17
+0.11
+0.15
+0.01
X-Axis
M_x
End Elev
A-A
Z-Axis
+Y
M_y
M_x
+X
M_z
+Z

(a) FOR M_x-COMPONENT

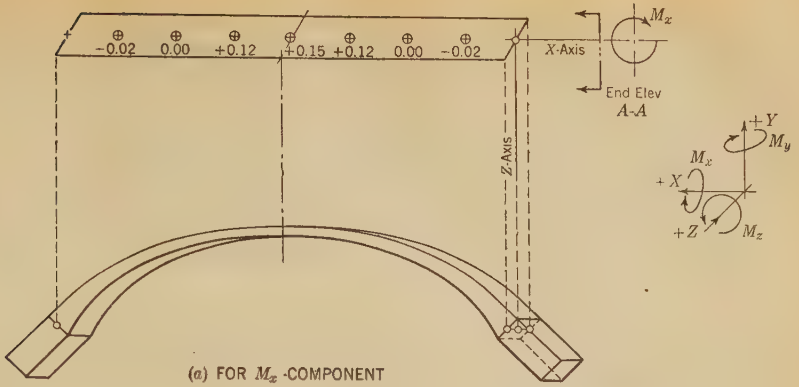

0.00
+0.07
+0.09
+0.21
+0.21
+0.13
−0.01
−0.02
X-Axis
M_y
Z-Axis
+Y
M_y
M_x
+X
M_z
+Z

(b) FOR M_y-COMPONENT

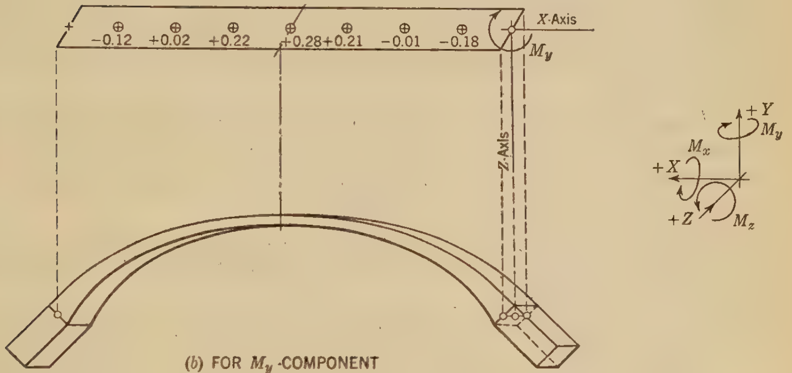

0.00
+0.37
+1.39
+2.08
+1.64
+0.10
−1.57
−1.73
X-Axis
Z-Axis
M_z
+Y
M_y
M_x
+X
M_z
+Z

(c) FOR M_z-COMPONENT

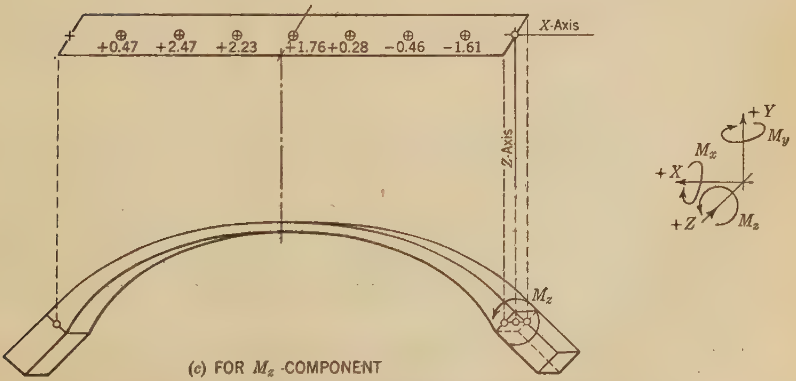
FIG. 107.—ARCH *D*: INFLUENCE VALUES FOR COMPONENTS OF REACTION AS DETERMINED BY MODEL.


+
−0.02
0.00
+0.12
+0.15
+0.12
0.00
−0.02
M_x
X-Axis
End Elev
A-A
Z-Axis
+Y
M_y
M_x
+X
+Z
M_z

(a) FOR M_x-COMPONENT


+
−0.12
+0.02
+0.22
+0.28
+0.21
−0.01
−0.18
X-Axis
M_y
Z-Axis
+Y
M_y
M_x
+X
+Z
M_z

(b) FOR M_y-COMPONENT


+
+0.47
+2.47
+2.23
+1.76
+0.28
−0.46
−1.61
X-Axis
Z-Axis
M_z
+Y
M_y
M_x
+X
+Z
M_z

(c) FOR M_z-COMPONENT

FIG. 108.—ARCH *D*: INFLUENCE VALUES FOR COMPONENTS OF REACTION AS DETERMINED BY THEORY

at once the desired component of the abutment reaction which the load would produce if the abutment were held rigid. By a complete series of such readings of arch deflections corresponding to each of the abutment movements, data were secured from which influence contours were plotted, precisely as topographical contours are plotted from a series of elevation readings. These contours are reproduced in the odd-numbered Figs. 93 to 107.

When all the data for one arch had been secured, the next narrower arch to be tested was produced by cutting the original model to the narrower width by sawing the material from its edges. In this manner Arches *B*, *C*, and *D* were made from the original Arch *A*.

Article 6.—Theoretical Results.—The influence contours corresponding to those determined by the model tests were computed theoretically by Professor Rathbun in accordance with his paper previously cited (18). In these calculations the value of Poisson's ratio was taken as 0.25. Later, a prism was accurately milled from the crown of the hard rubber arch model and carefully tested to determine its physical properties. The results showed the modulus of elasticity in compression to be 625 000 lb per sq in., and the value of Poisson's ratio to be 0.34. In the tests the compression was run as high as 529 lb per sq in., and the stress-strain diagram was found to be a straight line below this limit.

This change in Poisson's ratio from the value used by Professor Rathbun would change the results of his calculations by only a small amount. The usually accepted value of Poisson's ratio for concrete is not more than one-half that found for the hard rubber used in the models.

The theoretical contours as calculated by Professor Rathbun are shown by Figs. 94, 96, 98, 100, 102, 104, 106, and 108, on pages facing the corresponding results of the model tests.

Differences between several weighed values and corresponding values predicted by theory or from deflection measurements, where such differences appear significant, may be charged either to the difficulty of weighing reaction components correctly, as by preventing minute abutment displacement during weighing, or to secondary local deformations caused by the application of a single load, sharply concentrated at the load point, as shown in Fig. 90. Nevertheless, for arches of the proportions tested, the effect of secondary local deformations about a concentrated load do not appear to be of practical importance.

For the practical design of skew arches, the limited number of tests which have been conducted indicate that Professor Rathbun's theory may safely be applied for the computation of the six components of the abutment reactions.

Article 7.—Comparison of Results.—The model results are placed on the left-hand page and the corresponding theoretical results appear on the facing right-hand page (Figs. 93 to 108, inclusive). Contours have been drawn in all diagrams, except for Arch *D*, for which the numerical values of reaction components are recorded.

Reaction components were weighed for Arch *A* only, for several positions of the load along the center and edges of the model. The results obtained by weighing are shown in Fig. 93, by small circles.

Article 8.—Conclusion.—Comparison of the influence contours for the components of reactions as determined by model and by theory and recorded in Fig. 93 to 108, inclusive, indicate that within the limitations of the tests Professor Rathbun's theoretical method for computing the six components of the abutment reaction of a skew arch is reliable.

The comparison of results obtained by direct weighing with those derived by the deformation method also indicates that within the limitations of the test the latter method is correct for determining influence contours by experiment.

Acknowledgments.—The model tests described in this Chapter were planned by the Committee. The experimental work was done by Messrs. John H. Thatcher and Oscar L. Updike under the direction of Professor Beggs in the laboratory contributed by Princeton University. The theoretical computations were made under the supervision of Professor Rathbun.

A number of reconditioned dials were contributed for the laboratory work by the Ames Dial Company. Other dials were loaned by universities and technical schools. The United States Rubber Company co-operated in making many preliminary investigations of material and contributed the rubber for making the models, which were fabricated in its plant. The U. S. Bureau of Public Roads, through Mr. Davis, contributed valuable suggestions as a result of his earlier investigations of skew arch behavior.

X.—COMBINED ALGEBRAIC-AND-EXPERIMENTAL METHOD OF ANALYZING MULTIPLE-SPAN ARCH SERIES

Article 1.—Statement of Problem.—The action of a multiple-span series of arch ribs supported on high piers differs from that of a single span in that, for the latter, the ends of the arch are fixed, whereas, for the former, the fixed points are the outer ends of the end arches and the bases of the intermediate piers. A load on one arch adjacent to a pier, unless balanced by a similar load on the arch on the other side of the pier, causes the top of the pier to rotate and to move horizontally; and these movements introduce stresses in the arch rib.

An analysis of a multiple-span series of arch ribs takes into account the movement of the pier top due to the deformation of the pier. Many methods of making the analysis have been devised. The algebraic method presented in this Chapter, to be used in the combined algebraic-and-experimental method, is offered, not because it is new, or because it is a better method than the others, but because with it, the movements of the pier top are isolated, and the isolation and recognition of the component movements provide an excellent basis on which to develop the combined algebraic-and experimental method, which is the principal object of this Chapter. The method will be developed in detail when applied to a two-span structure and its application to three-span structures will be outlined. Because the method is similar in its conception to the slope-deflection method of analyzing rectangular frames and similar structures, it may well be designated as the slope-deflection method of analyzing multiple-span arches.

Article 2.—Slope-Deflection Method Applied to a Two-Span Arch Bridge.—Fig. 109 represents a two-span structure. Each arch and the structure as a whole are symmetrical about vertical center lines. The structure carries a load represented by *P*. It is required to determine the reaction components at *A*, *B*, and *C*.

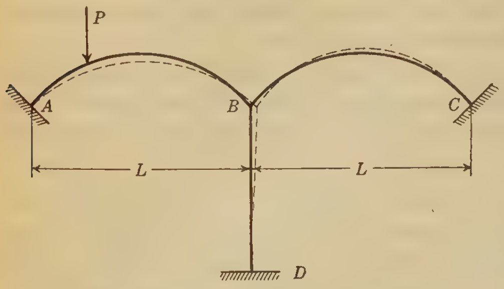


FIG. 109.—TWO-SPAN ARCH BRIDGE.

The dotted line represents the deflected position of the structure. The joints, *A*, *C*, and *D*, are fixed but Joint *B* has moved horizontally a distance represented by x and has rotated through an angle represented by θ. In order to determine the reaction components for a given load it is only necessary to know the elastic constants and the fixed-end reactions for the arch rib, as well as the values of x and θ.

The term, elastic constant, is used to designate the relation between a component of movement (x, y, or θ) of the free end of an arch and the reactions at that end which produce that component and which prevent the others. The force, H (Fig. 110(*a*)), applied on a horizontal line through the elastic center, gives Joint *B* a horizontal motion without either a vertical motion or a rotation. The force per unit spread is one of the elastic constants. If it is designated by H_A, then $H_A = \dfrac{H}{x}$. This force also produces

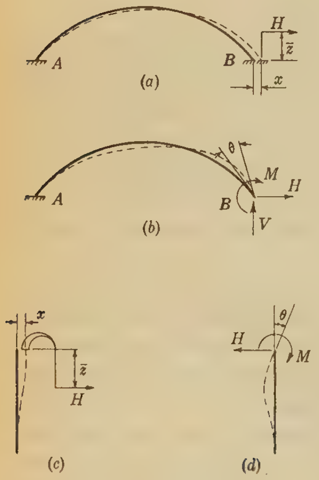

A
B
H
$\bar{z}$
x
(a)
A
B
θ
M
H
V
(b)
x
$\bar{z}$
H
(c)
θ
H
M
(d)

FIG. 110.—ELASTIC CONSTANTS.

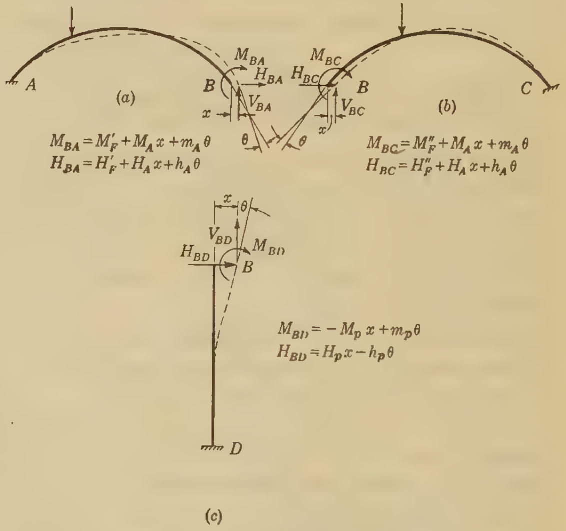

A
(a)
B
M_{BA}
H_{BA}
V_{BA}
x
θ
$M_{BA} = M'_F + M_A x + m_A \theta$
$H_{BA} = H'_F + H_A x + h_A \theta$
M_{BC}
H_{BC}
B
V_{BC}
x
θ
C
(b)
$M_{BC} = M''_F + M_A x + m_A \theta$
$H_{BC} = H''_F + H_A x + h_A \theta$
x
θ
V_{BD}
H_{BD}
M_{BD}
B
$M_{BD} = -M_p x + m_p \theta$
$H_{BD} = H_p x - h_p \theta$
D
(c)

FIG. 111.—REACTION COMPONENTS AT SPRINGINGS OF ARCHES, TWO-SPAN STRUCTURE.

a moment, $H\bar{z}$, and the moment due to unit spread, designated as M_A, is given by the equation, $M_A = \frac{H\bar{z}}{x}$. The quantity, M_A, is another elastic constant of the arch. Fig. 110(*b*) shows the reaction components that rotate Joint *B* through an angle, θ, without causing any translation. The elastic constants corresponding to these reaction components are: $m_A = \frac{M}{\theta}$; $v_A = \frac{V}{\theta}$; and $h_A = \frac{H}{\theta}$.

The moment at Joint *A* due to unit rotation at Joint *B*, designated as m_A', is equal in magnitude to the algebraic sum of the moments of *H*, *V*, and *M*, about Joint *A*, divided by θ. Its sense is such as to cause it to balance the moments of *H*, *V*, and *M*. The deformed pier is shown in Fig. 110(*c*) and Fig. 110(*d*), and the elastic constants are: $H_p = \frac{H}{x}$; $M_P = \frac{-H\bar{z}}{x}$; $h_P = \frac{-H}{\theta}$; and, $m_P = \frac{M}{\theta}$. The elastic constants may be summarized as follows:

H_A = horizontal thrust at end of arch due to a unit spread;
h_A = horizontal thrust at end of arch due to unit rotation;
M_A = moment at end of arch due to unit spread;
m_A = moment in arch at end rotated due to unit rotation;
m'_A = moment in arch at end opposite to the one rotated due to unit rotation;
H_P = horizontal thrust at pier top due to unit horizontal movement;
h_P = horizontal thrust at pier top due to unit rotation;
M_P = moment at pier top due to unit horizontal movement; and,
m_P = moment at pier top due to unit rotation.

As an aid in remembering these elastic constants: The subscripts, *A* and *P*, refer to the arch and pier; the capital letters refer to quantities resulting from a horizontal movement, and the small letters, to those resulting from a rotation; and *H* or *h* and *M* or *m* refer to horizontal thrust and moment. If forces are in pounds and distances in inches, then moments are in inch-pounds. Angles are expressed in radians; clockwise moments and rotations are positive, and forces and movements to the right are positive.

Fixed-end thrusts and moments are designated as follows:

H'_F = fixed-end horizontal thrust, left-hand span;
H''_F = fixed-end horizontal thrust, right-hand span;
M'_F = fixed-end moment at End *B* of *AB*; and,
M''_F = fixed-end moment at End *B* of *BC*.

Referring again to Fig. 109, if a load, *P*, is applied to the arch, the point, *B*, will move horizontally a distance, *x*, and will rotate through an angle, θ. Consider all members intersecting at Joint *B* to be cut as shown in Fig. 111. The reaction components at Joint *B* for the three free bodies are as given in Fig. 111. The arrows represent positive quantities. The equations are true when *x* and θ represent either positive or negative movements. The fixed-end thrust, H'_F, is negative and H''_F is positive, except for loads near Joint *B*;

the fixed-end moments, M'_F and M''_F may be either positive or negative, depending upon the location and distribution of the load.

Consider Point B a free body. It is in equilibrium under the action of forces equal and opposite to the foregoing. Summation H and Summation M, give $-H_{BA} - H_{BC} - H_{BD} = 0$ and $-M_{BA} - M_{BC} - M_{BD} = 0$. Substituting the values for these quantities as given in the preceding paragraph and multiplying by -1:

$$x\,[2\,H_A + H_P] + \theta\,[2\,h_A - h_P] = -H'_F - H''_F \ldots\ldots\ldots (1)$$

and,

$$x\,[2\,M_A - M_P] + \theta\,[2\,m_A + m_P] = -M'_F - M''_F \ldots\ldots\ldots (2)$$

In Equations (1) and (2), the coefficients of x and θ and the right-hand members of the equations are functions of the physical and geometrical properties of the pier and of the arch span and they, or their equivalents, must be known if the structure is to be analyzed by other than experimental methods. With these quantities known, x and θ can be determined from Equations (1) and (2). With θ and x known, the reaction components can be obtained by the use of equations given in Fig. 111.

Equations (1) and (2) are the basic slope-deflection equations for a two-span arch bridge for which each span and the structure as a whole are symmetrical about vertical center lines. Similar equations can be derived for unsymmetrical structures. The coefficients of x and θ are the elastic constants of the arch and pier, and the terms in the right-hand member of the equations are fixed-end reaction components for the arch.

The piers of a bridge are usually simple geometrical bodies and their elastic constants can be determined algebraically. Likewise, the elastic constants and the fixed-end reactions can be computed readily for an arch rib without deck. The elastic constants for both the rib and the pier, as determined by the elastic theory, are given by the following equations:

$$H_A = \frac{E}{\sum \frac{y^2\,ds}{I}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

$$\bar{z} = \frac{\sum \frac{y\,ds}{I}}{\sum \frac{ds}{I}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

$$M_A = h_A = \bar{z}\,H_A \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

$$v_A = \frac{L\,E}{2\sum \frac{x^2\,ds}{I}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

$$m_A = \frac{v_A\,L}{2} + \bar{z}\,h_A + \frac{E}{\sum \frac{ds}{I}} \ldots\ldots\ldots\ldots\ldots\ldots (7)$$

$$m'_A = v_A L - m_A \quad \text{(8)}$$

$$H_P = \frac{E}{\sum \frac{y^2\, ds}{I}} \quad \text{(9)}$$

$$M_P = h_P = \bar{z} H_P \quad \text{(10)}$$

and,

$$m_P = \frac{E}{\sum \frac{ds}{I}} + \bar{z}\, h_P \quad \text{(11)}$$

in which x and y are measured from horizontal and vertical axes through the elastic center of the arch or pier; and z and $\bar{z}$ are measured from a horizontal axis through the springings of the arch.

In Equation (5), M_A and h_A, and in Equation (10), M_P and h_P, are numerically equal, but are expressed in different units. The expressions for the fixed-end reaction components for an arch rib are given in texts on the analysis of arches.

The elastic constants for an arch span consisting of a rib and deck are not readily susceptible of mathematical analysis; but for these structures, the elastic constants and the fixed-end reactions can be obtained from tests of elastic models, and these experimentally determined values can be used in Equations (1) and (2) in the same manner as values determined by a mathematical analysis. Moreover, the values for the arch span may be determined experimentally and those for the piers by the elastic theory, providing the moduli of elasticity of the materials in the arch and in the model are known.

The analysis of a multiple-span arch by the use of formulas similar to Equations (1) and (2), in which experimentally determined elastic constants and fixed-end reaction components are used, is what has been designated the combined algebraic-and-experimental method of analyzing multiple-span arches.

An experimental method of determining the fixed-end reaction components by means of an elastic model has been described by Professor Beggs[8] and an experimental method of determining the elastic constants corresponding to changes in span has been described by Professor Wilson.[9] The other elastic constants can be determined in a similar manner.

Article 3.—Influence of Errors in the Elastic Constants Upon Stresses in Arch Rib.—The method of using experimentally determined elastic constants in the analysis of three-span arch series was used in connection with the tests of arches with decks described in *Bulletin No. 270* of the Engineering Experiment Station of the University of Illinois. Some studies were made which appeared to indicate that considerable variation in the elastic constants produces only small variations in the stress in the arch rib. Studies have since been made of two-span structures similar to the one shown in Fig. 109 to determine the effect of variations in each of the elastic constants upon the maximum stress at critical sections.

---

[8] *Proeedings*, Am. Concrete Inst., Vol. 18, p. 58.

[9] *Bulletin No. 226*, Univ. of Illinois Eng. Experiment Station.

The arch span for the structure used in this study was a rib without deck, identical with the rib of the structures used in the tests described in *Bulletin No. 270* previously cited, and the pier was identical with the 20-ft piers of the same structure (see Figs. 27 and 28). The critical sections are at End *A* of Span *AB* and at Load Point 3 of the same span. The elastic constants were increased, one at a time, successively, 25% over their normal value, and computations were made to determine the effect of each variation upon the maximum stress at End *A* of Span *AB* and at Load Point 3. The results are given in Table 31. From a study of this table it is apparent that H_A and M_A are the most influential elastic constants.

TABLE 31.—EFFECT OF VARIATIONS IN THE ELASTIC CONSTANTS UPON THE STRESS IN THE ARCH RIB OF A TWO-SPAN STRUCTURE

Elastic constant increased 25 per cent.	CHANGE IN MAXIMUM STRESS, IN PERCENTAGE, AT:		Elastic constant increased 25 per cent.	CHANGE IN MAXIMUM STRESS, IN PERCENTAGE, AT:	
	End A of Span *AB*	Load Point 3		End *A* of Span *AB*	Load Point 3
H_A......	—0.6	10.0	M_A.....	2.6	—9.9
H_P......	—3.5	—1.5	M_P.....	3.1	1.7
h_A......	0.5	5.9	m_A......	—3.6	—2.9
h_P......	2.9	1.2	m_A'.....	1.4	—2.7
			m_P......	—2.2	—1.2

The method of measuring these constants, referred to in Article 2, is very simple and will give accurate results if reasonable skill is exercised. Moreover, if both these constants are in error by the same amount, the effect of the error in one is almost exactly offset by the error in the other. The moment, M_A, is obtained from H_A and the two will be in error the same amount if the position of H is located accurately, and H can be located accurately. Of the other elastic constants, h_A is the only one that has a great influence upon the stress in the rib; and it, being numerically equal to M_A, can be determined with a high degree of accuracy. Of the remaining elastic constants that are to be determined experimentally, an error of 25% in the value of the constant produces an error in the stress of 3.6%, or less; that is, the error in the final objective, the stress in the rib at the critical section, is only about one-fourth as great as that in the quantity measured, in the case of h_A, and about one-eighth as great or less for all other elastic constants.

It would appear, therefore, that in determining the reaction components at the springings of the arches for a two-span structure having arch spans that are not readily susceptible of mathematical analysis, the algebraic method in which experimentally determined elastic constants are used, will give results which are accurate enough for purposes of design even though individual elastic constants may be in error 25 to 50 per cent. The experimental determination of these elastic constants is not particularly difficult and, if reasonable skill is exercised, the errors should be much less than 25%; but, if the analysis is to be used to determine the dead load stresses, then the fixed-end reaction components for the single span will need to be determined with a high degree of accuracy; but even for this part of the work, the required accuracy is not greater than in the tests of a three-span model.

Article 4.—Analysis of Multiple-Span Arches.—The analysis in Article 2 is for the two-span structure shown in Fig. 109. The same method can be used for structures having a greater number of spans, but the number of simultaneous equations increases with the number of spans, there being two independent equations and two unknowns, x and θ, for each pier. Except for flat arches with high slender piers, the stresses in one span are not greatly influenced by the deflection of the parts of the structure more than one span removed from the span being considered. Few structures will be encountered for which more than three adjacent arches need be considered in the analysis, and the two outside arches may be considered as fixed at their outer ends. Structures which have arches so flat and piers so high and slender as to make this treatment inadequate, will be rare. In fact, it would appear to be self-evident that such a combination is as unnecessary as it is undesirable because the rise of the arch can be increased and the height of the pier decreased without much loss of under-clearance.

The reaction components for a three-span series of arches are shown in Fig. 112. Applying the method given in Article 2, the basic equations for a three-span structure for which each span and the structure as a whole are symmetrical about vertical center lines, are as follows:

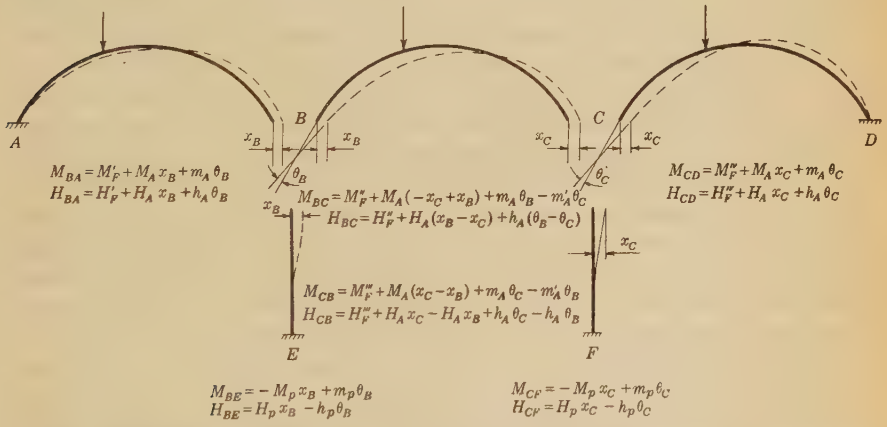

A
B
C
D
E
F
x_B
x_B
x_C
x_C
θ_B
θ_C
x_B
x_C
$M_{BA} = M'_F + M_A x_B + m_A \theta_B$
$H_{BA} = H'_F + H_A x_B + h_A \theta_B$
$M_{BC} = M''_F + M_A(-x_C + x_B) + m_A \theta_B - m'_A \theta_C$
$H_{BC} = H''_F + H_A(x_B - x_C) + h_A(\theta_B - \theta_C)$
$M_{CD} = M''''_F + M_A x_C + m_A \theta_C$
$H_{CD} = H''''_F + H_A x_C + h_A \theta_C$
$M_{CB} = M'''_F + M_A(x_C - x_B) + m_A \theta_C - m'_A \theta_B$
$H_{CB} = H'''_F + H_A x_C - H_A x_B + h_A \theta_C - h_A \theta_B$
$M_{BE} = -M_p x_B + m_p \theta_B$
$H_{BE} = H_p x_B - h_p \theta_B$
$M_{CF} = -M_p x_C + m_p \theta_C$
$H_{CF} = H_p x_C - h_p \theta_C$

Fig. 112.—Reaction Components at Springings of Arches, Three-Span Structure.

$$x_B\,[2\,M_A - M_P] - x_C\,M_A + \theta_B\,[2\,m_A + m_P] - \theta_C\,m'_A = -M'_F - M''_F. \quad (12)$$

$$x_B\,[2\,H_A + H_P] - x_C\,H_A + \theta_B\,[2\,h_A + h_P] - \theta_C\,h_A = -H'_F - H''_F. \quad (13)$$

$$-x_B M_A + x_C [2 M_A - M_P] - \theta_B m'_A + \theta_C [2 m_A + m_P] = - M'''_F - M^{IV}_F. \quad (14)$$

and

$$-x_B H_A + x_C [2 H_A - H_P] - \theta_B h_A + \theta_C [2 h_A - h_P] = - H'''_F - H^{IV}_F. \quad (15)$$

Equations (12) to 15) are the fundamental slope-deflection equations for a symmetrical three-span structure. Similar formulas can be developed for an unsymmetrical structure. The most practical method of solving these equations is to substitute the numerical values for the coefficients of the unknowns and for the right-hand member, and then determine the unknowns by a process of elimination, maintaining a check column in order that any mistakes that may be made can be detected immediately.

The solution can be made for any known system of forces. The only difference between solutions for different systems of forces occurs in the right-hand member of the equations. If a given structure is to be analyzed for a number of systems of forces the solutions for all systems can be made simultaneously by carrying a separate column containing the right-hand member of the equations for each system of loading.

The elastic constants and the fixed-end reaction components to be used in Equations (12), (13), (14), and (15), may be determined all algebraically, all experimentally, or part algebraically and part experimentally, as appears most convenient for any particular structure.

Article 5.—Summary.—As previously stated, the slope-deflection method of analyzing multiple-span arches has been described in this report because it appeared that, with this method, the elastic constants and fixed-end reactions are more completely isolated and the part which they play in the analysis is more apparent than with the other methods. For these reasons, the propriety of substituting experimentally determined elastic constants and fixed-end reactions for corresponding quantities determined by the elastic theory, seems more apparent with this method than with the others. The substitution can be made, however, with other methods of analysis. The experimental arches described in *Bulletin No. 270,* University of Illinois Engineering Experiment Station, were analyzed under the supervision of Professor Wilson by the combined algebraic-and-experimental method, introducing the experimental constants in the analysis developed by Professor Cross.[10]

The combined algebraic-and-experimental method has certain advantages over an all-experimental method, the tests being on elastic models in both instances. The models are much smaller and much simpler, and the amount of experimental work is much less for the former than for the latter. Moreover, a considerable error in the elastic constants produces a comparatively small error in the maximum stress at the critical sections of the rib. Furthermore, the reaction components are statically consistent with each other when they are all determined from a single set of elastic constants, whereas, if the reaction components are determined directly by a separate observation for each component, there are statical inconsistencies that are sometimes annoying.

---

[10] "Continuous Frames of Reinforced Concrete," by Hardy Cross and Newlin D. Morgan, Members Am. Soc. C. E., John Wiley & Sons, Inc.

The combined algebraic-and-experimental method is not recommended in the analysis of structures that are readily susceptible of analysis, such as a multiple-span series of arches each of which consists of a rib without a deck because, if the arch span is readily susceptible of analysis, the elastic constants and fixed-end reaction components can be computed more satisfactorily than they can be obtained by tests of elastic models. However, for a series of arch spans, each of which has a rib with spandrel columns and a deck, the combined algebraic-and-experimental method is useful for determining the reaction components at the springings of the arches.

Acknowledgments.—This Chapter constitutes a discussion of a method of analyzing multiple-span arch series by Professor Wilson.

MEMOIRS OF DECEASED MEMBERS

ARTHUR POWELL DAVIS, Past-President, Am. Soc. C. E.[1]

Died August 7, 1933

Arthur Powell Davis was born near Decatur, Ill., on February 9, 1861. His father, John Davis, was a Representative from Kansas in the 52d and 53d Congresses, owned and edited a weekly journal at Junction City, Kans., and was the first President of the Farmers' Cooperative Association of that State. His mother, Martha Ann (Powell) Davis, was the daughter of a Wesleyan minister who had emigrated from England. Upon graduation from the Kansas State Normal School, at Emporia, Mr. Davis entered Columbian (now George Washington) University, at Washington, D. C., from which he was graduated with the degree of Bachelor of Science in 1888. In 1917, he received from the latter institution the honorary degree of Doctor of Science, and, in 1920, from Iowa State College the degree of Doctor of Engineering.

Mr. Davis was appointed an Assistant Topographer in the United States Geological Survey in 1882. Rapidly rising to more responsible positions, he was shortly placed in charge of the topographic work in the Plateau Section; then, for two years, he directed the work of a section of the Irrigation Survey, following which he was for five years at the head of the topographic work of the U. S. Geological Survey in the Southwest Section. Transferring in 1894 to the Division of Hydrography, he had charge during 1896 and 1897 of all stream measurements in the United States carried on by the Geological Survey. From this post, he was detailed as United States Hydrographer in charge of the examination of rainfall, stream flow, flood control, and related observations, under the Isthmian Canal Commission, for both the Nicaragua and Panama Canal routes.

It is of interest in connection with this latter assignment to note that about ten years later, in 1909, Mr. Davis was one of the members of a board of seven distinguished engineers appointed to examine and report on the feasibility and type of canal to be built at Panama, with especial reference to the safety and design of high dams planned in connection with that undertaking, and, in 1915, he was a member of a committee of the National Academy of Sciences to study the slides in the Panama Canal at Culebra.

Upon the passage of the Reclamation Act and the organization of the United States Reclamation Service in 1902, Mr. Davis was appointed Assistant Chief Engineer of that Service. In 1907, he became Chief Engineer, and, from 1914 to 1923, served as Director of that organization, succeeding the late Frederick Haynes Newell, M. Am. Soc. C. E. During this time, Mr. Davis was responsible for the construction of the Shoshone and Arrowrock Dams, each in turn the "highest in the world"; the Elephant Butte Dam, creating the largest artificial irrigation reservoir in the United States; the 4-mile Strawberry Tunnel; the even more difficult 6-mile Gunnison Tunnel; and many other important irrigation works, which have been described in detail in articles in the popular and in the technical press, in papers before the Society, and, notably, in a book written by Mr. Davis, himself, entitled, "Irrigation Works Constructed by the United States Government," published in -1917. This book was, in the words of Mr. Davis, "Dedicated to John W. Powell, the far-seeing philosopher; Francis G. Newlands, the constructive statesman; and Frederick H. Newell, the faithful administrator; the pioneers who blazed the way for the beneficient work of National reclamation."

---

[1] Memoir prepared by Charles A. Bissell and F. E. Weymouth, Members, Am. Soc. C. E.

Although Mr. Davis' professional attainments covered a broad field outside the scope of his activities in connection with the Reclamation Service, as evidenced by his work on the Panama Canal, the Huai Conservancy Project, in China, and the East Bay Municipal Utility District, in California, it was primarily as an Irrigation Engineer that his talents found their fullest expression.

The art of irrigation was known and practiced by the prehistoric races who peopled parts of the arid regions of the American Continent and vanished centuries before the coming of the White Man. The remains of miles of ancient canals, built on as true a grade as those laid out by modern engineering skill, can still be traced on airplane photographs of the Salt River Valley, and other remains of irrigation works of this period can be found in parts of New Mexico and California. The Spaniard, when he settled in the arid Southwest, was not slow to adopt a system of irrigated agriculture which, in some regions, persists to the present day with little change from ancient custom.

As an enterprise of the English-speaking races, however, modern irrigation in the United States began in Utah, in 1847, when Brigham Young struck his staff upon the ground beside the stream flowing through the site of his future capital and gave utterance to his historic decision: "Here we will build our city". Paralleling the Mormon settlement as primarily a home-making community enterprise, but established as an outgrowth of a social rather than a religious enthusiasm, was the Greeley Colony, founded in 1870 in Colorado. Elsewhere early irrigation enterprises fall into one of two general classes: On the one hand the small ditches built with little or no cash outlay by the settlers themselves, individually or co-operatively, sometimes however combined and extended until they formed systems of surprising magnitude; and, on the other hand, the larger systems planned and constructed with large original capital expenditure in the expectation of great returns on an investment largely, if not entirely, contributed by outside interests.

Enterprises of the first class frequently attained ultimate success at a remarkably small outlay of money, but often at a cost to the original settlers of years of hardship and privation; enterprises of the latter type so frequently proved financial disappointments that, except in unusually well-favored regions, it became impossible to finance even soundly engineered irrigation projects at reasonable cost.

In 1902, the U. S. Reclamation Service reported that more than 7 500 000 acres were already under irrigation in the United States, but that while the development of the choice spots had been accomplished, and the easily available waters had been utilized, the larger public interests had not been guarded and the making of homes had not been carried on to the extent which the wisest statesmanship requires.

These conditions, coupled with the fact that practically all the public land which could be made to produce profitable crops without irrigation had passed into private ownership, produced a powerful incentive for the adoption by the Government of a policy for the expansion of the nation's arable area by the construction of irrigation works, and this movement culminated in the passage in 1902 of the Reclamation Act. This Act, which was the outcome of decades of engineering investigations, surveys, and debate, set apart in the United States Treasury a revolving fund made up of the proceeds from the sale of public lands and certain other receipts from the public domain in the Western States, to be used under the direction of the Secretary of the Interior in the construction of irrigation projects for the reclamation of arid lands in those States.

The need for a planned, orderly development of the irrigation resources of the West was first prominently brought before the public in a report made, in 1879, on "The Lands of the Arid Region", by Mr. Davis' uncle, Major J. W. Powell, the intrepid explorer of the canyons of the Colorado River, prophet and forerunner of the movement for the conservation of national resources, to the realization of the ideals of which Mr. Davis devoted a lifetime of outstanding service. In subsequent years, Major Powell, as the first Director of the U. S. Geological Survey, continued the accumulation of the vast mass of accurate data on the geology, topography, soils, and water supply of the Western United States, which was necessary to serve as a basis for a comprehensive scheme of development. Not less valuable than the physical data thus secured through these years of investigation and planning was the training of a personnel through years of experience in dealing with problems of water storage, stream measurements, and the design of irrigation works, for leadership in the new field opened up by the passage of the Reclamation Act.

The works of the U. S. Reclamation Service, constructed in the course of the seventeen years during which Mr. Davis was its Chief Engineer and, later, its Director, constitute a record of achievement of which the Engineering Profession may well be proud. Scattered over the seventeen Western States in difficult and often inaccessible locations, involving frequently the solution of problems for which no precedents existed, the engineering and construction work on these irrigation projects has received the commendation and excited the admiration of authorities not only of this country, but of the world at large. Indeed, irrigation in America during these years presented new problems for solution and evolved new solutions of old problems to such an extent that what might almost be called a new science was developed. Sir William Willcocks, the great English authority on irrigation, visiting this country after a lifetime of engineering experience in India and Egypt, said that he was so accustomed to look upon the irrigation industry as one absolutely dependent upon cheap labor like that of Asia and Africa that his chief interest in examining American irrigation was to learn how it was that it could be practiced at all in America. From the earliest days of the Reclamation Service, when the addition of 10 ft to the designed height of Roosevelt Dam was an engineering problem deserving the consideration of a Board of Engineers, to Boulder Dam, on the Colorado River, where at one step the height of the tallest previously existing dam was more than doubled, Mr. Davis kept that Service in the forefront, along with those leaders in other lines of endeavor who have made the early decades of the Twentieth Century outstanding in engineering achievement. During the twenty years between the construction of the Roosevelt Dam and the early designs of Boulder Dam—which Mr. Davis did not live to see completed, but for the general location and preliminary design of which he was responsible—the Reclamation Service under his direction constructed more than one hundred dams, many of them of outstanding importance, and all of them to-day successfully performing the functions for which they were created.

Preliminary designs and complete plans for the construction of Roosevelt Dam "from a report by A. P. Davis" were published in the First Annual Report of the U. S. Reclamation Service, covering operations from June 1 to December 1, 1902. In the same volume may also be found the results of a reconnaissance survey made that season of the Lower Colorado River, the canyon regions of which had for the first time been navigated successfully only thirty-three years previously by Major Powell, as noted.

Thirty irrigation projects were developed by the Reclamation Service during the ensuing twenty years; the equivalent in agricultural area of a new State was added to the nation's resources; a half million people found homes and employment on the project areas, or in adjacent urban developments dependent thereon. During these twenty years the harnessing of the Colorado — the Southwest's mightiest river — draining one-thirteenth of the continental area of the United States, dissipating in the wanton devastation of its annual summer floods the wealth of an empire, constituted a problem challenging the best thought and efforts of the engineers of the Reclamation Service. As the years passed, one after another of the individual projects and related reservoir sites of the Upper and Lower Colorado River and its tributaries were surveyed and studied, and Mr. Davis at length reached the conclusion that the secret of the mastery of the river lay hidden in its lower canyon reaches below the mouth of the Virgin River. The cost of the diamond drilling, topographic surveys, and other exploratory work necessary to determine the feasibility and cost of major storage in this region, however, was far in excess of the sum available for such purposes from the Reclamation Fund. At length, the growing realization of the flood menace to the Imperial Valley, a 500 000-acre irrigation project developed under State laws and situated in large part below sea level in the Colorado desert in Southern California, focused national attention upon the importance of the problem and, in 1920, Congress passed the Kincaid Act, calling for a comprehensive study and report.

The investigations were carried out under the direction of Mr. Davis. In his report on the "Problems of Imperial Valley and Vicinity", published as a Congressional document in 1922, Mr. Davis collected, summarized, and digested the results of all the previous investigations carried out by the Reclamation Service under his direction during the preceding twenty years, along with the results of the intensive investigations authorized under the Kincaid Act. The solution proposed by Mr. Davis is in all essentials that finally adopted, consisting of the Boulder Dam, raising the water surface about 600 ft and forming a reservoir with a capacity of approximately 30 000 000 acre-ft; the All-American Canal; and a power development at Boulder Dam of such magnitude as to amortize the cost of the development.

The situation, however, was one calling for the talents of a statesman, as well as those of an engineer. Seven States claimed interests in the river, interests which at the start appeared irreconcilable. To give these States an opportunity to work out their common problem, Congress, in 1921, had authorized them to negotiate an equitable division of the waters of the Colorado River System, and pursuant to this authorization the Colorado River Commission was formed upon which the Federal Government and each of the seven States of the Colorado Basin were represented. After meetings in Washington, D. C., Phoenix, Ariz., and Denver, Colo., and public hearings in each of the Basin States, this Commission held its final eighteen executive sessions at Santa Fé, N. Mex., whence it emerged on November 24, 1922, with the agreement known as the Colorado River Compact. As Technical Adviser of the Commission in its public hearings and in its executive sessions, Mr. Davis, through his intimate knowledge of the situation, his recognized technical skill and ability, his tact and diplomacy, rendered invaluable aid to the Commission and contributed in no small way to the successful outcome of the negotiations.

Years of additional surveys, planning, and research, of prolonged negotiations, discussions, and argument ensued before the Colorado River Compact, having been ratified by six of the seven States, was at length submitted to Congress for approval about five years after its execution. One year later the construction of the project was finally authorized by the Swing-Johnson Act, approved December 21, 1928. The conditions prescribed in that Act having been satisfied, the first appropriation for beginning actual construction in the amount of $10 660 000 was carried in the second Deficiency Appropriation Act, approved July 3, 1930. Perhaps the most complete summary extant in small compass of the engineering and economic justification for the Boulder Canyon Project is to be found in an article by

Mr. Davis on "The Development of the Colorado River", published in the *Atlantic Monthly* for February, 1929. In this article of eleven pages, Mr. Davis with characteristic modesty refers to his own part in this $165 000 000 engineering development, in one short sentence only, "the study was entrusted to the supervision of the author hereof".

During his busy years with the Reclamation Service, however, Mr. Davis found opportunity to give attention to many matters outside the scope of his official duties. In 1909, as previously mentioned, he was a member of a Board of Engineers appointed to examine and report on the feasibility and type of canal at Panama. In 1911, at the instance of the Imperial Tzarist Government, which desired to interest American capital in the development, he was engaged to investigate the irrigation of the Kara Kum Desert, lying to the south and west of the Amu Daria (River), in Turkestan. In 1914, he was a member of a board of three, the other members being Maj.-Gen. William L. Sibert, U. S. A. (*Retired*), M. Am. Soc. C. E., and D. W. Mead, Hon. M. Am. Soc. C. E, which was sent to China by the American National Red Cross to make a survey and investigation of the Huai River Conservancy Project in the Provinces of Honan, Anhwei, and Kiangsu. In 1915, he was a member of a committee appointed by the National Academy of Sciences to report on the Panama Canal slides. From 1912 to 1917, he was Vice-President of the Joint Conference on Standard Specifications for Portland Cement, which developed the first uniform Government specification for Portland cement. In 1922, and again in 1924, he was Lyman Lecturer on conservation of water resources before the Sheffield Scientific School of Yale University. In 1923, he was Technical Adviser of the United States before the Pecuniary Claims Arbitration, in London, England.

Upon the appointment of Dr. Hubert Work as Secretary of the Interior, in 1923, a change of policy was instituted in regard to the Reclamation Service, which Dr. Work said called for the appointment of a "business man" as the head of the Bureau, and after forty years of outstanding service as a Government engineer, Mr. Davis found his position as Director of the Service abolished. This action by Dr. Work, without giving Mr. Davis a hearing and without adequate explanation, aroused immediate and widespread indignation among friends of the Reclamation Service throughout the West and among engineers everywhere who were familiar with the outstanding ability as an engineer and administrator which had been displayed by Mr. Davis throughout his connection with the Service. At its Annual Convention in Chicago, Ill., the Society, on July 11, 1923, passed a unanimous resolution which stated:

> "The reported reason of better business administration advanced for the replacement of the Director [Mr. Davis] by a successor under the new title of Commissioner, appears to be a pretext which is refuted by the long and honorable business record of the Director dismissed * * *. The Board deplores the action taken by the Secretary of the Interior, for the following reasons:
>
> "(1) That it will work irreparable injury to the Public Service in the breakdown of morale and confidence of public employees.

"(2) That it is an injustice to a man who has given forty-one years of faithful and valuable service to the Government of the United States.

"(3) That arbitrary methods of removal are not creditable to a popular government based upon equality and fair dealing.

"(4) That the change now inaugurated bears evidence of an attack upon a worthy and highly creditable branch of the government service to serve political needs. * * *".

No charges of any nature were ever brought against Mr. Davis, even as an ostensible justification for this summary dismissal, and ten years later, in July, 1933, Secretary of the Interior Ickes announced the appointment of Mr. Davis as Consulting Engineer in the field service of the Bureau of Reclamation, and particularly upon the problems of the Boulder Dam Project. In this announcement, Mr. Davis is referred to as the "father" of the plan under which the Boulder Dam is being constructed, and particular stress is laid upon his enviable record of Government service. While Mr. Davis' failing health did not permit him to undertake the active discharge of the duties of this position, the appointment must have been a source of satisfaction to him in his last days.

Shortly after his dismissal from the Government service, Mr. Davis accepted the position of Chief Engineer and General Manager of the East Bay Municipal Utility District, comprising the Cities of Oakland, Berkeley, and seven other California cities along the eastern shores of San Francisco Bay. These cities, due to their rapid increase in population, had outgrown their available water supply and were facing imminent water shortage. Wherever they turned, the way to additional supply appeared blocked, except at prohibitive cost. A district was organized in May, 1923, and although no funds were immediately available for field work, Mr. Davis upon his appointment proceeded at once with office studies, based upon previous extensive investigations and Federal Government data, to such good effect that when funds became available in July, 1924, surveys, plans, and estimates could be pushed to completion and laid before the District at a bond election in November of that year. After a vigorous campaign the bonds were voted by a 3 to 1 majority. The bonds, themselves, were not validated in the Courts until September, 1925; yet, in June, 1929, less than four years later, a storage dam with a maximum height of 358 ft and containing 617 000 cu yd of concrete had been built, a 94-mile aqueduct had been completed, and Mokelumne River water was being delivered to the District.

Upon the completion of his work with the East Bay Municipal Utility District, Mr. Davis was engaged by the Union of Socialistic Soviet Republics as Chief Consulting Engineer for irrigation in Turkestan and Transcaucasia. A paper entitled "Irrigation in Turkestan," describing his observations in connection with this assignment, was published in *Civil Engineering* for January, 1932, after his return to the United States.

The following paragraphs on Mr. Davis' Russian work have been contributed by V. V. Tchikoff, M. Am. Soc. C. E., who was associated with Mr.

Davis as Consulting Engineer and acted as Liaison Officer between the Soviet Government and the American Bureau of which Mr. Davis was the head:

"In Tashkent, Mr. Davis was at the head of the Special American Consulting Bureau where under his supervision four American Consulting Engineers worked and there were also three Americans in Transcaucasia whose most important work was reviewed by him. A man, his personality and ability, is the best judged under trying circumstances. Such was the case sometimes in the work of Mr. Davis in the Soviet Union. In spite of freezing at home and even in the office, unaccustomed food, a foreign language, Mr. Davis never complained, showing a true Spartan character, always and ever a gentleman.

"Mr. Davis crossed and recrossed several times Turkestan and Transcaucasia, often on horseback, in regions where he needed a bodyguard. He was always first, choosing for himself the most spirited native horse, and this at seventy years of age! One can understand how he created around himself an atmosphere of respect and friendliness and the efficiency of work governed by his extensive experience and knowledge resulted in a remarkable amount of work being done.

"Mr. Davis' main duty in Tashkent was the review of plans of irrigation systems and related power plants. Many irrigation systems in Turkestan are by their size larger than our's and can be compared only to those of British India. Review of the Golodnaya Steppe Project, which he had visited on the occasion of his former trip to Turkestan, was his first task. Russian engineers had worked on the complete plans of irrigation for this Steppe since the time Mr. Davis was last there. A little less than a hundred volumes of typewritten documents and drawings were presented and the table of contents alone occupied one whole volume. Such an amount of material especially in a foreign language, was overwhelming, but two or three months were enough for Mr. Davis to write a comprehensive report with definite conclusions and answers to specific questions.

"Besides this review Mr. Davis was consulted on all questions that came up in connection with drainage, construction, and design, etc. Under the supervision of Mr. Davis and his assistants the typical structure was worked out as well as a special manual with instructions and data for designs."

Upon his return from Russia in 1931, Mr. Davis engaged in consulting practice in Oakland, Calif., and for a part of this period served as Consulting Engineer for The Metropolitan Water District of Southern California in connection with the construction of the Colorado River Aqueduct. He underwent a serious surgical operation in November, 1931, from which he made a good recovery, but a later operation in February, 1933, was not so successful, and he passed away August 7, 1933, after a long and painful illness, which he endured with typical patience and cheerfulness. He was buried in Rock Creek Cemetery, in Washington, D. C.

As an Engineer Mr. Davis combined outstanding technical ability with excellent judgment, both as to measures to be undertaken and the men to execute them. Perhaps the engineering works he constructed were themselves no greater achievement than the creation of the enduring organization that built them. All over the West to-day are engineers who are proud to have been at one time in their careers a part of the United States Reclamation Service. The outstanding record of that organization is in no small part due to Mr. Davis' ability to select and weld together an efficient personnel. Early in the Reclamation Service history, the Chief Engineer of the Southern Pacific Railroad Company expressed the view in a discussion before the Society that the Reclamation Service secured more per dollar expended than did the Southern Pacific Railroad itself. And this in spite of the handicaps of governmental "red tape" and considerations of political expediency which, up to the time of the Reclamation Service, were popularly supposed to be so prevalent and insurmountable as to preclude successful and efficient operation of a large construction organization under governmental auspices.

Nor was it alone in the selection of key men and subordinates within his organization that his genius displayed itself. Himself an engineer of high ability, Mr. Davis was able in spite of governmental limitations as to salaries and fees to secure the co-operation and service on his consulting boards of engineers in private practice of equally high rank and ability with varied and specialized experience, and he did not hesitate to avail himself liberally of such services in connection with every major feature. "An institution," it has been said, "is but the lengthened shadow of a man." The Reclamation Service during the years of his administration was to a marked degree an expression of Mr. Davis' engineering ideals and principles.

Personally, Mr. Davis was a man possessing the highest attributes of honor, straightforwardness, and sincerity. Practical to the last degree in material affairs, he was of the happy few who find in the work of their hands an opportunity to express in concrete form their aspirations for the welfare of their fellow men. The ideal which found expression in the formation of the Reclamation Service was humanitarian, to provide an opportunity to the man with small capital to develop his own farm and carve out his own destiny; to provide homes where men who loved the soil and the fundamentals of existence could raise their families in peace and comfort. As expressed by one who knew him well:

> "So practical was his [Mr. Davis'] humanitarianism, so hidden on the surface by his intense activity in making it practical and so taken for granted by himself, that it went commonly unrecognized by others, even some closely associated with him."

Mr. Davis was a deep and penetrating thinker, impressive and convincing as a speaker, and a writer of exceptional force and clarity. He was a contributor to the publications of the Society, the author of many articles in the technical press, and of numerous reports published in the earlier Water Supply Papers and Annual Reports of the United States Geological Survey and in Congressional documents and other governmental publications. He was also the author of "Elevation and Stadia Tables" (1901); "Irrigation Works Constructed by the United States Government" (1917); and "Irrigation Engineering" (1919).

Mr. Davis was a Fellow of the American Geographical Society; a Past-President and Honorary Member of the Washington (D. C.) Society of Engineers; a member of the Washington Academy of Sciences, the American Academy of Political and Social Sciences, and the American Philosophical Society. He was a member of the Commonwealth and Engineers'

Clubs, of San Francisco, and Past-President and member of the Cosmos Club, of Washington, D. C. At the time of his death, he was a member of the Unitarian Church, in Oakland, Calif.

He was married on June 20, 1888, to Elizabeth Brown, of Washington, D. C., who passed away on April 13, 1917, leaving four children, Mrs. Rena Peck, Mrs. Florence Eslin, Mrs. Dorothy Smith, and Mrs. Elizabeth Smith. On June 19, 1920, he was married to Marie MacNaughton, of Washington, D. C., who survives him.

Mr. Davis was elected an Associate Member of the American Society of Civil Engineers on June 7, 1893; and a Member on October 4, 1899. He served as a Director from 1917 to 1919, and as President of the Society in 1920.

CARL EWALD GRUNSKY, Past-President, Am. Soc. C. E.[1]

Died June 9, 1934

Carl Ewald Grunsky, the son of Albert and Josephine Frederica (Camerer) Grunsky, was born on April 4, 1855, in San Joaquin County, California. After some more or less successful business experiences on the part of Mr. Grunsky's father, the family settled on what was then known as the Columbus Ranch, about four miles east of Stockton, Calif. Later, they moved to Stockton, taking a house at the southwest corner of Sutter and Church Streets which, for a quarter of a century, was the Grunsky home place. In this environment Ewald grew up. He was a bashful, and not very sturdy, boy and his parents were fearful that they would not be able to raise him beyond childhood.

His ambition was to become a physician, and he devoted himself to his studies; he entered the Stockton High School from which he was graduated at the age of fifteen. He was handed a Teacher's Certificate on graduation and was at once offered a school. As he was anxious to earn money for his further education, he accepted the offer and taught successfully for one year. However, he declined a re-appointment and, with his younger brother, sailed for Germany in 1872.

When Carl Ewald Grunsky arrived there and was forced to make an immediate choice as to his future profession, he found that his savings from the year's teaching would not enable him to complete the course in Medicine. In order not to become a burden upon the family, which was already making sacrifices to send him to college, he decided to study engineering for which his love of mathematics and the sciences seemed to fit him. In 1876, he completed successfuly a course in Civil Engineering at the Stuttgart Polytechnikum, in Württemberg, then as now one of the outstanding technical schools in Germany. This institution honored itself and him by conferring on him in 1910 the degree of Doctor of Engineering. To this distinction was added a similar honorary degree of Doctor of Engineering, conferred by the famous American Engineering School, the Rensselaer Polytechnic Institute, at Troy, N. Y., in 1924.

---

[1] Memoir prepared by the following Committee: Charles D. Marx, Past-President and Hon. M. Am. Soc. C. E., *Chairman*, and Henry D. Dewell, F. C. Herrmann, Walter L. Huber, Thomas H. Means, and E. T. Thurston, Members, Am. Soc. C. E.

Soon after Mr. Grunsky's return to America, he entered the service of his home State. In the State Engineering Department he advanced from Topographer to Assistant, and Chief Assistant, from 1878 to 1888. Then followed a number of years of private practice largely on irrigation and sewerage problems. He served as a member of the Examining Commission on Rivers and Harbors for California, in 1889 and 1890; as a member of the Sewerage Commission of San Francisco, in 1892 and 1893; and as Consulting Engineer to the Commission of Public Works of San Francisco from 1893 to 1895.

Mr. Grunsky was best known in San Francisco as its first City Engineer, having served in that capacity during the four years after the Charter went into effect; that is, from 1900 to 1904. In this capacity he investigated and reported on the sources of water supply that could be made available to the City of San Francisco. He selected and recommended Tuolumne River as an auxiliary to near-by sources the acquisition of which he favored.

In 1904 he was called from the City Engineer's Office to Washington, D. C., by President Theodore Roosevelt, to serve as one of the four Engineer Members of the Panama Canal Commission. When the Commission was reorganized in 1905, Mr. Grunsky accepted appointment from President Roosevelt as Consulting Engineer in the United States Reclamation Service and Adviser to the Secretary of the Interior. From this position, he retired in 1907. For a short time, Mr. Grunsky had an office as a Consulting Engineer in New York, N. Y., but since 1908 he had been located in San Francisco as the head of the C. E. Grunsky Company. Many notable engineering projects in California, too many to be enumerated herein, have had the benefit of his advice as Consulting Engineer.

He was a member of the Technical Society of the Pacific Coast as early as 1884, and devoted much of his time to the welfare of that Society, which, for many years, was the leading technical organization in the Far West. He served as its President in 1893.

Mr. Grunsky was more than a successful engineer. He was a man of broad tastes and took a good citizen's interest in civic affairs. He early became a member of the Commonwealth Club of California, the largest and most influential non-partisan organization in the State devoted to the cause of good government and the development of a well-informed body politic, enjoying the respect and confidence of practically all factions. He had been its President, and was a member of its Board of Governors when he died. The California Academy of Sciences owes much to his valuable services; he was its President from 1912 to the time of his death, and during the last two years of his life he was also its Acting Director. Mr. Grunsky also acted as Vice-President and, later, as President of the Pacific Division of the American Association for the Advancement of Science. It was only natural that his San Francisco friends should honor him, and he was at various times President of the San Francisco Section of the Society and of the Consulting Engineers of the Pacific Coast. He was Vice-President of the American Delegation to the World Engineering Congress, at Tokyo, Japan, in 1929. In 1930, he was elected President of American Engineering Council, which office he held successfully for two years.

His devotion to the welfare of the Engineering Profession and to the Society was evidenced, not only by his prominence in the councils of the Society and as a contributor to its technical proceedings, but also by his continued personal attendance at the meetings of the San Francisco Section up to the time of his last brief illness, despite varied outside interests demanding his attention. He interested himself especially in the welfare of the younger generation of engineers, to whom his personal conduct was an inspiration.

Mr. Grunsky was an artist of some distinction and produced paintings in oil and water color of no mean merit. A few of his sketches of Sutter Fort were published in the first number of *Civil Engineering* in October, 1930. In this field appeared to lie his recreational hobby. He found a useful outlet for this talent in connection with the new Museum Building of the California Academy of Sciences, wherein he designed and executed several of the most impressive environmental settings for American wild life. It is not usual for an engineer to be gifted in this manner, nor for an engineer to devote so much attention freely, without personal ambition in that direction, to the problems of government; and rare indeed for an engineer to possess both these avocational traits.

His literary work was not confined to engineering; he was a thorough student of economics and, in his various duties as City Engineer of San Francisco, had to deal with many economic questions. He wrote *Water Supply Papers 17, 18,* and *19* for the United States Geological Survey, and to *Bulletin 100,* of the United States Department of Agriculture, he contributed the chapter on "Irrigation from King's River". Mr. Grunsky was the author of "Topographic Stadia Surveying", "Valuation Depreciation and the Rate Basis", "Public Utility Rate Fixing", and "Ways to National Prosperity". In 1910, he received the Norman Medal of the Society for his paper entitled "The Sewer System of San Francisco, and a Solution of the Storm-Water Flow Problem"[2]. Other important papers contributed to the publications of the Society by Mr. Grunsky were, as follows: "The Lower Colorado River and the Salton Basin"[3]; "Rain and Run-Off Near San Francisco, California"[4]; "The Low Stage of Lakes Huron and Michigan"[5]; "The Appraisal of Public Service Properties as a Basis for the Regulation of Rates"[6]; "Rainfall and Run-Off Studies"[7]; "Some Factors Affecting the

---

[2] *Transactions*, Am. Soc. C. E., Vol. LXV (1909), p. 294.

[3] *Loc. cit.*, Vol. LIX (1907), p. 1.

[4] *Loc. cit.*, Vol. LXI (1908), p. 496.

[5] *Loc. cit.*, Vol. LXIII (1909), p. 31.

[6] *Loc. cit.*, Vol. LXXV (1912), p. 770.

[7] *Loc. cit.*, Vol. 85 (1922), p. 66.

Problem of Flood Control"[8]; "Historical Notes on Engineers and Engineering Activities in California"; Presidential Address at the Annual Convention at Pasadena, California, June 18, 1924[9]; "The Head-Works of the Imperial Canal"[10]; "The Relief Outlets and By-Passes of the Sacramento Valley Control Project"[11]; and "Silt Transportation by the Sacramento and Colorado Rivers and by the Imperial Canal"[12]. He also contributed valuable discussions to many papers appearing in the *Transactions* of the Society. As a member of the Special Committee on Floods and Flood Prevention[13] of the Society and the Committees of the Irrigation Division on Interstate Water Matters[14] and on "A National Reclamation Policy"[15], Mr. Grunsky took part in their reports.

The Committee has passed in brief review the career of one of the leading American Civil Engineers; it has touched on his professional attainments and his success in the broad field of citizenship. What manner of man was he to have attained this eminence—a man respected and beloved? In 1884, Mr. Grunsky was married to Mattie Kate Powers, and they had a family of four children—two daughters and two sons. The latter are both members of the Society. Mr. Grunsky's daughter, Clotilde, well known to many members of the Society, writes of his family life:

> "Our home life was a uniformly happy one. We children always had unbounded respect and confidence in both our parents. Such rebukes as it was necessary to give a growing family were firm and brief and we did not question their justice. His pride and faith in us all was our greatest spur to achievement. I never (but once to the great amusement of the family) heard father use a swear word, and anything mean or vulgar was offensive to him; his sense of honor was strict and unfailing. Nothing but complete honesty would do. He believed in the natural happy enjoyment of life and, to the last, nothing gave him greater pleasure than a day in the country with his grand-children and the family pets. Undoubtedly, his unwillingness to admit restrictions of illness shortened his life, but who shall question his (I believe) reasoned choice that he would live normally and in the atmosphere of cheerfulness as long as he had life itself."

When Mr. Grunsky's mother died in 1862, he was only a small boy. The last admonition he received from her through a dear friend was "to be good, to avoid strife, to heed the little small voice within and follow its guidance, to travel the straight and narrow path". Referring to this Mr. Grunsky himself has said,

> "Perhaps it were better not recorded, but there has never been a moment of my life when this last admonition was not heeded, and it is with some satisfaction that I can record no regret at having endeavored to keep to the highest standard of living. I have never, even in those things concerning which there is so much difference of opinion, permitted myself to disregard

---

[8] *Transactions*, Am. Soc. C. E., Vol. 85 (1922), p. 1488.

[9] *Loc. cit.*, Vol. 87 (1924), p. 1225.

[10] *Loc. cit.*, Vol. 93 (1929), p. 262.

[11] *Loc. cit.*, Vol. 93 (1929), p. 791.

[12] *Loc. cit.*, Vol. 94 (1930), p. 1104.

[13] *Loc. cit.*, Vol. LXXXI (1917), p. 1218.

[14] *Loc. cit.*, Vol. 94 (1930), p. 1400.

[15] *Loc. cit.*, Vol. 95 (1931), pp. 1303, 1338.

the inner voice and have held sacred many of the man-made institutions the breach of which in our modern times is so readily overlooked. There is no other purpose in this declaration than the hope that a modest example may in turn have a good influence upon some one else."

As an engineer, Mr. Grunsky was eminent and honored among his professional brethren and a widely recognized authority in connection with water-works, irrigation, reclamation, and sanitation. As a professional man, he was active in the interchange of technical information and opinion and in the maintenance of his professional and ethical standards. As a citizen, he was a profound student of public affairs and gave generously of his time and talents in the interest of an enlightened citizenry and of good government. As a man, he was wise, honest, and conscientious, considerate of the interests and opinions of his fellow men, a devoted husband and father, and a sympathetic counselor and guide of the younger generation—verily, a "gentleman of the old school".

Such was Mr. Grunsky, such his guiding principles through a life rich in achievement and in devotion to his family, his profession, and his fellow man.

Mr. Grunsky was elected a Member of the American Society of Civil Engineers on October 5, 1898; and served as a Director of the Society, from 1919 to 1921; as a Vice-President in 1922 and 1923; and as President in 1924.

KOI FURUICHI, Hon. M. Am. Soc. C. E.[1]

Died January 28, 1934

Koi Furuichi was born in Yedo (Tokyo), Japan, on July 12, 1854, the first son of Takashi Furuichi, a Samurai of the former Himeji Clan.

From January, 1869, he studied in Kaiseigakko and Daigaku-nanko (later, Tokyo Imperial University) as a tribute scholar (excellent scholar sent from the former feudal clan to the Department of Education), until July, 1875, when he was sent to France.

Mr. Furuichi studied at l'Ecole Mongé, in Paris, and also at l'Ecole Centrale des Arts et Manufactures. He was graduated therefrom in August, 1879, with the degree of "Ingénieur des Arts et Manufactures". He then entered l'Université de Paris, from which he obtained the degree of "Licencie es Sciences" in July, 1880.

In October, 1880, he returned to Japan, and served as an Engineer in the Home Department, holding, in addition, the position of a Lecturer at Tokyo Daigaku.

In 1886, the Faculties of Tokyo Daigaku and other educational institutions were amalgamated, and the Tokyo Imperial University was organized. Mr. Furuichi became Professor and Dean of the College of Engineering of the Imperial University, with supplementary duties as Engineer of the Home Department, for which he superintended the notable river improvement works of Shinanogawa, Aganogawa, and Shogawa.

---

[1] Memoir prepared by Sakuro Tanabe, M. Am. Soc. C. E.

In May, 1888, the academic degree of "Kogakuhakushi" (Doctor of Engineering) was conferred upon him. The following year, he visited Europe in the suite of Count Yamagata, Minister of the Home Department, and investigated the various systems of engineering education.

In June, 1890, Dr. Furuichi became Director of the Bureau of Public Works of the Home Department, holding also the positions of Dean and Professor of the College of Engineering of the Imperial University, until his resignation in July, 1898.

In September, 1890, he became a Member of House of Peers, and, subsequently, served as Juryman and Councillor of the Domestic Industrial Exposition, Member of the Earthquake Investigation Committee, and of the River and Harbor Investigation Committee, as Adviser to Tokyo, Yokohama, and other harbor works.

In November, 1898, Dr. Furuichi was appointed Vice-Minister of the Department of Communications, Member of the Railway Nationalization Committee, and Chairman of the Investigation Committee for the Construction of the Imperial Steel Factory, the solution of an important National problem.

He then became President of the Imperial Railway Construction Bureau, and was elected Vice-President of the Imperial Railway Association, retaining that position until December, 1903. In the same year, the honorary title of Professor Emeritus of Toyko Imperial University was conferred on him.

Soon after his resignation from the Government Service, Dr. Furuichi became President of the Seoul-Fusan Railway Company, in Korea. The speedy construction of this railway line was of great advantage in the operations of the Russo-Japanese War, in 1904 and 1905.

In June, 1906, he became a Member of the Imperial Academy, and was President of the Second Section. On the transference of the Seoul-Fusan Railway to the Railway Bureau of the Korean Government in 1906, he was appointed President of the Bureau, remaining in that position until June, 1907.

Dr. Furuichi was then made Adviser to several important companies, and President of the Tokyo Subway Company. He also served on the Committee for the Railway-Gauge Question, construction of a new Parliament Building, and the Anglo-Japanese Exhibition. He was President of the Chemical Research Institute, and Chairman of the National Research Council.

For his long and distinguished engineering service, Dr. Furuichi was created a Baron in December, 1919.

He was a Councillor of the City Reconstruction Bureau of Tokyo, in connection with the destruction caused by the great earthquake of 1923, as well as an Honorary Member of the Imperial Railway Association. He was one of the founders of the first private school (now "Kogakuin") for training assistant engineers, and served as its Managing Director from 1903 to

1924. He was also President of the Japan Engineering Society ("Kogakkwai") for many years, until 1934. In September, 1914, he was elected President of the Civil Engineering Society of Japan, when that Society was founded, and in January, 1933, he was made an Honorary Member. In 1930, he was elected an Honorary Member of the Institution of Civil Engineers, of London, England.

In January, 1924, he was appointed a Privy Councillor of the Japanese Government, and remained in that position until his death on January 28, 1934.

Baron Furuichi held the Second Junior Court Rank, and the First-Class Decoration of the Grand Cordon of the Rising Sun with the Paulownia Flowers. He had many other decorations, such as those from Cambodia, Denmark, Belgium, and Korea; and, from France, the Legion d'Honneur, Second Class.

He was always a leader in all engineering and engineering education problems throughout his life. When the meetings of the World Engineering Congress and World Power Conference were held in Tokyo, in October and November, 1929, he was elected President.

Baron Furuichi was one of the best amateur singers and actors of "No Dance". He was married to Ko Kawana, a graduate of the Tokyo Woman's High School, who, with seven sons and three daughters, survives him.

Baron Furuichi was elected an Honorary Member of the American Society of Civil Engineers on July 8, 1929.

OCTAVIO AUGUSTO ACEVEDO, M. Am. Soc. C. E.[1]

DIED APRIL 30, 1933

Don Octavio Augusto Acevedo y Camarena was born in San Pedro de Macorís, Dominican Republic, on February 12, 1878, and there received his early education. His birthplace was only a fishing village at that time, and he was compelled to go to Santo Domingo City to continue his high school studies in the Normal School.

Having finished High School, Señor Acevedo returned to San Pedro de Macorís and was employed in the French Cable Office, where he remained until 1899, having been made head of that office. He then left for the United States, where, mainly through his own efforts, he continued his studies in engineering. He was graduated, in 1903, from the North Carolina State College, at Raleigh, N. C., with the degree of Highway Engineer and Surveyor.

Upon his return to his native country, the Department of Justice and Public Instruction granted him the license of Civil Engineer and Public Surveyor.

---

[1] Memoir prepared by Antonio S. Romero, M. Am. Soc. C. E., and Eduardo Soler, Jr., Assoc. M. Am. Soc. C. E.

At that time the sugar industry was beginning to flourish around San Pedro de Macorís, and it was there that Señor Acevedo began the practice of his chosen profession. Land surveying, railroad location and construction, and building and concrete design kept him busy until 1914.

During 1914 and 1915, Señor Acevedo was City Engineer of San Pedro de Macorís, which by that time had grown from a fishing village to an important sugar center. City planning, drainage systems, river walls—his native town owes Señor Acevedo a debt of gratitude for many improvements, for he was a tireless worker in its interests.

In 1916, he moved to Santo Domingo City where he was employed by the National Government as Technical Inspector of the Department of Fomento. No one in his country knew its natural resources as well as he did; nor was any one so interested in their development. At the beginning of his new work, the Dominican Republic was occupied by American military forces, and, as a vast public works and road construction program was started at that time, Señor Acevedo became closely allied with the development of his country, for which he was so desirous.

He held this post until 1922, when a Provisional Dominican Government was established, with Don J. B. Vicini Burgos as President. Under this Government, Señor Acevedo was appointed Secretary of State of Fomento and Communications. During this time, he had the opportunity to complete and open several important bridges and roads, among them, those which connect his native town with Santo Domingo City.

During 1925, Señor Acevedo made extensive studies of the Jimenoa River and designed a hydro-electric plant, which, however, was never built. He continued to engage in private work as a Consulting Engineer and, for some time, was Resident Engineer on the construction of the Hato Mayor-Sabana de la Mar Highway.

In 1929, Señor Acevedo was appointed Professor of Mathematics and Civil Engineering in the University of Santo Domingo. His work in the University was a constant struggle to raise the standards of Engineering Education in the Dominican Republic. From February, 1933, to the time of his sudden death, on April 30, 1933, Professor Acevedo held provisionally the office of Dean of the Faculty of Mathematics.

He was married in 1906 and is survived by his widow and four children—two daughters and two sons.

Señor Acevedo had been a member of the Sociedad de Ingenieros de Lima, Peru, since 1919; he was also a member of the Society of American Military Engineers; American Railway Engineering Association; and Sociedad Matemática Hispano Americana. In 1932, he organized the Instituto Dominicano de Ingenieros, of which he was the first President. The purpose of this Society was to improve the standards of engineering practice and education in the Dominican Republic. Unhappily, this institution did not survive him.

Señor Acevedo was an untiring and conscientious writer and the author of three books, namely "Vías de Comunicación," "Tópicos Técnicos," and

"Concreto Armado;" as well as several pamphlets and numerous newspaper articles on engineering subjects. He left unpublished a Manual de Carreteras, in which he presented a complete course in Highway Engineering; he also had in preparation a textbook on Railway Engineering.

Señor Acevedo was elected a Member of the American Society of Civil Engineers on November 11, 1929.

FREDERICK WHITNEY ADGATE, M. Am. Soc. C. E.[1]

DIED OCTOBER 23, 1934

Frederick Whitney Adgate was born on a farm near Keeseville, N. Y., on June 5, 1868. He was the son of George Adgate and Martha (Whitney) Adgate. He attended the public schools in Keeseville and, later, studied for three years at the Kansas State Agricultural College, at Manhattan, Kans.

Mr. Adgate's engineering experience covered a wide variety of work. From May, 1889, to August, 1890, he was Engineer and Superintendent for his grandfather, Captain Luther Whitney, on Government work on Lake Champlain. From August, 1890, to October, 1893, he was with General William Sooysmith, first in charge of machinery on the construction of Missouri River bridges, at Sioux City, Iowa, and Omaha, Nebr., and, later, as Superintendent of the concrete work on the foundations of the Stock Exchange Building, in Chicago, Ill. From January, 1894, to February, 1896, he was a Contractor on Lake Champlain, engaged in the construction of docks and foundation work.

In March, 1896, Mr. Adgate went with A. B. Clark and Company and was Superintendent on rock tunnel work at Jerome Park Reservoir, for the water supply of New York City, until September, 1897. He then returned to General Sooysmith and was Superintendent on bridges over the St. Regis and Paquet Rivers, until May, 1898. From May, 1898, until January, 1899, he was with the late Charles H. Deans, Assoc. M. Am. Soc. C. E., as Assistant Superintendent on the Intercolonial Bridge, at Ottawa, Ont., Canada. From January, 1899, to May, 1900, he was with the Engineering Contracting Company and was Superintendent, successively, on the Thames River Bridge, at Chatham, Ont., Canada, the Chicago and Alton Railroad Bridge over the Missouri River, at Glasgow, Mo., and the foundations for the Boone, Iowa, Viaduct. From December, 1900, to December, 1901, Mr. Adgate was Assistant Superintendent for Mr. John Pierce on the construction of the dry dock at Portsmouth, N. H., Navy Yard. From December, 1901, to October, 1902, he was Superintendent for the Ohio River Bridge and Ferry Company on the construction of a bridge across the Ohio River, at Marietta, Ohio, and from October, 1902, to November, 1903, he was Superintendent for the Foundation and Contracting Company, on a bridge over the Missouri River, at Omaha, Nebr.

---

[1] Memoir prepared by Louis J. Hotchkiss, M. Am. Soc. C. E.

From November, 1903, to January, 1910, Mr. Adgate was with The Foundation Company as Superintendent on many large foundation jobs for bridges and buildings and numerous mine shafts. In January, 1910, he was appointed Western Manager of The Foundation Company, in charge of all that Company's business in the Central West, and, with the exception of a brief leave of absence, he continued in that capacity until his death.

Mr. Adgate's home was at Wheaton, Ill., where he had lived for twenty-six years and where he was active in civic affairs. He had a wide circle of friends and his death, which occurred suddenly from angina pectoris, was a great shock to his family and to his friends and business associates. He was a man of kindly ways, always interested in the people he knew and in the things which concerned them, and always ready to lend a helping hand when needed. His passing marks the end of a useful life and he is sincerely mourned by all who knew him.

Mr. Adgate was a member of Ausable Lodge No. 149, A. F. & A. M., of Keeseville, N. Y.; Siloam Commandery No. 64, Knights Templars, of Oak Park, Ill., and Medinah Temple, A. A. O. N. M. S., of Chicago.

He is survived by his widow, Dolly M. Adgate, one son, George, two daughters, Mrs. Douglas Edward Huncke and Elizabeth Triplette Adgate, and by one sister, Mrs. John R. Dean, of Buffalo, N. Y.

Mr. Adgate was elected an Associate Member of the American Society of Civil Engineers on January 6, 1904, and a Member on May 28, 1912.

JULIUS BAIER, M. Am. Soc. C. E.[1]

Died May 8, 1905

Julius Baier, the son of Henry and Marie (Glenzer) Baier, was born on June 20, 1863, in Peoria, Ill. He was the second oldest of a family of seven children, four sons and three daughters.

He attended the public schools in Peoria and in Pekin, Ill., to which place the family moved in 1870. In 1875, the family went to St. Louis, Mo., and Julius attended the public schools there and was graduated from the St. Louis Public High School in 1881, receiving the honor award of a Scholarship to Washington University, in St. Louis. He was graduated from Washington University, in 1885, with the highest honors, receiving the degree of Civil Engineer.

Mr. Baier's first employment was in August, 1885, as Assistant Engineer in the office of the late Robert Moore, M. Am. Soc. C. E., in St. Louis. In March, 1886, he was employed as Rodman on the construction of the Batesville Branch of the St. Louis Iron Mountain and Southern Railway, in Arkansas. In September, 1886, he was engaged as Assistant Engineer in the Division Engineer's Office of the Kansas Division of the Union Pacific Railroad and, in March, 1887, he joined a field party as Leveler on the survey of the St. Louis and Paducah Railroad. He returned to St. Louis in June, 1887, as Assistant Engineer with Mr. Moore, and prepared the plans for the reconstruction of the floor system of the Eads Bridge. Later, he inspected the material and shopwork for this work at the shops of the Edge Moor Bridge Works, at Wilmington, Del., and then supervised its erection at St. Louis.

---

[1] Memoir prepared by E. B. Fay, M. Am. Soc. C. E.

From December, 1888, to September, 1893, Mr. Baier was Principal Assistant Engineer of the St. Louis Merchants Bridge Terminal Railway, of which Mr. Moore was Chief Engineer. In this position he was in responsible charge of the design and construction of $1\frac{2}{3}$ miles of double-track elevated structure for freight service, as well as the planning and construction of all tracks, yards, and terminals for the road. From September, 1893, to November, 1895, he was Principal Assistant Engineer of the Terminal Railroad Association of St. Louis, and prepared the designs and final plans and supervised the construction of the extension of the double-track elevated structure previously mentioned to reach the Union Station and for the reconstruction of the East Approach of the Eads Bridge.

In November, 1895, Mr. Baier opened an office in St. Louis and engaged in private practice as a Structural Engineer. During 1898, on account of poor health, he moved his office to Colorado Springs, Colo., where he continued in private practice until March, 1900. During this period he acted as Consulting Engineer for many architects, owners, and contractors, and prepared designs for the structural steelwork for many notable structures, among which were the Columbia Theatre and the Drill Shed, Battery, and Armory, at St. Louis; the cell wings for the Federal Penitentiary at Leavenworth, Kans.; and the steel arches for the old St. Louis Coliseum.

It was while he was engaged in consulting practice in St. Louis that the tornado of May 27, 1896, occurred. Mr. Baier made a careful study and investigation of the resulting damage to various structures and prepared a paper[2] entitled "Wind Pressures in the St. Louis Tornado, with Special Reference to the Necessity of Wind Bracing for High Buildings" which he presented before the Society on March 3, 1897. The paper was awarded the Norman Medal of the Society "for its merit as a contribution to engineering science."

While at Colorado Springs, Mr. Baier studied mining and became an expert in placer mining work. In March, 1900, he accepted a position with the Conrey Placer Mining Company, at Ruby, Mont., as Engineer in charge of plans and surveys. Within six months he was promoted to the position of Assistant General Manager and Engineer in responsible charge of all mining operations and properties of the Company. He was very successful in this work, greatly increasing the capacity and efficiency of the Company's dredges by remodeling them for electrical operation, and, later, he built new dredges to be operated electrically. While he was directing a survey on some of the outlying properties of the Company, he was caught in a severe storm and from the drenching and exposure to which he was subjected, contracted pneumonia from which he died in a hospital at Ruby.

---

[2] *Transactions*, Am. Soc. C. E., Vol. XXXVII (June, 1897), p. 221.

Mr. Baier was unmarried and, at the time of his death, was survived by his mother, a sister, and three brothers.

He was a man of sterling character. His exceptional ability, sound judgment, and spirit of fairness in dealing with those with whom he came in close contact, won their admiration and respect. Although mild in manner, Mr. Baier was firm in his decisions and insistent in carrying out work according to the best practice. His clear-sighted analysis of conditions enabled him to arrive quickly at the proper solution of a problem and to point out the fault or error in a poor design.

He is remembered with gratitude by many younger engineers for his kindly interest and assistance in their progress. He was studious and a lover of good literature and music, and his gentlemanly qualities, active interest in the good fortunes, and sure sympathy in times of loss, endeared him to his intimate friends.

Mr. Baier was elected a Junior of the American Society of Civil Engineers on September 7, 1887; an Associate Member on June 1, 1892; and a Member on February 6, 1901.

KORT BERLE, M. Am. Soc. C. E.[1]

Died May 5, 1934

Kort Berle, the son of Kort R. and Maria (Keyser) Berle, was born in Halden, Norway, on February 4, 1866. He received his early training in the public schools of his native town, and was graduated in 1886 from the Technical Institute of Oslo, Norway, with a degree in Mechanical Engineering.

After his graduation from the Technical Institute, Mr. Berle worked one year in Norway and, in 1888, came to the United States, where he secured a position as Draftsman with the Midvale Steel Works, at Philadelphia, Pa. His engineering experience for the next few years was, as follows: One year at Midvale; two years as Draftsman in the Supervising Architect's Office, of the United States Treasury Department, in Washington, D. C., designing the structural features of Post Office buildings; two years with the Phœnix Bridge Company, at Phœnixville, Pa.; and then, in 1894, back to the Supervising Architect's Office where he remained until 1905, the last two years of which period he served as Chief Engineer of the Structural Division of that Office. In addition to his duties as Chief Engineer for the Supervising Architect, Mr. Berle found time to engage in a large amount of consulting work with various architects, in Washington, and designed the structural features of the Mendota, Glenora, and Stonleigh Court Apartments, in that city, and a great number of other buildings.

---

[1] Memoir prepared by S. F. Holtzman, M. Am. Soc. C. E.

In 1905, he resigned from the Treasury Department to accept a position as Chief Engineer and Manager of the Structural Steel Department of Barber and Ross, in Washington, with which firm he remained for two years. For the next two years Mr. Berle was engaged in private consulting work in Washington. In 1909, he went to New York, N. Y., to enter the firm of Gunvald Aus Company, where he remained until his death. He was actively associated with the work done by this firm, and among the buildings with which he was connected were the Woolworth Building, New York City; Union Central Life Building, Cincinnati, Ohio; United States Supreme Court Building, at Washington; the new Federal Courts Building, in New York City; State Capitol Buildings for Arkansas, Missouri, Washington, and West Virginia; and many other monumental and business buildings.

Mr. Berle was a member of the New York State Society of Professional Engineers, the Structural Engineers Society of New York, the New York Rotary Club, and the New York Building Congress. He had a most engaging personality, a keen memory which always made his conversation interesting to his listeners, and a rich baritone voice which his friends loved to hear. His whole nature exuded optimism, and his genial smile made a pleasant atmosphere in which his business was transacted.

Mr. Berle had long been a sufferer from asthma, having been too ill to attend to his office duties for practically the entire last year of his life. This, ultimately, was the contributing cause of his death from heart failure. His body was cremated, and his ashes will probably be returned to Norway to be buried in a family plot with his ancestors.

He is survived by his widow, two sons, Fridtjof K. Berle, of Kew Gardens, N. Y., and Alf K. Berle, of New York City, and a daughter, Mrs. Sterling Peterson, of Kew Gardens.

Mr. Berle was elected an Associate Member of the American Society of Civil Engineers on December 6, 1899, and a Member on November 4, 1903.

LYMAN EDGAR BISHOP, M. Am. Soc. C. E.[1]

DIED MARCH 6, 1933

Lyman Edgar Bishop was born in Des Moines, Iowa, on January 13, 1884, the son of Edgar H. Bishop and Mary E. Bishop (*née* Van Horsen). His father was of Scotch-English ancestry, while his mother was a Hollander. In 1887 he removed, with his parents, to Denver, Colo., where his father pursued his occupation as a carpenter foreman, erecting mills and other buildings around Denver and Leadville, Colo.

Lyman Edgar Bishop was educated in the public schools of Denver. While attending the East Denver High School, he was made First Lieutenant of

---

[1] Memoir prepared by a Committee of the Colorado Section, consisting of William B. Freeman and W. W. Curtis, Members, Am. Soc. C. E., and J. H. Baily, Esq., Denver, Colo.

Cadets, and served two newspaper routes. From his earnings of these routes, together with their sale value when he finished High School, he secured enough money to enable him to start his course in the University of Colorado, at Boulder, Colo. Here he had the honor of being the first man elected from his class to Tau Beta Pi. He was graduated in 1908, with the degree of Bachelor of Science in Civil Engineering.

During the ensuing twenty years Mr. Bishop continued to live in Denver, while being occupied in various engineering work in the vicinity. He specialized in hydraulics and irrigation and was intimately connected with the water supply of Denver, both while the water-works system was in the hands of private parties and after it became the property of the City. The data he collected on the sources of supply for Denver were acquired after his death by the Water Board and constitute perhaps the best collection of facts thereon made thus far. In his earlier years Mr. Bishop was connected as an Assistant, with the Denver Reservoir and Irrigation Company on a project covering 200 000 acres and including four reservoirs, and with the Laramie Water Company, of Laramie, Wyo., serving 70 000 acres. In 1914, he was in private practice, during which time he located 22 miles of the Grand Valley Canal for the United States Reclamation Service.

From 1915 to 1917, Mr. Bishop was Deputy State Engineer of Colorado, under A. A. Weiland, M. Am. Soc. C. E., State Engineer. On the expiration of his term of office, he was appointed Construction Superintendent on the Costilla Dam, an earth structure 120 ft high. In July, 1917, he returned to private practice, covering the design and construction of hydraulic plants, investigations in connection with litigation, and testimony thereon. With C. A. Fisher, Geologist, he made a joint investigation and report on the geological conditions for the 100-ft masonry dam on the Bear River, at Alexander, Idaho, the investigation involving $75 000 of diamond drilling. In 1927 he presented a report on the water conditions of Denver—past, present, and future; the future requirements; and the possibilities of trans-mountain diversions, a report costing about $10 000.

In May, 1929, Mr. Bishop was appointed Consulting Irrigation Engineer for the Union of Socialistic Soviet Republics, with headquarters at Tashkent, Turkestan, Central Asia. He served in this capacity for two years, and was Consulting Engineer for the General Cotton Committee; and Consultant and Adviser on the operation and proposed irrigation and drainage projects of Turkestan, involving about 15 000 000 acres. While there, Mr. Bishop wrote some interesting letters home. In one, he describes a two weeks' field trip from which he had just returned inspecting conditions in the Ferghana Valley (about 200 miles from Tashkent):

> "This valley is egg-shaped about 60 miles wide by 140 miles long. There are now some 1 900 000 acres under irrigation and cultivation in this valley. Roughly two-thirds of the area is devoted to cotton. Some of this land has been under irrigation for over 1 200 years."

Mr. Bishop's service in Russia was varied by a visit from his wife and son in 1930. They went by way of Paris, London, and Berlin, to Moscow.

Then, after an 8-day wait to secure an opportunity to ride on the trains to Tashkent, they proceeded on their way in a compartment holding four, of whom two were burly Russians. Forced to live for four days on black bread, cucumbers, and hard-boiled eggs, they finally reached Tashkent and spent five months with Mr. Bishop, returning to America in January, 1931.

Mr. Bishop fulfilled his contract with the Soviet Government, but did not renew it on its termination in 1931. He returned to America by way of India, the Philippines, and the Pacific Coast, spending several months on the way, examining irrigation projects, and reaching Denver in the fall of 1931. Here, he found a world out of joint. His old clients had secured other advisers, if they had or could borrow money for new enterprises they dared undertake; a general lack of anything for which an engineer's services were desired; and a complete lack of confidence in things generally. He struggled against these conditions for eighteen months and then solved his difficulties with his own hand and passed out of life on March 6, 1933.

Lyman Edgar Bishop was a man largely endowed by Nature. To a good physique was added a fine mind, well and symmetrically educated. He was an exceptionally well-informed mathematician, thoroughly posted in his particular branch of engineering, namely, hydraulics.

He was a man of strong convictions and when he formed an opinion or conclusion it was difficult for him to compromise, or to alter his views, even in matters of small consequence. This inflexibility of mind was perhaps a handicap to him at times in his professional dealings as well as in his relations with the public. However, Mr. Bishop possessed many admirable qualities for which he was held in esteem by his associates. He was a most genial companion and was usually ready with a good story appropriate to the discussion. Among other things he was quite active in Masonry. He left many friends and admirers who regret his passing.

He was married in 1909, at Sterling, Colo., to Helen Clarissa Howell, who, with a son, Arthur, survives him.

Mr. Bishop was elected a Junior of the American Society of Civil Engineers on May 4, 1909; an Associate Member on October 31, 1911; and a Member on June 16, 1919.

GEORGE MEADE BOND, M. Am. Soc. C. E.[1]

DIED JANUARY 6, 1935

George Meade Bond was born in Newburyport, Mass., on July 17, 1852. The son of Daniel George and Wilhelmina (Kruger) Bond, he came from a line of New England ancestry, his father's forebears having landed at Newbury, Mass., from England, about 1638.

---

[1] Memoir prepared by Charles Coit Tyler, Esq., Hartford, Conn.

Soon after the death of his father, in 1854, his mother, with her two sons, moved to Grand Rapids, Mich., to be near her brother. His mother and his brother both died in 1862, and the orphan boy of ten then lived with his uncle and attended the local schools until he was seventeen years old when he became a teacher in the schools of near-by districts. Finding that teaching had no attraction for him, Mr. Bond decided to learn the machinist trade. He became an apprentice in one of the factories in Grand Rapids, and continued as a Machinist with the Phoenix Furniture Company in that city until his savings permitted him to enter Stevens Institute of Technology, at Hoboken, N. J., in 1876. By working during vacations he was enabled to continue his studies and to complete the four years' course, graduating in 1880 with the degree of Mechanical Engineer.

During 1879, in his Senior year, through the foresight of Professor James E. Denton, relations were brought about between Mr. Bond and Professor William A. Rogers, at that time Professor of Astronomy at Harvard College Observatory, Cambridge, Mass., and thus began his important work in connection with the establishment of a standard system of measurement.

As Assistant to Professor Rogers, and under his direction, Mr. Bond designed a comparator, which would enable the astronomer to conduct his investigations more efficiently in standards of length. Drawings were completed at Stevens Institute in 1880, and on July 1 of that year Mr. Bond entered the service of the Pratt and Whitney Company, of Hartford, Conn., to direct the building of the comparator, and to carry out the work of establishing standards for the Company, under the general direction of Professor Rogers, whose scientific work formed the basis for the system of measurement now generally recognized as the practical solution of the problem of accurate interchangeability in machine construction in the United States.

The need of uniformity in the sizes and threads of bolts and nuts for railroad service was the incentive for much of the preliminary work referred to, and its successful adoption, and the many other applications of standard interchangeability, not only in railroad service, but in manufacturing as well, has amply repaid the time and investigations required for it.

Mr. Bond was Manager of the Standards and Gauge Department of the Pratt and Whitney Company from 1880 to October, 1902, during which period he designed and the Company produced the line of cylindrical, caliper, snap, limit, thread, and other gauges which have been in common use throughout the country.

He also designed the Bond Standard Measuring Machine by the use of which it became possible to make gauges to a given size and to reproduce them to exactly the same length or diameter, no matter how many years apart the duplicates were made.

One or more of these measuring machines will be found in metal-working establishments throughout this country and abroad, particularly where products are manufactured upon an interchangeable basis—and to Mr. Bond's genius belongs the credit of being one of the outstanding factors in making that system practical.

Mr. Bond had been a member of the American Society of Mechanical Engineers since 1881, and served it as Manager from 1888 to 1891, and as Vice-President from 1908 to 1910. He presented papers at meetings of that Society upon the following subjects: "Standard Measurements"; "A Standard Gauge System," and "Standard Pipes and Pipe Threads"; and took part in the discussions of papers upon technical subjects. He also served the American Society of Mechanical Engineers as an active member of important committees, notably standard unions for pipes, standard proportions for machine screws, standard pipes and pipe threads, and the Committee appointed by that Society to appear before the Congressional Committee on Coinage, Weights, and Measures, in opposition to proposed legislation whereby the adoption of the metric system would be made compulsory, that system being already a legal standard by Act of Congress in 1866.

Mr. Bond was a Fellow of the American Association for the Advancement of Science; Tercentenary Member of the New England Historic Genealogical Society; Connecticut Historical Society; Alumni Association of Stevens Institute, and of the Hartford Club, of Hartford; The Engineers', Transportation, and the Union League Clubs, of New York, N. Y., and the Royal Societies Club, of London, England.

The honorary degree of Doctor of Engineering was conferred upon Mr. Bond by Stevens Institute of Technology in 1921, and that of Master of Science by Trinity College, Hartford, in 1927.

Mr. Bond had been a resident of Hartford for more than fifty-four years, his home being on Washington Street until 1926, and since that time on North Marshall Street.

The following tribute (published in the *Hartford Daily Times,* of January 9, 1935), to Mr. Bond was paid by Clayton R. Burt, President of Pratt and Whitney Company:

> "The passing of George M. Bond marks finis to the career of a man who has had a tremendous effect not only on Pratt & Whitney Company but indirectly on every person in the civilized world. Mr. Bond's work with Pratt & Whitney Company in the several years following 1879 in connection with Professor William A. Rogers marked the beginning of what we know today as interchangeable manufacture and mass production. It was the efforts of these men, combined with the response of Pratt & Whitney Company, which first standardized American methods of measuring and developed the standards of length which have been our basis ever since. The work made possible the development of standard gauges of all sorts. In the succeeding years it has made possible the large scale production of accurate parts, and has turned expensive luxuries into everyday necessities. Hundreds of things we use—automobiles, typewriters, telephones, sewing machines and many more—though they are the achievements of other men, would not have been possible as we now have them without the pioneer work which George M. Bond did. His contribution to the peace, happiness and comfort of the world is truly great.
>
> "We of Pratt & Whitney Company mourn his passing and can only hope to emulate his great example."

He had never married and had no near relatives living, but he had a large circle of friends in Hartford, New York City, and elsewhere, who will deeply regret his passing.

Mr. Bond was elected a Member of the American Society of Civil Engineers on February 2, 1887. He also served for a time as Chairman of the Special Committee of Units of Measurements of the Society.

ALFRED VICTOR BOUILLON, M. Am. Soc. C. E.[1]

Died January 14, 1935

Alfred Victor Bouillon, the son of Joseph Antoine and Victorine (Linchet) Bouillon, was born on April 13, 1869, at Custinne, a village in the picturesque Ardennes hills in the south-eastern part of Belgium, the ancestral home of the family being the old Town of Bouillon, near the present Franco-Belgian border, a famous historical spot intimately connected with the First Crusade. His branch of the family came to America in 1883, shortly thereafter moving to the Pacific Coast, and eventually settling in Seattle, Wash.

The period of his academic education was about equally divided between schools in Belgium, France, and Canada, including graduation from a business college. Being naturally studious and mechanically inclined, Mr. Bouillon combined home studies of engineering with practical field and office work, a system which he applied consistently throughout his career. In his engineering practice he was known to be very methodical, earnest, and resourceful, and possessed of a high sense of duty and responsibility, never venturing an opinion nor making a recommendation until after thorough analytical and practical studies of the subjects submitted for his consideration. He thus acquired a reputation for thoroughness, integrity, and dependability of judgment, highly valued by those with whom he became associated.

Early in his career Mr. Bouillon assisted his father, who was an Architect and Contractor, on the design and construction of buildings. From September, 1888, to September, 1890, he was on the staff of the City Engineer of Seattle, and was promoted to the position of Chief Draftsman shortly after his appointment. From September, 1890, to January 1892, he was in charge of the office of the late Henry Kinder Owens, M. Am. Soc. C. E., when he returned to the City's employ, successively, as Chief Draftsman, Office and Designing Engineer, and Assistant to the City Engineer, until September, 1895. He then accepted an offer as Assistant Chief Engineer with the Seattle Power Company, whose ambitious scheme for supplying power and mountain water to that City and adjoining territory was abandoned three months later because of the City's refusal to grant the required charter.

In December, 1895 Mr. Bouillon joined the staff of the Moran Brothers Company, Engineers and Shipbuilders, as Chief Draftsman. He was soon promoted to be Chief Engineer and Assistant to the General Manager, which position he held until the dissolution of the firm and the disposal of its entire plant to a large Eastern organization in April, 1908. During this period, the Moran Brothers Company had been very active, particularly in ship construction, which included the revenue cutter, *Golden Gate,* the torpedo-boat, *Roanoke,* the battleship *Nebraska,* for the United States Navy, etc. besides a flotilla of several dozen river steamboats built for Yukon and Alaskan waters, largely incidental to the Klondike gold rush. This firm also did considerable ship repair and salvage work and built a floating dry dock as well as a marine railway to meet these demands in connection with its plant. These additions and activities were also designed and largely supervised by Mr. Bouillon, whom Robert Moran, President of the Moran Brothers Company, referred to as "one of the most skillful and resourceful persons associated with me in my entire business career," adding that Mr. Bouillon's later activities with the United States Shipping Board, showed "skill, knowledge, and fairness of mind, which made it possible for the Government to come to a settlement with shipbuilders all over the United States on their claims for compensation arising out of the cancelling of ship contracts after the Armistice was signed."

---

[1] Memoir compiled by A. M. Bouillon, M. Am. Soc. C. E., from information supplied by R. H. Thomson, and W. Horace Williams, Members, Am. Soc. C. E., Messrs. Robert Moran, J. B. Woodward, Jr., and J. C. Peacock, and from information on file at the Headquarters of the Society.

Mr. Bouillon was appointed Superintendent of Public Utilities for the City of Seattle from April, 1908, after severing his connection with the Moran Brothers Company, until 1910; he was also elected Chairman of the Board of Public Works. In these capacities, he fought relentlessly against abuses and special privileges; he was a candidate for Mayor, but was defeated by an opponent backed by the powerful interests then in control, who, incidentally, was later recalled by public vote.

From May, 1910, to July, 1911, Mr. Bouillon was specially engaged as City Commissioner of Public Works and Utilities by the City of Edmonton, Alberta, Canada, in order to bring the city-owned utilities out of the chaos resulting from inexperienced and improper planning. In October, 1911, after his return to Seattle, he formed a partnership with Mr. Owens which firm was engaged in successful professional practice as Consulting Civil and Mechanical Engineers until late in 1917. During this period, Mr. Bouillon also acted, in the capacity of Expert and Assistant to the General Manager, with the Newport News Shipbuilding and Dry Dock Company on special assignments from 1915 until he closed his private practice and entered Government service in May, 1918.

It was the late Mr. William B. King, of King and King, internationally known Attorneys and Counsellors-at-Law, of Washington, D. C., who recommended Mr. Bouillon to the Newport News Shipbuilding and Dry Dock Company. In this connection, Mr. King wrote: "I am venturing to call to your attention the very striking ability of Mr. Bouillon;" and, also, "he is very industrious and earnest in his work, very straightforward and honorable. He is wonderfully exact in everything that he does, extremely ingenious in the consideration of methods of mathematical demonstration, an absolute master of detail." Speaking on behalf of the Newport News Shipbuilding and Dry Dock Company, Mr. J. B. Woodward, Jr., the present Assistant General Manager, added that the foregoing recommendations were fully sustained.

Mr. Bouillon entered the Government service in May, 1918, when he was sworn in as Assistant Manager, Contract Division, of the United States Shipping Board Emergency Fleet Corporation, remaining in Government employ until his death on January 14, 1935. During this period of 16½ years, his duties included that of Staff Specialist to the Committee on Financial Administration, involving the study of various shipyards, Staff Assistant to the Legal Division, and Assistant to the Manager of the Construction Department. After the World War, he was particularly engaged in the settlement of claims arising from the cancellation of contracts between the Government and various shipbuilding companies, acting also as Technical Assistant to the Vice-President of the Merchant Fleet Corporation and as an Alternate Member of the Wage Reimbursement Committee.

On September 1, 1925, Mr. Bouillon was appointed Secretary of the American Marine Standards Committee, the membership of which includes many of the largest American firms whose products are used in the construction and outfitting of all types of ships. One of his principal duties was to study and analyze the design, equipment, and operating mechanism of various types of shipping vessels, with the object of evolving new ideas leading toward simplification and standardization. His results in this connection have been of great benefit to the Shipping Industry and brought commendation in the United States as well as abroad. In listing Mr. Bouillon's varied responsibilities while in the service of the United States, Mr. J. C. Peacock, present Director of the U. S. Shipping Board Bureau, Department of Commerce, Washington, D. C., adds this testimonial. "Mr. Bouillon's services were at all times performed in the most satisfactory and thorough manner. His combined knowledge and experience relating to the various phases of the marine industry made him exceptionally valuable."

He was married, at Seattle, on December 31, 1888, to Mary Young, who survives him, with their two sons and two daughters: Victor J., Lincoln, Victorine, and Mrs. Lilian Baker, all living in the State of Washington. He is also survived by a brother, A. M. Bouillon, M. Am. Soc. C. E.

Mr. Bouillon was elected a Member of the American Society of Civil Engineers on April 2, 1913. He was also a member of the Society of Naval Architects and Marine Engineers.

WILBUR GAYLE BROWN, M. Am. Soc. C. E.[1]

Died July 14, 1934.

Wilbur Gayle Brown, the son of Wilbur Brown and Sallie (Gayle) Brown, was born in Selma, Ala., on March 8, 1872, and was graduated from the University of the South at Selma, Tenn., in 1894, with the degree of Bachelor of Science in Civil Engineering.

Mr. Brown entered the service of the Louisville and Nashville Railroad Company as Rodman in the Maintenance Engineer Corps, on June 1, 1895, and was advanced from time to time, as follows: January 1, 1897, to Assistant Engineer in charge of engineering work on the Mobile and Montgomery Division; and on July 1, 1900, to Roadmaster in charge of maintenance of roadway and track on the Mobile and New Orleans Division, and of the engineering work incidental thereto.

---

[1] Memoir prepared by J. E. Willoughby, M. Am. Soc. C. E.

Mr. Brown left the service of the Louisville and Nashville Railroad Company on January 30, 1912, to become Building Manager for the Brown-Marx and the Woodward Office Buildings, in Birmingham, Ala. During this engagement he was also Receiver for the Birmingham, Ensley and Bessemer Railroad. Although he continued as Building Manager for eight years, his interest was ever with railway work; he, therefore, sought and procured appointment as Valuation Engineer for the Jacksonville (Fla.) Terminal Company, a passenger terminal owned in part by the Florida East Coast Railway Company. On the completion of that task in 1923, he was transferred to the Florida East Coast Railway as Engineer of Roadway, in charge of maintenance of way and structures. The title was shortly afterward changed to Engineer, Maintenance of Way, and with the change came a slight modification of duties. This position, Mr. Brown held until his death in Johns Hopkins Hospital, at Baltimore, Md.

The characteristic features of Mr. Brown's professional engagements were the careful thought given to the work at hand, the intelligent conclusion reached after a thorough study of the problems, and the persistence with which accomplishment of the work was carried out in accord with predetermined plans and methods. For example, in his desire to procure uniform surface of rail on main track, Mr. Brown would first bring the rail to exact line, set grade stakes for each rail-joint for the exact elevation, and then procure precise conformity thereto by the trackman. Every engineering assignment undertaken by Mr. Brown was an engineering work well done.

He was likable, and his friends were wherever he went. All persons who worked under his direction were intensely loyal to him. He enjoyed social assemblies, and the social assembly that held him was the more agreeable by reason of his presence. Mr. Brown was never married. He gave up without complaint his desire for the companionship of wife and child so that he could devote himself to the care and comfort of his widowed mother.

He was a member of the American Railway Engineering Association, and also of the Florida Engineering Society.

Mr. Brown was elected a Member of the American Society of Civil Engineers on February 24, 1931.

JAMES PARKER BROWNELL, M. Am. Soc. C. E.[1]

DIED DECEMBER 17, 1933

James Parker Brownell was born at Croghan, N. Y., on March 25, 1868, a son of Joseph Parker and Parthenia (Caswell) Brownell. When about five years old, he moved to Carthage, N. Y., with his parents, and attended the local schools. His ability as a student in the Carthage High School, from which he was graduated in 1887, earned him a scholarship in Cornell University, at Ithaca, N. Y. He studied Civil Engineering at Cornell and was graduated with the degree of Civil Engineer in June, 1891.

---

[1] Memoir prepared by William P. Creager, M. Am. Soc. C. E.

After two years of employment with the late Henry Clarence Hodgkins, M. Am. Soc. C. E., an Engineer of Watertown, N. Y., Mr. Brownell returned to Carthage, in 1893, where he was associated with his father who was a Land Surveyor, until the latter's death a short time thereafter. During this period, he was engaged in the design and construction of paper mills and other engineering works.

In 1900, in partnership with the late Mr. Charles Eaton, Mr. Brownell went to Nova Scotia where he made various reports for projects for the Sissibo Pulp and Paper Company and designed and constructed a paper mill. After his return from Nova Scotia in 1901, he designed and built the Effley Falls Hydro-Electric Plant.

In 1902, he was appointed Chief Engineer of the railroads of the Glenfield and Western Railroad Company and, in 1903, was also made Chief Engineer of the railroads of the Pyrities Paper Company and the Malone Paper Company. In addition, he located and constructed railroads for the Lowville and Beaver River Railroad Company in 1904, the Carthage and Copenhagen Railroad Company and the R. W. Higbie Company, in 1906, the Norwood and St. Lawrence Railroad Company, in 1907, and the Mac-A-Mac Railroad Company, in 1913.

In 1902, Mr. Brownell was appointed City Engineer of Carthage, which position, in addition to his other activities, he held for twenty-one years. During that period he located and built a 15-mile pipe line and water supply for Carthage and also water supply systems for Black River, Edwards, Madrid, and Croghan, all in New York State.

In 1905, he dissolved his partnership with Mr. Eaton and opened an office as Consulting Engineer in Carthage. Between 1913 and 1923, while still in private practice, he designed and built hydro-electric plants at Taylorville, Elmer Falls, Black River, and Browns Falls, N. Y., as well as the extension of the plant at Belfort, N. Y.

In 1923, Mr. Brownell joined the Power Corporation of New York and the Northern New York Utilities as Assistant Chief Engineer, with offices in Watertown, engaging in the design and construction of the Effley Falls Extension, Herring, Flat Rock, Sewalls Island, Deferiet, Soft Maple, Eagle Falls Extension, Taylorville, Norfolk, Norwood, and Moshier, N. Y., hydro-electric power developments. In 1932, he retired to private practice and was retained by the Northern New York Utilities as Consulting Engineer until his death.

The foregoing list of Mr. Brownell's principal activities conveys only in a general way an indication of his extraordinary energy and ability. His initiative and ready grasp of intricate problems were a revelation to those whose good fortune it was to work with him. Most of all, however, he was a gentleman. Although adamant in his protection of his clients' interests, he was at all times gentle, courteous, and just. He was a devoted husband and father and, although much involved in important engineering matters, assisted materially in the practical education of his sons, all three of whom have embraced the Engineering Profession. A full account of Mr. Brownell's characteristics would not be complete without mention of his great loyalty and devotion to his superiors, which, in many respects, approached fanaticism.

He was married on June 28, 1893, to Frances Belle Stewart, of Watertown, and is survived by his widow, one daughter, Mrs. Helen Walker, and three sons, Parker J. Brownell, Osmyn C. Brownell, and Roland S. Brownell, all of Carthage.

Mr. Brownell was elected a Member of the American Society of Civil Engineers on June 24, 1914.

GEORGE BRYAN, Jr., M. Am. Soc. C. E.[1]

Died November 1, 1934

George Bryan, Jr., the son of George and Eugenie (Steele) Bryan, was born on January 21, 1886, at Peoria, Ill., where his family had resided for a number of years. He was a grandson of Judge William F. and Jane (Evans) Bryan, of Peoria. His family came of pioneer stock, and the name he bore was one of an unbroken line since the coming of his first ancestor, George Bryan, to America in 1798. The earliest member of the family became Chief Justice of the Court of Pennsylvania.

Mr. Bryan attended the White School, and then prepared for his higher education at Bradley College, at Peoria. He completed his engineering training at Rensselaer Polytechnic Institute, at Troy, N. Y., from which he was graduated as a Civil Engineer in 1908.

After his graduation from Rensselaer and until 1911, Mr. Bryan was employed, successively, by the Harman Engineering Company, of Peoria; the Central Illinois Construction Company, of Ottawa, Ill.; and by the Chicago and Northwestern Railway Company in its Bridge Department. In 1911, he joined the American Bridge Company, first in the Engineering Department and then in the Contracting Department. In 1916, he was sent to Duluth, Minn., as Sales Engineer for that firm. In 1922, he was made Assistant Contract Manager of the Company at the Duluth Office, which position he held until 1926, when he was made Contract Manager, the office he held at the time of his death.

Early in 1917 Mr. Bryan volunteered for service in the World War. He was appointed First Lieutenant in the Engineer Corps, with headquarters at Fort Humphrey, Virginia, and served until the close of the war.

Mr. Bryan was known to his friends and business associates as an unselfish, high principled, gentle character. He was the type who received most in the manner of personal pleasure in his home and with his immediate family. He was intensely interested in the finer activities of life, including the arts, music, literature, and the theatre. Rarely, if ever, did he indulge in any social or athletic activity except those shared with his family.

---

[1] Memoir prepared by a Committee of the Duluth Section, consisting of Wayne A. Clark, M. Am. Soc. C. E., and Gordon H. Butler, Sr., Assoc. M. Am. Soc. C. E.

He was married to Marie Elston, of Duluth, Minn., in 1920. At the time of his sudden death in Chicago, Ill., on November 1, 1934, he and Mrs. Bryan were returning from a vacation trip to New York, N. Y., Washington, D. C., and Eastern points. He is survived by his widow and by a sister who resides in the place of their birth. He was buried in the family plot at Peoria.

He was a member of the Christian Science Church. Mr. Bryan was a member of the Duluth Engineers Club, the Kitchi Gammi Club, the Ridgeview Golf Club, the Chamber of Commerce, and other organizations in the City of Duluth. He was also a member of the Duluth Section of the Society and a regular attendant at the monthly luncheons.

Mr. Bryan was elected a Member of the American Society of Civil Engineers on January 17, 1927.

EDWARD MORRIS BURD, M. Am. Soc. C. E.[1]

DIED NOVEMBER 5, 1934

Edward Morris Burd was born in Grand Rapids, Mich., on October 9, 1887, the son of Edward and Jennie M. Burd. He was graduated from the Central High School, in Grand Rapids, in 1905, and then attended Michigan State College, at East Lansing, Mich., from 1906 to 1908. He was graduated from the University of Michigan, at Ann Arbor, Mich., in 1911, with the degree of Bachelor of Civil Engineering.

Following his graduation Mr. Burd began his engineering career as Draftsman in the office of the City Engineer of Grand Rapids and, in 1912, was engaged as Inspector for the Fargo Engineering Company, of Jackson, Mich., at a steel fabricating and galvanizing plant on transmission-line towers, in Chicago, Ill. In this position, he showed his inherent qualities of initiative, inventiveness, and facility in research; among other things, he induced the owners of the plant to improve their methods of handling the steel to avoid injuring the coatings and otherwise improving the coating processes. In his spare time at this plant Mr. Burd made a set of drawings of the seventeen standard transmission towers under contract, each 60 to 80-ft tower being completely shown on a single sheet, 8½ by 11 in., with all principal dimensions, sizes, weights, surface areas, and other data. These drawings had the appearance of having been reduced by photography on a 1:3 ratio, yet all figures, etc., were legible to the naked eye. This illustrates the delight Mr. Burd took throughout his professional life in making highly detailed minute drawings, beautifully executed.

---

[1] Memoir prepared by W. G. Fargo, W. C. Giffels, and H. S. Hunt, Members, Am. Soc. C. E.

Later in 1912, Mr. Burd went with the Turners Falls Power Company, at Springfield, Mass., remaining there until June, 1915, in charge of transmission-line and sub-station construction and the preliminary field investigations of water power and storage projects on the Upper Connecticut River. Then followed two more years with the Fargo Engineering Company as Designer, Investigator, and Report Writer on the various projects pertinent to the practice of hydraulic engineering as applied to power plants, including tests and appraisals of power properties.

The outbreak of the World War drew him into the Army. He entered the Military Training Camp at Fort Monroe, Virginia, on August 27, 1917, as a civilian candidate for a commission. He accepted appointment as Captain in the Coast Artillery Corps, Officers' Reserve Corps, on November 27, 1917; was assigned to active duty on the same date; and was honorably discharged on April 8, 1919, at Fort Monroe. While at Fort Monroe, Captain Burd was an Instructor and, later, in charge of orientation, including mapping, position finding, pointing, fire control, etc., and wrote several military treatises on these subjects.

From April to August, 1919, Mr. Burd was employed as Structural Steel Designer and Estimator with the Blaw-Knox Company, at Pittsburgh, Pa.; then, until April, 1920, with the Consumers Power Company, at Jackson, Mich., as Assistant to the late William Wolcott Tefft, M. Am. Soc. C. E., then Hydraulic Engineer of the Company, on designs, economic studies, etc.

In 1920, Mr. Burd and William C. Giffels, M. Am. Soc. C. E., formed the engineering firm of Burd and Giffels, with offices at Grand Rapids, doing general power plant and municipal engineering work. In 1921, Mr. Burd returned to the Consumers Power Company and the Commonwealth Power Corporation and Affiliated Corporations, at Jackson, to be the Head of their Civil and Hydraulic Engineering Departments. When the engineering work of these Corporations was taken over by Stevens and Wood, Incorporated, in 1928, Mr. Burd continued as Head of the Civil and Hydraulic Engineering Department. When this firm was superseded by Allied Engineers, Incorporated, in 1930, Mr. Burd was appointed Assistant Manager of the Northern Division of the Company. On the dissolution of Allied Engineers, Incorporated, in October, 1931, he became Civil and Hydraulic Engineer for the Consumers Power Company, at Jackson, Mich. This position he filled until his sudden and tragic death, on November 5, 1934, when flying his own plane near Jackson he crashed to the ground, sustaining injuries that quickly proved fatal.

He was a member of the Detroit Engineering Society and of the Hydraulic Power Committee of the National Electric Light Association. In college, he was a member of Tau Beta Pi and Sigma Xi.

Although Mr. Burd was very busy with active engineering problems and construction, he made the following important contributions to engineering literature: "Hodenpyl Dam"[2]; "Hydro-Electric Developments During 1928"[3]; "Comparative Costs of Generating Electricity by Water Power and Steam," published by the National Electric Light Association and presented at its July, 1929, Convention; and "High Dams on Pervious Glacial Drift."[4] He also contributed various engineering articles for the house organs of the Public Utility Corporation with which he was connected, particularly for the *Au Sable News* of the Consumers Power Company. While with the Blaw-Knox Company, he wrote a "Treatise on Transmission Towers," for its Catalog No. 20. At the time of his death he was planning a paper for Society use on forecasting stream flow—a subject on which he had made long and intensive studies.

---

[2] *Power Plant Engineering*, July 1, 1926.

[3] *Loc. cit.*, December, 1928.

Mr. Burd's designing experience in hydraulics was extensive and he was equally competent in structural design. He was responsible for the design of the Hodenpyl Dam (71-ft head) on the Manistee River, in Michigan; the Hardy Dam (100-ft head), also on soil foundation, on the Muskegon River, in Michigan; and he designed the spillway, power house, and accessory structures of Blue Ridge Dam (150-ft head), near Blue Ridge, Ga., for the Toccoa Electric Power Company; this was an earth dam with a puddled core, on rock. The soil used in this dam was a clay that was very retentive of water and offered difficult construction problems. This plant involved a steel-lined concrete penstock, 14 ft in diameter, and 1 000 ft long, with a surge tank 30 ft in diameter and 158 ft high.

While Mr. Burd was essentially a Designing Engineer, he had extensive field experience on surveys and particularly on the construction of large hydro-electric power plants, transmission lines, sub-stations, and office buildings. He was Superintendent of Construction in charge of large forces of men and of various mechanical equipment at the Hodenpyl Dam, previously mentioned, which is located in an out-of-the-way section of Northern Michigan. While with the Consumers Power Company and Affiliated Companies, he designed six office buildings, one of six and another of eleven stories.

While Mr. Burd was a healthy and wholesome man of fine appearance and address, his casual, and even closer, acquaintances sometimes felt an apparent aloofness which prevented an insight to his true character. He was not unfriendly in spirit, although in common with many technical men whose minds are fully occupied by present and tentative problems, he was inclined to be reserved. He was a deep thinker, accurate in analysis, and his former instructor at the University of Michigan, John H. Griffith, M. Am. Soc. C. E., recently wrote, "Burd was one of the finest students I ever had."

Mr. Burd possessed a keen sense of humor, was always happy in being out of doors, and seemed to love Nature in all her aspects. He loved music and sometimes felt that he should have followed that profession, with special reference to the violin. He always liked pets—his riding horses, dogs, and cats. He was an expert photographer; an unusual swimmer with many rescues from drowning to his credit. He was an ingenious mechanic and delighted in craftmanship whether it might be in wood, metal, or masonry. The lot he selected on which to build his home in Jackson, commanded the widest views of any he could secure, and here on the end of a ridge, overlooking the city and the parks beyond, he built from his own designs and more or less with his own hands, a comfortable home with many unique contrivances of his own devising.

---

[4] *Transactions*, Am. Soc. C. E., Vol. 99 (1934), p. 792.

Mr. Burd's outstanding qualities were honesty, courage in abundance, with always plenty of ambition and initiative. He was a tireless worker and lived each day a full life, not too much concerned with what to-morrow might bring. Thus, he enjoyed life from day to day and took a proper pride in his own achievements without being in any sense an egotist. He was eminently self-reliant, perhaps too much so.

He was married to Florence Lee Harger, in Ann Arbor, Mich., on August 26, 1911. One daughter, Florence Montague Burd, was born to them. In 1926, he was married to Elizabeth Taylor who, with their two young sons, Robert Morris and Edward M. Burd, Jr., survives him.

Mr. Burd was elected an Associate Member of the American Society of Civil Engineers on October 10, 1916, and a Member on May 13, 1930.

WILLIAM HUBERT BURR, M. Am. Soc. C. E.[1]

Died December 13, 1934

William Hubert Burr, for many years Professor of Civil Engineering at Columbia University, New York, N. Y., and internationally known as a Consulting Engineer, was born at Watertown, Conn., on July 14, 1851, the son of George William and Marion Foote (Scoville) Burr, both descendants of old New England families. His early education was received at the Watertown Academy. He then entered Rensselaer Polytechnic Institute, at Troy, N. Y., from which he was graduated in 1872 with the degree of Civil Engineer.

Mr. Burr's first engagement after graduation was with the Phillipsburg Manufacturing Company, which was then engaged in building wrought-iron bridges. The engineer of to-day seldom realizes that the all-steel bridge was not introduced until the Eighties, and the replacement of the older, wrought-iron bridges by steel carried through into the present century. Professor Burr's career covered this change from wrought-iron to steel construction, and he played an important part throughout this transition period in the development of the standards of modern steel design.

About 1874, he became connected with the City Water-Works of Newark, N. J., another field in which he later achieved high standing as a Consulting Engineer. In 1875 his career as an Engineering Teacher and Educator began with his acceptance of a position as Instructor in Mechanics at Rensselaer Polytechnic Institute. An unusual honor followed in 1876 when, at the age of 25, he was appointed Professor of Rational and Technical Mechanics at the same institution, a position he held until 1884.

---

[1] Memoir prepared by J. K. Finch, M. Am. Soc. C. E.

In April, 1884, Professor Burr left Rensselaer to become Assistant to the Chief Engineer of the Phœnix Bridge Company, at Phœnixville, Pa., later becoming General Manager, in which position he superintended the design and construction of many of the important bridges built in the United States. Among these should be noted the three-span truss railroad bridge of the "camel-back," or broken-upper-chord truss type, over the Ohio River, at Cincinnati, Ohio, built in 1888. Its center span of 550 ft established a new truss-span record which was not materially exceeded until the construction of the Metropolis Bridge, over the Ohio River, at Metropolis, Ill., in 1917.

Leaving the Phœnix Bridge Company in 1891, Professor Burr became Vice-President of the pioneer firm of foundation and construction experts, Sooysmith and Company, of New York, N. Y. The following year, however, he returned to his university work and became Professor of Civil Engineering at Harvard University, Cambridge, Mass. From this position he was called to a similar appointment at Columbia University, in New York City, in 1893. He continued as Professor of Civil Engineering at Columbia until his retirement in 1916, from which time he was carried on the rolls of the University as Emeritus Professor of Civil Engineering. It was during this long period of service at Columbia that Professor Burr's extensive practice as a Consulting Engineer was developed, and he continued this practice after his retirement, opening offices in New York City.

Three phases of Professor Burr's career demand particular attention—his contributions to structural theory and design, his influence as an engineering teacher, and his activities as a consulting engineer.

Although he possessed to an almost uncanny degree that inherent structural sense and ability to check and design structures by judgment, on which the great engineers of earlier days had to depend so largely, he was a leader in the development and advancement of the present rational, scientific technique of design. As early as 1876 he presented before the Society his first paper[2] which dealt with "Approximate Determination of Stresses in the Eye-Bar Head." His contributions to the mechanical and mathematical phases of structural analysis and design continued throughout his life. His contributions to the subject of columns and struts are particularly interesting as they show the important part he played in the development of the famous "straight-line" column formula.

Many of his contributions to technical progress were contained in the various texts which he prepared for use in his classes. The "Theory of Bridges and Roof Trusses," first published in 1881, was long a standard work in this field. His monumental "Elasticity and Resistance of the Materials of Engineering," which first appeared in 1883, went through several editions. His "Graphic Methods by Influence Lines for Bridge and Roof Computations" (with Myron S. Falk, M. Am. Soc. C. E.) was one of the pioneer presentations of this modern method of analysis to American engineers. His "Suspension Bridges, Arch Ribs and Cantilevers" of 1913 was a similarly advanced treatment of such structures.

---

[2] *Transactions,* Am. Soc. C. E., Vol. VI (1877), p. 127.

In all these works, Professor Burr never side-stepped a problem because of its mathematical difficulties or the labor of calculation involved. He had no sympathy with the reader or student who refused to "dig out" for himself the often complicated and difficult analysis involved. In his younger days, at Troy, he had been forced to "find out for himself" the details as to facts and theories which were but briefly presented in class. He always insisted that the only kind of education was "self-education," and his classes were conducted on exactly this basis. As a teacher he gave no predigested, easily assimilated information to his classes; neither was his teaching of the inspirational type.

Nevertheless, Professor Burr occupied an outstanding position in earlier American engineering education. He stood for the most advanced development of technical theory, and he also was a staunch advocate of the broad and liberal training of the engineer—a training which would enable him to develop into a cultured and useful citizen. In his papers[3] on engineering education, written in his characteristically crisp, formal, and forceful manner, he always emphasized the fact that Engineering should be held to a professional parity with the professions of Law and Medicine.

His consulting practice may be said to have begun with his appointment in 1893, as Consulting Engineer in connection with two swing-bridges across the Harlem River, in New York City. This was followed, in 1895–97, by his successful completion of the construction of the Harlem River Driveway, later known as the "Speedway." This work involved the solution of a difficult foundation problem which had caused the abandonment of the earlier construction by the Park Department.

Professor Burr also served on the Board of Consulting Engineers of the Department of Docks of New York from 1895 to 1897. In the meantime, he had also been appointed by President Cleveland (1894) as a member of a Board of Engineers to investigate the possibility of a one-span bridge across the Hudson River, at New York City. This was followed by a similar appointment, in 1896, to a Board engaged to locate a deep-water harbor on the Pacific Coast.

Following the Act of 1899 he was appointed by President McKinley to a Board created to investigate both the Panama and the Nicaragua routes for an Isthmian Canal. In view of the difficulties of securing rights at Panama this Board advised the route through Nicaragua. In 1902, however, the old Panama Company made a new offer, the Board was again called together, and, with these changed conditions in view, advised the building of the Panama Canal. Later, in 1902, under the Spooner Act, President Roosevelt secured the Panama rights and a long argument began as to whether the canal should be built as a lock canal, the type planned by the earlier Board, or as a sea-level canal. In 1904, President Roosevelt appointed the famous International Board of Consulting Engineers to settle this question. Professor Burr was also a member of this Board and was among the majority favoring a sea-level plan. This appointment he considered one of his outstanding honors, and, always a staunch advocate of the sea-level plan for the canal, he later argued that the speed and ease with which a lock canal, the type finally adopted, was built proved that a sea-level canal would have been feasible.

---

[3] See "The Technical School and the University," *Columbia University Quarterly*, December, 1906; "The Study of Engineering," *Loc. cit.*, December, 1908 and "The Relation of Educational Training to the Practice of Engineering," *Loc. cit.*, December, 1915.

In 1911, he was appointed by Governor Dix as a member of the Board of Engineers to advise on the design of the New York State Barge Canal. His long connection as Consulting Engineer on the water supply of New York City dated from the days of the so-called Aqueduct Commissioners and included the first proposal of the Catskill project as embodied in the famous Burr-Hering-Freeman Report of 1903. When the States of New York and New Jersey undertook to build the Holland Tunnel, he was again summoned to give advice. He also served as Consultant to the Port of New York Authority, this service covering the plans for all the bridges built by this organization, including the famous George Washington Suspension Bridge over the Hudson River at New York City. In fact, there were few major American engineering undertakings of the Twentieth Century in connection with which his approval or advice was not sought. In addition, he was responsible for the design of many smaller projects. In 1900, his design for a Memorial Bridge across the Potomac received First Prize. In 1907, he presented a paper to the Society on the Sandy Hill Bridge[4] over the Hudson River, a graceful reinforced concrete structure, and, a few years later, his design for a huge reinforced concrete arch at Spuyten Duyvil, joining Manhattan Island with the Riverdale Section, also exhibited his keen sense of the æsthetic in structural design.

In 1891, Professor Burr was awarded the Thomas Fitch Rowland Prize of the Society for his paper[5] entitled "The River Spans of the Cincinnati and Covington Elevated Railway, Transfer and Bridge Company." Having joined the Society as a Junior in 1874, he was the oldest member, in years of affiliation with the Society, at the time of his death. He had many friends and advisers in Japan and was decorated by the Japanese Government with the Order of the Sacred Treasure (2d Degree). He also prized his membership in the Institution of Civil Engineers of Great Britain, and he was a Fellow of the American Academy of Arts and Sciences. His clubs included the Century Association and the University Club of New York City. He was also a member of the Pilgrims and of the Society of Cincinnati. Always a loyal and active Churchman, he served for many years as a Trustee of the Cathedral of St. John the Divine, in New York City, and was a member of St. Mark's Protestant Episcopal Church, in New Canaan, Conn., where he resided in the later years of his life.

In 1876, Professor Burr was married to Caroline Kent Seelye, of New York City, who died in 1894. The children of this marriage were George Lindsley, a banker residing in New York, Marion Elisabeth (Mrs. James A. Mars), and William Fairfield. In 1900, he was married to Gertrude Gold Shipman, a sister of the late Bishop Shipman, who survives him, with their daughter, Anne Louise (Mrs. S. Bayard Colgate).

---

[4] *Transactions*, Am. Soc. C. E., Vol. LIX (1907), p. 195.

[5] *Loc. cit.*, Vol. XXIII (1890), p. 47.

With the death of Professor Burr at the Doctors Hospital in New York City on December 13, 1934, the Engineering Profession lost one of the great leaders of an engineering generation whose record of achievement is written in the greatest engineering works the world has ever known.

Professor Burr was elected a Junior of the American Society of Civil Engineers on June 3, 1874; an Affiliate on May 5, 1880; and a Member on March 3, 1886. He served as a Director of the Society from 1894 to 1896.

MORTON LEWIS BYERS, M. Am. Soc. C. E.[1]

Died November 22, 1933

Morton Lewis Byers was born in Mauch Chunk, Pa., on March 22, 1868. He was the eldest child of John Morton and Caroline Mann (Lewis) Byers.

Mr. Byers came of a family interested in transportation. His grandfather, Joseph Byers, was a canal engineer connected with the James River Canal. His father, John Morton Byers, entered the service of the Pennsylvania Railway Company as a Rodman when the road was being built over the Allegheny Mountains. Later, he was Engineer of Construction with the same Company and built many of its branch lines. He was also Superintendent of the Pittsburg, Virginia, and Charleston Railway. Other members of Mr. Byers' family were four uncles: Joseph Byers who was engaged in railroad building in South America; Charles E. Byers was Chief Engineer of the Reading Railway Company; Henry W. Byers was Superintendent of the Erie and Ashtabula Division of the Pennsylvania Company; and Richard Barnard Lewis was a well-known civil engineer during the period following the Civil War. Mr. Byers' only brother, the late Maxwell Cunningham Byers, M. Am. Soc. C. E.,[2] whose tragic death occurred in 1930, was Chairman of the Board and President of the Western Maryland Railway Company. Thus, it will be seen that Mr. Byers was naturally interested in transportation.

His preliminary education was obtained in the private schools of Pittsburgh, Pa., and at Trinity Hall, Washington, Pa. At the age of sixteen, he entered Lehigh University, at Bethlehem, Pa., from which he was graduated in 1888 with the degree of Civil Engineer.

Upon leaving Lehigh, Mr. Byers entered the service of the Pennsylvania Railroad, Lines West, as Assistant Engineer on the Erie and Ashtabula Division, at New Castle, Pa. In 1891, he was transferred to the Indianapolis Division as Assistant Engineer, Maintenance of Way. In 1893, he spent a few months in the office of the Chief Engineer, after which he was appointed Engineer, Maintenance of Way, on the Cincinnati and Muskingum Division. Later, he held the same position on the Indianapolis and the

---

[1] Memoir prepared by W. G. Vollmer, Executive Asst., Mo. Pac. Lines, St. Louis, Mo., from information supplied by Mrs. H. A. Barbee, Point Pleasant, W. Va.

[2] *Transactions*, Am. Soc. C. E., Vol. 95 (1931), p. 1465.

Chicago Divisions, after which he was Secretary of the Transportation Officers' Association, Pennsylvania Railroad, Lines West.

At this time (1901), he left the Pennsylvania Railroad Company and, for a few months, was engaged as Assistant Manager of Erection for the Pittsburgh District of the American Bridge Company.

In 1902, he joined the Baltimore and Ohio Railway Company as Chief Engineer, Maintenance of Way, under his old chief, L. F. Loree, M. Am. Soc. C. E. Later, Mr. Byers became Assistant to the General Manager and, afterward, Assistant General Superintendent of Transportation.

In 1904, he left the Baltimore and Ohio Railway Company to become Assistant to Mr. Loree, then Chairman of the Rock Island-Frisco Lines. In 1905, Mr. Byers went to the Missouri Pacific System as Chief Engineer, Maintenance of Way, remaining in this capacity for six years. In 1911, he followed Mr. Loree to the Delaware and Hudson Railway Company as Consulting Engineer and Member of the Valuation Organization; at the same time he held a joint position with the Seaboard Air Line Railway Company as Chairman of the Valuation Committee, from which he resigned in 1927.

After spending several years traveling in Europe and Asia, Mr. Byers re-organized the West Virginia Malleable Iron Company, at Point Pleasant, W. Va., and was President of the Company until his death, on November 22, 1933.

He was a man of unsual character and ability. Good men were attracted to him. His associates respected his temperament and ability. His was an ever-ready spirit of helpfulness, especially to the young men in the Engineering Profession who became his associates at various times in his career.

Being intensely interested in his work, Mr. Byers applied himself diligently in perfecting the different branches with which he came in contact. He reorganized the Maintenance of Way Departments of the Baltimore and Ohio Railway and the Missouri Pacific Railway Companies. He also prepared a Disbursement Accounting System for the Baltimore and Ohio Railway, Pennsylvania Railroad, and the Rock Island-Frisco Lines all prior to 1905. He was also the author of the well-known technical publication, "Economics of Railway Operation."

During his extensive travels both in this country and abroad, Mr. Byers found great pleasure in his skill as an amateur photographer. His camera was his constant companion, and between his trips he spent many happy hours coloring his photographs and making lantern slides of his favorite subjects. About 900 of these slides were presented by him to the Newark, N. J., Public Library. He was a member of the Orange, N. J., Camera Club and was a frequent exhibitor at the Club's exhibitions.

He was married to Josephine Bringhurst, of Logansport, Ind., in 1907. They had one daughter, Anna. Mrs. Byers passed away a few weeks after the death of her husband.

Mr. Byers was elected a Junior of the American Society of Civil Engineers on January 2, 1890, and a Member on September 7, 1898. He was also a member of the American Railway Engineering Association.

WALTER JOHN CAHILL, M. Am. Soc. C. E.[1]

Died August 8, 1934

Walter John Cahill was born in Amboy, Ill., on June 17, 1872, the son of James J. and Mary Augusta (Gregory) Cahill. After receiving his preliminary education in Chicago, Ill., he entered the University of Michigan, at Ann Arbor, Mich., in 1892. While attending the University Mr. Cahill received the appointment and successfully passed the rigid entrance examination for the United States Military Academy, at West Point, N. Y., in 1894. However, he continued his studies at the University of Michigan from which he was graduated in 1896 with the degree of Bachelor of Science in Civil Engineering.

Upon graduation, Mr. Cahill was employed as an Engineer by the Illinois Central Railroad Company, being associated with Mr. Lawrence A. Downs, President of the Company since September, 1926. Later, he was engaged by the Chicago, Hammond and Western Railroad Company on track improvement work. In 1898, Mr. Cahill entered the employ of the City of Chicago as an Engineer on the construction of the Northwest Water Supply Land Tunnel, later becoming Assistant Engineer in the Division of Harbors, City of Chicago, in charge of river and land surveys, investigation, and reports. In this position he was in charge of the new water tunnel shaft and shore drift connections of the water mains at Adams Street and the Chicago River. Afterward, he had charge of the reconstruction of the 68th Street Pumping Station and was Assistant under the City Engineer, the late John Ericsson, Hon. M. Am. Soc. C. E., on tests of the 30 000 000-gal, "Lake Erie" pumping engine and boiler at the 14th Street Pumping Station. Mr. Cahill also made original investigations and calculations for new water tunnels and pumping stations for the City of Chicago, and, from 1900 to October, 1901, he was Engineer in charge for the City of Chicago of the construction of the 95th Street Bascule Bridge over the Calumet River.

In October, 1901, Mr. Cahill accepted a position as Engineer with Lydon and Drews Company, engaged in harbor improvements, for which firm he was in charge of estimating and engineering design. On the absorption of this Company by the Chicago and Great Lakes Dredge and Dock Company in 1903, Mr. Cahill was retained as Engineer in a similar capacity, but with enlarged duties as Vice-President and Chief Engineer, as with its successor, the Great Lakes Dredge and Dock Company, with which concern he was identified for more than thirty years, until the time of his death. During the past many years, Mr. Cahill, in addition to being Senior Vice-President and Chief Engineer of the Great Lakes Dredge and Dock Company, was also a Director and Member of the Executive Committee.

He was directly in charge of, and responsible for, a multitude of important marine construction projects, including bridges, breakwaters, lighthouses, coffer-dams, intakes, foundations, and other similar work for the Federal

---

[1] Memoir prepared by a Committee of the Illinois Section, consisting of John F. Cushing, W. W. DeBerard, and Hugh E. Young, Members, Am. Soc. C. E.

Government, as well as other governmental bodies, corporations, and private interests. His most recent work in and around Chicago included the construction of the Michigan Avenue Improvement, the Municipal Pier, the South Park and Lincoln Park Improvements, the Chicago River Straightening, and the North Robey Street, South Damen Avenue, and Ogden Avenue Improvements.

In addition to his arduous business duties, Mr. Cahill also found time for talking and writing to the younger generation on engineering subjects as is evidenced by two chapters in the book on "Foundations, Abutments, and Footings," by Hool and Kinne. As a specialist for more than three decades on the construction of marine and submarine works, Mr. Cahill's paper on "Water Terminals of Yesterday and Today"[2] which he presented before the Construction Division of the Society as part of the Symposium on "A Century of Progress in Construction Methods," at the Annual Convention, at Chicago, on June 29, 1933, was an example of his unusually clear thinking and statement of facts in careful and logical progression.

His intelligence, ability, and energy have found lasting expression in many monumental projects constructed under his supervision for the convenience, use, and enjoyment of the public. He was not only interested deeply in civic betterments, but graciously contributed largely of his time and labor in advancing comprehensive programs of municipal improvements, co-operating and advising whenever requested in such work as that of the Chicago Plan Commission and others. He contributed, by both written and spoken word, to the finer interests and the promotion of better physical and social conditions in his city.

Mr. Cahill's sense of humor and wholesome philosophy attracted to him many friends in the business and social world, all of whom deeply mourn his passing. He was considerate and generous with his numerous friends, associates, and fellow workers, going out of his way to help others, a noble trait to be recorded in the career of any gentleman.

In 1906, he was married to Irene M. Burke who, with his daughters, Dorothy, Marjorie, and Elspeth, and his sons, Walter and Richard, survives him. He is also survived by two sisters and a brother, James E. Cahill, M. Am. Soc. C. E.

Mr. Cahill was elected a Member of the American Society of Civil Engineers on June 6, 1906.

HENRY HALL CARTER, M. Am. Soc. C. E.[1]

DIED OCTOBER 4, 1934

Henry Hall Carter was born on October 5, 1857, on Beacon Hill, Boston, Mass., the son of Henry Carter, of Putney, Vt., and Eunice (Gorham) Carter. He received his early education in the public schools of his native city, and, in 1877, was graduated from the Massachusetts Institute of Technology, at Boston, with the degree of Bachelor of Science in Civil Engineering.

---

[2] *Civil Engineering*, August, 1933, p. 432.

[1] Memoir prepared from information supplied by the family and on file at the Headquarters of the Society.

Immediately after receiving his degree, and continuing until 1883, Mr. Carter was connected with the Boston Main Drainage Works in various engineering positions on tunnels, sewers, pumping stations, and outlets, including the works at Moon Island, the sanitation terminal for the City of Boston, etc.

From 1883 to 1887, he was employed as Engineer for the Boston Water-Works, in charge of the construction of the Farm Pond Conduit, in South Framingham, Mass.[2] For more than a year of this period, Mr. Carter was engaged in making surveys for the complete development of the water supply from the Sudbury River, and located the site of the Hopkinton Reservoir, and other reservoirs, for the City of Boston.

He was Chief Engineer of the Boston Sewer Department from 1887 to 1889, and was also Chief Engineer of the Stony Brook Improvement in charge of the work. In 1889, he was appointed Chief Engineer of the Boston Main Drainage Works and had charge, until 1891, of the extensions of sewers to Dorchester Lower Mills, South Boston, and the water-front of the city, and of the construction of the larger outlet sewer at Moon Island.

In 1891, Mr. Carter was appointed Superintendent of Streets of the City of Boston and had charge of the Paving, Sewer, Street Cleaning, Sanitary, and Bridge Divisions. He was also Commissioner for the Harvard and Cambridge Bridge. In 1894, he resigned the position as Superintendent of Streets and was appointed Consulting Engineer of the Boston Transit Commission, the West End Street Railway Company, and other corporations. He was also engaged in a general consulting engineering practice for the Metropolitan Construction Company, and others, in connection with contracting work.

In 1896, Mr. Carter gave up his active public work to engage in business with the late Mr. Charles L. Perrin, under the firm name of the Metropolitan Contracting Company, of which he became President. Among the works with which this Company was identified, were Sections 4 and 8 of the Boston Subway; sewers for the Metropolitan Sewerage Board; bridges at Saugus and Malden Rivers; Massachusetts Pipe Line Gas Company Works; lines of pipe for the Boston Pneumatic Tube Company; foundations, water supply, and dam for the Dominion Iron and Steel Company, at Sydney, Cape Breton; foundations for tracks of the Boston Elevated Railway; and, later, sewer contracts at Baltimore, Md., in connection with the outfall, approximating in value $1 000 000. This project involved some difficult work.

Mr. Carter was also engaged in the construction of heavy foundations at East Boston, Mass., and the many public works contracts which were completed in this section of the United States under his guidance placed him as one of the pioneer subway and tunnel builders and earned for him the title of a master in his chosen field.

---

[2] "Farm Pond Conduit—Boston Water Works", by Henry H. Carter, M. Am. Soc. C. E.. *Journal,* Assoc. of Eng. Societies, Vol. VIII (1889), p. 73.

As an expert, Mr. Carter testified in many important cases and because of his earnest and convincing manner, together with his clear presentation of established facts, assembled in a logical and able mind, he rarely lost a case for his client. Although not a member of the Legal Profession, he usually handled his own cases without assistance, and at a hearing in Washington, D. C., before the Interstate Commerce Commission, some years ago, he received most favorable comments for the manner in which he conducted his case against an array of able lawyers who were defendants for a railroad company.

For the last twenty-five years of his life, Mr. Carter had been retired from active business, and had found happiness in the outdoor play field of the world, traveling in many foreign countries, as well as in his own, spending many winters in Honolulu, summers in Maine, in and on the sea.

A lover of fishing (and his fish stories were measured by engineering accuracy and proved by photographic evidence), sailing, and duck shooting, he set aside a period of each year for these enjoyments in various parts of the South and North, and in the fall of each year spent several weeks at his shooting camp, in Saskatchewan, Canada.

With a mind always alert to current issues, he frequently submitted editorials and wrote various papers relating to professional work which were published in the *Journal* of the Association of Engineering Societies. Not infrequently he made charts and maps of property on land and sea which have been of great value to clubs and to the Government.

Although affiliated with many other clubs and associations in his active career, Mr. Carter always retained his membership with the Boston Society of Civil Engineers, the Boston Art Club, the Calumet Club of New York, N. Y., and the Santee Shooting Club, at Santee, S. C. He was a member of the Society of Mayflower Descendants.

On the eve of his seventy-seventh birthday, on October 4, 1934, Henry Hall Carter, died at his home in Brookline, Mass., bringing to a close an active and successful career as a Civil Engineer; leaving a record of work well done; a reputation for squareness and efficiency which would be difficult to duplicate, and the sacred heritage of a host of admiring friends whose tribute will ever be "that it was a privilege to have touched shoulders with a man of this character."

His widow, Mrs. Adelaide A. Carter, and a sister, Mrs. Horace Williams Fuller, both of Brookline, survive him.

A leader, yet interested in those who followed him, Mr. Carter always commanded respect and the admiration of all. He was a delightful companion to a host of friends who have paid tribute to his memory in genuine sincerity. And so passeth a distinguished man, an honor to his profession and a credit to himself, who in his love for the sea, the woods, and his home reminds us:

"Home is the sailor, home from the sea,
"And the hunter home from the hill"

Mr. Carter was elected a Member of the American Society of Civil Engineers on May 7, 1890.

WILLIAM ARNOLD CHRISTIAN, M. Am. Soc. C. E.[1]

Died February 4, 1934

William Arnold Christian, the son of William Howard and Martha Ann (Waite) Christian, was born in St. Louis, Mo., on November 21, 1870.

He received his early education in the Grammar Schools and High School of Cincinnati, Ohio. In 1893, he was graduated from the University of Cincinnati with the degree of Civil Engineer. Ambitious and versatile to an unusual degree, Mr. Christian then took up the study of medicine at night school, while employed as an engineer during the day, and, in 1899, was graduated from the University of Cincinnati with the degree of Doctor of Medicine.

Previous to 1894 he was engaged as Chainman, Rodman, and Instrumentman on railroad work; Draftsman and Estimator with the Stacey Gas Tank and Bridge Company, on tanks, bridges, and buildings; Assistant Engineer of Hamilton County, Ohio, on surveys, maps, and plans for roads and bridges; and Resident Engineer in charge of the construction of the Mitchell Avenue (steel) Aqueduct, in Hamilton County, Ohio.

From 1894 to 1902 Dr. Christian was employed on the improvement of the Ohio River under the District Engineer at Cincinnati, first, as an Inspector, and then as United States Assistant Engineer in charge of surveys, plans, estimates, and construction of dikes, ice piers, and the construction and repair of steel hulls for floating plant; later, he was put in charge of the construction of Movable Dam No. 4 and of dredging in that vicinity. This work was suspended in 1902 on account of the failure of a Congressional appropriation bill for river and harbor work.

From February to August, 1902, he served as Assistant Engineer, Maintenance-of-Way Department, of the Cleveland, Columbus, and Indianapolis Division of the "Big Four" Railroad, in charge of repairs, buildings, culverts, and double-tracking.

From 1902 to 1907, Dr. Christian was employed as an Assistant Engineer on the improvement of the Ohio River, in the Wheeling District; first, in charge of surveys, borings, and the acquisition of land and flowage easements for Dam No. 8; later, he assisted in the preparation of plans, estimates, and specifications for Locks and Dams Nos. 8 and 11; and, finally, in charge of the construction of Lock No. 8.

From 1907 to 1914, Dr. Christian served as First Assistant to the Chief Engineer of the Chicago Great Western Railroad Company; first, in charge of track construction and maintenance, land valuations, contracts, leases, etc.; then, in charge of the design and construction of bridges, buildings, and works to protect the railroad from the river. During a nine months' strike of shopmen, he was in charge of a system engine-house and repair shops.

From 1914 to 1920, he served as Senior Civil Engineer with the United States Interstate Commerce Commission, in charge of the valuation of all road-bed, track, and waterway construction, until that work was completed. He resigned from this position on March 31, 1920.

---

[1] Memoir prepared by John W. Woermann, M. Am. Soc. C. E.

From 1920 to 1921 Dr. Christian was Special Engineer for the Chairman of the Board of Directors of the Minneapolis and St. Louis Railroad Company, making detailed reports on the condition of the railroad, and in charge of claims of the Railroad Company against the Federal Railroad Administration.

From 1921 to 1924, he was employed by the City of Chicago, Ill., first, as Special Sanitary Engineer, reporting on bacterial and chemical contamination of the municipal water supply received from the various intakes in Lake Michigan, and treating them with chlorine; then, as a Structural Engineer with the Board of Education investigating and reporting upon reinforced concrete construction.

From 1924 to 1931 Dr. Christian was employed as Assistant Engineer with the Chicago and Western Indiana Railroad Company and the Belt Railway Company of Chicago, on the valuation of their railroad property, and on miscellaneous engineering assignments; also, on engineering investigations and estimates for private parties.

From April 27, 1931, until the date of his death, he was employed in the United States Engineer Office, at Chicago, as Office Engineer, supervising construction activities, including the completion of the locks and dams of the Illinois Waterway.

On June 11, 1917, the Secretary of War appointed Dr. Christian a Major of Engineers in the Officers Reserve Corps. He was assigned to the 311th Engineer Regiment, but was retained on duty with the Interstate Commerce Commission.

Dr. Christian was a member of the American Railway Engineering Association, the Western Society of Engineers, the Illinois Medical Association, the Ohio Medical Association, and the Chicago Medical Association; a Fellow of the American Medical Association, and Registered Physician in Ohio and Illinois. He maintained an active interest in medicine and surgery throughout his life, but limited his practice to evening hours.

Dr. Christian was a Thirty-second Degree Mason, a Knight Templar, and a Shriner. He was given a Masonic funeral.

He was married on September 16, 1908, to Mildred Mann, who survives him. He is survived, also, by his mother and a sister. Dr. Christian was justly proud of the scholastic records of his two sons, Arnold and Horace. The former is now (1935), an interne in the Illinois Research Hospital, at Chicago, while the younger son is attending the University of Illinois College of Medicine, at Chicago, Ill. Dr. Christian was devoted to his mother, whom he visited regularly every Sunday.

While Dr. Christian resided in Cincinnati, he was a student of voice and the piano at the Cincinnati Conservatory of Music. He also played the violin. He greatly enjoyed playing with his sons, one a violinist, and the other a pianist.

As an engineer, physician, and a man, Dr. Christian was held in high esteem. He was versatile and energetic to a remarkable degree until a few weeks before his death, the immediate cause of which was an operation for a tumor on the brain. Essentially a man of action, he prosecuted vigorously whatever he had in hand. As a physician he was sympathetic and generous. To his friends he was lovable, hearty, and cheerful. In his latter years he enjoyed nothing better than a luncheon with his friends and an exchange of reminiscences.

Dr. Christian was elected a Member of the American Society of Civil Engineers on October 3, 1911.

CHARLES SAMUEL CHURCHILL, M. Am. Soc. C. E[1]

Died January 25, 1934

Charles Samuel Churchill, who represented the eighth generation of an old New England family, was born in New Britain, Conn., on September 22, 1856. He was the son of Samuel William Churchill and Ellen L. (Hubbard) Churchill, the daughter of Dr. Henry E. Hubbard. His father was born on the old family homestead in Newington, Conn., on June 18, 1828.

Charles Samuel Churchill was graduated in 1874, with honors, from the New Britain High School, and was awarded a scholarship to Yale University. In the spring of 1878, he was graduated from Sheffield Scientific School, Yale University, New Haven, Conn., with the degree of Bachelor of Philosophy in Civil Engineering. On the day of his graduation, he was chosen Secretary of his Class and served in that capacity as long as he lived.

For approximately one year after graduation from college, Mr. Churchill taught school in one of the district schools near New Haven. In 1879, he entered his chosen profession, Civil Engineering, a profession by which he came naturally, for, in 1666, Josiah Churchill, the American ancestor of the Connecticut branch of the Churchill family, was chosen as Town Surveyor of Wethersfield, Conn. Charles Samuel Churchill's first position was as Transitman on what was known as the Parallel Railroad, a line intended to parallel the New York, New Haven, and Hartford Railroad, between New York, N. Y., and Providence, R. I. This road was never completed. In September, 1880, he was made Assistant Construction Engineer on the Meadville and Linesville Railroad (later a part of the Bessemer and Lake Erie Railroad). In 1881, he was appointed Division Engineer of the Pittsburg, McKeesport and Youghiogheny Railway Company, and was in charge of the construction of the line between Braddock and Newhaven, Pa.

In 1884, Mr. Churchill became Principal Assistant Engineer in charge of the location and construction of the Schuylkill Valley Division of the Pennsylvania Railroad Company between Hamburg, Pottsville, and Wilkes-

---

[1] Memoir prepared by Robert Carr Churchill, M. Am. Soc. C. E.

Barre, Pa. In 1887, he entered the Maintenance-of-Way Department of the Shenandoah Valley Railroad Company (later, the Norfolk and Western Railway Company), and the remainder of his active life was spent in the service of that Company. From Engineer, Maintenance of Way, he was promoted to the position of Chief Engineer in January, 1903. In March, 1914, he was made Assistant to the President and, in 1918, was promoted to be Vice-President in charge of Purchases, Real Estate, and Valuation, which position he held until he was retired from active service in 1926 at the age of seventy. After his retirement, Mr. Churchill opened an office as Consulting Engineer and, for several years, acted as Consultant on a large amount of construction work in and around Roanoke, Va.

During his entire lifetime, Mr. Churchill was active in building up the Engineering Profession to the high plane on which it now stands. He became a Charter Member of the American Railway Engineering and Maintenance of Way Association on May 12, 1899, and served as Director, Vice-President, and President. On June 1, 1931, in appreciation of his long years of service, he was made an Honorary Member of the Association. Mr. Churchill was also an active Member of the American Society for Testing Materials, and was elected an Honorary Member of that Society on January 1, 1927. On December 3, 1912, he was elected to membership in the Institution of Civil Engineers, London, England, and at the 1914–1915 Session of the Institution, he was awarded its Telford Premium for outstanding work in his profession.

Mr. Churchill was considered an authority on the ventilation of tunnels, both vehicular and railroad, and served as Consulting Engineer on the ventilation of many tunnels both in North America and in Europe. He was likewise an authority on the valuation of railroads and was the author of several technical papers dealing with these subjects.

He was married on November 11, 1885, to Anna D. Green, daughter of Robert Carr Green, of Pottsville, Pa. He is survived by his widow and the following children: Mrs. P. V. Littlejohn, Ethel Churchill, Robert Carr Churchill, M. Am. Soc. C. E., and Mrs. Helen C. Bertie.

He was a devoted husband and father, his home was his castle, and the happiest times of his life were with his loved ones, his flowers, his birds, and his trees.

Although as an official of a large railroad company, his spare moments were few, Mr. Churchill still found time for civic service, and served his country, his State, and his city in various capacities. He was an active member of the First Baptist Church, of Roanoke, and served for many years as a Deacon. His life was characterized by strict integrity, devotion to duty, and a punctilious regard for and observance of the highest standards of Christian life.

Mr. Churchill was elected a Member of the American Society of Civil Engineers on May 1, 1889. He served as a Director of the Society from 1908 to 1910, and as a Vice-President in 1912 and 1913.

GEORGE HALLETT CLARK, M. Am. Soc. C. E.[1]

Died December 6, 1934

George Hallett Clark, the son of Emmons and Augusta A. Clark, was born at Edgewater, N. J., on August 17, 1871. His father served as Colonel of the Seventh Regiment of Infantry, National Guard New York, for fifteen years, retiring from that position in 1889. The son later served as a private and non-commissioned officer in the same historic organization from 1891 to the outbreak of the Spanish-American War in 1898 when he was honorably discharged to permit him to accept a commission as First Lieutenant in the 201st Infantry, United States Volunteers. In that regiment he was made Battalion Adjutant and, later, was promoted to the rank of Captain, serving creditably as such until he was honorably discharged at the close of that war.

Mr. Clark's active life was spent principally in and about New York, N. Y., and he was identified with many of its construction activities, especially rapid transit subway construction. While a boy, he attended Public School No. 13, later, a private school for boys, and then the Charlier Institute, all of New York City. During this period, in 1888, he was employed as a Draftsman with the Edison Manufacturing Company in the same city. Then, he was matriculated in the School of Mines, Columbia University, from which he was graduated in 1893 with the degree of Civil Engineer.

Following his graduation Mr. Clark, in January, 1894, entered the service of the Metropolitan Street Railway Company, in New York City, as a Rodman. In April of the same year he was assigned to the construction of the Lexington Avenue Cable Road, between 63d and 23d Streets, and west through the latter street to Broadway. He remained in charge of all straight and special work of that Section until the completion of the road. Then, until May, 1896, he was in charge of the building of the first section of the "underground trolley" railway constructed in 23d Street, and also of a party making the surveys for similar roads in Sixth Avenue, Eighth Avenue, and West Broadway.

In May, 1896, he became an Assistant Engineer for the Commissioner of Street Improvements, 23d and 24th Wards, New York City, later the Borough of The Bronx, in the Bureau of Regulating, Grading, and Paving. From September, 1896, to May, 1900, Mr. Clark was in charge, as Assistant Engineer, of twenty-eight contracts, aggregating more than 16 miles of street regulating and grading, costing \$630 000; nine paving contracts, costing \$149 000; and the construction of two viaducts for grade-crossing eliminations, as well as a number of preliminary surveys and estimates for similar work.

In May, 1900, he entered the service of the Rapid Transit Railroad Commission of New York City (which was beginning the construction of the City's first Rapid Transit Subway), as a Senior Assistant Engineer, and was assigned to the First Division which supervised the building of the four-track structure from Park Row, at its junction with Broadway, north to Park Avenue and 41st Street. Of this pioneer work the late William Barclay Parsons, Hon. M. Am. Soc. C. E., was the Chief Engineer. On the resignation of the Division Engineer Mr. Clark, in March, 1905, was promoted to that position. The work described was completed under his supervision as was the two-track extension south under Broadway to Battery Park. In the meantime, he surveyed some of the proposed additional lines which were being studied for Manhattan. Later, when the construction of the so-called Center Street Loop was undertaken, its Manhattan Sections were assigned to the First Division for supervision. This was especially difficult work in the "soft ground" of Lower Manhattan, including the filled ground in the vicinity of the old Collect Pond where the care of the buildings founded on this material required especial attention and skillful work. When finished the Loop became in 1913 a part of the operating system of the Brooklyn-Manhattan Transit Corporation.

---

[1] Memoir prepared by Robert Ridgway, Past-President and Hon. M. Am. Soc. C. E.

In the meantime, in the summer of 1907, the Rapid Transit Railroad Commission was superseded by the then recently organized Public Service Commission for the First District, State of New York. Mr. Clark remained with the new organization as a Division Engineer until December, 1909, when he resigned to become Chief Engineer of the Holbrook, Cabot and Rollins Corporation, General Contractors, operating principally in the northeastern section of the United States. His work with this firm was of a varied and interesting character and included the completion of a dry dock in the United States Navy Yard, in Brooklyn, N. Y.; some compressed-air work in St. Louis, Mo.; and two important Rapid Transit Subway contracts at Times Square, Manhattan, known as Routes 4 and 38, Section 6A, and Routes 4 and 36, Section 3. The first of these contracts was for a short section of the four-track subway forming the connection between the original subway, operated by the Interborough Rapid Transit Company, and the new extension for it running south under Seventh Avenue. Joining the extension to the existing structure with trains operating in it at short intervals, day and night, involved unusual hazards and difficulties. Extraordinary care had to be exercised to insure safety of life and property, and its completion was a creditable performance to the engineers and the contractors.

The second contract required a greater amount of work, its construction cost amounting to $3 700 000. It was executed during the World War period when labor and material conditions were very trying, adding to the complications of an already difficult piece of work located in one of the most important centers of the city. The excavation of the wide cut was substantially all in rock, the attitude and character of which were such as to make the support of its sides, with abutting large buildings resting on them, of a hazardous nature. The difficulties were increased by the care necessary to support and maintain the heavy vehicle and pedestrian traffic, including the surface-car tracks, and the sub-service utility structures, especially while blasting was in progress, but the section was finished successfully, and since 1918 has been operated as part of the Brooklyn-Manhattan Transit Corporation System.

In 1923, Mr. Clark left the Holbrook, Cabot and Rollins Corporation and in 1924 and 1925 was identified with the Gotham Construction Corporation which installed the foundations for a section of the Elevated Railroad structure across the Flushing meadows—known as Route 52, Section 2—at a cost of $159 000.

In February, 1926, he accepted the position of Vice-President and Chief Engineer of the La Rocca Construction Corporation, of New York City, which, under his direction, built for the Board of Transportation two important sections of the Independent Subway System. One, known as Route 78, Section 4A, was under St. Nicholas Avenue between 141st and 149th Streets, Manhattan, and the other—Route 106, Section 2—was located under the Concourse and 161st Street in the Borough of The Bronx. Section 4A included the junction of the Washington Heights and the Concourse Branches of the System, and was of the double-deck type, providing seven tracks for part of the way and a large Express Station. The excavation of this rock cut was carried to a depth of about 60 ft, for nearly the entire width of the 100-ft avenue. Engineers can appreciate the difficulties faced by the constructors in executing a work of this character in a busy city street lined with buildings. Section 2, in The Bronx, was a three-track structure and much of it was in rock tunnel. The construction cost of these two Sections was about $10 500 000.

On the completion of these contracts, Mr. Clark, in January, 1930, retired from active work on account of declining health. Rest failed to restore his strength, and in spite of his long and courageous fight against the heart malady with which he was afflicted, and the devoted care of his wife, he gradually grew weaker and died on December 6, 1934.

Mr. Clark was a man of very positive characteristics and aggressive in dealing with his problems. He was quite active in the affairs of the General Contractors Association of New York. He was fond of sailing, and before his illness maintained a boat on Long Island Sound where he indulged in this sport.

One who for many years was associated with him in his work writes:

"Mr. Clark was an engineer of indefatigable energy, very keen in the conception of construction methods, always approachable by those engaged in work under him, and remarkably persuasive in establishing his point of view with others. His keenness of mind was combined with a brightness of humor that rendered him always a smiling, interesting companion who believed that 'all work and no play makes Jack a dull boy.' All who were associated with him are left with a memory of a forceful, pleasing, and lovable personality of the highest type, both personally and professionally.

* * * * * * *

"Mr. Clark has always stood for me as the highest type of engineer, professionally and personally, and I value, next to my training at West Point, the training I had under him. All who knew him must feel a real personal loss."

Mr. Clark was a member of the Alumni Association of the School of Mines of Columbia University, the Engineers Club of New York, and a Life Member of the New York Athletic Club.

He was married to Ella Van Blarcom, of New York City, on October 14, 1914, who, with his sister, Sophronia T. Clark of the same city, survives him.

Mr. Clark was elected a Junior of the American Society of Civil Engineers on October 3, 1893; an Associate Member on April 6, 1898; and a Member on October 31, 1905. He served as a Director of the Society from 1919 to 1921, and was active in Committee work.

JOHN ADAMS COLE, M. Am. Soc. C. E.[1]

Died November 16, 1932

John Adams Cole was born in Westmoreland, N. H., on December 16, 1838, the son of John and Elizabeth (Shaw) Cole. He was educated at Kimball Union Academy, at Meriden, N. H., and began his technical career as a student in the office of the late Thomas Doane, M. Am. Soc. C. E., in Boston, Mass.

In 1861 and 1862, Mr. Cole was with the late J. Herbert Shedd, M. Am. Soc. C. E. He was engaged also on the State survey of the Sudbury Meadows (Mystic) Water-Works.

In August, 1862, Mr. Cole volunteered as a Delegate of the United States Christian Commission, a voluntary organization for carrying on religious work among the soldiers and aid to the wounded, during the Civil War. He went to the Army of the Potomac then on the Peninsula. After the Battle of Antietam, he returned to New England and spent three months addressing public meetings in the larger cities, and organizing societies for Commission work. In January, 1863, Mr. Cole was appointed General Field Agent for the Commission for the Union Armies east of the Alleghenies, and was engaged in the field until the end of the war. More than $3 000 000 were donated and expended for supplies for the wounded and for hospital work, and hundreds of clergymen were delegates at the front. The course of his duties took Mr. Cole to the battlefields of Antietam, Gettysburg, The Wilderness, Cold Harbor, and many other historic engagements, and he was present at the surrender of Appomattox. His letters from these historic scenes, carefully preserved by the family, are of thrilling interest.

After the Civil War Mr. Cole opened an engineering office in Washington, D. C. He removed to Chicago, Ill., in 1872, and established himself in private engineering practice. Among other important works, he designed the original Lake View Pumping Station (later, a part of the Chicago Water-Works) and the Hyde Park Water-Works, including an inlet tunnel a mile out under Lake Michigan.

He also designed much of the sewerage system of the then separate municipalities of Hyde Park and Lake View; also, important Lake shore protection works, as well as other municipal improvements. Later, his practice was extended to many cities in the West and, for years, he was widely known as a Consulting Engineer of prominence. He resided in Ravenswood, Ill., for some years, but later removed to Hyde Park, a suburb of Chicago.

---

[1] Memoir prepared by Edward S. Cole, M. Am. Soc. C. E.

For many years Mr. Cole was engaged as a Consulting Engineer in making expert reports for bankers on public utilities throughout the Middle West and the Far West. In 1896, he became associated with his son, Edward S. Cole, M. Am. Soc. C. E., in the development of the pitometer.

Mr. Cole was always a devoted church worker. He served as Elder in the Hyde Park Presbyterian Church for more than forty years; and as an Honorary Trustee of Howard University, from which he received the Honorary degree of Doctor of Laws. He was also President of the Chicago Tract Society from 1911 to 1913; and of the Training School, in 1912 and 1913. He was a member of the Western Society of Engineers, and of the Chicago Literary Club.

He was survived by his widow, Mrs. Julia (Alvord) Cole, to whom he was married on December 15, 1870. Mrs. Cole died late in 1932. His immediate survivors are a son, Edward, and a daughter, Elizabeth (Mrs. D. J. Fleming).

Mr. Cole was elected a Member of the American Society of Civil Engineers on March 7, 1894.

EUGENE HUNTER COLEMAN, M. Am. Soc. C. E.[1]

DIED FEBRUARY 12, 1935

Eugene Hunter Coleman was born in New Orleans, La., on September 25, 1891. He was the only child of John Francis Coleman, Past-President, Am. Soc. C. E., and Annie Hardee (Hunter) Coleman. His maternal grandfather, Colonel Samuel Eugene Hunter, was Commanding Officer of the 4th Louisiana Regiment of the Confederate Army—a man greatly beloved by those who knew him best.

Hunter Coleman was graduated from Rugby Academy, in New Orleans, when he was only fourteen years of age. By a special dispensation he was allowed to enter Tulane University, and on the completion of the course and before he was twenty years of age, he received from that institution, in 1911, the degree of Bachelor of Engineering with highest honors. The next year was spent at Cornell University, at Ithaca, N. Y., where in June, 1912, he received the degree of Mechanical Engineer.

During his college career, Mr. Coleman's vacation periods were devoted to work which added to his engineering experience and outlook; one summer was spent on a large rice irrigation canal in Texas. Another summer he spent as a Rodman on the Southern Pacific Railroad. He was employed during still another summer as Instrumentman and Inspector on wharf and shed construction.

After his graduation from Cornell University, he was employed by the Illinois Central Railroad Company, on drafting, valuation, and the inspection of construction work. Before the end of 1912, Hunter Coleman became an Assistant Engineer in the J. F. Coleman Engineering Company, and at the beginning of 1917, he became a member of the firm.

---

[1] Memoir prepared by a Committee consisting of W. B. Gregory, *Chairman*, Edward S. Bres, and B. H. Grehan, Members, Am. Soc. C. E.

In 1915, when trouble with Mexico arose, he enlisted in the Washington Artillery, Battery C, and was ordered to duty on the Mexican border. Following this enlistment period, he was commissioned in the Engineer Officers Reserve Corps. Later, Mr. Coleman attended the First Officers Training Camp at Fort Roots, Arkansas, and Fort Leavenworth, Kansas. He was commissioned as a Captain in the Corps of Engineers, in August 1917, and was assigned to the 312th Combat Engineers of the 87th Division, stationed at Fort Pike, Arkansas.

With his regiment he served with the American Expeditionary Forces in France and returning to the United States was honorably discharged in July, 1919. His service in the Army tended to increase his interest in small arms. As a hobby, Mr. Coleman accumulated a notable collection of pistols and rifles and achieved local fame in amateur marksmanship.

In 1919, he became a member of the firm of Guerin and Coleman for a few months; but at the end of that year rejoined the firm of J. F. Coleman Engineering Company.

During the years that followed, the firm was notably outstanding in engineering work in port improvements at New Orleans, Houston, Tex., Charleston, S. C., Lake Charles, La., and Beaumont, Tex., as well as in the design and construction of foundations and various structures in steel, timber, and concrete, and in power plant and public utilities valuations.

When his father, Mr. J. F. Coleman, was appointed one of the Advisory Engineers of the Reconstruction Finance Corporation, the responsibility for the conduct of the work of the firm fell more heavily on Hunter Coleman. In meeting these responsibilities, he displayed his usual good judgment and won the admiration of his fellow engineers.

In 1923, he was honored with the Presidency of the Louisiana Section of the Society. He was also active in the affairs of the Louisiana Engineering Society. His affable and always cheerful personality won for him a host of friends, both among members of the Engineering Profession and in business and social relations. He will be missed from the local meetings of engineers.

Mr. Coleman was a member of the Delta Tau Delta Fraternity, the Boston, the Stratford, and the Round Table Clubs, and was prominently identified with the social activities of New Orleans.

Almost without previous warning, on February 12, 1935, Mr. Coleman was stricken with a heart attack while at his office, to which he succumbed a few hours later.

On August 18, 1917, he was married to Josephine Maginnis, of New Orleans, who, with his father and mother, survives him.

Mr. Coleman was elected a Junior of the American Society of Civil Engineers on October 1, 1913; an Associate Member on March 13, 1917; and a Member on March 16, 1925.

CHARLES FRANCIS CONN, M. Am. Soc. C. E.[1]

Died October 19, 1933

Charles Francis Conn, the son of Adna C. and Millicent (Colony) Conn, was born at Kenosha, Wis., on May 26, 1872. His childhood was spent in the South, and he attended school at Atlanta, Ga., and other places in that section. He was also a student at the Georgia School of Technology, at Atlanta, for two years.

Mr. Conn began his engineering career in 1891, with the Thomson-Houston Electric Company (General Electric Company). Having served his apprenticeship, he was appointed Construction Superintendent in the Isolated Lighting Department, installing lighting and power apparatus in industrial plants, office buildings, hotels, etc.

From January to September, 1895, he was engaged, in a private capacity, on the reconstruction of electric plants, etc., in the South, and from September, 1895, to January, 1896, he was employed as Inspector of Electrical Installation and Assistant Superintendent of Operation of the electric power plant at the Cotton States and International Exposition, at Atlanta, after which, for six months, he again engaged in private practice on electrical construction at various places in the South.

Coming North in June, 1896, Mr. Conn entered the employ of the Flatbush Gas Company, Brooklyn, N. Y., as Construction Foreman of the Electrical Department. He retained this position until February, 1898, when he was made Superintendent of the Yonkers, N. Y., Electric Light and Power Company in charge of plant operation, outside construction, etc.

Mr. Conn returned to the Flatbush Gas Company in 1899 and was employed first on the re-arrangement of the plant and the installation of new apparatus in the Electrical Department, until February, 1900, when he was appointed Superintendent of the Electrical Department in charge of operation, purchases, and contracts.

In February, 1903, Mr. Conn entered the employ of the General Electric Company, at Schenectady, N. Y., as Special Engineering Representative in the Transformer Department. In this position he assisted in handling competitive tests, complaints, etc., where matters of engineering and design were involved.

Mr. Conn became associated with The J. G. White and Company, Incorporated (later, The J. G. White Engineering Corporation), of New York, N. Y., in June, 1907, and continued with this Company until his death. From 1907 to October, 1909, he served as Assistant Engineering Manager in the New York Office. In this position, his duties included reporting on the conditions and operation of public utility properties, reviewing projects for the Financial Department, etc. He was then transferred to the San Francisco, Calif., Office where, as Assistant Manager and Manager, he had responsible charge and technical direction of the Engineering Staff, approval of designs, drawings, and installation of turbo-generators, hydro-electric plants, high-speed interurban railways, as well as reports and valuations of many of the principal utility properties in the State of California, etc.

---

[1] Memoir compiled from information on file at the Headquarters of the Society.

In 1916, Mr. Conn returned to the New York Office, and served as Secretary, Treasurer, and Director of the Corporation until his death from heart failure, on October 19, 1933, ended an association of twenty-seven years. He was also Secretary, Treasurer, and Director of the Engineers Corporation, and the Caibarien-Remedios Water-Works Company, and Treasurer and Director of the Garden Court Apartments, Incorporated.

He was a member of the American Society of Mechanical Engineers, and a Fellow of the American Institute of Electrical Engineers. He was also a member of the Engineers Clubs of Brooklyn and New York, and of the Mid-Day Club of New York, and had been a Director of the New York Board of Trade until his resignation in 1931.

Mr. Conn was an engineer of exceptional attainments of a technical nature, and although his principal occupation in recent years was along lines involving business and economic matters relating to public utility and industrial problems, he nevertheless retained great knowledge of, and interest in, the technical problems of power generation and electric transmission, and with the introduction of radio apparatus he took up, principally as an avocation, the many intricate problems involved in radio work. He was a man of sterling character and genial personality, with many friends both in and out of the Engineering Profession.

He had never married, and no member of his immediate family survives him. His nearest relative is a niece, Mary L. Bates, of Keene, N. H.

Mr. Conn was elected a Member of the American Society of Civil Engineers on October 7, 1914.

LORENZO DANA CORNISH, M. Am. Soc. C. E.[1]

Died May 12, 1934

Lorenzo Dana Cornish was the son of J. Bennett and Frances E. (Ward) Cornish, and was born at Lee Center, N. Y., on March 20, 1877. His first American ancestor married the daughter of the First Mate of the *Mayflower.*

After his academic schooling, Mr. Cornish entered Syracuse University, at Syracuse, N. Y., from which he was graduated in 1902, with the degree of Civil Engineer. From June, 1899, to September, 1902, in intervals of his college work, he was engaged in general engineering, consisting mostly of the design and construction of sewerage and water supply systems, with some work on the location and construction of railroads.

In September, 1902, Mr. Cornish entered the service of the Corps of Engineers, United States Army, as Junior Engineer, and remained in that employ until September, 1906. During this time he was engaged in the various positions of Surveyman, Inspector, and Superintendent of Construction on Locks and Dams Nos. 2 to 7 on the Ohio River. While still in the employ of the Corps of Engineers, Mr. Cornish was ordered to Washington, D. C., in October, 1905, by the Chief of Engineers, U. S. Army, on temporary duty to assist the International Board of Advisory Engineers appointed by President Theodore Roosevelt to study the Panama Canal situation and make recommendations as to the type of canal to be constructed.

---

[1] Memoir prepared by Walter M. Smith, M. Am. Soc. C. E.

On the completion of the estimates of cost and the submission of the report to President Roosevelt, Mr. Cornish returned to his work on the Ohio River where he remained until it had been decided to build a lock canal at Panama, when he was ordered back to this work by the Secretary of War in the late summer of 1906. He remained in the Washington Office of the Isthmian Canal Commission, engaged on the design of the locks and dams of the Panama Canal, until the summer of 1907 when the Designing Office was transferred to Panama. Mr. Cornish was put in complete charge of the design of the concrete portions of all the locks and dams and of reviewing the designs of the miter-gates, machinery, and protective devices. He remained on this work until August, 1913.

In the fall of 1913, when the Canal was thrown open to traffic, Mr. Cornish returned to the United States, and in the summer of 1914 he went to China as Principal Assistant Engineer to the American National Red Cross Board of Engineers appointed to investigate and report upon the Huai River Conservancy Project. He was in charge of field work, preparing estimates of cost, etc. He returned to the United States at the end of the year.

In the spring of 1915, Mr. Cornish entered the United States Engineer Office, Central Division, at Cincinnati, Ohio, as Principal Assistant Engineer, in charge of design and inspection for the Division Engineer, of improvements on the Ohio River and its tributaries.

Just after the United States entered the World War, Mr. Cornish was commissioned a Captain in the Corps of Engineers, U. S. Army, and assigned to construction work with the American Expeditionary Force in France. He was in France until July, 1919, a period of about two years, during which time he was engaged in the work of constructing emergency water supplies, railroads, roads, etc., for the American Expeditionary Force. For his able work with the American Army he was promoted to the rank of Major.

On his return to the United States in the summer of 1919, Mr. Cornish was engaged on railroad organization work until November of that year, when he was appointed to the position of Designing Engineer in the Division of Waterways, State of Illinois. As such, he was engaged on the design of the locks, dams, miter-gates, and other structures of the Illinois Waterway, a canal extending from the southern end of the Chicago Sanitary Canal, at Lockport, Ill., to the head of navigation in the Illinois River, at Starved Rock, Ill. As Designing Engineer, Mr. Cornish's services were very valuable in preparing the designs of the miter-gates. His most valuable work in this connection, however, was in developing a formula to be used to obtain the correct spacing of the ports from the culverts into the lock-chamber in order to prevent surging of the water in the lock during filling or emptying. In this, he was entirely successful as the operations of the locks have shown since completion.

In June, 1920, in recognition of his able work, Mr. Cornish was made Assistant Chief Engineer of the Division of Waterways. During his service as such there was a large amount of flood relief and prevention work along the various rivers in and bounding the State of Illinois. This work was put in the hands of Mr. Cornish, and was most ably carried through to completion.

In the spring of 1928, upon the resignation of the late Mortimer Grant Barnes, M. Am. Soc. C. E., the Chief Engineer, due to ill-health, Mr. Cornish was appointed Chief Engineer and vigorously carried on the work on the Illinois Waterway until the latter part of 1929, when, as it was impossible to finish the Waterway with the money that had been voted more than twenty years before, and to obtain any additional funds from the State of Illinois without an amendment to the Constitution which would necessitate long delay, Governor Emmerson requested the United States Government to take over the Waterway and complete and operate it. This, the Federal Government consented to do, provided the State of Illinois would rebuild the eleven bridges across the Waterway. This was agreed to, and during the incumbency of Mr. Cornish, four bascule bridges and one fixed bridge were completed, and one cantilever, one through truss, and one bascule bridge, nearly completed.

In the fall of 1932, Mr. Cornish began to suffer from ill health. In July, 1933, he resigned his position as Chief Engineer and went to various health resorts in Wisconsin and for a time he seemed to improve. In September, he went on an automobile trip through the East, visiting his old home in New York State. His health began to fail rapidly while on this trip and steadily declined after his return to his home in Chicago, until his death on May 12, 1934.

On January 15, 1901, Mr. Cornish was married to Mary Elizabeth Brodhead, at Syracuse, N. Y., who presented him with one son, Eugene Brodhead Cornish. Mrs. Cornish's health became impaired in Panama, and she died in 1911. On May 10, 1916, he was married to Mrs. Jeanette Welsh, at Cincinnati, Ohio, who survives him.

Mr. Cornish was elected a Junior of the American Society of Civil Engineers on April 5, 1904; an Associate Member on February 7, 1906; and a Member on June 30, 1910.

CLYDE MAXWELL CRAM, M. Am. Soc. C. E.[1]

DIED OCTOBER 20, 1934

Clyde Maxwell Cram was born at Zumbrota, Minn., on April 11, 1882, the son of James and Lucinda (Dickason) Cram. His father, James Cram, was a building contractor. His engineering training was received at the

---

[1] Memoir prepared by Arthur H. Adams, M. Am. Soc. C. E.

College of Civil Engineering, of the University of Minnesota, Minneapolis, Minn., from which he received the degree of Civil Engineer in 1907.

Soon after his graduation, he entered the employ of the United States Engineer Office at San Francisco, Calif., where he served three years as Junior Engineer in subordinate charge of river and harbor improvements.

In 1910, Mr. Cram was transferred to Fort Stevens, Oregon, where he was in charge (under the supervision of the non-resident Assistant Engineer) of the construction of the Columbia River South Jetty. The expenditure on this work was about \$2 500 000 during the period of his engagement.

Mr. Cram was appointed United States Assistant Engineer in December, 1913, and placed in charge of the U. S. Engineer Office at Fort Canby, Washington. In this capacity, he had direct charge of the construction of the Columbia River North Jetty, and other works on the Lower Columbia River. This job involved the expenditure of \$3 500 000.

On the completion of this work, in 1917, he was given various assignments in the United States Engineer Department in San Francisco and Eureka, Calif., and in Yellowstone National Park, which occupied his time for about five months, after which he was transferred to Los Angeles, Calif., as Assistant and Associate Engineer in the U. S. Engineer Department. While thus engaged he had charge, in the field and in the office, of all fortification and a part of the river and harbor design and construction of the Los Angeles District.

The City of Long Beach, Calif., started plans for some ocean-front structures in 1928, and succeeded in inducing Mr. Cram to take a leave of absence from the U. S. Engineer Department, to become Marine Construction Engineer for the City. In this capacity he designed, with his customary thoroughness and mastery over every detail of water-front construction—gained from his long experience in this line of work—a \$1 500 000 "Rainbow" pier and breakwater. This combination of rubble-mound breakwater and pleasure pier was designed as a protective work and also to enclose a still-water bathing and boating area on the ocean front adjacent to the main business section of the city. Enclosed within the area, thus created, is an earth-fill retained by a wooden bulkhead, constructed for the purpose of providing a site for the Municipal Auditorium. Mr. Cram not only prepared the plans and specifications for the improvement, but also supervised its construction. Subsequent storms have spent their fury upon the breakwater and pleasure pier without materially affecting their stability, so carefully was every eventuality foreseen and provided for by this truly great engineer.

On the completion of this work in 1930, Mr. Cram was engaged in private practice, as Consulting Engineer, for a short time, after which he was appointed Executive Engineer for Seacoast Protection in the Division of State Lands of the State of California, with headquarters in Los Angeles, in which capacity he aided in the formulation of rules and regulations for the construction of sea-coast protective works. To him is largely due the credit for the present enlightened control by this State, of the protection and improvement of its beaches and tidelands.

In 1933, he opened an office in Long Beach and was associated with Arthur H. Adams, M. Am. Soc. C. E., Charles D. Wailes, Jr., Assoc. M. Am. Soc. C. E., and Mr. J. H. Davies, all of whom had previously been connected with him in city work.

In 1934, Mr. Cram again entered the United States District Engineer Office in Los Angeles where, until his death, he was engaged principally upon the Newport Beach Harbor Project.

Honored and respected by all who knew him, Mr. Cram, affectionately called "Major" by his associates, was especially beloved by those who had the rare privilege of working with him. His remarkable ability to obtain for his employer, faithful performance of the work described in his specifications was generally known on the West Coast. However, in his insistence on strict conformity, it was not necessary for him to antagonize the contractors, as they too respected his honesty, ability, and spirit of fairness. His long experience on marine construction gave him a vast fund of knowledge on the subject, which he freely imparted to his fellow engineers whenever they had a knotty problem to solve.

Thus, in his long service to his Nation, State, municipality, and fellow man, he exemplified in his life the true ideals of the engineer, and, in passing, left behind many permanent structures which will stand as monuments to his genius.

On September 1, 1909, Mr. Cram was married to Erna E. March, of Stockton, Calif., who survives him.

His affiliations included The Central Christian Church, of Glendale, Calif.; Masonic Lodge, Knights of Pythias, and the Order of the Eastern Star, all of Zumbrota, Minn.; and the American Association of Engineers.

Mr. Cram was elected a Member of the American Society of Civil Engineers on September 9, 1919.

BENJAMIN JOHN CURTIS, M. Am. Soc. C. E.[1]

Died May 19, 1934

Benjamin John Curtis was born at Fairmont, Minn., on March 13, 1890, the son of Byron H. Curtis and Emily (Merritt) Curtis.

He received the degree of Bachelor of Science in Civil Engineering at the University of Minnesota, Minneapolis, Minn., in 1913, and a year later, in 1914, he obtained the degree of Civil Engineer.

Immediately upon his graduation, Mr. Curtis entered the employ of the Sanitary District of Chicago, Ill., on construction work as Instrumentman, Junior Engineer, Resident Engineer, and Assistant Engineer, to April,

---

[1] Memoir prepared by Charles E. Wheeler, Jr., Assoc. M. Am. Soc. C. E.

1921. During this period the Sanitary District was engaged in the construction of the Calumet Sag Channel, in which work Mr. Curtis had a very active part.

From April, 1921, to December, 1925, he was associated with the T. J. Forschner Construction Company, as Assistant to the Chief Engineer, Ralph H. Burke, M. Am. Soc. C. E. The Calumet Sewage Treatment Works, the 95th Street Sewage Pumping Station, and one section of the North Side Intercepting Sewer, were built by the Company during this period.

Mr. Curtis returned to the Sanitary District of Chicago in December, 1925, as Assistant Engineer on construction work, being in charge of field construction of the North Side Treatment Works from January, 1927, to July, 1929. At this time he was appointed Principal Construction Engineer and was in charge of the construction of the North Branch Pumping Station.

In July, 1930, he obtained leave of absence from the Sanitary District to become Assistant to the late George M. Wisner, M. Am. Soc. C. E., who was Chairman of the Advisory Subway Engineering Commission, of the City of Chicago. While with Mr. Wisner, Mr. Curtis served as the "eyes" of his Chief who had recognized his thoroughness for detail while Chief Engineer of the Sanitary District.

Mr. Curtis returned to the Sanitary District in June, 1931, and was engaged in preliminary design work for the extension of the Calumet Sewage Treatment Works. He also established a central accounting system for operating and maintenance cost records.

In May, 1933, his services were loaned to the Department of Public Works, City of Chicago, under the direction of Mr. Oscar Hewitt; in this position, his engineering studies of the taxicab business later became the foundation of the taxicab ordinance adopted by the City. He also directed the Civil Works Administration program of the Department of Public Works.

In recognition of his services, Mr. Curtis was appointed head of the Department of Streets and Electricity of the City of Chicago in December, 1933, by the Hon. Edward J. Kelly, M. Am. Soc. C. E., Mayor, who, as Chief Engineer of the Sanitary District of Chicago, was well acquainted with Mr. Curtis' ability. Shortly after assuming his new position, he suffered a fatal nervous breakdown.

His most outstanding characteristic was the complete thoroughness with which he handled assignments, a true disciple of his profession.

He is survived by his widow, Edna Wohlhuter, to whom he was married in March, 1916; also, a daughter, Mary Emily. He is also survived by his father and mother and by a brother, Thomas H. Curtis, Engineer with the Highway Commission, State of Missouri.

Mr. Curtis was elected a Junior of the American Society of Civil Engineers, on September 2, 1914; an Associate Member on June 16, 1919; and a Member on January 17, 1927. He also held membership in the Western Society of Engineers.

WILLIAM CLARENCE DAVIDSON, M. Am. Soc. C. E.[1]

Died September 1, 1934

William Clarence Davidson, the son of William Davidson and Esther (Smith) Davidson, was born in Grant City, Mo., on March 13, 1881. He gained his early and higher education in that State, having been graduated, in 1907, from the University of Missouri, at Columbia, Mo., with the degree of Bachelor of Science in Civil Engineering. He was engaged as an Instructor at the University from 1907 to 1911, during which time he pursued post-graduate work, and obtained the degree of Civil Engineer in 1909.

After leaving the University, Mr. Davidson served as Deputy Engineer for the Missouri State Highway Department until 1913, at which time he became Office Engineer with the James C. Travilla Engineering Company, of Fort Worth, Tex., with which Company he remained for two years. He then was appointed Assistant Engineer in McClennan County, Texas, which position he retained until 1916, when he went to New Mexico and became associated with the New Mexico State Highway Department.

Mr. Davidson's first assignment with the New Mexico State Highway Department was as Designing Engineer on bridge standards, at Santa Fé, N. Mex. He was transferred shortly to Roswell, N. Mex., as County Highway Superintendent, and, in 1917, when the Highway Commission Law went into effect, he became District Engineer for the Southeast District, with headquarters at Roswell. During this period he was in charge of the maintenance and construction of roads covering approximately one-fourth of New Mexico. Mr. Davidson's outstanding ability as District Engineer led to his selection as State Highway Engineer in January, 1927, which position he held until April, 1931. Had he remained until July 4 of that year, Mr. Davidson would have completed fifteen years' service with the New Mexico State Highway Department. During that period his sturdy uprightness of character and pleasing personality and his ability as an Engineer and Executive made him a host of friends in the Department, the State, and the Nation.

As State Highway Engineer for New Mexico, he was in charge of a highway construction program involving a total expenditure of $30 000 000, of which amount, $16 000 000 was expended upon the Federal Highway System.

In April, 1931, he engaged in private consulting engineering practice at Santa Fé, and, more recently, was Manager of the Associated Contractors of New Mexico. Early in June, 1934, he was stricken with serious heart trouble and went to California in the hope that the sea-level climate would assist him in regaining his health, but his fight was against odds too great to be overcome.

---

[1] Memoir prepared by Glenn D. Macy, M. Am. Soc. C. E.

During his incumbency as State Highway Engineer of New Mexico, Mr. Davidson was active in the affairs of both the American and the Western Associations of State Highway Officials, having served as a member of the Standing Committee on Road Design in the American Association and, for three years, as a member of the Executive Committee in the Western Association.

Mr. Davidson was a member of Tau Beta Pi, at Missouri University; he also belonged to the Masonic Fraternity, and was a member of the Shrine, and a Knight Templar.

He was married, on December 17, 1919, to Doris Parsons, of Roswell, N. Mex., and is survived by his widow and two sons, William C. Davidson, Jr., and Ralph Parsons Davidson.

Mr. Davidson was elected an Associate Member of the American Society of Civil Engineers on December 6, 1915, and a Member on June 2, 1920.

FREDERICK EATON, M. Am. Soc. C. E.[1]

Died March 11, 1934

Frederick Eaton was the son of Benjamin S. and Helen N. J. Eaton. The father rode horseback from St. Joseph, Mo., to Southern California in 1849, and was subsequently followed by his wife and young daughter who came West by way of the Isthmus of Panama. Benjamin S. Eaton, together with a Mr. Fletcher, a banker from Ohio, organized the Indiana Colony which grew into the City of Pasadena. They expended $25 000 for the purchase of 4 000 acres of land and $25 000 more in bringing the waters of the Arroyo Seco to that locality.

Frederick Eaton was born in Los Angeles, Calif., on September 23, 1855. After attending the Los Angeles schools, he went to work, at the age of 15 for the privately owned Los Angeles Water-Works. He devoted himself with energy to his work and educated himself technically by study at home.

He served the City Water-Works for thirteen years, becoming its General Superintendent, and then resigned to go into the private practice of engineering, building domestic and irrigation water-works. He was appointed City Engineer of Los Angeles in 1886, and initiated the sewerage system for the municipality. He retired from the City service for one year, after which he was elected as City Engineer and held that office for two years more.

Frederick Eaton subsequently became Chief Engineer of what is known as the Consolidated Electric Railway, which ultimately became the Los Angeles Street Railway Company. During his incumbency as City Engineer he also acted as Engineer for the Pico Street Electric Railroad Company which was a pioneer effort in this field.

---

[1] Memoir prepared by J. B. Lippincott, M. Am. Soc. C. E., assisted by the late William Mulholland, M. Am. Soc. C. E., and Burdick Eaton, Esq. Parts of the data were taken from a biography prepared by John Steven McGroarty, Esq.

He visited Owens Valley, in California, first in 1892 to investigate the possibilities of an irrigation project for that locality. Although this irrigation enterprise was not approved, Mr. Eaton at that time became impressed with the large quantity of water in that region, and was convinced that it should be put to use in some manner. This ultimately led to the conception of his idea of bringing this water to the City of Los Angeles.

He was elected Mayor of the City of Los Angeles in 1899, and served for two years. This was about the time of the termination of the private franchise of the City Water Department, and Mr. Eaton strongly advocated municipal ownership. After he retired from the position of Mayor, his mind reverted to the possibility of bringing the waters of Owens River to the City of Los Angeles. Mr. Eaton was a shrewd, practical, and energetic observer. He traversed the country between Owens Valley and Los Angeles practically alone and, with the aid of aneroid barometers, he satisfied himself that it was feasible to bring water from Owens River to Los Angeles by gravity. The only helpful maps available were the meager data furnished by the Wheeler surveys of the United States Corps of Engineers.

The late William Mulholland, M. Am. Soc. C. E., was first associated with Mr. Eaton in the privately owned Los Angeles City Water Company. After Mr. Eaton's retirement, Mr. Mulholland became its Superintendent and Engineer. These men were of the same age within a day. After the expiration of the 30-year franchise of the private Water Company, the City of Los Angeles acquired the local system. The municipality wisely retained the Operating Staff of the old Water Company, including its Superintendent. In 1902, Mr. Mulholland was planning the extension of the distribution system of the City and called in Mr. Eaton as a Consultant. During these conferences Mr. Eaton discussed with Mr. Mulholland the future water supply of Los Angeles, but did not at that time propose going to Owens Valley for additional water.

In 1903, the United States Reclamation Service began general investigations of all possible major irrigation projects in the West and among a large number of others the Owens Valley situation. All public lands in this region were withdrawn by the Department of the Interior pending this investigation, as was customary in such studies by the Department. In the meantime Mr. Eaton kept his own counsel and proceeded with his study of the Owens Valley Project, securing options on the large Rickey Cattle Ranch and the riparian lands below this ranch to Owens Lake. His idea was to organize a company for the purpose of constructing the project. The Rickey Cattle Ranch occupied a large section of the central part of Owens Valley and had as a high mountain range the Long Valley area in the same drainage basin.

In the summer of 1904, Mr. Eaton broached the subject of bringing water from Owens Valley to the City of Los Angeles as a municipal supply and in September, 1904, he and Mr. Mulholland made a reconnoissance of the route together. The results of these investigations were that Mr. Eaton proceeded with the acquiring of options on Owens Valley lands, and agreed to sell these options at their cost to him, the City to purchase the real estate, Mr. Eaton to retain the live stock and the Long Valley Mountain part of the ranch. He also endeavored to retain the right to develop hydro-electric power along the aqueduct for himself.

In order to carry out the municipal plan, it was necessary for the Reclamation Service to abandon the project which it had under investigation in Owens Valley. The officials of the Reclamation Service were requested to aid the City, and a conference resulted between the late Frederick Haynes Newell, M. Am. Soc. C. E., and J. B. Lippincott, M. Am. Soc. C. E., then of the Reclamation Service, with the Water Commissioners of the City of Los Angeles. At this conference these Federal representatives took the stand that the project must be municipally owned in whole, or they could not recommend the withdrawal of the Reclamation Service. They also urged that the City reserve an easement for a reservoir site which had been found on the Long Valley lands which were to be retained by Mr. Eaton. The Reclamation Service also required the payment to it of all monies that it had expended on its Owens Valley investigations, these surveys to be given to the City. Mr. Eaton finally very generously accepted these stipulations, his expenses being refunded, and turned his options over to the City of Los Angeles. This was a public-spirited action on his part in keeping with his general character of high citizenship. The lands covered by the options obtained by Mr. Eaton were purchased by the City, and the areas acquired have been expanded so that, at the present time, the City owns 271 171 acres, in the drainage basin of the Owens River, as of June 30, 1933. The municipality has also acquired 1 257 parcels of land in the small towns of that region, the total investment in these properties being in excess of $26 000 000. All these properties have been acquired by agreement with the owners and none by process of condemnation.

The withdrawal of the U. S. Reclamation Service from Owens Valley was done under recommendation of President Theodore Roosevelt, and approved by an Act of Congress on the theory that this domestication of water was its highest use. The project was built by the City of Los Angeles under the direction of Mr. Mulholland as Chief Engineer and is known as the Los Angeles Aqueduct. Credit is due to Mr. Eaton for the inception of the idea which has resulted in the great development of his native city.

Mr. Eaton was always active as a leading citizen in enterprises of importance. During the depression in the early Nineties he organized the unemployed of the city and began the first extensive park development for the municipality. He was active in many of the scientific organizations of the city.

His first wife was Nellie Burdick, by whom he had four children. His second wife was Alice S. Slosson, by whom he had two children.

Mr. Eaton was elected a Member of the American Society of Civil Engineers on May 5, 1886.

ERIC GUSTAF ERICSON, M. Am. Soc. C. E.[1]

Died September 22, 1934

Eric Gustaf Ericson was born in Ramdala, Sweden, on May 7, 1857, the son of Frans Rudolph and Johanna Gustafva (Weckholm) Ericson, both of whom were natives of Carlskrona, Sweden.

Eric Gustaf Ericson received his primary education at Ystad and Lund, Sweden, and took his student examinations in Lund. From 1876 to 1880, he was a student at the Royal Polytechnic Institute at Stockholm, Sweden, from which he was graduated in June, 1880, with the degree of Civil Engineer.

For two years after his graduation, Mr. Ericson was in charge of harbor improvements and was also engaged on surveys for canal projects in Sweden; and, for one year, he served as Resident Engineer in charge of the location and construction of a railroad on the west coast of Sweden.

Mr. Ericson came to America in August, 1883, and on September 1, 1883, he entered the employ of the Pennsylvania Company (operating the Northwest System of the Pennsylvania Lines West of Pittsburgh), at Pittsburgh, Pa., serving until April, 1886, as Draftsman and Transitman in the office of the Chief Engineer.

On April 1, 1886, he accepted the position of Supervisor of the Eastern Division of the Pittsburgh, Fort Wayne and Chicago Railroad, with headquarters at Wooster and Mansfield, Ohio. Mr. Ericson remained in this position until July 1, 1890, when he was made Engineer, Maintenance of Way, of the Cincinnati and Muskingum Valley Railway Company, at Zanesville, Ohio.

Three years later, on July 1, 1893, he was promoted to the position of Engineer, Maintenance of Way, on the Western Division of the Pittsburgh, Fort Wayne, and Chicago Railway, with headquarters at Fort Wayne, Ind., where he remained until January, 1901. During this time, extensive double-tracking was done on this Division under his supervision, and the first section of track elevation in Chicago, Ill., was constructed.

On January 1, 1901, Mr. Ericson was made Assistant Engineer in Charge of Construction and Maintenance, in the office of the Chief Engineer of the Pennsylvania Lines West of Pittsburgh, and, on January 1, 1903, he was promoted to the position of Principal Assistant Engineer, of the Northwest System of these Lines.

During the period of Government control of the railroads—1917 to 1920—following America's entry into the World War, Mr. Ericson was Principal Assistant Engineer of Construction of the entire Pennsylvania System West of Pittsburgh, and had direct responsibility (under the Chief Engineer of Construction) for the vast construction program then under way.

In 1920, the Lines West of Pittsburgh and the Lines East of Pittsburgh were consolidated, and, on March 1, 1920, Mr. Ericson was appointed Assistant to the Chief Engineer of the Pennsylvania Railroad System, remaining in this position until his retirement on May 31, 1927.

---

[1] Memoir prepared by L. J. Riegler, M. Am. Soc. C. E.

During the years 1901 to 1927, the construction work on the Pennsylvania Lines was of great magnitude, and a large share of the credit for its successful planning and execution belongs to Mr. Ericson. A part of this work consisted of the building of additional main tracks, the enlargement of yards, the revision of grades and alignment, and the separation of grades in Chicago, Ill., Cleveland, Ohio, Fort Wayne, Ind., Pittsburgh, Pa., Akron, Ohio, and other locations. Mr. Ericson also had large responsibility for the planning and construction of the System's modern freight terminal at Chicago, and solved many of the detail problems of the program for handling the traffic and carrying on the construction of the new Union Passenger Station in Chicago.

Mr. Ericson was also freely consulted on important maintenance-of-way matters, and at the time when extensive experiments and investigations were being made on the Pennsylvania Lines West of Pittsburgh, with a view to securing improved track appliances, he had direct charge of this work.

He was known as a master of detail, and one faithful to the interests of the Company, to which he gave a lifetime of service, and as a man who permitted no injustice to those serving under him. He enjoyed the affection, high regard, and respect of all his associates.

Mr. Ericson was a great lover of Nature and spent much of his spare time enjoying everything it had to offer. Being close to Nature, whether in his own garden, afield, on the water, or in the woods, was always the greatest privilege and pleasure that he enjoyed. He was fond of hunting and fishing and was fortunate to have enjoyed these sports in the once famous Kankakee marshes in Northwestern Indiana during his earlier life, when his work lay near this hunter's paradise.

From 1901 until his death, he resided in the Borough of Sewickley, Pa., and was an active member of St. Paul's Evangelical Lutheran Church of that Borough. He took a keen interest in all the activities of this Church, serving on its Council until ill-health forced his retirement.

Mr. Ericson was a member of the American Railway Engineering Association and of the Western Society of Engineers.

He is survived by his widow, Ebba, a daughter, Violetta A. Ericson, and a son, Carl E. Ericson, all of Sewickley.

Mr. Ericson was elected a Member of the American Society of Civil Engineers on December 31, 1913.

HARRY JACOB FINEBAUM, M. Am. Soc. C. E.[1]

DIED JULY 4, 1933

Harry Jacob Finebaum, the son of Barnett and Ray Finebaum, was born in London, England, on January 3, 1893. From early youth, he displayed a great interest in engineering and engineering projects, and, at the age of 9, had received a scholarship to a Preparatory School in Birmingham, England, tending toward engineering practice. Just prior to his leaving England for the United States, and at the age of 14, he had received a scholarship for Birmingham University.

---

[1] Memoir prepared by H. F. Finney, Esq., New York, N. Y.

Arriving in the United States Mr. Finebaum, after very little preparation, entered Cooper Union, in New York, N. Y., and was graduated therefrom in 1912. Post-graduate work was completed in the Polytechnic Institute of Brooklyn, N. Y., from which he was graduated with the degree of Civil Engineer, in 1914.

His first employment was with the Otis Elevator Company, in New York City, in connection with the design of the framework for special elevators requiring heavy loads. Mr. Finebaum was next employed with the Brooklyn Rapid Transit Company, with which Company he spent four years as Draftsman and Designer in connection with its elevated railroad system. A large number of stations now in use were of his design.

In 1917, Mr. Finebaum became associated as Assistant Engineer with the office of Clarence W. Hudson, M. Am. Soc. C. E., Consulting Engineer, of New York City, on the design of bridges. After spending a year with Mr. Hudson, he joined the United States Army in the Engineer Officers Training School, at Camp Humphreys, Va. Directly after the Armistice in 1919, Mr. Finebaum became Engineer at the Shipyard Plant of the Emergency Fleet Corporation, in Philadelphia, Pa., and specialized in the design of floating timber docks and graving docks. A year later, he was recalled by Mr. Hudson, in order to work on the design of the Hill-to-Hill Bridge, at Bethlehem, Pa. In July, 1921, active work was begun on the Hill-to-Hill Bridge and, for a period of more than five years, Mr. Finebaum acted as Resident Engineer and Chief Engineer in Charge of Supervision during its construction.

For a number of years, Mr. Finebaum had been a keen student of reinforced concrete, and the Hill-to-Hill Bridge is, to-day, representative of one of the finest types of reinforced concrete structures in the United States.

In spite of the time necessary for concentration on the activities of his profession, Mr. Finebaum also found time for lecturing and writing and was a frequent contributor to various technical publications. For a short period, he acted as an Instructor in the Engineering Department of Drexel Institute, at Philadelphia.

Upon the completion of the Hill-to-Hill Bridge in 1927, he became associated with the Potruch Construction Company, in Allentown, Pa., as Consulting Engineer, Vice-President, and Manager. All types of construction work were undertaken by this Company, and Mr. Finebaum's experience covered the entire field, from plot sub-division and erection of small homes to consultant work on hotels, warehouses, and miscellaneous structures.

In 1929, he became associated with the Gresham Construction Company, in New York City, as Project Manager, and under his direct supervision, the Company erected the 35-story office building at No. 19 Rector Street and the 235-family apartment development at Southgate, East 52d Street.

In 1931, Mr. Finebaum began private practice as a Civil and Consulting Engineer, specializing in building construction and in carrying out projects for owners from their inception to tenancy. In addition to consultant work, he also acted as Engineering Counselor and Consultant in cases involving building damages and liability suits resulting from defects in engineering construction.

While associated with Mr. Eugene Schoen as Vice-President and Project Engineer of the Building Improvement Associates, Mr. Finebaum was retained by Mr. George Colon and the State Dredging Corporation as Consultant in connection with private contracts issued by the Government for flood-control work in the Lake Ochochobee District, Florida. This work necessitated frequent trips to Florida, in order actively to supervise the work as it progressed. Traveling from Palm Beach, to Tampa, in order to attend a consultation in the latter city, he was killed, on July 4, 1933, by collision with an automobile driven by irresponsible persons.

In addition to his love for engineering work, Mr. Finebaum was a keen student of social and economic problems, and, at the time of his death, was engaged in preparing memoranda for a series of articles dealing with the depression period.

Surviving him are his widow, Blessing G. Finebaum, and three sons, Lee J., Robert H., and Harry Finebaum, Jr.

Mr. Finebaum was elected an Associate Member of the American Society of Civil Engineers on November 25, 1919, and a Member on January 18, 1932.

WALTER LINDER FOSTER, M. Am. Soc. C. E.[1]

Died May 24, 1934

Walter Linder Foster was born at Agency, Iowa, on March 13, 1883, the son of William F. and Harriet C. Foster. He received his early education in the public schools of Agency and was graduated from the High School in 1902. In 1906, he received the degree of Bachelor of Science in Civil Engineering from Iowa State College, at Ames, Iowa.

During the summer vacations of 1903 and 1905 he was employed by the City Engineer of Ottumwa, Iowa, and served as Rodman, Draftsman, and Instrumentman on municipal work.

Upon his graduation from college in 1906, Mr. Foster went West and entered the employ of the Oregon Short Line Railway Company, with which he served as Draftsman, Instrumentman, and, finally, as Assistant Engineer. He left the employ of the Railway Company in 1908, to engage in private practice in Sandpoint, Idaho.

While at Sandpoint, Mr. Foster had charge of the construction of the town's sewerage system, six miles of electric railway, and various water power surveys and design projects.

---

[1] Memoir prepared by R. A. Caughey, M. Am. Soc. C. E.

In June, 1912, Mr. Foster re-entered the railroad field by accepting a position as Inspector with the Great Northern Railway Company. He remained with that Company until March, 1913, during which time he was advanced to Assistant Engineer. The duties of this position involved supervision of the construction of snowsheds on the Rocky Mountain Lines of the Great Northern.

In March, 1913, he entered the employ of the Northern Pacific Railway Company as Assistant Engineer on grade separation at Spokane, Wash., returning to the service of the Great Northern in May, 1913, to become Assistant Engineer on track maintenance. He remained in this position until June, 1913, at which time he entered the contracting field.

Mr. Foster's first position in contracting work was with the A. Guthrie Company, Railroad Contractors, with which Company he served as General Foreman on track-laying, ballasting, and bridge and building construction. He was engaged in that capacity from June, 1913, to July, 1914.

Leaving the contracting field, Mr. Foster, in July, 1914, re-entered the employ of the Great Northern Railway Company, this time as Assistant Roadmaster on the Spokane Division, and remained in this position until May, 1915, when he returned to the Northern Pacific Railway Company as Assistant Engineer and Inspector on its Spokane Division. This engagement with the Northern Pacific ended in March, 1916, at which time he went to Alaska with the Kennecott Copper Corporation, serving for nine months as Shift Boss and for three years as Superintendent of Construction on mining and power plants, tramways, etc.

In March, 1920, Mr. Foster entered the teaching profession by accepting a position as Associate Professor of Civil Engineering at Iowa State College. From that time until his death he had charge of all instruction in Railway Engineering. He was advanced in rank to Professor of Railway Engineering in 1928. In addition to his work in railroads, he also developed courses in Engineering Construction in which he demonstrated that principles of scientific management can be applied to such construction. His wealth of experience in practical construction, together with his natural ability as engineer and teacher, enabled him to develop these courses in such a way that they were, in the opinion of those who were familiar with them, unexcelled by similar courses given anywhere.

At the time of his death he had in an advanced state of preparation manuscripts for two textbooks, one on Engineering Reports, on which he was collaborating with Thomas R. Agg, M. Am. Soc. C. E., and another on Railway Curves and Earthwork.

During the college year of 1922–23, Professor Foster secured a leave of absence to engage in highway construction, and entered the employ of the Empire Construction Company, of Des Moines, Iowa, with which he served as Superintendent of Construction for the year. This engagement brought him into close contact with highway work, and no doubt increased his ability and efficiency in teaching courses in Engineering Construction.

For several years preceding his death, Professor Foster served as a member of the Ballast Committee of the American Railway Engineering Association, and, through personal efforts in research, contributed much to the efficiency of that Committee by furnishing first-hand data on railroad ballast and the problems of ballasting.

Professor Foster's professional training and engineering ability, excellent as they were in themselves in qualifying him for the position of a teacher of engineering, did not excel those other qualities which a man in such a position should have, namely, the personal qualities — character, personality, a sense of humor, a personal interest in young men, and an ability to win their confidence and to inspire them to do their best.

All these qualities were possessed by him in abundance. Affectionately known by the students as "Pop", he held a unique position as counselor, ready to listen to a tale of woe at any time, and just as willing to rejoice with those who were rejoicing; he had the confidence of the students and his advice and suggestions were always welcome. Although very approachable and informal with his students, he insisted on a high standard of scholarship and received their hearty co-operation in the classroom and the laboratory.

Professor Foster was a member of the American Railway Engineering Association, Tau Beta Pi, and the Sigma Phi Fraternity. He was an active member of the First Methodist Episcopal Church, of Ames, Iowa, and, at the time of his death, was a member of the Official Board.

He was a member of the Athletic Council of Iowa State College from July, 1928, until his untimely death which was the result of an automobile accident near DeKalb, Ill., on the evening of May 23, 1934. He died on the morning of May 24, at the Glidden Memorial Hospital, at DeKalb.

He was married to Lucy Southmayd, at Victoria, B. C., Canada, on July 5, 1909, and is survived by his widow and three sons, William, John, and James.

Professor Foster was elected an Associate Member of the American Society of Civil Engineers on May 31, 1916, and a Member on January 20, 1922.

GEORGE WARREN FULLER, M. Am. Soc. C. E.[1]

DIED JUNE 15, 1934

George Warren Fuller was born at Franklin, Mass., on December 21, 1868. Both Mr. Fuller's parents came from a long line of Massachusetts ancestry, extending back to the early settlers, those on his father's side having originally come from England in 1642, while the Craigs, on his mother's side, were from Scotland.

---

[1] Memoir prepared by Arthur S. Tuttle, President, Am. Soc. C. E.

The first twenty-seven years of his life were devoted to preparation for leadership in his chosen field of Sanitary Engineering. Upon receiving the degree of Bachelor of Science at the Massachusetts Institute of Technology, at Boston, Mass., in 1890, he pursued graduate work at the University of Berlin, Germany, and was a member of the staff of Engineer of Water Filters Piefke.

After his return to the United States in 1891, Mr. Fuller served four years with the Massachusetts State Board of Health. During most of his service with this Board, he was in direct charge of the Lawrence Experiment Station, which was recognized as leading in research in the purification of water supply and treatment of sewage in this country; and Mr. Fuller's brilliant achievements in this field attracted such attention to his ability that he was selected in 1895 to take charge of the experiments at Louisville, Ky., in the use of rapid filtration for its water supply. Immediately after he had accomplished this work, he was offered a similar engagement in Cincinnati, Ohio.

These experiments served to remove the questions which had been raised with respect to the sufficiency of rapid filtration as compared with slow sand filtration for these municipalities, and, at the same time, established the value of mechanical filtration where conditions were such as to warrant its use.

With a reputation established as an expert on the treatment of water supply and sewage, Mr. Fuller came to New York, N. Y., in 1899, to establish himself in independent practice. In 1901, he joined the late Rudolph Hering, M. Am. Soc. C. E., as partner, and this association continued until 1911, when Mr. Fuller again engaged in private practice until 1916, at which time he formed a partnership with the late James Robinson McClintock, M. Am. Soc. C. E., as Fuller and McClintock, under which name the business has since been continued. Mr. McClintock died on April 11, 1933. The surviving partner is Fred G. Cunningham, M. Am. Soc. C. E., who became a member of the firm in 1927.

During his thirty-four years of active practice as a Consulting Engineer following the opening of his New York Office and, later, followed by the opening of branch offices in Kansas City, Mo., Toledo, Ohio, and Philadelphia, Pa., Mr. Fuller advised more than 150 cities, commissions, and corporations with respect to their water supply and sewerage problems, the outstanding engagements including, in addition to those hereafter mentioned, Washington, D. C., New Orleans, La., St. Louis, Mo., Indianapolis, Ind., Kansas City, Mo., Memphis, Tenn., Wilmington, Del., New Haven, Conn., Lexington, Ky., Minneapolis, and St. Paul, Minn., Montreal, Que., Canada, Shanghai, China, Water Company, the International Joint Commission (Canada and United States boundary waters), New Jersey Water Policy Commission, North Jersey Water Supply Commission, Hackensack Valley Sewerage Commission, and the Metropolitan Sewerage Commission of Rhode Island. For many of these engagements, his service included full control over all engineering work involved in the preparation of plans and contracts as well as the actual construction.

Mr. Fuller was intimately connected with the development of the plans for a modern system of sewage treatment for the City of Chicago, Ill. In 1924 and 1925, he was Chairman of the Engineering Board of Review of the Sanitary District of Chicago on the Lake Lowering Controversy and Program of Remedial Measures. In this capacity he worked with twenty-seven other experts on the problem of the advisability of installing huge new works for disposing of the sewage of the Sanitary District, to enable the quantity of water diverted from Lake Michigan into the Mississippi Valley for sewage disposal purposes to be decreased. He was also retained in 1934 by the Federal Government to advise a Special Board of Review on the processes to be used in proposed additions to Chicago's sewage disposal works. His views were also sought by the Metropolitan Sewerage Commission of New York City, which made a series of comprehensive reports during the period from 1910 to 1914, relative to the disposal of the city's sewage.

Over a long period of years, Mr. Fuller rendered important service to the City of New York in dealing with its water supply and sewerage problems. Upon joint recommendation made by him and by the late Allen Hazen, M. Am. Soc. C. E., the stripping of the Ashokan Reservoir was omitted, and the reservoir site was only cleared and grubbed, and the swamp areas covered with earth. From the latter part of 1923 to the time of his death, Mr. Fuller was a member of the staff of Consulting Engineers of the Board of Water Supply and was called in from time to time to advise with respect to its sanitary problems, including the improvement of the Schoharie Water-Shed. During this interval, he also advised with respect to the design of the sewage disposal plants at Tannersville, N. Y., and at Grand Gorge, N. Y., on the Schoharie Water-Shed.

In 1927 and 1828, he was also retained by the City of New York as Consulting Engineer to advise on the project for treating the raw sewage discharged into the Harlem River and into the upper part of the East River. His service in this respect was followed by the retention of his firm to prepare plans and specifications for an activated sludge plant at Wards Island, with a capacity of 180 000 000 gal per day. This service was satisfactorily completed in 1932, and was followed by the commencement of construction which, however, was halted owing to the difficulties experienced by the City in financing public improvements. It is expected that the work will be resumed in the near future, and that Mr. Fuller's plans will be carried out with little, if any, modification.

A short time before Mr. Fuller's death he was designated by the Federal Administrator of Public Works as a member of a board to be responsible for the engineering and designing of the proposed $18 000 000 St. Paul-Minneapolis Sewage Treatment Plant, a project which was to be financed with Federal funds.

During the World War Mr. Fuller rendered outstanding service to the United States as a member of a committee at Washington having to do with the engineering, planning, and sanitation of the various Army Camps, and also as an Adviser to the Council of National Defense. In this period, he was also Consulting Engineer for the United States Public Health Service and Consulting Engineer to the Construction Division of the Army. Following the war he was a member of the Franco-American Engineering Congress held in Paris directly after the Armistice to consider various reconstruction and economic problems in France, and, in 1929, he was a Delegate to the World Engineering Congress in Tokyo, Japan.

His keen mind and his recognized capacity for clear analysis not only of engineering projects, but of related fiancial problems, resulted in his advice being sought in valuation matters, and in this field Mr. Fuller performed noteworthy service in connection with the appraisal of many water-works properties and street railway systems, the latter including those operating in Pittsburgh, Pa., Philadelphia, and Buffalo, N. Y.

He always kept himself informed as to European developments in the field of sanitary engineering, and it was to him that many of the foreign engineers turned for information as to American progress.

Notwithstanding a busy life in active practice, Mr. Fuller gave freely of his time and energy to the upbuilding of his chosen profession through participation in the activities of technical societies, through contributions to the engineering press, and through educational activities. His record in this respect is outstanding. He was an active member of the American Water Works Association (Past-President); American Public Health Association (Past-President); Engineering Foundation (Chairman); American Institute of Consulting Engineers; American Society of Mechanical Engineers; Institution of Civil Engineers of Great Britain; American Chemical Society; American Society of Bacteriologists; Engineering Institute of Canada; the Vereines Deutscher Ingenieure; l'Association Générale des Hygienistes et Techniciens Municipaux, of France; and the Franklin Institute.

At the time of his death Mr. Fuller was an active member of the Co-Ordination Committee of the four Founder Societies, which had been appointed with a view to bringing about a more united participation in matters affecting the interests of the Engineering Profession as a whole, and, in this capacity, he had expended tireless effort.

Mr. Fuller always gave freely of his time in the interest of the Society. He served on the Executive Committee of the Board of Direction in 1928 and 1929, being Vice-Chairman in 1928. As stated, he was an Official Delegate of the Society at the World Engineering Congress in Tokyo, and one of his most recent appointments was membership on a committee to study the Co-Ordination of Engineering Organizations.

An outstanding contribution to the work of the Society was made in 1928, when as Chairman of the Committee on Publications and the Special Committee on Technical Expansion, Mr. Fuller made an exhaustive research into the relationship of Society operations and functions. The recommendations of this Committee were adopted by the Board of Direction in 1929, and resulted in placing the Society's publications on a higher standard of operation and establishing its research and committee program on an orderly and business-like basis.

Because of his familiarity with technical literature in his broad field, he was frequently consulted regarding Society publications. His time for such purposes was generously and promptly given, and many Society papers and reports bear the invisible imprint of his valuable advice. In 1903, he was awarded the Thomas Fitch Rowland Prize by the Society for his paper[2] entitled, "The Filtration Works of the East Jersey Water Company, at Little Falls, N. J."

At the time of his death, Mr. Fuller was one of the Society's representatives on Engineering Foundation, serving at the same time as the elected Chairman of that body. The Foundation directed that the following tribute to him be spread upon its records:

> "The Engineering Foundation records with sorrow its great loss caused by the sudden death of George Warren Fuller, its Chairman from 16 February 1933, Vice-Chairman from 18 February 1932, and member from 24 September 1931 to the day of his death, 15 June 1934. The Board counts itself, and the constituent societies of the Foundation, fortunate to have had his able and effective leadership of this research institution for engineers; regrets that his period of service was so brief; and expresses appreciation of his many good personal qualities and admiration for his distinguished achievements as an engineer."

Mr. Fuller's activities in the American Water Works Association are commented upon in a review of his life in the July, 1934, issue of its *Journal,* as follows:

> "During his life he had always taken a keen interest in activities aimed towards the standardization of practices related to the public health field. He contributed largely to the development and widespread adoption of the Standard Methods of Analysis for Water and Sewage sponsored by the American Public Health Association and more than any other individual, was responsible for the successful publication of the Manual of Water Works Practice by the American Water Works Association, as Chairman of its Council on Standardization."

The following is recorded in *The American Journal of Public Health,* for August, 1934:

> "His death * * * marks the passing of one who had probably more to do with the technical development and the public stimulation in the installation of municipal sanitary devices than almost any man in this country. * * * His loss to the public health profession in enthusiasm, in professional judgment, and in technical wisdom is irremediable."

In the *Journal* of the New England Water Works Association for September, 1934, appears the following note:

> "He contributed largely to the development and widespread adoption of the Standard Methods of Analysis for Water and Sewage, sponsored by the American Public Health Association. As Chairman of the Council on Standardization of the American Water Works Association, he, more than any other individual, was responsible for the successful publication of the Manual of Water Works Practice."

---

[2] *Transactions,* Am. Soc. C. E., Vol. L (June, 1903), p. 394.

Mr. Fuller was a frequent Lecturer before college students on subjects relating to his chosen field, and for a time delivered an annual series of lectures at Yale University on sewage disposal. At the time of his death, he was a member of the Advisory Committee of the New York University Council for the School of Engineering, to which he gave valuable service in connection with the development of special courses devoted to Sanitary Engineering. He was always ready to give helpful advice to young members of the profession and to him many owe their later success.

Besides a large number of professional papers credited to him, Mr. Fuller was the author of three books: "Water Purification at Louisville" (1898); "Sewage Disposal" (1912); and "Solving Sewage Problems" (1926). In the preparation of the latter volume he collaborated with his partner, the late Mr. McClintock.

In its issue of June 21, 1934, *Engineering News-Record* made the following editorial comment concerning Mr. Fuller's value to the profession:

> "History will be better able than we are to appraise the contributions of George W. Fuller to the art of water purification, but history will not be so well able to appraise Mr. Fuller's personal qualities of understanding, kindliness, sound judgment and tact as are we who have been fortunate enough to have frequent contact with him in our daily work. It is these qualities that first come to mind with the news of his passing, and it is they that should be first recorded in any note of appreciation of his contribution to human progress. Here also should be recorded an acknowledgment of the debt the profession owes to Mr. Fuller, especially his chosen branch of the profession, for his liberal contributions of time and energy to its professional societies. It can be said without fear of contradiction that it was chiefly through his efforts that the American Water Works Association has been raised from the level of a social group to its present high standing as a technical organization. Mr. Fuller's passing also serves to re-emphasize the youthfulness of sanitary engineering and the fundamental nature of the contributions made by a generation of notable men, now largely departed, work that centered around the Lawrence experiments and laid the foundation for present design methods and practices in water filtration. * * * Fuller's achievements and those of others of his generation are a legacy to be utilized by the present generation to carry the art forward to greater perfection."

In an editorial contained in its issue of July, 1934, *Municipal Sanitation* commented upon Mr. Fuller's career, as follows:

> "Little can be said about George W. Fuller without recalling a thousand and one connections which he has had with sanitary engineering practice in this country and abroad. Amazingly active mentally, he always catalyzed those individuals who were fortunate enough to work with him. An enthusiasm, tempered by seasoned judgment and reinforced by a remarkable technical equipment, accounted for the fact that his name is identified with almost every important sanitary advance in this country in the last four decades. * * * Many, however, are born at the right time who are either ill equipped or are lacking in sufficient vision to make the most of that good fortune. In Fuller's case, heredity and environmental influences, coupled with remarkable energy, all contributed to the development of a practitioner of outstanding stature. He will be remembered long in the future, as much for his distinctive personal characteristics as for his long list of contributions to sanitary science and practice.

"*Municipal Sanitation* records this last tribute to one of the important figures who have made this country safe to live in. His name will be among the first in public health achievements of this century."

Mr. Fuller was also keenly interested in human relations and in meeting his fellow men to discuss matters of general interest, and he was deeply devoted to the advancement of the arts. His alliances of this character are indicated by his club memberships which included the Engineers Club, Machinery Club, University Club (Chicago), Cosmos Club, Seaview Golf Club, Everglades Club, Canoe Brook Country Club, and the Pleiades Club.

Some of his fine qualities and interests are well brought out in an appreciation by Howard S. Neiman, Secretary of the Pleiades Club, to which Mr. Fuller was deeply devoted, as follows:

"While Mr. Fuller's scientific activities were directed to sanitary engineering, his recreational hours were spent with the several arts. A devoted lover of music and with the highest appreciation of the efforts and results of musical artists, he numbered the greatest exponents of that art among his most intimate friends.

"He was particularly interested in the young artists struggling along the rocky road toward the pinnacle of success and in those who, having attained success, were prevented from maintaining their deserved position before the public. His sympathetic and kindly words of encouragement, coupled with his personal material assistance, have engraved his name upon the hearts and memories of many artists whose lives have been made happier and more successful because of him.

"In order that he might be in constant intimate touch with those whom he delighted to serve, he was for many years a most active member of the Pleiades Club, one of the oldest New York organizations devoted to the encouragement and advancement of both the arts and artists, of which club he was President for the season of 1929–1930 and to which office he had been re-elected just prior to his death."

Perhaps the most intimate view of his character and outstanding qualities is expressed in the following tribute made by members of his Staff:

"Mr. Fuller's success in business life was quite largely due to his tremendous energy, his almost unerring judgment, his sense of fair play, and his pleasing and confident personality.

"In whatever he undertook he was absolutely tireless. Having once decided on a course of action, he made quick and accurate decisions, he was not turned aside but drove everything rapidly through to a final and satisfactory conclusion.

"His soundness of judgment was conspicuous. He had the ability to assemble in his mind and to analyze throughly all phases of any problem, and to reach quickly the correct solution. In this, as in many other things, he was aided by his amazingly retentive memory and by his ability to sense accurately the attitude and the reactions of others.

"He was broad-minded in recognizing the rights and privileges of others. 'Live and Let Live' was one of his mottoes. Among his most notable characteristics was his ability to deal with others in such a way as to bring their views into alignment and thus avoid or settle disagreements.

"His personality was such as to give him a high place among either strangers or friends. He inspired in others that confidence which he had in himself—a highly justifiable confidence. He was calm and unruffled in the most trying circumstances.

"To his employees he was loyal. Mistakes and troubles occur in attempting to carry on work of any kind, but whatever happened, he always adopted a sympathetic attitude; and much more than that, he took the responsibility upon his own broad shoulders. From responsibility he never shrank; it was a part of his life."

With his strong personality and convictions it was inevitable that Mr. Fuller should have encountered men of similar character and diverse views, but his engineering opinions were always accepted as entitled to most serious consideration. Those who knew him intimately were well aware of his executive ability, his keen power of analysis, his skill in harmonizing differences of opinion, as well as the fact that before reaching a conclusion he thoroughly familiarized himself with his subject.

His loss is deeply felt by an unusually large circle of friends and admirers, both in the United States and abroad. In the Engineering Profession, Mr. Fuller's name will always be held in highest esteem as one of a small group who pioneered the way for insuring the present high quality of water supply, partly by abatement of sewage pollution and partly by treatment of the sources of supply.

He is survived by his widow, Eleanor Todd Fuller; three sons by previous marriages: Myron E., Kemp G., and Asa W. Fuller; and three step-sons, Kenneth B., Gordon B., and George B. Fuller; and a sister, Mrs. Carl W. DeVoe, of Jerome, Idaho.

Mr. Fuller was elected an Associate Member of the American Society of Civil Engineers on March 1, 1899, and a Member on May 31, 1904. He served as a Director of the Society from 1914 to 1916, and as Vice-President in 1928 and 1929.

WILLIAM GORE, M. Am. Soc. C. E.[1]

Died June 7, 1934

William Gore was born at King's Lynn, Norfolk, England, on April 13, 1871, the son of John and Mary Ann (Barrett) Gore.

He was educated at King's Lynn Technical School and the City and Guilds Institute of London, England.

King's Lynn is situated in the Fenn country, a district largely below sea level and, like Holland, has been reclaimed from the sea by immense sea walls and dikes. In this district the rivers at high tide are below sea level and, consequently, have to be kept from overflowing their banks by a system of retaining walls. Throughout the district there is a great system of pumps and windmills to elevate the water into the rivers, where at low tide the pressure is great enough to force open the tidal gates and allow the rivers to discharge their accumulated waters into the sea. It was in such an atmosphere of irrigation, dikes, pumps, sea walls, and every conceivable problem concerning the control and management of water, that William Gore was raised.

---

[1] Memoir prepared by William Storrie, Esq., Toronto, Ont., Canada.

From 1888 to 1893 he was with the Highgate Iron Works and, afterward, became Chief Draftsman to the Western Electric Company, at Woolwich, England. In 1897, he joined the staff of the late George F. Deacon, Civil Engineer and Water-Works Expert. On the death of Dr. Deacon, in 1909, the business was taken over by Sir Alexander Binnie Son and Deacon and, from then until 1912, Mr. Gore was Chief Assistant in the firm, carrying out large works of water supply, sewerage, irrigation, and hydro-electric power, in London, Liverpool, and Birkenhead, England; Merthyr-Tydfil, in Wales; Petrograd, Russia; Genoa, Italy; Athens, Greece; Malta; and various places in India and Australia. Much of the work carried out at that time was connected with the design and construction of masonry dams, on which Mr. Gore wrote extensively. In 1912, he became Consulting Engineer to the Ransome-verMehr Machinery Company, of London, in the development of the Ransome Drifting Sand Filter, of which system Mr. Gore was the inventor.

In 1913, he went to Canada and installed this system of water purification for the City of Toronto, Ont., the plant having a capacity of 60 000 000 Imperial gal per day. In 1915, Mr. Gore developed a type of portable filter for water purification purposes which was used during the World War, by the British, French, and American Armies.

In 1919, he formed a partnership with Colonel George G. Nasmith and Mr. William Storrie, under the firm name of Gore, Nasmith and Storrie, with offices in Toronto, and this partnership continued until his death. During his twenty-one years sojourn in Canada, his firm carried out water supply and sewage disposal projects in Toronto, Ottawa, Windsor, Belleville, Kitchener, Hamilton, St. Thomas, Oshawa, York Township, and Port Hope, Ont., Calgary, Alta., and elsewhere.

Mr. Gore's experience in dealing with the problems in these municipalities covered a wide field. He had the inventive type of mind and in addition to the drifting sand filter, he had to his credit what is known as the "Gore Mixing Chamber" for use in water purification systems. This method of mixing chemicals with the water was first worked out in the experimental plant at Ottawa, Ont., and the results obtained in several of the filter plants since installed, are somewhat remarkable.

In 1893, Mr. Gore won the Norfolk County Scholarship; in 1894, the Whitworth Exhibition; in 1896, the Siemen Memorial Medal; and in 1907, the George Stephenson Gold Medal for research work.

Mr. Gore was a Fellow of the City and Guilds Institute of London, England; a member of the Institution of Civil Engineers, London; the Engineering Institute of Canada; the American Water Works Association, and other technical societies.

His varied professional experience, his wide knowledge of scientific subjects, his extraordinary memory for facts and details, and his keen insight and knowledge of construction work, made his advice thoroughly reliable on all engineering matters; but, above all, his extreme modesty and gentleness made him a great favorite with those who came in contact with him, and his willingness to assist and give advice to the younger members of his profession was appreciated by one and all.

It seemed somewhat fitting that Mother Nature should pay her tribute to one who spent so much of his life dealing with the forces in Nature for the benefit and convenience of mankind, for his mortal remains were placed in the grave accompanied by a sudden and heavy shower of rain.

In 1897, he was married to Kate Daisley, who survives him, with one son, George Gore, who is with the Canadian General Electric Company, of Peterboro, Ont.

Mr. Gore was elected a Member of the American Society of Civil Engineers on January 18, 1926.

WILLIAM ALBERT GROVER, M. Am. Soc. C. E.[1]

Died September 1, 1934

William Albert Grover was born in Lynn, Mass., on April 16, 1875, a son of Albert Jewett Grover and Kate (Sherman) Grover. He was graduated from the English High School, at Lynn, in 1893, and, in September of that year, entered the Massachusetts Institute of Technology, at Cambridge, Mass., with the Class of 1897.

After pursuing his studies at the Institute, Mr. Grover began active work as a Transitman for Aspinwall and Lincoln, of Boston, Mass. Following a two-year engagement with this firm, he became identified with engineering surveys for the Massachusetts Highway Commission. In 1899, he became Resident Engineer with the firm of McClintock and Woodfall, on sewer and filtration plants in Central Massachusetts.

In 1901, Mr. Grover moved to New Hampshire where he was employed as Resident Engineer on the construction of the Ocean Boulevard along the coast line of that State. In 1903, he was appointed City Engineer of Dover, N. H., which position he held for twenty-three years, carrying on, at the same time, a private practice, including land surveys and consulting engineering work on the construction of sewers, bridges, and highways. During this time he also served for five years as City Engineer of Portsmouth, N. H. Coincident with his work as City Engineer, from 1909 until his death, Mr. Grover was Division Engineer for the New Hampshire State Highway Department. In 1919, he was appointed Engineer Member of the Board of Engineers in charge of the Piscataqua River Memorial Bridge, built jointly by the Federal Government and the States of Maine and New Hampshire. On the completion of this project he acted as Superintendent of Maintenance of this bridge until his death.

---

[1] Memoir prepared by Frederic E. Everett, M. Am. Soc. C. E.

Mr. Grover was a Director of the American Shore and Beach Preservation Association, Past-President of the New Hampshire Good Roads Association, and on the day of his death, had been appointed a member of the New England Research Council.

In addition to his professional activities he found time to devote some attention to social life, being a Past-President of the Bellamy Club, of Dover, a Mason, and an Odd Fellow.

In his application to the various interests about which his life centered, Mr. Grover gained the confidence and admiration of all his associates. His courage and energy commanded the respect of all with whom he came in contact.

He is survived by his widow, Mrs. Anne (Rutledge) Grover, formerly of Portsmouth, N. H., a son, William Sherman Grover, and a daughter, Muriel Rutledge Grover, all of Dover.

Mr. Grover was elected a Member of the American Society of Civil Engineers on January 19, 1923.

JOHN WENDELL HALL, M. Am. Soc. C. E.[1]

Died November 1, 1934

John Wendell Hall was born on February 26, 1885, at Wellesley, Mass., the son of John and Rachel (Hunt) Hall. He received his early education in the Public and High Schools, at Wellesley, and was graduated from Harvard University in 1911 with the degree of Bachelor of Science. During his four years at Harvard, Mr. Hall was active in athletics, principally rowing and football, and also coached the rowing crews at Wellesley College.

During school vacations he was employed as Instrumentman and Inspector, by the Boston and Albany Railroad Company, at Boston, Mass., on maintenance-of-way, yard, and dock construction, and with Stone and Webster Engineering Corporation, of Boston, on steam power plant construction as Inspector of pile-driving, foundations, and masonry; also with the Boston Elevated Railway Company as Inspector in charge of building the Costodis radial brick chimney and compressed-air tunnel for the intake for the power-house condensing system. He was also employed during summer vacations in the Structural Department of the Boston and Albany Railroad Company and with the contractor on the Tunnel Section of the Cambridge, Mass., subway tunnel.

Following his graduation from Harvard University, Mr. Hall went to Idaho, where he was employed as Chief of Party on a survey of an irrigation project for the Downey (Idaho) Irrigation Company. In November, 1911, he entered the employ of the Mississippi River Power Company, at Keokuk, Iowa, under Hugh L. Cooper, M. Am. Soc. C. E., then Chief Engineer. He was engaged with this Company until January, 1914, first as Junior Engineer and, later, as Assistant to the Chief Engineer, in charge of engineering work in the power house, setting turbine machinery, pit liners, and masonry for the substructure of the power house. Following the completion of the Keokuk Hydro-Electric and Navigation Development, Mr. Hall was employed with W. S. Barstow and Company, of New York, N. Y., as Assistant Mechanical Engineer on the design of the penstock and trestles for a hydro-electric development at Rutland, Vt.

---

[1] Memoir prepared by Hugh L. Cooper, M. Am. Soc. C. E.

In June, 1914, he entered the employ of Hugh L. Cooper and Company, in charge of a preliminary field investigation survey for a hydro-electric development at Davis, W. Va. On the completion of this work in October, 1914, Mr. Hall was employed as an Assistant to the late Clifford M. Holland, M. Am. Soc. C. E., and as Shift Engineer on the East River (New York City) Tunnels for the Public Service Commission of New York City. Mr. Hall was continuously employed on the foregoing work until June, 1917, except for the period from August to October, 1916, when he served as Field Engineer with Mr. John Laury, Jr., of New York City, with the title of Assistant Superintendent, laying out and planning masonry structures on the estate of Mr. Charles M. Schwāb, at Loretta, Pa.

From June, 1917, to April, 1920, Mr. Hall represented Hugh L. Cooper and Company as Resident Engineer on the construction of the Zumbro Dam and hydro-electric plant for the City of Rochester, Minn., and from April, 1920, to November, 1926, he was employed as Resident Engineer for Hugh L. Cooper and Company, Consulting Engineers, in all matters pertaining to the construction of the Wilson Dam, Power Station, and Navigation Locks, at Muscle Shoals, Ala.

Upon the completion of the Wilson Dam development, Mr. Hall entered the employ of the Aluminum Company of America, at High Rock, N. C., representing that Company in the construction of the dam and power station on the Tallassee River at that place until June, 1927.

From July, 1927, to November, 1929, Mr. Hall represented Hugh L. Cooper and Company, at Kichkas, Ukraine, Union of the Socialistic Soviet Republics, on the construction of the Dnieper River hydro-electric development at that place. Mr. Hall's duties were principally those of a Supervisory Engineer with especial reference to the preparations of the foundations for the $100 000 000 Dnieprostroy Dam and Power Plant, a position of great responsibility because of the difficult foundation conditions which obtained at the site of the project. In November, 1929, he returned to the United States because of illness and, after a period of convalescence, in January, 1930, he re-entered the employ of Hugh L. Cooper and Company, Consulting Engineers on the Hawks Nest-Gauley Junction power development of the New-Kanawha Power Company, in West Virginia, where he served as Resident Engineer until his unfortunate death on November 1, 1934, as a result of the failure of a plate in a penstock under test.

Mr. Hall's death cut short an engineering career of exceptional value. He had built up an experience as a field engineering executive that will be difficult to replace in the hydro-electric field. Having served as Resident Engineer on works aggregating nearly 2 000 000 hp, he was entrusted with responsibilities of a major character. He had a deep sense of loyalty to his work and to his employer, and this, coupled with a thorough understanding of the details and general character of the work assigned to him, enabled him to produce uniform results of the highest quality. He was a real leader in whatever he was engaged upon, and no matter how firm the stand he had to take in compelling the fulfillment of specifications, both as to character of materials and workmanship, he could always succeed and, at the same time, retain the highest respect and admiration of every one concerned. He was a tireless worker, conscientious to an extreme, no detail of the work entrusted to his care being too trivial to command his attention. His thoroughness was reflected in the quality of the works built under his supervision. The Engineering Profession has sustained a great loss in his passing, not only because of the engineering ability he possessed, but also because of the sterling quality of mind and manhood his entire life exemplified.

He was married, in 1912, to Miriam Cole, a graduate of the University of Chicago, 1912, and to them were born three children, Miriam, Margaret, and Dorothy, all of whom survive him. Besides holding membership in the Blue Lodge of Masons he was a Past Priest in the Royal Arch Chapter, Past Commander of the Commandery, Member of the Zamora Shrine Temple, at Birmingham, Ala., and a Past Grand Patron, Order of Eastern Star, State of Alabama.

Mr. Hall's kindly and upright character attracted and won a host of friends wherever his engineering career took him. He won the esteem of people in many walks of life, and the abrupt and tragic end of his life is lamented by a multitude who knew him as a tried and true friend.

Mr. Hall was elected an Associate Member of the American Society of Civil Engineers on September 9, 1919, and a Member on October 12, 1925. He was also a member of the West Virginia Society of Engineers.

THOMAS CHALKLEY HATTON, M. Am. Soc. C. E.[1]

Died November 11, 1933

Thomas Chalkley Hatton was born in Avondale, Pa., on August 11, 1860, the son of Chalkley and Hannah H. Hatton. He was educated in the public schools of Delaware County, Pennsylvania, and at Maplewood Institute, in Concord, Pa.

Upon leaving school in 1878, he began his engineering career with the St. Paul, Minneapolis and Manitoba Railway Company (later, the Great Northern Railroad Company), as Rodman, being promoted subsequently to be Location and Construction Engineer. In 1881 and 1882, he was employed as Instrumentman by the Canadian Pacific Railway Company, and in 1882 and 1883 by the Department of Public Works of Canada, as Hydrographic Engineer upon survey work in the Hudson Bay District.

---

[1] Memoir prepared by James L. Ferebee, and Darwin W. Townsend, Members, Am. Soc. C. E.

In 1883, Mr. Hatton went to Wilmington, Del., where he was employed in the City Engineer's Office as Assistant City Engineer. In 1900, he was made Engineer in Charge of the Street and Sewer Department, which position he retained during the rebuilding and extension of the sewerage system of the City. Following this employment, Mr. Hatton was engaged in private consulting practice until 1914, during which time he served as Consulting Engineer to many municipalities and private industries throughout the East and South. Most notable among these engagements were those involving the design and construction of sewerage and storm-water drainage systems and sewage treatment works for the Cities of Atlantic City, N. J., Pensacola, Fla., Escanaba, Mich., Newark, Del., Schuylkill Haven, Pa., Ridgely, and Elkton, Md., and Browns Mills-in-the-Pines, N. J. He was also engaged upon the design and supervision of construction of water filtration plants and water distribution systems for the Cities of Asbury Park, N. J., Romney, W. Va., Berwick, Pa., Browns Mills, N. J., and Austin, Pa.

Among his engagements by industrial concerns may be mentioned those with the du Pont Powder Company, of Wilmington, Del., the Wycoff Wood and Pipe Company, of Elmira, N. Y., the R. D. Wood Company, of Camden, N. J., the Aberfoyle Manufacturing Company, of Chester, Pa., and others.

In 1914, Mr. Hatton went to Milwaukee, Wis., as Chief Engineer of the then recently created Sewerage Commission, and served in that capacity until 1927. From that time until his death, on November 11, 1933, he was engaged in private practice, having been appointed Consulting Engineer to the Milwaukee Sewerage Commission in October, 1930. At the time of his death (from injuries received in an automobile accident), he was serving the Sewerage Commission in the dual capacity of Consulting Engineer and Acting Chief Engineer.

The extent of the work carried on under Mr. Hatton's supervision as Chief Engineer of the Sewerage Commission is a matter of record, and represents the development of the activated sludge system in the United States, this work in brief having consisted of the design, construction, and operation of the intercepting sewerage system and the sewage treatment plant for the Metropolitan District of the County of Milwaukee. Too much credit cannot be given Mr. Hatton for his initiative and energy in connection with the work of advancing the activated sludge practice of sewage disposal which, since the time of its inception in 1915, has been developed from its early experimental stages into a practical and efficient working process, and is now used extensively as a method of sewage treatment by the larger cities in the United States, as well as by many smaller ones. Its use has also had general and extensive application to municipalities abroad.

Among other engagements of prominence undertaken by Mr. Hatton may be mentioned those in connection with the Great Lakes Water Diversion litigation, involving the Sanitary District of Chicago, Ill., particularly his professional engagement and association with the Engineering Board of Review of the Sanitary District. More recently, Mr. Hatton was engaged upon the work of designing an Activated Sludge Sewage Treatment Plant for the City of East Chicago, Ind.

Mr. Hatton was keenly interested in the work of Professional Societies. He was a Past-President of the American Society of Municipal Engineers; a member and Past Officer of the Institute of Consulting Engineers; a member and Past-President of the Engineers' Society of Milwaukee; a member of the Institute of Consulting Engineers, the Institution of Civil Engineers of Great Britain, the American Society of Military Engineers, the Franklin Institute, and an Honorary Member of the Sanitary Engineers of Great Britain. More recently, he was President of the Professional Men's Club of Milwaukee.

He was also a member of the Union League Club of Philadelphia, Pa., the Milwaukee Club, Professional Men's Club of Milwaukee, and the Milwaukee Country Club. He was a member of the Protestant Episcopal Church and of the Church Club of Milwaukee.

In 1916, the Pennsylvania Military College, of Chester, Pa., conferred upon Mr. Hatton the degree of Master of Civil Engineering in recognition of his outstanding work in connection with the development of the activated sludge process of sewage treatment.

He was the author of many papers on sanitation and sewerage published in technical journals of the United States and Great Britain. Among these was a paper[2] entitled "Deposition of Sludges Resulting from Sewage Disposal Plants," which was presented as part of a Symposium on "Stream Pollution and Sewage Disposal Plants," at a meeting of the Society on November 16, 1921.

In 1888, Mr. Hatton was married to Catherine E. Hinkson, daughter of William Hinkson, of Chester, Pa., who died in 1932. He is survived by his daughter, an only child, Mrs. Richard Horton Norris, Jr., of Milwaukee.

He was an outstanding engineer with vision and a keen conception of a problem; he was of a forceful character, accomplishing in rapid time whatever he attempted, allowing no obstacles to interfere with his plan. He was a friend to the younger members of his profession, and in his death the Engineering Profession has lost an active and aggressive leader.

Mr. Hatton was elected a Member of the American Society of Civil Engineers on March 6, 1895. He served as a Director of the Society from 1926 to 1928.

JOHN EDWARD HILL, M. Am. Soc. C. E.[1]

Died November 2, 1934

John Edward Hill, the son of Edward and Harriet (Smith) Hill, was born in New York, N. Y., on November 9, 1864, but from childhood made his home in Newark, N. J. His parents and grand-parents were born in New Brunswick, N. J.; they were of English and Dutch descent. One of his great-uncles, Thomas Hill, was President of Harvard University.

---

[2] *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), p. 448.

[1] Memoir prepared from information supplied by Professor Hill's family and the following former colleagues: Professor W. H. Kenerson, Dean O. E. Randall, Registrar F. T. Guild, and Hardy Cross, M. Am. Soc. C. E.

Following preparatory schooling at the Newark Academy, Mr. Hill matriculated at Rutgers College, at New Brunswick, N. J., from which in regular course he received the degree of Bachelor of Science in 1884. Later, his Alma Mater granted him the degree of Civil Engineer, followed by that of Master of Science. Still later, in 1905, in recognition of his success in the educational field, Rutgers College granted him the Honorary Degree of Doctor of Science. In addition to these academic honors, he studied at Cornell University, at Ithaca, N. Y., from which he received the degree of Master of Civil Engineering in 1895.

For several years, immediately following graduation, Mr. Hill held a number of engineering positions in New Jersey and in near-by States. For about three years he was in the employ of the United States Coast and Geodetic Survey and the United States Geological Survey in New Jersey. Another two years he spent in the service of the Pennsylvania Railroad Company, New York Division, as Assistant Engineer in the Maintenance-of-Way Office, engaged in construction work, especially near Trenton and Princeton, N. J. During a greater part of 1890 he was associated with the late Charles Haynes Haswell, Hon. M. Am. Soc. C. E., in New York City, on general surveying work.

All these engagements were preliminary to Mr. Hill's continuous connection, for the major part of his life, with engineering education. This began late in 1890 with his appointment as Instructor in Civil Engineering at Cornell University, a position which he held until 1894. During this time he carried on the studies for his Master's Degree from that University. His summers also were profitably spent. From July to December, 1892, he was in Brazil collecting data for the sanitation of the City of Santos under the direction of the late Estevan Antonio Fuertes, M. Am. Soc. C. E., then Director of the College of Civil Engineering at Cornell University. He was employed during his summer vacation of 1893 as a member of the firm of Hill and Ogden, with offices at Norfolk, Va., in association with Henry N. Ogden, M. Am. Soc. C. E., then, like Mr. Hill, an Instructor in Cornell but, later, Professor of Sanitary Engineering. This firm was engaged in general engineering work.

Mr. Hill's four years at Cornell were profitable in many ways. In his after life, he was wont to refer to those days with distinct pleasure. It was there that he was thrown into stimulating contact with an active teaching staff and student body. Pleasant and warm associations sprang up. During this time, also, he wrote much of his textbook on "Shades and Shadows and Perspective," which was published in 1896. It was his only essay into the field of textbook writing.

About this time Brown University, at Providence, R. I., decided to place its engineering courses in a separate department. Mr. Hill was asked to take charge of the Civil Engineering branches. He accepted an appointment as Instructor, effective in the fall of 1894, and spent the remainder of his life actively engaged in educational work at Brown. After a year, he received promotion as Assistant Professor and, in 1898, he was appointed Professor of Civil Engineering.

During the late Nineties, the University undertook to enlarge its athletic facilities by moving the playing fields from the Campus to a more commodious site a mile or two farther out. Andrews Field, long the scene of all college athletics, was thus brought into being. Professor Hill acted in a consulting capacity in its layout and in constructing its grand stands. On sabbatical leave, 1905–1906, he spent the year with the firm of Hering and Fuller, New York City, largely on preliminary studies for the sewage disposal works for Baltimore, Md.

The Department over which Professor Hill presided was never large either in teaching personnel or students. It was, therefore, incumbent on him personally in his teaching to cover many of the subdivisions in the Civil Engineering field. From the first, he conducted courses in Surveying, Railroad Engineering, and Sanitation, both Sewerage and Water Supply. Also, in later years, on occasion he taught Mechanics and Descriptive Geometry.

There was nothing either pedantic or frivolous about Professor Hill's teaching. His subject-matter was arranged logically and presented tersely. His policy was founded on an assumed educational hunger of the student. He resorted neither to honeyed diet nor to forcible feeding. The student by his own interest and effort alone determined the depth and breadth of his schooling. On Professor Hill's part there was no limit to advice and discussion, provided the attitude of the boy was healthy and sincere. He had no use for the mental sluggard. The slow student he would always help, but not the lazy one. Indeed, frequently, he might be seen in quiet talk with an inquiring boy or an earnest confrère, seated—always seated—on the nearest available stairway, chair, or table. He gave himself willingly and without stint to these conferences; and his advice was much sought, as indeed it might well be, because it was uncommonly sound.

This same trait of character was fully appreciated by his colleagues once they came to know the temper and caliber of his mind. As a result, Professor Hill was called on repeatedly for Faculty committee and conference work. Whenever the subject required clear thinking and calm judgment, then he was likely to be chosen. Numerous academic committees felt the impress of his mind. Only a few need be cited, such as those on Student Organizations which at that time controlled all college athletics; Academic Standing, of which he was Chairman for his last five years; Educational Advice and Direction; and Student Loans. He was always faithful to these responsibilities. For years he also served on the important Registration Committee, a difficult and onerous task for which he was especially fitted. While his work on this particular Committee had to do with engineering students particularly, most of his administrative committees concerned the general activities of the entire University.

In committee meetings Professor Hill's attitude was characteristic. He always had very definite opinions of his own at which he had arrived after careful study and deep thought. Furthermore, he never hestitated to express them and stand by them even in the face of fierce opposition. Apparently, he was indifferent as to the effect his statements might have upon his hearers. He felt it was his duty to give to those about him the full benefit of his convictions even though he might make bitter enemies by so doing. Indeed, with most men this would have resulted, but not with him; he was so honest and so conscientious that no one could hold any bitter feeling toward him.

He found a special field of service in connection with the records of the University and its Clerical Department. The Registrar leaned heavily on him, and Professor Hill did a vast amount of quiet unobtrusive work in this field. During the summer it was his wont to substitute in the capacity of Acting Registrar, in which position he was entirely at home. His decided flair for this type of work was indeed unusual; he seemed to find a keen satisfaction in the scrupulous detailing of administrative problems and records.

Outside the University, Professor Hill also had wide interests. Perhaps the widest, and certainly the deepest, was his garden, long recognized as one of the beauty spots of Providence. From spring until late fall it was a riot of color and glory, unmistakably revealing the handiwork of a master. The synchronizing of blossoms and the blending of colors, particularly at frequent critical periods during the season, showed the thought and care devoted to it. Locally, Professor Hill held a deserved reputation as an amateur horticulturist and was in demand on the programs of garden clubs and societies. He also had the honor to serve as one of the judges at the Boston (Mass.) Flower Show. He had collected and grown more than 500 varieties of iris, many of which he exhibited in the Providence Athenaeum. Indeed he was widely known especially throughout New England for his accomplishments in this field. His was a genuine love of flowers; he usually could be distinguished by a small boutonnière.

Another of his hobbies was the Providence Art Club. Although not himself an artist Professor Hill was genuinely interested and regularly attended the Club's meetings, exhibits, and parties. He also served generously in its Committee work and as an Officer. The Art Club was long one of his main social interests; here he was pretty sure to find many of his friends and here he exerted his considerable influence for genial culture.

Similarly, with the Brown Faculty Club: This organization benefited greatly when, in 1922, the University purchased for its use a fine Colonial mansion adjacent to the Campus. At various times, Professor Hill served as Treasurer of the Club and member of the House and other Committees. During 1931 and 1932, he was President of the Club.

With all these interests he found much time to devote to his family and his home, where he could usually be found in his leisure hours. He was essentially a student and in the fields of his interest he was an authority. Professor Hill was widely read and unusually well informed on matters of public or national interest. His views on educational questions were mature and thoughtful. When his family was young, it was his habit to spend the summer with his wife and four daughters on the family farm at Whitingham, in Southern Vermont.

In his college days, he interested himself somewhat in sports, showing proficiency in track athletics and holding records in the broad jump. He was also fond of archery. In his later years, however, bowling was his sole athletic activity.

Essentially quiet and reserved, Professor Hill to the casual observer might appear diffident and distant. He was far from effusive. The depths of his personality and character were found only by effort. Like a mine of somewhat indifferent aspect on the surface, he proved himself of richest value on deeper exploration. Only real intimates came to realize the fullness of his friendship, and they were amply repaid. For them his reserve was always drawn aside, revealing a cordiality that at times was almost jovial. Friendly banter and jocularity then came to the fore. Coupled with his innate good humor, this made Professor Hill a wonderful companion, whether for a week end or for only a few moments.

His death occurred from a heart ailment, only a week before he would have reached the age of 70. In accordance with college regulation, he was, therefore, in his last year of service. He was the oldest active member of the Faculty, although his appearance seemed to belie this.

By those who were fortunate enough to be brought close to Professor Hill, he was sincerely loved. It took some time to earn his intimacy and hence relatively few undergraduates were thus privileged. The Engineering Alumni however—and many of these among Society members—with maturer discretion came to appreciate Professor Hill fully. By this wide circle of admirers, he will be greatly missed. In many ways he was an engineer of the old school—unpretentious; mentally honest to a fault, even when it hurt; with an ethical idealism that was sharp and true; dignified and courteous in every dealing, be it with the humblest freshman or the most important master of industry. And, withal, he was blessed with a kindliness that smoothed the paths of every associate.

In short, he was the model of a gentleman in the old-fashioned sense. His thoughts were those of a gentleman and so it was inevitable that he always did just what a gentleman was expected to do. With his background of culture and consideration, he combined an ingrained faculty of doing the kindly but unobtrusive act when it was most appreciated. He was loyal to the highest ideals and traditions, and so it was worth any man's time to have contact with him, however brief.

As a result of his varied activities, Professor Hill was affiliated with many organizations. He was a Fellow of the American Association for the Advancement of Science and a member of the New England Water Works Association, the Providence Engineering Society, Brown Engineering Association, Society for the Promotion of Engineering Education, Phi Beta Kappa, Sigma Xi, Delta Phi Fraternity, the American Iris Society, the Massachusetts Horticultural Society, the Providence Garden Club, and the

Rhode Island Horticultural Society, of which he was a Past-President. He was also a member of the Board of Managers of the Providence Art Club for a considerable period.

On December 19, 1894, he was married to Jessie Louise Gould, of Newark, N. J. They had four children: Berenice, Audrey C., and Elsbeth, who made their home with Professor and Mrs. Hill, and Jessica, now Mrs. Eugene A. Bond, of Baltimore, Md.

Professor Hill was elected an Associate Member of the American Society of Civil Engineers on May 4, 1892, and a Member on March 31, 1908.

BURT BRADLEY HODGMAN, M. Am. Soc. C. E.[1]

Died January 20, 1935

Burt Bradley Hodgman was born at Coldwater, Mich., on May 6, 1877. His parents, Charles E. Hodgman and Mary (Bradley) Hodgman, were residents of that city, his father having been in the United States Postal Service for many years.

He prepared for college at Englewood High School, Chicago, Ill. He entered Princeton University, at Princeton, N. J., in 1899, from which he was graduated with the degree of Civil Engineer in 1903. His love for the game and his fitness for it, soon won him a place on the University football team.

After his graduation, Mr. Hodgman was associated with the Pitometer Company, Engineers, of New York, N. Y., serving as Engineer on water waste surveys until 1907. His most important work under this engagement was the pitometer survey in Philadelphia, Pa., during which he was in charge of a group of engineers, most of whom were previous acquaintances. His Engineering Corps was augmented with local Water Bureau men, and his work reduced materially the underground losses in the distribution system and resulted in a satisfactory saving. This work was under the direction of Major Gillette, Chief Engineer of the Bureau of Filtration, and, in 1907, Mr. Hodgman was appointed Assistant Engineer for the City of Philadelphia in charge of the distribution of filtered water under Major Gillette.

Later, in the same year, he became Chief Engineer for the National Water Main Cleaning Company of New York and as its Vice-President and Chief Engineer he remained with this Company until his death.

At the time of his appointment to this position, water-main cleaning was unknown as a practical operation in the United States. The devices first used were crude and their operation was very uncertain. It was necessary to develop both the machinery and the methods for doing this work, as well as to create a demand for the service. With his previous knowledge of hydraulic engineering, Mr. Hodgman was especially well fitted to engage in this pioneering work. His first extended water-main cleaning experience was at Little

---

[1] Memoir prepared by E. D. Case, M. Am. Soc. C. E., and Clinton Inglee, Esq., New York, N. Y.

Rock, Ark., in 1907. As a result of this work, he was able to suggest important improvements in cleaning devices, his ideas being incorporated in several patents issued under his name.

He succeeded in convincing the officials of many of the larger cities throughout the United States that systematic programs for water-main cleaning were as necessary as many other maintenance items. In this business he found his life work and established for himself an enviable position in the water-works field.

Mr. Hodgman presented many papers before the various water-works associations. He served as an expert witness in several valuation and rate cases where his extensive knowledge of main cleaning and hydraulics in connection with that service was valuable in determining valuations and the useful life of cast-iron pipe. In 1931, during a trip to Europe, he inspected the water mains more than 200 yr old and still in service, in the Versailles Gardens.

For many years, Mr. Hodgman acted as Chairman of Entertainment for the various Water-Works Association Conventions. Through this service and his extensive traveling for his Company throughout the United States and Canada, he established a wide acquaintanceship and popularity.

Following a serious operation in the spring of 1934, his health became steadily worse, and he died at the Presbyterian Hospital, New York City, on January 20, 1935.

In 1907, he was married to Grace Porter Wilding, of Fort Wayne, Ind., and with her established his permanent residence at East Orange, N. J. He is survived by his widow and a sister, Mrs. W. H. Compton, of California.

He was a member of the American Water Works Association, The New England Water Works Association, The Southwest Water Works Association, and The New England Engineering Society. His clubs were Princeton Quadrangle, Downtown Athletic, Missouri Athletic, Knickerbocker Whist, Buffalo University, and Machinery.

Mr. Hodgman was elected a Member of the American Society of Civil Engineers on November 25, 1919.

ELMER GUY HOOPER, M. Am. Soc. C. E.[1]

DIED OCTOBER 24, 1934

Elmer Guy Hooper was born at Lynn, Mass., on July 28, 1882. He was the son of William Henry and Kate Estelle (Frothingham) Hooper. His early education was obtained at the public and High Schools of Lynn. Subsequently, he entered the University of Maine, at Orono, Me., from which he was graduated in 1907 with the degree of Bachelor of Science in Civil Engineering. In 1911, he was awarded the degree of Civil Engineer by the same institution.

---

[1] Memoir prepared by C. T. Schwarze, M. Am. Soc. C. E.

During his vacations, while an undergraduate at Maine, Mr. Hooper served as Chief of Party on surveys for the Great Northern Paper Company and as Designing Engineer for the Howland Pulp and Paper Company. Upon graduation, he was employed by the late William Stone Johnson, M. Am. Soc. C. E., Consulting Engineer, in Boston, Mass., during the summer of 1907. In September, 1907, he became Instructor in Civil Engineering at Washington University, St. Louis, Mo., a position he held until the fall of 1912. During this time, from 1908 to 1911, he was engaged in hydraulic research for which he was awarded the degree of Civil Engineer by his Alma Mater.

From 1912 to the fall of 1919, Mr. Hooper served as Assistant Engineer in the Department of Water Supply, Gas, and Electricity of the City of New York. His special field during this time was the solution of problems of distribution and detection of waste, for which he had been eminently fitted by training. Descriptions of some of his work in the detection of leaks were published subsequently. In the fall of 1919, he was appointed to the Faculty of New York University, in the College of Engineering. Here, he served as Assistant Professor and Associate Professor of Civil Engineering until the fall of 1931, when he became Professor of Hydraulics. In this capacity he continued until his death.

During the decade between 1921 and 1931, Professor Hooper was associated with H. Eltinge Breed, M. Am. Soc. C. E., as a Consulting Engineer on various public works projects, including water supply, dams, and highway construction. In this capacity, he brought his fine mental equipment into the service of Nassau County, New York State, in the recent controversy over additional sources of water supply for New York City, and of the City of Beacon, N. Y., in the extensive road-construction projects of a few years ago. Many similar engagements, such as the research on State highways for the State of Alabama, be fulfilled with the same high intellectual integrity. He was the inventor of a type of highway pavement consisting of triangular slabs supported at three angle points.

During this period Professor Hooper was also engaged in extensive investigations. His research in the development of hydraulic theory was published in the *Journal* of the Society for the Promotion of Engineering Education. In the field of hydraulics he was eminently an authority. In addition, he was engaged in research, over a period of years, into the physical properties of concrete, in collaboration with Mr. Breed. These researches were published in the *Transactions* of the American Concrete Institute, and in the *Proceedings* of the Highway Research Board, National Research Council.

His pen was also in evidence in other matters. During the period of his connection with New York University, Professor Hooper was the author of papers in various technical magazines. Among these were discussions on highways, hydraulics, and kindred subjects in the publications of the Society, the American Society of Mechanical Engineers, and the American Water Works Association. One of his outstanding contributions was the editing of manuscripts of the monumental work, "Hydraulic Laboratory Practice", by the late John Ripley Freeman, Past-President and Hon. M. Am. Soc. C. E.

Professor Hooper had a remarkably keen, analytical mind. His work was characterized by thoroughness of research from every possible angle in order to make the solution of engineering problems brought to him as complete as human power could make it. Coupled with this ability was the highest standard of professional and personal ethics that won the admiration and appreciation of his associates on the Faculty and the sincerest respect of students and business associates. He was, moreover, of a kindly disposition, gladly helpful to any who came to him for assistance. He was a true friend to all his associates who found in him both sterling loyalty and an understanding devotion. His passing is a distinct and real loss to those who became accustomed to go to him for advice and friendly help.

Perhaps his qualifications as a man are best summed up in the words of his close associate, Mr. Breed:

> "It was Professor Hooper's character that made especially significant his achievements as an engineer. During the fifteen years that I worked with him, in the University and on many projects outside, the qualities that I came to recognize as essentially a part of him were his integrity and his consideration for others. Without prejudice or evasion he had worked through his personal problems to the place where he had effected a reconciliation with life and won a mellow understanding. Whatever he did was expressive of the whole of him, clear and lucid. In spite of years of fragile health, of grief and strain, he possessed to an exceptional degree that fine quality that the Romans described as 'integer vitae'.
>
> "This consideration for others seemed to grow out of his understanding of himself. He caught the vision of the whole, and no effort was too great for him to help others realize their part in it. In trifles, as in larger matters, he was sensitive to the rights and the desires of others.
>
> "His integrity was revealed in his work by his scrupulousness in regard to every detail. When he had approved a plan or an estimate, one could feel assured that every item in it was correct. When he decided on a course of work, one knew that every other course had been weighed and deemed inferior. Demanding rigorously of himself in work, he was tolerant of others, affable to diverse opinions, sympathetic with limitations. His alertness to perceive the worth and dignity of every man on a job secured for him willing co-operation. Because his work was basically sound, because his human relationships were finely wrought, he was distinguished as an engineer and as a man."

Professor Hooper's scholastic qualities were recognized by his election to various honor societies. He was a member of Phi Kappa Phi, Iota Alpha, Tau Beta Pi, and Sigma Xi. In addition, he was also a Member of the Society for the Promotion of Engineering Education, and of the American Association of University Professors.

On June 29, 1910, Professor Hooper was married to Alice Pote Wentworth, whom he survived by several years. Three children were born to this union, Louise Wentworth, Robert Frothingham, and Helen Maxim, who survive.

Professor Hooper was elected an Associate Member of the American Society of Civil Engineers on November 21, 1921, and a Member on July 5, 1933.

GEORGE LEONARD HOSMER, M. Am. Soc. C. E.[1]

DIED JANUARY 10, 1935

George Leonard Hosmer was born in Lynn, Mass., on December 31, 1874. His father, George Franklin Hosmer, was a descendant of James Hosmer who had settled in Concord, Mass., in 1635. His mother was Laura Caroline Tracy, the daughter of Cyrus Mason Tracy, a student, independent thinker, and philosopher of unusual talent.

After an uneventful period in the public schools of Lynn and Woburn, Mass., George Leonard Hosmer entered the Massachusetts Institute of Technology in the Class of 1897. For a decade following his term as an undergraduate in college, he was Assistant and Instructor in the Department of Civil Engineering at the Institute during the academic year, and spent the summer months in the practice of surveying. From the beginning, he combined the practice of his profession with his teaching career. By 1925, the Massachusetts Institute of Technology had appointed him to a full Professorship of Geodesy.

His work and his play carried Professor Hosmer into many divergent fields of activity and to different parts of the world: To Sumatra, in 1901, where he made astronomical and magnetic observations during a total eclipse of the sun; and to Labrador, in 1905, to make magnetic observations for the United States Coast and Geodetic Survey.

His trained eye and steady nerve gave him a fine background for developing into one of the excellent rifle and revolver shots of the State. He was a member of the Massachusetts Rifle Association and of the Boston Rifle and Revolver Club, of which he was at one time President.

He traced the records of the Hosmer family in America from James Hosmer, of Concord, Mass., and published a comprehensive genealogy of the family, a work of historic interest and value.

To the profession he was best known as an author of several authoritative texts. He wrote "Goedesy"; "Practical Astronomy"; "Navigation"; "Azimuth"; and "Principles and Practice of Surveying," the last in collaboration with Charles B. Breed, M. Am. Soc. C. E. Probably half the civil engineers in the United States have received their training in surveying through the study of his works.

For more than twenty years Professor Hosmer was in charge of the Technology Surveying Camp, in Maine. To his leadership its success may be largely attributed. There he established and operated a seismological station in co-operation with the U. S. Coast and Geodetic Survey.

He was a prolific reader in French, Spanish, Italian, German, as well as in his native tongue. From his mother, he may have inherited the habit of contemplation and of silence. His remarks were always the results of reflection and were worthy of the studious mind behind the thought.

---

[1] Memoir prepared by Charles B. Breed, M. Am. Soc. C. E.

Professor Hosmer was a member of the American Association for the Advancement of Science, the American Geophysical Society, the Seismological Society of America, the Society for the Promotion of Engineering Education, and the Boston Society of Civil Engineers. He was Chairman of the Committee of the Surveying and Mapping Division of the Society, on Definition of Surveying Terms, at the time of his death.

The Boy Scouts of America found in him a helpful companion and leader. He was a dependable supporter of the local Congregational Church, at Montvale, Mass. For three years he was on the School Board of Woburn and served as its Chairman in 1921.

Professor Hosmer was married on August 28, 1901, to Lucy Morse Harmon, of Randolph, Mass., who, with their three children, Chester Harmon Hosmer, Dr. Miriam Jewell Hosmer, and Elizabeth Tracy Hosmer, survives him.

Seldom does a man leave behind him such a depth of feeling in the hearts of his friends. Those who knew him well are grateful to have had the rare opportunity of close contact with his quiet nature, strong intellect, and his wholly genuine character.

Professor Hosmer was elected a Member of the American Society of Civil Engineers on April 20, 1925.

GEORGE EDWARD HOWE, M. Am. Soc. C. E.[1]

Died October 29, 1934

George Edward Howe was born at Waltham, Mass., on February 22, 1873, the eldest son of George and Harriet Howe. He studied Civil Engineering at the Massachusetts Institute of Technology, from which he was graduated, with the degree of Bachelor of Science, in June, 1895.

After six years in the employ of the Massachusetts Metropolitan Water Board, Distribution Department, Mr. Howe became an Assistant Engineer with the Hudson River Water Power Company, at Glens Falls, N. Y., and served as Resident Engineer in charge of line and grade work, estimates, and the design of miscellaneous details of the Spier Falls Dam and Power House, and the installation of the new water-wheels at the Mechanicsville, N. Y., Power House.

In 1903, Mr. Howe was engaged with the Commission on Additional Water Supply of New York City on field work and the design of filtration works. From 1904 to 1909, he was associated with John H. Gregory, M. Am.

---

[1] Memoir prepared by Langdon Pearse, M. Am. Soc. C. E.

Soc. C. E., on the improved water and sewerage works at Columbus, Ohio, on design. Later, he served as Resident Engineer in charge of the construction of the new water-works pumping station and purification works.

In 1911, Mr. Howe entered the flour milling business as a member of the firm of Lyon and Greenleaf, at Ligonier, Ind., serving in various capacities. He resided at Elkhart, Ind., until his death. He studied the milling business from a scientific basis, and became a member of the Cereal Chemists Association, but maintained an interest in engineering, serving as a member of the Water-Works Board at Elkhart. His interest in civil matters was wide; he served as an officer on boards connected with the Rotary Club, Boy Scouts, Community Chest, and the Elkhart General Hospital. He belonged to various Masonic Orders, including the Knights Templars and the Shrine.

Mr. Howe possessed an active, logical mind and good judgment, so that he was respected by his associates in the Engineering Profession and in business. As a public-spirited citizen, he participated whole-heartedly in the civic and social service activities of Elkhart. At the time of his death, he was President of the Elkhart General Hospital. Personally, Mr. Howe was a likable man, cheerful, genial, and friendly, and especially a favorite among young people.

In 1905, he was married to Alice M. Jennings, of Chicago, Ill., who survives him.

Mr. Howe was elected an Associate Member of the American Society of Civil Engineers on March 4, 1903, and a Member, on April 30, 1907.

GEORGE ARTHUR JOHNSON, M. Am. Soc. C. E.[1]

Died March 31, 1934

George Arthur Johnson, the son of Arthur and Lillian M. Johnson, was born at Auburn, Me., on May 26, 1874. He was educated in the public schools of Boston, Mass., including both the Boston Latin School and the English High School.

From the autum of 1895 until the end of 1898, Mr. Johnson was a member of the Technical Staffs of the Cities of Louisville, Ky., and Cincinnati, Ohio, where extensive investigations were conducted into methods of purifying the water of the Ohio River. The following year was devoted to tests of water filters at York, Pa., and Norfolk, Va., and in assisting in investigations of methods of purifying Ohio River water. In 1900, he was stationed at St. Louis, Mo., as Associate Director in conducting investigations on the influence of the Chicago Drainage Canal upon the local water supply. In 1901, he was engaged in Philadelphia, Pa., in conducting investigations into the filtration of local river water supplies. From June, 1902, until March, 1903, he was in local charge of the preliminary operations of the Filter Plant of the East Jersey Water Company, at Little Falls, N. J. The remainder of 1903 was spent as Field Director for the Burr-Hering-Freeman Commission in procuring data as to the quality of water of the Hudson River and its various tributaries in the general vicinity of Poughkeepsie and Kingston, N. Y.

---

[1] Memoir prepared by the late George Warren Fuller, M. Am. Soc. C. E., and by Harold C. Stevens, M. Am. Soc. C. E.

Beginning in May, 1904, Mr. Johnson was in charge of studies and tests on the purification of sewage and of the softening of the water supply for Columbus, Ohio, the results of which he recorded in an important report. He then spent a year on a trip around the world, to study the applicability of rapid filtration and coagulation in purifying water supplies in Japan, China, India, and Egypt.

From the autumn of 1906 until the autumn of 1910, Mr. Johnson was Principal Assistant Engineer and a member of the firm of Hering and Fuller, Consulting Engineers, of New York, N. Y. During this period he devoted himself to the operation of purification plants, particularly those at Reading, Pa., and Columbus, Ohio. His most notable work, however, was with reference to the application of hypochlorite of lime for the sterilization of water supplies of the stockyards at Chicago, Ill., and the Boonton Reservoir supply of Jersey City, N. J., which installations were the genesis of water sterilization by chlorine in the United States.

He then formed a partnership with the late William Barnard Fuller, M. Am. Soc. C. E., under the firm name of Johnson and Fuller. A short time later, Harold C. Stevens, M. Am. Soc. C. E., joined the firm which for several years continued to engage in work relative to projects for the purification of water supply and sewage.

In 1918, Mr. Johnson entered the military service of the United States, with the rank of Major in the Construction Division and, successively, became Lieutenant-Colonel and Colonel. He directed the technical management of all fixed properties and utilities in all the military establishments under the jurisdiction of the War Department in Continental United States, Alaska, Canal Zone, Hawaii, and The Philippines.

Colonel Johnson returned to private practice in New York City, in March, 1920, and was associated, in partnership, with Webster L. Benham, M. Am. Soc. C. E., and Mr. Harold C. Stevens, which partnership was succeeded by incorporation as the George A. Johnson Company. After 1924, he continued in consulting practice by himself.

His work in his special field brought him engagements from more than ninety cities, water companies, and State departments. These included a number of sizable cities for which he served as Adviser, or as a member of Advisory Boards. Mention may be appropriately made of the water purification works at Cambridge, Mass., and at Trenton, N. J.; the complete water supply system for the Manhasset-Lakeville Water District, Long Island, N. Y.; soil investigations for the Park Department, at Boston, Mass.; the sewage treatment works at Trenton, N. J., and Norwalk, Conn., and complete sewerage systems at Glen Cove, N. Y., and Matawan, N. J.; investigations and design of sewage treatment works, at Jersey City, N. J., Middletown, N. Y., and Bayonne, N. J.; services in connection with sewerage problems at Ridgewood, N. J., Lima, Ohio, New Rochelle, N. Y., Columbus, Ohio, Bridgeport, Conn., Reading and Allentown, Pa.; and services for the Bronx Valley, New York, and the Passaic Valley, New Jersey, Trunk Sewer Commissions, and for New York City.

He also served as Consulting Engineer to the State Conservation Commission of New York on shellfish pollution, and devoted considerable time to the investigation of garbage incineration and various mechanical means for clarifying sewage and for dewatering sewage and sewage sludge.

Colonel Johnson was the author of numerous technical papers and, in addition to his report on the Columbus, Ohio, tests, mention may be made of the "Romance of Water Storage", "The Typhoid Toll", "Water Filtration and Sterilization", "Municipal Water Works in The Far East", "Pure and Wholesome Water", "Purification of Public Water Supplies", "Present-Day Water Purification", etc.

He was an intensive reader of both technical and non-technical subjects, particularly those relating to travel and the history of various explorations beginning with the earliest appearance of Europeans in America. He was a member of the Explorers Club and the Circumnavigators Club. He was Treasurer and Editor of the latter club for many years, and served as its President during 1933. His contributions to *The Log* (a monthly publication of the Circumnavigators Club) attracted wide attention in various parts of the world, and several groups of his articles were published in book form.

Colonel Johnson was a devoted son and husband, generous to a fault. He was a man of marked and likeable personality, keen in his appreciation of human relations, and aggressive in advancing his views both on technical and non-technical subjects. At the same time, he had remarkable ability, both in speaking and in writing, for conciliating those holding opposite views. His achievements in advancing the cause of typhoid fever reduction, through the elimination of objectionable bacteria by means of chlorination, constitute one of the high spots of water supply engineering during the past generation.

He was an active worker in the New York Post of the American Legion and was its Commander for several years. He was a member of the Masonic Order and of the Benevolent and Protective Order of Elks.

Colonel Johnson died on March 31, 1934, of a heart attack while at work at his desk in the early evening, when he was alone in his office. Funeral services were conducted at his residence by the Masonic Order and by the New York Post of the American Legion, followed by a military funeral in Arlington National Cemetery where he was buried.

He is survived by his mother who resides in Southern California, and by his widow, the former Katherine Daubenspeck, of Allentown, Pa., to whom he was married in 1914. They had no children.

Colonel Johnson was elected an Associate Member of the American Society of Civil Engineers on February 6, 1907, and a Member on July 2, 1913.

GUY ROBERT JOHNSTON, M. Am. Soc. C. E.[1]

Died January 27, 1935

Guy Robert Johnston was born in Marianna, Ark., on January 30, 1886. He attended school at Marianna, but early in life found it necessary to go to work without completing his education. His first employment was as an Axeman and Rodman for the St. Louis, Iron Mountain and Southern Railroad Company, he having begun work in this capacity in July, 1902, at the age of sixteen.

From January, 1903, until September, 1905, Mr. Johnston served as Rodman and Instrumentman on drainage and land-boundary work with his father and this employment was immediately followed by service as Assistant City Engineer at Helena, Ark. Beginning in April, 1907, and until November, 1911, Mr. Johnston served as City Engineer and Plumbing Inspector in Marianna, and, at the same time, was engaged in private practice in drainage surveying and design. From December, 1911 to April, 1912, he was Foreman for the Midland Bridge Company on the construction of a steel highway bridge over the Brazos River, at West Columbia, Tex. He then spent five months as Assistant to the County Surveyor in Harris County, Texas, on roads, drainage, and subdivision—this work lasting until August, 1912.

From January to April, 1913, Mr. Johnston was Chief of Party of the State Highway Department of Arizona on bridge location. From August to December, 1913, he was Instrumentman for the Southern Pacific Railroad Company, and this employment was immediately followed by service with the firm of Marshall and Foss, on highway construction in the Santa Cruz Mountains, in California.

From January, 1915, to April, 1919, Mr. Johnston was with the United States Army Engineers—three years as Inspector and Chief of Party on concrete fortification construction and topographic surveys in the Philippine Islands, and one and one-sixth years at Washington Barracks, District of Columbia, training recruits and acting as Instructor of Surveying and Drafting in the United States Army Trade School.

While in the Philippine Islands, from January, 1915, to January, 1918, his service was as follows: Several months as Instrumentman and Chief of Party locating fire-control stations for the Coast Artillery, also anti-aircraft and machine gun emplacements; five months as Instrumentman and Concrete Inspector of mortar battery emplacements at Caballio Island; Chief of Party for three months making a survey of two proposed Army posts on

---

[1] **Memoir prepared by Gibb Gilchrist, M. Am. Soc. C. E.**

Betan Peninsula; and Instructor of Topographic Surveying for some time at Fort McKinley and at Fort Mills.

After a few months in private engineering practice at Desdemona, Tex., Mr. Johnston, in 1920, started his career on Texas highways that was to be his principal work. He began as an Assistant to the County Engineer of Tarrant County, Texas, on location and design of the State highways in that county. This work lasted until November, 1922, and for six months after the expiration of that period, Mr. Johnston was Superintendent of Construction with J. F. Hamer, General Contractor, of Amarillo, Tex. In April, 1923, he took charge of some work in Hill County, Texas, for the State Highway Department, but was transferred in January, 1924, to Jack County, where he lived until 1927, constructing a system of highways in that county during that time. From 1927 to 1929, Mr. Johnston was in charge of highway construction in Young and Archer Counties, Texas, being transferred in February, 1930, to Lubbock, Tex., where he served for two years as Resident Engineer, and for the three years preceding his death, as Division Engineer of Division No. 5, with headquarters at Lubbock. The crowning work of his career was at Lubbock, where he designed and planned, for the State Highway Department, a system of State highways that he did not live to see completed, but which will be constructed along the sound and progressive lines that he used.

Mr. Johnston's life was an inspiration. His technical ability shows what can be accomplished with determination. To those who have believed that a college education is essential to success and to efficiency, Mr. Johnston's technical life and ability should act as an inspiration and should lead those who have such a foundation to bigger and better things.

Guy Johnston was one of the ablest engineers in the Texas Highway Department and when it is remembered that his training came in the field of experience without the advantages that some engineers have, it is all the more creditable and outstanding.

Added to his technical ability, which was always, of course, in the forefront, he had a remarkable sense of public relations and had inspired the confidence of all worthy people with whom he associated or came in contact. He was a zealot in his loyalty to his employers, but not to the point of subservience, and could be depended upon at all time to give a correct analysis of any problems that might be on his mind or on those of his superiors. Mr. Johnston inspired in those working under him a loyalty and an affection seldom seen in public life. The result of this one man's ability was an organization second to none in the State of Texas. His passing is not only a loss to the immediate section in which he lived, but to the State Highway Department and to the State of Texas.

He is survived by his widow, Mrs. Charlotte (Tuttle) Johnston, and three children, Marjory, Guy Robert, Jr., and Charlotte; by his mother, Mrs. Sally B. Johnston, of Fort Worth, Tex.; and by two brothers and one sister.

Mr. Johnston was elected an Associate Member of the American Society of Civil Engineers on July 11, 1921, and a Member on April 23, 1928.

MAURICE EDWIN KERNOT, M. Am. Soc. C. E.[1]

Died January 13, 1934

Maurice Edwin Kernot was born at Geelong, Victoria, Australia, on June 10, 1852, the son of Charles Kernot, M. P., and Mary Wright (Archer) Kernot. The history of the Kernot family shows connection with the French Huguenots, who migrated to England to escape religious persecution, following the revocation of the "Edict of Nantes," in 1685. The name is a variant of the widely known French surname, "Carnot," which occupies a prominent place in the annals of France. Nicolas Léonard Sadi Carnot (1796–1832) has lasting scientific fame as the originator of the Theory of the Heat Engine (Carnot Cycle) and, in later years, Marie François Sadi Carnot (1837–1894) has been President of the French Republic.

On September 10, 1850, Charles Kernot, who had carried on his father's business as a chemist, migrated from Rochford, Essex, England, at the age of 34 years, leaving London in the sailing ship, *Duke of Wellington,* 600 tons (Captain Miller), with his wife and two children, and arrived in Victoria, after a voyage of ninety days. He finally settled in Geelong, practising as a chemist, and representing the District in the Legislative Assembly until his death, in 1882.

It was in this town that Maurice Edwin Kernot was born. He received his primary education at the Gheringhap Street School, and, later, at Melbourne University, in Melbourne, Victoria.

He began his career with the Victorian Water Supply Department in 1869, transferred to the Mines Department in 1871, and to the Railways Department in 1874, commencing work at Kyneton, in the office of the late W. H. Greene, then Resident Engineer, the Engineer-in-Chief at that time being the late Thomas Higinbotham. After eighteen months at Kyneton, Mr. Kernot was assigned to construction work on the Sandhurst to Inglewood Railway. He remained in the field on construction and surveys until 1888, when, during the "boom" period, he was called to Melbourne, and, as District Engineer, supervised contracts for fourteen new lines in the Western District.

Mr. Kernot was the first to apply the principles of technical analysis to railway location in Victoria, the economic results having a marked influence in reducing the total of capital costs and working expenses. The year, 1880, marked the first location carried out under these principles. When the collapse of the "boom" came in 1892, he took charge of the newly introduced method of construction, known for many years as the "Butty Gang," and, more recently, as the "Direct Labor" System, under which the letting of large contracts was abandoned, and the work was carried out by small direct labor contracts, piecework, and day labor. Under his direction, large savings were effected thereby.

---

[1] Memoir prepared by W. D. Chapman, Esq., Melbourne, Victoria, Australia, and Charles H. Kernot, M. Am. Soc. C. E.

In 1903, Mr. Kernot took charge of the Railway Construction Branch of the Victoria Railways Department as Acting Engineer in Chief. He was appointed Chief Engineer in 1907, and remained in that position until his retirement, in 1923. During the forty-nine years that he was associated with the Railways Department, approximately one-half the present railway mileage in Victoria was constructed, and during his period as Chief Engineer, 1 072 miles of railway were built at the remarkably low cost of £4 000 000, although this construction included many works of an important nature.

During his career, Mr. Kernot sat on various important Commissions: From 1903 to 1905, he was a member of the Conference of State Engineers in Chief on the Transcontinental Railway, to connect Adelaide, in South Australia, with Perth in Western Australia; and in 1908 and 1909, he was Chairman of a Royal Commission on the Sydney, New South Wales, Harbor Bridge or Tunnels. In 1914, he was sent by the Government on a mission to investigate railway practice in Europe and America.

On his retirement in August, 1923, being then 71 years of age, Mr. Kernot ceased active work in his profession, but took a live interest in the affairs of the Institution of Engineers of Australia. He had been associated with its establishment, being a Foundation Member and the first Chairman of the Melbourne Division. He was also a member of the Council of the Institution of Civil Engineers (London, England), and of the Faculty of Engineering of the University of Melbourne.

In 1928, Mr. Kernot was awarded the Kernot Memorial Medal (founded by his eldest brother, the late William Charles Kernot, M. Am. Soc. C. E.), and, in 1933, the Peter Nicol Russell Memorial Medal, which is the highest award of the Institution of Engineers of Australia.

In private life, he spent much of his leisure in church and philanthropic work. He was an Elder of the Presbyterian Church of Victoria for thirty-eight years, and took an active interest in its work to within a few days of his death.

His outstanding personal attributes were a fine sense of honor and the simplest of tastes, eschewing luxury as a weakness, and his methodical and business-like methods were brought to bear on all the public and private activities with which he was associated.

In 1882, Mr. Kernot was married to Caroline Grace Home, and is survived by his widow, a daughter, and a son, Charles H. Kernot, M. Am. Soc. C. E.

His brother, the late Professor William Charles Kernot, was practically the founder of the Engineering School at Melbourne University, and another brother, Professor W. N. Kernot, is Dean of the Faculty of Engineering at that University. Two other brothers, the late Mr. F. A. Kernot, and Mr. P. W. Kernot were associated with the Dental and Architectural Professions.

Mr. Kernot was elected a Member of the American Society of Civil Engineers on June 5, 1907.

VICTOR HUGO KRIEGSHABER, M. Am. Soc. C. E.[1]

Died November 10, 1934

Victor Hugo Kriegshaber, the son of William Kriegshaber and Regina (Sommers) Kriegshaber, was born in Louisville, Ky., on March 27, 1859.

His father came to America with Carl Schurz and his band of German revolutionists, exiled from Prussia on account of their activities in the Revolution of 1848. After a ninety-day boat trip to New Orleans, La., and an additional thirty-day journey to Kentucky, William Kriegshaber established himself as a citizen of the United States, in Louisville, and became a successful agent of general insurance.

Victor Hugo Kriegshaber was educated in private schools in Louisville until the age of sixteen. His early childhood was marked by the Civil War, and he witnessed the fiery cavalry dash of Morgan's Raiders through Bowling Green, Ky., on their way to Indiana. Later, he attended the Hailsman German-English Academy and was sent abroad to complete his education in the University Preparatory at Frankfort-on-the-Main and in the University of Darmstadt, in Germany, in which institutions of learning he remained from 1875 to 1880. At his graduation, he received the degree of Civil Engineer, a profession he afterward followed with success.

He returned to Louisville and was employed in the City Engineering Department. Being ambitious, Mr. Kriegshaber sought an outlet for his energies and talents, and secured a position in the Construction Department of the Louisville and Nashville Railroad Company, as Assistant Engineer and Draftsman, in which capacities he was connected with the building of the line to Livingston and Jellico, Tenn. Capable service in this first work led to employment by the Central of Georgia Railway Company which he served as Engineer in Charge of Maintenance of Way, and, later, as Assistant to the General Manager, the late Morris Sheppard Belknap, M. Am. Soc. C. E., with headquarters in Savannah, Ga.

He remained with the Central of Georgia Railway Company until 1889, when he decided to make his home in Atlanta. He became a Contractor, but having vision he saw the possibilities of the growing city and, one year later, established a building material business of which he was President for forty-three years. As a citizen of the growing City of Atlanta, he kept pace with its expansion, and his building material business enjoyed substantial success.

Mr. Kriegshaber was not purely a man of business, he enjoyed participation in the civic, philanthropic, and artistic life of the city. He became one of the founders and early Presidents of the Atlanta Terra Cotta Company and, in 1904, served as Secretary of the Atlanta Art Glass Company. Due to the rapid growth of the building material business in the South,

---

[1] Memoir prepared from information supplied by Mr. Kriegshaber's family and on file at the Headquarters of the Society.

Mr. Kriegshaber, interested in expanding this field for all dealers, was active in the National Builders' Supply Association and served as Vice-President. Following this work, he organized the South Eastern Building Supply Association, and was its first President.

Having a high sense of civic pride and obligation he looked for a channel of service and found in the Atlanta Associated Charities, an instrument worthy of his desires. As a Director of this organization he was instrumental in securing the first public playgrounds for children in the city. Later, he became the Vice-President of the Playground Association of America.

Combining an unusual degree of executive ability and a great humanitarian outlook on life, Mr. Kriegshaber interested himself for many years in the charitable work of Atlanta. His field of activity was large. In 1909, he became President of the Federation of Jewish Charities and, in 1911, served also as President of the Jewish Educational Alliance. This organization then newly housed in a modern building filled a great need, both educational and social, for the immigrant and orthodox Jewish settlement of the city.

Closest to his heart, however, were the orphaned children of the South, and after many years' affiliation with the Hebrew Orphans Home, he became its President in 1915. Under the guidance of Mr. Kriegshaber and the able Superintendent of the Home, Mr. Ralph Sonn, was established the "Mothers' Pension System," which enabled the Orphan Home to provide a subsidy to keep the child with its mother and prevent the breaking up of homes. This system, a pioneer in its field, has been widely used in other orphanages, as it gives the child a normal home life rather than institutional rearing.

In the same line of work, Mr. Kriegshaber, as a Director of the Atlanta Chamber of Commerce, in 1912, inaugurated medical examination of all the pupils in the public schools of Atlanta. He served also as a Life Director of the Local Council of Boy Scouts of America.

In 1913, the Atlanta Rotary Club was organized, and Mr. Kriegshaber was one of its Charter Members. During the World War, he did commendable and outstanding work as Chairman of the War Camp Community Service, to which post he was appointed by the Mayor of Atlanta.

In 1916 he was elected President of the Atlanta Chamber of Commerce and during his régime founded the *City Builder,* a monthly magazine of the Chamber, which still functions. An anti-wood shingle ordinance for the city was passed, and $500 000 was raised to bring Emory University to Atlanta.

Victor Kriegshaber was a pioneer in the true sense of the word. He came to Atlanta when it was feeling its "growing pains"; he was among those who shaped its growth and nurtured its resources and set it on the path of leadership. With every step of his personal success, he expanded his interest in and contributions to his community. He assumed more and more outside obligations, discharging all of them with a great-hearted competence that not only filled the needs of the moment, but effected lasting benefits to the cultural and civic life of the city.

In 1913, the Atlanta Music Festival Association was organized which brought to the city the great singers of the Metropolitan Opera Company, of New York, N. Y. Mr. Kriegshaber was a Director of this organization from its inception, and served for the twenty seasons of grand opera in Atlanta. He was instrumental later in bringing to the city several seasons of light opera. He was intensely interested in music and felt that, in its local artists, Atlanta had splendid material for a Symphony Orchestra and Choral Society. With the able assistance of the city's finest musicians, the Atlanta Philharmonic Society was organized of which Mr. Kriegshaber was President from its inception to the time of his death.

His love for and interest in the profession for which he was prepared, led him to membership in the Society. He was also a member of the Yaarab Temple of the Shrine, and of the Executive Committee of the Presidents' Club, an organization composed of the presidents of all the civic organizations of Atlanta. He was also a member of "The Temple"; the Tau Epsilon Phi Fraternity (Honorary Life President); and the B'nai B'rith, a fraternal organization devoted to welfare work.

His life's work was broad in scope and far-reaching in the beneficient results of his interests. He embodied in character the ability for leadership directed toward the highest and finest things of life and left a rare heritage of good works.

On November 10, 1934, after a brief illness, due to a heart attack, Mr. Kriegshaber passed to his eternal rest.

He made two successful and happy marriages. His first marriage was with Blanche Lieberman, who taught in the public schools of Atlanta and was a member of one of the most widely known families of the South. She died in 1902. His second wife was Adeline Mayer, of Cleveland, Ohio, a woman of high attainments in the field of social service work. They had twenty-eight years of happy married life. Two children were born to him by his first wife, William Victor and Marian Kriegshaber, who, with his widow, survive him.

Mr. Kriegshaber was elected a Member of the American Society of Civil Engineers on April 2, 1913.

OLIVER HOWARD LANG, M. Am. Soc. C. E.[1]

DIED AUGUST 25, 1934

Oliver Howard Lang was born in Waverly, Ga., on June 1, 1886, the son of Robert and Claudia (Ward) Lang. He received his preparatory education at South Georgia College, at McRae, Ga., and in 1906 he entered the Georgia School of Technology, at Atlanta, Ga., from which he was graduated with honor in 1910, with the degree of Bachelor of Science in Civil Engineering.

---

[1] Memoir prepared by John C. Parker, Esq., Moultrie, Ga.

From 1910 to 1912, Mr. Lang was employed as Assistant Professor of Experimental Engineering at the Georgia School of Technology. He then removed to Colquitt County, Georgia, and was made County Surveyor and County Engineer. In 1914, he was elected City Engineer of Moultrie, Ga., and, in 1915, he served in the same capacity for Quitman, Ga. In this position he was engaged on street paving, the construction of sanitary and storm sewers, and the regular engineering work connected with that office.

In 1916, Mr. Lang engaged in private consulting work with an office in Moultrie, which he continued until his death. In the course of his practice, he was employed on the design and construction of street paving, sewers, and bridges in the States of Georgia, Alabama, Florida, Mississippi, and Louisiana, and in South America.

In addition to his private practice Mr. Lang had also served as Consulting Engineer for Worth and Colquitt Counties, Georgia, on roads and bridges from 1920 until his death.

Always patient, kind, and unassuming, always charitable, pleasing, and public-spirited, Mr. Lang endeared himself to a host of friends and to the general public who knew him. Professionally, the results of his labor are the greatest monuments to his ability. His keen mind and thorough knowledge, coupled with his untiring energy, made for him a place high in the ranks of his profession.

He was married on January 11, 1913, to Murelle Culpepper, of Moultrie, and is survived by his widow and one son, Oliver Howard Lang, Jr.

Mr. Lang was elected a Member of the American Society of Civil Engineers on January 17, 1927.

WILLIAM CAMPBELL LANGFITT, M. Am. Soc. C. E.[1]

DIED APRIL 20, 1934

William Campbell Langfitt was born at Wellsburg, W. Va., on August 10, 1860, the son of Obadiah Wallingford and Virginia (Tarr) Langfitt. After preliminary schooling, he studied at Ohio State University, Columbus, Ohio, and then entered the United States Military Academy at West Point, N. Y., on July 1, 1878. He was graduated from West Point on June 13, 1883, standing second in a class of fifty-two.

Cadet Langfitt was commissioned a Second Lieutenant in the Corps of Engineers upon graduation, and assigned to station at Willet's Point, N. Y., for duty with the Battalion of Engineers, and to attend the Engineer School of Application, from which he was graduated in December, 1886. He rose steadily through the various grades, reaching that of Colonel on October 12, 1913. On August 5, 1917, he was appointed a Brigadier General, National

---

[1] Memoir prepared by Lt. Col. F. B. Wilby, Corps of Engrs., U. S. Army, Washington, D. C.

Army, and on December 17, 1917, a Major General. On May 31, 1920, he was retired at his own request after more than forty years of most distinguished service.

Upon graduation from the Engineer School of Application, Lieutenant Langfitt was assigned to duty as Chief Engineer, Department of the Columbia, where he remained until September, 1888. At this time, he was transferred to Galveston, Tex. Here, he became the Principal Assistant to the District Engineer and was actively engaged in the river and harbor and fortification work of the District. During this period of service he became particularly interested in the tides of Galveston Bay and conducted an extensive study of their phases in relation to the declination of the moon. His reports in this connection give early evidence of his professional ability and keen mind. In August, 1893, he was ordered to the Cincinnati (Ohio) District and became involved, for about two years, in the problems of improvement of the Ohio River and its tributaries. During his tour of duty at Cincinnati he was promoted to the grade of Captain.

In September, 1895, Captain Langfitt was transferred back to Willet's Point, N. Y., to command Company B of the Battalion of Engineers and to be an Instructor of Electrical Engineering in the Engineer School of Application. He remained at Willet's Point until May, 1898, at which time he was commissioned a Major in the Second United States Engineers, and ordered to Hawaii to assume charge of all Government engineering work, as well as to take command of the Engineer troops. In December, 1898, he became Commanding Officer of all United States forces in the Hawaiian Islands.

A six-year tour of duty, in charge of the Portland (Ore) River and Harbor District and the defenses of the Columbia River, followed his service in Hawaii. During four years of this period (July, 1899, to August, 1905), he was also Engineer of the Thirteenth Lighthouse District. While on this duty he planned and constructed the first United States aids to navigation in Alaskan waters.

From August to November, 1905, Major Langfitt was temporarily in charge of the River and Harbor Division of the Office of the Chief of Engineers, Washington, D. C. He was then assigned as Instructor in Civil Engineering at the Engineer Post-Graduate School at Washington Barracks, D. C. With the exception of the period from October, 1906, to June, 1907, he was continuously on duty at that station until May, 1910. He commanded the Post, the Engineer Depot, and the Engineer School; and was in charge of building construction on the Post from May to October, 1906, and from June, 1907, to May, 1910. He was Chairman of the Board on Engineer Equipment of troops from June to October, 1906. During the period, October, 1906, to June, 1907, Major Langfitt was Chief Engineer, Army of Cuban Pacification. Under his direction, and stimulated by his energy and leadership, a reconnaissance survey of the entire Island of Cuba (40 000 sq. miles) was completed in less than eight months. He was promoted to the grade of Lieutenant Colonel in June, 1908.

Following a short period of duty as Engineer Secretary of the Lighthouse Board, Lt. Col. Langfitt became District Engineer, at Washington,

D. C., and, as such, was in charge of the rivers, harbors, and fortifications of the District, and the water supply of Washington. As District Engineer in Washington he prepared and submitted the first report on the development of water power at the Great Falls of the Potomac in conjunction with increasing the water supply for the District of Columbia. During this tour of duty, he was promoted to the grade of Colonel. He remained as District Engineer at Washington until January, 1914, at which time he was transferred to duty as Division Engineer, Southeast Division, and as Engineer in Charge of the Savannah (Ga.) District.

In June, 1916, Colonel Langfitt was moved to San Antonio, Tex., to become Chief Engineer, Southern Department, and during the year in which he held this position, he was engaged particularly in railway planning in connection with the military operations in Northern Mexico.

Soon after the entry of the United States into the World War, Colonel Langfitt was ordered to Chicago, Ill., to organize the Thirteenth Engineer Regiment (Railway), and to command that organization. He reported to Chicago in May, and sailed for overseas in command of a well-trained and equipped regiment on July 23, 1917.

Colonel Langfitt received his promotion to Brigadier-General, National Army, soon after his arrival in France, and was detached from the Thirteenth Engineers to assume the duties of Chief of Staff, Line of Communications. New assignments of increased importance and involving increased responsibilities followed rapidly. On September 29, 1917, he became Manager of Light Railways under the Director General of Transportation; on October 15, he was assigned command of all American troops with the British Armies; his promotion to Major-General came in December, 1917. The forces under his command were among the first American troops to participate in actual fighting and to receive battle casualties; they were active in checking the Cambrai Drive in late November. General Langfitt's courage and leadership furnished a fine incentive to the officers and men under his command. He was cool under fire; and on one occasion saved the lives of two wounded "Tommies" by organizing their evacuation and personally assisting in carrying them to an aid station under heavy shell fire and at grave personal risk. With his characteristic modesty, he insisted that the incident should not be mentioned.

The ever-growing American Forces, with the accompanying large-scale engineering problems, resulted in his relief from his duties with the British Army, in March, 1918, and his assignment as Chief of Utilities, Service of Supply. He served in this capacity at General Headquarters, American Expeditionary Forces, in charge of Construction and Forestry, the Division of Light Railways and Roads, and of the Transportation and Motor Transportation Department. In July, 1918, he was designated Chief Engineer, American Expeditionary Forces. This assignment involved responsibility for the efficient prosecution of the work of the Divisions of Military Engineering and Engineer Supplies, of Construction and Forestry, and of Light Railways and Roads. He continued as Chief Engineer, American Expeditionary Forces, until his return to the United States in July, 1919.

General Langfitt's success in his high position was in no small part due to his complete knowledge of the details of the work being performed under him. He kept in constant touch with engineer operations by personal visits to the front and to other places of activity. On one occasion so close was he to the advancing troops that he ate a hot meal from a captured German kitchen in the Hinderburg line. His duties with the American Expeditionary Forces were performed uniformly with marked ability, and resulted in his being awarded decorations by various Foreign Governments as well as by his own. He received the decorations of Commander of the Legion of Honor (France); Companion of the Most Honorable Order of the Bath (England); Commander of the Order of the Crown (Belgium); and the Distinguished Service Medal (United States). The citation in the award of the latter decoration reads as follows:

"As Director of Light Railways and Roads, and, later as Chief of Utilities, he displayed great ability and marked breadth of vision. As Chief Engineer of the American Expeditionary Forces, his brilliant professional attainments, untiring energy and devotion to duty, placed his department in a state of efficiency and enabled it to perform its important function in the most satisfactory manner."

The following letter from General Pershing, is clearly indicative of the high degree of performance of the Engineer Department, American Expeditionary Forces, under its Chief Engineer:

"20 February, 1919

"MAJOR GENERAL WM. C. LANGFITT,
Chief Engineer, A.E.F.

"MY DEAR GENERAL LANGFITT:

"As the activities of our army in France draw to a close, I desire to express to you, and through you to the officers, enlisted men and civilian personnel of the Engineer Department, my appreciation of their loyal and energetic work, which contributed so greatly to our success.

"The various units attached to combat troops distinguished themselves at all times in the assistance which they rendered. The Division of Construction and Forestry, with limited resources at its disposal and under conditions of extreme severity, more than met the many demands made upon it. The Department of Light Railways and Roads furnished the indispensable link between the railheads and the front lines for the transportation of troops and supplies, and for the evacuation of sick and wounded. Its record in the construction and operation of light railways and roads has seldom been equalled.

"The many other services of the Engineer Department, connected with the acquisition and distribution of engineer supplies, particularly those needed for combat operations, were so conducted that our forces never lacked for any essential.

"The Engineer Department has made a proud record for itself, and it gives me pleasure to express to you my sincere thanks and admiration, and that of your comrades of the American Expeditionary Forces, for its splendid achievements.

"Sincerely yours,
JOHN J. PERSHING."

In August, 1919, General Langfitt reverted to his former rank of Colonel and assumed charge of the New York and Puerto Rico Districts, as well as the duties of Division Engineer, Northeast Division; he was also responsible for the supervision of construction and repairs within the Third and Ninth Lighthouse Districts. He remained on this duty until he was retired at his own request on May 31, 1920.

During his military career, General Langfitt was for many years a member of the Board of Engineers for Rivers and Harbors as well as a member of many Boards which conducted studies of engineering importance. He was a member of Special Boards in connection with the dam and reservoirs in the vicinity of Box Canyon, on the San Carlos Indian Reservation, in Arizona; the improvement of the Congaree River, in South Carolina; the Cape Fear River above Wilmington, N. C.; the Tennessee River, at Muscle Shoals, Ala.; the Tug and Levisa Forks of the Big Sandy River, in Kentucky and West Virginia; the inland waterway from Norfolk, Va., to Beaufort Inlet, N. C.; the East Coast Canal from the St. Johns River to Key West, Fla.; the Southwest Pass, Mississippi River; Boston Harbor Line; New York Harbor Line; and the construction of a mail tunnel in New York City.

Following his retirement in 1920, General Langfitt became associated with David Belais, Incorporated, New York City, and served as General Manager, Secretary, and Treasurer of the firm until 1926. From 1927 to 1929, he was Consulting Engineer on the Coolidge Dam, in Arizona.

General Langfitt was a soldier and leader of exceptional ability, an engineer well versed in his profession. He was genial, frank, loyal, and always loved and respected by the officers and men who served under him.

One of his outstanding characteristics was the ability to arrive quickly at the essentials of a proposition and to separate the wheat from the chaff in discussions. The unanimity with which his decisions were accepted as being right when once made was a tribute to his unerring judgment.

Another of his splendid qualifications was his innate courtesy and tact. As Chief Engineer of important activities in the American Expeditionary Forces, he was brought in contact with thousands of engineers from civil life—men without any previous military training, or experience—but men outstanding in their profession and occupying places of great responsibility in the United States. To induct men of this class into the requirements of military service in positions of subordinate rank, without friction, and to elicit their enthusiastic co-operation and admiration was one of his fine accomplishments.

Men in General Langfitt's Department soon realized they were being led by a commander superb in organization, and their association with him remains one of the pleasant memories of their service in France.

General Langfitt died in Washington, D. C., on April 20, 1934, and was buried in Arlington Cemetery with full military honors.

He was married in Flushing, Long Island, N. Y., in 1886, to Anne St. John Bemis. He is survived by his widow and a daughter, Dorothy Langfitt Wilby, the wife of Lt. Col. Francis B. Wilby, Corps of Engineers, U. S. Army.

He was a member of the American Association for the Advancement of Science, and was President of the Society of American Military Engineers. He was at one time Commander of the New York Chapter of the Military Order of the World War; and was also a member of the American Legion, the Chevy Chase Club, and the Army and Navy Club, of Washington, D. C.

General Langfitt was elected a Member of the American Society of Civil Engineers on June 6, 1921.

JAMES ROBINSON McCLINTOCK, M. Am. Soc. C. E.[1]

Died April 11, 1933

James Robinson McClintock was born in Malden, Mass., on February 5, 1883, the son of the late J. Y. McClintock, M. Am. Soc. C. E., and Mary (Robinson) McClintock. His father was for many years a Construction Engineer with the predecessor gas companies of the United Gas Improvement group; City Engineer and Commissioner of Public Works of Rochester, N. Y.; and, later, County Engineer of Monroe County, New York State. An uncle, the late William Edward McClintock, M. Am. Soc. C. E., was for many years Chairman of the Highway Commission of Massachusetts.

After his graduation from the High School, at Rochester, N. Y., in 1901, Mr. McClintock was a student at the University of Rochester. In 1902 he entered the Massachusetts Institute of Technology, at Boston, Mass., from which he was graduated in 1905, with the degree of Bachelor of Science.

His entire professional career was spent with the firms of Hering and Fuller, George W. Fuller, and Fuller and McClintock which was organized in 1916. In his earlier practice Mr. McClintock served as Resident Engineer, for the firm of Hering and Fuller, on pumping station and filtration projects at Burlington, Vt., Clarksburg, W. Va., and elsewhere. In 1909, he also designed filtration plants for Burlington, N. J., and Rock Island, Ill., and sewage disposal works for the Monroe County Tuberculosis Hospital, at Rochester.

The major portion of his professional career as a member of the firms of George W. Fuller, Hydraulic and Sanitary Engineers, and Fuller and McClintock, was devoted to design work and valuation appraisals in the fields of water supply, water purification, sewerage, and sewage disposal. He was in charge of the Toledo Office of Fuller and McClintock, directing design work for the Cities of Toledo and Lima, Ohio, and for several Florida projects. Most of Mr. McClintock's time from 1921 to 1925 was spent at Kansas City, Mo., in charge of the design of the new water supply works of that city. His most recent and best known work was in connection with the design of the sewage treatment project at Wards Island, in the East River, for the City of New York, of which he had immediate charge.

---

[1] Memoir prepared by the late George Warren Fuller, M. Am. Soc. C. E.

Mr. McClintock was a thorough-going student of engineering projects and had an unusually clear vision as to how work should be assembled in order to serve the objectives sought. When he had visualized a program to his satisfaction, it was with extraordinary energy that he made it of record and, on more than one occasion, he overtaxed the nervous energy which was required for the carrying on of his work in his characteristic way. His colleagues had a high appreciation of his technical skill as an Engineer.

He was much interested in the artistic side of life, and while he was still a High School student at Rochester, he received a first prize for the design of a school building. He was a man of great simplicity of character and friendliness, and all those who knew him admired him for these qualities and regret his passing in the prime of life.

Mr. McClintock was married in 1914, to Mary Musgrave, of Clarksburg, W. Va., who died in 1916.

He was a member of the American Society of Mechanical Engineers, the American Institute of Consulting Engineers, the American Water Works Association, and the American Public Health Association.

Mr. McClintock was elected a Junior of the American Society of Civil Engineers on October 2, 1906; an Associate Member, on May 3, 1910; and a Member, on June 24, 1914.

ARCHIBALD McLEAN, M. Am. Soc. C. E.[1]

Died January 12, 1935

Archibald McLean, the son of Archibald and Marian (Hamilton), McLean, was born in Greenock, Scotland, on August 3, 1856. His preliminary education was obtained in Highlanders Academy, at Greenock, and at the age of fourteen he became indentured with an architect and builder.

From July, 1875, to December, 1880, Mr. McLean had active charge (for the same builder) of the erection of masonry and carpentry portions of buildings, chiefly hospitals, schools, and stone villas in towns in England and Scotland. During this entire period he was a student in the evening classes of the Government Science and Art Department of Great Britain, taking many prizes in his work.

From December, 1880, to September, 1882, he was in charge of detailed construction of roofs, groined ceilings, and masonry moulded arches of two large churches, one in Portsmouth, England, and one in London, England.

Learning that his brother had been stricken with fever in Florida, Mr. McLean left for the United States and from October, 1882, to October, 1883, while caring for his brother, he designed and erected buildings under contract in Palatka, Fla.

He left Florida for New York, N. Y., and in November, 1883, obtained a position as Designer and Draftsman with J. B. Snook, Architect. He remained in New York until October, 1887, during which time he attended the evening classes at Cooper Union. An interesting sidelight of his determination and ambition is obtained from the fact that when he applied for admission to the school he was told there was no seat for him, whereat he replied "Well, can't I stand?" and that is what he did until a seat was available.

---

[1] Memoir prepared by Archibald McLean, Jr., New York, N. Y.

In 1887, Mr. McLean was employed, as a Designer and Draftsman in the Chief Engineer's Office, of the New York, Lake Erie and Western Railroad Company, assisting in the designs and detail work of stations, track systems, bridges, and warehouses.

He left the Erie Railroad Company in April, 1889, to enter the service of the Trustees of the New York and Brooklyn Bridge, and one of his first jobs with them was studies for railroad terminals at both the Brooklyn and the New York ends of the bridge, copies of which are contained in the published report for 1889.[2]

In 1893, Mr. McLean became Assistant Engineer and in this capacity, until 1901, he assisted in designing and supervising the erection of bridge railway terminals, elevated railroad connections, cable-driving plants, car equipment, tracks, etc., approaches, and, subsequently, a new power plant and equipment for the electrical operation of the bridge railway. He made numerous studies and plans for the enlargement of traffic facilities between New York and Brooklyn, including preliminary studies for, and the location of, the Manhattan Bridge and the Blackwell's Island Bridge over the East River. He also assisted in the design and was in charge of the installation of the through elevated railroad and trolley service, mail tubes, etc., upon the Brooklyn Bridge.

Mr. McLean's design of an arrangement of switches and platforms was adopted and, subsequently, made it possible that "the carrying of such a large number of people in an hour has never been accomplished by any other double track railway in the world."[3] His system has been adopted and used in other railroad terminals.

A copy of his design of the Manhattan Bridge made in 1898 was sent to the Paris Exposition in 1900. It might be interesting to note that his study of the Blackwell's Island Bridge (later known as the Queensboro Bridge), with his vision called for a 120-ft roadway, but this was not adopted much to the inconvenience of present-day traffic.

In September, 1901, Mr. McLean became Assistant Engineer in charge of the maintenance and repair of the Brooklyn Bridge and was aptly called "The Big Bridge's Nurse," for to him the bridge had become a pet, and the story has been written (when the thousands of users were grumbling at the service and the bridge was being criticized as useless) that he lovingly patted the cables and said "Well old girl you work for them every minute of every week without rest; you have accepted the ever-increasing loads and strains without grumbling, and you still will be performing your important function when all the present-day users are moulding in their graves."

---

[2] "The Traffic Capacity of the New York and Brooklyn Bridge Railway," by the late G. Leverich, M. Am. Soc. C. E.

[3] "Brooklyn Bridge 1883–1933," City of New York Dept. of Plant and Structures, p. 47.

In July, 1903, he became Engineer in Charge of the Brooklyn Bridge and under his direction and supervision the bridge was reinforced to take care of heavier train service. In December, 1909, the maintenance of the Manhattan Bridge was added to Mr. McLean's duties; in January, 1912, he became Engineer in Charge of the Queensboro Bridge; in 1913, Engineer in Charge of Bridges in the Boroughs of Brooklyn, Queens, and Richmond; in February, 1919, Assistant to the Chief Engineer of all bridges; and, in July, 1923, he became Deputy Chief Engineer of the Department of Plant and Structures of New York City, which position he held until his retirement on July 31, 1933.

Mr. McLean was a man of unusual character, vision, humor, and ability, endeared to all with whom he came in contact, and was always on hand to help in time of trouble; and, as has been said of him, he closed the eyes of many workmen in their last sleep and turning gave comfort to the grieving relatives by his quiet manner and appropriate words.

He was a religious man and for forty-two years served as an Elder in the Arlington Avenue Presbyterian Church, of Brooklyn. His was a workable religion which radiated from his actions and mannerisms, and his faith in its might gave strength to his calm deep-rooted judgment of right. He was a Thirty-second Degree Mason, a member of Covenant Lodge, in Brooklyn; a member of the Municipal Engineers; and a Registered Professional Engineer.

Mr. McLean was married in June, 1886, to Elizabeth Christie, of Dollar, Scotland, to whose devotion and encouragement he attributed his success. She passed away in June, 1931. They had five children, four of whom survive, and the cheerfulness of home life and the beauty of the love and self-sacrifice of their parents remains to them as an heritage not to be surpassed.

The admiration and esteem of the men with whom Mr. McLean was associated, was expressed in these words by a fellow member of the Society:

"Your father's bridges stand as witnesses
To the skill and art of Man
Another type of bridge he also built
Not made of steel or stone or earthly plan.
He crossed this bridge to meet his Chief
The work of weary years has dropped away
His patient eyes now shine with heavenly light
The Glorious Morning of the Perfect Day."

This admiration and esteem was also expressed in the resolution adopted by the Conference Committee consisting of the heads of the various Divisions of the Department of Plant and Structures of the City of New York:

> "He had endeared himself to his associates by his devotion to God and duty; his broad-minded sense of justice, his kindliness and humanity. The memory of his well spent and beautiful life will be a blessed heritage."

Mr. McLean was elected a Member of the American Society of Civil Engineers on May 6, 1903.

EDWARD CHARLES MURPHY, M. Am. Soc. C. E.[1]

DIED SEPTEMBER 18, 1934

Edward Charles Murphy was born in Croydon, Ont., Canada, on June 17, 1859. He was graduated in Civil Engineering by Cornell University, at Ithaca, N. Y., in 1884, with the degree of Bachelor in Civil Engineering. He was a Fellow at Cornell in 1885, and received the degrees of Master of Science in 1885, Civil Engineer in 1892, and Doctor of Philosophy in 1900, from that University.

Mr. Murphy was Examiner in Mathematics at Cornell, in 1886; Assistant Professor of Civil Engineering at the University of Kansas, at Lawrence, Kans., from 1887 to 1899; and Instructor at Cornell in 1901 and 1902. From 1892 to 1902, he was employed on a part-time basis by the United States Geological Survey, and from 1903 to 1926 on a full-time basis. He was retired on October 15, 1926, on account of failing health, and thereafter lived quietly with his children, first with his son, Arthur E., then Assistant Professor of Economics at the University of Chicago, and, later, with his daughter Gladys, wife of Malbone Watson Graham, Assistant Professor of Political Science at the University of California, Los Angeles.

During the ten years, 1892 to 1902, when Dr. Murphy was teaching at the University of Kansas and at Cornell University and was giving part-time service to the U. S. Geological Survey he was, perhaps unconsciously, fitting himself for the very important work that he was to do later. In Kansas, he was learning by actual field experience the difficulties of obtaining records of river discharge by means of the instruments and equipment then available, and the inaccuracies of the current meter under the varying conditions of river gauging. At Cornell, he conducted extensive experiments in the then new Hydraulic Laboratory relative to instruments and methods used in obtaining systematic records of river discharge under the variety of conditions pertaining to flowing water.

Dr. Murphy was well equipped, therefore, by field and laboratory work for the important accomplishments which were to follow his entrance on full-time service with the U. S. Geological Survey. Systematic recording of river flow was then in its pioneer stages and field methods had not been fully developed. Equipment was crude, causes of inaccuracies were little understood, and the records obtained were unsatisfactory. The late Frederick Haynes Newell, M. Am. Soc. C. E., who was then at the head of the water investigation work in the United States, gave Dr. Murphy the task of inspecting and improving the field program of stream gauging. During the years, 1903 to 1906, he was employed mostly in this work. His pioneer investigations contributed in large measure to the development of the present practice followed in obtaining records of river discharge.

---

[1] **Memoir prepared by N. C. Grover, M. Am. Soc. C. E.**

In 1906, Dr. Murphy was assigned to the investigation of the transportation of débris by flowing water, and was put in charge of the observational work in such an investigation for the U. S. Geological Survey at the Hydraulic Laboratory of the University of California, at Berkeley. He remained in this assignment until 1909 and obtained the basic information used by Dr. G. K. Gilbert for his report on "The Transportation of Débris by Running Water."[2]

From 1909 until his retirement in 1926 Dr. Murphy was engaged in special examinations related generally to the classification and utilization of public lands. In this assignment, he prepared many reports, largely unpublished, which were used by the Geological Survey in classifying lands with respect to the Public Land Laws.

He was a contributor to the publications of the Society, having presented two papers, "Current Meter and Weir Discharge Comparisons,"[3] and "The Flood of March, 1907, in the Sacramento and San Joaquin River Basins, California,"[4] and discussions of other papers; the author of many articles in the technical press, and of many reports published in the series of *Water Supply Papers* of the U. S. Geological Survey.

He was married in October, 1885, to Ida May Lambertson, of Ithaca, N. Y., who passed away in May, 1892, leaving four children, Lee Orlo Murphy, Assoc. M. Am. Soc. C. E., Mrs. Grace Elena Davis, Mrs. Carrie May Graff, and Charles Earle Murphy. Dr. Murphy was married a second time in July, 1893, to Emily Atkinson, of Lawrence, Kans., who survives him, and by whom he had two children, Mrs. Ethel Gladys Graham and Professor Arthur Edward Murphy, of Brown University, Providence, R. I.

Dr. Murphy was elected an Associate Member of the American Society of Civil Engineers on September 5, 1900, and a Member on March 31, 1908.

FLOYD AUGUST NAGLER, M. Am. Soc. C. E.[1]

Died November 10, 1933

Floyd August Nagler was born in Howard City, Mich., on January 11, 1892, the son of August F. and Carrie (Fox) Nagler. He attended the public schools of Jackson, Mich., and was an Assistant in the Jackson High School in 1908–1909, before he entered college. In 1914 he received the degree of Bachelor of Science at Michigan State College of Agriculture and Mechanics, at East Lansing, Mich. From October, 1914, to June, 1917, he was a Graduate Student at the University of Michigan, at Ann Arbor, Mich., where he was awarded, in 1915, the degree of Master of Science in Engineering. In 1917, he received the degree of Doctor of Philosophy from that University.

---

[2] *Professional Paper No. 86,* U. S. Geological Survey.

[3] *Transactions,* Am. Soc. C. E., Vol. XLVII (1902), p. 370.

[4] *Loc. cit.,* Vol. LXI (1908), p. 281.

[1] Memoir prepared by R. B. Kittredge and A. H. Holt, Members, Am. Soc. C. E.

During the summer months of 1914, 1915, and 1916, Mr. Nagler served as Assistant Engineer with the Fargo Engineering Company, Jackson, Mich., on a variety of projects, largely along the line of hydraulics, to which his lifework was to be devoted. In 1915 and 1916 he was a Teaching Assistant in Civil Engineering at the University of Michigan. Following the completion of his school work, he joined, in June 1917, the staff of Robert E. Horton, M. Am. Soc. C. E., and began in earnest the work in the field of water-power engineering which he continued so successfully throughout the remainder of his life.

In April, 1918, Mr. Nagler turned to his country's service, was commissioned a Second Lieutenant in the Meteorological Section, Science and Research Division, of the Signal Corps, United States Army, and served in that capacity until January, 1919. Upon his discharge from the Service, he returned to his work with Mr. Horton, where he remained until September, 1920.

In the fall of 1920, he joined the Staff of the State University of Iowa at Iowa City, Iowa, as Assistant Professor of Mechanics and Hydraulics. In 1922, he was promoted to be Associate Professor, and, in 1927, he was appointed Professor of Hydraulic Engineering. Upon the creation, in November, 1931, of the Iowa Institute of Hydraulic Research, he was made its Director. In these two positions, he continued until his death.

During the years in Iowa, from 1920 to 1933, Professor Nagler was frequently called upon by public and by private interests to give counsel or to direct some project in his chosen field. From 1921 to 1923, he served as Drainage Engineer for the United States Department of Agriculture in the making of certain tests concerning the flow of water past obstructions; and from 1928 to 1930, he was Senior Engineer with the United States War Department, in charge of stream investigations on the tributaries of the Upper Mississippi River.

Such is the bare outline of the professional undertakings to which Professor Nagler gave his time, but little suggestion is thus revealed of what he accomplished. Possessed of unlimited enthusiasm and tireless energy, coupled with the ability to inspire these qualities in those who worked with him, he aproached every task with an eagerness equalled only by the thoroughness with which he continued until the job was done.

Continually busy with his professional work, he nevertheless found time for productive activities in other fields, technical and non-technical. He was a member of a number of professional or scientific societies, and took an active interest in them. His important technical papers, numbering well over thirty, appeared for the most part in the publications of these societies. These papers, all in the field of hydraulics, were of such excellence as to receive world-wide recognition. Evidence of their high professional value is found in the recognition accorded them by the Society. In 1919, Professor Nagler was awarded the Collingwood Prize for Juniors for a paper on the "Obstruction of Bridge Piers to the Flow of Water"[2]; and in 1920, this award was again made to him for his paper entitled, "Verification of the Bazin Weir Formula by Hydro-Chemical Gaugings."[3] In 1931, the highest award of the Society for the recognition of technical writing, the Norman Medal, awarded to a paper "judged worthy of special commendation for its merit as a contribution to engineering science," was given to Professor Nagler and Albion Davis, M. Am. Soc. C. E., for their paper on "Experiments on Discharge over Spillways and Models, Keokuk Dam."[4] In 1932, the J. James R. Cross Medal, an award second in rank only to the Norman Medal, was presented to David L. Yarnell, M. Am. Soc. C. E. and Professor Nagler, for their paper on the "Effect of Turbulence on the Registration of Current Meters."[5]

---

[2] *Transactions*, Am. Soc. C. E., Vol. LXXXII (1918), p. 334.

Always able to derive pleasure from his work, Professor Nagler had made, as a by-product of his consulting practice over the State of Iowa, a thorough study of the history of water-power development in that State, and had accumulated a fund of information of great popular as well as historical and technical interest. In this connection, he had gathered and was constantly adding to an interesting collection of old water-wheels, mill-stones, and other related relics of the pioneer days of the industry.

While always occupied with professional matters, he found pleasure and satisfaction in other duties. In his Church, he was a dependable, tireless worker, inspiring others by the whole-hearted enthusiasm and faith which he carried into that work, as into every undertaking to which he turned his hand. To the Boy Scouts, to his social clubs, he contributed his full share of time and talent, always in a manner consistent with the high social ideals which he cherished and which he did his full part to maintain.

But, withal, his keenest interest was in his home. He was married, on September 1, 1921, to Marion Dell Truax, of Albany, N. Y., and into their home came three children, Robert Carlton, Phyllis Jane, and Donald Floyd. It was while working with and for this family that Professor Nagler seemed happiest; and great plans had been made and execution was well under way to increase their opportunities for happy times together. With the faculty of making his technical knowledge contribute to the pleasure of his pastimes, just as he was able to derive pleasure from his professional work, he was carrying forward the development of his physical home in a manner that made it an increasingly pleasant place in which to live.

The following is quoted from a statement made upon hearing of his death by one who knew him well:

> "A productive scientist, an inspiring teacher, an illustrious engineer, a loyal friend and neighbor, an ideal parent, a devoted Christian, a great citizen, he will long continue to be an influence in the lives of those who knew him."

Professor Nagler was elected a Junior of the American Society of Civil Engineers on March 14, 1916; an Associate Member on May 13, 1918; and a Member on July 12, 1926. He served as President of the Iowa Section of the Society in 1932.

---

[3] *Transactions*, Am. Soc. C. E., Vol. LXXXIII (1919–1920), p. 105.

[4] *Loc. cit.*, Vol. 94 (1930), p. 777.

[5] *Loc. cit.*, Vol. 95 (1931), p. 766.

LARS NETLAND, M. Am. Soc. C. E.[1]

DIED JULY 5, 1934

Lars Netland, the son of Thor and Louise Netland, was born in Flekkefjord, Norway, on June 30, 1869. He attended the common and High Schools of Flekkefjord from 1876 to 1886, and, upon the completion of his preparatory education, he entered the Trondhjems Tekniske Laere Anstalt (Polytechnic High School of Trondhjem), from which he was graduated as a Civil Engineer in 1889.

Shortly after his graduation, Mr. Netland came to the United States, and entered the employ of the Chester Machinery Company, at Chester, Pa., in March, 1890. He remained only a few months in this position, and in May of the same year, became a Draftsman with the Missouri Pacific Railroad Company. His work with the Railroad Company included the preparation of plans for a number of bridges throughout Kansas, Nebraska, Missouri, and Arkansas, the preparation of plans and the construction of an office building and warehouse at St. Louis, Mo., and the remodeling of about 150 stations in Arkansas to comply with the then recently enacted "Jim Crow Law."

In July, 1891, he was made Office Engineer for the Crozier Land Association, at Elkhorn, W. Va., and was engaged during the succeeding seven years in all the variety of work connected with the development of 20 000 acres of coal land. This involved topographic mapping and the subdivision of the land into leases; the investigation of titles; the design and construction of coal tipples, power plants, railroads, tramways, and coke ovens; and the layout of town sites, roads, and water supply systems.

It was in 1898 that Mr. Netland went to the Klondike, and began what must have been to him one of the most satisfactory periods of his career. Those who knew him will recall the enthusiasm with which he always spoke of his work and experiences in the Far North, his love of hunting and fishing and outdoor life, his hearty good humor, and that certain ruggedness of character, so in keeping with his physique, which must have shown to advantage against the background of virgin country and elemental forces.

For two years, from 1898 to 1900, he engaged in the private practice of mine and claim surveying at Dawson; from 1900 until 1903, he carried on exploration surveys of remote parts of the Yukon Territory, as Chief of Party for the Canadian Government; and from 1904 until 1910, he was employed in the same capacity by the United States Government on the survey of the Alaska-Canada boundary. During the six years he was engaged in this work, the boundary was surveyed and monumented from Latitude 54 to Latitude 66, and a 5-mile strip along the entire boundary was mapped.

In 1911, Mr. Netland became Resident Engineer and Superintendent at Cumberland, British Columbia, for the Canadian Collieries, Limited, of Vancouver. In this capacity he was in charge of the development of 10 000 acres of coal land, carrying out such work as the building of two complete towns, with water supply, sewers, and electric lights; the construction of a 25 000-hp hydro-electric project; 50 miles of 13 500-volt transmission line, with substations at four mines; 25 miles of standard gauge railroad; two 1 000-ft shafts, and two coal tipples; and the installation of lighting systems for seven towns.

---

[1] Memoir prepared by William C. Renshaw, Assoc. M. Am. Soc. C. E., assisted by Theodore J. B. Netland, Jun. Am. Soc. C. E.

Mr. Netland left the Canadian Colleries in 1915 and, during 1916 and 1917, was in charge of the inventory and valuation of the Southern Pacific Company's coal mine at Beaver Hill, Ore. During 1917 and 1918, he was Chief Engineer and Superintendent for the Chicaloon Coal Company, at Chicaloon, Alaska, then engaged in driving prospect tunnels and constructing power-plant buildings and transmission lines.

In 1919, he moved to California, and was employed, successively, by the Arden Salt Company, on surveys and plant construction at Newark; by the Berger and Carter Machinery Company, of Hayward; and by the Moore Drydock Company, of Oakland, as a Draftsman.

In 1924, Mr. Netland joined the forces of the East Bay Municipal Utility District, an authority organized to take over the water supply of Oakland and a number of other East Bay towns. Later, he was made Resident Engineer in charge of the Stockton Office, on the construction of the 80-mile Mokelumne pipe line from Pardee Dam to Walnut Creek.

During 1929 and 1930, he was employed for a short time by the City of Berkeley on storm-sewer construction, and, later, by the Standard Oil Company, in charge of right-of-way descriptions for its natural gas line from Kettleman Hills to Richmond, Calif.

From 1930 until his death in 1934, Mr. Netland was employed by the Water Department of San Francisco, Calif., in various capacities on pipe lines and aqueduct construction, and as a Transitman on location and land surveys.

He was married, in 1905, to Marie Bugge, who, with a son, Theodore J. B. Netland, Jun. Am. Soc. C. E., and a daughter, Mrs. J. A. Ofscnek, survives him.

Mr. Netland was elected a Member of the American Society of Civil Engineers on June 10, 1929.

MAURY NICHOLSON, M. Am. Soc. C. E.[1]

DIED OCTOBER 8, 1934

Maury Nicholson was born in Columbia, Tenn., on June 14, 1859. He was the son of Major Hunter Nicholson and Lottie (Stone) Nicholson. His father served in the Confederate Army throughout the Civil War and, later, occupied for many years the chair of Dean of the Agricultural School of the University of Tennessee, at Knoxville. His grandfather, A. O. P. Nicholson, was prominent in the political life of Tennessee in the days of Andrew Jackson and served his State as United States Senator and as the first Chief Justice of the State Supreme Court.

---

[1] Memoir prepared by Hugh A. Powell, Esq., Montgomery, Ala.

After attending private schools, Mr. Nicholson entered the University of Tennessee, at Knoxville, and was graduated in 1880 with the degree of Bachelor of Science. He spent one year at the University as a Chemistry Instructor and then began his engineering career with the East Tennessee, Virginia and Georgia Railroad Company as Assistant Engineer of Construction. Later, he served as Resident Engineer with the Louisville, New Orleans and Texas Pacific Railroad Company, his two assignments including some exceptionally heavy excavation work as well as extensive pile and bridge construction.

In 1884, Mr. Nicholson returned to Knoxville as City Engineer, holding this position for more than ten years. While handling the various engineering matters of the City, his ability as a municipal engineer began to be noted. However, on leaving the City of Knoxville he found it more lucrative to re-enter the railroad construction field, and spent several years on location and construction of railroads in various States in the Southeast.

In 1900, Mr. Nicholson went to Birmingham, Ala., as Assistant City Engineer. At that time, Birmingham was entering a period of remarkably rapid expansion which, in a very few years, was to make it the leading industrial city of the South. In absorbing a number of suburban areas the City was confronted with the problem of consolidating the many separate drainage, sewerage, and street systems into one adequate and comprehensive system. Mr. Nicholson handled these problems in such an able manner that he was elected by the citizens to the post of City Engineer in 1908. During the ensuing decade, in which he devoted his best energies to the City, his reputation as a competent municipal engineer became firmly established. He was known as one of the outstanding authorities in the South on sewerage and drainage matters.

Mr. Nicholson severed his long connection with the City of Birmingham in 1917. After having been rejected for military duty, he accepted a position as Chief Engineer of the Roads and Buildings Department of a subsidiary of the United States Steel Corporation, which was erecting a large ship-building plant at Mobile, Ala. However, this work was suspended shortly after the close of the World War and Mr. Nicholson then became interested in highway engineering work, which was just beginning to receive serious attention in the Southern States. Utilizing his previous railroad experience, he laid out a comprehensive highway system for the State of Arkansas, which was one of the first States in that section to finance road building with highway district bonds. Later, he was employed by the State Highway Department of Louisiana and supervised the location of a number of main highways through the difficult marsh and lake regions in the southern part of that State.

In 1924, Mr. Nicholson was recalled to Knoxville to aid in the design of important extensions to the storm sewer system of that city. On the completion of this work, he returned to Birmingham in 1926 and was retained by that City as Consulting Engineer in connection with an extensive drainage project, involving the straightening and widening of two creeks constituting the natural drainage system of the entire city, as well as the re-location and enlargement of the principal existing storm sewers. This project, estimated to cost about $4 000 000, was perhaps the largest and most important single engineering undertaking with which Mr. Nicholson was ever connected. His intimate knowledge of the city's existing facilities and of the many drainage problems which had arisen during his long service as City Engineer were of inestimable value in preparing plans for the new drainage project.

From the foregoing, it will be seen that Mr. Nicholson's long engineering career consisted for the most part in the planning and building of public works which were vital to the well-being of the community which he served. That his recommendations were accepted by public officials without question over a long period of years is evidence of the high esteem in which his technical judgment was held by his associates. His personal probity and professional ability were so far above reproach that he was able to weather the many and frequent changes in political administration.

Mr. Nicholson was married to Maggie McCampbell, of Knoxville, on January 1, 1885. He survived her by only four months. Their two daughters, Mrs. Henry L. Freeman and Mrs. Hugh A. Powell, survive him. In his private life, Mr. Nicholson enjoyed and assiduously developed a love of culture which naturally was his by inheritance. His keen interest in horticulture and landscape work, which he found necessary to cultivate in connection with his engineering duties, became one of his most absorbing hobbies in later life. He possessed a keen intelligence, a deeply philosophical nature, and an unfailing sense of humor. These attributes won for him many close friends whose devotion lasted throughout his long life.

In 1930, Mr. Nicholson attended the Fiftieth Annual Reunion of his Class, at Knoxville, in connection with the commencement exercises of his Alma Mater. The class of about a dozen survivors was the recipient of a number of honors from the Faculty and Alumni of the University.

He was a member of the American Society of Municipal Engineers and of the Engineers' Club of Birmingham.

Mr. Nicholson was elected a Member of the American Society of Civil Engineers on January 2, 1912.

EMIL LOUIS NUEBLING, M. Am. Soc. C. E.[1]

Died June 5, 1934

Emil Louis Nuebling was born in Reading, Pa., on May 11, 1868, the son of George E. and Agatha Nuebling. He was educated in the public schools of his native city and was graduated from the Boys' High School in 1885. He acquired his abilities as an Engineer by self-teaching and practical experience.

---

[1] Memoir prepared by Frederick H. Dechant, M. Am. Soc. C. E.

After his graduation, Mr. Nuebling entered the employ of the late A. Harvey Tyson, M. Am. Soc. C. E., a local Civil Engineer, and, later, the City Engineer's Office, serving as Rodman in both positions until 1889. From that time until 1891, he was connected with Mr. Howard E. Ahrens, a Contractor, as Engineer on the design and construction of municipal water-works systems. During 1891, Mr. Nuebling was Draftsman with the Schuylkill Navigation Company, and also with the late Edwin Foster Smith, M. Am. Soc. C. E., and William H. Dechant, Civil Engineer, assisting in the preparation of plans and reports for augmenting the water supply of Philadelphia, Pa. From 1891 to 1895, he was employed by C. Ph. Bassett, M. Am. Soc. C. E., of Orange, and Summit, N. J., on the design and construction of numerous municipal water-works in Northern New Jersey.

In 1895, Mr. Nuebling became Superintendent and Engineer of the Water Department of Reading, after having served as Draftsman for Mr. William H. Dechant, in the preparation of the first comprehensive plans prepared for the future development of the water supply system for that city. Mr. Nuebling has been rightfully called the "Father of Reading's Water Supply," because, until 1924, he devoted his entire time to the development of the system, executing, during that period, major improvements and extensions to the service.

In 1924, he left the employ of the City and entered into private practice, organizing the Hydraulic Equipment Company, with headquarters in Reading. This Company was engaged in the design and construction of water-works systems throughout Southeastern Pennsylvania.

Mr. Nuebling was Engineer Consultant to the Reading City Planning Commission for a period of ten years, and was elected a member of the City Council on November 7, 1933. During his political campaign he was taken ill, and, unfortunately for the community, he was prevented from fulfilling his duties as Councilman. He was confined to his home from the time of his inauguration until his death.

As a man, Mr. Nuebling was highly respected for his kindly disposition, his affable manner, and his integrity. His deep love of Nature was evidenced by his delight in walking the mountain trails of his native County of Berks and by his collection of rare plants and wild flowers. He established the Tree Nursery of the City of Reading and began the planting of the watersheds twenty years ago.

He rose entirely through his own force of character and personal effort, to the forefront in the field of Hydraulic Engineering.

His whole life was devoted to his family and his work, and his passing is a distinct loss to the Engineering Profession and to the community in which he lived and which he served.

Mr. Nuebling was a member of the American Water Works Association, the New England Water Works Association, and the Pennsylvania Water Works Operators Association. Among his affiliations, other than those of a professional nature, were Reading Lodge No. 115, Benevolent and Protective

Order of Elks; the Lake View Country Club; the Mountain Springs Association; and the Blue Mountain Climbing Club.

He was married in March, 1891, to Mary Doreman, of Reading, Pa., and is survived by his widow and four sons and two daughters.

Mr. Nuebling was elected a Member of the American Society of Civil Engineers on December 7, 1904.

FRANCIS JOSEPH O'HARA, M. Am. Soc. C. E.[1]

Died July 23, 1934

Francis Joseph O'Hara was born at Fort Niagara, N. Y., on January 30, 1877, the son of Col. James J. and Katherine (Ransome) O'Hara. After his preparatory education, he entered Notre Dame University, at South Bend, Ind., from which he was graduated in 1899, with the degree of Civil Engineer.

During his vacations from July, 1893, to July, 1896, Mr. O'Hara served as Computer and Instrumentman with the United States Corps of Engineers on hydrographic and topographic surveys, jetty construction, and fortifications.

Immediately after his graduation from Notre Dame University, he went West, and from January, 1900, to October 1904, was engaged as Computer, Draftsman, and Office Engineer with the Southern Pacific Railroad Company on computing and drafting in the office of the Chief Engineer, and with the San Pedro, Los Angeles, and Salt Lake Railroad Company, in charge of construction in the office of the Division Engineer.

Leaving railroad work, Mr. O'Hara served as Civil Engineer and Superintendent of Construction, in the Quartermaster Department, U. S. Army, on the design and construction of buildings, roads, wharves, and water and sewerage systems, from October, 1904, to October, 1906.

In January, 1907, he re-entered the service of the Southern Pacific Railroad Company, as Assistant Engineer in charge of hydro-electric investigations, design, and heavy railroad construction in the Sierra Nevadas, and special studies in railroad economics.

Mr. O'Hara entered the employ of Dockweiler and Cragin, Consulting Engineers, in San Francisco, Calif., in August, 1913, as Chief Assistant Engineer. In this capacity he was engaged, until December, 1917, on the design and construction of irrigation projects, reports on water supplies, etc., including studies of the various sources of supply from the City of San Francisco. He also made an appraisal of the properties of the Spring Valley Water Company which later were acquired by the municipality.

During the World War, from December, 1917, to July, 1920, Mr. O'Hara served as Major in the Ordnance Department, United States Army, and rendered distinguished services to the Government. During this period he was engaged on the design and supervision of construction of office buildings, warehouses, etc., adjustment of war contracts and claims, as well as special work for the Chief of the Engineering Division.

---

[1] Memoir prepared by Alfred J. Cleary, M. Am. Soc. C. E.

After his honorable discharge from the Army in July, 1920, Mr. O'Hara was appointed Chief Assistant Engineer on the Salt River Project, in Arizona, and was instrumental in developing adequate storage and a modern up-to-date distribution system for the Desert Irrigation District, adjacent to Phœnix, Ariz.

Later, he engaged in private practice as a Consulting Engineer, with headquarters at San Francisco, and, until his death, on July 23, 1934, was identified with some of the large water and power projects of the West and Southwest.

Mr. O'Hara was married on April 9, 1910, to Mary Stark, of San Francisco, and is survived by his widow, a son, John Francis, and two brothers and two sisters.

He was a man of forceful character, high ideals, and steadfastness of purpose which resulted in a life of notable achievement. Above all, he was a friend to the young men in the profession and he was never too busy to lend a helping hand to a fellow engineer. He was a member of the Roman Catholic Church.

Mr. O'Hara was elected a Member of the American Society of Civil Engineers on July 11, 1921.

JOHN FRANCIS O'ROURKE, M. Am. Soc. C. E.[1]

Died July 28, 1934

John Francis O'Rourke was born in Tipperary, Ireland, on October 3, 1854, the oldest of seven children, four boys and three girls, and was brought to the United States in his infancy.

After completing his Grammar School education in the public schools of New York, N. Y., he spent four years in the office of Charles Guidet, one of the best-known contractors of that period, during which time he became well acquainted with the construction of all kinds of public works in New York City. Mr. O'Rourke's studies and Mr. Guidet's work on the New York Elevated Railway, on West Broadway, of which Mr. O'Rourke was in charge, went on together.

After completing his work with Mr. Guidet, Mr. O'Rourke entered the office of Edward Boyle, Civil Engineer and City Surveyor, as a student, and at the end of two years he joined the Engineering Corps of the New York Elevated Railway Company. In the meantime he was graduated from Cooper Union with the degree of Bachelor of Science. Four years of post-graduate study followed, and, subsequently, he received degrees in Civil and Mechanical Engineering.

---

[1] Memoir prepared by Innis O'Rourke, Esq., Great Neck, N. Y.

Mr. O'Rourke was next engaged on the location of the West Shore Railway between Congers and Fort Montgomery, N. Y., and when construction was started he was appointed Assistant Engineer in charge between Congers and Tompkins Cove, N. Y. This section included many bridges, trestles, and the Haverstraw Tunnel. After the completion of the Haverstraw Section, Mr. O'Rourke was made Resident Engineer of the railway line through Genesee County, New York State.

Then followed work on the construction of St. Croix River Bridge; the shop inspection of steel for the Ohio River Bridge, at Louisville, Ky.; the Prairie du Chien Bridge across the Wisconsin River; and the Chignecto Ship Railway, a project planned to save the long trip around Nova Scotia by connecting the Bay of Fundy with the Strait of Northumberland and the Gulf of St. Lawrence. Mr. O'Rourke was Chief Engineer for the Union Bridge Company on the construction of the Hudson River Railroad Bridge, at Poughkeepsie, N. Y., one of the largest railway bridges in the world at that time.

In 1891, he had charge of the surveys and maps for the Rapid Transit Commission, New York City, under the late William Barclay Parsons, Hon. M. Am. Soc. C. E., and the following year he was made Deputy Chief Engineer. He held this position for several years, during which time various methods of subway construction were studied and the open-cut, structural-steel method was originated and adopted by Mr. Parsons.

While engaged with the Rapid Transit Commission, Mr. O'Rourke was also Assistant to the late Thomas Curtis Clarke, Past-President, Am. Soc. C. E., Consulting Engineer in charge of the Third Avenue and Willis Avenue Bridges over the Harlem River. In this position he designed the pneumatic caisson foundations and masonry for both structures and supervised their construction. Mr. O'Rourke resigned from the Commission in 1894, in order to give all his time to consulting practice, which included the preparation of plans for the new structural steel buildings then coming into vogue and the designs of the foundations required by these heavy buildings. He designed the pneumatic caisson foundations for the Commercial Cable Building, in New York City, the first work of its kind. In 1896, he organized the firm of Stephens and O'Rourke and constructed the pneumatic caissons for the Gillender Building, at Wall and Nassau Streets, New York City. Among other improvements in pneumatic caissons, he originated the method of connecting individual caissons, after they had been sunk to solid rock, into a continuous masonry dam for the purpose of extending a water-tight cellar through quicksand to any desired depth. The Stock Exchange, Hudson Terminal Buildings, Whitehall Building, Equitable Building, Federal Reserve Building, and the New York Telephone Building, on West Street, New York City, are a few of the many examples in which this method was used to obtain four or more stories below street level for boiler rooms, machinery, safe deposit vaults, etc., in quicksand, where only a single basement was possible before.

In 1902, the O'Rourke Engineering Construction Company, of which Mr. O'Rourke was President and Chief Engineer, was organized, and in

1904 the Company commenced the Hudson River Tunnels and Approaches for the Pennsylvania Railroad from Weehawken, N. J. to 33d Street, New York City, completing the driving of the tunnels in 1906. Subsequently, the Company constructed the Grand Central Yard improvements on Park Avenue, south of 57th Street, New York City, and much pneumatic caisson, subway, railway, and tunnel work, including the foundations of some of the buildings previously mentioned.

In 1913, Mr. O'Rourke originated the method of blowing gravel through holes in tunnel lining into the space outside shield-driven tunnels. In 1914, the Flinn-O'Rourke Company, of which he was Vice-President and Chief Engineer, commenced the contracts for the Whitehall-Montague Street and Old Slip-Clark Street East River Tunnels, with their tunnel extensions through Montague and Willoughby Streets to Flatbush Avenue, in Brooklyn, N. Y. and through Fulton Street under the Elevated Railway to Borough Hall. The gravel packing prevented all settlement of streets, buildings, or Elevated Railway structures. This method of preventing settlements was used subsequently by the contractors for the Fulton-Cranberry Street and Rutgers-Jay Street East River Tunnels of the new Municipal Subway System of the City of New York and in the driving of the Boston, Mass., Vehicular Tunnel in 1932.

In 1912, Mr. O'Rourke invented concrete blocks by means of which the thrust from shield-jacks is successfully resisted and; in recent years, he had developed and perfected a system of constructing tunnels of pre-cast reinforced concrete blocks. The more notable examples of this system are the tunnels of the new Municipal Subway System under Prospect Avenue and Prospect Park, in Brooklyn, and the Ford Water Tunnel, at Detroit, Mich.

After the disastrous sinking of the submarine, S-4, off Provincetown, Mass., on December 18, 1927, Mr. O'Rourke invented a submarine air-lock by means of which entrapped men could be taken through a hatch, into the air-lock, and brought to the surface. This air-lock was built and successfully tested by the United States Navy Department.

Mr. O'Rourke was a member of the Roman Catholic Church, a Trustee of St. Patrick's Cathedral, and a Knight of St. Gregory. He was a member of the Engineering Institute of Canada, and the General Society of Mechanics and Tradesmen, and the Cooper Union General Science Alumni Association, of New York City. He was also a member of the Garden City, Nassau Country, and other golf clubs.

While living in Poughkeepsie, Mr. O'Rourke was married on January 7, 1890, to Katharine Innis, daughter of Aaron Innis, a leading manufacturer of that city, by whom he is survived. He also leaves two sons, Innis O'Rourke and John F. O'Rourke, Jr., and a daughter, Mrs. Katharine (O'Rourke) Tweddell.

Mr. O'Rourke was elected a Member of the American Society of Civil Engineers on April 2, 1884. He served as a Director of the Society from 1900 to 1902.

MICHAEL MAURICE O'SHAUGHNESSY, M. Am. Soc. C. E.[1]

Died October 12, 1934

Michael Maurice O'Shaughnessy was born in Limerick, Ireland, on May 28, 1864, the son of Patrick and Margaret (O'Donnell) O'Shaughnessy. At the age of twenty he was graduated with honors from the Royal University, at Dublin, receiving the degree of Bachelor of Engineering.

Shortly after graduation Mr. O'Shaughnessy came to the United States and followed here the usual course of a young engineer in the early part of his professional life. One assignment, in 1885–1886, was that of Assistant Engineer on a small railroad in California—the Sierra Valley and Mohawk Railroad. For two years, 1886–1888, he was Assistant Engineer with the Southern Pacific Railroad at various locations. Following these engagements he was employed in surveys and engineering of land developments in California, laying out a number of small towns. For four years, 1890 to 1894, he was in general engineering practice in California with an office in San Francisco. In this period he acted as Chief Engineer of the California Midwinter International Exposition, held in San Francisco in 1893.

During 1895, Mr. O'Shaughnessy, as Chief Engineer, built twelve miles of narrow-gauge railroad for the Mountain Copper Company in California. This, with other work for various corporations, including the Spring Valley Water Company, occupied his time through 1896 to 1898. From 1899 to 1906, he was actively engaged in the design and construction of works for the water supply of about twenty sugar plantations in the Hawaiian Islands. Among these may be mentioned the Olokele Aqueduct, Kauai, ten miles long, with tunnels; the Koolau, Keanaii, Aqueduct, of 90 000 000 gal per day capacity, also ten miles long with tunnels; and the Kohola Aqueduct, Hawaii; in all, about thirty miles of conduit were built.

From 1907 to 1912, Mr. O'Shaughnessy acted as Chief and Consulting Engineer of the Southern California Mountain Water Company, at San Diego, Calif., at the same time being engaged on other works. The work on the San Diego water supply included the Dulzura Conduit of 44 000 000 gal per day capacity, 13.38 miles long, and the Morena Rock Fill Dam. On the conduit are seventeen tunnels aggregating more than 1.75 miles in length. The Morena Rock Fill Dam, of a maximum height of 262 ft, was completed in 1911. It remains a notable example of this type of dam. Other works executed in this period include the rectification of the channel of the Salmas River for the Shreelo Sugar Company, and the building of a masonry dam on the Merced River for the Crocker-Huffman Land Company, and waterworks at Port Costa, Calif.

On September 1, 1912, Mr. O'Shaughnessy was called upon to accept appointment as City Engineer of San Francisco, which position he held for twenty years, until a City Charter was adopted which separated the ordinary work of the City Engineer from that having to do with its public utilities, including the municipal water supply, the principal element of which at that time was the building of the Hetch Hetchy Aqueduct from the Sierra Nevada Mountains. At that time, he was made Consulting Engineer, a position he held until his death.

---

[1] Memoir prepared by John D. Galloway and Nelson A. Eckart, Members, Am. Soc. C. E.

During his incumbency of the office of City Engineer Mr. O'Shaughnessy had charge of the numerous types of construction that fall to the lot of that official in a large and growing city. He had responsible charge of the design and construction of public works during this period aggregating approximately $180 000 000. This work included, in addition to the ordinary extensions of streets and sewers, the design and construction of boulevards, tunnels, bridges, and municipally-owned utilities, including a street railway system and water supply and hydro-electric power projects. He built the 9 by 11-ft main sewer under Golden Gate Park and Sutro Heights and thence out into the waters of the Pacific Ocean He completed the auxiliary high-pressure fire system, including the pumping station at Fort Mason. From 1912 to 1916, the Municipal Railway System was built, made up of 68 miles of electric street railway, with car barns, 220 street cars, and other equipment, the expenditures reaching a total of $8 000 000. The Stockton Street Tunnel, a difficult piece of work, was built in 1913–14 at a cost of $650 000. The tunnel is 885 ft long, 50-ft span of arch, and 19 ft high. Another tunnel, the Twin Peaks, was built in 1914–17. This is a double-track street railway tunnel, 25 ft wide, 18 ft high, and 12 000 ft long, the cost being $4 300 000. Of the boulevards built under his charge may be mentioned Twin Peaks, Ocean Beach, Sloat, Junipero Serra, and Telegraph Hill. The sea-wall that protects the Esplanade along the edge of the Pacific Ocean for a distance of approximately a mile was constructed at intervals during this period.

In 1927, Mr. O'Shaughnessy served as a member of the first Board of Engineers on the Transbay Bridge across San Francisco Bay, with Robert Ridgway, Past-President and Hon. M. Am. Soc. C. E., Arthur N. Talbot, Past-President and Hon. M. Am. Soc. C. E., and John D. Galloway, M. Am. Soc. C. E.

Above all these engagements, the work that occupied the major part of Mr. O'Shaughnessy's time and interest was the Hetch Hetchy Water Supply for the city. Some description of the works as built, together with notes on the history, must be given in order to appreciate the magnitude of the project.

Water is stored in reservoirs in the Sierra Nevada and carried through 37.7 miles of mountain tunnels and pipe to the border of the San Joaquin Valley, thence in a steel pipe line 47.5 miles across the Valley thence by tunnels through the Coast Range, 29.1 miles long, thence by pipes to a tunnel across the valley borders and under an arm of San Francisco Bay, 23.0 miles, into the Crystal Springs Reservoir. The total constructed length of the main aqueduct is 137.4 miles. Plans for the ultimate development contemplate the construction of 12 miles of tunnel aqueduct between Hetch Hetchy Reservoir and the Early Intake Diversion to provide a power drop, and 19.6

miles of tunnel and pipe by-passing the Crystal Springs Reservoir and delivering Hetch Hetchy water directly into a terminal reservoir in San Francisco.

The selection of the Tuolumne River as a source of water supply for San Francisco, which had been mentioned by other engineers, was definitely made by the late Carl Ewald Grunsky, Past-President, Am. Soc. C. E., when he was City Engineer. This selection was made after an extensive review of all possible sources of supply. Interminable delays took place, due to opposition from many sources. The project was kept alive by the late Marsden Manson, M. Am. Soc. C. E., when he was City Engineer. In 1912, a complete report on the project was made by the late John R. Freeman, Past-President and Hon. M. Am. Soc. C. E., but it was not until the end of 1913 that Congressional approval of the grant of lands to San Francisco was made under President Wilson.

Construction work was started in 1914 in the mountains at the beginning, as required by the Congressional grant, and for twenty years it was prosecuted to completion in 1934 when the water first reached the peninsula reservoirs. The work required the building of a standard gauge railroad, 68 miles long, a construction power plant of 3 300-kva capacity, a storage dam at Lake Eleanor, the larger storage dam at Hetch Hetchy Valley, the aqueduct as described previously, a regulatory reservoir with penstocks and a 70 000-kw power plant at Moccasin Creek, and a transmission line, 98 miles long, to the Newark Sub-Station of the Pacific Gas and Electric Company. The dam at Lake Eleanor is of concrete, of the multiple-arch type, 1 260 ft long, and with a maximum head of water of 70 ft; it impounds 8 200 000 000 gal. The main storage dam at the Hetch Hetchy Valley, as built, is of concrete of gravity section, 344 ft in height, and contains 399 000 cu yd of concrete. The foundations were built to permit an increase in height. It impounds 67 000 000 000 gal of water.

The expenditures on the entire project amounted to $86 000 000, exclusive of interest. The dam at Hetch Hetchy Valley was officially given the name, O'Shaughnessy Dam, in honor of the man who had direct charge throughout the entire period of construction. The successful completion of this project is as much a tribute to Mr. O'Shaughnessy's courage and indomitable will as it is to his engineering ability.

Throughout the entire period of construction he was called upon to resist the attacks of opponents of the project who for political or other purposes continually sought to stop the progress of the work or otherwise thwart the plans of the engineer. Again he was forced to carry on through periods of financial difficulty which at times threatened to shut down the work. His stout heart and contagious optimism enabled him to overcome all these obstacles and to finish the work in accordance with his plans.

Mr. O'Shaughnessy's ability as an engineer was widely recognized, and his services were sought by many private and public organizations throughout the United States. While carrying on his official duties he found time to act in an advisory capacity in a number of important projects among which may be mentioned: In 1914, on the Strawberry Rock Fill Dam, on the Stanislaus River, of the Pacific Gas and Electric Company; in 1916, at San Diego, on the building of the new Lower Otay Dam and the Barrett Dams; in 1918, at Seattle, Wash., on the Power Plant on the Skagit River; from 1916 to 1918, on the building of the Alpine Dam and distribution works of the Marin Municipal Water District on the peninsula north of San Francisco; in 1919, at Detroit, Mich., on the purchase of the street railway systems by the City of Detroit; in 1919 at Tacoma, Wash., on the street railways problem; in 1924 and 1925, with L. F. Harza, M. Am. Soc. C. E., on the Dix River Dam, in Kentucky; and, in 1926, on the New Bowman Rock Fill Dam on the Yuba River, California, with Fred H. Tibbetts, M. Am. Soc. C. E.

A few days before the mountain water flowed into the reservoirs, Mr. O'Shaughnessy died of heart failure. His work was done, but it is regrettable that a few years could not have been given him to realize the success of his labors.

The outstanding characteristic of Mr. O'Shaughnessy was his undeviating honesty in all that he undertook, coupled with an Irish temperament that was willing at all times to fight vigorously for what he deemed the right. Always outspoken, he never hesitated to answer any political attacks or reply to his critics, and believed in calling a "spade a spade."

Mr. O'Shaughnessy was a contributor to the publications of the Society. His two most notable papers were the one on the "Construction of the Morena Rock Fill Dam, San Diego County, California,"[2] in 1912, for which he received the James Laurie Prize of the Society in 1913, and the one on the "Construction Progress of the Hetch Hetchy Water Supply of San Francisco, California,"[3] in 1922. Shortly before his death he published a book on the Hetch Hetchy Project written in his characteristic manner.

He was a member of the New England Water Works Association; the American Water Works Association; the Commercial, Engineers, University, Olympic, and Chit-Chat Clubs and of the Presidio Golf Club. He was throughout his life a consistent member of the Roman Catholic Church.

He was married, in 1890, to Mary Spottiswood, of San Francisco, who, with four daughters and a son, survives him.

Mr. O'Shaughnessy was elected a Member of the American Society of Civil Engineers on June 4, 1902. He was also a member of the San Francisco Section and served as its President in 1920.

WILLIAM BANCROFT POTTER, M. Am. Soc. C. E.[1]

Died January 15, 1934

William Bancroft Potter was born on February 19, 1863, in Litchfield County, Connecticut, near Thomaston. His parents, Horace A. and Charlotte S. (Pierce) Potter, were prosperous farmers, and he spent his boyhood days mostly in a small farming community. Even at that time he gave evidence of a fondness for mechanical matters, and made a number of devices that revealed his ingenuity in this direction. In 1877, he moved to Thomaston, Conn., and at the age of 16, he was attending school in the winter and working in the Seth Thomas Clock Factory in the summer. He was graduated from the Thomaston High School in 1881, and shortly afterward he became apprenticed to the firm of Sawtelle and Judd, of Hartford, Conn., having made up his mind that he wanted to be an engineer.

---

[2] *Transactions*, Am. Soc. C. E., Vol. LXXV (1912), p. 27.

[3] *Loc. cit.*, Vol. LXXXV (1922), p. 869.

[1] Memoir compiled from information supplied by W. D. Bearce, Statistician, Transportation Eng. Dept., General Electric Co., Erie, Pa.

Mr. Potter's work with this firm had to do largely with the repairing of stationary engines, and while thus engaged at the plant of the Hartford Electric Light Company his interest in electrical apparatus was aroused. He foresaw that the development of the electric dynamo would have a marked influence upon the future of the steam engine, and this trend in his thinking led him to study dynamos and electric generating stations. Subsequently, he obtained employment with the old Thomson-Houston Electric Company at its Lynn, Mass., plant, where he met the late Charles A. Coffin and also Edwin W. Rice, Jr., both of whom were his professional associates for years afterward.

It was in 1887 that Mr. Potter began working for the Thomson-Houston Company. He acquired experience and knowledge very rapidly and soon was in the field correcting trouble in electric lighting plants in various places in Massachusetts and North Carolina. In the fall of 1889, having seen his first electric street car, in Washington, D. C., he returned to Lynn and became a Shop Student in electric traction work. This was again supplemented by practical experience in the field, particularly on the West End Street Railway Company's System in Boston, Mass., the Alamo Electric Railway in San Antonio, Tex., and others of less importance.

Later, in the Lynn plant, Mr. Potter aided in the development of the Thomson-Houston single-reduction motor for street cars, and when the late Walter H. Knight was appointed Engineer of the Railway Department, he became Mr. Knight's Principal Assistant. Mr. Potter was active in working out railway circuit breakers, fuse blocks, lightning arresters, and switches. The most serious problem in electric railway work at that period was the matter of an efficient controller for electric cars. Messrs. Knight and Potter both worked on this, and ultimately they solved the problem by the invention of the series-parallel controller, which became the basic method in use all over the United States from that time forward. It was a joint invention, which led to Mr. Potter being placed in charge of all control work for street railways.

In 1894 Mr. Potter was transferred to Schenectady, N. Y., when the main offices of General Electric Company were established. The following year he succeeded Mr. Knight as Engineer of the Railway Department and continued in that position until his retirement from active work in 1930. In that long interval Mr. Potter witnessed and aided materially the tremendous expansion of electric traction in the street-railway and interurban field, and also participated in the earliest installations of electrical equipment upon the lines of steam railroads.

The major projects with which he was associated were the electrification of the Manhattan Elevated Railway, of New York, N. Y., the electrification of the Grand Central Terminal, also in New York City, and of the Paris-Orleans Railway, in France. The projects concerning the Detroit Tunnel, the New York, New Haven, and Hartford, the West Jersey and Seashore, Great Northern, Southern Pacific, Butte, Anaconda and Pacific, and Chicago, Milwaukee, St. Paul, and Pacific Railroads, were also among the big installations with which he was prominently identified. More recently, he had been connected with the electrical engineering problems of the Reading, Illinois Central, Delaware Lackwanna and Western, and the Pennsylvania Systems.

Possibly his most valuable work within the General Electric Company during later years had been his organization and development of a great engineering staff for electric railway work. His supervision of the Department of which he was the head, was practically an unbroken stretch of harmonious relations, and the Department came to have a wonderful *esprit de corps* that many times won out against formidable odds. His knowledge of both men and engineering enabled him to develop and exercise a faculty for meeting all situations with foresight and fortitude. His personal energy was always extreme and his enthusiasm once aroused, carried him forward unflaggingly and imparted itself to others.

In the spring of 1896, Mr. Potter made the suggestion to Mr. Rice, which resulted in the establishment of the old General Electric test railroad, or test tracks, along the "berm bank" of the Erie Canal, stretching outward for about 1½ miles from the local General Electric plant. All tests of electric railway equipment were conducted on this track for many years until a new track was established at Erie, Pa. On this track also were tested early types of gas-electric rail cars, some of which later went into commercial service on Eastern railroad lines. Mr. Potter participated in many of these test trips, and had an active part in directing the development of the propulsion equipment for these cars.

He was one of the best known electric traction engineers in America. His career went back prior to 1890, when electric street cars were still struggling to overcome technical difficulties and public prejudice. His contributions to the electric traction art were substantial and long continuing. Many of them were covered by the 130 patents which had been issued in his name.

In Schenectady, Mr. Potter was interested for many years in community life and development. He particularly followed the fortunes of the Schenectady Civic Players. He was a man of various hobbies, which gave him a means on many occasions of delighting his friends. He was a consistent radio enthusiast from the early days of the art, and had done independent research and developmental work on a type of radio receiver.

He was a member of the American Institute of Electrical Engineers, the American Society of Mechanical Engineers, and the Society of Naval Architects and Marine Engineers. His clubs were the Engineers Club, the Transportation Club, and the New York Railroad Club, of New York City; the Mohawk Club, the Mohawk Golf Club, and the Edison Club, of Schenectady; and the Griswold Club, of Erie, Pa.

Mr. Potter had been a member of St. George's Lodge, F. and A. M., since July 8, 1897; he was also a member of St. George's Chapter; St. George's Commandery; and a Shriner.

The following is quoted from a tribute to him by Mr. John Winthrop Hammond, which was published in the *General Electric Review* for March, 1934:

> "Those who were much in contact with Mr. Potter invariably remarked his knowledge of both men and engineering. His breadth of mental range, great perseverance, and extraordinary diligence raised him in the minds of many to the height of a genius. But, unlike too many geniuses, his disposition was cheery and pleasant, his personality friendly to the point of right goodfellowship. He had a way about him that inspired his subordinates and encouraged his colleagues. He will be remembered for those rare qualities of thought which so endeared him to his associates and which were an inspiration to all those with whom he came in contact. These will far outlive his material contributions to the age in which he lived.
>
> * * * * * * *
>
> "Above all else he was a student of modern transportation and a promoter of transportation's usefulness to mankind. His gifts and aptitude were such that, as Owen D. Young observed in speaking with keen regret of his death, 'he could reach with one hand into the achievements and illimitable possibilities of research and with the other into the practical side of definitive business application'."

Mr. Potter died at his home in Schenectady on January 15, 1934, as the result of a heart attack. He had been confined to the house with a cold, but there was no intimation that his condition was at all serious.

In 1890, he was married to Loretta Harward, of Raleigh, N. C., and, in 1912, to Rose Hubbard, of Sandusky, Ohio. In addition to his widow, he is survived by a daughter, Mrs. Dorothy (Potter) Bird, of Kent, England, and a son, Harward Bancroft Potter.

Mr. Potter was elected a Member of the American Society of Civil Engineers on March 2, 1904.

HENRY SEWALL PRICHARD, M. Am. Soc. C. E.[1]

Died October 17, 1934

Henry Sewall Prichard, the son of Abraham and Frances (Sawyer) Prichard, of Boston, Mass., was born in Philadelphia, Pa., on June 26, 1859. He was of early Colonial ancestry and a member of the General Society of Mayflower Descendants.

---

[1] Memoir prepared by Marshall Williams, M. Am. Soc. C. E.

He attended the public schools of Philadelphia, and, upon the death of his father, he terminated his connection with prescribed schooling at the age of 15 years. This did not terminate his effort toward higher education, as during his entire life he was a profound student and an omnivorous reader, to the end that he made a distinctive place for himself in the Engineering Profession. During the seven years after leaving school he was occupied in clerical endeavors, during which time he devoted his spare hours to unremitting study.

From November, 1881, to January, 1884, Mr. Prichard was engaged with the Pennsylvania Railroad Company in various positions on railroad survey. location, and construction; during most of the time he was in charge of a party engaged in topographic surveys. From January 10, 1884, to 1887, he was a Draftsman in the Bridge Department of the New Jersey Steel and Iron Company, at Trenton, N. J. In 1887, he was engaged in private practice as a Consulting Engineer and as an Inspector of structural iron work, in Philadelphia. In the latter part of the same year, he re-entered the service of the New Jersey Steel and Iron Company, and was placed in charge of the Drawing-Room. In 1891, he was appointed Chief Engineer of the Company, and was serving in that capacity when the New Jersey Steel and Iron Company merged with the American Bridge Company in 1900. At this time Mr. Prichard was transferred to the General Offices of the Engineering Department of the American Bridge Company, at Wissahickon, Philadelphia, where he remained for a time. During the process of seething and sedimentation that usually occurs in the formation of a large organization from the personnel of a number of smaller organizations, he was transferred to the Accounting Department, where he was engaged in analytical and statistical work. He remained in that Department until his retirement on May 1, 1931. From December 1, 1915, to January 1, 1920, Mr. Prichard was Efficiency Engineer, and from January 1, 1920, to the date of his retirement, he served as Analytical Engineer of the Company.

During his service in analytical and statistical work, Mr. Prichard did not lag in his endeavor to keep up to the times in structural engineering advancement. He was keenly alive to all the progress of the day, as is evidenced by his frequent contribution of papers on engineering subjects and in his participation in discussions of the contributions of others. In 1917, he was awarded the J. James R. Croes Medal by the Society for his paper, "The Effects of Straining Structural Steel and Wrought Iron".[2] In addition Mr. Prichard presented three other papers before the Society, on "Insufficient Provision for Counterstresses in Railroad Bridges"[3]; "Faults in the Theory of Flexure, and an Epitome of Certain I-Beam Tests Made at Ambridge, Pa.";[4] and "Utilizable Capacity of Steel Members of Structures".[5]

In 1907, the Engineers Society of Western Pennsylvania awarded him a Gold Medal for his paper "Proportioning of Steel Railway Bridge Members".[6]

---

[2] *Transactions*, Am. Soc. C. E., Vol. LXXX (1916), p. 1429.

[3] *Loc. cit.*, Vol. XLII (1899), p. 547.

[4] *Loc. cit.*, Vol. LXXV (1912), p. 895.

[5] *Loc. cit.*, Vol. 89 (1926), p. 1213.

[6] *Transactions*, Engrs. Soc. of Western Pennsylvania, July, 1907.

This was the first of three Gold Medals awarded by that Society in a term of fifty-four years. In 1919, Mr. Prichard became a Member of the American Association for the Advancement of Science, and was elected a Fellow on May 21, 1925.

He was an indefatigable worker, clear-headed and careful, and was endowed with a wonderful intellect which absorbed and analyzed all human activities and beliefs. In all, Mr. Prichard was a many-sided man of keen perception, acute analysis, and apt application. He had a marvelous command of the English language, and his observations were couched in terse phrases that hit hard and meant much. He was a charming, honorable gentleman, who was always of interest to his friends and associates.

He was married to Bertha Bowers Wright, of Trenton, N. J., on October 29, 1889, and is survived by his widow, three daughters, and a sister, who mourn a devoted husband, father, and brother.

Mr. Prichard was elected a Member of the American Society of Civil Engineers on January 2, 1895.

JAMES RITCHIE, M. Am. Soc. C. E.[1]

DIED NOVEMBER 15, 1934

James Ritchie was born in Roxbury, Mass., on August 1, 1850, of a well-known family, his father having been an educator and, at one time, Mayor of that city. James Ritchie was a graduate of the Roxbury Latin School and, in 1878, of the Massachusetts Institute of Technology, Boston, Mass., with the degree of Bachelor of Science in Civil Engineering.

From 1878 to 1884, he was with Aspinwall and Lincoln, Engineers, of Boston, Mass., the Chicago, Burlington and Quincy Railroad Company, the United States Coast and Geodetic Survey, the Rich Hill, Mo., Water-Works, and the Mississippi River Commission, all as an Assistant Engineer.

In 1885 and 1886 Mr. Ritchie was Assistant Engineer of Parks in the City of Cleveland, Ohio, where he resided almost continuously thereafter and where he served as City Engineer and also for a time as Consulting Engineer for the Grade Crossing Elimination Commission of Cleveland.

From 1886 to 1889, he was engaged as Engineer of Construction for the Erie Railroad Company on second-track construction from Cleveland to Youngstown, Ohio. From 1889 to 1890 he was employed as General Superintendent of the McKeesport and Belle-Vernon Railroad. About this time (1887), Mr. Ritchie served as Instructor in Civil Engineering at Case School of Applied Science in Cleveland, and, later, as Chief Engineer of the Pittsburgh Testing Laboratories. During 1892 and 1893, he was Principal Assistant Engineer of the Cleveland, Cincinnati, Chicago and St. Louis Railway Company ("Big Four"), later a part of the New York Central System.

From 1893 for a number of years he was in private practice as Civil and Consulting Engineer, in Cleveland, and during this period he was engaged in the design and supervision of construction of dry docks for the American Shipbuilding Company, at Cleveland, Lorain, Ohio, and Buffalo, N. Y., and at Baltimore, Md., and Bay City, Mich.

---

[1] Memoir prepared by Wendell P. Brown, M. Am. Soc. C. E.

His clients also included the Aurora, Elgin, and Chicago, the Fort Wayne and Springfield, and the Cleveland, Painesville, and Eastern Electric Railways. At a later period, about 1910 to 1930, he was an Officer and Engineer for the National Foundation and Engineering Company which firm served as contractors, among other important undertakings, for the construction of the Cherry Street and the Ash-Consaul Bridges, over the Maumee River, at Toledo, Ohio, the Clark Avenue Viaduct, and the Baltimore and Ohio Railroad Bascule Lift Bridge, at Cleveland, and for the foundations of the Lincoln Memorial, at Washington, D. C. He was also engaged as an Engineer for the contractors for the Leader-News, Hanna, Bulkley, and the Federal Reserve Bank Buildings, in Cleveland.

Mr. Ritchie was married in 1886 to Mrs. Sarah Elizabeth Ruple, of Cleveland, who died in 1926, an estimable and cultured lady, long an active member of the First Baptist Church of Cleveland, of which Mr. Ritchie was a regular attendant.

He had been a member of the Cleveland Engineering Society for nearly fifty years and had served that Society as Secretary, Vice-President, and President.

His contributions to the publications of the American Society of Civil Engineers included a paper entitled the "Construction of the Lorain Dry Dock and Shipyard of the Cleveland Ship-Building Company,"[2] and discussions on bridges and dry docks.

Mr. Ritchie was elected a Member of the American Society of Civil Engineers on November 5, 1890.

CLARENCE MORD ROGERS, M. Am. Soc. C. E.[1]

DIED OCTOBER 7, 1933

Clarence Mord Rogers was born in Plainfield, N. J., on May 20, 1874, the son of David D. and Julia F. (Davis) Rogers. In September, 1874, he moved with his parents to Daytona Beach, Fla. After securing his primary and preparatory education, he entered the Rensselaer Polytechnic Institute, at Troy, N. Y., working his way through, and was graduated in 1903 with the degree of Civil Engineer. While at the Institute he was elected a member of the Rensselaer Society of Engineers.

After completing his studies at Rensselaer, Mr. Rogers became Assistant to F. A. Dunham, Engineer, at Plainfield, N. J., during 1895 and 1896. He also served as Assistant Engineer to the late Frank James Hubbard, M. Am. Soc. C. E., County Engineer, at Plainfield, in 1896, 1897, and 1898.

---

[2] *Transactions*, Am. Soc. C. E., Vol. XXXIX (1898), p. 323.

[1] Memoir prepared from information on file at the Headquarters of the Society.

In 1904, Mr. Rogers was Assistant Engineer of Union County, New Jersey, but in the fall of that year, he returned to Florida where, from January, 1905, to February, 1907, he was engaged on the Key West Extension of the Florida East Coast Railway, with headquarters at Miami. During this engagement he was, first, Assistant Office Engineer, preparing maps for the United States War Department; after September, 1905, he was Division Engineer on Key Largo, Lower Matecumbe Division, on right-of-way clearings, location of camps, rock cuts, and embankments. Later, from July, 1907, to January, 1908, he served as Resident Engineer on Long Key Viaduct of the Key West Extension, in charge of field engineering. From July to December, 1908, Mr. Rogers was Assistant Resident Engineer on the Knights' Key Bridge, on the line of the Florida East Coast Railway, with the same duties as on the previous job. His work for this Railroad Company, covering a period of three years, included some of the largest and most difficult concrete work done in Florida.

In 1907, he was appointed Locating Engineer, for the Florida Coast Canal and Transportation Company, on the location of the Canal from St. Augustine to Mayport, Fla., on the St. John's River, near Jacksonville. In 1909, he became Chief Engineer on the design and construction of the Daytona Beach (Fla.) City Water-Works, Mainland Plant (costing approximately \$50 000). From January to July, 1910, as Engineer, Mr. Rogers had charge of the surveying of 200 000 acres of land in Osceola County, Florida. In 1910 and 1911, he was Engineer in charge of the estimates for the Florida Coast Line Canal and Transportation Company.

Mr. Rogers, in 1911 and 1912, served as Chief Engineer in St. Lucie County, Florida, in charge of the laying of a system of hard-surface roads. In 1912 and 1913, he was Designing Engineer and Engineer for the construction of water-works, electric light plant, and a sewerage system for the City of Fort Pierce, Fla. In 1912 and part of 1913, he was employed as Designing Engineer for a system of canals for the drainage of 40 000 acres of land in St. Lucie County, and from November, 1912, to July, 1913, he was engaged as Designing and Chief Construction Engineer on a reinforced concrete bridge, 2 000 ft long and costing more than \$100 000, spanning the Halifax River, at Daytona Beach. In 1914, he was Engineer in Charge, for the Board of Public Works of Daytona Beach, on the construction of a complete sewerage system for that city.

Many other engineering projects were the products of his skill, including the Cocoa Bridge, at Cocoa, Fla.; and the Mantanzas Inlet Bridge, and the Fairview Avenue Bridge, at Daytona Beach (2 520 ft long). In his own community, he had served as Engineer for nearly seven miles of bridges. He was Engineer for the Drainage District of Vero, Fla., and the Halifax Drainage District, in Volusia County.

Mr. Rogers was a patriotic citizen as well as a successful engineer. When war was declared against Germany in 1917, he was preparing for military duty and, at the time of the signing of the Armistice, was enrolled for service. After the World War, he was appointed a Captain in the Officers Reserve

Corps, U. S. Army, Engineer Division. Mr. Rogers also served his own community in numerous ways most efficiently. For two terms he was a member of the local School Board, and had been City Engineer of Daytona Beach, his home town, from January 1, 1930, to January 1, 1932. He was a leader in the investigation of suitable sites for the Daytona Beach Airport. Many other local projects were aided and approved of him during the years of his residence and public life in the City of Daytona Beach.

During the incumbency of Governor Doyle Carlton, Mr. Rogers was appointed by the Governor to the Board of Commissioners of the Florida Inland Navigation. After serving the term of the appointment, he was elected to that position by the citizens of the State. He was Commissioner of his District at the time of his death. Not only efficient as a member of his profession, but interested in all public business and financial projects that might be beneficial to his home community, he was at the time of his death President of the Daytona Beach Bond and Mortgage Company; at one time a Director of one of the banks; and Surveyor for the New York Title and Mortgage Company, of New York, N. Y., the Mortgage Bond Title Corporation, of Baltimore, Md., and the Southern Title and Trust Company, of Daytona Beach. The positions held by Mr. Rogers, and for which he was ably qualified, were those of trust that required integrity of character. His opinions and his work as an Engineer were sought by National, State, and community leaders in all professions, as he was widely known as a man of the highest type of skill in his profession and one whose ethical standards were above question.

He was a Christian gentleman, and during 1933 he acted as Chairman of the Building Committee for the erection of the Seventh Day Baptist Church, in Daytona Beach, of which he was a consistent member. Mr. Rogers was a man who loved his home and family. In his death, the community and State have lost an outstanding citizen and Engineer.

He was married on August 23, 1905, at Troy, N. Y., to Louise F. Mannl, and to them were born the following sons and daughters: Mary K., Frances R., Elizabeth L., David E., Crosby U., Clarence M., Julia D., Lewis T., who died in infancy, and Ruth T. Rogers.

Mr. Rogers was elected a Member of the American Society of Civil Engineers on January 6. 1915.

HIRAM ABIFF SCHOFIELD, M. Am. Soc. C. E.[1]

DIED OCTOBER 18, 1934

Hiram Abiff Schofield was born in Elwyn, Pa., on May 8, 1860, a son of Hiram and Esther (Burke) Schofield. His early education was obtained in the public schools at Elwyn, Miss Walter's Private School, at Media, Pa., and the Lauderbach Academy, at Philadelphia, Pa. He finally entered the University of Pennsylvania, Department of Civil Engineering, from which he was graduated in 1883, receiving the degree of Bachelor of Science.

---

[1] Memoir prepared by Benjamin Franklin and Marshall Williams, Members, Am. Soc. C. E.

Almost immediately after his graduation Mr. Schofield obtained an appointment as Chainman with the Water Department of Philadelphia, on the survey made to secure a better water supply for the City, of which the late Rudolph Hering, M. Am. Soc. C. E., was in charge. During the few months that Mr. Schofield was engaged on this project, he advanced through the grades of Rodman and Levelman.

At the conclusion of this survey, his ability and reliability were so well established, that he was among the few who were retained. He was placed in charge of gauging streams and measuring the rainfall in the Pennsylvania area of the Delaware River water-shed, above Philadelphia. He also rendered valuable assistance in determining the relation between rainfall and stream flow.

In the spring of 1886 Mr. Schofield resigned from this position to accept that of Assistant Engineer on the location of the Washington and St. Mary's Railroad, in Maryland. At the completion of this work in October, 1886, he was engaged as Engineer in charge of surveys for the Tidewater Pipe Line Company. In 1887 and 1888, he was engaged as Engineer in charge of of the construction of the lower section of the Washington and St. Mary's Railroad.

In March, 1888, Mr. Schofield was made Resident Engineer for the Pittsburgh (Pa.) Testing Laboratory, associated with F. H. Lewis, M. Am. Soc. C. E. This Corporation was re-organized in 1895, and Mr. Schofield's ability and tact were so well recognized that he was made a member of the firm, and became Manager and Resident Engineer in charge of the Eastern District. This District included some of the largest steel fabricators in the United States, among which were Midvale, Phœnix, Bethlehem, Worth's, Lukens, Pencoyd, and others. It also included all the large Eastern cement manufacturing plants. It was a position requiring peculiar tact, judgment, and a thorough practical knowledge of the most important branches of engineering.

During his connection with the Pittsburgh Testing Laboratory, although many difficult problems in inspection came before Mr. Schofield for his judgment, his decisions were invariably fair to both owners and manufacturers, and were made almost always without causing friction or complaint from either party.

Mr. Schofield resigned from the service of the Pittsburgh Testing Laboratory on April 10, 1923, after thirty-five years of association with the Company as Manager, and engaged in business on his own account in the same line of professional service that had been his specialty for nearly all his professional career.

He was active in the practice of his profession until two years previous to his death, when owing to ill-health he was forced to retire. He was outstanding in his line of work; was of a cheery disposition, but always conducted himself with a dignity that made a lasting impression. He aimed constantly to protect his client; but his spirit of fairness and his diplomatic approach to all difficult matters of discussion, always resulted in an equable settlement of disputed questions that was acceptable and agreeable to all parties. This phase of Mr. Schofield's character impressed itself on the workmen with whom he came into contact at the various plants, and secured to him their respect and co-operation, and established his true worth. He was very earnest in impressing his assistants with these elements of his character, and was so successful that an inspector who served under him was considered fortunate.

Mr. Schofield was married on October 10, 1901, to Maude Crout, daughter of John J. and Susan (Beal) Crout, of Germantown, Philadelphia, Pa. He is survived by his widow and four children, the oldest of whom is a practicing attorney in Philadelphia.

He was an active member of the Masonic Order for many years, and a member of St. Mary's Commandery, Knights Templars.

Mr. Schofield was elected a Member of the American Society of Civil Engineers on October 7, 1896.

PAUL ALBERT NICOLAS SEUROT, M. Am. Soc. C. E.[1]

DIED NOVEMBER 18, 1934

In the death of Paul Albert Nicolas Seurot, M. Am. Soc. C. E., on November 18, 1934, the Engineering Profession has lost one of its most capable members, Montreal has lost a distinguished French citizen, and the Montreal Tramways Company an outstanding official. He leaves a host of friends and business associates to mourn his loss.

Paul Albert Nicolas Seurot was born in Paris, France, on August 26, 1869, the son of Henri Paul Marie Seurot and Pauline Josphini (Bouvier) Seurot.

He was educated at l'Ecole Turgot de Paris, and was graduated as Ingénieur diplômé with the "Certificate d'Etudes des Supérieures," in 1887. For the succeeding three years he was engaged in post-graduate work and as a tutor at l'Ecole Fénelon. In 1890, when 21 years of age, Mr. Seurot came to the United States, and was employed first by the late Charles M. Jacobs, M. Am. Soc. C. E., Consulting Engineer, as Rodman on triangulation and various surveys of New York Harbor. Subsequently, for five years, he was engaged with the Elmira, Cortland, and Northern Railroad Company, and the Long Island Railroad Company, successively, as Rodman, Draftsman, Instrumentman, and Chief of Surveying Corps, upon preliminary and relocation surveys, and as Assistant Engineer in charge of the construction of new lines; double-tracking of roadway; erection of bridges; and the renewal of wharves and piers for the Long Island Railroad System.

Between 1895 and 1908, Mr. Seurot as a much respected and valued member of the staff of Jacobs and Davies, Incorporated, Consulting Engineers, of

---

[1] Memoir prepared by W. McG. Gardner, Assoc. M. Am. Soc. C. E., with the co-operation of James H. Brace and J. Vipond Davies, Members, Am. Soc. C. E.

New York, was occupied in a wide variety of engineering enterprises. His activities comprised the analyses of many important projects—a proceeding for which his capabilities were eminently fitted—besides designing and supervising notable engineering construction, especially some large essential structures in and about New York City and the water-front. For these he prepared the surveys, designs, and reports, and likewise supervised the execution of the extensive plans for land reclamation and shore protection.

In connection with the development and extension of the Long Island Railroad, Mr. Seurot was associated with the dredging and wharf construction along the ocean frontage at Manhattan Beach, and in Montauk Harbor. In the interest of the same road he was engaged progressively upon the pumping plant of the Montauk Water Company—a subsidiary of the Long Island Railroad Company—and the design of the coffer-dams for a proposed railroad bridge, to cross the East River and Blackwell's Island to Long Island, at East 64th Street, New York City. Although this latter project was contemplated as an entry for the Long Island Railroad into the City of New York, it was never completed.

In the same interval of years, Mr. Seurot was occupied upon the design of the Ravenswood Tunnel, later owned and operated by the Consolidated Gas Company of New York, for conveying gas from Ravenswood, Long Island, under the East River, into the City of New York. He also assisted in preparing designs and propositions for a project that was submitted later by the Brooklyn Rapid Transit Company, for the Battery Tunnel, between Broadway, Borough of Manhattan, and Flatbush Avenue, Brooklyn, prior to the final award made to the Interborough Railroad Company.

Following these engagements, Mr. Seurot became Designing Engineer on the preparation of plans for the Atlantic Avenue Improvement of the Long Island Railroad, in Brooklyn. In part, this work concerned the construction of an underground subway and, in part, an elevated railroad structure, the entire length—5½ miles—of which was intended for the primary purpose of eliminating about ninety-two grade crossings in the streets of the Borough of Brooklyn.

In the same period of his active professional career, Mr. Seurot was engaged by the Dutton Engineering Company upon the design of basins, locks, lock-gates, and other improvements for the Erie Canal, and thereafter his services were retained by the new Panama Canal Company in its negotiations, in Washington, D. C., with the Interstate Commerce Committee.

Subsequently, he became Office Engineer for a considerable period on the tunnel construction of the Hudson and Manhattan Railroad Company, until he was assigned, by Mr. Jacobs, the Chief Engineer, to be Office Engineer for the North River Division of the Pennsylvania, New York, and Long Island Railroad, which Company at that time was proceeding with the construction of the North River Tunnels of the Pennsylvania Railroad Company. During the course of this engagement, Mr. Seurot was detailed, for a short interval, to make a study and report for the construction of a proposed tunnel under the Elizabeth River, between Norfolk, and Portsmouth, Va., in connection with a projected rapid transit plan which, however, was never executed.

In 1906, Mr. Seurot was detailed to proceed to France to make a study and to examine, as the basis for a report tendered to the Minister of Public Works, in October, 1906, on behalf of the governmental authorities of the Chemin de Fer de l'Etat, the feasibility of constructing a tunnel under the estuary of the Seine, at Tancarville. This project never reached construction.

While Mr. Seurot was in France upon this assignment, he was approached by Messrs. Dayde and Pille, Contractors for the construction of a section of the Paris Metropolitan Railroad under the River Seine and known as the Place de la Concorde Tunnel. The contract for this work was awarded to Messrs. Dayde and Pille, in 1907, and the firm retained Mr. Jacobs as Consulting Engineer. Thereupon, Mr. Seurot returned to France to assume the position of Resident Engineer on the execution of this important enterprise and remained there until 1911, when the shield-driven tunnel was completed successfully. His assignment being ended, Mr. Seurot returned to New York.

In the following February, 1912, Messrs. Jacobs and Davies decided to open an office in Montreal, Que., Canada, with Mr. Seurot in charge, and he continued to direct this Montreal Office for his principals until some time after the outbreak of the World War when the office was assigned to Mr. Seurot personally in November, 1915. From that date, he carried on a private consulting engineering practice.

In the twenty-six year period, from 1890 to 1916, Mr. Seurot had the opportunity of being engaged upon projects relating to almost every department of Civil Engineering and of an association with many of the most prominent construction men of the time. Besides his employers, these included Messrs. Austin Corbin, Alexander Cassatt, the late Samuel Rea, Hon. M. Am. Soc. C. E., the late Alfred Noble, Past-President, Am. Soc. C. E., and many others. In this period of strenuous activity, Mr. Seurot contributed his full share to the accomplishment of some of the most difficult and complicated engineering works that have ever been undertaken.

On his arrival in Montreal, Mr. Seurot was engaged as Chief Engineer of the Montreal Tunnel Company and was retained, successively, during the years, 1912 to 1916, by the City of Montreal, to report upon the location and to design a tunnel under the Lachine Canal to replace the Wellington Street Bridge; by the Canadian Northern Railway Company as an Expert Consultant during the condemnation proceedings and the suits for damages resulting from the construction of the Mount Royal Tunnel; and by the Cook Construction Company as its expert witness in its law case with the City of Montreal over a serious break in the Montreal Aqueduct which caused a water famine in that city for several weeks. He was also attached to the Canadian delegation sent to France by the Canadian Ministry of Trade and Commerce, and for some time acted as Professor of Mathematics in l' Ecole Téchnique, in Montreal.

Although granted exemption in 1889 from French military service and its requirements until the repeal of the Conscription Act, and although this exemption was sustained in September, 1914, immediately upon the declaration of war, Mr. Seurot volunteered his services in the office of the French Consul General in Canada and was appointed to supervise the award of contracts for the French Government to Canadian firms outside of Montreal. He was also engaged to act as an intermediary between French citizens in Canada and Military Headquarters. In this office he corresponded with and visited the French patriotic agencies in Canada, outside of Montreal, for the Department of Lands, Homestead, and Municipalities.

Subsequently, Mr. Seurot was requested to negotiate with the Canadian Government and the Bank of Montreal upon the purchase of Lewis machine guns sold by the Dominion of Canada to France. He was assigned to prepare a special report of the sale and export of asbestos and mica to Holland and the German-American refineries in New Jersey. Likewise, he edited other technical reports for the use of the French Minister of Defense, and was charged with the preparation of naval reports for the Office of Foreign Affairs, by the French Government. In 1916, he was dispatched on a special visit to France as attaché to the Canadian Economic Mission. He prepared an account for this Mission and reported upon his visit to the French front between Belfort to Rheims, dealing with the battlefields of Gerbeviller, Nancy, Toul, Chalôns, Termaise, Suiffe, and Epernay. While on this mission, Mr. Seurot organized assistance for the French prisoners of war who had enlisted from Montreal, and made arrangements with the Army Photographic Service in Paris for French propaganda in Canada. On returning to Canada from this mission, Mr. Seurot supervised the collection of aid for the French prisoners of war enlisted in Canada which he had arranged while in Paris, and was appointed representative in Montreal for the enlistment of American aviators volunteering for service prior to the entrance of the United States into the war.

Following the termination of the World War in 1918 and on the creation of the Montreal Tramways Commission by the Government of the Province of Quebec, Mr. Seurot was named its Chief Engineer and, until 1927, advised that body upon all engineering features relating to the operation of the Montreal Tramways Company's electric street railway system. This work involved the review of property valuations; the estimates, designs, and construction of track work, buildings, sub-stations, car barns, electrical equipment, and rolling stock; the study of transportation details, routes, schedules, loadings, and the analysis of reports upon traffic conditions.

In the autumn of 1927, Mr. Seurot was named Engineer of Rapid Transit for the Montreal Tramways Company, to prepare a concrete proposal with route plans, designs, and estimates, for the construction of an extensive underground railway system in Montreal, that would be completely co-ordinated with the surface railway lines. Although a comprehensive design was prepared, actual construction was never commenced owing to the abrupt change in the financial situation. Acting then in a Consulting Engineering capacity to the Management of the Montreal Tramways Company, in his office of Engineer of Rapid Transit, Mr. Seurot reported upon many phases of tramway operation, particularly those concerning street traffic congestion, the selection and operation of traffic light signals, the re-routing of car service, and the physical valuation of the Company's assets. He conducted a special survey of the City's transportation facilities in conjunction with representatives of the steam railroads, with a view to co-ordinating the Montreal Tramways Company's System with the proposed Canadian National Railway's terminal development. He co-operated with Engineers of the City of Montreal in organizing public works for unemployment relief and made a general survey for the selection of an adequate traffic-control light system for adoption in the Montreal Metropolitan Area.

During an active life, Mr. Seurot found time to write a number of important papers dealing with engineering problems. Among his published papers are the following: "The Seine Concorde Tunnel of the Paris Metropolitan" (*Minutes of Proceedings* of the Institution of Civil Engineers of Great Britain); "Sub-Aqueous Tunnelling" (*Proceedings*, Engineering Institute of Canada); "Fuel Conservation and Coal Briquetting" (*Proceedings*, Engineering Institute of Canada); "The Montreal Tramways System and Service at Cost Operations" (*The Electric News*); "Transportation and the Development of Modern Cities" (*Journal* of the Town Planning Institute); "Freezing as an Aid to Excavation in Unstable Material," in collaboration with J. H. Brace, M. Am. Soc. C. E. (*Transactions*, Am. Soc. C. E., Vol. LII (1904), p. 365; and the translation of French papers submitted to the St. Louis Engineering Congress (*Transactions*, Am. Soc. C. E. (1905), Vol. LIV, Parts A, C, E, and F).

Mr. Seurot was particularly active in the service of the French Colony in Montreal, being interested in many French patriotic and welfare societies. For thirteen years he presided as President of l'Union Nationale Française de Montreal. He was a Counsellor of the French Chamber of Commerce; Director of l'Alliance Française of Montreal; and, in 1923, served as a Counsellor of the French Department of the Exterior. In recognition of these valuable services to France and his devotion to her interests, the French Government awarded him the Gold Medal of Honor in 1920; made him an Officer d'Académie in 1923; and a Chevalier de la Légion d'honneur in 1927. For his contributions to engineering knowledge, he was honored by the Institution of Civil Engineers of Great Britain in 1912, by the award of the Telford Premium, and, in 1914, he received the Gzowski Medal as the highest award of the Engineering Institute of Canada, for his paper upon "Sub-Aqueous Tunnelling."

He was a member of the Institution of Civil Engineers of Great Britain, the Engineering Institute of Canada, and the Corporation of Professional Engineers, Province of Quebec.

In social spheres he was a member of the Canadian Railway Club, the Canadian Club of Montreal, the St. Denis Club, and the Montreal Orchestra Society. He was intensely interested in music and was a lover of animals, particularly dogs. He was devoted to his home.

In character, Mr. Seurot was gifted and capable, kindly and charitable, delighting to help others at sacrifice to himself. Throughout his life he loved and served his native country, France, although during most of its duration he was absent from her shores. By diligent study and wide experience he acquired a broad knowledge of every aspect of Civil Engineering, but especially that branch relating to the construction of underground tunnels in which he was recognized as an authority. He was skilled in the preparation of technical reports, expressing himself with equal versatility, exactness, and fluency in both the French and English languages. As a result, his services were constantly in demand for the translation of engineering papers and articles, from one language to the other, by the companies and societies with which he was associated. All his technical reports were meticulously thorough in their preparation, beautifully phrased, and absolutely reliable. Modest and reserved in mien, nevertheless he was tenacious in the fulfillment of his purposes. Diligent throughout his entire life, it was his constant pleasure to help others. In his passing, the Engineering Profession loses a most valued associate and his countless friends from every walk of life a loyal companion.

He is survived by his widow, formerly Catherine Laprelle, of Brooklyn, N. Y., to whom he was married on September 5, 1894. He leaves no children. He is also survived by his brother, Rene Alphonse Seurot, who is Chief Engineer of the Municipal Gas Works, Paris, France.

In religion Mr. Seurot was a Roman Catholic and attended the Church of St. Germaine, at Outremont.

He enjoyed motoring, but his chief interest in life lay in his technical engineering studies his French societies, and his philanthropies. On his death, l' Union National Française postponed the annual ball which Mr. Seurot had himself organized, and a reception which he had planned for the French Minister to Canada, as a token of its esteem to its President.

He is remembered by his associates and friends as a man of sterling integrity, and high attainments, one faithful and efficient in all his work, the possessor of a keen analytical mind and gifted and charming personality which captivated the warm regard of all those who entered into his public and private life.

Mr. Seurot was elected a Junior of the American Society of Civil Engineers on April 30, 1895; and Associate Member on October 4, 1899; and a Member on November 1, 1904.

GEORGE HARRY THORNTON SHAW, M. Am. Soc. C. E.[1]

Died April 17, 1934

George Harry Thornton Shaw was born at Lee Center, Ill., on June 14, 1869, the son of James Monroe and Mary Rebecca (Linn) Shaw, both of whom were born at Lee Center. His paternal grandparents had come from Western New York to Lee County, Illinois, by ox-wagon in 1836, and his maternal grandparents came a few years later from the State of Maine.

---

[1] Memoir prepared by E. E. R. Tratman, Assoc. M. Am. Soc. C. E.

His father, a farmer and village storekeeper, was a veteran of the Civil War, a member of the 13th Illinois Infantry, known as the "Fighting Thirteenth."

The primary education of Mr. Shaw was received in the Lee Center District School, which in the previous generation had been known as the Lee Center Academy. It was noted for its educational standing, and attracted students from a considerable territory. It still retained the traditions and good features imparted to it by its New England founders. In 1886 and 1887 and, again early in 1890, he took courses in Civil Engineering at the Northern Illinois Normal School, at Dixon, Ill. From November, 1890, to the spring of 1891, he was a Student in Engineering at Cornell College, Mount Vernon, Iowa; and from September, 1891, to March, 1892, he was a Special Student in Bridge Engineering at the University of Michigan, Ann Arbor, Mich.

Mr. Shaw began his engineering career with a wide range of experience, starting as a Rodman and then as Instrumentman on the Lake Shore and Michigan Southern Railway, being located at Dunkirk, N. Y., from 1887 to 1889. During part of this time he was in charge of maintenance of way. In 1890, he was Instrumentman on the double-tracking of the Chicago and Northwestern Railway, in Illinois, and in the spring of 1891 he was Instrumentman on the location of the Burlington and Missouri River Railroad, with headquarters at Plattsmouth, Nebr.

Mr. Shaw then went to Colombia, South America, and from April, 1892, to the spring of 1893, he was, successively, Instrumentman, Chief of Party, and Assistant Locating Engineer on the surveys for the Cauca Railway (later a part of the main line of the Pacific Railway). This line, of 3-ft gauge, starts from the Port of Buenaventura, on the Pacific Coast, and follows the River Dagua for about 52 miles to Dagua (or Caldas), at an approximate elevation of 2 700 ft. This part of the line had been begun in 1877. From Dagua, the survey fought its way by a developed line about 82 miles up the steep and rugged western face of the Cordillera to the summit at Cresta del Gallo (Elevation 5 220). The construction of this section was begun in 1907. The maximum grades are 4%, with numerous curves of 230 to 260-ft radius. From the summit, the line descended the eastern slopes by grades of 3% to Yumbo, 98 miles (158 km), at the foot of the range in the Cauca Valley, and thence to Cali (which was reached by the construction work in 1915), 108 miles (174 km), and at about the same elevation as at Yumbo. In later years, the line has been extended along the valley, but Mr. Shaw's survey ended at Cali, having effected the difficult crossing of the range.

The survey work was done under the unfavorable conditions which at that time obtained generally on projects in tropical countries, little consideration being given by promoters to health or sanitary conditions as affecting the effectiveness of the engineers. Thus, many of the force—natives as well as foreigners—were ill with fever much of the time. In addition, the climate was trying, especially around Buenaventura and in the Valley of the Dagua; dangers from snakes and wild animals were frequent; and there was continual threat of attack by natives.

Returning from Colombia about the end of 1893, Mr. Shaw settled at Dixon, Ill., and, until 1905, was engaged in private practice, which included the variety of work that falls to an engineering "general practitioner." There was no city engineer at Dixon, and Mr. Shaw handled practically all the municipal work, such as surveys, plans, and construction. His outside activities covered land drainage, subdivision surveys, interurban railways, and other miscellaneous work. He drained the swamps west of Necedah, Wis., laid out the City of Oelwein, Iowa, and was engaged on land drainage work in Florida.

During this 12-year period he was also, at different times, on the Klamath Falls Irrigation Project, in Oregon; laying out the Salinas Valley Irrigation Project, at King City, Calif.; the pumping and irrigation plant of the Southwestern Rice Company, at Sour Lake, Tex.; various rice-land irrigation projects near Beaumont, Tex.; the improvement of the canals of the Inlet Swamp Drainage District, in Lee County, Illinois; the laying out and construction of the Little Yellow River Drainage District, in Juneau County, Wisconsin; and the drainage and irrigation works of the Old Plantation Company, of Harvey, La. From 1902 to 1905, Mr. Shaw was directly interested in the development of electric interurban railways, as President of the Northern Illinois Electric Railway Company, of Dixon, Ill.; the DeKalb and Southwestern Electric Railway Company; and the Rock Falls and Southwestern Railway Company. Besides planning these three lines, he carried out the preliminary location and construction surveys for the Northern Illinois Railway.

In May, 1905, Mr. Shaw went to Mexico as Chief Engineer for the Mexico, Toluca and Pacific Railway Company, to find a practicable location for the proposed line which was to connect the City of Mexico with the excellent but isolated Port of Acapulco. This project was promoted by General Fernando Gonzalez, Governor of the State of Mexico, as a monument to himself and his family, but the project was approved also by President Porfirio Diaz and the National Government, although the expenses were borne mainly by General Gonzalez and the State Government.

Location of a practicable route was a difficult matter, owing to the wild and mountainous country and the steep descents to the Pacific Coast. Various preliminary surveys had been made, but they all included maximum grades of 3 per cent. Mr. Shaw's experience in locating the Cauca Railway under somewhat similar topographical conditions aided him greatly on this work, in the course of which he made many reconnoissance trips, practically alone.

As described by him in an article written for *Engineering News-Record* (April 26, 1928), he located three routes, all connecting with the existing lines out of the City of Mexico. They were, respectively, 306 miles long, with maximum grades of 3%; 322 miles, with 2% maximum grades; and

650 miles, with 2% maximum grades. This last route was adopted, as it opened up a new territory having great mining, agricultural, and timber resources, while the line had but little adverse grade in its descent to the coast. Beyond the City of Mexico it crossed the ridge at El Salazar, 10 000 ft above sea level, and then passed through Toluca, Temascaltepec, Tejupilco, Zacualpan, and Taxco; then following down the Balsas River, it reached the coast near Zihuatanejo, whence it kept along the coast to Acapulco. Although the grade limit was nominally 2%, the maximum used on the preliminary location was 1.75%, in order that on final location the grades could be compensated for curvature without exceeding the 2% limit.

By 1910, the final location had been completed, plans were drawn and approved, and arrangements were made to begin construction. Then came the Madero Revolution, and General Gonzalez, who was Chief of Staff for President Diaz, went with the latter on his retirement to Paris, France. All work on the railway, therefore, was stopped. In 1911, however, the project was taken up by English capitalists, and Mr. Shaw became Chief Engineer for the Mexican Pacific Railway (starting from Acapulco) and Chief Locating Engineer of the Balsas and Pacific Railway. By 1913, five miles of line were built and in operation from Acapulco, including a tunnel, one bridge, and some heavy grading, as well as temporary docks at Acapulco for the landing of construction material.

During this period, Mr. Shaw acted not only as Chief Engineer, but also had charge of the financing and other business affairs, which necessitated several trips to New York, N. Y., London, England, and Paris. Then came more revolutionary troubles, and the Zapatista guerillas became so hostile that the railway work was abandoned. It has never been resumed, owing to the continually troubled conditions and the numerous changes in the Government. In recent years, however, the National Government has been active in the promotion of highway development, and a highway from the City of Mexico to Acapulco has been completed, using part of the location and constructed grade of the proposed railway. Mr. Shaw was engaged on surveys for other railway projects in Mexico, but the Acapulco project was the most important.

In 1913, Mr. Shaw returned to the United States, and was engaged in consulting and general engineering practice, mainly at Chicago, Ill. In 1914, when unfriendly relations prevailed between the United States and Mexico, he was called to Washington, D. C., to furnish information to the War Department, owing to his intimate knowledge of Mexican affairs. Between 1917 and 1920, he was interested in the oil developments in the Southwest. In 1921, he became Locating Engineer for the Joyce Lumber Company, at Harvey, La., and, later, for the Tremont Lumber Company, of Minneapolis, Minn., which had lumber properties in Mexico and found it necessary to clear up titles to the lands. From 1922 to 1928, he was with the Guerrero Land and Timber Company, engaged in adjusting contested titles, making boundary contracts, and preparing maps of a 1 000 000-acre timber tract in the State of Guerrero, Mexico. In this work, he made many trips in the region practically unattended, although at that time the country was disturbed by a succession of revolutions. His intimate knowledge of the language and the people enabled him to get along without interference. In 1931, he became a Director in the Mexican Hardwoods Company, for which he carried on similar title adjustments and other business affairs.

After a year of private practice in Chicago (1928–1929), Mr. Shaw was appointed, in May, 1929, by Governor Emmerson to the engineering staff of the Illinois Waterway, a 9-ft navigable route under development from the Chicago Drainage Canal to the Illinois River and the Mississippi. This work was being built by the Waterways Division of the Illinois Department of Public Works and Buildings, with the late Lorenzo Dana Cornish, M. Am. Soc. C. E., as Chief Engineer. Mr. Shaw was Principal Assistant Engineer of the Marseilles Division, in charge of construction, until June, 1930, when he was made Acting Division Engineer. In February, 1931, he was transferred to Joliet, Ill., as Resident Engineer, in charge of heavy construction work, and the Joliet bridges. With a change in political administration in Illinois, Mr. Shaw resigned, in January, 1933, and soon after went on a short trip to Mexico, although he had no definite engagements.

Returning to the United States, he went to New Orleans, La., for the winter in order to escape the cold weather of the North. His health had been failing for some months, and in April, 1934, he became seriously ill. and was taken to a hospital, where he died on April 17, 1934.

Mr. Shaw was a man of quiet manners, far-sighted as to future developments and prospects, resourceful, and with a self-confidence that made him ready to accept heavy responsibilities. He was able also to inspire confidence in others. In his work abroad, he was singularly successful in dealing with people of all classes, owing to his thorough knowledge of colloquial Spanish and his consequent ability to converse with them in the vernacular; while, at the same time, he had an intimate knowledge of local manners and customs. With these abilities he was friendly with and trusted by those with whom he had dealings, whether peons or officials. Furthermore, reading and speaking Spanish fluently, in his examinations of property titles he was well fitted to delve into the old Mexican land records, some of which go back to the days of the conquest of Mexico by Spain, so that the titles are extremely involved and complicated.

On October 5, 1899, Mr. Shaw was married to Sarah Clark, at Lee Center, Ill. His wife died in August, 1932. He is survived by two children, Mary Gwen Shaw and Clarke M. Shaw; also, two brothers and a sister, Sherman L. Shaw, of Lee Center; Arthur M. Shaw, M. Am. Soc. C. E., of New Orleans; and Mrs. C. T. Leonard, of Chicago.

Mr. Shaw was elected a Member of the American Society of Civil Engineers on October 4, 1910. He was also a member of the Western Society of Engineers.

JOSEPH MANSFIELD SLATER, M. Am. Soc. C. E.[1]

DIED SEPTEMBER 14, 1934

Joseph Mansfield Slater was born at Van Wert, Ohio, on May 29, 1876, the youngest son of Mr. and Mrs. Joseph Slater, pioneer residents of Van Wert. He was educated in the public schools of that city and was graduated from the Van Wert High School, in 1895. He was then taken into the engineering office of Mr. G. L. McKibben, a Civil Engineer, where he was trained as an engineer and expert draftsman.

In August, 1899, Mr. Slater entered the service of the Pennsylvania Lines West of Pittsburgh. He was employed as Chainman and Rodman on double-track work, until September, 1900, when he was made Draftsman and Topographer on the change of alignment from Ravenna, Ohio, to Hudson, Pa.

From May, 1902, until September, 1903, he was engaged as Resident Engineer on the Wybark Branch of the Missouri-Kansas and Oklahoma Railroad (later, a part of the Missouri, Kansas and Texas Railway), on the construction of road-bed, bridges, and buildings.

Mr. Slater then went to work for the Wabash Railroad Company, starting as a Draftsman. He was promoted through the various grades, in rapid succession, to the position of Assistant Engineer, Maintenance of Way, on the Decatur Division, and, later, Assistant Engineer and Chief Draftsman at the General Office.

He resigned this position in May, 1908, to enter into partnership, with A. P. Poirot, Assoc. M. Am. Soc. C. E. This partnership operated as Engineers and Contractors under the firm name of Slater and Poirot, at Belleville, Ill., for approximately three years, principally on bridge design and construction.

In February, 1912, Mr. Slater was appointed Principal Assistant Engineer of the Wabash Railroad Company, continuing in that position until he was appointed Street Commissioner of the City of St. Louis, Mo., in 1915. He served in this capacity under Charles M. Talbert, M. Am. Soc. C. E., who was Director of Streets and Sewers, for four years; Mr. Talbert's successor, Mr. Clinton H. Fisk, for the following six years; and, subsequently, an additional eight years under the writer.

Mr. Slater's duties in this position made him responsible for the maintenance of approximately 1 100 miles of streets and 500 miles of alleys and sidewalks. Under his jurisdiction the work was carried on by the following sections: Bituminous Pavement, Block Pavement, Bridge Maintenance, Excavation Inspection, Harbor and Wharf, House Numbering, Refuse Collection, Garbage Reduction, Sidewalk Inspection, Stable, Street Cleaning, and Telford-Macadam. That his work was well done was proved by the fact that he served the City of St. Louis for a period of eighteen years under various administrations.

---

[1] Memoir prepared by Robert B. Brooks, M. Am. Soc. C. E.

Mr. Slater was an affable gentleman who enjoyed the highest respect and esteem of his associates in business and social life. He was a member of the York and Scottish Rite of Free Masonry, as well as a member of the Shrine. He was also affiliated with the Benevolent and Protective Order of Elks. He thoroughly enjoyed music and was one of the sponsors for the St. Louis Symphony Orchestra over a period of years. He enjoyed baseball immensely and took a great interest in both the American League and National League teams of St. Louis.

He was married in September, 1933, to Delia Smythe, who survives him.

Mr. Slater was elected a Member of the American Society of Civil Engineers on September 3, 1912.

KENNETH IRVING SMALL, M. Am. Soc. C. E.[1]

Died October 10, 1934

Kenneth Irving Small, the son of Frederick A. and Lydia (Trimble) Small, was born at Dexter, Iowa, on July 1, 1879.

Mr. Small received his early education in the public schools of Blue Island, Ill., to which place his family had moved in 1886. Later, he attended and was graduated from the Chicago, Ill., Manual Training School.

For a short time thereafter he worked as Draftsman on the design of special machinery and sprinkler surveys. In 1898, he entered the employ of the Chicago Bridge and Iron Works, which he left in 1901 to accept a position with Ralph Modjeski, M. Am. Soc. C. E., Consulting Engineer. Later, he went to Roanoke, Va., where he was in the service of the Virginia Bridge and Iron Company.

In 1904, Mr. Small returned to the Chicago Bridge and Iron Works, with which firm he continued until his death. His first major work after rejoining this firm was as Superintendent of the Drafting-Room, but he was shortly transferred to the Contracting Department, where his keen judgment as a Designer, coupled with his art as a Draftsman in the portrayal of his ideas, made his services invaluable. When, in 1910, the Company opened a branch office in New York City, Mr. Small's selection as Manager there naturally followed.

His earlier experience was mostly in the field of bridge construction, but he became much interested in the trend toward steel tank construction, and progressed so far as to become recognized as very much of an authority on all classes of plate metal design, particularly elevated steel tanks.

He was a master in the art of engineering, and was a man of good sense and sound judgment. A high sense of honor—an inflexible honesty—was his outstanding characteristic.

A cultured, sympathetic gentleman, of pleasing personality, Mr. Small was much loved by his associates. During the years many of the young engineers had opportunity to work under his tutelage, and every one so fortunate, benefited thereby. Without exception, they acclaim how much he gave them professionally—never too busy to study their problems and offer sound suggestions. His kindly manner and semi-humorous ways inspired confidence, and his counsel on the many intimate and personal problems of life was often sought.

---

[1] Memoir prepared by George T. Horton, M. Am. Soc. C. E.

He was fond of sports, particularly golf, and invariably excelled in those which captured his interest.

In 1902, Mr. Small married Elsie Irene Kauffman, who survives him. He also leaves three children—a daughter, Mrs. Leonard Crable Webber, and two sons (twins), Frederick Trimble and John Eggert Small. Another daughter, Eleanor, died in childhood.

Mr. Small's death occurred at Portland, Me. In some inexplicable manner, he was struck by what may have been a projection of some kind on a passing truck. The accident took place in the morning, and at mid-afternoon he died from the resultant skull fracture.

He was a member of the Masonic Fraternity, the Engineers' Club of New York, and the Oakland Golf Club.

Mr. Small was elected a Member of the American Society of Civil Engineers on July 14, 1930.

THEODORE SPEIDEN, M. Am. Soc. C. E.[1]

DIED APRIL 17, 1934

Theodore Speiden was born in Louisville, Ky., on March 25, 1881, the son of Theodore Speiden and Rosalie Mauzy (Latham) Speiden. After attending the public schools, the boy entered the University of Public Schools, at Louisville, from which he was graduated in 1899, with the degree of Bachelor of Arts. Subsequently, he took up the study of engineering. In later years, he studied law, and having passed the examination before the Board of Law Examiners of the State of Virginia, was admitted to the bar of that State on July 5, 1927.

From May, 1899, to June, 1914, Mr. Speiden was with the Louisville and Nashville Railroad Company, as follows: Until April, 1902, successively, as Rodman, Instrumentman, and Masonry Inspector, on the construction of yards, terminals, bridges, buildings, shops, and round-houses; from April, 1902, to July, 1905, he was Assistant Engineer in the Maintenance-of-Way Department; from July, 1905, to April, 1912, he served as Roadmaster on the Nashville Division in charge of maintenance of track and signals; and from April, 1912, to April, 1914, in addition to his work as Roadmaster of the Nashville Division, he served as Roadmaster and Manager of Construction, in charge of the Company's forces reducing grade under traffic and double-tracking. From April, 1914, to April, 1916, he was Division Superintendent of the Chattanooga Division, and from April, 1916, to July, 1918, he served as Assistant General Manager of the Nashville, Chattanooga and St. Louis Railway Company.

---

[1] Memoir prepared from information on file at the Headquarters of the Society.

In September, 1918, Mr. Speiden was appointed Terminal Engineer with the Port and Harbor Facilities Commission of the United States Shipping Board, at Washington, D. C. This was a war-time activity and his duties consisted of the preparation of preliminary studies effecting the design of port terminals. He retained this position until March, 1919, when he was appointed Supervisor of Tracks and Signals for the Washington (D. C.) Terminal Company, in its passenger terminals.

In April, 1920, Mr. Speiden went to the Baltimore and Ohio Railroad Company, as Division Engineer of the Philadelphia (Pa.) Division on maintenance of road, structures, and signals; and from July, 1922, to January, 1923, he served as Assistant Engineer of the Pittsburgh (Pa.) Division in charge of the reconstruction of a number of small concrete bridges.

As Valuation Engineer and Chief Engineer of the Illinois Appraisal Company of Chicago, Ill., Mr. Speiden was employed for approximately one year (1923–1924) on the appraisal of industrial plants and on the valuation of public utilities.

In 1924, he entered the service of the Southern Railway Company, as Office Engineer of the Piedmont District, with headquarters at Danville, Va. Mr. Speiden continued in this position until his death on April 17, 1934. He was on his way from Danville to Washington, D. C., when he met with an automobile accident from which he died in the Petersburg Hospital, Petersburg, Va. He was buried in Cave Hill Cemetery, at Louisville. Mrs. Speiden who was accompanying him, was also seriously injured.

Mr. Speiden was a Member of the American Railway Engineering Association. He was also a Member of the Baptist Church, and a Thirty-second Degree Mason.

He was married on November 23, 1915, to Olive Lee, of Nashville, Tenn., who survives him.

Throughout his professional career of thirty-five years Mr. Speiden was noted for his keen analysis of engineering and economic problems, and for meticulous accuracy in assembling factual data and in stating conclusions.

Mr. Speiden was elected a Member of the American Society of Civil Engineers on October 15, 1923.

FRANK JULIAN SPRAGUE, M. Am. Soc. C. E.[1]

DIED OCTOBER 25, 1934.

Frank Julian Sprague was born in Milford, Conn., on July 25, 1857, in a house which was reputed to have been the refuge of the English regicides in 1661. He was the son of David Cummings and Frances Julia (King)

---

[1] Memoir prepared by Robert C. Sprague, Esq., North Adams, Mass.

Sprague, and was a descendant of Ralph Sprague, who came to Salem, Mass., in 1628, from Upwey, Dorset County, England, was a member of the First Church of Boston, and one of the original settlers of Charlestown, Mass. The Sprague line contains many substantial New England citizens, but no man of scientific or inventive distinction other than Frank Julian Sprague.

His mother died in 1864, and Frank Sprague and his younger brother were taken to North Adams, Mass., and left in the care of a maiden aunt (later Mrs. Anne Parker), while his father sought fortune in the West. Mrs. Parker was a woman of the finest New England type and of striking beauty. Living in a modest, frugal way, as an occasional teacher, she devoted herself with great self-sacrifice and with sanity of judgment to her young charges. In July, 1932, there was published in the *Milford News* a reminiscent sketch of their early school life, by Mrs. Susan Amelia Shove. She said:

> "One day word came that sudden death had taken the mother of one of our little boys. Soon after, the father decided to move his family from Milford and the little fellow came for his books. I can see him now, a pathetic figure standing in the doorway, with spelling book, reader and slate under his arm, while we at the teacher's bidding all shouted in unison: 'Good-bye Frank!' That boy was Frank J. Sprague, seven years old, just my age."

Frank Sprague went to the North Adams Public School and, later, to Drury High School. He was a good student especially in mathematics, and he was urged by the Superintendent to enter a competitive examination held in Springfield, Mass., for entrance to the United States Naval Academy. He won this examination and borrowing $400 from various sources he started on his journey to Annapolis, Md.

In a recent paper[2] Mr. Sprague wrote of his first visit to New York City en route to the Naval Academy:

> "I landed here in '74. The New York of that day was not the great metropolis of the present. There were no bridges, no river tunnels and no subways. There were no telephones, electric lights, no electric cars or elevators. Transportation was by horse-drawn street cars or buses, while automobiles were still of the dream world. On the corner of 42nd Street and 5th Avenue was a great stone reservoir, and the vast territory running north of 72nd Street was largely barren and the home of goats and squatters.
>
> "I little dreamed that I should ever in any way be a factor in the city's growth, but determined to make the most of this, my first visit, I climbed half-way up Trinity steeple to get a panoramic view of the city. Now that territory is occupied by a forest of skyscrapers, and all one can see from that vantage point would be across the cemetery."

All through his life Mr. Sprague had a great love for the beautiful in art and architecture—this was early evidenced by his interested admiration of St. Patrick's Cathedral, in New York City, just finished but not open to the public. The green country boy of seventeen must have surprised the workmen by his urgent request to see the interior. When this was refused, he asked if there was any way he could get permission, and was told that only the architect or the Cardinal could grant it. Persistence, always one of his dominant characteristics, prompted him to ring the Cardinal's bell and ask the attendant Priest-Secretary if he could see the Cardinal. The kindly Cardinal McCloskey won his heart by his personal interest and granted the request, giving him a card which he proudly showed to the astonished workmen, so that he was permitted to roam through the Cathedral to his heart's content.

---

[2] *Proceedings*, A. E. R. A., August, 1932.

Four years of hard study at Annapolis followed and further developed his aptitude for mathematics, and a keen interest in Naval Architecture and Physics, the latter under the teaching of the great Admiral William T. Sampson, one of the Navy's most brilliant officers.

Then came orders to the U. S. S. *Richmond*, flagship of the Asiatic Squadron, detailed to pick up General Grant on his famous tour around the world, and young Midshipman Sprague acted as Special Correspondent for *The Boston Herald*. All the time he was feeling the urge for invention. He filled a notebook with detailed drawings and descriptions of different inventions; even then, he was dreaming of motors. Among these inventions were a duplex telephone, a time fuse, quadruplex and octoplex telegraph systems, a motor, means for transmitting pictures by wire, gyroscopic control of the mercury horizon and torpedo direction, regulation of incandescent lights, and a water-cooling apparatus.

Returning to New York in 1880, he began experimental work at Stevens Institute, at Hoboken, N. J., and the Brooklyn Navy Yard. In the months following he was privileged to work with Professor Moses G. Farmer, Government Electrician at the Naval Station at Newport, R. I., and to use the machine-shop equipment, resulting in the Sprague invention of the "inverted" type of dynamos and series-paralleling of armature circuits, in the summer of 1881.

Late in 1881, Ensign Sprague was ordered to the U. S. S. *Lancaster* and during this duty busied himself with installing the first crude electric call system in the Service. Early in 1882, came his coveted detail to the Crystal Palace Electrical Exhibition, in London, England. Here, Ensign Sprague was appointed a member of the Jury of Award and counted himself most fortunate to have the opportunity of association with so many eminent scientists. He was appointed Secretary of the Jury, making it possible to initiate series of tests of dynamos, electric lights, and gas engines, on which he made a report of such importance that it probably saved him from a court-martial for overstaying his orders for six months. This report, considered a most remarkable one for a young ensign of twenty-five, was in 1884 published in full by the Government, a report of 169 pages, with many charts and diagrams.

During his London stay he indulged in a wild dream for electrifying the Metropolitan District Railway, and using for conductors the tracks and a center-line over-head network, with universal under contact, the forerunner of the modern trolley.

In 1883, came his resignation from the Service, and a year's leave of absence, most of which time was spent with the late Thomas A. Edison, acting as Technical Assistant for some months, until his interest in the development of the electric motor caused his resignation. While with Mr. Edison, however, he had perfected a mathematical determination of electrical distribution which was of value to the entire electric-light development, in greatly increased accuracy and saving of time. This Sprague patent was assigned by him to the Edison Company.

In 1884, Mr. Sprague formed his own Motor Company, and became the electrician, office boy, treasurer, mechanic, and administrator, but he made good headway, and in the fall of that year had a well commended exhibition at Philadelphia, Pa. The new enterprise grew apace, with official endorsement of the constant-speed motors by the parent and other Edison Companies in the spring of 1895. After getting well started, Mr. Sprague's mind went back to the question of transportation. Then, again, came the opportunity when promoters in New York City got a franchise in Richmond, Va., to build and equip a railroad. He entered a bid and took a rather foolish contract to build the Richmond Union Passenger Railway, to be completed in ninety days, with payment of $110 000, "if satisfactory." This equipment was equal to that of the combined experimental work throughout the world. It was not child's play, but Mr. Sprague had a splendid crew of helpers, who knew no hours, and no cessation of work, who never lacked enthusiasm, and never lacked encouragement, and who were always faithful. The romance of the enterprise has been sung many times and is fairly recognized now as the prototype of the modern trolley, the beginning of modern electric transportation.

Within two years of its opening, one hundred and ten other electric railroads using Sprague equipment were in operation, or under contract, among them traction systems in Florence, Italy, and Halle, Germany.

Mr. Sprague then took up vertical transportation, the replacement of the steam elevator by the electric elevator. A crude experiment at the first large installation, that of the Postal Telegraph Building, in New York City, about 1893, was primarily responsible for the multiple-unit system of train control. There were constructed nearly 600 elevators, the equipment of the highest buildings as well as the largest installation—that for the Central London Railway—then the business was transferred to his largest competitor in a new combination.

Meanwhile, in 1895, what may fairly be called an inspiration—the multiple-unit system—had blossomed into full growth in theory and plan of practice. This made possible the combination of any number of equipped units, regardless of sequence or end relation, all controlled from any desired point, but for two years no effort on Mr. Sprague's part could win a chance to prove its possibilities. He had long since committed himself to the practicability of an electrically-operated underground system for urban transportation, and had often offered to back his belief that the first essential to determine was the method of operation, and that it must be electrical. Then followed another milestone, the successful equipment, under most difficult conditions, of the South Side Elevated Railroad, in Chicago, Ill.

In 1903 came duties as member of the Commission for Electrification of the New York Central Terminal. The invention of the Sprague Automatic Train Control followed in 1906, and the formation of the Sprague Safety Control and Signal Corporation some time later. During the World War further responsibilities came to Mr. Sprague as a member of the Naval Consulting Board, and the development of depth charges and delay-action fuses. In 1927, he invented the Dual Elevator System (for running two elevators, local and express, in a single shaft) and, in 1929, the Sprague Electric Sign.

Many distinguished awards were granted Mr. Sprague, and his work has been broadly recognized among fellow scientists and engineers. Among such awards were the Gold Medal by the Paris Electrical Exhibition, 1889; the Grand Prize, by the St. Louis (Mo.) Exhibition, 1905; the Edison Medal, by the American Institute of Electrical Engineers, 1910; and the Elliot Cresson Medal (1904) and the Franklin Medal (1921), by the Franklin Institute. Mr. Sprague was also honored by degrees by educational institutions, as follows: Doctor of Engineering by Stevens Institute; Doctor of Science by Columbia University; and Doctor of Laws by the University of Pennsylvania. He was an Honorary Member and Past-President of the American Institute of Electrical Engineers; Past-President of the New York Electrical Society, the American Institute of Consulting Engineers, and the Inventors Guild; an Honorary Member of the National Electric Light Association, and the Franklin Institute, and of the Engineers Club of New York City.

All through his life and up to his last day, Frank Sprague had a prodigious capacity for work, and once having made up his mind on a new invention, or new line of work, he was tireless and always striving for improvement. He had a brilliantly alert mind and was impatient of any half-way compromise.

For many years he maintained a beautiful country home in Sharon, Conn., and became an enthusiastic and expert amateur gardener. He had a great love of the country, particularly for flowers and trees.

Aside from his engrossing scientific work, he was deeply interested in art and music, and for many years was a regular subscriber to the various orchestral concerts, and the opera. He helped and encouraged many artists and musicians; he was, for a time, President of the School of Applied Design for Women, and served as a Trustee to the time of his death.

At the time of his seventy-fifth birthday, July 25, 1932, a most unusual tribute was tendered Mr. Sprague. Many former colleagues and friends had gathered together more than five hundred letters from friends and admirers all over the world, and these, together with autographed photographs of the writers, were mounted in six splendid large books. These were presented to him at a largely attended meeting at the Engineering Societies Building, in New York City.

In 1885, Mr. Sprague was married to Mary Keatinge, of New Orleans, La., who had one son, Frank Desmond Sprague. In 1899, he was married to Harriet Chapman Jones, of New Hartford, Conn.; they had three children, Robert Chapman, Julian King, and Frances Althea, wife of John Jerome Tucker. He is survived by his widow, four children, and seven grand-children. He was a generous, unselfish family man, and a most devoted, indulgent father. An unusually hard and demanding task-master, he still had the unwavering admiration, respect, and affection of his helpers and co-workers, some of whom had rounded out a quarter of a century with him. His interest in his work never ceased; only a few hours before the end, he asked to have a newly designed model of his latest invention brought to his bedside.

The *New York Herald-Tribune,* of October 27, 1934, two days after Mr. Sprague's death, published an editorial from which is quoted:

> "The death of Frank J. Sprague removes the third of a remarkable trio of American inventors who made notable the closing quarter of the last century. Perhaps no three men in all human history have done more to change the daily lives of human kind. Certainly no three have been responsible for greater progress in use of electricity. Edison, the telegrapher, made it possible for human beings to work or play at night with cheap electric light. Bell, the singing teacher, took the first steps in that vast development of telephony and radio which now makes it possible for one person almost anywhere in the world to converse with another person almost anywhere else. Sprague, the young naval officer, dreamed of railways and street railways run cheaply and safely by this motive power of electricity and made his dream come true. All three men might stand as models of the peculiar practical genius of America. There were electric lamps before Edison. There were telephones of a sort before Bell. There were electric motors before Sprague. But these three men made these three things work. * * * Not merely as father of the trolley, but of all transportation by electricity, his [Mr. Sprague's] name is firm."

Mr. Sprague was elected a Member of the American Society of Civil Engineers on June 1, 1904.

GUSTAVE ARNOLD STIERLIN, M. Am. Soc. C. E.[1]

Died May 16, 1934

Gustave Arnold Stierlin, the son of Gustave A. and Ida (Zuercher) Stierlin, was born in St. Gall, Switzerland, on May 23, 1866. He was graduated from the Polytechnical School, in Zurich, Switzerland, with the degree of Mechanical Engineer, in 1885, and served as Apprentice in the Locomotive Works, of Winterthur, Switzerland.

In 1887, Mr. Stierlin went to Sumatra to serve as Assistant to the Manager of the Krusi Tobacco Plantation, on drainage, roads, buildings, bridges, etc. Leaving Sumatra in 1892, he came to the United States and obtained employment with the George F. Ott Ice Machines and Refrigerating Company. He remained with this Company until 1897, when he resigned to become Assistant Chief Draftsman for the Pennsylvania Iron Works, in Philadelphia, Pa.

---

[1] Memoir prepared by A. Kusebach, Esq., Avalon, Pa.

In 1901, Mr. Stierlin went to Pittsburgh, Pa., as Designer of Power Plants with Mr. James Bryan, and, shortly afterward, became Chief Engineer of the Expanded Metal Fireproofing Company, in which capacity he remained until 1918. It was during this period that he gained his valuable experience in the field of concrete and reinforced concrete work, which he also had an opportunity to apply in his next position as Engineer and Treasurer of the E. M. Wichert Company, of Pittsburgh, designers of sea walls, pump houses, shafts, and subaqueous work. He held this position from 1918 to 1923, when he became Engineer in charge of all concrete work for S. Diesher and Sons, Consulting Engineers, in Pittsburgh.

In 1925, Mr. Stierlin entered the service of the Pennsylvania Railroad Company, and, in 1930, left that Company to become Assistant Engineer in the Bridge Department of the Planning Commission of Allegheny County, Pennsylvania, which position he held until his death.

He was a member of the German-American Technical Society, which was organized in 1888, and served as Secretary and, later, as President of that organization.

In November, 1894, he was married to Adele Jaquet. His deep devotion to his wife won him the admiration and respect of all intimate friends, and his pleasing personality made him a desirable member of any social gathering.

Mr. Stierlin was elected a Member of the American Society of Civil Engineers on August 4, 1924.

AARON HOWELL VAN CLEVE, M. Am. Soc. C. E.[1]

Died March 31, 1934

Aaron Howell Van Cleve was born at South Amboy, N J., on October 21, 1868. His father, Aaron Van Cleve, was also a native of South Amboy, and served in the Civil War as Second Lieutenant, 11th New Jersey Infantry, a Company of which he helped to organize. His mother was Ella (Muirheid) Van Cleve.

Aaron Howell Van Cleve was graduated from the Model High School at Trenton, N. J., at the age of fifteen, and from Lehigh University, at Bethlehem, Pa., in 1890, with the degree of Civil Engineer. He was elected to Phi Beta Kappa at the age of eighteen, and to Tau Beta Pi when he was graduated. He was always an able and an earnest student, and his scholarship was very high.

After his graduation from Lehigh University, Mr. Van Cleve was employed for two years by the Brooklyn Elevated Railway Company as Draftsman, Computer, and Field Assistant, in charge of damage suit surveys, and also of surveys for new lines.

---

[1] Memoir prepared by Frank C. Boes, M. Am. Soc. C. E.

In April, 1892, he entered the employ of the Niagara Falls Power Company as Draftsman, being continuously promoted until he held the position of Engineer, for both that Company and the Canadian Niagara Falls Power Company. Upon the completion of the important construction work in hand, at the close of 1908, the Construction Department was abolished and Mr. Van Cleve was appointed Consulting Engineer for these Companies, advising in regard to maintenance and betterments. During his seventeen years of service, he was engaged on the following works:

Niagara Junction Railway Company: Construction and maintenance of masonry, trestles, steel bridges, docks, buildings, and tracks.

Niagara Development Company: Building and maintaining a town containing 112 dwellings; this work included several miles of macadam road, sewerage and drainage systems, a sewage disposal plant, etc.

The Niagara Falls Water Works Company: Complete water-works system for a town of about 10 000 people and numerous factories, including steam pumping plant, hydraulic pumping plant, mechanical filtration plant, and stand-pipe.

The Niagara Falls Power Company: A hydro-electric plant with an estimated capacity of 110 000 hp; installed machinery, costing upward of $10 000 000; the discharge tunnel for this plant was about 1½ miles long, and each of the two wheel-pits was 180 ft deep and 500 ft long; the auxiliary work consisted of the development of the Company's extensive land holdings, and included transmission lines, conduit systems, paving, water-works, drains, and sewers; estimates on separate hydraulic power developments, aggregating $31 000 000, were also included in this work.

Canadian Niagara Power Company: A hydro-electric development having a capacity in tunnel, canal, and wheel-pit of 110 000 hp, five units having been installed at the close of 1908; the work included the design and construction of the power house, transformer houses, transmission lines to Buffalo, N. Y., miles of electric conduit systems, paving, water-works, etc.

Utica Electric Light and Power Company: A 6 600-hp hydro-electric plant at Trenton Falls, N. Y., with a concrete gravity dam about 60 ft high, penstocks, ¾ mile long, leading into the power house, which was made of stone masonry. Mr. Van Cleve was associated in the design and construction of this plant.

In connection with all these engagements, Mr. Van Cleve wrote the specifications covering the great variety of work involved. Most of this construction was let by contract, but he organized and had charge of the Company's forces engaged in building macadam roads, conduits, sewers, water-works, transmission lines, machinery, and gravity oiling systems, at a cost of $350 000. He had an intimate connection with the financial administration of both contract and Company work. Furthermore, in this important capacity, he wrote and let the contracts and laid out the accounting and cost-keeping systems, his talents being brought to bear on every phase of the operation, from preliminary design to completion of construction.

From 1909 until 1911 Mr. Van Cleve was engaged in private practice, specializing in hydro-electric work, making investigations and reports on developments involving large amounts of power. He acted as Resident Engineer and Manager on the construction of a 6 000-hp hydro-electric and pneumatic plant at Cobalt, Ont., Canada. He was also at this time Engineer for the City of London, Ont., in charge of paving, sewers, electric installation, and water-works.

From April, 1911, to October, 1916, he was Assistant to the Secretary of the Society, the late Charles Warren Hunt, M. Am. Soc. C. E., in which position he did executive work under the direction of Mr. Hunt. Here, Mr. Van Cleve lost no opportunity of being of the greatest possible and painstaking service to every member of the Society with whom he came in contact.

He left the employ of the Society to take charge of the design and construction of a large plant for the Sizer Forge Company, in Buffalo, N. Y., which Company was then engaged in making heavy marine forgings for the Allies, and was later taken over by the United States Government. The position was offered to Mr. Van Cleve by a former acquaintance who knew of the excellent work he had done for the Niagara Falls Power Company.

The plant of the Sizer Forge Company covered about 4½ acres, and included a large machine shop, storage yard, forge shop, and electric steel plant, fully equipped with the most modern heavy machinery of the time, the plant and equipment costing approximately $3 000 000. The new plant was built over and around an existing one which had to be kept in full operation day and night, seven days in the week. It was Mr. Van Cleve's duty to design the new buildings, supervise their erection, and install the complicated equipment involved. Later, he had charge of employment, inspection, shipping, purchasing, and maintenance. He was an Officer and Director of this Company for about two years.

It was during this connection that, imbued with patriotic fervor, he overtaxed his powers and undermined his health to such a degree that he contracted the illness that later caused his death. His loyalty to his country and to his Company would not permit him to spare himself for the sake of his own health during the time of war. He resigned in 1920, partly because of poor health, and partly to look after personal business matters. In October, 1920, he moved to Lynn Haven, Fla.

In the fall of 1921 Mr. Van Cleve was called in consultation on a proposed development of about 275 000 hp in the Lower Gorge of the Niagara River. In 1921 and 1922, he was a Director and Vice-President of the Lynn Haven Land and Plantation Company, taking an active part in its development and management. From 1924 to 1928, he was with the United States Engineers on a survey of the Tennessee River, as Hydro-Electric Expert on a dam project.

Mr. Van Cleve was co-author with Viktor Gelpke of "Turbines, Their Design and Installation," published in 1911. He was also the author of a paper[2] presented before the Society entitled, "The Hydro-Electric Development and Transmission Lines of the Canadian Niagara Power Company," as well as of discussions of other papers.

---

[2] *Transactions*, Am. Soc. C. E., Vol. LXII (1909), p. 199.

His business judgment, keen and far-seeing, was much sought after by his friends, often to their financial profit; Mr. Van Cleve, however, seldom used his knowledge and study of the general business situation, or of any particular security as a means of personal profit. He seldom "played the market." To him this was largely an interesting study—a means of diversion. Other small interests were the collection of sea shells and postage stamps. In these hobbies, he was never the mere superficial dabbler, but an earnest student who sought the scientific facts and the correct information concerning these interests.

Although he was comfortably well-to-do, with ample means to care for and educate his family, it is certain that he could have won a greater financial gain from this life if he had not held himself so sternly to his extremely high code of personal and professional ideals.

The only child of highly cultured and intellectual parents, he valued the society of refined and intelligent people. Mr. Van Cleve was characterized by a deep and sincere interest in the personal welfare of his friends, and he left no deed undone when it was possible to serve them in any way. Modest and self-effacing himself, he was loud in the praise of his friends, and his loyalty to them was practically without bounds, as the writer has good reason to know. In a letter written just prior to his death, occur these words, "As a money maker I have not been much of a success, but I have been wonderfully fortunate in my friends; it is a joy to reflect on them." Always the cultured gentleman, he yet took keen delight in a good story and he told many a one in his dry and whimsical manner with side-splitting effect on his hearers. His sense of humor was delightful, but usually carried a note of pathos.

On engineering subjects Mr. Van Cleve's language was precise and accurate, committed to paper only after careful personal investigation. His letters to contractors on projects with which he was connected, written to call attention to their lapses and shortcomings on the work, were often literary gems, much sought after by his associates when in search of merriment. Even his victims quoted from them with glee, and loved him for it.

His meticulous honesty was proverbial, and his itemized expense accounts when on trips for his employers aroused wonder and amazement in those accustomed to a more liberal interpretation of the term, "legitimate expenses." One such expense account, rendered after an important business trip for a wealthy corporation, during the World War, was framed by the auditor who received it. On it were such items as "carfare, 5 cents; sandwich, 10 cents; beer, 5 cents; hotel room, $2.00," etc. Seldom did he spend anything for Pullman travel and never anything for entertainment, lavish food, expensive hotel accommodations, or the like. Anything except bare necessities he considered should be met from his own purse. Inclined to be abstemious and careful on his own account, he was positively puritanical in his use of company funds.

Somewhat frail physically, he possessed real and great courage, for nothing and no one daunted him once he had decided what was the right course to pursue. For the same reason, there was no hesitancy in admitting an error in his own judgment, should subsequent circumstances indicate that one had been made. It goes without saying that such a man, even with the great personal charm which Mr. Van Cleve possessed, did not make intimate friends quickly, but those who were fortunate enough to be accepted by him, soon learned his real worth and valued him as a jewel among jewels. They will long honor the memory of Aaron Howell Van Cleve.

He died of heart trouble at Fort Myers, Fla., where he had been going during the winter season for several years, his health being such that he could not endure, without extreme discomfort, even moderately cold weather.

On July 2, 1894, he was married at Niagara Falls, Ont., Canada, to Agnes McKinnon, who was born there and who, with one son, Howell M. Van Cleve, of Geneva, N. Y., and one daughter, Mrs. Donald S. Fraser, of Panama City, Fla., survives him.

Mr. Van Cleve was elected a Junior of the American Society of Civil Engineers on May 31, 1892; an Associate Member on December 5, 1894; and a Member on November 1, 1904.

ALEXANDER WILSON VARS, M. Am. Soc. C. E.[1]

Died January 14, 1934

Alexander Wilson Vars, the son of Nathan Vars and Elizabeth (Wilson) Vars, was born in Dunellen, N. J., on February 24, 1873. His education was secured in the public schools of Dunellen, and of Plainfield, N. J.

Mr. Vars' first engagement in the engineering field was as a Rodman, in 1890, for F. A. Dunham, Civil Engineer, of Plainfield. He worked with Mr. Dunham until 1899 in various capacities, on real estate developments, street and highway improvements, sewers, water supply, and general municipal work.

From 1899 to 1906, Mr. Vars was affiliated with H. C. Van Emburgh, Civil Engineer, also of Plainfield, during which period he was engaged largely in municipal engineering work in its various phases, as well as in the construction of street railways. He assisted in the construction of the Union County, New Jersey, Road System and the Plainfield-Elizabeth Trolley Line.

In 1906, Mr. Vars became Town Engineer of Westfield, N. J., and served in that capacity until 1920. Under his direction, streets and sewers were built in the new section of the town, the disposal works were enlarged and modernized, the railroad grade crossing near the station was eliminated, and the adjacent areas were landscaped and beautified. During the latter years of that service, he also acted as Consulting Engineer for the City of Plainfield, as well as for the Borough of Somerville, N. J.

---

[1] Memoir prepared by Willard W. James, M. Am. Soc. C. E.

Mr. Vars was appointed City Engineer of Plainfield on February 2, 1920, and served in that capacity until his death. Many important engineering projects were handled under his direction, including large storm sewers, sanitary sewers, and extensive street widenings.

It is interesting to note that he was Town Engineer of Westfield, for fourteen years (1906 to 1920), and City Engineer of Plainfield, for fourteen years (1920 to 1934). Similarly, he resided in Dunellen for thirty years and in Plainfield for thirty years, having moved from Dunellen to Plainfield with his bride in 1903.

Mr. Vars was an honest, efficient public servant, modest but courageous. He was a man of strict integrity and square dealing. To know him was to love and respect him. His patience and tact in explaining complicated engineering projects to the many lay members of the governing bodies of the municipality for which he labored, could not be surpassed. As one prominent engineer so well said "for integrity, loyalty, and downright ability, I have yet to meet his peer."

The following quotation from an editoral appearing in the *Plainfield Courier-News* (which was quoted by the Rev. Alva J. C. Bond at Mr. Vars' funeral service), vividly pictures some of his qualities:

> "Because of his rugged honesty and his fairness toward friend and foe, he was outstanding. His devoted service to the City covered many administrations during which the growth of the City has been steady and its problems more and more complicated. Such a period makes great demands upon those occupying key positions such as his. Mr. Vars met those demands fully and capably.
>
> "A man of exceptional qualifications * * * he was ever zealous of the interests of the public and because of his faithfulness and probity he retained the respect and esteem of all who knew him."

In the words of Dr. Bond: "He was a man of logical mind and a retentive memory; he was conscientious and above reproach and his humor was a spontaneous one."

Mr. Vars was of a deeply religious nature, being an active member of the Seventh Day Baptist Church, of Plainfield, the services of which he attended the day before his death. He served that Church as Trustee and Treasurer and was a regular attendant at its services; he will be greatly missed by his many friends.

He was a member of the Board of Directors of the American Sabbath Tract Society; the Rotary Club; the New Jersey State Board of Professional Engineers and Land Surveyors; the Engineers' Club, of Plainfield, of which he served as Secretary for many years; and other organizations.

Probably the high spot in Mr. Vars' life, which left memories that he cherished to the day of his death, was a testimonial dinner that was given him by the Engineers' Club of Plainfield on November 6, 1931. Congressman Charles A. Eaton presided, and many glowing tributes were paid Mr. Vars at that time by outstanding leaders in engineering, industry, and law. He was presented with an engraved gold watch, the following resolution being unanimously adopted:

"*Resolved:* That the Engineers' Club of Plainfield assembled in recognition of the long service of Alexander W. Vars, its Secretary, in presenting him with a watch suitably engraved, does so express appreciation of his worth as a man, his engineering ability of high order, and his service to the community."

In response, Mr. Vars said in part: "A man can have no greater satisfaction than knowing that he has the confidence of his friends—and I have that satisfaction to-night."

He was married on October 6, 1903, to Mildred Titsworth, of Dunellen, who survives him, as does their only daughter, Esther.

Mr. Vars was elected a Member of the American Society of Civil Engineers on October 15, 1923.

ISAAC SPURR VOORHEES, M. Am. Soc. C. E.[1]

Died March 14, 1935

Isaac Spurr Voorhees was born in New York, N. Y., on September 18, 1872, the son of Gilbert H. Voorhees and Mary Ella (Spurr) Voorhees. He received his education at the Flushing High School, Long Island, N. Y., and at the Brooklyn Polytechnic Institute from which he received the degree of Bachelor of Science in Civil Engineering in 1901. He was a member of the Phi Kappa Psi Fraternity.

Following his graduation, Mr. Voorhees was employed by the Central Railway of New Jersey from June to August, 1901, as Inspector and Instrumentman. From September, 1901, to August, 1902, he served as Draftsman and Computer on the Fourth Avenue Subway, Rapid Transit Railway Commission of New York City.

From January to December, 1903, he was Engineering Aid, and from January, 1904, to March, 1917, Assistant Engineer, with the United States Reclamation Service, and was engaged as follows: January, 1903, to April, 1904, on triangulation and topographic surveys on the Salt River Valley and Gila Valley Project in Arizona; from May to November 1904, on topographical surveys, including the investigation of reservoir sites on the Malheur Project in Oregon; from December, 1904, to June, 1914, Mr. Voorhees was on the Klamath River Project, in Oregon, for two years on triangulation, lake, and topographic surveys, canal location, and in charge of canal construction, including the installation of structures; for seven years, as Principal Assistant to the Project Engineer, on estimates of design of gravity and pumping canal systems, rockfill and hollow concrete dams, flumes, bridges, buildings, and reinforced concrete structures; and for six months as Office Engineer. From July to December, 1914, Mr. Voorhees was Assistant Engineer in charge of construction of the Three Mile Falls Diversion Works, Umatilla Project, west extension; and from March, 1916, to March, 1917, he was with the Technical Section in the Denver Office on structural design.

---

[1] Memoir prepared by Earl T. Scott, Esq., Los Angeles, Calif.

In March, 1917, Mr. Voorhees was employed by the California Highway Commission, District VII, with headquarters at Los Angeles, where he served as Office Engineer until October, 1918, at which time he was made General Superintendent of Maintenance and continued in that capacity until March, 1924. From March, 1924 to June, 1925, he was Assistant Maintenance Engineer, and, after July, 1927, was District Maintenance Engineer, Department of Public Works, Division of Highways, in which capacity he served until his death. Mr. Voohees supervised and directed all maintenance work in District VII of the Division of Highways, including the preparation of estimates, control of expenditures, and the planning and execution of improvements.

Mr. Voorhees was a hard worker, exceptionally conscientious, and a thoroughly competent engineer. He was held in high esteem not only by the engineering and highway organizations, but also by the general public.

He was a member of the Oneonta Congregational Church, of South Pasadena, Calif.

In 1906, he was married to Jennie L. Newnham, who survives him, with two daughters, Edna Elizabeth and Madeline L. Voorhees.

Mr. Voorhees was elected a Member of the American Society of Civil Engineers on June 24, 1914.

JOHN DOUGLAS WALDROP, M. Am. Soc. C. E.[1]

Died August 20, 1934

John Douglas Waldrop, the son of Richard Woolfolk Waldrop and Virginia Robertson (Taylor) Waldrop, was born in Norfolk, Va., on November 16, 1887. His father was Traveling Passenger Agent for the Baltimore Steam Packet Company. His ancestry may be traced back through his grandfather, Christopher Waldrop, his great-grandfather, John Waldrop, to his great-great-grandfather, Alexander Waldrop, who came to Virginia from Scotland.

John Douglas Waldrop's early education was obtained at Mrs. Hunter's Private School, in Norfolk, from 1894 to 1899, followed by preparatory schooling at the Norfolk Academy, from 1899 to 1903. In September, 1903, he entered the Virginia Polytechnic Institute, at Blacksburg, Va., registering in the Mechanical Engineering Course; he went through the Freshman, Sophomore, Junior, and Senior grades, adding post-graduate work in the Senior grade, and in June, 1908, was awarded two degrees, that of Bachelor of Science in Mechanical Engineering, and that of Mechanical Engineer. His academic record was good and his conduct record excellent. He took a prominent part in student activities; was Fourth Lieutenant in the Institute Band; Literary Editory of *The Bugle* in 1904 and 1905; Secretary-Treasurer of the Norfolk-Portsmouth Club, in 1905 and 1906; Secretary-Treasurer of the Mechanical Engineering Club, in 1906 and 1907; and Vice-President of the Norfolk Academy Club in 1906 and 1907. He was a member of the German Club, and was on the Junior-Senior German Committee in 1905 and 1906.

---

[1] Memoir prepared by O. B. Bestor, M. Am. Soc. C. E.

During the session from September, 1908, to June, 1909, immediately following his graduation, Mr. Waldrop was Principal of Toms Brook High School, at Toms Brook, Va. He then began the practice of his Engineering Profession and pursued it continuously until his death.

From September, 1909, to June, 1910, he was employed as Draftsman by Benson and Drummond, Consulting Engineers, of Norfolk. His duties required field work as well, and included land surveys, real estate developments, paving, sea-wall construction, creosote timber inspection, machine design, and general drafting.

From June, 1910, to March, 1911, he was, successively, Levelman, Transitman, and Chief of Party, for the Richmond Cedar Works, engaged in the location of a lumber road across the Dismal Swamp, in Virginia. From June, 1911, to March, 1915, as Assistant Engineer, Maintenance of Way, for the Virginia Railway and Power Company, of Norfolk, he was in the office in charge of surveys, bridge and trestle design, maintenance of track and bridges, design of layouts for storage tracks and car barns, and also of work-pits and special track work.

Mr. Waldrop then moved to North Carolina, and turned his attention to highways. In April, 1915, he became County Engineer for Guilford County, in charge of the construction of hard surfaced roads. He prepared all specifications, contracts, estimates, and pay-rolls, and supervised the construction of twenty-three miles of highway, consisting of "Warrenite" surfacing on a concrete base. The Greensboro-High Point Highway, fifteen miles long, formed part of the program. This work occupied him for a little more than two years.

In July, 1917, Mr. Waldrop began his connection with the North Carolina State Highway Commission, in charge of a field party on the location of State roads in Guilford and Rockingham Counties. About seven months later, in February, 1918, he was promoted to the position of Division Office Engineer, at Greensboro, engaged in the preparation of road plans and estimates for Federal Aid projects, including office accounts and the inspection of road locations. Nine months later, in November, 1919, he was appointed Division Engineer for the North Carolina State Highway Commission, in charge of all State construction and location of highways in Division No. 2, embracing thirty-two counties, covering about sixty-five projects, at an estimated cost of about $5 000 000. He served in this capacity until May, 1921.

At this time, the first $50 000 000 State bond issue for roads was authorized, and all State roads were taken over by the State Highway Commission for maintenance. In the re-organization necessitated by the magnitude of the undertaking, the State was divided into nine districts instead of four, and Mr. Waldrop was appointed District Engineer for the Fifth District, with headquarters at Greensboro, in charge of all construction, maintenance, and the location of new highways, in the nine counties of the District, covering construction valued at about $10 000 000, and the maintenance of about 700 miles of highways. He continued as District Engineer for five years through the period of heavy highway construction, which was augmented by later bond issues.

In June, 1926, he was promoted to be Assistant State Highway Engineer, in direct charge of all State maintenance and surface treatment, and moved his headquarters to Raleigh, N. C. He served in that capacity until January, 1929, and was then made Acting State Highway Engineer from January to June, 1929.

In June, 1929, Mr. Waldrop was appointed State Highway Engineer, and continued as such until May, 1931. From May, 1931, until August, 1933, he was State Construction Engineer, with headquarters still in Raleigh. In August, 1933, he was re-appointed State Highway Engineer, and handled the responsibilities of that position during the last year of his life.

Mr. Waldrop was a member of the North Carolina Society of Engineers, and of the Raleigh Engineers' Club. He was also a member of the American Association of State Highway Officials, and was elected Vice-President of the Association at its Annual Convention in Pittsburgh, Pa., November 17–20, 1930. He was also a member of the American Road Builders' Association.

He took an active and prominent part in civic and social organizations. During his stay in Greensboro, he was a leading member of the Kiwanis Club, and served a term as President. His originality and energy are still vividly recalled by that organization, which, after he left for Raleigh, went on record that it would take six new members to fill his place. He was also a member of the Chamber of Commerce, the Merchants' and Manufacturers' Club, and the Greensboro Country Club. When he moved to Raleigh, he became a member of the Carolina Country Club, and the Capital Club.

John Waldrop was endowed with unusual natural gifts. An expert draftsman, he displayed artistic talent in assembling drawings and in lettering. He could letter equally well with either hand whether the letters were normal to his position at the drawing-board, or upside down.

He was a born actor, with an inimitable falsetto voice which he could use with telling effect on occasion. He appeared in a number of amateur performances in Greensboro and, later, in Raleigh, and was a joy to his audience, when he chose to take a comedian's part.

He was extraordinarily quick and alert, with marked ability to anticipate emergencies and prepare for them. He dispatched business with ease and promptitude, and his desk was practically always "clear". The same characteristics enabled him to carry the responsibilities of State Highway Engineer with capability and apparently little effort.

"John D" was a graceful and fluent public speaker. He delivered addresses on a great variety of subjects; among those relating to engineering subjects were, "The Surface Treatment of Earth Roads in North Carolina", "The Maintenance of the Highway System", "Concrete Highways", and "The Business End of a State Highway Department", delivered on sundry occasions; also, "The Surface Treatment of Gravel, Sand-Clay, and Dirt Roads", delivered before the American Association of State Highway Officials at its meeting in November, 1926, at Pinehurst, N. C.; and "Service Results from the Use of Asphalt in North Carolina", delivered at the Seventh Annual Asphalt Paving Conference, at New Orleans, La., in December, 1928. He wrote a short article on "Shoulder Widths, Construction and Protection", which was published in the January 2, 1930, issue of *Engineering News-Record.*

The Greensboro-High Point Highway, constructed under Mr. Waldrop's supervision when he was County Engineer of Guilford County, has been regarded as the beginning of the extension of modern paved highways into the rural sections of North Carolina, and as contributing to the creation of a public demand for good roads which resulted in the $50 000 000 bond issue of 1921. This road was widened when Mr. Waldrop was District Engineer, and now forms a link in State Highway No. 10, North Carolina's longest paved thoroughfare.

As District Engineer, during the great State highway construction program of 1921 to 1926, he developed an efficient and smoothly working organization. The monthly and final estimates for the payment of contractors, aggregating millions of dollars, forwarded to the main office, were found not only free from even the smallest errors, but indicated that he had kept the work within the costs determined by carefully prepared preliminary estimates.

Some of Mr. Waldrop's most effective work was done on committees and in conference. During the seventeen years that he was identified with the State Highway Commission, scarcely a single important conference upon details of highway construction and maintenance was held without his participation. If he had any particular predilection, it was for highway signs. Those in North Carolina were the products of a committee of which he was Chairman, and gave expression to his artistic ability. He stressed the advantage of pointers at cross-roads and road forks to strangers, and reminded his hearers that while they might know the way, even in the dark, their visitors did not. The early signs designed by him and installed throughout the State gave rise to the slogan that "it was as difficult to get lost in North Carolina as it was to find the way in some States".

Mr. Waldrop was an Episcopalian, and a member of the Church of the Good Shepherd, at Raleigh, N. C. He took special interest in the financial condition of the Church, and was active in drives for funds.

In 1914, Mr. Waldrop was married to Frances S. Barringer, daughter of Col. John Alston Barringer and Mattie Barringer, of Greensboro, who died about four years later. In November, 1924, he was married to Jenny Lind Penn, daughter of Mr. and Mrs. Thomas J. Penn, of Greensboro.

He loved his home, which was a delightful gathering place for his friends in the evenings. Handsome and debonair, with a certain jauntiness that not all the responsibilities of his office could suppress, he was an inspiration to cheerfulness and courage.

Mr. Waldrop died from injuries received from an automobile colliding with his car, at Newbern, N. C., while he was en route to join his family at the beach. His sudden passing was a profound shock to hosts of friends and associates. From the Governor of the State, from the Hon. E. B. Jeffress, Chairman of the State Highway and Public Works Commission, from the Secretary of State, and from others prominent in civic and business life, came messages expressing sorrow at his loss and the warm regard in which he was held.

He is survived by his widow and his two daughters, Virginia Penn Waldrop, aged 5½ years, and Eloise Taylor Waldrop, aged 7 months; and by two sisters, Eloise Taylor Waldrop, of Staunton, Va., and Mrs. Douglas S. Brooke, of Norfolk, Va.

"John D." commanded the admiration of all who knew him. As an Engineer, he attained the highest position in the great organization to which he had dedicated the vigorous years of his life. As a man, "he was the kindred of all clean and genial and earnest spirits. His talents were the pride of his friends; his personality was their delight."

Resolutions were passed by Division D and Division E of the State Highway and Public Works Commission, testifying to his worth to the State and to their sense of personal loss; and the following resolution was passed by the North Carolina State Highway and Public Works Commission:

"IN MEMORIAM: Upon motion by Commissioner Sigmon, seconded by Commissioner Miller, the following resolution was adopted:

"The Commission wishes to express its sense of profound loss at the tragic death of State Highway Engineer Waldrop on August 20, 1934, which was caused by an automobile accident.

"Mr. Waldrop has been with this Commission since its organization, and in the various capacities in which he has served he has given a service that has always been inspired by high ideals, excellent capacity, and a fine sense of loyalty. The Members of this Commission shall miss him as a co-laborer, and as a cherished friend, and they desire to express to his wife and the other members of his family their deepest sympathy.

"It is ordered that this resolution be spread upon the minutes of this Commission, and a copy be furnished Mrs. Waldrop and the press of the State".

The resolution passed by the Kiwanis Club of Greensboro is as follows:

"*Whereas*, the whole State was shocked and grieved by the tragic death on August 20, 1934, of John D. Waldrop, Chief Engineer of the State Highway and Public Works Commission, a man so skilled, efficient and courageous in his profession that he had rapidly advanced to the head of the Engineering Department of the State, and yet so genial, courteous and friendly that he was loved by all with whom he came in contact; and

"*Whereas,* he was long a member of the Greensboro Kiwanis Club and perhaps its most beloved *ex*-President, and so faithful to it and its ideals that he was made an honorary member when it became necessary for him to move his residence; and

"*Whereas,* we have been deprived of a friend, and the Community of a valuable citizen:

"*Be it Resolved* by the Greensboro Kiwanis Club:

"First: That we pause a moment with bowed heads as a memorial of respect and love for our departed brother;

"Second: That this Club, through its Secretary, express to the family of the deceased its sympathy;

"Third: That this resolution be spread upon the record of minutes of this meeting."

The following resolution was passed by the Raleigh Engineers Club:

"We, the members of the Raleigh Engineers' Club, are deeply moved by the passing of our beloved member,

JOHN DOUGLAS WALDROP

and record our testimony as to his work and worth.

"We deplore his loss because of his ability as an engineer, his leadership, and his high character as a man and citizen.

"The State of North Carolina has lost a valuable public servant, the State Highway and Public Works Commission an exceptionally capable and efficient executive, and we a prominent member. His counsel and advice will be sorely missed.

"We extend our sincere sympathy to his family, and join in their grief; direct that a copy of these resolutions be sent to them; and that a copy be spread on the minutes of the Club."

Mr. Waldrop was elected an Associate Member of the American Society of Civil Engineers on April 25, 1921, and a Member on January 17, 1927.

EDWIN JOHN WEIGAND, M. Am. Soc. C. E.[1]

DIED JUNE 12, 1934

Edwin John Weigand was born of German parentage, at Cincinnati, Ohio, on November 8, 1896, where his family resided until he was five years of age. They then moved to Dayton, Ohio, which remained his home until March, 1923, when his work took him to Louisville, Ky., where he lived until his death on June 12, 1934.

Educated in the public schools of Dayton and at Stivers Manual Training High School of that city, from which he was graduated in 1914, Mr. Weigand entered the Ohio State University, at Columbus, Ohio, in September, 1914. Completing his education there, he was graduated with the degree of Bachelor of Science in Civil Engineering in June, 1918.

---

[1] Memoir prepared by Woolsey M. Caye, M. Am. Soc. C. E.

At the outbreak of the World War and almost immediately after graduating from the Ohio State University, he enrolled in the United States Naval Reserve Forces, and entered the United States Navy Steam Engineering School, Stevens Institute of Technology, at Hoboken, N. J., in July, 1918. Upon completing the course, he was appointed, December 21, 1918, an Ensign (Engineering) in the United States Navy and served aboard the U. S. S. *Kittery,* as Junior Watch Engineer in Overseas Transport Service. He was released from active duty on January 18, 1919, and on September 30, 1921, he was honorably discharged.

Following his World War service, Mr. Weigand, in March, 1919, was employed by C. D. Putnam, M. Am. Soc. C. E., Landscape Architect, at Dayton, Ohio, as Instrumentman on surveys until October, 1919. Severing this connection at this time, he accepted employment with the Miami Conservancy District, at Dayton, as a member of the Headquarters Engineering Staff where he remained until February, 1923, as an Assistant Engineer on this flood-prevention undertaking. His work with this organization was on the design of steel and concrete bridges spanning the Great Miami River, flood-control structures, and hydraulic studies in conjunction with the flood-prevention work.

Leaving Dayton in March, 1923, he accepted a position as Junior Engineer with the Commissioners of Sewerage, of Louisville, Ky., on sewer design. On January 1, 1925, he was promoted to be Senior Engineer in the Engineering Department of the Commission and on May 17, 1933, he was made Designing Engineer, which position he held until his death. The work of Mr. Weigand, while with the Commissioners of Sewerage, was directing the design of large trunk sewers, and, as head of the Design and Drafting Department, it was under his supervision that hydraulic experiments and earth load tests as effecting the design of large sewer structures were conducted, including the necessary comprehensive preliminary investigations made in connection with a large and extensive sewer-construction program which has extended over a period of fourteen years.

Mr. Weigand at the time of his death was a member of St. Peter's Evangelical Church, of Louisville. He was also a member of Stillwater Lodge, No. 616, F. and A. M.; Victor Chapter, No. 210, R. A. M.; A. and A. Scottish Rite, and Antioch Temple Shrine, all of Dayton. He was also a member of Phi Beta Chapter of Alpha Tau Omega Fraternity.

He is survived by an only living relative, his mother, Mrs. Louise Weigand, of Louisville. His father, John J. Weigand, died on September 14, 1924. The remains of Edwin John Weigand are interred in Woodland Cemetery in the family burying lot in Dayton.

His death terminated a career of great promise and a life that reached high levels in the esteem of his many personal and professional friends, associates, and fellow men.

Mr. Weigand was elected a Member of the American Society of Civil Engineers on February 23, 1932.

PAUL WILLIS, M. Am. Soc. C. E.[1]

Died January 30, 1935

Paul Willis was born in East Orange, N. J., on December 21, 1863, the son of George Hall Willis, of Portland, Me., and Harriet (Hammond) Willis, of Boston, Mass. After his preparatory schooling, he attended Stevens Institute, in Hoboken, N. J., and on his graduation therefrom in 1885, he entered the employ of the Wallis Iron Works, of Jersey City, N. J. Not long thereafter, he joined the staff of the late George Shattuck Morison, Past-President, Am. Soc. C. E., as Engineer and Draftsman, also acting as Inspector for many of the large bridges then being built in the historic plants at Athens, Pa., and Edgemoor, Del.

In 1890, with Messrs William Morava and F. W. Barker, Mr. Willis organized and became Secretary and Chief Engineer of the Kenwood Bridge Company, and from the first was its guiding spirit. This was a highly successful organization from its beginning to its close in 1925, when it was absorbed by the McClintic-Marshall Cmpany, which was doubly fortunate, as it at that time secured not only the business but also the services of Mr. Willis, as Vice-President in charge of the Chicago, Ill., District.

When the McClintic-Marshall Company became a subsidiary of the Bethlehem Steel Corporation, in 1931, Mr. Willis retired from active business to devote his time to civic pursuits.

In the forty years of his career his work was confined principally to the construction of mill and factory buildings, and the many fine structures erected for practically all the outstanding firms in and about Chicago bear mute testimony to his ability as a builder.

Although so active in business, he managed more nearly to approach a perfectly balanced life—simple, yet complete—than do most men, for he did not allow his professional interests so completely to absorb him that he could not reserve for himself annual and regular seasons for recreation. His host of friends will miss him at golf and at bridge, both of which he played remarkably well.

His church—St. Paul's—will long remember its beloved Senior Warden. Kindly, generous, charitable, Mr. Willis gave freely and gladly of his time, attention, and financial aid to charity, church, and society. His firm handclasp and friendly smile have many times cheered those—and they were legion—who were fortunate enough to call him friend.

He was a man who can ill be spared in this day that sees the passing of his school. His genial manner, great personal charm, and outstanding integrity made him not only invaluable to his companies, but won him the personal confidence and friendship of every one with whom he came into either business or social contact.

---

[1] Memoir prepared by George T. Horton, M. Am. Soc. C. E.

A pleasant acquaintance; a loyal friend, sought out with confidence by all who were in trouble or in need of helpful counsel; a perfect host; an ever-welcome guest—Mr. Willis might be called the Twentieth Century personification of Chaucer's "parfit gentil knight"— a truly Christian gentleman.

He was married, in 1900, to Daisy Hubbard, who survives him, together with three children, a daughter, Marian (Mrs. Rufus R. Jeffris), and two sons, George Hall, and Paul.

Mr. Willis was elected a Member of the American Society of Civil Engineers on October 2, 1901.

HERBERT ALVA WILSON, M. Am. Soc. C. E.[1]

Died May 7, 1934

Herbert Alva Wilson, the eldest son of Alva J. and Helen Frances (Hagar) Wilson, was born in Brighton (later a part of Boston), Mass., on November 27, 1870. He was educated in the public schools of Boston, and immediately upon his graduation from the Brighton High School in 1889, he began his engineering work as a Rodman in the City Surveyor's Office.

From February, 1890, to January, 1910, Mr. Wilson was with the Engineering Department, City of Boston, as Transitman and Assistant Engineer. Meanwhile, he continued his technical education for three years in the city's evening schools, after which for four years he studied civil engineering in the Lowell Institute evening courses at the Massachusetts Institute of Technology.

During his twenty years of service in the City Engineer's Office, Mr. Wilson was engaged principally on the construction of bridges and water-front structures. Among the tide-water crossings built under his charge were the Malden Bridge, between Boston and Everett, Mass., the Chelsea Street Bridge between Boston and Chelsea, Mass., and the Essex Street and the West Boston Temporary Bridges between Boston and Cambridge, Mass., all of which were over navigable waterways and were provided with draw-spans. He was also in charge of the construction of several other municipal highway bridges spanning railroads. His water-front work was varied and included the building of wharves, sea walls, and bulkheads; dredging; construction of coal storage plants, including coal-pockets, trestles, towers, and unloading machinery; and the building and maintenance of landings, slips, and other facilities of the municipal ferries. In this particular field of water-front work he came to be recognized as a highly competent engineer.

Mr. Wilson resigned from the City's service in January, 1910, to enter private practice as a Consulting Civil Engineer, specializing in water-front work, with an office in Boston. Included among his clients were steamship companies, coal companies, one of the New England railroads, and industrial firms. He continued in private practice until administrative work as a public official demanded his sole attention.

---

[1] Memoir prepared by Frederic H. Fay, M. Am. Soc. C. E.

It was in the field of public service that Mr. Wilson came to be most widely known. He exemplified the engineer mindful of his civic duties and responsibilities. Always deeply interested in community welfare, he was long active in neighborhood improvement associations and other civic organizations. His political career began in the fall of 1912 with his election as Representative from the Brighton District in the Massachusetts Legislature, where he served continuously for six years—four years (1913, 1914, 1915, and 1916) in the House of Representatives, and two years (1917 and 1918) in the Senate. In the Legislature he served at different times as Chairman of the Committees on Banks and Banking and Metropolitan Affairs and of the Special Committee for Redistricting the State, and as a member of the Committees on Rules, Water Supply, Education, and Social Welfare. His earlier engineering work in Boston peculiarly fitted him for service, in 1915 and 1916, on the Boston Terminal Commission, composed in part of members of the Legislature and in part of members appointed by the Governor and the Mayor of Boston. The comprehensive report of this Commission upon recommended co-ordination and improvement of railroad and water-front facilities at Boston, made after a thorough technical investigation of conditions there and in other principal cities of the United States and Canada, toward which Mr. Wilson gave constructive and valuable assistance, was an outstanding public document and an example of wide, long-range planning for public improvements.

More than seventeen years of Mr. Wilson's life were given to public service, six years as a Legislator and nearly twelve years to administrative work. In August, 1918, he was appointed Building Commissioner of Boston by Mayor Andrew J. Peters. This position he held until April, 1922, when Governor Channing H. Cox appointed him as Police Commissioner for Boston. He served as Police Commissioner for eight years until he resigned in May, 1930.

Mr. Wilson became a member of the Boston Society of Civil Engineers on May 19, 1897. He was also a member of the Massachusetts Society, Sons of the American Revolution; the Sons of Union Veterans of the Civil War; the Republican Club of Masachusetts; and the Roosevelt Club of Boston; a Corporator and Trustee of the Brighton Five Cents Savings Bank and a member of its Investment Committee; Past-Master of Bethesda Lodge, A. F. and A. M.; and a member of the First Parish Brighton (Unitarian) Church.

He was married, in 1896, to Georgia D. Livermore, of Brighton, and they resided in the Brighton District of Boston until Mr. Wilson's death, after a protracted illness, on May 7, 1934. Quiet and unassuming, genial and kindly, Herbert Wilson gave a lifetime to useful, loyal service and gained a wide circle of steadfast friends.

Mr. Wilson was elected a Member of the American Society of Civil Engineers on October 1, 1912.

GEORGE MARSHALL AMES, Assoc. M. Am. Soc. C. E.[1]

Died January 16, 1935

George Marshall Ames was born at Elgin, Ill., on July 18, 1858, the son of Bela and Martha M. Ames. Three years later, his parents moved to Michigan, and settled on a farm between Pokagon and Cassopolis. Educated in the rural schools of Cass County, and then at the Northern Indiana Normal, he taught two school terms before entering the University of Michigan in 1881 as a student in Civil Engineering. He remained in college for four years making an excellent record, but left in June, 1885, to work on a Michigan Central Railroad survey between St. Clair and Detroit, Mich., without submitting the thesis which was then required. Years later, in 1914, in recognition of his eminently successful professional career Mr. Ames was given his degree as of the Class of 1885.

A special survey of Grand River for the City of Grand Rapids, Mich., from August to December, 1885, was Mr. Ames' first engagement in his future home town to which he gave so many years of service. In 1886, he was employed as Levelman on a location survey for the Atchison, Topeka, and Santa Fé Railroad Company, but soon afterward returned to Grand Rapids as Assistant City Engineer.

In 1894, legislative action fixed the dock lines along Grand River. Mr. Ames surveyed their location in the field, and the preservation of the river's natural beauty engaged his persistent effort ever after. He served as City Engineer of Grand Rapids from 1896 to 1900, and wrote the first specifications for concrete curbing which at 25 cts per ft replaced Medina sandstone at 45 cts. Pioneering for increased efficiency, he sanctioned the first local use of concrete for sewer construction. He never gave up his interest in civic building and guidance, serving on many commissions, but never in an elective office.

Mr. Ames left the employ of the City in 1900 to purchase the interest of William J. Hayden, Building Contractor, in the firm of Hauser, Hayden and Owen. Later, this leading building firm was known as the Owen-Ames-Kimball Company and constructed many of the largest buildings in Grand Rapids and Western Michigan. Mr. Ames' particular aptitude was in supervising outside construction; but because of a heart ailment he was compelled to give up any physically strenuous activity after 1920, although he retained undiminished interest in official counsel to his firm.

Without complaint he said of these later years that he was doing "the things he had always wanted to do." He served a variety of commercial interests—Vice-President of the Owen-Ames-Kimball Company; Director of the Michigan Hardware Company, also of the Grand Rapids Forging and Iron

---

[1] Memoir prepared by his Fellow Members of the Society residing in Grand Rapids Mich.

Company; and for some years he was President of the Michigan Paper Company, of Plainwell, Mich. His best energies, however, were constantly devoted to the upbuilding of his beloved city, and the improvement of the lives of its people. He had an abiding faith in his fellow men. His sweet personality endeared him to all. The following lines well described George Ames, who caught a vision of social values beyond the mere materials in his structures:

> "Could I but choose between the milling plaudits of the throng,
> The smiles of fortune—the all too fickle glare of fame,
> And what I wish to leave when I have passed beyond
> Then would I choose to build—yea, build with might and main
> The portals of a gate beyond which all who pass along my way
> May find a richer life, less griefs to bear and more of love;
> Less sorrow and dispair, less of the ills that bend and break—
> A gate through which I hope to pass when my day's work is done."

Modestly, unselfishly, he lived his good citizenship and did not boast of it. When he thought more land was needed for a public park he induced others to donate it or bought it himself and quietly dedicated the tract to public use. If he disagreed with the conclusions of public reports, he often made additional engineering investigation at his own expense, so sincere was his advocacy of what he felt to be the best future policy.

As President of the City Planning Department Mr. Ames attended all National conventions, State planning conferences, and was a member of the County Regional Planning Board. Crusader for reforestation, he was Chairman of the Park and Boulevard Association. He gave unsparingly of his time on behalf of intelligent city development. Sometimes winning, sometimes losing, but never discouraged, Mr. Ames regarded it his self-assumed task to meet indifference and overcome it. He thought his plans through to definite conclusions before he moved and, therefore, his efforts were never misdirected.

Mr. Ames regarded utilitarian considerations as subordinate to those of beauty. A market or oil station was not permitted to destroy natural beauty, yet he was practical enough at all times to desire that commercial and industrial structures should develop in their proper zones.

The Michigan Engineering Society met in Grand Rapids on January 25, 1887, and Mr. Ames joined at this first opportunity; in 1932, he became its President. He was a Charter Member of the Grand Rapids Engineers' Club and, later, served as President. He joined and retained an active interest in the Western Society of Engineers. The upbuilding of engineers and of his vocation was dear to his heart and he always urged the younger men whom he often addressed as his "boys" to join and be active in their local, State, and National professional societies. On November 11, 1927, at Jackson, Mich., the Michigan Engineering Society held a testimonial dinner in his honor, "an event the like of which comes into the lives of few men" for few have given the devoted unselfish community service of George Ames.

Recognized for years as Dean of Civil Engineers and Building Contractors in Western Michigan, Mr. Ames succumbed at the age of 76, to an attack of pneumonia at his home in Grand Rapids. A Thirty-second Degree Mason, an escort of Knights Templars attended the burial at Oak Hill Cemetery; the pall-bearers were fellow Past-Presidents of the Grand Rapids Engineers' Club of which he was a Charter Member. Eulogized by the City Commission for activity in developing zoning and the City Plan, he had well earned the rank of "one of its most useful and prominent citizens."

On November 22, 1887, he was married at Des Moines, Iowa, to Eva Farley whom he had met at El Dorado, Kans., while serving as Levelman for the Atchison, Topeka, and Santa Fé Railroad Company. Surviving besides his widow are a sister, Mary J. Ames, of Grand Rapids, and a brother, Robert L. Ames, of Chicago, Ill.

As a descendant of an old Boston (Mass.) family, he was a member of the Sons of the American Revolution.

A news editorial said of Mr. Ames:

"Grand Rapids has had no more lovable and far-sighted servant of its welfare than this loyal citizen who, not deterred by advancing years, actually increased his voluntary responsibilities in developing the city plan he had sponsored. Acquisitions for the beautification of the city's environs, for recreation facilities, the campaign for flood protection, efforts to carry out the better features of the city plan, in a host of sound proposals for civic betterment, his judgment based upon a lifetime of competent engineering service was a vital factor. The influences of George Ames' intelligent leadership are not lost; many of his recommendations will be carried forward, a mighty legacy to a better planned city."

The following tribute is contained in a letter from Henry Earle Riggs, M. Am. Soc. C. E., a life-long friend:

"Personally I feel that the passing of this dear friend of more than forty years is a severe loss to the Society. He was distinctly our 'No. 1 Member' in Western Michigan. A fine engineer, a man who placed contracting on a high professional plane, an outstanding public-spirited citizen, a wise and valued counselor of his Alma Mater, and a splendid Christian gentleman. Such a man brought great honor to the Society."

Mr. Ames always maintained the keenest interest in the welfare of the Society and promptly responded to every call for service. He did not ask for transfer to the grade of Member because he retired from the City Engineer's office in 1900 and had never since then maintained an office as Consulting Engineer, but had been identified with the Construction Industry as a Contractor. In 1934, when friends urged him to apply for transfer to full membership he did so, but said "I consider Associate Membership in this great Society a very high honor and probably all that I merit." His application for transfer had passed to the ballot which was canvassed a few weeks after his death.

Mr. Ames was elected an Associate Member of the American Society of Civil Engineers on June 7, 1899.

FREDERIC MORRIS FAUDE, Assoc. M. Am. Soc. C. E.[1]

Died November 28, 1934

Frederic Morris Faude, the son of the Rev. John J. Faude and Florence (Holland) Faude, was born in Plymouth, Ind., on August 31, 1876. He received his preparatory education in the Grammar and High Schools of Minneapolis, Minn., and was a student in Civil Engineering at the University of Minnesota, from September, 1895, to June, 1897. He was a member of the Beta Theta Phi Fraternity.

Upon leaving college Mr. Faude was employed for ten and one-half years with the Northern Pacific Railway Company in charge of the design of bridges and buildings, terminal yards, estimates of construction and maintenance work, and the valuation of railroad lines in Minnesota.

Leaving the employ of the Northern Pacific, he was engaged, for four years, in work with the United States Reclamation Service, on mapping, designing irrigation structures, construction work, and the layout of a canal and reservoir system for the irrigation of 30 000 acres.

Following this, Mr. Faude was employed as Assistant Engineer with the Railroad Commission of the State of California, resigning to work with P. E. Harroun, M. Am. Soc. C. E., Consulting Engineer, of San Francisco, Calif., on a valuation of the Spring Valley Water Company's System.

For two and one-half years he was Engineer and Assistant Manager of the Cuyamaca Water Company, San Diego County, California, after which he became Engineer for the Cardiff and San Luis Rey Irrigation Districts, comprising an area of 70 000 acres in San Diego County.

Again entering the employ of the Railroad Commission of the State of California, Mr. Faude served with ability and distinction as Chief Engineer of the Hydraulic Division until 1925, in which year he joined the staff of The Loveland Engineers, Incorporated, as Vice-President and Director, in which position his knowledge, ability, and wide experience were fully recognized and appreciated. He continued in this capacity until his death.

Mr. Faude was a member of the Commonwealth Club of California, the Corinthian Yacht Club, and the American Water Works Association.

He enjoyed an enviable reputation for his loyalty to his friends and associates and for his meticulous care and detail in preparing and carrying out any work in which he was engaged. He had many friends numbered among all classes, and his untimely passing is sincerely mourned by all with whom he came in contact.

He is survived by his widow, Mrs. Anne E. Faude, a brother, Frank Faude, a newspaper man of Los Angeles, Calif., and a sister, Mrs. Alice Banks, of Minneapolis, Minn.

Mr. Faude was elected an Associate Member of the American Society of Civil Engineers on December 6, 1915.

---

[1] Memoir prepared by Members of the Staff of The Loveland Engineers, Inc., San Francisco, Calif.

HARRY FOSTER FERGUSON, Assoc. M. Am. Soc. C. E.[1]

DIED JANUARY 16, 1935

Harry Foster Ferguson was born at Adams, Mass., on March 12, 1889, the son of William Ferguson and Elizabeth (Donaldson) Ferguson. He received his early education in the public schools of his native city, then attended Trinity College, at Hartford, Conn., where he was awarded the McKay-Smith Mathematical Prize, and, in 1912, was graduated from the Massachusetts Institute of Technology (to which he had been also awarded a scholarship), with the degree of Bachelor of Science in Civil Engineering. He was a member of the Alpha Chi Rho Fraternity. He was always an able and an earnest student as evidenced by his awards.

During summer vacations, from June to September, 1910, and from June to September, 1911, Mr. Ferguson was employed as a Rodman with the Engineering Department, Sewer Division, of the City of Boston. After his graduation, from July to September, 1912, he was engaged as Assistant in the Engineering Corps of the Pennsylvania Railroad Company, which employment consisted of field work in connection with track elevation at Fort Wayne, Ind., and maintenance of way.

From October, 1912, to May, 1917, Mr. Ferguson served as Third, Second, and First Assistant Engineer in the State Water Survey at Urbana, Ill. In this capacity his work required the examinations of and reports on public water supplies, water-purification plants, sewerage systems, and sewage disposal plants.

On May 13, 1917, he enlisted in the United States Army and was made First Lieutenant (Engineers) on June 13, 1917, having supervision over water supplies and sanitary conditions in the camps. Mr. Ferguson returned from France, on May 13, 1918, and was honorably discharged from the service on January 31, 1919. He returned to his position with the State Water Survey of Illinois in March, 1919.

Mr. Ferguson joined the Sanitary Engineering Staff of the Illinois State Department of Public Health in July, 1919, and served as Principal Assistant Engineer in that Department until he was appointed Chief Sanitary Engineer in May, 1920, and held that position until the time of his death.

In this position he was in charge of the investigations, supervision, and analyses of existing and proposed water supplies, including purification works; investigation and supervision of existing and proposed sewerage projects, including sewage treatment works, and engineering, bacterial, chemical, and biological studies of stream pollution (in co-operation with the State Sanitary Water Board); supervision of the installation and maintenance of swimming pools; review, issuance of approvals, and permits for plans and specifications for water and sewerage projects, including treatment works and swimming pools; supervision of milk pasteurization plants; advice and assistance in rural sanitation, municipal plumbing ordinances, malaria prevention by mosquito control, school and camp sanitation, and various other items involving sanitary engineering that may have directly or indirectly affected the lives of the people of the State of Illinois.

---

[1] Memoir prepared by Clarence W. Klassen, Acting Chf. San. Engr., State Dept. of Public Health, Springfield, Ill.

Mr. Ferguson attended many meetings of various city councils and other civic organizations, advising on public works or other sanitary improvements for their municipalities. After the enactment of the Sanitary Water Board Law, he served as *ex officio* Technical Secretary of that Board. He was one of the first Trustees of the newly organized Springfield Sanitary District, to which he was appointed on April 24, 1924. He was re-appointed several times by the various succeeding county judges and served in rotation as President and Secretary of that Board. He was a member of the Board of Trustees at the time of his death.

The Illinois Sanitary Water Board came into existence and functioned effectively largely through the efforts and influence of Mr. Ferguson, who drew up the bill, and planned its passage in the 1929 session of the Illinois General Assembly. He was chiefly responsible for the law enacted in 1925 which placed under the State Department of Public Health the supervision of pasteurized milk supplies, and principally upon his initiative and efforts laws were enacted which provided for the establishment of local sanitary districts, local mosquito abatement districts, and the sanitary regulations concerning tourist camps, roadside wells, and food establishments at State Fairs. The bill passed in 1931 giving the State Department of Public Health supervision over swimming pools and bathing places was sponsored by him.

Locally, he took an active part and gave considerable technical advice regarding the problems involving the development of Lake Springfield, a local water supply and recreational project, and these activities, together with the considerable volume of work he did in connection with the local Sanitary District, serve only to illustrate how, although even carrying on his State-wide and often National duties, he still had time for, and gave his services to, his local community.

Needless to say, Mr. Ferguson was nationally known as an authority on Sanitary Engineering. Few men have contributed as much as he did to progress and improvement in the sanitary conditions of Illinois, particularly in relation to water and milk supplies, to the streams and lakes, and to the disposal of sewage. Endowed alike with adminstrative talent, clear vision, accurate judgment, and a high sense of integrity, he applied his faculties industriously and effectively. Seeking legislation, however, was with Mr. Ferguson only a means to an end and that was always and steadfastly for the public benefit in the form of a healthier environment. To this purpose he unreservedly dedicated his life and his talents and toward that end he labored effectively in season and out. He was a practicable realist who saw clearly not only his duty but through it the public good. He applied himself to his work with a courage and devotion that commanded respect and admiration from the members of his profession as well as from every one acquainted with his activities. Scores of public water supply and sewage disposal systems will continue to function efficiently in the preservation of public health for many years to come as monuments to the ability, integrity, and industriousness of Mr. Ferguson. His record is one of faithful service; his death at the very prime of life and in the midst of his fine, humanitarian service, is an inestimable loss. His memory will long be sacred to those who knew him intimately and understood his character, purpose, and achievements.

He passed away January 16, 1935, at the Illinois Research Hospital, Chicago, Ill., after an illness of several months. His remains are in the abbey at Oak Ridge Cemetery, at Springfield, Ill.

In 1920, Mr. Ferguson was married to Zelda Henson, of Villa Grove, Ill., and she, with one daughter, Nadine, age 9, survives him. He is also survived by his mother, Mrs. William Ferguson, of Winthrop, Mass., and a brother, Craig, of Walla Walla, Wash.

He was a member of the American Public Health Association; American Water Works Association; Illinois Society of Engineers (Past-President); Illinois State Academy of Science (Past-President); Illinois Association of Sanitary Districts (Past-President); Conference of State Sanitary Engineers; Central States Sewage Works Association (Past-President); American Chemical Society; American Association for Promoting Hygiene and Public Baths; Izaak Walton League of America; Associate Professor of the University of Illinois, College of Medicine, in Chicago (in this capacity he lectured on problems in the field of public health and sanitation); and a member of the American Legion.

He was the author of many articles on sanitary engineering problems, appearing in current periodicals, among which are the following: *American Journal of Public Health, Journal* of the American Water Works Association, Illinois State Water Survey *Bulletins, Illinois Municipal Review, Illinois Medical Journal, Illinois Health News,* etc.

Mr. Ferguson was elected a Junior of the American Society of Civil Engineers, on June 23, 1916, and an Associate Member on October 9, 1917.

ELBERT ALLAN GIBBS, Assoc. M. Am. Soc. C. E.[1]

DIED MARCH 6, 1934

Elbert Allan Gibbs, the son of Salisbury C. and Elsie (Duncan) Gibbs, was born in Kasson, Minn., on December 12, 1881. After the acquisition of a scholastic training, he attended the University of Minnesota, at Minneapolis, Minn., and, later, the University of California, at Berkeley, Calif., from which he was graduated in 1905 with the degree of Bachelor of Science in Civil Engineering.

From 1899 until 1901, Mr. Gibbs was a member of engineering parties engaged in various projects throughout the Northwest. From 1901 until 1903 he was Rodman for the Bismarck, Washburn and Great Falls Railway Company; Instrumentman for the Chicago, Milwaukee and St. Paul Railway Company; and Chief Engineer for the Eastern Texas Railroad Company. For short periods he was Assistant Engineer with Waddell and Hedrick; Draftsman at the Judson Iron Works, at San Francisco, Calif.; Assistant to the late Samuel Fortier, M. Am. Soc. C. E.; and Assistant and Instructor at Cornell University, Ithaca, N. Y.

---

[1] Memoir prepared by M. O. Fuller, M. Am. Soc. C. E.

Mr. Gibbs spent the summer of 1906 in the Estimating and Designing Department of the McClintic-Marshall Company, and, on February 1, 1907, accepted a permanent position with that Company, being promoted to Assistant to the Engineer in Charge of the Department on August 1, 1907. In that capacity he served until May 1, 1908, when he was appointed Engineer in Charge of Estimating and Designing, discharging these duties until July 1, 1910, when he was made Manager of Erection, in charge of all erection operations west of Harrisburg, Pa.

When the United States entered the World War, Mr. Gibbs enlisted in the service of his country, and for more than two years served with distinction on foreign soil. On May 1, 1917, he joined a Corps of Engineers of the United States Army, and while a member of the American Expeditionary Force, served successfully as Captain, Major, Lieutenant Colonel, and Colonel. After attaining the latter rank, he was in charge of troops in the field for a period of one year. He also served as Chief of General Construction for one year, this branch of service being affiliated with the Division of Construction and Forestry, which embraced all American engineering construction in France, pertaining to roads, warehouses, hospitals, water supply, and electrical, mechanical, and aviation construction. He was awarded the Distinguished Service Medal by the United States Government, and was made a member of the French Legion of Honor.

On August 1, 1919, Colonel Gibbs became General Manager of McClintic-Marshall Company, in charge of all fabricating and erecting operations, and, as such, efficiently discharged the duties of that position until August 1, 1929, when he was elected Vice-President and General Manager of the Company, with the same duties as prescribed in the foregoing. On February 10, 1931, Colonel Gibbs was elected Vice-President in charge of operations of the McClintic-Marshall Corporation, which position he held at the time of his death.

He was Colonel in the Engineers Reserve Corps, commanding the 324th Engineers, and President of the American Society of Military Engineers. He was affiliated with the American Legion, Military Order of Foreign Wars, the University Club and the Duquesne Club, of Pittsburgh; the University Club, of Chicago, Ill.; and the Saucon Valley Country Club, of Bethlehem, Pa.

He is survived by four sisters, Mrs. George Mock, of Whitewater, Colo.; Mrs. H. L. Stedman, of Rochester, Minn.; Mrs. Lou Nelson, of West Concord, Minn.; and Mrs. J. R. Nelson, of Zumbrota, Minn.

Colonel Gibbs was elected an Associate Member of the American Society of Civil Engineers on July 1, 1909. He was also a member of the Lehigh Valley Section of the Society.

ELMER CLARK GOODWIN, Assoc. M. Am. Soc. C. E.[1]

DIED AUGUST 18, 1934

Elmer Clark Goodwin was born in New York, N. Y., on May 9, 1888, the son of Frank J. Goodwin and Anna (Campbell) Goodwin. He received his preliminary education in the Grammar and High Schools of New York City, and his professional education at Columbia University, from which he was graduated with the degree of Civil Engineer in 1910.

After his graduation, Mr. Goodwin was employed as a Rodman with the Erie Railroad Company, and, in 1911, as a Rodman with the New York Central Railroad Company. During a greater part of 1912, he served as a Temporary Assistant Engineer with the Board of Water Supply of New York City, on the construction of the Catskill Aqueduct, earning by his record the respect and esteem of his associates.

In December, 1912, Mr. Goodwin joined the Public Service Commission for the First District, State of New York, as a Junior Engineer on subway construction work in New York City. He remained on this work until June, 1918. During the World War, from July, 1918, to July, 1919, he served as Lieutenant (Junior Grade) in the Civil Engineer Corps, United States Naval Reserve Force. In July, 1919, he returned to the Public Service Commission for a few months, resigning on December 20, 1919.

From 1919 until his death, Mr. Goodwin served as Examining Engineer of the Department of Street Cleaning (later, the Department of Sanitation), of New York City. He was in charge of the Engineering Bureau of the Department and was Engineer-Adviser to the Commissioner of Sanitation.

Mr. Goodwin had charge of the preparation of plans and specifications for all automotive equipment purchased by the Department, consisting of dump trucks, power flushers, sweeping machines, snow loaders, automobile snow plows, and other miscellaneous equipment. Under his supervision the motor equipment grew from less than one hundred to more than twenty-five hundred pieces Many of the specifications for this equipment, originally prepared by Mr. Goodwin, have been adopted as standard by other large cities in the United States.

In addition to this automotive work, he supervised the drawing of plans for water-front dumping boards, self-dumping scows, storehouses, garages, and other departmental buildings. A large part of Mr. Goodwin's time was given to investigations and reports on the various phases of collection and disposal of municipal refuse and of snow removal. When he joined the Department, in 1919, the application of engineering principles to the work of street cleaning and refuse disposal was just beginning. During his incumbency the Engineering Bureau was expanded, and it was largely through his efforts that street-cleaning methods were studied and unified, not only in New York City, but in other cities and towns.

---

[1] Memoir prepared by Sigmund Muenz, Esq., New York, N. Y., and Robert Ridgway, Past-President and Hon. M. Am. Soc. C. E.

As a member of the Association of Public Works Officials, Mr. Goodwin was on the Committee appointed to study "Costs and Methods of Various Garbage Disposal Systems." A paper by Mr. Goodwin entitled. "Snow Removal Methods in New York," was presented at the meeting of the Highway Division of the Society on January 20, 1927, and published in the October, 1927, *Proceedings*. On March 17, 1927, at the meeting of the Massachusetts Civic League Public Service Institute, Mr. Goodwin was assigned the subject of "Administration of Street Cleaning and Disposal of Community Waste." This paper received commendation from the Institute. Several other papers bearing on street cleaning and sanitation were prepared and delivered by him before civic societies and associations, and his paper on "Keeping Streets Clean in Our Largest City," was published in *Municipal Sanitation* in June, 1930. Other short articles on "Snow Removal" have appeared in *The American City*.

He was studious, quiet, and unobtrusive, yet had keen judgment of men and methods. He endeared himself to all his associates and subordinates by his kind and lovable disposition, and his passing was felt as a personal loss by those with whom he came in contact.

Mr. Goodwin was married at New York City, in 1914, to Madeleine Hickey, who died in 1931. Their three children, Madeleine, Frank J., and Peter J. Goodwin, survive him. In 1933, he was married to Regina Masterson, who also survives him.

He was a member of the Society of Automotive Engineers, and of the International Association of Public Works Officials, of which he was a Past-President. He was also a member of the Friendly Sons of St. Patrick, Monroe Country Club, Knights of Columbus, and of the Phi Sigma Kappa Fraternity.

Mr. Goodwin was elected an Associate Member of the American Society of Civil Engineers on October 1, 1926.

JOHN FRANCIS GRADY, Assoc. M. Am. Soc. C. E.[1]

Died August 4, 1934

John Francis Grady, the son of James J. Grady and Mary (Donovan) Grady, was born at Fairmont, Kans., on December 19, 1896. His grade and High School education was secured in the public schools of Lansing, Kans. After teaching in a county school in Western Kansas for two years, Mr. Grady entered the Kansas State Agricultural College, at Manhattan, Kans., from which he received his degree of Bachelor of Science in Civil Engineering in 1920, and his professional degree in 1923. During the period from July to December, 1918, Mr. Grady saw service with the Student Army Training Corps.

---

[1] Memoir prepared by H. D. Barnes, Assoc. M. Am. Soc. C. E.

Mr. Grady's first engagement in the engineering field was as an Instrumentman in a survey party during the summer of 1919. After his graduation from college in 1920, he was employed by the Kansas Highway Commission as Assistant Resident Engineer on highway construction work, in which capacity he was engaged until the spring of 1922. During this period, Mr. Grady gained valuable experience in road and bridge construction.

In the spring of 1922, he was called back to the Kansas State Agricultural College to serve as a Designer and Resident Engineer during the construction of the Memorial Stadium. He was engaged in this work until the fall of 1923. Upon the completion of this engagement Mr. Grady accepted a position as Assistant County Engineer of Labette County, Kansas, in which capacity he served until February, 1926, when he was promoted to be County Engineer. It was during Mr. Grady's service with Labette County that the county unit system was adopted there, and it was largely through his efforts that this system became a success.

It is interesting to note that Labette County was the first in Southeastern Kansas to adopt this system, and due to its success under the skillful development and management of Mr. Grady, it has since been adopted by many other counties throughout the State.

During his seven years of work in Labette County he became one of the leading County Engineers in the State, and his knowledge of administration, maintenance, and construction problems made him outstanding in his line.

In July, 1930, Mr. Grady again became associated with the State Highway Commission as a Resident Engineer in charge of concrete pavement construction. From this position, he advanced to that of Superintendent of Maintenance in charge of maintenance operations in Division 6 in Southwestern Kansas. Mr. Grady was promoted to the position of Division Engineer, with headquarters at Chanute, in April, 1931, in which capacity he served until his death.

He was an honest and efficient public servant and was well known and very popular with his acquaintances throughout the State. To know him was to love and respect him. One of his most outstanding qualities was his patience and tact in dealing with members of his organization and with the public at large.

The following quotation from an editorial appearing in the Chanute, Kans., *Tribune,* vividly pictures some of his qualities:

> "Comparatively few men command as great respect from their associates and acquaintances as did John F. Grady. Neither is the government and its various subdivisions always so fortunate in its selection of capable men to serve the public.
>
> "Trained in the fundamentals of civil engineering, John Grady began his professional experience as County Engineer at Oswego. Seven years later, he entered the service of the State as a Resident Engineer. His advancement in the State Highway Department was rapid, from Resident Engineer to Division Engineer, the position he held here more than three years before his death Saturday night.
>
> "In John Grady was character, ability, personality. He maintained pleasant contact with the public he served and with his associates in the High-

way Department. This community shares with the State of Kansas the loss of a valuable engineer and executive, because in the three years he had been a resident of Chanute, Mr. Grady had bcome known as a most desirable citizen whose counsel and support in civic affairs were sought in many instances."

Mr. Grady was a Registered Professional Engineer, a member of the Rotary Club, the Kansas Engineering Society, the American Legion, and other organizations. He was an untiring worker and, although his first concern was in his home and his family, he was greatly interested in civic affairs, and gave a good measure of his time to such work.

He was married, in 1921, to Marian Clarke, of Paola, Kans. He is survived by his widow and six children, two girls and four boys.

Mr. Grady was elected an Associate Member of the American Society of Civil Engineers on October 26, 1931.

FRANCIS VINCENT HARNEY, Assoc. M. Am. Soc. C. E.[1]

DIED APRIL 11, 1935

Francis Vincent Harney, the son of Frank Harney and Ellen (Butler) Harney, was born in New York, N. Y., on May 30, 1893. After completing his Grammar and High School education, he entered a course of Architecture at the Mechanics Institute of New York City, from which he was graduated in 1916.

Shortly thereafter, Mr. Harney became associated with the office of Mr. William H. Fissell, New York City, engaged in the construction of Federal buildings throughout the United States. He served as Estimating Engineer for Mr. Fissell until the entrance of the United States into the World War in 1917.

It was at this stage of his career that Mr. Harney played an unwilling part in one of the war's minor comedies. His eagerness to serve in a combat unit impelled him to early enlistment as a private at the nearest recruiting station. His impatience for this service could not tolerate the brief training period which would qualify him for a commission in the United States Engineer Corps. Actually, he was detailed to an Army Engineer unit in Washington, D. C., and worked there on quantity surveys for cantonment construction for the duration of the war; and the several applications he submitted for overseas combat service were disapproved.

In 1919, after demobilization, he was engaged by the building construction firm of Valentine Lynch Company, Incorporated, of New York City, as Assistant Estimating Engineer and rose to the position of Chief Estimating Engineer for that Company in 1921. He severed his connection with the firm late in 1922. During this engagement, he completed a two-year extension course on engineering subjects at Columbia University, New York City.

---

[1] Memoir prepared by D. A. O'Sullivan, Esq., New York, N. Y.

Early in 1923, he assisted in the organization of the building construction firm of O'Sullivan-Harney Company, Incorporated, took office as Vice-President and held that office until his death, on April 11, 1935.

Mr. Harney was active in New York City politics. He was appointed Superintendent of Parks during the administration of Mayor Joseph V. McKee, and resigned at the close of that administration.

He was Past-President of the Mechanics Institute Alumni Association and a member of the General Society of Mechanics and Tradesmen. He was serving a three-year term on the Finance Committee of the latter Society at the time of his death. He was also a member of the Board of Governors of the New York County Democratic Veterans Association and a member of the County Committee of the Democratic Party in New York County.

On September 8, 1917, he was married to Ann Genevieve Dalton, of Naugatuck, Conn., who survives him.

Mr. Harney was elected an Associate Member of the American Society of Civil Engineers on December 15, 1924.

GEORGE WALDO HILLMAN, Assoc. M. Am. Soc. C. E.[1]

Died November 11, 1933

George Waldo Hillman, the son of John Wesley Hillman and Rosalie (Fry) Hillman, was born in New Orleans, La., on May 21, 1876. After his academic education in the public schools at New Orleans, he entered the Louisiana State University, at Baton Rouge, La., from which he was graduated in 1897 with the degree of Bachelor of Science.

Mr. Hillman entered the employ of the Federal Government in 1897 as a Rodman with the United States Corps of Engineers. In 1898, he was promoted to the position of Instrumentman, and was employed on the construction of foundations for forts and on inspection work for two batteries at Sabine Pass, Tex. From the latter part of 1898 to January, 1899, he served as Inspector on the dredging of a channel by the Government at Calcasieu Lake, Louisiana. After the completion of this work, he was made Chief of Party on levee construction on the Mississippi River until 1901, when he was given charge of bank-protection work on that river, near Natchez, Miss.

In 1901, Mr. Hillman entered the Engineering Department of the Texas and Pacific Railway Company and remained continuously in the service of that Company, filling various positions in the Engineering Department, from Rodman to Division Engineer, until 1912. He then resigned to become Assistant Engineer with the Trans-Mississippi Terminal Company, in which position he assisted in the design and construction of terminals in New Orleans, La.

He re-entered the service of the Federal Government in 1914, and, until 1917, was engaged in the supervision of levee construction, in the Fourth

---

[1] Memoir prepared by A. F. Barclay, M. Am. Soc. C. E.

Mississippi Levee District, on the Mississippi River, between Vicksburg, Miss., and the mouth of the river.

Mr. Hillman was then employed by Ford, Bacon and Davis, Consulting Engineers, in making surveys and supervising wharf construction at New Orleans, and as Assistant Engineer with the Board of Commissioners of the Port of New Orleans, on wharf construction.

In June, 1917, he entered the service of the New Orleans Public Belt Railroad Company as Assistant Engineer, in which position he remained until his death. During his employment with this Company, he was engaged in various construction work and in the maintenance of track and structures.

As an Engineer, Mr. Hillman's ability, knowledge, close attention to detail, and unfailing loyalty, made him a valuable man. High ideals of honor and integrity, loyalty to his friends, and charity to all, were his outstanding characteristics as a man.

Mr. Hillman was married, in Beaumont, Tex., on February 20, 1901, to Maude L. Watson. From this marriage there was born a daughter, Marjorie Hildegarde, who died in 1931. He is survived by his widow.

He was a member of the Masonic Fraternity, and of the Sons of the American Revolution.

Mr. Hillman was elected an Associate Member of the American Society of Civil Engineers on April 1, 1908. He served as Treasurer of the Louisiana Section of the Society for several terms.

JOHN JAY LAFAYETTE HOUSTON, Assoc. M. Am. Soc. C. E.[1]

Died March 7, 1933

John Jay Lafayette Houston was born in Christiana, Pa., on September 2, 1860, the son of Dr. John Houston, a former surgeon in the Union Forces during the Civil War, and Caroline M. Houston. The boy attended the Belmont Grammar School and the Central High School, in Philadelphia, Pa., and enrolled in Civil Engineering at the University of Pennsylvania about 1878. After the death of his father, Mr. Houston left college and, in 1881, entered the employ of the Pennsylvania Railroad Company as Draftsman and Chainman, rising to become Assistant Supervisor, in the Maintenance-of-Way Department. He returned to the University on leave of absence in September, 1888, and completed his engineering studies, receiving the degrees of Bachelor of Science and Civil Engineer, and acting as Assistant Instructor for a brief period in 1891, following his graduation.

During 1892, Mr. Houston was engaged principally as Instrumentman and Inspector, first with the New York, New Haven, and Hartford Railroad Company, and, later, with the Pennsylvania Railroad Company. From 1893 to 1896, he was in business with others in Chicago, Ill., promoting an early type of air-lift pump, but this venture was not successful and the firm failed. Mr. Houston demonstrated unusual strength of character by single-handed payment of the firm's debts, but in so doing, exhausted what savings he had made up to that time.

---

[1] Memoir prepared by Philip H. Carlin, Assoc. M. Am. Soc. C. E.

In 1897 he went back to railroading, serving until 1899 in various field capacities with the Chicago and Northwestern Railroad Company; New York, New Haven and Hartford Railroad Company; and the Norfolk and Portsmouth Belt Line. From July to October, 1898, he was employed as Instrumentman with the United States Board of Engineers on Deep Waterways. The year, 1899, was spent as Assistant Engineer and Assistant Supervisor of Track with the Chicago and Northwestern and the Lehigh Valley Railroad Companies.

Mr. Houston returned to Philadelphia in 1900, entering the employ of the City as Draftsman in the Bureau of Surveys, Sewer Department, and, after a brief engagement with the Pennsylvania Railroad Company in charge of a change of line near Claymont, Del., continued until May, 1902, as Draftsman with the Eighth District (Manayunk) Surveyor and Regulator. From October, 1902, until March, 1906, he was Draftsman with the Philadelphia Rapid Transit Company on the Market Street Subway and Elevated Line, the first high-speed railway to be constructed in that city.

Upon completion of the design work on the Market Street Line and until January, 1907, Mr. Houston filled engagements with the Long Island Railroad Company as Draftsman and in charge of a survey party, and with the Hudson Companies, New York City, designing track connections for new tubes under the Hudson River. From 1907 to 1911, he was employed by the Department of Parks, Boroughs of Brooklyn and Queens, New York, N. Y., as Topographical Draftsman, and, later, as Assistant Engineer. During 1912, he worked as Draftsman in the United States District Engineer's Office in New York City, and from April, 1913, until April, 1915, held similar positions with the New York Municipal Railway Corporation and with the firm of Jacobs and Davies, New York City. Failing health about this time forced Mr. Houston to seek outdoor work, and the following ten months were spent as Chainman with the New York Central Railroad Company, at Oswego, N. Y.

About the middle of 1916, Mr. Houston returned once more to Philadelphia, still seeking outdoor work, and on September 6, 1916, he entered the employ of the Philadelphia Department of City Transit as Transitman on field work in connection with the Frankford Elevated Railway. Upon the start of active construction work on the Broad Street Subway in 1924, he was made Assistant Engineer, and continued in this position until March, 1932, when municipal conditions forced his dismissal together with that of many others. At that time he had been with the Department for almost sixteen years, and this, in conjunction with his earlier employment by the Philadelphia Rapid Transit Company on the Market Street Line, made him familiar with high-speed transit developments in Philadelphia from their earliest beginnings.

In later years, Mr. Houston had become more and more interested in Philadelphia's early Colonial background, and after March, 1932, he devoted almost all his time to historical research. By strict attention to diet and constant outdoor exercise, he had in the meantime acquired excellent health, despite a misfortune when, at seventy years of age, he was attacked from behind by a thug, struck down unconscious by a blow on the head, and robbed.

On the evening of March 6, 1933, he attended a dinner given by an association in which he was interested, where he appeared to be in good health, although not as active as formerly. The following morning he was found dead in bed, the end having come with little warning. He had never married, living alone in a small apartment with his books and documents.

At the time of his death, Mr. Houston was in his seventy-second year and had been engaged in various types of engineering for more than fifty-two years—a longer period than engineering, as now designated, has itself existed. While his station in the profession was modest, a great part of his life lay outside its scope, in the fields of history, social science, and music, in each of which he had unusual knowledge and keen interest. He was also the possessor of a fund of general information on a wide variety of subjects, which he unfailingly placed at the disposal of his friends, both old and young.

During his long residence in Philadelphia, Mr. Houston had collected numerous early maps and historical documents of local interest, which he later presented to the various historical societies of which he was an active member. These included the Historical Society of Pennsylvania, the City History Society of Philadelphia, the Historical Society of Frankford (Philadelphia), and the Camden County (New Jersey) Historical Society.

His interest in physical science is evidenced by membership in the Franklin Institute of Philadelphia; he took part also, however, in the activities of the Philadelphia Academy of Natural Sciences and of the Wanderlust Club. He was a member of the Theosophical Society; a member of University Lodge No. 610, F. and A. M., and of the Philadelphia Consistory; a Past-Commander of the Sons of Union Veterans of the Civil War, and a member of the Loyal Legion; a former member of the Engineers' Club of Philadelphia; and a participant in the activities of a number of other scientific, social, and musical organizations. He will long be remembered for his integrity, sincerity, and kindly warm-hearted qualities, which endeared him to his numerous friends.

Mr. Houston was elected an Associate Member of the American Society of Civil Engineers on January 2, 1912. He had been a member of the Philadelphia Section of the Society almost since its inception.

WARREN RAYMOND KING, Assoc. M. Am. Soc. C. E.[1]

Died July 26, 1934

Warren Raymond King, the son of Albert and May Ella (Boies) King, was born on April 29, 1888, at Laramie, Wyo. He was educated in the public schools of Cheyenne, Wyo., and Salt Lake City, Utah, and was graduated from the University of Utah, in 1911, with degree of Bachelor of Science in General Engineering.

From July, 1911, to June, 1912, Mr. King was employed as Instrumentman with the Oregon Short Line Railroad Company. In July, 1912, he was appointed Junior Engineer with the United States Geological Survey, Water Resources Branch, and remained with that organization until August, 1916, with headquarters in Washington, D. C., Newport, Ky., Salt Lake City, Santa Fé, N. Mex., and Denver, Colo.

He resigned from the Government Service in August, 1916, to take a position as Assistant Engineer with the Colorado Power Company, which he retained until April, 1919, when he again entered the Federal Service as Chief of Party with the United States Bureau of Public Roads, in Colorado.

In December, 1919, Mr. King transferred back to the U. S. Geological Survey, and was assigned temporarily to Washington, D. C. Early in 1920, he was appointed District Engineer and moved to Nashville, Tenn., in charge of stream-gauging work in the Southeastern States. In October, 1920, he moved the District Office to Chattanooga, Tenn., and remained in charge of that office until his death.

During his eighteen years of service in the U. S. Geological Survey, Mr. King made an enviable record as an Engineer and as an Executive. Due to his convincing reasoning, diplomacy, and unfailing record of accomplishment, he was able to build from a small beginning, one of the largest and most important districts in the Survey.

Mr. King's chief duty in Tennessee was the collection and preparation for publication of daily stream-flow records at various stations on rivers in his District, and, in this, he was eminently successful, with an achievement of constant growth and improvement of stations and records. In 1920, when he took charge, there were 33 gauging stations in the District, all but 3 of which were of the staff or chain-gauge type. At the time of his death, the District was operating 110 stations, 84 of which were equipped with the latest recording gauges, housed in permanent structures. In addition to his contributions to the annual *Water Supply Papers* of the U. S. Geological Survey, he was the author of two Tennessee State *Bulletins* and one Kentucky *Bulletin*, covering all phases of information relative to the rivers of this section. This information, which is basic for water-pawer development, flood control, water supply, and navigation, has proved of great value to this section by assisting the various governmental agencies in their development plans for the Tennessee River Valley.

---

[1] Memoir prepared by O. J. Miller, Assoc. M. Am. Soc. C. E.

Mr. King did not confine his activities to the official duties of his office, but was keenly interested in all things pertaining to this region. He spent much time assisting in the solution of many perplexing problems. He was familiar with all the various activities undertaken by the Federal Government to relieve unemployment, and was responsible for the employment of many engineers and others who came to him for help.

An attractive personality and love of his fellow man were among Mr. King's outstanding characteristics. He enjoyed the society of others and was willing to do more than his share to promote and organize gatherings for purposes of work or of pleasure. Mr. King had served as Secretary-Treasurer of the Chattanooga Section of the Society in 1932 and, at the time of his death, was the first President of the Tennessee Valley Section organized in 1934. He was also one of the organizers of the Chattanooga Engineers' Club.

The following resolution on the death of Mr. King, was passed by the members of the Tennessee Valley Section resident in Knoxville, Tenn.,

"*Whereas* Almighty Providence has seen fit to suddenly remove from our midst, our esteemed president and beloved co-worker, Warren Raymond King;

"*Therefore Be It Resolved* that we, members of the Tennessee Valley Section of the American Society of Civil Engineers resident in the City of Knoxville, pay tribute to the high professional standing and willing co-operation of our esteemed fellow member, and that we hereby extend to the widow and immediate family in their hour of bereavement, our heartfelt sympathy and deepest condolence.

"*Be It Further Resolved* that a copy of this resolution be suitably inscribed and transmitted to Mrs. King, and that a copy be forwarded to the Secretary of the Society."

Mr. King served as President of the Glenwood Civic Club, was a member of the Civitan Club, and was active in Community Chest drives, membership campaigns for the Young Men's Christian Association, Chamber of Commerce, and other civic organizations. He was a Beta Theta Pi, a Mason, and an active member of the Methodist Episcopal Church.

In 1913, he was married to Mabelle L. Casady, of Salt Lake City, Utah, and is survived by his widow, a son, Warren R. King, a daughter, Mary Elizabeth King, and by his father, a brother, and a sister.

Mr. King was elected an Associate Member of the American Society of Civil Engineers on July 9, 1923.

ISAAC HENRY KIRBY, Assoc. M. Am. Soc. C. E.[1]

Died March 15, 1934

Isaac Henry Kirby, the son of Wallace W. and Susan E. Kirby, was born in Roslyn, Long Island, N. Y., on December 14, 1871. He lived in the house in which he was born, and in an adjoining house, throughout his entire life. He attended the University Grammar Schools, in New York, N. Y., and then entered New York University, from which he was graduated in 1895 with the degree of Bachelor of Science. He took a Post-Graduate Course and, from 1895 to 1896, was Demonstrator in Analytical Chemistry. In 1896, he received the degree of Civil Engineer. He was a member of Zeta Psi Fraternity and was elected to the Phi Beta Kappa (Honorary) Fraternity in 1895.

---

[1] Memoir prepared by J. C. Riedel, M. Am. Soc. C. E.

From October, 1896, to April, 1900, Mr. Kirby was a member of the Engineering Staff of the Department of Water Supply in the former City of Brooklyn, engaged on weir, boiler, and engine tests. He became a member of the Engineering Staff of the Bureau of Sewers, Borough of Brooklyn, City of New York, in April, 1900, and remained with the Bureau until his death.

During the earlier years of his connection with the Bureau of Sewers, Mr. Kirby was Assistant to the Engineer in Charge of Construction on many miles of sewers, including the Huron Street Sewer, the Bedford Avenue Sewer, and the 92d Street Tunnel Sewer.

In 1905, the need for the testing of materials used in sewer construction became manifest, and Mr. Kirby was entrusted with the problem of devising a satisfactory means of accomplishing this purpose. He devised ingenious methods of testing pipe used for sewer construction and was a pioneer in this field. Many of his devices and methods have since been adopted in standard practice.

About 1907, he became Resident Engineer in charge of some of the largest sewers that have ever been constructed, notably those in Ralph Avenue and in Flatlands Avenue where their rated size is 226 in. in diameter. The value of the work supervised by Mr. Kirby exceeded $30 000 000. He was painstaking, industrious, talented, able to co-operate with others, and to obtain from contractors work of very high character. He became an expert in the determination of the condition of the largest of the sewers in Brooklyn, many of which exceeded 12 ft in diameter, and on his recommendation, millions of dollars were expended for repair and construction work.

He was one of the early members of the Brooklyn Engineers Club, and served upon Committees of that Club for many years. In his spare time, he acted as a Consultant upon engineering projects in Roslyn. He was highly regarded by all those who knew him, and commanded the respect of every one with whom he came in contact.

Mr. Kirby rarely suffered from illness and was active in his work until a short time before his death.

He is survived by his widow, Mrs. Marion (Ludlum) Kirby, to whom he was married on November 25, 1916. Mrs. Kirby is the daughter of Mr. Charles H. Ludlum, of Hempstead, Long Island, who was a member of the Class of 1863 of New York University and who was also a member of Phi Beta Kappa. Mr. Kirby is also survived by a brother, Ralph, who lives in Roslyn. His mother died in 1933, having passed her ninetieth birthday. His father, an Alumnus of Union College, Schenectady, N. Y., died in 1901.

Mr. Kirby was elected an Associate Member of the American Society of Civil Engineers on November 1, 1905.

JAMES BROWNSON McCLAIN, Assoc. M. Am. Soc. C. E.[1]

Died April 19, 1934

James Brownson McClain, the son of John and Anna (Gordon) McClain, was born at Washington, Pa., on June 29, 1877. Receiving his early education in the public schools in Washington, he continued throughout his career the study of the principles and practices of his profession.

In 1905, Mr. McClain began his engineering career and, for several years, was engaged in land and mining surveys, and investigations for water supply and city planning. In 1906, he entered the employ of the Seaboard Air Line Railway Company as Masonry Inspector in the Bridge Department, and continued with that Company for the remainder of his life.

In May, 1918, he was appointed Engineer of Bridges of the Seaboard, and thereafter had charge of the replacement and maintenance of many important structures along this line. In this position, Mr. McClain's experience, engineering skill, and perseverance enabled him to solve difficult problems of replacement of obsolete structures and to adopt in each case the best structure obtainable. Willing always to approve better methods and materials developed by progress in engineering, his conservative character led him to require proof of their value. He was a member of the American Railway Engineering Association.

Mr. McClain was married in Baltimore, Md., on June 26, 1907, to Mildred Edith Kerr, of Freeport, Pa., who survives him. He also leaves one brother.

He was an unusual character, strong, persevering, with very definite opinions, especially on professional matters. Respected for his integrity and ability, he was appreciated and admired for his willingness to help others. He preferred an out-of-doors life and, during his earlier years, was fond of shooting, winning several prizes at rifle matches. By his death the Engineering Profession has lost a most useful member.

Mr. McClain was elected an Associate Member of the American Society of Civil Engineers on September 2, 1914.

ARTHUR SAWYER MAHONY, Assoc. M. Am. Soc. C. E.[2]

Died April 18, 1933

Arthur Sawyer Mahony was born on October 8, 1891, at Passaic, N. J., the son of Dennis and Letitia (Finn) Mahony. His father was Editor, in turn, of both the local newspapers and, later, served as Postmaster of Passaic until his death.

---

[1] Memoir prepared by a Committee of the Virginia Section, consisting of E. A. Frink, M. Am. Soc. C. E., and T. W. Roby, Assoc. M. Am. Soc. C. E.

[2] Memoir prepared by Paul L. Troast, Esq., Passaic, N. J.

Mr. Mahony received his early education in the public schools and the High School, at Passaic, and, in 1914, was graduated from Bucknell University, at Lewisburg, Pa., with the degree of Bachelor of Science in Civil Engineering. Within a few years he received the degree of Civil Engineer from Bucknell. He was a member of Sigma Chi Fraternity.

During school vacations, Mr. Mahony was employed as Inspector of Public Works for the Borough of Manhattan, New York City, and as Topographical Draftsman for the same Borough on design and layout, including calculations for sewerage systems. After his graduation in June, 1914, and for a period of one year, he was Assistant Engineer with the Board of Water Supply of New York City, in charge of a field party giving lines, grades, and data for estimates on Kensico Dam, at Valhalla, N. Y. From June, 1915, to June, 1916, he was Junior Assistant Engineer for the Public Service Commission of New York State, in charge of a field party on the Seventh Avenue Subway.

In June, 1916, Mr. Mahony entered the building construction field as Superintendent of Construction for the American Concrete Steel Company. Until he entered the Naval Service during the World War, he was in full charge of the construction of numerous reinforced concrete buildings, involving a total cost of $1 500 000.

From August, 1918, to February, 1919, he was in the service of the United States Government, attached to the Naval Reserve Force as Inspector on the construction of the Navy Hospital at Wards Island, New York. With the termination of this service, Mr. Mahony again re-entered the building construction field as Superintendent of Construction with the C. R. Hedden Company and built and laid out the $1 500 000 Chemical Plant of the Lehn and Fink Chemical Company, at Bloomfield, N. J. From November, 1920, to June, 1921, he was with Cauldwell Wingate Company, of New York, as Mechanical Engineer, inspecting and estimating mechanical work on several large commercial buildings, including the Borden Building, the American Surety Company Building, and Mt. Sinai Hospital, with a total value in excess of $12 000 000.

In June, 1921, Mr. Mahony left the building construction field to accept an appointment as City Engineer of Clifton, N. J. This position he held until January, 1928. He was placed in charge of the organization of the Engineering Bureau of that city at a time when its major improvements were about to be planned and executed. During his term as City Engineer, the City planned and constructed its entire sanitary sewerage system, numerous storm-water sewers, its own water distribution system, and several highway improvements. He was in entire charge of the design, estimating, and supervision of all these improvements. The City, during this period, spent approximately $4 000 000 on this work.

In February, 1928, Mr. Mahony again entered the building construction field and, with Mr. Paul L. Troast, organized the Mahony-Troast Construction Company, with main offices at Passaic. He was Vice-President and General Manager of this Company from the date of its formation until his death. During this period of approximately six years, the Company became one of the outstanding building construction firms in Northern New Jersey. The firm did not participate in public work of any character, but confined all its efforts to the construction of buildings for banking institutions, public utility companies, and commercial and industrial companies of high National standing.

Mr. Mahony's thorough knowledge of the basic principles underlying engineering is rather forcefully shown by his success in both the designing and the contracting branches of the profession. It is rather unusual for one to be so successful in both these branches and, at the same time, to have a thorough and practical knowledge of such diversified engineering problems as the actual design of public works projects, such as sewerage and water developments, and also to attain such a high degree of success in the building construction field. In all his contacts, Mr. Mahony impressed those associated with him with his thorough familiarity with basic engineering principles and with his keen mental capacity in applying these principles in the most efficient manner to the practical problems at hand.

His kindly disposition, keen sense of humor, and his unerring fairness and honesty endeared him to all those with whom he came in contact. These contacts embraced the rather wide field from the day laborer on the construction job to the nationally known engineer and architect, as well as municipal officials and officers of large corporations. His untimely and sudden death is sincerely regretted by all who had the privilege of his acquaintance and association.

He was married in 1920, to Julia Walsh, of Fitchburg, Mass. His widow and one son survive him.

Mr. Mahony was elected an Associate Member of the American Society of Civil Engineers on August 28, 1922.

THOMAS VINCENT MOORE, Assoc. M. Am. Soc. C. E.[1]

Died May 28, 1934

Thomas Vincent Moore was born in Baltimore, Md., on November 25, 1897, the son of William and Celia (Connor) Moore. He received his preliminary education in the Grammar and High Schools of Baltimore, and, after three years of preparatory school, he entered Loyola College, at Baltimore, from which he was graduated in 1912. Later, he studied at Georgetown University, at Washington, D. C.

After leaving college, Mr. Moore was employed for a few years by the Baltimore and Ohio Railroad Company and also by the Empire Engineering Company, of Baltimore, in an engineering capacity.

In September, 1919, he entered the employ of the Raymond Concrete Pile Company, of New York, N. Y., as Field Engineer and Assistant Engineer on its construction projects. He continued with this Company until his death. From September, 1919, to January, 1921, he was engaged as Instrumentman and, later, as Field Engineer on the construction of shipways, craneways, etc., at Sparrows Point Lighthouse, on the Rappahannock River at Sharps, Va.; and from September, 1921, to March, 1922, as Engineer in charge of the construction of the quay wall at the Norfolk Navy Yard, at Portsmouth, Va.

---

[1] Memoir prepared by R. A. McMenimen, Assoc. M. Am. Soc. C. E.

From March, 1922, to October, 1923, he served as Field Engineer for the Rollin Construction Corporation, of New York City, on the construction of a 5-mile causeway through the Santee Swamps, in South Carolina, including the concrete bridges over the Santee River, with headquarters at St. Stephen, S. C. In 1923 and 1924, Mr. Moore was in charge of engineering on pile-driving work for the Ford Motor Company, at Detroit, Mich., and in 1924 and 1925, he was engaged as Superintendent on dock construction for the Humble Oil Company, at Baytown, Tex.

From May, 1925, to April, 1926, Mr. Moore was Superintendent in charge of construction of the City Dock and the Municipal Pier, at St. Petersburg, Fla. He was then sent to South America, where he served as Superintendent in charge of the port development at Maracaibo, Venezuela, from May to October, 1926. In November, 1926, he was engaged as Superintendent on the Lake Pontchartrain Bridge, at New Orleans, La., and from November, 1927, to December, 1928, he superintended the work on the San Francisco Bay Bridge, at San Mateo, Calif. From that date until his death, he was in charge of several jobs, among them being the pier for the Gulf Refining Company, at Philadelphia, Pa.; foundation work for the new steel mills of the Great Lakes Steel Corporation, at Ecorse, Mich.; and various pile-driving jobs.

Mr. Moore was a member of the following organizations: American Association of Engineers; South Carolina State Board of Civil Engineers; Florida State Board of Civil Engineers; Virginia State Board of Civil Engineers; and the Elks Club, at St. Petersburg, Fla.

On August 11, 1925, he was married to Harriet Marian Floyd, of St. Stephen, S. C. He is survived by his widow and two sons, Thomas Vincent, Jr., and William Gregg Moore. His passing has saddened a host of friends all over the United States and in South America.

Mr. Moore was elected an Associate Member of the American Society of Civil Engineers in July 12, 1926.

GEORGE ABEL PIERCE, Assoc. M. Am. Soc. C. E.[1]

DIED SEPTEMBER 12, 1934

George Abel Pierce, the son of Freeman A. and Olive (Ranney) Pierce, was born at St. Johnsbury, Vt., on February 2, 1880. After receiving his public school education in his native town, he entered the University of

---

[1] Memoir prepared from information on file at the Headquarters of the Society.

Vermont, at Burlington, Vt., where he was a student in Civil Engineering from September, 1899, to June, 1900, and from September, 1901, to June, 1903.

For one year from September, 1900, to September, 1901, Mr. Pierce was in the employ of the Boston and Maine Railroad Company as Assistant in the Division Engineer's Office and as Rodman. Following his two years of college, he re-entered the service of the Boston and Maine Railroad Company and from June, 1903, to June, 1905, he was Assistant Engineer in the office and in the field on construction and maintenance of way. During this period he served as Engineer on the construction of the arch bridge at Bellows Falls, Vt.

In August, 1905, Mr. Pierce entered the service of the Lake Shore and Michigan Southern Railway Company and until February, 1906, was with the Division Engineer as Transitman on drafting and designing of masonry. From February, 1906, to January, 1907, he served as Resident Engineer in charge of eight miles of construction.

Mr. Pierce then removed to New York, N. Y., where from January to November, 1907, he was Assistant Engineer with the Westinghouse, Church, Kerr, and Company, in charge of a field party as well as drafting, estimating, designing, etc. In August, 1907, he was sent by the Company to Sharon, Vt., as Superintendent of Construction on a concrete dam and power house, substations, and transmission line for which he made all the surveys, including the location of the transmission line.

In January, 1908, Mr. Pierce was appointed Deputy County Engineer of Champaign County, Ohio, with headquarters at Urbana, Ohio. He remained in this position until December, 1909, and was engaged in all the engineering work that pertains to that office.

He then moved to St. Louis, Mo., where, until 1914, he was employed as a Draftsman on Construction with the Missouri Pacific Railroad Company. He was then made Civil Engineer in the Valuation Department of the Company where he remained until 1916.

From 1917 to 1928, Mr. Pierce was in the employ of the New York Central Railroad Company, as Pilot Engineer and as Assistant Engineer in the Valuation Department, successively, with headquarters at Cleveland and Berea, Ohio.

Following this engagement Mr. Pierce was appointed Assistant Engineer and Designer with the Union Terminal Company, of Cincinnati, Ohio, where he remained until 1933, when he again took up his residence in St. Johnsbury, Vt. Since that time, he had been engaged as Engineer with the United States Forest Service, at the Civil Conservation Camp, at Rutland, Vt., and, at the time of his death, was serving as Federal Resident Engineer-Inspector for the Public Works Administration, at Brattleboro, Vt.

George Abel Pierce was a civil engineer of considerable ability. He was a man of deep sincerity and was always extremely conscientious about his work, a fact indeed so true that it was the actual cause of his death. He had very few interests outside his work and his family life, being a real home man; and those who knew and loved him for his fine character, remember that he gave up his whole life to providing all that he could for those he loved. There was never a more unselfish man, and no kinder husband and father ever lived. He was among the finest and most honest, and, in intellectual capacity, among the best. His experience was of a wide and varied nature, and his work carried him to many different places and situations.

In 1907, he was married to Nora Blanche Dale, by whom he is survived. He also leaves his mother, and three children, two sons, Carl and Louis, and a daughter, Dorothy.

Mr. Pierce was elected an Associate Member of the American Society of Civil Engineers on January 5, 1909.

HERBERT ALLAN RICE, Assoc. M. Am. Soc. C. E.[1]

Died July 28, 1934

The question has often been discussed as to what constitutes a great teacher. Many qualities have been listed, such as knowledge of subject, ability to present each point in logical order, patience, sympathy, a knowledge of human nature, and many others, all of which seem to fall short of a satisfactory definition. At any rate, whatever is required, the late Herbert Allan Rice had it, and the proof of this is best shown by the many young men who have gone through his classes, and gone out into their professional life, with the stamp of his personality, and an ever-increasing appreciation of his work. For twenty-nine years he served the University of Kansas and the State in an efficient and unselfish manner, and his record as a great teacher is secure.

Herbert Allan Rice was born on October 22, 1875, in Jackson, Ohio. His father, James Abram Rice, and his mother, Katharine (Radcliffe) Rice, were natives of Jackson, but his grandparents and great-grandparents on both sides were natives of Virginia.

The boy attended the public schools at Jackson, and when he was 16 years of age, before completing the High School course, took a teacher's examination, and received a three-year teacher's certificate. He then taught one year in a district school (the Kessinger School), named for his great-grandfather, William Kessinger. In 1893, the family moved to Columbus, Ohio, where his father was engaged in business, and Herbert Allan Rice entered the Preparatory School for Ohio State University. He was graduated from the University, with the degree of Bachelor of Science in Civil Engineering, in 1897. During the summer vacations of his student years, he worked as a Draftsman for an architectural firm in Columbus.

After his graduation, Mr. Rice was employed, as follows: August to December, 1897, on stream gauging for the State Board of Health of Ohio; December, 1897, to January, 1898, on mine surveys for the Sunday Creek Coal Company, Corning, Ohio; February to September, 1898, on structural drafting for Yost and Packard, Architects, Columbus; September, 1898, to August, 1901, in private practice as an Architect, and as a Teacher of Mathematics in the Jackson, Ohio, High School; September, 1901, to September, 1902, in complete charge of the construction of a $200 000 Portland cement plant at Ironton, Ohio; and September, 1902, to June, 1905, as an Instructor in Civil Engineering at Lehigh University, at Bethlehem, Pa.

---

[1] Memoir prepared by Frank A. Russell, M. Am. Soc. C. E.

In the fall of 1905, Mr. Rice accepted a position with the University of Kansas, at Lawrence, Kans., teaching mechanics, strength of materials, and bridge design, subjects previously taught by the late Frank Olin Marvin, M. Am. Soc. C. E. From 1905 to 1906, he was Assistant Professor in Civil Engineering; from 1906 to 1913, Associate Professor; from 1913 to 1922, Professor of Mechanics and Structural Engineering; and from 1922 until his death on July 28, 1934, Head of the Department of Civil Engineering at the University.

Professor Rice was engaged in a number of other projects aside from his teaching. Immediately following the close of the World War, he was active in promoting plans for flood control of rivers in the State of Kansas, both for the benefit to be derived from conserving water, and as a means of providing work for returned soldiers. At that time, he recommended Federal participation in bearing the cost of flood control, supporting the project of the National Drainage Council for a Federal appropriation of $1 000 000 000. Governor Allen appointed Professor Rice as Secretary of the Kansas Water Commission.

Soon after coming to the University of Kansas, in 1905, he was appointed a member of the Athletic Board, and took great interest in that activity. On several occasions, he served as the representative of the University at the Big Six Conference. He was particularly interested in football, attending as many games as his work permitted. He was also quite interested in student affairs, sponsoring many of the annual inspection trips of the Engineering Students to industrial plants in Kansas City, St. Louis, Mo., and Chicago, Ill.

Professor Rice assisted in the organization of Sachem, the Senior Honor Society at the University, and was President of the Kansas Engineering Society in 1924. He was a member of the Society for the Promotion of Engineering Education, The Kansas Engineering Society, The University Club, Sigma Xi, Tau Beta Pi, and Theta Tau Engineering Fraternities, and Phi Gamma Delta, a social fraternity. He was not a member of any church, but held a high regard for its work and, in later years, expressed himself as "seeing no reason why he should not belong to some church." He was always more than generous in supporting the Young Men's Christian Association, religious projects, and charitable institutions, and was always ready to make a financial contribution.

Professor Rice was an independent thinker, and was never afraid to voice his protest on any point, the principle of which he did not approve, even when it would have seemed good policy to "line up" with those in power. He was never in favor of self-advertising in any way, and often questioned the practice of writing textbooks by men who had but little to add to the knowledge of a subject, and could only hope to gain self-publicity. He was the author of no textbook, but at one time he had a manuscript practically prepared on a text on Reinforced Concrete Structures, but lost the work by a fire which completely destroyed his house and its contents, including his entire library. He did not attempt to rewrite the work.

As a teacher, Professor Rice had the ability to present his subject clearly, the sense of humor to insert a story at the right point, the faculty of stimulating the student to work, and the patience and sympathy to give advice whether of a technical or a personal nature. He had been in poor health for the last few years and in the spring of 1933 had missed several weeks of school. He entered the hospital in the summer of 1934 for examination and observation, and decided to submit to what was considered a very minor operation, but which proved fatal. He is sadly missed by the students and Faculty of the University of Kansas as well as by many former students and friends.

Professor Rice was married in 1906 to Edith Sweezy, a graduate of the University of Kansas in 1907, who died in February, 1917. In November, 1920, he was married to Bertha Mix, also a graduate of the University, Class of 1912, who died on June 20, 1927. In November, 1929, Professor Rice was married to Elenor McDonald, of Topeka, Kans., who survives him. He is also survived by his mother and two sisters, Mrs. Ester (Rice) Smith, and Bertha Rice, of Columbus, Ohio.

Professor Rice was elected an Associate Member of the American Society of Civil Engineers on June 7, 1905.

EDSON OLIVER SESSIONS, Assoc. M. Am. Soc. C. E.[1]

DIED APRIL 15, 1934

Edson Oliver Sessions was born in Louisville, Ky., on December 23, 1871, the son of William and Priscilla (Allen) Sessions. He was a descendant of Ethan Allen of Revolutionary fame.

He was a student from 1886 to 1889 at l'Ecole de Polytechnique, Paris, France, and, in 1891 and 1892, he was a Post-Graduate Student in Mathematics, at the Steinharte Technical College.

His business career was begun with the Thomson-Huston Electric Company, of Lynn, Mass., in 1889. The position of Construction Superintendent was attained in 1893, but, in 1896, Mr. Sessions left this firm to act as Consulting Engineer for the Frank Jones Properties in New Hampshire.

Upon the completion of multifarious engineering projects for Mr. Jones, in 1898, Mr. Sessions was employed as Sales Engineer for the General Electric Company, and assisted in its merger of the interests of the Stanley Electric Company. It was while with the General Electric Company that he took up his permanent residence in Chicago, Ill.

Leaving the General Electric Company in 1907, he assisted in the formation of the consulting engineering firm of Woodmansee, Davidson, and Sessions. In this connection, his contributions to the field of power distribution were so noteworthy as to bring him national prominence, with clients numbered among the largest industrial companies of the country. About twenty patents in cable-end bells and bus-bar supports found such a national demand that, in 1911, together with Mr. Fay Woodmansee, he organized the firm of Electrical Engineers Equipment Company, with offices and plant in Chicago.

---

[1] Memoir prepared by William P. Sessions, Esq., Chicago, Ill.

In 1914, Mr. Sessions disposed of his active interests in this firm, while retaining his patent rights, and inaugurated the General Devices and Fittings Company. This Company specialized in the sale of high-tension switch gear and bus-bar supports, under patents to the number of thirteen, all inventions of Mr. Sessions, who was President of the Company until 1920.

In 1917, his tremendous energies found additional expansion in the formation of the Sessions Engineering Company, a national organization of consultants, designers, and builders, and in his acceptance of the Vice-Presidency of the Chicago Rolling Mills.

Appointed an executive of the United States Shipping Board, by President Wilson, in 1918, Mr. Sessions, as a "dollar-a-year man," devoted his entire energies to the Government until 1920, at which time he concluded his interests with the General Devices and Fittings Company, and concentrated his abilities on the development of the Sessions Engineering Company.

The next eleven years found him building his reputation as a sound, careful engineer-economist, whose meticulous desire to devote a substantial amount of his own time to every project, undoubtedly was the cause of the sapping of the tremendous energies that were his, and contributed directly to wearing down even his iron-like constitution.

In 1931, at the invitation of the Supreme Economic Council of the Union of Socialistic Soviet Republics, Mr. Sessions spent several months in Russia in an intensive study of the heavy industries. His report, submitted upon his return to the United States, acted as an important adjunct to the later engineering phases of the Five-Year Plan adopted by the Soviet Government.

Late in 1931, his health failed him, and he severed his interests with the Sessions Engineering Company. After a period of convalescence in the hospital, he did some private consulting work; and, in 1933, organized the firm of E. O. Sessions Company, Limited, embracing, largely, general consulting services. He passed away, on April 15, 1934, following an operation at St. Joseph's Hospital, in Chicago.

Mr. Sessions was responsible for numerous articles in technical society and trade journals. To a large extent, his writings concerned power transmission and the conversion of energy to power.

In addition to his many basic patents in cable-end bells, bus-bar supports, locking switches, and insulators, his more recent patents included a modern electrical melting furnace, and a water-cooled conveyor chain for high-temperature industrial furnaces.

Mr. Sessions was a Fellow of the American Institute of Electrical Engineers, a member of the American Society of Mechanical Engineers, and held the degree of Doctor of Engineering from l'Ecole de Polytechnique, of Paris, France. He belonged to the Union League Club and the Mid-Day Club of Chicago.

In 1899, he was married to Helen Potter, of Toledo, Ohio, who, with three children, William P., Edson Oliver, Jr., and Helen Diana, survives him.

Mr. Sessions was elected an Associate Member of the American Society of Civil Engineers on July 9, 1912.

BURTIS PAUL THOMAS, Assoc. M. Am. Soc. C. E.[1]

Died June 28, 1934

Burtis Paul Thomas, the son of Joseph A. and Mary Thomas, was born on June 29, 1884, in Scipio Township, La Porte County, Indiana. Mr. Thomas attended the city schools of La Porte, Ind. He was very fond of athletics and outdoor sports and while in High School was a member of the football team. In one of the games, he was seriously injured; in consequence of this injury, he did not remain to be graduated and resumed his place on the farm.

In 1906, Mr. Thomas began his career in engineering. From 1906 to 1908, he worked as Instrumentman under Frank Scholes, Civil Engineer, of Fort Wayne, Ind., and from 1908 to 1911, he served as Assistant City Engineer of La Porte, Ind. In 1911, he was elected County Surveyor of La Porte County, and was re-elected in 1913, serving two full terms. From 1915 until 1918, he was engaged in the private practice of surveying and engineering work.

In 1918, Mr. Thomas was appointed City Engineer of La Porte, and to that position, he gave all his professional time and energy until his death. In the same year a mastoid operation left him deaf in one ear. A second mastoid operation in 1922 took his hearing entirely and then, in 1932, another mastoid operation was performed, not to help his hearing, but to save his life. From the time of his last operation until his death, his health gradually failed.

Mr. Thomas was a leader in the communities in which he lived, due to his sterling qualities of character. He was a member of Excelsior Lodge No. 11, F. and A. M.; La Porte Chapter No. 15, R. A. M.; La Porte Council No. 32, R. and S. M.; La Porte Chapter No. 280, Eastern Star; and Lodge No. 396, Benevolent and Protective Order of Elks. He was a member of St. Paul's Protestant Episcopal Church, of La Porte.

On October 17, 1921, Mr. Thomas received his State License as a Registered Professional Engineer.

He was married in 1909 to Ella C. Seidler, of La Porte, and is survived by his widow, one daughter, Valerie, and one son, Delos, his mother, and one brother.

---

[1] Memoir prepared by C. Ken. Schnable, City Civ. Engr., La Porte, Ind.

Mr. Thomas was elected an Associate Member of the American Society of Civil Engineers on January 14, 1924.

RALPH PENNY THOMPSON, Assoc. M. Am. Soc. C. E.[1]

Died February 16, 1935

Ralph Penny Thompson, the son of James H. Thompson and Julia (Penny) Thompson, was born at Patterson, N. Y., on January 7, 1878.

He was educated at the Patterson High School and the Vermont Academy, at Saxton's River, Vt. After graduation from the latter institution, he studied with the International Correspondence Schools, at Scranton, Pa.

Mr. Thompson's early experience in the engineering field consisted of surveying in connection with a widely diversified list of projects and locations throughout the United States.

When in the capacity of Assistant Engineer and Assistant Superintendent with the Fraser, Brace Company, Contractors, Mr. Thompson participated in the construction of the Federal Shipbuilding Plant, at Kearny, N. J., and in the following year (1919) he was connected with the Canal Construction Company as an Engineer on dredging work for harbor and dry docks. In 1920, he was Superintendent of Construction of a concrete pool at Columbia Park, N. J. From 1920 to 1923, Mr. Thompson was in charge of party and Resident Engineer on the construction of bridges and on railroad location, at Norton, Va. He was Chief of Party in the following year, serving with the North Carolina Highway Commission, at Raleigh, on highway construction projects.

In 1924, Mr. Thompson was in Cuba and Florida as Chief of Party concerned with the location of transmission lines, spotting of poles and towers, and property and water-works surveys for the Electric Bond and Share Company of New York, N. Y. This work kept him in the field until 1926. From 1927 to 1929 he was with the Consolidated Gas Company of New York as Field Engineer in charge of the preparation of sub-surface maps and the laying of gas mains. From 1929 to 1930, Mr. Thompson was associated with the New Jersey Highway Commission on highway location surveys and concrete road construction. Stationed in New York City from September 18, 1933, until his death, Mr. Thompson served as a Computer with the United States Coast and Geodetic Survey.

A reticent, quiet man, of New England stock, Mr. Thompson's considerate ways endeared him to every one with whom he had contact. Like countless other Americans, he was particularly interested in baseball and college football activities.

He had never married, and is survived by a sister, Mrs. Frederic Timmerman, of Morrisville, Vt.

Mr. Thompson was elected an Associate Member of the American Society of Civil Engineers on November 21, 1921.

---

[1] Memoir prepared by Francis A. Boyle, Assoc. M. Am. Soc. C. E.

JOHN ALEXANDER JAMESON, Jr., Jun. Am. Soc. C. E.[1]

DIED SEPTEMBER 26, 1934

John Alexander Jameson, Jr., was born in Chicago, Ill., on November 14. 1903. From 1904 to 1919, his home was in Hubbard Woods, Ill., although several of these years were spent at Ojai, Calif., where he attended the Thacher School.

Mr. Jameson received the degree of Bachelor of Arts, magna cum laude, at Williams College, Williamstown, Mass., in 1925; Bachelor of Science at the Massachusetts Institute of Technology, at Cambridge, Mass., in 1928; and Master of Science at the University of California, at Berkeley, Calif., in 1933.

Although his short career was spent largely in academic work, Mr. Jameson was not without practical experience. In 1928, he was employed by the Central Maine Power Company on the construction of a dam at Bingham, Me., and the following year found him engaged as Assistant Engineer on the building of a dam for the Montecito County Water District of Santa Barbara, Calif. During 1930 and 1931, he was employed by the Starrett Company on construction work in New York, N. Y.

After receiving his Master's degree, Mr. Jameson received an appointment as Research Assistant at the University of California and spent nearly a year on hydraulic research.

Early in 1934, he was engaged by the United States War Department to finish the design of a tidal-model laboratory to be built at the University of California, and, later, he represented the University during the period of its construction. Death called him before this laboratory was completed.

In 1930, Mr. Jameson was married to Helen Williams, of Philadelphia, Pa., and Santa Barbara, Calif., who, with their infant daughter, Janet, survives him. The young couple moved to California in 1931, living for short periods in Burlingame and Berkeley before purchasing a small farm at Danville.

At Williams, Mr. Jameson was a member of the football, baseball, and track teams, and this interest in sport continued throughout his life.

His broad cultural background combined with technical proficiency, poise, and sound judgment would have carried him far in his chosen profession. Interested in the best in music and literature, and in current political and social questions, he was a delightful companion. His quiet manner, cheerful disposition, and unfailing consideration of others endeared him to all his associates.

He was a member of the Sigma Phi, Phi Beta Kappa, Tau Beta Pi, and Chi Epsilon Fraternities.

Mr. Jameson was elected a Junior of the American Society of Civil Engineers on March 11, 1929.

---

[1] Memoir prepared by Morrough P. O'Brien, Assoc. M. Am. Soc. C. E.

OLIVER BERTRAND JOHNSON, Jun. Am. Soc. C. E.[1]

Died December 21, 1934

Oliver Bertrand Johnson, the son of Chris B. Johnson and Anna Margaret (Husby) Johnson, was born at Seattle, Wash., on December 28, 1903. He received his early education in the public schools and High School at Seattle, and, in 1927, was graduated from the University of Washington with the degree of Bachelor of Science in Civil Engineering.

He was employed from January, 1923, to September, 1924, as Chainman for the Northern Pacific Railway Company. From July, 1926, to April, 1927, Mr. Johnson served as Locator for the Puget Sound Title Insurance Company, and from June to October, 1927, he was employed by the United States Bureau of Reclamation as Levelman and Inspector on canal excavation and embankment.

From July, 1928, until his death in December, 1934, Mr. Johnson was a member of the United States Geological Survey, engaged as a Junior and Assistant Engineer in surface-water investigations in Arkansas, Washington, and Minnesota. His thorough knowledge of hydraulics, his keen analytical mind, and his complete devotion to the interest of his work, assured him success in his chosen profession had his life been spared.

He was married in 1932 to Retta Wooden, of Kent, Wash., who, with a daughter, survives him.

His kindly disposition, pleasing personality, and keen sense of humor endeared him to all those with whom he came in contact, and his death is sincerely regretted by his many friends and associates.

Mr. Johnson was elected a Junior of the American Society of Civil Engineers on June 4, 1928.

HERMAN GREIG VEEDER, Jr., Jun. Am. Soc. C. E.[2]

Died June 11, 1934

Herman Greig Veeder, Jr., the son of Herman Greig and Mabel (Marshall) Veeder, was born at Pittsburgh, Pa., on September 10, 1903. He was the great-grandson of John Peter Veeder, a pioneer builder of New York State, who constructed a part of the Erie Canal and some of the early railroads. He received his early education in the public schools and Thurston School, at Pittsburgh; prepared for college at the Hill School, Pottstown, Pa.; and attended Cornell University, Ithaca, N. Y., from which he was graduated, in 1925, with the degree of Civil Engineer. At Cornell, he was a member of the Kappa Alpha Fraternity and Tau Beta Pi.

---

[1] Memoir prepared by C. L. Batchelder, Esq., St. Paul, Minn.

[2] Memoir prepared by Robert W. Eiler, Esq., Pittsburgh, Pa.

After his graduation, Mr. Veeder started work in Pittsburgh for the McClintic-Marshall Corporation, taking a College Apprentice Course. In 1925, he went to Bath, Me., as Chief Timekeeper and Assistant Engineer on the Carlton Bridge, a lift-span structure crossing the Kennebec River and having a total length of 3 090 ft.

In 1928 and 1929, he was Resident Engineer in charge of the erection of the Mt. Hope Bridge, a large suspension bridge over Mt. Hope Bay, between Portsmouth and Bristol, R. I.

Following work in England on an erection estimate for the Irrawaddy River Bridge in India, Mr. Veeder's next project took him to Bristol, Pa., where, for a time, he was Resident Engineer in charge of the Burlington-Bristol Bridge, the longest lift-span bridge in the world, connecting the States of New Jersey and Pennsylvania.

In 1932, he was transferred to the Leetsdale (Pa.) Plant of the McClintic-Marshall Corporation, as Assistant Manager of the Erection Department, where he gained valuable experience in plate work. Thereafter, he supervised the erection of a tank at West Haverstraw, N. Y., and of a water pipe line at Tamaqua, Pa.

In September, 1933, Mr. Veeder sailed for Buenos Aires, Argentine Republic, to take charge of the construction of 163 oil-storage tanks. This task completed, he was on his way home, where his sterling work was to be rewarded with a more important post, when the crash of a passenger airplane into a shallow lake near Junin, Argentine Republic, on June 11, 1934, brought to an untimely end his brilliant engineering career.

He is survived by his parents and two younger brothers, James M. and Nicholas P. Veeder, all of Pittsburgh.

Mr. Veeder was elected a Junior of the American Society of Civil Engineers on October 1, 1928.

PATRICK McGOVERN, Affiliate, Am. Soc. C. E.[1]

Died February 22, 1933

Patrick McGovern was born on August 15, 1871, in the village of Black Lion, County Cavan, Ireland, the son of Patrick and Mary McGovern.

He attended the district school for a few years, working on the home farm when the school day was over. The energy and force of the man manifested themselves in those school days, making the lessons and chores irksome to him, and he felt the urge to go in search of broader horizons. He found employment in the village store, serving in many capacities until he reached his nineteenth year, saving the small earnings from his hard labor with the idea of using them to fulfill his dream of coming to America, which he looked upon as the land of opportunity and promise. He arrived in New York City in the spring of 1890, empty in pocket, but richly endowed with vigor, strong purpose, and character.

---

[1] Memoir prepared by Robert Ridgway, Past-President and Hon. M. Am. Soc. C. E.

Mr. McGovern's first years in America were colorful. To satisfy his strong young power he traveled from city to city in search of employment. Reaching the West Coast he went to Alaska during the exciting days of the gold rush. The country was filled with the movement and struggle that he craved; he absorbed it all during these first years, and found it satisfying. Back of it all, however, was the desire to find his place in the world of men and deeds, and to become settled.

Coming back East he found work in Philadelphia, Pa., with a firm of contractors, where he earned such recognition for his ability and capacity for hard work that he was promoted to a Foreman's job. On the completion of this work, he went to Boston, Mass., where he had been previously employed, obtaining work and positions of increasing responsibility. It was there that Mr. McGovern, about 1896, secured his first contract—a paving job. This was the small beginning of the successful career that followed, and he was soon recognized for his tirelessness and the faithful performance of his obligations. This first contract was followed by another paving job, and, in 1901, by a contract under the Metropolitan Water Board of Massachusetts for the construction of a section of the Weston Aqueduct and other work.

When the municipal authorities of Boston were ready to develop a subway system and asked for bids for its construction, Mr. McGovern entered the field. He was the low bidder in 1901 for a section of the Tremont Street Subway and, later, for the approach to the East Boston Tunnel and for a part of the Washington Street Subway, and was awarded the contracts, completing them with the efficiency and system which characterized his work. These were projects of a pioneer character, involving the care of the adjacent buildings and other responsibilities incident to working in a narrow street crowded with traffic. These projects were followed by contracts for a section of cut-and-cover aqueduct for the water supply of Montreal, Que., Canada, and in 1909 for the Beacon Hill Tunnel, in Boston—a part of the Cambridge Subway.

In February, 1912, in partnership with Mr. Charles L. Perrin, of Boston, Mr. McGovern was the low bidder for Section 9, Route 5, of the Lexington Avenue Subway, in New York City—a 4-track structure extending from East 67th Street to East 79th Street, Manhattan—and was given the contract amounting to about $2 500 000. This was the first of the notable contracts of magnitude which he executed in New York City, involving many difficult features of construction, all of them skillfully and vigorously carried to completion, reflecting credit on his management and on his ability to do work successfully under the difficult conditions incident to operations under the traffic-laden streets of New York, with the care of all the buildings and the complicated sub-surface structures.

Before the completion of this job, Mr. McGovern undertook the construction of four miles of the Champlain Branch of the New York State Barge Canal, between Mechanicsville and Waterford, including a dam and other appurtenant structures. A contract followed for the building of the Fort

Point Channel Tunnels under the harbor from Boston to Dorchester, Mass. These were two 24-ft span tunnels, shield-driven under compressed air, using a lining of wooden blocks. The cost was about $2 500 000.

In August, 1916, he secured the contract for the twin-tube rapid-transit tunnel under the East River at 60th Street, New York City, with its approaches. This was also a compressed-air job, the pressure reaching a maximum of about 48 lb per sq in. under the West Channel. A unique feature of this tunnel was that a portion of the river bottom was more than 100 ft below mean high water, about 12 ft lower than the tops of the tubes making it necessary to deposit a heavy clay blanket supported by rock-revetment walls across part of the West Channel, and through this artificial bottom or submerged weir, to drive the shields—the blanket being left as a permanent cover and protection. Some engineers doubted the practicability of doing this, but in spite of the difficulties the job was carried through to a successful completion.

Later, between 1920 and 1922, Mr. McGovern completed a 2-track cut-and-cover subway on 14th Street, New York City, between Irving Place and Avenue B, forming the west approach to the under-river tunnel at that point. The material excavated was a fine saturated sand flowing readily when opportunity offered, and extraordinary care was required to prevent damage to near-by buildings and the street. The cut in this material reached a depth of nearly 20 ft below the water level. Mention is also made of the building of the foundations of the Boston Army Base constructed during the World War; of the installation, in 1919 and 1920, of six miles of a 6-ft, riveted, steel pipe line for the water supply of Jersey City, N. J.; and, later, of two sewer tunnel jobs in the Borough of Manhattan, New York City, one under 26th Street and the other under Carmine Street. They were followed about 1922 by a contract for the completion of a causeway across Jamaica Bay, New York City, involving the driving of reinforced concrete piles of a maximum length of 108 ft, some of them weighing as much as 40 tons. Concrete piles of this length had to be handled carefully to avoid the risk of injury, and the method of accomplishing this, designed by the contractor's organization, worked successfully. In 1922 and 1923, he constructed two large shafts for a proposed standard railroad tunnel between Brooklyn and Staten Island, New York Harbor, at a cost of about $1 250 000.

Mr. McGovern was the successful bidder for two sections of the 4-track subway under North Broad Street, Philadelphia, Pa., aggregating about 5 miles in length, between Clearfield Street and Olney Avenue, including the Terminal Yard at the latter thoroughfare. He adopted novel methods of construction for this work which proved to be quite successful; the cost approximated $25 000 000. Then, between March, 1927, and July, 1930, he constructed for the Board of Transportation, City of New York, the so-called 53d Street Subway, under that street, extending from Eighth Avenue, Manhattan, east to and under the East River, together with the approach section in the Borough of Queens, a length of about 11 300 ft. Except for the 4-track portion between Eighth and Sixth Avenues, this was a 2-track structure, most of it in tunnel, the under-river part being cast iron and concrete lined, and excavated by the shield method under compressed air. The work, as a whole, was of a complicated and difficult character. The bottom of the West Channel of the river was below the subgrade of the tunnel and, as at 60th Street, it was necessary to deposit a heavy blanket of clay protected by rip-rap through which the two shields were driven successfully. The land section under 53d Street was in rock. The plans called for the excavations to be made sometimes in single separate tubes and sometimes with all the tubes in one excavation—all of this under a narrow street flanked with buildings and with an elevated and a surface railroad to be supported and protected. Three stations, each 600 ft long, were included and required a substantial enlargement of the excavation. One experienced in such work realizes its difficulties and hazards. The value of the contract was nearly $20 000 000.

On October 5, 1928, Mr. McGovern was awarded, by the Board of Water Supply of New York City, contracts for the construction of what is known as City Tunnel No. 2, but because of litigation the work could not begin until the following spring. The tunnel, about 20 miles long, extended from Hill View Reservoir, in Yonkers, to and under the Borough of The Bronx, the East River, and through the Borough of Queens into Brooklyn. It was at that time the longest true tunnel in the world. It averaged 650 ft below the surface of the ground and was circular in section, most of it 17-ft finished diameter. It involved the sinking of nineteen shafts, all concrete lined, nine of them requiring pneumatic caissons to reach the rock which attained a maximum depth of more than 100 ft below the ground surface. At the time it was let, it was in value the largest construction contract ever awarded by the City of New York, its estimated cost being about $42 000 000. Notwithstanding the difficulties encountered with bad ground and water at several points, the tunnel work was pushed through to completion in advance of the contract requirements. In one place a large volume of water, under high pressure, was encountered under the Bronx River. It was necessary to inject more than 25 000 bags of cement in the ground before the excavation could be resumed. A novel and ingenious method, devised by the contractor, for overcoming the difficulty there was the driving of horizontal steel sheet-piling through the disintegrated and broken rock to insure safety. It took about seven months to advance approximately 300 ft at this point.

This brief summary of the major contracts executed by Mr. McGovern and amounting to about $125 000 000 in value, does not attempt to describe and gives but little idea of the many interesting and difficult details of his work. That would make an instructive and fascinating story of accomplishment and a history of continued improvements in the art of construction which Mr. McGovern was always ready to adopt. To have built successfully so many and such great structures designed for and contributing so much to the good of mankind was a matter of great pride to him, and it gave him much satisfaction. No task was too great or too difficult for him to undertake—the more difficult it was the more it appealed to him. He mastered the conditions and details of any work he undertook, and knew the strong points and limitations of his assistants. He felt that his place to guide the work was primarily in the field, and, in consequence, made a point of being on the work early in the morning and inspecting it. An excellent executive, he was instinctively a Constructing Engineer, and fully deserved the title of "Master Builder."

To his friends it was inspiration to watch the development of this masterful and human man. He preferred association with those who did things—for a weakling and the man with a dishonest mind he had no tolerance. His hobby was work, and with the magnificent physique with which he was blessed he never seemed to tire. His sound common sense and his knowledge of affairs made time spent with him satisfying mentally. He was very generous in his habits and instincts, and liberal with his contributions to charities and other good causes. Few, however, knew of his many benefactions. He died with the respect and esteem of many good friends who miss his inspiring personality and his sound constructive counsel.

Mr. McGovern was a member of the General Contractors Association of New York, having served as its President in 1924. Following his death the Executive Committee of that Association passed a resolution which is quoted herein in part:

> "*Resolved,* That as Members of the Executive Committee and his associates for many years, we record this expression of sorrow at the death of our beloved member and our former President, Patrick McGovern, on Wednesday, February 22nd, last.
>
> * * * * * * * *
>
> "Possessed of a keen mind, his exceptional ability, his clear judgment and wise counsel, combined with his high sense of honor, commanded the respect of all those associated or who had contact with him, and the memory of his cheerful, kind and lovable qualities and sterling character will be an abiding inspiration to all who enjoyed his friendship."

Mr. McGovern was also a member of the New York Athletic Club, the Boston Engineers Club, the Boston Society of Civil Engineers, the Engineers Club of New York, Society of the Friendly Sons of St. Patrick, American Irish Historical Society, Catholic Club of the City of New York, and the Knights of Columbus. He was a member of the Roman Catholic Church to which he was deeply attached, and was proud of the fact that he had been decorated by it with the orders of Grand Cross Knight of the Order of St. Gregory the Great, and Knight Grand Cross of the Order of the Holy Sepulchre.

He is survived by his widow, Mrs. Mary (McCaffrey) McGovern, to whom he was married in April, 1898, and by three daughters, Mrs. F. L. Kane, Mrs. J. V. Gallagher, and Mrs. H. V. Roden, and several grandchildren.

Mr. McGovern was elected an Affiliate of the American Society of Civil Engineers on July 6, 1920.

ALFRED WATTS KIDDLE, Fellow, Am. Soc. C. E.[1]

DIED JANUARY 7, 1935

Alfred Watts Kiddle was born in New York, N. Y., on August 9, 1865. His father, Henry Kiddle, was at one time Superintendent of Schools of the City of New York; his mother was Jane (Wray) Kiddle. He was educated in the public schools of New York City, the College of the City of New York, and the Law School of Columbia University from which he was graduated in the Class of 1887. He was admitted to the practice of law on February 17, 1887.

After his admission to the Bar, Mr. Kiddle specialized in Patent Law in which he was greatly aided by a knowledge of scientific and engineering subjects. One of his earlier affiliations was with the firm of Dyer and Seely, Patent Attorneys for the late Thomas A. Edison for whom he solicited many of his patents. Later, he was Patent Attorney for the Edison Electric Light Company.

Mr. Kiddle was for many years a member of the law firm of Redding, Kiddle, and Greeley, and, later, was Senior Member of the law firms of Kiddle, Wendell, and Varney; Kiddle, Wendell, and Margeson; Kiddle, Margeson, and Hornidge; at the time of his death he was Senior Member of the firm of Kiddle, Bethell, and Montgomery. He was also Special Counsel for the City of New York in patent matters.

His association with inventors led him to become affiliated with many engineering organizations. He was a Charter Member and Past-President of The Engineers' Club; Past-President of The Lawyers' Club; and Charter Member and Past Vice-President of The Edison Pioneers.

Mr. Kiddle was also a member of the following associations: Alpha Delta Phi Fraternity; American Bar Association; American Patent Law Association; Association of the Bar of the City of New York; Merchants' Association of the City of New York; Pilgrims of the United States; Honorary Member of John Ericsson Society; anl member of the New York Patent Law Association, in which latter organization he served on several committees.

His kindliness, genial nature, and ready wit, as well as his conscientious application to activities at his various clubs, won for him great popularity among his fellow members, and he was highly esteemed by all who came in contact with him whether in professional or in social relations.

Mr. Kiddle was a man of unusually high character and enjoyed the reputation among his friends of being a most delightful companion and a true friend. He was one of few men who had the art of conversation.

His philosophy of life was to spread cheer wherever he could, and he took an unusual pleasure in doing a favor for a friend.

He is survived by his widow, Florence (Dunham) Kiddle, his son Alfred M. F. Kiddle, and his sister, Bessie A. Kiddle.

Mr. Kiddle was elected a Fellow of the American Society of Civil Engineers on October 3, 1893.

---

[1] Memoir prepared by Joseph Montgomery, Esq., New York, N. Y., assisted by Charles W. Guttzeit, Esq., New York, N. Y.

TRANSACTIONS

OF THE

AMERICAN SOCIETY OF CIVIL ENGINEERS

INDEX

VOLUME 100

1935

SUBJECT INDEX, PAGE 1798
AUTHOR INDEX, PAGE 1810

Titles of papers are in quotation marks when given with the author's name.

VOLUME 100

SUBJECT INDEX

ADDRESSES.

Presidential Address at the Annual Convention at Los Angeles, California, July 3, 1935. Arthur S. Tuttle. 1407.

AERATION.

"Loss of Head in Activated Sludge Aeration Channels." Darwin Wadsworth Townsend. (With Discussion.) 558.

AMERICAN SOCIETY OF CIVIL ENGINEERS.

"The Development of the American Society of Civil Engineers": Address at the Annual Convention, Los Angeles, California, July 3, 1935. Arthur S. Tuttle. 1407.

ARCHES.

See BRIDGES.

BEAMS.

"Experiments with Concrete in Torsion." Paul Andersen. 949. Discussion: E. Mirabelli, Frank Russell, Leslie Turner, A. W. Fischer, H. J. Gilkey, and A. A. Eremin. 961.

"Investigation of Web Buckling in Steel Beams." Inge Lyse and H. J. Godfrey. 675. Discussion: R. L. Moore and E. C. Hartmann. 696.

BEARINGS.

"Renewal of Miter-Gate Bearings. Panama Canal." Clinton Morse. (With Discussion.) 471.

BIBLIOGRAPHY.

Bibliography on street thoroughfares compiled by the Committee of the City Planning Division on Street Thoroughfares Manual. 1076.

BREAKWATERS.

"Wave Pressures on Sea-Walls and Breakwaters." David A. Molitor. 984. Discussion: Charles E. Fowler, Charles T. Leeds, A. A. Eremin. and T. L. Condron and Chester L. Post. 1003.

BRIDGES.

"A Generalized Deflection Theory for Suspension Bridges." D. B. Steinman. 1133. Discussion: E. L. Pavlo, Jonathan Jones, A. Müllenhoff, H. Cecil Booth, Jacob Feld, Glenn B. Woodruff, Howard C. Wood, and Ralph A. Tudor, L. J. Mensch, A. A. Eremin, Hans H. Bleich, F. H. Frankland, Gustav Lindenthal, Julian W. Shields, A. W. Fischer, J. M. Frankland, Fredrik Vogt, Leon S. Moisseiff, A. Mitchell and G. T. Parkin, Sterling Johnston, Harold E. Wessman, C. H. Gronquist, W. R. Frederick, Jr., and William H. Yates. 1171.

"Concrete and Reinforced Concrete Arches. Final Report of the Special Committee." 1427.

"Laboratory Tests of Multiple-Span Reinforced Concrete Arch Bridges." Wilbur M. Wilson. 424. Discussion: C. B. McCullough, Carroll L. Mann, Jr., and M. Hirschthal. 455.

BUILDINGS.

"Flexible 'First-Story' Construction for Earthquake Resistance." Norman B. Green. 645. Discussion: Lee H. Johnson, Edward J. Bednarski, Merit P. White and Paul L. Kartzke, Howard G. Smits, and H. A. Williams. 653.

BULKHEAD WALLS.

See RETAINING WALLS.

BUTTRESSES.

"Water-Bearing Members of Articulated Buttress Dams." Hakan D. Birke. (With Discussion.) 55.

CANALS.

"Renewal of Miter-Gate Bearings, Panama Canal." Clinton Morse. (With Discussion.) 471.

CONCRETE.

"Experiments with Concrete in Torsion." Paul Andersen. (With Discussion.) 949.

CRIBS.

See FOUNDATIONS.

CUT-OFFS.

"Security from Under-Seepage—Masonry Dams on Earth Foundations." E. W. Lane. (With Discussion.) 1235.

"Uplift and Seepage Under Dams on Sand." L. F. Harza. (With Discussion.) 1352.

DAMS.

"Model of Calderwood Arch Dam." A. V. Karpov and R. L. Templin. 185. Discussion: A. W. Simonds, Lars R. Jorgensen, A. C. Janni, P. Wilhelm Werner, Eldred D. Smith, Elmer O. Bergman. and E. Probst. 220.

"Security from Under-Seepage—Masonry Dams on Earth Foundations." E. W. Lane, 1235. Discussion: William P. Creager, L. F. Harza, Joel D. Justin, Louis E. Ayres. W. M. Griffith. E. Mckenzie Taylor, Donald J. Hebert, Arthur Casagrande, Calvin V. Davis. C. H. Eiffert, E. H. Burroughs, Jr., Alexander Potter, Robert E. Kennedy, A. N. Khosla, and A. Streiff. 1273.

"Some Soil Pressure Tests." H. de B. Parsons. (With Dscussion.) 1.

"Uplift and Seepage Under Dams on Sand." L. F. Harza. 1352. Discussion: Edward Godfrey, Hibbert M. Hill, Joel D. Justin, Charles Terzaghi, Harry H. Hatch, Donald J. Hebert, and C. A. Mockmore and John W. Dougherty. 1386.

"Water-Bearing Members of Articulated Buttress Dams." Hakan D. Birke. 55. Discussion: R. A. Sutherland, P. Wilhelm Werner, Charles P. Williams, Paul Baumann, and A. Floris. 70.

DRAINAGE.

"Rainfall Studies for New York, N. Y." S. D. Bleich. (With Discussion.) 609.

EARTH PRESSURES.

"Analysis of Sheet-Pile Bulkheads." Paul Baumann. (With Discussion.) 707.

"Some Soil Pressure Tests." H. de B. Parsons. 1. Discussion: J. C. Meem, Eugene E. Halmos, L. C. Wilcoxen, O. K. Froehlich, H. L. Thackwell, Jacob Feld, T. Farrance Davey, D. P. Krynine, Charles Terzaghi, R. L. Vaughn, and M. Hirschthal. 14.

EARTHQUAKES.

"Flexible 'First-Story' Construction for Earthquake Resistance." Norman B. Green. (With Discussion.) 645.

ECONOMICS.

"Water-Power Development on the St. Lawrence River." Daniel W. Mead. (With Discussion.) 498.

EROSION.

"Modifying the Physiographical Balance by Conservation Measures." A. L. Sonderegger. (With Discussion.) 284.

FINANCE.

Paying for street thoroughfares. 1068.

FLOODS.

"Modifying the Physiographical Balance by Conservation Measures." A. L. Sonderegger. (With Discussion.) 284.

"The Reservoir as a Flood-Control Structure." George R. Clemens. (With Discussion.) 879.

FORESTS.

"Modifying the Physiographical Balance by Conservation Measures." A. L. Sonderegger. (With Discussion.) 284.

FOUNDATIONS.

"Analysis of Sheet-Pile Bulkheads." Paul Baumann. (With Discussion.) 707.
"Lincoln Highway from Jersey City to Elizabeth, New Jersey." Sigvald Johannesson. (With Discussion.) 91.
"Security from Under-Seepage—Masonry Dams on Earth Foundations." E. W. Lane. (With Discussion.) 1235.
"Some Soil Pressure Tests." H. de B. Parsons. (With Discussion.) 1.
"Uplift and Seepage Under Dams on Sand." L. F. Harza. (With Discussion.) 1352.
"Wave Pressures on Sea-Walls and Breakwaters." David A. Molitor. (With Discussion.) 984.

GRAPHICAL CHARTS.

"Eccentric Riveted Connections." Eugene A. Dubin. 1086. Discussion: Carlton T. Bishop, James R. Bole, Albert Wertheimer, Kenneth L. DeBlois, Jonathan Jones, Armin Rozman, A. E. R. de Jonge, Fang-Yin Tsai, and Odd Albert. 1094.

HIGHWAYS AND ROADS.

Bibliography on street thoroughfares compiled by the Committee of the City Planning Division on Street Thoroughfares Manual. 1076.
"Lincoln Highway from Jersey City to Elizabeth, New Jersey." Sigvald Johannesson. 91. Discussion: Fred Lavis, Theodore Belzner, and J. B. French. 115.
"Street Thoroughfares": A Symposium. George H. Herrold, W. W. Crosby, Jacob L. Crane, Jr., Jay Downer, J. P. Hallihan, W. W. Horner, and Daniel L. Turner. 1045. Discussion: H. Shifrin, Donald M. Baker, Frederic Bass, and Hugh E. Young. 1079.

JOINTS.

Expansion joints on high-level steel viaduct structure of Lincoln Highway, New Jersey. 106.

LOCK-GATES.

"Renewal of Miter-Gate Bearings, Panama Canal." Clinton Morse. 471. Discussion: E. S. Randolph. 496.

MEMOIRS OF DECEASED MEMBERS.

Acevedo, Octavio Augusto. 1597.
Adgate, Frederick Whitney. 1599.
Ames, George Marshall. 1759.
Baier, Julius. 1600.
Berle, Kort. 1602.
Bishop, Lyman Edgar. 1603.
Bond, George Meade. 1605.
Bouillon, Alfred Victor. 1608.
Brown, Wilbur Gayle. 1610.
Brownell, James Parker. 1611.

Bryan, George, Jr. 1613.
Burd, Edward Morris. 1614.
Burr, William Hubert. 1617.
Byers, Morton Lewis. 1621.
Cahill, Walter John. 1623.
Carter, Henry Hall. 1624.
Christian, William Arnold. 1627.
Churchill, Charles Samuel. 1629.
Clark, George Hallett. 1631.
Cole, John Adams. 1634.
Coleman, Eugene Hunter. 1635.
Conn, Charles Francis. 1637.
Cornish, Lorenzo Dana. 1638.
Cram, Clyde Maxwell. 1640.
Curtis, Benjamin John. 1642.
Davidson, William Clarence. 1644.
Davis, Arthur Powell. 1582.
Eaton, Frederick. 1645.
Ericson, Eric Gustaf. 1648.
Faude, Frederic Morris. 1762.
Ferguson, Harry Foster. 1763.
Finebaum, Harry Jacob. 1649.
Foster, Walter Linder. 1651.
Fuller, George Warren. 1653.
Furuichi, Koi. 1595.
Gibbs, Elbert Allan. 1765.
Goodwin, Elmer Clark. 1767.
Gore, William. 1660.
Grady, John Francis. 1768.
Grover, William Albert. 1662.
Grunsky, Carl Ewald. 1591.
Hall, John Wendell. 1663.
Harney, Francis Vincent. 1770.
Hatton, Thomas Chalkley. 1665.
Hill, John Edward. 1667.
Hillman, George Waldo. 1771.
Hodgman, Burt Bradley. 1672.
Hooper, Elmer Guy. 1673.
Hosmer, George Leonard. 1676.
Houston, John Jay Lafayette. 1772.
Howe, George Edward. 1677.
Jameson, John Alexander, Jr. 1789.
Johnson, George Arthur. 1678.
Johnson, Oliver Bertrand. 1790.
Johnston, Guy Robert. 1681.
Kernot, Maurice Edwin. 1683.
Kiddle, Alfred Watts. 1796.
King, Warren Raymond. 1775.
Kirby, Isaac Henry. 1776.
Kriegshaber, Victor Hugo. 1685.
Lang, Oliver Howard. 1687.

Langfitt, William Campbell. 1688.
McClain, James Bronson. 1778.
McClintock, James Robinson. 1693.
McGovern, Patrick. 1791.
McLean, Archibald. 1694.
Mahony, Arthur Sawyer. 1778.
Moore, Thomas Vincent. 1780.
Murphy, Edward Charles. 1697.
Nagler, Floyd August. 1698.
Netland, Lars. 1701.
Nicholson, Maury. 1702.
Nuebling, Emil Louis. 1704.
O'Hara, Francis Joseph. 1706.
O'Rourke, John Francis. 1707.
O'Shaughnessy, Michael Maurice. 1710.
Pierce, George Abel. 1781.
Potter, William Bancroft. 1713.
Prichard, Henry Sewall. 1716.
Rice, Herbert Allan. 1783.
Ritchie, James. 1718.
Rogers, Clarence Mord. 1719.
Schofield, Hiram Abiff. 1721.
Sessions, Edson Oliver. 1785.
Seurot, Paul Albert Nicolas. 1723.
Shaw, George Harry Thornton. 1728.
Slater, Joseph Mansfield. 1733.
Small, Kenneth Irving. 1734.
Speiden, Theodore. 1735.
Sprague, Frank Julian. 1736.
Stierlin, Gustave Arnold. 1741.
Thomas, Burtis Paul. 1787.
Thompson, Ralph Penny. 1788.
Van Cleve, Aaron Howell. 1742.
Vars, Alexander Wilson. 1746.
Veeder, Herman Greig, Jr. 1790.
Voorhees, Isaac Spurr. 1748.
Waldrop, John Douglas. 1749.
Weigand, Edwin John. 1754.
Willis, Paul. 1756.
Wilson, Herbert Alva. 1757.

MODELS.

"Flexible 'First-Story' Construction for Earthquake Resistance." Norman B. Green. (With Discussion.) 645.
"Laboratory Tests of Multiple-Span Reinforced Concrete Arch Bridges." Wilbur M. Wilson. (With Discussion.) 424.
"Model of Calderwood Arch Dam." A. B. Karpov and R. L. Templin. (With Discussion.) 185.
"Practical River Laboratory Hydraulics." Herbert D. Vogel. (With Discussion.) 118.

Reliability of conclusions based on models or electric analogy experiments. 1343.

"Sand Mixtures and Sand Movement in Fluvial Models." Hans Kramer. (With Discussion.) 798.

"Uplift and Seepage Under Dams on Sand." L. F. Harza. (With Discussion.) 1352.

PARKS AND PARKWAYS.

Design of boulevards and parkways. 1060.

PAVEMENT.

Roadway surfaces, Lincoln Highway, New Jersey. 108.

PERCOLATION.

See **SEEPAGE.**

PILES.

"Analysis of Sheet-Pile Bulkheads." Paul Baumann. (With Discussion.) 707.

RAINFALL.

"An Approach to Determinate Stream Flow." Merrill M. Bernard. (With Discussion.) 347.

"Rainfall Studies for New York, N. Y." S. D. Bleich. 609. Discussion: H. Alden Foster, J. J. Slade, Jr., Charles W. Sherman, Merrill M. Bernard, Clifford Seaver, Jose Garcia Montes, Jr., and Rhodes E. Rule. 628.

REINFORCED CONCRETE.

"Concrete and Reinforced Concrete Arches. Final Report of the Special Committee." 1427.

"Laboratory Tests of Multiple-Span Reinforced Concrete Arch Bridges." Wilbur M. Wilson. (With Discussion.) 424.

"Water-Bearing Members of Articulated Buttress Dams." Hakan D. Birke. (With Discussion.) 55.

REPORTS OF COMMITTEES.

"Concrete and Reinforced Concrete Arches. Final Report of the Special Committee." Clyde T. Morris, George E. Beggs, E. H. Harder, A. C. Janni, and Wilbur M. Wilson. 1427.

Definitions, general principles and recommendations. Report of the Committee of the City Planning Division on Street Thoroughfares Manual. 1048.

RESERVOIRS.

"The Reservoir as a Flood-Control Structure." George R. Clemens. (With Discussion.) 879.

RETAINING WALLS.

"Analysis of Sheet-Pile Bulkheads." Paul Baumann. 707. Discussion: Jacob Feld, R. L. Vaughn, M. A. Drucker, Raymond P. Pennoyer, D. P. Krynine, E. h. O. Franzius, Theodor von Kármán, and E. Lohmeyer. 741.

"Determination of Trapezoidal Profiles for Retaining Walls." A. J. Sutton Pippard. 1116. Discussion: E. S. Lindley, Kenneth L. DeBlois, and M. A. Drucker. 1128.
"Some Soil Pressure Tests." H. de B. Parsons. (With Discussion.) 1.
"Wave Pressures on Sea-Walls and Breakwaters." David A. Molitor. (With Discussion.) 984.

RIVER REGULATION.
See **RIVERS.**

RIVERS.
"An Approach to Determinate Stream Flow." Merrill M. Bernard. (With Discussion.) 347.
"Modifying the Physiographical Balance by Conservation Measures." A. L. Sonderegger. 284. Discussion: H. H. Chapman, E. B. Debler, Frank E. Bonner, C. S. Jarvis, W. P. Rowe, J. C. Stevens, Gerard H. Matthes, J. B. Lippincott, and Rhodes R. Rule. 305.
"Practical River Laboratory Hydraulics." Herbert D. Vogel. (With Discussion.) 118.
"Sand Mixtures and Sand Movement in Fluvial Models." Hans Kramer. (With Discussion.) 798.
"The Reservoir as a Flood-Control Structure." George R. Clemens. 879. Discussion: C. Maxwell Stanley, F. Knapp, Charles S. Bennett, and Gerard H. Matthes. 914.
"Water-Power Development on the St. Lawrence River." Daniel W. Mead. (With Discussion.) 498.

RIVET SPACING.
"Eccentric Riveted Connections." Eugene A. Dubin. (With Discussion.) 1086.

RIVETED JOINTS.
"Eccentric Riveted Connections." Eugene A. Dubin. (With Discussion.) 1086.

ROADS.
See **HIGHWAYS AND ROADS.**

RUN-OFF.
"An Approach to Determinate Stream Flow." Merrill M. Bernard. (With Discussion.) 347.
"Rainfall Studies for New York, N. Y." S. D. Bleich. (With Discussion.) 609.
"The Reservoir as a Flood-Control Structure." George R. Clemens. (With Discussion.) 879.

SEA-WALLS.
See **RETAINING WALLS.**

SEEPAGE.
"Security from Under-Seepage—Masonry Dams on Earth Foundations." E. W. Lane. (With Discussion.) 1235.

"Some Soil Pressure Tests." H. de B. Parsons. (With Discussion.) 1.

"Uplift and Seepage Under Dams on Sand." L. F. Harza. (With Discussion.) 1352.

SEWAGE DISPOSAL.

"Loss of Head in Activated Sludge Aeration Channels." Darwin Wadsworth Townsend. (With Discussion.) 558.

SEWAGE TREATMENT.

See **SEWAGE DISPOSAL.**

SEWERAGE.

"Rainfall Studies for New York, N. Y." S. D. Bleich. (With Discussion.) 609.

SHEATHING.

"Analysis of Sheet-Pile Bulkheads." Paul Baumann. (With Discussion.) 707.

SHEET-PILING.

See **PILES.**

SHIP CANALS.

See **CANALS.**

SILT.

"Modifying the Physiographical Balance by Conservation Measures." A. L. Sonderegger. (With Discussion.) 284.

"Sand Mixtures and Sand Movement in Fluvial Models." Hans Kramer. 798. Discussion: John Leighly, Paul W. Thompson, Gerard H. Matthes, R. H. Keays, F. T. Mavis, V. V. Tchikoff, Morrough P. O'Brien and Bruce D. Rindlaub, Herbert D. Vogel, Joseph B. Tiffany, Jr., and Carl E. Bentzel, and Lorenz G. Straub. 839.

SIMILARITY, THEORY OF.

See **MODELS.**

SOILS.

"Some Soil Pressure Tests." H. de B. Parsons. (With Discussion.) 1.

STREAMS.

See **RIVERS.**

STREETS.

"Street Thoroughfares": A Symposium. George H. Herrold, W. W. Crosby, Jacob L. Crane, Jr., Jay Downer, J. P. Hallihan, W. W. Horner, and Daniel L. Turner. (With Discussion.) 1045.

STRESS AND STRAIN.

"A Generalized Deflection Theory for Suspension Bridges." D. B. Steinman. (With Discussion.) 1133.

"Analysis of Sheet-Pile Bulkheads." Paul Baumann. (With Discussion.) 707.

"Experiments with Concrete in Torsion." Paul Andersen. (With Discussion.) 949.

"Flexible 'First-Story' Construction for Earthquake Resistance." Norman B. Green. (With Discussion.) 645.

"Investigation of Web Buckling in Steel Beams." Inge Lyse and H. J. Godfrey. (With Discussion.) 675.

"Laboratory Tests of Multiple-Span Reinforced Concrete Arch Bridges." Wilbur M. Wilson. (With Discussion.) 424.

"Model of Calderwood Arch Dam." A. V. Karpov and R. L. Templin. (With Discussion.) 185.

"Stresses in Space Structures." F. H. Constant. (With Discussion.) 928.

"Water-Bearing Members of Articulated Buttress Dams." Hakan D. Birke. (With Discussion.) 55.

"Wave Pressures on Sea-Walls and Breakwaters." David A. Molitor. (With Discussion.) 984.

"Williot Equations for Statically Indeterminate Structures in Combination with Moment Equations in Terms of Angular Displacements." Charles A. Ellis. (With Discussion.) 580.

STRUCTURES, THEORY OF.

"A Generalized Deflection Theory for Suspension Bridges." D. B. Steinman. (With Discussion.) 1133.

"Stresses in Space Structures." F. H. Constant. 928. Discussion: William R. Osgood, L. E. Grinter, Charles M. Spofford, and A. H. Finlay. 935.

"Williot Equations for Statically Indeterminate Structures in Combination with Moment Equations in Terms of Angular Displacements." Charles A. Ellis. 580. Discussion: Ralph A. Tudor, Leon S. Moisseiff, and A. A. Eremin. 599.

SUSPENSION BRIDGES.

See BRIDGES.

TEMPERATURE.

"Model of Calderwood Arch Dam." A. V. Karpov and R. L. Templin. (With Discussion.) 185.

TRAFFIC CIRCLES.

Design of traffic circle on Lincoln Highway, New Jersey. 97.

UPLIFT (HYDROSTATIC).

"Security from Under-Seepage—Masonry Dams on Earth Foundations." E. W. Lane. (With Discussion.) 1235.

"Some Soil Pressure Tests." H. de B. Parsons. (With Discussion.) 1.

"Uplift and Seepage Under Dams on Sand." L. F. Harza. (With Discussion.) 1352.

VIADUCTS.

"Lincoln Highway from Jersey City to Elizabeth, New Jersey." Sigvald Johannesson. (With Discussion.) 91.

VIBRATION.

"Flexible 'First-Story' Construction for Earthquake Resistance." Norman B. Green. (With Discussion.) 645.

WATER.

"Formation of Floc by Ferric Coagulants." Edward Bartow, A. P. Black, and Walter E. Sansbury. 263. Discussion: Edward S. Hopkins, W. D. Hatfield, L. B. Miller, Linn H. Enslow, and C. J. Brockman. 277.

WATER, FLOW OF, IN OPEN CHANNELS.

"An Approach to Determinate Stream Flow." Merrill M. Bernard. 347. Discussion: C. S. Jarvis, LeRoy K. Sherman, W. W. Horner, C. H. Eiffert, Charles S. Bennett, R. L. Gregory and C. E. Arnold, Gordon R. Williams, and W. G. Hoyt, L. L. Harrold, and F. F. Snyder. 363.

"Loss of Head in Activated Sludge Aeration Channels." Darwin Wadsworth Townsend. 558. Discussion: H. L. Thackwell, Henry R. King, and M. H. Klegerman. 567.

"Practical River Laboratory Hydraulics." Herbert D. Vogel. 118. Discussion: Lorenz G. Straub, Paul W. Thompson, Ralph W. Powell, K. D. Nichols, Frank W. Edwards, I. H. Patty, Charles S. Bennett, Kenneth C. Reynolds, V. V. Tchikoff, Samuel Shulits, Charles D. Curran, and J. B. Egiazaroff. 145.

WATER, FLOW OF, IN PIPES.

"Flow of Water Around Bends in Pipes." David L. Yarnell and Floyd A. Nagler. 1018. Discussion: D. Benjamen Gumensky, Wallace M. Lansford, and F. T. Mavis. 1033.

WATER, FLOW OF, OVER DAMS AND WEIRS.

"Discharge Formula and Tables for Sharp-Crested Suppressed Weirs." C. G. Cline. 396. Discussion: Jasper O. Draffin, and W. Bruce McMillan. 414.

WATER POWER.

"Water-Power Development on the St. Lawrence River." Daniel W. Mead. 498. Discussion: T. H. Hogg, Theron M. Ripley, W. S. Lee, Frank E. Bonner, Rufus W. Putnam, James W. Rickey, Walter M. Smith, J. W. Beardsley, L. F. Harza, and Adolph J. Ackerman. 517.

WATER PRESSURES.

"Flow of Water Around Bends in Pipes." David L. Yarnell and Floyd A. Nagler. (With Discussion.) 1018.

"Security from Under-Seepage—Masonry Dams on Earth Foundations." E. W. Lane. (With Discussion.) 1235.

"Some Soil Pressure Tests." H. de B. Parsons. (With Discussion.) 1.
"Uplift and Seepage Under Dams on Sand." L. F. Harza. (With Discussion.) 1352.
"Wave Pressures on Sea-Walls and Breakwaters." David A. Molitor. (With Discussion.) 984.

WATER PURIFICATION.
See **WATER**

WAVES.
"Wave Pressures on Sea-Walls and Breakwaters." David A. Molitor. (With Discussion.) 984.

WEIRS.
See **WATER, FLOW OF, OVER DAMS AND WEIRS.**

WIND.
Wind records for the Great Lakes. 998.

AUTHOR INDEX

ACEVEDO, OCTAVIO AUGUSTO.
Memoir of. 1597.

ACKERMAN, ADOLPH J.
St. Lawrence power development. 551.

ADGATE, FREDERICK WHITNEY.
Memoir of. 1599.

ALBERT, ODD.
Eccentric riveted connections. 1104.

AMES, GEORGE MARSHALL.
Memoir of. 1759.

ANDERSEN, PAUL.
"Experiments with Concrete in Torsion." 949.

ARNOLD, C. E.
An aproach to determinate stream flow. 372.

AYRES, LOUIS E.
Security from under-seepage. 1278.

BAIER, JULIUS.
Memoir of. 1600.

BAKER, DONALD M.
Street thoroughfares. 1080.

BARTOW, EDWARD.
"Formation of Floc by Ferric Coagulants." 263.

BASS, FREDERIC.
Street thoroughfares. 1083.

BAUMANN, PAUL.
"Analysis of Sheet-Pile Bulkheads." 707.
Articulated buttress dams. 81.

BEARDSLEY, J. W.
St. Lawrence power development. 545.

BEDNARSKI, EDWARD J.

Flexible first-story construction. 657.

BEGGS, GEORGE E.

"Final Report of the Special Committee on Concrete and Reinforced Concrete Arches." 1427.

BELZNER, THEODORE.

Lincoln Highway in New Jersey. 116.

BENNETT, CHARLES S.

An approach to determinate stream flow. 371.
Reservoirs for flood control. 918.
River laboratory hydraulics. 168.

BENTZEL, CARL E.

Sand movement in fluvial models. 861.

BERGMAN, ELMER O.

Model of Calderwood Dam. 240.

BERLE, KORT.

Memoir of. 1602.

BERNARD, MERRILL M.

"An Approach to Determinate Stream Flow." 347.
Rainfall studies for New York, N. Y. 632.

BIRKE, HAKAN D.

"Water-Bearing Members of Articulated Buttress Dams." 55.

BISHOP, CARLTON T.

Eccentric riveted connections. 1094.

BISHOP, LYMAN EDGAR.

Memoir of. 1603.

BLACK, A. P.

"Formation of Floc by Ferric Coagulants." 263.

BLEICH, HANS H.

Deflection theory for suspension bridges. 1187.

BLEICH, S. D.

"Rainfall Studies for New York, N. Y." 609.

BOLE, JAMES R.

Eccentric riveted connections. 1094.

BOND, GEORGE MEADE.

Memoir of. 1605.

BONNER, FRANK E.

Modifying the physiographical balance. 307.
St. Lawrence power development. 532.

BOOTH, H. CECIL.

Deflection theory for suspension bridges. 1177.

BOUILLON, ALFRED VICTOR.

Memoir of. 1608.

BROCKMAN, C. J.

Formation of floc by ferric coagulants. 282.

BROWN, WILBUR GAYLE.

Memoir of. 1610.

BROWNELL, JAMES PARKER.

Memoir of. 1611.

BRYAN, GEORGE, JR.

Memoir of. 1613.

BURD, EDWARD MORRIS.

Memoir of. 1614.

BURR, WILLIAM HUBERT.

Memoir of. 1617.

BURROUGHS, E. H., JR.

Security from under-seepage. 1300.

BYERS, MORTON LEWIS.

Memoir of. 1621.

CAHILL, WALTER JOHN.

Memoir of. 1623.

CARTER, HENRY HALL.

Memoir of. 1624.

CASAGRANDE, ARTHUR.

Security from under-seepage. 1289.

CHAPMAN, H. H.

Modifying the physiographical balance. 305.

CHRISTIAN, WILLIAM ARNOLD.

Memoir of. 1627.

CHURCHILL, CHARLES SAMUEL.

Memoir of. 1629.

CLARK, GEORGE HALLETT.

Memoir of. 1631.

CLEMENS, GEORGE R.

"The Reservoir as a Flood-Control Structure." 879.

CLINE, C. G.

"Discharge Formula and Tables for Sharp-Crested Suppressed Weirs." 396.

COLE, JOHN ADAMS.

Memoir of. 1634.

COLEMAN, EUGENE HUNTER.

Memoir of. 1635.

CONDRON, T. L.

Wave pressures on sea-walls and breakwaters. 1010.

CONN, CHARLES FRANCIS.

Memoir of. 1637.

CONSTANT, F. H.

"Stresses in Space Structures." 928.

CORNISH, LORENZO DANA.

Memoir of. 1638.

CRAM, CLYDE MAXWELL.

Memoir of. 1640.

CRANE, JACOB L., JR.

"Bibliography on Street Thoroughfares." 1076.
"Definitions, General Principles, and Recommendations: Report of the Committee of the City Planning Division on Street Thoroughfares Manual." 1048.
"Relation of Street Thoroughfares to Other Arteries of Transportation." 1056.

CREAGER, WILLIAM P.

Security from under-seepage. 1273.

CROSBY, W. W.

"Bibliography on Street Thoroughfares." 1076.
"Definitions, General Principles, and Recommendations: Report of the Committee of the City Planning Division on Street Thoroughfares Manual." 1048.

CURRAN, CHARLES D.

River laboratory hydraulics. 177.

CURTIS, BENJAMIN JOHN.

Memoir of. 1642.

DAVEY, T. FARRANCE.

Some soil pressure tests. 27.

DAVIDSON, WILLIAM CLARENCE.

Memoir of. 1644.

DAVIS, ARTHUR POWELL.

Memoir of. 1582.

DAVIS, CALVIN V.

Security from under-seepage. 1294.

DEBLER, E. B.

Modifying the physiographical balance. 306.

DeBLOIS, KENNETH L.

Eccentric riveted connections. 1096.
Profiles for retaining walls. 1128.

DOUGHERTY, JOHN W.

Uplift and seepage under dams on sand. 1396.

DOWNER, JAY.

"Bibliography on Street Thoroughfares." 1076.
"Definitions, General Principles, and Recommendations: Report of the Committee of the City Planning Division on Street Thoroughfares Manual." 1048.
"Design of Boulevards and Parkways." 1060.

DRAFFIN, JASPER O.

Discharge formula for weirs. 414.

DRUCKER, M. A.

Analysis of sheet-pile bulkheads. 753.
Profiles for retaining walls. 1129.

DUBIN, EUGENE A.

"Eccentric Riveted Connections." 1086.

EATON, FREDERICK.

Memoir of. 1645.

EDWARDS, FRANK W.

River laboratory hydraulics. 163.

EGIAZAROFF, J. B.

River laboratory hydraulics. 178.

EIFFERT, C. H.

An approach to determinate stream flow. 370.
Security from under-seepage. 1299.

ELLIS, CHARLES A.

"Williot Equations for Statically Indeterminate Structures in Combination with Moment Equations in Terms of Angular Displacements." 580.

ENSLOW, LINN H.

Formation of floc by ferric coagulants. 279.

EREMIN, A. A.

Deflection theory for suspension bridges. 1185.
Experiments with concrete in torsion. 980.
Wave pressures on sea-walls and breakwaters. 1009.
Williot equations for statically indeterminate structures. 604.

ERICSON, ERIC GUSTAF.

Memoir of. 1648.

FAUDE, FREDERIC MORRIS.

Memoir of. 1762.

FELD, JACOB.

Analysis of sheet-pile bulkheads. 741.
Deflection theory for suspension bridges. 1178.
Some soil pressure tests. 26.

FERGUSON, HARRY FOSTER.

Memoir of. 1763.

FINEBAUM, HARRY JACOB.

Memoir of. 1649.

FINLAY, A. H.

Stresses in space structures. 942.

FISCHER, A. W.

Deflection theory for suspension bridges. 1197.
Experiments with concrete in torsion. 969.

FLORIS, A.

Articulated buttress dams. 84.

FOSTER, H. ALDEN.

Rainfall studies for New York, N. Y. 628.

FOSTER, WALTER LINDER.

Memoir of. 1651.

FOWLER, CHARLES E.

Wave pressures on sea-walls and breakwaters. 1003.

FRANKLAND, F. H.

Deflection theory for suspension bridges. 1190.

FRANKLAND, J. M.

Deflection theory for suspension bridges. 1201.

FRANZIUS, E. h. O.

Analysis of sheet-pile bulkheads. 771.

FREDERICK, W. R., JR.

Deflection theory for suspension bridges. 1215.

FRENCH, J. B.

Lincoln Highway in New Jersey. 116.

FROEHLICH, O. K.

Some soil pressure tests. 21.

FULLER, GEORGE WARREN.

Memoir of. 1653.

FURUICHI, KOI.

Memoir of. 1595.

GIBBS, ELBERT ALLAN.

Memoir of. 1765.

GILKEY, H. J.

Experiments with concrete in torsion. 970.

GODFREY, EDWARD.

Uplift and seepage under dams on sand. 1386.

GODFREY, H. J.

"Investigation of Web Buckling in Steel Beams." 675.

GOODWIN, ELMER CLARK.

Memoir of. 1767.

GORE, WILLIAM.

Memoir of. 1660.

GRADY, JOHN FRANCIS.

Memoir of. 1768.

GREEN, NORMAN B.

"Flexible 'First-Story' Construction for Earthquake Resistance." 645.

GREGORY, R. L.

An approach to determinate stream flow. 372.

GRIFFITH, W. M.

Security from under-seepage. 1280.

GRINTER, L. E.

Stresses in space structures. 936.

GRONQUIST, C. H.

Deflection theory for suspension bridges. 1214.

GROVER, WILLIAM ALBERT.

Memoir of. 1662.

GRUNSKY, CARL EWALD.

Memoir of. 1591.

GUMENSKY, D. BENJAMEN.

Flow of water around bends in pipes. 1033.

HALL, JOHN WENDELL.

Memoir of. 1663.

HALLIHAN, J. P.

"Bibliography on Street Thoroughfares." 1076.
"Definitions, General Principles, and Recommendations: Report of the Committee of the City Planning Division on Street Thoroughfares Manual." 1048.
"Super-Highways." 1062.

HALMOS, EUGENE E.

Some soil pressure tests. 17.

HARDER, E. H.

"Final Report of the Special Committee on Concrete and Reinforced Concrete Arches." 1427.

HARNEY, FRANCIS VINCENT.

Memoir of. 1770.

HARROLD, L. L.

An approach to determinate stream flow. 384.

HARTMANN, E. C.

Web buckling in steel beams. 696.

HARZA, L. F.

St. Lawrence power development. 547.
Security from under-seepage. 1275.
"Uplift and Seepage Under Dams on Sand." 1352.

HATCH, HARRY H.

Uplift and seepage under dams on sand. 1395.

HATFIELD, W. D.

Formation of floc by ferric coagulants. 278.

HATTON, THOMAS CHALKLEY.

Memoir of. 1665.

HEBERT, DONALD J.

Security from under-seepage. 1285.
Uplift and seepage under dams on sand. 1396.

HERROLD, GEORGE H.

"Bibliography on Street Thoroughfares." 1076.
"Definitions, General Principles, and Recommendations: Report of the Committee of the City Planning Division on Street Thoroughfares Manual." 1048.
"Paying for Street Thoroughfares." 1068.
"Street Thoroughfares: Foreword." 1046.

HILL, HIBBERT M.

Uplift and seepage under dams on sand. 1386.

HILL, JOHN EDWARD.

Memoir of. 1667.

HILLMAN, GEORGE WALDO.

Memoir of. 1771.

HIRSCHTHAL, M.

Some soil pressure tests. 43.
Tests of multiple-span arch bridges. 465.

HODGMAN, BURT BRADLEY.

Memoir of. 1672.

HOGG, T. H.

St. Lawrence power development. 517.

HOOPER, ELMER GUY.

Memoir of. 1673.

HOPKINS, EDWARD S.

Formation of floc by ferric coagulants. 277.

HORNER, W. W.

An approach to determinate stream flow. 368.
"Bibliography on Street Thoroughfares." 1076.
"Definitions, General Principles, and Recommendations: Report of the Committee of the City Planning Division on Street Thoroughfares Manual." 1048.
"Thoroughfare Layouts in Relation to Drainage Systems." 1064.

HOSMER, GEORGE LEONARD.

Memoir of. 1676.

HOUSTON, JOHN JAY LAFAYETTE.

Memoir of. 1772.

HOWE, GEORGE EDWARD.

Memoir of. 1677.

HOYT, W. G.

An aproach to determinate stream flow. 384.

JAMESON, JOHN ALEXANDER, JR.

Memoir of. 1789.

JANNI, A. C.

"Final Report of the Special Committee on Concrete and Reinforced Concrete Arches." 1427.
Model of Calderwood Dam. 227.

JARVIS, C. S.

An approach to determinate stream flow. 363.
Modifying the physiographical balance. 315.

JOHANNESSON, SIGVALD.

"Lincoln Highway from Jersey City to Elizabeth, New Jersey." 91.

JOHNSON, GEORGE ARTHUR.

Memoir of. 1678.

JOHNSON, LEE H.

Flexible first-story construction. 653.

JOHNSON, OLIVER BERTRAND.

Memoir of. 1790.

JOHNSTON, GUY ROBERT.

Memoir of. 1681.

JOHNSTON, STERLING.

Deflection theory for suspension bridges. 1210.

JONES, JONATHAN.

Deflection theory for suspension bridges. 1174.
Eccentric riveted connections. 1097.

JONGE, A. E. R. de.

Eccentric riveted connections. 1099.

JORGENSEN, LARS R.

Model of Calderwood Dam. 224.

JUSTIN, JOEL D.

Security from under-seepage. 1276.
Uplift and seepage under dams on sand. 1388.

KARPOV, A. V.

"Model of Calderwood Arch Dam." 185.

KARTZKE, PAUL L.

Flexible first-story construction. 660.

KEAYS, R. H.

Sand movement in fluvial models. 848.

KENNEDY, ROBERT E.

Security from under-seepage. 1320.

KERNOT, MAURICE EDWIN.

Memoir of. 1683.

KHOSLA, A. N.

Security from under-seepage. 1320.

KIDDLE, ALFRED WATTS.

Memoir of. 1796.

KING, HENRY R.

Activated sludge aeration channels. 573.

KING, WARREN RAYMOND.

Memoir of. 1775.

KIRBY, ISAAC HENRY.

Memoir of. 1776.

KLEGERMAN, M. H.

Activated sludge aeration channels. 575.

KNAPP, F.

Reservoirs for flood control. 917.

KRAMER, HANS.

"Sand Mixtures and Sand Movement in Fluvial Models." 798.

KRIEGSHABER, VICTOR HUGO.

Memoir of. 1685.

KRYNINE, D. P.

Analysis of sheet-pile bulkheads. 766.
Some soil pressure tests. 30.

LANE, E. W.

"Security from Under-Seepage—Masonry Dams on Earth Foundations." 1235.

LANG, OLIVER HOWARD.

Memoir of. 1687.

LANGFITT, WILLIAM CAMPBELL.

Memoir of. 1688.

LANSFORD, WALLACE M.

Flow of water around bends in pipes. 1036.

LAVIS, FRED.

Lincoln Highway in New Jersey. 115.

LEE, W. S.

St. Lawrence power development. 531.

LEEDS, CHARLES T.

Wave pressures on sea-walls and breakwaters. 1004.

LEIGHLY, JOHN.

Sand movement in fluvial models. 839.

LINDENTHAL, GUSTAV.

Deflection theory for suspension bridges. 1190.

LINDLEY, E. S.

Profiles for retaining walls. 1128.

LIPPINCOTT, J. B.

Modifying the physiographical balance. 328.

LOHMEYER, E.

Analysis of sheet-pile bulkheads. 776.

LYSE, INGE.
"Investigation of Web Buckling in Steel Beams." 675.

McCLAIN, JAMES BROWNSON.
Memoir of. 1778.

McCLINTOCK, JAMES ROBINSON.
Memoir of. 1693.

McCULLOUGH, C. B.
Tests of multiple-span arch bridges. 455.

McGOVERN, PATRICK.
Memoir of. 1791.

McLEAN, ARCHIBALD.
Memoir of. 1694.

McMILLAN, W. BRUCE.
Discharge formula for weirs. 415.

MAHONY, ARTHUR SAWYER.
Memoir of. 1778.

MANN, CARROLL L., JR.
Tests of multiple-span arch bridges. 461.

MATTHES, GERARD H.
Modifying the physiographical balance. 326.
Reservoirs for flood control. 919.
Sand movement in fluvial models. 844.

MAVIS, F. T.
Flow of water around bends in pipes. 1040.
Sand movement in fluvial models. 849.

MEAD, DANIEL W.
"Water-Power Development of the St. Lawrence River." 498.

MEEM, J. C.
Some soil pressure tests. 14.

MENSCH, L. J.
Deflection theory for suspension bridges. 1185.

MILLER, L. B.
Formation of floc by ferric coagulants. 279.

MIRABELLI, E.

Experiments with concrete in torsion. 961.

MITCHELL, A.

Deflection theory for suspension bridges. 1209.

MOCKMORE, C. A.

Uplift and seepage under dams on sand. 1396.

MOISSEIFF, LEON S.

Deflection theory for suspension bridges. 1205.
Williot equations for statically indeterminate structures. 601.

MOLITOR, DAVID A.

"Wave Pressures on Sea-Walls and Breakwaters." 984.

MONTES, JOSE GARCIA, JR.

Rainfall studies for New York, N. Y. 638.

MOORE, R. L.

Web buckling in steel beams. 696.

MOORE, THOMAS VINCENT.

Memoir of. 1780.

MORRIS, CLYDE T.

"Final Report of the Special Committee on Concrete and Reinforced Concrete Arches." 1427.

MORSE, CLINTON.

"Renewal of Miter-Gate Bearings, Panama Canal." 471.

MÜLLENHOFF, A.

Deflection theory for suspension bridges. 1175.

MURPHY, EDWARD CHARLES.

Memoir of. 1697.

NAGLER, FLOYD A.

"Flow of Water Around Bends in Pipes." 1018.
Memoir of. 1698.

NETLAND, LARS.

Memoir of. 1701.

NICHOLS, K. D.

River laboratory hydraulics. 159.

NICHOLSON, MAURY.

Memoir of. 1702.

NUEBLING, EMIL LOUIS.

Memoir of. 1704.

O'BRIEN, MORROUGH P.

Sand movement in fluvial models. 858.

O'HARA, FRANCIS JOSEPH.

Memoir of. 1706.

O'ROURKE, JOHN FRANCIS.

Memoir of. 1707.

OSGOOD, WILLIAM R.

Stresses in space structures. 935.

O'SHAUGHNESSY, MICHAEL MAURICE.

Memoir of. 1710.

PARKIN, G. T.

Deflection theory for suspension bridges. 1209.

PARSONS, H. de B.

"Some Soil Pressure Tests." 1.

PATTY, I. H.

River laboratory hydraulics. 166.

PAVLO, E. L.

Deflection theory for suspension bridges. 1171.

PENNOYER, RAYMOND P.

Analysis of sheet-pile bulkheads. 761.

PIERCE, GEORGE ABEL.

Memoir of. 1781.

PIPPARD, A. J. SUTTON.

"Determination of Trapezoidal Profiles for Retaining Walls." 1116.

POST, CHESTER L.

Wave pressures on sea-walls and breakwaters. 1010.

POTTER, ALEXANDER.

Security from under-seepage. 1318.

POTTER, WILLIAM BANCROFT.
Memoir of. 1713.

POWELL, RALPH W.
River laboratory hydraulics. 155.

PRICHARD, HENRY SEWALL.
Memoir of. 1716.

PROBST, E.
Model of Calderwood Dam. 244.

PUTNAM, RUFUS W.
St. Lawrence power development. 539.

RANDOLPH, E. S.
Renewal of miter-gate bearings, Panama Canal. 496.

REYNOLDS, KENNETH C.
River laboratory hydraulics. 170.

RICE, HERBERT ALLAN.
Memoir of. 1783.

RICKEY, JAMES W.
St. Lawrence power development. 540.

RINDLAUB, BRUCE D.
Sand movement in fluvial models. 858.

RIPLEY, THERON M.
St. Lawrence power development. 529.

RITCHIE, JAMES.
Memoir of. 1718.

ROGERS, CLARENCE MORD.
Memoir of. 1719.

ROWE, W. P.
Modifying the physiographical balance. 317.

ROZMAN, ARMIN.
Eccentric riveted connections. 1097.

RULE, RHODES E.
Modifying the physiographical balance. 335.
Rainfall studies for New York, N. Y. 640.

RUSSELL, FRANK M.

Experiments with concrete in torsion. 964.

SANSBURY, WALTER E.

"Formation of Floc by Ferric Coagulants." 263.

SCHOFIELD, HIRAM ABIFF.

Memoir of. 1721.

SEAVER, CLIFFORD.

Rainfall studies for New York, N. Y. 634.

SESSIONS, EDSON OLIVER.

Memoir of. 1785.

SEUROT, PAUL ALBERT NICOLAS.

Memoir of. 1723.

SHAW, GEORGE HARRY THORNTON.

Memoir of. 1728.

SIMONDS, A. W.

Model of Calderwood Dam. 220.

SHERMAN, CHARLES W.

Rainfall studies for New York, N. Y. 630.

SHERMAN, LeROY K.

An approach to determinate stream flow. 365.

SHIELDS, JULIAN W.

Deflection theory for suspension bridges. 1195.

SHIFRIN, H.

Street thoroughfares. 1079.

SHULITS, SAMUEL.

River laboratory hydraulics. 175.

SLADE, J. J., JR.

Rainfall studies for New York, N. Y. 628.

SLATER, JOSEPH MANSFIELD.

Memoir of. 1733.

SMALL, KENNETH IRVING.

Memoir of. 1734.

SMITH, ELDRED D.

Model of Calderwood Dam. 235.

SMITH, WALTER M.

St. Lawrence power development. 541.

SMITS, HOWARD G.

Flexible first-story construction. 662.

SNYDER, F. F.

An approach to determinate stream flow. 384.

SONDEREGGER, A. L.

"Modifying the Physiographical Balance by Conservation Measures." 284.

SPEIDEN, THEODORE.

Memoir of. 1735.

SPOFFORD, CHARLES M.

Stresses in space structures. 941.

SPRAGUE, FRANK JULIAN.

Memoir of. 1736.

STANLEY, C. MAXWELL.

Reservoirs for flood control. 914.

STEINMAN, D. B.

"A Generalized Deflection Theory for Suspension Bridges." 1133.

STEVENS, J. C.

Modifying the physiographical balance. 324.

STIERLIN, GUSTAVE ARNOLD.

Memoir of. 1741.

STRAUB, LORENZ G.

River laboratory hydraulics. 145.
Sand movement in fluvial models. 867.

STREIFF, A.

Security from under-seepage. 1325.

SUTHERLAND, R. A.

Articulated buttress dams. 70.

TAYLOR, E. MCKENZIE.

Security from under-seepage. 1283.

TCHIKOFF, V. V.

River laboratory hydraulics. 170.
Sand movement in fluvial models. 852.

TEMPLIN, R. L.

"Model of Calderwood Arch Dam." 185.

TERZAGHI, CHARLES.

Some soil pressure tests. 35.
Uplift and seepage under dams on sand. 1391.

THACKWELL, H. L.

Activated sludge aeration channels. 567.
Some soil pressure tests. 24.

THOMAS, BURTIS PAUL.

Memoir of. 1787.

THOMPSON, PAUL W.

River laboratory hydraulics. 151.
Sand movement in fluvial models. 842.

THOMPSON, RALPH PENNY.

Memoir of. 1788.

TIFFANY, JOSEPH B., JR.

Sand movement in fluvial models. 861.

TOWNSEND, DARWIN WADSWORTH.

"Loss of Head in Activated Sludge Aeration Channels." 558.

TSAI, FANG-YIN.

Eccentric riveted connections. 1103.

TUDOR, RALPH A.

Deflection theory for suspension bridges. 1179.
Williot equations for statically indeterminate structures. 599.

TURNER, DANIEL L.

"Bibliography on Street Thoroughfares." 1076.
"Definitions, General Principles, and Recommendations: Report of the Committee of the City Planning Division on Street Thoroughfares Manual." 1048.
"Express Highways and By-Pass Routes." 1066.

TURNER, LESLIE.

Experiments with concrete in torsion. 967.

TUTTLE, ARTHUR S.

"The Development of the American Society of Civil Engineers:" Address at the Annual Convention, Los Angeles, California, July 3, 1935. 1407.

VAN CLEVE, AARON HOWELL.

Memoir of. 1742.

VARS, ALEXANDER WILSON.

Memoir of. 1746.

VAUGHN, R. L.

Analysis of sheet-pile bulkheads. 744.
Some soil pressure tests. 39.

VEEDER, HERMAN GREIG, JR.

Memoir of. 1790.

VOGEL, HERBERT D.

"Practical River Laboratory Hydraulics." 118.
Sand movement in fluvial models. 860.

VOGT, FREDRIK.

Deflection theory for suspension bridges. 1204.

von KÁRMÁN, THEODOR.

Analysis of sheet-pile bulkheads. 774.

VOORHEES, ISAAC SPURR.

Memoir of. 1748.

WALDROP, JOHN DOUGLAS.

Memoir of. 1749.

WEIGAND, EDWIN JOHN.

Memoir of. 1754.

WERNER, P. WILHELM.

Articulated buttress dams. 71.
Model of Calderwood Dam. 231.

WERTHEIMER, ALBERT.

Eccentric riveted connections. 1095.

WESSMAN, HAROLD E.

Deflection theory for suspension bridges. 1213.

WHITE, MERIT P.

Flexible first-story construction. 660.

WILCOXEN, L. C.

Some soil pressure tests. 19.

WILLIAMS, CHARLES P.

Articulated buttress dams. 76.

WILLIAMS, GORDON R.

An approach to determinate stream flow. 383.

WILLIAMS, H. A.

Flexible first-story construction. 665.

WILLIS, PAUL.

Memoir of. 1756.

WILSON, HERBERT ALVA.

Memoir of. 1757.

WILSON, WILBUR M.

"Final Report of the Special Committee on Concrete and Reinforced Concrete Arches." 1427.

"Laboratory Tests of Multiple-Span Reinforced Concrete Arch Bridges." 424.

WOOD, HOWARD C.

Deflection theory for suspension bridges. 1179.

WOODRUFF, GLENN B.

Deflection theory for suspension bridges. 1179.

YARNELL, DAVID L.

"Flow of Water Around Bends in Pipes." 1018.

YATES, WILLIAM H.

Deflection theory for suspension bridges. 1222.

YOUNG, HUGH E.

Street thoroughfares. 1083.